Conflict, the paradox of power, and income redistribution: a game-theoretic analysis

by

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Abstract

In this paper, we present a conflict perspective on income inequality by analyzing issues on the *Paradox of Power* as addressed by Hirshleifer. Specifically, we use a standard model of conflict to analyze the incentives of fighting between two parties with income disparity when conflict's destructiveness is an endogenous function of fighting effort. We find that when destructiveness is high, the more-endowed and the less-endowed individuals are better off by reducing the costly fighting. But when the destructiveness is low, fighting for more resources becomes more severe across different parties with income disparity. We further look at how income re-distribution policies affect each party's incentives for fighting under the shadow of conflict.

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Dedication

To Mom, Dad, Ian, Chloe, Monte, and Opal.

Chapter 1 - Introduction

Can conflict serve as an "income equalizer" in an economy with initial wealth disparities across individuals or ethnic groups? In characterizing an optimal allocation of limited resources between production and appropriation, Hirshleifer (1991a) shows that initially less-endowed contenders tend to fight harder (by allocating proportionally more effort to appropriative activities) and hence reduce their initial income disparities relative to the more-endowed opponents when the "decisiveness parameter" of conflict increases. This fact is what Hirshleifer called the 'Paradox of Power' (POP). That is, poorer groups often improve their position relative to their richer counterparts by the means of fighting.

The body of research on issues related to conflict and the distribution of power through fighting is extensive. Hirshleifer (1988) provides a general-equilibrium model of conflict between groups that contrasts with earlier economic theory by allowing for hostile interactions in equilibrium. Hirshleifer (1991a) expands upon this general-equilibrium model of conflict between groups by formally introducing the notion of the POP. Further research on the Paradox of Power comes from Durham, Hirshleifer, and Smith (1998). The authors show that the theoretical predictions of the POP (that conflict constitutes as a mechanism for reducing or eliminating income disparity) are broadly supported by experimental evidence.

In this paper, we present a conflict perspective on income inequality by re-examining issues on the Paradox of Power as originally addressed by Hirshleifer. Specifically, we introduce endogenous destruction into a standard model of conflict to analyze the incentives of fighting between two parties with income disparity. Our research interest goes beyond showing the presence of the Paradox of Power when conflict is destructive and conflict technology takes the difference form. Several questions we wish to analyze include the following: What are the incentives of conflicting parties to fight when costly fighting is destructive as an endogenous and increasing function of their fighting efforts? How would the level of decisiveness in conflict affect their incentives to fight when fighting is destructive? How would their incentives be affected by actions undertaken by public policy-makers? These actions include a redistribution of income by taxing the high-income party and then transferring the taxes to the low-income party.

Destruction as a consequence of conflict is a topic that has not been extensively studied until recently. Grossman and Kim (1995) develop a general equilibrium model for the allocation of scarce resources between appropriative and productive activities in the presence of destruction. Garfinkel and Skaperdas (2000) examine how parties engaged in conflictual relations will choose whether to fight or to reach a peaceful settlement in equilibrium depending on the level of destruction. Chang and Luo (2017) analyzes bargaining and equilibrium outcomes when endogenous destruction exists. To our knowledge, however, no research has been conducted specifically related to the existence of the Paradox of Power when conflict is destructive.

In conflict literature, it is standard to model the technology of conflict through a Contest Success Function (CSF). These CSFs have been employed in a wide variety of contexts to analyze economic issues, including: rent-seeking behavior (e.g., Tullock 1980; Nitzan 1991), third-party interventions in conflict (e.g., Chang, Potter, and Sanders 2007; Chang and Sanders 2009) and even competitive sports (e.g., Fort and Winfree 2009). Hirshleifer (1991b) gives a comprehensive overview of the various functional forms and analytic properties of contest success functions. Meanwhile, Skaperdas (1996) axiomatizes both the ratio-form CSF and difference-form CSF. Ratio-form contest success functions are employed when the outcome of conflict is determined by the ratio each parties' combative effort (e.g., Hirshleifer 1991a; Chang and Sanders 2009; Spolaore 2008). By contrast, difference-form contest success functions are used when the outcome of conflict is determined by the difference in effort between each parties' combative effort (e.g. Hirshleifer 1988, 1989; Baik 1998).

In choosing a functional form for the contest success function used in this paper, we turn to recent developments in the empirical conflict literature. Jia, Skaperdas, and Vaidya (2013) provide an overview of the significant econometric issues surrounding the empirical estimation of contest success functions (namely, the availability of data and endogeneity concerns). Hwang (2012) uses battle data from both 17th-century European and World War II battles to estimate CSF parameters. For 17th-century European battles, a ratio-form CSF better described the outcomes of conflict, while for WWII a difference-form CSF provided a better fit for the data. Finally, Mildenberger and Pietri (2018) compares different forms of contest success functions for conflict in the virtual world *Eve Online* and found both ratio-form and difference-form CSFs are outperformed by the 'relative difference' CSF (where differences in combative efforts are scaled by the size of the conflict) as proposed by Beviá and Corchón (2015). It is clear that the question of which functional form of CSF is appropriate when describing conflict is dependent on the nature of the conflict itself. In some cases, a ratio-form CSF may be suitable, while in others a difference-form CSF may be better. Because there is no 'correct' functional form for the CSF, in our model we opt to consider conflict when it is characterized by a difference-form contest success function. However, many of the results we derive are robust to changes in the functional form of the CSF.

The idea that wealthy groups may seek to maintain their power by enacting policies to hurt low-income groups is supported by Sonin (2003). In an economy where property rights are poorly protected, wealthy groups will invest in private protection of property rights and are incentivized to engage in rent-seeking activities to maintain the status-quo, depriving low-income groups of property rights and driving low growth rates and high levels of income inequality. A modern example of this type of “bad equilibrium” is the Russian oligarchs of the 1990s. Conditions under which low-income groups may have incentive to combat income inequality is addressed in Falkinger (1999). The author shows that the stability of wealth distributions between high-income and low-income socio-economic groups vary across stages of economic development. In developing economies there may exist no stable wealth distribution, while in developed economies a stable wealth distribution exists. This result may help explain why we see high income countries engage in redistributive policies in order to achieve social stabilization.

Conflict is not an activity that is reserved exclusively for interactions between groups, however. For example, countries often face internal strife while also having to engage foreign adversaries and companies regularly must negotiate with worker’s unions while competing in an open marketplace. This distinction between internal and external conflict is addressed in Cruz and Torrens (2019), which develops a model of conflict that finds the POP holds *within* groups but not necessarily *across* groups.

We summarize the key findings of our analysis as follows. (i) When fighting's destructiveness is high, the more-endowed and the less-endowed individuals are better off by allocating less of their resources to fighting. But when the destructiveness is low, fighting for more resources becomes more severe across different parties with income disparity. (ii) Our analysis indicates that efforts by policy-makers to deal with initial income disparities are met with mixed results. While a redistributive income policy (such as a lump-sum or proportional tax) will cause the low-income group to allocate relatively less of its endowed resource to fighting, the high-income group allocates relatively *more* of its endowed resource to fighting. Consequently, the overall fighting effort may remain the same in equilibrium.

The remainder of this paper proceeds as follows. In section 2, we incorporate destruction into an analytical framework two-party conflict when fighting's destructiveness is endogenous. In Section 3, we first derive the equilibrium levels of fighting effort, productive effort, and each party's expected income after fighting. We then analyze how each party's incentive for fighting changes in respond to changes in destruction, the decisiveness of conflict, and various public policy actions. Finally, Section 4 concludes our work.

Chapter 2 - The Analytical Framework

Following the framework set forth by Hirshleifer (1991a, 1995), we consider two players $(i = 1, 2)$ engaging in conflictual interaction. Player i is endowed with an exogenously determined resource R_i, which is divided between productive effort E_i and fighting effort F_i. That is, $R_i = E_i + F_i$. Prior to fighting, we assume player 1 is the wealthier player ($R_1 > R_2 > 0$). Each side is fighting to obtain a share of the total civilian good available for consumption, I, given by:

$$I = E_1 + E_2. \tag{1}$$

This expression can be rewritten in terms of the initial resource R_i and fighting effort F_i to show the total civilian good available for consumption before fighting:

$$I = R_1 + R_2 - F_1 - F_2 \tag{2}$$

As in the literature, conflict technology or the outcome of fighting is categorized by a Contest Success Function (CSF).[1] This function takes combative efforts F_1 and F_2 as inputs and returns a probability p_i as the output. This probability p_i determines the proportion of the total civilian good that each player obtains after fighting.

Broadly speaking, there are two types of contest success functions: (1) ratio form[2] and (2) difference form. We choose to use the difference form. In difference-form CSFs, the outcome of

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[1] We use a canonical "contest success function" (CSF) to reflect the technology of conflict (see, e.g., Tullock 1980; Hirshleifer 1989; Skaperdas 1996).

[2] In ratio-form contest success functions, the outcome of conflict depends on the *ratio* of the two parties' fighting efforts F_1 and F_2. So for a given party i and combative efforts F_1 and F_2, the ratio-form CSF is:

$$p_i = \frac{F_i^m}{F_1^m + F_2^m} \text{ for } i = 1, 2.$$

conflict depends on the *difference* between the two parties' fighting efforts F_1 and F_2. For party 1, the probability of victory in fighting is specified as:

$$p_1 = \frac{e^{mF_1}}{e^{mF_1} + e^{mF_2}} = \frac{1}{1 + e^{m(F_2 - F_1)}}. \tag{3a}$$

where $m(>0)$ represents the *mass effect parameter* that determines the level of *decisiveness* of conflict. Similarly, for party 2:

$$p_2 = \frac{e^{mF_2}}{e^{mF_1} + e^{mF_2}} = \frac{1}{1 + e^{m(F_1 - F_2)}}. \tag{3b}$$

Unlike the analysis in Hirshleifer (1991a, 1995), we take into account the existence of *conflict-related destruction*. We assume when two players engage in conflictual interaction, the overall destruction of the civilian good depends on their fighting effort. For analytical simplicity, total destruction (*D*) is taken to be a linear function of the combative efforts F_1 and F_2. That is,

$$D = d(F_1 + F_2), \tag{4}$$

where d represents the (constant) marginal cost of fighting. If the allocation of resources R_i from productive effort E_i to fighting effort F_i is treated as the *direct* cost of conflict, then the conflict-related destruction can be considered as the *indirect* cost of fighting.[3]

[3] For studies on conflict that takes into account the endogeneity of destruction costs see, e.g., Sanders and Walia (2014), and Chang, Sanders, and Walia (2015), and Chang and Luo (2013, 2017).

Chapter 3 - Equilibrium Outcomes and Their Implications

Conflict in our model is a simultaneous-move game. Each player chooses an optimal level of fighting effort F_i (and hence productive effort E_i) to maximize his/her share of the total civilian good available for consumption I, given the initial resource constraint R_i and the opposing player's optimal allocation of fighting effort and productive effort. For player 1 this optimization problem is:

$$\text{Max } I_1 = p_1\left(E_1 + E_2 - d(F_1 + F_2)\right) \text{ subject to } E_1 + F_1 = R_1. \tag{5a}$$

Similarly, player 2's optimization problem is:

$$\text{Max } I_2 = p_2\left(E_1 + E_2 - d(F_1 + F_2)\right) \text{ subject to } E_2 + F_2 = R_2. \tag{5b}$$

Each of these optimization problems can be rewritten by plugging the resource constraint directly into the objective function.

Player 1's optimization problem becomes:

$$\text{Max } I_1 = \frac{1}{1+e^{m(F_2-F_1)}}(R_1 + R_2 - F_1 - F_2 - d(F_1 + F_2)). \tag{6a}$$

Player 2's optimization problem, as with player 1, becomes:

$$\text{Max } I_2 = \frac{1}{1+e^{m(F_1-F_2)}}(R_1 + R_2 - F_1 - F_2 - d(F_1 + F_2)). \tag{6b}$$

Using standard optimization techniques, we have the first-order conditions (FOCs) for the two parties:

$$\frac{\partial I_1}{\partial F_1} = \frac{(-1-d)(1+e^{m(F_2-F_1)}) - (R_1 + R_2 - F_1 - F_2 - d(F_1 + F_2))(-me^{m(F_2-F_1)})}{(1+e^{m(F_2-F_1)})^2} = 0, \tag{7a}$$

$$\frac{\partial I_2}{\partial F_2} = \frac{(-1-d)(1+e^{m(F_1-F_2)}) - (R_1 + R_2 - F_1 - F_2 - d(F_1 + F_2))(-me^{m(F_1-F_2)})}{(1+e^{m(F_1-F_2)})^2} = 0. \tag{7b}$$

From these FOCs, we obtain the two parties' reaction curves.

$$\text{RC1: } (1+d)(1+e^{m(F_2-F_1)}) = (R_1+R_2-F_1-F_2-d(F_1+F_2))(me^{m(F_2-F_1)}) \tag{8a}$$

$$\text{RC2: } (1+d)(1+e^{m(F_1-F_2)}) = (R_1+R_2-F_1-F_2-d(F_1+F_2))(me^{m(F_1-F_2)})\,. \tag{8b}$$

It is important to note that these reaction curves are only valid in the cases where *interior* solutions exist. That is, $R_1 > F_1$ and $R_2 > F_2$.[4] When an interior solution exists, it is easy to verify that $F_1 = F_2 = F$[5] in equilibrium. Plugging this condition into RC1 and RC2, we solve for the equilibrium value of F. It follows from RC1 and RC2 in (8) that

$$(1+d)(1+e^{m(F-F)}) = (R_1+R_2-F-F-d(F+F))(me^{m(F-F)}) \tag{9a}$$

$$(1+d)(2) = (R_1+R_2-2F-2dF)(m) \tag{9b}$$

both of which imply that

$$F = \frac{m(R_1+R_2)-2(1+d)}{2m(1+d)}\,. \tag{9c}$$

It follows immediately that we have the following Lemma:

LEMMA 1: *If the destructiveness of fighting is critically high (i.e., $d \geq \frac{m(R_1+R_2)}{2}-1$), then each party's fighting effort is zero. As such, conflict is not an option of solving the income inequality problem.*

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[4] In this paper we will not consider the cases where *corner* solutions exist (i.e. $R_1 > F_1$ and $R_2 = F_2$).

[5] Dividing (8a) by (8b) we get: $\frac{(1+e^{m(F_2-F_1)})}{(1+e^{m(F_1-F_2)})} = \frac{e^{m(F_2-F_1)}}{e^{m(F_1-F_2)}} \Longrightarrow (1+e^{m(F_2-F_1)})e^{m(F_1-F_2)} = (1+e^{m(F_1-F_2)})e^{m(F_2-F_1)}$ Simplifying: $e^{m(F_1-F_2)}+1 = e^{m(F_2-F_1)}+1$. From this, we can see that $e^{m(F_1-F_2)} = e^{m(F_2-F_1)}$ which can be written as $\frac{e^{m(F_1)}}{e^{m(F_2)}} = \frac{e^{m(F_2)}}{e^{m(F_1)}} \Longrightarrow e^{m(2F_1)} = e^{m(2F_2)}$. This implies $F_1 = F_2 = F$.

The result in Lemma 1 indicates there does not exist an effective mechanism in distributing the total contestable civilian good between the two parties and hence income disparity is persistent. We rule out this case by assuming $0 < d < \frac{m(R_1 + R_2)}{2} - 1$ such that $F > 0$. This assumption allows us to focus on the frequently observed scenarios where two conflicting parties choose to fight for more resources (Falkinger 1999; Sonin 2003).

Fighting arises when the equilibrium value of the fighting effort is strictly positive. It follows from (9c) that $F > 0$ if, and only if,

$$(R_1 + R_2) > \frac{2(1+d)}{m}. \tag{9d}$$

We assume that this condition holds throughout the analysis. This assumption implies that total initial endowment is greater than total equilibrium income as illustrated below.

Plugging the equilibrium fighting effort F from (9c) back into each player's expected payoff functions in (6a) and (6b) yields:

$$I_1 = I_2 = \frac{1+d}{m}. \tag{10}$$

This indicates $R_1 + R_2 > I_1 + I_2$ in equilibrium.

It follows that, regardless of initial wealth disparities ($R_1 > R_2$), each player obtains the *same* level of post-conflict civilian good ($I_1 = I_2$) after fighting. This fact is the key to comprehend what Hirshleifer referred to as the *strong-form* Paradox of Power. That is, $\frac{R_1}{R_2} > \frac{I_1}{I_2} = 1$. [6] The result as shown in (10) indicates that the *strong-form* Paradox of Power holds.

Our research interest goes beyond the presence of the Paradox of Power when conflict technology takes the difference form. We wish to explore the following questions: What are the incentives of the players to fight when conflict is costly and increasing with fighting effort? How does the degree of decisiveness in conflict affect incentives to fight when fighting is destructive? How would their incentives be affected by actions undertaken by public policy-makers? These actions include a redistribution of income by taxing the high-income party and then transferring the taxes to the low-income party. In the following sections we will answer these questions.

Because each party has the same level of fighting effort in equilibrium ($F_1 = F_2 = F$) and same share of final income equilibrium ($I_1 = I_2 = I$), we will consider how these variables change as a *proportion* of initial resource. In equilibrium, player 1 chooses an optimal level of fighting effort F_1 such that:

$$\frac{F_1}{R_1} = \frac{m(R_1 + R_2) - 2(1+d)}{2m(1+d)R_1} \tag{11a}$$

and the equilibrium income-endowment ratio is:

$$\frac{I_1}{R_1} = \frac{1+d}{mR_1}. \tag{11b}$$

[6] In contrast, Hirshleifer (1991a) also examines the *weak-form* Paradox of Power which arises when conflict reduces the initial income disparity between groups, though it does not close that gap completely. This is, $\frac{R_1}{R_2} > \frac{I_1}{I_2} > 1$. Hirshleifer (1991a) shows numerical examples to illustrate this case which emerges for corner solutions. In the present study, we concentrate our analysis solely on the *strong-form* Paradox of Power.

Likewise, for player 2, we have:

$$\frac{F_2}{R_2} = \frac{m(R_1 + R_2) - 2(1+d)}{2m(1+d)R_2} \tag{11c}$$

$$\frac{I_2}{R_2} = \frac{1+d}{mR_2}. \tag{11d}$$

Since $R_1 > R_2$, it is clear that $\frac{F_2}{R_2} > \frac{F_1}{R_1}$ and $\frac{I_2}{R_2} > \frac{I_1}{R_1}$. Both players' commit the same effort towards fighting and hence receive the same share of final civilian good. However, player 2's initial resource endowment is less than player 1's, so player 2 dedicates a relatively *larger* share of their resources to fighting. Likewise, player 2's final income to initial resource ratio is also larger than player 1's.

3.1 *Changes in Incentives to Fight in Response to Changes in* d

The effect that changes in the marginal cost of destruction d have on each party's optimal level of fighting effort (as a proportion of their initial resource) can be analyzed as follows:

$$\frac{\partial}{\partial d}\left(\frac{F_1}{R_1}\right) = -\frac{R_1 + R_2}{2R_1(1+d)^2} < 0 \tag{12a}$$

$$\frac{\partial}{\partial d}\left(\frac{F_2}{R_2}\right) = -\frac{R_1 + R_2}{2R_2(1+d)^2} < 0 \tag{12b}$$

It follows (12a) and (12b) that

$$\frac{\partial}{\partial d}\left(\frac{F_1}{R_1}\right) - \frac{\partial}{\partial d}\left(\frac{F_2}{R_2}\right) = \left(-\frac{R_1 + R_2}{2R_1(d+1)^2}\right) - \left(-\frac{R_1 + R_2}{2R_2(d+1)^2}\right) = \frac{R_1^2 - R_2^2}{2R_1R_2(d+1)^2} > 0 \tag{12c}$$

since $R_1 > R_2$.

As the marginal cost of destruction d *increases,* each party allocates relatively *less* of their initial resources towards fighting effort. In this case, an increase in destruction acts as a deterrent for fighting for *both* parties. In addition, we see that $\frac{\partial}{\partial d}\left(\frac{F_1}{R_1}\right)-\frac{\partial}{\partial d}\left(\frac{F_2}{R_2}\right)>0$. This implies party 2 decreases their fighting effort as a proportion of their initial resource *more* than party 1 decreases their fighting effort as a proportion of their initial resource when the marginal cost of destruction increases.

We can analyze the effect changes in the marginal cost of destruction d have on each parties' final share of the total civilian good as a proportion of their initial resource:

$$\frac{\partial}{\partial d}\left(\frac{I_1}{R_1}\right)=\frac{\partial}{\partial d}\left(\frac{1+d}{mR_1}\right)=\frac{1}{mR_1}>0 \tag{13a}$$

$$\frac{\partial}{\partial d}\left(\frac{I_2}{R_2}\right)=\frac{\partial}{\partial d}\left(\frac{1+d}{mR_2}\right)=\frac{1}{mR_2}>0 \tag{13b}$$

$$\frac{\partial}{\partial d}\left(\frac{I_1}{R_1}\right)-\frac{\partial}{\partial d}\left(\frac{I_2}{R_2}\right)=\frac{1}{mR_1}-\frac{1}{mR_2}=\frac{R_2-R_1}{mR_1R_2}<0 \text{ as } R_1>R_2. \tag{13c}$$

As the marginal cost of destruction increases, we can see that both parties obtain a larger share of the total civilian good relative to their initial resource. However, this increase in income as a proportion of initial resource is *larger* for player 2, since $\frac{\partial}{\partial d}\left(\frac{I_1}{R_1}\right)-\frac{\partial}{\partial d}\left(\frac{I_2}{R_2}\right)<0$. These results permit us to establish the first proposition:

PROPOSITION 1. *When fighting is more destructive (i.e., the value of d increases), other things being equal, both parties allocate relatively less of their initial resources towards fighting effort. But the low-income party reduces its fighting effort proportionally more than the high-income party. In equilibrium, both parties' expected payoffs increase but the low-income group has a relatively higher income-endowment ratio than that of the high-income group. Destruction thus acts as a deterrent for fighting and affects each party's equilibrium income positively.*

3.2 *Changes in Incentives to Fight in Response to Changes in* m

The effect that changes in the mass effect parameter m has on each parties' optimal level of fighting effort as a proportion of their initial resource can be analyzed as follows:

$$\frac{\partial}{\partial m}\left(\frac{F_1}{R_1}\right)=\frac{1}{m^2 R_1}>0 \tag{14a}$$

$$\frac{\partial}{\partial m}\left(\frac{F_2}{R_2}\right)=\frac{1}{m^2 R_2}>0 \tag{14b}$$

$$\frac{\partial}{\partial m}\left(\frac{F_1}{R_1}\right)-\frac{\partial}{\partial m}\left(\frac{F_2}{R_2}\right)=\frac{1}{m^2 R_1}-\frac{1}{m^2 R_2}=-\frac{(R_1-R_2)}{m^2 R_1 R_2}<0 \text{ as } R_1>R_2. \tag{14c}$$

As the mass effect parameter m *increases,* each party allocates relatively *more* of their initial resource towards fighting effort. That is, parties are induced to fight *more* when the mass effect parameter increases. This increased level of fighting effort is not uniform across groups, however. Because $\frac{\partial}{\partial m}\left(\frac{F_1}{R_1}\right)-\frac{\partial}{\partial m}\left(\frac{F_2}{R_2}\right)<0$ we can see that an increase in m causes party 2 to increase their level of fighting effort relatively *more* than that of party 1.

We now consider how a change in the mass effect parameter m affects each party's share of the total civilian good as a proportion of initial resource.

$$\frac{\partial}{\partial m}\left(\frac{I_1}{R_1}\right)=-\frac{(d+1)}{m^2 R_1}<0 \tag{15a}$$

$$\frac{\partial}{\partial m}\left(\frac{I_2}{R_2}\right)=-\frac{(d+1)}{m^2 R_2}<0 \tag{15b}$$

$$\frac{\partial}{\partial m}\left(\frac{I_1}{R_1}\right)-\frac{\partial}{\partial m}\left(\frac{I_2}{R_2}\right)=\left(-\frac{(d+1)}{m^2 R_1}\right)-\left(-\frac{(d+1)}{m^2 R_2}\right)=\left(\frac{(d+1)\left(R_1+R_2\right)}{m^2 R_1 R_2}\right)>0 \text{ as } R_1>R_2. \tag{15c}$$

When the mass effect parameter increases, both parties' share of the total civilian good as a proportion of their initial resource decrease. However, the decrease in total civilian good as a proportion of the initial resource is *larger* for player 2 than that for player 1. We, therefore, have

PROPOSITION 2. *When fighting is less decisive (i.e., the value of m increases), other things being equal, both parties allocate relatively* <u>*more*</u> *of their initial resources towards fighting effort. But the low-income party raises its fighting effort proportionally more than the high-income party. In equilibrium, both parties are worse off but the low-income group has a relatively lower income-endowment ratio than that of the high-income group. Decisiveness thus facilitates fighting and affects each party's equilibrium income negatively.*

3.3 *Changes in Incentives to Fight in Response to Changes in Initial Resource Disparity*

Next, we consider an exogenous increase in initial resource disparity (denoted by δ) such that:

$$R_1 = R + \delta, \tag{16a}$$

$$R_2 = R - \delta, \tag{16b}$$

where $R(\equiv \frac{R_1 + R_2}{2})$ is the average amount of the two parties' endowed resources. It follows that

$$R_1 + R_2 = (R+\delta) + (R-\delta) = 2R. \tag{16c}$$

An *increase* in δ implies an *increase* in the initial endowment disparity. Likewise, a *decrease* in δ implies a *decrease* in the initial endowment disparity. However, changes in δ do *not* lead to any variations in the total societal endowment. As such, a variation in δ will not affect the equilibrium fighting effort. That is,

$$F = \frac{m(R_1 + R_2) - 2(1+d)}{2m(1+d)} = \frac{2mR - 2(1+d)}{2m(1+d)}. \tag{16d}$$

Analyzing the effect that changes in the initial resource disparity δ has on each player's optimal level of combative effort yields the following:

$$\frac{\partial}{\partial \delta}\left(\frac{F_1}{R_1}\right)=\frac{\partial}{\partial \delta}\left(\frac{mR-2(1+d)}{2m(1+d)(R+\delta)}\right)=-\frac{Rm-(d+1)}{m(R+\delta)^2(d+1)}<0, \tag{17a}$$

$$\frac{\partial}{\partial \delta}\left(\frac{F_2}{R_2}\right)=\frac{\partial}{\partial \delta}\left(\frac{mR-2(1+d)}{2m(1+d)(R-\delta)}\right)=\frac{Rm-(d+1)}{m(R-\delta)^2(d+1)}>0. \tag{17b}$$

since $R>\frac{(d+1)}{m}$. When initial income disparity increases, player 1 decreases its fighting effort relative to initial resource endowment. Meanwhile, player 2 increases its fighting effort relative to initial resource endowment. While an increase in initial income disparity does not change the total amount of resources dedicated to fighting, it *does* cause does cause the distribution of fighting effort across parties to change. The wealthier party will fight relatively *less* intensely, and the poorer side will fight relatively *more* intensely.

The finding of the above analysis is in line with Hirshleifer (1991a, 1995), which explains the Paradox of Power as a situation with the low-income party being *rationally motivated* to fight harder than the high-income party. An increase in the initial resource disparity accentuates this effect, causing the low-income group to have 'less to lose' and therefore more to gain from engaging in conflictual interaction.

When the initial resource disparity between parties increases, we also observe changes in each parties' income-endowment ratio:

$$\frac{\partial}{\partial \delta}\left(\frac{I_1}{R_1}\right)=-\frac{d+1}{m\left(R+\delta\right)^2}<0 \tag{18a}$$

$$\frac{\partial}{\partial \delta}\left(\frac{I_2}{R_2}\right)=\frac{d+1}{m\left(R-\delta\right)^2}>0. \tag{18b}$$

When the initial income disparities increase, party 1 allocates a smaller share of its initial resource towards fighting effort and as a result its final income to endowment ratio falls. Likewise, party 2 will allocate a larger share of initial resource towards fighting effort and as a result its final income to endowment ratio rises.

PROPOSITION 3. *When the disparity in endowed incomes increases (i.e., the value of* δ *increases), other things being equal, the low-income party allocates relatively* <u>*more*</u> *of its endowed income towards fighting effort and captures a higher income-endowment ratio, whereas the high-income party allocates relatively* <u>*less*</u> *of its endowed income towards fighting effort and captures a lower income-endowment ratio. Income disparity thus facilitates an asymmetric increase in fighting effort by the low-income party and doesn't affect final income levels.*

3.4 *Changes in Incentives to Fight in Response to Changes Lump-Sum Taxation*

A government may be interested in addressing the initial income disparities between groups through a redistributive lump-sum tax. For analytical simplicity, we consider a lump-sum tax (T) such that the post-tax endowments of the two parties are given, respectively, as:

$$\tilde{R}_1 = R_1 - T \text{ and } \tilde{R}_2 = R_2 + T .$$

It follows that

$$\tilde{R}_1 + \tilde{R}_2 = R_1 + R_2 .$$

The lump-sum tax will increase the initial resource endowment of the low-income party by the size of the tax while simultaneously decreasing the initial resource endowment of the high-income party by the size of the tax. However, total initial resource endowment remains unaffected by the lump-sum tax, as does the equilibrium level of total fighting effort. As a result, final income levels will also be unchanged. That is,

$$F = \frac{m(R_1 + R_2) - 2(1+d)}{2m(1+d)} = \frac{m(\tilde{R}_1 + \tilde{R}_2) - 2(1+d)}{2m(1+d)} \tag{19}$$

Analyzing the effect that changes in the lump-sum tax T has on each player's optimal level of combative effort yields

$$\frac{\partial}{\partial T}\left(\frac{F_1}{R_1 - T}\right) = \frac{\partial}{\partial T}\left(\frac{m(R_1 + R_2) - 2(1+d)}{2m(1+d)(R_1 - T)}\right) = \frac{m(R_1 + R_2) - 2(d+1)}{2m(R_1 - T)^2(d+1)} > 0 \tag{20a}$$

$$\frac{\partial}{\partial T}\left(\frac{F_2}{R_1 + T}\right) = \frac{\partial}{\partial T}\left(\frac{m(R_1 + R_2) - 2(1+d)}{2m(1+d)(R_2 + T)}\right) = -\frac{m(R_1 + R_2) - 2(d+1)}{2m(R_1 + T)^2(d+1)} < 0 \tag{20b}$$

under the plausible assumption in (9d) that $R_1 + R_2 > \frac{2(d+1)}{m}$. An increase in the lump-sum tax T induces the high-income group to *raise* its fighting effort to post-tax endowment ratio. Moreover, the low-income group *lowers* its fighting effort to post-tax endowment ratio.

This finding parallels the result of Hirshleifer (1991a, 1995), which explains the Paradox of Power due to the low-income party being *rationally motivated* to fight harder than the high-income party. An increase in the lump-sum tax *reduces* the initial resource disparity, accentuating this effect. As a consequence, low-income group to has 'more to lose' and thus less to gain from engaging in conflictual interaction.

Turning our focus to the impact that the lump-sum tax has on both parties' equilibrium income levels:

$$\frac{\partial}{\partial T}\left(\frac{I_1}{R_1 - T}\right) = \frac{d+1}{m(T - R_1)^2} > 0 \tag{21a}$$

$$\frac{\partial}{\partial T}\left(\frac{I_2}{R_2 + T}\right) = -\frac{d+1}{m(T + R_1)^2} < 0\,. \tag{21b}$$

When the lump-sum tax increases, and therefore initial income disparities decrease, party 1 will allocate a larger share of its initial resource towards fighting effort and as a result its final income to resource ratio will rise. Likewise, party 2 will allocate a smaller share of initial resource towards fighting effort and as a result its final income to resource ratio will fall.

PROPOSITION 4. *When the lump-sum tax increases (i.e., the value of T increases), other things being equal, the low-income party allocates relatively <u>less</u> of its initial resources towards fighting effort and captures a lower income-endowment ratio, whereas the high-income party allocates relatively <u>more</u> of its initial resources towards fighting effort and captures a higher income-endowment ratio. A lump-sum tax thus facilitates an asymmetric increase in fighting effort by the high-income party but doesn't affect the parties' equilibrium income levels.*

3.5 *Changes in Incentives to Fight in Response to Changes in a Proportional Tax*

We now consider an alternative policy where the government addresses disparities in endowed incomes through a redistributive *proportional* tax. That is, the government taxes the high-income party an amount of τR_1 and then transfers this amount to the low-income party such that

$$\hat{R}_1 = R_1 - \tau R_1 \text{ and } \hat{R}_2 = R_2 + \tau R_1.$$

It follows that

$$\hat{R}_1 + \hat{R}_2 = R_1 + R_2.$$

The *proportional* tax will increase the initial resource endowment of the low-income party by some constant proportion t of the high-income party's endowment while simultaneously decreasing the initial resource endowment of the high-income party by some constant proportion of its own endowment. However, total initial resource endowment remains unaffected by the proportional tax, as does the equilibrium level of total fighting effort. As a result, final income levels will also be unchanged. That is,

$$F = \frac{m(R_1 + R_2) - 2(1+d)}{2m(1+d)} = \frac{m\left[(R_1 - \tau R_1) + \left((R_2 + \tau R_1)\right)\right] - 2(1+d)}{2m(1+d)}$$

$$= \frac{m(\hat{R}_1 + \hat{R}_2) - 2(1+d)}{2m(1+d)}. \tag{22d}$$

Analyzing the effect changes in proportional tax τ has on each player's optimal level of combative effort:

$$\frac{\partial}{\partial \tau}\left(\frac{F_1}{R_1 - \tau R_1}\right) = \frac{m(R_1 + R_2) - 2(d+1)}{2mR_1(d+1)(\tau - 1)^2} > 0, \tag{23a}$$

$$\frac{\partial}{\partial \tau}\left(\frac{F_2}{R_2 + \tau R_1}\right) = -\frac{m(R_1 + R_2) - 2(d+1)}{2m(d+1)(R_2 + \tau R_1)^2} < 0. \tag{23b}$$

An increase in the proportional tax τ induces the high-income party to *raise* its fighting effort to initial resource ratio. Moreover, the low-income group *lowers* its fighting effort to initial resource ratio.

As with the lump-sum tax, this result is in line with Hirshleifer (1991a, 1995), which explains the Paradox of Power as a result of the low-income party being *rationally motivated* to fight harder than the high-income party. An increase in the lump-sum tax *reduces* the initial resource disparity, accentuating this effect. Consequently, the low-income party has 'more to lose' and hence less to gain from engaging in conflictual interaction.

We can now see how a change in the proportional tax affects each party's equilibrium income (as a proportion of post-tax endowment):

$$\frac{\partial}{\partial \tau}\left(\frac{I_1}{R_1 - \tau R_1}\right) = \frac{d+1}{mR_1(1-\tau)^2} > 0, \tag{24a}$$

$$\frac{\partial}{\partial \tau}\left(\frac{I_2}{(R_2 + \tau R_1)}\right) = -\frac{R_1(d+1)}{m(R_2 + \tau R_1)^2} < 0. \tag{24b}$$

When the proportional tax increases, and therefore initial income disparities decrease, the high-income party allocates a larger share of its initial resource towards fighting effort and as a result its final income to post-tax endowment ratio rises. Likewise, the low-income party allocates a smaller share of initial resource towards fighting effort and as a result its final income to post-tax endowment ratio falls. We thus have

PROPOSITION 5. *When the redistributive proportional tax increases (i.e., the value of* τ *increases), other things being equal, the low-income party allocates relatively less of its initial resources towards fighting effort and captures a lower income-endowment ratio, whereas the high-income party allocates relatively more of its initial resources towards fighting effort and captures a higher income-endowment ratio. A redistributive proportional tax thus facilitates an increase in fighting effort asymmetrically by the high-income party but doesn't affect the parties' equilibrium income levels.*

Chapter 4 - Concluding Remarks

This paper contributes to the theoretical conflict literature by introducing *destruction* into the standard model of two-party conflict as presented by Hirshliefer (1991a). We identify several factors that influence each party's incentive to fight and their equilibrium shares of the overall civilian good. When conflict becomes more destructive, agents choose to reduce their equilibrium allocation of fighting effort. As a result, both parties' equilibrium shares of the final civilian good increase. Conversely, when the destructiveness of fighting decreases, both parties increase their fighting effort, causing their shares of final civilian good to decline.

Our analysis indicates that efforts by policy-makers to address initial resource disparities are met with mixed results. While both a redistributive lump-sum tax and proportional tax will cause the low-income group to allocate relatively less of their endowed resources towards fighting, the high-income group allocates relatively *more* of their initial endowment towards fighting effort. As a result, total societal fighting effort remains unchanged in equilibrium.

Some caveats about the analysis in this paper should be mentioned. We only consider equilibrium outcomes where each party chooses to fight (i.e. non-corner solutions). However, the presence of conflict and the absence of cooperation need not be an equilibrium outcome under the productive and appropriative effort framework. Skaperdas (1992) shows that under certain conditions (most notably, when conflict is ineffective), agents may cooperate in equilibrium. We also only consider conflict as a static game. That is, combatants choose optimal levels of fighting effort for a single time period. Maxwell and Reuveny (2005) expand upon this framework by modeling conflict in a dynamic two-period model and showing the preconditions for the paradox of power (i.e. initial wealth disparities) may disappear in the steady state. This result suggests that persistent wealth disparities between groups may be driven by exogenous factors, such as laws enacted by the dominant group to disenfranchise the weaker group. Further extensions of the results found in this paper would be to expand our model into the multi-period case or to allow for non-combative equilibrium outcomes.

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