STATISTICAL INFERENCE FOR RANKINGS IN THE PRESENCE OF PANEL SEGMENTATION

by

LIN XIE

M.S., University of Arkansas, 2001

M.S., University of Arkansas, 2003

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Statistics
College of Arts and Sciences

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Abstract

Panels of judges are often used to estimate consumer preferences for *m* items such as food products. Judges can either evaluate each item on several ordinal scales and indirectly produce an overall ranking, or directly report a ranking of the items. A complete ranking orders all the items from best to worst. A partial ranking, as we use the term, only reports rankings of the best *q* out of *m* items. Direct ranking, the subject of this report, does not require the widespread but questionable practice of treating ordinal measurement as though they were on ratio or interval scales. Here, we develop and study segmentation models in which the panel may consist of relatively homogeneous subgroups, the segments. Judges within a subgroup will tend to agree among themselves and differ from judges in the other subgroups. We develop and study the statistical analysis of mixture models where it is not known to which segment a judge belongs or in some cases how many segments there are. Viewing segment membership indicator variables as latent data, an E-M algorithm was used to find the maximum likelihood estimators of the parameters specifying a mixture of Mallow's (1957) distance models for complete and partial rankings. A simulation study was conducted to evaluate the behavior of the E-M algorithm in terms of such issues as the fraction of data sets for which the algorithm fails to converge and the sensitivity of initial values to the convergence rate and the performance of the maximum likelihood estimators in terms of bias and mean square error, where applicable.

A Bayesian approach was developed and credible set estimators was constructed. Simulation was used to evaluate the performance of these credible sets as confidence sets.

A method for predicting segment membership from covariates measured on a judge was derived using a logistic model applied to a mixture of Mallows probability distance models. The effects of covariates on segment membership were assessed.

Likelihood sets for parameters specifying mixtures of Mallows distance models were constructed and explored.

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List of Notation

$\mathbf{y}$: $(y_1, y_2, \ldots, y_n)$, a realized value of the random ranking

$\mathbf{y}^*$: a ranking of size n represented as a permutation of the integer $\{1,2,\ldots,n\}$

$\mathbf{w}_0$: baseline ranking for panel consisting of one segment

$\mathbf{w}_i$: baseline ranking for segmentation i

$\{\mathbf{w}_i\}$: set estimation for segmentation i

$\mathbf{w}^{\#}$: $(\mathbf{w}_1, \mathbf{w}_2)$

$\{\mathbf{w}_i\}_{i=1}^{K}$: set estimation for K segmentation

n : the number of judges

m : the number of items

K : the number of segments

θ : a parameter lying in the unit interval

$\mathbf{S}_{qm}$: the collection of all permutation of q out of the m integers {1,2,…,m}

π_i : proportion of consumers from whole population in segmentation i

d_k : Kendall's distance

d_s : Spearman's distance

iid : independent and identical distribute

q: partial rankings q out of m items

MLE: Maximum Likelihood Estimate

EM: Expectation-Maximization

d : true distance from $\mathbf{w_1}$ to $\mathbf{w_2}$

d_1: sample mean distance from $\mathbf{w_1}$ to $\hat{\mathbf{w}}_1$

sd_1: standard error of d_1

d_2: sample mean distance from $\mathbf{w}_2$ to $\hat{\mathbf{w}}_2$

sd_2: standard error of d_2

$m(\hat{\theta})$: sample mean of $\hat{\theta} = \frac{1}{R}\sum_{i=1}^{R}\hat{\theta}_i = \overline{\theta}$

$m(\hat{\pi})$ = sample mean of $\hat{\pi} = \frac{1}{R}\sum_{i=1}^{R}\hat{\pi}_i = \overline{\pi}$

$m(\hat{\beta}_0)$ = sample mean of $\hat{\beta}_0 = \frac{1}{R}\sum_{i=1}^{R}\hat{\beta}_{0i} = \overline{\beta_0}$

$m(\hat{\beta}_1)$ = sample mean of $\hat{\beta}_1 = \frac{1}{R}\sum_{i=1}^{R}\hat{\beta}_{1i} = \overline{\beta_1}$

$s(\hat{\theta})$: standard error of $m(\hat{\theta}) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\theta}_i - \overline{\theta})^2}{R(R-1)}}$

$$s(\hat{\pi}) = \text{standard error of } m(\hat{\pi}) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\pi}_i - \overline{\pi})^2}{R(R-1)}}$$

$$s(\hat{\beta}_0) = \text{standard error of } m(\hat{\beta}_0) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\beta}_{0i} - \overline{\beta_0})^2}{R(R-1)}}$$

$$s(\hat{\beta}_1) = \text{standard error of } m(\hat{\beta}_1) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\beta}_{1i} - \overline{\beta_1})^2}{R(R-1)}}$$

$$mse\_t: \text{ estimated MSE of } \hat{\theta} = \hat{MSE}(\hat{\theta}) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\theta}_i - \theta)^2 = \frac{\sum_{i=1}^{R} y_i}{R} = \overline{y}, \text{ where } y_i = (\hat{\theta}_i - \theta)^2$$

$$mse\_\pi: \text{ estimated MSE of } \hat{\pi} = \hat{MSE}(\hat{\pi}) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\theta}_i - \theta)^2 = \frac{\sum_{i=1}^{R} x_i}{R} = \overline{x}, \text{ where } x_i = (\hat{\pi}_i - \pi)^2$$

$$mse\ \beta_0 = \text{estimated MSE of } \hat{\beta}_0 = \hat{MSE}(\hat{\beta}_0) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\beta}_{0i} - \beta_0)^2 = \frac{\sum_{i=1}^{R} z_i}{R} = \overline{z}, \text{ where } z_i = (\hat{\beta}_{0i} - \beta_0)^2$$

$$mse\ \beta_1 = \text{estimated MSE of } \hat{\beta}_1 = \hat{MSE}(\hat{\beta}_1) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\beta}_{1i} - \beta_1)^2 = \frac{\sum_{i=1}^{R} v_i}{R} = \overline{v}, \text{ where } v_i = (\hat{\beta}_{1i} - \beta_1)^2$$

$$se\_mse\_t: \text{ standard error of } mse\_t = Se(\hat{MSE}(\hat{\theta})) = \sqrt{\frac{\sum_{i=1}^{R}(y_i - \overline{y})^2}{R(R-1)}}$$

$$se\_mse\_\pi: \text{ standard error of } mse\_\pi = Se(\hat{MSE}(\hat{\pi})) = \sqrt{\frac{\sum_{i=1}^{R}(x_i - \overline{x})^2}{R(R-1)}}$$

$$se\_mse\_\beta_0 = \text{standard error of } mse\_\beta_0 = Se(\hat{MSE}(\hat{\beta}_0)) = \sqrt{\frac{\sum_{i=1}^{B}(z_i - \bar{z})^2}{R(R-1)}}$$

$$se\_mse\_\beta_1 = \text{standard error of } mse\_\beta_1 = Se(\hat{MSE}(\hat{\beta}_1)) = \sqrt{\frac{\sum_{i=1}^{B}(v_i - \bar{v})^2}{R(R-1)}}$$

t_wid : estimated mean width of profile interval estimate

se_t_wid : standard error of t_wid

cov_t : estimated coverage rate of profile interval estimate

MRV: mean relative volume

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CHAPTER 1 – Introduction

My research will develop and explore new statistical models, methods of analysis and modes of inference for discrete multivariate data consisting of rankings reported by non-homogeneous panels of judges. Most product preference studies ask consumer-judges to score products on a variety of scales which are at best ordinal. These scales are then often incorrectly analyzed as though the data were actually interval or ratio scales. A rating of '4' for the flavor of a cookie, for example, is then often mistakenly interpreted as being twice as good as a rating of '2.' These ordinal scores may then be converted using arithmetic operations such as averaging to produce a consensus ranking of the items, from best to worst. Asking each judge to directly rank products without having to say exactly how much better one item is than another offers the possibility of more accurately assessing consumer preferences. However, the use of direct rankings has been limited by the difficulty of modeling and analyzing such data. The text by Marden (1995) compiled and extended the substantial advances in the analysis of rankings that had taken place in the previous two decades and stimulated increased interest in these types of data. My research will focus on the use of rankings in sensory analysis.

Most products created today are designed to satisfy the needs of consumers (ASTM, 1979). Each year, hundreds of food products are introduced and at least 90% quickly disappear, not be seen again for at least another few years (Stone, 1988).

"Sensory evaluation is very important in product development. It provides guidance for product improvement, optimization, and maintenance" (Resurreccion, 1997). Sensory evaluation is a scientific discipline used to evoke, measure, analyze and interpret reactions to those characteristics of food and materials as they are perceived by the sense of sight, smell, taste, touch and hearing (Anonymous, 1975). Sensory evaluation has evolved over a period of more than 100 years and has become a scientific discipline. The early work in sensory evaluation carried out by experimental psychologists and others attempted to measure subjective phenomena, including the determination of taste and odor thresholds. Modern sensory evaluation had its origin at the Quartermaster Food and Container Institute for the Armed Force (QMFCI) in Chicago in the late 1940's through the mid 1950's (Schutz, 1998). A group of psychologists and food technologists at QMFCI studied basic tastes, olfaction, appetite and hunger and developed reliable and valid methods for sensory evaluation. Since then, many methods have been developed for sensory evaluation, including descriptive sensory analysis (DSA), quantitative descriptive analysis (QDA), texture profile method (TPM), the spectrum method and free-choice profiling (FCP).

There are four types of sensory measurements: nominal, ordinal, interval and ratio scales Nominal scales are used for classification, which is seldom used in sensory analysis. Ordinal scales are used for ranking or ordering. Interval scales are used for measuring magnitude, assuming equal distances between points on the scale. Ratio scales are used to measure magnitude, assuming equality ratio between points, where the zero level is not arbitrary and will reflect relative proportions.

To begin describing the statistical analysis of rankings, consider a number n of consumers-judges, each of whom ranks the same set of m objects according to some particular attribute(s). Variation among the judges is modeled by viewing the reported rankings as random variables. Assume that each judge ranks independently of the other judges. See Daniels (1950) and Kendall (1970) for discussions of this model. A complete ranking assigns a complete ordering to the objects, from the best object to the worst, as described in Marden(1995). Rankings can be obtained by what are called *directed* methods and *indirected* methods. For a directed method, consumers are forced to assign ranks to products with respect to their intensity of specified attributes. Complete ranking forces the judges to implicitly compare all objects so that a preference order can be produced. For the undirected method, ranking can be obtained by converting data from paired comparison tests provided that the multiple pairwise judgments are consistent. Consistency means that there are no circular triads of comparisons such as A is preferred to B, B is preferred to C and C is preferred to A, as described in Vigneau, Courcoux and $S\dot{e}m\dot{e}nou$ (1999). In a paired comparison test, two products are usually presented simultaneously and the consumers are asked to indicate which of the two products is preferred. In general, for m objects, there are m*(m-1)/2 comparisons, which is difficult and tiring for a judge to consistently carry out when m is large. In order to overcome sensory fatigue and interactions, several strategies are widely used to trim the amount of time required for complete ranking, such as “a round robin” procedure. The “round robin” procedure can be considered as an incomplete paired comparison method. Moskowitz (1983) described a round robin paired comparison experiment in which 10 judges was asked to rank 6 peach ice creams, denoted A-F, with respective to flavor intensity. In the first round, 3 sections were set up: A versus B, C versus D, and E versus F. In the first section, 7 out of 10 judges chose A as stronger. In the second section, 8 out of 10 judges chose D as stronger. In the third section, 6 out of 10 judges chose F as stronger. A, D, and F, who had the higher rank in each section were selected for inclusion in follow-up sensory testing. In the second round, one section was set up: A versus D. In the final round, A, the winner in the second round, paired with F. A was selected as the final choice for the strongest peach flavor. This procedure can not directly generate a complete ranking. Further work is needed to obtain a complete ranking.

Ranking can also be obtained by a rating test. The 9-point hedonic scale is the most popular rating test. Rating tests reflect a consumer's perceived intensity of a specified attribute. A ranking is then commonly produced by averaging the reported scores to each item.

When a sensory evaluation is conducted, a group of consumers (or judges) is selected as a sample of some larger population from which the sensory scientist wishes to draw conclusions. In consumer sensory testing experiments when a large number of judges are involved, it may happen that one group of judges likes some product and another group dislikes the same product. For example, a younger judge might like cakes that taste sweet, and an older judge might like cake that is sugar free. If such groups or segments of the potential market exist we say there exists 'market segmentation'. If market segments exist, we would want to know the target population for some particular attribute. Identification of the target population is the goal for many sensory evaluation tests. The target population can be defined as the segment of the population that regularly purchases, or is expected to use, the product or the product category (Resurreccion, 1998). It is clear that research on sensory preference segmentation may contribute to the innovation of confectionery, as it helps both the R & D and the Marketing Departments to develop appropriate strategies (Januszewska and Viaene, 2001). Segmented populations may be represented as mixtures of distributions, as described by McLachlan and Basford (1988). I will follow the view of Everitt (1993) who defined a Latent class analysis, in which units, judges in our case, are assumed to belong to one of a set of K latent classes, with the number K of classes, but not their membership lists, known a priori. Units belonging to the same class are similar with respect to some underlying scale, which may not be directly observable, and are assumed to come from the same probability distribution. Accurately estimating *K*, the number of segments in a mixture distribution would be a very difficult problem, especially in small samples of the type generally available in sensory analysis. For example, the probability that units from some segments may not be present in small samples can be large. One way of approaching the problem of making inferences about *K* is to use the likelihood ratio test statistic, denoted λ, to sequentially test for the smallest positive integer *t*, which does not lead to rejection of the hypothesis that *t* groups suffice. Unfortunately with mixture models, regularity conditions do not hold for $-2\log\lambda$ to have its usual asymptotic null distribution of chi-squared with degrees of freedom equal to the difference in the number of parameters in the two hypotheses (McLachlan and Basford, 1988). McLachlan et al (1988, 2001) state that hypothesis test for the number of component in mixture model has not been completely resolved.

There is undoubtedly a limit to how many items a judge can reliably distinguish in a specified time period, especially when the objects are very similar. For example, in taste testing, a judge may only be able to reliably report her/his top 3 out of *m* because sensory fatigue. In situations like this, *partial ranking* where only the *q* best items are reported could provide more information than an inaccurate complete ranking. On the other hand, if a judge has good discriminating ability, information will be lost when only a partial ranking is reported. I will explore this issue, about which I have been unable to find any references.

There is variability among and within judges. A judge who repeats the same ranking over and over again won't necessarily yield the same result if the labels of the products are switched. Judges vary even within the same segment. The probability models I will use provide a means for quantifying and describing this variability.

CHAPTER 2 – Literature Review

There is a large body of literature on analyzing complete rankings from one segment. See Goldberg (1976), Feigin and Cohen (1978), Cohen (1980,1982), Cohen and Mallows (1983), Fligner (1986), Croon (1989), Marden (1995), Lim (1999), Nelson and Kemp (2000), Olivares (2005). Statistical analysis can be carried out through well-established procedures for these data. Some studies have been reported on partial ranking in one segment. See Critchlow (1985), Diaconis (1989), Beckett (1993), Kemp and Nelson (2006). Other studies have employed mixture model in two segments on complete ranking includes Thomas and Donal (2002), Murphy and Martin (2003), Gormley and Murphy (2006, 2007, 2008a, 2008b, 2008c), Busse et al (2007), among others.

The study of segment membership based on complete and partial rankings has not been well developed so far. Vigneau (1999) et al. reported a sensory experiment in which 56 consumers ranked seven extruded snacks according to their overall preference. In his paper the choice of a relevant number of segments was performed using the likelihood ratio test. The statistic of this test is proportional to the difference between the maximal log-likelihood values obtained with (S+1) classes and S classes. Croon (1989) suggested that this test statistics asymptotically follows a chi-square distribution under the null hypothesis of S latent classes. Unfortunately with mixture models, regularity conditions sufficient for the test statistics to have its usual asymptotic chi-square distribution do not hold, McLachlan and Basford (1988, 2001)

Murphy and Martin (2003) developed E-M algorithm for mixtures of distance-based models for ranking data. The selection of the best mixture model was obtained by using the Bayesian information criterion (BIC).

Distances between Rankings

My research concerns the statistical modeling and analysis of discrete multivariate data $\{\mathbf{y_i} = (\mathbf{y_{i1}}, \mathbf{y_{i2}}, \ldots \mathbf{y_{ik}}), \mathbf{i} = \mathbf{1,2,\ldots n}\}$, viewed as realizations of independent, vector random variables $\{\mathbf{Y_i}, \mathbf{i} = \mathbf{1,2,\ldots n}\}$, each of which is an element of $\mathbf{S}_{qm}$, the collection of all permutations of *q* out of the *m* integers **{1,2,…,m}.** As just indicated, I will use bold type to designate vectors. This setup is used to represent the preferences shown by *n* consumer-judges for their best *q* out of *m* objects. When *q*= *m*, the observations will be interpreted as *complete rankings* of the objects so that $\mathbf{y_{ij}} = l$ means that object j is the *l*- th best item as evaluated by judge *i*; *l, j* = 1,2,…,*m* . Otherwise, the observations are to be interpreted as *orderings* so that $\mathbf{y_{ij}} = l$ means that object *l* is the *j*- th best object evaluated by judge *i; i*=1,2,…,*n, l, j* = 1,2,…,*k*. In both cases, I will assume that ties do not occur and simply refer to the data $\{\mathbf{y_i}, \mathbf{i} = \mathbf{1,2,\ldots n}\}$ as *rankings*.

Two special distances, Kendall's and Spearman's distance will be included in my study. For $\mathbf{y}*$ and $\mathbf{y}$ in S_m, Kendall's distance is based on the discordant pairs. A discordant pair is one in which $\mathbf{y}*$and have opposite relative preferences, that is, for $\mathbf{i} \neq \mathbf{j}$,

$$(\mathbf{y_i^*} - \mathbf{y_j^*})(\mathbf{y_i} - \mathbf{y_j}) < 0$$

The Kendall distance between two complete rank vectors is the number of discordant pairs,

$$\mathrm{d_{Ken}}(\mathbf{y}^*, \mathbf{y}) = \sum\sum_{1 \le i \le j \le m} \mathrm{I}[(\mathbf{y_i^*} - \mathbf{y_j^*})(\mathbf{y_i} - \mathbf{y_j}) < 0\mathbf{]}$$

Spearman's Distance:

A natural set of spatial distance between two complete rankings is the d_p distance, which is given as following:

$$d_p(\mathbf{y}^*, \mathbf{y}) = \sum_{i=1}^{m} |\mathbf{y}_i^* - \mathbf{y}_i|^p, \text{ where } 0 < p < \infty.$$

The Spearman distances use $\mathbf{p} = \mathbf{2}$ and is therefore the square of Euclidean distance.

Critchlow (1985) and Marden present two methods for defining distance between partial rankings consisting of the best *q* out of *m* objects. Let $\mathbf{u}$ and $\mathbf{v}$ be two such partial rankings and *d* a distance between two complete rankings in $S_m \equiv S_{mm}$. Following Critchlow (1985) I will use Hausdorf distances between $\mathbf{u}$ and $\mathbf{v}$ defined by:

$$d_{Haus}(\mathbf{u}, \mathbf{v}) = \max\{\max_{\mathbf{x}\in\mathbf{u}} d^*(\mathbf{x}, \mathbf{v}), \max_{\mathbf{y}\in\mathbf{v}} d^*(\mathbf{u}, \mathbf{y})\},$$

$$d^*(\mathbf{x}, \mathbf{v}) = d^*(\mathbf{v}, \mathbf{x}) = \inf_{\mathbf{y}\in\mathbf{v}} d(\mathbf{x}, \mathbf{y}),$$

where '$\mathbf{x}\ \varepsilon\ \mathbf{u}$' denotes that the complete ranking $\mathbf{x}$ is *consistent* with $\mathbf{u}$ in the sense that it orders the k elements of $\mathbf{u}$ just as $\mathbf{u}$ does. For example, suppose a judge is asked to rank the intensity of lemon flavor for her/his best 3 out of 5 syrups. A illustrative data set may be summarized as follows.

Table 2-1. Partial Rankings

Judge	Ranking for the following products				
	1	2	3	4	5
1	3	2		1	
2	1		3		2

Lim (1999) pointed out that when we are faced with incomplete rankings, one approach is to assign a rank for those objects that are left unranked by a given judge. Therefore, the combination of the possible ways to assign the unused ranks to the judge's unranked objects for the above partial rankings is as follow:

Table 2-2. Complete Rankings

Judge	Ranking for the following products				
	1	2	3	4	5
1	3	2	**4**	1	**5**
1	3	2	**5**	1	**4**
2	1	**5**	3	**4**	2
2	1	**4**	3	**5**	2

Let $\mathbf{u} = \mathbf{(3,2,,1,)}$ and $\mathbf{v} = \mathbf{(1,,3,,2)}$. In this example,

$$\begin{matrix} \mathbf{x}_1 = \mathbf{(3,2,4,1,5)} \\ \mathbf{x}_2 = \mathbf{(3,2,5,1,4)} \end{matrix}, \quad \begin{matrix} \mathbf{y}_1 = (1,5,\mathbf{3},4,2) \\ \mathbf{y}_2 = (1,4,3,5,2) \end{matrix}.$$

Since **x** and **y** are complete rankings, computing the distance between them is straightforward using either Kendall's or Spearman's distance. I will use Spearman's distance to illustrate how to compute the Hausdorf distances between **u** and **v**.

$$\begin{matrix} d(\mathbf{x}_1, \mathbf{y}_1) = 39 \\ d(\mathbf{x}_1, \mathbf{y}_2) = 34 \end{matrix}, \quad \begin{matrix} d(\mathbf{x}_2, \mathbf{y}_1) = 35 \\ d(\mathbf{x}_2, \mathbf{y}_2) = 32 \end{matrix}$$

$$d^*(\mathbf{x}_1, \mathbf{v}) = d^*(\mathbf{v}, \mathbf{x}_1) = \inf_{\mathbf{y} \in \mathbf{v}} d(\mathbf{x}_1, \mathbf{y}) = 34$$

$$d^*(\mathbf{x}_2, \mathbf{v}) = d^*(\mathbf{v}, \mathbf{x}_1) = \inf_{\mathbf{y} \in \mathbf{v}} d(\mathbf{x}_2, \mathbf{y}) = 32$$

$$\max_{\mathbf{x} \in \mathbf{u}} d^*(\mathbf{x}, \mathbf{v}) = 34$$

Similarly,

$$\begin{array}{l} d(\mathbf{x}_1, \mathbf{y}_1) = 39 \quad d(\mathbf{x}_1, \mathbf{y}_2) = 34 \\ d(\mathbf{x}_2, \mathbf{y}_1) = 35 \,, \; d(\mathbf{x}_2, \mathbf{y}_2) = 32 \end{array}$$

$$d^*(\mathbf{y}_1, \mathbf{u}) = d^*(\mathbf{u}, \mathbf{y}_1) = \inf_{\mathbf{y}_1 \in \mathbf{v}} d(\mathbf{y}_1, \mathbf{x}) = 35$$

$$d^*(\mathbf{y}_2, \mathbf{u}) = d^*(\mathbf{u}, \mathbf{y}_2) = \inf_{\mathbf{y}_2 \in \mathbf{v}} d(\mathbf{y}_2, \mathbf{x}) = 32$$

$$\max_{\mathbf{y} \in \mathbf{v}} d^*(\mathbf{y}, \mathbf{u}) = 35$$

Therefore, $d_{\text{Haus}}(\mathbf{u}, \mathbf{v}) = \max\{\max_{\mathbf{x} \in \mathbf{u}} d^*(\mathbf{x}, \mathbf{v}), \max_{\mathbf{y} \in \mathbf{v}} d^*(\mathbf{u}, \mathbf{y})\} = 35$

Studies Resulting in Rankings

Ranking *m* test objects is relatively easy for consumer judges if *m* is small or moderate. Ranking a large number of items is difficult, especially if some of them are similar or in the case of sensory evaluations if the food has extreme attributes such as being very sweet or very spicy, which could dull the taster's sensitivity to change. The following are examples of a ballot and instructions to the judges used in a sensory analysis.

Example#1:

Name____________ Session Code___________ Date___________

Please rinse your mouth with water before starting, before each sample and anytime you need to.

In front of you are 6 samples. Beginning with the sample on the left, taste each one.

After you taste all samples, you may re-taste as often as you need to. **No ties are allowed.**

Rank the samples from the most preferred (=1) to least preferred (=5)

Sample Numbers: 436 188 251 754 869

Rank	Sample Number
1	______________
2	______________
3	______________
4	______________
5	______________

Fig 2-1. An Example of A Ballot to Be Used in A Ranking Test.

Example#2:

Panelist Code_______________

Session No_________________

Take a piece of sample 873 using a fork and place the whole piece into your mouth.

Please answer the following questions by completely filling in the square that best reflects your feelings about this sample.

Overall, how would you rate this sample?

Dislike Extremely	Dislike Very much	Dislike Moderately	Dislike Slightly	Neither Like nor Dislike	Like Slightly	Like Moderately	Like Very much	Like Extremely

Fig 2-2. An example of a hedonic scale for overall acceptance.

Example#3:

Taste the following samples; circle the sweeter sample.

483 695

Fig 2-3. An example of paired comparison test (preference)

As described above, *segmentation* refers to situations where the panel consists of sub-groups of judges whose rankings tend to agree. Segmentation is important when judges within the segments are very similar and the segments differ relatively greatly among themselves. The advantage of a ranking test is that it does not require the testers to learn to use an external scale and then to produce ratings with it. What they only need to do is to preference order all or some of the objects. One of my research goals is to increase the use of direct ranking in sensory science.

Computing Maximum Likelihood Estimators-One Segment

Probability models supported on $\mathbf{S}_{qm}$ have been developed and explored from a variety of perspectives. See Marden (1995) and Critchlow (1985) for a description of many of these approaches. I will concentrate on Mallows type models which are based on a distance *d* between two complete or partial rankings. Specifically, *d* assigns a nonnegative number to any pair of rankings which is zero only if the rankings are identical. Within this large class, I will further limit my study to the widely used Kendall and Spearman Distances. The distance model based on Kendall distance is often called Mallows model (Mallows, 1957). For a population consisting of one segment, the Mallows model is a two parameter family of mass functions given by;

$$P(\mathbf{Y}=\mathbf{y}) = p(\mathbf{y} \mid \theta, \mathbf{w_0}) = \frac{\theta^{d(\mathbf{y},\mathbf{w_0})}}{\psi(\theta)}, \mathbf{y} \text{ in } \mathbf{S}_{qm} \qquad (2.1)$$

where $\mathbf{y}\ \varepsilon\ \mathbf{S}_{qm}$, θ is a spread parameter lying in the unit interval, $\psi(\theta)$is a norming constant making the probabilities sum to one and $\mathbf{w_0}$ is a baseline ranking lying in $\mathbf{S}_{qm}$. The probability given in (2.1) is uniformly distributed over all *m!/(m-q)!* possible 'rankings' when $\theta = 1$. On the other hand, the probability is increasingly more concentrated about the mode $\mathbf{w_0}$ as θ decrease toward 0. Hence, this model may be interpreted as positing that a judge tries to reproduce her/his true ranking $\mathbf{w_0}$ and that increasing θ is associated with increasing variability in rankings actually reported by the judge and decreasing ability to report his/her true ranking.

The log-likelihood $l(\theta, \mathbf{w})$ based on data $\{\mathbf{y}_i; i = 1, 2, ..., n\}$ consisting of observed values of independent rankings sampled from the mass function $p(\mathbf{y} \mid \theta, \mathbf{w}) = \frac{\theta^{d(\mathbf{y},\mathbf{w})}}{\psi(\theta)}$ defined in (2.1) is given by

$$l(\theta, \mathbf{w}) \;=\; D(\mathbf{w}) \log(\theta) - n \log(\psi(\theta)),$$

where $D(\mathbf{w}) = \sum_{i=1}^{n} d(\mathbf{y}_{\mathbf{i},} \mathbf{w})$. I wrote a program in the Software Package *R* to find maximum likelihood estimates (MLE'S) of θ and $\mathbf{w}$ using both Kendall's distance and Spearman's distance. First, an MLE $\hat{\mathbf{w}}$, which may not be unique, can easily be found by searching through $\mathbf{S}_{qm}$ to find vectors $\mathbf{w}$ that minimize $D(\mathbf{w})$. Plugging $D(\hat{\mathbf{w}})$ into the likelihood converts the problem of finding the MLE of θ to finding an MLE in a one parameter exponential family. Noting that $\theta\psi'(\theta)/\psi(\theta) = E_\theta(d(\mathbf{Y}, \theta)) \equiv H(\theta)$, this is accomplished by solving the equation

$$H(\theta) = \frac{1}{n} D(\hat{\mathbf{w}})$$

Since the MLE $\hat{\theta}$ is the root of an increasing function, it can be found using bisection. Bisection places the points of evaluation in the middle of an interval in which the root lies and cuts the interval of uncertainty in half. It continues in the same manner until a desired accuracy, expressed in terms of powers of ½ , is achieved. Specifically, to solve $H(\theta) - \frac{1}{n} D(\hat{\mathbf{w}}) = 0$, first let $\theta_L = 0,\ \theta_L = 1$, and start with an initial "guess" $\theta_0 \in (0,1)$.

Then,

(1) If $\frac{1}{n}\sum_{i=1}^{n} d(\mathbf{y_i}, \hat{\mathbf{w}}) = 0,$ set $\hat{\theta} = 0$

(2) If $\frac{1}{n}\sum_{i=1}^{n} d(\mathbf{y_i}, \hat{\mathbf{w}}) = \max_{\theta} H(\theta),$ set $\hat{\theta} = 1$

Otherwise

(3) If $H(\theta_0) < \frac{1}{n}\sum_{i=1}^{n} d(\mathbf{y_i}, \hat{\mathbf{w}}) = 0,$ set $\theta_L = \theta_0,\ \theta_1 = \frac{\theta_U + \theta_0}{2}$

(4) If $H(\theta_0) > \frac{1}{n}\sum_{i=1}^{n} d(\mathbf{y_i}, \hat{\mathbf{w}}),$ set $\theta_u = \theta_0,\ \theta_1 = \frac{\theta_l + \theta_0}{2}$

(5) Set $\theta_0 = \theta_1$ and return to (3)

Iterating M times leads to an approximate MLE within 2^{-M} of the true MLE.

Example: Goldberg Data:

The data for this example can be found in Goldberg (1976). It was from a sociological survey of graduates of the Faculty of Industrial Engineering and Management. Each graduate was asked to rank 10 occupations (from 1 to 10) with respective to the degree of social prestige.

This is an example of a complete ranking and Spearman distance function was used to find the MLE of $\mathbf{w_0}$. There were 143 responses in total. The MLE of $\mathbf{w_0}$ was found to be the ranking (1 7 4 9 8 2 10 5 3). $\sum_{i=1}^{n} d(\mathbf{y}, \mathbf{w_0}) = 1200$, $\hat{\theta} = 0.5$, which very close to the results analyzed by Feigin and Cohen (1978). They also found that 3 judges who misunderstood the ranking procedure gave completely different rankings from the others. Their rankings can be treated as outliers. After removing the 3 outliers, The MLE of $\mathbf{w}_0$ was found totally different from the previous one, which was (1 6 10 3 8 9 2 5 4 7), which is identical to the results obtained by Cohen (1982). $\sum_{i=1}^{n} d(\mathbf{y}, \mathbf{w_0}) = 1208$ and The MLE $\hat{\theta} = 0.5$.

Finding MLE's for a Mixture Having Two Segments

Let π denote the proportion of the population from segment one. Then, a mixture model for two segments is given by:

$$P(\mathbf{y} \mid \theta, \mathbf{w_1}, \mathbf{w_2}) = \pi P(\mathbf{y} \mid \theta, \mathbf{w_1}) + (1-\pi) P(\mathbf{y} \mid \theta, \mathbf{w_2}),$$

where $0 < \pi < 1$, $0 < \theta < 1$, $\mathbf{w}_1$ and $\mathbf{w}_2$ are distinct elements of S_{qm}. Let $\boldsymbol{\xi} = (\pi, \theta, \mathbf{w_1}, \mathbf{w_2})$ be the vector of unknown parameters. Note that the parameter θ is assumed to be the same in both segments. Therefore, The EM algorithm can be used to the find the maximum likelihood estimates $\hat{\mathbf{w}}_1$, $\hat{\mathbf{w}}_2$, $\hat{\theta}$ and $\hat{\pi}$ from data $\{\mathbf{y_j}\}_{j=1}^{n}$, which are realizations of *iid* random vectors having probability mass function of the form $P(y \mid \theta, \mathbf{w_1}, \mathbf{w_2})$ given in (2.1). The EM algorithm is a method for finding maximum likelihood estimates of parameters by alternating two steps, that is, the E-step and the M-step. In the E-step, an expectation of the complete-data loglikelihood is computed given the observed data and the current estimates of the parameters. In the M-step, the parameter estimates are updated by maximizing the expected complete-data log-likelihood (Tanner, 1996).

Let $\{z_j\}$ be unobserved segment membership variables of the *n* judges so that we can write

$$z_j = \begin{cases} 1 & if\ \mathbf{w} = \mathbf{w_1} \\ 0 & if\ \mathbf{w} = \mathbf{w_2} \end{cases},$$

j = 1,2,…,n. The MLE's would be easy to find if the membership variables were known. This is a situation where the EM algorithm, which iteratively replaces these unobserved variables by their conditional expectations with respect to the data. First, note that the conditional distribution of z_j is a binomial distribution specified by

$$\mathrm{z_j} \mid \mathbf{y_j}, \theta, \mathbf{w_2}, \pi \sim \text{Binomial}(1, \frac{\pi \mathrm{P}(\mathbf{y_j} \mid \theta, \mathbf{w_1})}{\pi \mathrm{P}(\mathbf{y_j} \mid \theta, \mathbf{w_1}) + (1-\pi)\, \mathrm{P}(\mathbf{y_j} \mid \theta, \mathbf{w_2})}).$$

Hence, we see that

$$E_\xi(z_j \mid \mathbf{y_j}) = \frac{\pi \mathrm{P}(\mathbf{y_j} \mid \theta, \mathbf{w_1})}{\pi \mathrm{P}(\mathbf{y_j} \mid \theta, \mathbf{w_1}) + (1-\pi)\, \mathrm{P}(\mathbf{y_j} \mid \theta, \mathbf{w_2})}$$

$$\equiv \lambda_j(\xi),\ \mathrm{j} = 1,2,\ldots,\mathrm{n}.$$

The conditional likelihood based on the augmented data is given by:

$$\underset{\{\mathbf{y},z\}}{L}(\pi, \theta, \mathbf{w}_1, \mathbf{w_2}) = \psi^{-n}(\theta)\theta^{D(\mathbf{w})}\pi^{\sum_{i=1}^{n} z_i}(1-\pi)^{n-\sum_{i=1}^{n} z_i}, \tag{2.2}$$

where $\mathbf{w}^{\#} = (\mathbf{w_1}, \mathbf{w_2})$ and $D(\mathbf{w}^{\#}, \{z_j\})$ is defined as:

$$D(\mathbf{w}^{\#},\{z_j\}) = \sum_{i=1}^{n} z_i d(\mathbf{y_i},\mathbf{w_1}) + \sum_{i=1}^{n}(1-z_i)d(\mathbf{y_i},\mathbf{w_2})$$

Plugging $D(\mathbf{w}^{\#},\{z_j\})$ in equation (2.2), we then have

$$\log_{\{\mathbf{y},\mathbf{z}\}}(\pi,\theta,\mathbf{w}_1,\mathbf{w_2}) = -n\log\psi(\theta) + D(\mathbf{w}^{\#},\{z_j\})\log\theta + \sum_{i=1}^{n} z_i \log\pi + (n-\sum_{i=1}^{n} z_i)\log(1-\pi)$$

The E-M algorithm:

E step:

After the ith iteration where we have estimated parameter values $\boldsymbol{\xi}^{\mathbf{i}}$, compute the conditional expectation of the log-likelihood based on the augmented data with respect to the observed data, yielding:

$$\log_{\{\mathbf{y},z\}}(\pi,\theta,\mathbf{w}_1,\mathbf{w_2}) = -n\log\psi(\theta) + D(\mathbf{w}^{\#},\{z_j\})\log\theta + \sum_{i=1}^{n} z_i \log\pi + (n-\sum_{i=1}^{n} z_i)\log(1-\pi)$$

$$\begin{aligned}
&Q(\xi,\boldsymbol{\xi}^{\mathbf{i}}) = E_{\xi^i} l_{\{\mathbf{z}\}|\{\mathbf{y}\}}(\xi) \\
&= -n\log\psi(\theta) + D(\mathbf{w}^{\#}, E_{\xi^{\mathbf{i}}}\{z_j\}\,|\,\{\mathbf{y_j}\})\log\theta + \sum_{j=1}^{n} E_{\xi^{\mathbf{i}}}(z_j\,|\,\mathbf{y_j})\log\pi + (n-\sum_{i=1}^{n} E_{\xi^{\mathbf{i}}}(z_j\,|\,\mathbf{y_j})\log(1-\pi) \\
&= -n\log\psi(\theta) + D(\underline{\mathbf{w}^{\#},\{\lambda_j}\{\boldsymbol{\xi}^{\mathbf{i}}\})\log\theta + \sum_{i=1}^{n}\lambda_j(\boldsymbol{\xi}^{\mathbf{i}})\log\pi + (n-\sum_{i=1}^{n}\lambda_j)(\boldsymbol{\xi}^{\mathbf{i}})\log(1-\pi)
\end{aligned}$$

(1). In order to maximize the conditional expectation of log likelihood $Q(\xi,\boldsymbol{\xi}^{\mathbf{i}})$, we first search over the permutations of the first n integers to find $\mathbf{w_1},\mathbf{w_2}$ which minimize $D(\mathbf{w}^{\#},\boldsymbol{\xi}^{\mathbf{i}})$,

$\hat{\mathbf{w}}_1$ *and* $\hat{\mathbf{w}}_2$ be such values so that for all $\mathbf{w}^{\#}$, $D(\mathbf{w}^{\#},\boldsymbol{\xi}^{\mathbf{i}}) \geq D(\hat{\mathbf{w}}^{\#},\boldsymbol{\xi}^{\mathbf{i}})$ with the starting value of π_i is set equal to its true value.

M step:

(1). Once $\hat{\mathbf{w}}_1$ *and* $\hat{\mathbf{w}}_2$ are found, then update $\hat{\theta}$, and $\hat{\pi}$, continues these iterations until there is relatively change in function $Q(\xi,\xi^{\mathbf{i}})$ and the stopping rule is given as:

$$|Q(\xi^{\mathbf{i+1}},\xi^{\mathbf{i}})-Q(\xi^{\mathbf{i}},\xi^{\mathbf{i-1}})|=10^{-2}$$

(2). To estimate θ we have to solve the following equation:

$$\frac{\partial\ell(\theta,\hat{\mathbf{w}}^{\#})}{\partial\theta}=0$$

$$\frac{\theta\,\psi'(\theta)}{\psi(\theta)}=\frac{\mathrm{D}(\hat{\mathbf{w}}^{\#})}{n}$$

Note: $$\frac{\theta\,\psi'(\theta)}{\psi(\theta)}=\frac{\mathrm{D}(\hat{\mathbf{w}}^{\#})}{n}=E_{\theta,w}(d(\mathbf{y_i},\mathbf{w}))=H(\theta)$$

As described earlier, bisection can be used to solve $H(\theta)-\frac{1}{n}D(\hat{\mathbf{w}}^{\#})=0$,

Note that the mean distance $H(\theta)$ between a ranking $\mathbf{Y}$ generated from the mixture probability model and fixed baseline rankings $\mathbf{w}^{\#}$ will have to be computed since an explicit expression for it is only available in a few special cases for complete rankings. Specifically, for any $\mathbf{w}$,

$$H(\theta)=\sum d(\mathbf{y_i},\mathbf{w})\frac{\theta^{d(\mathbf{y_i},\mathbf{w})}}{\psi(\theta)}. \qquad (3).$$

To determines the new estimate π^{i+1} as the solution of

$$\frac{\partial Q(\xi,\xi^{\mathbf{i}})}{\partial\pi}=\sum_{j=1}^{n}\lambda_j(\xi)\frac{1}{\pi}-(n-\sum_{j=1}^{n}\lambda_j(\xi))\frac{1}{1-\pi}=0 \quad \Longrightarrow \quad \pi^{i+1}$$

$$(1-\pi)\sum_{j=1}^{n}\lambda_j(\xi)-\pi(n-\sum_{j=1}^{n}\lambda_j(\xi))=0$$

$$\sum_{i=1}^{n}\lambda_j(\xi)-\pi\sum_{j=1}^{n}\lambda_j(\xi)-n\pi+\pi\sum_{j=1}^{n}\lambda_j(\xi)=0$$

Hence, $$\hat{\pi} = \frac{\sum_{j=1}^{n} \lambda_j(\xi)}{n}$$

Consequently, $$\hat{\pi}^{i+1} = \frac{\sum_{j=1}^{n} \lambda_j^i(\xi)}{n}$$

CHAPTER 3 – Point and Profile Likelihood Set Estimation

Given fixed data $\underline{\mathbf{y}}=\{\underline{\mathbf{y}}_i\}$ whose joint density p($\underline{\mathbf{y}}|\underline{\psi}$) is specified up to a an unknown vector of parameters $\underline{\psi}$, the likelihood $L(\underline{\psi})$ = { c($\underline{\mathbf{y}}$)p($\underline{\mathbf{y}}|\underline{\psi}$) }, a class of functions of $\underline{\psi}$ where c($\underline{\mathbf{y}}$) > 0. Likelihood function for a probability density can be defined as the likelihood as the function of unknown parameters. While the definition of likelihood covers the multiparameter models, the resulting multidimensional likelihood function can be difficult to describe. Even when we are interested in several parameters, it is always easier to describe one parameter at a time. The likelihood approach to eliminate a nuisance parameter which we are not interested in is to replace it by its MLE at each fixed value of the parameter of interests. The resulting likelihood is called the profile likelihood (Pawitan, Y., 2001). Since the parameter space can be high dimensional, I developed Profile Likelihood inference for θ, π and $\{\mathbf{w}_i\}$, (when the number of segments is known) emphasizing joint set estimators of each of these parameters. Keep in mind that constructing traditional confidence sets for the baseline rankings $\{\mathbf{w}_i\}$, which may be the most interesting parameters in sensory analysis, will be very difficult since these parameters are multivariate and discrete. A profile likelihood for $\{\mathbf{w}_i\}$ is then given by

$$L^{[p]}(\{\mathbf{w}_i\}) = L(\{\mathbf{w}_i\}, \hat{\theta}\{\mathbf{w}_i\}\hat{\pi}\{\mathbf{w}_i\}) . \tag{3.1}$$

where , $\hat{\theta}(\{\mathbf{w}_i\}, \hat{\pi}_j(\{\mathbf{w}_i\}, j=1,2,\ldots\ldots t$ are the MLE's of θ and $\{\pi_j\}$ when $\{\mathbf{w}_i\}$ are fixed. I will construct an EM Algorithm to find $\hat{\theta}(\{\mathbf{w}_i\}, \hat{\pi}_j(\{\mathbf{w}_i\}, j=1,2,\ldots\ldots t$ given fixed $\{\mathbf{w}_i\}$ in $\mathbf{S}_{qm}$.

Then, a joint profile likelihood set for $\{\mathbf{w}_i\}$ has the form

$$Ls(\{\mathbf{w}_i\}) = \{\{\mathbf{w}_i\}, \frac{L^P\{\mathbf{w}_i, i=1,\ 2,..K\}}{L^P\{\hat{\mathbf{w}}_i, i=1,\ 2,..K\}} \geq C\} \text{ for some } C \in (0,1).$$

As noted above, under certain conditions, the profile log-likelihood may be used just like any other log-likelihood. Accordingly, I studied a chi-square calibration of these likelihood sets to choose *C* so that they have close to a desired probability content. Similarly, I also carried out profile likelihood inference for θ and π and exploited the fact that the maximum profile likelihood estimate is equal to the overall maximum likelihood estimate.

3.1 Point Estimation

For a given data, the maximum likelihood estimates $\hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2$, $\hat{\theta}$, and $\hat{\pi}$ can be found using the EM algorithm described in the previous chapter. Here, the number of judges, n, the number of objects, m, and the number of segments *K* are all known quantities. Here we take *K* = 2. Recall that in the EM algorithm, initial guesses were set for θ, and π. Then, a grid search over the permutations of the first m integers taken q a time was used to find the $\mathbf{w}_1, \mathbf{w}_2$ which minimize $D(\mathbf{w}^{\#}, \boldsymbol{\xi}^{\mathbf{i}})$, $\hat{\mathbf{w}}_1$ *and* $\hat{\mathbf{w}}_2$ be such that for all $\mathbf{w}^{\#} = (\mathbf{w}_1, \mathbf{w}_2)$, $D(\mathbf{w}^{\#}, \boldsymbol{\xi}^{\mathbf{i}}) \geq D(\hat{\mathbf{w}}^{\#}, \boldsymbol{\xi}^{\mathbf{i}})$.

Simulation studies were conducted to assess the overall performance of MLES. Comparing the estimates from the simulated data to the true parameters value allows us objectively evaluate the bias and precision of those estimators. In our simulation studies, we consider the following varying conditions:

$\pi = 0.1,\ 0.3,\ 0.5;$

$\theta = 0.1,\ 0.3,\ 0.5,\ 0.7,\ 0.9;$

$\mathbf{w}_2 = 132,\ 312,\ 321;$

n=10, 20, 30 (number of judges);

m=5, 6, 7 (number of items);

Distance function includes Kendall's and Spearman's distance functions,

where $\mathbf{w}_2 = 132$ is the medium distance from $\mathbf{w}_1 = 123$ with distance=1 for Kendall distance, 2 for Spearman distance; $\mathbf{w}_2 = 312$ is somewhat far from w1 with distance=2 for Kendall distance, 6 for Spearman distance ; and $\mathbf{w}_2 = 321$ is far from w1 with distance=3 for Kendall distance, 8 for Spearman distance.

One hundred data sets (R=100) for each of the 3*5*3*3*3*2=810 scenarios were generated in Software Package *R*. The EM algorithm was used to find MLE $\hat{\mathbf{w}}_1$, $\hat{\mathbf{w}}_2$, $\hat{\theta}$ and $\hat{\pi}$.

The initial values for π and θ were set at their true values.

The following is the algorithm used to simulate desired partial ranking data:

$$P(\mathbf{y} \mid \theta, \mathbf{w}_1, \mathbf{w}_2) = \pi P(\mathbf{y} \mid \theta, \mathbf{w}_1) + (1-\pi) P(\mathbf{y} \mid \theta, \mathbf{w}_2),$$
$$P(\mathbf{y} \mid \theta, \mathbf{w}_1, \mathbf{w}_2, \pi) = \frac{\pi \theta^{d(\mathbf{y}_k, \mathbf{w}_1)} + (1-\pi)\theta^{d(\mathbf{y}_l, \mathbf{w}_2)}}{\Psi(\theta)}$$

First generate U~ U(0,1)

If $U \le \pi$ generate $\mathbf{y}$ from $P(\mathbf{y} \mid \theta, \mathbf{w}_1)$

If $U > \pi$ generate $\mathbf{y}$ from $P(\mathbf{y} \mid \theta, \mathbf{w}_2)$

Let $F(\mathbf{y_k}) = \sum_{l=1}^{k} P(\mathbf{y_i} \mid \theta, \mathbf{w})$, where $P(\mathbf{y_l}) = \frac{1}{\Psi(\theta)} \theta^{d(\mathbf{y}_1, \mathbf{w})}$ (if $U \le \pi$, $\mathbf{w} = \mathbf{w_1}$, otherwise $U > \pi$, $\mathbf{w} = \mathbf{w_2}$).

Hence, $F(\mathbf{y_l}) = \sum_{l=1}^{n!} \frac{\theta^{d(\mathbf{y}_1, \mathbf{w})}}{\Psi(\theta)}$

Here $\psi(\theta) = \sum_{l=1}^{n!} \theta^{d(\mathbf{y}_1, \mathbf{w})}$, we specify $\theta = (0.1, 0.3, 0.5, 0.7, 0.9)$, respectively

$\mathbf{w} = (\mathbf{1,2,\ldots,n}) \rightarrow \{F(\mathbf{y_l})\}$, we compute $\{F(\mathbf{y_l})\}$, respectively.

Eventually, let $F^{-1}(u) = y_2 = \inf\{k, F(\mathbf{y_k}) \ge u\}$

If $F(\mathbf{y_1}) < u < F(\mathbf{y_2})$, $u = y_2$

.....

If $F(\mathbf{y_k}) < u < F(\mathbf{y_{k+1}})$, $u = y_{k+1}$, etc

The following statistics were computed for each of the 810 investigated scenarios:

R=number of simulations or replications.

d=true distance from $\mathbf{w_1}$ to $\mathbf{w_2}$

d_1=sample mean distance from $\mathbf{w_1}$ to $\hat{\mathbf{w}}_1$

sd_1=standard error of d_1

d_2=sample mean distance from $\mathbf{w}_2$ to $\hat{\mathbf{w}}_2$

sd_2=standard error of d_2

$\hat{m}(\theta)$ = sample mean of $\hat{\theta} = \frac{1}{R}\sum_{i=1}^{R}\hat{\theta}_i = \overline{\theta}$.

$\hat{m}(\pi)$ = sample mean of $\hat{\pi} = \frac{1}{R}\sum_{i=1}^{R}\hat{\pi}_i = \overline{\pi}$.

$s(\hat{\theta})$ =standard error of $\hat{m}(\theta) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\theta}_i - \overline{\theta})^2}{R(R-1)}}$

$s(\hat{\pi})$ =standard error of $\hat{m}(\pi) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\pi}_i - \overline{\pi})^2}{R(R-1)}}$

mse_t = estimated MSE of $\hat{\theta} = \hat{MSE}(\hat{\theta}) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\theta}_i - \theta)^2 = \frac{\sum_{i=1}^{R} y_i}{R} = \overline{y}$, where $y_i = (\hat{\theta}_i - \theta)^2$

mse_π = estimated MSE of $\hat{\pi} = \hat{MSE}(\hat{\pi}) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\theta}_i - \theta)^2 = \frac{\sum_{i=1}^{R} x_i}{R} = \overline{x}$, where $x_i = (\hat{\pi}_i - \pi)^2$

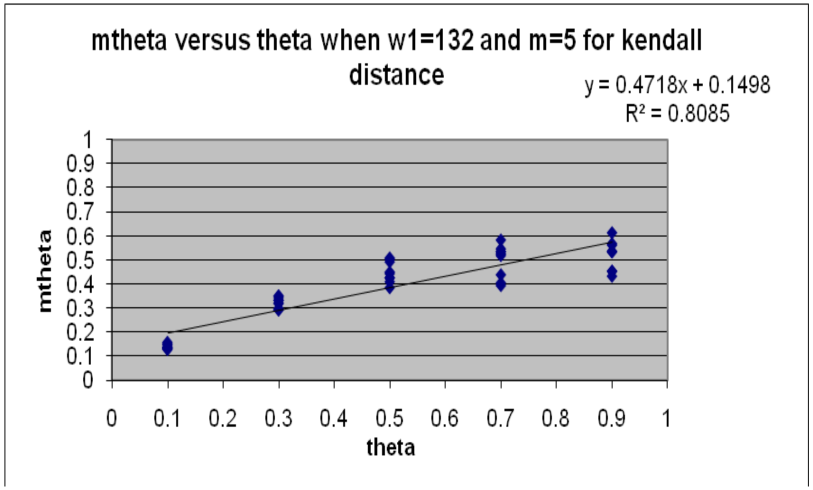
se_mse_t = standard error of $mse\_t = Se(\hat{MSE}(\hat{\theta})) = \sqrt{\frac{\sum_{i=1}^{R}(y_i - \overline{y})^2}{R(R-1)}}$

se_mse_π = standard error of $mse\_\pi = Se(\hat{MSE}(\hat{\pi})) = \sqrt{\frac{\sum_{i=1}^{R}(x_i - \overline{x})^2}{R(R-1)}}$

3.1.1 Point Estimation for θ

Appendix 1 exhibits point estimate $\hat{m}(\theta)$, standard error of $\hat{m}(\theta)$, MSE of $\hat{\theta}$, standard error of MSE of $\hat{\theta}$. The scatter plots in Figures 3-1 to 3-18 based on the data from Appendix 1 show the relationship between the simulated sample means of the $\hat{\theta}$'s and the true values of θ. Keep in mind that the values of θ as measures of concentration around the baseline rankings cannot be compared across different values of m, the total number of times from which the best q = 3 are being selected. If $\hat{\theta}$ is an unbiased estimator of θ, we would expect to see the points fall close to a 45 degree line. The least squares line is given in each graph. We observed that as m increased, R^2 decreased, and indicating increasing bias of the estimators. The R^2 ranged from 0.74 to 0.96.

For all cases, the estimators based on the Spearman distance performed better when the range for θ was $0.7 \ge \theta \ge 0.3$, For the Kendall distance, they performed best when $\theta = 0.1$. Using Spearman's distance, the bias of the estimators appeared to be negative for both $\theta \le 0.1$ and $\theta \ge 0.9$. For Kendall's distances, the estimator of θ was heavily biased when $\theta = 0.9$. Even though $\hat{\theta}$ is biased, it had very small standard error and small MSE as well as small standard error of MSE. Recall: Bias is defined as the difference between an estimator's expected value and the true value of the parameter being estimated. The means of 100 replications for $\hat{\theta}$ were not exactly equal to the corresponding population values θ. However, the large-sample theory explains that the maximum likelihood estimator is unbiased only asymptotically or as sample size tends to infinity. Because the sample size of our simulation study is only 100, biased estimates could be expected.



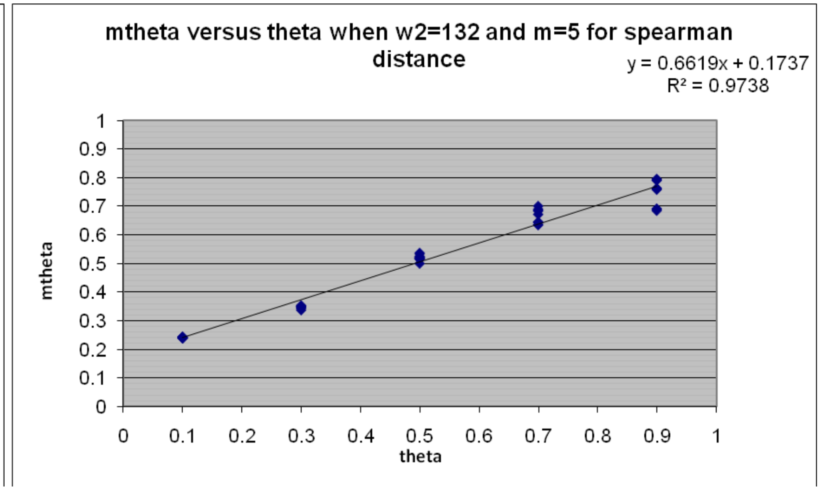
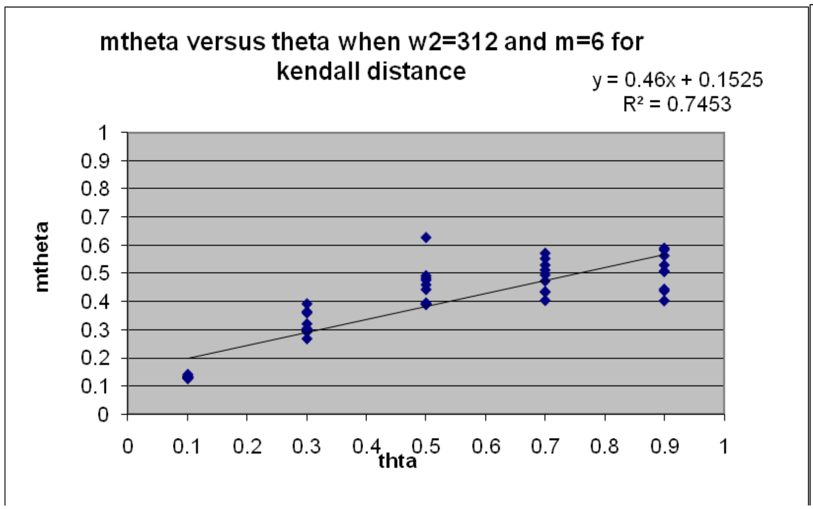


Fig 3-1: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 132$ and m=5 for Kendall distance

Fig 3-2: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 132$ and m=5 for Spearman distance



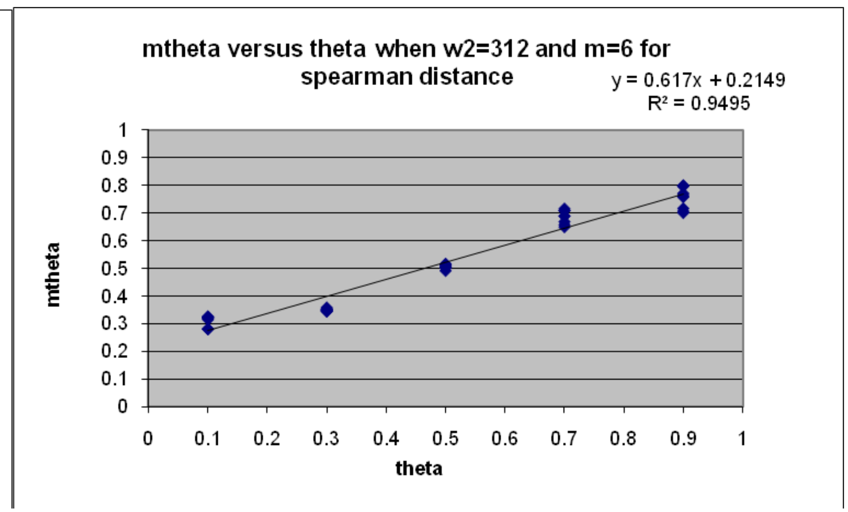


Fig 3-3: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 132$ and m=6 for Kendall distance

Fig 3-4: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 132$ and m=6 for Spearman distance

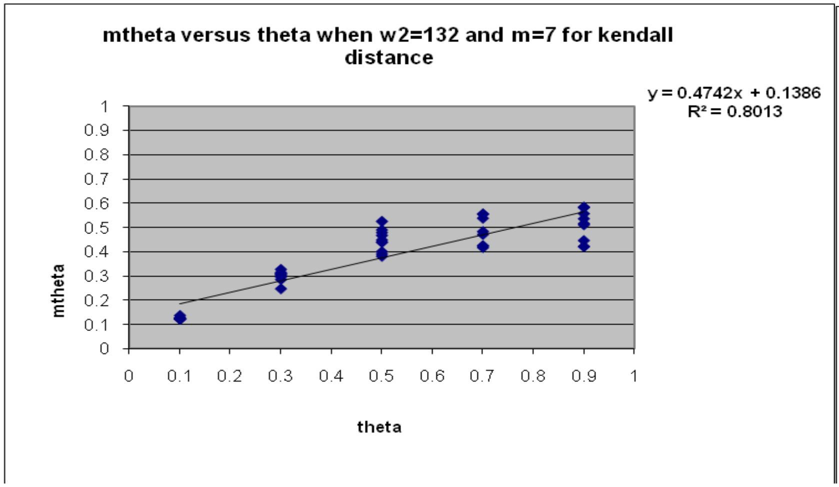


Fig 3-5: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 132$ and m=7 for Kendall distance

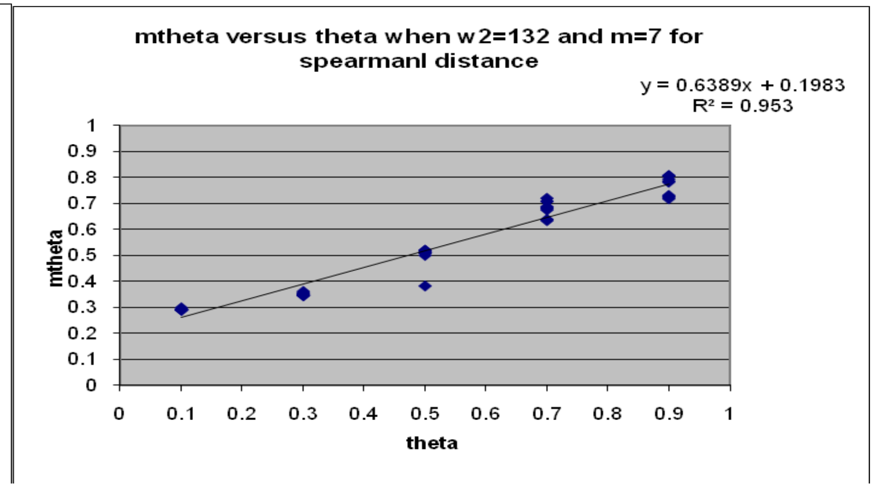


Fig 3-6: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 132$ and m=7 for Spearman distance

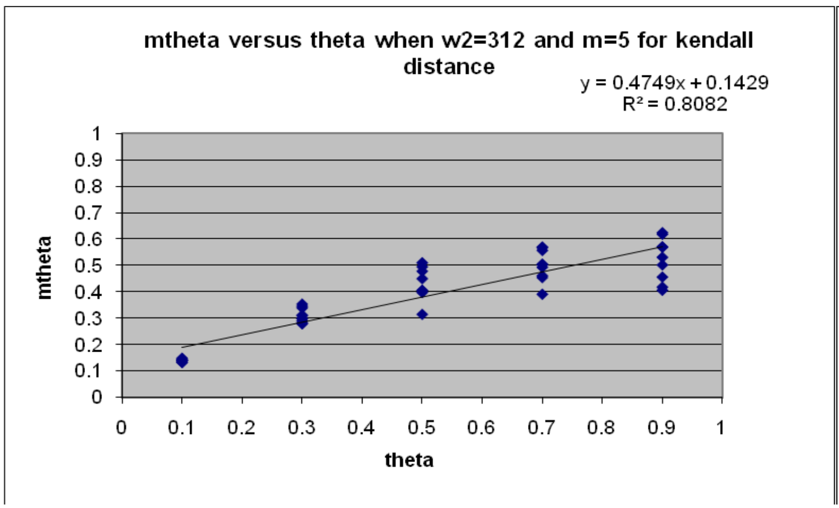


Fig 3-7: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 312$ and m=5 for Kendall distance

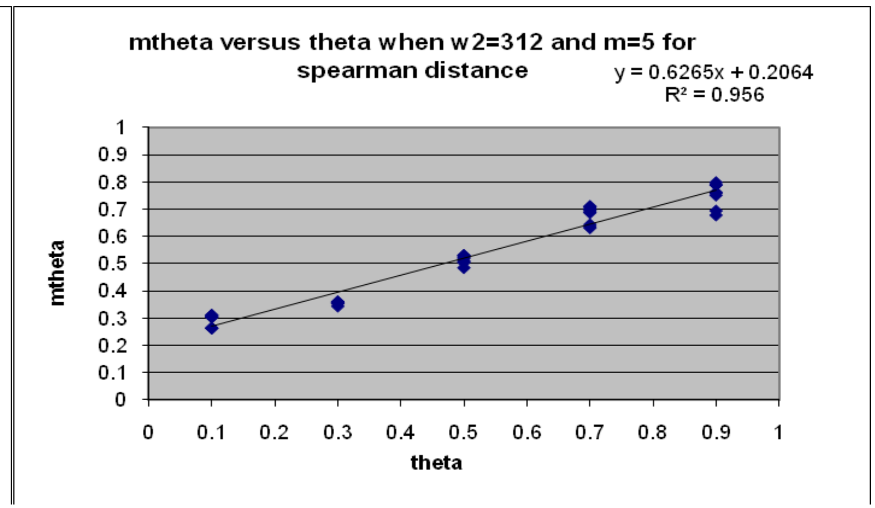


Fig 3-8: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 312$ and m=5 for Spearman distance

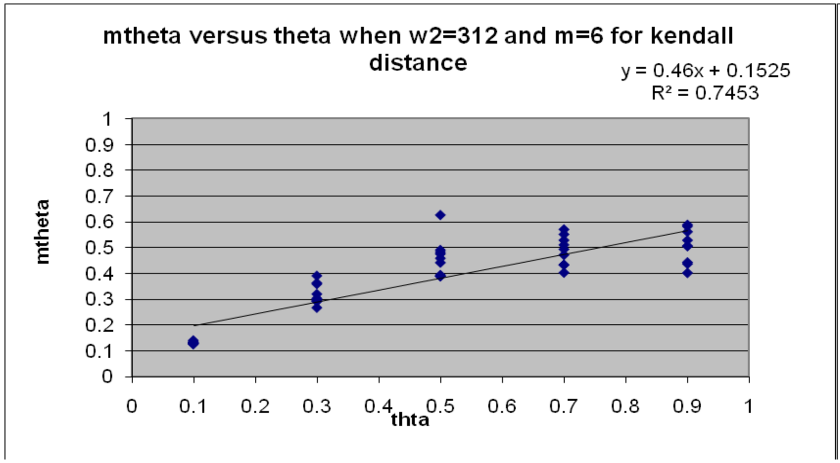


Fig 3-9: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 312$ and m=6 for Kendall distance

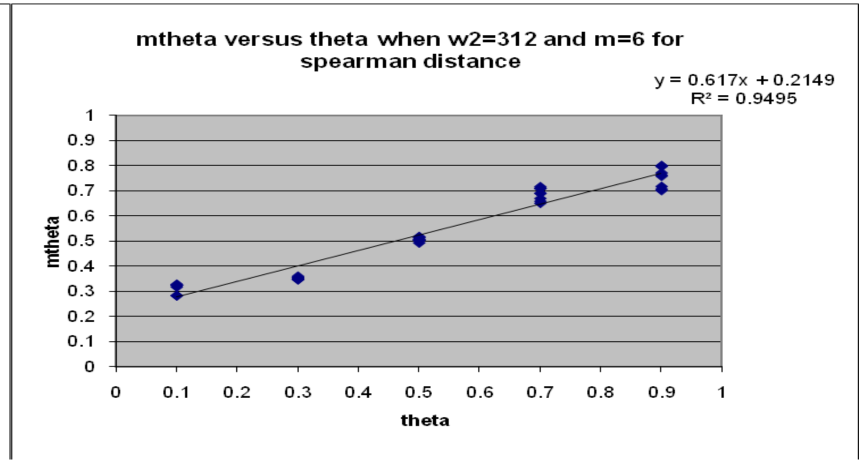


Fig 3-10: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 312$ and m=6 for Spearman distance

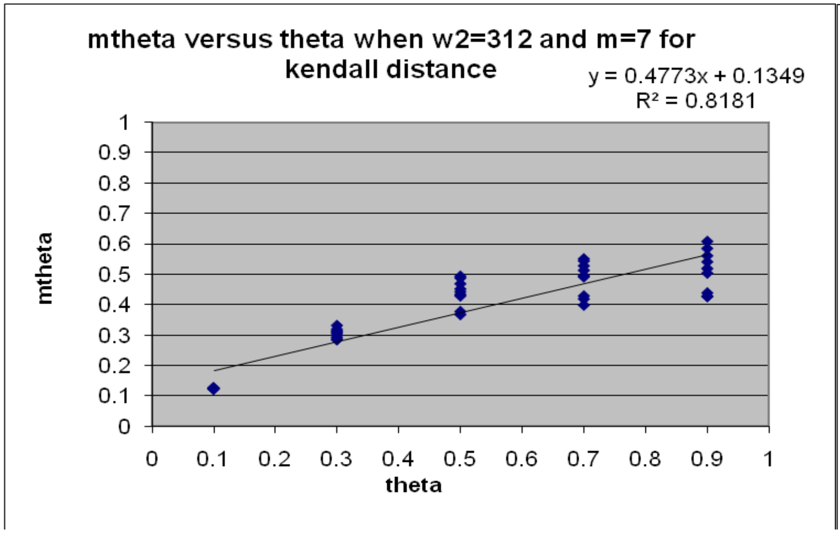


Fig 3-11: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 312$ and m=7 for Kendall distance

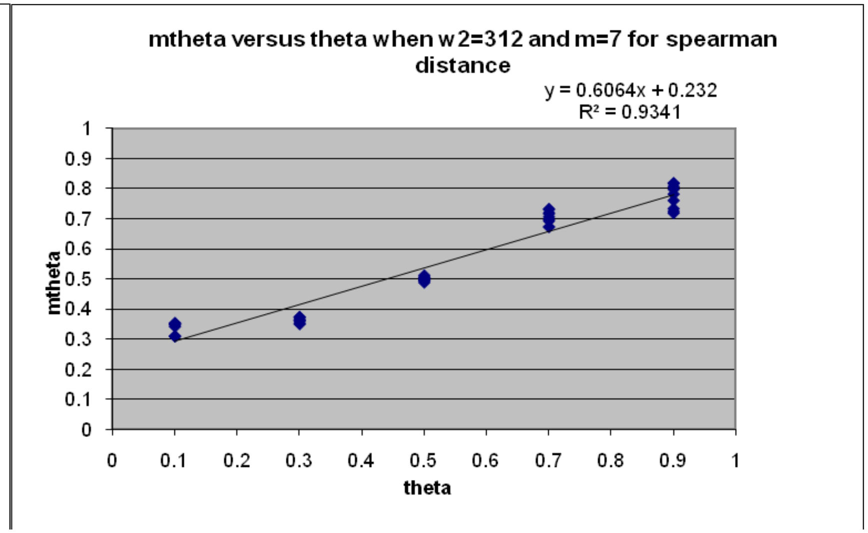


Fig 3-12: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 312$ and m=7 for Spearman distance

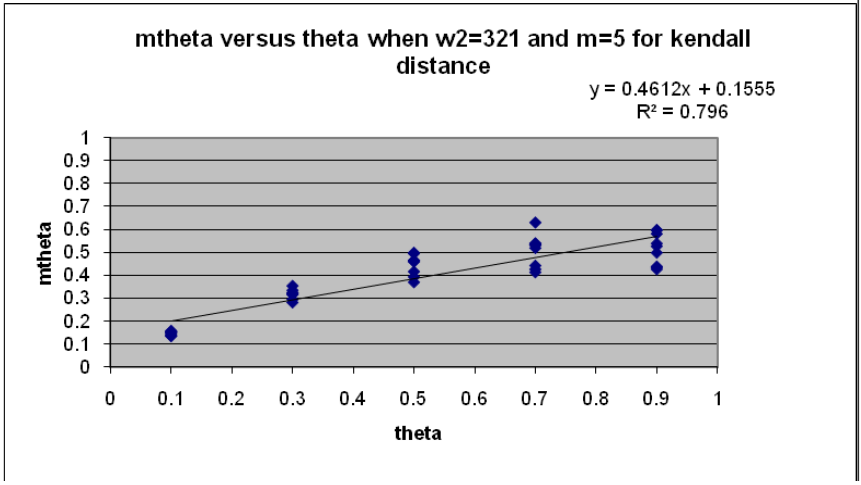


Fig 3-13: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 321$ and m=5 for Kendall distance

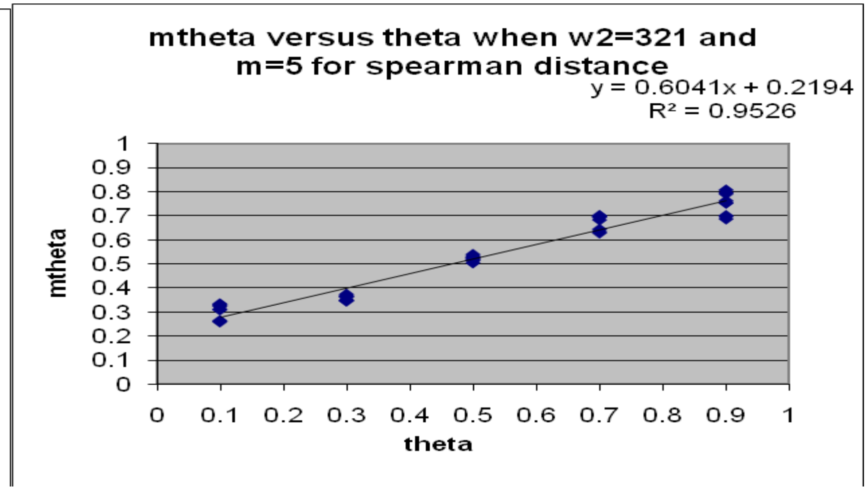


Fig 3-14: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 321$ and m=5 for Spearman distance

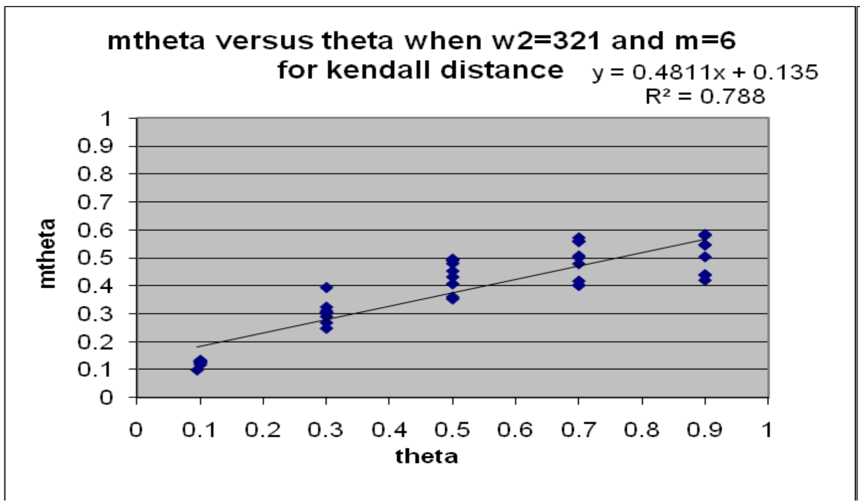


Fig 3-15: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 321$ and m=6 for Kendall distance

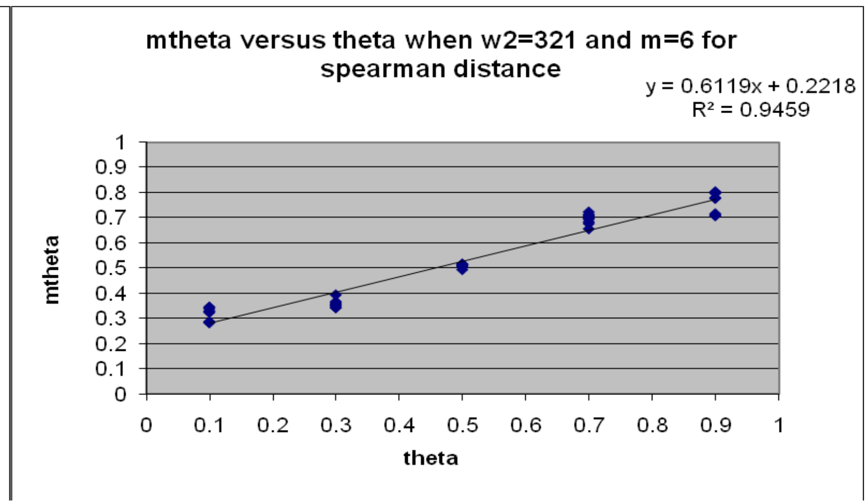
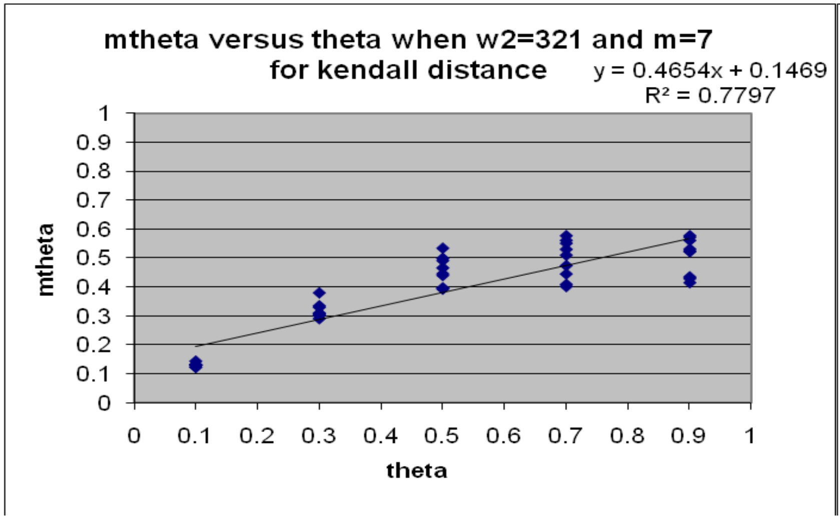


Fig 3-16: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 321$ and m=6 for Spearman distance



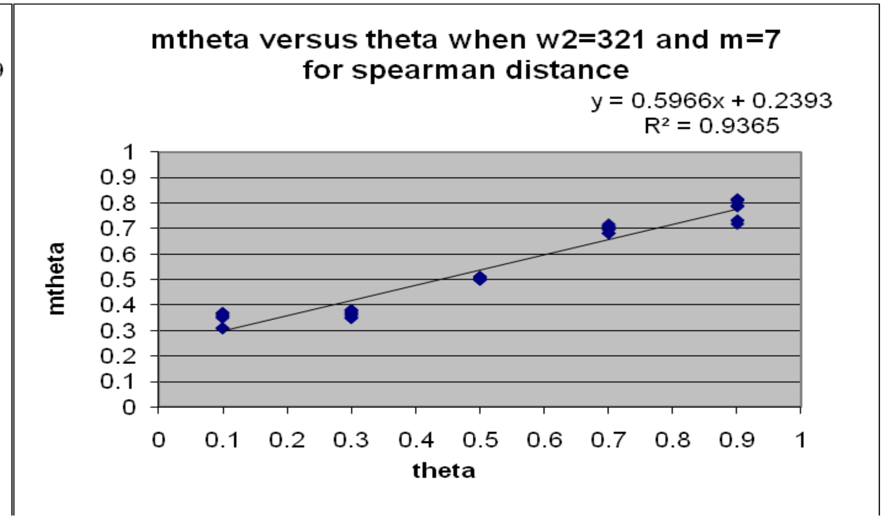


Fig 3-17: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 321$ and m=7 for Kendall distance

Fig 3-18: $\hat{m}(\theta)$ versus θ when $\mathbf{w}_2 = 321$ and m=7 for Spearman distance

A multiple regression model was used to estimate the relationship between the sample mean $\hat{m}(\theta)$ and a set of explanatory variables.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots..+ \beta_p x_p + \varepsilon$$

Where,

y = Dependent variable,

$\beta_0, \beta_1, \beta_2, \ldots.., \beta_p$ are the parameters to be estimated.

The dependent (response) variable was the sample mean $\hat{m}(\theta)$. The quantitative relationships were sought between the dependent variable and the following sets of independent (explanatory) variables:

X_1 = n (the number of judges)

X_2 = m, (the number of items),

$X_3 = \theta$ (a parameter lying in the unit interval, true value)

$X_4 = \pi$ (proportion of consumers from whole population in segmentation 1)

X_5 =Distance (Distance=0 for Kendall, Distance=1 for Spearman)

Two dummy variables were used for three distinct orderings in $\mathbf{w}_2$. Let D_1=0, D_2=0 for

$\mathbf{w}_2 = 132$; D_1=1 D_2=0 for $\mathbf{w}_2 = 312$; D_1=0, D_2=1for $\mathbf{w}_2 = 321$.

X_6= D_1 (distance from $\mathbf{w}_1$ is somewhat far)

X_7 =D_2 (distance from $\mathbf{w}_1$ is far)

Base line $\mathbf{w}_2 = 132$ with D_1=0and D_2=0 (medium distance from $\mathbf{w}_1$)

ε = Error term which is assumed to follow a normal distribution with mean zero and variance σ_ε^2.

JMP statistical software (SAS Institute, Cary, NC, USA) was used to perform the analysis. A multiple regression model defined above was fit to the data in appendix 1 using least squares with sample mean $\hat{m}(\theta)$ as the dependent variable and m, n and true values of θ, and π, the type of distance (Distance=0 for Kendall, Distance=1 for Spearman) and D_1, D_2 (the form of $\mathbf{w}_2$) as independent variables. A summary of the fit given in appendix 4 shows that n (the number of judges), m (the number of items), θ, Distance (distance function) have statistically significant effects on the sample mean $\hat{m}(\theta)$. There is not enough evidence to conclude that D_1, D_2 (the form of $\mathbf{w}_2$) or π influence the sample mean $\hat{m}(\theta)$. As n (the number of judges) increased and m decreases, everything else remaining fixed, the sample mean $\hat{m}(\theta)$ increases by small quantities.

Since $MSE(\hat{\theta})$ was very smaller for most scenarios, simulation results were also computed in terms of $'CV' = \sqrt{MSE(\hat{\theta})} / \theta$. The distribution of $'CV' = \sqrt{MSE(\hat{\theta})} / \theta$ in Fig 2 in appendix 4 is skewed to the left. For both Kendall and Spearman distance, the five number summary for $'CV'$ is 0, 0.006, 0.0209, 0.0519, 0.5965. 50% of $'CV'$ has values of 0.0209 or less. 25% had a relatively low $'CV'$ value of 0.006 or less. The top 25% of $'CV'$ has values of 0.0519 or more. For Kendall distance, the five number summary for $'CV'$ is 0.00007, 0.0147, 0.0369, 0.0553, 0.5965. 50% of $'CV'$ has values of 0.0369 or less. 25% had a relatively low $'CV'$ value of 0.0147 or less. The top 25% of $'CV'$ has values of 0.0553 or more. For Spearman distance, the five number summary for $'CV'$ is 0, 0.00288, 0.0147, 0.0232, 0.5567. 50% of $'CV'$ has values of 0.0147 or less. 25% had a relatively low $'CV'$ value of 0.00288 or less. The top 25% of $'CV'$ has values of 0.0232 or more. The $'CV'$ value had smaller five number summary for Spearman's distance than corresponding five number summary for Kendall's distance, suggesting that bias was smaller for Spearman's distance than for Kendall's distance.

3.1.2 Point estimation for π

Appendix 2 displays point estimate $m(\hat{\pi})$, standard error of $m(\hat{\pi})$, MSE of $\hat{\pi}$, standard error of MSE of $\hat{\pi}$. The scatter plots in Fig 3-19 to 3-36 based on the data from Appendix 2 show the relationship between the simulated sample means of $\hat{\pi}$ and the true values of π. If $\hat{\pi}$ is an unbiased estimator of π, we would expect to see the points fall close to a 45 degree line. The least squares line is given in each graph. We observed that as m increased, R^2 decreased, and suggesting increasing bias of the estimators. The R^2 value ranged 0.62 to 0.87.

For most cases, the estimators based on the Spearman distance performed better than Kendall distance. The bias of the estimators appeared to be negative for most cases. Even though $\hat{\pi}$ is biased, it had very small standard error and small MSE as well as small standard error of MSE.

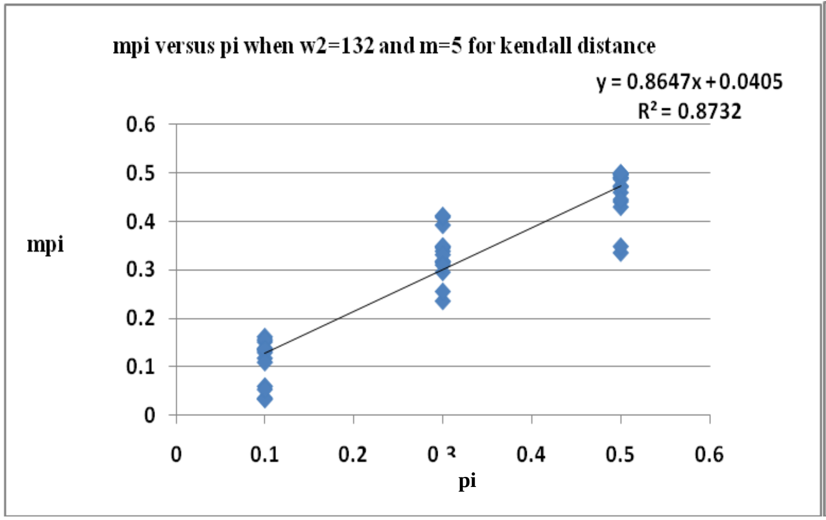


Fig 3-19: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 132$ and m=5 for Kendall distance

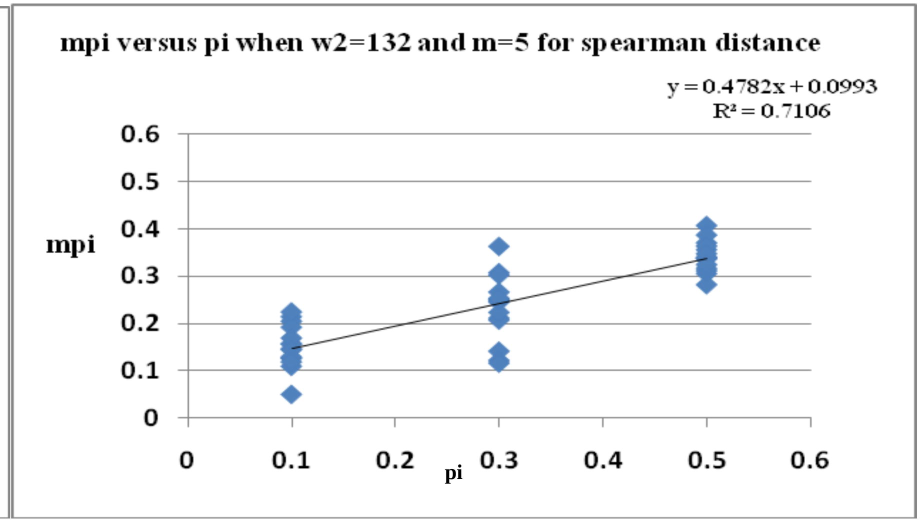


Fig 3-20: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 132$ and m=5 for Spearman distance

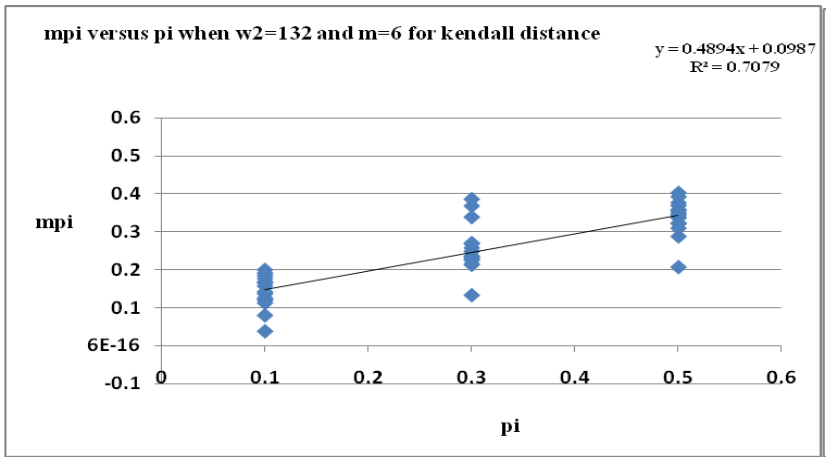


Fig 3-21: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 132$ and m=6 for Kendall distance

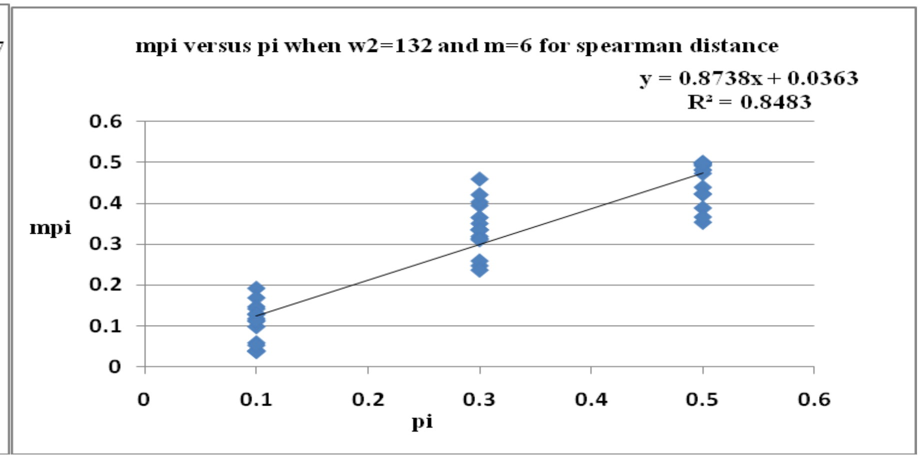
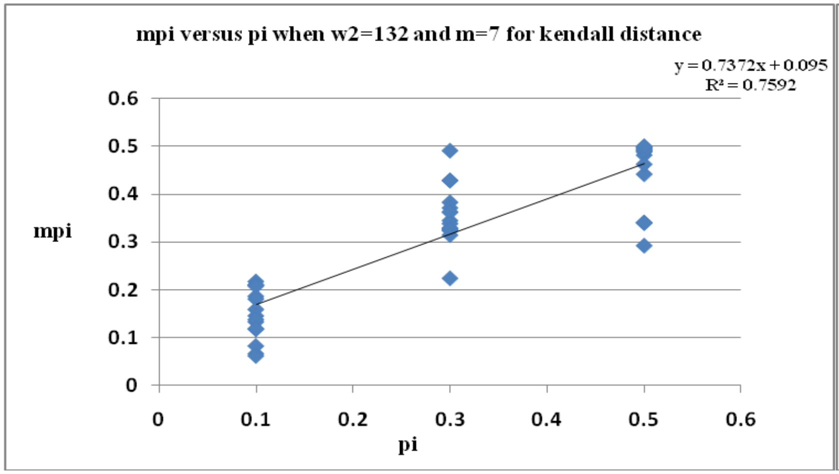


Fig 3-22: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 132$ and m=6 for Spearman distance



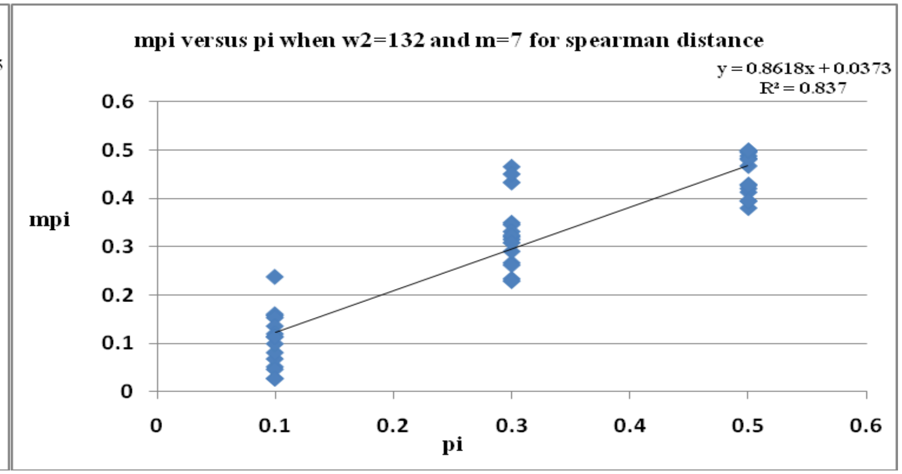
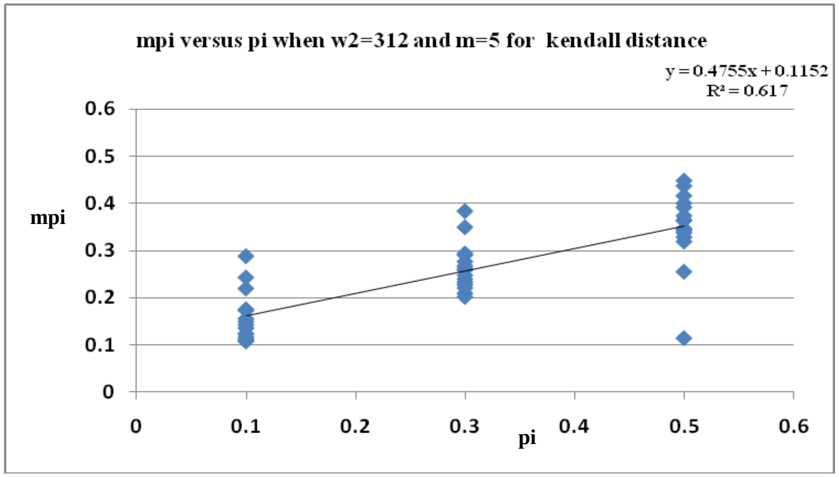


Fig 3-23: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 132$ and m=7 for Kendall distance

Fig 3-24: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 132$ and m=7 for Spearman distance



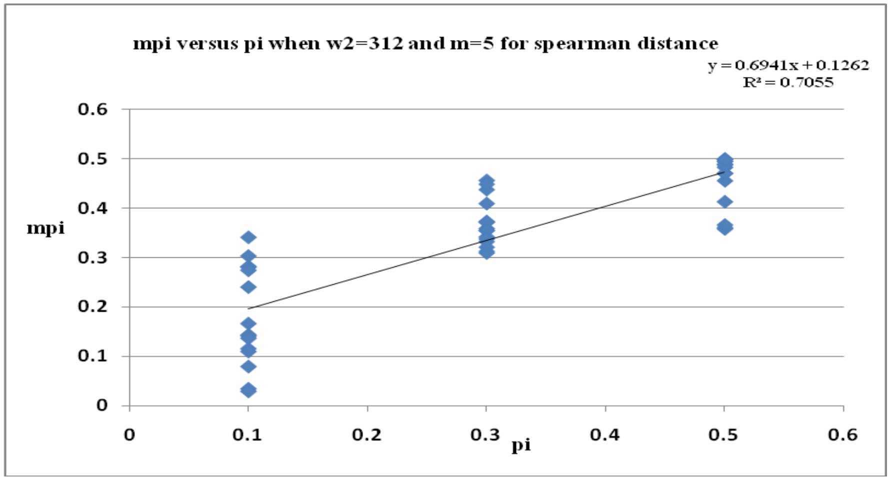


Fig 3-25: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 312$ and m=5 for Kendall distance

Fig 3-26: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 312$ and m=5 for Spearman distance

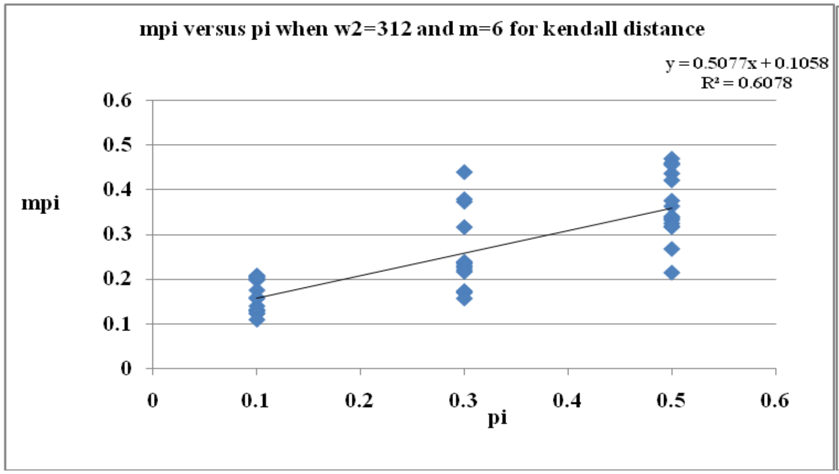


Fig 3-27: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 312$ and m=6 for Kendall distance

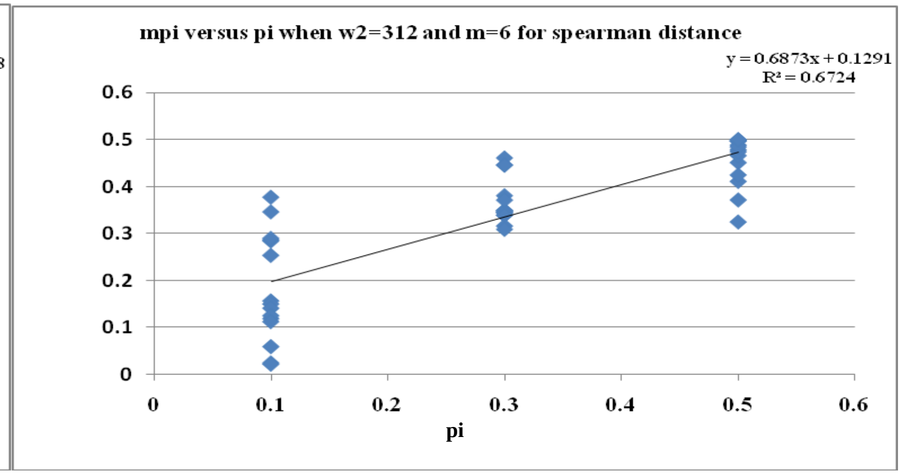


Fig 3-28: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 312$ and m=6 for Spearman distance

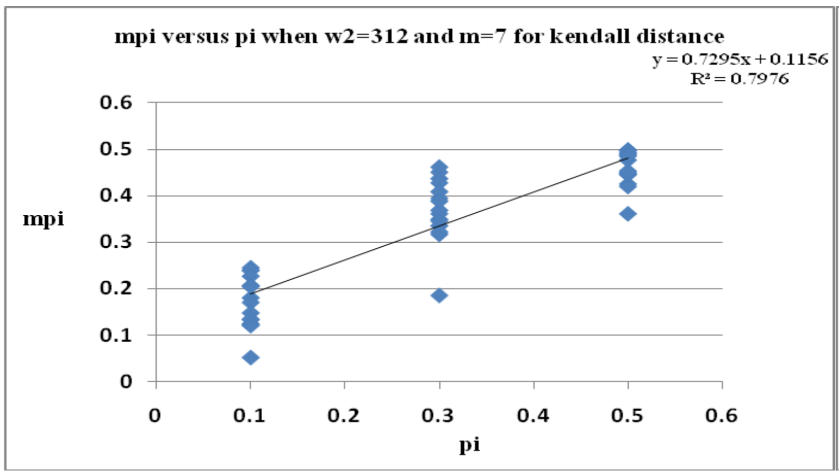


Fig 3-29: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 312$ and m=7 for Kendall distance

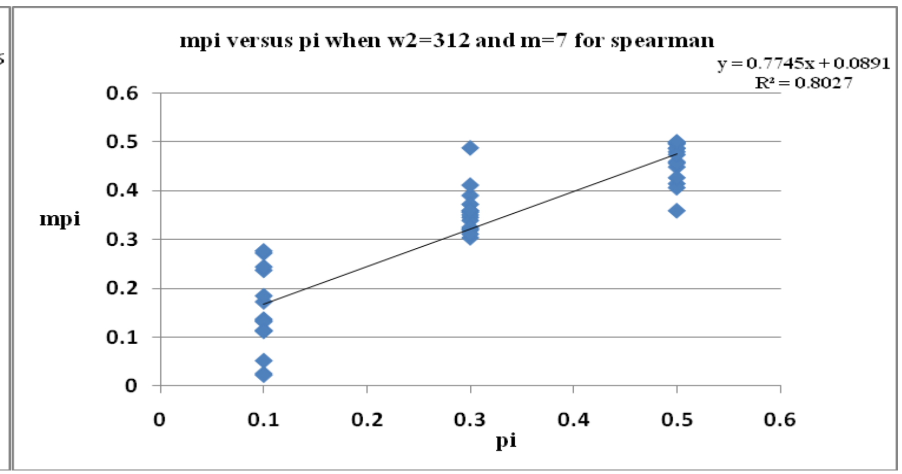
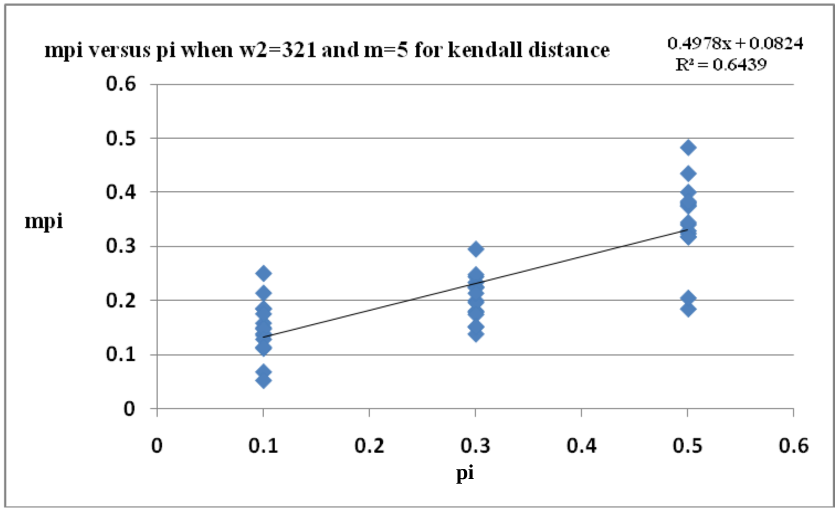


Fig 3-30: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 312$ and m=7 for Spearman distance



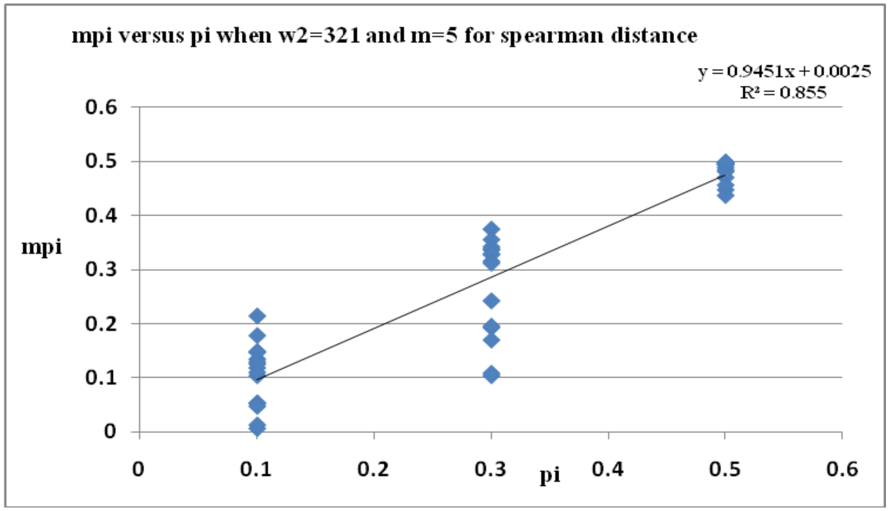
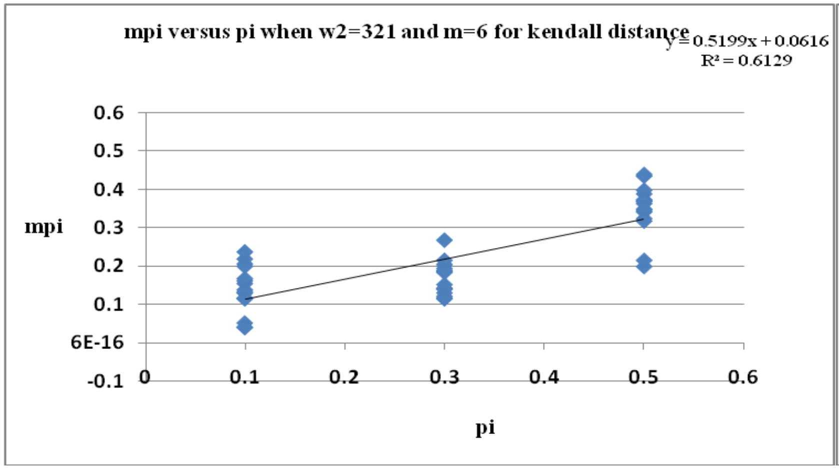


Fig 3-31: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 321$ and m=5 for Kendall distance

Fig 3-32: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 321$ and m=5 for Spearman distance



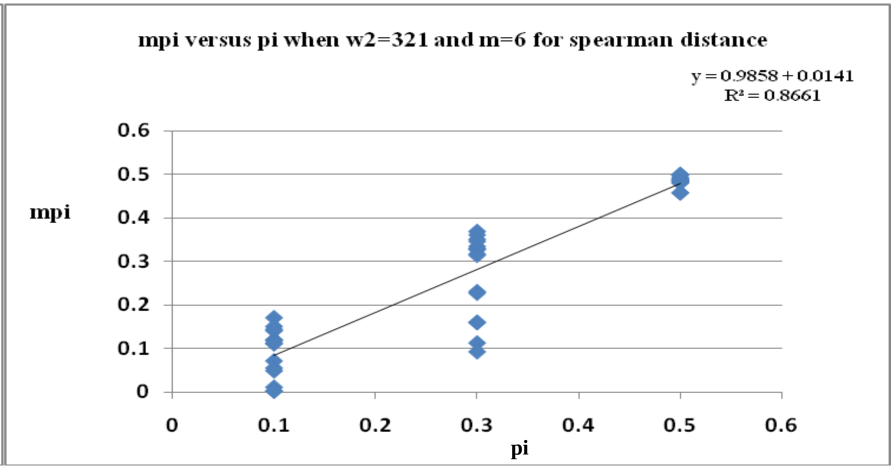
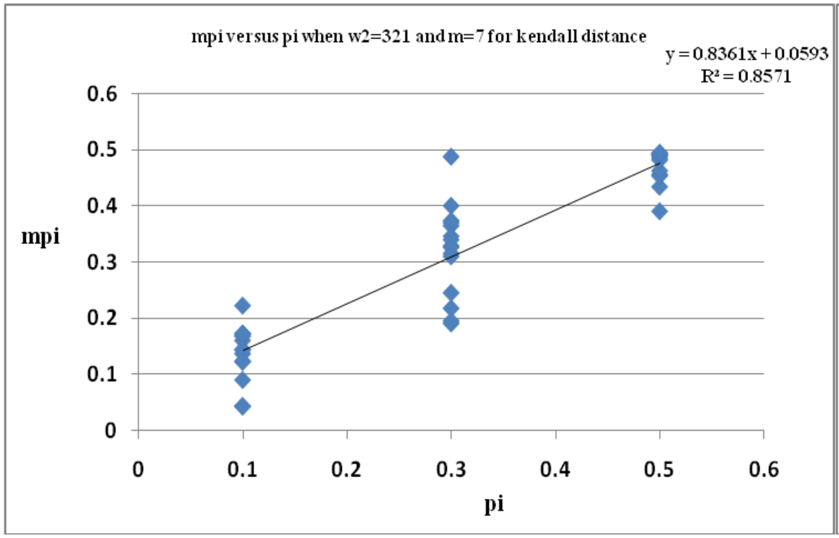


Fig 3-33: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 321$ and m=6 for Kendall distance

Fig 3-34: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 321$ and m=6 for Spearman distance



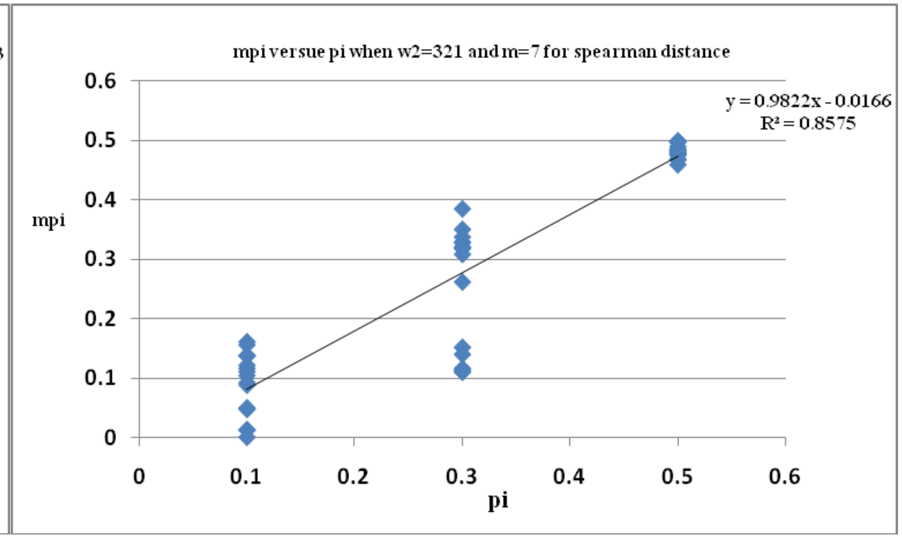


Fig 3-35: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 321$ and m=7 for Kendall distance

Fig 3-36: $\hat{m}(\pi)$ versus π when $\mathbf{w}_2 = 321$ and m=7 for Spearman distance

A multiple regression model was fit to the data in Appendix 2 using least squares with sample mean $\hat{m}(\pi)$ as the dependent variable and m, n and true values of θ, and π, the type of distance (Distance=0 for Kendall, Distance=1 for Spearman) and D_1, D_2 (the form of $\mathbf{w}_2$) as independent variables. A summary of the fit given in appendix 5 shows that all independent variables have statistically significant effects on the sample mean $\hat{m}(\pi)$. As n (the number of judges) increases and m decreases, everything else remaining fixed, the sample mean $\hat{m}(\pi)$ increases by small quantities. Similarly, since $MSE(\hat{\pi})$ was very smaller for most scenarios, simulation results were also computed in terms of $'CV' = \sqrt{MSE(\hat{\pi})}/\pi$. The distribution of $'CV' = \sqrt{MSE(\hat{\pi})}/\pi$ in Fig 2 in appendix 17 is skewed to the left. The range of $'CV'$ is from 0 to 1.063. The median of $'CV'$ is 0.0209. For both Kendall and Spearman distance, the five number summary for $'CV'$ is 0.00002, 0.00983, 0.0277, 0.0634, 1.063. 50% of $'CV'$ has values of 0.0277 or less. 25% had a relatively low $'CV'$ value of 0.00983 or less. The top 25% of $'CV'$ has values of 0.0634 or more. For Kendall distance, the five number summary for $'CV'$ is 4.29e-5, 0.0145, 0.0387 0.0849, 1.063. 50% of $'CV'$ has values of 0.0387 or less. 25% had a relatively low $'CV'$ value of 0.0145 or less. The top 25% of $'CV'$ has values of 0.0849 or more. For Spearman distance, the five number summary for $'CV'$ is 0.00002, 0.00535, 0.0173, 0.0496, 0.886. 50% of $'CV'$ has values of 0.0 173 or less. 25% had a relatively low $'CV'$ value of 0.00535 or less. The top 25% of $'CV'$ has values of 0.0496 or more. Again, the $'CV'$ value had smaller five number summary for Spearman's distance than corresponding five number summary for Kendall's distance, suggesting that bias was smaller for Spearman's distance than for Kendall's distance.

3. 2 Profile Likelihood Set Estimation of θ, π and $\{\mathbf{w}_i\}$

Under standard regularity conditions, which may not hold for mixture models of the type I am investigating, a nominal $(1-\alpha)100\% = 95\%$ profile likelihood confidence set for θ is given by the values

$$\{\theta,\ \ 2[LogL(\hat{\theta},\hat{\pi},\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2) - LogL^P(\theta)] \le 3.84\}. \tag{3.1}$$

Values of these θ satisfying this condition were estimated by incrementing θ from 0.05 to 1 by increment of 0.05 and computing $LogL^P(\theta)$ for each value of θ

Note that

$$L^P(\theta) \le \max L(\theta,\pi,\mathbf{w}_1,\mathbf{w}_2) = L(\hat{\theta},\hat{\pi},\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2) \tag{3.2}$$

Similarly, a standard profile likelihood set estimate with nominal confidence $(1-\alpha)100\% = 95\%$ for the mixing proportion π is given by the set of values π such that

$$\{\pi,\ \ 2[LogL(\hat{\theta},\hat{\pi},\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2) - LogL^P(\pi)] \le 3.84\} \tag{3.3}$$

The baseline rankings $\mathbf{w}_i$ are discrete vectors for which one would not expected the usual likelihood based results to hold. Nonetheless, I investigated profile likelihood set estimates for these baseline rankings. A nominal 95% profile likelihood set for $LS\{\mathbf{w}_1,\mathbf{w}_2\}$ for $\{\mathbf{w}_1,\mathbf{w}_2\}$ is given by

$$\{\mathbf{w}_i,\ \ 2[LogL(\hat{\theta},\hat{\pi},\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2) - LogL^P(\mathbf{w}_i)] \le 3.84\},\ \ i=1,\ 2 \tag{3.4}$$

where

$$L^P(\mathbf{w}_1,\mathbf{w}_2) = \max_{\theta,\pi} L(\theta,\pi,\mathbf{w}_1,\mathbf{w}_2) \le L(\hat{\theta},\hat{\pi},\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2) \tag{3.5}$$

The following algorithm can be used to approximate $LS\{\mathbf{w}_1, \mathbf{w}_2\}$.

(1). First update $\hat{\theta}$ with the initial guess for θ, and, π, then update $\{\hat{\pi}_i\}$, and $\hat{\theta}$,…., until convergence, yielding

$$L^P(\{\mathbf{w}_i\}) = L(\{\mathbf{w}_i\}, \hat{\theta}\{\mathbf{w}_i\}, \hat{\pi}\{\mathbf{w}_i\}) \tag{3.6}$$

(2). Search through $S_{qm} * S_{qm} * \ldots * S_{qm}$ to find $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k$ such that

$$[A] \qquad \frac{L^P\{\mathbf{w}_i, i = 1,\ 2,\ldots K\}}{L^P\{\hat{\mathbf{w}}_i, i = 1,\ 2,\ldots K\}} \geq C$$

$LS(\mathbf{w}^{\#}) = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k\}$ *such that* $[A]$ *holds*

The estimating set $LS(\mathbf{w}^{\#})$ is evaluated using the following two criteria:

(1). Estimated Coverage rate, denoted cov_t: Proportion of simulated data sets for which $LS(\mathbf{w}^{\#})$ contains the true baseline rankings.

(2) Mean relative volume: $MRV = \dfrac{\overline{V}\{LS(\mathbf{w}^{\#})\}}{\left[\dfrac{n!}{(n-k)!}\right]}$,

where $V\{LS(\mathbf{w}^{\#})\}$ = the number of rankings $\in LS(\mathbf{w}^{\#})$ and $\overline{V}\{LS(\mathbf{w}^{\#})\}$ is the average of these volumes over simulated data sets.

This approach did not work for π and these results were not presented**.**

3.2.1 Profile Likelihood Set Estimation for θ

Two simulation studies were conducted to examine the relative operating characteristics of likelihood set estimation for θ. Large replication (R=100) with varieties of varying conditions were used in the first simulation. We were not able to combine the first and second simulation together due to extensive computation time in the second simulation. Large replication (R=100) with few varying conditions were used in the second simulation.

The first simulation was done on a total of 3*5*3*3*3=810 scenarios with one hundred data sets for each scenario. The data generated in section 3.1 for point estimation was used to study the behavior of MLE's with respective to the set estimation. The initial value for θ were set from 0.05 to 1 with increment by 0.05. For each increment for θ, a modified EM algorithm was used to find the MLES for $\hat{\pi}$, $\hat{\mathbf{w}}_1$, and $\hat{\mathbf{w}}_2$ with θ fixed , Once those MLES were found, then update $\hat{\pi}$, $\hat{\mathbf{w}}_1$, and $\hat{\mathbf{w}}_2$, continues these iterations until relatively change in function $Q(\xi,\xi^{\mathbf{i}})$ and the stopping rule is given as: $|Q(\xi^{\mathbf{i+1}},\xi^{\mathbf{i}})-Q(\xi^{\mathbf{i}},\xi^{\mathbf{i-1}})|=10^{-2}$. In the last iterations, compute the log likelihood with the updated $\hat{\pi}$, $\hat{\mathbf{w}}_1$, $\hat{\mathbf{w}}_2$, and the fixed θ value. For each data set, log likelihood values were collected for each fixed θ, where θ was set from 0.05 to 1 with increment by 0.05. At the same time, MLES for $\hat{\pi}$, $\hat{\mathbf{w}}_1$, $\hat{\mathbf{w}}_2$, and $\hat{\theta}$ for each data set were obtained using the EM algorithm described in the previous chapter.

We define:

t_wid = estimated mean width of profile interval estimate of θ

se_t_wid = standard error of t_wid

cov_t = estimated coverage rate of profile interval estimate of θ.

Table 3-1 shows the average time in hours required to complete a profile likelihood interval estimate for θ when m=5, 6, 7. From Table 3-1, it can be seen that as n and m increase, the average time in hours required completing a profile likelihood interval estimate for θ increases, as expected.

Table 3-1 Average time in hours required to complete a profile likelihood interval estimate for θ

	m=5	m=6	m=7
n=10	0.5	1.4	8
n=20	1	3	12
n=30	1.5	6	24

Fig 3-37 to 3-40 shows the changes of profile likelihood as θ changes while true θ=0.1, 0,3, 0.5, and 0.7, respectively. It can be seen from those plots that the MLEs of θ occur close to the true values of θ.

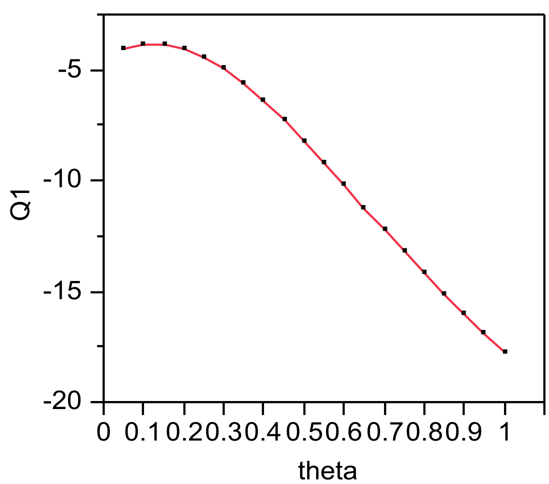


Fig 3-37. Profile likelihood for θ when the true θ=0.1.

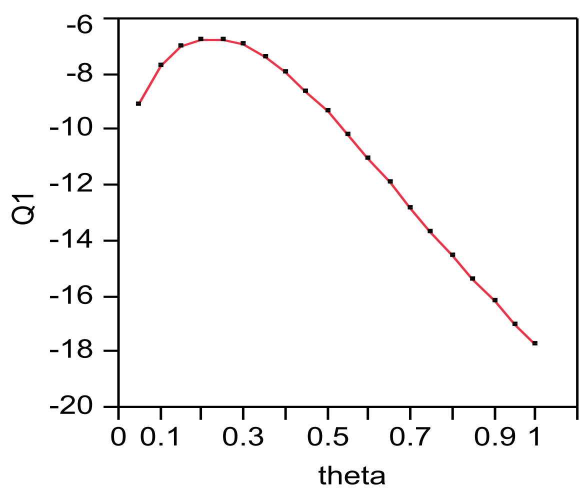


Fig 3-38. Profile likelihood for θ when the true θ=0.3.

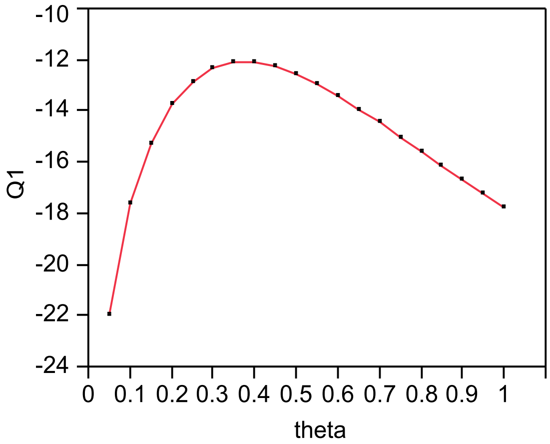


Fig 3-39. Profile likelihood for θ when the true θ=0.5.

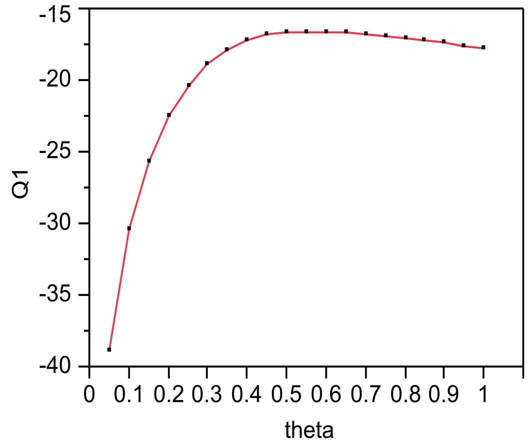


Fig 3-40. Profile likelihood for θ when the true θ=0.7.

Appendix 1shows that profile likelihood interval estimate and sample mean coverage rate for $\hat{\theta}$ when the number of items m=5, 6, 7. Overall, the coverage rate performed better for Spearman's distance than for Kendall's distance, especially for small m.

By using a multiple regression method, it is easy to find the relationship between the interval width and the various facts including true values of θ, π, n, m, the type of distance (Distance=0 for Kendall, Distance=1 for Spearman) and D_1, D_2 (the form of $\mathbf{w}_2$) according to the data in appendix 1. The summary of fit from JMP output in Appendix 6 shows that the true values of π, and D_1 and D_2 do not have significant effect on the sample mean interval width estimate for θ when m=5, 6, 7. The true value of θ, distance function, m (number of items), and n (number of judge) did have significant effect on sample mean interval width estimate for θ when m=5, 6, 7. As θ increased, everything else remaining fixed, interval width for $\hat{\theta}$ increased. As m (number of items) decreased and *n* (number of judges) increased, everything else remaining fixed, interval width for $\hat{\theta}$ increased.

A logistic regression model was used to estimate the relationship between the sample mean coverage rate and a set of explanatory variables for the data in appendix 1.

$$\log it\left(\frac{y}{100}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots.. + \beta_p x_p$$

Where,

y =number of coverages out of 100 simulation data sets,

$\beta_0, \beta_1, \beta_2, \ldots.., \beta_p$ are the parameters to be estimated.

The dependent (response) variable was the sample coverage rate. The quantitative relationships were sought between the dependent variable and the following sets of independent (explanatory) variables:

X_1 = n (the number of judges)

X_2 = m (the number of items),

$X_3 = \theta$ (a parameter lying in the unit interval, true value)

$X_4 = \pi$ (proportion of consumers from whole population in segmentation 1)

X_5 =Distance (Distance=0 for Kendall, Distance=1 for Spearman)

Two dummy variables were used for three distinct orderings in $\mathbf{w}_2$.

Let D_1=0, D_2=0 for $\mathbf{w}_2 = 132$; D_1=1 D_2=0 for $\mathbf{w}_2 = 312$; D_1=0, D_2=1for $\mathbf{w}_2 = 321$.

X_6= D_1

X_7 = D_2

JMP output is listed in appendix 6. It can be seen that all explanatory variables had significant effect on sample mean coverage rate estimate. As θ increased, everything else remaining fixed, coverage rate for $\hat{\theta}$ increased since coverage width increased as θ increased. As m (number of items) increased and n (number of judges) increased, the probability that the interval would contain the true θ decreased provided that all remaining predictor variables for the coverage rate being compared have identical values. The odds ratio for D_1 was 0.8854, suggesting that comparing $\mathbf{w}_2 = 312$ (somehow far distance from $\mathbf{w}_1 = 123$) to $\mathbf{w}_2 = 132$ (medium distance from $\mathbf{w}_1 = 123$) holding other explanatory variables constant, the odds of reducing the coverage rate by a factor of 0.88. Similarly, the odds ratio for D_2 was 0.8275, suggesting that comparing $\mathbf{w}_2 = 321$ (far distance from $\mathbf{w}_1 = 123$) to $\mathbf{w}_2 = 132$ (medium distance from $\mathbf{w}_1 = 123$), the odds of coverage rate would be reduced by a factor of 0.8275 while other variables were identical for the simulation studies being compared. The statistical significance of the parameter estimate of -0.1954 corresponding to n (the number of judges) imply that the odds of coverage rate would be reduced by a factor of 0.8225 with every judge increase by one unit. m (the number of items) was statistically significant and had an estimated value of -0.0175, indicating that the odds of coverage rate would be reduced by a factor of 0.9827 with every item increase by one unit. The average coverage rates for θ with m=5, 6, 7 were 0.97, 0.92 and 0.95, respectively.

Table 3-2 Average time in hours required to complete a profile likelihood interval estimate for θ

	m=8	m=9
n=10	30	86

Table 3-2 shows the average time in hours required to complete a profile likelihood interval estimate for θ when m=8 and 9 with n=10. Beocat program in CIS at K-State provided space for running large sets of simulation. Due to the limitation of space, profile likelihood interval estimate for the combination of n=20, 30 (numbers of judges) and m=8, 9 (number of items) were excluded from the second simulation. We only consider cases where n=10 and m=8, 9.

In the second simulation, we examined the performance of the set estimator across 240 scenarios for large m. The performance of the interval estimate was investigated for one hundred data sets per scenario generated in Software Package *R*.

The data generation scenarios were taken the following parameters values:

$\pi = 0.1, \; 0.3, \; 0.5;$

$\theta = 0.1, \; 0.3, \; 0.5, \; 0.7, \; 0.9;$

$\mathbf{w}_2 = 132, \; 312, \; 321;$

n=10 (number of judges);

m=8, 9 (number of items);

Distance function includes Kendall's and Spearman's distance functions;

Appendix 3 displays interval estimate, and sample mean coverage rate for θ when m=8, 9 (the numbers of items). The set estimator performed unsatisfactorily for θ when $\theta = 0.9$. The coverage rate performed better for Spearman's distance than for Kendall's distance.

A multiple regression model was fit to data in appendix 3 with sample mean interval width for m=8, 9 as dependent variable and true values of θ, π, *n* (the numbers of judges), the type of distance (Distance=0 for Kendall, Distance=1 for Spearman) and D_1, D_2 (the form of $\mathbf{w}_2$) as independent variables. The summary of fit from JMP output in appendix 6 shows that the true values of π, n and D_1 and D_2 do not have significant effect on the sample mean interval width estimate for θ. The true value of θ and distance function had significant effect on sample mean interval width estimate for θ.

A logistic regression model was fitted to the data in Appendix 3 with sample mean coverage rate estimate as dependent variable and true values of θ, π, and n, the type of distance

(Distance=0 for Kendall, Distance=1 for Spearman) and D_1, D_2 (the form of $\mathbf{w}_2$) as independent variables. The true value of θ, π, Distance, D_1, D_2, n does have significant effect on sample mean coverage rate estimate for θ. As θ increases, everything else remaining fixed, coverage rate for θ increases since coverage width increases as θ increases. As n (number of judges) increases, the probability that the interval will contain the true θ decreases provided that all remaining predictor variables for the coverage rate being compared have identical values. The odds ratio for D_1, is 0.4049, suggesting that comparing $\mathbf{w}_2 = 312$ (somehow far distance from $\mathbf{w}_1 = 123$) to $\mathbf{w}_2 = 132$ (medium distance from $\mathbf{w}_1 = 123$) holding other explanatory variables constant, the odds of the coverage rate will be reduced by a factor of 0.4049. Similarly, the odds ratio for D_2 is 0.6347, suggesting that comparing $\mathbf{w}_2 = 321$ (far distance from $\mathbf{w}_1 = 123$) to $\mathbf{w}_2 = 132$ (medium distance from $\mathbf{w}_1 = 123$), the odds of coverage rate will be reduced by a factor of 0.6347 while other variables are identical for the simulation studies being compared. The statistical significance of the parameter estimate of -0.2587 corresponding to n (the number of judges) imply that the odds of coverage rate will be reduced by a factor of 0.7721 with every judge increase by one unit.

3.2.2 Profile likelihood set estimation for $\mathbf{w}_1$ and $\mathbf{w}_2$

Table 3-3 shows the average time in hours required to complete a likelihood interval estimate for $\{\mathbf{w}_i\}_{i=1}^{2}$ when m=5 and n=10, 20, 30. From Table 3-3, it can be seen that as n increase, the average time in hours required completing a profile likelihood interval estimate for

$\{\mathbf{w}_i\}_{i=1}^{2}$ increases, as expected. Taking intensive computation time into consideration, profile likelihood set estimation with *m* = 6, 7, 8, 9 for $\{\mathbf{w}_i\}_{i=1}^{2}$ were excluded from the simulation study.

Table 3-3 Average time in hours required to complete a likelihood interval estimate for $\{\mathbf{w}_i\}_{i=1}^{2}$

	m=5
n=10	68
n=20	98
n=30	198

In this section, we conducted three simulation studies. The purpose of the first simulation was to find the 'right' cutoff point "C" such that the coverage rate is close to 95% nominal level whether θ is close to 0 or 1 with smaller SMRV in all cases. After the cutoff point C was chosen, the second and third simulation were conducted to examine the performance of likelihood set estimator for $\mathbf{w}_1$ and $\mathbf{w}_2$ with respective to mean relative volume (MRV) and coverage rate Scov_t using the cutoff point C value obtained from the first simulation study. Small replication (R=20) with varieties of varying conditions and large replication (R=100) with few varying conditions were used in the second and third simulation study, respectively.

The following scenarios are considered in our first simulation with varying parameters values:

π =0.1, θ=0.1, *n*=10, *m*=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#1

π =0.1, θ=0.1, *n*=20, *m*=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#2

π =0.1, θ=0.1, *n*=30, *m*=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#3

π =0.1, θ=0.3, *n*=10, *m*=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#4

π =0.1, θ=0.3, *n*=20, *m*=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#5

π =0.1, θ=0.3, *n*=30, *m*=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#6

π =0.1, θ=0.5, n=10, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#7

π =0.1, θ=0.5, n=20, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#8

π =0.1, θ=0.5, n=30, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#9

π =0.1, θ=0.7, n=10, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#10

π =0.1, θ=0.7, n=20, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#11

π =0.1, θ=0.7, n=30, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#12

π =0.1, θ=0.9, n=10, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#13

π =0.1, θ=0.9, n=20, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#14

π =0.1, θ=0.9, n=30, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#15

The initial values for π and θ were set at their true values. $\mathbf{w}_1 \in \mathbf{S}_{qm} - \{\mathbf{w}_2\}$ and $\mathbf{w}_2 \in \mathbf{S}_{qm}$. For m=5 (numbers of items), there are total 5*4*3=60 distinct ranking. So, the domain for $\{\underline{\mathbf{w_i}}\}_{i=1}^{2}$ are 60*59 =3540 distinct rankings (assumed $\mathbf{w}_1$ is different from $\mathbf{w}_2$, the basic criterion for two segments ranking). For each distinct ranking for $\{\mathbf{w}_i\}_{i=1}^{2}$, a modified EM algorithm was used to find the MLES for $\hat{\pi}$, and $\hat{\theta}$ with $\{\mathbf{w}_i\}_{i=1}^{2}$ fixed , Once those MLE'S were found, then update $\hat{\pi}$, and $\hat{\theta}$, continues these iterations until relatively change in function $Q(\boldsymbol{\xi},\boldsymbol{\xi}^{\mathbf{i}})$ and the stopping rule is given as: $|Q(\boldsymbol{\xi}^{\mathbf{i+1}},\boldsymbol{\xi}^{\mathbf{i}}) - Q(\boldsymbol{\xi}^{\mathbf{i}},\boldsymbol{\xi}^{\mathbf{i-1}})| = 10^{-2}$. In the last iterations, compute the log likelihood with the updated $\hat{\pi}$, $\hat{\theta}$, and the fixed $\{\mathbf{w}_i\}_{i=1}^{2}$ value. For each data set, log likelihood values were collected for each fixed $\{\mathbf{w}_i\}_{i=1}^{2}$, where $\{\mathbf{w}_i\}_{i=1}^{2}$ was set as one of 3540 distinct rankings. At the same time, MLE'S for $\hat{\pi}$, $\hat{\mathbf{w}}_1$, $\hat{\mathbf{w}}_2$,and $\hat{\theta}$ for each data set were obtained using the EM algorithm described in the previous chapter.

The following is the procedure to find likelihood set for $\{\mathbf{w}_i\}_{i=1}^2$ given a data set:

(1). First update $\hat{\theta}$ with the initial guess for θ, and π , then update $\{\hat{\pi}_i\}$, and $\hat{\theta}$,…., until converge.

$$L^{[p]}(\{\mathbf{w}_i\}) = L(\{\mathbf{w}_i\}, \hat{\theta}\{\mathbf{w}_i\}\hat{\pi}\{\mathbf{w}_i\}) \tag{3.7}$$

Same value obtained even if $\{\mathbf{w}_i\}_{i=1}^2$ is not unique.

(2). Search through $S_{qm} * S_{qm} * \ldots * S_{qm}$ to find $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k$ such that

$$[A] \qquad \frac{L^{[P]}\{\mathbf{w}_i, i = 1,2,\ldots,k\}}{L^{[P]}\{\hat{\mathbf{w}}_i, i = 1,2,\ldots,k\}} \geq C$$

$LS(\mathbf{w}^{\#}) = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k\}$ *such that* $[A]$ *holds* with some cutoff point C. Fisher provided a specific example for a binomial parameter. How to choose the cutoff point remained open, but Fisher suggested that parameter values with less than 1/15 or 6.7% likelihood 'are obviously open to grave suspicion'. In general, there is a calibration issue that we have to deal with (Pawitan, Y., 2001).

Table 3-4. The effect of different cutoff points C on profile likelihood set and interval estimate for $\{\mathbf{w}_i\}_{i=1}^2$ with 20 replications

part	C	π	θ	m	SMRV	Scov_t
a	0.05	0.1	0.1	10	0.033	1
a	0.10	0.1	0.1	10	0.033	1
a	0.15	0.1	0.1	10	0.033	1
a	0.20	0.1	0.1	10	0.033	1

b	0.25	0.1	0.1	10	0.033	1
b	0.30	0.1	0.1	10	0.033	1
b	0.35	0.1	0.1	10	0.033	1
b	0.4	0.1	0.1	10	0.033	1
b	0.45	0.1	0.1	10	0.033	1
b	0.05	0.1	0.1	20	0.033	1
b	0.10	0.1	0.1	20	0.033	1
b	0.15	0.1	0.1	20	0.033	1
b	0.20	0.1	0.1	20	0.033	1
b	0.25	0.1	0.1	20	0.033	1
b	0.30	0.1	0.1	20	0.033	1
b	0.35	0.1	0.1	20	0.033	1
b	0.4	0.1	0.1	20	0.033	1
b	0.45	0.1	0.1	20	0.033	1
b	0.05	0.1	0.1	30	0.033	1
b	0.10	0.1	0.1	30	0.033	1
b	0.15	0.1	0.1	30	0.033	1
b	0.20	0.1	0.1	30	0.033	1
b	0.25	0.1	0.1	30	0.033	1
b	0.30	0.1	0.1	30	0.033	1
b	0.35	0.1	0.1	30	0.033	1
b	0.4	0.1	0.1	30	0.033	1
b	0.45	0.1	0.1	30	0.033	1
b	0.05	0.1	0.3	10	0.057	1
b	0.10	0.1	0.3	10	0.046	1
b	0.15	0.1	0.3	10	0.040	1
b	0.20	0.1	0.3	10	0.038	1
b	0.25	0.1	0.3	10	0.038	1
b	0.30	0.1	0.3	10	0.038	1
b	0.35	0.1	0.3	10	0.037	1
b	0.4	0.1	0.3	10	0.036	1
b	0.45	0.1	0.3	10	0.034	1

c	0.05	0.1	0.3	20	0.035	1
c	0.10	0.1	0.3	20	0.035	1
c	0.15	0.1	0.3	20	0.035	1
c	0.20	0.1	0.3	20	0.035	1
c	0.25	0.1	0.3	20	0.035	1
c	0.30	0.1	0.3	20	0.035	1
c	0.35	0.1	0.3	20	0.035	1
c	0.4	0.1	0.3	20	0.034	1
c	0.45	0.1	0.3	20	0.034	1
c	0.05	0.1	0.3	30	0.033	1
c	0.10	0.1	0.3	30	0.033	1
c	0.15	0.1	0.3	30	0.033	1
c	0.20	0.1	0.3	30	0.033	1
c	0.25	0.1	0.3	30	0.033	1
c	0.30	0.1	0.3	30	0.033	1
c	0.35	0.1	0.3	30	0.033	1
c	0.4	0.1	0.3	30	0.033	1
c	0.45	0.1	0.3	30	0.033	1
c	0.05	0.1	0.5	10	0.404	1
c	0.10	0.1	0.5	10	0.314	1
c	0.15	0.1	0.5	10	0.252	1
c	0.20	0.1	0.5	10	0.201	1
c	0.25	0.1	0.5	10	0.167	1
c	0.30	0.1	0.5	10	0.140	0.95
c	0.35	0.1	0.5	10	0.117	0.9
c	0.4	0.1	0.5	10	0.099	0.9
c	0.45	0.1	0.5	10	0.084	0.85
c	0.05	0.1	0.5	20	0.159	1
c	0.10	0.1	0.5	20	0.120	1
c	0.15	0.1	0.5	20	0.098	1
c	0.20	0.1	0.5	20	0.083	1
c	0.25	0.1	0.5	20	0.071	1

d	0.30	0.1	0.5	20	0.063	1
d	0.35	0.1	0.5	20	0.058	1
d	0.4	0.1	0.5	20	0.051	1
d	0.45	0.1	0.5	20	0.045	0.9
d	0.05	0.1	0.5	30	0.069	0.95
d	0.10	0.1	0.5	30	0.054	0.95
d	0.15	0.1	0.5	30	0.045	0.9
d	0.20	0.1	0.5	30	0.039	0.9
d	0.25	0.1	0.5	30	0.036	0.9
d	0.30	0.1	0.5	30	0.032	0.9
d	0.35	0.1	0.5	30	0.030	0.9
d	0.4	0.1	0.5	30	0.029	0.9
d	0.45	0.1	0.5	30	0.027	0.85
d	0.05	0.1	0.7	10	0.774	1
d	0.10	0.1	0.7	10	0.633	1
d	0.15	0.1	0.7	10	0.532	0.95
d	0.20	0.1	0.7	10	0.453	0.95
d	0.25	0.1	0.7	10	0.388	0.85
d	0.30	0.1	0.7	10	0.335	0.85
d	0.35	0.1	0.7	10	0.289	0.85
d	0.4	0.1	0.7	10	0.248	0.80
d	0.45	0.1	0.7	10	0.213	0.80
d	0.05	0.1	0.7	20	0.720	1
d	0.10	0.1	0.7	20	0.601	0.95
d	0.15	0.1	0.7	20	0.513	0.90
d	0.20	0.1	0.7	20	0.438	0.90
d	0.25	0.1	0.7	20	0.371	0.85
d	0.30	0.1	0.7	20	0.316	0.85
d	0.35	0.1	0.7	20	0.268	0.85
d	0.4	0.1	0.7	20	0.228	0.80
d	0.45	0.1	0.7	20	0.198	0.80
d	0.05	0.1	0.7	30	0.544	0.95

e	0.10	0.1	0.7	30	0.418	0.95
e	0.15	0.1	0.7	30	0.340	0.85
e	0.20	0.1	0.7	30	0.274	0.85
e	0.25	0.1	0.7	30	0.220	0.85
e	0.30	0.1	0.7	30	0.183	0.85
e	0.35	0.1	0.7	30	0.159	0.85
e	0.4	0.1	0.7	30	0.137	0.85
e	0.45	0.1	0.7	30	0.117	0.85
e	0.05	0.1	0.9	10	0.907	0.9
e	0.10	0.1	0.9	10	0.828	0.85
e	0.15	0.1	0.9	10	0.746	0.85
e	0.20	0.1	0.9	10	0.662	0.85
e	0.25	0.1	0.9	10	0.581	0.8
e	0.30	0.1	0.9	10	0.504	0.75
e	0.35	0.1	0.9	10	0.432	0.7
e	0.4	0.1	0.9	10	0.367	0.5
e	0.45	0.1	0.9	10	0.300	0.4
e	0.05	0.1	0.9	20	0.900	0.85
e	0.10	0.1	0.9	20	0.818	0.85
e	0.15	0.1	0.9	20	0.753	0.80
e	0.20	0.1	0.9	20	0.695	0.80
e	0.25	0.1	0.9	20	0.637	0.75
e	0.30	0.1	0.9	20	0.577	0.70
e	0.35	0.1	0.9	20	0.519	0.70
e	0.4	0.1	0.9	20	0.451	0.65
e	0.45	0.1	0.9	20	0.392	0.55
e	0.05	0.1	0.9	30	0.939	1
e	0.10	0.1	0.9	30	0.844	0.9
e	0.15	0.1	0.9	30	0.749	0.8
e	0.20	0.1	0.9	30	0.676	0.75
e	0.25	0.1	0.9	30	0.614	0.65
e	0.30	0.1	0.9	30	0.553	0.65

f	0.35	0.1	0.9	30	0.497	0.65
f	0.4	0.1	0.9	30	0.444	0.65
f	0.45	0.1	0.9	30	0.386	0.50

Note: SMRV=MRV for Spearman's distance

Considering the suggested "C" value=0.067 by Fisher, the "C" values were chosen between 0.05 and 0.45 by increment of 0.05. It can be seen in table 3-4 that the coverage rates were obtained 100% and SMRV was smaller for all C values when $\theta \leq 0.3$. The SMRV was quite large and coverage rate was close to 95% nominal level when $\theta \geq 0.7$. As mentioned previously, the criteria for selecting C is: Scov_t close to 95% nominal level with smaller SMRV. Considering all factors, C=0.1 was selected as the cutoff point for any simulation study that involved in calculating coverage rate for $\{\mathbf{w}_i\}_{i=1}^{2}$, where i=1, 2.

The following values of π, θ, $\mathbf{w}_1$, $\mathbf{w}_2$, m, n and distance function were selected for the second simulation study:

$\pi = 0.1,\ 0.3,\ 0.5;$

$\theta = 0.1,\ 0.3,\ 0.5,\ 0.7,\ 0.9;$

$\mathbf{w}_2 = 132,\ 312,\ 321;$

n=10, 20, 30 (number of judges);

m=5 (number of items);

Distance function includes Kendall's and Spearman's distance functions;

Twenty data sets for each of the 3*5*3*3*2=270 scenarios were generated in Software Package *R* to study the behavior of MLES with respective to the likelihood set estimation for $\{\mathbf{w}_i\}_{i=1}^{2}$.

Assessing Performances:

The estimating sets for $\{\mathbf{w}_i\}_{i=1}^{k}$ is evaluated using the following two criteria:

(1). Coverage rate Scov_t: Record proportion of data sets for which

$\{\mathbf{w}_1^*, \mathbf{w}_2^*, \ldots, \mathbf{w}_k^*\} \in LS(\mathbf{w}^\#)$ where $\{\mathbf{w}_1^*, \mathbf{w}_2^*, \ldots, \mathbf{w}_k^*\}$ are true baseline rankings.

(2) For each scenario, compute Mean relative volume: $MRV = \dfrac{\overline{V}\{LS(\mathbf{w}^\#)\}}{\left\lfloor \dfrac{n!}{(n-k)!} \right\rfloor}$,

where $V\{LS(\mathbf{w}^\#)\}$ = the number of rankings $\in LS(\mathbf{w}^\#)$ and $\overline{V}\{LS(\mathbf{w}^\#)\}$ is the average of these volumes over the simulated data sets.

KMRV= estimated mean relative V value for Kendall distance

SMRV= estimated mean relative V value for Spearman distance

Kcov_t=estimated coverage rate for Kendall distance

Scov_t= estimated coverage rate for Spearman distance

Table 3-5. Profile likelihood set and interval estimate for $\{\mathbf{w}_i\}_{i=1}^{2}$ with replication=20

part	π	θ	D_1	D_2	n	KMRV	SMRV	Kcov_t	Scov_t
a	0.1	0.1	0	0	10	0.040	0.033	1	1
a	0.1	0.3	0	0	10	0.333	0.046	1	1
a	0.1	0.5	0	0	10	0.769	0.314	1	1
a	0.1	0.7	0	0	10	0.781	0.614	0.85	1
a	0.1	0.9	0	0	10	0.848	0.834	0.85	0.9
a	0.3	0.1	0	0	10	0.06	0.034	1	1
a	0.3	0.3	0	0	10	0.523	0.048	1	1
a	0.3	0.5	0	0	10	0.808	0.309	1	1
a	0.3	0.7	0	0	10	0.772	0.713	0.85	0.85
a	0.3	0.9	0	0	10	0.892	0.929	0.9	0.9
a	0.5	0.1	0	0	10	0.051	0.048	1	1

b	0.5	0.3	0	0	10	0.362	0.035	1	1
b	0.5	0.5	0	0	10	0.719	0.281	0.95	1
b	0.5	0.7	0	0	10	0.845	0.672	0.85	1
b	0.5	0.9	0	0	10	0.844	0.926	0.85	0.9
b	0.1	0.1	1	0	10	0.054	0.033	1	1
b	0.1	0.3	1	0	10	0.417	0.056	1	1
b	0.1	0.5	1	0	10	0.731	0.311	1	0.95
b	0.1	0.7	1	0	10	0.858	0.761	0.85	0.95
b	0.1	0.9	1	0	10	0.716	0.83	0.75	0.9
b	0.3	0.1	1	0	10	0.027	0.027	1	1
b	0.3	0.3	1	0	10	0.322	0.052	1	1
b	0.3	0.5	1	0	10	0.750	0.164	1	1
b	0.3	0.7	1	0	10	0.676	0.702	0.9	0.95
b	0.3	0.9	1	0	10	0.889	0.926	0.8	0.9
b	0.5	0.1	1	0	10	0.030	0.025	1	1
b	0.5	0.3	1	0	10	0.349	0.055	1	1
b	0.5	0.5	1	0	10	0.717	0.211	0.95	1
b	0.5	0.7	1	0	10	0.933	0.735	0.85	1
b	0.5	0.9	1	0	10	0.840	0.858	0.85	0.85
b	0.1	0.1	0	1	10	0.041	0.031	1	1
b	0.1	0.3	0	1	10	0.432	0.048	1	1
b	0.1	0.5	0	1	10	0.807	0.255	1	1
b	0.1	0.7	0	1	10	0.796	0.749	0.95	1
b	0.1	0.9	0	1	10	0.883	0.836	0.9	1
b	0.3	0.1	0	1	10	0.024	0.019	1	1
b	0.3	0.3	0	1	10	0.284	0.037	1	1
b	0.3	0.5	0	1	10	0.577	0.241	1	1
b	0.3	0.7	0	1	10	0.777	0.715	0.8	1
b	0.3	0.9	0	1	10	0.902	0.902	0.9	1
b	0.5	0.1	0	1	10	0.022	0.018	1	1
b	0.5	0.3	0	1	10	0.329	0.033	1	1
b	0.5	0.5	0	1	10	0.773	0.286	1	1

c	0.5	0.7	0	1	10	0.689	0.674	0.85	1
c	0.5	0.9	0	1	10	0.823	0.813	0.85	0.85
c	0.1	0.1	0	0	20	0.033	0.033	1	1
c	0.1	0.3	0	0	20	0.181	0.035	1	1
c	0.1	0.5	0	0	20	0.545	0.121	0.95	1
c	0.1	0.7	0	0	20	0.775	0.59	0.85	1
c	0.1	0.9	0	0	20	0.862	0.819	0.9	0.85
c	0.3	0.1	0	0	20	0.026	0.026	1	1
c	0.3	0.3	0	0	20	0.272	0.039	1	1
c	0.3	0.5	0	0	20	0.506	0.156	1	1
c	0.3	0.7	0	0	20	0.905	0.436	1	1
c	0.3	0.9	0	0	20	0.789	0.886	0.9	0.9
c	0.5	0.1	0	0	20	0.038	0.012	1	1
c	0.5	0.3	0	0	20	0.217	0.036	1	1
c	0.5	0.5	0	0	20	0.648	0.135	0.9	1
c	0.5	0.7	0	0	20	0.779	0.591	0.95	1
c	0.5	0.9	0	0	20	0.854	0.789	0.85	0.95
c	0.1	0.1	1	0	20	0.032	0.033	1	1
c	0.1	0.3	1	0	20	0.248	0.035	1	1
c	0.1	0.5	1	0	20	0.615	0.182	1	1
c	0.1	0.7	1	0	20	0.755	0.576	0.95	1
c	0.1	0.9	1	0	20	0.914	0.891	0.9	0.9
c	0.3	0.1	1	0	20	0.01	0.015	1	1
c	0.3	0.3	1	0	20	0.093	0.028	1	1
c	0.3	0.5	1	0	20	0.506	0.0823	0.9	0.95
c	0.3	0.7	1	0	20	0.683	0.508	0.95	0.95
c	0.3	0.9	1	0	20	0.812	0.867	0.95	0.9
c	0.5	0.1	1	0	20	0.007	0.033	1	1
c	0.5	0.3	1	0	20	0.159	0.065	1	1
c	0.5	0.5	1	0	20	0.621	0.142	0.95	0.95
c	0.5	0.7	1	0	20	0.813	0.541	0.9	1
c	0.5	0.9	1	0	20	0.686	0.826	0.8	0.95

d	0.1	0.1	0	1	20	0.031	0.033	1	1
d	0.1	0.3	0	1	20	0.269	0.034	1	1
d	0.1	0.5	0	1	20	0.562	0.100	0.95	1
d	0.1	0.7	0	1	20	0.837	0.597	0.95	1
d	0.1	0.9	0	1	20	0.803	0.843	0.8	0.85
d	0.3	0.1	0	1	20	0.018	0.024	1	1
d	0.3	0.3	0	1	20	0.165	0.018	1	1
d	0.3	0.5	0	1	20	0.549	0.095	1	1
d	0.3	0.7	0	1	20	0.839	0.75	1	1
d	0.3	0.9	0	1	20	0.819	0.845	0.9	0.9
d	0.5	0.1	0	1	20	0.01	0.010	1	1
d	0.5	0.3	0	1	20	0.159	0.013	1	1
d	0.5	0.5	0	1	20	0.553	0.156	1	1
d	0.5	0.7	0	1	20	0.851	0.514	0.95	1
d	0.5	0.9	0	1	20	0.858	0.928	0.9	0.9
d	0.1	0.1	0	0	30	0.033	0.030	1	1
d	0.1	0.3	0	0	30	0.066	0.033	1	1
d	0.1	0.5	0	0	30	0.562	0.054	1	0.95
d	0.1	0.7	0	0	30	0.648	0.418	0.85	0.95
d	0.1	0.9	0	0	30	0.865	0.845	0.9	1
d	0.3	0.1	0	0	30	0.015	0.015	1	1
d	0.3	0.3	0	0	30	0.127	0.036	1	1
d	0.3	0.5	0	0	30	0.546	0.082	0.95	1
d	0.3	0.7	0	0	30	0.692	0.389	0.95	1
d	0.3	0.9	0	0	30	0.752	0.868	0.85	1
d	0.5	0.1	0	0	30	0.027	0.004	1	1
d	0.5	0.3	0	0	30	0.139	0.034	1	1
d	0.5	0.5	0	0	30	0.557	0.075	1	1
d	0.5	0.7	0	0	30	0.877	0.483	1	1
d	0.5	0.9	0	0	30	0.757	0.846	0.85	0.9
d	0.1	0.1	1	0	30	0.033	0.025	1	1
d	0.1	0.3	1	0	30	0.131	0.034	1	1

e	0.1	0.5	1	0	30	0.482	0.097	1	1
e	0.1	0.7	1	0	30	0.708	0.534	0.95	1
e	0.1	0.9	1	0	30	0.926	0.911	0.95	1
e	0.3	0.1	1	0	30	0.011	0.018	1	1
e	0.3	0.3	1	0	30	0.127	0.020	1	1
e	0.3	0.5	1	0	30	0.447	0.055	1	1
e	0.3	0.7	1	0	30	0.779	0.472	1	1
e	0.3	0.9	1	0	30	0.764	0.763	0.85	0.9
e	0.5	0.1	1	0	30	0.009	0.005	1	1
e	0.5	0.3	1	0	30	0.068	0.022	1	1
e	0.5	0.5	1	0	30	0.599	0.090	1	1
e	0.5	0.7	1	0	30	0.831	0.513	0.95	1
e	0.5	0.9	1	0	30	0.907	0.819	0.9	0.95
e	0.1	0.1	0	1	30	0.033	0.033	1	1
e	0.1	0.3	0	1	30	0.099	0.030	1	1
e	0.1	0.5	0	1	30	0.546	0.068	1	1
e	0.1	0.7	0	1	30	0.602	0.371	1	1
e	0.1	0.9	0	1	30	0.889	0.916	0.9	1
e	0.3	0.1	0	1	30	0.018	0.030	1	1
e	0.3	0.3	0	1	30	0.110	0.025	1	1
e	0.3	0.5	0	1	30	0.472	0.053	1	1
e	0.3	0.7	0	1	30	0.584	0.367	1	0.9
e	0.3	0.9	0	1	30	0.875	0.909	0.95	0.9
e	0.5	0.1	0	1	30	0.010	0.010	1	1
e	0.5	0.3	0	1	30	0.071	0.013	1	1
e	0.5	0.5	0	1	30	0.405	0.068	1	1
e	0.5	0.7	0	1	30	0.838	0.401	0.95	1
e	0.5	0.9	0	1	30	0.819	0.922	0.85	1

An examination of Table 3-5 revealed that sample mean relative volumes MRV was around 1.2% to 6% of the total volume and coverage rate cov_t was 100% when $\theta \leq 0.3$. MRV was ranged from 5.3% to 31.4% of the total volume and the cov_t was above the coverage probability 0.95 when $\theta = 0.5$. MRV was extremely large for when $\theta = 0.9$. It is also noted that the estimator cov_t performs best for $\theta \le 0.7$ than for $\theta > 0.7$. Both Spearman's and Kendall's distance performed better when θ was small than when θ was large. The findings were similar to the study conducted by Nelson and Kemp (2000). In their study for one segment the simulated coverage rate was above the nominal 0.95 rate for small θ values and is below 0.85 when θ exceeds 0.80. Recall that the judges randomly assign a ranking from $\mathbf{S}_{qm}$ when $\theta = 1$ and perfectly reproduce the baseline ranking with probability=1 when $\theta = 0$ according to the model in (2.1). In other words, small values of θ reflect relatively well trained judges and large values of θ for poor judges.

There is a tradeoff between coverage rate and MRV. In general large MRV will yield relatively large coverage rate, especially for large values of θ. But Large MRV is not desirable and the price is so high. This study confirmed the conclusion made by Nelson and Kemp (2000) that the relatively important of training for the judges to correctly reproduce their baseline rankings.

A multiple regression was fit to the data in Table 3-5 with sample mean relative volumes MRV as the dependent variable and n, true values of θ, and π, the type of distance (Distance=0 for Kendall, Distance=1 for Spearman) and D_1, D_2 (the form of $\mathbf{w}_2$) as independent variables. A summary of the fit given in appendix 7 shows that θ, Distance (distance function) , and n have statistically significant effects on MRV. As θ increased, everything else remaining fixed, the sample mean volumes MRV increased steeply. This is what we are expected to see because with θ increasing toward 1, the probability is decreasingly more concentrated about the mode $\mathbf{w}_0$ and the volume is expected to increase. MRV for Spearman distance was smaller than those for Kendall distance with everything else remaining fixed. MRV increased as n increased with everything else remaining fixed.

A logistic model was fit to the data in Table 3-5 using maximum likelihood with sample mean coverage rate cov_t as the dependent variable and n, true values of θ, π, the type of distance (Distance=0 for Kendall, Distance=1 for Spearman) and D_1, D_2 (the form of $\mathbf{w}_2$) as independent variables. A summary of the fit given appendix 7 shows that the model is statistical significant at 0.061. θ has statistically significant effects on the sample mean coverage rate cov_t.. As θ increased, everything else remaining fixed, the sample mean coverage rate cov_t decreased.

The SMRV and Scov_t obtained from the third simulation with 100 replications are provided in Table 3-6. The trends for SMRV and Scov_t in table 3-5 were similar to the corresponding values in Table 3-6. The coverage rates for $\{\mathbf{w}_i\}_{i=1}^{2}$ was above to 95% nominal level, indicating that the program was robust in estimating the baseline ranking even when the sample size is small.

Table 3-6. Profile likelihood set and interval estimate for $\{\mathbf{w}_i\}_{i=1}^{2}$ with replication=100

π	θ	D_1	D_2	N	SMRV	Scov_t
0.1	0.1	0	0	10	0.087	1
0.1	0.5	0	0	10	0.267	0.98
0.1	0.9	0	0	10	0.781	0.96

3.2.3 Profile likelihood set estimation for w_1:

We constructed likelihood confident set for four scenarios using Spearman's distance with parameters values taken as follows:

π =0.3, θ=0.1, n=10; 20; 30, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 321$ scenario#1

π =0.3, θ=0.9, n=10; 20; 30, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#2

π =0.1, θ=0.1, n=10; 20; 30, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 312$ scenario#3

π =0.1, θ=0.9, n=10; 20; 30, m=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 321$ scenario#4

The initial values for π and θ were set at their true values. $\mathbf{w}_1 \in \mathbf{S}_{qm} - \{\mathbf{w}_2\}$, and $\mathbf{w}_2$ was fixed at its true value. For m=5 (numbers of items), there are total 5*4*3=60 distinct ranking. So the domain for $\{\mathbf{w}_1\}$ are 59 distinct rankings (assumed $\mathbf{w}_1$ is different from $\mathbf{w}_2$, the basic criterion for two segments ranking). For each distinct ranking for $\{\mathbf{w}_1\}$, a modified EM algorithm was used to find the MLES for $\hat{\pi}$, and $\hat{\theta}$ with $\{\mathbf{w}_2\}$ fixed , Once those MLE'S were found, then update $\hat{\pi}$, and $\hat{\theta}$, continues these iterations until relatively change in function $Q(\xi, \xi^{\mathbf{i}})$ and the stopping rule is given as: $|Q(\xi^{\mathbf{i+1}}, \xi^{\mathbf{i}}) - Q(\xi^{\mathbf{i}}, \xi^{\mathbf{i-1}})| = 10^{-2}$. In the last iterations, compute the log likelihood with the updated $\hat{\pi}, \hat{\theta}$, and the fixed $\{\mathbf{w}_2\}$ value. For each data set, log likelihood values were collected for each distinct $\{\mathbf{w}_1\}$, where $\{\mathbf{w}_1\}$ was set as one of 59 distinct rankings and fixed $\{\mathbf{w}_2\}$, at the same time, MLES for $\hat{\pi}$, $\hat{\mathbf{w}}_1$, $\hat{\mathbf{w}}_2$,and $\hat{\theta}$ for each data set were obtained using the EM algorithm described in the previous chapter.

Table 3-7. Profile likelihood set and interval estimate for $\{\mathbf{w}_1\}$ with 20 replications

π	θ	D1	D2	n	SMRV	Scov_t
0.1	0.1	1	0	10	0.083	0.95
0.1	0.1	1	0	20	0.07	0.9
0.1	0.1	1	0	30	0.211	1
0.3	0.1	0	1	10	0.192	1
0.3	0.1	0	1	20	0.083	1
0.3	0.1	0	1	30	0.085	1
0.3	0.9	0	0	10	0.673	0.8
0.3	0.9	0	0	20	0.589	0.75
0.3	0.9	0	0	30	0.587	0.75
0.1	0.9	0	1	10	0.603	0.75
0.1	0.9	0	1	20	0.519	0.8
0.1	0.9	0	1	30	0.826	0.85

The profile likelihood method seem to give confident sets with coverage probability above 95% whenever $\theta = 0.1$. But performed poorly when $\theta = 0.9$. The mean relative volume SMRV was extremely larger for $\theta = 0.9$ than for $\theta = 0.1$.

A multiple regression model was fit to the pooled simulation results in Table 3-7 using maximum likelihood with sample mean SMRV (volumes) $\mathbf{w}_1$ as the dependent variable and n, true values of θ and π as independent variables. A summary of the fit given in Appendix 8 that θ has statistically significant effects on the sample mean coverage rate Scov_t for $\mathbf{w}_1$. There is not enough evidence to conclude that n (the number of judges) or π influence the sample mean coverage rate Scov_t for $\mathbf{w}_1$. As θ increased, everything else remaining fixed, the sample mean relative mean volume SMRV increased.

A logistic regression model was fit to the pooled simulation results in Table 3-7 using maximum likelihood with sample mean coverage rate Scov_t for $\mathbf{w}_1$ as the dependent variable and m, true values of θ and π as independent variables. A summary of the fit given in Appendix 8 shows that none of the independent variables have significant effect on the sample mean coverage rate for Spearman's distance.

Table 3-8. Profile likelihood set and interval estimate for $\{\mathbf{w}_1\}$ with 100 replications

π	θ	D1	D2	n	SMRV	Scov_t
0.1	0.1	0	0	10	0.108	0.93
0.1	0.5	0	0	10	0.402	0.91
0.1	0.9	0	0	10	0.507	0.76

Likelihood set estimator for $\mathbf{w}_1$ exhibited consistency in patterns in both twenty and one hundred replication. The simulation results in Table 3-8 agree quite nicely that the coverage probability was close to nominal level when $0.1 \le \theta \le 0.5$. The SMRV increased as θ increased.

3.2.4 Profile likelihood set estimation for $\mathbf{w}_2$:

Profile likelihood set estimation for $\mathbf{w}_2$ is very similar to the profile likelihood set estimation for $\mathbf{w}_1$ described in 3.2.3. Four scenarios in 3.2.3 were included in this part of simulation. 20 data sets for each scenario were considered. The initial values for π and θ were set at their true values. $\mathbf{w}_2 \in \mathbf{S}_{qm} - \{\mathbf{w}_1\}$, and $\mathbf{w}_1$ was fixed at its true value. For m=5 (numbers of items), there are total 5*4*3=60 distinct ranking. So the domain for $\{\mathbf{w}_2\}$ are 59 distinct rankings (assumed $\mathbf{w}_1$ is different from $\mathbf{w}_2$, the basic criterion for two segments ranking). For each distinct ranking for $\{\mathbf{w}_2\}$, a modified EM algorithm was used to find the MLES for $\hat{\pi}$, and $\hat{\theta}$ with $\{\mathbf{w}_1\}$ fixed , Once those MLES were found, then update $\hat{\pi}$, and $\hat{\theta}$, continues these iterations until relatively change in function $Q(\xi,\xi^{\mathbf{i}})$ and the stopping rule is given as: $|Q(\xi^{\mathbf{i+1}},\xi^{\mathbf{i}})-Q(\xi^{\mathbf{i}},\xi^{\mathbf{i-1}})|=10^{-2}$. In the last iterations, compute the log likelihood with the updated $\hat{\pi}$, $\hat{\theta}$, and the fixed $\{\mathbf{w}_1\}$ value. For each data set, log likelihood values were collected for each distinct $\{\mathbf{w}_2\}$, where $\{\mathbf{w}_2\}$ was set as one of 59 distinct rankings and fixed $\{\mathbf{w}_1\}$, At the same time, MLES for $\hat{\pi}$, $\hat{\mathbf{w}}_1$, $\hat{\mathbf{w}}_2$, and $\hat{\theta}$ for each data set were obtained using the EM algorithm described in the previous chapter.

Table 3-9. Profile likelihood set and interval estimate for $\{\mathbf{w}_2\}$ with replications=20

part	π	θ	D1	D2	n	SMRV	Scov_t
a	0.1	0.1	1	0	10	0.032	1
a	0.1	0.1	1	0	20	0.032	1
a	0.1	0.1	1	0	30	0.032	1
a	0.3	0.1	0	1	10	0.045	1
a	0.3	0.1	0	1	20	0.032	1
b	0.3	0.1	0	1	30	0.032	1
b	0.3	0.9	0	0	10	0.655	0.75
b	0.3	0.9	0	0	20	0.501	0.75
b	0.3	0.9	0	0	30	0.528	0.75
b	0.1	0.9	0	1	10	0.549	0.75
b	0.1	0.9	0	1	20	0.425	0.8
b	0.1	0.9	0	1	30	0.663	0.9

Table 3-9 shows the set estimates for $\{\mathbf{w}_2\}$ at few varying parameters settings. The coverage rates for $\mathbf{w}_2$ was above 95% nominal level when $\theta = 0.1$. The sample mean coverage rate decreased as θ increased, the likelihood set estimator performed poorly when $\theta = 0.9$.

A model was fit to the data in Table 3-9 using least squares with sample mean SMRV for $\mathbf{w}_2$ as the dependent variable and m, true values of θ and π as independent variables. A summary of the fit given in appendix 9 shows that θ has statistically significant effects on the sample mean volumes for $\mathbf{w}_2$. There is not enough evidence to conclude that n (the number of judges) or π influence the sample mean volumes for $\mathbf{w}_2$. As θ increased, everything else remaining fixed, the sample mean volumes decreased.

A logistic regression model was fit to the data in Table 3-9 using maximum likelihood with sample mean coverage rate Scov_t for $\mathbf{w}_2$ as the dependent variable and m, true values of θ and π as independent variables. A summary of the fit given in appendix 9 shows that θ, n (the number of judges), and π does not have statistically significant effects on the sample mean coverage rate Scov_t for $\mathbf{w}_2$.

Table 3-10. Profile likelihood set and interval estimate for $\{\mathbf{w}_2\}$ with 100 replications

π	θ	D1	D2	n	SMRV	Scov_t
0.1	0.1	0	0	10	0.0318	1
0.1	0.5	0	0	10	0.245	0.94
0.1	0.9	0	0	10	0.382	0.75

One-hundred replications were performed for each of case in Table 3-10 to confirm the findings in Table 3-9. Results had exactly the same pattern as with twenty replications and only numbers were a little different. The 95% confidence interval set was nearly adequate for

$\theta = 0.1$. The SMRV was very small for $\theta = 0.1$ and increased as θ increased. The likelihood set estimator performed poorly for $\theta = 0.9$.

CHAPTER 4 – Bayesian Analysis

Rankings, being at best ordinal, may not contain sufficient information in the small sample sizes, which are typical in sensory analysis, to provide useful inference for the complex segmented model we are using. The Bayesian paradigm provides a mechanism to augment the sample and hopefully improve this situation. Following Kemp and Nelson (2000, 2006), I developed and investigated Bayesian inference for the probability model in (2.1) viewed as being conditional on the parameters. Let $q_{\alpha}(\boldsymbol{\xi})$ be a prior density on $\boldsymbol{\xi} = (\theta, \boldsymbol{\pi}, \{\mathbf{w}_i\})$, where $\boldsymbol{\alpha}$ is a vector of hyperparameters. Note that $\boldsymbol{\xi}$ consists of both discrete and continuous parameters, where θ lies in the unit interval, $\boldsymbol{\pi}$ lies in a K-dimensional simplex and each $\mathbf{w}_i$ lies in $\mathbf{S}_{qm}$. Using Bayes' rule, $L(\theta, \boldsymbol{\pi}, \{\mathbf{w}_i\}) = \Pi p(\mathbf{y}_j \mid \theta, \boldsymbol{\pi}, \{\mathbf{w}_i\})$, and having observed $\{\mathbf{y}_i\}$, the posterior density of $\boldsymbol{\xi}$ is given by

$$p(\xi \mid \{\mathbf{y}_i\}) = c(\{\mathbf{y}_i\}) L(\boldsymbol{\xi}) q_{\alpha}(\boldsymbol{\xi}), \tag{4.1}$$

where $c(\{\mathbf{y}_i\})$ is a norming function. Here, I use (4.1) to construct credible sets for the components of $\boldsymbol{\xi}$ and use simulation to investigate how well these estimators behave as confidence sets. The mixed posterior in (4.1) is too complicated to yield to an analytic approach. Instead, I developed simulation algorithms to sample from (4.1) and following Kemp and Nelson (2000, 2006), who worked in the setting of the one segment model, use simulated parameter values to construct approximate credible sets. This plan was difficult to implement because of constraints on the parameter space and the cardinality of $\mathbf{S}_{qm}$, which can be very large. I used independent uniform priors on θ, $\{\mathbf{w}_i\}$ and π. In order to ensure that the segments are well defined, the baseline rankings $\{\mathbf{w}_i\}$ need to be distinct. We only use simulation to investigate the case where there are two segments.

4.1 Bayesian Point and Set Estimation for θ

Again letting $\mathbf{S}_{qm}$ denote the collection of permutations of m objects taken q = 3 at a time. The likelihood of the two segment model based on Data = $\{\mathbf{y}_{\mathbf{i}}, i = 1, 2, \ldots\ldots, n\}$, independent rankings lying in $\mathbf{S}_{qm}$ is

$$L(\theta, \mathbf{w_1}, \mathbf{w_2}, \pi)$$

$$= \prod \{\pi\theta^{d(y_i, \mathbf{w}_1)} + (1-\pi)\theta^{d(y_i, \mathbf{w}_2}\} / \psi^n(\theta)\{I_{\mathbf{S}}(\mathbf{w_1}) I_{\{\mathbf{S}-\mathbf{w_1}\}}(\mathbf{w_2}) I_{(0,1)}(\theta) I_{(0,0.5]}(\pi)\} \qquad (4.2)$$

Letting C represent a norming constant and using uniform priors, the continuous posterior density of θ is given by:

$$p(\theta \mid Data) = \mathrm{C} \sum_{w_1 \in \mathbf{S}} \sum_{w_2 \in \mathbf{S}-\{w_1\}} \int_0^{0.5} L(.) d\pi \qquad (4.3)$$

A built in program in the Software Package *R* was used to implement the numerical integration in (4.3). First, I integrated the likelihood function with respect to π in the domain (0, 0.5). Then I summed over $\mathbf{w_1}$ and $\mathbf{w_2}$ in the domain $\mathbf{S}_{qm}$ and $\mathbf{S}_{qm} - \{\mathbf{w}_1\}$, respectively.

Sampling from the above posterior distribution is conceptually simple but can be difficult to carry out. As we know, the distribution of a continuous parameter can be approximated by a discrete distribution on a grid of points. The simplest discrete approximation can be obtained by computing the target density { $p(\theta_i \mid Data)$ } at a set of evenly spaced values θ_1, …, θ_M, that cover the enter parameter space of θ. The continuous distribution with density $p^{\#}(\theta \mid Data)$ is then approximated arbitrarily closely by the discrete distribution with mass function

$$p^{\#}(\theta \mid Data) = p(\theta_j \mid Data) / \sum_{j}^{M} p(\theta_j \mid Data) ,$$

supported on the values $\theta_1, \ldots, \theta_M$. Note that the normalizing constant C in (1) cancels out and does not have to be computed, a potentially big advantage of this approach.

Since the probabilities assigned to { $\theta_1, \ldots, \theta_M$ } can be computed in a straight forward manner, a random sample from $p^{\#}(\theta \mid Data)$ can be drawn with replacement using an *R* Software Package routine. These simulated observations $\{\theta_j^*\}$ can then be used to provide estimates of properties of the continuous density $p^{\#}(\theta \mid Data)$ such as moments, quartiles and Credible Interval. The procedure for constructing a nominal (1-α) Credible Interval for θ is:

1). Compute the posterior probabilities for { $\theta_1, \ldots, \theta_M$ }.

2). Take a random sample with replacement of size B=1000, { $\theta_1^*, \ldots, \theta_B^*$ }, from { $\theta_1, \ldots,$ θ_M }with respective to their posterior probabilities.

3). Sort the { $\theta_1^*, \ldots, \theta_B^*$ } from low to high, low bound is α/2 in low tail and upper bound is α/2 in the upper tail.

Our simulation study for θ is structured as follows. Fifty-four scenarios were used with twenty data sets for each scenario generated using the true parameters values as follows: n=10, 20, 30; m=5, 6, 7; π =0.1; θ=0.1, 0.9; $\mathbf{w_1} = (\mathbf{1,2,3})$; $\mathbf{w}_2 = (1,3,2)$, $\mathbf{w}_2 = (3,1,2)$, $\mathbf{w}_2 = (3,2,1)$; This procedure yielded a subtotal 3*3*1*2*3=54 scenarios. Twelve scenarios from π =0.3 for m=5 were also investigated using Spearman's distance and 20 replications. A total 68 scenarios were conducted using Spearman's distance in this section. Table 4-1 shows Bayesian point and set estimation for θ.

We define:

R=number of simulations or replications

$\hat{m}(\theta)$ = sample mean of $\hat{\theta}=\frac{1}{R}\sum_{i=1}^{R}\hat{\theta}_i=\bar{\theta}$.

t_wid = estimated mean width of Bayesian interval estimate of θ

Scov_t = estimated coverage rate of Bayesian interval estimate of θ for Spearman's distance

Two dummy variables were used for three distinct orderings in $\mathbf{w}_2$. Let $D_1=0$, $D_2=0$ for $\mathbf{w}_2=132$; $D_1=1$, $D_2=0$ for $\mathbf{w}_2=312$; $D_1=0$, $D_2=1$ for $\mathbf{w}_2=321$.

Table 4-1 Bayesian Point and Set Estimation for θ With 20 Replications

part	m	n	True π	True θ	$\hat{m}(\theta)$	t_wid	Scov_t	D_1	D_2
a	5	10	0.1	0.1	0.092	0.333	1	0	0
a	5	20	0.1	0.1	0.093	0.233	1	0	0
a	5	30	0.1	0.1	0.070	0.210	1	0	0
a	5	10	0.1	0.9	0.759	0.370	0.95	0	0
a	5	20	0.1	0.9	0.785	0.263	1	0	0
a	5	30	0.1	0.9	0.789	0.250	0.90	0	0
a	5	10	0.1	0.1	0.151	0.345	1	1	0
a	5	20	0.1	0.1	0.119	0.176	1	1	0
a	5	30	0.1	0.1	0.102	0.142	1	1	0
a	5	10	0.1	0.9	0.774	0.285	0.90	1	0
a	5	20	0.1	0.9	0.776	0.265	0.95	1	0
a	5	30	0.1	0.9	0.818	0.240	1	1	0
a	5	10	0.1	0.1	0.137	0.360	1	0	1
a	5	20	0.1	0.1	0.131	0.258	1	0	1
a	5	30	0.1	0.1	0.084	0.191	1	0	1
a	5	10	0.1	0.9	0.784	0.325	0.95	0	1
a	5	20	0.1	0.9	0.776	0.270	0.90	0	1
a	5	30	0.1	0.9	0.792	0.260	0.95	0	1
a	5	10	0.3	0.1	0.187	0.290	1	1	0
a	5	20	0.3	0.1	0.142	0.331	1	1	0
a	5	30	0.3	0.1	0.148	0.276	1	1	0
a	5	10	0.3	0.9	0.772	0.285	0.95	0	0
a	5	20	0.3	0.9	0.771	0.276	0.95	0	0
a	5	30	0.3	0.9	0.771	0.236	0.90	0	0
a	5	10	0.3	0.1	0.190	0.396	1	0	1
a	5	20	0.3	0.1	0.172	0.393	1	0	1
a	5	30	0.3	0.1	0.149	0.293	1	0	1
a	5	10	0.3	0.9	0.789	0.370	1	0	1
a	5	20	0.3	0.9	0.772	0.282	0.90	0	1
a	5	30	0.3	0.9	0.799	0.220	0.95	0	1

b	6	10	0.1	0.1	0.077	0.152	1	0	0
b	6	20	0.1	0.1	0.096	0.148	1	0	0
b	6	30	0.1	0.1	0.042	0.120	1	0	0
b	6	10	0.1	0.9	0.652	0.296	0.95	0	0
b	6	20	0.1	0.9	0.672	0.227	0.80	0	0
b	6	30	0.1	0.9	0.819	0.155	0.95	0	0
b	6	10	0.1	0.1	0.129	0.276	1	1	0
b	6	20	0.1	0.1	0.118	0.195	1	1	0
b	6	30	0.1	0.1	0.111	0.153	1	1	0
b	6	10	0.1	0.9	0.590	0.260	0.70	1	0
b	6	20	0.1	0.9	0.718	0.201	0.80	1	0
b	6	30	0.1	0.9	0.809	0.178	1	1	0
b	6	10	0.1	0.1	0.187	0.315	1	0	1
b	6	20	0.1	0.1	0.101	0.255	1	0	1
b	6	30	0.1	0.1	0.077	0.171	1	0	1
b	6	10	0.1	0.9	0.579	0.277	0.75	0	1
b	6	20	0.1	0.9	0.673	0.250	0.80	0	1
b	6	30	0.1	0.9	0.796	0.201	0.90	0	1
b	7	10	0.1	0.1	0.141	0.204	0.95	0	0
b	7	20	0.1	0.1	0.068	0.117	1	0	0
b	7	30	0.1	0.1	0.358	0.135	1	0	0
b	7	10	0.1	0.9	0.706	0.115	0.80	0	0
b	7	20	0.1	0.9	0.703	0.253	0.84	0	0
b	7	30	0.1	0.9	0.778	0.168	0.90	0	0
b	7	10	0.1	0.1	0.171	0.317	0.95	1	0
b	7	20	0.1	0.1	0.134	0.176	1	1	0
b	7	30	0.1	0.1	0.109	0.157	1	1	0
b	7	10	0.1	0.9	0.624	0.230	0.75	1	0
b	7	20	0.1	0.9	0.665	0.252	0.85	1	0
b	7	30	0.1	0.9	0.800	0.175	0.90	1	0
b	7	10	0.1	0.1	0.163	0.331	1	0	1
b	7	20	0.1	0.1	0.117	0.271	1	0	1
b	7	30	0.1	0.1	0.074	0.215	1	0	1
b	7	10	0.1	0.9	0.507	0.215	0.67	0	1
b	7	20	0.1	0.9	0.595	0.270	0.75	0	1
b	7	30	0.1	0.9	0.700	0.245	1	0	1

One purpose of this simulation is to demonstrate the impact on parameter estimation that results from changing the number of judges n, the number of items m, and differing choices of $\mathbf{w}_2$, D_1 and D_2. As can be seen from Table 4-1, the estimator $\hat{m}(\theta)$ was close to the true values of θ regardless of the values of π, m, and n when θ is equal to 0.1, but the estimator underestimated the true value when θ =0.9 for all values of π, m and n. Coverage rate for θ was close to nominal level when θ =0.1 and far below nominal level when θ =0.9. As number of judges *n* increased, coverage rate showed little improvement for most of cases.

A multiple regression model was fitted to the data in Table 4-1 with sample mean $\hat{m}(\theta)$ as dependent variables, and true values of θ, m, n, D_1, and D_2 as independent variables. The summary of fit from the JMP output in Appendix 10 indicates that the number of judges *n* and true values of θ played a statistically significant role in estimating θ. Surprisingly, m has little effect on $\hat{m}(\theta)$. Just as in profile likelihood point estimation, *n* and true values of θ had positive effects on $\hat{m}(\theta)$.

A multiple regression model was fitted to the data in Table 4-1 with sample mean interval width as dependent variables, and true values of θ, m, n, D_1, and D_2 as independent variables. The summary of the fit from JMP output in Appendix 10 indicates that m, n, D_1 and D_2 but not true value of θ contributed significantly in estimating the mean interval width.

A logistic regression model was fit to the pooled simulation results in Table 4-1 using maximum likelihood with sample mean coverage rate Scov_t for θ as the dependent variable and n, true values of θ, π, D_1, and D_2 independent variables. A summary of the fit given Appendix 10 shows that coverage rate was only related to true value of θ. As θ increased, the odds for the coverage rate decreased.

Table 4-2 below reports the results of set estimation for θ with 100 replications. This simulation was used to confirm the findings observed in Table 4-1 in which 20 replications was conducted for each case. The finding was consistent with the results reported in Table 4-1. The estimators performed better when the range for θ is $0.5 \geq \theta \geq 0.1$, Just as the profile likelihood estimator, the Bayesian estimator of θ is heavily biased when $\theta = 0.9$. The coverage rate was close to nominal level when θ is small.

Table 4-2 Bayesian Point and Set Estimation for θ With 100 Replications

m	n	True π	True θ	$\hat{m}(\theta)$	t_wid	Scov_t	D1	D2
5	10	0.1	0.1	0.156	0.194	0.98	0	0
5	10	0.1	0.5	0.466	0.369	0.93	0	0
5	10	0.1	0.9	0.717	0.433	0.92	0	0
5	10	0.3	0.1	0.179	0.353	1	0	0
5	10	0.3	0.5	0.451	0.378	0.91	0	0
5	10	0.3	0.9	0.745	0.439	0.89	0	0

The large amount of CPU time required to carry out these simulations was a major factor in limiting their scope. Table 4-3 shows the average time in hours required to complete a Bayesian set estimation for θ when m=5, 6, 7. From Table 4-3, it can be seen that as *n* and *m* increase, the average time in hours required completing a Bayesian interval estimate for θ increases, as expected. Comparing to Table 3-1 (average time in hours required to complete a profile likelihood interval estimate for θ), it takes longer time for a Bayesian set estimation than for a profile likelihood interval estimate under similar conditions.

Table 4-3. Average Time in Hours Required to Complete A Bayesian Analysis for θ

	m=5	m=6	m=7
n=10	4	9	10
n=20	6	21	72
n=30	30	48	168

In summary, parameter estimates from Bayesian were generally very similar to those from profile likelihood estimates. Bayesian point and set estimation for θ performed satisfactorily whenever $0.5 \geq \theta \geq 0.1$. There are a number of factors that affect Bayesian sample mean interval width for θ such as number of items m, the number of judges n, D_1 and D_2 (the form of $\mathbf{w}_2$) . The coverage rate for θ was related to θ and close to nominal level when θ is small. Compared to computing profile likelihood estimators, the Bayesian approach is more time consuming since the likelihood function has to be integrated respective to $\mathbf{w}_1$ and $\mathbf{w}_2$ in the domain $\mathbf{S}_{qm}$ and $\mathbf{S}_{qm} - \{\mathbf{w}_1\}$, respectively. There are 3540 distinct rankings for partial ranking 3 out of 5. Other than that, the Bayesian approach is compatible with profile likelihood approach in the performance of set estimation for θ.

4.2 Bayesian Point and Set Estimation for π

Letting C represent a norming constant and using uniform priors, the continuous posterior density of π is given by:

$$p(\pi \mid Data) = \mathrm{C} \sum_{\mathbf{w}_1 \in \mathbf{S}} \sum_{w_2 \in \mathbf{S} - \{\mathbf{w}_1\}} \int_0^1 L(.) d\theta \qquad (4.4)$$

A numerical integration program in the Software Package *R* was used to integrate the likelihood function with respective to π in the domain of (0,1); then with respective to $\mathbf{w_2}$ in the domain of $\mathbf{S}_{qm} - \{\mathbf{w}_1\}$, and finally with respect to $\mathbf{w_1}$ in the domain of $\mathbf{S}_{qm}$.

The simplest discrete approximation can be obtained by computing the target density $\{ p(\pi_i \mid Data) \}$ at a set of evenly spaced values $\pi_1, \ldots, \pi_M$, that cover the enter parameter space of π. The continuous distribution with density $p^{\#}(\pi \mid Data)$ is then approximated arbitrarily closely by the discrete distribution with mass function supported

$$p^{\#}(\pi \mid Data) = p(\pi_j \mid Data) / \sum_{j}^{M} p(\pi_j \mid Data) ,$$

on the values $\pi_1, \ldots, \pi_M$. Note that the normalizing constant C in (4.4) cancels out and does not have to be computed, a potentially big advantage of this approach.

Since the probabilities assigned to $\{ \pi_1, \ldots, \pi_M \}$ can be computed in a straight forward manner, a random sample from $p^{\#}(\pi \mid Data)$ can be drawn with replacement using an R routine.

These simulated observations $\{\pi_j^*\}$ can then be used to provide estimates of properties of the continuous density $p^{\#}(\pi \mid Data)$ such as moments and quartiles.

The procedure for constructing a nominal (1-α) Credible Interval for π is:

1). Compute the posterior probabilities for { $\pi_1, \ldots, \pi_M$ }.

2). Take a random sample of size with replacement of size B=1000, { $\pi_1^*, \ldots, \pi_B^*$ } from { $\pi_1, \ldots, \pi_M$ }with respective to their posterior probabilities.

3). Sort the { $\pi_1^*, \ldots, \pi_B^*$ } from low to high, low bound is α/2 in low tail and upper bound is α/2 in the upper tail.

In this study, the simulation design is based on 109 scenarios, 20 data sets for each scenario generated using Spearman's distance with the true parameters values as follows: n=10, 20, 30; m=5, 6, 7; π =0.1, 0.3; θ=0.1, 0.9; $\mathbf{w_1} = \mathbf{(1,2,3)}$; $\mathbf{w}_2 = (1,3,2)$, $\mathbf{w}_2 = (3,1,2)$, $\mathbf{w}_2 = (3,2,1)$; A total 3*3*2*2*3=108 scenarios was conducted using Spearman's distance in this section. Table 4-4 shows Bayesian point and set estimation for π .

Table 4-4 Bayesian Point and Set Estimation for π With 20 Replications

part	m	n	True θ	True π	$\hat{m}(\pi)$	$\hat{s}(\pi)$	mse_ π	se_mse_ π	t_wid	se_t_wid	Scov_t	D_1	D_2
a	5	10	0.1	0.1	0.180	1.22E-02	3.21E-04	3.1E-04	0.390	0.016	1	0	0
a	5	20	0.1	0.1	0.316	1.05E-02	2.32E-03	2.3E-03	0.415	0.012	0.8	0	0
a	5	30	0.1	0.1	0.178	1.65E-03	3.04E-04	3.0E-04	0.450	0.000	1	0	0
a	5	10	0.9	0.1	0.239	1.97E-02	9.66E-04	9.4E-04	0.390	0.018	0.9	0	0
a	5	20	0.9	0.1	0.206	2.09E-02	5.57E-04	5.4E-04	0.368	0.025	0.95	0	0
a	5	30	0.9	0.1	0.187	1.94E-03	3.82E-04	3.7E-04	0.450	0.000	1	0	0
a	5	10	0.1	0.1	0.156	1.06E-02	1.66E-04	1.6E-04	0.371	0.020	1	1	0
a	5	20	0.1	0.1	0.175	1.73E-02	2.79E-04	2.7E-04	0.315	0.026	1	1	0
a	5	30	0.1	0.1	0.165	1.88E-02	2.10E-04	2.1E-04	0.245	0.026	0.95	1	0

b	5	10	0.9	0.1	0.205	1.27E-02	5.80E-04	5.6E-04	0.424	0.010	1	1	0
b	5	20	0.9	0.1	0.237	1.85E-02	9.45E-04	9.2E-04	0.393	0.022	0.95	1	0
b	5	30	0.9	0.1	0.219	1.81E-02	7.07E-04	6.9E-04	0.323	0.022	0.85	1	0
b	5	10	0.1	0.1	0.154	1.18E-02	1.48E-04	1.4E-04	0.345	0.020	1	0	1
b	5	20	0.1	0.1	0.201	1.92E-02	5.71E-04	5.6E-04	0.344	0.028	1	0	1
b	5	30	0.1	0.1	0.168	3.84E-03	2.31E-04	2.3E-04	0.448	0.003	1	0	1
b	5	10	0.9	0.1	0.222	1.27E-02	7.43E-04	7.2E-04	0.425	0.010	1	0	1
b	5	20	0.9	0.1	0.269	1.72E-02	1.58E-03	1.5E-03	0.394	0.016	0.9	0	1
b	5	30	0.9	0.1	0.216	7.11E-03	6.68E-04	6.5E-04	0.448	0.003	1	0	1
b	5	10	0.1	0.3	0.317	1.05E-02	1.37E-05	1.3E-05	0.415	0.011	1	0	0
b	5	20	0.1	0.3	0.296	8.51E-03	8.58E-07	8.3E-07	0.437	0.009	1	0	0
b	5	30	0.1	0.3	0.282	6.05E-03	1.56E-05	1.5E-05	0.443	0.008	1	0	0
b	5	10	0.9	0.3	0.301	9.25E-03	7.08E-08	6.9E-08	0.433	0.008	1	0	0
b	5	20	0.9	0.3	0.299	9.73E-03	1.00E-07	9.7E-08	0.426	0.013	1	0	0
b	5	30	0.9	0.3	0.285	8.37E-03	1.17E-05	1.1E-05	0.438	0.013	1	0	0
b	5	10	0.1	0.3	0.307	9.83E-03	2.75E-06	2.7E-06	0.426	0.010	1	1	0
b	5	20	0.1	0.3	0.306	1.05E-02	1.78E-06	1.7E-06	0.423	0.012	1	1	0
b	5	30	0.1	0.3	0.307	1.14E-02	2.37E-06	2.3E-06	0.425	0.016	1	1	0
b	5	10	0.9	0.3	0.328	1.00E-02	4.18E-05	4.1E-05	0.413	0.009	1	1	0
b	5	20	0.9	0.3	0.317	9.61E-03	1.51E-05	1.5E-05	0.423	0.014	1	1	0
b	5	30	0.9	0.3	0.298	6.08E-03	2.43E-07	2.4E-07	0.420	0.011	1	1	0
b	5	10	0.1	0.3	0.309	9.74E-03	3.68E-06	3.6E-06	0.425	0.009	1	0	1
b	5	20	0.1	0.3	0.309	1.15E-02	4.26E-06	4.1E-06	0.422	0.014	1	0	1
b	5	30	0.1	0.3	0.289	7.53E-03	6.43E-06	6.3E-06	0.438	0.010	1	0	1
b	5	10	0.9	0.3	0.316	1.05E-02	1.22E-05	1.2E-05	0.415	0.012	1	0	1
b	5	20	0.9	0.3	0.301	9.29E-03	1.11E-07	1.1E-07	0.431	0.012	1	0	1
b	5	30	0.9	0.3	0.278	3.34E-03	2.49E-05	2.4E-05	0.450	0.000	1	0	1
b	6	10	0.1	0.1	0.272	9.02E-05	1.48E-03	1.4E-03	0.450	0.000	1	0	0
b	6	20	0.1	0.1	0.202	1.22E-03	6.12E-04	5.9E-04	0.450	0.000	1	0	0
b	6	30	0.1	0.1	0.187	2.10E-02	3.82E-04	3.7E-04	0.285	0.032	1	0	0
b	6	10	0.9	0.1	0.303	9.42E-03	2.07E-03	2.0E-03	0.420	0.012	0.8	0	0
b	6	20	0.9	0.1	0.208	1.57E-03	6.89E-04	6.7E-04	0.450	0.000	1	0	0
b	6	30	0.9	0.1	0.295	8.82E-03	1.90E-03	1.9E-03	0.433	0.012	0.9	0	0
b	6	10	0.1	0.1	0.266	1.92E-03	1.38E-03	1.3E-03	0.450	0.000	1	1	0
b	6	20	0.1	0.1	0.189	6.49E-03	4.40E-04	4.3E-04	0.433	0.017	1	1	0
b	6	30	0.1	0.1	0.158	2.78E-03	1.75E-04	1.7E-04	0.445	0.004	1	1	0
b	6	10	0.9	0.1	0.306	1.08E-02	2.13E-03	2.1E-03	0.420	0.012	0.8	1	0
b	6	20	0.9	0.1	0.208	5.04E-03	6.47E-04	6.3E-04	0.439	0.011	1	1	0
b	6	30	0.9	0.1	0.218	1.25E-02	7.36E-04	7.2E-04	0.450	0.000	1	1	0
b	6	10	0.1	0.1	0.202	1.81E-02	5.22E-04	5.1E-04	0.383	0.017	1	0	1

c	6	20	0.1	0.1	0.189	1.31E-02	3.92E-04	3.8E-04	0.390	0.029	1	0	1
c	6	30	0.1	0.1	0.167	1.91E-02	2.25E-04	2.2E-04	0.230	0.018	0.85	0	1
c	6	10	0.9	0.1	0.294	7.93E-03	1.88E-03	1.8E-03	0.438	0.007	0.9	0	1
c	6	20	0.9	0.1	0.243	1.04E-02	1.02E-03	1.0E-03	0.443	0.005	0.95	0	1
c	6	30	0.9	0.1	0.244	1.73E-02	1.03E-03	1.0E-03	0.313	0.021	0.75	0	1
c	6	10	0.1	0.3	0.273	1.73E-04	3.59E-05	3.5E-05	0.450	0.000	1	0	0
c	6	20	0.1	0.3	0.251	8.51E-03	7.12E-05	7.0E-05	0.447	0.002	1	0	0
c	6	30	0.1	0.3	0.303	1.27E-02	3.84E-07	3.7E-07	0.423	0.016	1	0	0
c	6	10	0.9	0.3	0.301	8.26E-03	6.33E-08	6.2E-08	0.425	0.010	1	0	0
c	6	20	0.9	0.3	0.285	4.95E-03	1.36E-05	1.3E-05	0.444	0.006	1	0	0
c	6	30	0.9	0.3	0.187	1.44E-02	6.41E-04	6.2E-04	0.395	0.016	1	0	0
c	6	10	0.1	0.3	0.275	4.94E-04	3.03E-05	3.0E-05	0.450	0.000	1	1	0
c	6	20	0.1	0.3	0.288	8.68E-03	7.87E-06	7.6E-06	0.439	0.011	1	1	0
c	6	30	0.1	0.3	0.246	1.19E-02	1.53E-04	1.5E-04	0.450	0.000	1	1	0
c	6	10	0.9	0.3	0.303	9.63E-03	3.57E-07	3.5E-07	0.430	0.008	1	1	0
c	6	20	0.9	0.3	0.289	7.66E-03	6.35E-06	6.2E-06	0.442	0.008	1	1	0
c	6	30	0.9	0.3	0.291	9.41E-03	3.97E-06	3.9E-06	0.437	0.011	1	1	0
c	6	10	0.1	0.3	0.282	4.37E-03	8.16E-06	8.1E-06	0.444	0.004	1	0	1
c	6	20	0.1	0.3	0.296	1.08E-02	9.81E-07	9.6E-07	0.435	0.010	1	0	1
c	6	30	0.1	0.3	0.308	1.10E-02	2.93E-06	2.9E-06	0.423	0.014	1	0	1
c	6	10	0.9	0.3	0.295	8.47E-03	1.22E-06	1.2E-06	0.433	0.008	1	0	1
c	6	20	0.9	0.3	0.289	6.58E-03	6.28E-06	6.1E-06	0.438	0.009	1	0	1
c	6	30	0.9	0.3	0.304	9.66E-03	7.05E-07	6.9E-07	0.433	0.010	1	0	1
c	7	10	0.1	0.1	0.261	6.96E-04	1.30E-03	1.3E-03	0.450	0.000	1	0	0
c	7	20	0.1	0.1	0.283	3.02E-03	1.59E-03	1.6E-03	0.448	0.002	0.95	0	0
c	7	30	0.1	0.1	0.186	1.37E-03	3.71E-04	3.6E-04	0.450	0.000	1	0	0
c	7	10	0.9	0.1	0.298	9.72E-03	1.97E-03	1.9E-03	0.430	0.010	0.85	0	0
c	7	20	0.9	0.1	0.277	2.17E-03	1.83E-03	1.8E-03	0.450	0.000	1	0	0
c	7	30	0.9	0.1	0.281	3.67E-03	1.64E-03	1.6E-03	0.450	0.000	1	0	0
c	7	10	0.1	0.1	0.264	1.15E-03	1.35E-03	1.3E-03	0.450	0.000	1	1	0
c	7	20	0.1	0.1	0.238	1.53E-03	1.19E-03	1.2E-03	0.450	0.000	1	1	0
c	7	30	0.1	0.1	0.180	3.94E-03	3.78E-04	3.7E-04	0.432	0.006	1	1	0
c	7	10	0.9	0.1	0.286	6.62E-03	1.73E-03	1.7E-03	0.440	0.006	0.95	1	0
c	7	20	0.9	0.1	0.282	2.31E-03	2.08E-03	2.0E-03	0.450	0.000	1	1	0
c	7	30	0.9	0.1	0.280	2.22E-03	1.91E-03	1.9E-03	0.450	0.000	1	1	0
c	7	10	0.1	0.1	0.266	8.77E-04	1.32E-03	1.3E-03	0.450	0.000	1	0	1
c	7	20	0.1	0.1	0.242	1.46E-03	1.18E-03	1.1E-03	0.450	0.000	1	0	1
c	7	30	0.1	0.1	0.209	2.27E-03	5.98E-04	5.8E-04	0.450	0.000	1	0	1
c	7	10	0.9	0.1	0.289	6.49E-03	1.69E-03	1.7E-03	0.438	0.006	0.95	0	1
c	7	20	0.9	0.1	0.284	5.79E-03	1.98E-03	1.9E-03	0.444	0.006	0.95	0	1

d	7	30	0.9	0.1	0.288	7.41E-03	1.77E-03	1.7E-03	0.440	0.007	1	0	1
d	7	10	0.1	0.3	0.257	1.85E-03	9.04E-05	8.8E-05	0.450	0.000	1	0	0
d	7	20	0.1	0.3	0.244	1.42E-02	1.88E-04	1.8E-04	0.429	0.011	1	0	0
d	7	30	0.1	0.3	0.189	1.55E-03	6.17E-04	6.0E-04	0.450	0.000	1	0	0
d	7	10	0.9	0.3	0.297	8.94E-03	4.53E-07	4.4E-07	0.435	0.008	1	0	0
d	7	20	0.9	0.3	0.284	5.06E-03	1.45E-05	1.4E-05	0.450	0.000	1	0	0
d	7	30	0.9	0.3	0.274	4.59E-03	3.28E-05	3.2E-05	0.450	0.000	1	0	0
d	7	10	0.1	0.3	0.267	5.81E-04	5.44E-05	5.3E-05	0.450	0.000	1	1	0
d	7	20	0.1	0.3	0.258	9.74E-04	1.08E-04	1.1E-04	0.450	0.000	1	1	0
d	7	30	0.1	0.3	0.218	4.39E-03	3.99E-04	3.9E-04	0.450	0.000	1	1	0
d	7	10	0.9	0.3	0.294	8.12E-03	2.07E-06	2.0E-06	0.438	0.007	1	1	0
d	7	20	0.9	0.3	0.278	1.55E-03	3.13E-05	3.0E-05	0.450	0.000	1	1	0
d	7	30	0.9	0.3	0.285	5.40E-03	1.37E-05	1.3E-05	0.450	0.000	1	1	0
d	7	10	0.1	0.3	0.272	1.50E-03	3.84E-05	3.7E-05	0.450	0.000	1	0	1
d	7	20	0.1	0.3	0.255	2.21E-03	1.18E-04	1.1E-04	0.450	0.000	1	0	1
d	7	30	0.1	0.3	0.237	3.21E-03	1.96E-04	1.9E-04	0.450	0.000	1	0	1
d	7	10	0.9	0.3	0.283	3.02E-03	1.42E-05	1.4E-05	0.448	0.002	1	0	1
d	7	20	0.9	0.3	0.283	7.69E-03	1.63E-05	1.6E-05	0.444	0.006	1	0	1
d	7	30	0.9	0.3	0.278	3.45E-03	2.42E-05	2.4E-05	0.450	0.000	1	0	1

In this simulation, we observed that the estimator $\hat{m}(\pi)$ was close to the true values of π regardless of the values of m, and n when π is equal to 0.3, but the estimator overestimated the true value when θ =0.9 and π =0.1. Coverage rate for π was close to nominal level when θ =0.1 and far below nominal level when θ =0.9. As the number of items *m* increased, interval width increased in most of cases. Interval width was wider than for θ =0.9 for =0.1 Mean square error and standard error of the mean square error (se_mse) for the estimators $\hat{\pi}$ was less than 0.03.

A multiple regression model is fitted to the data in Table 4-4 with sample mean $\hat{m}(\pi)$ as dependent variables and true values of θ, m, n, D_1 and D_2 (the form of $\mathbf{w_2}$) as independent variables. The summary of fit from JMP output in Appendix 11 indicates that true values of

π and θ, and n played a significant role in estimating π. Surprisingly, m did not have significant relationship with $\hat{m}(\pi)$.

Similarly, a multiple regression model is fitted to the data in Table 4-4 with sample mean credible interval widths for π as dependent variables and true values of π and θ, m, n, D_1, and D_2 as independent variables. The summary of fit from JMP output in Appendix 11 indicates that m, true values of π and θ, but not n contributed significantly in sample mean interval width estimate for π.

A logistic regression model was fit to the pooled simulation results using maximum likelihood in Table 4-4 with sample mean coverage rate Scov_t for π as the dependent variable and m, n, true values of π and θ, D1, and D2 independent variables. A summary of the fit given in Appendix 11 shows that coverage rate was not related to any explanatory variables.

Table 4-5 Bayesian Point and Set Estimation for π With 100 Replications

m	n	True π	True θ	$\hat{m}(\pi)$	t_wid	Scov_t	D1	D2
5	10	0.1	0.1	0.139	0.327	1	0	0
5	10	0.1	0.5	0.249	0.436	0.85	0	0
5	10	0.1	0.9	0.248	0.433	0.9	0	0
5	10	0.3	0.1	0.282	0.442	1	0	0
5	10	0.3	0.5	0.308	0.418	1	0	0
5	10	0.3	0.9	0.311	0.418	1	0	0

Table 4-5 reports the results of set estimation for π with 100 replications, five times as many as the replications used in Table 4-4. This simulation was used to confirm the findings observed in Table 4-4 in which 20 replications were conducted for each case. The findings were consistent with the results reported in Table 4-4. The estimator $\hat{m}(\pi)$ underestimated the true value when π =0.1. The interval width was wider whenever θ is large. The coverage rate was close to nominal level.

In summary, Bayesian approach has the advantage over the profile likelihood method of being able to estimating π . Overall, using a regression analysis, the results of the simulation studies in this section reveal that true values of π and θ, D1 and D2 (the form of w2) contribute statistically significantly to the point estimate and confidence interval width of π . In addition to the above factors, the number of judges *n* also played a important role in the set estimation. The true value of θ was found to be related in the coverage rate for π.

4.3 Bayesian Point and Set Estimation for $\mathbf{w}_1$ and $\mathbf{w}_2$

Letting C represent a norming constant and using uniform priors, the discrete posterior density of $\mathbf{w}_1$ is given by:

$$p(\mathbf{w}_1, \mathbf{w}_2 \mid Data) = \mathrm{C} \int_0^{0.5} \int_0^1 L(.) d\theta d\pi \qquad (4.5)$$

A numerical integration program in the Software Package *R* was used to integrate the likelihood function with respective to θ in the domain of (0,1) and then with respective to π in the domain of (0,0.5).

A discrete approximation to the posterior can be obtained by computing the target density $\{ p(\mathbf{w}_1, \mathbf{w}_2 \mid Data) \}$ at collection of permutations of m objects taken q = 3 at a time that cover the enter parameter space of $\mathbf{S}_{qm}$. For m=5 (numbers of items), q=3, there are total 5*4*3=60 distinct ranking. So the domain $\mathbf{S}_{qm}$ for $\{\underline{w_i}\}_{i=1}^{2}$ are 60*59 =3540 distinct rankings (assumed $\mathbf{w}_1$ is different from $\mathbf{w}_2$, the basic criterion for two segments ranking).The discrete distribution with density $p(\mathbf{w_1}, \mathbf{w}_2 \mid Data)$ is then approximated arbitrarily closely by the discrete distribution with mass function

$$\tau^{\#}(\mathbf{w_1}, \mathbf{w}_2 \mid Data) = p(\mathbf{w_1}, \mathbf{w}_2 \mid Data) / \sum_s p(\mathbf{w_1}, \mathbf{w}_2 \mid Data) \ ,$$

A nominal 1-α joint HPD region for $\mathbf{w_1}$ and $\mathbf{w}_2$ is found by sorting the posteriors from high to low and including rankings until the sum of the posteriors first exceed 1-α (Nelson and Kemp, 2000).

Table 4-6. Bayesian Point and Set Estimation for $\mathbf{w_1}$ **and** $\mathbf{w}_2$ With 100 Replications

m	n	True π	True θ	Scov_t	D1	D2
5	10	0.1	0.1	1	0	0
5	10	0.1	0.5	0.91	0	0
5	10	0.1	0.9	0.63	0	0
5	10	0.3	0.1	1	0	0
5	10	0.3	0.5	0.92	0	0
5	10	0.3	0.9	0.71	0	0

In this section, we simulated R=100 data sets using Spearman's distance with n=5, m=10, parameter $\pi = 0.1,\ 0.3,\ 0.5,$ and $\theta = 0.1,\ 0.5,\ 0.9.$ We can see in Table 4-6 that, the coverage rate was poor when =0.9, The coverage rate was close to nominal level whenever $0.1 \le \theta \le 0.5$. Recall from Chapter 2, θ is a spread parameter lying in the unit interval, the probability is increasingly more concentrated about the mode $\mathbf{w_0}$ as θ decrease toward 0 or it can be interpreted as positing that a judge tries to reproduce her/his true ranking $\mathbf{w_0}$ and that increasing θ is associated with increasing variability in rankings actually reported by the judge and decreasing ability to report his/her true ranking. This simulation study suggests as judges get more training, fewer judges are required to reach a true 95 % credible regions for the true baseline rankings $\mathbf{w_1}$ and $\mathbf{w}_2$. If the judges randomly assign his/her ranking, more judges are needed to arrive at a 95% credible regions for the true baseline ranking. Similar conclusions were reached by Nelson and Kemp (2000) who performed a simulation study on one segment. The credible regions for $\mathbf{w}_0$ in their study perform satisfactorily when the rankings are based from reliable judges, when the judges are not consistent, they are of little practical value.

A logistic regression model was fitted to the data in Table 4-6 with sample mean coverage rate for $\mathbf{w_1}$ and $\mathbf{w}_2$ as dependent variables, and true values of θ, and π as independent variables. There is not enough evidence to show that the sample mean coverage rate for $\mathbf{w_1}$ and $\mathbf{w}_2$ is related to true values of θ, and π (Appendix 12).

In summary, Bayesian set estimation for $\mathbf{w_1}$ and $\mathbf{w}_2$ did poorly when θ=0.9, but performed satisfactorily when $0.1 \le \theta \le 0.5.$ A pattern similar to the one observed in profile likelihood set estimation was also observed here. Coverage rate decreased as θ increased even thought the coverage rate was not statistically significantly related to $\theta.$ In profile likelihood set estimation for $\mathbf{w_1}$ and $\mathbf{w}_2$, the logistic model for coverage rate was a statistically significant with p = 0.061 and θ was also significant. The difference could be due to the fact that the estimators for Bayesian approach were calculated from 12 cases, whereas 90 cases were conducted in the profile likelihood approach. Despite these facts, Bayesian approach is computationally faster than profile likelihood approach. It took around 68 hours to conduct a profile likelihood set estimation for $\mathbf{w_1}$ and $\mathbf{w}_2$, but only 16 hours for a Bayesian set estimation for $\mathbf{w_1}$ and $\mathbf{w}_2$. The main problem in conducting our simulation is the very long computation times required. As the number of judges *n* and number of items *m* increased, the numbers of possible rankings will quickly become very large and unmanageable in a profile likelihood set estimation for $\mathbf{w_1}$ and $\mathbf{w}_2$. Therefore, the Bayesian approach could be very beneficial in future simulation studies of estimation of $\mathbf{w_1}$ and $\mathbf{w}_2$.

4.4 Bayesian Point and Set Estimation for $\mathbf{w_1}$

Letting C represent a norming constant and using uniform priors, the discrete posterior density of $\mathbf{w_1}$ is given by:

$$p(\mathbf{w_1} \mid Data) = \mathrm{C} \sum_{w_2 \in \mathbf{S}-\mathbf{w}_1} \int_0^{0.5}\int_0^{1} L(.)d\theta d\pi \tag{4.5}$$

A numerical integration program in the Software Package *R* was used to integrate the likelihood function with respective to θ in the domain of (0,1); then with respective to π in the domain of (0,0.5) , and finally with respective to $\mathbf{w_2}$ in the domain of $\mathbf{S}_{qm} - \mathbf{w}_1$. A discrete approximation can be obtained by computing the target density { $p(\mathbf{w_1} \mid Data)$ } at the collection of permutations of m objects taken q = 3 at a time that cover the enter parameter space of $\mathbf{S}_{qm}$. For m=5 (numbers of items), there are total 5*4*3=60 distinct ranking. So the domain $\mathbf{S}_{qm}$ for $\mathbf{w}_1$ are 59 distinct rankings (assumed $\mathbf{w}_1$ is different from $\mathbf{w}_2$, the basic criterion for two segments ranking).The discrete distribution with density $p(\mathbf{w_1} \mid Data)$ is then approximated arbitrarily closely by the discrete distribution with mass function

$$\tau^{\#}(\mathbf{w_1} \mid Data) = p(\mathbf{w_1} \mid Data) / \sum_{s} p(\mathbf{w_1} \mid Data) ,$$

A nominal 1-α HPD region for $\mathbf{w_1}$ is found by sorting these posteriors from high to low and including rankings until the sum of the posteriors first exceed 1-α (Nelson and Kemp, 2000).

In this section, we simulated 36 scenarios consisting of 20 data sets for each scenario using Spearman distance. We let n=10, 20, 30; m=5; π =0.1, 0.3; θ=0.1, 0.9; $\mathbf{w_1} = \mathbf{(1,2,3)}$; $\mathbf{w}_2 = (1,3,2)$, $\mathbf{w}_2 = (3,1,2)$, $\mathbf{w}_2 = (3,2,1)$. Simulation results were presented in Table 4-7.

Table 4-7. Bayesian Point and Set Estimation for $\mathbf{w_1}$ With 20 Replications

m	n	True π	True θ	Scov_t	D_1	D_2
5	10	0.1	0.1	1	0	0
5	20	0.1	0.1	1	0	0
5	30	0.1	0.1	1	0	0
5	10	0.1	0.9	0.80	0	0
5	20	0.1	0.9	0.85	0	0
5	30	0.1	0.9	0.95	0	0
5	10	0.1	0.1	1	1	0
5	20	0.1	0.1	1	1	0
5	30	0.1	0.1	1	1	0
5	10	0.1	0.9	0.8	1	0
5	20	0.1	0.9	0.90	1	0
5	30	0.1	0.9	0.95	1	0
5	10	0.1	0.1	1	0	1
5	20	0.1	0.1	1	0	1
5	30	0.1	0.1	1	0	1
5	10	0.1	0.9	0.75	0	1
5	20	0.1	0.9	0.90	0	1
5	30	0.1	0.9	0.90	0	1
5	10	0.3	0.1	1	0	0
5	20	0.3	0.1	1	0	0
5	30	0.3	0.1	1	0	0
5	10	0.3	0.9	0.85	0	0
5	20	0.3	0.9	0.90	0	0
5	30	0.3	0.9	0.95	0	0
5	10	0.3	0.1	1	1	0
5	20	0.3	0.1	1	1	0
5	30	0.3	0.1	1	1	0
5	10	0.3	0.9	0.80	1	0
5	20	0.3	0.9	0.95	1	0
5	30	0.3	0.9	0.95	1	0
5	10	0.3	0.1	1	1	0
5	20	0.3	0.1	1	1	0
5	30	0.3	0.1	1	1	0
5	10	0.3	0.9	0.80	1	0
5	20	0.3	0.9	0.90	1	0
5	30	0.3	0.9	0.95	1	0

It can be seen from Table 4.7 that as θ increased, the sample mean coverage rate decreased by a small amount. The coverage rate reached 100% when $\theta = 0.1$, regardless of the number of judges n. The coverage rate fell below nominal level when $\theta = 0.9$. As the number of judges increased, the coverage rate increased slightly. Even thought we assume that $\mathbf{w}_2$ is fixed and known, similar pattern observed in simulation study in 4.3 for $\mathbf{w}_1$ and $\mathbf{w}_2$ was observed here. This simulation study also suggests as judges get more training, fewer judges are required to reach a 95 % credible regions for the true baseline ranking for $\mathbf{w}_1$. If the judges are randomly guessing, more judges are needed to reach a 95% credible regions for the true baseline ranking.

A logistic regression model is fitted to the data in Table 4-7 with sample mean coverage rate for $\mathbf{w}_1$ as dependent variables, and true values of θ, π, m, n, D_1, and D_2 as independent variables. The summary of fit from JMP output in appendix 13 indicates that the performance of sample mean coverage rate for $\mathbf{w}_1$ was not affected by any of the explanatory variables. The same conclusion we reached in previous chapters likelihood set estimation for $\mathbf{w}_1$.

Table 4-8. Bayesian point and set estimation for $\mathbf{w}_1$ with 100 replications

m	n	True π	True θ	Scov_t	D1	D2
5	10	0.1	0.1	1	0	0
5	10	0.1	0.5	0.95	0	0
5	10	0.1	0.9	0.74	0	0
5	10	0.3	0.1	1	0	0
5	10	0.3	0.5	0.88	0	0
5	10	0.3	0.9	0.71	0	0

One-hundred replications were performed for each of case to confirm the findings in Table 4-8. Similar pattern in Table 4-7 was observed in Table 4-8. The coverage rate was close to nominal level whenever $0.1 \le \theta \le 0.5$.

In summary, parameter estimates from Bayesian were generally very similar to those from profile likelihood estimates. The coverage rate was close to the nominal level whenever $0.1 \le \theta \le 0.5$. The sample mean coverage rate decreased as θ increased, but θ was not significantly related to the sample mean coverage rate.

4.5 Bayesian Point and Set Estimation for $\mathbf{w}_2$

Letting C represent a norming constant and using uniform priors, the discrete posterior density of $\mathbf{w}_2$ is given by:

$$p(\mathbf{w}_2 \mid Data) = \mathrm{C} \sum_{w_1 \in \mathbf{S} - \mathbf{w}_2} \int_0^{0.5} \int_0^{1} L(.) d\theta d\pi \tag{4.5}$$

Numerical integration in Software Package *R* was used to integrate the likelihood function with respective to θ in the domain of (0,1); then with respective to π in the domain of (0,0.5) , in the end with respective to $\mathbf{w}_1$ in the domain of $\mathbf{S}_{qm} - \mathbf{w}_2$.

The simplest discrete approximation can be obtained by computing the target density $\{ p(\mathbf{w}_2 \mid Data) \}$ at the collection of permutations of m objects taken q = 3 at a time that cover the enter parameter space of $\mathbf{S}_{qm}$. For m=5 (numbers of items), there are total 5*4*3=60 distinct ranking. So the domain $\mathbf{S}_{qm}$ for $\mathbf{w}_2$ are 59 distinct rankings (assuming $\mathbf{w}_1$ is different from $\mathbf{w}_2$, the basic criterion for two segments ranking).The discrete distribution with density $p(\mathbf{w}_2 \mid Data)$ is then approximated arbitrarily closely by the discrete distribution with mass function

$$\tau^{\#}(\mathbf{w}_2 \mid Data) = p(\mathbf{w}_2 \mid Data) / \sum_{s} p(\mathbf{w}_2 \mid Data) \ ,$$

A nominal 1-α HPD region for $\mathbf{w}_2$ is found by sorting the posteriors from high to low and including rankings until the sum of the posteriors first exceed 1-α (Nelson and Kemp, 2000).

Similar simulation design used in Bayesian point and set estimation for $\mathbf{w}_1$ was used here. The impact of true values of θ, π, the number of items m, the number of judges n, D_1, and D_2 on coverage rate for is showed in Table 4-9.

Table 4-9. Bayesian Point and Set Estimation for $\mathbf{w_2}$ With 20 Replications

m	n	True π	True θ	Scov_t	D_1	D_2
5	10	0.1	0.1	1	0	0
5	20	0.1	0.1	1	0	0
5	30	0.1	0.1	1	0	0
5	10	0.1	0.9	0.80	0	0
5	20	0.1	0.9	0.9	0	0
5	30	0.1	0.9	0.95	0	0
5	10	0.1	0.1	1	1	0
5	20	0.1	0.1	1	1	0
5	30	0.1	0.1	1	1	0
5	10	0.1	0.9	0.75	1	0
5	20	0.1	0.9	0.85	1	0
5	30	0.1	0.9	0.90	1	0
5	10	0.1	0.1	1	0	1
5	20	0.1	0.1	1	0	1
5	30	0.1	0.1	1	0	1
5	10	0.1	0.9	0.75	0	1
5	20	0.1	0.9	0.8	0	1
5	30	0.1	0.9	0.95	0	1
5	10	0.3	0.1	1	0	0
5	20	0.3	0.1	1	0	0
5	30	0.3	0.1	1	0	0
5	10	0.3	0.9	0.85	0	0
5	20	0.3	0.9	0.85	0	0
5	30	0.3	0.9	0.95	0	0
5	10	0.3	0.1	1	1	0
5	20	0.3	0.1	1	1	0
5	30	0.3	0.1	1	1	0
5	10	0.3	0.9	0.80	1	0
5	20	0.3	0.9	0.80	1	0
5	30	0.3	0.9	0.90	1	0
5	10	0.3	0.1	1	1	0
5	20	0.3	0.1	1	1	0
5	30	0.3	0.1	1	1	0
5	10	0.3	0.9	0.85	1	0
5	20	0.3	0.9	0.85	1	0
5	30	0.3	0.9	0.90	1	0

A pattern similar to the one observed for $\mathbf{w_1}$ occurs here for estimation of $\mathbf{w_2}$. Here we assume that $\mathbf{w}_1$ is fixed and known. It can be seen from Table 4.9 that as θ increased, the sample mean coverage rate decreased by a small amount. The coverage rate reached 100% whenever $\theta = 0.1$ regardless of the number of judges n. The coverage rate fell below nominal level whenever $\theta = 0.9$. As the number of judges increased, the coverage rate increased slightly.

The simulation study suggests as judges get more training, fewer judges are required to reach a 95 % credible regions for the true baseline ranking for $\mathbf{w}_2$. If the judges randomly assign his/her ranking, more judges are needed to reach a 95 confidence for the true baseline ranking.

A logistic regression model was fit to the data in Table 4-9 with sample mean coverage rate for $\mathbf{w}_2$ as dependent variables, and true values of θ, π, m, n, D_1, and D_2 as independent variables. The summary of fit from JMP output in appendix 14 indicates that all explanatory variables have not effect in estimating sample mean coverage rate for $\mathbf{w}_2$. The same conclusion we reached in previous chapters likelihood set estimation for $\mathbf{w}_2$.

Table 4-10. Bayesian Point and Set Estimation for $\mathbf{w}_2$ With 100 Replications

m	n	True π	True θ	Scov_t	D_1	D_2
5	10	0.1	0.1	1	0	0
5	10	0.1	0.5	0.96	0	0
5	10	0.1	0.9	0.91	0	0
5	10	0.3	0.1	1	0	0
5	10	0.3	0.5	0.90	0	0
5	10	0.3	0.9	0.72	0	0

Table 4-10 presented Bayesian point and set estimation for $\mathbf{w}_2$ with 100 replications for each case. The coverage rates for $\mathbf{w}_2$ was close to 95% nominal level when θ is small.

Similarly, parameter estimates from Bayesian were generally very similar to those from profile likelihood estimates. The coverage rate was close to nominal level whenever $0.1 \le \theta \le 0.5$. The sample mean coverage rate decreased as θ increased, but θ was found not significantly related to the sample mean coverage rate.

CHAPTER 5 – Covariate Model for Segment Membership

It is generally unknown to which segment a judge belongs and we do not know how many judges in the sample are in each segment. Here, we assume that there are two segments. Let $\mathbf{Y}_j$ be the ranking of judge j and $\mathbf{x}_j$ a corresponding covariate vector of covariates, j = 1,2,…,n. Suppose that for a judge with covariate vector $\mathbf{x} = (1, x)'$, the probability of lying in Segment 1 is given by $\pi(\mathbf{x}, \boldsymbol{\beta})$, where $\boldsymbol{\beta} = (\beta_0, \beta_1)$ is a vector of unknown parameters. We assume a logistic model of the form.

$$\eta(\mathbf{x}, \boldsymbol{\beta}) = \log\left(\frac{\pi(\mathbf{x}, \boldsymbol{\beta})}{1 - \pi(\mathbf{x}, \boldsymbol{\beta})}\right) = \boldsymbol{\beta}'\mathbf{x} \qquad ,$$

$$\pi(\mathbf{x}, \boldsymbol{\beta}) = \frac{\exp(\eta(\mathbf{x}, \boldsymbol{\beta})}{(1 + \eta(\mathbf{x}, \boldsymbol{\beta}))} \tag{5.1}$$

The likelihood function incorporating $\boldsymbol{\beta}$ in (2.7) for two segments is:

$$L(\theta, \boldsymbol{\beta}, \{\mathbf{w}_i\}) = \prod p(y_j \mid \theta, \boldsymbol{\beta}, \{\mathbf{w}_i\}) = \prod \{\pi(\mathbf{x}, \boldsymbol{\beta})\theta^{d(\mathbf{y}_j, \mathbf{w}_1)} + (1 - \pi(\mathbf{x}, \boldsymbol{\beta})\theta^{d(\mathbf{y}_j, \mathbf{w}_2)}\} / \psi^n(\theta)$$

The *MLE*'s for the covariate model with segment membership can be found by modifying the E-M given in Chapter 2. The conditional distribution of the unobserved membership variable z_j of judge j is binomial, specified by

$$z_j \mid \mathbf{y}_j, \theta, \mathbf{w}_1, \mathbf{w}_2, \boldsymbol{\beta} \sim Binomial(1, \frac{\pi(\mathbf{x}, \boldsymbol{\beta})P(\mathbf{y}_j \mid \theta, \mathbf{w}_1)}{\pi(\mathbf{x}, \boldsymbol{\beta})P(\mathbf{y}_j \mid \theta, \mathbf{w}_1) + (1 - \pi(\mathbf{x}, \boldsymbol{\beta})P(\mathbf{y}_j \mid \theta, \mathbf{w}_2)})$$

In this setting, letting $\mathbf{w_1}$ and $\mathbf{w_2}$ denote the unknown baseline rankings of segments one and two respectively, j = 1,2,…,n, and letting $\mathbf{w}$ denote the baseline ranking of judge *j*, we have that

$$z_j = \begin{cases} 1 & if\ \mathbf{w} = \mathbf{w_1} \\ 0 & if\ \mathbf{w} = \mathbf{w_2} \end{cases}$$

$$E_{\xi}(z_j \mid \mathbf{y_j}) = \frac{\pi(\mathbf{x},\boldsymbol{\beta})P(\mathbf{y}_j \mid \theta, \mathbf{w}_1)}{\pi(\mathbf{x},\boldsymbol{\beta})P(\mathbf{y}_j \mid \theta, \mathbf{w}_1) + (1-\pi(\mathbf{x},\boldsymbol{\beta})P(\mathbf{y}_j \mid \theta, \mathbf{w}_2)}$$

$$\equiv \lambda_j(\boldsymbol{\xi}), \text{ j = 1,2,…,n.}$$

The likelihood based on the augmented data is then given by:

$$\underset{\{\mathbf{y},\mathbf{z}\}}{L}(\ \pi(\mathbf{x},\boldsymbol{\beta}), \theta, \mathbf{w}_1, \mathbf{w_2}) = \psi^{-n}(\theta)\theta^{D(\mathrm{w})}\prod(\pi(\mathbf{x},\boldsymbol{\beta})^{z_j}(1-\pi(\mathbf{x},\boldsymbol{\beta})^{1-z_j}) \tag{5.2}$$

where $\mathbf{w}^{\#} = (\mathbf{w_1}, \mathbf{w_2})$ and $D(\mathbf{w}^{\#},\{z_j\})$ $D(\mathbf{w}^{\#},\{z_j\})$ is defined as:

$$D(\mathbf{w}^{\#},\{z_j\}) = \sum_{j=1}^{n} z_j d(\mathbf{y_j}, \mathbf{w_1}) + \sum_{j=1}^{n}(1-z_j)d(\mathbf{y_j}, \mathbf{w_2})$$

Plugging $D(\mathbf{w}^{\#},\{z_j\})$ and $\pi(x_j, \boldsymbol{\beta}) = \dfrac{e^{\beta_0+\beta_1 x_j}}{1+e^{\beta_0+\beta_1 x_j}}$ into equation (5.2), we then have:

$$\log\underset{\{\mathbf{y},\mathbf{z}\}}{(}\ \pi(x_j,\boldsymbol{\beta}), \theta, \mathbf{w}_1, \mathbf{w_2}) = -n\log\psi(\theta) + D(\mathbf{w}^{\#},\{z_j\})\log\theta + \log(\prod(\pi(x_j,\boldsymbol{\beta})^{z_i}(1-\pi(x_j,\boldsymbol{\beta})^{1-z_i})$$

The Modified E-M Algorithm:

E step:

After the ith iteration where we have estimated parameter values $\xi^{\mathbf{i}}$, compute the conditional expectation of the log-likelihood based on the augmented data with respect to the observed data. In the one covariate, two segment cases being considered here, $Q(\xi, \xi^{\mathbf{i}})$ becomes:

$$\log_{\{y,z\}} L\left(\pi(\beta, x_j), \theta, \mathbf{w_1}, \mathbf{w_2}\right) = -n\log\psi(\theta) + D(\mathbf{w}^{\#}, \{z_j\})\log\theta + \log(\prod\left(\frac{e^{\beta_0+\beta_1 x_j}}{1+e^{\beta_0+\beta_1 x_j}}\right)^{z_j}\left(\frac{1}{1+e^{\beta_0+\beta_1 x_j}}\right)^{1-z_j})$$

$$= -n\log\psi(\theta) + D(\mathbf{w}^{\#}, \{z_j\})\log\theta + \log\left(\prod\frac{(e^{\beta_0+\beta_1 x_j})^{z_j}}{1+e^{\beta_0+\beta_1 x_j}}\right)$$

$$= -n\log\psi(\theta) + D(\mathbf{w}^{\#}, \{z_j\})\log\theta + \log\left(\prod(e^{\beta_0+\beta_1 x_j})^{z_j}\right) - \log\left(\prod(1+e^{\beta_0+\beta_1 x_j})\right)$$

$$= -n\log\psi(\theta) + D(\mathbf{w}^{\#}, \{z_j\})\log\theta + z_1 * (\beta_0 + \beta_1 x_1) + \ldots + z_n * (\beta_0 + \beta_1 x_n) - \log\left(\prod(1+e^{\beta_0+\beta_1 x_j})\right)$$

$$Q(\boldsymbol{\xi}, \boldsymbol{\xi}^{\mathbf{i}}) = E_{\xi^i} Log(L_{\{\mathbf{z}\}|\{\mathbf{y}\}}(\boldsymbol{\xi}))$$

$$= -n\log\psi(\theta) + D(\mathbf{w}^{\#}, E_{\boldsymbol{\xi}^{\mathbf{i}}}\{z_j\} | \{\mathbf{y_j}\})\log\theta + E_{\xi}(z_1 | \mathbf{y_j}) * (\beta_0 + \beta_1 x_1) + \ldots E_{\xi}(z_j | \mathbf{y_j}) * (\beta_0 + \beta_1 x_j) - \log\left(\prod(1+e^{\beta_0+\beta_1 x_j})\right)$$

$$= -n\log\psi(\theta) + D(\underline{\mathbf{w}^{\#}, \{\lambda_j}\{\boldsymbol{\xi}^{\mathbf{i}}\})\log\theta + \lambda_1(\boldsymbol{\xi}^{\mathbf{i}}) * (\beta_0 + \beta_1 x_1) + \ldots + \lambda_j(\boldsymbol{\xi}^{\mathbf{i}}) * (\beta_0 + \beta_1 x_j) - \log\left(\prod(1+e^{\beta_0+\beta_1 x_j})\right)$$

M step:

1). Once $\hat{\mathbf{w}}_1^{(i)}$ *and* $\hat{\mathbf{w}}_2^{(i)}$ are found, update $\hat{\theta}$, and $\hat{\beta}_0, \hat{\beta}_1$ and continue these iterations until there is relatively change in $Q(\boldsymbol{\xi}, \boldsymbol{\xi}^{\mathbf{i}})$ and the say, $|Q(\boldsymbol{\xi}^{\mathbf{i+1}}, \boldsymbol{\xi}^{\mathbf{i}}) - Q(\boldsymbol{\xi}^{\mathbf{i}}, \boldsymbol{\xi}^{\mathbf{i-1}})| \le 10^{-2}$

(2). To estimate θ we have to solve the following equation: $\frac{\partial Log(L(\theta, \hat{\boldsymbol{\beta}}\, \hat{\mathbf{w}}^{\#}))}{\partial\theta} = 0$

which becomes

$$\frac{\theta\,\psi'(\theta)}{\psi(\theta)} = \frac{\mathrm{D}(\hat{\mathbf{w}}^{\#})}{n}$$

Note that $E_{\theta,\mathbf{w}}(d(\mathbf{y},\mathbf{w}))$ = $\dfrac{\theta\,\psi'(\theta)}{\psi(\theta)} = H(\theta)$, say.

The method of Bisection was used to solve $H(\theta) = \dfrac{1}{n} D(\hat{\mathbf{w}}^{\#})$,

For Kendall's distance:

$$E_{\theta,\mathbf{w}}(d(\mathbf{y},\mathbf{w})) = \frac{m\theta}{1-\theta} - \sum_{i=1}^{m} \frac{i\theta^i}{1-\theta^i}$$, where m= the number of objects

(3) For Spearman's distance $E_{\theta,\mathbf{w}}(d(\mathbf{y},\mathbf{w}))$ does not have a closed form expression and must be computed on a case by case basis.

To determine the new estimate β_0^{i+1} as the solution of

$$\frac{\partial Q(\boldsymbol{\xi},\boldsymbol{\xi}^{\mathbf{i}})}{\partial \beta_0} = \lambda_1(\boldsymbol{\xi}^{\mathbf{i}}) + \ldots + \lambda_n(\boldsymbol{\xi}^{\mathbf{i}}) - \{\frac{e^{\beta_0+\beta_1 x_1}}{1+e^{\beta_0+\beta_1 x_1}} + \ldots + \frac{e^{\beta_0+\beta_1 x_n}}{1+e^{\beta_0+\beta_1 x_n}}\} = 0 \Longrightarrow \beta_0^{\,i+1}$$

$$\sum_{j=1}^{n} \lambda_j(\boldsymbol{\xi}^{\mathbf{i}}) = \sum_{j=1}^{n} \frac{e^{\beta_0+\beta_1 x_j}}{1+e^{\beta_0+\beta_1 x_j}} \Longrightarrow \beta_0^{\,i+1}$$

Let *N* be the random variable denoting the number out of the *n* judges in the sample who lie in segment 1. Considering the n covariates $\{x_j\}$ as fixed values, the above equation may be interpreted as:

$$E_{\varsigma^i}(N \mid Data) = \mathrm{E(N)}$$

Simultaneously, to determine the new estimate β_1^{i+1} as the solution of

$$\frac{\partial Q(\xi,\xi^{\mathbf{i}})}{\partial \beta_1} = \lambda_1(\xi^{\mathbf{i}})*(x_1)+\ldots+\lambda_m(\xi^{\mathbf{i}})*(x_m)-\{\frac{x_1*(e^{\beta_0+\beta_1 x_1})}{1+e^{\beta_0+\beta_1 x_1}}+\ldots+\frac{x_m*(e^{\beta_0+\beta_1 x_m})}{1+e^{\beta_0+\beta_1 x_m}}\}=0 \Longrightarrow \beta_1^{i+1}$$

$$\sum_{j=1}^{m}\lambda_j(\xi^{\mathbf{i}})*x_j = \sum_{j=1}^{m}\frac{x_j e^{\beta_0+\beta_1 x_j}}{1+e^{\beta_0+\beta_1 x_j}} \Longrightarrow \beta_1^{i+1}$$

There is no closed form solution to the above equations. I used the Newton-Raphson approximation method because the derivatives for the above equations are relatively easy to handle. The following notation helps implement this process.

Let

$e^{\beta_0+\beta_1 x_j} = A$, $1+e^{\beta_0+\beta_1 x_j} = B$,

$xe^{\beta_0+\beta_1 x} = C$, $1+e^{\beta_0+\beta_1 x} = D$,

$A'(\beta_0)$ be the first derivatives of A with respective to β_0;

$B'(\beta_0)$ be the first derivatives of B with respective to β_0;

$C'(\beta_1)$ be the first derivatives of C with respective to β_1;

$D'(\beta_1)$ be the first derivatives of D with respective to β_1;

$A'(\beta_1)$ be the first derivatives of A with respective to β_1;

$B'(\beta_1)$ be the first derivatives of B with respective to β_1;

$$A'(\beta_0) = e^{\beta_0+\beta_1 x}\ \frac{d}{d\beta_0}(\beta_0+\beta_1 x) = e^{\beta_0+\beta_1 x}$$

$$B'(\beta_0) = e^{\beta_0+\beta_1 x}\ \frac{d}{d\beta_0}(\beta_0+\beta_1 x) = e^{\beta_0+\beta_1 x}$$

$$\frac{d}{d\beta_0}(\frac{A}{B}) = \frac{A'(\beta_0)*B - B'(\beta_0)*A}{B^2}$$

$$(A'(\beta_0)*B - B'(\beta_0)*A) = (e^{\beta_0+\beta_1 x})(1+e^{\beta_0+\beta_1 x}) - (e^{\beta_0+\beta_1 x})^2 \; = e^{\beta_0+\beta_1 x}$$

$$\frac{\partial Q^2(\xi,\xi^{\mathbf{i}})}{\partial \beta_0^2} = 0 - \{\frac{e^{\beta_0+\beta_1 x_1}}{(1+e^{\beta_0+\beta_1 x_1})^2} + \ldots + \frac{e^{\beta_0+\beta_1 x_m}}{(1+e^{\beta_0+\beta_1 x_m})^2}\}$$

$$C'(\beta_1) = xe^{\beta_0+\beta_1 x} \;\; \frac{d}{d\beta_1}(\beta_0 + \beta_1 x) = x^2 e^{\beta_0+\beta_1 x}$$

$$D'(\beta_1) = e^{\beta_0+\beta_1 x} \frac{d}{d\beta_1}(\beta_0 + \beta_1 x) = xe^{\beta_0+\beta_1 x}$$

$$\frac{d}{d\beta_1}(\frac{C}{D}) = \frac{C'(\beta_1)*D - D'(\beta_1)*C}{D^2}$$

$$C'(\beta_1)*D - D'(\beta_1)*C = x^2 e^{\beta_0+\beta_1 x}(1+e^{\beta_0+\beta_1 x}) - x\; e^{\beta_0+\beta_1 x} \;\; (1+e^{\beta_0+\beta_1 x})$$

$$\frac{d}{d\beta_1}(\frac{C}{D}) = \frac{C'(\beta_1)*D - D'(\beta_1)*C}{D^2}$$

$$\frac{d}{d\beta_1}(\frac{C}{D}) = \frac{x^2 e^{\beta_0+\beta_1 x}(1+e^{\beta_0+\beta_1 x}) - xe^{\beta_0+\beta_1 x_j}(1+e^{\beta_0+\beta_1 x}\;)}{(1+e^{\beta_0+\beta_1 x}\;)^2} = \frac{x^2 e^{\beta_0+\beta_1 x} - x\; e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

$$\frac{\partial^2 Q(\xi,\xi^{\mathbf{i}})}{\partial \beta_1^2} = 0 - \{\frac{x_1^2 * (e^{\beta_0+\beta_1 x_1}) - x_1 * (e^{\beta_0+\beta_1 x_1})}{1+e^{\beta_0+\beta_1 x_1}} + \ldots + \frac{x_n^2 * (e^{\beta_0+\beta_1 x_n}) - x_n * (e^{\beta_0+\beta_1 x_n})}{1+e^{\beta_0+\beta_1 x_m}}\}$$

$$A'(\beta_1) = e^{\beta_0+\beta_1 x} \;\; \frac{d}{d\beta_0}(\beta_0 + \beta_1 x) = x\; e^{\beta_0+\beta_1 x}$$

$$B'(\beta_1) = e^{\beta_0+\beta_1 x} \;\; \frac{d}{d\beta_1}(\beta_0 + \beta_1 x\;) = xe^{\beta_0+\beta_1 x}$$

$$\frac{d}{d\beta_1}(\frac{A}{B}) = \frac{A'(\beta_1)*B - B'(\beta_1)*E}{B^2}$$

$$\frac{d}{d\beta_1}(\frac{A}{B}) = \frac{x\; e^{\beta_0+\beta_1 x}\;\;(1+e^{\beta_0+\beta_1 x}\;) - x\; e^{\beta_0+\beta_1 x}\;\;(e^{\beta_0+\beta_1 x}\;)}{(1+e^{\beta_0+\beta_1 x}\;)^2} = \frac{xe^{\beta_0+\beta_1 x}}{(1+e^{\beta_0+\beta_1 x})^2}$$

$$\frac{\partial Q^2(\boldsymbol{\xi},\boldsymbol{\xi}^{\mathbf{i}})}{\partial\beta_0\partial\beta_1} = 0 - \{\frac{x_1 e^{\beta_0+\beta_1 x_1}}{(1+e^{\beta_0+\beta_1 x_1})^2} + \ldots + \frac{x_n e^{\beta_0+\beta_1 x_n}}{(1+e^{\beta_0+\beta_1 x_n})^2}\}$$

Therefore, we have

$$\frac{\partial Q(\boldsymbol{\xi},\boldsymbol{\xi}^{\mathbf{i}})}{\partial\beta_0} = \sum_{j=1}^{n}\lambda_j(\boldsymbol{\xi}^{\mathbf{i}}) - \sum_{j=1}^{n}\frac{e^{\beta_0+\beta_1 x_j}}{1+e^{\beta_0+\beta_1 x_j}}$$

$$\frac{\partial Q(\boldsymbol{\xi},\boldsymbol{\xi}^{\mathbf{i}})}{\partial\beta_1} = \sum_{j=1}^{n}\lambda_j(\boldsymbol{\xi}^{\mathbf{i}}) * x_j - \sum_{j=1}^{n}\frac{x_j e^{\beta_0+\beta_1 x_j}}{1+e^{\beta_0+\beta_1 x_j}}$$

$$\frac{\partial Q^2(\boldsymbol{\xi},\boldsymbol{\xi}^{\mathbf{i}})}{\partial\beta_0^2} = -\sum_{j=1}^{n}\frac{e^{\beta_0+\beta_1 x_j}}{1+e^{\beta_0+\beta_1 x_j}}$$

$$\frac{\partial^2 Q(\boldsymbol{\xi},\boldsymbol{\xi}^{\mathbf{i}})}{\partial\beta_1^2} = -\sum_{j=1}^{n}\frac{x_j^2 e^{\beta_0+\beta_1 x_j} - x_j e^{\beta_0+\beta_1 x_j}}{1+e^{\beta_0+\beta_1 x_j}}$$

$$\frac{\partial Q^2(\boldsymbol{\xi},\boldsymbol{\xi}^{\mathbf{i}})}{\partial\beta_0\partial\beta_1} = -\sum_{j=1}^{n}\frac{x_j e^{\beta_0+\beta_1 x_j}}{(1+e^{\beta_0+\beta_1 x_j})^2}$$

Simulation Study for Covariate Model in Two Segments:

I simulated two segments, partial ranking data with some specific parameter values for a one covariate, two segment model. The simulation algorithm is very similar to the one used to simulate two segment partial ranking data in Chapter (3) except that the probability of segment one membership is potentially different for each judge and depends on a covariate, as given in (5.1)., Specifically, the probability mass function of ranking $\mathbf{y}$ for a judge having covariate vector $\mathbf{x}$ is given by

$$p(\mathbf{y}\,|\,\theta,\mathbf{w_1},\mathbf{w_2},\pi(\mathbf{x},\boldsymbol{\beta})) = \frac{\pi(\mathbf{x},\boldsymbol{\beta})\theta^{d(\mathbf{y},\mathbf{w}_1)} + (1-\pi(\mathbf{x},\boldsymbol{\beta}))\theta^{d(\mathbf{y},\mathbf{w}_2)}}{\Psi(\theta)}$$

I used the following simulation design:

First, we took the covariates for the j judges, $\{x_j\}$, to be independent , uniformly distributed on the unit interval. The Software Package R has a built-in function for generating random variates from a variety of popular distributions, including the uniform distribution. This built-in function was used to generate x_j over the interval (0,1) for each of the n judges.

Step 1: First take $\beta_0 = 0,\ \beta_1 = 1$ and $\theta = (0.1, 0.5, 0.9)$, respectively. Then generate $\{x_j\} \overset{iid}{\sim} U(0,1)$.

Compute $\pi(x_j, \boldsymbol{\beta}) = \dfrac{e^{\beta_0+\beta_1 x_j}}{1+e^{\beta_0+\beta_1 x_j}}$ and

$P(\mathbf{y}_i \mid \theta, \mathbf{w_1}, \mathbf{w_2}, \pi(\mathbf{x}, \boldsymbol{\beta})) = \dfrac{\pi(\mathbf{x}, \boldsymbol{\beta})\theta^{d(\mathbf{y}_i, \mathbf{w}_1)} + (1-\pi(\mathbf{x}, \boldsymbol{\beta}))\theta^{d(\mathbf{y}_i, \mathbf{w}_2)}}{\Psi(\theta)}$ for each judge with covariate vector x_j.

Step 2: generate U~ U(0,1)

If $U \le \pi(\mathbf{x}, \boldsymbol{\beta})$ generate $\mathbf{y}_j$ from $P(\mathbf{y}_j \mid \theta, \mathbf{w_1})$

If $U > \pi(\mathbf{x}, \boldsymbol{\beta})$ generate $\mathbf{y}_j$ from $P(\mathbf{y}_j \mid \theta, \mathbf{w}_2)$,

Independently repeat this procedure for each of the n judges. The rest of the simulation is the same as was used when no covariates were recorded in Chapter (3).

5.1 Point and Set Estimation for β and θ

The reliability of the performance of **β** and θ in covariate models were investigated, we present simulations results of that were undertaken for the following cases with 50 data sets for each case. We took $\beta_0 = 0$, $\beta_1 = 0,\ 2,\ 4,$ $\theta = 0.1,\ 0.5,\ 0.9,$ $n = 10, m = 5,$ $\mathbf{w_1} = (1,2,3)$, $\mathbf{w}_2 = (1,3,2)$ using Spearman's distance.

Under standard regularity conditions, which may not hold for mixture models of the type I investigated, a nominal $(1-\alpha)100\% = 95\%$ profile likelihood confidence set for θ is given by the values

$$\{\theta,\ \ 2[LogL(\hat{\theta},\hat{\boldsymbol{\beta}} = (\hat{\beta}_0,\hat{\beta}_1),\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2) - LogL^P(\theta)] \le 3.84\}.$$

Values of these θ satisfying this condition were estimated by incrementing θ from 0.05 to 1 in steps of 3.1.1 and computing $LogL^P(\theta)$ for each value of θ

Note that

$$L^P(\theta) \le \max L(\theta,\hat{\boldsymbol{\beta}} = (\hat{\beta}_0,\hat{\beta}_1),\mathbf{w}_1,\mathbf{w}_2) = L(\hat{\theta},\hat{\boldsymbol{\beta}} = (\hat{\beta}_0,\hat{\beta}_1),\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2)$$

The following statistics were computed for each of the 9 investigated scenarios:

R=number of simulations or replications.

$\hat{m}(\theta)$ = sample mean of $\hat{\theta} = \dfrac{1}{R}\sum_{i=1}^{R}\hat{\theta}_i = \bar{\theta}$.

$s(\hat{\theta})$ =standard error of $\hat{m}(\theta) = \sqrt{\dfrac{\sum_{i=1}^{R}(\hat{\theta}_i - \bar{\theta})^2}{R(R-1)}}$

$m(\hat{\beta}_0)$ = sample mean of $\hat{\beta}_0 = \frac{1}{R}\sum_{i=1}^{R}\hat{\beta}_{0i} = \overline{\beta_0}$.

$s(\hat{\beta}_0)$ =standard error of $m(\hat{\beta}_0) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\beta}_{0i} - \overline{\beta_0})^2}{R(R-1)}}$

$m(\hat{\beta}_1)$ = sample mean of $\hat{\beta}_1 = \frac{1}{R}\sum_{i=1}^{R}\hat{\beta}_{1i} = \overline{\beta_1}$.

$s(\hat{\beta}_1)$ =standard error of $m(\hat{\beta}_1) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\beta}_{1i} - \overline{\beta_1})^2}{R(R-1)}}$

mse_t = estimated MSE of $\hat{\theta} = \hat{MSE}(\hat{\theta}) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\theta}_i - \theta)^2 = \frac{\sum_{i=1}^{R} y_i}{R} = \bar{y}$, where $y_i = (\hat{\theta}_i - \theta)^2$

$mse\ \beta_0$ = estimated MSE of $\hat{\beta}_0 = \hat{MSE}(\hat{\beta}_0) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\beta}_{0i} - \beta_0)^2 = \frac{\sum_{i=1}^{R} z_i}{R} = \bar{z}$, where

$z_i = (\hat{\beta}_{0i} - \beta_0)^2$

$mse\ \beta_1$ = estimated MSE of $\hat{\beta}_1 = \hat{MSE}(\hat{\beta}_1) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\beta}_{1i} - \beta_1)^2 = \frac{\sum_{i=1}^{R} v_i}{R} = \bar{v}$, where $v_i = (\hat{\beta}_{1i} - \beta_1)^2$

se_mse_t = standard error of $mse\_t = Se(\hat{MSE}(\hat{\theta})) = \sqrt{\frac{\sum_{i=1}^{R}(y_i - \bar{y})^2}{R(R-1)}}$

$se\_mse\_\beta_0$ = standard error of $mse\_\beta_0 = Se(\hat{MSE}(\hat{\beta}_0)) = \sqrt{\frac{\sum_{i=1}^{B}(z_i - \bar{z})^2}{R(R-1)}}$

$se\_mse\_\beta_1$ = standard error of $mse\_\beta_1 = Se(\hat{MSE}(\hat{\beta}_1)) = \sqrt{\frac{\sum_{i=1}^{B}(v_i - \bar{v})^2}{R(R-1)}}$

t_wid = estimated mean width of interval estimate of θ in covariate model

se_t_wid = estimated standard error for t_wid in covariate model

Scov_t = estimated coverage rate of interval estimate of θ for spearman's distance in covariate model

Table 5-1. The Performance of Point and Set Estimation of $\hat{\beta}_0, \hat{\beta}_1$ and $\hat{\theta}$ with 50 Replications

True θ	$m(\hat{\theta})$	$s(\hat{\theta})$	mse_t	se_mse_t	t_wid	se_t_wid	Scov_t	β_0	β_1	$m(\hat{\beta}_0)$	$m(\hat{\beta}_1)$	$s(\hat{\beta}_0)$	$s(\hat{\beta}_1)$	mse β_0	se_mse β_0	mse β_1	se_mse β_1
0.1	0.193	0.011	2.3E-4	2.3E-4	0.194	0.0103	1	0	0	-0.04	-0.474	0.144	0.114	2.6E-5	2.5E-5	6.1E-3	5.9E-3
0.5	0.481	0.010	7.1E-6	7E-6	0.537	0.013	0.92	0	0	-0.719	0.301	0.0301	0.0	0.0106	0.0105	4.3E-4	4.3E-4
0.9	0.659	0.009	1.3E-3	1.3E-3	0.357	0.0135	1	0	0	0.582	-0.52	0.256	0.194	7E-3	7E-3	6.1E-3	6.1E-3
0.1	0.198	0.015	2.3E-4	2.3E-4	0.205	0.0131	1	0	2	0.037	1.88	0.090	0.0553	3.2E-5	3.2E-5	3.4E-4	3.4E-4
0.5	0.488	0.013	3E-6	3E-6	0.543	0.0165	0.91	0	2	-0.16	1.44	0.440	0.127	15.8E-4	5.7E-4	7E-3	7E-3
0.9	0.681	0.008	9.9E-4	9.9E-4	0.311	0.0100	1	0	2	0.646	1.36	0.322	0.111	8.7E-3	8.7E-3	8.6E-3	8.5E-3
0.1	0.150	0.016	6.3E-5	6.3E-5	0.167	0.0162	0.95	0	4	-0.189	3.25	0.178	0.307	8.9E-4	8.9E-4	1.4E-2	1.3E-2
0.5	0.480	0.010	7.9E-6	7.9E-6	0.543	0.0136	0.92	0	4	0.22	3.43	0.334	0.0479	2.2E-3	2.2E-3	6.5E-3	6.5E-3
0.9	0.662	0.011	1,2E-3	1.23-4	0.330	0.0124	1	0	4	0.473	2.81	0.414	0.493	4.6E-3	4.6E-3	2.9E-2	2.9E-2

Table 5.1 shows the point and set estimation for $\hat{\beta}_0, \hat{\beta}_1$ and $\hat{\theta}$. The coverage rate for θ was close to the nominal level. Mean square error (mse) and standard error of the mean square error (se_mse) for the estimators $\hat{\beta}_0, \hat{\beta}_1$ and $\hat{\theta}$ were less than 0.03. The estimator $\hat{\theta}$ perform better when the range for θ is $0.5 \ge \theta \ge 0.1$. When θ=0.9, $\hat{\theta}$ underestimated θ. A multiple regression was fitted to $m(\hat{\theta})$ as dependent variables and true values of θ and β_1 as independent variables. Summaries of the fit given in Appendix 15 show that only θ has a statistically effect on $m(\hat{\theta})$. Similarly, β_1 has significant effect on $m(\hat{\beta}_1)$.

Multiple regressions were fitted to mse and se_mse β_1 as dependent variables and true values of θ and β_1 as independent variables, respectively. Summaries of the fit given in Appendix 15 show that β_1 does not have a statistically significant effect on the mse and se_mse β_1. But θ does have a statistically significant effect on the mse, and se_mse of $\hat{\theta}$.

5.2 Set Estimation for $\mathbf{w}_1$ and $\mathbf{w}_2$

The performance of confidence intervals for $\{\mathbf{w}_i\}_{i=1}^2$ in covariate models was investigated for the following cases with 50 data sets for each case. The following is
the procedure to find likelihood set for $\{\mathbf{w}_i\}_{i=1}^2$ in the covariate model in a data set:

(1). First update $\hat{\theta}$ with the initial guess for θ, then update $\{\hat{\boldsymbol{\beta}} = (\hat{\beta}_0, \hat{\beta}_1)\}$ and $\{\hat{\mathbf{w}}_i\}$,…., until the stopping rule leads to termination.

$$L^{[p]}(\{\mathbf{w}_i\}) = L(\{\mathbf{w}_i\}, \hat{\theta}\{\mathbf{w}_i\}, \hat{\boldsymbol{\beta}}\{\mathbf{w}_i\}) \qquad (5.3)$$

(2). Search through $S_{km} * S_{km} * \ldots * S_{km}$ to find $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k$ such that

$$[A] \qquad \frac{L^{[P]}\{\mathbf{w}_i, i = 1,2,\ldots,k\}}{L^{[P]}\{\hat{\mathbf{w}}_i, i = 1,2,\ldots,k\}} \geq C$$

$LS(\mathbf{w}^{\#}) = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k\}$ *such that* $[A]$ *holds* with some cutoff point C. The cutoff point C=0.1 was chosen. We only evaluated k=2 in our simulation.

<u>Assessing Performance</u>:

The estimating sets for $\{\mathbf{w}_i\}_{i=1}^{k}$ is evaluated using the following two criteria:

(1). Coverage rate Scov_t: Proportion of simulated data sets for which $LS(\mathbf{w}^{\#})$ contains the true baseline rankings.

(2) For each scenario, compute Mean relative volume: $MRV = \dfrac{\overline{V}\{LS(\mathbf{w}^{\#})\}}{\left[\dfrac{n!}{(n-k)!}\right]}$,

where $V\{LS(\mathbf{w}^{\#})\}$ = the number of rankings $\in LS(\mathbf{w}^{\#})$ and $\overline{V}\{LS(\mathbf{w}^{\#})\}$ is the average of these volumes over the simulated data sets.

SMRV= estimated mean relative V value for Spearman distance

Scov_t= estimated coverage rate for Spearman distance

In the simulation study, we took $\beta_0 = 0$, $\beta_1 = 0,\ 2,\ 4,$ $\theta = 0.1,\ 0.5,\ 0.9,$ $n = 10,$ $m = 5,$ $\mathbf{w}_1 = (1,2,3),$ $\mathbf{w}_2 = (1,3,2).$

Table 5.2 The performance of set estimation for $\{\mathbf{w}_i\}_{i=1}^{2}$ with 50 replications

True θ	True β_0	True β_1	SMRV	Scov_t
0.1	0	0	0.062	1
0.5	0	0	0.527	0.98
0.9	0	0	0.934	0.98
0.1	0	2	0.052	1
0.5	0	2	0.643	1
0.9	0	2	0.944	0.96
0.1	0	4	0.015	1
0.5	0	4	0.686	1
0.9	0	4	0.946	1

From table 5.2, it can be seen that the coverage rates for $\mathbf{w}_1$ and $\mathbf{w}_2$ were above the nominal level whether θ is small or large, indicating that the program was robust in estimating the baseline ranking. The mean relative volume increases with θ and is quite large when $\theta = 0.9$. A multiple regression and a logistic regression model were fit to the pooled simulation results with sample mean relative volume SMRV and sample mean coverage rate Scov_t for $\mathbf{w}_1$ and $\mathbf{w}_2$ as the dependent variable and true values of θ and β_1 as independent variables, respectively. Summaries of the fit given in Appendix 16 shows that only θ has a statistically significant effect on the sample mean relative volume as well as sample mean coverage rate for $\mathbf{w}_1$ and $\mathbf{w}_2$ (Appendix 16). The results in Table 5.2 suggest that the proposed method performed fairly well in the set estimation for $\mathbf{w}_1$ and $\mathbf{w}_2$. Just like set estimation using profile likelihood, θ plays an important role in both sample mean relative volume and sample mean coverage rate. It seems that covariates *x* plays an important role in the point estimates in the covariate model, as expected. The formulation in (5.1) might be of interest, for example, in cases where the rate at which consumers prefer cooked vegetables, lie in segment one, to raw ones increases with age. Then, the age where 100π% of consumers belong to segment one would be given by $\log((\pi/(1-\pi)) - \beta_0)/\beta_1$, which could be estimated by $\log((\pi/(1-\pi)) - \hat{\beta}_0)/\hat{\beta}_1$. The performance of these estimators and their standard errors obtained from applying the delta-method to variance estimates of the regression parameters obtained from inverting the information matrix will be investigated in future research. Gormley and Murphy (2008c) proposed a mixture of experts model for complete ranking data and social factors such as "age" and "government satisfaction" were found influence voting bloc in the president of Ireland election. It is critical to have a good method or algorithm to correctly analyse data. The covariate model has the potential to greatly improve the ability to model and interpret rank data, an issue that will be investigated in future research.

CHAPTER 6 – The Effect of Initial Values

In carrying out my simulations, which involved iterative estimation procedures, it is important to know what effects the initial guess for π and θ had on the estimates produced by the EM algorithm. To investigate this issue, in this chapter 20 data sets for each of the following 3 scenarios were generated using Spearman's distance function under a given set of conditions. I investigated how the results produced by the EM algorithm changed as the starting values changed. All the parameters, $\{\theta,\ \pi,\ \mathbf{w_1}, \mathbf{w}_2, n,\ m\}$ were fixed at representative chosen design points and I used the EM algorithm to estimate $\{\theta,\ \pi,\ \mathbf{w_1}, \mathbf{w}_2\}$. This simulation study is based on the following parameter settings:

π =0.5, θ=0.7, m=10, n=5, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 132$ scenario#1

π =0.1, θ=0.3, m=20, n=6, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 312$ scenario#2

π =0.3, θ=0.5, m=30, n=7, $\mathbf{w_1} = 123$, $\mathbf{w}_2 = 321$ scenario#3

A total 3*5=15 combinations (3 different π values, 5 different θ values, 20 data sets for each 3*5 combinations) for each scenario including the initial values $\pi = 0.1,\ 0.3,\ 0.5$, and $\theta = 0.1,\ 0.3,\ 0.5,\ 0.7,\ 0.9$ were investigated, totaling 15*3=45 combinations.

In the EM algorithm, initial values are not required for $\mathbf{w_1}$ and $\mathbf{w_2}$. Instead we first searched over the permutations of the first *n* integers taken three at a time to find $\mathbf{w_1}, \mathbf{w_2}$ which minimize the sum of distances and hence maximize the conditional log likelihood $Q(\xi, \xi^{\mathbf{i}})$. Let $\hat{\mathbf{w}}_1$ and $\hat{\mathbf{w}}_2$ be such values so that for all $\mathbf{w}^{\#}$, $D(\mathbf{w}^{\#}, \xi^{\mathbf{i}}) \geq D(\hat{\mathbf{w}}^{\#}, \xi^{\mathbf{i}})$.

The following statistics were computed for each of the 45 combinations:

R=number of simulations or replications.

d= true Spearman's distance from $\mathbf{w_1}$ to $\mathbf{w_2}$

d_1 = sample mean distance from $\mathbf{w_1}$ to $\hat{\mathbf{w}}_1$

sd_1 = standard error of d_1

d_2 = sample mean distance from $\mathbf{w}_2$ to $\hat{\mathbf{w}}_2$

sd_2 = standard error of d_2

$m(\hat{\theta})$ = sample mean of $\hat{\theta} = \frac{1}{R}\sum_{i=1}^{R}\hat{\theta}_i = \overline{\theta}$.

$s(\hat{\theta})$ =standard error of $m(\hat{\theta}) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\theta}_i - \overline{\theta})^2}{R(R-1)}}$

mse_t = estimated MSE of $\hat{\theta} = M\hat{S}E(\hat{\theta}) = \frac{1}{R}\sum_{i=1}^{R}(\hat{\theta}_i - \theta)^2 = \frac{\sum_{i=1}^{R} y_i}{R} = \overline{y}$, where $y_i = (\hat{\theta}_i - \theta)^2$

se_mse_t = standard error of $mse\_t = Se(M\hat{S}E(\hat{\theta})) = \sqrt{\frac{\sum_{i=1}^{R}(y_i - \overline{y})^2}{R(R-1)}}$

.

Table 6-1: The Effect of Various Initial Values θ and π on Point Estimates of $\hat{\theta}$ and $\hat{\pi}$

part	Initial π	Initial θ	m	n	D_1	D_2	True θ	$m(\hat{\theta})$	$s(\hat{\theta})$	mse_t	se_mse_t	True π	$m(\hat{\pi})$	$s(\hat{\pi})$	mse_π	se_mse_π
a	0.1	0.1	5	10	0	0	0.7	0.504	0.017	0.0019	0.0019	0.5	0.484	0.031	0.031	1.2E-05
a	0.1	0.3	5	10	0	0	0.7	0.492	0.012	0.0022	0.0021	0.5	0.406	0.036	0.036	0.0004
a	0.1	0.5	5	10	0	0	0.7	0.498	0.012	0.0019	0.0021	0.5	0.408	0.036	0.023	0.0009
a	0.1	0.7	5	10	0	0	0.7	0.522	0.011	0.0016	0.0015	0.5	0.260	0.027	0.027	0.0028
a	0.1	0.9	5	10	0	0	0.7	0.520	0.011	0.0016	0.0016	0.5	0.260	0.024	0.024	0.0029
a	0.3	0.1	5	10	0	0	0.7	0.620	0.007	0.0012	0.0016	0.5	0.466	0.034	0.024	0.0029
a	0.3	0.3	5	10	0	0	0.7	0.583	0.009	0.0014	0.0014	0.5	0.498	0.032	0.032	0.0001
a	0.3	0.5	5	10	0	0	0.7	0.586	0.009	0.0014	0.0014	0.5	0.459	0.036	0.036	0.0001
a	0.3	0.7	5	10	0	0	0.7	0.588	0.009	0.0014	0.0014	0.5	0.469	0.035	0.030	0.0001
a	0.3	0.9	5	10	0	0	0.7	0.588	0.009	0.0013	0.0014	0.5	0.469	0.035	0.030	0.0001
a	0.5	0.1	5	10	0	0	0.7	0.583	0.009	0.0013	0.0014	0.5	0.456	0.036	0.034	0.0001
a	0.5	0.3	5	10	0	0	0.7	0.583	0.009	0.0013	0.0014	0.5	0.456	0.036	0.037	0.0001
a	0.5	0.5	5	10	0	0	0.7	0.623	0.008	0.0010	0.0010	0.5	0.446	0.038	0.038	0.0001
a	0.5	0.7	5	10	0	0	0.7	0.624	0.008	0.0010	0.0010	0.5	0.459	0.036	0.033	0.0001
a	0.5	0.9	5	10	0	0	0.7	0.584	0.009	0.0013	0.0014	0.5	0.461	0.036	0.031	0.0001
a	0.1	0.1	6	20	1	0	0.3	0.231	0.017	0.0002	0.0002	0.1	0.081	0.018	0.0001	0.0001
a	0.1	0.3	6	20	1	0	0.3	0.232	0.014	0.0002	0.0002	0.1	0.136	0.042	0.0002	0.0001
a	0.1	0.5	6	20	1	0	0.3	0.233	0.014	0.0002	0.0002	0.1	0.134	0.042	0.0002	0.0001
a	0.1	0.7	6	20	1	0	0.3	0.233	0.014	0.0002	0.0002	0.1	0.133	0.042	0.0002	0.0001
a	0.1	0.9	6	20	1	0	0.3	0.233	0.014	0.0002	0.0002	0.1	0.134	0.042	0.0002	0.0001
a	0.3	0.1	6	20	1	0	0.3	0.230	0.014	0.0002	0.0002	0.1	0.099	0.036	0.0001	0.0001
a	0.3	0.3	6	20	1	0	0.3	0.231	0.014	0.0002	0.0002	0.1	0.082	0.018	0.0001	0.0001
a	0.3	0.5	6	20	1	0	0.3	0.231	0.014	0.0002	0.0002	0.1	0.081	0.018	0.0001	0.0001
a	0.3	0.7	6	20	1	0	0.3	0.231	0.014	0.0002	0.0002	0.1	0.082	0.018	0.0001	0.0001
a	0.3	0.9	6	20	1	0	0.3	0.231	0.014	0.0002	0.0002	0.1	0.082	0.018	0.0001	0.0001
a	0.5	0.1	6	20	1	0	0.3	0.230	0.014	0.0002	0.0002	0.1	0.129	0.031	0.0001	0.0001
a	0.5	0.3	6	20	1	0	0.3	0.230	0.014	0.0002	0.0002	0.1	0.065	0.029	0.0001	0.0001
a	0.5	0.5	6	20	1	0	0.3	0.235	0.013	0.0002	0.0002	0.1	0.222	0.074	0.0008	0.0008
a	0.5	0.7	6	20	1	0	0.3	0.231	0.014	0.0002	0.0002	0.1	0.272	0.093	0.0009	0.0009
a	0.5	0.9	6	20	1	0	0.3	0.231	0.014	0.0002	0.0002	0.1	0.191	0.071	0.0008	0.0008
a	0.1	0.1	7	30	0	1	0.5	0.474	0.0078	0.0001	0.0001	0.3	0.396	0.059	0.004	0.004
a	0.1	0.3	7	30	0	1	0.5	0.474	0.0080	0.0001	0.0001	0.3	0.407	0.060	0.006	0.006
a	0.1	0.5	7	30	0	1	0.5	0.477	0.0081	0.0001	0.0001	0.3	0.280	0.052	0.0001	0.0001
a	0.1	0.7	7	30	0	1	0.5	0.477	0.0080	0.0001	0.0001	0.3	0.265	0.050	0.0001	0.0001
a	0.1	0.9	7	30	0	1	0.5	0.477	0.0079	0.0001	0.0001	0.3	0.262	0.049	0.0001	0.0001
a	0.3	0.1	7	30	0	1	0.5	0.473	0.0080	0.0001	0.0001	0.3	0.375	0.059	0.003	0.003
a	0.3	0.3	7	30	0	1	0.5	0.471	0.0082	0.0001	0.0001	0.3	0.394	0.059	0.003	0.003
a	0.3	0.5	7	30	0	1	0.5	0.474	0.0079	0.0001	0.0001	0.3	0.342	0.049	0.0001	0.0001

b	0.3	0.7	7	30	0	1	0.5	0.474	0.0079	0.0001	0.0001	0.3	0.340	0.048	0.0001	0.0001
b	0.3	0.9	7	30	0	1	0.5	0.474	0.0079	0.0001	0.0001	0.3	0.338	0.047	0.0001	0.0001
b	0.5	0.1	7	30	0	1	0.5	0.473	0.0080	0.0001	0.0001	0.3	0.350	0.093	0.0003	0.003
b	0.5	0.3	7	30	0	1	0.5	0.472	0.0083	0.0001	0.0001	0.3	0.358	0.094	0.001	0.001
b	0.5	0.5	7	30	0	1	0.5	0.473	0.0080	0.0001	0.0001	0.3	0.351	0.088	0.001	0.001
b	0.5	0.7	7	30	0	1	0.5	0.473	0.0080	0.0001	0.0001	0.3	0.354	0.090	0.001	0.001
b	0.5	0.9	7	30	0	1	0.5	0.473	0.0079	0.0001	0.0001	0.3	0.391	0.058	0.004	0.004

Table 6-1 shows the point estimates for θ and π at various initial values. Multiple regressions were fitted to the data in Table 6-1 with sample mean $\hat{m}(\theta)$ and $\hat{m}(\pi)$ as dependent variables and initial values of θ, π, and true value of θ as independent variables, respectively. From Table 1 in appendix 30, it can be seen that $R^2 = 0.91$ indicated a good fit for the model. Table 3 shows that there is not enough evidence to say that initial values of θ and π had significant effect on the sample mean $\hat{m}(\theta)$ at $\alpha = 0.05$ level of significance (appendix 30). But the true value of θ played an important role in sample mean estimation for θ and π. The initial value of π did affect $\hat{m}(\pi)$. Since the initial values of π had some effect on point estimation for π, the initial values of π and θ were set at the true values in the EM algorithm thereafter.

From this simulation, it is clear that the starting values of θ and π had little effect on the point estimation for θ. But initial value of π was significantly related to $\hat{m}(\pi)$.

CHAPTER 7 – Summary and Future Study

A partial ranking is a ranking of *q* out of *m* items. Usually, one assumes a top-q ranking, i.e. subjects have ranked their *q* favorites out of a larger number of m items, Busse (2007), and that data consists of choices made by a sample selected at random form a homogeneous population of judges. The main contribution of this thesis is the development and investigation of inference based on random samples of partial rankings selected from a population of judges that fall into strata called segments. It is implicitly assumed that the segments differ in at least one important characteristic and that the judges within each segment have common traits. This thesis has concentrated on the most widely used cases, two segments and partial rankings based on the best three out of five, q=3 and m=5.

A number of studies have investigated inference based on Mallows (1953) parametric model for partial rankings in the setting of a single setting. These references include Critchlow (1985), Diaconis (1989), Kemp and Nelson (2006).To the best of our knowledge, no research has been conducted on mixture model with two segments on partial ranking using Mallows type distance models. Consumer preference models, an important example of segmentation, can be powerful tools for identifying and filling unmet customer needs. Companies that identify an underserved segment can, for example outperform the competition by developing uniquely targeted products and services, as described in Bain &Company Guide (2010). Knowledge of segment-specific parameters involved in Mallows models for partial rankings may prove valuable in making strategy decision where market segmentation exits. According to Smith (1993), a market research analyst needs reliable information as to how consumer preferences differ among the segments for a variety of products. In the Mallows model, these preferences are assessed by the baseline rankings, segment proportions, and scale like parameters among the segments. We also introduced inference for a Mallows model where segmentation membership depends on judge specific covariates through a logistic regression relationship involving unknown parameters $\{\beta_i\}$. We proposed and investigated properties of point and set estimators of $\boldsymbol{\xi} = (\theta, \boldsymbol{\pi}, \{\mathbf{w}_i\}, \{\beta_j\})$ based on Bayesian and likelihood modes of inference, both applied to mixture models and implemented using an E-M algorithm. My study used extensive simulations, many never carried out before.

In sum, the unique contributes of this study were (1) Mallows models was extended to two segment mixture models for partial rankings; (2) Bayesian and likelihood modes of inference for constructing point and set estimators were developed; (3) Logistic regression was used to model and estimate segment membership; (4) Extensive simulations were used to evaluate the performance the estimators.

In Chapter 3, we proposed point and profile likelihood set estimation. An EM algorithm was developed and used to evaluate the performance of MLE's of the parameters θ, $\boldsymbol{\pi}$, $\mathbf{w}_1$ and $\mathbf{w}_2$. In Chapter 4, we derived the posterior density for θ, π, $\mathbf{w}_1$ and $\mathbf{w}_2$ based on a uniform prior. In Chapter 5, we modeled and studied mixture models where segment membership depended on judge specific covariates.

In conclusion, my simulation study showed that the Bayesian credible regions have, in frequentist terms, close to their nominal coverage rates, except when θ is large. Moreover, across all simulation studies, θ played an very important role in the point and set estimation for $\boldsymbol{\xi}$. Kemp and Nelson (2006) pointed out that θ is measure of consistency, in modeling a judge's accuracy, a term that used to describe how well a judge does in reproducing $\mathbf{w}_0$. A small value of θ indicated a high level of consistency and a large value of θ resulted in a low level of consistency. Our simulation studies also suggest as judges get more training, fewer judges are required to reach a 95 % credible regions for the true baseline ranking for $\mathbf{w}_1$ and $\mathbf{w}_2$. If the judges are randomly guessing, more judges are needed to reach a 95% credible regions for the true baseline ranking. But how many more? Moreover, how many gain from 10 judges to 30 judges? Does training really help? Biases might not improve. For example, biases might differ among young and old judge. Some of the questions might be answered by future research.

In general, the estimation for the base line ranking was quite computationally intensively, especially for large m (number of item) and n (number of judge). Computation time has already been highlighted throughout this thesis. For the same m and n, the computation time increases as θ increased. As the number of judges *n* and number of items *m* increased, the numbers of possible rankings will quickly become very large and unmanageable in a profile likelihood set estimation for $\mathbf{w}_1$ and $\mathbf{w}_2$. Therefore, the Bayesian approach could be very beneficial in future simulation studies of estimation of $\mathbf{w}_1$ and $\mathbf{w}_2$.

Although computer simulations showed that all three methods were capable of providing good performance, Bayesian approach appears to have advantages in computational efficiency (less computation time), and ability of providing set estimation for important variable π. The choice of method, however, be application dependent so further investigation is necessary.

More computation power is needed for future research. In this study, only a limited number of scenarios were considered (q=3 and m=5 for most scenarios) due to limited computing space. According to ASTM (1979), no less than 50 responses per product should be involved in a consumer test. Carpenter et al (2000) suggested that thirty is the minimum number of assessors for a ranking test and Tomlins et al. (2003) used 100 consumers to rank consumer acceptability of locally available sweet potato. Future research can consider a broader range of simulation scenarios such as including some other values of θ, $\boldsymbol{\pi}$, large number of q, the number of items m, and the number of judges n from 50 to 100. But partial ranking for large number of items m might not be practical in most situations such as sensory evolution due to the sensory fatigues.

Another limitation of this study is that only R=20 to 100 replications were employed in our simulation study. Future research should include large replication such as R=1000. It is not known whether sample size will play a role. Our coverage rates for $\mathbf{w}_1$ and $\mathbf{w}_2$ were generally above their nominal levels and low, around 0.8, when $\theta = 0.9$. Similar conclusions were reached by Nelson and Kemp (2000) who performed a simulation study on one segment with R=1000.

Identification problems associated with MLE are another challenge associated with mixture models. Previous studies have shown the meaning of covariate in finding latent classes (Lubke and Muthén, 2007) or voting bloc membership in the president of Ireland election (Gormley and Murphy, 2008c). Therefore, in further research, the effect of covariate on identifying group membership should be taken into consideration.

CHAPTER 8 – References

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Appendices:

Appendix 1: Point and Profile likelihood Set Estimation for θ when m=5, 6, 7

Part	m	n	D_1	D_2	True π	Distance	d	d_1	sd_1	d_2	sd_2	True θ	$m(\hat{\theta})$	$s(\hat{\theta})$	mse_t	se_mse_t	t_wid	se_t_wid	cov_t
a	5	10	0	0	0.1	0	2	0.087	0.049	0.15	0.085	0.1	0.132	0.004	0	0	0.199	0.011	1
a	5	10	0	0	0.1	1	2	0	0	0	0	0.1	0.241	0.002	0	0	0.297	0.003	1
a	5	10	0	0	0.1	0	2	0.933	0.192	1.409	0.256	0.3	0.292	0.013	0	0	0.676	0.028	0.95
a	5	10	0	0	0.1	1	2	0.057	0.028	0.126	0.052	0.3	0.35	0.005	0	0	0.375	0.003	0.97
a	5	10	0	0	0.1	0	2	2.279	0.227	3.237	0.218	0.5	0.41	0.013	0	0	0.823	0.015	1
a	5	10	0	0	0.1	1	2	1.164	0.118	1.495	0.13	0.5	0.515	0.008	0	0	0.53	0.009	1
a	5	10	0	0	0.1	0	2	2.879	0.225	4.081	0.171	0.7	0.405	0.01	0.002	0.002	0.847	0.007	1
a	5	10	0	0	0.1	1	2	2.35	0.162	2.934	0.167	0.7	0.643	0.007	0	0	0.621	0.004	1
a	5	10	0	0	0.1	0	2	3.718	0.25	3.786	0.274	0.9	0.433	0.008	0.004	0.004	0.881	0.004	1
a	5	10	0	0	0.1	1	2	3.772	0.143	4.244	0.142	0.9	0.684	0.006	0	0	0.614	0.004	0.99
a	5	10	0	0	0.3	0	2	0	0	0	0	0.1	0.157	0.005	0	0	0.272	0.012	0.92
a	5	10	0	0	0.3	1	2	0	0	0	0	0.1	0.241	0.002	0	0	0.308	0.002	1
a	5	10	0	0	0.3	0	2	0.947	0.24	2.384	0.265	0.3	0.332	0.009	0	0	0.783	0.023	0.98
a	5	10	0	0	0.3	1	2	0.042	0.024	0.13	0.057	0.3	0.338	0.006	0	0	0.374	0.004	0.9
a	5	10	0	0	0.3	0	2	2.188	0.267	3.815	0.225	0.5	0.426	0.013	0	0	0.856	0.011	1
a	5	10	0	0	0.3	1	2	0.766	0.131	1.393	0.156	0.5	0.525	0.007	0	0	0.548	0.009	1
a	5	10	0	0	0.3	0	2	2.965	0.24	4.019	0.216	0.7	0.394	0.009	0.002	0.002	0.83	0.018	0.94
a	5	10	0	0	0.3	1	2	2.59	0.174	3.187	0.174	0.7	0.645	0.008	0	0	0.612	0.005	1
a	5	10	0	0	0.3	0	2	4.141	0.213	4.688	0.175	0.9	0.454	0.012	0.004	0.004	0.845	0.006	0.96
a	5	10	0	0	0.3	1	2	3.416	0.162	4.152	0.156	0.9	0.691	0.007	0	0	0.62	0.005	0.98
a	5	10	0	0	0.5	0	2	0	0	0	0	0.1	0.127	0.002	0	0	0.17	0.006	1
a	5	10	0	0	0.5	1	2	0	0	0	0	0.1	0.24	0.002	0	0	0.308	0.002	1
a	5	10	0	0	0.5	0	2	0.989	0.226	2.058	0.252	0.3	0.294	0.008	0	0	0.706	0.029	0.95
a	5	10	0	0	0.5	1	2	0.073	0.042	0.243	0.06	0.3	0.352	0.005	0	0	0.381	0.004	1
a	5	10	0	0	0.5	0	2	2.864	0.266	3.716	0.272	0.5	0.385	0.011	0	0	0.848	0.011	0.9
a	5	10	0	0	0.5	1	2	0.522	0.1	1.073	0.144	0.5	0.516	0.007	0	0	0.524	0.008	0.95
a	5	10	0	0	0.5	0	2	3.398	0.23	4.466	0.223	0.7	0.439	0.012	0.001	0.001	0.867	0.003	1
a	5	10	0	0	0.5	1	2	2.071	0.161	3.079	0.143	0.7	0.635	0.007	0	0	0.613	0.005	1
a	5	10	0	0	0.5	0	2	4.253	0.148	4.569	0.137	0.9	0.454	0.015	0.004	0.004	0.833	0.013	0.88
a	5	10	0	0	0.5	1	2	3.758	0.146	4.49	0.109	0.9	0.689	0.007	0	0	0.611	0.005	0.96
a	5	20	0	0	0.1	0	2	1.08	0.178	1.145	0.172	0.1	0.136	0.004	0	0	0.135	0.006	1
a	5	20	0	0	0.1	1	2	0	0	0	0	0.1	0.243	0.001	0	0	0.243	0.002	1
a	5	20	0	0	0.1	0	2	0.607	0.11	1.025	0.18	0.3	0.319	0.009	0	0	0.5	0.023	1
a	5	20	0	0	0.1	1	2	0.014	0.014	0.024	0.024	0.3	0.351	0.004	0	0	0.254	0.002	0.99
a	5	20	0	0	0.1	0	2	1.636	0.265	3.255	0.206	0.5	0.45	0.009	0	0	0.783	0.009	1

b	5	20	0	0	0.1	1	2	0.552	0.1	0.678	0.113	0.5	0.521	0.005	0	0	0.354	0.005	1
b	5	20	0	0	0.1	0	2	3.582	0.152	4.399	0.108	0.7	0.519	0.013	0.001	0.001	0.789	0.006	1
b	5	20	0	0	0.1	1	2	1.448	0.16	2.23	0.167	0.7	0.689	0.007	0	0	0.503	0.005	0.93
b	5	20	0	0	0.1	0	2	3.322	0.146	4.656	0.149	0.9	0.537	0.009	0.003	0.003	0.787	0.005	1
b	5	20	0	0	0.1	1	2	3.299	0.183	4.184	0.14	0.9	0.757	0.006	0	0	0.504	0.005	0.98
b	5	20	0	0	0.3	0	2	0	0	0	0	0.1	0.132	0.003	0	0	0.135	0.006	0.96
b	5	20	0	0	0.3	1	2	0	0	0	0	0.1	0.245	0.001	0	0	0.255	0.002	1
b	5	20	0	0	0.3	0	2	0.783	0.189	1.277	0.192	0.3	0.344	0.011	0	0	0.57	0.025	1
b	5	20	0	0	0.3	1	2	0	0	0	0	0.3	0.352	0.004	0	0	0.263	0.003	0.93
b	5	20	0	0	0.3	0	2	1.103	0.237	1.806	0.282	0.5	0.426	0.013	0	0	0.697	0.02	1
b	5	20	0	0	0.3	1	2	0.273	0.079	0.549	0.107	0.5	0.537	0.006	0	0	0.376	0.006	1
b	5	20	0	0	0.3	0	2	2.776	0.282	3.202	0.278	0.7	0.526	0.01	0.001	0.001	0.776	0.007	1
b	5	20	0	0	0.3	1	2	1.201	0.148	2.155	0.172	0.7	0.685	0.007	0	0	0.505	0.005	1
b	5	20	0	0	0.3	0	2	3.676	0.249	4.64	0.145	0.9	0.535	0.014	0.003	0.003	0.795	0.012	0.94
b	5	20	0	0	0.3	1	2	3.274	0.172	4.018	0.155	0.9	0.762	0.005	0	0	0.497	0.005	0.98
b	5	20	0	0	0.5	0	2	0	0	0	0	0.1	0.15	0.004	0	0	0.15	0.006	1
b	5	20	0	0	0.5	1	2	0	0	0	0	0.1	0.241	0.001	0	0	0.253	0.001	0.92
b	5	20	0	0	0.5	0	2	0.481	0.172	1.104	0.215	0.3	0.333	0.015	0	0	0.506	0.022	0.94
b	5	20	0	0	0.5	1	2	0	0	0	0	0.3	0.344	0.004	0	0	0.257	0.002	0.95
b	5	20	0	0	0.5	0	2	3.156	0.177	3.31	0.147	0.5	0.444	0.009	0	0	0.742	0.015	1
b	5	20	0	0	0.5	1	2	0.073	0.044	0.364	0.094	0.5	0.519	0.006	0	0	0.352	0.006	0.95
b	5	20	0	0	0.5	0	2	3.32	0.235	4.471	0.087	0.7	0.534	0.015	0.001	0.001	0.775	0.009	1
b	5	20	0	0	0.5	1	2	1.168	0.139	2.15	0.157	0.7	0.671	0.006	0	0	0.5	0.005	1
b	5	20	0	0	0.5	0	2	3.782	0.188	4.648	0.174	0.9	0.535	0.01	0.003	0.003	0.803	0.006	1
b	5	20	0	0	0.5	1	2	3.254	0.16	4.064	0.13	0.9	0.761	0.006	0	0	0.497	0.004	0.96
b	5	30	0	0	0.1	0	2	0.845	0.164	1.045	0.171	0.1	0.145	0.003	0	0	0.121	0.004	0.98
b	5	30	0	0	0.1	1	2	0	0	0	0	0.1	0.244	0.001	0	0	0.217	0.002	1
b	5	30	0	0	0.1	0	2	0.255	0.078	0.526	0.137	0.3	0.299	0.006	0	0	0.343	0.011	1
b	5	30	0	0	0.1	1	2	0	0	0	0	0.3	0.351	0.003	0	0	0.206	0.002	1
b	5	30	0	0	0.1	0	2	1.593	0.195	2.828	0.159	0.5	0.509	0.009	0	0	0.739	0.006	1
b	5	30	0	0	0.1	1	2	0.19	0.054	0.302	0.082	0.5	0.521	0.004	0	0	0.272	0.004	0.9
b	5	30	0	0	0.1	0	2	2.286	0.213	3.693	0.194	0.7	0.536	0.011	0.001	0.001	0.746	0.011	1
b	5	30	0	0	0.1	1	2	0.769	0.12	1.268	0.142	0.7	0.634	0.009	0	0	0.384	0.008	0.81
b	5	30	0	0	0.1	0	2	3.299	0.196	4.666	0.13	0.9	0.614	0.011	0.002	0.002	0.74	0.024	0.98
b	5	30	0	0	0.1	1	2	3.045	0.17	3.925	0.14	0.9	0.794	0.005	0	0	0.437	0.004	0.99
b	5	30	0	0	0.3	0	2	0	0	0	0	0.1	0.134	0.002	0	0	0.113	0.003	1
b	5	30	0	0	0.3	1	2	0	0	0	0	0.1	0.245	0.001	0	0	0.226	0.003	1
b	5	30	0	0	0.3	0	2	0.098	0.069	0.268	0.091	0.3	0.32	0.01	0	0	0.406	0.021	0.89
b	5	30	0	0	0.3	1	2	0	0	0	0	0.3	0.348	0.003	0	0	0.21	0.002	1
b	5	30	0	0	0.3	0	2	1.466	0.233	2.914	0.252	0.5	0.504	0.011	0	0	0.732	0.008	1
b	5	30	0	0	0.3	1	2	0.091	0.042	0.333	0.087	0.5	0.534	0.004	0	0	0.32	0.003	1
b	5	30	0	0	0.3	0	2	2.079	0.242	3.028	0.272	0.7	0.547	0.013	0	0	0.727	0.008	1
b	5	30	0	0	0.3	1	2	0.753	0.111	1.686	0.161	0.7	0.7	0.007	0	0	0.427	0.005	0.99
b	5	30	0	0	0.3	0	2	3.431	0.255	4.291	0.219	0.9	0.569	0.011	0.002	0.002	0.736	0.005	1
b	5	30	0	0	0.3	1	2	2.829	0.167	3.6	0.158	0.9	0.789	0.005	0	0	0.443	0.004	1

c	5	30	0	0	0.5	0	2	0	0	0	0	0.1	0.135	0.002	0	0	0.102	0.002	1
c	5	30	0	0	0.5	1	2	0	0	0	0	0.1	0.242	0.001	0	0	0.222	0.002	0.99
c	5	30	0	0	0.5	0	2	0.155	0.078	0.744	0.186	0.3	0.351	0.009	0	0	0.477	0.023	1
c	5	30	0	0	0.5	1	2	0.042	0.042	0.042	0.042	0.3	0.354	0.003	0	0	0.207	0.002	0.99
c	5	30	0	0	0.5	0	2	2.217	0.205	2.983	0.17	0.5	0.493	0.012	0	0	0.703	0.015	1
c	5	30	0	0	0.5	1	2	0.024	0.024	0.109	0.046	0.5	0.5	0	0	0	0.291	0.016	1
c	5	30	0	0	0.5	0	2	2.625	0.247	3.924	0.222	0.7	0.584	0.011	0	0	0.733	0.006	1
c	5	30	0	0	0.5	1	2	0.835	0.128	1.573	0.173	0.7	0.683	0.007	0	0	0.416	0.006	1
c	5	30	0	0	0.5	0	2	3.654	0.234	4.521	0.148	0.9	0.563	0.013	0.002	0.002	0.739	0.007	0.94
c	5	30	0	0	0.5	1	2	3.177	0.168	4.122	0.134	0.9	0.79	0.006	0	0	0.436	0.004	0.98
c	6	10	0	0	0.1	0	2	0	0	0	0	0.1	0.123	0.002	0	0	0.165	0.006	0.98
c	6	10	0	0	0.1	1	2	0	0	0	0	0.1	0.265	0.001	0	0	0.316	0.002	1
c	6	10	0	0	0.1	0	2	1.228	0.231	1.605	0.272	0.3	0.32	0.014	0	0	0.641	0.031	1
c	6	10	0	0	0.1	1	2	0	0	0	0	0.3	0.337	0.005	0	0	0.364	0.003	0.95
c	6	10	0	0	0.1	0	2	3.155	0.225	4.041	0.18	0.5	0.378	0.009	0	0	0.748	0.017	0.97
c	6	10	0	0	0.1	1	2	0.662	0.105	0.927	0.131	0.5	0.5	0.006	0	0	0.393	0.006	1
c	6	10	0	0	0.1	0	2	3.683	0.166	4.293	0.147	0.7	0.423	0.01	0.002	0.002	0.816	0.012	0.98
c	6	10	0	0	0.1	1	2	3.012	0.216	3.695	0.207	0.7	0.658	0.006	0	0	0.533	0.005	1
c	6	10	0	0	0.1	0	2	4.253	0.237	4.789	0.157	0.9	0.42	0.013	0.005	0.005	0.814	0.014	0.85
c	6	10	0	0	0.1	1	2	4.712	0.198	5.441	0.164	0.9	0.718	0.005	0	0	0.541	0.004	0.98
c	6	10	0	0	0.3	0	2	0	0	0	0	0.1	0.124	0.004	0	0	0.162	0.008	0.96
c	6	10	0	0	0.3	1	2	0	0	0	0	0.1	0.266	0.002	0	0	0.32	0.003	0.99
c	6	10	0	0	0.3	0	2	0.808	0.189	1.756	0.251	0.3	0.303	0.01	0	0	0.632	0.026	1
c	6	10	0	0	0.3	1	2	0.024	0.024	0.024	0.024	0.3	0.338	0.004	0	0	0.369	0.003	1
c	6	10	0	0	0.3	0	2	3.166	0.329	4.076	0.242	0.5	0.422	0.01	0	0	0.83	0.007	1
c	6	10	0	0	0.3	1	2	0.36	0.091	0.825	0.125	0.5	0.511	0.006	0	0	0.404	0.006	0.95
c	6	10	0	0	0.3	0	2	4.146	0.174	4.662	0.136	0.7	0.434	0.009	0.001	0.001	0.831	0.007	1
c	6	10	0	0	0.3	1	2	2.54	0.22	3.618	0.197	0.7	0.66	0.007	0	0	0.524	0.005	0.99
c	6	10	0	0	0.3	0	2	4.055	0.225	4.761	0.195	0.9	0.41	0.009	0.005	0.005	0.835	0.007	0.9
c	6	10	0	0	0.3	1	2	4.821	0.189	5.673	0.161	0.9	0.706	0.005	0	0	0.539	0.005	0.98
c	6	10	0	0	0.5	0	2	0.294	0.114	1.227	0.104	0.1	0.135	0.003	0	0	0.17	0.006	0.94
c	6	10	0	0	0.5	1	2	0	0	0	0	0.1	0.265	0.001	0	0	0.322	0.003	1
c	6	10	0	0	0.5	0	2	1.509	0.26	2.54	0.258	0.3	0.311	0.009	0	0	0.664	0.024	0.96
c	6	10	0	0	0.5	1	2	0.025	0.025	0.068	0.04	0.3	0.341	0.004	0	0	0.373	0.003	0.97
c	6	10	0	0	0.5	0	2	2.049	0.287	3.757	0.216	0.5	0.389	0.008	0	0	0.82	0.01	1
c	6	10	0	0	0.5	1	2	0.504	0.109	1.008	0.16	0.5	0.514	0.007	0	0	0.405	0.007	0.99
c	6	10	0	0	0.5	0	2	3.524	0.219	4.172	0.173	0.7	0.4	0.009	0.002	0.002	0.807	0.015	0.94
c	6	10	0	0	0.5	1	2	3.139	0.226	4.51	0.192	0.7	0.67	0.006	0	0	0.536	0.004	1
c	6	10	0	0	0.5	0	2	3.79	0.253	3.835	0.281	0.9	0.433	0.009	0.004	0.004	0.882	0.004	1
c	6	10	0	0	0.5	1	2	5.428	0.187	5.93	0.15	0.9	0.706	0.005	0	0	0.54	0.004	0.99
c	6	20	0	0	0.1	0	2	0	0	0	0	0.1	0.134	0.003	0	0	0.109	0.004	1
c	6	20	0	0	0.1	1	2	0	0	0	0	0.1	0.265	0.001	0	0	0.265	0.002	1
c	6	20	0	0	0.1	0	2	2.015	0.221	2.876	0.3	0.3	0.595	0.013	0.002	0.002	0.331	0.013	0.02
c	6	20	0	0	0.1	1	2	0	0	0	0	0.3	0.349	0.003	0	0	0.237	0.003	0.9
c	6	20	0	0	0.1	0	2	1.962	0.235	2.499	0.237	0.5	0.476	0.012	0	0	0.734	0.015	1

d	6	20	0	0	0.1	1	2	0.169	0.056	0.3	0.082	0.5	0.519	0.004	0	0	0.252	0.004	0.99
d	6	20	0	0	0.1	0	2	3.119	0.239	3.743	0.217	0.7	0.501	0.007	0.001	0.001	0.762	0.006	0.94
d	6	20	0	0	0.1	1	2	1.927	0.188	2.572	0.212	0.7	0.696	0.006	0	0	0.418	0.006	1
d	6	20	0	0	0.1	0	2	4.236	0.207	4.509	0.221	0.9	0.537	0.01	0.003	0.003	0.767	0.004	0.95
d	6	20	0	0	0.1	1	2	4.905	0.168	5.547	0.162	0.9	0.769	0.005	0	0	0.437	0.004	1
d	6	20	0	0	0.3	0	2	4.905	0.168	5.547	0.162	0.9	0.769	0.005	0	0	0.437	0.004	1
d	6	20	0	0	0.3	1	2	0	0	0	0	0.1	0.264	0.001	0	0	0.279	0.003	0.91
d	6	20	0	0	0.3	0	2	0.403	0.151	0.894	0.19	0.3	0.637	0.008	0.002	0.002	0.37	0.013	0.02
d	6	20	0	0	0.3	1	2	0	0	0	0	0.3	0.347	0.003	0	0	0.24	0.003	1
d	6	20	0	0	0.3	0	2	2.862	0.222	3.686	0.224	0.5	0.476	0.014	0	0	0.699	0.013	1
d	6	20	0	0	0.3	1	2	0.126	0.052	0.36	0.085	0.5	0.524	0.004	0	0	0.263	0.004	0.97
d	6	20	0	0	0.3	0	2	3.392	0.245	4.121	0.19	0.7	0.498	0.009	0.001	0.001	0.748	0.009	0.95
d	6	20	0	0	0.3	1	2	1.535	0.187	2.486	0.216	0.7	0.687	0.006	0	0	0.406	0.006	1
d	6	20	0	0	0.3	0	2	4.093	0.233	4.881	0.177	0.9	0.507	0.012	0.003	0.003	0.762	0.007	1
d	6	20	0	0	0.3	1	2	4.945	0.191	5.577	0.16	0.9	0.769	0.004	0	0	0.434	0.004	0.96
d	6	20	0	0	0.5	0	2	0	0	0	0	0.1	0.127	0.002	0	0	0.103	0.004	1
d	6	20	0	0	0.5	1	2	0	0	0	0	0.1	0.265	0.001	0	0	0.279	0.002	1
d	6	20	0	0	0.5	0	2	0.317	0.101	1.119	0.199	0.3	0.592	0.008	0.002	0.002	0.321	0.008	0.02
d	6	20	0	0	0.5	1	2	0	0	0	0	0.3	0.339	0.003	0	0	0.249	0.003	1
d	6	20	0	0	0.5	0	2	0.342	0.128	0.754	0.169	0.5	0.596	0.01	0.002	0.002	0.675	0.007	0.9
d	6	20	0	0	0.5	1	2	0.102	0.046	0.312	0.082	0.5	0.522	0.004	0	0	0.258	0.004	1
d	6	20	0	0	0.5	0	2	3.531	0.209	4.297	0.157	0.7	0.52	0.011	0.001	0.001	0.758	0.011	0.94
d	6	20	0	0	0.5	1	2	1.725	0.192	2.736	0.216	0.7	0.7	0.006	0	0	0.419	0.005	1
d	6	20	0	0	0.5	0	2	4.307	0.162	4.668	0.156	0.9	0.505	0.01	0.003	0.003	0.749	0.009	0.92
d	6	20	0	0	0.5	1	2	4.489	0.192	5.453	0.159	0.9	0.77	0.005	0	0	0.44	0.003	1
d	6	30	0	0	0.1	0	2	1.763	0.275	3.082	0.256	0.1	0.522	0.008	0.004	0.004	0.008	0.009	0
d	6	30	0	0	0.1	1	2	0	0	0	0	0.1	0.264	0.001	0	0	0.251	0.001	1
d	6	30	0	0	0.1	0	2	0.058	0.04	0.339	0.131	0.3	0.271	0.005	0	0	0.293	0.007	0.95
d	6	30	0	0	0.1	1	2	0	0	0	0	0.3	0.345	0.002	0	0	0.195	0.003	1
d	6	30	0	0	0.1	0	2	2.069	0.263	3.426	0.207	0.5	0.514	0.01	0	0	0.677	0.012	0.9
d	6	30	0	0	0.1	1	2	0	0	0.014	0.014	0.5	0.508	0.004	0	0	0.191	0.003	0.94
d	6	30	0	0	0.1	0	2	3.069	0.263	4.298	0.192	0.7	0.572	0.01	0	0	0.73	0.004	1
d	6	30	0	0	0.1	1	2	1.127	0.16	1.859	0.188	0.7	0.708	0.005	0	0	0.349	0.005	1
d	6	30	0	0	0.1	0	2	4.77	0.129	5.007	0.112	0.9	0.575	0.011	0.002	0.002	0.705	0.008	0.89
d	6	30	0	0	0.1	1	2	4.55	0.221	5.057	0.213	0.9	0.805	0.005	0	0	0.381	0.005	1
d	6	30	0	0	0.3	0	2	4.55	0.221	5.057	0.213	0.9	0.805	0.005	0	0	0.381	0.005	1
d	6	30	0	0	0.3	1	2	0	0	0	0	0.1	0.265	0.001	0	0	0.251	0.001	0.91
d	6	30	0	0	0.3	0	2	0.085	0.048	0.771	0.185	0.3	0.318	0.009	0	0	0.344	0.02	0.85
d	6	30	0	0	0.3	1	2	4.115	0.055	4.403	0.037	0.3	0.33	0.003	0.001	0.001	0.194	0.003	1
d	6	30	0	0	0.3	0	2	1.457	0.189	2.699	0.217	0.5	0.484	0.011	0	0	0.67	0.016	0.66
d	6	30	0	0	0.3	1	2	0.04	0.029	0.119	0.051	0.5	0.527	0.004	0	0	0.203	0.003	0.92
d	6	30	0	0	0.3	0	2	3.032	0.256	4.232	0.182	0.7	0.564	0.01	0	0	0.727	0.007	1
d	6	30	0	0	0.3	1	2	0.932	0.147	1.572	0.188	0.7	0.703	0.006	0	0	0.337	0.005	1
d	6	30	0	0	0.3	0	2	4.008	0.127	4.668	0.11	0.9	0.573	0.012	0.002	0.002	0.705	0.008	0.89
d	6	30	0	0	0.3	1	2	4.388	0.207	4.97	0.196	0.9	0.793	0.004	0	0	0.387	0.003	1

e	6	30	0	0	0.5	0	2	0	0	0	0	0.1	0.122	0.002	0	0	0.086	0.003	0.95
e	6	30	0	0	0.5	1	2	0	0	0	0	0.1	0.266	0.001	0	0	0.251	0.001	1
e	6	30	0	0	0.5	0	2	0	0	0.811	0.153	0.3	0.311	0.007	0	0	0.327	0.012	0.92
e	6	30	0	0	0.5	1	2	0	0	0	0	0.3	0.346	0.003	0	0	0.199	0.003	0.94
e	6	30	0	0	0.5	0	2	1.684	0.241	2.571	0.295	0.5	0.481	0.016	0	0	0.61	0.021	1
e	6	30	0	0	0.5	1	2	0.024	0.024	0.067	0.04	0.5	0.521	0.004	0	0	0.199	0.003	0.88
e	6	30	0	0	0.5	0	2	3.198	0.072	3.63	0.133	0.7	0.539	0.01	0.001	0.001	0.704	0.01	1
e	6	30	0	0	0.5	1	2	0.857	0.162	2.066	0.235	0.7	0.712	0.006	0	0	0.344	0.006	1
e	6	30	0	0	0.5	0	2	4.166	0.223	4.505	0.167	0.9	0.577	0.013	0.002	0.002	0.685	0.006	0.9
e	6	30	0	0	0.5	1	2	4.458	0.205	5.038	0.193	0.9	0.798	0.005	0	0	0.384	0.004	0.98
e	7	10	0	0	0.1	0	2	0	0	0	0	0.1	0.129	0.004	0	0	0.151	0.006	1
e	7	10	0	0	0.1	1	2	0	0	0	0	0.1	0.291	0.001	0	0	0.357	0.002	1
e	7	10	0	0	0.1	0	2	0.904	0.216	1.454	0.249	0.3	0.249	0.007	0	0	0.407	0.019	0.95
e	7	10	0	0	0.1	1	2	0	0	0	0	0.3	0.348	0.004	0	0	0.358	0.003	1
e	7	10	0	0	0.1	0	2	2.946	0.234	3.571	0.251	0.5	0.402	0.01	0	0	0.752	0.018	1
e	7	10	0	0	0.1	1	2	0.454	0.087	0.619	0.107	0.5	0.51	0.005	0	0	0.31	0.003	1
e	7	10	0	0	0.1	0	2	3.893	0.179	4.761	0.152	0.7	0.425	0.009	0.002	0.001	0.811	0.009	1
e	7	10	0	0	0.1	1	2	2.635	0.228	3.239	0.238	0.7	0.684	0.005	0	0	0.458	0.005	0.96
e	7	10	0	0	0.1	0	2	4.465	0.106	4.949	0.142	0.9	0.423	0.007	0.005	0.005	0.81	0.009	0.9
e	7	10	0	0	0.1	1	2	4.992	0.21	5.929	0.148	0.9	0.732	0.005	0	0	0.477	0.006	0.94
e	7	10	0	0	0.3	0	2	0	0	0.057	0.04	0.1	0.139	0.005	0	0	0.18	0.009	0.9
e	7	10	0	0	0.3	1	2	0.072	0.059	0.129	0.092	0.1	0.299	0.007	0	0	0.364	0.003	0.97
e	7	10	0	0	0.3	0	2	1.465	0.265	2.269	0.303	0.3	0.316	0.008	0	0	0.603	0.023	1
e	7	10	0	0	0.3	1	2	0.025	0.025	0.025	0.025	0.3	0.35	0.004	0	0	0.357	0.004	1
e	7	10	0	0	0.3	0	2	3.176	0.244	4.017	0.182	0.5	0.383	0.008	0	0	0.751	0.017	1
e	7	10	0	0	0.3	1	2	3.176	0.244	4.017	0.182	0.5	0.383	0.008	0	0	0.751	0.017	0.95
e	7	10	0	0	0.3	0	2	3.893	0.239	4.829	0.221	0.7	0.427	0.009	0.001	0.001	0.8	0.009	1
e	7	10	0	0	0.3	1	2	2.66	0.273	3.505	0.299	0.7	0.676	0.006	0	0	0.453	0.006	0.96
e	7	10	0	0	0.3	0	2	4.84	0.149	5.549	0.087	0.9	0.448	0.009	0.004	0.004	0.818	0.006	0.94
e	7	10	0	0	0.3	1	2	6.268	0.255	7.336	0.171	0.9	0.728	0.005	0	0	0.479	0.004	0.99
e	7	10	0	0	0.5	0	2	0	0	0	0	0.1	0.124	0.003	0	0	0.153	0.006	1
e	7	10	0	0	0.5	1	2	0	0	0	0	0.1	0.289	0.001	0	0	0.365	0.002	1
e	7	10	0	0	0.5	0	2	1.053	0.267	1.681	0.294	0.3	0.298	0.011	0	0	0.516	0.027	1
e	7	10	0	0	0.5	1	2	0.025	0.025	0.054	0.038	0.3	0.346	0.003	0	0	0.363	0.004	0.97
e	7	10	0	0	0.5	0	2	3.517	0.233	4.42	0.166	0.5	0.392	0.01	0	0	0.755	0.014	1
e	7	10	0	0	0.5	1	2	0.246	0.067	0.567	0.101	0.5	0.521	0.006	0	0	0.314	0.003	0.99
e	7	10	0	0	0.5	0	2	3.705	0.225	4.538	0.256	0.7	0.419	0.008	0.002	0.002	0.801	0.007	1
e	7	10	0	0	0.5	1	2	2.921	0.29	4.213	0.277	0.7	0.682	0.005	0	0	0.452	0.005	1
e	7	10	0	0	0.5	0	2	4.683	0.113	5.124	0.123	0.9	0.423	0.007	0.005	0.005	0.764	0.025	0.9
e	7	10	0	0	0.5	1	2	6.796	0.227	7.446	0.187	0.9	0.719	0.005	0	0	0.472	0.004	0.97
e	7	20	0	0	0.1	0	2	0	0	0	0	0.1	0.124	0.002	0	0	0.1	0.002	0.95
e	7	20	0	0	0.1	1	2	0	0	0	0	0.1	0.291	0.001	0	0	0.301	0.001	1
e	7	20	0	0	0.1	0	2	0.293	0.124	0.581	0.168	0.3	0.287	0.009	0	0	0.326	0.02	1
e	7	20	0	0	0.1	1	2	0.245	0.074	0.245	0.074	0.3	0.347	0.003	0	0	0.225	0.005	0.99
e	7	20	0	0	0.1	0	2	2.722	0.272	3.319	0.238	0.5	0.45	0.01	0	0	0.626	0.018	1

f	7	20	0	0	0.1	1	2	0.275	0.068	0.401	0.093	0.5	0.512	0.004	0	0	0.201	0.002	1
f	7	20	0	0	0.1	0	2	3.53	0.267	4.249	0.212	0.7	0.481	0.011	0	0	0.706	0.014	0.95
f	7	20	0	0	0.1	1	2	0.348	0.079	0.509	0.106	0.7	0.635	0.008	0	0	0.216	0.007	0.22
f	7	20	0	0	0.1	0	2	4.58	0.145	4.952	0.154	0.9	0.513	0.007	0.003	0.003	0.722	0.009	0.9
f	7	20	0	0	0.1	1	2	5.246	0.188	5.646	0.176	0.9	0.795	0.003	0	0	0.384	0.004	0.98
f	7	20	0	0	0.3	0	2	0	0	0	0	0.1	0.127	0.003	0	0	0.096	0.003	1
f	7	20	0	0	0.3	1	2	0	0	0	0	0.1	0.291	0.001	0	0	0.304	0.001	1
f	7	20	0	0	0.3	0	2	0.147	0.083	0.956	0.181	0.3	0.31	0.009	0	0	0.361	0.018	0.93
f	7	20	0	0	0.3	1	2	0	0	0	0	0.3	0.353	0.003	0	0	0.229	0.004	0.95
f	7	20	0	0	0.3	0	2	2.373	0.234	3.466	0.193	0.5	0.442	0.011	0	0	0.612	0.02	0.96
f	7	20	0	0	0.3	1	2	0	0	0.123	0.123	0.5	0.514	0.008	0	0	0.2	0.004	0.9
f	7	20	0	0	0.3	0	2	3.408	0.252	4.002	0.213	0.7	0.473	0.01	0.001	0.001	0.698	0.013	0.96
f	7	20	0	0	0.3	1	2	1.58	0.213	2.295	0.251	0.7	0.689	0.005	0	0	0.322	0.005	1
f	7	20	0	0	0.3	0	2	4.823	0.194	5.236	0.121	0.9	0.538	0.009	0.003	0.003	0.727	0.008	0.9
f	7	20	0	0	0.3	1	2	6.281	0.221	6.839	0.181	0.9	0.783	0.004	0	0	0.383	0.003	1
f	7	20	0	0	0.5	0	2	0	0	0	0	0.1	0.125	0.002	0	0	0.098	0.001	1
f	7	20	0	0	0.5	1	2	0	0	0	0	0.1	0.291	0.001	0	0	0.304	0.001	1
f	7	20	0	0	0.5	0	2	0.202	0.071	0.971	0.191	0.3	0.311	0.006	0	0	0.365	0.017	0.96
f	7	20	0	0	0.5	1	2	0	0	0	0	0.3	0.345	0.003	0	0	0.252	0.004	0.91
f	7	20	0	0	0.5	0	2	2.43	0.245	3.188	0.274	0.5	0.44	0.01	0	0	0.635	0.016	1
f	7	20	0	0	0.5	1	2	0.114	0.051	0.244	0.067	0.5	0.511	0.003	0	0	0.196	0.002	1
f	7	20	0	0	0.5	0	2	3.37	0.27	3.822	0.206	0.7	0.485	0.011	0.001	0.001	0.682	0.017	0.9
f	7	20	0	0	0.5	1	2	0.421	0.131	1.092	0.139	0.7	0.64	0.015	0.001	0.001	0.31	0.006	0.2
f	7	20	0	0	0.5	0	2	4.618	0.146	5.061	0.131	0.9	0.518	0.01	0.003	0.003	0.728	0.006	0.92
f	7	20	0	0	0.5	1	2	6.557	0.164	7.016	0.163	0.9	0.787	0.004	0	0	0.378	0.006	0.98
f	7	30	0	0	0.1	0	2	0	0	0	0	0.1	0.125	0.002	0	0	0.078	0.004	1
f	7	30	0	0	0.1	1	2	0	0	0	0	0.1	0.291	0.001	0	0	0.278	0.003	1
f	7	30	0	0	0.1	0	2	0.17	0.066	0.464	0.142	0.3	0.304	0.006	0	0	0.28	0.012	1
f	7	30	0	0	0.1	1	2	0.23	0.092	0.268	0.107	0.3	0.362	0.005	0	0	0.19	0.004	0.94
f	7	30	0	0	0.1	0	2	3.513	0.285	4.297	0.211	0.5	0.527	0.011	0	0	0.655	0.012	0.98
f	7	30	0	0	0.1	1	2	0.091	0.042	0.165	0.058	0.5	0.51	0.003	0	0	0.145	0.002	0.95
f	7	30	0	0	0.1	0	2	3.965	0.235	4.798	0.168	0.7	0.557	0.009	0	0	0.666	0.007	1
f	7	30	0	0	0.1	1	2	0.724	0.184	0.836	0.173	0.7	0.721	0.004	0	0	0.178	0.003	1
f	7	30	0	0	0.1	0	2	4.864	0.166	5.359	0.191	0.9	0.586	0.011	0.002	0.002	0.675	0.008	0.94
f	7	30	0	0	0.1	1	2	4.443	0.227	4.92	0.21	0.9	0.725	0.019	0.002	0.002	0.369	0.014	0.9
f	7	30	0	0	0.3	0	2	0	0	0	0	0.1	0.128	0.002	0	0	0.072	0.004	1
f	7	30	0	0	0.3	1	2	0	0	0	0	0.1	0.291	0.001	0	0	0.287	0.002	0.95
f	7	30	0	0	0.3	0	2	0.118	0.057	0.793	0.19	0.3	0.329	0.009	0	0	0.309	0.017	1
f	7	30	0	0	0.3	1	2	0	0	0	0	0.3	0.351	0.002	0	0	0.188	0.004	1
f	7	30	0	0	0.3	0	2	2.167	0.255	2.998	0.264	0.5	0.492	0.009	0	0	0.616	0.013	1
f	7	30	0	0	0.3	1	2	0.075	0.043	0.147	0.056	0.5	0.52	0.003	0	0	0.154	0.002	0.98
f	7	30	0	0	0.3	0	2	3.663	0.187	4.299	0.154	0.7	0.54	0.01	0.001	0.001	0.662	0.01	1
f	7	30	0	0	0.3	1	2	1.202	0.186	1.612	0.223	0.7	0.709	0.005	0	0	0.273	0.005	1
f	7	30	0	0	0.3	0	2	4.972	0.118	5.354	0.108	0.9	0.559	0.011	0.002	0.002	0.68	0.008	0.84
f	7	30	0	0	0.3	1	2	5.86	0.256	6.309	0.237	0.9	0.809	0.004	0	0	0.332	0.003	0.91

g	7	30	0	0	0.5	0	2	0	0	0	0	0.1	0.123	0.002	0	0	0.079	0.004	1
g	7	30	0	0	0.5	1	2	0	0	0	0	0.1	0.294	0.003	0	0	0.287	0.003	0.98
g	7	30	0	0	0.5	0	2	0.338	0.1	1.114	0.187	0.3	0.311	0.007	0	0	0.281	0.013	1
g	7	30	0	0	0.5	1	2	0	0	0	0	0.3	0.356	0.004	0	0	0.206	0.004	0.98
g	7	30	0	0	0.5	0	2	1.478	0.207	2.173	0.266	0.5	0.469	0.01	0	0	0.58	0.016	0.97
g	7	30	0	0	0.5	1	2	0	0	0.042	0.024	0.5	0.501	0.003	0	0	0.144	0.002	1
g	7	30	0	0	0.5	0	2	3.748	0.226	4.901	0.22	0.7	0.558	0.013	0	0	0.647	0.018	0.88
g	7	30	0	0	0.5	1	2	1.139	0.17	2.075	0.226	0.7	0.709	0.004	0	0	0.268	0.004	1
g	7	30	0	0	0.5	0	2	4.775	0.128	5.266	0.079	0.9	0.584	0.009	0.002	0.002	0.691	0.006	0.94
g	7	30	0	0	0.5	1	2	6.557	0.163	7.016	0.161	0.9	0.807	0.005	0	0	0.337	0.002	0.97
g	5	10	1	0	0.1	0	6	1.715	0.16	1.075	0.086	0.1	0.149	0.007	0	0	0.235	0.027	1
g	5	10	1	0	0.1	1	6	2.278	0.063	1.315	0.036	0.1	0.262	0.003	0	0	0.307	0.004	1
g	5	10	1	0	0.1	0	6	2.04	0.192	1.69	0.222	0.3	0.312	0.013	0	0	0.647	0.03	1
g	5	10	1	0	0.1	1	6	2.259	0.073	1.269	0.046	0.3	0.356	0.005	0	0	0.37	0.004	1
g	5	10	1	0	0.1	0	6	2.554	0.314	3.538	0.244	0.5	0.398	0.017	0	0	0.794	0.025	0.95
g	5	10	1	0	0.1	1	6	1.974	0.122	1.368	0.09	0.5	0.509	0.008	0	0	0.535	0.009	0.99
g	5	10	1	0	0.1	0	6	3.403	0.218	4.366	0.205	0.7	0.454	0.011	0.001	0.001	0.861	0.003	1
g	5	10	1	0	0.1	1	6	2.578	0.164	3.097	0.165	0.7	0.643	0.008	0	0	0.626	0.005	1
g	5	10	1	0	0.1	0	6	3.816	0.216	4.632	0.133	0.9	0.406	0.01	0.005	0.005	0.877	0.004	1
g	5	10	1	0	0.1	1	6	3.513	0.167	4.196	0.148	0.9	0.677	0.007	0	0	0.617	0.005	0.98
g	5	10	1	0	0.3	0	6	0.833	0.166	0.566	0.099	0.1	0.133	0.003	0	0	0.173	0.006	1
g	5	10	1	0	0.3	1	6	1.188	0.124	0.829	0.07	0.1	0.302	0.002	0	0	0.349	0.002	0.98
g	5	10	1	0	0.3	0	6	1.024	0.193	1.231	0.181	0.3	0.279	0.009	0	0	0.639	0.028	0.97
g	5	10	1	0	0.3	1	6	1.311	0.123	0.871	0.069	0.3	0.343	0.004	0	0	0.372	0.003	0.91
g	5	10	1	0	0.3	0	6	2.831	0.217	3.156	0.206	0.5	0.405	0.011	0	0	0.857	0.008	1
g	5	10	1	0	0.3	1	6	1.268	0.129	1.246	0.088	0.5	0.484	0.007	0	0	0.501	0.008	0.99
g	5	10	1	0	0.3	0	6	2.988	0.285	4.087	0.216	0.7	0.391	0.011	0.002	0.002	0.849	0.009	1
g	5	10	1	0	0.3	1	6	2.309	0.157	2.624	0.154	0.7	0.635	0.009	0	0	0.609	0.005	0.98
g	5	10	1	0	0.3	0	6	3.964	0.227	4.585	0.182	0.9	0.456	0.011	0.004	0.004	0.872	0.004	1
g	5	10	1	0	0.3	1	6	3.547	0.163	4.144	0.149	0.7	0.695	0.006	0	0	0.621	0.005	1
g	5	10	1	0	0.5	0	6	0	0	1.329	0.048	0.1	0.145	0.005	0	0	0.233	0.013	0.94
g	5	10	1	0	0.5	1	6	0.171	0.063	0.608	0.07	0.1	0.305	0.002	0	0	0.354	0.002	0.94
g	5	10	1	0	0.5	0	6	1.068	0.188	2.284	0.17	0.3	0.3	0.01	0	0	0.712	0.028	1
g	5	10	1	0	0.5	1	6	0.294	0.08	0.735	0.071	0.3	0.36	0.004	0	0	0.389	0.003	1
g	5	10	1	0	0.5	0	6	2.326	0.252	3.81	0.188	0.5	0.406	0.013	0	0	0.845	0.01	1
g	5	10	1	0	0.5	1	6	0.954	0.128	1.71	0.103	0.5	0.507	0.007	0	0	0.521	0.009	0.84
g	5	10	1	0	0.5	0	6	3.617	0.212	4.569	0.174	0.7	0.461	0.01	0.001	0.001	0.873	0.004	1
g	5	10	1	0	0.5	1	6	2.371	0.167	3.278	0.151	0.7	0.63	0.007	0	0	0.613	0.005	0.99
g	5	10	1	0	0.5	0	6	4.16	0.224	4.706	0.215	0.9	0.418	0.011	0.005	0.005	0.85	0.01	0.9
g	5	10	1	0	0.5	1	6	3.624	0.16	4.399	0.118	0.9	0.693	0.007	0	0	0.614	0.004	0.98
g	5	20	1	0	0.1	0	6	2.009	0.134	1.201	0.085	0.1	0.142	0.004	0	0	0.129	0.005	0.99
g	5	20	1	0	0.1	1	6	2.4	0.035	1.386	0.02	0.1	0.263	0.002	0	0	0.244	0.002	0.97
g	5	20	1	0	0.1	0	6	2.182	0.162	1.279	0.143	0.3	0.346	0.011	0	0	0.577	0.022	1
g	5	20	1	0	0.1	1	6	2.425	0.024	1.41	0.018	0.3	0.353	0.004	0	0	0.248	0.003	1
g	5	20	1	0	0.1	0	6	1.636	0.265	3.515	0.169	0.5	0.45	0.009	0	0	0.783	0.009	1

h	5	20	1	0	0.1	1	6	1.961	0.104	1.242	0.069	0.5	0.527	0.006	0	0	0.379	0.006	1
h	5	20	1	0	0.1	0	6	2.653	0.223	3.509	0.192	0.7	0.503	0.01	0.001	0.001	0.791	0.008	0.97
h	5	20	1	0	0.1	1	6	2.375	0.158	2.621	0.161	0.7	0.707	0.007	0	0	0.513	0.005	1
h	5	20	1	0	0.1	0	6	3.635	0.219	4.585	0.152	0.9	0.569	0.012	0.002	0.002	0.798	0.005	1
h	5	20	1	0	0.1	1	6	3.632	0.16	4.246	0.135	0.9	0.75	0.005	0	0	0.497	0.004	0.92
h	5	20	1	0	0.3	0	6	0.49	0.14	0.283	0.081	0.1	0.14	0.003	0	0	0.121	0.007	1
h	5	20	1	0	0.3	1	6	1.125	0.124	0.649	0.072	0.1	0.304	0.002	0	0	0.278	0.003	0.89
h	5	20	1	0	0.3	0	6	0.931	0.17	1.046	0.156	0.3	0.283	0.006	0	0	0.405	0.019	0.94
h	5	20	1	0	0.3	1	6	1.364	0.124	0.802	0.072	0.3	0.354	0.003	0	0	0.278	0.004	0.94
h	5	20	1	0	0.3	0	6	1.364	0.124	0.802	0.072	0.3	0.354	0.003	0	0	0.728	0.004	0.99
h	5	20	1	0	0.3	1	6	1.014	0.122	0.881	0.078	0.5	0.519	0.005	0	0	0.357	0.006	1
h	5	20	1	0	0.3	0	6	2.192	0.246	3.6	0.268	0.7	0.493	0.009	0.001	0.001	0.797	0.006	1
h	5	20	1	0	0.3	1	6	2.044	0.153	2.305	0.124	0.7	0.686	0.006	0	0	0.501	0.005	1
h	5	20	1	0	0.3	0	6	3.928	0.234	4.356	0.153	0.9	0.531	0.011	0.003	0.003	0.794	0.006	0.98
h	5	20	1	0	0.3	1	6	3.236	0.166	4.106	0.144	0.9	0.761	0.006	0	0	0.511	0.005	0.91
h	5	20	1	0	0.5	0	6	0	0	0.141	0.061	0.1	0.139	0.002	0	0	0.12	0.003	1
h	5	20	1	0	0.5	1	6	0.073	0.042	0.354	0.062	0.1	0.31	0.001	0	0	0.289	0.002	0.83
h	5	20	1	0	0.5	0	6	0.391	0.154	1.563	0.137	0.3	0.31	0.008	0	0	0.502	0.023	1
h	5	20	1	0	0.5	1	6	0.099	0.049	0.786	0.071	0.3	0.359	0.003	0	0	0.281	0.004	0.91
h	5	20	1	0	0.5	0	6	0.776	0.214	1.707	0.111	0.5	0.315	0.013	0	0	0.76	0.025	0.97
h	5	20	1	0	0.5	1	6	0.515	0.097	1.443	0.06	0.5	0.522	0.006	0	0	0.357	0.006	0.9
h	5	20	1	0	0.5	0	6	3.128	0.218	3.912	0.168	0.7	0.505	0.011	0.001	0.001	0.778	0.007	1
h	5	20	1	0	0.5	1	6	1.345	0.153	2.825	0.14	0.7	0.689	0.006	0	0	0.502	0.005	1
h	5	20	1	0	0.5	0	6	3.382	0.229	4.508	0.171	0.9	0.502	0.012	0.003	0.003	0.793	0.008	0.94
h	5	20	1	0	0.5	1	6	3.652	0.163	4.439	0.121	0.9	0.758	0.006	0	0	0.501	0.005	0.98
h	5	30	1	0	0.1	0	6	2.352	0.069	1.358	0.04	0.1	0.133	0.002	0	0	0.099	0.003	1
h	5	30	1	0	0.1	1	6	2.425	0.025	1.4	0.014	0.1	0.263	0.002	0	0	0.22	0.003	0.97
h	5	30	1	0	0.1	0	6	1.555	0.163	1.08	0.102	0.3	0.339	0.009	0	0	0.387	0.026	1
h	5	30	1	0	0.1	1	6	2.449	0	1.414	0	0.3	0.356	0.003	0	0	0.197	0.002	0.96
h	5	30	1	0	0.1	0	6	2.123	0.203	1.573	0.188	0.5	0.495	0.012	0	0	0.704	0.016	1
h	5	30	1	0	0.1	1	6	2.019	0.096	1.313	0.058	0.5	0.53	0.005	0	0	0.295	0.005	1
h	5	30	1	0	0.1	0	6	3.286	0.23	3.754	0.223	0.7	0.557	0.01	0	0	0.753	0.004	1
h	5	30	1	0	0.1	1	6	2.278	0.13	2.042	0.139	0.7	0.71	0.006	0	0	0.438	0.005	1
h	5	30	1	0	0.1	0	6	4.536	0.153	4.853	0.202	0.9	0.624	0.008	0.002	0.002	0.732	0.005	1
h	5	30	1	0	0.1	1	6	3.499	0.151	3.987	0.131	0.9	0.788	0.005	0	0	0.431	0.004	0.97
h	5	30	1	0	0.3	0	6	0.637	0.153	0.368	0.089	0.1	0.139	0.002	0	0	0.101	0.003	1
h	5	30	1	0	0.3	1	6	0.965	0.121	0.557	0.07	0.1	0.306	0.001	0	0	0.255	0.004	0.78
h	5	30	1	0	0.3	0	6	0.784	0.163	0.725	0.13	0.3	0.29	0.008	0	0	0.319	0.017	0.98
h	5	30	1	0	0.3	1	6	1.435	0.122	0.843	0.07	0.3	0.353	0.002	0	0	0.238	0.006	1
h	5	30	1	0	0.3	0	6	2.154	0.205	2.39	0.267	0.5	0.478	0.012	0	0	0.669	0.017	1
h	5	30	1	0	0.3	1	6	1.25	0.126	0.909	0.071	0.5	0.503	0.005	0	0	0.305	0.003	0.97
h	5	30	1	0	0.3	0	6	3.196	0.225	3.497	0.26	0.7	0.57	0.011	0	0	0.727	0.007	1
h	5	30	1	0	0.3	1	6	1.973	0.143	2.002	0.125	0.7	0.699	0.007	0	0	0.425	0.005	0.98
h	5	30	1	0	0.3	0	6	3.873	0.166	4.095	0.142	0.9	0.571	0.012	0.002	0.002	0.74	0.007	0.94
h	5	30	1	0	0.3	1	6	3.136	0.172	3.771	0.164	0.9	0.788	0.005	0	0	0.443	0.004	1

i	5	30	1	0	0.5	0	6	0	0	0.368	0.089	0.1	0.139	0.002	0	0	0.105	0.003	1
i	5	30	1	0	0.5	1	6	0.024	0.024	0.255	0.055	0.1	0.311	0.001	0	0	0.272	0.004	0.84
i	5	30	1	0	0.5	0	6	0.098	0.069	1.216	0.07	0.3	0.298	0.005	0	0	0.356	0.015	0.9
i	5	30	1	0	0.5	1	6	0.073	0.042	0.82	0.07	0.3	0.357	0.002	0	0	0.237	0.005	1
i	5	30	1	0	0.5	0	6	1.779	0.26	3.341	0.194	0.5	0.51	0.01	0	0	0.724	0.012	1
i	5	30	1	0	0.5	1	6	0.515	0.097	1.443	0.06	0.5	0.529	0.005	0	0	0.357	0.006	0.93
i	5	30	1	0	0.5	0	6	3.211	0.236	4.339	0.138	0.7	0.567	0.01	0	0	0.739	0.007	0.98
i	5	30	1	0	0.5	1	6	1.239	0.154	2.579	0.128	0.7	0.706	0.006	0	0	0.434	0.005	0.91
i	5	30	1	0	0.5	0	6	3.175	0.131	4.059	0.171	0.9	0.618	0.012	0.002	0.002	0.742	0.007	1
i	5	30	1	0	0.5	1	6	3.301	0.17	4.195	0.138	0.9	0.797	0.005	0	0	0.434	0.004	1
i	6	10	1	0	0.1	0	6	2.505	0.105	2.346	0.044	0.1	0.141	0.004	0	0	0.173	0.009	0.95
i	6	10	1	0	0.1	1	6	2.352	0.048	1.358	0.028	0.1	0.283	0.002	0	0	0.329	0.003	1
i	6	10	1	0	0.1	0	6	2.243	0.198	2.104	0.194	0.3	0.293	0.007	0	0	0.594	0.022	0.93
i	6	10	1	0	0.1	1	6	2.103	0.086	1.214	0.05	0.3	0.346	0.004	0	0	0.357	0.003	0.9
i	6	10	1	0	0.1	0	6	3.131	0.229	3.968	0.231	0.5	0.395	0.007	0	0	0.783	0.005	1
i	6	10	1	0	0.1	1	6	2.105	0.118	1.506	0.101	0.5	0.508	0.006	0	0	0.406	0.006	0.99
i	6	10	1	0	0.1	0	6	4.22	0.166	4.573	0.177	0.7	0.404	0.008	0.002	0.002	0.832	0.008	1
i	6	10	1	0	0.1	1	6	2.96	0.227	3.212	0.232	0.7	0.651	0.007	0	0	0.526	0.005	0.97
i	6	10	1	0	0.1	0	6	3.836	0.185	4.504	0.171	0.9	0.403	0.011	0.005	0.005	0.808	0.016	0.78
i	6	10	1	0	0.1	1	6	5.02	0.195	5.284	0.172	0.9	0.702	0.005	0	0	0.544	0.004	0.99
i	6	10	1	0	0.3	0	6	0.441	0.134	0.311	0.084	0.1	0.135	0.004	0	0	0.164	0.008	1
i	6	10	1	0	0.3	1	6	1.151	0.123	0.693	0.071	0.1	0.317	0.002	0	0	0.346	0.002	0.96
i	6	10	1	0	0.3	0	6	1.7	0.232	1.883	0.23	0.3	0.268	0.009	0	0	0.512	0.029	0.93
i	6	10	1	0	0.3	1	6	1.37	0.127	0.821	0.073	0.3	0.349	0.004	0	0	0.362	0.003	1
i	6	10	1	0	0.3	0	6	2.964	0.259	3.492	0.244	0.5	0.39	0.01	0	0	0.786	0.023	0.94
i	6	10	1	0	0.3	1	6	1.117	0.137	1.075	0.093	0.5	0.504	0.006	0	0	0.399	0.006	1
i	6	10	1	0	0.3	0	6	3.968	0.16	4.288	0.193	0.7	0.434	0.008	0.001	0.001	0.85	0.004	0.95
i	6	10	1	0	0.3	1	6	3.001	0.227	3.734	0.194	0.7	0.659	0.006	0	0	0.527	0.005	0.99
i	6	10	1	0	0.3	0	6	4.643	0.112	4.877	0.178	0.9	0.438	0.011	0.004	0.004	0.821	0.011	0.93
i	6	10	1	0	0.3	1	6	5.133	0.169	5.644	0.171	0.9	0.708	0.006	0	0	0.542	0.004	0.98
i	6	10	1	0	0.5	0	6	1.429	0.24	1.602	0.18	0.1	0.133	0.002	0	0	0.164	0.005	0.95
i	6	10	1	0	0.5	1	6	0.294	0.08	0.735	0.071	0.1	0.324	0.001	0.001	0	0.349	0.002	0.93
i	6	10	1	0	0.5	0	6	0.745	0.163	2.781	0.185	0.3	0.321	0.009	0	0	0.681	0.025	1
i	6	10	1	0	0.5	1	6	0.45	0.096	0.851	0.07	0.3	0.355	0.003	0	0	0.374	0.003	1
i	6	10	1	0	0.5	0	6	2.623	0.271	3.893	0.227	0.5	0.391	0.01	0	0	0.789	0.015	1
i	6	10	1	0	0.5	1	6	0.75	0.12	1.455	0.078	0.5	0.515	0.006	0	0	0.395	0.006	0.99
i	6	10	1	0	0.5	0	6	3.605	0.22	4.545	0.142	0.7	0.435	0.011	0.001	0.001	0.825	0.008	0.96
i	6	10	1	0	0.5	1	6	2.93	0.215	4.131	0.186	0.7	0.67	0.006	0	0	0.527	0.004	1
i	6	10	1	0	0.5	0	6	3.798	0.163	4.225	0.2	0.9	0.444	0.009	0.004	0.004	0.84	0.006	0.98
i	6	10	1	0	0.5	1	6	4.998	0.186	5.839	0.152	0.9	0.718	0.005	0	0	0.537	0.004	0.97
i	6	20	1	0	0.1	0	6	2.449	0	1.414	0	0.1	0.132	0.004	0	0	0.107	0.005	0.92
i	6	20	1	0	0.1	1	6	2.449	0	1.414	0	0.1	0.282	0.002	0	0	0.267	0.002	1
i	6	20	1	0	0.1	0	6	1.555	0.163	1.418	0.085	0.3	0.36	0.009	0.002	0.002	0.33	0.007	0.02
i	6	20	1	0	0.1	1	6	2.352	0.048	1.372	0.024	0.3	0.348	0.003	0	0	0.236	0.003	1
i	6	20	1	0	0.1	0	6	1.592	0.209	2.723	0.215	0.5	0.628	0.02	0	0	0.756	0.006	1

j	6	20	1	0	0.1	1	6	2.033	0.092	1.223	0.056	0.5	0.505	0.004	0	0	0.258	0.003	0.9
j	6	20	1	0	0.1	0	6	3.436	0.219	4.084	0.216	0.7	0.495	0.012	0.001	0.001	0.752	0.007	1
j	6	20	1	0	0.1	1	6	4.324	0.123	4.057	0.147	0.7	0.69	0.006	0	0	0.412	0.006	0.98
j	6	20	1	0	0.1	0	6	4.11	0.161	4.568	0.148	0.9	0.53	0.011	0.003	0.003	0.741	0.006	0.92
j	6	20	1	0	0.1	1	6	4.913	0.189	5.364	0.166	0.9	0.764	0.005	0	0	0.432	0.004	0.98
j	6	20	1	0	0.3	0	6	0.588	0.149	0.339	0.086	0.1	0.13	0.002	0	0	0.1	0.004	1
j	6	20	1	0	0.3	1	6	1.089	0.123	0.629	0.071	0.1	0.323	0.001	0.001	0	0.282	0.003	0.81
j	6	20	1	0	0.3	0	6	0.539	0.145	0.651	0.101	0.3	0.364	0.009	0.001	0.001	0.297	0.005	0.02
j	6	20	1	0	0.3	1	6	1.445	0.121	0.863	0.069	0.3	0.348	0.003	0	0	0.267	0.005	0.99
j	6	20	1	0	0.3	0	6	1.517	0.253	2.932	0.208	0.5	0.46	0.011	0	0	0.71	0.013	1
j	6	20	1	0	0.3	1	6	0.975	0.122	0.765	0.072	0.5	0.493	0.004	0	0	0.251	0.003	1
j	6	20	1	0	0.3	0	6	2.963	0.226	4.219	0.202	0.7	0.473	0.011	0.001	0.001	0.738	0.015	0.94
j	6	20	1	0	0.3	1	6	2.339	0.169	2.78	0.173	0.7	0.691	0.006	0	0	0.408	0.006	0.9
j	6	20	1	0	0.3	0	6	4.368	0.159	4.902	0.176	0.9	0.507	0.013	0.003	0.003	0.753	0.008	0.89
j	6	20	1	0	0.3	1	6	4.495	0.196	5.056	0.179	0.9	0.759	0.005	0	0	0.436	0.004	0.98
j	6	20	1	0	0.5	0	6	0	0	0.226	0.074	0.1	0.128	0.002	0	0	0.1	0.003	1
j	6	20	1	0	0.5	1	6	0.049	0.034	0.339	0.061	0.1	0.325	0.001	0.001	0.001	0.293	0.002	0.84
j	6	20	1	0	0.5	0	6	1.525	0.229	2.936	0.145	0.3	0.392	0.008	0.002	0.002	0.321	0.008	0.02
j	6	20	1	0	0.5	1	6	0.148	0.059	0.543	0.069	0.3	0.355	0.002	0	0	0.282	0.005	1
j	6	20	1	0	0.5	0	6	1.905	0.254	3.395	0.187	0.5	0.443	0.01	0	0	0.7	0.016	0.93
j	6	20	1	0	0.5	1	6	0.385	0.089	1.286	0.041	0.5	0.515	0.004	0	0	0.247	0.004	0.9
j	6	20	1	0	0.5	0	6	3.61	0.201	4.32	0.22	0.7	0.512	0.01	0.001	0.001	0.749	0.009	0.95
j	6	20	1	0	0.5	1	6	1.887	0.206	3.374	0.192	0.7	0.71	0.006	0	0	0.416	0.005	1
j	6	20	1	0	0.5	0	6	4.167	0.192	4.897	0.144	0.9	0.508	0.008	0.003	0.003	0.771	0.006	0.98
j	6	20	1	0	0.5	1	6	4.991	0.202	5.739	0.177	0.9	0.772	0.006	0	0	0.435	0.004	0.98
j	6	30	1	0	0.1	0	6	2.352	0.069	1.358	0.04	0.1	0.13	0.003	0	0	0.086	0.004	0.98
j	6	30	1	0	0.1	1	6	2.449	0	1.414	0	0.1	0.283	0.001	0	0	0.242	0.002	0.95
j	6	30	1	0	0.1	0	6	0.98	0.171	0.707	0.101	0.3	0.294	0.005	0	0	0.296	0.012	0.9
j	6	30	1	0	0.1	1	6	2.449	0	1.414	0	0.3	0.35	0.002	0	0	0.191	0.002	1
j	6	30	1	0	0.1	0	6	2.025	0.255	2.64	0.204	0.5	0.492	0.008	0	0	0.68	0.009	1
j	6	30	1	0	0.1	1	6	2.177	0.078	1.257	0.045	0.5	0.507	0.004	0	0	0.202	0.002	0.97
j	6	30	1	0	0.1	0	6	3.25	0.213	3.769	0.239	0.7	0.572	0.01	0	0	0.71	0.004	0.95
j	6	30	1	0	0.1	1	6	2.321	0.165	2.344	0.158	0.7	0.711	0.005	0	0	0.347	0.005	1
j	6	30	1	0	0.1	0	6	4.498	0.176	4.8	0.153	0.9	0.59	0.011	0.002	0.002	0.716	0.005	1
j	6	30	1	0	0.1	1	6	6.11	0.529	5.917	0.566	0.9	0.798	0.01	0.001	0.001	0.388	0.008	0.95
j	6	30	1	0	0.3	0	6	0.147	0.083	0.085	0.048	0.1	0.129	0.002	0	0	0.086	0.003	0.92
j	6	30	1	0	0.3	1	6	1.2	0.123	0.707	0.071	0.1	0.325	0.001	0.001	0.001	0.253	0.005	0.64
j	6	30	1	0	0.3	0	6	0.784	0.163	0.877	0.098	0.3	0.298	0.007	0	0	0.296	0.015	0.96
j	6	30	1	0	0.3	1	6	1.396	0.122	0.806	0.07	0.3	0.354	0.002	0	0	0.234	0.006	0.95
j	6	30	1	0	0.3	0	6	1.457	0.189	3.16	0.192	0.5	0.484	0.011	0	0	0.65	0.016	0.66
j	6	30	1	0	0.3	1	6	1.029	0.122	0.764	0.071	0.5	0.503	0.004	0	0	0.2	0.002	0.97
j	6	30	1	0	0.3	0	6	3.051	0.142	4.112	0.173	0.7	0.553	0.011	0	0	0.706	0.011	1
j	6	30	1	0	0.3	1	6	1.806	0.169	2.373	0.177	0.7	0.71	0.006	0	0	0.346	0.006	1
j	6	30	1	0	0.3	0	6	4.287	0.222	4.931	0.084	0.9	0.584	0.013	0.002	0.002	0.731	0.054	0.04
j	6	30	1	0	0.3	1	6	4.418	0.204	4.949	0.175	0.9	0.801	0.005	0	0	0.383	0.004	0.97

k	6	30	1	0	0.5	0	6	0	0	0.566	0.099	0.1	0.126	0.002	0	0	0.092	0.004	1
k	6	30	1	0	0.5	1	6	0	0	0.211	0.052	0.1	0.328	0.001	0.001	0.001	0.285	0.004	0.85
k	6	30	1	0	0.5	0	6	0.845	0.164	2.711	0.155	0.3	0.304	0.006	0	0	0.315	0.011	0.91
k	6	30	1	0	0.5	1	6	0.024	0.024	0.481	0.067	0.3	0.36	0.002	0	0	0.256	0.006	0.94
k	6	30	1	0	0.5	0	6	1.382	0.234	3.383	0.2	0.5	0.477	0.01	0	0	0.639	0.015	1
k	6	30	1	0	0.5	1	6	0.161	0.06	1.216	0.049	0.5	0.518	0.003	0	0	0.199	0.002	1
k	6	30	1	0	0.5	0	6	3.112	0.081	3.733	0.156	0.7	0.53	0.009	0.001	0.001	0.702	0.008	0.94
k	6	30	1	0	0.5	1	6	1.323	0.162	3.256	0.187	0.7	0.717	0.006	0	0	0.348	0.006	0.98
k	6	30	1	0	0.5	0	6	3.752	0.212	4.59	0.133	0.9	0.563	0.011	0.002	0.002	0.709	0.003	0.95
k	6	30	1	0	0.5	1	6	4.347	0.205	5.333	0.176	0.9	0.799	0.005	0	0	0.387	0.004	1
k	7	10	1	0	0.1	0	6	1.911	0.145	1.103	0.084	0.1	0.123	0.004	0	0	0.138	0.007	0.95
k	7	10	1	0	0.1	1	6	2.376	0.042	1.372	0.024	0.1	0.309	0.002	0	0	0.359	0.002	0.98
k	7	10	1	0	0.1	0	6	1.308	0.239	0.966	0.18	0.3	0.3	0.007	0	0	0.548	0.023	1
k	7	10	1	0	0.1	1	6	2.325	0.055	1.342	0.032	0.3	0.35	0.004	0	0	0.35	0.004	1
k	7	10	1	0	0.1	0	6	3.069	0.219	4.435	0.195	0.5	0.378	0.009	0	0	0.738	0.016	1
k	7	10	1	0	0.1	1	6	1.88	0.102	1.16	0.055	0.5	0.495	0.005	0	0	0.316	0.003	0.99
k	7	10	1	0	0.1	0	6	3.507	0.256	4.274	0.238	0.7	0.399	0.01	0.002	0.002	0.758	0.016	0.94
k	7	10	1	0	0.1	1	6	3.508	0.253	4.007	0.254	0.7	0.673	0.006	0	0	0.452	0.005	1
k	7	10	1	0	0.1	0	6	4.879	0.117	5.217	0.093	0.9	0.439	0.011	0.004	0.004	0.785	0.013	0.84
k	7	10	1	0	0.1	1	6	6.599	0.211	7.051	0.187	0.9	0.733	0.005	0	0	0.473	0.004	0.98
k	7	10	1	0	0.3	0	6	0.95	0.172	0.548	0.099	0.1	0.126	0.003	0	0	0.138	0.005	0.97
k	7	10	1	0	0.3	1	6	1.293	0.207	0.864	0.117	0.1	0.342	0.003	0.009	0.009	0.374	0.005	0.98
k	7	10	1	0	0.3	0	6	1.445	0.252	1.792	0.265	0.3	0.306	0.011	0	0	0.566	0.032	0.94
k	7	10	1	0	0.3	1	6	1.47	0.121	0.905	0.068	0.3	0.362	0.004	0	0	0.366	0.004	1
k	7	10	1	0	0.3	0	6	3.248	0.19	4.218	0.196	0.5	0.368	0.01	0	0	0.755	0.017	1
k	7	10	1	0	0.3	1	6	1.053	0.122	0.912	0.072	0.5	0.488	0.005	0	0	0.315	0.003	0.98
k	7	10	1	0	0.3	0	6	4.918	0.177	5.27	0.113	0.7	0.429	0.008	0.001	0.001	0.833	0.005	1
k	7	10	1	0	0.3	1	6	3.211	0.272	3.663	0.274	0.7	0.672	0.006	0	0	0.444	0.006	0.98
k	7	10	1	0	0.3	0	6	4.513	0.134	4.696	0.155	0.9	0.428	0.008	0.004	0.004	0.802	0.01	0.86
k	7	10	1	0	0.3	1	6	6.195	0.236	7.063	0.176	0.9	0.724	0.005	0	0	0.474	0.004	0.93
k	7	10	1	0	0.5	0	6	0.25	0.107	0.202	0.071	0.1	0.123	0.003	0	0	0.137	0.005	1
k	7	10	1	0	0.5	1	6	0.22	0.07	0.552	0.069	0.1	0.35	0.001	0.001	0.001	0.392	0.002	0.91
k	7	10	1	0	0.5	0	6	1.348	0.243	2.605	0.198	0.3	0.332	0.009	0	0	0.638	0.028	1
k	7	10	1	0	0.5	1	6	0.514	0.1	0.863	0.069	0.3	0.372	0.003	0	0	0.364	0.005	0.99
k	7	10	1	0	0.5	0	6	4.918	0.177	5.27	0.113	0.5	0.429	0.008	0.001	0.001	0.833	0.005	1
k	7	10	1	0	0.5	1	6	0.529	0.101	1.303	0.053	0.5	0.497	0.005	0	0	0.314	0.003	0.99
k	7	10	1	0	0.5	0	6	4.308	0.143	4.888	0.139	0.7	0.419	0.006	0.002	0.002	0.808	0.011	1
k	7	10	1	0	0.5	1	6	6.771	0.231	7.464	0.155	0.7	0.719	0.005	0	0	0.473	0.005	1
k	7	10	1	0	0.5	0	6	4.616	0.21	4.733	0.221	0.9	0.427	0.01	0.005	0.005	0.798	0.012	0.83
k	7	10	1	0	0.5	1	6	6.771	0.231	7.464	0.155	0.9	0.718	0.005	0	0	0.471	0.005	0.91
k	7	20	1	0	0.1	0	6	2.303	0.083	1.329	0.048	0.1	0.124	0.002	0	0	0.096	0.003	1
k	7	20	1	0	0.1	1	6	2.449	0	1.414	0	0.1	0.31	0.002	0	0	0.294	0.002	0.96
k	7	20	1	0	0.1	0	6	0.7	0.16	0.52	0.098	0.3	0.287	0.005	0	0	0.313	0.013	0.95
k	7	20	1	0	0.1	1	6	2.449	0	1.414	0	0.3	0.352	0.003	0	0	0.233	0.005	1
k	7	20	1	0	0.1	0	6	2.191	0.274	2.918	0.231	0.5	0.443	0.012	0	0	0.597	0.022	1

l	7	20	1	0	0.1	1	6	2.029	0.093	1.257	0.045	0.5	0.512	0.004	0	0	0.205	0.002	1
l	7	20	1	0	0.1	0	6	3.738	0.219	4.143	0.225	0.7	0.528	0.007	0.001	0.001	0.751	0.004	1
l	7	20	1	0	0.1	1	6	2.086	0.196	2.247	0.205	0.7	0.706	0.006	0	0	0.341	0.006	0.99
l	7	20	1	0	0.1	0	6	4.523	0.175	4.843	0.161	0.9	0.541	0.01	0.003	0.003	0.732	0.01	0.9
l	7	20	1	0	0.1	1	6	5.976	0.236	6.514	0.237	0.9	0.797	0.004	0	0	0.383	0.004	1
l	7	20	1	0	0.3	0	6	0.95	0.172	0.548	0.099	0.1	0.125	0.002	0	0	0.09	0.004	1
l	7	20	1	0	0.3	1	6	0.863	0.125	0.537	0.069	0.1	0.352	0.004	0.001	0.001	0.313	0.009	0.78
l	7	20	1	0	0.3	0	6	0.599	0.147	0.825	0.102	0.3	0.293	0.009	0	0	0.334	0.02	0.9
l	7	20	1	0	0.3	1	6	1.617	0.117	0.933	0.067	0.3	0.364	0.002	0	0	0.254	0.006	1
l	7	20	1	0	0.3	0	6	2.1	0.254	2.987	0.239	0.5	0.434	0.013	0	0	0.584	0.02	1
l	7	20	1	0	0.3	1	6	1.311	0.123	0.953	0.069	0.5	0.494	0.004	0	0	0.204	0.002	0.98
l	7	20	1	0	0.3	0	6	3.364	0.218	4.232	0.241	0.7	0.495	0.01	0.001	0.001	0.722	0.011	1
l	7	20	1	0	0.3	1	6	3.785	0.323	4.562	0.3	0.7	0.732	0.008	0	0	0.349	0.006	0.98
l	7	20	1	0	0.3	0	6	5.044	0.146	5.265	0.157	0.9	0.504	0.008	0.003	0.003	0.725	0.008	0.96
l	7	20	1	0	0.3	1	6	4.508	0.341	5.043	0.347	0.9	0.76	0.02	0.001	0.001	0.368	0.004	0.69
l	7	20	1	0	0.5	0	6	0	0	0.085	0.048	0.1	0.127	0.002	0	0	0.096	0.005	0.9
l	7	20	1	0	0.5	1	6	0.122	0.054	0.396	0.064	0.1	0.353	0.001	0.001	0.001	0.323	0.005	0.73
l	7	20	1	0	0.5	0	6	0.215	0.087	1.535	0.053	0.3	0.312	0.007	0	0	0.28	0.012	0.959
l	7	20	1	0	0.5	1	6	0.15	0.06	0.447	0.067	0.3	0.374	0.001	0	0	0.281	0.008	1
l	7	20	1	0	0.5	0	6	2.244	0.193	3.382	0.237	0.5	0.452	0.011	0	0	0.657	0.018	1
l	7	20	1	0	0.5	1	6	0.392	0.09	1.188	0.052	0.5	0.505	0.004	0	0	0.197	0.002	0.99
l	7	20	1	0	0.5	0	6	3.915	0.206	4.51	0.168	0.7	0.513	0.011	0.001	0.001	0.722	0.01	1
l	7	20	1	0	0.5	1	6	1.485	0.205	3.46	0.234	0.7	0.716	0.005	0	0	0.35	0.005	1
l	7	20	1	0	0.5	0	6	4.704	0.145	4.995	0.151	0.9	0.519	0.012	0.003	0.003	0.692	0.012	0.75
l	7	20	1	0	0.5	1	6	5.909	0.246	6.646	0.203	0.9	0.781	0.004	0	0	0.388	0.003	0.98
l	7	30	1	0	0.1	0	6	2.352	0.069	1.358	0.04	0.1	0.127	0.002	0	0	0.066	0.003	1
l	7	30	1	0	0.1	1	6	2.449	0	1.414	0	0.1	0.311	0.001	0	0	0.272	0.003	0.93
l	7	30	1	0	0.1	0	6	1.715	0.16	0.939	0.111	0.3	0.319	0.005	0	0	0.306	0.011	1
l	7	30	1	0	0.1	1	6	2.449	0	1.414	0	0.3	0.358	0.003	0	0	0.196	0.004	0.99
l	7	30	1	0	0.1	0	6	2.3	0.272	2.553	0.268	0.5	0.488	0.011	0	0	0.6	0.018	0.9
l	7	30	1	0	0.1	1	6	2.303	0.058	1.329	0.034	0.5	0.505	0.003	0	0	0.156	0.002	1
l	7	30	1	0	0.1	0	6	3.281	0.278	3.921	0.235	0.7	0.544	0.008	0	0	0.699	0.007	1
l	7	30	1	0	0.1	1	6	1.952	0.156	2.063	0.171	0.7	0.697	0.004	0	0	0.259	0.004	1
l	7	30	1	0	0.1	0	6	4.625	0.16	5.067	0.184	0.9	0.608	0.009	0.002	0.002	0.694	0.006	1
l	7	30	1	0	0.1	1	6	6.012	0.265	6.497	0.227	0.9	0.806	0.004	0	0	0.343	0.004	0.98
l	7	30	1	0	0.3	0	6	0.3	0.116	0.173	0.067	0.1	0.127	0.002	0	0	0.084	0.003	0.96
l	7	30	1	0	0.3	1	6	1.225	0.123	0.707	0.071	0.1	0.347	0.001	0.001	0.001	0.288	0.005	0.71
l	7	30	1	0	0.3	0	6	0.55	0.148	0.548	0.099	0.3	0.285	0.006	0	0	0.23	0.011	0.9
l	7	30	1	0	0.3	1	6	1.575	0.119	0.909	0.069	0.3	0.365	0.002	0	0	0.229	0.007	1
l	7	30	1	0	0.3	0	6	2.458	0.181	2.725	0.257	0.5	0.494	0.012	0	0	0.603	0.023	1
l	7	30	1	0	0.3	1	6	1.064	0.123	0.71	0.073	0.5	0.498	0.003	0	0	0.158	0.002	0.96
l	7	30	1	0	0.3	0	6	4.213	0.191	4.272	0.172	0.7	0.551	0.01	0	0	0.663	0.011	0.97
l	7	30	1	0	0.3	1	6	1.106	0.152	1.518	0.124	0.7	0.692	0.006	0	0	0.256	0.005	0.98
l	7	30	1	0	0.3	0	6	4.33	0.128	4.974	0.123	0.9	0.561	0.009	0.002	0.002	0.681	0.005	1
l	7	30	1	0	0.3	1	6	5.472	0.267	6.207	0.239	0.9	0.818	0.004	0	0	0.332	0.003	1

m	7	30	1	0	0.5	0	6	0	0	0.255	0.078	0.1	0.126	0.001	0	0	0.081	0.003	0.95
m	7	30	1	0	0.5	1	6	0.848	0.156	2.025	0.195	0.1	0.354	0.016	0.003	0.003	0.29	0.005	0.17
m	7	30	1	0	0.5	0	6	0.215	0.087	1.535	0.053	0.3	0.312	0.007	0	0	0.28	0.012	0.95
m	7	30	1	0	0.5	1	6	0.15	0.06	0.447	0.067	0.3	0.374	0.001	0	0	0.221	0.008	1
m	7	30	1	0	0.5	0	6	1.478	0.207	2.596	0.199	0.5	0.469	0.01	0	0	0.58	0.016	1
m	7	30	1	0	0.5	1	6	0.147	0.058	1.117	0.058	0.5	0.503	0.003	0	0	0.149	0.002	1
m	7	30	1	0	0.5	0	6	2.006	0.25	3.102	0.238	0.7	0.492	0.013	0	0	0.602	0.018	0.98
m	7	30	1	0	0.5	1	6	0.958	0.168	2.442	0.19	0.7	0.73	0.005	0	0	0.293	0.005	1
m	7	30	1	0	0.5	0	6	4.908	0.171	5.613	0.157	0.9	0.585	0.011	0.002	0.002	0.672	0.008	0.9
m	7	30	1	0	0.5	1	6	5.561	0.265	6.341	0.239	0.9	0.816	0.004	0	0	0.338	0.003	1
m	5	10	0	1	0.1	0	8	2.416	0.03	0.077	0.056	0.1	0.14	0.004	0	0	0.224	0.01	1
m	5	10	0	1	0.1	1	8	2.425	0.024	0	0	0.1	0.259	0.003	0	0	0.296	0.004	1
m	5	10	0	1	0.1	0	8	2.678	0.201	1.692	0.24	0.3	0.322	0.01	0	0	0.742	0.026	1
m	5	10	0	1	0.1	1	8	2.484	0.039	0.122	0.054	0.3	0.349	0.005	0	0	0.379	0.004	0.96
m	5	10	0	1	0.1	0	8	2.35	0.232	2.932	0.279	0.5	0.392	0.013	0	0	0.785	0.025	1
m	5	10	0	1	0.1	1	8	2.379	0.098	0.817	0.118	0.5	0.504	0.008	0	0	0.516	0.009	1
m	5	10	0	1	0.1	0	8	3.176	0.234	3.479	0.303	0.7	0.425	0.011	0.002	0.002	0.868	0.004	1
m	5	10	0	1	0.1	1	8	2.882	0.158	2.514	0.18	0.7	0.633	0.008	0	0	0.621	0.006	1
m	5	10	0	1	0.1	0	8	3.296	0.258	4.278	0.199	0.9	0.425	0.009	0.005	0.004	0.87	0.005	0.96
m	5	10	0	1	0.1	1	8	3.697	0.165	4.102	0.157	0.9	0.686	0.007	0	0	0.625	0.006	0.98
m	5	10	0	1	0.3	0	8	1.47	0.171	0	0	0.1	0.155	0.005	0	0	0.223	0.013	1
m	5	10	0	1	0.3	1	8	1.519	0.119	0.049	0.034	0.1	0.309	0.003	0	0	0.345	0.002	0.98
m	5	10	0	1	0.3	0	8	1.9	0.233	1.649	0.28	0.3	0.281	0.01	0	0	0.622	0.029	0.95
m	5	10	0	1	0.3	1	8	1.559	0.119	0.099	0.049	0.3	0.35	0.004	0	0	0.376	0.004	0.94
m	5	10	0	1	0.3	0	8	3.26	0.164	4.199	0.155	0.5	0.369	0.011	0	0	0.83	0.015	1
m	5	10	0	1	0.3	1	8	1.502	0.134	0.875	0.134	0.5	0.514	0.008	0	0	0.526	0.009	1
m	5	10	0	1	0.3	0	8	3.517	0.193	4.566	0.185	0.7	0.441	0.01	0.001	0.001	0.86	0.005	1
m	5	10	0	1	0.3	1	8	2.378	0.177	2.664	0.189	0.7	0.643	0.009	0	0	0.615	0.005	1
m	5	10	0	1	0.3	0	8	3.679	0.183	4.808	0.15	0.9	0.437	0.01	0.004	0.004	0.867	0.003	1
m	5	10	0	1	0.3	1	8	3.895	0.141	4.534	0.139	0.9	0.7	0.007	0	0	0.618	0.004	1
m	5	10	0	1	0.5	0	8	0.392	0.128	0.931	0.17	0.1	0.148	0.005	0	0	0.187	0.01	0.99
m	5	10	0	1	0.5	1	8	0.465	0.097	0.431	0.093	0.1	0.324	0.002	0.001	0	0.35	0.001	0.94
m	5	10	0	1	0.5	0	8	1.198	0.2	2.202	0.211	0.3	0.294	0.009	0	0	0.66	0.029	0.91
m	5	10	0	1	0.5	1	8	0.396	0.091	0.612	0.107	0.3	0.362	0.004	0	0	0.387	0.004	1
m	5	10	0	1	0.5	0	8	2.975	0.252	3.642	0.266	0.5	0.415	0.01	0	0	0.878	0.004	1
m	5	10	0	1	0.5	1	8	0.914	0.131	1.531	0.146	0.5	0.519	0.008	0	0	0.528	0.008	1
m	5	10	0	1	0.5	0	8	3.544	0.264	4.084	0.178	0.7	0.412	0.016	0.002	0.002	0.809	0.018	0.88
m	5	10	0	1	0.5	1	8	1.88	0.173	3.072	0.176	0.7	0.628	0.007	0	0	0.615	0.006	0.97
m	5	10	0	1	0.5	0	8	3.979	0.177	4.575	0.203	0.9	0.433	0.011	0.004	0.004	0.863	0.005	1
m	5	10	0	1	0.5	1	8	3.628	0.17	4.504	0.128	0.9	0.696	0.007	0	0	0.614	0.005	0.96
m	5	20	0	1	0.1	0	8	2.449	0	0	0	0.1	0.135	0.003	0	0	0.132	0.004	1
m	5	20	0	1	0.1	1	8	2.449	0	0	0	0.1	0.263	0.002	0	0	0.254	0.003	0.93
m	5	20	0	1	0.1	0	8	2.392	0.183	0.964	0.189	0.3	0.353	0.012	0	0	0.563	0.028	0.9
m	5	20	0	1	0.1	1	8	2.425	0.024	0	0	0.3	0.346	0.004	0	0	0.256	0.002	1
m	5	20	0	1	0.1	0	8	1.641	0.235	1.942	0.209	0.5	0.462	0.011	0	0	0.761	0.012	1

n	5	20	0	1	0.1	1	8	2.258	0.073	0.446	0.09	0.5	0.525	0.005	0	0	0.357	0.006	0.93
n	5	20	0	1	0.1	0	8	3.142	0.146	4.018	0.187	0.7	0.536	0.01	0.001	0.001	0.811	0.006	1
n	5	20	0	1	0.1	1	8	2.517	0.143	1.97	0.168	0.7	0.699	0.008	0	0	0.504	0.005	0.99
n	5	20	0	1	0.1	0	8	3.598	0.277	4.522	0.193	0.9	0.497	0.012	0.003	0.003	0.814	0.005	0.94
n	5	20	0	1	0.1	1	8	3.587	0.153	4.029	0.15	0.9	0.752	0.006	0	0	0.508	0.005	1
n	5	20	0	1	0.3	0	8	1.323	0.174	0	0	0.1	0.152	0.003	0	0	0.148	0.004	1
n	5	20	0	1	0.3	1	8	2.009	0.095	0	0	0.1	0.312	0.002	0	0	0.253	0.004	0.42
n	5	20	0	1	0.3	0	8	1.299	0.195	0.791	0.16	0.3	0.319	0.009	0	0	0.505	0.026	1
n	5	20	0	1	0.3	1	8	1.786	0.112	0	0	0.3	0.367	0.003	0	0	0.277	0.005	0.94
n	5	20	0	1	0.3	0	8	2.975	0.128	4.077	0.125	0.5	0.462	0.014	0	0	0.738	0.021	0.95
n	5	20	0	1	0.3	1	8	1.506	0.121	0.317	0.083	0.5	0.518	0.006	0	0	0.358	0.006	1
n	5	20	0	1	0.3	0	8	3.098	0.217	4.412	0.153	0.7	0.53	0.011	0.001	0.001	0.801	0.005	1
n	5	20	0	1	0.3	1	8	2.6	0.166	2.235	0.179	0.7	0.696	0.006	0	0	0.506	0.005	1
n	5	20	0	1	0.3	0	8	3.392	0.248	4.247	0.216	0.9	0.525	0.01	0.003	0.003	0.79	0.005	1
n	5	20	0	1	0.3	1	8	3.318	0.178	4.079	0.151	0.9	0.752	0.006	0	0	0.504	0.004	0.99
n	5	20	0	1	0.5	0	8	0.098	0.069	0.343	0.121	0.1	0.158	0.003	0	0	0.136	0.005	1
n	5	20	0	1	0.5	1	8	0.392	0.09	0.22	0.07	0.1	0.331	0.001	0.001	0.001	0.292	0.003	0.76
n	5	20	0	1	0.5	0	8	2.734	0.075	3.978	0.109	0.3	0.315	0.013	0	0	0.46	0.025	1
n	5	20	0	1	0.5	1	8	0.171	0.063	0.465	0.097	0.3	0.374	0.004	0	0	0.327	0.006	0.99
n	5	20	0	1	0.5	0	8	1.667	0.245	3.512	0.145	0.5	0.464	0.011	0	0	0.767	0.016	1
n	5	20	0	1	0.5	1	8	0.425	0.086	1.532	0.126	0.5	0.539	0.006	0	0	0.37	0.006	1
n	5	20	0	1	0.5	0	8	3.255	0.22	4.144	0.195	0.7	0.538	0.012	0.001	0.001	0.781	0.004	1
n	5	20	0	1	0.5	1	8	1.823	0.15	2.501	0.165	0.7	0.682	0.007	0	0	0.498	0.005	0.99
n	5	20	0	1	0.5	0	8	3.378	0.219	4.051	0.281	0.9	0.537	0.011	0.003	0.003	0.762	0.031	0.93
n	5	20	0	1	0.5	1	8	3.26	0.165	4.305	0.117	0.9	0.761	0.006	0	0	0.506	0.005	0.98
n	5	30	0	1	0.1	0	8	2.401	0.049	0	0	0.1	0.135	0.003	0	0	0.103	0.004	1
n	5	30	0	1	0.1	1	8	2.449	0	0	0	0.1	0.264	0.002	0	0	0.235	0.002	0.95
n	5	30	0	1	0.1	0	8	1.851	0.16	0.443	0.131	0.3	0.334	0.009	0	0	0.403	0.018	0.84
n	5	30	0	1	0.1	1	8	2.449	0	0	0	0.3	0.347	0.003	0	0	0.204	0.001	1
n	5	30	0	1	0.1	0	8	2.298	0.208	2.237	0.214	0.5	0.493	0.012	0	0	0.695	0.014	1
n	5	30	0	1	0.1	1	8	2.259	0.056	0.312	0.075	0.5	0.525	0.005	0	0	0.279	0.005	1
n	5	30	0	1	0.1	0	8	2.662	0.232	2.571	0.239	0.7	0.53	0.012	0.001	0.001	0.727	0.008	1
n	5	30	0	1	0.1	1	8	2.415	0.126	1.561	0.154	0.7	0.697	0.006	0	0	0.432	0.006	1
n	5	30	0	1	0.1	0	8	3.681	0.207	4.136	0.158	0.9	0.596	0.01	0.002	0.002	0.729	0.004	0.96
n	5	30	0	1	0.1	1	8	3.565	0.143	3.702	0.164	0.9	0.79	0.005	0	0	0.435	0.005	0.92
n	5	30	0	1	0.3	0	8	1.47	0.171	0	0	0.1	0.153	0.002	0	0	0.117	0.003	0.98
n	5	30	0	1	0.3	1	8	2.01	0.1	0.093	0.061	0.1	0.327	0.009	0.001	0.001	0.223	0.006	0.41
n	5	30	0	1	0.3	0	8	1.021	0.176	0.359	0.107	0.3	0.324	0.011	0	0	0.368	0.02	0.95
n	5	30	0	1	0.3	1	8	1.93	0.101	0	0	0.3	0.365	0.003	0	0	0.225	0.006	0.98
n	5	30	0	1	0.3	0	8	3.035	0.193	3.562	0.187	0.5	0.497	0.013	0	0	0.69	0.015	1
n	5	30	0	1	0.3	1	8	1.418	0.122	0.267	0.08	0.5	0.53	0.005	0	0	0.319	0.004	1
n	5	30	0	1	0.3	0	8	3.498	0.18	3.966	0.166	0.7	0.517	0.011	0.001	0.001	0.727	0.007	1
n	5	30	0	1	0.3	1	8	1.942	0.147	1.753	0.173	0.7	0.693	0.006	0	0	0.424	0.005	0.99
n	5	30	0	1	0.3	0	8	3.256	0.23	4.493	0.155	0.9	0.595	0.011	0.002	0.002	0.728	0.007	0.96
n	5	30	0	1	0.3	1	8	3.134	0.172	3.733	0.168	0.9	0.795	0.005	0	0	0.445	0.005	0.99

o	5	30	0	1	0.5	0	8	0	0	0.459	0.139	0.1	0.146	0.002	0	0	0.108	0.003	1
o	5	30	0	1	0.5	1	8	0.099	0.049	0.099	0.049	0.1	0.334	0.001	0.001	0.001	0.288	0.004	0.91
o	5	30	0	1	0.5	0	8	0.226	0.074	1.555	0.247	0.3	0.317	0.008	0	0	0.373	0.017	1
o	5	30	0	1	0.5	1	8	0.147	0.058	0.367	0.088	0.3	0.367	0.002	0	0	0.304	0.006	1
o	5	30	0	1	0.5	0	8	1.331	0.197	2.019	0.222	0.5	0.457	0.011	0	0	0.671	0.015	1
o	5	30	0	1	0.5	1	8	0.555	0.106	1.196	0.134	0.5	0.531	0.005	0	0	0.29	0.006	0.97
o	5	30	0	1	0.5	0	8	2.62	0.256	3.748	0.213	0.7	0.628	0.011	0	0	0.741	0.007	1
o	5	30	0	1	0.5	1	8	1.23	0.139	2.165	0.177	0.7	0.7	0.006	0	0	0.434	0.006	0.98
o	5	30	0	1	0.5	0	8	3.25	0.148	5.094	0.092	0.9	0.579	0.011	0.002	0.002	0.756	0.006	1
o	5	30	0	1	0.5	1	8	3.63	0.134	4.55	0.099	0.9	0.806	0.005	0	0	0.44	0.004	1
o	6	10	0	1	0.1	0	8	2.918	0.092	1.486	0.049	0.1	0.119	0.003	0	0	0.151	0.007	1
o	6	10	0	1	0.1	1	8	2.449	0	0	0	0.1	0.282	0.002	0	0	0.33	0.002	0.93
o	6	10	0	1	0.1	0	8	1.904	0.135	1.011	0.207	0.3	0.269	0.009	0	0	0.519	0.023	1
o	6	10	0	1	0.1	1	8	2.403	0.029	0.014	0.014	0.3	0.342	0.004	0	0	0.367	0.002	0.8
o	6	10	0	1	0.1	0	8	2.424	0.29	3.218	0.246	0.5	0.354	0.009	0	0	0.76	0.025	0.95
o	6	10	0	1	0.1	1	8	2.182	0.088	0.416	0.1	0.5	0.507	0.006	0	0	0.397	0.007	0.99
o	6	10	0	1	0.1	0	8	3.587	0.226	4.031	0.25	0.7	0.403	0.012	0.002	0.002	0.783	0.02	0.84
o	6	10	0	1	0.1	1	8	2.687	0.233	3.007	0.223	0.7	0.654	0.006	0	0	0.525	0.005	0.98
o	6	10	0	1	0.1	0	8	4.393	0.15	4.896	0.13	0.9	0.441	0.01	0.004	0.004	0.828	0.009	0.96
o	6	10	0	1	0.1	1	8	5.028	0.168	5.62	0.166	0.9	0.706	0.006	0	0	0.541	0.004	0.99
o	6	10	0	1	0.3	0	8	1.939	0.145	0	0	0.1	0.129	0.003	0	0	0.158	0.007	1
o	6	10	0	1	0.3	1	8	1.592	0.117	0	0	0.1	0.323	0.002	0	0	0.326	0.003	0.77
o	6	10	0	1	0.3	0	8	1.283	0.169	0.848	0.159	0.3	0.249	0.009	0	0	0.469	0.026	0.94
o	6	10	0	1	0.3	1	8	1.445	0.121	0.039	0.028	0.3	0.352	0.004	0	0	0.372	0.004	0.95
o	6	10	0	1	0.3	0	8	2.555	0.258	3.118	0.286	0.5	0.361	0.01	0	0	0.776	0.016	1
o	6	10	0	1	0.3	1	8	1.575	0.138	0.668	0.124	0.5	0.493	0.007	0	0	0.384	0.006	0.99
o	6	10	0	1	0.3	0	8	3.95	0.199	4.288	0.159	0.7	0.403	0.01	0.002	0.002	0.821	0.014	0.93
o	6	10	0	1	0.3	1	8	3.687	0.223	4.062	0.237	0.7	0.675	0.006	0	0	0.528	0.005	1
o	6	10	0	1	0.3	0	8	4.326	0.223	4.616	0.212	0.9	0.421	0.009	0.005	0.005	0.822	0.012	0.88
o	6	10	0	1	0.3	1	8	4.865	0.2	5.438	0.178	0.9	0.709	0.006	0	0	0.537	0.004	0.98
o	6	10	0	1	0.5	0	8	0.637	0.153	0	0.196	0.095	0.1	0.134	0.003	0	0	0.152	0.95
o	6	10	0	1	0.5	1	8	0.416	0.092	0.441	0.095	0.1	0.338	0.001	0.001	0.001	0.334	0.002	0.76
o	6	10	0	1	0.5	0	8	0.918	0.205	2.27	0.25	0.3	0.311	0.012	0	0	0.618	0.031	1
o	6	10	0	1	0.5	1	8	0.563	0.104	0.588	0.105	0.3	0.358	0.003	0	0	0.383	0.004	1
o	6	10	0	1	0.5	0	8	2.727	0.304	4.15	0.172	0.5	0.408	0.012	0	0	0.789	0.017	0.9
o	6	10	0	1	0.5	1	8	0.663	0.106	1.378	0.134	0.5	0.515	0.007	0	0	0.406	0.007	1
o	6	10	0	1	0.5	0	8	3.723	0.197	4.757	0.195	0.7	0.418	0.011	0.002	0.002	0.841	0.009	0.94
o	6	10	0	1	0.5	1	8	2.945	0.223	4.631	0.172	0.7	0.682	0.007	0	0	0.534	0.005	1
o	6	10	0	1	0.5	0	8	4.459	0.213	5.161	0.114	0.9	0.439	0.009	0.004	0.004	0.852	0.004	0.97
o	6	10	0	1	0.5	1	8	4.816	0.193	5.378	0.175	0.9	0.714	0.005	0	0	0.54	0.004	0.99
o	6	20	0	1	0.1	0	8	2.449	0	1.414	0	0.1	0.128	0.003	0	0	0.102	0.004	1
o	6	20	0	1	0.1	1	8	2.449	0	0	0	0.1	0.287	0.002	0	0	0.268	0.002	0.94
o	6	20	0	1	0.1	0	8	1.747	0.156	0.775	0.156	0.3	0.304	0.011	0.002	0.002	0.327	0.009	0.02
o	6	20	0	1	0.1	1	8	2.449	0	0	0	0.3	0.342	0.003	0	0	0.244	0.003	1
o	6	20	0	1	0.1	0	8	2.564	0.23	1.987	0.236	0.5	0.433	0.007	0	0	0.72	0.017	0.94

p	6	20	0	1	0.1	1	8	2.149	0.062	0.309	0.072	0.5	0.512	0.004	0	0	0.257	0.004	0.97
p	6	20	0	1	0.1	0	8	3.064	0.292	3.943	0.203	0.7	0.48	0.011	0.001	0.001	0.736	0.016	0.96
p	6	20	0	1	0.1	1	8	3.034	0.187	2.588	0.226	0.7	0.697	0.006	0	0	0.413	0.005	1
p	6	20	0	1	0.1	0	8	3.688	0.249	4.372	0.227	0.9	0.505	0.011	0.003	0.003	0.743	0.01	0.88
p	6	20	0	1	0.1	1	8	4.839	0.182	5.251	0.189	0.9	0.776	0.005	0	0	0.439	0.005	0.97
p	6	20	0	1	0.3	0	8	1.421	0.173	0	0	0.1	0.134	0.002	0	0	0.11	0.004	1
p	6	20	0	1	0.3	1	8	1.935	0.1	0	0	0.1	0.327	0.002	0.001	0.001	0.256	0.003	0.47
p	6	20	0	1	0.3	0	8	1.281	0.171	0.479	0.115	0.3	0.395	0.01	0.002	0.002	0.317	0.008	0.02
p	6	20	0	1	0.3	1	8	1.911	0.102	0.024	0.024	0.3	0.355	0.003	0	0	0.252	0.005	0.96
p	6	20	0	1	0.3	0	8	2.455	0.238	2.625	0.319	0.5	0.454	0.014	0	0	0.69	0.018	1
p	6	20	0	1	0.3	1	8	1.347	0.122	0.305	0.076	0.5	0.513	0.005	0	0	0.26	0.003	0.99
p	6	20	0	1	0.3	0	8	3.004	0.226	3.515	0.27	0.7	0.502	0.007	0.001	0.001	0.772	0.008	0.98
p	6	20	0	1	0.3	1	8	2.643	0.195	3.036	0.241	0.7	0.711	0.007	0	0	0.422	0.005	1
p	6	20	0	1	0.3	0	8	4.039	0.167	4.53	0.206	0.9	0.548	0.008	0.002	0.002	0.763	0.005	0.97
p	6	20	0	1	0.3	1	8	4.674	0.224	5.118	0.219	0.9	0.777	0.005	0	0	0.436	0.004	0.98
p	6	20	0	1	0.5	0	8	0	0	0.245	0.105	0.1	0.133	0.002	0	0	0.095	0.004	1
p	6	20	0	1	0.5	1	8	0.416	0.092	0.367	0.088	0.1	0.343	0.001	0.001	0.001	0.276	0.004	0.7
p	6	20	0	1	0.5	0	8	0.543	0.113	1.255	0.161	0.3	0.303	0.011	0.002	0.002	0.325	0.009	0.02
p	6	20	0	1	0.5	1	8	0.294	0.08	0.367	0.088	0.3	0.363	0.002	0	0	0.303	0.005	0.94
p	6	20	0	1	0.5	0	8	2.217	0.3	3.532	0.248	0.5	0.492	0.011	0	0	0.765	0.007	0.92
p	6	20	0	1	0.5	1	8	0.628	0.102	1.053	0.122	0.5	0.512	0.005	0	0	0.259	0.004	1
p	6	20	0	1	0.5	0	8	4.07	0.208	4.018	0.214	0.7	0.508	0.01	0.001	0.001	0.753	0.006	1
p	6	20	0	1	0.5	1	8	2.221	0.192	3.534	0.198	0.7	0.706	0.006	0	0	0.417	0.005	0.91
p	6	20	0	1	0.5	0	8	4.038	0.184	4.903	0.133	0.9	0.547	0.011	0.003	0.003	0.776	0.006	0.98
p	6	20	0	1	0.5	1	8	4.463	0.216	5.495	0.171	0.9	0.775	0.005	0	0	0.442	0.004	1
p	6	30	0	1	0.1	0	8	2.449	0	0	0	0.1	0.129	0.002	0	0	0.093	0.003	0.97
p	6	30	0	1	0.1	1	8	2.449	0	0	0	0.1	0.285	0.001	0	0	0.246	0.001	0.92
p	6	30	0	1	0.1	0	8	2.13	0.158	0.141	0.061	0.3	0.325	0.006	0	0	0.333	0.013	0.95
p	6	30	0	1	0.1	1	8	2.449	0	0	0	0.3	0.347	0.002	0	0	0.192	0.003	1
p	6	30	0	1	0.1	0	8	1.925	0.213	2.307	0.305	0.5	0.493	0.013	0	0	0.641	0.016	1
p	6	30	0	1	0.1	1	8	2.164	0.061	0.118	0.04	0.5	0.507	0.004	0	0	0.19	0.003	1
p	6	30	0	1	0.1	0	8	3.234	0.252	3.63	0.284	0.7	0.559	0.008	0	0	0.711	0.005	1
p	6	30	0	1	0.1	1	8	2.112	0.144	1.259	0.149	0.7	0.693	0.005	0	0	0.332	0.005	0.92
p	6	30	0	1	0.1	0	8	3.878	0.217	4.514	0.215	0.9	0.585	0.013	0.002	0.002	0.702	0.007	0.91
p	6	30	0	1	0.1	1	8	5.04	0.435	5.116	0.544	0.9	0.798	0.01	0.001	0.001	0.388	0.008	0.95
p	6	30	0	1	0.3	0	8	1.617	0.166	0	0	0.1	0.125	0.001	0	0	0.087	0.003	1
p	6	30	0	1	0.3	1	8	2.009	0.095	0	0	0.1	0.328	0.002	0.001	0.001	0.225	0.004	0.31
p	6	30	0	1	0.3	0	8	1.296	0.164	0.884	0.155	0.3	0.305	0.008	0	0	0.322	0.016	0.9
p	6	30	0	1	0.3	1	8	1.825	0.108	0.126	0.044	0.3	0.393	0.007	0	0	0.209	0.006	0.82
p	6	30	0	1	0.3	0	8	1.417	0.191	3.123	0.276	0.5	0.497	0.01	0	0	0.657	0.013	1
p	6	30	0	1	0.3	1	8	1.396	0.122	0.126	0.052	0.5	0.505	0.003	0	0	0.2	0.003	1
p	6	30	0	1	0.3	0	8	2.656	0.265	4.291	0.212	0.7	0.573	0.012	0	0	0.707	0.007	0.97
p	6	30	0	1	0.3	1	8	1.756	0.169	1.861	0.199	0.7	0.708	0.005	0	0	0.343	0.005	1
p	6	30	0	1	0.3	0	8	4.324	0.198	4.706	0.188	0.9	0.581	0.006	0.002	0.002	0.718	0.004	0.95
p	6	30	0	1	0.3	1	8	4.336	0.19	4.534	0.221	0.9	0.795	0.004	0	0	0.387	0.003	1

q	6	30	0	1	0.5	0	8	0	0	0.931	0.17	0.1	0.134	0.001	0	0	0.095	0.002	1
q	6	30	0	1	0.5	1	8	0	0	0.177	0.062	0.1	0.345	0	0.001	0.001	0.293	0.003	0.94
q	6	30	0	1	0.5	0	8	0.17	0.066	1.96	0.14	0.3	0.291	0.003	0	0	0.295	0.006	0.92
q	6	30	0	1	0.5	1	8	0	0	0.225	0.072	0.3	0.366	0.002	0	0	0.31	0.005	0.97
q	6	30	0	1	0.5	0	8	1.645	0.19	2.939	0.204	0.5	0.481	0.015	0	0	0.609	0.019	1
q	6	30	0	1	0.5	1	8	0.27	0.07	1.127	0.123	0.5	0.513	0.004	0	0	0.205	0.003	1
q	6	30	0	1	0.5	0	8	3.125	0.24	3.335	0.267	0.7	0.561	0.009	0	0	0.709	0.005	0.9
q	6	30	0	1	0.5	1	8	1.639	0.179	3.157	0.181	0.7	0.721	0.006	0	0	0.354	0.006	1
q	6	30	0	1	0.5	0	8	4.224	0.2	4.721	0.202	0.9	0.541	0.006	0.003	0.003	0.695	0.005	0.89
q	6	30	0	1	0.5	1	8	4.672	0.195	5.477	0.153	0.9	0.801	0.004	0	0	0.386	0.003	0.98
q	7	10	0	1	0.1	0	8	2.286	0.077	0	0	0.1	0.128	0.005	0	0	0.157	0.009	1
q	7	10	0	1	0.1	1	8	2.376	0.042	0	0	0.1	0.311	0.003	0	0	0.352	0.001	0.95
q	7	10	0	1	0.1	0	8	2.1	0.117	0.972	0.205	0.3	0.309	0.013	0	0	0.557	0.03	1
q	7	10	0	1	0.1	1	8	2.315	0.055	0.043	0.024	0.3	0.349	0.004	0	0	0.359	0.003	1
q	7	10	0	1	0.1	0	8	2.76	0.253	3.242	0.315	0.5	0.395	0.01	0	0	0.762	0.015	0.98
q	7	10	0	1	0.1	1	8	2.139	0.086	0.302	0.07	0.5	0.499	0.006	0	0	0.315	0.004	0.98
q	7	10	0	1	0.1	0	8	4.229	0.18	4.399	0.21	0.7	0.401	0.009	0.002	0.002	0.768	0.013	0.95
q	7	10	0	1	0.1	1	8	3.531	0.279	3.636	0.282	0.7	0.68	0.006	0	0	0.458	0.013	0.98
q	7	10	0	1	0.1	0	8	5.066	0.115	5.235	0.185	0.9	0.428	0.01	0.004	0.004	0.779	0.014	0.76
q	7	10	0	1	0.1	1	8	6.603	0.21	7.151	0.18	0.9	0.729	0.005	0	0	0.472	0.004	0.95
q	7	10	0	1	0.3	0	8	1.029	0.173	0	0	0.1	0.122	0.003	0	0	0.126	0.005	1
q	7	10	0	1	0.3	1	8	1.675	0.116	0	0	0.1	0.35	0.002	0.001	0.001	0.366	0.002	0.71
q	7	10	0	1	0.3	0	8	1.494	0.239	2.564	0.262	0.3	0.334	0.01	0	0	0.595	0.025	1
q	7	10	0	1	0.3	1	8	1.111	0.124	0	0	0.3	0.364	0.003	0	0	0.36	0.003	1
q	7	10	0	1	0.3	0	8	2.883	0.293	3.672	0.278	0.5	0.392	0.01	0	0	0.743	0.02	1
q	7	10	0	1	0.3	1	8	1.332	0.133	0.508	0.095	0.5	0.499	0.006	0	0	0.307	0.006	0.94
q	7	10	0	1	0.3	0	8	4.016	0.148	4.508	0.244	0.7	0.444	0.009	0.001	0.001	0.802	0.014	0.96
q	7	10	0	1	0.3	1	8	3.024	0.262	3.659	0.309	0.7	0.68	0.007	0	0	0.448	0.006	0.98
q	7	10	0	1	0.3	0	8	5.025	0.132	5.344	0.099	0.9	0.414	0.01	0.005	0.005	0.758	0.016	0.71
q	7	10	0	1	0.3	1	8	6.834	0.214	7.227	0.207	0.9	0.732	0.004	0	0	0.478	0.004	0.97
q	7	10	0	1	0.5	0	8	0.098	0.069	0.686	0.157	0.1	0.13	0.003	0.012	0.012	0.14	0.005	1
q	7	10	0	1	0.5	1	8	0.47	0.097	0.346	0.086	0.1	0.364	0.001	0.001	0.001	0.375	0.004	0.83
q	7	10	0	1	0.5	0	8	2.266	0.299	2.769	0.238	0.3	0.307	0.013	0	0	0.553	0.03	1
q	7	10	0	1	0.5	1	8	0.343	0.085	0.514	0.1	0.3	0.375	0.003	0	0	0.378	0.004	0.99
q	7	10	0	1	0.5	0	8	3.584	0.193	4.337	0.173	0.5	0.395	0.011	0	0	0.729	0.019	1
q	7	10	0	1	0.5	1	8	0.715	0.106	1.085	0.133	0.5	0.507	0.005	0	0	0.319	0.004	1
q	7	10	0	1	0.5	0	8	5.477	0	6	0	0.7	0.408	0.005	0.001	0.001	0.756	0.009	1
q	7	10	0	1	0.5	1	8	3.256	0.298	4.637	0.275	0.7	0.682	0.006	0	0	0.449	0.005	1
q	7	10	0	1	0.5	0	8	4.959	0.174	5.318	0.157	0.9	0.434	0.008	0.004	0.004	0.783	0.012	0.84
q	7	10	0	1	0.5	1	8	6.129	0.269	6.957	0.208	0.9	0.717	0.004	0	0	0.473	0.004	0.93
q	7	20	0	1	0.1	0	8	2.449	0	0	0	0.1	0.122	0.002	0	0	0.093	0.002	1
q	7	20	0	1	0.1	1	8	2.449	0	0	0	0.1	0.309	0.002	0	0	0.299	0.001	0.97
q	7	20	0	1	0.1	0	8	1.737	0.199	0.311	0.084	0.3	0.304	0.005	0	0	0.331	0.009	0.9
q	7	20	0	1	0.1	1	8	2.449	0	0	0	0.3	0.35	0.003	0	0	0.236	0.004	0.94
q	7	20	0	1	0.1	0	8	2.719	0.229	2.076	0.241	0.5	0.439	0.011	0	0	0.633	0.017	1

r	7	20	0	1	0.1	1	8	2.148	0.064	0.139	0.046	0.5	0.511	0.004	0	0	0.194	0.003	0.98
r	7	20	0	1	0.1	0	8	4.35	0.21	4.801	0.127	0.7	0.473	0.009	0.001	0.001	0.707	0.011	1
r	7	20	0	1	0.1	1	8	2.657	0.203	2.487	0.244	0.7	0.694	0.005	0	0	0.331	0.006	0.22
r	7	20	0	1	0.1	0	8	4.615	0.16	4.661	0.151	0.9	0.53	0.009	0.003	0.003	0.74	0.008	0.9
r	7	20	0	1	0.1	1	8	6.027	0.248	6.448	0.263	0.9	0.785	0.004	0	0	0.382	0.004	0.99
r	7	20	0	1	0.3	0	8	1.421	0.173	0	0	0.1	0.129	0.002	0	0	0.1	0.002	1
r	7	20	0	1	0.3	1	8	1.984	0.097	0	0	0.1	0.357	0.003	0.001	0.001	0.279	0.004	1
r	7	20	0	1	0.3	0	8	1.163	0.17	0.652	0.147	0.3	0.29	0.008	0	0	0.325	0.018	0.94
r	7	20	0	1	0.3	1	8	2.075	0.09	0	0	0.3	0.366	0.003	0	0	0.247	0.005	1
r	7	20	0	1	0.3	0	8	2.217	0.257	3.083	0.288	0.5	0.445	0.012	0	0	0.624	0.018	1
r	7	20	0	1	0.3	1	8	1.46	0.121	0.234	0.066	0.5	0.507	0.003	0	0	0.201	0.002	0.98
r	7	20	0	1	0.3	0	8	3.446	0.241	3.816	0.263	0.7	0.509	0.009	0.001	0.001	0.717	0.008	1
r	7	20	0	1	0.3	1	8	2.287	0.23	2.474	0.243	0.7	0.697	0.005	0	0	0.321	0.006	1
r	7	20	0	1	0.3	0	8	4.853	0.17	5.457	0.084	0.9	0.524	0.011	0.003	0.003	0.714	0.008	0.88
r	7	20	0	1	0.3	1	8	6.072	0.253	6.634	0.235	0.9	0.788	0.004	0	0	0.39	0.004	0.96
r	7	20	0	1	0.5	0	8	0	0	0.343	0.121	0.1	0.131	0.002	0	0	0.097	0.003	1
r	7	20	0	1	0.5	1	8	0.421	0.093	0.322	0.084	0.1	0.368	0.001	0.001	0.001	0.314	0.006	0.7
r	7	20	0	1	0.5	0	8	0.774	0.121	1.64	0.206	0.3	0.329	0.008	0	0	0.396	0.021	0.97
r	7	20	0	1	0.5	1	8	0.41	0.092	0.148	0.059	0.3	0.382	0.003	0	0	0.302	0.006	0.97
r	7	20	0	1	0.5	0	8	3.189	0.294	3.487	0.313	0.5	0.498	0.011	0	0	0.714	0.011	1
r	7	20	0	1	0.5	1	8	0.294	0.08	1.948	0.083	0.5	0.503	0.003	0	0	0.196	0.002	1
r	7	20	0	1	0.5	0	8	3.897	0.168	4.282	0.197	0.7	0.529	0.012	0.001	0.001	0.732	0.013	1
r	7	20	0	1	0.5	1	8	1.886	0.235	3.623	0.227	0.7	0.709	0.005	0	0	0.341	0.005	0.97
r	7	20	0	1	0.5	0	8	4.791	0.126	5.07	0.128	0.9	0.521	0.005	0.003	0.003	0.741	0.003	1
r	7	20	0	1	0.5	1	8	6.344	0.214	6.808	0.229	0.9	0.788	0.004	0	0	0.389	0.004	0.97
r	7	30	0	1	0.1	0	8	2.594	0.097	0.375	0.212	0.1	0.143	0.011	0	0	0.065	0.01	0.93
r	7	30	0	1	0.1	1	8	2.449	0	0	0	0.1	0.31	0.001	0	0	0.275	0.003	0.93
r	7	30	0	1	0.1	0	8	1.396	0.152	0.289	0.082	0.3	0.305	0.005	0	0	0.276	0.01	1
r	7	30	0	1	0.1	1	8	2.449	0	0	0	0.3	0.356	0.003	0	0	0.187	0.004	0.94
r	7	30	0	1	0.1	0	8	2.381	0.226	2.248	0.265	0.5	0.465	0.009	0	0	0.583	0.017	1
r	7	30	0	1	0.1	1	8	2.241	0.053	0.085	0.034	0.5	0.511	0.003	0	0	0.149	0.002	0.99
r	7	30	0	1	0.1	0	8	4.231	0.17	4.265	0.108	0.7	0.55	0.009	0	0	0.675	0.007	0.96
r	7	30	0	1	0.1	1	8	2.331	0.158	1.751	0.211	0.7	0.706	0.005	0	0	0.271	0.005	1
r	7	30	0	1	0.1	0	8	4.835	0.186	5.147	0.155	0.9	0.571	0.009	0.002	0.002	0.685	0.005	0.97
r	7	30	0	1	0.1	1	8	6.127	0.195	6.556	0.208	0.9	0.814	0.004	0	0	0.337	0.004	0.9
r	7	30	0	1	0.3	0	8	1.45	0.174	0	0	0.1	0.13	0.002	0	0	0.065	0.003	1
r	7	30	0	1	0.3	1	8	1.876	0.104	0.024	0.024	0.1	0.356	0.001	0.001	0.001	0.249	0.006	1
r	7	30	0	1	0.3	0	8	1.45	0.174	0.729	0.16	0.3	0.3	0.005	0	0	0.268	0.009	0.95
r	7	30	0	1	0.3	1	8	1.905	0.103	0	0	0.3	0.369	0.002	0	0	0.19	0.007	1
r	7	30	0	1	0.3	0	8	2.389	0.235	2.403	0.28	0.5	0.49	0.011	0	0	0.585	0.023	1
r	7	30	0	1	0.3	1	8	1.494	0.12	0.14	0.053	0.5	0.511	0.003	0	0	0.167	0.007	0.98
r	7	30	0	1	0.3	0	8	3.212	0.202	3.928	0.233	0.7	0.56	0.011	0	0	0.689	0.012	0.96
r	7	30	0	1	0.3	1	8	1.475	0.182	1.831	0.208	0.7	0.71	0.005	0	0	0.272	0.005	1
r	7	30	0	1	0.3	0	8	4.792	0.139	5.154	0.135	0.9	0.576	0.009	0.002	0.002	0.681	0.005	1
r	7	30	0	1	0.3	1	8	6.136	0.25	6.632	0.251	0.9	0.808	0.004	0	0	0.34	0.003	0.97

s	7	30	0	1	0.5	0	8	0	0	0.343	0.121	0.1	0.131	0.002	0	0	0.097	0.003	0.98
s	7	30	0	1	0.5	1	8	0.148	0.059	0.049	0.035	0.1	0.371	0	0.001	0.001	0.338	0.004	0.92
s	7	30	0	1	0.5	0	8	0.076	0.043	0.227	0.073	0.3	0.379	0.001	0	0	0.33	0.005	0.9
s	7	30	0	1	0.5	1	8	0.076	0.043	0.227	0.073	0.3	0.379	0.001	0	0	0.29	0.005	1
s	7	30	0	1	0.5	0	8	0	0	0	0	0.5	0.533	0.002	0	0	0.082	0.003	1
s	7	30	0	1	0.5	1	8	0.544	0.096	1.078	0.122	0.5	0.508	0.003	0	0	0.157	0.002	0.99
s	7	30	0	1	0.5	0	8	3.818	0.258	4.357	0.238	0.7	0.576	0.007	0	0	0.693	0.005	0.9
s	7	30	0	1	0.5	1	8	1.012	0.165	3.182	0.206	0.7	0.716	0.005	0	0	0.304	0.005	0.97
s	7	30	0	1	0.5	0	8	4.381	0.143	4.815	0.197	0.9	0.56	0.007	0.002	0.002	0.683	0.009	0.88
s	7	30	0	1	0.5	1	8	5.618	0.253	6.741	0.222	0.9	0.812	0.004	0	0	0.338	0.003	0.98

Appendix 2: Point Estimation for π

part	n	m	D1	D2	True θ	Distance	True π	$m(\hat{\pi})$	$s(\hat{\theta})$	mse_ π	se_mse_ π
a	5	10	0	0	0.1	0	0.1	0.048	0.007	0.000	0.000
a	5	10	0	0	0.3	0	0.1	0.092	0.015	0.000	0.000
a	5	10	0	0	0.5	0	0.1	0.151	0.017	0.000	0.000
a	5	10	0	0	0.7	0	0.1	0.162	0.006	0.000	0.000
a	5	10	0	0	0.9	0	0.1	0.138	0.004	0.000	0.000
a	5	20	0	0	0.1	0	0.1	0.036	0.004	0.000	0.000
a	5	20	0	0	0.3	0	0.1	0.060	0.008	0.000	0.000
a	5	20	0	0	0.5	0	0.1	0.156	0.012	0.000	0.000
a	5	20	0	0	0.7	0	0.1	0.134	0.003	0.000	0.000
a	5	20	0	0	0.9	0	0.1	0.118	0.002	0.000	0.000
a	5	30	0	0	0.1	0	0.1	0.033	0.004	0.000	0.000
a	5	30	0	0	0.3	0	0.1	0.053	0.004	0.000	0.000
a	5	30	0	0	0.5	0	0.1	0.130	0.011	0.000	0.000
a	5	30	0	0	0.7	0	0.1	0.136	0.006	0.000	0.000
a	5	30	0	0	0.9	0	0.1	0.109	0.001	0.000	0.000
a	5	10	0	0	0.1	1	0.1	0.146	0.026	0.000	0.000
a	5	10	0	0	0.3	1	0.1	0.215	0.013	0.000	0.000
a	5	10	0	0	0.5	1	0.1	0.205	0.013	0.000	0.000
a	5	10	0	0	0.7	1	0.1	0.157	0.008	0.000	0.000
a	5	10	0	0	0.9	1	0.1	0.192	0.007	0.000	0.000
a	5	20	0	0	0.1	1	0.1	0.109	0.023	0.000	0.000
a	5	20	0	0	0.3	1	0.1	0.225	0.015	0.000	0.000
a	5	20	0	0	0.5	1	0.1	0.157	0.007	0.000	0.000
a	5	20	0	0	0.7	1	0.1	0.127	0.004	0.000	0.000
a	5	20	0	0	0.9	1	0.1	0.126	0.004	0.000	0.000
a	5	30	0	0	0.1	1	0.1	0.050	0.006	0.000	0.000
a	5	30	0	0	0.3	1	0.1	0.169	0.013	0.000	0.000
a	5	30	0	0	0.5	1	0.1	0.145	0.005	0.000	0.000
a	5	30	0	0	0.7	1	0.1	0.129	0.004	0.000	0.000
a	5	30	0	0	0.9	1	0.1	*0.119*	0.002	0.000	0.000
a	5	10	0	0	0.1	0	0.3	0.295	0.020	0.000	0.000
a	5	10	0	0	0.3	0	0.3	0.316	0.016	0.000	0.000
a	5	10	0	0	0.5	0	0.3	0.392	0.008	0.000	0.000
a	5	10	0	0	0.7	0	0.3	0.347	0.006	0.000	0.000
a	5	10	0	0	0.9	0	0.3	0.345	0.005	0.000	0.000
a	5	20	0	0	0.1	0	0.3	0.236	0.013	0.000	0.000
a	5	20	0	0	0.3	0	0.3	0.317	0.014	0.000	0.000
a	5	20	0	0	0.5	0	0.3	0.409	0.007	0.000	0.000
a	5	20	0	0	0.7	0	0.3	0.338	0.004	0.000	0.000
a	5	20	0	0	0.9	0	0.3	0.317	0.002	0.000	0.000
a	5	30	0	0	0.1	0	0.3	0.255	0.011	0.000	0.000
a	5	30	0	0	0.3	0	0.3	0.348	0.011	0.000	0.000
a	5	30	0	0	0.5	0	0.3	0.411	0.007	0.000	0.000
a	5	30	0	0	0.7	0	0.3	0.331	0.004	0.000	0.000
a	5	30	0	0	0.9	0	0.3	0.310	0.001	0.000	0.000
a	5	10	0	0	0.1	1	0.3	0.308	0.031	0.001	0.000
a	5	10	0	0	0.3	1	0.3	0.251	0.015	0.000	0.000
a	5	10	0	0	0.5	1	0.3	0.212	0.011	0.000	0.000
a	5	10	0	0	0.7	1	0.3	0.207	0.015	0.000	0.000
a	5	10	0	0	0.9	1	0.3	0.148	0.007	0.000	0.000
a	5	20	0	0	0.1	1	0.3	0.303	0.020	0.001	0.001
a	5	20	0	0	0.3	1	0.3	0.246	0.011	0.000	0.000
a	5	20	0	0	0.5	1	0.3	0.167	0.007	0.000	0.000
a	5	20	0	0	0.7	1	0.3	0.124	0.003	0.000	0.000
a	5	20	0	0	0.9	1	0.3	0.151	0.009	0.000	0.000
a	5	30	0	0	0.1	1	0.3	0.364	0.015	0.001	0.001
a	5	30	0	0	0.3	1	0.3	0.254	0.012	0.000	0.000
a	5	30	0	0	0.5	1	0.3	0.141	0.005	0.000	0.000
a	5	30	0	0	0.7	1	0.3	0.122	0.002	0.000	0.000
a	5	30	0	0	0.9	1	0.3	0.116	0.002	0.000	0.000

b	5	10	0	0	0.1	0	0.5	0.473	0.019	0.000	0.000
b	5	10	0	0	0.3	0	0.5	0.459	0.024	0.000	0.000
b	5	10	0	0	0.5	0	0.5	0.440	0.013	0.000	0.000
b	5	10	0	0	0.7	0	0.5	0.490	0.007	0.000	0.000
b	5	10	0	0	0.9	0	0.5	0.498	0.004	0.000	0.000
b	5	20	0	0	0.1	0	0.5	0.470	0.012	0.000	0.000
b	5	20	0	0	0.3	0	0.5	0.335	0.013	0.000	0.000
b	5	20	0	0	0.5	0	0.5	0.430	0.011	0.000	0.000
b	5	20	0	0	0.7	0	0.5	0.490	0.005	0.000	0.000
b	5	20	0	0	0.9	0	0.5	0.499	0.001	0.000	0.000
b	5	30	0	0	0.1	0	0.5	0.487	0.011	0.000	0.000
b	5	30	0	0	0.3	0	0.5	0.348	0.013	0.000	0.000
b	5	30	0	0	0.5	0	0.5	0.445	0.010	0.000	0.000
b	5	30	0	0	0.7	0	0.5	0.496	0.004	0.000	0.000
b	5	30	0	0	0.9	0	0.5	0.499	0.001	0.000	0.000
b	5	10	0	0	0.1	1	0.5	0.339	0.029	0.006	0.006
b	5	10	0	0	0.3	1	0.5	0.306	0.020	0.001	0.001
b	5	10	0	0	0.5	1	0.5	0.208	0.015	0.000	0.000
b	5	10	0	0	0.7	1	0.5	0.172	0.009	0.000	0.000
b	5	10	0	0	0.9	1	0.5	0.142	0.005	0.000	0.000
b	5	20	0	0	0.1	1	0.5	0.388	0.021	0.005	0.005
b	5	20	0	0	0.3	1	0.5	0.283	0.014	0.001	0.001
b	5	20	0	0	0.5	1	0.5	0.157	0.011	0.000	0.000
b	5	20	0	0	0.7	1	0.5	0.138	0.006	0.000	0.000
b	5	20	0	0	0.9	1	0.5	0.139	0.003	0.000	0.000
b	5	30	0	0	0.1	1	0.5	0.365	0.013	0.006	0.006
b	5	30	0	0	0.3	1	0.5	0.212	0.009	0.000	0.000
b	5	30	0	0	0.5	1	0.5	0.148	0.006	0.000	0.000
b	5	30	0	0	0.7	1	0.5	0.117	0.002	0.000	0.000
b	5	30	0	0	0.9	1	0.5	0.125	0.004	0.000	0.000
b	5	10	1	0	0.1	0	0.1	0.080	0.018	0.007	0.007
b	5	10	1	0	0.3	0	0.1	0.341	0.023	0.003	0.003
b	5	10	1	0	0.5	0	0.1	0.303	0.015	0.000	0.000
b	5	10	1	0	0.7	0	0.1	0.167	0.006	0.000	0.000
b	5	10	1	0	0.9	0	0.1	0.144	0.004	0.000	0.000
b	5	20	1	0	0.1	0	0.1	0.035	0.010	0.008	0.008
b	5	20	1	0	0.3	0	0.1	0.282	0.015	0.004	0.004
b	5	20	1	0	0.5	0	0.1	0.241	0.008	0.000	0.000
b	5	20	1	0	0.7	0	0.1	0.141	0.004	0.000	0.000
b	5	20	1	0	0.9	0	0.1	0.116	0.002	0.000	0.000
b	5	30	1	0	0.1	0	0.1	0.029	0.008	0.008	0.008
b	5	30	1	0	0.3	0	0.1	0.281	0.012	0.004	0.004
b	5	30	1	0	0.5	0	0.1	0.275	0.012	0.000	0.000
b	5	30	1	0	0.7	0	0.1	0.137	0.003	0.000	0.000
b	5	30	1	0	0.9	0	0.1	0.110	0.001	0.000	0.000
b	5	10	1	0	0.1	1	0.1	0.337	0.039	0.006	0.006
b	5	10	1	0	0.3	1	0.1	0.309	0.021	0.001	0.001
b	5	10	1	0	0.5	1	0.1	0.212	0.018	0.000	0.000
b	5	10	1	0	0.7	1	0.1	0.176	0.011	0.000	0.000
b	5	10	1	0	0.9	1	0.1	0.174	0.007	0.000	0.000
b	5	10	1	0	0.1	1	0.1	0.289	0.035	0.007	0.007
b	5	20	1	0	0.3	1	0.1	0.244	0.012	0.000	0.000
b	5	20	1	0	0.5	1	0.1	0.157	0.007	0.000	0.000
b	5	20	1	0	0.7	1	0.1	0.150	0.005	0.000	0.000
b	5	20	1	0	0.9	1	0.1	0.125	0.003	0.000	0.000
b	5	30	1	0	0.1	1	0.1	0.418	0.031	0.002	0.002
b	5	30	1	0	0.3	1	0.1	0.220	0.010	0.000	0.000
b	5	30	1	0	0.5	1	0.1	0.144	0.005	0.000	0.000
b	5	30	1	0	0.7	1	0.1	0.125	0.002	0.000	0.000
b	5	30	1	0	0.9	1	0.1	0.112	0.001	0.000	0.000
b	5	10	1	0	0.1	0	0.3	0.313	0.030	0.002	0.002
b	5	10	1	0	0.3	0	0.3	0.359	0.023	0.001	0.001
b	5	10	1	0	0.5	0	0.3	0.456	0.013	0.000	0.000
b	5	10	1	0	0.7	0	0.3	0.355	0.007	0.000	0.000
b	5	10	1	0	0.9	0	0.3	0.342	0.005	0.000	0.000

c	5	20	1	0	0.1	0	0.3	0.372	0.033	0.001	0.001
c	5	20	1	0	0.3	0	0.3	0.373	0.023	0.001	0.001
c	5	20	1	0	0.5	0	0.3	0.438	0.010	0.000	0.000
c	5	20	1	0	0.7	0	0.3	0.332	0.004	0.000	0.000
c	5	20	1	0	0.9	0	0.3	0.321	0.002	0.000	0.000
c	5	30	1	0	0.1	0	0.3	0.410	0.032	0.001	0.001
c	5	30	1	0	0.3	0	0.3	0.355	0.024	0.001	0.001
c	5	30	1	0	0.5	0	0.3	0.449	0.011	0.000	0.000
c	5	30	1	0	0.7	0	0.3	0.338	0.004	0.000	0.000
c	5	30	1	0	0.9	0	0.3	0.310	0.002	0.000	0.000
c	5	10	1	0	0.1	1	0.3	0.450	0.043	0.004	0.004
c	5	10	1	0	0.3	1	0.3	0.294	0.018	0.001	0.001
c	5	10	1	0	0.5	1	0.3	0.210	0.011	0.000	0.000
c	5	10	1	0	0.7	1	0.3	0.203	0.012	0.000	0.000
c	5	10	1	0	0.9	1	0.3	0.163	0.005	0.000	0.000
c	5	20	1	0	0.1	1	0.3	0.459	0.032	0.003	0.003
c	5	20	1	0	0.3	1	0.3	0.277	0.015	0.001	0.001
c	5	20	1	0	0.5	1	0.3	0.168	0.008	0.000	0.000
c	5	20	1	0	0.7	1	0.3	0.140	0.008	0.000	0.000
c	5	20	1	0	0.9	1	0.3	0.133	0.004	0.000	0.000
c	5	30	1	0	0.1	1	0.3	0.484	0.036	0.003	0.003
c	5	30	1	0	0.3	1	0.3	0.291	0.013	0.001	0.001
c	5	30	1	0	0.5	1	0.3	0.148	0.005	0.000	0.000
c	5	30	1	0	0.7	1	0.3	0.118	0.002	0.000	0.000
c	5	30	1	0	0.9	1	0.3	0.121	0.002	0.000	0.000
c	5	10	1	0	0.1	0	0.5	0.413	0.022	0.000	0.000
c	5	10	1	0	0.3	0	0.5	0.361	0.020	0.000	0.000
c	5	10	1	0	0.5	0	0.5	0.456	0.011	0.000	0.000
c	5	10	1	0	0.7	0	0.5	0.483	0.008	0.000	0.000
c	5	10	1	0	0.9	0	0.5	0.500	0.004	0.000	0.000
c	5	20	1	0	0.1	0	0.5	0.471	0.017	0.000	0.000
c	5	20	1	0	0.3	0	0.5	0.358	0.016	0.000	0.000
c	5	20	1	0	0.5	0	0.5	0.499	0.011	0.000	0.000
c	5	20	1	0	0.7	0	0.5	0.489	0.004	0.000	0.000
c	5	20	1	0	0.9	0	0.5	0.495	0.002	0.000	0.000
c	5	30	1	0	0.1	0	0.5	0.471	0.014	0.000	0.000
c	5	30	1	0	0.3	0	0.5	0.366	0.015	0.000	0.000
c	5	30	1	0	0.5	0	0.5	0.495	0.009	0.000	0.000
c	5	30	1	0	0.7	0	0.5	0.488	0.004	0.000	0.000
c	5	30	1	0	0.9	0	0.5	0.499	0.001	0.000	0.000
c	5	10	1	0	0.1	1	0.5	0.437	0.031	0.004	0.004
c	5	10	1	0	0.3	1	0.5	0.300	0.018	0.001	0.001
c	5	10	1	0	0.5	1	0.5	0.192	0.013	0.000	0.000
c	5	10	1	0	0.7	1	0.5	0.164	0.006	0.000	0.000
c	5	10	1	0	0.9	1	0.5	0.165	0.007	0.000	0.000
c	5	20	1	0	0.1	1	0.5	0.448	0.017	0.004	0.004
c	5	20	1	0	0.3	1	0.5	0.247	0.013	0.000	0.000
c	5	20	1	0	0.5	1	0.5	0.274	0.015	0.001	0.001
c	5	20	1	0	0.7	1	0.5	0.129	0.003	0.000	0.000
c	5	20	1	0	0.9	1	0.5	0.143	0.006	0.000	0.000
c	5	30	1	0	0.1	1	0.5	0.416	0.023	0.005	0.005
c	5	30	1	0	0.3	1	0.5	0.256	0.010	0.000	0.000
c	5	30	1	0	0.5	1	0.5	0.138	0.004	0.000	0.000
c	5	30	1	0	0.7	1	0.5	0.119	0.002	0.000	0.000
c	5	30	1	0	0.9	1	0.5	0.116	0.001	0.000	0.000
c	5	10	0	1	0.1	0	0.1	0.006	0.004	0.000	0.000
c	5	10	0	1	0.3	0	0.1	0.104	0.016	0.000	0.000
c	5	10	0	1	0.5	0	0.1	0.214	0.015	0.000	0.000
c	5	10	0	1	0.7	0	0.1	0.178	0.008	0.000	0.000
c	5	10	0	1	0.9	0	0.1	0.148	0.005	0.000	0.000
c	5	20	0	1	0.1	0	0.1	0.012	0.011	0.000	0.000
c	5	20	0	1	0.3	0	0.1	0.053	0.006	0.000	0.000
c	5	20	0	1	0.5	0	0.1	0.147	0.013	0.000	0.000
c	5	20	0	1	0.7	0	0.1	0.134	0.004	0.000	0.000
c	5	20	0	1	0.9	0	0.1	0.118	0.002	0.000	0.000

d	5	30	0	1	0.1	0	0.1	0.052	0.022	0.000	0.000
d	5	30	0	1	0.3	0	0.1	0.047	0.003	0.000	0.000
d	5	30	0	1	0.5	0	0.1	0.129	0.011	0.000	0.000
d	5	30	0	1	0.7	0	0.1	0.125	0.003	0.000	0.000
d	5	30	0	1	0.9	0	0.1	0.110	0.001	0.000	0.000
d	5	10	0	1	0.1	1	0.1	0.111	0.019	0.000	0.000
d	5	10	0	1	0.3	1	0.1	0.250	0.017	0.000	0.000
d	5	10	0	1	0.5	1	0.1	0.213	0.014	0.000	0.000
d	5	10	0	1	0.7	1	0.1	0.157	0.007	0.000	0.000
d	5	10	0	1	0.9	1	0.1	0.184	0.008	0.000	0.000
d	5	10	0	1	0.1	1	0.1	0.067	0.010	0.000	0.000
d	5	20	0	1	0.3	1	0.1	0.184	0.014	0.000	0.000
d	5	20	0	1	0.5	1	0.1	0.135	0.005	0.000	0.000
d	5	20	0	1	0.7	1	0.1	0.137	0.005	0.000	0.000
d	5	20	0	1	0.9	1	0.1	0.149	0.005	0.000	0.000
d	5	30	0	1	0.1	1	0.1	0.051	0.008	0.000	0.000
d	5	30	0	1	0.3	1	0.1	0.174	0.014	0.000	0.000
d	5	30	0	1	0.5	1	0.1	0.147	0.005	0.000	0.000
d	5	30	0	1	0.7	1	0.1	0.127	0.003	0.000	0.000
d	5	30	0	1	0.9	1	0.1	0.113	0.001	0.000	0.000
d	5	10	0	1	0.1	0	0.3	0.193	0.024	0.000	0.000
d	5	10	0	1	0.3	0	0.3	0.242	0.023	0.000	0.000
d	5	10	0	1	0.5	0	0.3	0.375	0.014	0.000	0.000
d	5	10	0	1	0.7	0	0.3	0.356	0.007	0.000	0.000
d	5	10	0	1	0.9	0	0.3	0.338	0.004	0.000	0.000
d	5	20	0	1	0.1	0	0.3	0.104	0.017	0.000	0.000
d	5	20	0	1	0.3	0	0.3	0.195	0.016	0.000	0.000
d	5	20	0	1	0.5	0	0.3	0.342	0.011	0.000	0.000
d	5	20	0	1	0.7	0	0.3	0.328	0.004	0.000	0.000
d	5	20	0	1	0.9	0	0.3	0.315	0.002	0.000	0.000
d	5	30	0	1	0.1	0	0.3	0.108	0.016	0.000	0.000
d	5	30	0	1	0.3	0	0.3	0.170	0.016	0.000	0.000
d	5	30	0	1	0.5	0	0.3	0.337	0.012	0.000	0.000
d	5	30	0	1	0.7	0	0.3	0.329	0.004	0.000	0.000
d	5	30	0	1	0.9	0	0.3	0.312	0.002	0.000	0.000
d	5	10	0	1	0.1	1	0.3	0.180	0.029	0.000	0.000
d	5	10	0	1	0.3	1	0.3	0.294	0.021	0.001	0.001
d	5	10	0	1	0.5	1	0.3	0.233	0.012	0.000	0.000
d	5	10	0	1	0.7	1	0.3	0.178	0.011	0.000	0.000
d	5	10	0	1	0.9	1	0.3	0.150	0.007	0.000	0.000
d	5	20	0	1	0.1	1	0.3	0.199	0.029	0.000	0.000
d	5	20	0	1	0.3	1	0.3	0.243	0.016	0.000	0.000
d	5	20	0	1	0.5	1	0.3	0.195	0.011	0.000	0.000
d	5	20	0	1	0.7	1	0.3	0.137	0.004	0.000	0.000
d	5	20	0	1	0.9	1	0.3	0.125	0.003	0.000	0.000
d	5	30	0	1	0.1	1	0.3	0.173	0.027	0.000	0.000
d	5	30	0	1	0.3	1	0.3	0.247	0.017	0.000	0.000
d	5	30	0	1	0.5	1	0.3	0.151	0.005	0.000	0.000
d	5	30	0	1	0.7	1	0.3	0.123	0.002	0.000	0.000
d	5	30	0	1	0.9	1	0.3	0.113	0.002	0.000	0.000
d	5	10	0	1	0.1	0	0.5	0.497	0.029	0.000	0.000
d	5	10	0	1	0.3	0	0.5	0.448	0.026	0.000	0.000
d	5	10	0	1	0.5	0	0.5	0.490	0.015	0.000	0.000
d	5	10	0	1	0.7	0	0.5	0.485	0.008	0.000	0.000
d	5	10	0	1	0.9	0	0.5	0.496	0.004	0.000	0.000
d	5	20	0	1	0.1	0	0.5	0.482	0.023	0.000	0.000
d	5	20	0	1	0.3	0	0.5	0.438	0.021	0.000	0.000
d	5	20	0	1	0.5	0	0.5	0.471	0.011	0.000	0.000
d	5	20	0	1	0.7	0	0.5	0.495	0.005	0.000	0.000
d	5	20	0	1	0.9	0	0.5	0.500	0.002	0.000	0.000
d	5	30	0	1	0.1	0	0.5	0.498	0.014	0.000	0.000
d	5	30	0	1	0.3	0	0.5	0.457	0.020	0.000	0.000
d	5	30	0	1	0.5	0	0.5	0.498	0.011	0.000	0.000
d	5	30	0	1	0.7	0	0.5	0.495	0.004	0.000	0.000
d	5	30	0	1	0.9	0	0.5	0.499	0.001	0.000	0.000

e	5	10	0	1	0.1	1	0.5	0.434	0.042	0.004	0.004
e	5	10	0	1	0.3	1	0.5	0.340	0.020	0.001	0.001
e	5	10	0	1	0.5	1	0.5	0.184	0.008	0.000	0.000
e	5	10	0	1	0.7	1	0.5	0.204	0.015	0.000	0.000
e	5	10	0	1	0.9	1	0.5	0.177	0.011	0.000	0.000
e	5	20	0	1	0.1	1	0.5	0.482	0.024	0.003	0.003
e	5	20	0	1	0.3	1	0.5	0.274	0.015	0.001	0.001
e	5	20	0	1	0.5	1	0.5	0.180	0.010	0.000	0.000
e	5	20	0	1	0.7	1	0.5	0.123	0.004	0.000	0.000
e	5	20	0	1	0.9	1	0.5	0.128	0.004	0.000	0.000
e	5	30	0	1	0.1	1	0.5	0.400	0.023	0.005	0.005
e	5	30	0	1	0.3	1	0.5	0.283	0.014	0.001	0.001
e	5	30	0	1	0.5	1	0.5	0.144	0.005	0.000	0.000
e	5	30	0	1	0.7	1	0.5	0.117	0.002	0.000	0.000
e	5	30	0	1	0.9	1	0.5	0.117	0.002	0.000	0.000
e	6	10	0	0	0.1	0	0.1	0.038	0.005	0.000	0.000
e	6	10	0	0	0.3	0	0.1	0.127	0.022	0.000	0.000
e	6	10	0	0	0.5	0	0.1	0.191	0.015	0.000	0.000
e	6	10	0	0	0.7	0	0.1	0.168	0.008	0.000	0.000
e	6	10	0	0	0.9	0	0.1	0.146	0.006	0.000	0.000
e	6	20	0	0	0.1	0	0.1	0.036	0.004	0.000	0.000
e	6	20	0	0	0.3	0	0.1	0.057	0.005	0.000	0.000
e	6	20	0	0	0.5	0	0.1	0.112	0.009	0.000	0.000
e	6	20	0	0	0.7	0	0.1	0.141	0.004	0.000	0.000
e	6	20	0	0	0.9	0	0.1	0.117	0.002	0.000	0.000
e	6	30	0	0	0.1	0	0.1	0.037	0.004	0.000	0.000
e	6	30	0	0	0.3	0	0.1	0.051	0.004	0.000	0.000
e	6	30	0	0	0.5	0	0.1	0.097	0.004	0.000	0.000
e	6	30	0	0	0.7	0	0.1	0.128	0.003	0.000	0.000
e	6	30	0	0	0.9	0	0.1	0.110	0.001	0.000	0.000
e	6	10	0	0	0.1	1	0.1	0.081	0.012	0.000	0.000
e	6	10	0	0	0.3	1	0.1	0.200	0.012	0.000	0.000
e	6	10	0	0	0.5	1	0.1	0.226	0.015	0.000	0.000
e	6	10	0	0	0.7	1	0.1	0.185	0.009	0.000	0.000
e	6	10	0	0	0.9	1	0.1	0.169	0.007	0.000	0.000
e	6	10	0	0	0.1	1	0.1	0.039	0.004	0.000	0.000
e	6	20	0	0	0.3	1	0.1	0.177	0.009	0.000	0.000
e	6	20	0	0	0.5	1	0.1	0.157	0.008	0.000	0.000
e	6	20	0	0	0.7	1	0.1	0.137	0.005	0.000	0.000
e	6	20	0	0	0.9	1	0.1	0.121	0.002	0.000	0.000
e	6	30	0	0	0.1	1	0.1	0.191	0.007	0.000	0.000
e	6	30	0	0	0.3	1	0.1	0.165	0.012	0.000	0.000
e	6	30	0	0	0.5	1	0.1	0.142	0.005	0.000	0.000
e	6	30	0	0	0.7	1	0.1	0.122	0.002	0.000	0.000
e	6	30	0	0	0.9	1	0.1	0.113	0.002	0.000	0.000
e	6	10	0	0	0.1	0	0.3	0.246	0.017	0.000	0.000
e	6	10	0	0	0.3	0	0.3	0.335	0.020	0.000	0.000
e	6	10	0	0	0.5	0	0.3	0.394	0.008	0.000	0.000
e	6	10	0	0	0.7	0	0.3	0.364	0.008	0.000	0.000
e	6	10	0	0	0.9	0	0.3	0.334	0.005	0.000	0.000
e	6	20	0	0	0.1	0	0.3	0.258	0.013	0.000	0.000
e	6	20	0	0	0.3	0	0.3	0.313	0.013	0.000	0.000
e	6	20	0	0	0.5	0	0.3	0.402	0.007	0.000	0.000
e	6	20	0	0	0.7	0	0.3	0.350	0.005	0.000	0.000
e	6	20	0	0	0.9	0	0.3	0.318	0.003	0.000	0.000
e	6	30	0	0	0.1	0	0.3	0.236	0.011	0.000	0.000
e	6	30	0	0	0.3	0	0.3	0.459	0.018	0.001	0.001
e	6	30	0	0	0.5	0	0.3	0.420	0.008	0.000	0.000
e	6	30	0	0	0.7	0	0.3	0.334	0.003	0.000	0.000
e	6	30	0	0	0.9	0	0.3	0.310	0.002	0.000	0.000
e	6	10	0	0	0.1	1	0.3	0.367	0.032	0.001	0.001
e	6	10	0	0	0.3	1	0.3	0.258	0.023	0.000	0.000
e	6	10	0	0	0.5	1	0.3	0.215	0.010	0.000	0.000
e	6	10	0	0	0.7	1	0.3	0.168	0.008	0.000	0.000
e	6	10	0	0	0.9	1	0.3	0.170	0.009	0.000	0.000

f	6	20	0	0	0.1	1	0.3	0.385	0.015	0.002	0.002
f	6	20	0	0	0.3	1	0.3	0.229	0.010	0.000	0.000
f	6	20	0	0	0.5	1	0.3	0.148	0.005	0.000	0.000
f	6	20	0	0	0.7	1	0.3	0.131	0.006	0.000	0.000
f	6	20	0	0	0.9	1	0.3	0.134	0.005	0.000	0.000
f	6	30	0	0	0.1	1	0.3	0.339	0.012	0.001	0.001
f	6	30	0	0	0.3	1	0.3	0.236	0.013	0.000	0.000
f	6	30	0	0	0.5	1	0.3	0.135	0.005	0.000	0.000
f	6	30	0	0	0.7	1	0.3	0.125	0.003	0.000	0.000
f	6	30	0	0	0.9	1	0.3	0.114	0.002	0.000	0.000
f	6	10	0	0	0.1	0	0.5	0.472	0.018	0.000	0.000
f	6	10	0	0	0.3	0	0.5	0.388	0.018	0.000	0.000
f	6	10	0	0	0.5	0	0.5	0.422	0.012	0.000	0.000
f	6	10	0	0	0.7	0	0.5	0.495	0.005	0.000	0.000
f	6	10	0	0	0.9	0	0.5	0.497	0.004	0.000	0.000
f	6	20	0	0	0.1	0	0.5	0.492	0.016	0.000	0.000
f	6	20	0	0	0.3	0	0.5	0.353	0.015	0.000	0.000
f	6	20	0	0	0.5	0	0.5	0.439	0.010	0.000	0.000
f	6	20	0	0	0.7	0	0.5	0.481	0.004	0.000	0.000
f	6	20	0	0	0.9	0	0.5	0.500	0.001	0.000	0.000
f	6	30	0	0	0.1	0	0.5	0.496	0.012	0.000	0.000
f	6	30	0	0	0.3	0	0.5	0.366	0.012	0.000	0.000
f	6	30	0	0	0.5	0	0.5	0.422	0.008	0.000	0.000
f	6	30	0	0	0.7	0	0.5	0.481	0.005	0.000	0.000
f	6	30	0	0	0.9	0	0.5	0.498	0.002	0.000	0.000
f	6	10	0	0	0.1	1	0.5	0.402	0.024	0.005	0.005
f	6	10	0	0	0.3	1	0.5	0.288	0.016	0.001	0.001
f	6	10	0	0	0.5	1	0.5	0.207	0.011	0.000	0.000
f	6	10	0	0	0.7	1	0.5	0.176	0.010	0.000	0.000
f	6	10	0	0	0.9	1	0.5	0.192	0.008	0.000	0.000
f	6	20	0	0	0.1	1	0.5	0.337	0.025	0.006	0.006
f	6	20	0	0	0.3	1	0.5	0.237	0.014	0.000	0.000
f	6	20	0	0	0.5	1	0.5	0.257	0.011	0.000	0.000
f	6	20	0	0	0.7	1	0.5	0.144	0.006	0.000	0.000
f	6	20	0	0	0.9	1	0.5	0.122	0.002	0.000	0.000
f	6	30	0	0	0.1	1	0.5	0.268	0.016	0.008	0.008
f	6	30	0	0	0.3	1	0.5	0.255	0.007	0.000	0.000
f	6	30	0	0	0.5	1	0.5	0.149	0.007	0.000	0.000
f	6	30	0	0	0.7	1	0.5	0.121	0.003	0.000	0.000
f	6	30	0	0	0.9	1	0.5	0.109	0.002	0.000	0.000
f	6	10	1	0	0.1	0	0.1	0.059	0.014	0.007	0.007
f	6	10	1	0	0.3	0	0.1	0.346	0.024	0.003	0.003
f	6	10	1	0	0.5	0	0.1	0.286	0.015	0.000	0.000
f	6	10	1	0	0.7	0	0.1	0.156	0.005	0.000	0.000
f	6	10	1	0	0.9	0	0.1	0.149	0.006	0.000	0.000
f	6	20	1	0	0.1	0	0.1	0.024	0.002	0.008	0.008
f	6	20	1	0	0.3	0	0.1	0.253	0.016	0.004	0.004
f	6	20	1	0	0.5	0	0.1	0.284	0.011	0.000	0.000
f	6	20	1	0	0.7	0	0.1	0.140	0.004	0.000	0.000
f	6	20	1	0	0.9	0	0.1	0.118	0.002	0.000	0.000
f	6	30	1	0	0.1	0	0.1	0.021	0.002	0.008	0.008
f	6	30	1	0	0.3	0	0.1	0.377	0.030	0.003	0.003
f	6	30	1	0	0.5	0	0.1	0.289	0.010	0.000	0.000
f	6	30	1	0	0.7	0	0.1	0.125	0.003	0.000	0.000
f	6	30	1	0	0.9	0	0.1	0.112	0.002	0.000	0.000
f	6	10	1	0	0.1	1	0.1	0.201	0.032	0.010	0.010
f	6	10	1	0	0.3	1	0.1	0.340	0.017	0.001	0.001
f	6	10	1	0	0.5	1	0.1	0.198	0.009	0.000	0.000
f	6	10	1	0	0.7	1	0.1	0.208	0.010	0.000	0.000
f	6	10	1	0	0.9	1	0.1	0.207	0.010	0.000	0.000
f	6	20	1	0	0.1	1	0.1	0.156	0.011	0.011	0.011
f	6	20	1	0	0.3	1	0.1	0.204	0.011	0.000	0.000
f	6	20	1	0	0.5	1	0.1	0.123	0.003	0.000	0.000
f	6	20	1	0	0.7	1	0.1	0.129	0.003	0.000	0.000
f	6	20	1	0	0.9	1	0.1	0.129	0.004	0.000	0.000

g	6	30	1	0	0.1	1	0.1	0.175	0.021	0.011	0.010
g	6	30	1	0	0.3	1	0.1	0.259	0.008	0.001	0.000
g	6	30	1	0	0.5	1	0.1	0.131	0.004	0.000	0.000
g	6	30	1	0	0.7	1	0.1	0.123	0.002	0.000	0.000
g	6	30	1	0	0.9	1	0.1	0.109	0.002	0.000	0.000
g	6	10	1	0	0.1	0	0.3	0.348	0.030	0.001	0.001
g	6	10	1	0	0.3	0	0.3	0.349	0.025	0.001	0.001
g	6	10	1	0	0.5	0	0.3	0.461	0.011	0.000	0.000
g	6	10	1	0	0.7	0	0.3	0.345	0.007	0.000	0.000
g	6	10	1	0	0.9	0	0.3	0.339	0.005	0.000	0.000
g	6	20	1	0	0.1	0	0.3	0.380	0.032	0.001	0.001
g	6	20	1	0	0.3	0	0.3	0.349	0.023	0.001	0.001
g	6	20	1	0	0.5	0	0.3	0.446	0.011	0.000	0.000
g	6	20	1	0	0.7	0	0.3	0.346	0.005	0.000	0.000
g	6	20	1	0	0.9	0	0.3	0.316	0.003	0.000	0.000
g	6	30	1	0	0.1	0	0.3	0.344	0.032	0.001	0.001
g	6	30	1	0	0.3	0	0.3	0.371	0.025	0.001	0.001
g	6	30	1	0	0.5	0	0.3	0.446	0.009	0.000	0.000
g	6	30	1	0	0.7	0	0.3	0.339	0.004	0.000	0.000
g	6	30	1	0	0.9	0	0.3	0.309	0.001	0.000	0.000
g	6	10	1	0	0.1	1	0.3	0.440	0.026	0.002	0.002
g	6	10	1	0	0.3	1	0.3	0.317	0.019	0.001	0.001
g	6	10	1	0	0.5	1	0.3	0.221	0.015	0.000	0.000
g	6	10	1	0	0.7	1	0.3	0.170	0.009	0.000	0.000
g	6	10	1	0	0.9	1	0.3	0.172	0.007	0.000	0.000
g	6	20	1	0	0.1	1	0.3	0.473	0.030	0.003	0.003
g	6	20	1	0	0.3	1	0.3	0.239	0.010	0.000	0.000
g	6	20	1	0	0.5	1	0.3	0.173	0.007	0.000	0.000
g	6	20	1	0	0.7	1	0.3	0.157	0.006	0.000	0.000
g	6	20	1	0	0.9	1	0.3	0.136	0.005	0.000	0.000
g	6	30	1	0	0.1	1	0.3	0.379	0.022	0.002	0.002
g	6	30	1	0	0.3	1	0.3	0.230	0.013	0.000	0.000
g	6	30	1	0	0.5	1	0.3	0.135	0.005	0.000	0.000
g	6	30	1	0	0.7	1	0.3	0.127	0.003	0.000	0.000
g	6	30	1	0	0.9	1	0.3	0.116	0.002	0.000	0.000
g	6	10	1	0	0.1	0	0.5	0.371	0.022	0.000	0.000
g	6	10	1	0	0.3	0	0.5	0.324	0.020	0.000	0.000
g	6	10	1	0	0.5	0	0.5	0.451	0.013	0.000	0.000
g	6	10	1	0	0.7	0	0.5	0.497	0.006	0.000	0.000
g	6	10	1	0	0.9	0	0.5	0.497	0.003	0.000	0.000
g	6	20	1	0	0.1	0	0.5	0.466	0.018	0.000	0.000
g	6	20	1	0	0.3	0	0.5	0.411	0.018	0.000	0.000
g	6	20	1	0	0.5	0	0.5	0.478	0.012	0.000	0.000
g	6	20	1	0	0.7	0	0.5	0.488	0.004	0.000	0.000
g	6	20	1	0	0.9	0	0.5	0.500	0.002	0.000	0.000
g	6	30	1	0	0.1	0	0.5	0.474	0.012	0.000	0.000
g	6	30	1	0	0.3	0	0.5	0.425	0.015	0.000	0.000
g	6	30	1	0	0.5	0	0.5	0.497	0.008	0.000	0.000
g	6	30	1	0	0.7	0	0.5	0.484	0.004	0.000	0.000
g	6	30	1	0	0.9	0	0.5	0.500	0.001	0.000	0.000
g	6	10	1	0	0.1	1	0.5	0.340	0.027	0.006	0.006
g	6	10	1	0	0.3	1	0.5	0.268	0.014	0.001	0.001
g	6	10	1	0	0.5	1	0.5	0.215	0.011	0.000	0.000
g	6	10	1	0	0.7	1	0.5	0.164	0.008	0.000	0.000
g	6	10	1	0	0.9	1	0.5	0.176	0.006	0.000	0.000
g	6	20	1	0	0.1	1	0.5	0.460	0.026	0.004	0.004
g	6	20	1	0	0.3	1	0.5	0.237	0.014	0.000	0.000
g	6	20	1	0	0.5	1	0.5	0.170	0.008	0.000	0.000
g	6	20	1	0	0.7	1	0.5	0.133	0.004	0.000	0.000
g	6	20	1	0	0.9	1	0.5	0.137	0.003	0.000	0.000
g	6	30	1	0	0.1	1	0.5	0.322	0.020	0.007	0.007
g	6	30	1	0	0.3	1	0.5	0.257	0.007	0.000	0.000
g	6	30	1	0	0.5	1	0.5	0.145	0.005	0.000	0.000
g	6	30	1	0	0.7	1	0.5	0.117	0.003	0.000	0.000
g	6	30	1	0	0.9	1	0.5	0.118	0.002	0.000	0.000

h	6	10	0	1	0.1	0	0.1	0.010	0.008	0.000	0.000
h	6	10	0	1	0.3	0	0.1	0.071	0.009	0.000	0.000
h	6	10	0	1	0.5	0	0.1	0.143	0.011	0.000	0.000
h	6	10	0	1	0.7	0	0.1	0.170	0.009	0.000	0.000
h	6	10	0	1	0.9	0	0.1	0.150	0.006	0.000	0.000
h	6	20	0	1	0.1	0	0.1	0.002	0.000	0.000	0.000
h	6	20	0	1	0.3	0	0.1	0.054	0.006	0.000	0.000
h	6	20	0	1	0.5	0	0.1	0.119	0.009	0.000	0.000
h	6	20	0	1	0.7	0	0.1	0.140	0.006	0.000	0.000
h	6	20	0	1	0.9	0	0.1	0.118	0.003	0.000	0.000
h	6	30	0	1	0.1	0	0.1	0.001	0.000	0.000	0.000
h	6	30	0	1	0.3	0	0.1	0.048	0.004	0.000	0.000
h	6	30	0	1	0.5	0	0.1	0.110	0.007	0.000	0.000
h	6	30	0	1	0.7	0	0.1	0.118	0.003	0.000	0.000
h	6	30	0	1	0.9	0	0.1	0.112	0.002	0.000	0.000
h	6	10	0	1	0.1	1	0.1	0.039	0.007	0.000	0.000
h	6	10	0	1	0.3	1	0.1	0.218	0.018	0.000	0.000
h	6	10	0	1	0.5	1	0.1	0.236	0.012	0.000	0.000
h	6	10	0	1	0.7	1	0.1	0.205	0.011	0.000	0.000
h	6	10	0	1	0.9	1	0.1	0.166	0.006	0.000	0.000
h	6	20	0	1	0.1	1	0.1	0.051	0.005	0.000	0.000
h	6	20	0	1	0.3	1	0.1	0.199	0.018	0.000	0.000
h	6	20	0	1	0.5	1	0.1	0.154	0.008	0.000	0.000
h	6	20	0	1	0.7	1	0.1	0.161	0.009	0.000	0.000
h	6	20	0	1	0.9	1	0.1	0.129	0.004	0.000	0.000
h	6	30	0	1	0.1	1	0.1	0.040	0.005	0.000	0.000
h	6	30	0	1	0.3	1	0.1	0.137	0.013	0.000	0.000
h	6	30	0	1	0.5	1	0.1	0.131	0.004	0.000	0.000
h	6	30	0	1	0.7	1	0.1	0.114	0.003	0.000	0.000
h	6	30	0	1	0.9	1	0.1	0.116	0.003	0.000	0.000
h	6	10	0	1	0.1	0	0.3	0.160	0.020	0.000	0.000
h	6	10	0	1	0.3	0	0.3	0.230	0.020	0.000	0.000
h	6	10	0	1	0.5	0	0.3	0.369	0.015	0.000	0.000
h	6	10	0	1	0.7	0	0.3	0.350	0.007	0.000	0.000
h	6	10	0	1	0.9	0	0.3	0.334	0.005	0.000	0.000
h	6	20	0	1	0.1	0	0.3	0.112	0.017	0.000	0.000
h	6	20	0	1	0.3	0	0.3	0.159	0.017	0.000	0.000
h	6	20	0	1	0.5	0	0.3	0.361	0.012	0.000	0.000
h	6	20	0	1	0.7	0	0.3	0.327	0.004	0.000	0.000
h	6	20	0	1	0.9	0	0.3	0.316	0.002	0.000	0.000
h	6	30	0	1	0.1	0	0.3	0.092	0.014	0.000	0.000
h	6	30	0	1	0.3	0	0.3	0.227	0.020	0.000	0.000
h	6	30	0	1	0.5	0	0.3	0.346	0.012	0.000	0.000
h	6	30	0	1	0.7	0	0.3	0.328	0.003	0.000	0.000
h	6	30	0	1	0.9	0	0.3	0.314	0.002	0.000	0.000
h	6	10	0	1	0.1	1	0.3	0.114	0.028	0.000	0.000
h	6	10	0	1	0.3	1	0.3	0.267	0.020	0.001	0.001
h	6	10	0	1	0.5	1	0.3	0.204	0.012	0.000	0.000
h	6	10	0	1	0.7	1	0.3	0.193	0.008	0.000	0.000
h	6	10	0	1	0.9	1	0.3	0.185	0.009	0.000	0.000
h	6	20	0	1	0.1	1	0.3	0.183	0.029	0.000	0.000
h	6	20	0	1	0.3	1	0.3	0.214	0.015	0.000	0.000
h	6	20	0	1	0.5	1	0.3	0.141	0.006	0.000	0.000
h	6	20	0	1	0.7	1	0.3	0.141	0.006	0.000	0.000
h	6	20	0	1	0.9	1	0.3	0.129	0.004	0.000	0.000
h	6	30	0	1	0.1	1	0.3	0.151	0.028	0.000	0.000
h	6	30	0	1	0.3	1	0.3	0.188	0.010	0.000	0.000
h	6	30	0	1	0.5	1	0.3	0.139	0.005	0.000	0.000
h	6	30	0	1	0.7	1	0.3	0.120	0.002	0.000	0.000
h	6	30	0	1	0.9	1	0.3	0.115	0.001	0.000	0.000
h	6	10	0	1	0.1	0	0.5	0.487	0.028	0.000	0.000
h	6	10	0	1	0.3	0	0.5	0.490	0.030	0.000	0.000
h	6	10	0	1	0.5	0	0.5	0.487	0.015	0.000	0.000
h	6	10	0	1	0.7	0	0.5	0.483	0.007	0.000	0.000
h	6	10	0	1	0.9	0	0.5	0.499	0.003	0.000	0.000

i	6	20	0	1	0.1	0	0.5	0.484	0.026	0.000	0.000
i	6	20	0	1	0.3	0	0.5	0.492	0.023	0.000	0.000
i	6	20	0	1	0.5	0	0.5	0.498	0.014	0.000	0.000
i	6	20	0	1	0.7	0	0.5	0.480	0.005	0.000	0.000
i	6	20	0	1	0.9	0	0.5	0.500	0.001	0.000	0.000
i	6	30	0	1	0.1	0	0.5	0.482	0.012	0.000	0.000
i	6	30	0	1	0.3	0	0.5	0.457	0.014	0.000	0.000
i	6	30	0	1	0.5	0	0.5	0.457	0.009	0.000	0.000
i	6	30	0	1	0.7	0	0.5	0.486	0.003	0.000	0.000
i	6	30	0	1	0.9	0	0.5	0.499	0.001	0.000	0.000
i	6	10	0	1	0.1	1	0.5	0.434	0.037	0.002	0.002
i	6	10	0	1	0.3	1	0.5	0.288	0.017	0.001	0.001
i	6	10	0	1	0.5	1	0.5	0.197	0.011	0.000	0.000
i	6	10	0	1	0.7	1	0.5	0.198	0.008	0.000	0.000
i	6	10	0	1	0.9	1	0.5	0.214	0.009	0.000	0.000
i	6	20	0	1	0.1	1	0.5	0.438	0.020	0.004	0.004
i	6	20	0	1	0.3	1	0.5	0.272	0.015	0.001	0.001
i	6	20	0	1	0.5	1	0.5	0.162	0.005	0.000	0.000
i	6	20	0	1	0.7	1	0.5	0.140	0.004	0.000	0.000
i	6	20	0	1	0.9	1	0.5	0.145	0.005	0.000	0.000
i	6	30	0	1	0.1	1	0.5	0.369	0.028	0.006	0.006
i	6	30	0	1	0.3	1	0.5	0.268	0.010	0.001	0.001
i	6	30	0	1	0.5	1	0.5	0.148	0.007	0.000	0.000
i	6	30	0	1	0.7	1	0.5	0.123	0.004	0.000	0.000
i	6	30	0	1	0.9	1	0.5	0.117	0.002	0.000	0.000
i	7	10	0	0	0.1	0	0.1	0.052	0.008	0.000	0.000
i	7	10	0	0	0.3	0	0.1	0.069	0.008	0.000	0.000
i	7	10	0	0	0.5	0	0.1	0.158	0.012	0.000	0.000
i	7	10	0	0	0.7	0	0.1	0.161	0.008	0.000	0.000
i	7	10	0	0	0.9	0	0.1	0.154	0.007	0.000	0.000
i	7	20	0	0	0.1	0	0.1	0.029	0.004	0.000	0.000
i	7	20	0	0	0.3	0	0.1	0.047	0.005	0.000	0.000
i	7	20	0	0	0.5	0	0.1	0.136	0.012	0.000	0.000
i	7	20	0	0	0.7	0	0.1	0.121	0.009	0.000	0.000
i	7	20	0	0	0.9	0	0.1	0.115	0.002	0.000	0.000
i	7	30	0	0	0.1	0	0.1	0.028	0.003	0.000	0.000
i	7	30	0	0	0.3	0	0.1	0.082	0.013	0.000	0.000
i	7	30	0	0	0.5	0	0.1	0.100	0.007	0.000	0.000
i	7	30	0	0	0.7	0	0.1	0.114	0.002	0.000	0.000
i	7	30	0	0	0.9	0	0.1	0.238	0.027	0.000	0.000
i	7	10	0	0	0.1	1	0.1	0.066	0.011	0.000	0.000
i	7	10	0	0	0.3	1	0.1	0.217	0.019	0.000	0.000
i	7	10	0	0	0.5	1	0.1	0.210	0.014	0.000	0.000
i	7	10	0	0	0.7	1	0.1	0.210	0.014	0.000	0.000
i	7	10	0	0	0.9	1	0.1	0.186	0.007	0.000	0.000
i	7	20	0	0	0.1	1	0.1	0.082	0.009	0.000	0.000
i	7	20	0	0	0.3	1	0.1	0.180	0.010	0.000	0.000
i	7	20	0	0	0.5	1	0.1	0.207	0.025	0.000	0.000
i	7	20	0	0	0.7	1	0.1	0.158	0.013	0.002	0.002
i	7	20	0	0	0.9	1	0.1	0.132	0.007	0.000	0.000
i	7	30	0	0	0.1	1	0.1	0.061	0.007	0.000	0.000
i	7	30	0	0	0.3	1	0.1	0.145	0.007	0.000	0.000
i	7	30	0	0	0.5	1	0.1	0.137	0.006	0.000	0.000
i	7	30	0	0	0.7	1	0.1	0.117	0.003	0.000	0.000
i	7	30	0	0	0.9	1	0.1	0.118	0.002	0.000	0.000
i	7	10	0	0	0.1	0	0.3	0.267	0.018	0.000	0.000
i	7	10	0	0	0.3	0	0.3	0.321	0.020	0.000	0.000
i	7	10	0	0	0.5	0	0.3	0.466	0.016	0.000	0.000
i	7	10	0	0	0.7	0	0.3	0.350	0.008	0.000	0.000
i	7	10	0	0	0.9	0	0.3	0.346	0.005	0.000	0.000
i	7	20	0	0	0.1	0	0.3	0.234	0.011	0.000	0.000
i	7	20	0	0	0.3	0	0.3	0.263	0.012	0.000	0.000
i	7	20	0	0	0.5	0	0.3	0.434	0.037	0.001	0.001
i	7	20	0	0	0.7	0	0.3	0.332	0.005	0.000	0.000
i	7	20	0	0	0.9	0	0.3	0.316	0.003	0.000	0.000

j	7	30	0	0	0.1	0	0.3	0.230	0.010	0.000	0.000
i	7	30	0	0	0.3	0	0.3	0.291	0.011	0.000	0.000
j	7	30	0	0	0.5	0	0.3	0.451	0.010	0.000	0.000
i	7	30	0	0	0.7	0	0.3	0.324	0.003	0.000	0.000
j	7	30	0	0	0.9	0	0.3	0.309	0.002	0.000	0.000
i	7	10	0	0	0.1	1	0.3	0.344	0.020	0.000	0.000
j	7	10	0	0	0.3	1	0.3	0.383	0.010	0.000	0.000
i	7	10	0	0	0.5	1	0.3	0.224	0.012	0.000	0.000
j	7	10	0	0	0.7	1	0.3	0.371	0.008	0.000	0.000
i	7	10	0	0	0.9	1	0.3	0.362	0.007	0.000	0.000
j	7	20	0	0	0.1	1	0.3	0.362	0.020	0.000	0.000
i	7	20	0	0	0.3	1	0.3	0.428	0.013	0.000	0.000
j	7	20	0	0	0.5	1	0.3	0.428	0.013	0.000	0.000
i	7	20	0	0	0.7	1	0.3	0.491	0.009	0.000	0.000
j	7	20	0	0	0.9	1	0.3	0.338	0.005	0.000	0.000
i	7	30	0	0	0.1	1	0.3	0.325	0.005	0.000	0.000
j	7	30	0	0	0.3	1	0.3	0.328	0.014	0.000	0.000
i	7	30	0	0	0.5	1	0.3	0.328	0.005	0.000	0.000
j	7	30	0	0	0.7	1	0.3	0.323	0.004	0.000	0.000
i	7	30	0	0	0.9	1	0.3	0.314	0.002	0.000	0.000
j	7	10	0	0	0.1	0	0.5	0.468	0.020	0.000	0.000
i	7	10	0	0	0.3	0	0.5	0.413	0.019	0.000	0.000
j	7	10	0	0	0.5	0	0.5	0.421	0.016	0.000	0.000
i	7	10	0	0	0.7	0	0.5	0.488	0.005	0.000	0.000
j	7	10	0	0	0.9	0	0.5	0.496	0.004	0.000	0.000
i	7	20	0	0	0.1	0	0.5	0.499	0.017	0.000	0.000
j	7	20	0	0	0.3	0	0.5	0.381	0.013	0.000	0.000
i	7	20	0	0	0.5	0	0.5	0.429	0.011	0.000	0.000
j	7	20	0	0	0.7	0	0.5	0.427	0.014	0.000	0.000
i	7	20	0	0	0.9	0	0.5	0.498	0.002	0.000	0.000
j	7	30	0	0	0.1	0	0.5	0.483	0.013	0.000	0.000
i	7	30	0	0	0.3	0	0.5	0.395	0.011	0.000	0.000
j	7	30	0	0	0.5	0	0.5	0.395	0.011	0.000	0.000
i	7	30	0	0	0.7	0	0.5	0.481	0.003	0.000	0.000
j	7	30	0	0	0.9	0	0.5	0.500	0.000	0.000	0.000
i	7	10	0	0	0.1	1	0.5	0.339	0.020	0.001	0.001
j	7	10	0	0	0.3	1	0.5	0.441	0.024	0.000	0.000
i	7	10	0	0	0.5	1	0.5	0.481	0.012	0.000	0.000
j	7	10	0	0	0.7	1	0.5	0.499	0.006	0.000	0.000
i	7	10	0	0	0.9	1	0.5	0.495	0.004	0.000	0.000
j	7	20	0	0	0.1	1	0.5	0.292	0.019	0.001	0.001
i	7	20	0	0	0.3	1	0.5	0.462	0.015	0.000	0.000
j	7	20	0	0	0.5	1	0.5	0.488	0.006	0.000	0.000
i	7	20	0	0	0.7	1	0.5	0.489	0.004	0.000	0.000
j	7	20	0	0	0.9	1	0.5	0.497	0.002	0.000	0.000
i	7	30	0	0	0.1	1	0.5	0.340	0.016	0.001	0.001
j	7	30	0	0	0.3	1	0.5	0.340	0.016	0.001	0.001
i	7	30	0	0	0.5	1	0.5	0.493	0.005	0.000	0.000
j	7	30	0	0	0.7	1	0.5	0.492	0.002	0.000	0.000
i	7	30	0	0	0.9	1	0.5	0.500	0.000	0.000	0.000
j	7	10	1	0	0.1	0	0.1	0.052	0.013	0.007	0.007
i	7	10	1	0	0.3	0	0.1	0.272	0.021	0.004	0.004
j	7	10	1	0	0.5	0	0.1	0.313	0.015	0.000	0.000
i	7	10	1	0	0.7	0	0.1	0.173	0.008	0.000	0.000
j	7	10	1	0	0.9	0	0.1	0.135	0.005	0.000	0.000
i	7	20	1	0	0.1	0	0.1	0.026	0.003	0.008	0.008
j	7	20	1	0	0.3	0	0.1	0.237	0.013	0.004	0.004
i	7	20	1	0	0.5	0	0.1	0.285	0.012	0.000	0.000
j	7	20	1	0	0.7	0	0.1	0.137	0.005	0.000	0.000
i	7	20	1	0	0.9	0	0.1	0.115	0.002	0.000	0.000
j	7	30	1	0	0.1	0	0.1	0.023	0.002	0.008	0.008
i	7	30	1	0	0.3	0	0.1	0.244	0.011	0.004	0.004
j	7	30	1	0	0.5	0	0.1	0.277	0.008	0.000	0.000
i	7	30	1	0	0.7	0	0.1	0.132	0.004	0.000	0.000
j	7	30	1	0	0.9	0	0.1	0.113	0.002	0.000	0.000

k	7	10	1	0	0.1	1	0.1	0.290	0.040	0.007	0.007
k	7	10	1	0	0.3	1	0.1	0.246	0.016	0.000	0.000
k	7	10	1	0	0.5	1	0.1	0.240	0.015	0.000	0.000
k	7	10	1	0	0.7	1	0.1	0.240	0.015	0.000	0.000
k	7	10	1	0	0.9	1	0.1	0.181	0.010	0.000	0.000
k	7	20	1	0	0.1	1	0.1	0.171	0.024	0.011	0.011
k	7	20	1	0	0.3	1	0.1	0.227	0.011	0.000	0.000
k	7	20	1	0	0.5	1	0.1	0.207	0.007	0.000	0.000
k	7	20	1	0	0.7	1	0.1	0.125	0.003	0.000	0.000
k	7	20	1	0	0.9	1	0.1	0.123	0.004	0.000	0.000
k	7	30	1	0	0.1	1	0.1	0.148	0.020	0.011	0.011
k	7	30	1	0	0.3	1	0.1	0.205	0.008	0.000	0.000
k	7	30	1	0	0.5	1	0.1	0.134	0.003	0.000	0.000
k	7	30	1	0	0.7	1	0.1	0.135	0.004	0.000	0.000
k	7	30	1	0	0.9	1	0.1	0.121	0.003	0.000	0.000
k	7	10	1	0	0.1	0	0.3	0.304	0.045	0.004	0.004
k	7	10	1	0	0.3	0	0.3	0.358	0.024	0.001	0.001
k	7	10	1	0	0.5	0	0.3	0.490	0.014	0.000	0.000
k	7	10	1	0	0.7	0	0.3	0.356	0.007	0.000	0.000
k	7	10	1	0	0.9	0	0.3	0.345	0.006	0.000	0.000
k	7	20	1	0	0.1	0	0.3	0.411	0.029	0.001	0.001
k	7	20	1	0	0.3	0	0.3	0.319	0.023	0.001	0.001
k	7	20	1	0	0.5	0	0.3	0.487	0.012	0.000	0.000
k	7	20	1	0	0.7	0	0.3	0.322	0.005	0.000	0.000
k	7	20	1	0	0.9	0	0.3	0.372	0.011	0.000	0.000
k	7	30	1	0	0.1	0	0.3	0.350	0.033	0.001	0.001
k	7	30	1	0	0.3	0	0.3	0.339	0.023	0.001	0.001
k	7	30	1	0	0.5	0	0.3	0.459	0.010	0.000	0.000
k	7	30	1	0	0.7	0	0.3	0.325	0.004	0.000	0.000
k	7	30	1	0	0.9	0	0.3	0.311	0.311	0.000	0.000
k	7	10	1	0	0.1	1	0.3	0.437	0.038	0.001	0.001
k	7	10	1	0	0.3	1	0.3	0.451	0.017	0.000	0.000
k	7	10	1	0	0.5	1	0.3	0.186	0.010	0.000	0.000
k	7	10	1	0	0.7	1	0.3	0.362	0.006	0.000	0.000
k	7	10	1	0	0.9	1	0.3	0.369	0.008	0.000	0.000
k	7	20	1	0	0.1	1	0.3	0.428	0.038	0.002	0.001
k	7	20	1	0	0.3	1	0.3	0.395	0.014	0.000	0.000
k	7	20	1	0	0.5	1	0.3	0.462	0.011	0.000	0.000
k	7	20	1	0	0.7	1	0.3	0.350	0.004	0.000	0.000
k	7	20	1	0	0.9	1	0.3	0.318	0.003	0.000	0.000
k	7	30	1	0	0.1	1	0.3	0.409	0.028	0.000	0.000
k	7	30	1	0	0.3	1	0.3	0.389	0.015	0.000	0.000
k	7	30	1	0	0.5	1	0.3	0.346	0.008	0.000	0.000
k	7	30	1	0	0.7	1	0.3	0.336	0.004	0.000	0.000
k	7	30	1	0	0.9	1	0.3	0.323	0.003	0.000	0.000
k	7	10	1	0	0.1	0	0.5	0.414	0.022	0.000	0.000
k	7	10	1	0	0.3	0	0.5	0.359	0.019	0.000	0.000
k	7	10	1	0	0.5	0	0.5	0.459	0.015	0.000	0.000
k	7	10	1	0	0.7	0	0.5	0.497	0.006	0.000	0.000
k	7	10	1	0	0.9	0	0.5	0.495	0.005	0.000	0.000
k	7	20	1	0	0.1	0	0.5	0.448	0.019	0.000	0.000
k	7	20	1	0	0.3	0	0.5	0.406	0.016	0.000	0.000
k	7	20	1	0	0.5	0	0.5	0.479	0.012	0.000	0.000
k	7	20	1	0	0.7	0	0.5	0.487	0.003	0.000	0.000
k	7	20	1	0	0.9	0	0.5	0.498	0.001	0.000	0.000
k	7	30	1	0	0.1	0	0.5	0.456	0.010	0.000	0.000
k	7	30	1	0	0.3	0	0.5	0.426	0.016	0.000	0.000
k	7	30	1	0	0.5	0	0.5	0.486	0.009	0.000	0.000
k	7	30	1	0	0.7	0	0.5	0.473	0.004	0.000	0.000
k	7	30	1	0	0.9	0	0.5	0.499	0.001	0.000	0.000
k	7	10	1	0	0.1	1	0.5	0.426	0.021	0.000	0.000
k	7	10	1	0	0.3	1	0.5	0.421	0.013	0.000	0.000
k	7	10	1	0	0.5	1	0.5	0.362	0.006	0.000	0.000
k	7	10	1	0	0.7	1	0.5	0.499	0.004	0.000	0.000
k	7	10	1	0	0.9	1	0.5	0.486	0.005	0.000	0.000

l	7	20	1	0	0.1	1	0.5	0.454	0.022	0.000	0.000
l	7	20	1	0	0.3	1	0.5	0.446	0.013	0.000	0.000
l	7	20	1	0	0.5	1	0.5	0.490	0.005	0.000	0.000
l	7	20	1	0	0.7	1	0.5	0.499	0.003	0.000	0.000
l	7	20	1	0	0.9	1	0.5	0.498	0.002	0.000	0.000
l	7	30	1	0	0.1	1	0.5	0.449	0.013	0.000	0.000
l	7	30	1	0	0.3	1	0.5	0.449	0.017	0.000	0.000
l	7	30	1	0	0.5	1	0.5	0.493	0.005	0.000	0.000
l	7	30	1	0	0.7	1	0.5	0.477	0.008	0.000	0.000
l	7	30	1	0	0.9	1	0.5	0.487	0.004	0.000	0.000
l	7	10	0	1	0.1	0	0.1	0.014	0.007	0.000	0.000
l	7	10	0	1	0.3	0	0.1	0.093	0.012	0.000	0.000
l	7	10	0	1	0.5	0	0.1	0.162	0.014	0.000	0.000
l	7	10	0	1	0.7	0	0.1	0.157	0.007	0.000	0.000
l	7	10	0	1	0.9	0	0.1	0.138	0.005	0.000	0.000
l	7	20	0	1	0.1	0	0.1	0.001	0.000	0.000	0.000
l	7	20	0	1	0.3	0	0.1	0.048	0.005	0.000	0.000
l	7	20	0	1	0.5	0	0.1	0.105	0.008	0.000	0.000
l	7	20	0	1	0.7	0	0.1	0.139	0.006	0.000	0.000
l	7	20	0	1	0.9	0	0.1	0.117	0.002	0.000	0.000
l	7	30	0	1	0.1	0	0.1	0.001	0.000	0.000	0.000
l	7	30	0	1	0.3	0	0.1	0.051	0.005	0.000	0.000
l	7	30	0	1	0.5	0	0.1	0.089	0.006	0.000	0.000
l	7	30	0	1	0.7	0	0.1	0.122	0.004	0.000	0.000
l	7	30	0	1	0.9	0	0.1	0.111	0.002	0.000	0.000
l	7	10	0	1	0.1	1	0.1	0.043	0.006	0.000	0.000
l	7	10	0	1	0.3	1	0.1	0.159	0.015	0.000	0.000
l	7	10	0	1	0.5	1	0.1	0.222	0.014	0.000	0.000
l	7	10	0	1	0.7	1	0.1	0.172	0.010	0.000	0.000
l	7	10	0	1	0.9	1	0.1	0.172	0.010	0.000	0.000
l	7	20	0	1	0.1	1	0.1	0.042	0.010	0.000	0.000
l	7	20	0	1	0.3	1	0.1	0.168	0.012	0.000	0.000
l	7	20	0	1	0.5	1	0.1	0.168	0.012	0.000	0.000
l	7	20	0	1	0.7	1	0.1	0.143	0.007	0.000	0.000
l	7	20	0	1	0.9	1	0.1	0.136	0.004	0.000	0.000
l	7	30	0	1	0.1	1	0.1	0.089	0.020	0.000	0.000
l	7	30	0	1	0.3	1	0.1	0.171	0.010	0.000	0.000
l	7	30	0	1	0.5	1	0.1	0.143	0.006	0.000	0.000
l	7	30	0	1	0.7	1	0.1	0.123	0.003	0.000	0.000
l	7	30	0	1	0.9	1	0.1	0.122	0.002	0.000	0.000
l	7	10	0	1	0.1	0	0.3	0.152	0.020	0.000	0.000
l	7	10	0	1	0.3	0	0.3	0.263	0.019	0.000	0.000
l	7	10	0	1	0.5	0	0.3	0.386	0.018	0.000	0.000
l	7	10	0	1	0.7	0	0.3	0.351	0.006	0.000	0.000
l	7	10	0	1	0.9	0	0.3	0.329	0.004	0.000	0.000
l	7	20	0	1	0.1	0	0.3	0.111	0.017	0.000	0.000
l	7	20	0	1	0.3	0	0.3	0.117	0.014	0.000	0.000
l	7	20	0	1	0.5	0	0.3	0.330	0.015	0.000	0.000
l	7	20	0	1	0.7	0	0.3	0.319	0.005	0.000	0.000
l	7	20	0	1	0.9	0	0.3	0.321	0.003	0.000	0.000
l	7	30	0	1	0.1	0	0.3	0.113	0.016	0.000	0.000
l	7	30	0	1	0.3	0	0.3	0.141	0.015	0.000	0.000
l	7	30	0	1	0.5	0	0.3	0.338	0.012	0.000	0.000
l	7	30	0	1	0.7	0	0.3	0.320	0.004	0.000	0.000
l	7	30	0	1	0.9	0	0.3	0.309	0.002	0.000	0.000
l	7	10	0	1	0.1	1	0.3	0.244	0.028	0.000	0.000
l	7	10	0	1	0.3	1	0.3	0.400	0.011	0.000	0.000
l	7	10	0	1	0.5	1	0.3	0.217	0.015	0.000	0.000
l	7	10	0	1	0.7	1	0.3	0.373	0.008	0.000	0.000
l	7	10	0	1	0.9	1	0.3	0.364	0.008	0.000	0.000
l	7	20	0	1	0.1	1	0.3	0.190	0.026	0.000	0.000
l	7	20	0	1	0.3	1	0.3	0.369	0.012	0.000	0.000
l	7	20	0	1	0.5	1	0.3	0.487	0.009	0.000	0.000
l	7	20	0	1	0.7	1	0.3	0.328	0.004	0.000	0.000
l	7	20	0	1	0.9	1	0.3	0.315	0.004	0.000	0.000

m	7	30	0	1	0.1	1	0.3	0.194	0.027	0.000	0.000
m	7	30	0	1	0.3	1	0.3	0.345	0.013	0.000	0.000
m	7	30	0	1	0.5	1	0.3	0.339	0.007	0.000	0.000
m	7	30	0	1	0.7	1	0.3	0.326	0.002	0.000	0.000
m	7	30	0	1	0.9	1	0.3	0.309	0.002	0.000	0.000
m	7	10	0	1	0.1	0	0.5	0.479	0.028	0.000	0.000
m	7	10	0	1	0.3	0	0.5	0.468	0.027	0.000	0.000
m	7	10	0	1	0.5	0	0.5	0.499	0.017	0.000	0.000
m	7	10	0	1	0.7	0	0.5	0.484	0.006	0.000	0.000
m	7	10	0	1	0.9	0	0.5	0.498	0.003	0.000	0.000
m	7	20	0	1	0.1	0	0.5	0.476	0.025	0.000	0.000
m	7	20	0	1	0.3	0	0.5	0.460	0.021	0.000	0.000
m	7	20	0	1	0.5	0	0.5	0.490	0.012	0.000	0.000
m	7	20	0	1	0.7	0	0.5	0.485	0.005	0.000	0.000
m	7	20	0	1	0.9	0	0.5	0.500	0.001	0.000	0.000
m	7	30	0	1	0.1	0	0.5	0.479	0.015	0.000	0.000
m	7	30	0	1	0.3	0	0.5	0.470	0.016	0.000	0.000
m	7	30	0	1	0.5	0	0.5	0.480	0.014	0.000	0.000
m	7	30	0	1	0.7	0	0.5	0.485	0.003	0.000	0.000
m	7	30	0	1	0.9	0	0.5	0.500	0.000	0.000	0.000
m	7	10	0	1	0.1	1	0.5	0.390	0.030	0.000	0.000
m	7	10	0	1	0.3	1	0.5	0.433	0.019	0.000	0.000
m	7	10	0	1	0.5	1	0.5	0.456	0.013	0.000	0.000
m	7	10	0	1	0.7	1	0.5	0.453	0.013	0.000	0.000
m	7	10	0	1	0.9	1	0.5	0.462	0.011	0.000	0.000
m	7	20	0	1	0.1	1	0.5	0.484	0.009	0.000	0.000
m	7	20	0	1	0.3	1	0.5	0.488	0.008	0.000	0.000
m	7	20	0	1	0.5	1	0.5	0.489	0.008	0.000	0.000
m	7	20	0	1	0.7	1	0.5	0.490	0.007	0.000	0.000
m	7	20	0	1	0.9	1	0.5	0.491	0.006	0.000	0.000
m	7	30	0	1	0.1	1	0.5	0.481	0.007	0.000	0.000
m	7	30	0	1	0.3	1	0.5	0.491	0.006	0.000	0.000
m	7	30	0	1	0.5	1	0.5	0.493	0.006	0.000	0.000
m	7	30	0	1	0.7	1	0.5	0.493	0.006	0.000	0.000
m	7	30	0	1	0.9	1	0.5	0.494	0.005	0.000	0.000

Appendix 3: Point and Profile likelihood Set Estimation for θ when m=8, 9

part	m	D_1	D_2	True π	Distance	d	d_1	sd_1	d_2	sd_2	True θ	$m(\hat{\theta})$	$s(\hat{\theta})$	mse_t	se_mse_t	t_wid	se_t_wid	cov_t
a	8	0	0	0.1	0	2	0	0	0	0	0.1	0.111	0.003	0	0	0.12	0.006	1
a	8	0	0	0.1	1	2	0	0	0	0	0.1	0.304	0	0.002	0.002	0.4	0	1
a	8	0	0	0.1	0	2	4.365	0.266	4.434	0.38	0.5	0.401	0.016	0	0	0.74	0.028	0.85
a	8	0	0	0.1	1	2	0	0	0.495	0.155	0.5	0.483	0.011	0	0	0.28	0.011	1
a	8	0	0	0.1	0	2	5.571	0.112	6.015	0.041	0.9	0.434	0.011	0.011	0.011	0.8	0.006	0.8
a	8	0	0	0.1	1	2	4.179	0.311	4.172	0.29	0.9	0.77	0.012	0.001	0.001	0.44	0.005	1
a	8	0	0	0.3	0	2	5.826	0.067	4.372	0.196	0.1	0.111	0.003	0	0	0.12	0.006	1
a	8	0	0	0.3	1	2	0	0	0	0	0.1	0.304	0	0.002	0.002	0.4	0	1
a	8	0	0	0.3	0	2	5.619	0.088	4.547	0.228	0.5	0.421	0.017	0	0	0.75	0.024	0.6
a	8	0	0	0.3	1	2	0	0	0.521	0.161	0.5	0.481	0.011	0	0	0.28	0.011	1
a	8	0	0	0.3	0	2	5.347	0.091	6.05	0.043	0.9	0.42	0.005	0.012	0.012	0.81	0.004	0.75
a	8	0	0	0.3	1	2	4.806	0.145	5.099	0	0.9	0.801	0.003	0	0	0.44	0.005	1
a	8	0	0	0.5	0	2	0	0	0.283	0.13	0.1	0.117	0.002	0	0	0.13	0.006	1
a	8	0	0	0.5	1	2	0	0	0	0	0.1	0.304	0	0.002	0.002	0.4	0	1
a	8	0	0	0.5	0	2	2.924	0.401	4.054	0.56	0.5	0.378	0.008	0.001	0.001	0.6	0.019	0.85
a	8	0	0	0.5	1	2	0	0	0.495	0.155	0.5	0.503	0.008	0	0	0.27	0.005	1
a	8	0	0	0.5	0	2	5.219	0.087	5.414	0.121	0.9	0.414	0.01	0.012	0.012	0.79	0.012	0.8
a	8	0	0	0.5	1	2	4.468	0.043	4.656	0.079	0.9	0.782	0.003	0.001	0.001	0.4	0	0.9
a	9	0	0	0.1	0	2	2.449	0	2.449	0	0.1	0.115	0.004	0	0	0.13	0.009	1
a	9	0	0	0.1	1	2	0	0	0	0	0.1	0.322	0	0.003	0.003	0.4	0	1
a	9	0	0	0.1	0	2	5.274	0.123	5.764	0.098	0.5	0.412	0.009	0.005	0.005	0.74	0.012	1
a	9	0	0	0.1	1	2	0	0	0	0	0.5	0.47	0.007	0	0	0.27	0.01	1
a	9	0	0	0.1	0	2	5.102	0.147	5.613	0.17	0.9	0.459	0.004	0.007	0.007	0.78	0.008	0.85
a	9	0	0	0.1	1	2	4.915	0.135	4.747	0.092	0.9	0.773	0.005	0.001	0.001	0.41	0.006	0.9
a	9	0	0	0.3	0	2	1.031	0.285	1.418	0.285	0.1	0.121	0.003	0	0	0.12	0.006	1
a	9	0	0	0.3	1	2	0	0	0	0	0.1	0.322	0	0.002	0.002	0.4	0	1
a	9	0	0	0.3	0	2	4.656	0.261	5.257	0.217	0.5	0.431	0.018	0.005	0.005	0.74	0.03	0.9
a	9	0	0	0.3	1	2	0	0	0	0	0.5	0.471	0.008	0	0	0.28	0.018	0.85
a	9	0	0	0.3	0	2	5.663	0.083	5.734	0.108	0.9	0.454	0.005	0.006	0.006	0.76	0.011	0.7
a	9	0	0	0.3	1	2	3.695	0.211	4.842	0.09	0.9	0.76	0.007	0.001	0.001	0.37	0.005	0.95
a	9	0	0	0.5	0	2	0.49	0.225	1.592	0.268	0.1	0.131	0.005	0	0	0.14	0.008	1
a	9	0	0	0.5	1	2	0	0	0	0	0.1	0.322	0	0.002	0.002	0.4	0	1
a	9	0	0	0.5	0	2	3.952	0.378	4.76	0.385	0.5	0.372	0.027	0.004	0.003	0.55	0.053	0.85
a	9	0	0	0.5	1	2	0	0	0	0	0.5	0.47	0.007	0	0	0.25	0.01	1
a	9	0	0	0.5	0	2	5.688	0.109	5.95	0.094	0.9	0.433	0.011	0.006	0.006	0.76	0.019	0.7
a	9	0	0	0.5	1	2	3.691	0.23	4.847	0.098	0.9	0.759	0.008	0.001	0.001	0.37	0.006	1
a	8	0	1	0.1	0	6	1.934	0.235	1.116	0.136	0.1	0.115	0.004	0	0	0.12	0.006	1
a	8	0	1	0.1	1	6	1.592	0.268	2.087	0.113	0.1	0.361	0.004	0.003	0.003	0.4	0	1
a	8	0	1	0.1	0	6	4.28	0.215	5.265	0.12	0.5	0.441	0.005	0	0	0.71	0.005	0.9
a	8	0	1	0.1	1	6	0.735	0.258	0.919	0.155	0.5	0.495	0.003	0	0	0.27	0.005	0.8
a	8	0	1	0.1	0	6	4.877	0.231	5.215	0.206	0.9	0.449	0.016	0.01	0.01	0.74	0.022	0.8

b	8	0	1	0.1	1	6	3.234	0.245	4.02	0.113	0.9	0.732	0.008	0.001	0.001	0.45	0.009	0.95
b	8	0	1	0.3	0	6	0.857	0.268	0.495	0.155	0.1	0.121	0.004	0	0	0.13	0.006	1
b	8	0	1	0.3	1	6	0.857	0.268	0.919	0.155	0.1	0.373	0.002	0.004	0.004	0.4	0	0.8
b	8	0	1	0.3	0	6	3.503	0.264	5.222	0.132	0.5	0.434	0.007	0	0	0.74	0.044	0.9
b	8	0	1	0.3	1	6	0.735	0.258	0.919	0.155	0.5	0.45	0.011	0	0	0.3	0.009	0.65
b	8	0	1	0.3	0	6	4.493	0.18	5.619	0.132	0.9	0.447	0.012	0.01	0.01	0.75	0.019	0.6
b	8	0	1	0.3	1	6	3.366	0.226	4.404	0.163	0.9	0.732	0.007	0.001	0.001	0.44	0.005	1
b	8	0	1	0.5	0	6	0	0	0	0	0.1	0.125	0.004	0	0	0.13	0.006	1
b	8	0	1	0.5	1	6	0	0	0.919	0.155	0.1	0.369	0.004	0.004	0.004	0.42	0.031	0.9
b	8	0	1	0.5	0	6	4.007	0.285	5.342	0.118	0.5	0.415	0.012	0	0	0.79	0.009	1
b	8	0	1	0.5	1	6	0	0	1.414	0	0.5	0.45	0.01	0	0	0.32	0.005	1
b	8	0	1	0.5	0	6	4.699	0.165	5.457	0.165	0.9	0.404	0.014	0.012	0.012	0.67	0.029	0.4
b	8	0	1	0.5	1	6	2.257	0.459	3.832	0.412	0.9	0.744	0.007	0.001	0.001	0.4	0.006	0.9
b	9	0	1	0.1	0	6	2.449	0	1.414	0	0.1	0.14	0.004	0	0	0.15	0	1
b	9	0	1	0.1	1	6	2.449	0	1.414	0	0.1	0.381	0.003	0.005	0.005	0.43	0.006	1
b	9	0	1	0.1	0	6	4.005	0.439	3.599	0.245	0.5	0.377	0.015	0.004	0.004	0.61	0.034	0.9
b	9	0	1	0.1	1	6	1.715	0.258	0.99	0.149	0.5	0.543	0.005	0	0	0.23	0.005	1
b	9	0	1	0.1	0	6	4.244	0.19	4.808	0.142	0.9	0.427	0.003	0.005	0.005	0.72	0	0.8
b	9	0	1	0.1	1	6	4.781	0.234	4.489	0.198	0.9	0.738	0.007	0.001	0.001	0.35	0	0.85
b	9	0	1	0.3	0	6	0	0	0	0	0.1	0.137	0.006	0	0	0.13	0.005	1
b	9	0	1	0.3	1	6	0.735	0.258	0.424	0.149	0.1	0.392	0.005	0.004	0.004	0.44	0.005	1
b	9	0	1	0.3	0	6	4.599	0.211	4.12	0.167	0.5	0.402	0.009	0.005	0.004	0.74	0.021	0.85
b	9	0	1	0.3	1	6	1.805	0.254	1.042	0.147	0.5	0.517	0.008	0	0	0.25	0.019	1
b	9	0	1	0.3	0	6	5.653	0.223	5.84	0.177	0.9	0.445	0.009	0.006	0.006	0.73	0.012	0.8
b	9	0	1	0.3	1	6	4.352	0.277	4.58	0.181	0.9	0.729	0.004	0.001	0.001	0.37	0.005	0.75
b	9	0	1	0.5	0	6	0	0	0.915	0.169	0.1	0.129	0.006	0	0	0.13	0.006	1
b	9	0	1	0.5	1	6	0	0	0.447	0.155	0.1	0.4	0.002	0.005	0.005	0.43	0.005	1
b	9	0	1	0.5	0	6	4.525	0.051	4.381	0.197	0.5	0.399	0.006	0.001	0.001	0.8	0.01	0.9
b	9	0	1	0.5	1	6	0.857	0.268	0.919	0.155	0.5	0.501	0.01	0	0	0.28	0.005	0.85
b	9	0	1	0.5	0	6	4.885	0.444	5.353	0.396	0.9	0.47	0.03	0.023	0.022	0.78	0.009	0.8
b	9	0	1	0.5	1	6	4.265	0.278	4.523	0.182	0.9	0.742	0.007	0.001	0.001	0.37	0.011	0.7
b	8	1	0	0.1	0	8	2.449	0	1.414	0	0.1	0.108	0.005	0	0	0.12	0.009	1
b	8	1	0	0.1	1	8	2.449	0	0	0	0.1	0.322	0.006	0.002	0.002	0.37	0.005	1
b	8	1	0	0.1	0	8	3.986	0.268	4.982	0.208	0.5	0.432	0.005	0	0	0.76	0.005	1
b	8	1	0	0.1	1	8	2.087	0.113	0	0	0.5	0.496	0.011	0	0	0.28	0.005	1
b	8	1	0	0.1	0	8	4.923	0.239	5.307	0.194	0.9	0.459	0.02	0.01	0.01	0.74	0.023	0.8
b	8	1	0	0.1	1	8	5.859	0.096	5.734	0.108	0.9	0.743	0.003	0.001	0.001	0.45	0	1
b	8	1	0	0.3	0	8	1.47	0.275	1.414	0	0.1	0.122	0.005	0	0	0.12	0.006	1
b	8	1	0	0.3	1	8	2.449	0	0	0	0.1	0.362	0.004	0.003	0.003	0.39	0.005	0.7
b	8	1	0	0.3	0	8	3.693	0.355	5.306	0.132	0.5	0.433	0.009	0	0	0.75	0.012	1
b	8	1	0	0.3	1	8	1.592	0.268	0	0	0.5	0.504	0.013	0	0	0.31	0.005	1
b	8	1	0	0.3	0	8	4.575	0.187	5.719	0.069	0.9	0.446	0.011	0.01	0.01	0.75	0.02	0.6
b	8	1	0	0.3	1	8	5.792	0.117	5.792	0.117	0.9	0.739	0.002	0.001	0.001	0.43	0.005	1
b	8	1	0	0.5	0	8	0.98	0.275	1.414	0	0.1	0.131	0.003	0	0	0.14	0.005	1
b	8	1	0	0.5	1	8	0	0	0.68	0.266	0.1	0.395	0.001	0.005	0.005	0.42	0.011	0.7
b	8	1	0	0.5	0	8	4.007	0.285	5.309	0.106	0.5	0.405	0.01	0	0	0.76	0.013	1

c	8	1	0	0.5	1	8	0.495	0.155	1.592	0.268	0.5	0.513	0.002	0	0	0.29	0.005	1
c	8	1	0	0.5	0	8	4.493	0.18	5.317	0.222	0.9	0.434	0.011	0.011	0.011	0.75	0.015	0.6
c	8	1	0	0.5	1	8	5.286	0.249	5.304	0.211	0.9	0.735	0.002	0.002	0.001	0.43	0.006	0.65
c	9	1	0	0.1	0	8	2.449	0	0	0	0.1	0.115	0.005	0	0	0.13	0.011	1
c	9	1	0	0.1	1	8	2.449	0	0	0	0.1	0.381	0.003	0.005	0.005	0.42	0.006	1
c	9	1	0	0.1	0	8	5.477	0	6	0	0.5	0.415	0.006	0.002	0.002	0.75	0.006	0.75
c	9	1	0	0.1	1	8	2.139	0.109	0.424	0.149	0.5	0.501	0.007	0	0	0.28	0.005	0.8
c	9	1	0	0.1	0	8	5.443	0.187	5.086	0.315	0.9	0.444	0.02	0.007	0.006	0.72	0.039	0.55
c	9	1	0	0.1	1	8	4.717	0.238	4.544	0.214	0.9	0.741	0.008	0.001	0.001	0.35	0	0.9
c	9	1	0	0.3	0	8	0	0	1.414	0	0.1	0.136	0.007	0	0	0.13	0.006	1
c	9	1	0	0.3	1	8	1.592	0.268	0	0	0.1	0.401	0.006	0.005	0.004	0.43	0.005	0.65
c	9	1	0	0.3	0	8	4.599	0.211	4.329	0.225	0.5	0.402	0.009	0.005	0.004	0.74	0.021	1
c	9	1	0	0.3	1	8	1.805	0.254	0	0	0.5	0.505	0.007	0	0	0.28	0.018	1
c	9	1	0	0.3	0	8	5.653	0.223	5.927	0.12	0.9	0.445	0.009	0.006	0.006	0.74	0.012	0
c	9	1	0	0.3	1	8	4.352	0.277	5.029	0.106	0.9	0.729	0.004	0.001	0.001	0.37	0.005	0.8
c	9	1	0	0.5	0	8	0.857	0.268	0	0	0.1	0.134	0.003	0	0	0.13	0.005	1
c	9	1	0	0.5	1	8	0.735	0.258	0.857	0.268	0.1	0.416	0.001	0.005	0.005	0.42	0.005	0.35
c	9	1	0	0.5	0	8	4.725	0.169	5.479	0.15	0.5	0.466	0.013	0.007	0.007	0.82	0.006	1
c	9	1	0	0.5	1	8	0.857	0.268	0.735	0.258	0.5	0.485	0.008	0	0	0.28	0.005	1
c	9	1	0	0.5	0	8	5.477	0	6	0	0.9	0.353	0.022	0.002	0.002	0.54	0.047	0.45
c	9	1	0	0.5	1	8	4.265	0.278	4.996	0.106	0.9	0.742	0.007	0.001	0.001	0.37	0.011	0.7

Appendix 4 Point Estimation for θ

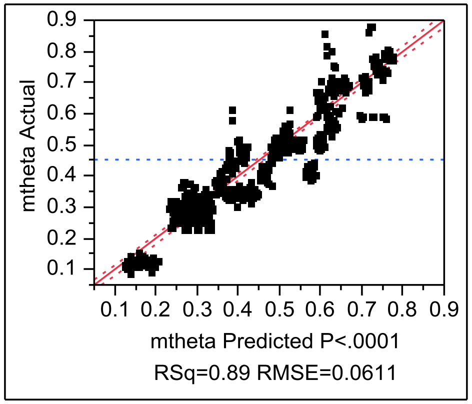

Fig 1 Actual by Predicted Plot for $m(\hat{\theta})$

Table 1 Summary of Fit for $m(\hat{\theta})$

RSquare	0.886887
RSquare Adj	0.8859
Root Mean Square Error	0.061115
Mean of Response	0.450621
Observations (or Sum Wgts)	810

Table 2 Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	7	23.487207	3.35532	898.3203
Error	802	2.995549	0.00374	**Prob > F**
C. Total	809	26.482757		0.0000*

Table 3 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0813429	0.01803	4.51	<.0001*
m	-0.002683	0.000263	-10.20	<.0001*
n	0.005424	0.00263	2.06	0.0395*
π	0.0173148	0.01315	1.32	0.1883
θ	0.5469846	0.007592	72.05	0.0000*
Distance	0.1347037	0.004295	31.36	<.0001*
D_1	0.0079556	0.00526	1.51	0.1308
D_2	-0.001581	0.00526	-0.30	0.7637

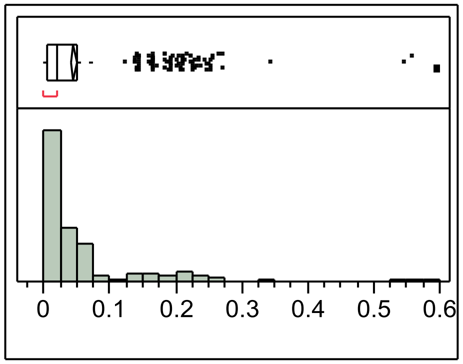

Fig 2 Distributions $'CV'$ for Both Spearman and Kendall Distance

Table 4 Quantiles of $'CV'$ for Both Spearman and Kendall Distance

100.0%	Maximum	0.59653

99.5%		0.34041
97.5%		0.23091
90.0%		0.14276
75.0%	Quartile	0.05193
50.0%	Median	0.02091
25.0%	Quartile	0.00602
10.0%		0.00164
2.5%		0.0005
0.5%		0.00004
0.0%	Minimum	0

Table 5 Moments of 'CV' for Both Spearman and Kendall Distance

Mean	0.0452689
Std Dev	0.0672631
Std Err Mean	0.0023634
Upper 95% Mean	0.049908
Lower 95% Mean	0.0406298
N	810

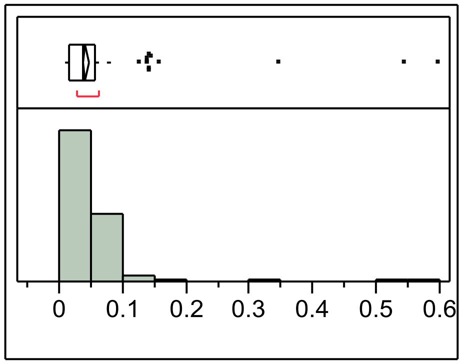

Fig 3 Distributions 'CV' for Kendall Distance

Table 6 Quantiles of $'CV'$ for Kendall Distance

100.0%	maximum	0.59653
99.5%		0.53892
97.5%		0.13787
90.0%		0.07228
75.0%	quartile	0.05533
50.0%	median	0.03693
25.0%	quartile	0.01476
10.0%		0.00435
2.5%		0.00101
0.5%		9.4e-5
0.0%	minimum	0.00007

Table 7 Moments of $'CV'$ for Kendall Distance

Mean	0.041282
Std Dev	0.0486746
Std Err Mean	0.0024187
Upper 95% Mean	0.0460367
Lower 95% Mean	0.0365273
N	405

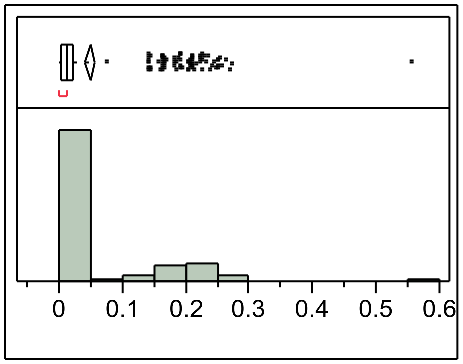

Fig 4 Distributions $'CV'$ for Spearman Distance

Table 8 Quantiles of $'CV'$ for Spearman Distance

100.0%	Maximum	0.55676
99.5%		0.27181
97.5%		0.24956
90.0%		0.20509
75.0%	Quartile	0.02326
50.0%	Median	0.01413
25.0%	Quartile	0.00288
10.0%		0.00113
2.5%		0.00031
0.5%		4.82e-6
0.0%	Minimum	0

Table 9 Moments of $'CV'$ for Spearman Distance

Mean	0.0492559
Std Dev	0.0816013
Std Err Mean	0.0040548
Upper 95% Mean	0.057227
Lower 95% Mean	0.0412848

N	405

Appendix 5 Point Estimation for π

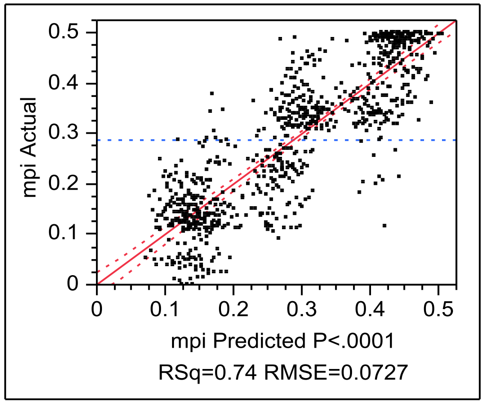

Fig 1 Actual by Predicted Plot for $m(\hat{\pi})$

Table 1Summary of Fit for $m(\hat{\pi})$

RSquare	0.736596
RSquare Adj	0.734297
Root Mean Square Error	0.072656
Mean of Response	0.28601
Observations (or Sum Wgts)	810

Table 2Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	7	11.839230	1.69132	320.3930
Error	802	4.233668	0.00528	**Prob > F**
C. Total	809	16.072898		<.0001*

Table 3Parameter Estimates

Term	**Estimate**	**Std Error**	**t Ratio**	**Prob>|t|**
Intercept	-0.054737	0.021435	-2.55	0.0108*
m	-0.00077	0.000313	-2.46	0.0141*
n	0.0176789	0.003127	5.65	<.0001*
θ	0.0281649	0.009026	3.12	0.0019*
π	0.718162	0.015633	45.94	<.0001*
Distance	0.0344358	0.005106	6.74	<.0001*
D_1	0.0250819	0.006259	4.01	<.0001*
D_2	-0.014987	0.006248	-2.40	0.0167*

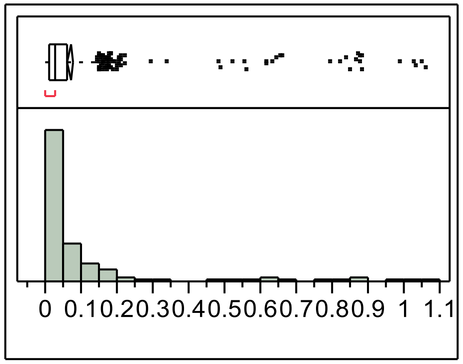

Fig 2 Distributions $'CV'$ for Both Kendall and Spearman Distance

Tabel 4 Quantiles of $'CV'$ for Both Kendall and Spearman Distance

100.0%	maximum	1.06289
99.5%		1.02311
97.5%		0.64336
90.0%		0.14085
75.0%	quartile	0.06339
50.0%	median	0.02768
25.0%	quartile	0.00983
10.0%		0.00269
2.5%		0.00032
0.5%		3.52e-5
0.0%	minimum	0.00002

Table 5 Moments of $'CV'$ for Both Kendall and Spearman Distance

Mean	0.0694755
Std Dev	0.1472136
Std Err Mean	0.0051726
Upper 95% Mean	0.0796287
Lower 95% Mean	0.0593223
N	810

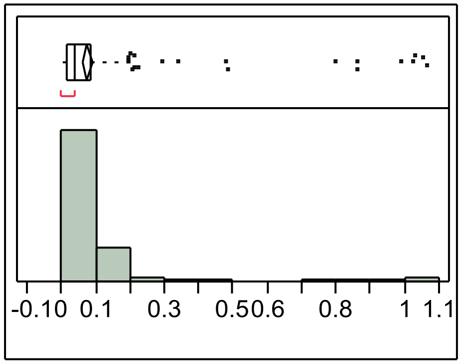

Fig 3 Distributions $'CV'$ for Kendall Distance

Table 6Quantiles of $'CV'$ for Kendall Distance

100.0%	maximum	1.06289
99.5%		1.05094
97.5%		0.46074
90.0%		0.15511
75.0%	quartile	0.08497
50.0%	median	0.03873
25.0%	quartile	0.01449
10.0%		0.00601
2.5%		0.00074
0.5%		0.00033
0.0%	minimum	4.29e-5

Table 7 Moments of $'CV'$ for Kendall Distance

Mean	0.0757206
Std Dev	0.1409412
Std Err Mean	0.0070034

Upper 95% Mean	0.0894883
Lower 95% Mean	0.0619529
N	405

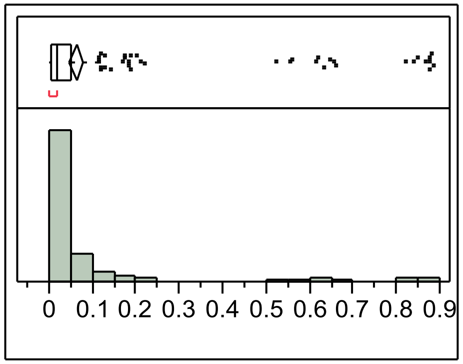

Fig 4 Distributions $'CV'$ for Spearman Distance

Table 8Quantiles of $'CV'$ for Spearman Distance

100.0%	maximum	0.88566
99.5%		0.8806
97.5%		0.66178
90.0%		0.11426
75.0%	quartile	0.04959
50.0%	median	0.01733
25.0%	quartile	0.00535
10.0%		0.00107
2.5%		8.44e-5
0.5%		3.1e-5
0.0%	minimum	0.00002

Table 9 Moments of $'CV'$ for Spearman Distance

Mean	0.0632304
Std Dev	0.1531494
Std Err Mean	0.0076101
Upper 95% Mean	0.0781906
Lower 95% Mean	0.0482701
N	810

Appendix 6 Profile Likelihood Set Estimation for θ

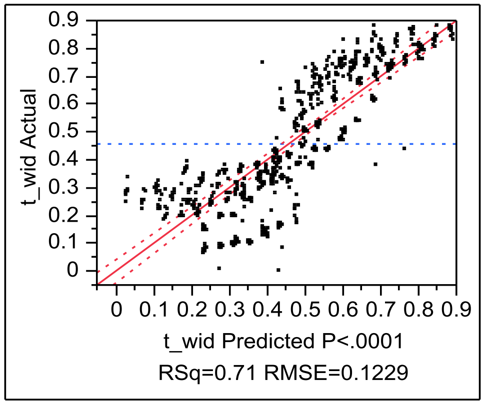

Fig 1 Actual by Predicted Plot for t_wid for θ When m=5, 6, 7

Table 1 Summary of Fit for t_wid for θ When m=5, 6, 7

RSquare	0.711554
RSquare Adj	0.709036
Root Mean Square Error	0.122917
Mean of Response	0.457622
Observations (or Sum Wgts)	810

Table 2 Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	7	29.891261	4.27018	282.6310
Error	802	12.117160	0.01511	**Prob > F**
C. Total	809	42.008420		<.0001*

Table 4 Parameter Estimates

Term	**Estimate**	**Std Error**	**t Ratio**	**Prob>|t|**
Intercept	0.7123964	0.036263	19.65	<.0001*
π	0.0153386	0.026448	0.58	0.5621
Distance	-0.205156	0.008638	-23.75	<.0001*
θ	0.5127319	0.01528	33.56	<.0001*
D_1	0.0016351	0.01058	0.15	0.8772
D_2	-0.001478	0.010579	-0.14	0.8889
n	0.043024	0.00529	8.13	<.0001*
m	-0.007791	0.000529	-14.73	<.0001*

Table 5 Whole Model Test for cov_t for θ When m=5, 6, 7

Model	**-LogLikelihood**	**L-R ChiSquare**	**DF**	**Prob>ChiSq**
Difference	250.075391	500.1508	7	<.0001*
Full	7457.99343			
Reduced	7708.06882			

Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	25294.94	802	0.0000*	
Deviance	12923.45	802	0.0000*	
AICc				
14932.167				

Table 6 Effect Tests

Source	**DF**	**L-R ChiSquare**	**Prob>ChiSq**
π	1	8.4896475	0.0036*
Distance	1	64.083639	<.0001*
θ	1	210.93896	<.0001*
D_1	1	10.058442	0.0015*
D_2	1	25.076319	<.0001*
n	1	107.498	<.0001*
m	1	86.260533	<.0001*

Table 7 Parameter Estimates

Term	**Estimate**	**Std Error**	**L-R ChiSquare**	**Prob>ChiSq**
Intercept	4.0677153	0.1318305	1010.0812	<.0001*
π	-0.273792	0.0939982	8.4896475	0.0036*
Distance	0.2464112	0.0308771	64.083639	<.0001*
θ	0.7930443	0.0550031	210.93896	<.0001*
D_1	-0.121704	0.0384059	10.058442	0.0015*
D_2	-0.189371	0.0378893	25.076319	<.0001*
n	-0.195439	0.0189297	107.498	<.0001*
m	-0.017507	0.0018913	86.260533	<.0001*

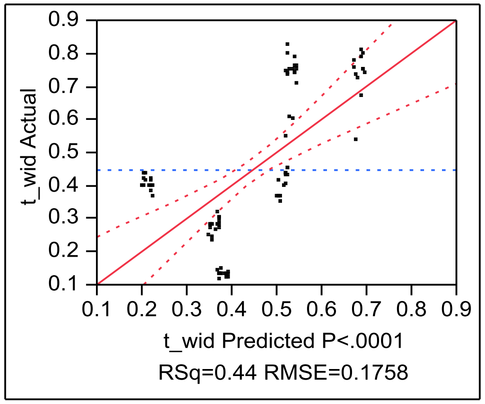

Fig 2.Actual by Predicted Plot for t_wid for θ when m=8, 9

Table 8 Summary of Fit for t_wid for θ when m=8, 9

RSquare	0.436451
RSquare Adj	0.402973
Root Mean Square Error	0.175788
Mean of Response	0.447157
Observations (or Sum Wgts)	108

Table 9 Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	6	2.4171639	0.402861	13.0369
Error	101	3.1210584	0.030902	**Prob > F**
C. Total	107	5.5382223		<.0001*

Table 10 Parameter Estimates

Term	**Estimate**	**Std Error**	**t Ratio**	**Prob>|t|**
Intercept	0.4835567	0.292354	1.65	0.1012

π	-0.009167	0.103584	-0.09	0.9297
Distance	-0.170167	0.03383	-5.03	<.0001*
D_1	0.0043611	0.041434	0.11	0.9164
D_2	0.0031111	0.041434	0.08	0.9403
θ	0.3758681	0.051792	7.26	<.0001*
n	-0.016352	0.03383	-0.48	0.6299

Table 11Whole Model Test for cov_t for θ When m=8, 9

Model	**-LogLikelihood**	**L-R ChiSquare**	**DF**	**Prob>ChiSq**
Difference	417.612829	835.2257	6	<.0001*
Full	1059.37153			
Reduced	1476.98436			
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	2265.541	101	0.0000*	
Deviance	1805.419	101	<.0001*	
AICc				
2133.8631				

Table 12 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
π	1	64.143537	<.0001*
θ	1	547.72228	<.0001*
Distance	1	82.361031	<.0001*
D1	1	141.61266	<.0001*
D2	1	30.975063	<.0001*
n	1	17.818198	<.0001*

Table 13Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq
Intercept	6.2352294	0.539817	137.47435	<.0001*
π	-1.506849	0.1896822	64.143537	<.0001*
θ	2.293458	0.105566	547.72228	<.0001*
Distance	0.5588795	0.0622939	82.361031	<.0001*
D1	-0.903949	0.0782302	141.61266	<.0001*
D2	-0.45465	0.0823589	30.975063	<.0001*
n	-0.258655	0.0614257	17.818198	<.0001*

Appendix 7 Profile Likelihood Set Estimation for $\mathbf{w}_1$ and $\mathbf{w}_2$

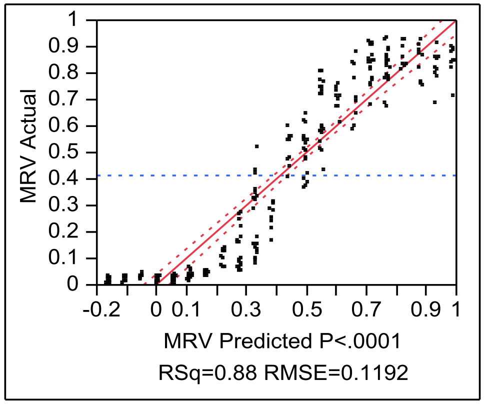

Fig 1 Actual by Predicted Plot for MRV for $\mathbf{w}_1$ and $\mathbf{w}_2$

Table 1 Whole Model Test for MRV for $\mathbf{w}_1$ and $\mathbf{w}_2$

RSquare	0.883371
RSquare Adj	0.88071
Root Mean Square Error	0.119168
Mean of Response	0.411894
Observations (or Sum Wgts)	270

Table 2 Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	6	28.288477	4.71475	332.0012
Error	263	3.734861	0.01420	**Prob > F**
C. Total	269	32.023338		<.0001*

Table 3 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0616554	0.029459	2.09	0.0373*
π	-0.011222	0.044411	-0.25	0.8007
θ	1.0959907	0.025641	42.74	<.0001*
D_1	-0.005097	0.017765	-0.29	0.7744
D_2	-0.008956	0.017765	-0.50	0.6146
n	-0.005384	0.000888	-6.06	<.0001*
Distance	-0.164035	0.014505	-11.31	<.0001*

Table 4 Whole Model Test for cov_t for $\mathbf{w}_1$ and $\mathbf{w}_2$

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	6.01972236	12.0394	6	0.0611
Full	27.1882718			
Reduced	33.2079941			
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	5.6550	263	1.0000	
Deviance	5.2934	263	1.0000	
AICc				
3427.6508				

Table 5 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
π	1	0.0093945	0.9228
θ	1	10.221858	0.0014*
D_1	1	9.8816e-5	0.9921
D_2	1	0.1159457	0.7335
n	1	0.55546	0.4561
Distance	1	1.1899545	0.2753

Table 6 Parameter Estimates

Term	**Estimate**	**Std Error**	**L-R ChiSquare**	**Prob>ChiSq**
Intercept	5.3017126	1.7454844	13.406191	0.0003*
π	-0.199905	2.0628197	0.0093945	0.9228
θ	-4.551662	1.7367664	10.221858	0.0014*
D_1	0.0079053	0.7952518	9.8816e-5	0.9921
D_2	0.2870514	0.8463943	0.1159457	0.7335
n	0.0309828	0.0419952	0.55546	0.4561
Distance	0.7517141	0.7084921	1.1899545	0.2753

Appendix 8 Profile Likelihood Set Estimation for $\mathbf{w}_1$

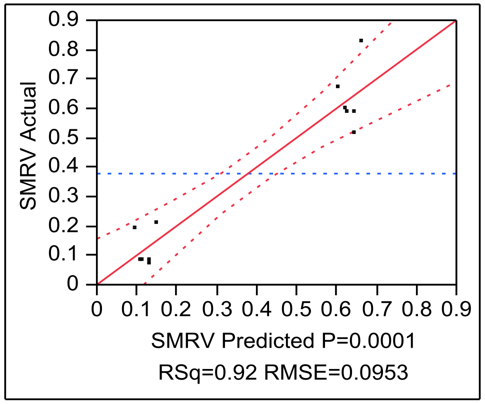

Fig 1 Actual by Predicted Plot for SMRV for $\mathbf{w}_1$

Table 1 Summary of Fit for SMRV for w_1

RSquare	0.915804
RSquare Adj	0.88423
Root Mean Square Error	0.09534
Mean of Response	0.37675
Observations (or Sum Wgts)	12

Table 2 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	0.79094867	0.263650	29.0053
Error	8	0.07271758	0.009090	**Prob > F**
C. Total	11	0.86366625		0.0001*

Table 3 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0343125	0.097549	0.35	0.7341
π	-0.085833	0.275223	-0.31	0.7631
θ	0.6402083	0.068806	9.30	<.0001*
n	0.001975	0.003371	0.59	0.5741

Table 4 Whole Model Test for Scov_t for w_1

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	0.55958098	1.1192	3	0.7725
Full	2.76085403			
Reduced	3.32043501			
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	0.3178	8	1.0000	Pearson
Deviance	0.2858	8	1.0000	Deviance
AICc				
19.2360				

Table 5 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
π	1	0.001354	0.9706
θ	1	1.1074172	0.2926
n	1	0.01145	0.9148

Table 6 Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq
Intercept	3.5125964	3.9574119	1.0407641	0.3076
π	0.3385374	9.2016248	0.001354	0.9706
θ	-2.814769	3.3168098	1.1074172	0.2926
n	0.0120629	0.1128757	0.01145	0.9148

Appendix 9 Profile Likelihood Set Estimation for $\mathbf{w}_2$

Whole Model

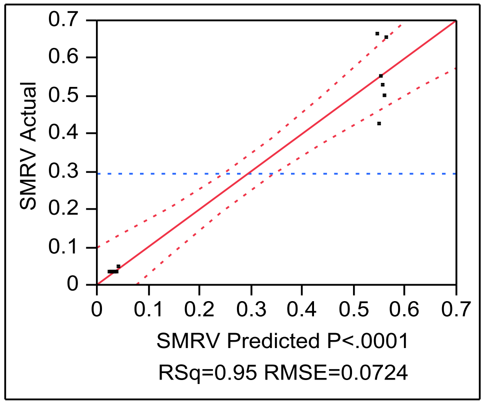

Fig 1 Actual by Predicted Plot for SMRV for $\mathbf{w}_2$

Table 1 Summary of Fit for SMRV for $\mathbf{w}_2$

RSquare	0.950689
RSquare Adj	0.932198
Root Mean Square Error	0.072446
Mean of Response	0.293833
Observations (or Sum Wgts)	12

Table 2 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	0.80950583	0.269835	51.4121
Error	8	0.04198783	0.005248	**Prob > F**
C. Total	11	0.85149367		<.0001*

Table 3 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.03425	0.074125	-0.46	0.6563
π	0.05	0.209135	0.24	0.8171
θ	0.6491667	0.052284	12.42	<.0001*
n	-0.000325	0.002561	-0.13	0.9022

Table 4 Whole Model Test for cov_t for w_2

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	0.88961364	1.7792	3	0.6195
Full	2.28360033			
Reduced	3.17321397			
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	0.0495	8	1.0000	
Deviance	0.0515	8	1.0000	
AICc				
18.2815				

Table 5 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
π	1	0.0384752	0.8445
θ	1	1.7165991	0.1901
n	1	0.0324778	0.8570

Table 6 Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq
Intercept	5.7817206	7.2146837	1.6202315	0.2031
π	-1.929771	9.8844221	0.0384752	0.8445
θ	-5.028232	7.3439603	1.7165991	0.1901
n	0.0217086	0.120932	0.0324778	0.8570

Appendix 10 Bayesian Point and Set Estimation for θ

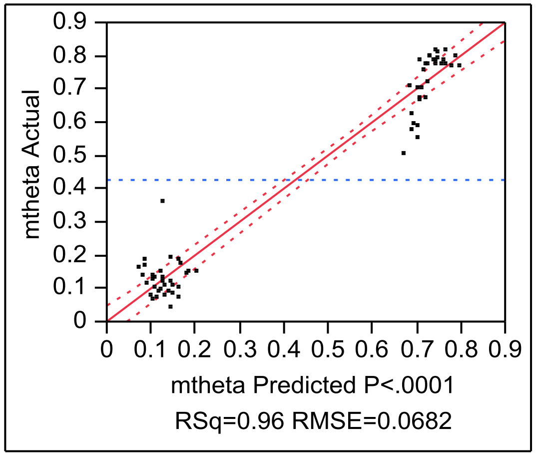

Fig 1 Actual by Predicted Plot for $\hat{m}(\theta)$

Table 1 Summary of Fit for $\hat{m}(\theta)$

RSquare	0.956198
RSquare Adj	0.951744
Root Mean Square Error	0.068171
Mean of Response	0.428076
Observations (or Sum Wgts)	66

Table 2 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	6	5.9856611	0.997610	214.6638
Error	59	0.2741916	0.004647	**Prob > F**
C. Total	65	6.2598526		<.0001*

Table 3 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.087944	0.080222	1.10	0.2774
m	-0.017194	0.011362	-1.51	0.1355
n	0.0021227	0.001028	2.07	0.0433*
π	0.1935093	0.123596	1.57	0.1228
θ	0.7494079	0.021116	35.49	<.0001*
D_1	0.0037895	0.021176	0.18	0.8586
D_2	-0.012594	0.020541	-0.61	0.5421

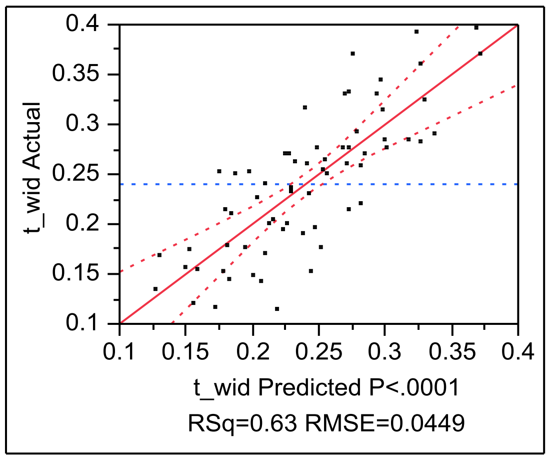

Fig 2 Actual by Predicted Plot for t_wid for θ

Table 4 Summary of Fit for t_wid for θ

RSquare	0.634074
RSquare Adj	0.596861
Root Mean Square Error	0.044935
Mean of Response	0.240909
Observations (or Sum Wgts)	66

Table 5 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	6	0.20642372	0.034404	17.0391
Error	59	0.11912773	0.002019	**Prob > F**
C. Total	65	0.32555145		<.0001*

Table 6 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.4394063	0.052877	8.31	<.0001*
m	-0.028611	0.007489	-3.82	0.0003*
n	-0.004445	0.000677	-6.56	<.0001*
π	0.2077222	0.081467	2.55	0.0134*
θ	0.00375	0.013918	0.27	0.7885
D_1	0.023	0.013958	1.65	0.1047
D_2	0.0532333	0.013539	3.93	0.0002*

Table 7 Whole Model Test for Scov_t or θ

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	2.63491104	5.2698	6	0.5097
Full	9.25138246			
Reduced	11.8862935			
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	1.4593	59	1.0000	
Deviance	1.5919	59	1.0000	
AICc				
34.4338				

Table 8 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
m	1	0.6133695	0.4335
n	1	0.488249	0.4847
π	1	0.0075378	0.9308
θ	1	3.9679371	0.0464*
D_1	1	0.0224536	0.8809
D_2	1	0.0469423	0.8285

Table 9 Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq
Intercept	7.7653538	5.8522723	2.1104418	0.1463
m	-0.55603	0.7310411	0.6133695	0.4335
n	0.0467152	0.068277	0.488249	0.4847
π	0.8703496	10.119986	0.0075378	0.9308
θ	-3.576206	2.5888643	3.9679371	0.0464*
D_1	-0.208256	1.390315	0.0224536	0.8809
D_2	-0.281905	1.3057316	0.0469423	0.8285

Appendix 11 Bayesian Point and Set Estimation for π

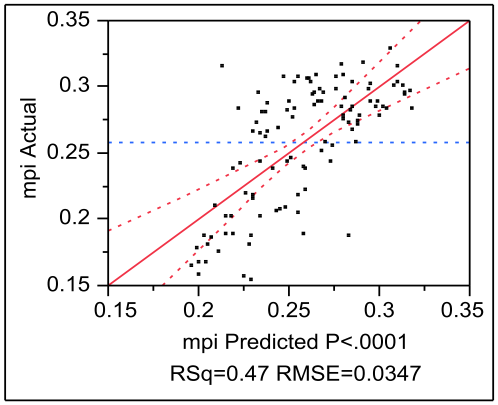

Fig 1. Actual by Predicted Plot for $\hat{m}(\pi)$

Table 1 Summary of Fit for $\hat{m}(\pi)$

RSquare	0.473912
RSquare Adj	0.442659
Root Mean Square Error	0.03474
Mean of Response	0.257407
Observations (or Sum Wgts)	108

Table 2 Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	6	0.10980421	0.018301	15.1639
Error	101	0.12189323	0.001207	**Prob > F**
C. Total	107	0.23169744		<.0001*

Table 3 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.1923153	0.02768	6.95	<.0001*
m	0.0043176	0.004094	1.05	0.2941
n	-0.001469	0.000409	-3.59	0.0005*
θ	0.0372812	0.008357	4.46	<.0001*
π	0.2520732	0.033429	7.54	<.0001*
D_1	-0.002665	0.008188	-0.33	0.7455
D_2	0.0012007	0.008188	0.15	0.8837

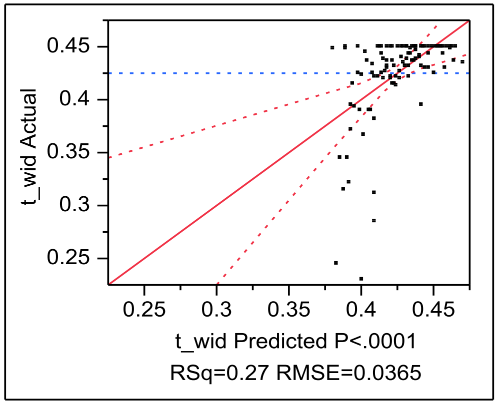

Fig 2. Actual by Predicted Plot for t_wid for π .

Table 4 Summary of Fit for t_wid for π

RSquare	0.266498
RSquare Adj	0.222923
Root Mean Square Error	0.03647
Mean of Response	0.424599
Observations (or Sum Wgts)	108

Table 5 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	6	0.04880720	0.008135	6.1159
Error	101	0.13433597	0.001330	**Prob > F**
C. Total	107	0.18314317		<.0001*

Table 6 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.2905363	0.029058	10.00	<.0001*
m	0.0198514	0.004298	4.62	<.0001*
n	-0.000458	0.00043	-1.07	0.2892
θ	0.0100832	0.008773	1.15	0.2531
π	0.1204734	0.035093	3.43	0.0009*
D_1	-0.006193	0.008596	-0.72	0.4729
D_2	-0.008875	0.008596	-1.03	0.3043

Table 7 Whole Model Test for Scov_t for π

Model	1.2803952	2.5608	6	0.8616
Difference	8.57000112			
Full	9.85039632			
Reduced	1.2803952	2.5608	6	0.8616
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	1	0.071763	0.7888	
Deviance	1	0.0121464	0.9122	
AICc	1	0.5558639	0.4559	
33.8899	1	1.8554504	0.1732	

Table 8 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
m	1	0.071763	0.7888
n	1	0.0121464	0.9122
θ	1	0.5558639	0.4559
π	1	1.8554504	0.1732
D_1	1	0.0739461	0.7857
D_2	1	0.005587	0.9404

Table 9 Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq
Intercept	1.178905	5.3107718	0.0495049	0.8239
m	0.2117857	0.7946138	0.071763	0.7888
n	0.0086809	0.0788343	0.0121464	0.9122
θ	-1.25903	1.7842296	0.5558639	0.4559
π	10.90222	10.23834	1.8554504	0.1732
D_1	0.4380028	1.627552	0.0739461	0.7857
D_2	0.1117905	1.4965918	0.005587	0.9404

Appendix 12 Bayesian Point and Set Estimation for $\mathbf{w}_1$ and $\mathbf{w}_2$

Table 1 Whole Model Test for Scov_t for $\mathbf{w}_1$ and $\mathbf{w}_2$

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	0.85499811	1.7100	2	0.4253
Full	1.84324007			
Reduced	2.69823818			
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	0.0820	6	1.0000	Pearson
Deviance	0.1019	6	1.0000	Deviance
AICc				
22.5091				

Table 2 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
θ	1	1.6907515	0.1935
π	1	0.0235523	0.8780

Table 3 Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq
Intercept	5.2541562	5.073886	2.4413674	0.1182
θ	-5.412946	5.746247	1.6907515	0.1935
π	1.0262415	6.7088687	0.0235523	0.8780

Appendix 13 Bayesian Point and Set Estimation for $\mathbf{w}_1$

Table 1 Whole Model Test for Scov_t for $\mathbf{w}_1$

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	1.25905444	2.5181	4	0.6414
Full	5.25454164			
Reduced	6.51359608			
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	0.6784	31	1.0000	
Deviance	0.6757	31	1.0000	
AICc				
22.5091				

Table 2 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
θ	1	2.4785776	0.1154
π	1	0.0119393	0.9130
D_1	1	0.0001874	0.9891
D_2	1	0.007855	0.9294

Table 3 Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq
Intercept	5.5580635	4.1978713	4.7504712	0.0293*
θ	-4.122648	4.2753354	2.4785776	0.1154
π	0.8923209	8.1414093	0.0119393	0.9130
D_1	0.022479	1.6411316	0.0001874	0.9891
D_2	-0.191227	2.1511057	0.007855	0.9294

Appendix 14 Bayesian Point and Set Estimation for $\mathbf{w}_2$

Table 1 Whole Model Test for Scov_t or $\mathbf{w}_2$

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	1.59506399	3.1901	4	0.5265
Full	5.7658606			
Reduced	7.36092459			
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	0.4997	31	1.0000	
Deviance	0.5346	31	1.0000	
AICc				
23.5317				

Table 2 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
θ	1	3.1340501	0.0767
π	1	0.0025474	0.9597
D_1	1	0.0465595	0.8292
D_2	1	0.0293654	0.8639

Table 3 Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq
Intercept	5.8961126	4.1841436	5.7923528	0.0161*
θ	-4.397429	4.2610211	3.1340501	0.0767
π	0.3814598	7.5454319	0.0025474	0.9597
D_1	-0.332125	1.5576615	0.0465595	0.8292
D_2	-0.360924	2.0965167	0.0293654	0.8639

Appendix 15 Point and Set Estimation for $\boldsymbol{\beta}$ and θ in Covariate Model

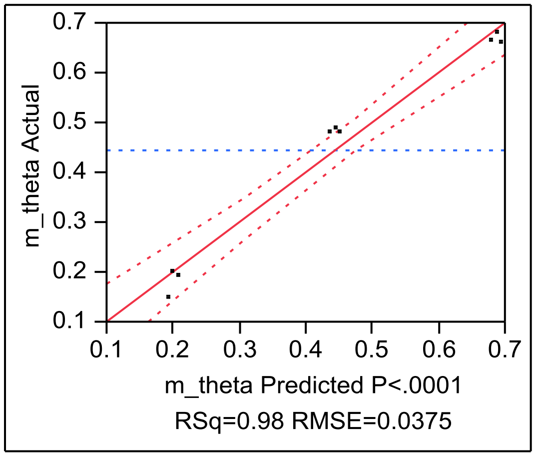

Fig 1. Actual by Predicted Plot for $\hat{m}(\theta)$

Table 1 Summary of Fit for $\hat{m}(\theta)$

RSquare	0.976853
RSquare Adj	0.969137
Root Mean Square Error	0.037498
Mean of Response	0.443556
Observations (or Sum Wgts)	9

Table 2 Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	2	0.35603367	0.178017	126.6039
Error	6	0.00843656	0.001406	**Prob > F**
C. Total	8	0.36447022		<.0001*

Table 3 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.1460139	0.027509	5.31	0.0018*
θ	0.60875	0.038271	15.91	<.0001*
β_1	-0.003417	0.007654	-0.45	0.6710

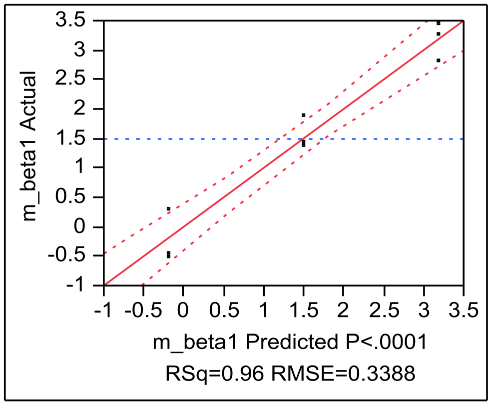

Fig 2. Actual by Predicted Plot for $m(\hat{\beta}_1)$

Table 4 Summary of Fit for $m(\hat{\beta}_1)$

RSquare	0.955574
RSquare Adj	0.949228
Root Mean Square Error	0.338794
Mean of Response	1.497444
Observations (or Sum Wgts)	9

Table 5 Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	1	17.282248	17.2822	150.5666
Error	7	0.803470	0.1148	**Prob > F**
C. Total	8	18.085718		<.0001*

Table 6 Parameter Estimates

Term	**Estimate**	**Std Error**	**t Ratio**	**Prob>|t|**
Intercept	-0.199722	0.17856	-1.12	0.3003
β_1	0.8485833	0.069156	12.27	<.0001*

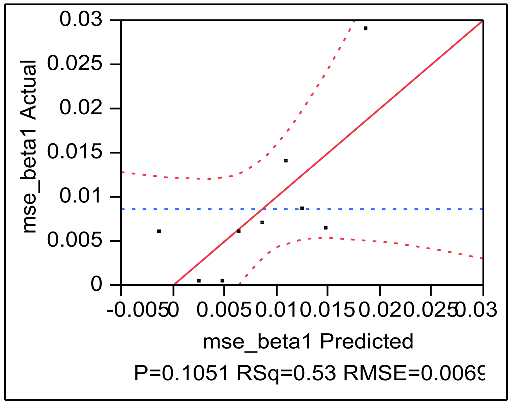

Fig 3. Actual by Predicted Plot for mse for β_1

Tabel 7 Summary of Fit for mse for β_1

RSquare	0.528145
RSquare Adj	0.37086
Root Mean Square Error	0.006868
Mean of Response	0.008674
Observations (or Sum Wgts)	9

Table 8 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	0.00031674	0.000158	3.3579
Error	6	0.00028298	0.000047	**Prob > F**
C. Total	8	0.00059972		0.1051

Table 9 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.002316	0.005038	-0.46	0.6619
θ	0.0096917	0.007009	1.38	0.2160
β_1	0.0030725	0.001402	2.19	0.0709

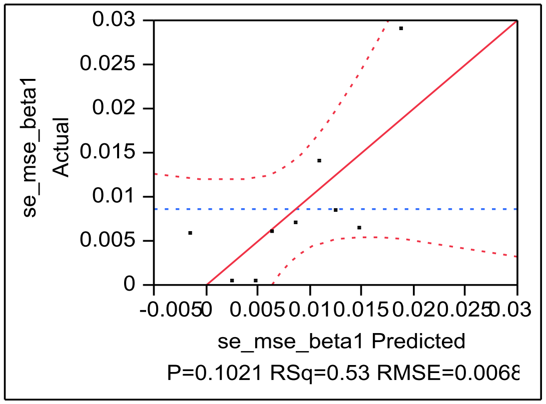

Fig 4. Actual by Predicted Plot for mse_se for β_1

Table 10 Summary of Fit for mse_se for β_1

RSquare	0.532587
RSquare Adj	0.376783
Root Mean Square Error	0.006841
Mean of Response	0.008641
Observations (or Sum Wgts)	9

Table 11 Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	2	0.00031998	0.000160	3.4183
Error	6	0.00028082	0.000047	**Prob > F**
C. Total	8	0.00060080		0.1021

Table 12 Parameter Estimates

Term	**Estimate**	**Std Error**	**t Ratio**	**Prob>|t|**
Intercept	-0.002404	0.005019	-0.48	0.6489
θ	0.0097333	0.006982	1.39	0.2128
β_1	0.0030892	0.001396	2.21	0.0689

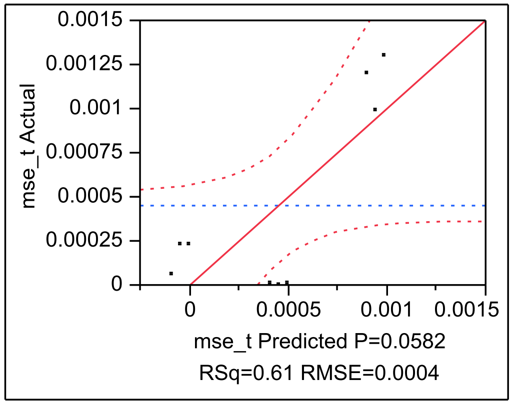

Fig 5. Actual by Predicted Plot for sample mse_t for θ

Table 13 Summary of Fit for mse_t for θ

RSquare	0.612533
RSquare Adj	0.483377
Root Mean Square Error	0.000395
Mean of Response	0.000448
Observations (or Sum Wgts)	9

Table 14 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	1.47899e-6	7.395e-7	4.7426
Error	6	9.35559e-7	1.5593e-7	**Prob > F**
C. Total	8	2.41455e-6		0.0582

Table 15 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.000126	0.00029	-0.43	0.6791
θ	0.0012363	0.000403	3.07	0.0220*
β_1	-2.218e-5	8.06e-5	-0.28	0.7924

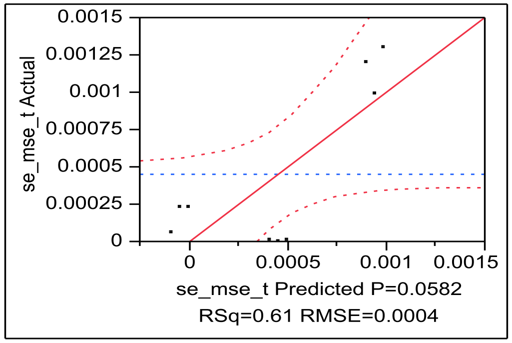

Fig 6. Actual by Predicted Plot for se_mse_t for θ

Table 16 Summary of Fit for se_mse_t for θ

RSquare	0.612507
RSquare Adj	0.483343
Root Mean Square Error	0.000395
Mean of Response	0.000448
Observations (or Sum Wgts)	9

Table 17 Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	2	1.47898e-6	7.3949e-7	4.7421
Error	6	9.35656e-7	1.5594e-7	**Prob > F**
C. Total	8	2.41464e-6		0.0582

Table 18 Parameter Estimates

Term	**Estimate**	**Std Error**	**t Ratio**	**Prob>|t|**
Intercept	-0.000126	0.00029	-0.43	0.6791
θ	0.0012363	0.000403	3.07	0.0220*
β_1	-2.218e-5	8.061e-5	-0.28	0.7925

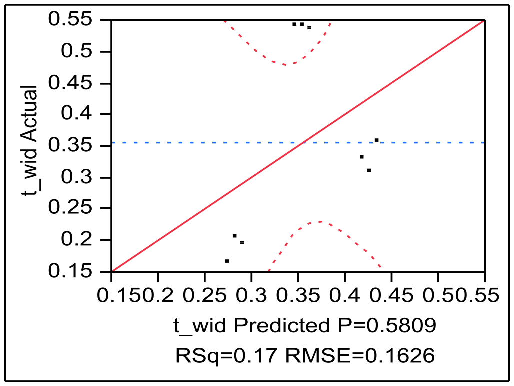

Fig 7. Actual by Predicted Plot for t_wid for θ

Table 19 Summary of Fit for t_wid for θ

RSquare	0.165609
RSquare Adj	-0.11252
Root Mean Square Error	0.162607
Mean of Response	0.354111
Observations (or Sum Wgts)	9

Table 20 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	0.03148800	0.015744	0.5954
Error	6	0.15864689	0.026441	**Prob > F**
C. Total	8	0.19013489		0.5809

Table 1 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.2721111	0.119292	2.28	0.0627
θ	0.18	0.16596	1.08	0.3197
β_1	-0.004	0.033192	-0.12	0.9080

Appendix 16 Set Estimation for $\mathbf{w}_1$ and $\mathbf{w}_2$ in Covariate Model

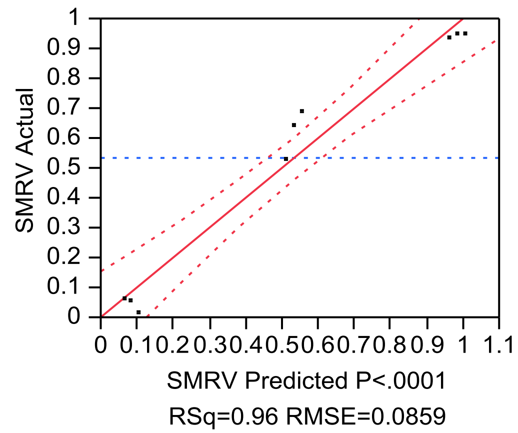

Fig 1. Actual by Predicted Plot for SMRV for $\mathbf{w}_1$ and $\mathbf{w}_2$

Table 1 Summary of Fit for SMRV for w_1 and w_2

RSquare	0.964784
RSquare Adj	0.953045
Root Mean Square Error	0.085906
Mean of Response	0.534333
Observations (or Sum Wgts)	9

Table 2 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	1.2130668	0.606533	82.1876
Error	6	0.0442792	0.007380	**Prob > F**
C. Total	8	1.2573460		<.0001*

Table 3 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.047792	0.063022	-0.76	0.4770
θ	1.1229167	0.087678	12.81	<.0001*
β_1	0.0103333	0.017536	0.59	0.5772

Table 4 Whole Model Test for Scov_t for w_1 and w_2

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	4.93235759	9.8647	2	0.0072*
Full	10.9313838			
Reduced	15.8637414			
Goodness Of Fit Statistic	**ChiSquare**	**DF**	**Prob>ChiSq**	
Pearson	2.6642	6	0.8497	
Deviance	2.7884	6	0.8349	
AICc				
32.6628				

Table 5 Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
θ	1	9.6427668	0.0019*
β_1	1	0.2243432	0.6358

Table 6 Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq
Intercept	5.9554916	0.873868	168.21844	<.0001*
θ	-2.77908	1.0221107	9.6427668	0.0019*
β_1	-0.074913	0.158582	0.2243432	0.6358

Appendix 17 Effect of Initial Values on Point Estimation for θ and π

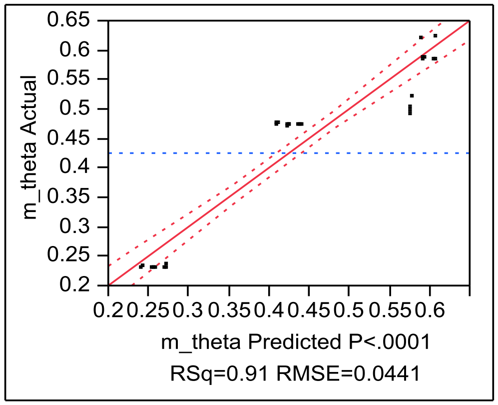

Fig 1.Actual by Predicted Plot for $m(\hat{\theta})$

Table 1 Summary of Fit for $m(\hat{\theta})$

RSquare	0.914096
RSquare Adj	0.90781
Root Mean Square Error	0.044094
Mean of Response	0.424
Observations (or Sum Wgts)	45

Table 2 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	0.84822621	0.282742	145.4256
Error	41	0.07971379	0.001944	**Prob > F**
C. Total	44	0.92794000		<.0001*

Table 3 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.018772	0.027002	-0.70	0.4908
θ	0.8375	0.040252	20.81	<.0001*
Initial θ	0.0735	0.040252	1.83	0.0751
initial π	0.0039444	0.023239	0.17	0.8661

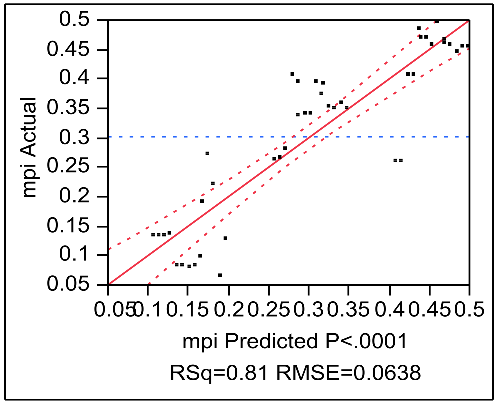

Fig 2.Actual by Predicted Plot for $m(\hat{\pi})$

Table 4 Summary of Fit for $m(\hat{\pi})$

RSquare	0.811508
RSquare Adj	0.797716
Root Mean Square Error	0.06378
Mean of Response	0.301844
Observations (or Sum Wgts)	45

Table 5 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	0.71804488	0.239348	58.8386
Error	41	0.16678303	0.004068	**Prob > F**
C. Total	44	0.88482791		<.0001*

Table 6 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.103294	0.039057	-2.64	0.0115*
θ	0.7556667	0.058223	12.98	<.0001*
Initial θ	-0.036889	0.033615	-1.10	0.2789
initial π	0.1525	0.058223	2.62	0.0123*