BENDING VIBRATIONS OF A PIPE LINE CONTAINING FLOWING FLUID

by

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NOMENCLATURE

m	Total mass, pipe plus fluid, per unit length
I	Moment of inertia of pipe
E	Modulus of elasticity in tension
ρ	Mass of fluid per unit length
v	Velocity of flowing fluid relative to the pipe
x	Axial coordinate parallel to the pipe
z	Vertical coordinate perpendicular the pipe
L	Length of single-span pipe
t	Time
T	Total kinetic energy
T_f	Kinetic energy of the flowing fluid
T_p	Kinetic energy of the pipe
V	Total potential energy of mechanical system, $V=U+\Omega$
U	The potential energy of internal forces
Ω	The potential energy of external forces
δ	Variation
W_n	Natural frequency in rad per sec
f	Natural frequency in cps
F	Applied force
A	Coefficient of normal mode
n, k, r, s, i	Subscripts (integers)
$f(t), f(x)$	Periodic function
b	Coefficient of Fourier series
d_0, d_i	Outside and inside diameter of the pipe
P	Axial compressive force
p	Distributed uniformly load or lateral load per unit length

P_{cr} Critical load

$\phi(x)$, Φ Characteristic function

B Coefficient

q^4 $\frac{m\omega_n^2}{EI}$

z_p Particular integral for forced vibration with zero fluid velocity

f_{ci} Frequency of ith mode including the corrections for rotatory inertia and shear

f_i Frequency of ith mode as determined by the Bernoulli-Euler flexure theory

r Radius of gyration

α_o Shear-deflection coefficient

G Modulus of elasticity in shear

γ, α, β Constant coefficients

ε Error function

$|E|$ Norm $= \varepsilon^2$

I $= \int_0^1 \varepsilon^2 \, dx$

INTRODUCTION

Both free vibrations and forced motions for the fundamental mode of a simple supported pipe containing a flowing fluid may create important problems in the design of pipe lines supported above ground. The trans-Arabian pipe line is a 30-in diameter pipe which over a portion of its length is above ground. The pipe is supported at 66-ft intervals and has approximately 20 inches clearance between the bottom of the pipe and the ground. At certain wind velocities the pipe line has been observed to vibrate in the following manner:

At a wind velocity of approximately 20 mph, the pipe begins to vibrate with a frequency of vibration of approximately 1.2 cycles per second and attains a maximum deflection of 3/16 inch at mid-span. After about 20 seconds the frequency increases and amplitude decreases so that in approximately 10 seconds the pipe has come to rest and the vibration then repeats. During vibration the adjacent spans move in opposite directions with nodes at supports. The computed natural frequency of vibration of the pipe is 2.5 cps as compared with the observed 1.2 cps.

G. W. Housner (1)* derived the correct differential equation of motion by utilizing Hamilton's principle. Holt Ashley and George Haviland (2) derived not exactly correct differential equation of motion on which contains insufficient terms. The solution indicates a slight decrease in frequency with an increase in flow rate. R. H. Long, Jr, (3) did experimental work by using tubes. The experimental results agree reasonably well with these solutions. It should be noted that at low fluid velocities there is negligible effect upon the vibration of the pipe line.

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*Numbers in parenthesis refer to biobliography.

In this report, the problems of aerodynamically induced vibrations will be discussed in detail. However, the purpose of this report is to obtain the effect of flowing fluid upon the pipe line according to simple supported beam theory.

DIFFERENTIAL EQUATION OF MOTION

The trans-Arabian pipe line is installed as shown in Fig. 1.

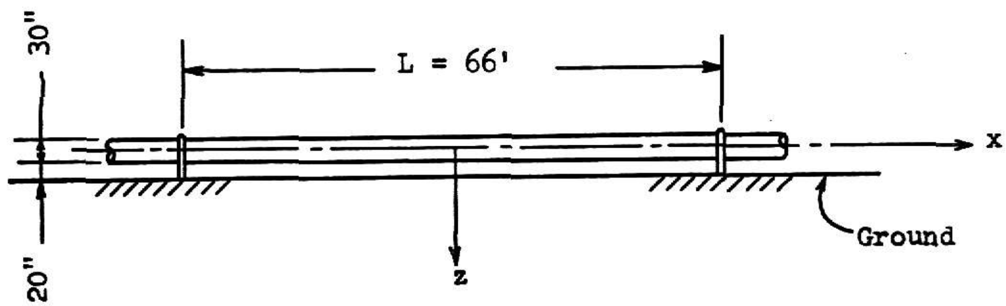

Fig. 1.

To derive the differential equation consider a pipe full of fluid, whose total mass, pipe plus fluid, per unit length is m, moment of inertia of pipe I, and modulus of elasticity E. Let ρ be the mass per unit length of the fluid and let it have a constant velocity v, relative to the pipe, thus neglecting the effect upon the velocity of the change in length of the pipe during vibration. Let the coordinate measured along the pipe be x, and the vertical displacement of the pipe z. It is desired to derive the differential equation describing the motion of the pipe. As the fluid flows along the curve described by the center line of the pipe, it has a rather complicated motion owing to the fact that each point on the center line has a vertical acceleration, an angular acceleration, and a changing curvature. With a problem of this complexity it is convenient to utilize Hamiliton's principle* to insure that no part of the motion is overlooked.

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*See reference (4)

As an element of fluid flows along the pipe it has a velocity v tangent to the pipe axis and a velocity $\dot{z} = \partial z/\partial t$ in a vertical direction. Assuming small displacements of the pipe, the x-component of the fluid velocity is v, and the z-component is

$$\dot{z} + vz' = \frac{\partial z}{\partial t} + v\,\frac{\partial z}{\partial x}\ , \tag{1}$$

where $\frac{\partial z}{\partial t}$ is the vertical velocity of the pipe.

$\frac{\partial z}{\partial t} + v\,\frac{\partial z}{\partial x}$ is the vertical velocity of the fluid.

The kinetic energy of the pipe in a length dx is

$$dT_p = \frac{1}{2}\,(m-\rho)\,\dot{z}^2\,dx \tag{2}$$

The kinetic energy of the fluid in a length dx is

$$dT_f = \frac{1}{2}\,\rho\,[v^2 + (\dot{z} + vz')^2]dx \tag{3}$$

The total kinetic energy in a length dx of the pipe is

$$dT = \left\{\frac{1}{2}(m-\rho)\dot{z}^2 + \frac{1}{2}\rho[v^2 + (\dot{z} + vz')^2]\right\}dx \tag{4}$$

The strain energy in the length dx is, according to the beam theory

$$dU = \frac{1}{2}\,EI(z'')^2\,dx \tag{5}$$

Hamilton's principle is formally expressed by

$$\int_{t_0}^{t_1} (\delta T - \delta V)dt = \delta \int_{t_0}^{t_1} (T - V)dt = 0, \tag{6}$$

where T represents the total kinetic energy, V the total potential energy of the system and δ the first variation.

The total potential energy can be expressed as $V = U + \Omega$, where U is the potential energy of internal forces, or the strain energy of the system, and Ω is the potential energy of external forces. Since there is no external forces upon the system, so is equal to zero. The total potential energy becomes

$$V = U = \frac{1}{2} EI \int_0^L (z'')^2 \, dx. \tag{7}$$

The total kinetic energy can be expressed as

$$T = \int_0^L \left\{\frac{1}{2} (m-\rho)\dot{z}^2 + \frac{1}{2}\rho[v^2 + (\dot{z}+vz')^2]\right\}dx. \tag{8}$$

Variations of T and V are evaluated as follows

$$\delta T = \int_0^L \left\{(m-\rho)\dot{z} \; \delta\dot{z} + \rho(\dot{z} + vz')(\delta\dot{z} + v \; \delta z')\right\}dx$$

$$= \int_0^L \left\{m\dot{z} \; \delta\dot{z} - \rho\dot{z} \; \delta\dot{z} + \rho\dot{z} \; \delta\dot{z} + \rho vz' \; \delta\dot{z} + \rho vz \; \delta z'\right.$$

$$\left. + \rho v^2 \; z' \; \delta z'\right\}dx$$

$$= \int_0^L \left\{m\dot{z} \; \delta\dot{z} + \rho vz' \; \delta\dot{z} + \rho v\dot{z} \; \delta z' + \rho v^2 z' \; \delta z'\right\}dx$$

$$\delta V = EI \int_0^L z'' \; \delta(z'')dx$$

Substitution of δT and δV into Eq. (6) yields

$$\int_{t_0}^{t_1}\int_0^L \left\{m\dot{z}\ \delta\dot{z} + \rho v z'\ \delta\dot{z} + \rho v\dot{z}\ \delta z' + \rho v^2 z'\ \delta z' - EIz''\ \delta z''\right\} dx\ dt$$

$$= 0 \tag{9}$$

Each term of Eq. (9) can be integrated by parts as follows*

$$\int_{t_0}^{t_1}\int_0^L m\dot{z}(\delta\dot{z})dx\ dt = \int_0^L\int_{t_0}^{t_1} m\dot{z}(\frac{\partial}{\partial t}\ \delta z)dt\ dx = \int_0^L \left\{\int_{t_0}^{t_1} m\dot{z}\ d(\delta z)\right\}dx$$

$$= \int_0^L [m\dot{z}(\delta z)]_{t_0}^{t_1}\ dx - \int_0^L \left\{\int_{t_0}^{t_1} m(\delta z)d\dot{z}\right\}dx = 0 - \int_0^L\int_{t_0}^{t_1} m(\delta z)\ddot{z}\ dt\ dx$$

$$= -\int_{t_0}^{t_1}\int_0^L m\ddot{z}(\delta z)dx\ dt, \tag{9a}$$

$$\int_{t_0}^{t_1}\int_0^L \rho v z'(\delta\dot{z})dx\ dt = \int_0^L\int_{t_0}^{t_1} \rho v z'\ (\frac{\partial}{\partial t}\ \delta z)dt\ dx$$

$$= \int_0^L \left\{\int_{t_0}^{t_1} \rho v z'\ d(\delta z)\right\}dx = \int_0^L [\rho v z'\ (\delta z)]_{t_0}^{t_1}\ dx - \int_0^L\int_{t_0}^{t_1} \rho v(\delta z)dz'\cdot dx$$

$$= 0 - \int_0^L\int_{t_0}^{t_1} \rho v(\delta z)\dot{z}'\ dt\ dx = -\int_{t_0}^{t_1}\int_0^L \rho v\dot{z}'\ (\delta z)dx\ dt, \tag{9b}$$

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*See reference (5)

$$\int_{t_0}^{t_1}\int_0^L \rho v\dot{z}(\delta z')dx\,dt = \int_{t_0}^{t_1}\int_0^L \rho v\dot{z}(\frac{\partial}{\partial x}\,\delta z)dx\,dt = \int_{t_0}^{t_1}\left\{\int_0^L \rho v\dot{z}\;d(\delta z)\right\}dt$$

$$= \int_{t_0}^{t_1} [\rho v\dot{z}(\delta z)]_0^L \;dt - \int_{t_0}^{t_1}\int_0^L \rho v(\delta z)d\dot{z}\;dt = 0 - \int_{t_0}^{t_1}\int_0^L \rho v(\delta z)\dot{z}'\;dx\,dt$$

$$= -\int_{t_0}^{t_1}\int_0^L \rho v\dot{z}'(\delta z)dx\,dt, \tag{9c}$$

$$\int_{t_0}^{t_1}\int_0^L \rho v^2 z'\;(\delta z')dx\,dt = \int_{t_0}^{t_1}\int_0^L \rho v^2 z'(\frac{\partial}{\partial x}\,\delta z)dx\,dt$$

$$= \int_{t_0}^{t_1}\int_0^L \rho v^2 z'\;d(\delta z)\;dt = \int_{t_0}^{t_1} [\rho v^2 z'(\delta z)]_0^L\;dt - \int_{t_0}^{t_1}\int_0^L \rho v^2\;(\delta z)dz'\;dt$$

$$= 0 - \int_{t_0}^{t_1}\int_0^L \rho v^2(\delta z)z''\;dx\,dt = -\int_{t_0}^{t_1}\int_0^L \rho v^2 z''\;(\delta z)dx\,dt, \tag{9d}$$

$$\int_{t_0}^{t_1}\int_0^L \rho v^2 z'\;(\delta z')dx\,dt = \int_{t_0}^{t_1}\int_0^L \rho v^2 z'\;(\frac{\partial}{\partial x}\,\delta z)dx\,dt = \int_{t_0}^{t_1}\int_0^L \rho v^2 z' d(\delta z)dt$$

$$= \int_{t_0}^{t_1} [\rho\, v^2 z'(\delta z)]_0^L\;dt - \int_{t_0}^{t_1}\int_0^L \rho v^2\;(\delta z)dz'\;dt = 0 -$$

$$\int_{t_0}^{t_1}\int_0^L \rho v^2(\delta z)z''\;dx\,dt = -\int_{t_0}^{t_1}\int_0^L \rho v^2 z''(\delta z)dx\,dt, \tag{9d}$$

$$\int_{t_0}^{t_1}\int_0^L EIz''(\delta z'')\,dx\,dt = \int_{t_0}^{t_1}\int_0^L EIz''\left(\frac{\partial^2}{\partial x^2}\delta z\right)dx\,dt$$

$$= \int_{t_0}^{t_1}\int_0^L EIz'' \frac{\partial\frac{\partial(\delta z)}{\partial x}}{\partial x}\,dx\,dt = \int_{t_0}^{t_1}\left\{\int_0^L EIz''\,d\,\frac{\partial(\delta z)}{\partial x}\right\}dt$$

$$= \int_{t_0}^{t_1}\left[EIz''\frac{\partial(\delta z)}{\partial x}\right]_0^L dt - \int_{t_0}^{t_1}\int_0^L EI\frac{\partial(\delta z)}{\partial x}\,dz''\,dt$$

$$= 0 - \int_{t_0}^{t_1}\int_0^L EI\frac{\partial(\delta z)}{\partial x}z'''\,dx\,dt = -\int_{t_0}^{t_1}\left\{\int_0^L EIz'''\,d(\delta z)\right\}dt$$

$$= -\int_{t_0}^{t_1}\left[EIz'''(\delta z)\right]_0^L dt + \int_{t_0}^{t_1}\int_0^L EI(\delta z)\,dz'''\,dt$$

$$= 0 + \int_{t_0}^{t_1}\int_0^L EI(\delta z)z''''\,dx\,dt = \int_{t_0}^{t_1}\int_0^L EIz''''(\delta z)\,dx\,dt, \tag{9e}$$

Substitution of Eqs. (9a), (9b), (9c), (9d), and (9e) into Eq. (9) yields

$$-\int_{t_0}^{t_1}\int_0^L\left\{m\ddot{z} + 2\rho v\dot{z}' + \rho v^2 z'' + EIz''''\right\}\delta z\,dx\,dt = 0. \tag{10}$$

Since δz is arbitrary the only way for the integral to be zero is for the expression in square brackets to be identically zero. Equating this to zero gives the differential equation of motion

$$EI\frac{\partial^4 z}{\partial x^4} = -\rho v^2\frac{\partial^2 z}{\partial x^2} - 2\rho v\frac{\partial^2 z}{\partial t\partial x} - m\frac{\partial^2 z}{\partial t^2}. \tag{11}$$

This equation states that the beam is acted upon by three different inertia forces. The first term on the right can be regarded as the inerita force associated with the change in direction of v, enforced by the curvature of the beam; that is, the fluid experiences an acceleration because it travels along a curved path. The second term is referred to as inertia force associated with the Coriolis acceleration which arises because the fluid is flowing with velocity v relative to the pipe, while the pipe itself has an angular velocity $(\partial^2 z/\partial t \partial x)$ at any point along its length. The last term represents the inertia force associated with the vertical acceleration of the pipe.

An investigation of this problem also derived by Mr. Holt Ashley and Mr. George Haviland (2). The differential equation of motion studied there was

$$EI \frac{\partial^4 z}{\partial x^4} = - \rho v \frac{\partial^2 z}{\partial x \partial t} - m \frac{\partial^2 z}{\partial t^2} . \tag{12}$$

This equation neglects the inertia forces produced by the curvature of the pipe and includes only one half of the inertia forces due to the Coriolis accleration, so that it underestimates the influence of the fluid velocity.

FREE VIBRATIONS

The governing differential equation can be written in the following form:

$$EI \frac{\partial^4 z}{\partial x^4} + \rho v^2 \frac{\partial^2 z}{\partial x^2} + 2\rho v \frac{\partial^2 z}{\partial t \partial x} + m \frac{\partial^2 z}{\partial t^2} = 0. \tag{13}$$

This equation differs from the more usual vibration equations in that it contains a mixed derivative ($\partial^2 z/\partial x \partial t$). Although this term contains a first derivative with respect to time, its influence is not the same as a viscous damping term which has the form $\partial z/\partial t$. This can be seen by writing the equation in the form

$$EI \frac{\partial^4 z}{\partial x^4} + \rho v^2 \frac{\partial^2 z}{\partial x^2} + m \frac{\partial^2 z}{\partial t^2} = F(x,t), \tag{14}$$

where $F(x,t)$ is considered to be an applied force. The free vibrations are then the normal modes which are the solutions of Eq. (14) with $F(x,t)$ set equal to zero. For a simple supported beam these are

$$z_n = A_n \sin \frac{n\pi}{L} x \sin w_n t. \tag{15}$$

Assume that the beam is constrained so as to move in the nth mode only. Where w_n is natural frequency of the system.

The first partial derivative of z_n with respect to time is

$$\frac{\partial z_n}{\partial t} = A_n w_n \sin \frac{n\pi}{L} x \cos w_n t, \tag{16a}$$

the second partial derivative of z_n with respect to t and x is

$$\frac{\partial^2 z_n}{\partial t \partial x} = A_n w_n \frac{n\pi}{L} \cos \frac{n\pi}{L} x \cos w_n t. \quad (16b)$$

If F(x,t) is proportional to $-\partial z/\partial t$ then the applied force is proportional to

$$-A_n w_n \sin \frac{n\pi}{L} x \cos w_n t,$$

if F(x,t) is proportial to $-\partial^2 z/\partial t \partial x$, then the applied force is proportional to

$$-A_n w_n \frac{n\pi}{L} \cos \frac{n\pi}{L} x \cos w_n t.$$

It is seen that if $\sin(n\pi/L)x$ is symmetrical about the mid-point of the beam then F(x,t) is antisymmetrical and there F(x, t) does no work on the mode and is not associated with damping. The significance of $-(\partial^2 z/\partial t \partial x)$ is that it represents dynamic coupling, and it is impossible for a symmetrical mode of Eq. (14) to exist by itself in a free vibration, for corresponding with this mode there is an antisymmetrical force, 90 degree out of phase, which excites all the antisymmetrical modes, and an antisymmetrical mode produces an F(x,t) which excite all of the symmetrical modes. Thus a normal mode of Equation (13) must be a linear combination of all the normal modes of the homogeneous form of Equation (14) with symmetrical and antisymmetrical modes 90 degree out of phase.

A simply supported span thus has normal modes z_i satisfying Equation (13)

$$z_i = \sum_n A_{2n-1} \sin(2n-1) \frac{\pi}{L} x \sin w_i t +$$

$$\sum_k A_{2k} \sin 2k \frac{\pi}{L} x \cos w_i t, \tag{17}$$

the normal mode z_i is composed of terms which interact with one another through the mixed derivative term

$$2\rho v \frac{\partial^2 z}{\partial x \partial t},$$

An odd term, say

$$A_{2n-1} \sin(2n-1) \frac{\pi}{L} x$$

must be in dynamic balance with the coupling forces exerted by all the even terms, and even term must be in balance with the coupling forces exerted by all the odd terms. Each coefficient A_i is coupled with every other coefficient.

The coefficients and the natural frequency w_i are determined as follows

$$\frac{\partial z_i}{\partial x} = \sum_n A_{2n-1} (2n-1)(\frac{\pi}{L}) \cos(2n-1) \frac{\pi}{L} x \sin w_i t +$$

$$\sum_k A_{2k} (2k)(\frac{\pi}{L}) \cos 2k \frac{\pi}{L} x \cos w_i t \tag{18a}$$

$$\frac{\partial^2 z_i}{\partial x^2} = \sum_n A_{2n-1} -(2n-1)^2(\frac{\pi}{L})^2 \sin(2n-1) \frac{\pi}{L} x \sin w_i t +$$

$$\sum_k A_{2k} - (2k)^2(\frac{\pi}{L})^2 \sin 2k \frac{\pi}{L} x \cos w_i t \tag{18b}$$

$$\frac{\partial^3 z_i}{\partial x^3} = \sum_n A_{2n-1} - (2n-1)^3 (\frac{\pi}{L})^3 \cos(2n-1) \frac{\pi}{L} x \sin w_i t +$$

$$\sum_k A_{2k} - (2k)^3 (\frac{\pi}{L})^3 \cos 2k \frac{\pi}{L} x \cos w_i t \qquad (18c)$$

$$\frac{\partial^4 z_i}{\partial x^4} = \sum_n A_{2n-1} (2n-1)^4 (\frac{\pi}{L})^4 \sin(2n-1) \frac{\pi}{L} x \sin w_i t +$$

$$\sum_k A_{2k} (2k)^4 (\frac{\pi}{L})^4 \sin 2k \frac{\pi}{L} x \cos w_i t \qquad (18d)$$

$$\frac{\partial^z i}{\partial t} = \sum_n A_{2n-1} (w_i) \sin(2n-1) \frac{\pi}{L} x \cos w_i t +$$

$$\sum_k A_{2k} (-w_i) \sin 2k \frac{\pi}{L} x \sin w_i t \qquad (18e)$$

$$\frac{\partial^2 z_i}{\partial t \, \partial x} = \sum_n A_{2n-1} (2n-1)(\frac{\pi}{L})(w_i) \cos(2n-1) \frac{\pi}{L} x \cos w_i t +$$

$$\sum_k A_{2k} (2k)(\frac{\pi}{L})(-w_i) \cos(2k \frac{\pi}{L} x) \sin(w_i t), \qquad (18f)$$

$$\frac{\partial^2 z_i}{\partial t^2} = \sum_n A_{2n-1} (-w_i^2) \sin(2n-1) \frac{\pi}{L} x \sin(w_i t) +$$

$$\sum_k A_{2k} (-w_i^2) \sin(2k \frac{\pi}{L} x) \cos(w_i t), \qquad (18g)$$

Substitution of Eqs. (18) into Eq. (13) yields

$$EI\left\{\sum_n A_{2n-1}(2n-1)^4(\tfrac{\pi}{L})^4 \sin(2n-1)\tfrac{\pi}{L}x \sin(w_i t) + \right.$$

$$\sum_k A_{2k}(2k)^4(\tfrac{\pi}{L})^4 \sin 2k\tfrac{\pi}{L}x \cos(w_i t)$$

$$+ \rho v^2\left\{\sum_n A_{2n-1} - (2n-1)^2(\tfrac{\pi}{L})^2 \sin(2n-1)\tfrac{\pi}{L}x \sin(w_i t) + \right.$$

$$\left. \sum_k A_{2k} - (2k)^2(\tfrac{\pi}{L})^2 \sin 2k\tfrac{\pi}{L}x \cos(w_i t)\right\}$$

$$+ 2\rho v\left\{\sum_n A_{2n-1}(2n-1)(\tfrac{\pi}{L})\ (w_i)\cos(2n-1)\tfrac{\pi}{L}x \cos(w_i t) + \right.$$

$$\left. \sum_k A_{2k}\ (2k)(\tfrac{\pi}{L})(-w_i)\cos 2k\tfrac{\pi}{L}x \sin(w_i t)\right\}$$

$$+ m\left\{\sum_n A_{2n-1}(-w_i^2)\sin(2n-1)\tfrac{\pi}{L}x \sin(w_i t) + \right.$$

$$\left. A_{2k}(-w_i^2)\sin 2k\tfrac{\pi}{L}x \cos(w_i t)\right\} = 0. \qquad (19)$$

The terms $\cos(2n-1)\frac{\pi}{L}x$ and $\cos(2k\frac{\pi}{L}x)$ may be expanded in a Fourier series*

For the fundamental mode, the period is equal to 2L. Assuming the period function f is odd, we obtain the Fourier sine series. The general form is

*See reference (6)

$$f(t) = \sum_{n=1}^{\infty} b_n \sin(\frac{n\pi}{L} t), \tag{20}$$

where

$$b_n = \frac{2}{L} \int_0^L f(t) \sin \frac{n}{L} t(dt) \qquad n = 1, 2, 3, \ldots$$

For $\quad f(x) = \cos(2n-1) \frac{\pi}{L} x$

Then, $$\cos(2n-1) \frac{\pi}{L} x = \sum_{r=1}^{\infty} b_r \sin \frac{r\pi}{L} x \tag{22}$$

$$b_r = \frac{2}{L} \int_0^L \cos(2n-1) \frac{\pi}{L} x \cdot \sin \frac{r\pi}{L} x dx$$

$$= \frac{1}{L} \int_0^L [\sin(r+2n-1) \frac{\pi}{L} x + \sin(r-2n+1) \frac{\pi}{L} x] dx$$

$$= \frac{1}{\pi} \left[- \frac{\cos(r+2n-1) \frac{\pi}{L} x}{r+2n-1} - \frac{\cos(r-2n+1) \frac{\pi}{L} x}{r-2n+1}\right]_0^L$$

$$= \frac{1}{\pi} \left\{- \frac{\cos[r+(2n-1)]\pi}{r+(2n-1)} + \frac{1}{r+(2n-1)} - \frac{\cos[r-(2n-1)]\pi}{r-(2n-1)} + \frac{1}{r-(2n-1)}\right\} \tag{22a}$$

where,

$$\cos[r \pm (2n-1)] = \cos r\pi \cos(2n-1)\pi \mp \sin r\pi \sin(2n-1)\pi$$

$$= -\cos r\pi = \begin{cases} 1 & \text{for odd } r, \\ -1 & \text{for even } r, \end{cases} \tag{22b}$$

Substitution of Eq. (22b) into Eq. (22a) yields

$$b_r = \frac{2}{\pi}\left[\frac{1}{r+(2n-1)} + \frac{1}{r-(2n-1)}\right] \qquad r = 2, 4, 6, \ldots$$

$$= \frac{2}{\pi}\left[\frac{r-(2n-1)+r+(2n-1)}{r^2-(2n-1)^2}\right.$$

$$= \frac{4}{\pi}\left[\frac{r}{r^2-(2n-1)^2}\right] \tag{22c}$$

Let $r=2k$ $\qquad k = 1, 2, 3, \ldots \qquad$ (22d)

Substitution of Eq. (22d) into Eq. (22c) yields

$$b_{2k} = \frac{4}{\pi}\left[\frac{2k}{(2k)^2 - (2n-1)^2}\right] \tag{22e}$$

Substitution of Eq. (22e) into Eq. (22) yields

$$\cos(2n-1)\frac{\pi}{L}x = \frac{4}{\pi}\sum_{k}^{\infty}\frac{2k}{(2k)^2 - (2n-1)^2}\sin\frac{2k\pi}{L}n \tag{23}$$

where $\quad n = 1, 2, 3, \ldots$

$\quad k = 1, 2, 3, \ldots$

For $f(x) = \cos 2k\frac{\pi}{L}x$

then, $$\cos 2k\frac{\pi}{L}x = \sum_{s=1}^{\infty} b_s \sin\frac{s\pi}{L}x \tag{24}$$

$$b_s = \frac{2}{L} \int_0^L \cos 2k \frac{\pi}{L} x \sin \frac{s\pi}{L} x \, dx$$

$$= \frac{1}{L} \int_0^L [\sin(s+2k) \frac{\pi}{L} x + \sin(s-2k) \frac{\pi}{L} x] \, dx$$

$$= \frac{1}{\pi} [- \frac{\cos(s+2k) \frac{\pi}{L} x}{s+2k} - \frac{\cos(s-2k) \frac{\pi}{L} x}{s-2k}]_0^L$$

$$= \frac{1}{\pi} [- \frac{\cos(s+2k)\pi}{s+2k} + \frac{1}{s+2k} - \frac{\cos(s-2k)\pi}{s-2k} + \frac{1}{s-2k}] \tag{24a}$$

where

$$\cos(s \pm 2k)\pi = \cos(s\pi)\cos 2k\pi \mp \sin(s\pi)\sin 2k\pi = \cos(s\pi)$$

$$= \cos(s\pi) = \begin{cases} -1 & \text{for odd } s, \\ 1 & \text{for even } s, \end{cases} \tag{24b}$$

Substitution of Eq. (24b) into Eq. (24a) yields

$$b_s = \frac{2}{\pi} [\frac{1}{s+2k} + \frac{1}{s-2k}] \qquad s = 1, 3, 5, \ldots$$

$$= \frac{2}{\pi} [\frac{s-2k+s+2k}{s^2-(2k)^2}]$$

$$= \frac{4}{\pi} [\frac{s}{s^2-(2k)^2}] \tag{24c}$$

Let $s = 2n-1$ $\qquad n = 1, 2, 3, \ldots \qquad$ (24d)

Substitution of Eq. (24d) into Eq. (24c) yields

$$b_{2n-1} = \frac{4}{\pi} [\frac{2n-1}{(2n-1)^2 - (2k)^2}] \tag{24e}$$

Substitution of Eq. (24e) into Eq. (24) yields

$$\cos 2k \frac{\pi}{L} x = \frac{4}{\pi} \sum_{n}^{\infty} \frac{2n-1}{(2n-1)^2-(2k)^2} \sin \frac{(2n-1)\pi}{L} x \tag{25}$$

where $k = 1, 2, 3, \ldots$

$n = 1, 2, 3, \ldots$

Substitution of Eq. (23) and (25) into Eq. (19) yields

$$EI\left\{\sum_n A_{2n-1}(2n-1)^4\left(\frac{\pi}{L}\right)^4 \sin(2n-1)\frac{\pi}{L} x \sin(w_i t) + \right.$$

$$\left.\sum_k A_{2k}(2k)^4\left(\frac{\pi}{L}\right)^4 \sin 2k \frac{\pi}{L} x \cos(w_i t)\right\} - \rho v^2\left\{\sum_n A_{2n-1}(2n-1)^2\left(\frac{\pi}{L}\right)^2 \cdot\right.$$

$$\left.\sin(2n-1)\frac{\pi}{L} w_i t + \sum_k A_{2k}(2k)^2\left(\frac{\pi}{L}\right)^2 \sin 2k \frac{\pi}{L} x \cos(w_i t)\right\} +$$

$$2\rho v\left\{\sum_n A_{2n-1}(2n-1)\left(\frac{\pi}{L}\right)(w_i)\frac{4}{\pi}\sum_k \frac{2k}{(2k)^2-(2n-1)^2} \sin\frac{2k\pi}{L} x \cos(w_i t)\right.$$

$$\left.-\sum_n A_{2k}(2k)\left(\frac{\pi}{L}\right)(w_i)\frac{4}{\pi}\sum_n \frac{2n-1}{(2n-1)^2-(2k)^2} \sin\frac{(2n-1)\pi}{L} x \sin(w_i t)\right\}$$

$$-m\left\{\sum_n A_{2n-1}(w_i^2)\sin(2n-1)\frac{\pi}{L} x \sin(w_i t) + \sum_k A_{2k}(w_i^2)\sin 2k\frac{\pi}{L} x\right.$$

$$\cos(w_i t) = 0 \tag{26}$$

Collection of like groups of $\sin(2n-1)\frac{\pi}{L} x \sin(w_i t)$ or $\sin 2k \frac{\pi}{L} x \cos(w_i t)$ results in

$$\sum_n \sin(2n-1)\frac{\pi}{L}\sin w_i t\Big\{A_{2n-1}[EI(2n-1)^4(\frac{\pi}{L})^4 - \rho v^2(2n-1)^2(\frac{\pi}{L})^2$$

$$- m(w_i)^2] = \sum_k A_{2k}\,[\frac{8\rho v}{\pi}\,(2k)(\frac{\pi}{L})(w_i)\,\frac{2n-1}{(2n-1)^2-(2k)^2}]\Big\} +$$

$$\sum_k \sin 2k\frac{\pi}{L}\,x\,\cos w_i t\Big\{A_{2k}[EI(2k)^4(\frac{\pi}{L})^4 - \rho v^2(2k)^2(\frac{\pi}{L})^2] - m(w_i)^2]$$

$$+ \sum_n A_{2n-1}[\frac{8\rho v}{\pi}\,(2n-1)(\frac{\pi}{L})(w_i)\,\frac{2k}{(2k)^2-(2n-1)^2}]\Big\} = 0 \tag{27}$$

The coefficients of these groups must then be equated to zero in order to satisfy Eq. (13). This gives the following set of equations.

$$A_{2n-1}[EI(2n-1)^4(\frac{\pi}{L})^2 - \rho v^2(2n-1)^2(\frac{\pi}{L})^2 - m(w_i)^2] -$$

$$\sum_k A_{2k}\,[\frac{8\rho v}{\pi}\,(2k)(\frac{\pi}{L})(w_i)\,\frac{2n-1}{(2n-1)^2-(2k)^2}] = 0 \tag{28a}$$

$$A_{2k}[EI(2k)^4(\frac{\pi}{L})^4 - \rho v^2(2k)^2(\frac{\pi}{L})^2 - m(w_i)^2] +$$

$$\sum_n A_{2n-1}[\frac{8\rho v}{\pi}\,(2n-1)(\frac{\pi}{L})(w_i)\,\frac{2k}{(2k)^2-(2n-1)^2}] = 0 \tag{28b}$$

where $n = 1, 2, 3, \ldots$

$k = 1, 2, 3, \ldots$

Eqs. (28) can be changed into the following forms

$$A_{2n-1} = \sum_k A_{2k}\,\frac{16\rho v w_i(\frac{k}{L})(2n-1)/[(2n-1)^2-(2k)^2]}{EI(2k)^4(\frac{\pi}{L})^4 - \rho v^2(2k)^2(\frac{\pi}{L})^2 - m(w_i)^2} \tag{29a}$$

$$A_{2k} = \sum_n A_{2n-1} \frac{16\rho v w_i (\frac{k}{L})(2n-1)/[(2n-1)^2-(2k)^2]}{EI(2k)^4(\frac{\pi}{L})^4-\rho v^2(2k)^2(\frac{\pi}{L})^2-m(w_i)^2} \tag{29b}$$

Expand Eq. (29a) into the following form.

for n = 1,

$$A_1 = \sum_k A_{2k} \frac{16(\frac{\rho v w}{L})[\frac{k}{1-(2k)^2}]}{EI(\frac{\pi}{L})^4-\rho v^2(\frac{\pi}{L})^2-mw^2}$$

$$= \frac{16\rho vw/L}{EI(\frac{\pi}{L})^4-\rho v^2(\frac{\pi}{L})^2-mw^2}[-\frac{A_2}{3} - \frac{2A_4}{15} - \frac{3A_6}{35} - \cdot\cdot\cdot], \tag{30a}$$

for n = 2,

$$A_3 = \sum_k A_{2k} \frac{48(\frac{\rho v w}{L})[\frac{k}{9-(2k)^2}]}{81EI(\frac{\pi}{L})^4-9\rho v^2(\frac{\pi}{L})^2-mw^2}$$

$$= \frac{48\rho vw/L}{81EI(\frac{\pi}{L})^4-9\rho v^2(\frac{\pi}{L})^2-mw^2}[\frac{A_2}{5} - \frac{2A_4}{7} - \frac{3A_6}{27} - \cdot\cdot\cdot], \tag{30b}$$

for n = 3,

$$A_5 = \sum_k A_{2k} \frac{80(\frac{\rho v w}{L})[\frac{k}{25-(2k)^2}]}{625EI(\frac{\pi}{L})^4-25\rho v^2(\frac{\pi}{L})^2-mw^2}$$

$$= \frac{80\rho vw/L}{625EI(\frac{\pi}{L})^4-25\rho v^2(\frac{\pi}{L})^2-mw^2}[\frac{A_2}{21} + \frac{2A_4}{9} - \frac{3A_6}{11} - \cdot\cdot\cdot], \tag{30c}$$

Expand Eq. (29b) into the following form

for k = 1,

$$A_2 = \sum_n A_{2n-1} \frac{16(\frac{\rho vw}{L})[\frac{2n-1}{(2n-1)^2-4}]}{16EI(\frac{\pi}{L})^4-4\rho v^2(\frac{\pi}{L})^2 - mw^2}$$

$$= \frac{16\rho vw/L}{16EI(\frac{\pi}{L})^4-4\rho v^2(\frac{\pi}{L})^2 - mw^2}\left[-\frac{A_1}{3}+\frac{3A_3}{5}+\frac{5A_5}{21}+\ldots\right], \qquad (30d)$$

for k = 2,

$$A_4 = \sum_n A_{2n-1} \frac{32(\frac{\rho vw}{L})\frac{2n-1}{(2n-1)^2-16}}{256EI(\frac{\pi}{L})^4-16\rho v^2(\frac{\pi}{L})^2 - mw^2}$$

$$= \frac{32\rho vw/L}{256EI(\frac{\pi}{L})^4-16\rho v^2(\frac{\pi}{L})^2 - mw^2}\left[-\frac{A_1}{15}-\frac{3A_3}{7}+\frac{5A_5}{9}+\ldots\right], \qquad (30e)$$

for k = 3,

$$A_6 = \sum_n A_{2n-1} \frac{48(\frac{\rho vw}{L})\frac{2n-1}{(2n-1)^2-36}}{1296EI(\frac{\pi}{L})^4-36\rho v^2(\frac{\pi}{L})^2 - mw^2}$$

$$= \frac{48\rho vw/L}{1296EI(\frac{\pi}{L})^4-36\rho v^2(\frac{\pi}{L})^2 - mw^2}\left[-\frac{A_1}{35}-\frac{3A_3}{27}-\frac{5A_3}{11}+\ldots\right], \qquad (30f)$$

If all the modes are suppressed except the first two, say A_1 and A_2, that is let n and k each be equal to one, the equations are then solved readily

$$A_1 = A_2 \frac{-\frac{16}{3}\rho v(w_i)/L}{EI(\frac{\pi}{L})^4-\rho v^2(\frac{\pi}{L})^2 - m(w_i)^2}, \tag{31a}$$

$$A_2 = A_1 \frac{-\frac{16}{3}\rho v(w_i)/L}{16EI(\frac{\pi}{L})^4 - 4\rho v^2(\frac{\pi}{L})^2 - m(w_i)^2}, \tag{31b}$$

Substitution of Eq. (31b) into Eq. (31a) yields

$$A_1 = A_1 \frac{-\frac{16}{3}\rho v(w_i)/L}{16EI(\frac{\pi}{L})^4 -4\rho v^2(\frac{\pi}{L})^2 - m(w_i)^2} \cdot \frac{-\frac{16}{3}\rho v(w_i)/L}{EI(\frac{\pi}{L})^4-\rho v^2(\frac{\pi}{L})^2 - m(w_i)^2},$$

elimination A_1, we obtain the frequency equation

$$[16EI(\frac{\pi}{L})^4 -4\rho v^2(\frac{\pi}{L})^2 -m(w_i)^2][EI(\frac{\pi}{L})^4 - \rho v^2(\frac{\pi}{L})^2 - m(w_i)^2]$$

$$= (\frac{16\rho v\, w_i}{3L})^2 . \tag{32}$$

The parameters of the trans-Arabian pipe line are given below

$$v = 15 \text{ fps}, \tag{33a}$$

$$I = \frac{\pi}{64}(d_o{}^4 - d_i{}^4) \text{ in}^4 = \frac{\pi}{64}(30^4 - 29.5^4) \text{ in}^4$$

$$= \frac{\pi}{64}(53{,}000) \text{ in}^4 = 0.125 \text{ ft}^4, \tag{33b}$$

$$E = 30\text{x}10^6 \text{ psi} = 30\text{x}10^6\text{x}144 \text{ psf},$$

$$EI = 30\text{x}10^6\text{x}114\text{x}0.125 = 5.4\text{x}10^8 \text{ ib-ft}^2,$$

$$= 1.75 \text{ slugs/ft}, \tag{33c}$$

$$m = 11 \text{ slugs/ft}, \tag{33d}$$

$$l = 66 \text{ ft}, \tag{33e}$$

$$(\frac{\pi}{L})^2 = 2.26\text{x}10^{-3} \text{ ft}^{-2}, \tag{33f}$$

$$(\frac{\pi}{L})^4 = 5.1\text{x}10^{-6} \text{ ft}^{-4}, \tag{33g}$$

Substitution of Eqs. (33) into Eq. (32) yields

$$[16\text{x}5.4\text{x}10^8\text{x}5.1\text{x}10^{-6} - 4\text{x}1.75\text{x}15^2\text{x}2.26\text{x}10^{-3} - 11w^2] \cdot$$

$$[5.4\text{x}10^8\text{x}5.1\text{x}10^{-6} - 1.75\text{x}15^2\text{x}2.26\text{x}10^{-3} - 11w^2]$$

$$= (\frac{16\text{x}1.75\text{x}15 \ w}{3\text{x}66})^2, \tag{34a}$$

$$[43\text{x}10^3 - 3.56 - 11\ w^2][27.54\text{x}10^2 - 0.89 - 11\ w^2]$$

$$= 4.5\ w^2, \tag{34b}$$

omitting 3.56 and 0.89 by comparing with $43\text{x}10^3$ and $27.54\text{x}10^2$, we get

$$118.2\text{x}10^6 - 503.3\text{x}10^3\ w^2 + 121\ w^4 = 4.5\ w^2, \tag{34c}$$

and omitting $4.5\ w^2$ by comparing with $503.3\text{x}10^3\ w^2$, we obtain the frequency equation of the normal mode

$$121\ w^4 - 503.3\text{x}10^3\ w^2 + 118.2\text{x}10^6 = 0, \tag{35}$$

$$w^2 = \frac{503.3\text{x}10^3 \pm \sqrt{(503.3)^2\text{x}10^6 - 4\text{x}121\text{x}118.2\text{x}10^6}}{2\text{x}121}$$

$$= \frac{503.3\text{x}10^3 \pm \sqrt{19.61\text{x}10^{10}}}{242},$$

only the minus sign is reasonable for the fundamental mode, then

$$w^2 = \frac{5.93\text{x}10^4}{242} = 2.44\text{x}10^2.$$

The natural frequency for the fundamental mode is obtained

$$w_1 = 15.6 \text{ rad/sec} = 2.5 \text{ cps}. \tag{36}$$

Divided Eq. (31a) by Eq. (31b)

$$\frac{A_1}{A_2} = \frac{A_2}{A_1} \frac{16EI(\frac{\pi}{L})^4 - 4\rho v^2(\frac{\pi}{L})^2 - m\,w^2}{EI(\frac{\pi}{L})^4 - \rho v^2(\frac{\pi}{L})^2 - m\,w^2}. \tag{37}$$

Substituting Eq. (36) and Eqs. (33) into Eq. (37) yields

$$(\frac{A_1}{A_2})^2 = 583, \tag{38a}$$

$$\frac{A_1}{A_2} = \pm\ 24.19, \tag{38b}$$

The minus sign is reasonable for dynamic balance, so

$$A_1 = -\ 24.19\ A_2\ . \tag{38}$$

If $A_1=1$, (39a)

$$A_2 = -\frac{1}{24.19} = -\ 0.0414. \tag{39b}$$

Above values are based on a fluid velocity of 15 fps. It is seen that the 15 fps velocity is not large enough to excite the unsymmetrical A_2 component appreciably. It may be verified that the strength of the coupling decreases rapidly, so that if A_1, A_2, and A_3 are retained, the magnitude of A_3 is small compared to that of A_2. In general, the dynamic coupling is such that in the ith mode of vibration the coefficient A_i is the largest and the coefficients A_{i+1}, A_{i+2}, . . . and A_{i-1}, A_{i-2}, . . . decrease in magnitude very rapidly as the subscript goes away from i.

Assuming zero fluid velocity in Eq. (13), we obtain the Euler's differential equation*

$$EI \frac{\partial^4 z}{\partial x^4} + m \frac{\partial^2 z}{\partial t^2} = 0, \tag{40}$$

To determine the normal modes of vibration, the solution in the form

$$z(x,t) = \phi_i(x) e^{jw_n t} \tag{41}$$

is substituted into Eq. (40) to obtain the linear differential equation with constant coefficient.

$$\phi_i^{iv}(x) - q^4 \phi_i(x) = 0, \tag{42}$$

where:

$\phi_i(x)$ = characteristic function describing the deflection of the ith mode

$q^4 = mw_n^2/(EI)$.

*See reference (7) and (8)

The solution of Eq. (42) is

$$\phi = A_1e^{qx} + A_2e^{-qx} + A_3e^{jqx} + A_4e^{-jqx} \tag{43a}$$

or, $$\phi = B_1\sinh qx + B_2\cosh qx + B_3\sin qx + B_4\cos qx \tag{43b}$$

For simply supported beam of uniform section, the natural frequency and mode shapes are

$$(w_n)_i = i^2\pi^2\sqrt{\frac{EI}{mL^4}} \tag{44a}$$

$$z(x,t) = B_3\sin\frac{i\pi}{L}x\ e^{jw_nt} \tag{44b}$$

The value of the coefficient B_3 is the maximum displacement or the amplitude of the free vibration, and is dependent on the initial (t=0) conditions of displacement and velocity. The $\sin(i\pi/L)x$ term represents the shape normal mode i, or the characteristic function.

For the fundamental mode of zero fluid velocity, we utilize Eq. (44a) and obtain

$$w_n = \pi^2\sqrt{\frac{EI}{mL^4}} = (3.1416)^2\sqrt{\frac{5.4\text{x}10^8}{11\text{x}66^4}}$$

$$= 15.9 \text{ rad/sec} = 2.538 \text{ cps.} \tag{45}$$

Comparing Eq. (45) with Eq. (36), we find that the influence of fluid in the pipe line is small.

The effect of shear distortion and of rotatory inertia on natural frequencies of simple supported beams is to reduce the tabulated natural frequencies. To estimate the magnitude of this reduction, reference is made in the paper to an adaptation of the well-known formula of Timoshenko*

$$\frac{f_{ci}}{f_i} = 1-i^2 \left(\frac{r}{L}\right)^2\left(\frac{\pi^2}{2}\right)(1 + \alpha_o E/G) \tag{46}$$

where:

f_{ci} = frequency of ith mode (in cycles per second) including the corrections for rotatory inertia and shear

f_i = frequency of ith mode as determined by the Bernoulli--Euler flexure theory

i = mode number (1,2,3 ...)

r = radius of gyration

α_0 = shear-deflection coefficient

G = modulus of elasticity in shear

For the fundamental mode, we substitute the parameters of the trans-Arabian pipe line into Eq. (46), and let α_o = 1.2 and E/G = 8/3

$$\frac{f_{c1}}{2.5} = 1 - 1^2 \frac{\frac{1}{2}(30)^2}{(12\text{x}66)^2}\left(\frac{\pi^2}{2}\right)(1+1.2\text{x}\frac{8}{3})$$

$$= 1-0.015 = 0.985$$

$$\therefore\ f_{c1} = 2.5\text{x}0.985 = 2.46 \text{ cps}, \tag{47}$$

therefore, we find out that the influence of shear distortion and rotatory inertia on natural frequencies is small.

The natural frequency for the free transverse vibrations of a simple

*See reference (10)

supported pipe containing a flowing fluid can be solved with a least-squares method*. Because an approximate solution of the differential equation of motion is made, so care must be taken in evaluation the ordinary differential equation. First, we change the partial differential equation into ordinary differential equation as follows:

The Eq. (13) may be put into nondimensional form.

$$Z=z/L, \qquad X=x/L, \qquad \tau = t\sqrt{EI/(mL^4)} = t\gamma,$$

hence,

$$z=LX, \tag{48a}$$

$$x=LX, \tag{48b}$$

$$t=\tau/\gamma, \tag{48c}$$

where,

$$\gamma = \sqrt{EI/(mL^4)} \qquad (\mathrm{sec}^{-1}).$$

Therefore,

$$\frac{\partial z}{\partial x} = \frac{\partial (LZ)}{\partial (LX)} = \frac{\partial Z}{\partial X}, \tag{49a}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial (\frac{\partial z}{\partial x})}{\partial x} = \frac{\partial (\frac{\partial Z}{\partial X})}{\partial (LX)} = \frac{1}{L}\frac{\partial^2 Z}{\partial X^2}, \tag{49b}$$

$$\frac{\partial^3 z}{\partial x^3} = \frac{\partial (\frac{\partial^2 z}{\partial x^2})}{\partial x} = \frac{\partial (\frac{1}{L}\frac{\partial^2 Z}{\partial X^2})}{\partial (LX)} = \frac{1}{L^2}\frac{\partial^3 Z}{\partial X^3} \tag{49c}$$

*See reference (11)

$$\frac{\partial^4 z}{\partial x^4} = \frac{\partial(\frac{\partial^3 z}{\partial x^3})}{\partial x} = \frac{\partial(\frac{1}{L^2}\frac{\partial^3 Z}{\partial X^3})}{\partial(LX)} = \frac{1}{L^3}\frac{\partial^4 Z}{\partial X^4} \tag{49d}$$

$$\frac{\partial z}{\partial t} = \frac{\partial(LZ)}{\partial(\tau/\gamma)} = L\gamma\frac{\partial Z}{\partial \tau} \tag{49e}$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial(\frac{\partial z}{\partial t})}{\partial t} = \frac{\partial(L\gamma\frac{\partial Z}{\partial \tau})}{\partial(\tau/\gamma)} = L\gamma^2\frac{\partial^2 Z}{\partial \tau^2} \tag{49f}$$

$$\frac{\partial^2 z}{\partial x \partial t} = \frac{\partial(\frac{\partial z}{\partial t})}{\partial x} = \frac{\partial(L\gamma\frac{\partial Z}{\partial \tau})}{\partial(LX)} = \gamma\frac{\partial^2 Z}{\partial X \partial \tau} \tag{49g}$$

Substituting Eqs. (49) into Eq. (13) leads

$$\frac{\partial^4 Z}{\partial X^4} + \alpha\frac{\partial^2 Z}{\partial X^2} + \beta\frac{\partial^2 Z}{\partial X \partial \tau} + \frac{\partial^2 Z}{\partial \tau 2} = 0 \tag{50}$$

where,

$$\alpha = \frac{\rho v^2 L^2}{EI} \tag{51a}$$

$$\beta = \frac{2\rho v L}{\sqrt{mEI}} \tag{51b}$$

Owing to the great difficulty of obtaining a solution of Eq. (50) which will fit the boundary conditions a approximate method of least-square is used. Assume

$$Z(X,\tau) = \Phi(X)\, e^{jwt} = \Phi(X)\, e^{j(\frac{w}{\gamma})\tau} = \Phi(X)\, e^{jD\tau} \tag{52}$$

where $\Phi(X)$ is characteristic function

$$D = w/\gamma \tag{52a}$$

Substituting of Eq. (52) into Eq. (50) gives an ordinary differential equation

$$\frac{d^4\Phi}{dX^4} + \alpha\frac{d^2\Phi}{dX^2} + j\beta D\,\frac{d\Phi}{dX} - D^2\Phi = 0 \tag{53}$$

Choose

$$\Phi(X) = \frac{AX}{12}\,(X^3-2X^2 + 1) \tag{54a}$$

$$= \frac{A}{12}X^4 - \frac{A}{6}X^3 + \frac{A}{12}X \tag{54b}$$

then,

$$\Phi'(X) = \frac{A}{3}X^3 - \frac{A}{2}X^2 + \frac{A}{12} \tag{54c}$$

$$\Phi''(X) = AX^2 - AX \tag{54d}$$

$$\Phi'''(X) = 2A\,X - A \tag{54e}$$

$$\Phi''''(X) = 2A \tag{54f}$$

where A is an arbitrary coefficient.

For simply supported beam the boundary conditions are

$$X = 0, \qquad Z = \frac{\partial^2 Z}{\partial X^2} = 0 \tag{55a}$$

$$X = 1, \qquad Z = \frac{\partial^2 Z}{\partial X^2} = 0, \tag{55b}$$

from Eq. (52) we get

$$Z(0,\tau) = \Phi(0)\ e^{jD\tau} = 0$$

$$Z(1,\tau) = \Phi(1)\ e^{jD\tau} = 0$$

from Eq. (54d) we get

$$Z''(0,\tau) = \Phi''(0)\ e^{jD\tau} = 0$$

$$A''(1,\tau) = \Phi''(1)\ e^{jD\tau} = 0$$

so the Eq. (54a) satisfies the boundary conditions.

Substituting Eqs. (54) into Eq. (53) leads the error function

$$\varepsilon = \Phi'^{V}(X) + \alpha\Phi''(X) + j\beta D\Phi'(X) - D^2\Phi(X) \tag{56}$$

$$= 2A + \alpha(AX^2 - AX) + j\beta D(\frac{A}{3}X^3 - \frac{A}{2}X^2 + \frac{A}{12}) - \frac{AD^2}{12}(X^4 - 2X^3 + X)$$

$$= (2A + \frac{j\beta DA}{12}) - (A\alpha + \frac{AD^2}{12})X + (A\alpha - \frac{Aj\beta D}{2})X^2 + (\frac{Aj\beta D}{3} + \frac{AD^2}{6})X^3 - (\frac{AD^2}{12})X^4$$

The value of norm is

$$|E| = L\varepsilon J\{\varepsilon\} = \varepsilon^2 \tag{57a}$$

$$|E| = (2A+\frac{Aj\beta D}{12})^2 + (A\alpha+\frac{AD^2}{12})^2X^2 + (A\alpha - \frac{Aj\beta D}{2})^2X^4 + (\frac{Aj\beta D}{3} + \frac{AD^2}{6})^2X^6$$

$$+ (\frac{AD^2}{12})^2X^8 - 2(2A+\frac{Aj\beta D}{12})(A\alpha + \frac{AD^2}{12})X + 2(2A+\frac{Aj\beta D}{12})(A\alpha - \frac{Aj\beta D}{2})X^2$$

$$+ 2(2A+\frac{Aj\beta D}{12})(\frac{Aj\beta D}{3}+\frac{AD^2}{6})X^3 - 2(2A+\frac{Aj\beta D}{12})(\frac{AD^2}{12})X^4 - 2(A\alpha+\frac{AD^2}{12})(A\alpha - \frac{Aj\beta D}{2})X^3$$

$$- 2(A\alpha+\frac{AD^2}{12})(\frac{Aj\beta D}{3}+\frac{AD^2}{6})X^4 + 2(A\alpha + \frac{AD^2}{12})(\frac{AD^2}{12})X^5$$

$$+ 2(A\alpha - \frac{Aj\beta D}{2})(\frac{Aj\beta D}{3}+\frac{AD^2}{6})X^5 - 2(A\alpha - \frac{Aj\beta D}{2})(\frac{AD^2}{12})X^6$$

$$- 2(\frac{Aj\beta D}{3} + \frac{AD^2}{6})(\frac{AD^2}{12})X^7 \tag{57b}$$

$$= (2A+ \frac{Aj\beta D}{12})^2 - (4A + \frac{Aj\beta D}{6})(A\alpha+ \frac{AD^2}{12})X + [(A\alpha+ \frac{AD^2}{12})^2 +$$

$$(4A+ \frac{Aj\beta D}{6})(A\alpha- \frac{Aj\beta D}{2})]X^2 + [(4A + \frac{Aj\beta D}{6})(\frac{Aj\beta D}{3})(\frac{Aj\beta D}{3} + \frac{AD^2}{6}) -$$

$$(2A\alpha+ \frac{AD^2}{6})(A - \frac{Aj\beta D}{2})]X^3 + [(A\alpha- \frac{Aj\beta D}{2})^2 - (4A+ \frac{Aj\beta D}{6})(\frac{AD^2}{12}) -$$

$$(2A\alpha+ \frac{AD^2}{6})(\frac{Aj\beta D}{3} + \frac{AD^2}{6})]X^4 + [(2A\alpha+ \frac{AD^2}{6})(\frac{AD^2}{12}) +$$

$$(2A\alpha-Aj\beta D)(\frac{Aj\beta D}{3}+\frac{AD^2}{6})]X^5 + [(\frac{Aj\beta D}{3}+ \frac{AD^2}{6})^2 - (2A\alpha-Aj\beta D)(\frac{AD^2}{12})]X^6$$

$$- 2(\frac{Aj\beta D}{3}+\frac{AD^2}{6})(\frac{AD^2}{12})X^7 + (\frac{AD^2}{12})^2X^8 \tag{57c}$$

We require that the relation holds:

$$I = \int_0^1 \varepsilon^2 \, dx = \min \tag{58a}$$

then,

$$I = A^2\Big\{(2 + \frac{j\beta D}{12})^2 - (2+ \frac{j\beta D}{12})(\alpha+ \frac{D^2}{12}) + \frac{1}{3}[(\alpha+ \frac{D^2}{12})^2 + (4 + \frac{j\beta D}{6})(\alpha- \frac{j\beta D}{2})]$$

$$+ \frac{1}{4}[(4 + \frac{j\beta D}{6})(\frac{j\beta D}{3} + \frac{D^2}{6}) - (2\alpha + \frac{D^2}{6})(\alpha- \frac{j\beta D}{2})] + \frac{1}{5}[(\alpha- \frac{j\beta D}{2})^2 -$$

$$(4 + \frac{j\beta D}{6})(\frac{D^2}{12}) - (2\alpha+ \frac{D^2}{6})(\frac{j\beta D}{3} + \frac{D^2}{6})] + \frac{1}{6}[(2\alpha+ \frac{D^2}{6})(\frac{D^2}{12}) +$$

$$(2\alpha-j\beta D)(\frac{j\beta D}{3} + \frac{D^2}{6})] + \frac{1}{7}[(\frac{j\beta D}{3} + \frac{D^2}{6})^2 - (2\alpha-j\beta D)(\frac{D^2}{12})$$

$$- \frac{1}{4}(\frac{j\beta D}{3} + \frac{D^2}{6})(\frac{D^2}{12}) + \frac{1}{9}(\frac{D^2}{12})^2\Big\} = 0 \tag{58b}$$

The necessary conditions to satisfy Eq. (58a) is

$$\frac{\partial I}{\partial A} = 0 \tag{59a}$$

that is,

$$\left\{(2 + \frac{j\beta D}{12})^2 - (2 + \frac{j\beta D}{12})(\alpha + \frac{D^2}{12}) + \dots\dots\dots + \frac{1}{9}(\frac{D^2}{12})^2\right\} = 0 \tag{59b}$$

Expand Eq. (59b) and rearrange it:

$$[4 - \frac{D^2}{15} + (\frac{5}{2592} + \frac{1}{252} - \frac{1}{180})D^4 + \frac{\alpha^2}{30} + \frac{17}{2520}\alpha D^2 - \frac{2}{3}\alpha] + (-\frac{1}{16} + \frac{1}{20} + \frac{1}{63})(j\beta D)^2$$

$$+ (-\frac{1}{36} + \frac{1}{63} + \frac{1}{84})j\beta D^3 = 0 \tag{59c}$$

The values α and β can be calculated from Eqs. (51), and we get

$$\alpha = 3.176\text{x}10^{-3} \tag{60a}$$

$$\beta = 4.494\text{x}10^{-2} \tag{60b}$$

Substituting Eqs. (60) into Eq. (59c) leads

$$(3.998 - 6.666714\text{x}10^{-2}D^2 + 3.417\text{x}10^{-4}D^4) + (4.494\text{x}10^{-7}D^3 j) = 0 \tag{61a}$$

The imaginary part of Eq. (61a) approaches zero. Then,

$$3.998 - 6.666714\text{x}10^{-2}D^2 + 3.417\text{x}10^{-4}D^4 = 0 \tag{61b}$$

Solve Eq. (61b), we get

$$D^2 = 106.1 + 50.8j \tag{61c}$$

Let

$$D = a + bj \tag{61d}$$

$$D^2 = (a^2 - b^2) + 2abj \tag{61e}$$

then,

$$a^2 - b^2 = 106.1 \tag{62a}$$

$$ab = 25.4 \tag{62b}$$

Solve Eqs. (62), we get

$$a = 10.58 \qquad\qquad b = 2.4 \tag{62c}$$

Substituting Eqs. (62c) into Eq. (61d) leads

$$D = 10.58 + 2.4j \tag{63}$$

Substituting Eq. (63) and Eq. (54a) into Eq. (52) leads

$$Z(X,\tau) = \frac{A}{12}\,(X^4-2X^3+X)\;e^{j(10.58+2.4j)\tau}$$

$$= \frac{A}{12e^{2.4\tau}}(X^4-2X^3+X)\;e^{10.58j\tau} \tag{64}$$

Therefore the natural frequency can be obtained from Eq. (52a)

$$W_n = 10.58\gamma$$

where,

$$\gamma = \sqrt{\frac{EI}{mL^4}} = 1.605$$

hence,

$$W_n = 17 \text{ rad/sec} = 2.7 \text{ cps} \tag{66}$$

The coefficient A can be obtained if the initial condition is known. Comparing Eq. (66) with Eq. (36), we know the approximate solution to get the natural frequency is reasonable.

FORCED VIBRATIONS

If the aerodynamic forces on the pipe are assumed to be distributed uniformly along the span and to vary sinusoidally with time, the differential equation of motion for steady-state forced vibrations is

$$EI \frac{\partial^4 z}{\partial x^4} + \rho v^2 \frac{\partial^2 z}{\partial x^2} + m \frac{\partial^2 z}{\partial t^2} + 2\rho v \frac{\partial^2 z}{\partial x \partial t} = p \sin wt \tag{67}$$

where the load p sinwt may be expanded in Fourier series:

Utilizing Eqs. (20) and (21)

$$b_s = \frac{2}{L} \int_0^L f(x) \sin \frac{s\pi}{L} x \, dx \tag{21}$$

$$= \frac{2}{L} \int_0^L p \sin wt \sin \frac{s\pi}{L} x \, dx$$

$$= \frac{2p \sin wt}{L} \int_0^L \sin \frac{s\pi}{L} x \, dx = \frac{2p \sin wt}{L} \left[- \frac{\cos \frac{s\pi}{L} x}{s\pi/L}\right]_0^L$$

$$= \frac{2p \sin wt}{s\pi} [- \cos s\pi + 1]$$

$$= \frac{4}{\pi} \frac{p \sin wt}{s} \qquad s = 1, 3, 5, \ldots \tag{68a}$$

$$\text{Let } s = 2n-1 \qquad n = 1, 2, 3, \ldots \tag{68b}$$

Substituting of Eq. (68b) into Eq. (68a) yields

$$b_{2n-1} = \frac{4}{\pi} \frac{p \sin wt}{2n-1} \tag{68c}$$

So the period function is

$$p \sin wt = \sum_s b_s \sin \frac{s\pi}{L} x \tag{20}$$

$$= \frac{4}{\pi} \sum_n \frac{p \sin wt}{2n-1} \sin (2n-1) \frac{\pi}{L} x \tag{69}$$

The differential equation of motion for steady-state forced vibrations becomes

$$EI \frac{\partial^4 z}{\partial x^4} + \rho v^2 \frac{\partial^2 z}{\partial x^2} + m \frac{\partial^2 z}{\partial t^2} + 2\rho v \frac{\partial^2 z}{\partial x \partial t}$$

$$= \frac{4p}{\pi(2n-1)} \sum_n \sin(2n-1) \frac{\pi}{L} x \sin wt \tag{70}$$

Substituting the normal modes of Equation (17) into Equation (70) leads to the equation

$$\sum_n \sin(2n-1) \frac{\pi}{L} x \sin wt \Big\{ A_{2n-1} [EI(2n-1)^4 (\tfrac{\pi}{L})^4 - \rho v^2 (2n-1)^2 (\tfrac{\pi}{L})^2$$

$$-mw^2] + \sum_k A_{2k} \frac{8\rho vw}{L} \frac{(2k)(2n-1)}{(2k)^2-(2n-1)^2} \Big\} + \sum_k \sin 2k \frac{\pi}{L} x \cos wt \cdot$$

$$\Big\{ A_{2k} [EI(2k)^4 (\tfrac{\pi}{L})^4 - \rho v^2 (2k)^2 (\tfrac{\pi}{L})^2 - mw^2] + \sum_n A_{2n-1} [\frac{8\rho vw}{L} \frac{(2k)(2n-1)}{(2k)^2 - (2n-1)^2}] \Big\}$$

$$= \frac{4\,p}{\pi(2n-1)} \sum_n \sin(2n-1) \frac{\pi}{L} x \sin wt \tag{71}$$

And hence,

$$A_{2n-1} [EI(2n-1)^4 (\tfrac{\pi}{L})^4 - \rho v^2 (2n-1)^2 (\tfrac{\pi}{L})^2 - mw^2] +$$

$$\sum_k A_{2k} \frac{8\rho vw}{L} \frac{(2k)(2n-1)}{(2k)^2 - (2n-1)^2} = \frac{4p}{\pi(2n-1)} , \tag{72a}$$

$$A_{2k}\left[EI(2k)^4\left(\frac{\pi}{L}\right)^4 - \rho v^2 (2k)^2\left(\frac{\pi}{L}\right)^2 - mw^2\right] + \sum_n A_{2n-1}\left[\frac{8\rho vw}{L}\,\frac{(2k)(2n-1)}{(2k)^2 - (2n-1)^2}\right]$$

$$= 0 \tag{72b}$$

Eqs. (72) determine the coefficients A_{2n-1} and A_{2k}, thus giving the complete solution for the steady-state vibrations. When v is such that the Eqs. (72) give $A = \infty$ it means that if the pipe line is undamped, resonance will occur when the frequency of the aerodynamic force coincides with a natural frequency of vibration, and the amplitude of unlimited magnitude will develop.

There is a singular case when the amplitudes may become infinite. The basic differential equation for bending of beam-columns is*

$$EI\,\frac{d^4y}{dx^4} + P\,\frac{d^2y}{dx^2} = p, \tag{73}$$

where P is the axial compressive force,

p is the lateral load per unit length,

then, the limiting value of the compressive force is called the critical load and is denoted by P_{cr}:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{74}$$

When the axial compressive force approaches the limiting value given by Eq. (74), even the smallest lateral load will produce considerable lateral deflection. Now, we consider the differential equation for static loading

*See Reference (9)

which is

$$EI \frac{\partial^4 z}{\partial x^4} + \rho v^2 \frac{\partial^2 z}{\partial x^2} = p. \qquad (75)$$

This is the same as the Eq. (73), if the axial load P is equal to ρv^2. It follows that the pipe will buckle when P reaches Euler load, that is

$$\rho v^2 = \frac{\pi^2 EI}{L^2} . \qquad (76)$$

Thus, when the fluid velocity has the critical value

$$v = \frac{\pi}{L}\sqrt{\frac{EI}{\rho}} \qquad (77)$$

the amplitudes may become infinite.

The term

$$\rho v^2 \frac{\partial^2 z}{\partial x^2}$$

which gives rise to the buckling, represents the inertia force produced by the curvature. The critical velocity is, of course, very large and is not of importance for pipe lines. In the actual pipe line there will be, of course, some damping and consequently the amplitude will remain finite. The amplitude attained will depend on the amount of damping percent and the magnitude of the exciting force.

Assuming an applied force p sinwt acting upon a simply supported span and setting v=0 results in the steady-state forced vibration

$$EI \frac{\partial^4 z}{\partial x^4} + m \frac{\partial^2 z}{\partial t^2} = p \text{ sinwt}. \qquad (78)$$

The solution of Eq. (78) is the sum of complementary solution and particular solution. To determine the particular solution, the solution in the form

$$z_p = A \sin wt \tag{79a}$$

is substituted into Eq. (78) to obtain the equation

$$-Amw^2 \sin wt = p \sin wt, \tag{79b}$$

and hence,

$$A = -\frac{p}{mw^2} \tag{79c}$$

Substituting Eq. (79c) into Eq. (79a) leads the particular solution

$$z_p = -\frac{p}{mw^2} \sin wt \tag{80}$$

The general solution of Eq. (78) is then

$$\begin{aligned} z(x,t) &= \phi_i(x)e^{jwt} - \frac{p}{mw^2} \sin wt \\ &= (B_1 \sinh qx + B_2 \cosh qx + B_3 \sin qx + B_4 \cos qx)e^{jwt} \\ &\quad - \frac{p}{mw^2} \sin wt \end{aligned} \tag{81}$$

The boundary conditions for simple supported beam are $z=0$ and $d^2z/dx^2 = 0$, when $x=0$ and $x=1$.

$$z(0,t) = (B_2 + B_4)e^{jwt} - \frac{p}{mw^2} \sin wt = 0, \tag{82a}$$

$$z''(0,t) = (B_2 - B_4)q^2 e^{jwt} = 0, \tag{82b}$$

from Eq. (82b), we find that

$$B_2 = B_4$$

Substitution Eq. (82c) into Eq. (82a), we find that

$$B_2 = B_4 = \frac{p \sin wt}{2mw^2 e^{jwt}} \tag{82d}$$

$$z(L,t) = (B_1 \sinh qL + B_2 \cosh qL + B_3 \sin qL + B_4 \cos qL)e^{jwt}$$

$$- \frac{p}{mw^2} \sin wt = 0, \tag{82e}$$

$$z''(L,t) = (B_1 \sinh qL + B_2 \cosh qL - B_3 \sin qL -$$

$$B_4 \cos qL)q^2 e^{jwt} = 0, \tag{82f}$$

from Eq. (82e), and Eq. (82d), we find that

$$B_1 \sinh qL + B_2 \cosh qL + B_3 \sin qL + B_4 \cos qL$$

$$= \frac{p}{mw^2 e^{jwt}} \sin wt = 2B_2 = 2B_4, \tag{82g}$$

from Eq. (82f), we find that

$$B_1 \sinh qL + B_2 \cosh qL - B_3 \sin qL - B_4 \cos qL = 0. \tag{82h}$$

Adding Eq. (82g) and (82h), we find that

$$2(B_1 \sinh qL + B_2 \cosh qL) = 2B_2, \tag{82i}$$

and hence,

$$B_1 = \frac{1-\cosh qL}{\sinh qL} B_2 = \frac{1-\cosh qL}{\sinh qL} \frac{p \sin wt}{2mw^2 e^{jwt}} . \tag{82j}$$

Substracting Eq. (82h) from Eq. (82g), we find that

$$2(B_3 \sin qL + B_4 \cos qL) = \frac{p \sin wt}{mw^2 e^{jwt}} = 2B_4, \tag{82k}$$

and hence,

$$B_3 = \frac{1-\cos qL}{\sin qL} B_4 = \frac{1-\cos qL}{\sin qL} \frac{p \sin wt}{2mw^2 e^{jwt}} . \tag{821}$$

Substitution Eq. (82d), Eq. (82j), and Eq. (821) into Eq. (81) leads the general solution for steady-state forced vibration with zero fluid velocity

$$z = \frac{p \sin wt}{2mw^2} [\frac{1-\cosh qL}{\sinh qL} \sinh qx + \cosh qx +$$

$$\frac{1-\cos qL}{\sin qL} \sin qx + \cos qx - 2] . \tag{83}$$

where,

$$q = \sqrt[4]{mw_n^2/EI}$$

The foregoing analysis does not take into account an axial force which likely is in the pipe. The effect of an axial force is to decrease the natural frequency of vibration. The fluid pressure in the pipe is not sufficient to produce an appreciable change in frequency, but if the pipe line is fabricated so that large axial compressive stresses exist, the frequency can be change greatly. For example*, if the fluid pressure in the trans-Arabian pipe line is 500 psi and the net axial compressive stress stresses are 20,000 psi, then the natural frequency is reduced from

*See reference (1).

2.5 cps to 1.2 cps which coincides with the frequency of the aerodynamic force. It is not unlikely that the axial force in the trans-Arabian pipe line is much larger than it is thought to be, and a relatively small aerodynamic force is approximately in resonance with the pipe. If this is the case, the amplitude of vibration can be controlled by the installation of vibration dampers along the line.

SUMMARY

In this report, an analytical investigation, based on simple beam theory, shows that the flow of fluid in such a pipe line has no benefical effect upon the vibrations. The influence upon the vibration system at low fluid velocities is negligible.

The solution is presented for free vibrations and for steady-state forced vibrations. It is shown that large amplitudes may be developed if the amount of damping is too small, and that at a certain high critical velocity the fluid flow may cause a dynamic instability. The effect of shear distortion and rotatory inertia is small.

It should be mentioned that the analysis has neglected the viscosity of the fluid, the internal friction of the pipe material, and the support damping.

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BENDING VIBRATIONS OF PIPE LINE
CONTAINING FLOWING FLUID

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In this report the problems of aerodynamically induced vibrations of a pipe line containing flowing fluid for free vibrations and for steady-state forced vibrations are considered. This report consists of deriving the governing differential equation of motion, finding the fundamental natural frequency, and discussing the effect of fluid flow in the pipe line. An analytical investigation, based on simple beam theory, shows that the fluid flow in the pipe line has no beneficial effect upon the vibrations. The solutions are presented for free vibrations and for steady-state forced vibrations. It is shown that large amplitudes may be developed if the amount of damping is too small, and at a certain high critical velocity the fluid flow causes a dynamic instability. Owing to the great difficulty of solving the fourth order partial differential equation to obtain the fundamental natural frequency, an additional approximate solution based on the least-square technique given.