ANALYSIS OF PRESTRESSED AND REINFORCED CONCRETE FOLDED PLATES

by

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SYNOPSIS

The simplified methods of analysis for long span prestressed and reinforced concrete folded plate structures are presented herein.

A reinforced concrete folded plate is analyzed by resolving the applied load into two directions, one vertical and the other parallel to the plate on which it acts. The vertical load will induce "slab" action and the parallel load will induce "plate" action. Slab action can be analyzed by assuming non-yielding supports and then applying a correction for deflection, from which total plate loads are found. "Plate" action causes a deflection of the longitudinal edge and introduces longitudinal stresses. To satisfy the compatibility condition, a stress distribution is done. Moments are computed due to relative displacements. From these moments, plate loads and longitudinal stresses are calculated.

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To analyze the prestressed plate, the stress distribution along the longitudinal axis is analyzed in the same manner as for any homogenous beam of rectangular section.

Transverse bending in successive plates is analyzed by the conventional moment distribution method utilized for continuous structures.

A typical problem is outlined and the simplified method of analysis is discussed in this report.

INTRODUCTION

Folded plates, or as they are sometimes called, prismatic shells or hipped plates, provide a useful and economical method of construction for roof and floor systems in a wide variety of structures. They are competitive with other construction methods for short spans and have proven exceptionally economical where relatively large spans are needed as for auditoriums, gymnasiums, industrial buildings, hangers, department stores and parking garages. The folded plate shape of roof structure has come into wide usage because of its low cost of construction for long span, high load carrying capacity, rigidity, and aesthetic interest. Selection of concrete for the shell material furnishes a high degree of fire resistivity, ease of molding to the desired alinement and profile, a great degree of permanence, and low construction and maintenance costs.

Folded plate floor or roof construction consists of a series of repeated units, each of which is formed by two or more flat plates intersecting at an angle. The plates act as a continuous slab transversely and as beams in their own planes. Their structural behaviour resembles that of shells. In fact, a cylindrical shell can be thought of as a folded plate in limit. The structural action of a folded plate consists of transverse "slab action" by which the loads are carried to the joints and longitudinal "plate action" by which the loads are finally transmitted to the transverses. Because of its great depth and small thickness, each plate offers considerable resistance to bending in its own plane. This "plate action" explains the remarkable rigidity of folded plate construction.

Folded plates have certain advantages over shells. These advantages are:

(1) The shuttering required is relatively simpler as it involves only straight planks.

(2) Shuttering can be stripped at the end of seven days, if not earlier, because of their greater rigidity; this results in quicker turnover which, in turn, cuts down construction time.

(3) The design involves only simple calculations which do not call for a knowledge of higher mathematics.

(4) Movable formwork can be employed for their construction with greater ease than with cylindrical shells.

(5) Simple rectangular diaphragms take the place of complicated transverses required for shells.

(6) Their light reflecting geometry and pleasing outlines make them comparable with shells in their aesthetic appeal.

Interest in and use of this type of roof has increased considerably in this country. Considerable additions have been made to our analytical and experimental knowledge in the last decade. The purpose of this report is to solve typical problems and discuss procedures which may be employed for the analysis of a single-span folded plate structure.

DEFINITIONS

The following definitions are used as a basis for the discussion in this report:

(1) A plate is an individual planar element of the structure.

(2) The length of a plate is the dimension between transverse supports. (Fig. 1, "L")

(3) The width of a plate is the transverse dimention between longitudinal edges. (Fig. 1, "W")

(4) The height of the structure is the vertical dimension of the upper and lower extremes of a transverse cross section. (Fig. 1, "h")

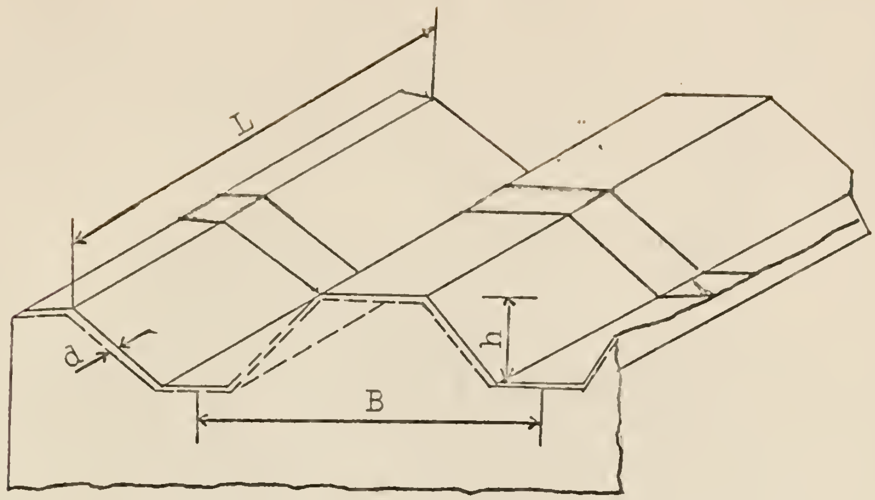


FIG. 1. - END PORTION OF A FOLDED PLATE ROOF.

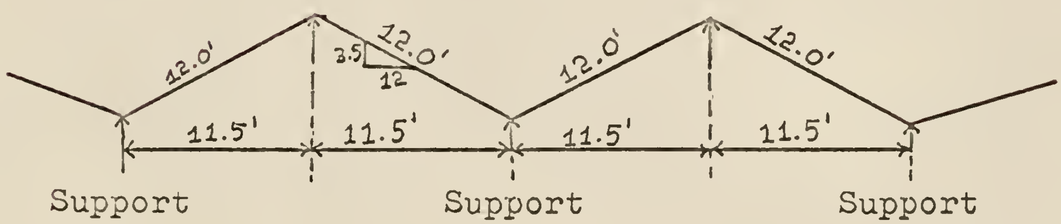


FIG. 2. - TYPICAL CROSS SECTION THROUGH MULTIPLE FOLDS

STATEMENT OF PROBLEM

Consider a simple span of 60 feet in which we shall arbitrarily assume a column spacing along the ends of this longer span of 23 feet. The slope of each plate will be assumed to be 3 1/2 on 12, vertical to horizontal. (Fig. 2.) A trial analysis will indicate this corrugated configuration to be unsuitable for conventionally reinforced concrete due to the very small area of concrete available at the ridge, causing high compressive stresses and a very high percentage of compressive steel reinforcement. This can be primarily attributed to the shallow ratio of depth from ridge to valley for the long span of 60 feet and the absence of sufficient concrete area in the ridge.

Several remedies for this condition become readily apparent:

(1) Steepen the slope of the plate and add a third plate horizontally at the ridge in order to provide sufficient concrete area to reduce the high compressive stresses.

(2) Steepen the slope of the plates in order to provide a deeper section from ridge to valley.

(3) Provide a combination of each of those stated in (1) and (2).

(4) Prestress the structure in order to utilize all of the concrete area as homogenous section.

This report shall advance methods of analysis for design of folded plates by the above mentioned procedures (2) and (4).

(a) To steepen the slope of the plates in order to provide a deeper section from ridge to valley. The folded plate is analyzed by considering the effects of relative displacements of the longitudinal edges.

(b) To prestress the structure in order to utilize all of the concrete area as homogenous section.

ANALYSIS OF FOLDED PLATES CONSIDERING JOINT DISPLACEMENT

Assumptions

The following general assumptions are made in analyzing a folded plate structure:

1. The material is homogenous, uncracked and elastic.
2. Longitudinal edge joints are fully monolithic and continuous; there is no relative rotation or translation of two adjoining plates at their common boundary.
3. The principal of superposition holds, that is, the structure may be analyzed separately for the effects of its redundants and various external loadings and the results combined algebraically.
4. Individual plates possess negligible torsional resistance and torsional stresses due to twisting of the plates can be neglected.
5. The function of the supporting diaphragms or bents is to supply the end reactions for the plate action and

for the longitudinal slab action. They are assumed incapable of providing restraint against rotation of the ends of the plates in their own planes, but may provide some restraint for longitudinal slab bending.

6. Longitudinal strain due to plate action varies linearly across the width of each plate (plane section remains plane). The rate of change of strain with respect to width ordinarily will differ from plate to plate from which it can be inferred that there will be some relative displacement of the joints of a cross section.
7. Longitudinal slab action can be neglected; that is, slab bending carries the load applied to the surface of a plate to the longitudinal edges only, as in a one way slab.

ANALYSIS

Primary Stresses

The folded plate for the structure, as discussed in "Statement of Problem", is deepened at ridge and valley as shown in the line diagram, Fig. 3.

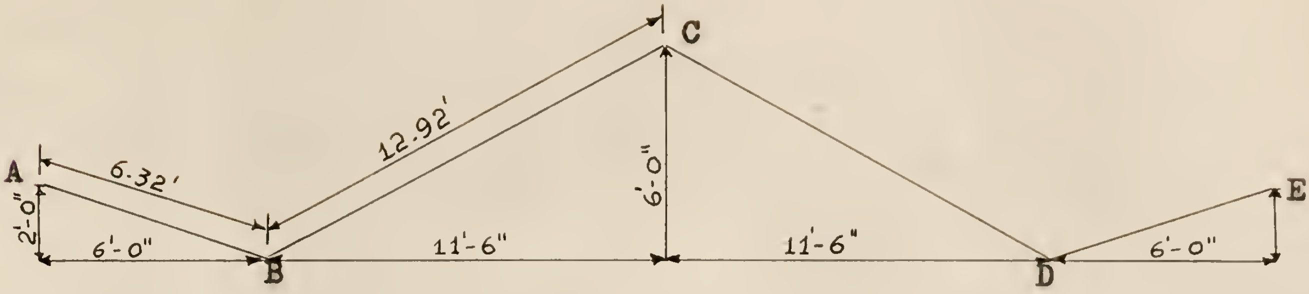


FIG. 3. - LINE DIAGRAM OF FOLDED PLATE STRUCTURES.

The data for this sample solution are as follows:

Slab thickness = ... 4 1/2"

Span between transverses = . . . 60' 0"

Column spacing = ... 23' 0"

<u>Loading</u>:

Roofing . . . . 5 psf

Snow load, insulation, and accoustatics . . . 30 psf

The loads on the inclined surfaces are as follows:

<u>For Plate AB</u> (Fig. 4)

<u>Snow Load</u>:

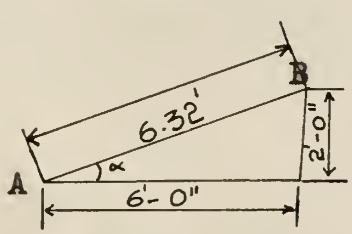


FIG. 4. - PLATE AB.

$$\cos\alpha = \frac{6}{6.32} = 0.949$$

$$\sin\alpha = \frac{2}{6.32} = 0.317$$

Snow load on inclined surface

$= 30 \times \cos\alpha = 30 \times 0.949 = 29.47$ lbs/sq. in.

Dead Load:

Roofing material = 5.00 lbs/sq. in.

Slab . . . . . . $\frac{4.5 \times 150}{12}$ = 55.33 lbs/sq. in.

Total Load:

Sum of above three

$= 29.47 + 5.00 + 55.33$

$= 89.80$ say 90.00 lbs/sq. in.

Components of Total Load: (Fig. 5.)

This load can be resolved into normal and tangential components, Wn and Wt.

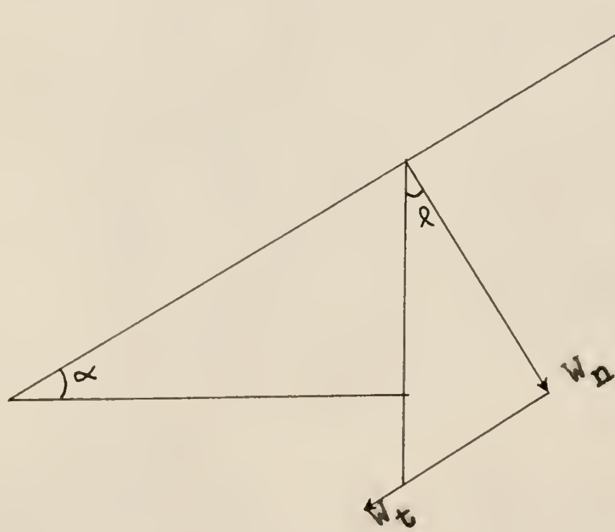


FIG. 5. - NORMAL AND TANGENTIAL COMPONENTS ON PLATE AB.

$W_n = W \times \cos\alpha$

$= 90 \times 0.949$

$= 85.41$ lbs/sq. in.

$W_t = W \times \sin\alpha$

$= 90 \times 0.317$

$= 28.53$ lbs/sq. in.

For Plate BC (Fig. 6.)

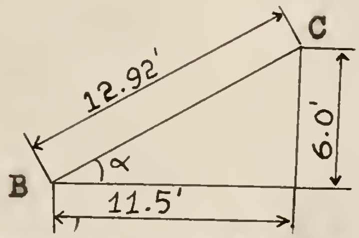


FIG. 6. - PLATE BC.

Snow Load:

$$= \frac{30 \times 11.5}{12.92}$$

$= 27.62$ lbs/sq. in.

Dead Load:

Roofing	= 5.00 lbs/sq. in.
Dead load of slab	= 55.33 lbs/sq. in.

Total Load:

= 87.95 lbs/sq. ft.

Components of Total Load:

Normal component $W_n = \dfrac{87.95 \times 11.5}{12.95}$

$= 77.5$ lbs/sq. ft.

Tangential component $= W_t = \dfrac{87.95 \times 6}{12.92}$

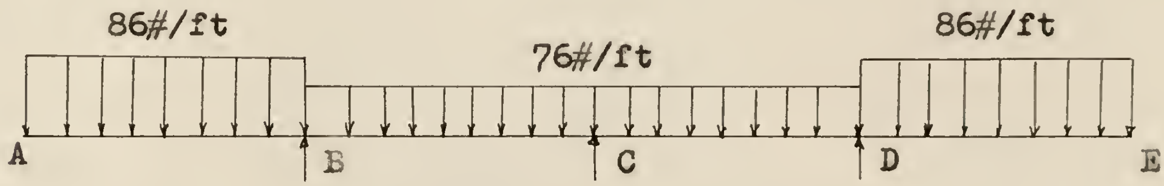
$= 40.60$ lbs/sq. ft.

The load on DE is by observation the same as the load on BC, that is:

W_n = 77.5 lbs./sq. ft.   say 78 lbs./sq. ft.

W_t = 40.6 lbs./sq. ft.   say 41 lbs./sq. ft.

For the purpose of analysis, assume fictitious supports at joints B, C, D, and calculate the moments and reactions. It is assumed that the relative displacement of joints is not present. A transverse strip one foot wide is considered, treating it as continuous slab supported at the joints by non-yielding supports. The moment distribution for this condition is performed in Fig. 7.



	A — B	B — C		C — D		D — E
Dist. F.	0	1	0.5	0.5	1	0
FEM	+1725	-1090	+1090	-1090	+1090	
C.O.		-635	-317.5	+317.5	+625	
Final M	+1725	-1725	+772.5	-772.5	+1725	-1725

FIG. 7. - SLAB MOMENT DUE TO EXTERNAL LOAD.

Reactions are as follows:

Total reaction at B = 1124 lbs.

Total reaction at C = 868 lbs.

Total reaction at D = 1124 lbs.

These reactions were obtained on the assumption that fictitious supports exist at B, C, and D and hence, on removal, they give equal and opposite reactions. These reactions are placed on the plates as shown in Fig. 8.

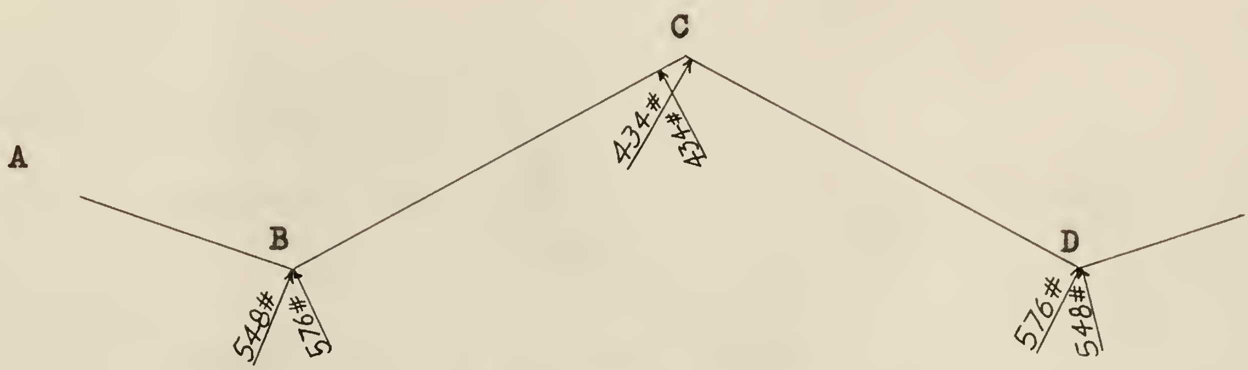


FIG. 8. - PLATE SHOWING REACTIONS DUE TO SLAB ACTION.

These plate loads due to the above reactions can be obtained as follows (see Fig. 9):

$$\tan \theta_1 = \frac{2}{6} = 0.3333$$

$$\theta_1 = 18° - 26'$$

$$\tan \theta_2 = \frac{6}{11.5} = 0.521$$

$$\theta_2 = 27° - 40'$$

Considering joint B, (see Fig. 10)

$$\frac{\text{Comp. along AB}}{\sin(90 + \theta_1 + \theta_2)} = \frac{\text{Comp. along BC}}{\sin 90} = \frac{548}{\sin(180 - \theta_1 - \theta_2)}$$

$$\text{Comp. along AB} = \frac{548 \times \sin(90 + \theta_1 + \theta_2)}{\sin(180 - \theta_1 - \theta_2)}$$

$$= 548 \times 0.96 = 526 \text{ lbs.}$$

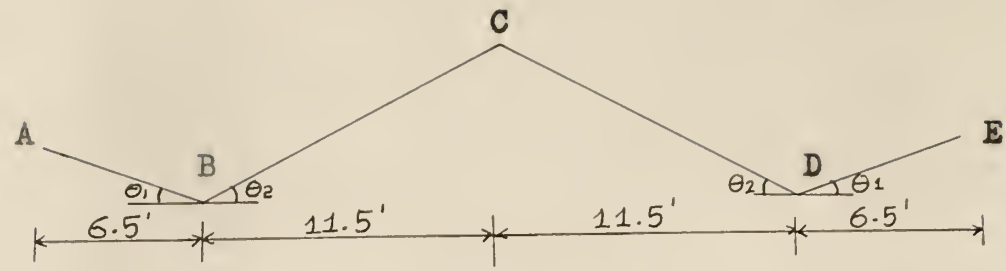


FIG. 9. - INCLINATION OF PLATES.

$$\text{Comp. along BC} = \frac{548 \times \sin 90}{\sin (180 - \theta_1 - \theta_2)}$$

$$= \frac{548}{0.71}$$

$$= 770 \text{ lbs.}$$

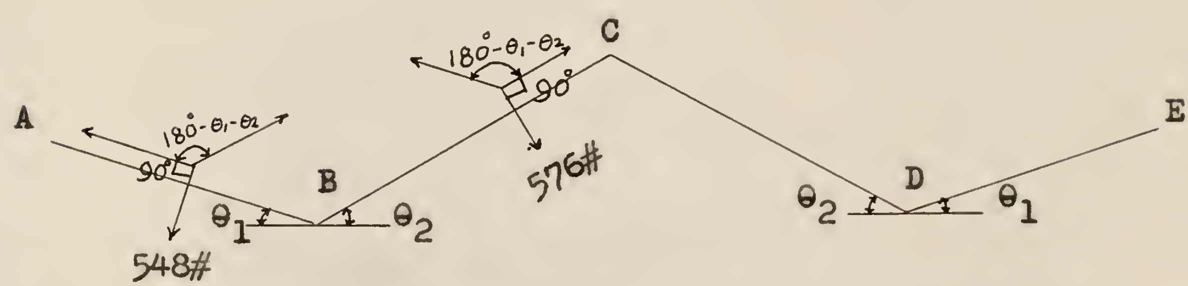


FIG. 10. - COMPONENTS ALONG AB AND BC DUE TO REACTIONS.

Similarly due to the load of 576 lbs. the components along AB and BC can be obtained as follows:

$$\frac{\text{Comp. along AB}}{\sin 90} = \frac{\text{Comp. along BC}}{\sin (90 + \theta_1 + \theta_2)} = \frac{576}{\sin (180 - \theta_1 - \theta_2)}$$

$$\text{Comp. along AB} = \frac{576 \times \sin 90}{\sin (180 - \theta_1 - \theta_2)}$$

$$= \frac{576}{0.71} = 812 \text{ lbs.}$$

$$\text{Comp. along BC} = \frac{576 \times \sin(90 + \theta_1 + \theta_2)}{\sin(180 - \theta_1 - \theta_2)}$$

$$= 576 \cot(\theta_1 + \theta_2)$$

$$= 576 \times 0.96$$

$$= 553 \text{ lbs.}$$

Considering joint C, see Fig. 11.

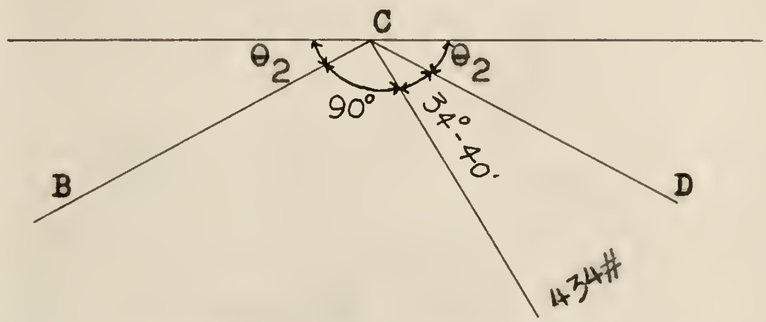


FIG. 11. - COMPONENTS DUE TO REACTIONS AT C.

$$\frac{\text{Comp. BC}}{\sin 34° - 40'} = \frac{\text{Comp. along CD}}{\sin 90°} = \frac{434}{\sin(180 + 2\,\theta_2)}$$

$$\text{Comp. along BC} = 434 \times \frac{\sin 34° - 40'}{\sin 55° - 20'}$$

$$= 434 \times \frac{0.56}{0.82}$$

$$= 297 \text{ lbs.}$$

$$\text{Comp. along CD} = \frac{434}{\sin 55° - 20'}$$

$$= \frac{434}{0.82}$$

$$= 528 \text{ lbs.}$$

Due to 434 lbs. reaction

Comp. along BC = 528 lbs.

Comp. along CD = 297 lbs.

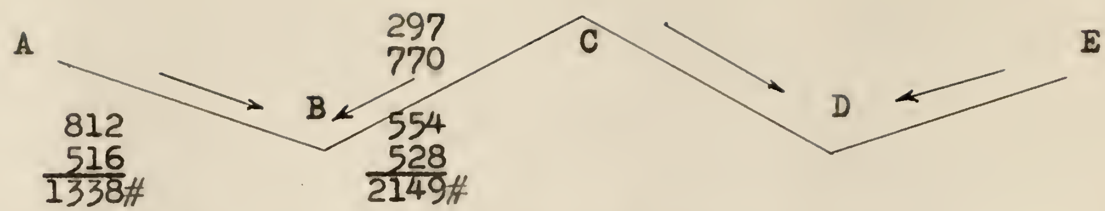
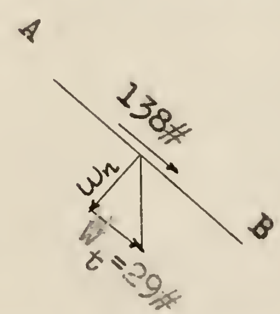


FIG. 12. - TOTAL FORCES AT THE JOINTS.

The total forces at the joints are shown in Fig. 12.

The total load along AB can be calculated as follows

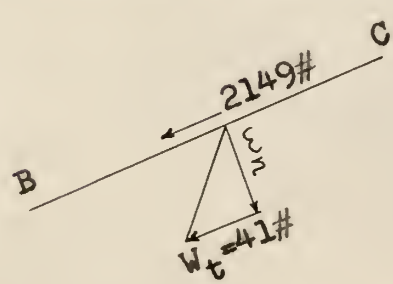


Load along plate AB = 1338 + Wt x 6.32

= 1338 + 29 x 6.32

= 1521 lbs.

Similarly total load along plate BC



= 2149 + Wt x 12.92

= 2149 + 528

= 2677 lbs.

From these loadings we can find longitudinal stresses on each plate. In the analyses one treats each plate as a beam carrying loads and spanning between end diaphragms with no edge shear along joints.

Plate AB (Fig. 13.)

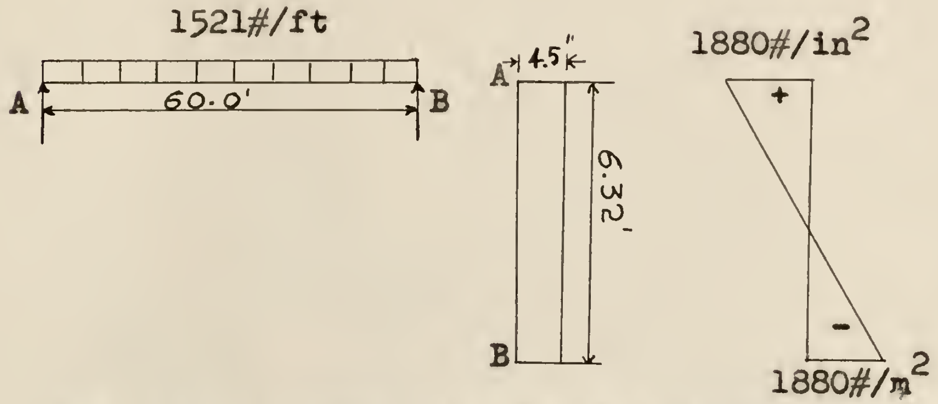


FIG. 13. - STRESS DISTRIBUTION ALONG PLATE AB.

$$f = \frac{M \times c}{1}$$

$$M = \frac{w \times L^2}{8}$$

$$f = \frac{w \times L^2 \times h \times 12_1}{8 \times 2 \times b \times h^3}$$

$$= \frac{3 \times w \times L^2}{4 \times b \times h^2}$$

$$= \frac{3 \times 1521 \times 60^2 \times 12}{4 \times 4.5 \times 6.32 \times 6.32 \times 144}$$

$$= 1880 \text{ lbs/sq. in.}$$

The stresses in top and bottom fiber are

$$f_t = + 1880 \text{ lbs/sq. in.}$$

$$f_b = - 1880 \text{ lbs/sq. in.}$$

Plate BC. (Fig. 14.)

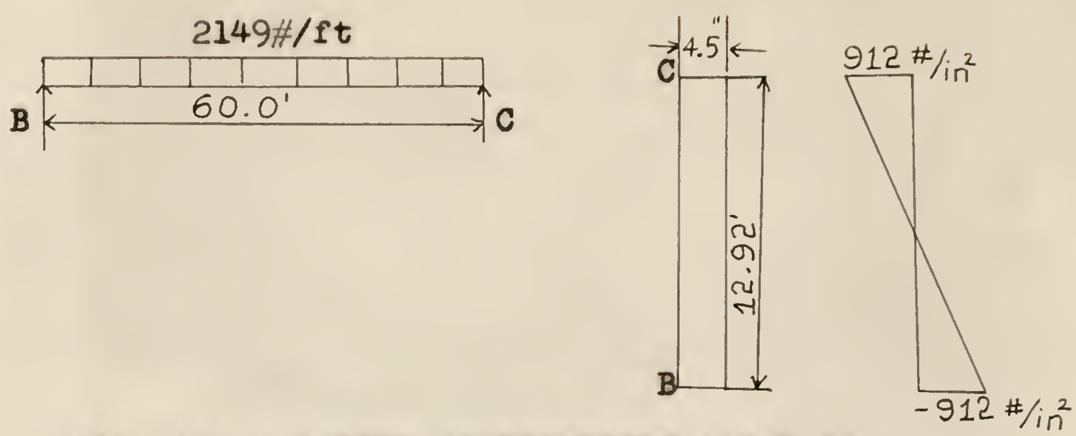


FIG. 14. - STRESS DISTRIBUTION ALONG BC.

$$f = \frac{3 \times w \times L^2}{4 \times b \times h^2}$$

$$= \frac{3 \times 2.49 \times 60 \times 60 \times 12}{4 \times 4.5 \times 12.92 \times 12.92}$$

$$= 912 \text{ lbs/sq. in.}$$

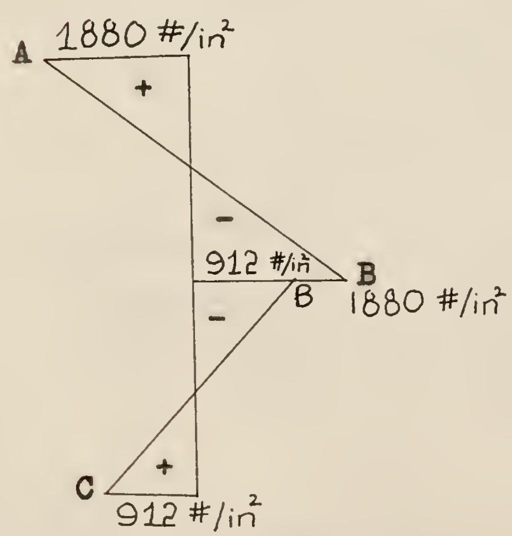


FIG. 15. - STRESSES AT JUNCTIONS OF PLATES.

From the preceding analysis, it is seen that the stresses at the junctions of plates AB and BC at B are different as shown in Fig. 15, but the compatibility condition requires them to be equal. To satisfy the compatibility condition, a stress distribution similar to the process of moment distribution is done where,

Stiffness . . . . $\propto \frac{1}{bh}$

Carry over factor . . . $= -\frac{1}{2}$

Since the thickness of AB and BC are equal, the stiffness factors are as follows:

$$K_{ab} = \frac{1}{6.32}$$

$$= 0.160$$

$$K_{ab} = \frac{1}{12.92}$$

$$= 0.078$$

The stress distribution is as follows:

	A	½	B	½	C	½	D	½ E
DF	0.672		0.328	0.5	0.5	0.328	0.672	
Stresses	+1880	-1880	-912	+912	+912	-912	-1880	+1880
C.O.	-325	+650	-318	+159	+159	-318	+650	-325
Final Stress	+1555	-1230	-230	+1071	+1071	-1230	-1230	+1555

Sign Convention:

Compression $+ V_e$

Tention $- V_e$

Secondary Stresses

Assume that plates AB, BC, CD and DE are not rigidly jointed and solve for the free edge deflections. There will be stresses due to rotations of those plates which do not have a free edge. Considering a transverse strip one foot wide at the middle of the structure

$$\delta = \frac{-5 \times L^2 (f_t - f_b)}{48 \times E \times h}$$

where f_b = stresses at left edge of plate

f_t = stresses at right edge of plate.

The deflection of plate BC can be computed as follows:

$$\delta_{bc} = \frac{-5 \times L^2 (f_t - f_b)}{48 \times E \times h}$$

$$= \frac{5 \times 60 \times 60 \quad 1071 - (-1230) \quad \times 144}{48 \times E \times 12.92}$$

$$= \frac{5 \times 3600 \times 2300 \times 144}{48 \times 12.92 \times E}$$

$$= \frac{18 \times 23 \times 144 \times 10^5}{48 \times 12.92 \times E}$$

$$\delta_{bc} = \frac{-96.0 \times 10^5}{E}$$

Similarly for AB

$$\delta_{max} = \frac{-5x\ 60 \times 60\ (-1230 - 155) \times 144}{48 \times E \times 6.32}$$

$$= + \frac{5 \times 3600 \times 2785 \times 144}{48 \times 6.32}$$

$$\delta_{ab} = \frac{236 \times 10^5}{E} \text{ inches.}$$

From these individual deflections, the composite deflection is found out by graphical construction as shown in Figure 17. From Figure 17, we obtained the deflection of $BC = \frac{170 \times 10^5}{E}$. This deflection is obtained on the assumption that edges A, B, D, D, and E, are all free but actually this is not the case and the deflection will be very, very small due to the fixity of the joints. Therefore, moments are calculated which are produced by this relative displacement. In case of a fixed end beam, moments induced at A and B by sinking of supports B will be $-\frac{6EI\delta}{L^2}$, see Fig. 16.

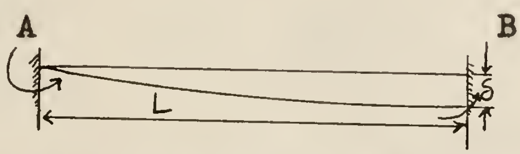
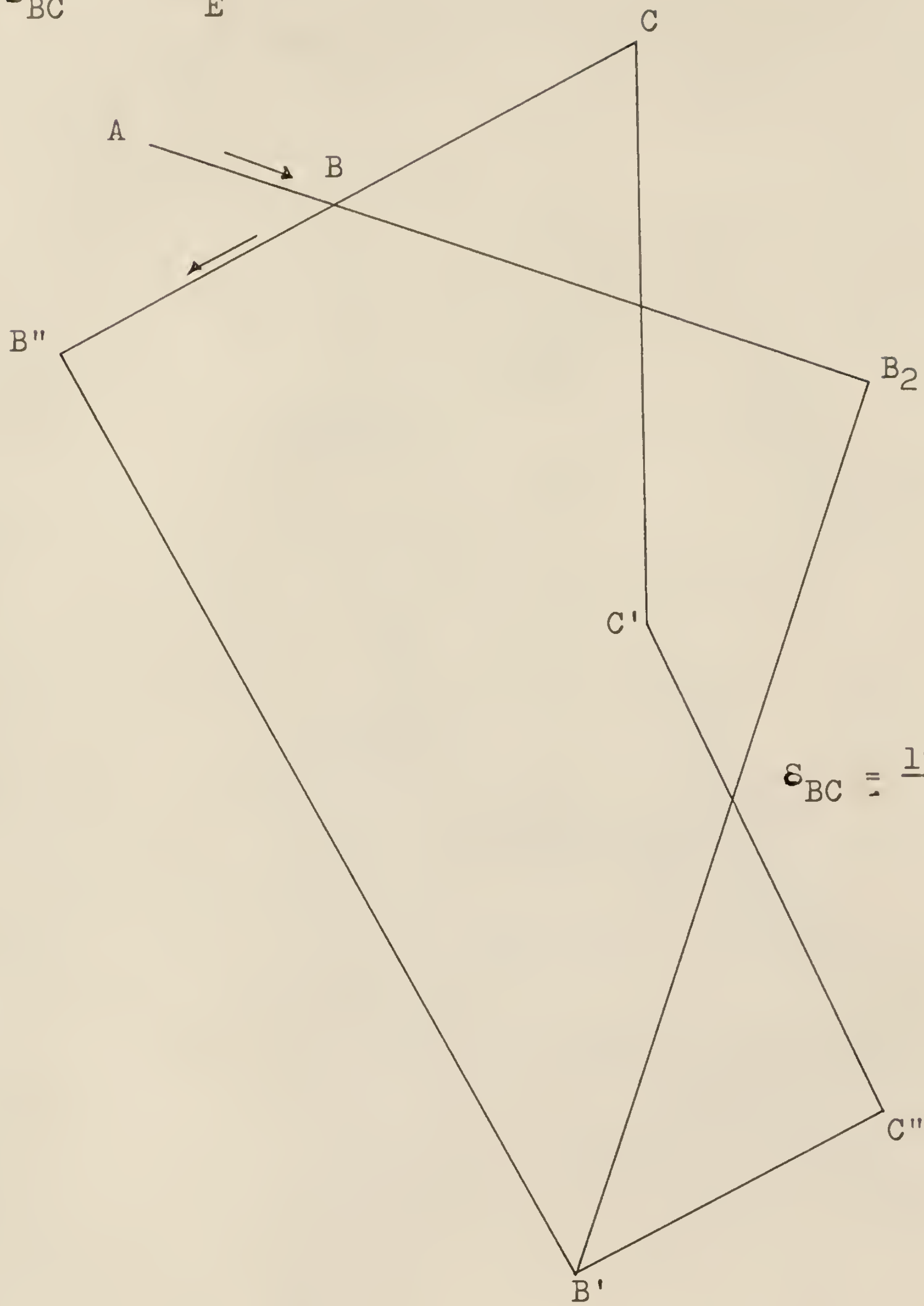


FIG. 16. - MOMENT DUE TO DISPLACEMENT.

Considering edge C as the fixed end and end B as the end free to rotate, the bending moment at C = $\frac{3\ EI\ \delta}{L^2}$.

$$\delta_{AB} = \frac{236 \times 10^5}{E} \text{ in units}$$

$$\delta_{BC} = \frac{96 \times 10^5}{E} \text{ in units}$$



$$\delta_{BC} = \frac{170 \times 10^5}{E} \text{ in units}$$

Scale: 1" = 60"

FIG. 17. - COMPOSITE DEFLECTION ALONG BC.

$$I = \frac{b \times t^3}{12}$$

$$= \frac{1 \times 4.5^3}{12^3 \times 12}$$

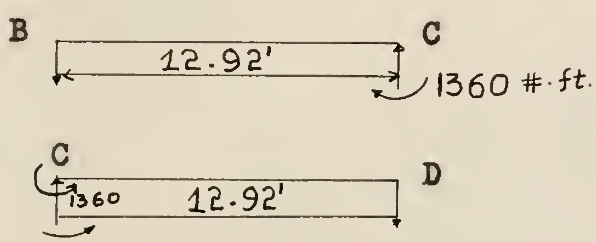
$$I = \frac{1}{19 \times 12}$$

$$M_c = \frac{3 \times E \times 170 \times 10^5}{19 \times 12 \times E \times 12.92 \times 12.92}$$

where $s = \dfrac{170 \times 10^5}{E}$ $\qquad L = 12.92$ ft.

$M_c = 1360$ lb. ft.

The calculation of shears is as follows:



$$R_b = R_c = \frac{1360}{12.92}$$

$$= 105 \text{ lbs.}$$

The components of the loads in the direction parallel to plates are

Plate load along AB $= \dfrac{105}{0.71}$

$= 148$ lbs.

Plate load along BC $= 105 \times 0.96$

$= 101$ lbs.

Plate load along BC $= 105 \times \dfrac{0.56}{0.82}$

$= 71$ lbs.

Plate load along BC is therefore = 101 - 71

= 30 lbs.

Plate loads along AB and BC are, (see Fig. 18.)

Plate load along AB = 148 lbs.

Plate load along BC = 30 lbs.

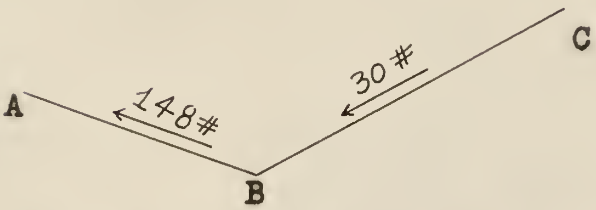
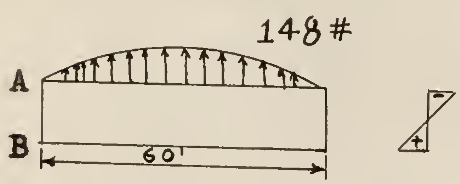


FIG. 18. - PLATE LOADS ON AB AND BC.

From this the moment on plate AB can be obtained as follows:



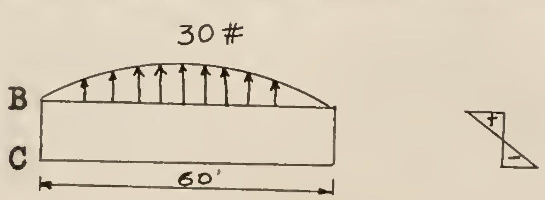
With tension as (-) and compression as (+)

R_a x 60 = 2/3 x 60 x 148 x 30 and

M_o = 2/3 x 148 x 30 x 30 x 5/8

= 55500 ft. lbs.

For plate BC the moment is as follows:



$$M_o = 2/3 \times 30 \times 30 \times 30 \times 5/8$$

$$= 75 \times 150$$

$$= 11250 \text{ ft. lbs.}$$

$$\text{Stress in plate AB} = \frac{M}{Z}$$

$$Z = \frac{b \times h^2}{6}$$

$$Z = \frac{4.5 \times 6.32^2 \times 144}{6}$$

$$= 108 \times 40$$

$$= 4320 \text{ in}^3$$

$$\text{Stress in plate AB} = \frac{55500 \times 12}{4320}$$

$$= 154 \text{ psi}$$

$$\text{Stress in plate BC} = \frac{M}{Z}$$

$$= \frac{11250 \times 12}{Z}$$

$$Z = \frac{b \times h^2}{6}$$

$$= \frac{4.5 \times 12.92 \times 12.92 \times 144}{6}$$

$$Z = 18000 \text{ in}^3$$

$$\text{Stress in plate BC} = \frac{11250 \times 12}{18000}$$

$$= 7.5 \text{ psi}$$

Stress condition for plates AB and BC are as shown in Figure 19.

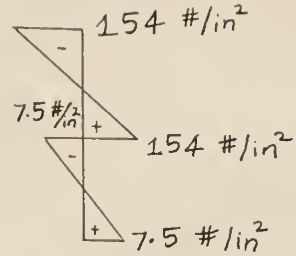


FIG. 19. - STRESSES IN AB AND BC.

Compatibility requires stresses at the junction to be equal and hence stress distribution is carried out as follows:

	A	B		C
		0.672	0.328	
Stresses	-154	+154	-7.5	+7.5
C.O.	+54.25	-109.5	+53	-26.50
Final Check	-99.75	+45.5	+45.5	-19.0

The maximum deflection is obtained once again from which relative deflection is computed by the graphical procedure as in Fig. 21.

$$\delta_{ab} = -5/48 \times L^2 \times (f_t - f_b)$$

$$= +5/48 \times 60 \times \frac{60\ (45.5 + 99.75)}{E \times 6.32}$$

$$= \frac{12.25 \times 10^5}{E} \text{ inches}$$

$$\delta_{bc} = -5/48 \times 60 \times 60\ \frac{(45.5 + 19.0) \times 144}{12.92 \times E}$$

$$= \frac{5/48 \times 3600 \times 64.5 \times 144}{12.92}$$

$$= \frac{-2.70 \times 10^5}{E} \text{ inches}$$

Individual deflections are as shown in Fig. 20, from which relative deflection at BC can be found out graphically as shown in Fig. 21. From Fig. 21, $\delta_{bc} = \frac{11.0 \times 10^5}{E}$ inches

We see that correction applied for a displacement of $\frac{170 \times 10^5}{E}$ causes itself a displacement of $\frac{11.0 \times 10^5}{E}$. The correction will have to apply for a displacement of $\frac{11.0 \times 10^5}{E}$ which will be in form of geometric series, such as

$$a, ar, ar^2, \ldots\ldots ar^n$$

$$\text{Summation} = \frac{a}{1 \mp r}$$

In the above case the series will be as shown below after taking out common factor of $\frac{10^5}{E}$

$$170,\ 170 \times \frac{11.0}{170},\ 170 \times \frac{(11.0^2)}{170^2} \ \ldots\ldots$$

$$\text{Summation} = \frac{170}{1 \mp \frac{11.0}{170}}$$

$$= \frac{170 \times 170}{181}$$

$$= 160$$

$$\text{Correct } \delta_{bc} = \frac{160 \times 10^5}{E}$$

$$\frac{\text{Actual } \delta_{bc}}{\text{Initial } \delta_{bc}} = \frac{\text{Actual Longitudinal stress}}{\text{Initial Longitudinal stress}} = \frac{\text{Actual Moment}}{\text{Initial Moment}}$$

$$= \frac{160.0}{170.0}$$

$$= 0.94$$

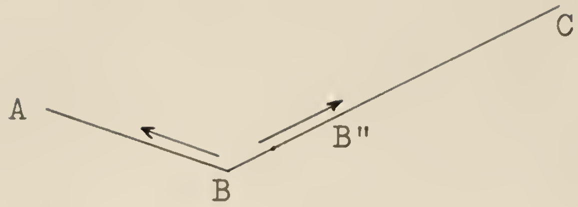
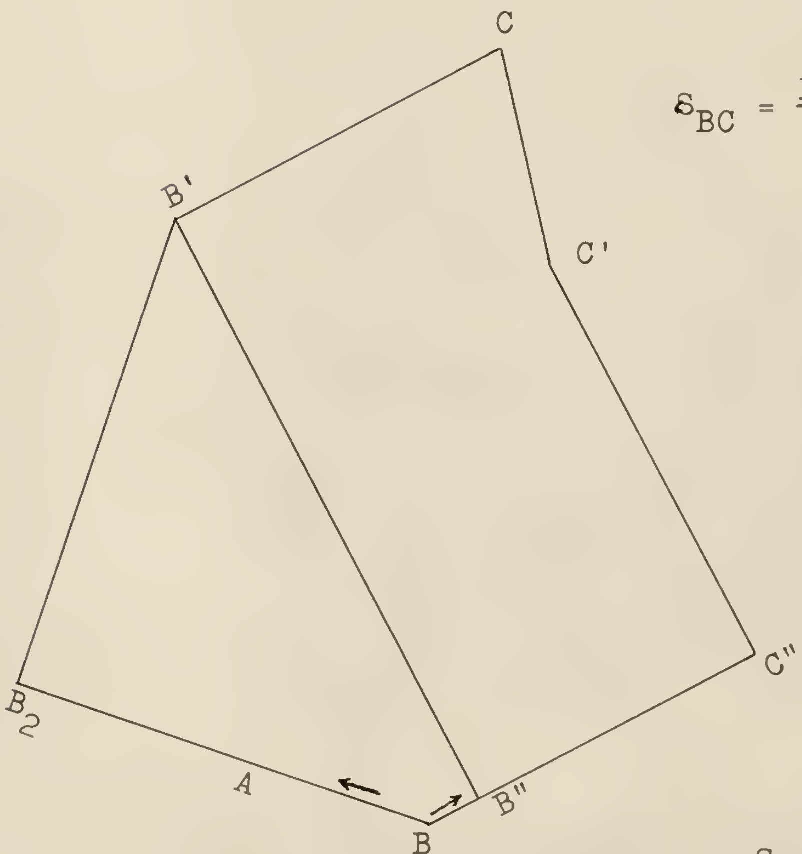


FIG. 20. - INDIVIDUAL DEFLECTION AT AB AND BC.

$$\delta_{AB} = \frac{12.25 \times 10^5}{E} \text{ in units}$$

$$\delta_{BC} = \frac{2.70 \times 10^5}{E} \text{ in units}$$

$$\delta_{BC} = \frac{11.0 \times 10^5}{E} \text{ in units}$$



Scale: 1" = 10"

FIG. 21. - RELATIVE DISPLACEMENT ALONG BC.

The longitudinal stresses at center line of structure due to deflection are multiplied by 0.94 by which we obtain the actual longitudinal stresses.

Longitudinal stresses at the center line of the structure due to deflection are:

	A		B	C
Initial stresses	-99.75	45.5	45.5	-19.0
Final stresses = Initial xo 94	-93.75	+42.8	+42.8	-17.8

The total longitudinal stresses at center line are:

A		B	C	
+1550.00	-1230	-1230	+1071	Stresses due to load (psi)
-93.75	+42.80	+42.80	-17.80	Stresses due to deflections in psi
+1461.25	-1187.20	-1187.20	+1053.20	Final stresses in psi

Final end moments:

A	B		C	D	
+1725	-1725	+77.50	-772.50	+1725	Final end moment due to loading
0	0	+1360	-1360	0	Final end moment due to deflection without correction
0	0	+1280	-1280	0	Final end moment due to deflection with correction c. 1360 x 0.94 = 1280

Moments at center of long span (60'),

$$\text{Moments due to load} = \frac{w \times L^2}{8}$$

$$= \frac{78 \times 12.92 \times 12.92}{8}$$

$$= 1620 \text{ lb. ft.}$$

The moment diagram at center of long span is shown in Fig. 22.

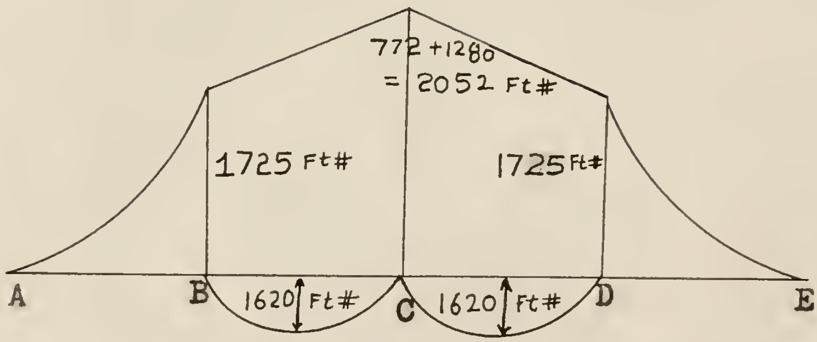


FIG. 22. - MOMENT AT CENTER OF LONG SPAN.

Moment at quarter point of long span (60') due to deflection (Fig. 23):

The moment due to deflection is assumed to have parabolic variation along the span of structure.

$$\text{Ordinate ab} = \frac{1280 \times 15 \times 15}{30 \times 30}$$

$$= 320 \text{ units}$$

Moment at quarter point of long span due to deflection = 1280 -- 320 = 960 ft. lb. (See Fig. 23.)

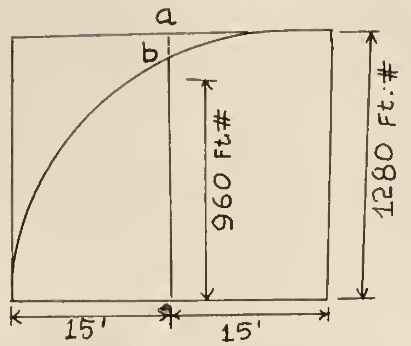


FIG. 23. - MOMENT AT QUARTER POINT DUE TO DEFLECTION.

Moment diagram at quarter point: (Fig. 24.)

Moment at C = 772 + 1280 - 320

= 1732 ft. lbs.

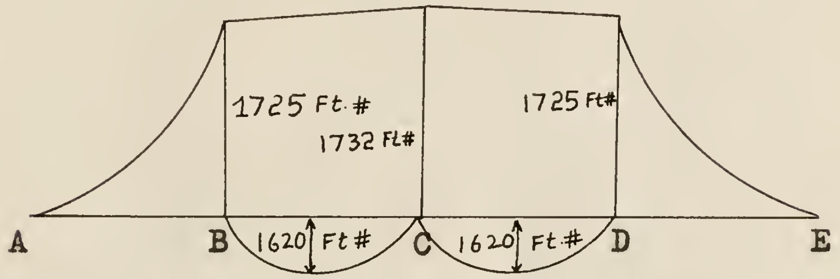


FIG. 24. - MOMENT DIAGRAM AT QUARTER POINT OF LONG SPAN (60').

Moment diagram at end of long span (60'): (Fig. 25.)

Moment due to deflection = 0

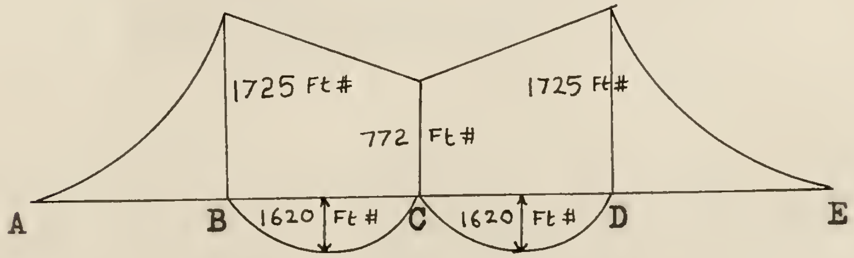


FIG. 25 - MOMENT DIAGRAM AT END.

Shear force at center line of long span: (Fig. 26.)

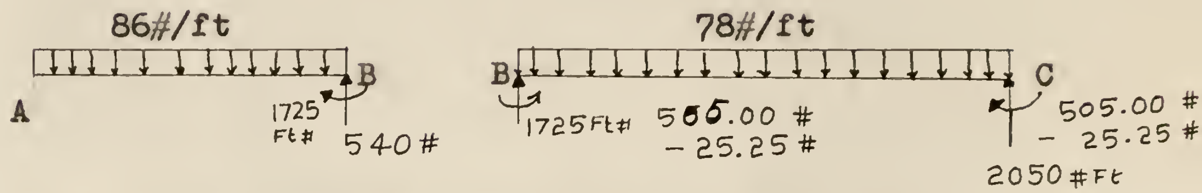


FIG. 26. - SHEAR FORCE AT CENTER LINE OF LONG SPAN.

For Span BC:

Reactions due to Loadings:

$$R_B = R_C = 78 \times \frac{12.92}{2} = 505 \text{ lbs.}$$

Reactions due to Moment:

$$R_B = R_C = \frac{1725 - 2025}{12.92} = - 25.25 \text{ lbs.}$$

For Span AB:

$$R_B = 540 \text{ lbs.}$$

Total shear force at

C = 505 - 25.25 = 479.75 lbs.

B = 505 - 25.25 + 540 = 1011.75 lbs.

Shear force at quarter point of long span: (Fig. 27.)

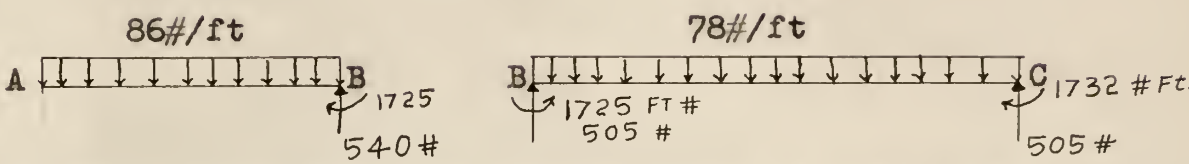


FIG. 27. - SHEAR FORCE AT QUARTER POINT OF LONG SPAN.

For span BC:

Reactions due to loading

$R_B = R_C = 505$ lbs.

Reaction due to moment = negligible

For span AB:

$R_b \times 6.32 = 1725 + 86 \times 6.32 \times \frac{6.32}{2}$

$R_b = 540$ lbs.

Total shear force at

C = 505 lbs.

B = 505 + 540 = 1045 lbs.

Shear force at end of long span (60'): (Fig. 28.)

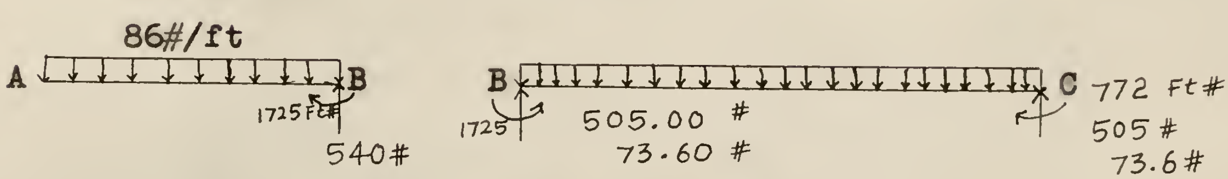


FIG. 28. - SHEAR FORCE AT END.

For span BC:

Reaction due to loading = 505 lbs.

Reaction due to moment $= \frac{1725 - 772}{12.92}$

= 73.6 lbs.

Total shear force at

C = 505 + 73.6 = 578.6 lbs.

B = 540 + 578.6 = 1118.6 lbs.

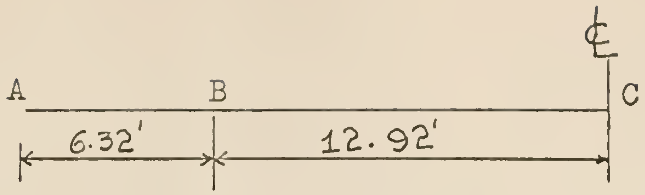
Final results may be summarized as follows:

Maximum longitudinal stresses (consider compression as positive and tension as negative):

A	B	C
+1461.50 psi	-1187.20 psi	+1053.20 psi

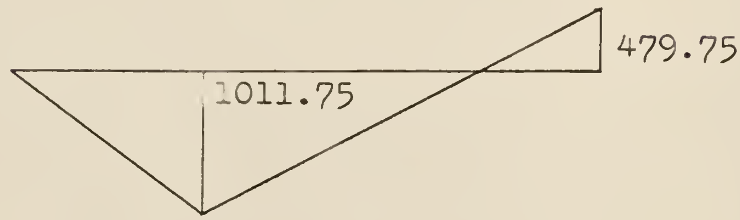
Maximum moments (moments in ft. lbs.):

B	C	D	Location
-1725	-2052	-1725	@ mid span
-1725	-1732	-1725	@ 1/4th span
-1725	-772	-1725	at end

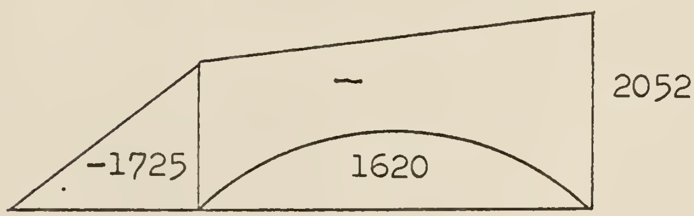


B.M. Diagram is plotted on Tension Side.

PLATES AB AND BC.

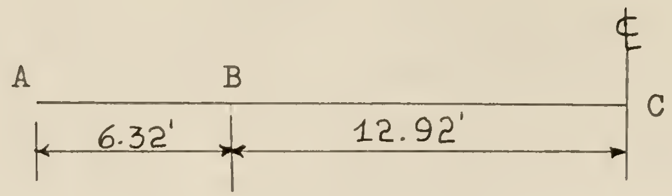


SHEAR FORCE AT CENTER OF LONG SPAN.

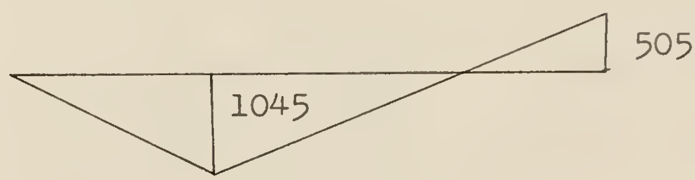


BENDING MOMENT AT CENTER OF LONG SPAN

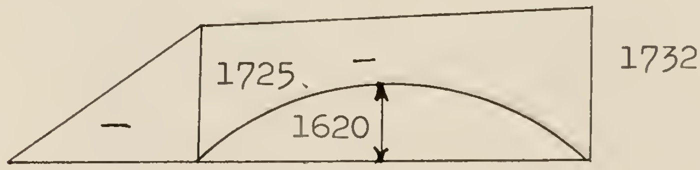
FIG. 29. - BENDING MOMENT AND SHEAR FORCE DIAGRAMS AT CENTER OF LONG SPAN.



PLATES AB AND BC.

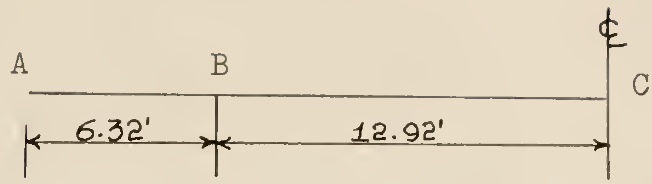


SHEAR FORCE AT QUARTER POINT.

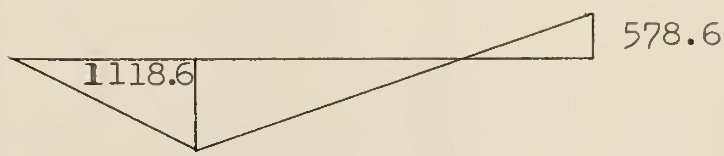


BENDING MOMENT AT QUARTER POINT.

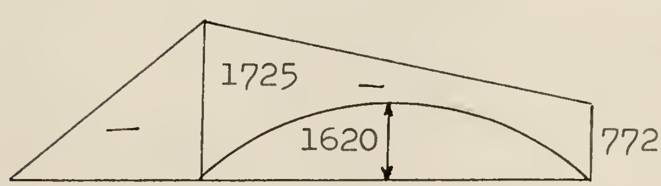
FIG. 30. - SHEAR FORCE AND BENDING MOMENT AT QUARTER POINT.



PLATES AB AND BC.



SHEAR FORCE AT END OF SPAN.



BENDING MOMENT AT END OF SPAN.

FIG. 31. - BENDING MOMENT AND SHEAR FORCE AT END OF SPAN.

ANALYSIS OF POST-TENSIONED PRESTRESSED CONCRETE FOLDED PLATE

The stress distribution acting axially along the longitudinal axis is the same for any homogeneous beam of rectangular section. The top surface will be in compression, the bottom in tension under gravity loads. Application of a sufficient prestressing force at a suitable distance below the center of gravity of the section can eliminate the tension stress.[1]

The transverse direction of the plate is analyzed as a continuous slab of length equal to the width of the plate and supported at the folded lines of the ridges and valleys. These spans being quite short thus require only nominal steel reinforcing as dictated by slab thickness and bending moment. Reductions of the plate action deflections along the ridges, and valleys, due to application of prestressing force, permit analysis of the transverse slab by moment distribution with a reasonable degree of accuracy. Distribution bars are provided in the longitudinal direction in an amount ordinarily utilized for temperature steel.

The design of the structure is more easily understood with the assumption of one individual leaf isolated from the others. One may consider the plate oriented in its working position and compute its sectional properties about the horizontal axis

[1]"Long Span Prestressed Concrete Folded Plate Roofs", by J. Brough and B. Stephens, Proc. Am. Soc. C. E., Jan., 1960, p. 95.

extending through its center of gravity. Figure 32 indicates the terms used in the following presentation.

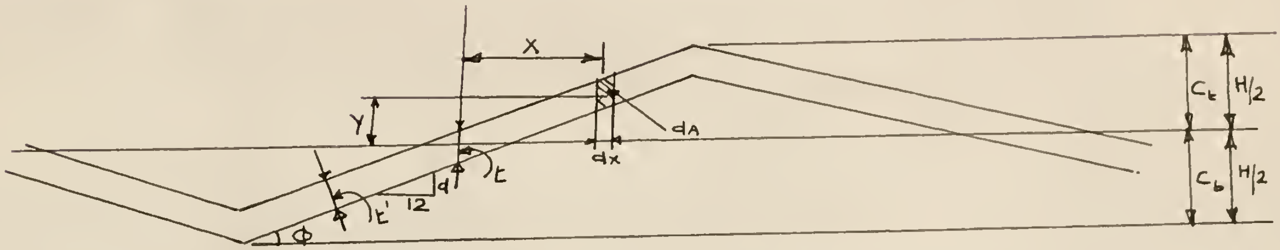


FIG. 32. - SECTION PROPERTIES OF ONE FOLDED PLATE.

First a slab thickness is assumed and then the moment of inertia of the cross section for the inclined position to the vertical axis is computed. The moment of inertia of one plate is derived as follows with a slab thickness of 4½ inches, normal to the slope, and with the slope of the plate being 3.5 vertical to 12 horizontal.

Thus, in Fig. 32:

$$dA = tdx$$

$$y = \tan\phi$$

$$\frac{dy}{dx} = \frac{3.5}{12}$$

$$dx = \frac{12\,dy}{3.5}$$

$$I = \int_{-H/2}^{+H/2} y^2 t\,dx$$

$$= \int_{-H/2}^{+H/2} y^2 t\,\frac{12\,dy}{3.5}$$

$$= \frac{12\ t}{3.5}\ \frac{(y^3)}{3}\Big|_{-H/2}^{+H/2}$$

$$= \frac{12\ t}{3.5}\left[\frac{(H/2)^3}{3} + \frac{(H/2)^3}{3}\right]$$

$$= \frac{12\ t}{3.5}\left(\frac{H}{24} + \frac{H}{24}\right)\Big|_{-H/2}^{+H/2}$$

$$I = \frac{H^3\ t}{3.5}$$

$$I = \frac{H^3\ t}{i}$$

Where t is vertical thickness of the slab

i is the slope rise on 12 which is 3.5

H is height of valley to ridge center to center

t' is the slab thickness normal to slope

$$\text{Thus } t = \frac{t'}{\cos\phi} = \frac{4.5}{12} \times \sqrt{12^2 + 3.5^2}$$

$$= 4.5/0.98$$

$$= 4.6875 \text{ in.}$$

$$I = \frac{4.6875}{3.5} \times (40.25)^3$$

$$I = 87300 \text{ in}^4$$

The tensioning force required for zero stress in the bottom under working load, at midspan is

$$F = \frac{M_t}{e + k_t}$$

Where F is the prestress force

e is the ecentricity of prestress force from the center of gravity.

k is the kern point

The top fiber stress, at midspan under working load is

$$f_{ct} = \frac{FH}{A_c C_b}$$

Where f_c is the allowable concrete unit stress.

H is the vertical height of the section valley to ridge

A_c is the area of concrete.

The bottom fiber stress at initial condition at transfer is

$$f_{cb} = \frac{F_i}{A_c}\left[1 + e - \frac{M_g/F_i}{k_t}\right]$$

Where subscript i indicates the condition at transfer of prestress.

Area of one plate = 4.6875 x 11.5 x 12

$= 647 \text{ in}^2$

$$r^2 = \frac{I}{A} = \frac{87300}{647} = 135$$

$$C_t = C_b = c = \frac{H + t}{2} = \frac{40.25 + 4.6875}{2} = 22.47 \text{ in.}$$

$$k_t = k_b = \frac{r^2}{c} = 6.0 \text{ in.}$$

Where k_t is top kern distance

k_b is bottom kern distance

r is the radius of gyration

If one assumes the center of gravity of the tendons as being e' above bottom of section at midspan, the prestress force eccentric arm is

$e = c - e' = 16.47$ in.

Assume $e' = 6.0$ in.

The longitudinal bending moments acting on the section for the dead load of the slab, the applied dead loads such as the roofing, insulation and ceiling, and the live load for which the structure shall be designed are computed as follows:

Dead load for slab

$\frac{4.5 \times 144}{12} = 54$ PSF on inclined surface

Vertical load

$\frac{54}{\cos\phi} = \frac{54}{0.96} = 56.25$

Total load per plate

$11.25 \times 56.25 = 646$ PSF.

Additional applied dead load:

Roofing (five ply built up asphalt and felt) = 5.0 PSF

Concrete insulation and ½ in. acoustical plaster = 5.0 PSF

Total applied dead load:

$10 \times \cos\phi = 9.6$

Applied dead load per plate

$9.6 \times 11.5 = 111$ lbs.

Snow load per plate

$25 \times 11.5 = 287.5$ lbs.

Moments can be computed as follows:

$$M_{dl} \text{ slab} = 0.646 \times \frac{60^2}{8} = 291.0 \text{ ft-kip}$$

$$M_{dl} \text{ applied} = 0.111 \times \frac{60^2}{8} = 50.0 \text{ ft-kip}$$

$$M_{ll} = .2875 \times \frac{60^2}{8} = 129.6 \text{ ft-kip}$$

$$M_{tl} = 470.6 \text{ ft-kip}$$

The required prestressing force to furnish zero stress in the bottom at mid span with total load moment is

$$F = \frac{M_{tl}}{e + K_t} = \frac{470.6 \times 12000}{16.47 + 6.0} = 250.0 \text{ kips}$$

$$F_o = \frac{250}{0.85} = 294 \text{ kips}$$

F = Total effective prestress after deducting losses.

F_o = Total prestress just after transfer, using say 4 cables, requires $F = 250/4 = 62.5$ kips per cable.

F_i = 73.5 kips per tendon.

Initial tensioning force required for each cable should increase adequately for overcoming tendon friction and wobble of conduit.

Unit stress at mid span $= F_t = \dfrac{F_x H}{A_c C_b}$

$$= \frac{250 \times 44.94 \times 1000}{647 \times 22.47}$$

= 770 psi top comp.

= 0 psi bottom comp.

Under the slab load only, the unit stresses at time of transfer or prestress force will be

$$F_b = \frac{F_e}{A_c}\left[1 + \frac{e - \frac{M_g}{F_i}}{K_t}\right]$$

$$= \frac{294}{647}\left[1 + \frac{16.47 - \frac{291.0}{294}}{6.0}\right]$$

$$= \frac{294}{647} \times 2.54 \times 1000$$

$$= 1155 \text{ psi comp.}$$

Using allowable steel and concrete stresses specified by the Joint Committee 323 ACI, ASCE for prestressed concrete for tensioned member.[2]

Prestressing steel f's = 240,000 psi

f's yield = 210,000 psi

f'_s = ultimate strength of steel

f'_c at design load = 3750 psi

determine the area of prestress steel

$$A_s = \frac{F}{f_s} = \frac{250,000}{144,000} = 1.736 \text{ sq. in., say } 1.75 \text{ sq. in.}$$

Check the unit shear stress:

$$V = \frac{1.1 \times 60}{2} = 33.3 \text{ kips}$$

Assume tendons placed in parabolic curve with c-k = e

= 22.47 - 6.0 = 16.47 in above bottom at ends

[2]Joint Committee 323 ACI, ASCE, for prestressed concrete for tensioned members.

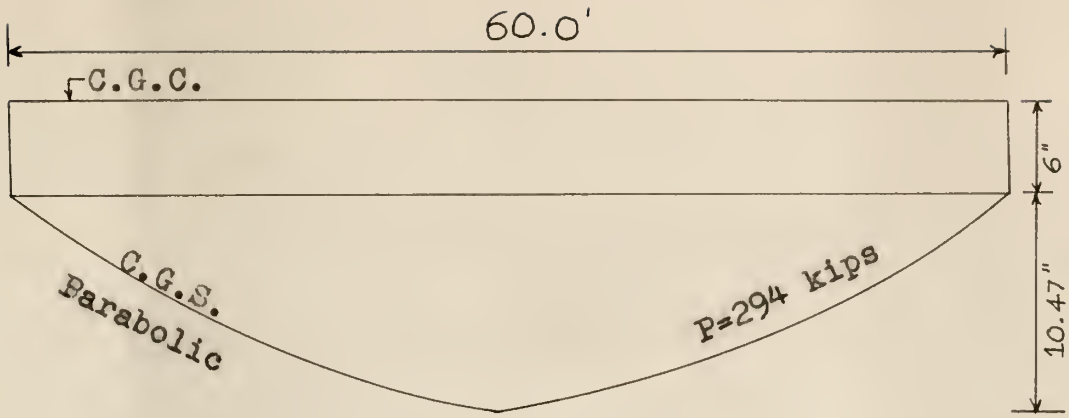


FIG. 33. - LOCATION OF PRESTRESS FORCE ARRANGED IN PARABOLIC CURVE.

Shear at cracking load in steel:

$$V_s = \frac{4 \times F \times h}{L}$$

Where V_s is shear in steel

F is prestressing force after losses

L is length of span, center to center of supports; 61.5'

h is vertical distance below center of gravity of concrete section, 10.45 in.

$$V_s = \frac{4 \times 286 \times 10.45}{61.5 \times 12}$$

$$= 142 \text{ kips}$$

Shear force in concrete:

$$v_c = \frac{V_c Q}{I_b}$$

Where Q is statical moment about center of gravity of concrete section

I is moment of inertia of section

b is horizontal projection of plate width

$$v_c = \frac{19.1 \times 2560}{87300 \times 4.5} = 125 \text{ psi}$$

The principal tensile stress is given by

$$S_t = \sqrt{v_c^2 + (f_c/2)^2} - f_{c/2}$$

Where f_c is allowable concrete unit stress

$$f_c = \frac{250{,}000}{647} = 386 \text{ psi}$$

$$S_t = \sqrt{125^2 + 193^2} - 193$$

= 37 psi, which is satisfactory, since the allowable tensile stress is 110 psi.

Check shear at ultimate load:

$$v_u = \frac{V_c \cdot Q}{I \cdot b}$$

Where V_c is ultimate concrete shear

$2V - v_s = 66.6 - 14.2 = 52.4$ kips

v_u is ultimate shear stress

$$v_u = \frac{52.4 \times 2560}{87300 \times 4.5}$$

= 351 psi

Ultimate principal tensile stress is given by

$$S_t' = \sqrt{v_u^2 + (f_{c/2})^2} - f_{c/2}$$

$$= \sqrt{351^2 + 193^2} - 193$$

= 199 psi, which is satisfactory, since the allowable tensile stress is 300 psi, thus design is satisfactory.

Check section for cracking moment:

The factor of safety against cracking moment

$$= M/M_s$$

Where $M = M_s + M_c$

$$M_s = F\ (e + k_t)$$

$$= \frac{286\ (16.47 + 6.0)}{12}$$

$$= 514.10 \text{ ft. kip.}$$

$$M_c = \frac{F_c . I}{c_b} = \frac{0.14\ f_c' \times I}{c_b}$$

$$M_c = \frac{0.14 \times 3750 \times 87300}{22.47 \times 12} = 170 \text{ ft. kip.}$$

The factor of safety against cracking:

$$= \frac{514.10 - 170}{525} = 1.34$$

The factor of safety against live load cracking:

$$= \frac{M - M_{dl}}{M_{ll}}$$

Where $M = M_s + M_c = 684.10$ ft. kip.

M_{dl} is total dead load moment = 341.0 ft. kip.

M_{ll} is live load moment . . . = 129.6 ft. kip.

The factor of safety against live load cracking:

$$= \frac{684.10 - 341.0}{129.6}$$

$$= 2.65$$

The factor of safety considering ultimate moment:

The factor of safety for total load

$$= \frac{M_u}{M_s}$$

Where $M_u = \dfrac{\text{ultimate tensile force x ultimate lever arm}}{M_s}$

Ultimate tensile force = $f'_c \cdot A_s$

= 240,000 x 1.75 x 0.001

= 420 kips.

Ultimate lever arm can be computed as below: (See Fig. 34.)

Ultimate lever arm = 44.94 - 6.0 - distance of center of concrete resistance below top

Center of concrete resistance if found as below: (See Fig. 34.)

Equivalent width of top section = $\dfrac{12 \times 4.5}{3.5}$ = 15.43 in.

Area of top triangle . . . . . = $\dfrac{4.69 \times 15.43}{2}$ = 36.2 sq.in.

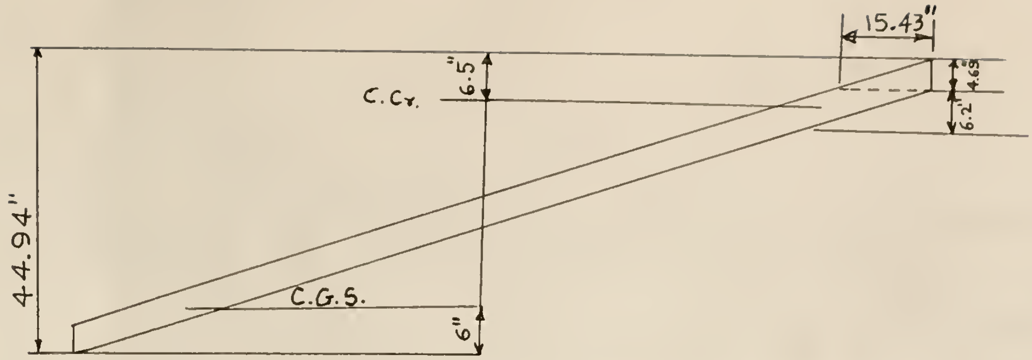


FIG. 34. - ULTIMATE MOMENT STRESS CONDITION.

Additional compressive area of stem necessary is

= Total area under compression - area of top triangle

$$= \frac{420 \times 1000}{3190} - 36.2$$

= 95.3 sq. in.

Additional vertical depth of stem required is

$$= \frac{95.3}{15.43} = 6.2 \text{ in.}$$

Taking moment about top for finding center of concrete resistance "C_{cr}".

$$C_{cr} = \frac{36.2 \times 2/3 \times 4.69 \quad 95.3\ (6.2/2+4.69)}{131.5}$$

= 6.5 in. below top

Ultimate lever arm is

= 44.94 - 6.0 - 6.5

= 32.44 in.

Ultimate moment

$$= \frac{420 \times 32.44}{12} = 113 \text{ ft. kip.}$$

The factor of safety for total load

$$= \frac{M_s}{M_s} = \frac{1138}{514}$$

$$= 2.2$$

The factor of safety for live load

$$= \frac{M - M_{dl}}{M_{ll}}$$

$$= \frac{1138 - 341}{129.6}$$

$$= 6.15$$

Transverse bending in successive plates is analyzed by the conventional moment distribution method utilized for continuous structures. The valley and ridge serve as support for the slab. This being a short span, 11" - 6", moments will be no greater than would be encountered in a level slab with supports at the same frequency.

Analysis of a 1 ft. wide strip transverse to the main span:

$$W_{dl} = 54 + 10$$

$$= 64 \text{ lbs.}$$

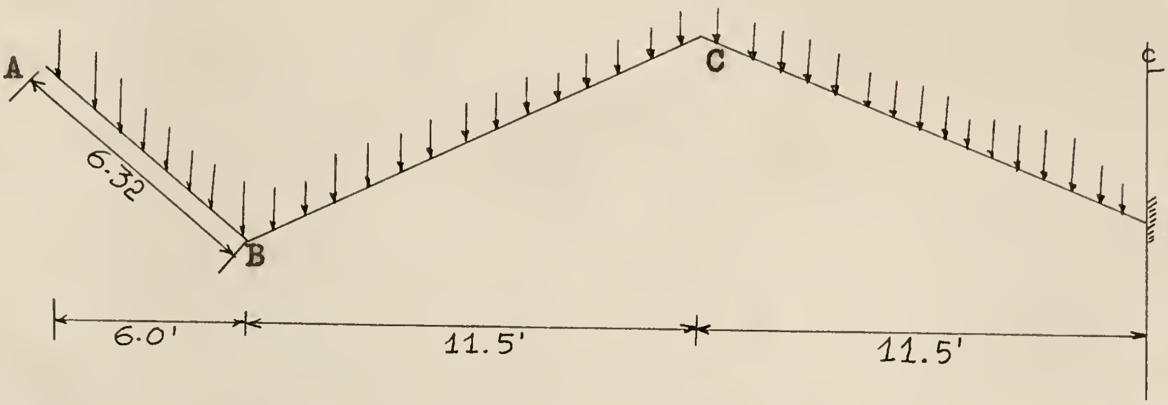
$$W_{ll} = 25 \cos \phi$$

$$= 24.5 \text{ lbs.}$$

$$W \text{ total load} = 64 + 24.5$$

$$= 88.5 \text{ lbs.}$$

Assuming a strip of unit width of cross section of the structure to act as a continuous one way slab on unyielding supports, the ridge moments are determined by the moment distribution. Because of symmetry, the slab may be assumed as rigidly fixed at ridge.[3]



A	B	C	C	D
0	1	3/7	4/7	
+1725	-1060	+1060	-1060	+1060
	-665	-333		
		+143	+190	+95
+1725	-1725	+870	-870	+1155

FIG. 35. - RIDGE MOMENTS.

Ridge moments are computed in Figure 35.

Maximum bending moment at the center of span occurs when that span and every other alternate span is loaded. Thus, bending moment at center due to uniformly distributed loading can be

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[3]"Design of Folded Plates", by Eliahu Tranm, Proc. Am. Soc. C.E., Vol. 85, Oct., 1959, p. 87.

obtained by the following formula:

$$\text{Midspan positive bending moment} = \frac{1\ x}{12}\,(W_d + W_{1/2})\,L^2$$

$$= \frac{1}{12} \times (64 + 12.25) \times 12^2$$

$$= 912 \text{ lb. ft.}$$

Distribution bars are provided in the longitudinal direction in an amount ordinarily utilized for temperature steel. The positive moment reinforcement, extending transversely, was placed on top of the distribution steel. The prestressing tendons were placed directly on top of the bottom steel and tied to the bottom mat at the correct location.

After all tendons were placed, the end bearing plates were substantially anchored to the end form.

Reinforcing bars for slab negative moments at valleys and ridges, stirrups, ties and grids for resistance of bursting of concrete at the end anchorage were placed last.

Deflection:

The moment due to prestress is

$P \times e = 294 \times \frac{6}{12} = 147$ kip-ft for uniform loading

plus $294 \times \frac{10.47}{2} = 256$ kip-ft for parabolic prestress.

M_{dl} slab = 29.10 ft-kip

M_{dl} applied = 50.0 kip-ft.

M_{ll} = 130.6 ft-kip.

Upward deflection due to uniform prestress of 147 kip-ft:

$$= \frac{W \times L^2}{8\ EI}$$

$$= \frac{147 \times 60^2 \times 12^2 \times 12000}{8 \times 3{,}500{,}000 \times 87300}$$

$$= 0.374 \text{ in.}$$

Upward deflection due to parabolic prestress is

$$= \frac{5 \times W \times L^2}{48\ EI}$$

$$= \frac{5 \times 256 \times 60^2 \times 12^2 \times 12000}{3{,}500{,}000 \times 87300}$$

$$= 0.543 \text{ in.}$$

Total instantaneous upward deflection due to prestress

$$= 0.374 + 0.543$$

$$= 0.917 \text{ in.}$$

Deflection after losses

$$= 0.85 \times 0.917$$

$$= 0.78 \text{ in.}$$

Downward deflection due to slab load

$$= \frac{5 \times 291 \times 60^2 \times 12^2 \times 12000}{48 \times 3{,}500{,}000 \times 87300}$$

$$= 0.617 \text{ in.}$$

Hence immediate upward deflection at transfer is

$$0.917 - 0.617$$

$$= 0.300$$

Dead load deflection downward

$$= \frac{50 \times 0.617}{391.0}$$

$$= 0.162 \text{ in.}$$

Net downward deflection after losses of prestress and effect of creep is

$$= 0.300 - 0.162$$

$$= 0.138 \text{ in.}$$

Instantaneous downward deflection due to live load moment is

$$\frac{130.6 \times 0.617}{291.0}$$

$$= 0.275 \text{ in.}$$

The resulting camber due to application of prestressing force supports the plate in its geometrical position thus eliminating critical secondary stresses caused by rotation of the ends at the supports that would be inevitable with deflections encountered in conventionally reinforced concrete.

The ends of the plates behind the anchorage are thickened to provide for distribution of the concentrated force applied by the tendoms. Additional reinforcing in the form of transverse and tie steel is provided in the area immediately behind the tendon anchorages to resist the bursting of the concrete created by the prestress force.[4]

The end leaves of a transverse section through the

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[4] "Long span prestressed concrete folded plate roofs", by J. Brough and B. Stephens, Proc. Am. Soc. C. E., Jan. 1960, Vol. 85, p. 91.

structure must be provided with tension ties to resist the horizontal force component. This may be accomplished by providing a tie beam, gable wall or other means, at or near the support, that is adequate for resistance of horizontal reaction.

CONCLUSIONS

The first structures of this kind were large coal bunkers designed and erected by G. Ehlers of Germany in 1924-25. The first paper on the subject was published by him in 1930. His analysis assumed the longitudinal joints to be hinged, neglecting the transverse moments at the junction of the plates. The displacement of joints was also ignored. This theory was improved upon in 1932 by E. Gruber who included the effects of transverse continuity and joint displacements. Assuming the joints to be hinged as a first approximation, he developed a solution in the form of simultaneous differential equations of the fourth order, which were solved by the use of rapidly converging series. This approach involves (7n + 2) unknowns for (n + 1) plates; thus, a roof of 5 plates would involve 30 unknowns. Although solution proposed by Gruber was very laborious, his conclusion that the assumption of hinged or rigid joints would considerably affect the final results was significant. This work was followed by that of Cramer who published a paper in 1933. He laid down rough limits in terms of the length to width ratio of individual plates for their classification as "long" and "short". The paper by Winter and Pei published in 1947 is a landmark in the theory on the subject as they, for the first time, reduced the algebraic solution into a stress distribution procedure analogous to the well-known moment distribution method. However, they neglected the displacement of the joints. For short folded plates their approach offers a very simple design procedure. However, for long plates the joint deflections cannot be ignored. Girkmann, in his book published in 1948, takes into account joint displacements. Treating transverse moments at the joints as the unknowns, he formulated conditions for the compatibility of longitudinal stresses and displacements at joints. The method leads to as many simultaneous equations as the unknown transverse moments. The paper presented by Whitney at the joint ASCE-IABSE meeting in New York is a presentation in English of the Girkmann method with some modification. Gaafar in 1953 published a modification of the Winter and Pie method extended to include the effect of joint displacements.

Among available methods, the Winter and Pie procedure is the simplest. It is applicable only to short folded plates for which the joint displacements can be ignored without appreciable error. Of the methods that are applicable to folded plates of all proportions, those due to Gaafar which is considered in this report seems to be most suitable.

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The writer wishes to express his sincere gratitude to Dr. John Mc Entyre for his kind guidance and assistance in the preparation of this report.

APPENDIX I - EXPLANATION OF TERMS

a = Slope rise on 12

A_c = Area of concrete

A_s = Area of steel

c.g. = Center of gravity

dl = Dead load

e = Ecentricity of prestress force from the c.g.

e' = Distance from surface to center of prestress force

F = Prestress force after losses

F_c = Modulus of rupture of concrete

F_o = Prestress force at time of transfer

f_c = Allowable concrete unit stress

f'_c = Design concrete strength

f_s = Allowable unit stress

H = Vertical height of section valley to ridge

h = Vertical distance above or below c.g. of concrete section

I = Moment of inertia of section

i = Subscript indicating condition at transfer of prestress

k = Kern point

ll = Live load

M = Bending moment

Q = Statical moment about c.g.

r = Radius of gyration

S = Principal tensile stress

S' = Principal tensile stress for ultimate

S_t = Principal tensile stress

S_t' = Ultimate principal tensile stress

t = Thickness of slab normal to section

t' = Thickness of slab vertically

tl = Total load

v = Unit shear

v_c = Unit shear stress in concrete

v_u = Ultimate shear stress

V_c = Ultimate concrete shear

APPENDIX II - BIBLIOGRAPHY

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ANALYSIS OF PRESTRESSED AND REINFORCED
CONCRETE FOLDED PLATES

by

RAMESH GAMI

B. S., Bombay University, 1958

AN ABSTRACT OF
A MASTER'S REPORT

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Approved by:

John G. Mc Entyre
Major Professor

The simplified methods of analysis for long span prestressed and reinforced concrete folded plate structures are presented herein.

A reinforced concrete folded plate is analyzed by solving the applied load into two directions, one vertical and the other parallel to the plate on which it acts. Vertical load will induce "slab" action and the parallel load will induce "plate" action. Slab action can be analyzed by assuming nonyielding supports and then applying a correction for deflection, from which total loads are found. "Plate" action causes a deflection of the longitudinal edge and introduces longitudinal stresses. To satisfy the compatibility condition a stress distribution is done. Moments are computed due to relative displacements. From these moments plate loads and longitudinal stresses are calculated. This process is repeatedly carried out and every time correction is applied. This correction will be in form of geometric series. Summation of this series gives the actual deflection. The ratio of actual deflection and initial deflection gives the multiplier which, when multiplied with initial longitudinal stresses, gives actual longitudinal stresses. The final longitudinal stress is calculated by adding stresses due to loading and stresses due to deflections. Moments are calculated at quarter point, end, and at half distance of the longitudinal edge.

To analyze the prestressed plate, the stress distribution along the longitudinal axis is analyzed in the same manner as for any homogenous beam of rectangular section.

Transverse bending in successive plates is analyzed by the conventional moment distribution method utilized for continuous structures.

A typical problem is outlined and the simplified method of analysis is discussed in this report.