VIBRATION IN MACHINE TOOLS

by 580

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Nomenclature

s - chip thickness

s_0 - feed mm/rev

v_0 - cutting speed mm/sec.

R - Radius of the work

N - Rotational speed in r.p.s.

Ω - Angular velo. of the tool

T - Time for one rev.

r_0 - feed rate mm/sec. $r_0 = \frac{s_0\Omega}{2\Pi}$

P_0 - cutting force

dP - Increment in the force P_0

ds - Chip thickness variation (variation of the rate of penetration)

dr - feed rate variation

k_s - cutting force co-efficient - $(\frac{\partial P_0}{\partial s_0})_{dv=0}$

k_Ω - $(\frac{\partial P_0}{\partial \Omega_0})_{ds=0} = RK_v$

k_1 - $(\frac{\partial P}{\partial r})_{dr=d\Omega=0}$ chip thickness coefficient

k_2 - $(\frac{\partial P}{\partial r})_{ds=d\Omega=0}$

k_3 - $(\frac{\partial P}{\partial \Omega})_{ds=dr=0}$

$K = k_s - k_1$ = Penetration co-efficient

$k_\Omega - \frac{(k_s - k_1)s_0}{\Omega}$ = cutting speed co-efficient

m - Equivalent mass

λ - Equivalent spring

ρ - Equivalent damper

ν - displacement coefficient

x - amplitudes

$P_z = F_H$ = Tangential force

τ_s - shear stress

F_s - shear force

F_V - vertical force

A_0 - s.d

s_1 - feed in in/rev

d - depth of cut

ϕ - shear angle

β - Friction angle

α - rake angle

γ_e - true rake angle

c - side cutting edge angle

ϕ_n - normal shear angle

ζ_n - normal chip reduction coefficient

CHAPTER I

Introduction

Much emphasis has been placed upon vibrations in machine tools during recent years because many people have recognized that accuracy, surface finish and, last but not least, production costs are considerably influenced by them. Today an arsenal of sophisticated instruments is available for the investigation of machine tool vibration. However, in the final analysis, the finished surface itself will reflect the dynamic behavior of the machine tool.

Machine tools have always vibrated and will continue to do so. We strive to control these vibrations and keep them at or below a tolerable level. This was easier to do in the past than it is today. The older machine tools had fewer auxiliary mechanisms, lower speed and feed ranges, and wide sliding ways which provided plenty of friction and also acted as vibration dampers. Newer machine tools often have sliding ways which have been designed for reduced friction in order to keep servo-mechanisms small in size. Some friction dampening effects of metal to metal sliders have been eliminated because of the introduction of many anti-friction bearing design features. While higher cutting speeds generally contribute to an improvement of the surface finish obtained, they often excite components of the machine tool at their natural frequency. Such resonance conditions can usually be avoided by changing the spindle speed. If the cutting tool and the machine tool were infinitely stiff, it would be possible to predict surface finishes and accuracy of rigid workpieces.

Only the shape of the tool and the feed per revolution or per tooth would have to be taken into consideration. Tests carried out by Okushims have shown that if machining can take place under conditions of great rigidity, no effect of cutting speed upon the surface finish is noticeable. However, to build such machines for production is highly unlikely.

A single-point cutting tool in a lathe will generate a helix or thread whose pitch is determined by the feed per revolution. (vide fig. 1) The influence of the size of the nose radius of the lathe tool upon the surface finish obtained was investigated by Fees in 1939 and recently reported by Wetzel. The following relationship was established:

$$H = \frac{S^2}{8R}$$

in which H is the height of feed mark, S the feed per revolution and R the nose radius.

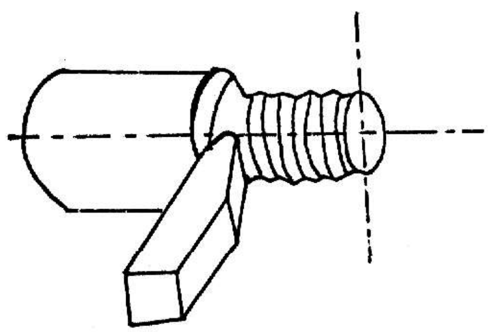


Fig. 1. Single-point tool cutting in a lathe.

The above formula will give a value of surface roughness which can be obtained under good machining conditions when using feeds larger than 0.010 in. per revolution. This relationship also holds true approximately for slab milling if R is considered to be the radius of the cutter.

It can be seen that a theoretically flat surface can be obtained with a tool which has a straight nose. This is often used in planing a finishing cut and sometimes in face milling. In a face mill, a flat region is ground on the tool face, which is in contact with the freshly produced workpiece surface.

Although under certain conditions the surface finish is predictable from a knowledge of the feed and nose radius, the depth of cut and cutting speed have the most decisive influence. The deeper cut and heavier feed may cause deflections in the tool and the workpiece giving rise to a wavy pattern and chatter marks. Cutting speeds which coincide with a natural frequency of the machine tool will create excessive motion between the machine tool elements and naturally will be reflected in a poorer surface finish. Even if all the mechanical conditions are properly taken care of, the metal cutting process itself will contribute a number of unbalancing factors.

Mechanical work is transformed into heat when metal is cut and the temperature in the shear zone and chip-tool interface will be non-uniform. This can be associated with fluctuations in the cutting force and thus be another source of chatter vibrations. Many of the newly developed alloys with high temperature resistant properties will accentuate this effect through their pronounced work-hardening tendencies and contribute further to those perturbations. Pitz and Kob have shown instances in which oscillations in cutting force and temperatures corresponded to segmentations in the chip, and the highest temperatures occurred simultaneously with the lowest forces and vice versa. These fluctuations were more pronounced at lower cutting speeds than at higher speeds, resulting in periodic variations of the cutting force during the formation of a single chip.

Tool wear causes a change in the dimensions of the workpiece and sometimes can contribute either to the stability or instability of the machine tool system. Some materials weld to the face forming, what is commonly called, a built-up edge. Other materials may adhere to the clearance flanks of the tool causing objectionable interferences with the cutting process. All of these tool factors have an effect upon surface finish too and are usually quite visible to the naked eye.

Quietness in machine tools sometimes is an important factor, especially in those shops which have established operating noise levels. Loose gears, bearings and slides can cause a noisy machine tool. However, workpiece configuration and mounting are the contributing factors.

In certain cases, an analysis of the surface finish will also give an indication of the amplitude of vibration as well as its frequency. There can be other sources of trouble, which may include microstructure and flaws in the workpiece, stiffness of tool and workpiece assembly, anti-friction bearings, gears, motors, pumps, pulleys, splines, keys and numerous disturbances coming into the machine tool from the surrounding floors, walls and adjacent machinery. Proper lubrication of sliding ways and bearings have an influence upon the dynamic behavior of a machine tool and its many mechanisms.

Thus vibration in machining of metals is detrimental to cutting tool life, surface finish and the accuracy of the machined components. Vibrations set up frequent stress cycles and produce rather high stress levels, particularly when resonance occurs. The effect of vibration also causes fatigue failure. Machine tools are elastic bodies and due to various causes discussed later, vibrations are produced in the machine tool structure. Although the structure of a machine tool is rather complex, with many modes of vibration, frequency curves show an equivalent number of resonance peaks which are usually well separated, and in the neighbourhood of a resonance peak, the curve resembles that obtained with a simple mass-spring system of single degree of freedom having viscous damping. For many purposes, therefore, a machine tool structure can be considered to be a collection of damped single degree of freedom systems, each of which resonates at one of the resonance frequencies of the real structure.

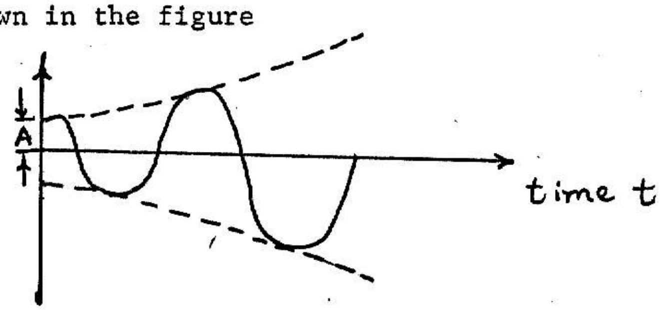
The purpose of this report is to study the forced vibration phenomenon in machine tools, more particularly those which use single-point cutting tools. In the study of forced vibration, a mathematical model was formed on the basis of a simplified physical model of the machine tool system. In the formulation of the mathematical model, it was found that forced vibration in machines is mainly due to a forcing function, which is varying with time. The author, in this report, has tried to investigate primarily the variation of this forcing function under various cutting conditions. This investigation was carried out both theoretically and experimentally. In the theoretical analysis, fundamentals of mechanics of chip formation were considered as given by Ernst, Merchant, Lee, Shaffer and Loladze.

The theoretical formula of cutting force variation with various cutting conditions was correlated with the experimental data. Unfortunately, present theory in metal cutting has not given the variation of cutting force with time; but experimentally, it was shown that the cutting force varies with time, which is mainly due to vibration in machine tools.

CHAPTER II

Sources and Causes of Vibrations in Machine Tools

The vibrations that occur in the machining of metals are ordinarily of two types. Forced vibrations are those which occur under the action of a periodically varying force on the cutting tool arising out of mechanical causes. The frequency of such vibration depends upon the frequency of force variations at the source, which may be quite different from the natural frequencies of the vibrating members and 'self excited vibrations' are those which occur because of dynamic instability of the vibrating member as shown in the figure



and once started by some mechanical means they are then self perpetuating. Self excited vibrations, as stated earlier, occur at a frequency very close to the natural frequency of the vibrating member.

Of the two types of vibrations, the self excited type is ordinarily the more severe and gives most of the trouble.

Forced vibrations may be due to one or more of the following causes:

(1) Out of balance of rotating or reciprocating machine components, faulty gears, belts, ball and roller bearings, and mechanisms which transfer energy in uniformly timed impulses. These vibrations can be eliminated by careful static and dynamic

balancing of the faulty components, by improving gear and belt quality, etc.

(2) Vibrations transmitted from other machines through foundations: such vibrations can be suppressed by applying vibration isolators or flexible supporting layers.

(3) Vibrations caused by chip formation: when a discontinuous type of chip is formed (this is very often found in the machining of brittle materials or in the cutting of ductile materials at very low speed) the recurring fractures of the metal in the shear plane ahead of the tool produce periodic variations in the cutting force. Similarly, in the case of machining operations which produce a continuous chip with built-up edge (as generally found in the cutting of ductile materials with high speed tools at ordinary cutting speeds), as the fragments of the built-up edge pass off with the chip of the work surface, a variation in the force on the cutting tool results.

Yet another source of force variation may be caused by formation of chips of varying thickness as obtained, for example, in cylindrical milling. These periodic variations in the cutting force, the frequency of which depends upon the frequency of discontinuity in the chip, of shedding of fragments of the built-up edge, or the number of teeth in the milling cutter, as the case may be, give rise to forced vibrations.

'Self excited vibrations' are mainly caused by the dynamic instability of the cutting. Experiments indicate that in the case of single-point cutting tools, an important source of dynamic instability lies in the variation of the cutting force with the cutting speed. Usually, for most materials, the cutting force decreases with increasing cutting speed, a condition which is favorable for self-excited vibrations to occur.

A vibration once started by mechanical means, for example by a hard spot in the material, vibration due to out-of-balance components or any other cause, will perpetuate itself at a frequency very close to the natural frequency of the cutting tool.

Theoretically, the amplitude of self-excited vibration may be expected to increase to infinity but it is found that the amplitude of vibration cannot exceed a value in excess of $xo = \frac{V}{2nf}$ where V is the cutting speed and f the natural frequency of the cutting tool in cycles per second.

It is also found that the wear on the clearance face of the cutting tool appreciably increases the tendency to vibrate, and that there is a limit of the cutting speed, which increases as the natural frequency of the cutting tool increases, below which vibration will not occur.

Chatter Vibrations. A vibration of large amplitude can be produced in a system only by a harmonic force, the frequency of which is equal to, or very nearly equal to, the natural frequency or frequencies of the system. In most cases, the forced vibration produced in a machine tool structure is not harmonic but only periodic. Not every periodic motion is harmonic, but any periodic motion can be split up into a series of fourier components which are harmonic by nature. Resonance, therefore, still may occur if any of the fourier component frequencies is equal to the natural frequency of the system.

A machine tool, being a complex structure, may vibrate in many ways or 'modes' with the structure and its components subject to bending and/or torsion. A mode may be defined as the amplitude distribution of the structure when vibrating at one of its natural frequencies. It does not always follow that every large absolute vibration recorded on the individual parts of a machine tool will seriously affect the machining performance. It is the relative vibration between the cutting tool and the workpiece commonly known as 'chatter' which is of main importance as far as the machining performance of a machine tool is concerned. Chatter causes a perceptible irregularity in the tool marks on a finished surface. The excited amplitudes of chatter vibration are usually very high, i.e., 100 to 1,000 times greater than the average, and a typical build-up of amplitudes can be observed in most cases.

The presence of chatter is not desirable for good surface finish and accuracy of the finished product. Sometimes, chatter vibration of low intensity may be advantageous in rough cuts. Chatter is definitely objectionable when tungsten carbide or ceramic tools are used as they will chip and severe chatter tends to cause excessive wear on feed screws, bearings, etc. of the machine tool and to loosen all fastenings.

The theory of chatter is complex and a large number of parameters are involved in the chatter phenomena. The individual effects of these parameters are not sufficiently known and at present time no general theory exists which would cover all types of chatter. These difficulties are often due to limitations in measuring techniques and available instruments. Following his earlier work with Fishwick, Tobias has made important contributions in the study of machine tool chatter and in particular with the type known as 'regenerative chatter'. Regenerative chatter occurs when a part of the chip removed by a cutting edge has been cut previously, either by the same cutting edge (single-edge tools such as lathe tools or grinding wheels), or by another cutting edge which is solidly connected to the first (multi-edge tools such as drills, face milling cutters, etc.). In such cases, the forces acting on the cutting edge at a certain instant are partly determined by the form of the chip removed earlier. One interesting phenomenon that occurs when regenerative chatter takes place is that certain speed ranges are stable, these being separated by unstable speed ranges. The cutting process is regarded as being dynamic in nature and the principal parameters involved are the nominal feed and the cutting velocity.

One interesting result of Tobias and Fishwick's investigation is the development of the concept of "stability charts". These charts are, in fact, graphical representations of the differential equations of the vibrating system and show those rotational speeds at which the system is stable, unstable or at the threshold of stability, for a given structure and a given tool, workpiece material and machining condition. Such charts can be of much use in making proper choice of the machining conditions and comparing the dynamic performance of competitive designs of new machine tools.

The assumptions made by Tobias and Fishwick make the model of the cutting force variations somewhat complex and, so far, the investigations made by them were limited to cases where structural vibrations of the models were sufficiently separated to permit their interaction to be neglected. Their theory is essentially an adaptation and development of Arnold's work on the chatter of tools, but beyond the basic ideas, it diverges from Arnold's approach.

Thusty and Polacek, who have also done much useful work independently, made the rather simpler assumption that cutting force is proportional to chip thickness. This enabled them to consider the machine tool structure in greater detail, particularly with regard to the superposition of structural modes when determining the relative movement between the tool and the workpiece.

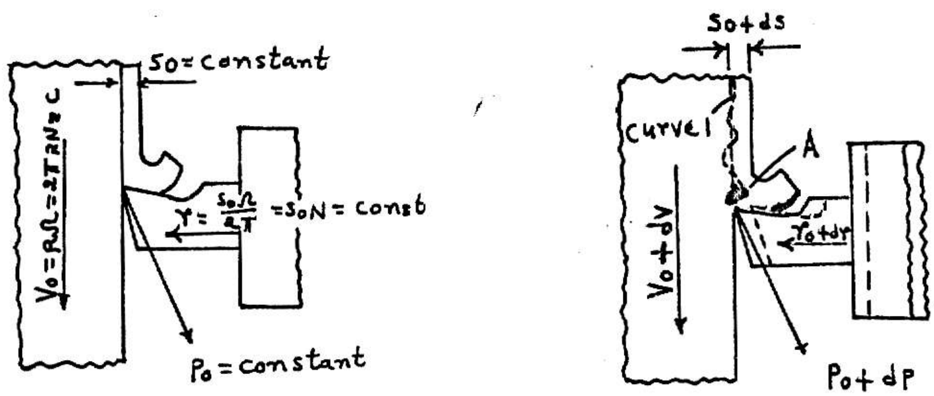
CHAPTER 3

Analytical Approach to Predict Vibration in Machine Tools

General Phenomenon of Vibrations in Machine Tools

The conventional theory of metal cutting deals entirely with the steady state cutting process in which the cutting takes place under vibration free conditions. Thus it treats the problem of the cutting process from a purely static view point.

Chatter itself is a cutting process, having a dynamic character. Chatter theory thus deals with metal cutting under non-steady state conditions.



Static dynamic (A hard grain)

Fig. 1

Figure (1) shows a lathe tool performing orthogonal cutting. Under steady state conditions, the chip thickness s is equal to the feed s_0 [mm/rev]. The cutting speed v_0 is constant and $v_0 = R\Omega = 2\Pi RN$, where R is the radius of work and $N = \frac{\Omega}{2\Pi}$ is the rotational speed in revolutions per second. $\Omega = \frac{2\Pi}{T}$ is the angular velocity of the tool. T is the time for one revolution, $N = 1/T$). The feed rate is $r_0 = \frac{s_0\Omega}{2\Pi} = s_0N$ and it is sufficient to give s_0 and v_0 because these determine r_0. The independent variables are therefore s_0 and Ω (apart from chip width which is not involved). Thus, the cutting force P_0 is a function of s_0 and v_0 for static investigation.

Now let the tool strike a hard grain in the material and as a result the cutting force P_0 is suddenly increased by dP. Force P_0 is absorbed by static deformation of the machine frame. The increase in cutting force gives rise to further deformation. Assume the further deformation to take place along the longitudinal axis of the tool. The torque P_0R acting on the work is increased by dPR and this is absorbed by elastic torsional deformation in the drive and results in reduction of the cutting speed.

Suppose the hard grain now breaks out at time t = 0. A sudden drop in cutting force and torque will occur and the potential energy stored in the machine frame and the drive will be released to throw the system into vibration. It is clear that the hard grain has modified the cutting conditions and the static cutting process will not be resumed until the vibration has decayed. At time t = 0 the cutting edge of the tool starts to move along a damped vibration curve superimposed on the steady state cutting path indicated in figure (1) by curve 1. The chip thickness s is now not equal to nominal feed s_0. The instantaneous chip thickness is therefore $s = s_0 + ds$ where ds and hence s are time dependent. The quantity ds is the chip thickness variation. Similarly the feed rate also becomes time-dependent.

$$r = r_0 + dr = \frac{s_0 \Omega}{2\Pi} + dr = s_0 N + dr$$

where dr is the feed rate variation or variation of the rate of penetration. If the work undergoes torsional vibration the cutting speed likewise will vary to give

$$v = v_0 + dv = v_0 + R d\Omega = v_0 + 2\Pi R N$$

Under dynamic conditions ds, dr and dv (i.e. $d\Omega$) and hence s, r and v (i.e. Ω) are independent of each other, so that the expression for the cutting force contains three independent parameters [i.e. $P(s,r,\Omega)$].

Now the cutting force functions are $P_0(s_0, v_0)$ and $P(s, r, \Omega)$. Let us see how the cutting force changes when small variations in these factors take place. Under static conditions the cutting force variation is defined by

$$dP_0 = k_s\, ds_0 + k_v\, dv_0 = k_s\, ds_0 + k_\Omega\, d\Omega$$

where $\quad k_s = \left(\frac{\partial P_0}{\partial s_0}\right)_{dv=0} \quad$ and $k = \left(\frac{\partial P_0}{\partial \Omega_0}\right)_{ds=0}$

Under dynamic cutting conditions the cutting force is a function of three independent factors $P(r,s,\Omega)$ and hence the cutting force variation for small changes in these factors is

$$dP = k_1\, ds + k_2 dr + k_3\, d\Omega$$

Mathematically

$$k_1 = \left(\frac{\partial P}{\partial s}\right)_{dr=d\Omega\,=\,0}$$

$$k_2 = \left(\frac{\partial P}{\partial r}\right)_{ds\,=\,d\Omega\,=\,0}$$

$$k_3 = \left(\frac{\partial P}{\partial \Omega}\right)_{ds\,=\,dr\,=\,0}$$

The coefficients of the equation for the dynamic cutting-force variation are tied to the coefficients of the static equation and the equation for the dynamic cutting-force variation becomes

$$dP = k_1\, ds + (k_s - k_1)\,\frac{2\Pi}{\Omega}\, dr + \left[k_\Omega - (k_s - k_1)\,\frac{s_0}{\Omega}\right] d\Omega \qquad (1)$$

This equation contains only one dynamic coefficient namely k_1, which is termed the chip-thickness coefficient. The quantity $k_s - k_1 = K$ is a penetration coefficient, while $k \; - (k_s - k_1)\, s_0/\Omega$ is termed the cutting speed coefficient and k_s is the cutting force coefficient.

Now the disturbance affecting the cutting process is time-dependent and therefore the cutting force element dP is also a function of time. If a time-variable force acts on an elastic system (machine frame) the latter will be thrown into vibration. If chatter occurs, the time-dependence of dP will not alone give rise to instability. It is important to know that dP depends not only on the displacement brought about by the disturbance but also on its velocity. Forces which are velocity dependent can be regarded as damping forces and they may either add to or subtract from the damping forces contained in the system. If the damping introduces by dP is positive it will increase the structural damping with the result that the prevailing disturbance will rapidly decay. If the damping force brought about by the disturbance will rapidly decay. If the damping force brought about by the disturbance is negative, it will reduce the damping of the system and may even overcome it to make the overall damping negative. Positive damping is negative, it will reduce the damping of the system and may even overcome it to make the overall damping negative. Positive damping is energy-absorbing. Negative damping introduces energy in the system where it is used for building up vibration and maintaining it. The machine tool drive always acts as the energy reservoir for this purpose.

It is obvious that the dynamic behaviour of the machine frame is of great importance. The cutting process will remain free of disturbance if, for example, the stiffness between the work and the tool and the drive is infinitely great. The cutting process can only be disturbed if the steady-state relative motion of the tool is altered as a result of tool deflection, twist in a transmission train or some similar cause.

The dynamic cutting-force element dP acts on the machine frame and forces the frame into vibration, this effects a change in the relative position of the cutting edge, which in turn leads to a change in dP. The disturbance forces the frame to vibrate in one or more of its natural modes of vibration. The natural mode determines the relative motion of the cutting edge with reference to the work and is also partly responsible for deciding the form and characteristic of the dynamic cutting-force element dP. Figure (2a) shows a situation in which the natural mode of vibration of the frame permits vibration in a direction perpendicular to the cutting speed. (direction of principal vibration). If the speed of rotation is constant, no variation in the cutting speed can take place and so $d\Omega = 0$ in equation (1). The only component of dP to exert any influence in the line with the motion is dP_x. In figure (2b) the vibration direction is parallel to the cutting speed. This gives a constant chip thickness $s(ds = 0)$ but a variable degree of penetration $r(dr = \frac{s_0(d\Omega)}{2\Pi})$ and hence cutting speed is variable.

Thus dP depends upon the relative motion between cutting edge and work and on the angle between the cutting force P_0 and the direction of principal vibration say relative vibration.

The frame has distributed mass and elasticity and so it possesses an infinite number of natural modes of vibration. To each natural mode of vibration there belongs a direction of principal vibration for the point at which the cutting takes place and this direction determines the direction of relative motion between the cutting edge and the work.

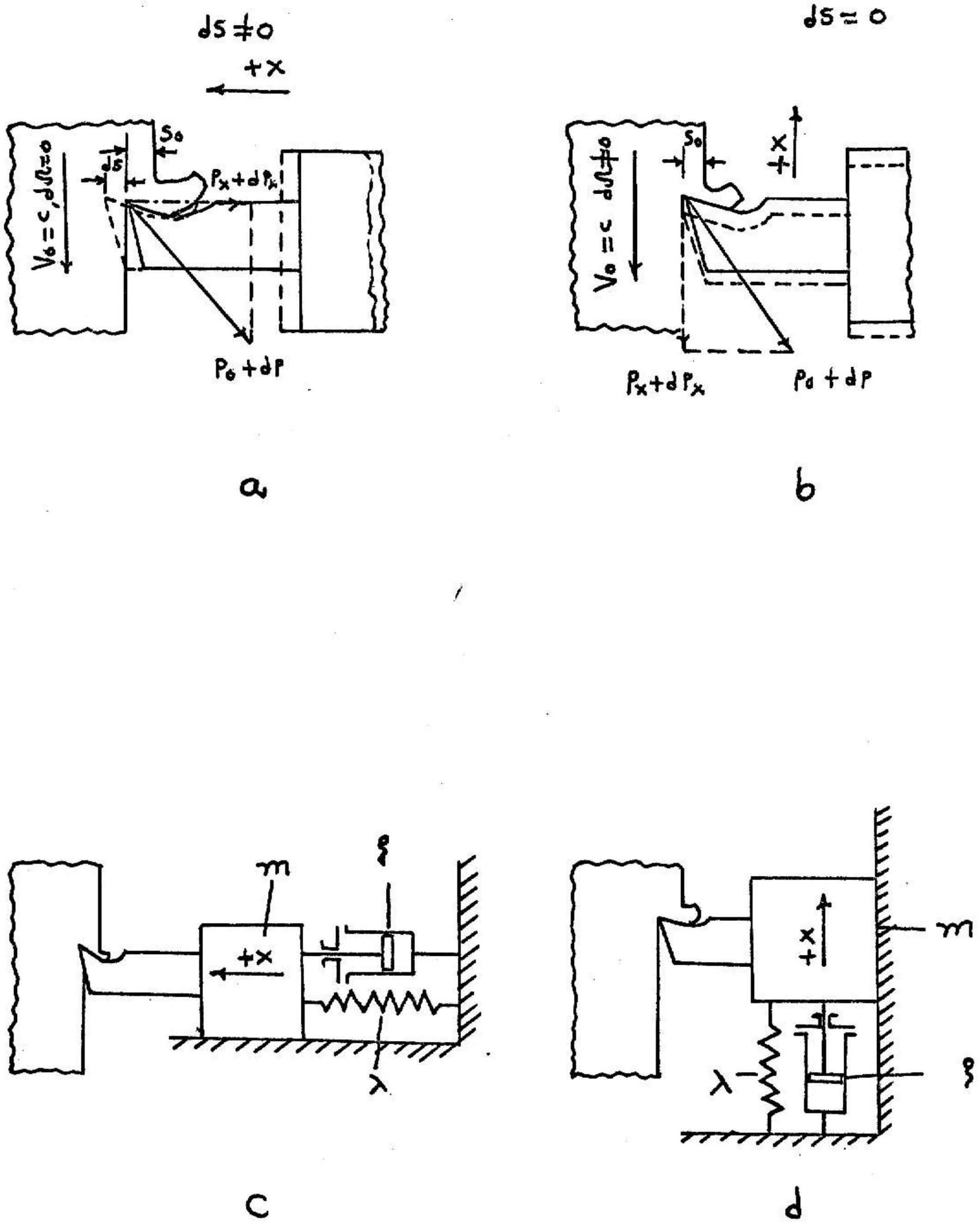

ds ≠ 0
+x
S_0
ds
$V_0 = c, d\Omega = 0$
$P_x + dP_x$
$P_0 + dP$
a
ds = 0
S_0
+x
$V_0 = c\ d\Omega \neq 0$
$P_x + dP_x$
$P_0 + dP$
b
m
ξ
+x
λ
c
m
+x
λ
ξ
d

Fig. 2

This assumes the natural frequencies of the frame are not very close together. In order to study the dynamic behavior we can imagine the frame to be replaced by a number of elementary vibratory systems consisting of an equivalent mass m, and equivalent spring λ and an equivalent damper ρ. The equivalent constants can be found from the resonance diagram of the frame.

Considering the dynamic characteristics of the frame jointly with the dynamic cutting conditions, we get the two models c and d for the direction of principal vibration shown in a and b. The stability of the cutting process is decided by the stability behavior of all the models.

The theoretical investigation of the chatter process is carried out in the following manner: (1) it is assumed that the steady state cutting process has been disturbed by a relative vibration x occuring between work and tool and having the direction of a principal vibration, (2) considering the influence of x on the cutting process, the cutting force element dP_x can be found, (3) from dP_x the equivalent damping of the dynamic cutting process can be calculated and should be added to the frame damping, (4) cutting conditions for which the overall damping is negative are unstable and will cause chatter.

The calculation should be carried out for all modes of vibration but practically only the low modes are of importance. In many cutting processes certain modes of vibration are prevented from occurring through the geometry of the tools and similar factors.

Analysis of vibrations during the cutting process in a lathe: -

Further critical analysis of the chatter of machine tools discussed above can be classified into two types (i.e. type A and type B). Type A occurs when the amplitudes lie in the plane perpendicular to the cutting direction, regardless of whether the amplitudes are co-directional with the feed or perpendicular to it. This type of chatter occurs in drilling, end milling and spot facing. Type B occurs when the chatter amplitudes have a component co-directional with the cutting speed. The latter is generally found in lathes and grinders. We will confine the discussions to vibrations occurring in the cutting process in lathes in which the chip thickness changes only if the cutting edge is diverted away from the plane of cutting speed as a result of dynamic deflection of the tool shank in the same direction as the cutting speed. This is evident from figure (3).

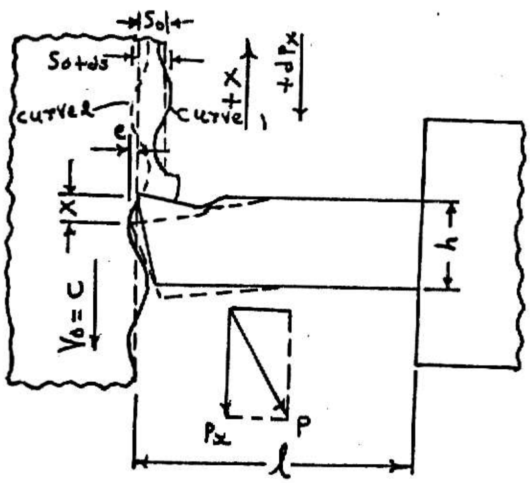


Fig - 3

Fig. 3

The downward bending of the cutting edge (deflection x) is accompanied by a horizontal displacement defined by $e = \nu x$, where ν is the displacement coefficient. It is negative under the condition shown above because $+x$ and $+dP_x$ are opposite in direction. The cutting force component P_x and hence also $+dP_x$ point in the same direction as the cutting speed v_0 (i.e. downwards), while $+x$ points upwards. Any downward deflection of the tool makes x negative. This has the effect of increasing the cutting depth, however dP_x is positive and hence the condition $\nu < 0$ is required.

Positive ν occurs when the chip thickness decreases as a result of deflection of the tool in the same direction as the cutting speed. The displacement coefficient ν is determined to some extent by characteristic design features. From the above figure, in the event of type B chatter a chip thickness variation effect can occur only if deflection of the tool is accompanied by drift of the cutting edge away from the cutting plane. For single point cutting tools the chip thickness variation is again given by

$$ds = \nu[x(t) - \mu x(t-T)] \qquad (2)$$

In the above equation all the possibilities for ν are $\nu < 0$ (cutting edge advances into work) or $\nu > 0$ (cutting edge backs out of the work) or $\nu = 0$ (cutting edge stays in the plane of cutting speed) where μ is the overlap factor.

From figure (3) the variation in the rate of penetration of the cutting edge is

$$dr = \frac{de}{dt} = \nu \frac{dx}{dt} \tag{3}$$

Now the dynamic cutting-force variation

$$dP = k_1 \, ds + (k_s - k_1) \frac{2\Pi}{\Omega} dr + [k_\Omega - (k_s - k_1) \frac{s_0}{\Omega}] \, d\Omega \tag{4}$$

on substituting ds from equation (2) and dr from equation (3) in equation (4) and introducing $d\Omega$ we get the following expression for the dynamic force element.

$$dP_x = \nu k_1 [x(t) - \mu x(t-T)] + [\frac{k_\Omega}{R} - k(\nu - \frac{s_0}{2\Pi R}) \frac{2\Pi}{\Omega}] \frac{dx}{dt} \tag{5}$$

Assuming the form of disturbance as follows

$$x(t) = Ae^{\delta t} \cos\omega t \tag{6}$$

where A is an indefinite amplitude constant, ω is the chatter frequency and δ is the decay constant. Equation (6) represents a positively damped $\delta < 0$ or negatively damped vibration.

If the overall damping of the system is positive, the analysis will yield $\delta < 0$ and this means that the disturbance x will be damped out after sufficient time has elapsed and hence the cutting process is stable. For negative overall damping the calculation will yield $\delta > 0$ which means that the amplitude of the disturbance x grows exponentially so that the cutting process is unstable.

It is not necessary that x be of the above form but it must be some harmonic function of time.

Substituting equation (6) into equation (5) gives

$$dP_x = \nu k_1 F_1 x + [\nu k_1 F_2 + k(\nu - \frac{s_0}{2\Pi R}) \frac{2\Pi}{\Omega} + \frac{k_\Omega}{R}] \frac{dx}{dt} \tag{7}$$

where F_1 and F_2 are of the following form

$$F_1 = [1 - \mu \exp \frac{-2\Pi\delta}{\Omega} (\cos \frac{2\Pi\omega}{\Omega} + \frac{\delta}{\omega} \sin \frac{2\Pi\omega}{\Omega})]$$

$$F_2 = \frac{\mu}{\omega} \exp \frac{-2\delta}{\Omega} \sin \frac{\Pi\omega}{\Omega}$$

obtained by Tobias.

It is to be noted that dP_x depends not only on the disturbance x(t) but also on the velocity of the disturbance $\frac{dx}{dt}$ and it is this velocity dependent term in equation (7) which may bring about dynamic instability.

The dynamic force element dP_x acts on the elastic frame of the machine and may induce vibration in it. When the machine frame is vibrating in one of its natural modes it may be represented by an equivalent mass m damping ρ and spring λ. We can assume that these constants are already known because we can find them from the resonance curves obtained during the experimental study preceding the theoretical treatment of the chatter process.

The modes of vibration determine the direction of the disturbance x(t) and must therefore be found prior to the theoretical analysis. From the resonance curve it is possible to obtain directly the natural frequency ω_0, the amplification factor Q (or the damping ratio D) and the equivalent static stiffness λ which are linked to the equivalent constants of the elementary vibratory system in the following manner

$$\omega_0^2 = \frac{\lambda}{m} \quad \text{and} \quad Q = \frac{\lambda}{\rho\omega_0} \tag{8}$$

The natural frequency ω_0 is simply put equal to the resonance frequency. Q or D is found from the width of the resonance curve

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{1}{2D} \tag{9}$$

and the equivalent static stiffness as

$$\lambda = \text{exciting force} \times \frac{Q}{\text{resonance amplitude}} \tag{10}$$

The elementary equivalent vibratory system is now acted on by the force element dP_x and hence the equation of motion for the mode of vibration is found to be

$$m\ddot{x} + \rho\dot{x} + \lambda x = - dP_x \tag{11}$$

substituting equation (7) in equation (11)

$$\frac{d^2x}{dt^2} + \omega_0^2 \left(\frac{1}{Q\omega_0} + \frac{\nu k_1}{\lambda} F_2 + \frac{K}{\lambda} (\nu - \frac{s_0}{2\Pi R}) \frac{2\Pi}{\Omega} + \frac{k_\Omega}{\lambda} \frac{1}{R}\right) \frac{dx}{dt}$$

$$+ \omega_0^2 \left(1 + \frac{\nu k_1}{\lambda} F_1\right) x = 0 \tag{12}$$

under stable conditions x(t) is a damped vibration i.e. $\delta < 0$ in equation (6). This is so when the damping factor of equation (12) (i.e. the factor of $\frac{dx}{dt}$ term) is positive. The disturbance x(t) will increase if the damping factor has a negative value i.e. ($\delta > 0$) and the cutting process is then unstable.

Now we know that $\delta = \frac{b}{2a}$ from the characteristic equation $aP^2 + bP + c = 0$ so the factor of the velocity dependent term is equal to -2δ

The sign of the factor -2δ

$$-2\delta = \omega_0^2\left(\frac{1}{Q\omega_0} + \frac{\nu k_1}{\lambda} F_2 + \frac{k}{\lambda} (\nu - \frac{s_0}{2\Pi R}) \frac{2\Pi}{\Omega} + \frac{k_\Omega}{\lambda} \frac{1}{R}\right) \begin{array}{l} > 0 \text{ stable} \\ < 0 \text{ unstable} \end{array} \tag{13}$$

This factor, also contains the frequency of the disturbance ω (chatter frequency) and its sign cannot be found without ω. The disturbing frequency is given by the coefficients of the third part of equation (12)

$$\omega^2 = \omega_0^2 \left(1 + \frac{\nu k_1}{\lambda} F_1\right) \tag{14}$$

equation (13) and (14) are the stability conditions for the mode of vibration.

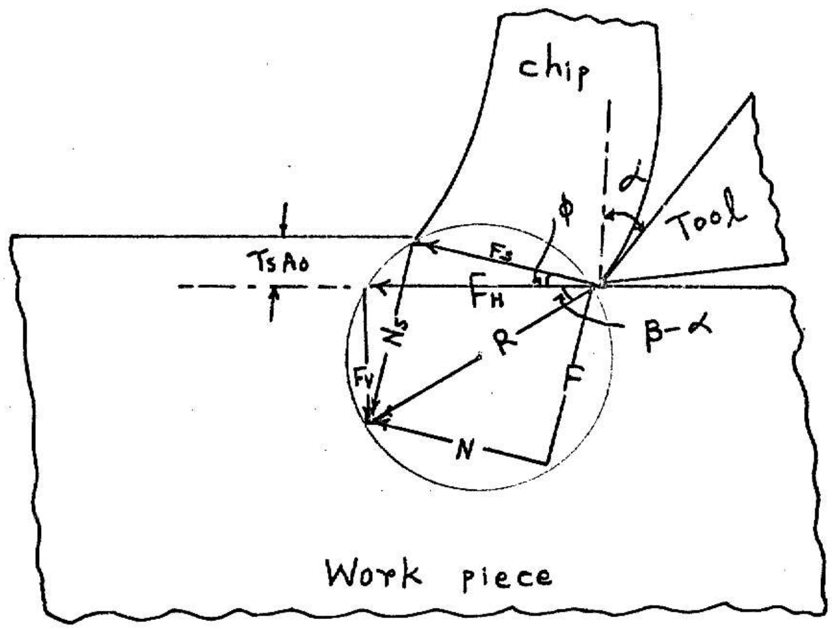
CHAPTER 4

Force Analysis of the Machine Tool

A theoretical method for estimating the cutting forces without the help of any force measuring dynamometer has long been one of the focal points of research in metal cutting. Before analyzing the forces acting on the surface of the tool during the metal cutting process it is essential to see how the mechanism of plastic deformation of metal in the orthogonal cutting process can be explained by considering the following diagram.

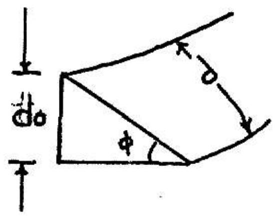
The metal removed from the surface of the work is in the form of continuous chips. It has been found that most of the plastic deformation takes place along a narrow band which extends from the cutting edge to the workpiece surface.

This narrow band is the shear zone, since the deformation in this region is principally by shear. The chip, after separation from the workpiece by the cutting edge, slides over the rake face of the tool. During this event the chip may undergo some additional deformation which may be called secondary flow and finally curls away from the tool, thereby breaking contact with it. The plastic deformation process of metal in the vicinity of the tool edge is very complex. Consequently certain simplifications must be made for the analytical approach to the mechanics of chip formation.



Consider the state of stress of a point on the shear surface for which a partial theoretical analysis is possible. In the simplified cutting process the tool removes the layer d_0 from the workpiece and the strain in the deformation process is assumed to be only by shear along the shear surface. The shear mechanism along the shear surface is assumed to be similar to a stack of cards gliding one over the other as the undeformed chip of thickness d_0 transforms into the chip of thickness d. The shear stress therefore is a maximum and the shear surface which is a line in the two dimensional picture is coincident with a slip line.

Equilibrium of the forces associated with metal cutting may be obtained by considering the chip as the free body as shown in the above figure. In this diagram the sizes of undeformed chip thickness are related to the magnitudes of forces by taking the scale of $\tau_s A_0$ for the distance corresponding to the undeformed chip thickness. Thus the vector of the shearing force coincides with the shear plane and the force circle passes through the end points of the shear plane where F_H and F_V are force components in the directions parallel and perpendicular to the motion of the tool respectively, τ_s is the shearing stress on the shear plane, A_0 is the cross-sectional area of the undeformed chip thickness and ϕ, β and α are the shear angle, friction angle and rake angle respectively



Considering unit length in the transverse direction

$$A_0 = d_0 \times 1$$

and

$$A = d \times 1$$

where A is the area in the shear plane.

Considering the shear angle ϕ in the above triangle, we know the basic trigonometrical result

$$\sin \phi = \frac{d_0}{d}$$

But from the above equation we can say that

$$\sin \phi = \frac{A_0}{A}$$

so

$$A = \frac{A_0}{\sin \phi}$$

Analyzing the force diagram, shear force F_s = shear stress x area

$$F_s = \tau_s \times \frac{A_0}{\sin \phi} \tag{1}$$

Resolving in horizontal direction

$$F_H = R \cos (\beta-\alpha) \tag{2}$$

We also know from the diagram that

$$F_s = R \cos (\phi+\beta-\alpha) \tag{3}$$

From equation (1) and (3)

$$R = \frac{\tau_s \times A_0}{\sin \phi \cos (\phi+\beta-\alpha)}$$

substituting for R in equation (2)

we get

$$F_H = \frac{\tau_s \times A_0 \cos (\beta-\alpha)}{\sin \phi \cos (\phi+\beta-\alpha)} \tag{4}$$

Resolving in vertical direction

$$F_v = R \sin (\beta - \alpha)$$

substituting for R

we get

$$F_v = \frac{\tau_s \times A_0 \sin (\beta-\alpha)}{\sin \phi \cos (\phi+\beta-\alpha)} \tag{5}$$

Several shear angle relationships have been developed by various research workers for evaluation of the trigonometrical term in equations (4) and (5). The most important shear angle relationships are the following:

Ernst and Merchant

$$2\phi + \beta - \alpha = \Pi/2 \tag{6}$$

Lee and Shaffer

$$\phi + \beta - \alpha = \Pi/4 \tag{7}$$

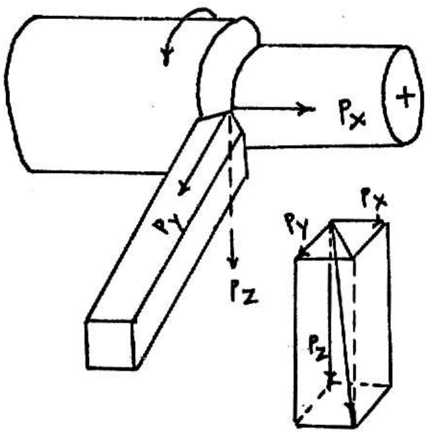
Loladze

$$\tan (\phi+\beta-\alpha) = 1 + (\phi-\alpha) + \frac{1}{2} \sin 2(\phi-\alpha) \tag{8}$$

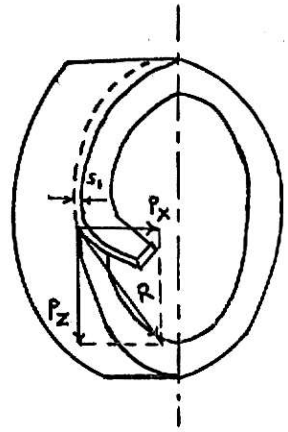
Of these Laladze's equation gives consistently correct results for all materials.

Forces on Tool in Lathe Turning

Application of the above analysis can be made by theoretically estimating the forces acting on the tool in lathe turning operations.



Oblique



Orthogonal

Referring to the above figure, the resulting load at the cutting edge can be conveniently resolved into three co-ordinates. In a conventional lathe turning operation the three components expressed as coordinates are

P_z = the tangential force in the direction of the surface velocity of the workpiece.

P_x = the feed force in the direction opposite to the feed motion.

P_y = the radial force perpendicular to P_x and P_z, acting along the tool axis.

The tangential cutting force P_z is estimated from equation (4) which was derived for orthogonal cutting conditions.

τ_s - shear stress

A_0 - $s_1 \cdot d$

s_1 - feed in in/rev.

d - depth of cut in inch

so the formula becomes

$$P_z = \tau_s \cdot s_1 \cdot d \cdot \frac{\cos (\beta-\alpha)}{\sin \phi \ \cos (\phi+\beta-\alpha)} \tag{9}$$

Since the cutting process is oblique equation (9) cannot be used for determining the tangential force during the actual cutting process. Loladze, Lee and Shaffer suggested the modification of equation (9) as shown below, which is valid for oblique cutting process.

$$P_z = \tau_s \cdot s_1 \cdot d \cdot \frac{\cos (\beta-\gamma_e)}{\sin \phi_n \ \cos (\phi_n+\beta-\gamma_e)} \tag{10}$$

where $\gamma_e = \sin^{-1} [\sin \alpha \cos^2 (\lambda_1+\psi') + \sin^2 (\lambda_1+\psi')]$ (11)

λ_1 = Inclination angle

$$\psi' = \tan^{-1} \frac{\sin (\theta+\phi_1)}{(\frac{2d}{s} \sin \theta) + \cos (\theta+\phi_1)} \tag{12}$$

ψ' - chip deviation from normal to cutting edge.

ψ' is affected by feed s_1 and depth of cut and so the true rake angle γ_e also becomes a function of feed s_1 and depth of cut d_1 and so they vary as feed and depth of cut vary.

$$\tan \phi_n = \frac{\cos \gamma_e}{\zeta_n - \sin \gamma_e} \tag{13}$$

ζ_n - normal chip reduction coefficient $= \frac{d}{d_0}$

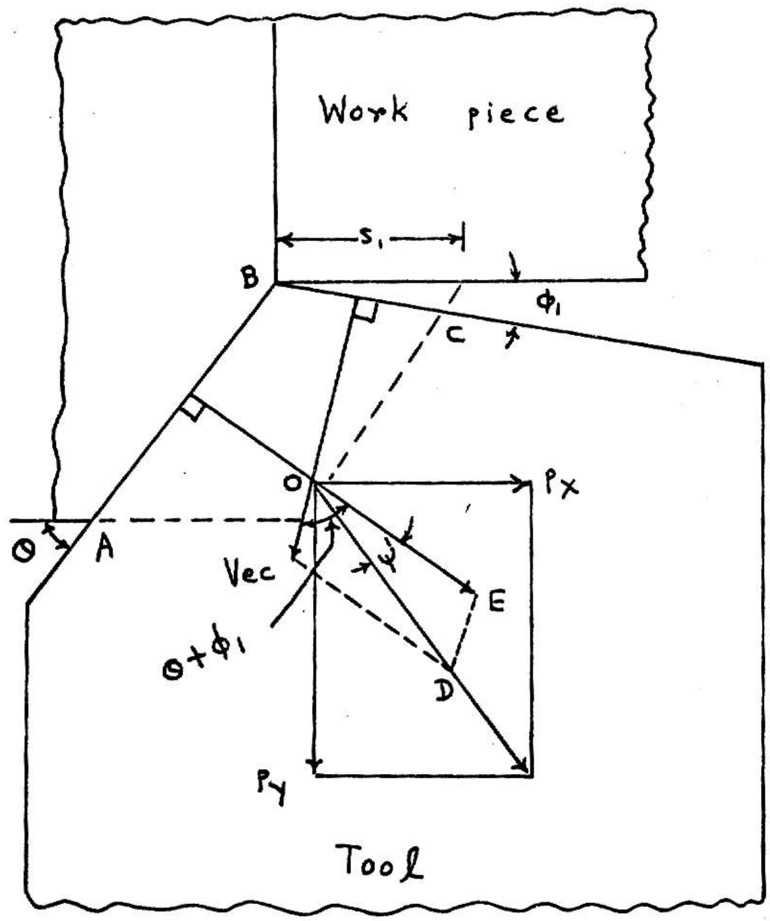
ϕ_n - normal shear angle.

This figure will explain all the terms used in all the empirical relations shown in the above equations. These are the empirical relations verified by the experimentation.

s_1 = feed in. per rev. ϕ_1 = end cutting edge angle

θ = 90° - side cutting edge angle

ψ' = chip deviation from normal to cutting edge

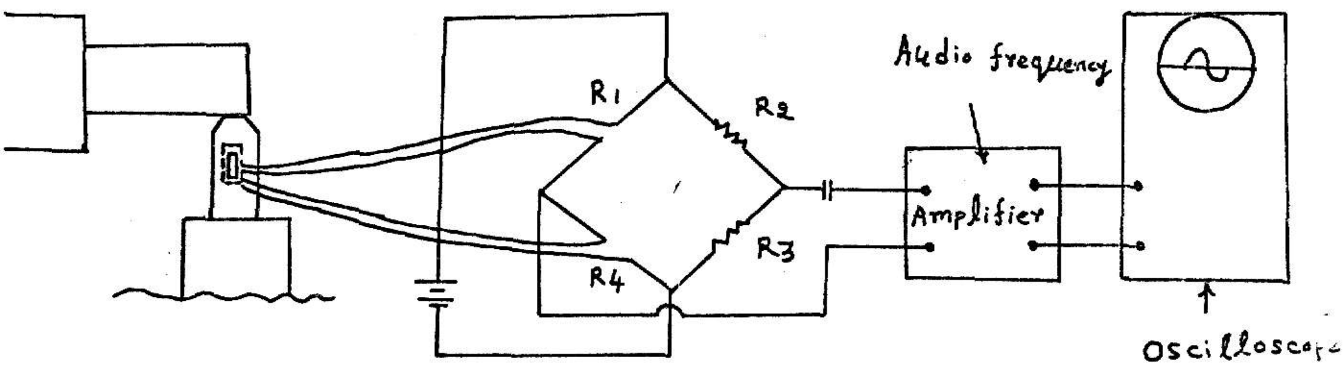


Work piece
s_1
B
ϕ_1
C
O
P_x
θ
A
Vec
ψ
E
$\theta+\phi_1$
D
P_y
Tool

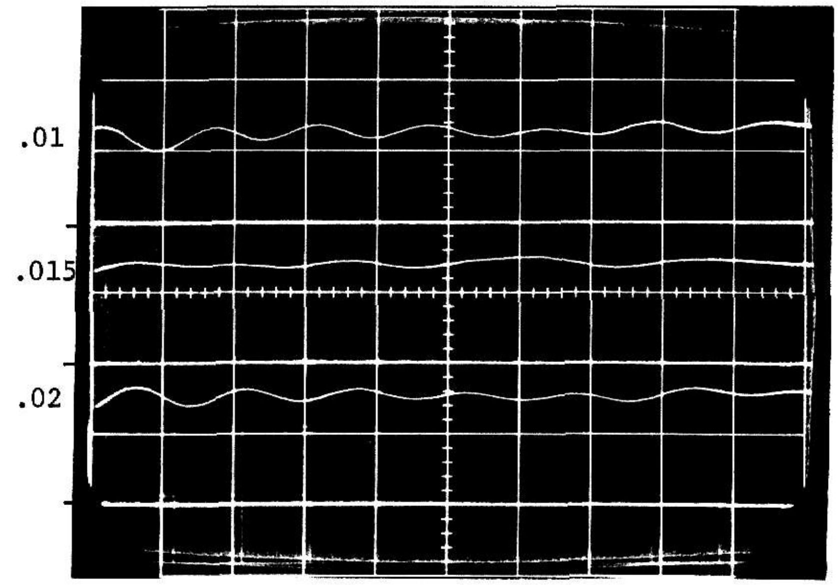
CHAPTER 5

Experimental Details

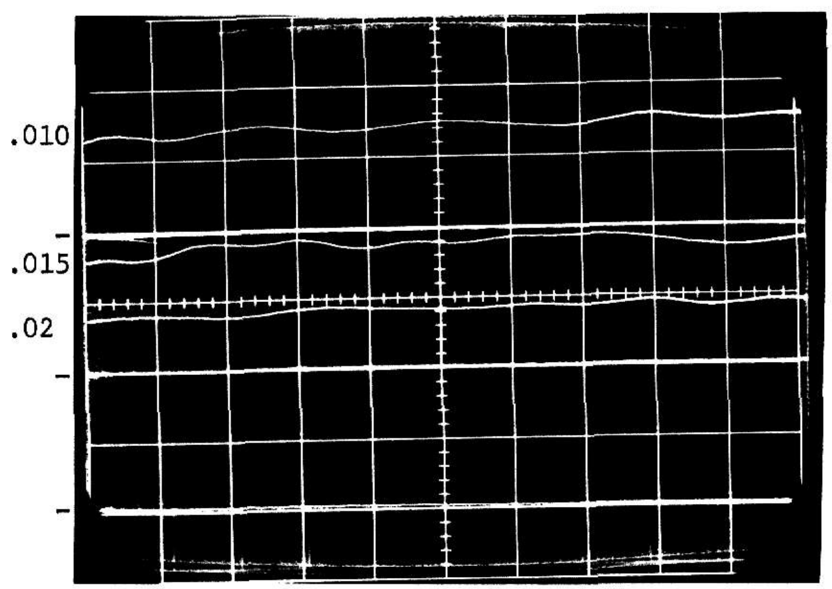
The experiment was conducted with a single point tool which was designed on the basis of maximum expected force on the tool during the cutting process. The force was measured at various cutting conditions by transducers fixed on the tool. The signals from the transducers were fed to the oscilloscope and variation of force as indicated on the oscilloscope was photographed using a polaroid camera. The experimental set up is shown schematically in the figure



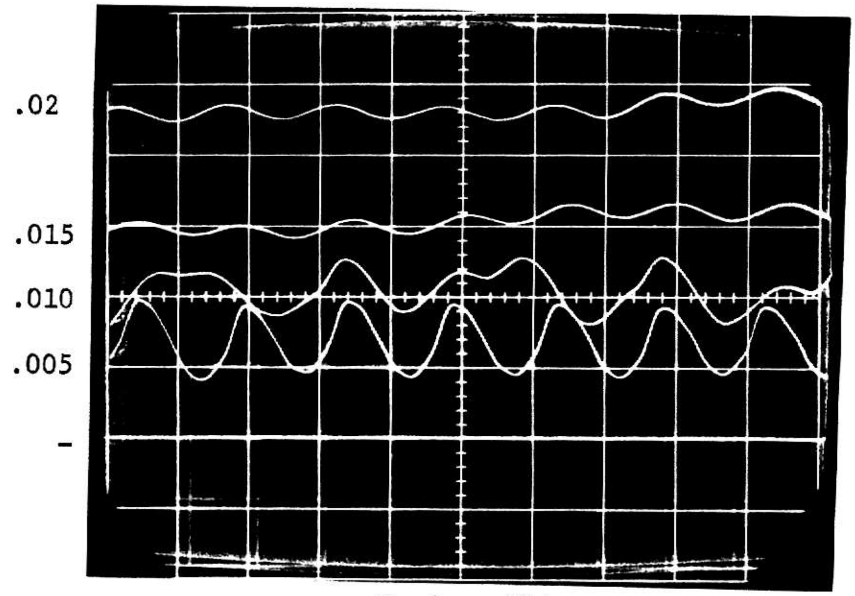
The various cutting conditions were obtained by changing feed and depth of cut while r.p.m. of machine was kept constant. The force variation at various cutting conditions as obtained experimentally was determined by taking the average of force variation during cutting process. The results of the same are given in the table. The theoretical force was also calculated by using equation (10). The theoretical



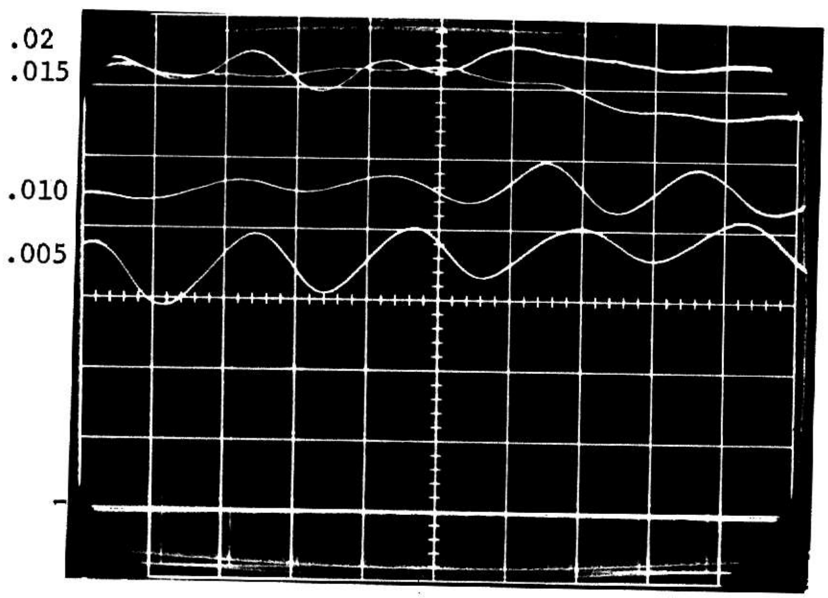
Feed = .006



Feed = .012



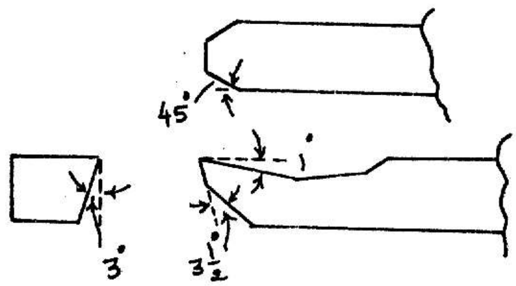
Feed = .024



Feed = .0421

force obtained from the use of the formula is also given in the table. The details of the tool and specimen and instrumentation is given below.

Three views of tool indicating various angles on the tool



Tool specifications: -

End relief angle - 3° material - tool steel

front clearance angle - $3\frac{1}{2}^{\circ}$

rake angle - 1°

side cutting edge angle c - 45°

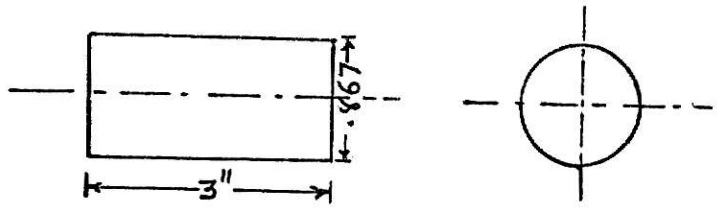
$\theta = 45° = 90° - c$

$\lambda = 0$

$\phi_1 = 0$

Work piece specifications: -

material - mild steel, ultimate shearstress - 78000 psi



Instrumentation:-

strain gage specification:-

type - BUDD METALFILM 120Ω

gage factor - 2.08 ± 1/2 %

Oscilloscope specification:-

TEKTRONIX TYPE 564 STORATE OSCILLOSCOPE

Amplifier specification:-

TYPE 3C66 CARRIER AMPLIFIER

Calibration process:- A known load was applied on the tool tip by a bucket half full of sand with the use of rope and pointed bar. The oscilloscope was calibrated in that way initially. The calibration result was 1 Div = 25 lbs. at 50 μ strain div.

The time base was .5 millisecond/division

The results of the experimental work and theoretical work are given in the tabular form.

Theoretical versus experimental results

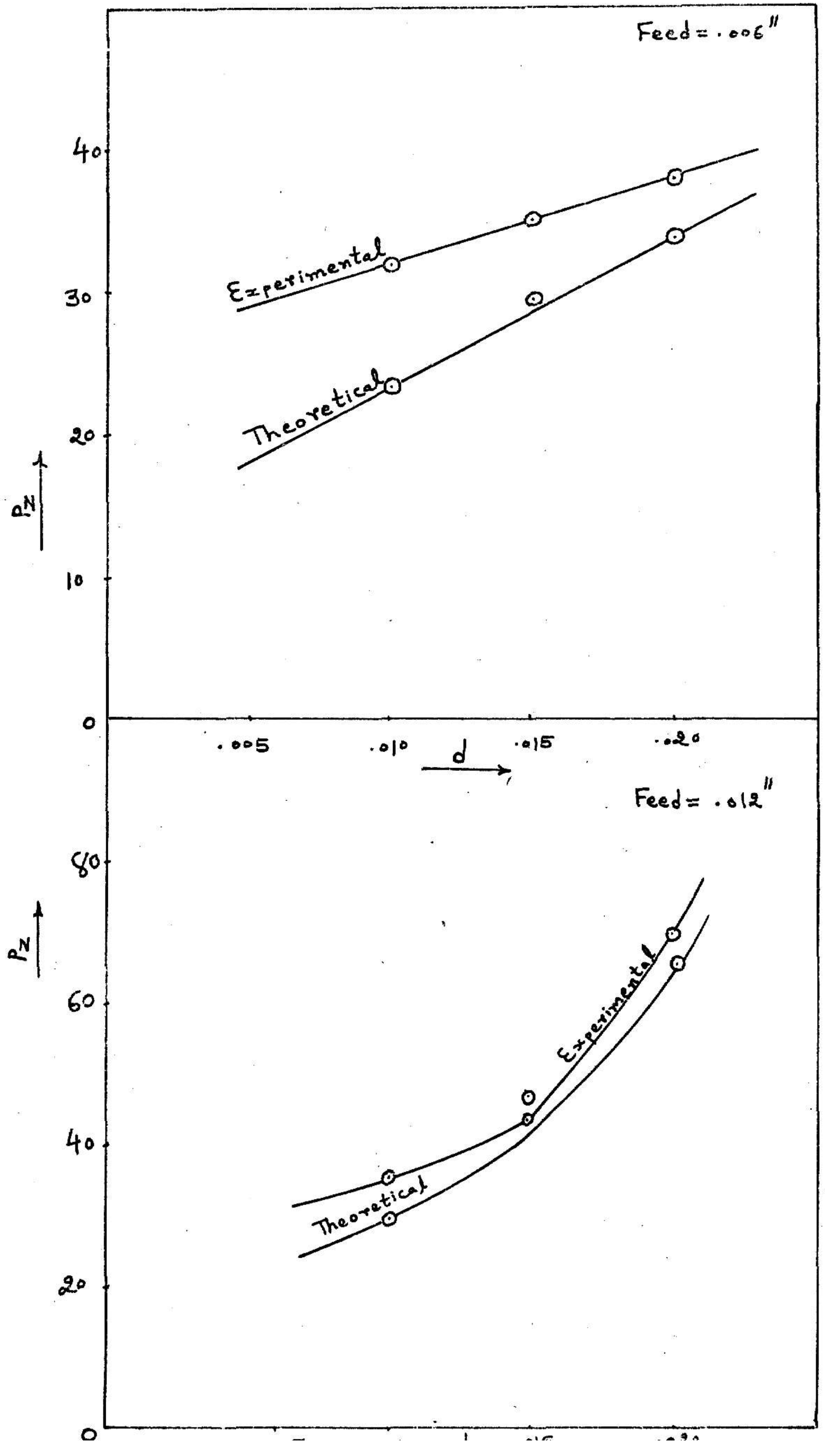
Sr.	Feed	Depth of cut	ζ_n	ϕ_n	$\beta-\gamma_e$	$\phi_n+\beta-\gamma_e$	k_1	P_z experimental	P_z theoretical
1	.006	.020	2.57	21.5	23.5	45	3.538	38.63	34.1156
2	"	.015	2.46	16.4	28.6	"	4.39	35.61	29.71
3	"	.010	2.38	17.6	27.4	"	4.15	32.08	23.210
4	.012	.020	2.61	21.5	23.5	"	3.53	70.8	66.08
5	"	.015	2.47	22.7	22.3	"	3.39	43.21	47.59
6	"	.010	2.31	24.5	20.5	"	3.19	35.77	29.85
7	.024	.020	2.41	23.7	21.3	"	3.068	116.12	114.86
8	"	.015	1.915	29.5	15.5	"	2.76	76.86	77.50
9	"	.010	1.76	32.5	12.5	"	2.57	52.52	48.11
10	"	.005	1.58	27.1	8.1	"	2.32	34.05	21.71
11	.0421	.020	2.25	25.7	18.3	"	3.09	158.3	172.93
12	"	.015	2.03	28.7	16.3	"	2.82	151.28	138.90
13	"	.010	1.92	30.6	14.4	"	2.69	114.31	88.31
14	"	.005	1.80	33.2	11.8	"	2.12	91.05	41.54

<u>Conclusion</u>: - Comparison of calculated results with experimental results shows that the computations are accurate within $\pm$ 20% for the tangential force P_z. The results are absurd at two to three places that may be considered as experimental error. The chip reduction coefficient goes on decreasing as we decrease the depth of cut or increase the feed.

The graph is drawn

cutting force vs depth of cut.

The results of experimental and theoretical cutting force are shown in the graphs.


Feed = .006"
40
30
20
10
0
P_N
Experimental
Theoretical
.005
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Feed = .012"
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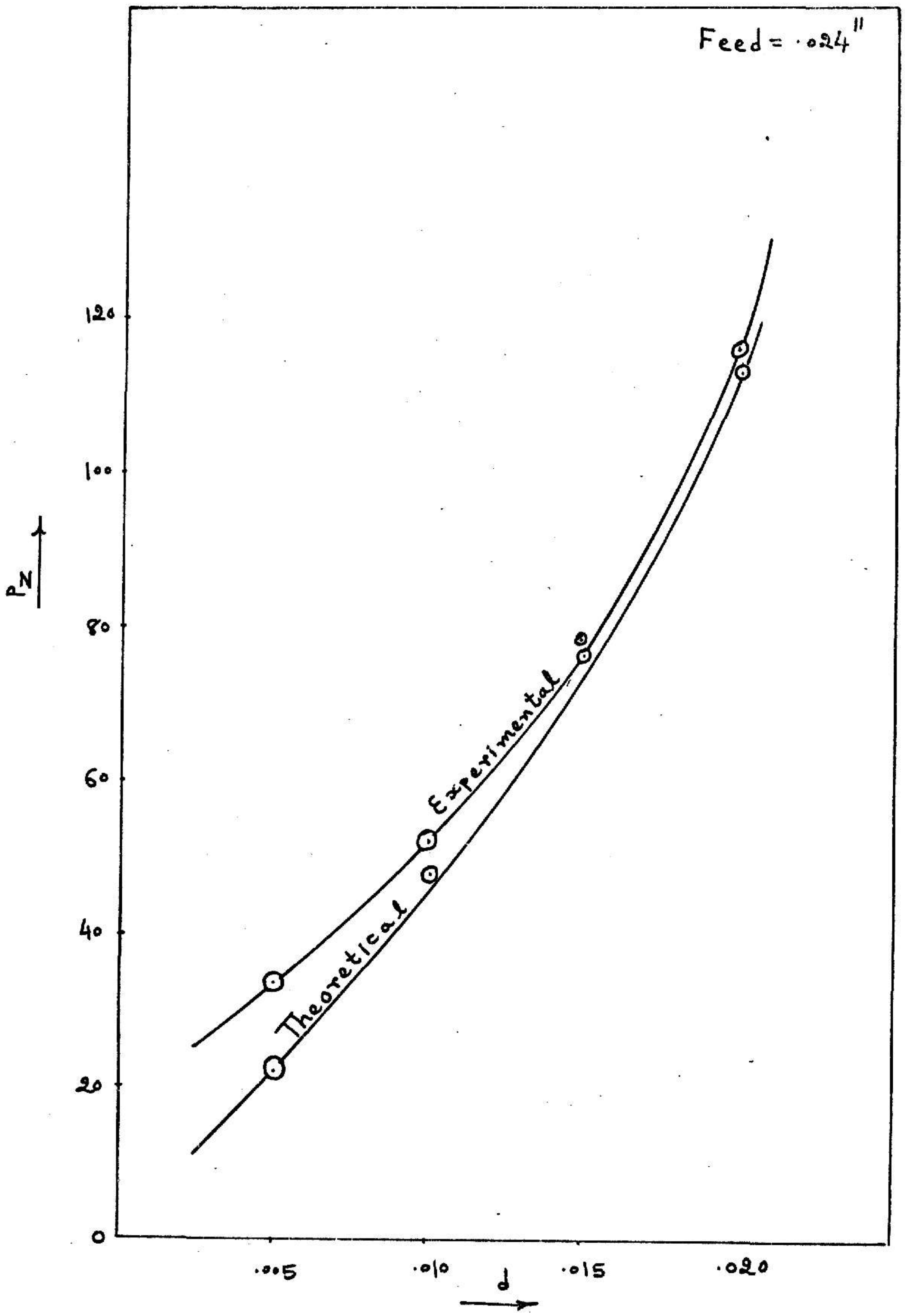
P_Z
Experimental
Theoretical


Feed = ·024"
Experimental
Theoretical
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80
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0
·005
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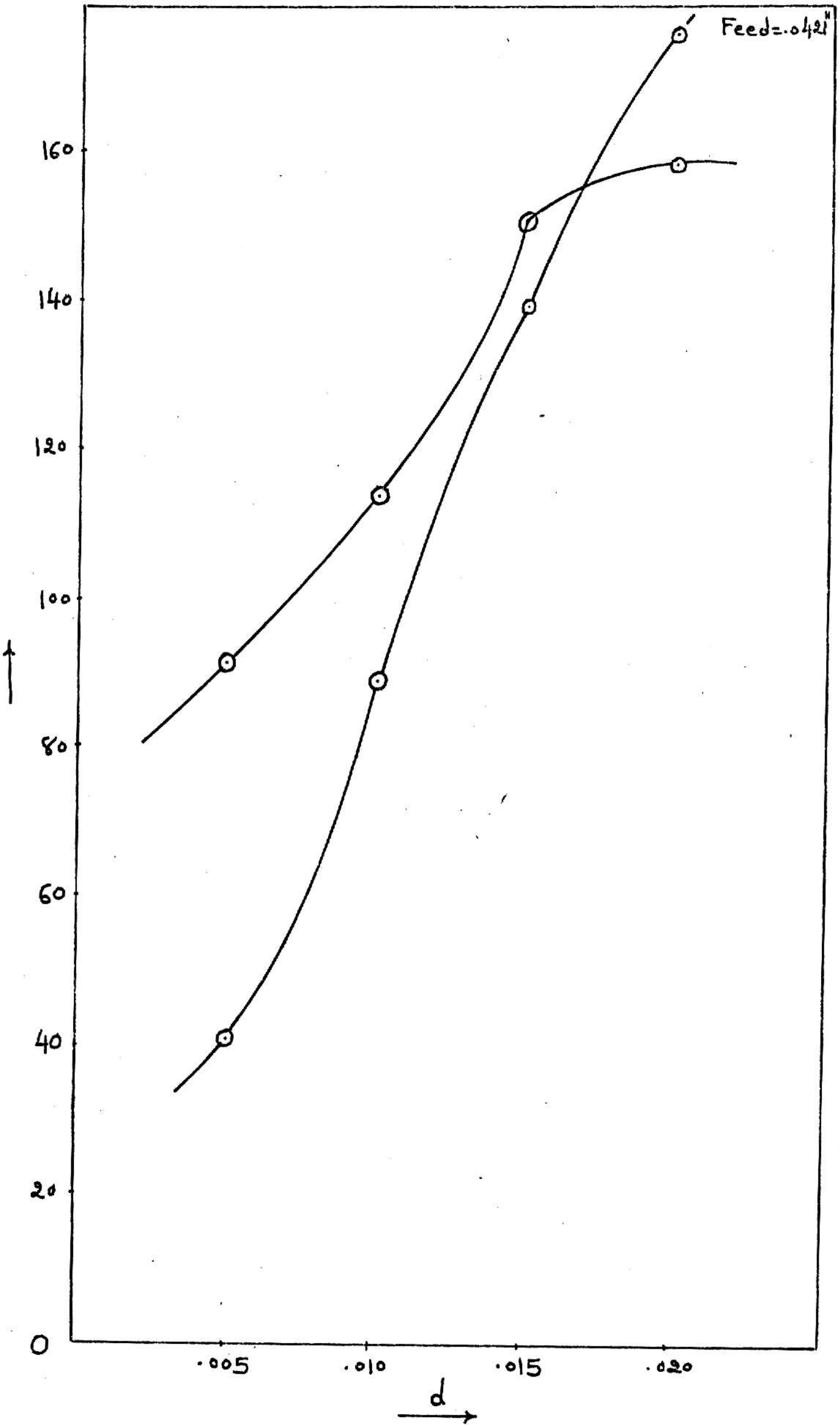
d
P_N



Feed=.0421"
160
140
120
100
80
60
40
20
0
.005
.010
.015
.020
d

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VIBRATION IN MACHINE TOOLS

by

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Ahmedabad, India, 1966

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1969

In the past the machine tool industry faced great difficulty in controlling the problem of vibration in machine tools. It was mainly due to a lack of knowledge of exact analysis of mechanics of chip formation in the cutting process. The causes of vibration in machine tools was recognized long ago but an analytic approach to predict the exact nature of vibration was not known until 1942 when Ernst and Merchant gave the theory of mechanics of chip formation and an analytic approach to find the forces acting on the tool during the cutting process.

The author has given in this report, the analysis of forces acting on the machine tools during the cutting process and it is also discussed how this analysis helps in analyzing the vibration in machine tools. Both physical and mathematical models were discussed. Theoretical analysis of forces were verified experimentally and the results of the experiment verify the close relationship with the theoretical analysis.

This force analysis could give exact information to know the frequency of vibration, if other parameters e.g. damping coefficient and spring constant of the machine tool were known. These parameters were discussed while formulation of the mathematical model of the vibration system.