# DESIGN OF A MACHINE FOR TESTING THE FATIGUE STRENGTH OF STEEL

by

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During a discussion with one of the author's professors at Kansas State College, the question was raised whether the principle of resonant lateral vibration of an elastic body could be applied to the design of a machine for testing fatigue strength of steel, and whether such a machine could result in a saving of time in the testing process. It was decided that such a machine would be designed and built by the author, to test the practicability of this idea.

A search of the literature revealed that several machines using this principle have been designed in the past (1, 2, 3). The highest speed obtained, however, mentioned only in (2), was 18,000 cycles per minute. At this rate, one test of 2,000,000 cycles consumes nearly two hours. The highest recorded speed attained in fatigue testing was 30,000 cycles per minute (4), using a rotating-beam testing machine driven by an air turbine. This reference described the effect of high speed on fatigue strengths, citing the case of a sample of rail steel whose endurance limit was two percent higher than normal when tested at the above speed. Similarly, a sample of brass tested at 30,000 cycles per minute showed a limit which was thirty-five percent above normal. Other steels showed varying strengths, all above normal.

In view of the foregoing, it was decided to continue with development of the new machine, in the hope that, if successful, it might enable further studies of the speed

effect.

### PRINCIPLE OF OPERATION

In an elastic body, vibrating at its resonant frequency, the effect of a small driving force is magnified to produce large stresses in the material. It was decided to apply the driving force by mounting an electromagnet in close proximity to the test specimen, and to supply the magnet with alternating current whose frequency was one-half the resonant frequency of the specimen. The magnet would then produce a pulsating force of the proper frequency to maintain vibration. Stresses could be calculated by measuring deflections, on the basic assumption that stress is proportional to strain.

Lateral vibration was selected over longitudinal vibration because of higher deflections per unit of stress, hence more accurate measurements would be obtainable.

Several methods of mounting the test coupon suggested themselves. Mounting as a cantilever beam, however, requires a very heavy clamping device, and it would have been exceedingly difficult to correlate values of deflection with bending moment for stress calculations. Mounting as a fixed beam was rejected for the same reasons. Mounting as a hinged-end beam would have simplified stress calculations, but would have required high driving power because of energy loss through the supports. However, mounting as a free-free beam entailed none of these objections. The only forces acting on the supports are reactions of the driving force, therefore, the supports

need not be so firm. These forces are very small compared to the total energy of the beam, and were assumed negligible, enabling calculation of the stress-deflection relationship direct from the equation of vibration. The only power loss from the beam is due to internal friction of the material and to air resistance. The free-free mounting was therefore selected. In a free-free mounting, the beam is supported only at the nodes, where the lateral motion is zero. Since there is rotary motion at these points, and a very small longitudinal motion, the supports must be pivots, and must be arranged to accommodate this slight longitudinal motion.

### MATHEMATICS OF VIBRATING ELASTIC BAR

Following is a summary of the mathematical solution of a prismatic elastic bar vibrating as a free-free beam.

### Differential Equations

Several involved differential equations have been proposed, none of which is exact for all conditions. Following is the most precise of these equations (5):

I. 
$$\frac{Er^2}{p} \frac{d^4y}{dx^4} + \frac{d^2y}{dt^2} - r^2(1 + \frac{E}{K'G}) \frac{d^4y}{dx^2 dt^2} + \frac{r^2p}{K'G} \frac{d^4y}{dt^4} = 0$$

in which:

- d indicates partial derivative.
- x coordinate in direction of length, inches,
- y = lateral displacement, inches,

r = radius of gyration of cross-section with respect to centroidal z axis, inches,

p = mass per unit volume, slugs,

t = time, seconds.

G = modulus of elasticity in shear,

K' constant relating to effect of shear on slope of elastic line, = 5/6.

In this equation, the first two terms comprise the fundamental differential equation of motion of a vibrating body, the third term corrects for the effect of rotatory inertia, and the last term corrects for shear. It can be shown, however, that for L/r ratios of 70 or greater, the frequency correction due to these two terms is less than 1 percent, becoming less as the ratio increases. It was therefore neglected, and the solution was based on the following equation.

### Solution of Approximate Equation

The fundamental differential equation, with partial solutions, is given in books by Timoshenko (6) and Rayleigh (7):

II. 
$$\frac{d^2y}{dt^2} + a^2 \frac{d^4y}{dx^4} = 0$$

where:

$$a^2 = \frac{EI}{AD}$$

I is moment of inertia of cross-section about centroidal axis, inches,

A is area of cross-section, inches2.

If we let  $k^4 = \frac{M^2}{a^7}$ , where w is the circular frequency of vibration in radians per second, the general solution of Eq. II is:

III. y = [A sin kx + B cos kx + C sinh kx + D cosh kx] (F cos wt + G sin wt),

of which the bracketed term denotes the shape of the deflected body, and the remaining term denotes the variation of amplitude with time. This variation with time will be neglected in the following solution, assuming t = 0.

The constants may be evaluated by noting that, for a free-free beam, at each end, where x = 0 or x = L, the bending moment  $\frac{d^2y}{dx^2} \quad \text{and shear} \quad \frac{d^3y}{dx^3} \quad \text{must be zero.} \quad \text{This gives:}$ 

A = C, B = D, cos kL cosh kL = 1,

so that kL = 0, 4.730, 7.853, 10.996, etc., where each root represents a different mode of vibration, 4.730 corresponding to vibration with two nodes. Substituting this root in the second derivative and evaluating gives:

B = -1.0178A.

If  $y_m$  is the maximum deflection at x = L/2, substitution of these values in Eq. III and evaluation gives:

A = .80815ym.

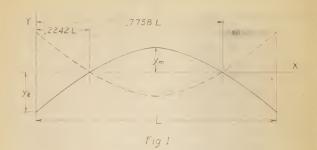
Eq. III now becomes:

IV.  $y = .80815y_m$  [(sinh kx + sin kx) - 1.0178(cosh kx + cos kx)]. See graph on page 7, Figs. 1 and 2.

### EXPLANATION OF PLATE I

- Fig. 1 shows the curve of Eq. IV, plotted on rectangular coordinates, for kL = 4.730, corresponding to fundamental frequency, or first mode of vibration.
- Fig. 2 shows curves of Eq. IV for kL = 7.853 and kL = 10.996 respectively, corresponding to second and third modes of vibration. Nodal points marked with small circles.
- Fig. 3 shows a bar vibrating in first mode. (d) represents total movement of a point on the surface of bar at midlength.

# PLATE I





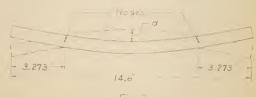


Fig 3

To locate the nodes, it is only necessary to set y equal to zero and solve for the value of kx which makes the function equal to zero. kx = 1.0602 or 3.6698, which satisfies this condition, making x = .22421 or .77581.

Stress and Bending Moment

Taking unit stress = 
$$\frac{Mc}{I} = \frac{Mt}{2I}$$
, and M = EI  $\frac{d^2y}{dx^2}$ ;

V. 
$$s = (\frac{Et}{2})(.80815y_m)k^2 [(\sinh kx - \sin kx) - 1.0178(\cosh kx - \cos kx)]$$
.

The maximum bending moment occurs at L/2.

 $kL = 4.730, \frac{kL}{2} = 2.365, k^2 = 22.373/L^2.$ 

Taking E = 30,000,000, for steel, and making these substitutions, Eq. V reduces to:

VI. s = 4.3869 x  $10^8 (\frac{t}{L^2}) y_m$ , p.s.i, where t is thickness of bar, inches.

Frequency

Since 
$$k^4 = \frac{w^2}{a^2}$$
, and  $a^2 = \frac{EI}{AD}$ ;  
 $f = \frac{w}{2\pi} = \frac{k^2}{2\pi} \frac{EI}{AD}$ ;

Taking E = 30,000,000 and p = .28183/g, for steel: VII.  $f = 208.520(t/L^2)$ .

The calculations were originally based upon an approximate equation obtained by what is called "Rayleigh's Method"(8), by which the shape of the vibrating bar is assumed to be: VIII.  $y = y_m \sin(\pi x/L) - A$ 

This equation produces a frequency expression similar to Eq. VII, but about 1 percent higher.

### Test Coupon Size

As the machine was to test samples of material, rather than specimens of structural shapes, welded or riveted joints, etc., a free choice of coupon size was possible. An electromagnet was on hand from a previous experiment, in which the distance between pole centers was 2-inches. As it was deemed advisable to apply the driving force as near the test coupon's center as practically feasible, the length of test coupon was arbitrarily fixed at 14.5-inches. Eq. VIII gave the nodal positions as .2196L and .7804L, or 5.19-inches from each end. The frequency of a coupon ½-inch thick, from Eq. VII; f = 208,520(½/14.5<sup>2</sup>) = 495 cycles per second, or 29,700 cycles per minute, very near the operating frequency hoped for.

As it happens, this relatively large size of test coupon minimizes the "size effect" (9), since the relative proportions of material defects and inhomogenities are reduced.

The width has no appreciable effect on frequencies or on stresses developed. It was believed, however, that making

width greater than thickness would eliminate any tendency to vibration in the horizontal plane through resonant coupling, since the natural horizontal frequency would then be greater than any operational frequency. Also, it was believed that test bar width should be made equal to the width of the magnet poles, to obtain as much magnetic flux as possible. Accordingly, it was decided to make the test coupons one inch wide.

As the mathematical equations used assume a constant section modulus and a constant mass per unit length, it was decided to set tolerances for parallelism in both width and thickness at .0005 x dimension. These tolerances can be easily met in production by finishing the bars on a surface grinder.

No trouble with harmonic frequencies was expected in either vertical or horizontal vibration, because, as shown in Fig. 2, the nodal positions corresponding to higher modes of vibration are different from the positions for the fundamental mode, and the supports would tend to damp the vibration. Furthermore, the resonant frequencies for the higher modes, being proportional to the higher values of kL, (p.5), are not integral multiples of the fundamental frequency, and since all harmonics contained in alternating current are integral multiples of the fundamental, there should have been no forces of the proper frequency to excite the higher modes.

The shape of the vibrating test bar is shown in Fig. 3, p.7.

Base

In the design of the base and its associated parts, the

expectation was that only one specimen of the machine would be constructed. The emphasis, therefore, was upon simplicity and ease of fabrication rather than upon economical use of materials.

The completed machine is shown on page 13. For the sake of clarity, dimensions are omitted from this illustration.

The base was designed to carry the test bar supports, the magnet mounting, and the deflection measuring device. It was also desired to provide room for possible later addition of a photoelectric cell, (for continuous measurement and control), in any location. As a margin of 2½-inches was estimated to be ample for the latter requirement, the size of the base was taken to be: size of test bar + 5-, or 6-inches in width and 20-inches in length.

Any vibration of the base itself would have interfered with accurate deflection measurements, and been otherwise objectionable. As the weight of the electromagnet was uncertain, and the weight of future additions was unpredictable, the resonant frequency could not be calculated. However, any alternating force generated by the electromagnet would have an opposite reaction at the test bar supports. Since the base was to be 5%-inches longer than the test bar, the supports would be nearer the center of the base than the nodal positions, and the driving forces would tend to cancel out.

It was decided to make the base of steel  $\frac{1}{2}$ -inch thick, and to increase the longitudinal stiffness by welding steel bars 5/8-inch wide and  $1\frac{1}{4}$ -inches high to the bottom side along each edge.

### EXPLANATION OF PLATE II

### Testing machine built by author.

- 1. Test bar
- 2. Movable test bar support
- 3. Fixed test bar support
- 4. Electromagnet
- 5. Electromagnet support
- 6. Deflection measuring micrometer
- 7. Test bar clamping screw
- 8. Test bar clamping screw lock

# 

To permit shortening of the test bar supports, a rectangular hole 3-inches wide x 5-inches long was to be out in the center of the base plate into which the electromagnet was to extend.

### Test Bar Supports

To minimize loss of energy through the supports, the test bar was to be supported at the points of least motion, the nodal points. As the motion here is rotation about the transverse centroidal axis, it was decided to clamp the bar between pointed screws at each node. The screws were to be made from SAE 1090 steel, with the points fully hardened to minimize wear, and ground after hardening, the cone angle to be 120°. The method of mounting these clamping screws is shown in the drawing, p. 13. The movable support was designed to allow for the slight displacement of nodal points occurring when the test bar was deflected, also to provide for a slight variation in length of test bars.

### Electromagnet

The design of the electromagnet presented a problem. For greatest efficiency, the reactance of the winding had to be matched to the power supply used. The reactance, however, varies with the size of test bar used, with the length of the air gap between pole faces and test bar, and with the type of winding (10). After consultation with an instructor in the Department of Electrical Engineering, a discarded power trans-

former of 15 watte rating was obtained. The old windings were discarded and part of the core cut away, leaving the shape shown on p. 13, part no. 4,  $2\frac{1}{8}$ -inches long, 2-inches high, and 1-inch wide. The center section of this core was wound with 1800 turns of No. 32 enamel coated magnet wire, the layers being separated by thin ciled paper. It was decided to make the magnet supports of aluminum strip,  $\frac{1}{8}$ -inch thick  $\frac{1}{8}$ -inch wide, bent to permit fastening to the core with machine screws. Non-magnetic material was necessary to prevent diverting the magnetic flux from its intended path. To provide vertical adjustment these supports were to be clamped between nuts on stud bolts projecting upwardly from the base.

### Deflection Measuring Micrometer

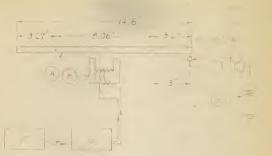
It was apparent that anything allowed to contact the test bar very forcefully at a point of large deflection would interfere with the vibration. However, the design of a suitable electrical device such as mentioned on p. 24, would have been beyond the scope of this project, and suitable optical devices were not available. The solution was to provide an electrical circuit between the test bar and a contact point carried on a micrometer movement, (part 6, p. 13, and Fig. 4, p. 17). The circuit would normally be open, with the contact point retracted beyond the range of motion of the test bar. When a deflection reading was desired, the micrometer could be advanced until the point was just touched by the bar at maximum deflection, thus

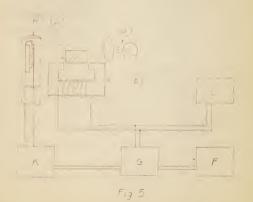
### EXPLANATION OF PLATE III

- Fig. 4 shows a block diagram of apparatus as assembled for first tests.
  - A Test bar, showing location of nodes
  - B Electromagnet
  - C Electrical contact point for measuring deflection
  - D Earphones
  - E Dry-cell flashlight battery
  - F Hewlett-Packard audio oscillator, model 200 B
  - G Audio signal amplifier, Challenger model C 14 Arrows show direction of signal flow.

## Fig. 5 Proposed control and measurement apparatus

- A Test bar, end view
- B Electromagnet, mounted transversly
- F Audio frequency signal generator (commercial)
- G Audio signal amplifier 30 watts power output (commercial)
- H Photoelectric cell
- J Lamp and lens for operating photoelectric cell
- K Preamplifier incorporating signal strength meter, automatic amplitude control, and phase shifter. This piece of apparatus would probably have to be specially designed, but would be easily accomplished by a radio engineer.
- L Cycle counter and automatic shutdown device.





closing the circuit once during each operating cycle and generating a signal which could be heard in earphones.

It was hoped that the small amount of contact necessary to close the electrical circuit would not be great enough to interfere with vibration of the test bar, and to this end the micrometer was designed to be capable of visually controlled movements as small as .0005-inch. Thread specification was American Standard form, 40 threads per inch, 1-inch major diameter, class 4 fit. This large diameter was necessary to provide space through the spindle center for an insulated electrical conductor from the contact point at the top of the barrel to a pickup brush beneath the base plate. The micrometer barrel was to be 3/4-inch in external diameter, with 25 equal divisions on the circumference. Thus each division would represent .001-inch of movement, and with .094-inch distance between each graduation, interpolation to the nearest .0002-inch would be possible.

Because of the obvious difficulty of locating this micrometer device at the center of the test bar, and to take advantage of the larger deflection of the ends, the location shown on p. 13 was selected. It now became necessary to evaluate Eq. IV, to obtain the proportional deflection at this point. As kL = 4.730, kx = 4.73x/L = (4.73)(.273)/14.6 = .0884. Evaluation gave  $y = -1.5024y_m$ , and substitution of this quantity and L = 14.6 in Eq. VI produced:

IX. s = 1,362,000ty

where t is thickness of the test bar, inches, and y is deflection at a point .273-inch from the end.

### Power Supply

Electronic equipment was selected because of availability. A Challenger model C 14 public address amplifier was borrowed from the Department of Machine Design. This amplifier was rated at 14 watts output power, and provided output impedances of 4, 8, 15, 250, and 500 ohms.

It was ascertained from the Department of Electrical Engineering that a Hewlett-Packard model 200 B audio oscillator would be available at such times as not needed for instructional purposes. Frequency of this oscillator was continuously variable from 10 to 20,000 cycles per second.

The arrangement of the power supply is shown in Fig. 4, p.17.

### CONSTRUCTION

The machine was constructed by the author, using facilities of the Department of Shop Practice. Construction occupied a large part of the time spent on the project. A few of the high-lights are presented here.

It was important that the test bar mounting axes be as truly parallel as possible. To this end, the holes for the clamping screws and for the movable support pivot pin were to be drilled and tapped after welding was completed. A horizontal boring mill would have been the proper machine for this operation, but none was then available, so a Browne & Sharp No. 3 milling machine was substituted. To enable threading both holes in a support without moving the work, the size was

enlarged to 7/16-inch, as all the smaller taps had shanks too large to follow through the threaded hole.

Points of the clamping screws were to be ground to 120° after hardening. As the only available lathe at this time was too small to permit mounting a standard tool-post grinder, a small handcraft grinder was clamped to the compound rest and used as a tool-post grinder.

The magnet was wound on a lathe, the end of the core being clamped in a four-jaw chuck. It was impossible to arrange a mechanical guide for the wire, as no feed was available corresponding to the wire size, so the winding was guided by hand, and was slightly uneven as a result. Later, when the magnet was rewound with larger wire, a guide was fastened in the toolpost as close to the work as possible. The wire was fed from a reel, over the guide and onto the revolving magnet. A very even winding resulted.

Threads specified for the deflection micrometer were exceedingly fine in relation to the diameter, and some difficulty was expected in machining to the small tolerance allowed. To minimize this difficulty, finishing was accomplished by lapping the parts together with grade A-2 grinding compound.

### TRIAL RESULTS

On the first trial, the magnet winding became overheated. This was judged to be due to the small wire used, and accordingly the magnet was rewound with 1800 turns of No. 26 enamel coated wire, whose resistance is one-fourth as much as that of

No. 32 wire. No heating was evident in any later trials.

The new winding was provided with taps at 900, 1200, 1500, and 1800 turns, to enable selection of different impedances. A test was run to determine the proper tap for use with the amplifier. At the start of the test, the frequency control was adjusted as close as possible to the resonant point, and the power reduced until vibration amplitude was about 2/3 maximum. These controls were unchanged during the remainder of the test. With the amplifier impedance set at 500 chms, the power leads were connected to each magnet winding tap in turn, the amplitude being measured at each step. The greatest amplitude occurred when the connection was to the highest tap. The process was repeated with the amplifier impedance set at 250 chms, and greatest amplitude was found with the connection on the lowest tap, that is, with half the winding in the circuit.

This seems to indicate that the total magnet impedance was approximately matched to the amplifier setting of 500 ohms. Trials were made with the amplifier impedance set at 4, 8, and 15 ohms, but the amplitudes were so small as to be insignificant.

Early tests were carried out using a test bar 3/8-inch thick, on which the nodal points were marked lightly with a center punch to aid in placing the clamps. This method was found to be too inaccurate, however, and a jig was constructed for holding the test bar in fixed relationship to the screws during clamping. This jig provided adjustments for clamping position and for varying bar size.

A test was run with a 1-inch bar to determine the effect

of slight longitudinal variations in clamping position. As the sorew points made small indentations in the bar, each subsequent position had to be changed by .025-inch to avoid previous marks. Clamping positions at each end were displaced equally to maintain symmetry. Four trials were made, with the results listed here.

Trial 1 2 3 Displacement, inches -.025 0 +.025 +.050 Deflection reading, inches .014 .016 .017 .016 Stress developed\*, p.s.1. 9,500 10,900 11,600 "The accuracy of these stress values is doubtful, because of conditions.

Plus displacements were toward the center of the bar, zero being the calculated node position. Power was set at maximum. Frequency had to be changed slightly for each trial, but the change was too slight to be read on the dial. Trial 3 gave smoothest operation, while during trials 1 and 4 the movable test bar support vibrated objectionably. These results seemed to indicate that the distance between supports was too great in proportion to the length of bar. In subsequent tests, the bar length was increased to 14.6-inches and much smoother operation resulted.

A later comparison between Eq. VIII and Eq. IV indicates a more proper length of test bar would have been 14.73-inches, and one test with this length was made.

Trials with relatively thin bars gave higher stress values, with results as shown in the following tabulation:

Bar Dimensions, Inches t L		Calculated Frequency Cycles per Min.	Maximum Deflection, Inches	Calculated Stress p.s.i.
.122	14.6	7,160	•114	19,000
.185	do	10,600	.094	23,700
.248	do	14,600	.056	19,000
.314	14.73	18,100	.042	18,000
.500	14.5	29,800	.017	11,600

In all cases, except the last, amplitude was limited by the spacing of air gaps. Attempts to increase amplitude by increasing power resulted in the bar striking the magnet poles, yet when the air gap was increased, use of full power failed to produce amplitudes as great as those listed. This fact seems to indicate an optimum length of air gap, beyond which the magnetic attraction is insufficient to produce the desired amplitudes.

During part of these tests, operation was hampered by use of a different audic oscillator whose frequency could not be controlled closely enough. Maximum amplitude was then measured by tuning through the resonant point as slowly as possible, the micrometer point being retracted a few thousandths each time, until no further signal was heard when the amplitude reached its height. The last reading was then taken to be the maximum.

### Frequency Control

The amplitude of vibration was found to be so very sensitive to frequency control that it was difficult to set the dial at the proper value. When set at resonant frequency, with vibration amplitude at maximum, a dial movement so slight as to be nearly imperceptible, less than is cycle per second, was sufficient to lower the amplitude by one-half. This sensitivity seemed more pronounced with the smaller test bars, and where higher stresses were developed. It seems doubtful that a true resonant frequency setting was attained for the larger test bars, (higher frequencies).

### CONCLUSIONS AND SUGGESTED IMPROVEMENTS

A suggested modification in magnet shape and mounting is shown in Figs. 6 and 7, p. 26, in which the air gap is independent of vibration amplitude and can be adjusted vertically to produce maximum force as the test bar reaches its position of greatest velocity. This should enable the transfer of more energy to the bar. Transverse mounting of the magnet would result in application of driving force at the center of the test bar, and the space thus saved would allow shortening of the bar if it seemed desirable.

The need for a more precise method of frequency control is apparent. It is significant that the commercial machines referred to (1,2) and some others of a slightly different type (11,12) use automatic devices for frequency and amplitude control, similar in nature to the system shown in Fig. 5, p. 17, where a signal generated by the movement of the bar is amplified and used to operate the electromagnet. None of these articles showed use of a photoelectric cell for signal pickup,

### EXPLANATION OF PLATE IV

- Fig. 6. Proposed new design of electromagnet. Winding is not shown in left hand view. While the distance between magnetic poles is shown as 21/32", it is to be understood that this dimension should be 1/32" greater than the width of test bars used, so as to provide a clearance of about 1/64" at each side of the test bar.
- Fig. 7. Supports for above electromagnet. 2 required;
  material, cast or wrought aluminum; finish all over.
  Fillets 2" radius.
  All tolerances ±.015 unless specified.

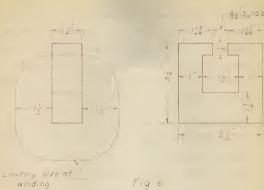
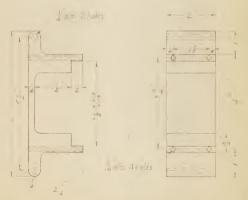


Fig 6



The test bar clamping screws proved to be a source of great damping when tight, because of the rotary movement at the nodes. It was necessary to clamp a test bar tightly enough to produce a small indentation in its surface, then loosen the screws slightly, before operating the machine. It is thought that making the cone angle 90° instead of 120° would have given better holding ability with less frictional losses.

The tests described, plus a few other trials made, are hardly extensive enough to warrant definite conclusions, and should be taken as indications of the direction of further testing. Stresses were developed which were nearly in the fatigue range of mild steel, and were apparently limited by the magnetic requirements.

### ACKNOWLEDGMENT

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