64

The Limiting Reliability of a Complex System of Components Subject to Wearout Failures

by

Robert Lee Lyon

B.A., Southwestern College,

Winfield, Kansas, 1970

9589

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

Master of Science

Department of Industrial Engineering

Kansas State University

Manhattan, Kansas

1972

Approved:

Doris Grosh

Major Professor

TABLE OF CONTENTS

Chapter 1

Introduction

We review here basic concepts and formulas of reliability theory.

The reliability of a component for a stipulated interval of time T is the probability of continuous satisfactory operation during the interval. That is, a component is put into operation for a time T until it fails. The random variable, T, is called the time to failure or the life length of a component. It is assumed here that T is defined as a non-negative real number. Let F(t) designate the probability that the component will fail in the interval from 0 to t, that is

$$F(t) = P(T \leq t).$$

F(t) is called the <u>cumulative distribution function</u>. The probability that the component will not fail in time t (the reliability of a component) is

$$R(t) = 1 - F(t) \tag{1.1}$$

where R(t) is called the <u>reliability function</u>. If F(t) is continuous and differentiable, the <u>probability density function</u> f(t) is defined by the relationship

$$f(t) = F'(t),$$

so that

$$F(t) = \int_0^t f(x)dx \tag{1.2}$$

and

$$R(t) = \int_{t}^{\infty} f(x)dx. \tag{1.3}$$

Using the Mean Value theorem, it can be proved that the probability that the component will fail between t and t + Δt is approximately f(t)dt.

The distribution of the time to failure of the component can alternatively be described by the <u>instantaneous</u> <u>failure</u> <u>rate</u> (or merely "failure rate") or the <u>hazard</u> <u>function</u>, Z(t). The hazard function of a component is the conditional density for the component to fail "at" time t, given that the component is functioning satisfactorily at time t. Thus, the formula of the hazard function is

$$Z(t) = \frac{F'(t)}{R(t)} = \frac{f(t)}{R(t)} = \frac{f(t)}{1-F(t)} \; . \tag{1.4}$$

The failure density can be obtained if the hazard function is known, by means of the relationship

$$f(t) = Z(t)\, e^{-\int_{0}^{t} Z(x)dx} \tag{1.5}$$

This is true because the relationship (1.1),

$$R(t) - 1 - F(t),$$

when differentiated with respect to t, yields

$$R'(t) = - F'(t) = - f(t).$$

Thus, from (1.4)

$$Z(t) = \frac{f(t)}{R(t)} = \frac{-R'(t)}{R(t)} .$$

Solving this differential equation by separation of variables and definite integration gives

$$-\int_0^t Z(x)dx = -\int_0^t -\frac{R'(x)}{R(x)} dx = \ell n\ R(x) \Big|_0^t = \ell n R(t),$$

since

$$\ell n\ R(0) = 0.$$

Hence

$$e^{\ell n\ R(t)} = R(t) = e^{-\int_0^t Z(x)dx} .$$

Thus

$$f(t) = Z(t)\ R(t) = Z(t)\ e^{-\int_0^t Z(x)dx} .$$

Therefore the failure rate determines the failure density.

Some components fail due to wearout only. A common model to use for this type of failure is the normal or Gaussian distribution. The normal probability density is:

$$f(t) = \frac{1}{\sqrt{2\pi}\ \sigma} \exp^{-\frac{1}{2}\left(\frac{t-M}{\sigma}\right)^2} , \qquad -\infty < t < \infty .$$

where M is the mean failure time, σ is the standard deviation of the normal distribution and t is the operation time. The cumulative distribution function, the reliability function, and the hazard function of the normal failure law are:

$$F(t) = \int_{-\infty}^{t} f(x)dx = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{t} e^{-\frac{1}{2}\left(\frac{x-M}{\sigma}\right)^2} dx,$$

$$R(t) = 1 - F(t) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{t}^{\infty} e^{-\frac{1}{2}\left(\frac{x-M}{\sigma}\right)^2} dx,$$

and

$$Z(t) = \frac{f(t)}{R(t)} = \frac{e^{-\frac{1}{2}\left(\frac{t-M}{\sigma}\right)^2}}{\int_{t}^{\infty} e^{-\frac{1}{2}\left(\frac{x-M}{\sigma}\right)^2} dx} .$$

No convenient closed form is available for these functions.

Probably due to the inconvenience of the formulae, the normal failure law is not as commonly assumed in the reliability literature as is the exponential failure law, whose density function is

$$f(t) = \alpha e^{-\alpha t}, \quad t > 0$$

where $\alpha > 0$ is a scale parameter. The cumulative distribution function, the reliability function, and the hazard function of the exponential distribution are as follows:

$$F(t) = \int_{0}^{t} \alpha e^{-\alpha x}\, dx = 1 - e^{-\alpha t},$$

$$R(t) = 1 - F(t) = 1 - (1 - e^{-\alpha t}) = e^{-\alpha t},$$

and

$$Z(t) = \frac{f(t)}{R(t)} = \frac{\alpha e^{-\alpha t}}{e^{-\alpha t}} = \alpha.$$

Thus if the time to failure distribution is exponential, the component has a constant failure rate. Conversely, if the failure rate is constant, then the distribution is an exponential distribution. This can be shown by denoting the constant failure rate by α, where $\alpha > 0$, and by substituting α for $Z(t)$ in formula (1.5) yielding

$$f(t) = \alpha e^{-\int_0^t \alpha dx} = \alpha e^{-\alpha t}, \qquad t > 0.$$

In the case of the normal failure law, the older the component, the greater the probability of its failing. But in the exponential case a component can be considered new until it fails. In other words, age does not affect its performance and the failures are described as random.

Chapter II

Bazovsky's Example

Igor Bazovsky, in his book, Reliability Theory and Practice, used an example to show graphically what happens in a system containing a large number of components operating simultaneously when components are replaced immediately upon failure. He assumed that the components can fail only by wearout; random failures are excluded. He assumed that the failure time of the individual components is $N(M,\sigma^2)$, that is, normally distributed with mean time to failure M and standard deviation σ. His system contained 10,000 lamps with M equal to 7200 hours and σ equal to 600 hours. His graphs showed the number of lamps failing per day (failure density) plotted against the time in hours. In his system, when the first generation of components failed, the second generation started to come into service. Since the second generation lamps do not begin operation simultaneously like the first generation, he asserted that the peak of the number of lamps failing in the second generation will not be as high as the peak of the first generation. And that the peak of each succeeding generation will be lower than that of the previous one. He stated that the wearout peak (mean life) for the Kth generation occurred at time KM with standard deviation was equal to $K\sigma$. He plotted the superimposed failure frequency curves for several generations, showing the increasing spread and the shift to the right on the time axis for succeeding generations. (The failure frequency curve is the failure density curve with a vertical scale change, so that the area under each curve is N = 10,000). His graph is reproduced here as Figure 1. After a certain period of time has elapsed, the failures which occur in a given interval may represent several generations. For example, a bulb failing at the end of 20,000 hours may be a second generation failure, if the first generation lamp in that socket lasted 12,000 hours and its replacement lasted 8,000 hours. Or it might be a fourth generation failure, if the initial bulb in that socket lasted 4,000 hours, the first replacement lasted 5,000 hours, the second replacement lasted 9,000 hours, and the 3rd replacement lasted 2,000 hours. The total number of failures in a given interval is therefore the sum of all the failure frequency curves. Bazovsky asserted that this sum smooths out to a constant level by about the fourth generation. His argument was presented graphically and is reproduced here as Figure 2. He also stated that this system would stabilize and the failure rate would become constant at time $T = nM$, where n is the number of generations to achieve stability and M is the mean life. He used the formula

$$n = M/3\sigma$$

to calculate n, giving $n = \frac{7200}{3(600)} = 4$. His assertion is that the graph in Figure 2 shows a constant failure rate after the fourth generation. This constant failure rate implies that the system behaved exponentially. It was felt that his heuristic method, although appealing, was incorrect, (in essence his graphs showed a superposition of normal failure densities) and that the leveling off process, if it does occur, should refer to the instantaneous failure rate curve rather than the density curve.

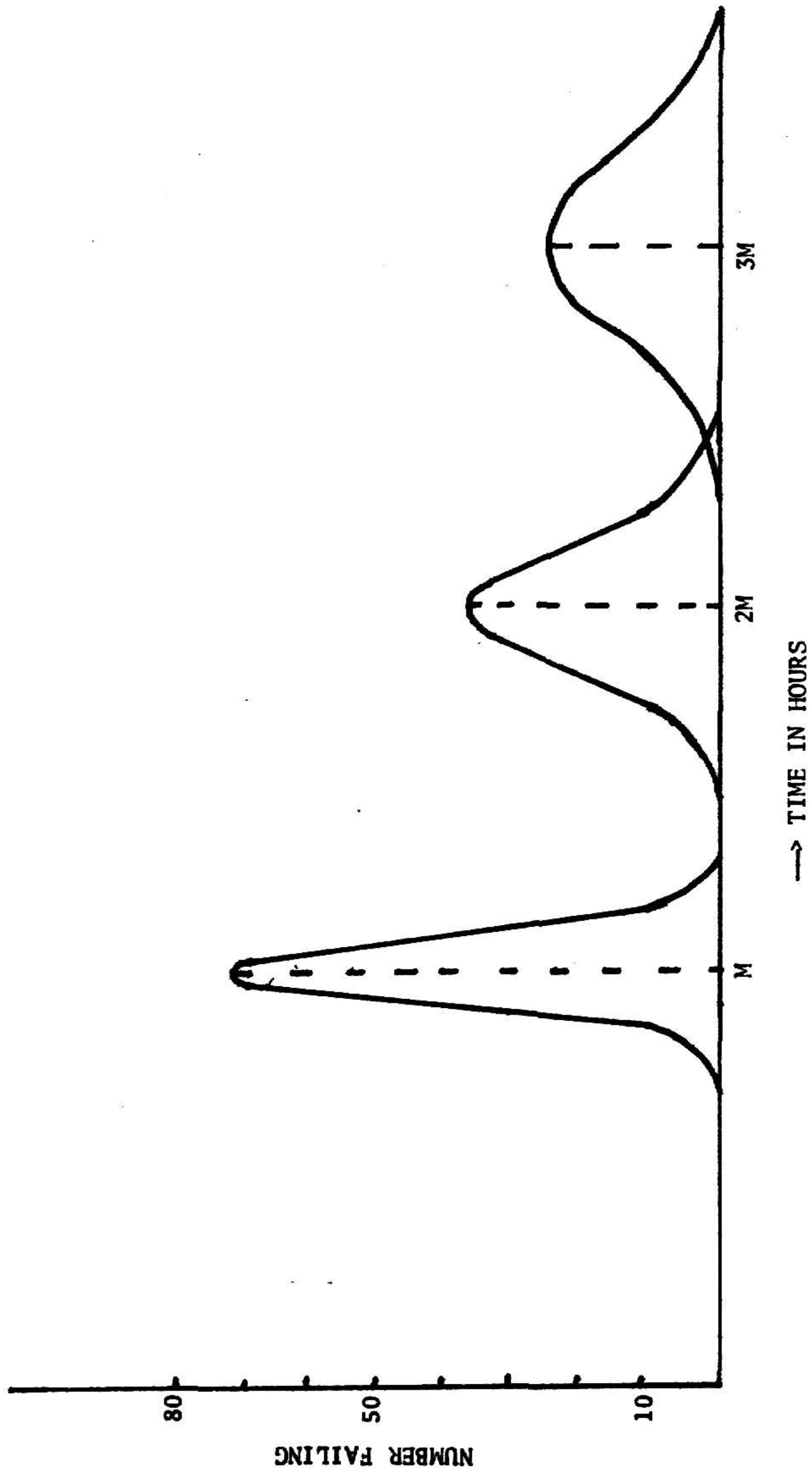


FIGURE 1. WEAROUT CURVE FOR THREE GENERATION

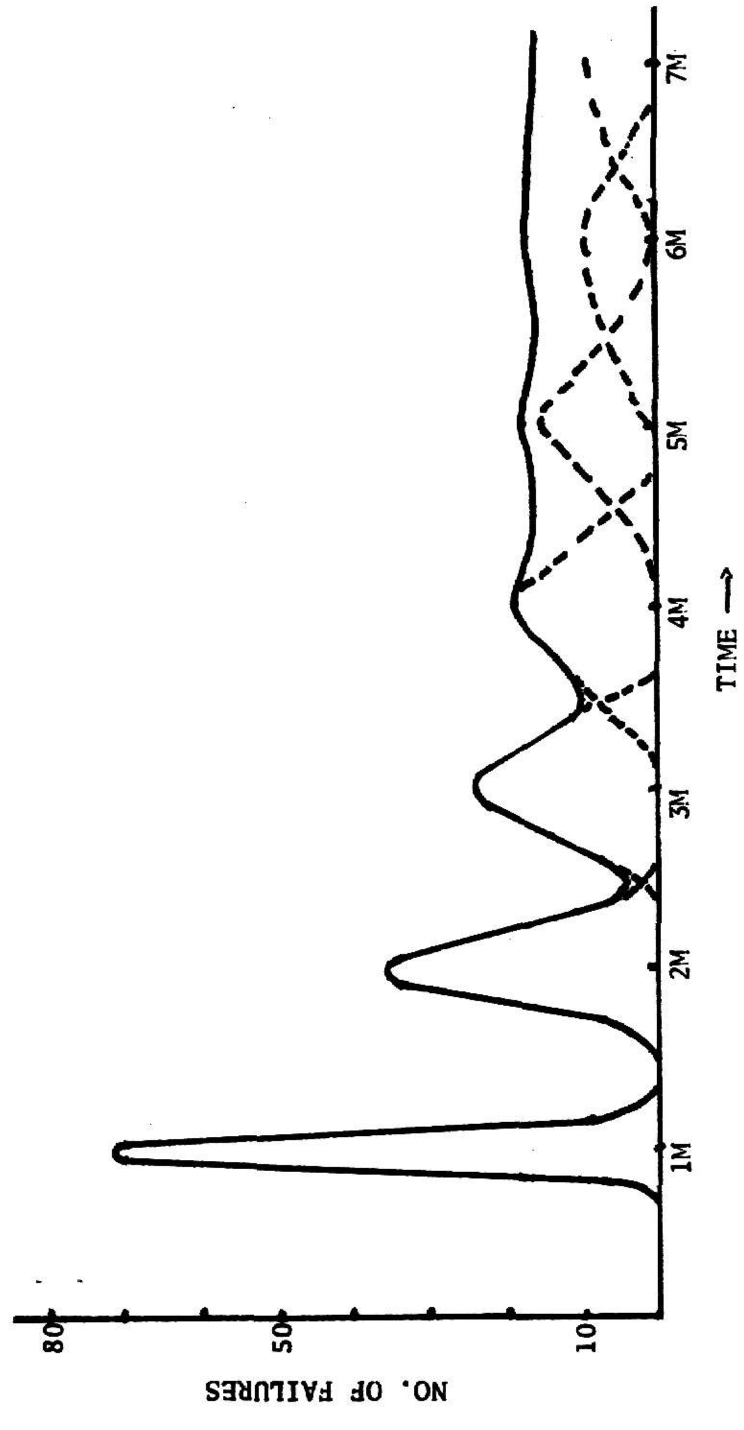


FIGURE 2. STABILIZATION OF A COMPLEX SYSTEM WITH 10,000 COMPONENTS

It was felt that Bazovsky's information was correct -- that a large complex system of identical components failing (with replacement) according to the normal law -- might indeed exhibit the failure pattern he suggests, but that his reasoning was based more on wishful thinking than on rigor. One difficulty encountered was the problem of how to define the hazard function (instantaneous failure rate) in the problem at hand. The usual definition of hazard function

$$Z(t) = \frac{f(t)}{1-F(t)}$$

can be approximated over the interval Δt_i by an empirical hazard function $Z_e(t)$ defined as (see [5, p. 161])

$$Z_e(t) = \frac{n(t_i) - n(t_i + \Delta t_i)}{\Delta t_i \; n(t_i)} ,$$

where N items are placed on test at time $t = 0$, and $n(t)$ is the number of survivors remaining at time t. In the current problem, since items are replaced as they fail, the usual concept of survivors does not apply, nor does it appear possible to make a meaningful definition of hazard function. However if the process does approach the behavior postulated by Bazovsky, the failures occur <u>as if</u> they were generated with exponential inter-arrival times (with mean $1/\lambda$, say) and hence the number of failure for a constant interval length will be a Poisson random variate with parameter λt, where t is the length of the interval and λ is the "intensity" of the Poisson process. To test if this is a valid assumption the Chi-square goodness of fit test can be used when the number of failures starts to stabilize. The value of λ can be estimated by the fact that the mean number of failures for fixed interval length t is a maximum likelihood estimator of λt. Equating λt to the observed average number $\bar{f}$, the estimate of λ is obtained, $\hat{\lambda} = \frac{\bar{f}}{t}$. The theoretical value of λ of the limiting process as pointed out by Bazovsky, is N, the number of components of the first generation, divided by the mean life, M. This is because for a single component with mean life M hours the expected number of failures per hour is 1/M. Then for N components, the expected number of failures per hour is N/M. A "running estimate" of λ, designated as λ^*, can be obtained from the generated data from the beginning to see how the system behaves and to help decide when the system approaches stability. Although it makes no sense to talk about failure rate or hazard function for the replacement system, still the population of exponential interarrival times corresponding to the stabilized system does have a constant hazard function $Z(t) = \lambda$. In a sense, then, any sensibly chosen set of estimates of λ can be considered as an indicator of system stability. We use the phrase pseudo-hazard rate to indicate λ^*, a cumulative estimate of λ based on all observation up to the time of calculation. The statistical properties of λ^* are not known, since it represents a transient phase. As soon as the number of failures (for a fixed interval of time) starts to stabilize, an unbiased estimate of lambda, say $\hat{\lambda}$, will be calculated based on the steady state observations. If the system does behave exponentially, then $\hat{\lambda}$ should approximate closely the theoretical λ given by N/M.

David K. Lloyd and Myron Lipow in Appendix 9B of their book, Reliability: Management, Methods and Mathematics, treated a more general problem. Instead of using a heuristic graphical approach they used mathematical methods to show what happens to a complex system when components are replaced immediately upon failure. They showed that the reliability of a complex system tends to approach that which corresponds to the exponential failure law. That is, the failure density of the system becomes exponential and hence the instantaneous failure rate or hazard function becomes constant. Lloyd and Lipow did not state at what time or in what generation the hazard function would become constant; their theorem was the result of a limiting process where the age of the system is allowed to become infinite. Their derivation is summarized in Appendix 1.

The purpose of this report is to examine Bazovsky's assertion that the failure rate becomes constant and at what time or in what generation will this occur. A Fortran based computer program was used to simulate the complex system which he discussed.

Chapter III

Procedure

A Fortran based computer program was used to simulate Bazavsky's example as well as the others. (See Appendix 2 for print-out of the program.) He treated a complex system with 10,000 components each of which is immediately replaced upon failure. The time to failure of each component was assumed to be normally distributed with mean life M equal to 7200 hours and with a standard deviation σ equal to 600 hours. In the program scientific subroutines or generators for the IBM 360/50 were used to generate a normally distributed random failure time of the components with the given mean and standard deviation. The generated data was depicted as a histogram by dimensioning the cells of the histogram in the storage area of the computer. One cell represented 24 hours or one day. The computer program was made general enough so that any number of components (subject to computer storage) of the complex system can be run for any length of time or for any number of generations. When a component is replaced by another component upon failure, a string of component replacements is formed. The length of the string is the number of generations. Thus, the total number of components in the system is the product of the original number of components and the number of generations of replacement. The parameters of the system, (namely the number of components in a generation, the number of generations desired, the desired number of cells for the histograms and the cell index to start printing the output) need to be read in as input data. All variables were initialized in order to simulate the system. The failure time of a component was generated. To make the system continuous, the failure time of a particular component in one generation was added to the failure time of the component it replaced. From this new time the appropriate cell of the histogram was determined and one was added to the cell frequency count. This process was repeated until the desired number of generations for each string of component replacement was achieved. At this point the cell count showed the number of failures that occurred for the day. (Note that a cell may contain observations from different generations. It is reasonable to describe a cell as within the Kth generation if it is within three standard deviations of KM, the mean life of the Kth generation.) To help determine if the failure density was approaching the exponential, a cumulative running estimate of λ was calculated from the data. Because the histograms were printed in rows of 24 cells, for format convenience, the estimator λ^* was calculated from row to row instead of from cell to cell. The calculation of λ^* was determined by the following rule: The cell counts were summed up by rows, from the beginning to the end of the current row (where the cell index number is a multiple of 24). Since the first few generations, as indicated by Bazovsky, do not behave exponentially, and hence are of little interest, a row in the "middle" of the histogram could be picked to start calculating the estimator λ^*. Since a cell represented the number of failures per day, the row total was divided by the time t corresponding to the row (24 days) to get the current estimator λ^*. The new current λ^* was calculated by adding the following row count to the previous row total and by dividing the new accumulated row total, CM, by the product of t and R, where R is the number of rows that has been accumulated. Then, using the formula,

$$\lambda^* = \frac{CM}{t \cdot R},$$

the current estimation of λ, was determined from row to row. These λ^* values are not true estimates of the parameter λ of an exponential distribution because a true estimator of λ should not be attempted until the system starts to stabilize, i.e. until the number of failures in a standard interval appears to reach uniformity. This procedure was repeated until the input number of cells of the histogram was reached (this also being a multiple of 24). The output of the program contained the following information: echo checks of the number of components contained in one generation, the number of generations, and the index cell number. Following this, headings were printed for the row total frequency count, the accumulated frequency count (total number of failures), and the estimator λ^*. The data were printed in columns under the appropriate headings. Table 1 shows an example of the output of the program.

50 2 0

ROW TOTAL	FAILURES CUM. SUM	CURRENT EST.
1.00000	1.00000	0.00007
8.00000	9.00000	0.00033
10.00000	19.00000	0.00046
17.00000	36.00000	0.00065
11.00000	47.00000	0.00068
1.00000	48.00000	0.00058
2.00000	50.00000	0.00052
0.00000	50.00000	0.00045
0.00000	50.00000	0.00040
0.00000	50.00000	0.00036
0.00000	50.00000	0.00033
0.00000	50.00000	0.00030
1.00000	51.00000	0.00028
4.00000	55.00000	0.00028
12.00000	67.00000	0.00032
16.00000	83.00000	0.00038
8.00000	91.00000	0.00039
7.00000	98.00000	0.00039
1.00000	99.00000	0.00038
1.00000	100.00000	0.00036

TABLE 1 OUTPUT OF COMPUTER PROGRAM

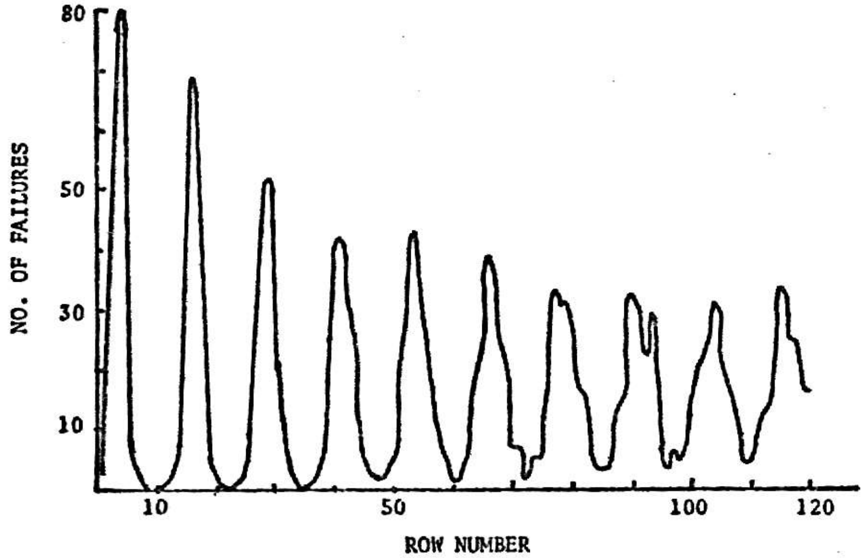
Chapter IV

Results and Conclusion

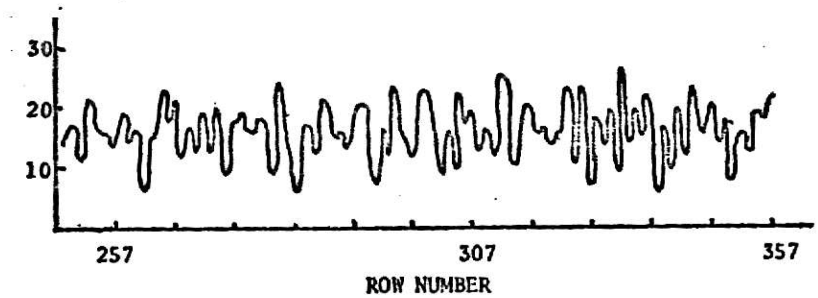
The computer program of a complex system was run varying the number of components and the number of generations. See Table 2 for the different runs and Appendices 3-8 for the print-out of the output for each run. By inspecting the row total column (which represents the number of failures for a 24 day period) it was soon discovered that basically the shape of the histogram remained the same for each run, but the frequency of each row increased as the number of the components increased. This implied that the number of components in the system did not substantially affect the stabilization of the system. (See Figure 6.) Using 200 components the system was simulated to see if the system's behavior could be described as exponential, to see if the number of failures stabilized and to see if the pseudo-hazard function became constant. The simulation of the complex system with 200 components was finally run with 50 generations. In terms of time, this corresponds to total system operating time of 450,000 hours. Appendix 7 shows the print-out of the output of this run. Graphs were made of the number of failures listed in the column labeled under ROW TOTAL to see if the number of failures approached uniformity. (See Figure 3.) Figure 3a shows the first 120 row totals from time 0 (cell 1) to time 74,204 hours (cell 3,096). Figure 3b shows the number of failures per 24 days from 141,696 hours (cell 5,904) to 210,816 hours (cell 8784) and Figure 3c shows the number of failures from 245,400 hours (cell 10,225) to 314,496 hours (cell 13,104). Figure 3a shows that

Case	Appendix No.	No. of Components	No. of Generations	Page No.
1	3	50	7	34
2	4	850	8	37
3	5	1000	20	40
4	6	200	20	46
5	7	200	50	52
6	8	10,000	7	64

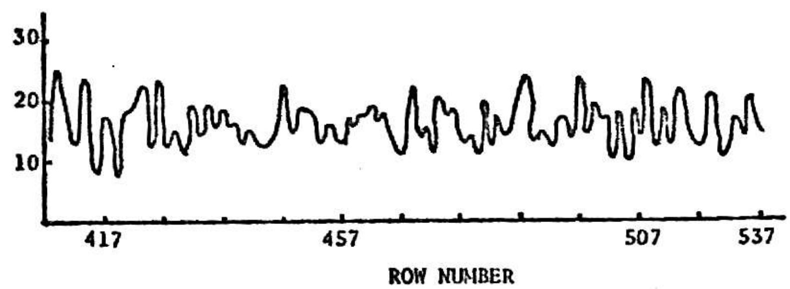
TABLE 2 CASES INVESTIGATED IN THIS REPORT



(a) Early (transient) stage)



(b) Middle stage, showing fair stability



(c) Later stage, showing more stability

FIGURE 3. BEHAVIOR OF A SYSTEM WITH 200 COMPONENTS WITH 50 GENERATIONS

the number of failures per row of the system was very unstable, while Figures 3b & 3c shows that the number of failures per row had started to stabilize. Thus, the system was assumed to be approaching exponentiality. Assuming that the system did approach exponentiality, the number of failures for a constant time interval was recognized to be a Poisson random variate with parameter λt where t was the length of the interval (row) and λ was the intensity of the system. To test this assumption a Chi-square goodness of fit test was made on the number of failures starting at time 141,696 hours (cell 5904 or within the 19th generation) and ending at time 245,376 hours (cell 10224, or within the 34th generation). Appendix 7 shows the raw data that was used. The assumed value of the Poisson parameter was $\lambda t = 16$. This is because, for a single component with mean life M hours, the expected number of failures per hour is 1/M. Then for N components, the expected number of failures per hour is N/M. For a system with 200 components with a mean life of 7200 hours, the theoretical λ was 200/7200 = .0277. The value t = 576 results from the fact that each row contains 24 cells, each representing 24 hours. The Poisson probabilities were obtained from Molina's table [4]. In order to have larger degrees of freedom for the test 60 row totals were added to the 120 row totals that were used to make the graphs (Figure 3b and 3c). The observations were grouped into 16 classes as shown in Table 3. The critical value of Chi-square with 15 degrees of freedom with $\alpha = .05$ is 25.0. The hypothesis of the Chi-square test (set up more formally) is

H_o: The distribution is Poisson with $\lambda t = 16$.

H_1: The distribution is not Poisson with $\lambda t = 16$.

Classes i	Theoretical Frequency E	Observed Frequency O_1	Observed Frequency O_2	$(O_1-E)^2/E$	$(O_2-E)^2/E$
9 or less	7.79	16	4	8.65	1.84
10	6.14	8	3	.56	1.61
11	8.92	1	7	7.03	.41
12	11.90	11	17	.01	2.19
13	14.65	10	16	1.48	.12
14	16.74	14	20	.45	.63
15	17.86	21	18	.55	.00
16	17.86	24	16	2.11	.19
17	16.81	10	16	2.76	.04
18	14.94	13	15	.25	.00
19	12.58	16	17	.93	1.55
20	10.07	11	9	.09	.11
21	7.67	8	9	.01	.23
22	5.58	4	4	.45	.45
23	3.88	6	4	1.16	.00
24 or more	6.61	7	5	.02	.39
	180.00	180	180	26.51	9.76

TABLE 3 COMPUTATION OF CHI-SQUARE

The critical value of Chi-square for this test was 26.51. Thus, the Chi-square test rejected the hypothesis H_o that the parameter of this set of data does arise from a Poisson distribution with parameter 16.

A second Chi-square goodness of fit test was made on the number of failures starting at time 245,400 hours (cell 10225, or within the 34th generation) and ending at time 349,056 hours (cell 14544 or within the 48th generation). The value of the Chi-square on this data was 9.76. Thus, the second Chi-square test accepted the hypothesis H_o. This means that the data generated from time before 245,376 hours still behaved somewhat normally, but as time went on (after the 34th generation) the generated data approached exponentiality. (See Table 3 for the computation of Chi-square).

The cumulative estimator λ^* shown on the third column of output under λ^* CURRENT EST. in Appendix 7, showed in another form the stabilization of the process because the estimations decrease very little from one row to the next. A true unbiased estimator of λ, designated $\hat{\lambda}$, was calculated for the interval where the data behaved exponentially (from 245,400 hours to 349,056 hours). The value of λ can be estimated by the fact that the average number of failures per row is a maximum likelihood estimator of λt. The estimate of λ was calculated by equating λt to the obtained average of failures for the interval, i.e. setting $\lambda t = \bar{f}$. Thus, $\hat{\lambda} = \bar{f}/t$. The average number of failures for the interval was calculated by adding the 180 rows of data, by dividing the sum by 180 and then by dividing the quotient by the time of the interval of a row which was 24 days. The result was of $\hat{\lambda} = .02790$. The theoretical limiting value of λ was $\lambda = N/M = .02777$. This was considered good agreement. This also was taken as good supporting evidence for the contention that the system had become exponential.

Finally, Bazovsky's example with 10,000 components was run for 7 generations; the output can be seen in Appendix 8. A graph was made by plotting the number of failures per row, (given by the first column of the output under ROW TOTAL), against time. See Figure 4 for this graph. Inspection of the graph indicates that the number of failures per row of the system did not stabilize in the 4th generation or anytime afterwards as Bazovsky asserted. Next, a graph was made of λ^*, the pseudo-hazard function (the 3rd column of output labeled λ^* CURRENT EST.) plotted against time. (See Figure 5). This graph was compared with Bazovsky's graph, reproduced here as Figure 2. The curves showed similarity and the psuedo-hazard function appears to stabilize within the 4th generation. Referring to Bazovsky's graphs (reproduced here as Figures 1 and 2) it appears that he confused the number of failures (the failure density) with the failure rate. By changing the dimensioning of the vertical axis, the curve of his graph would correspond to the output of this report or Figure 5.

In general terms, Bazovsky stated that the mean life and the standard deviation of the Kth generation were KM and Kσ, where M was the mean life of the basic distribution and σ is the standard deviation of the basic distribution. The assertion about mean life is true because of the following: Let t_i be the length of life of the i^{th} component, and $t = t_1 + t_2 + \cdots + t_k$ be the length of life of a string of k components representing successive replacements. Then, since $E(t_i) = M$ and $Var(t_i) = \sigma^2$ and the t_i are assumed to be independent,

$$E(t) = E(t_1 + t_2 + \cdots + t_k) = E(t_1) + \cdots + E(t_k) = KM$$

and

$$\mathrm{Var}(t) = \mathrm{Var}(t_1 + t_2 + \cdots + t_k) = \mathrm{Var}(t_1) + \cdots + \mathrm{Var}(t_k) = K\sigma^2.$$

This implies that the standard deviation of the Kth generation is $\sqrt{K}\,\sigma$ and not $K\,\sigma$ as Bazovsky stated.

In addition, Bazovsky stated that the type of complex system discussed in this paper would stabilize and the failure rate would become constant within the nth generation, where n is obtained from the formula $n = M/3\sigma$. For his example this would given $n = 7200/3(600) = 4$. Bazovsky does not give any reference or reasoning for this formula, and this report showed that it is defective.

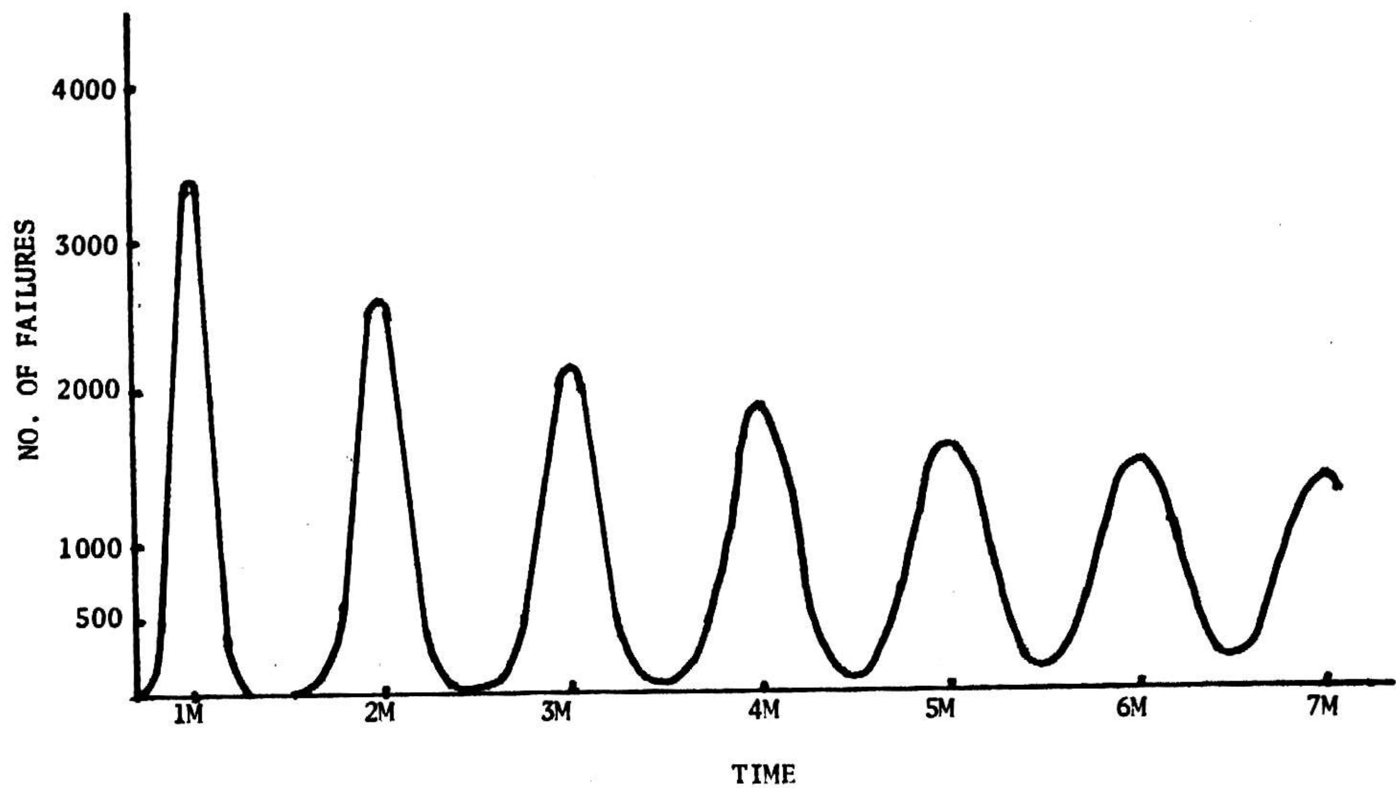


FIGURE 4. BAZOVSKY'S EXAMPLE WITH NUMBER OF FAILURES PLOTTED AGAINST TIME

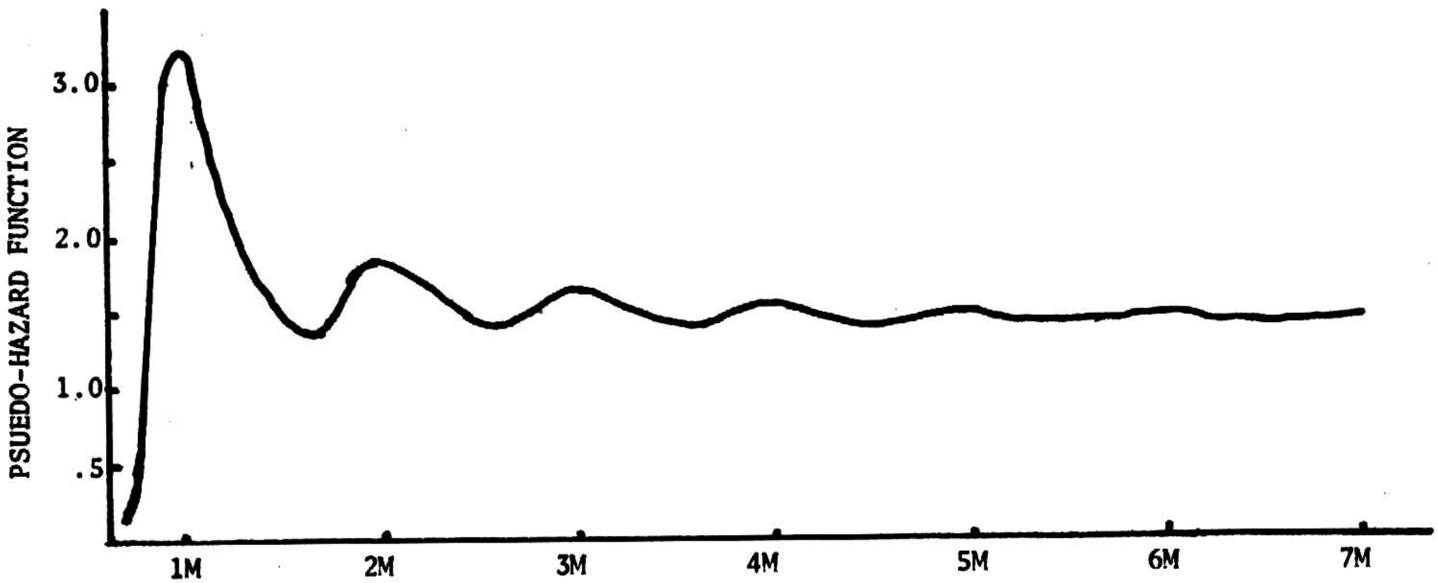


FIGURE 5. THE PSEUDO-HAZARD FUNCTION FOR BAZOVSKY'S EXAMPLE

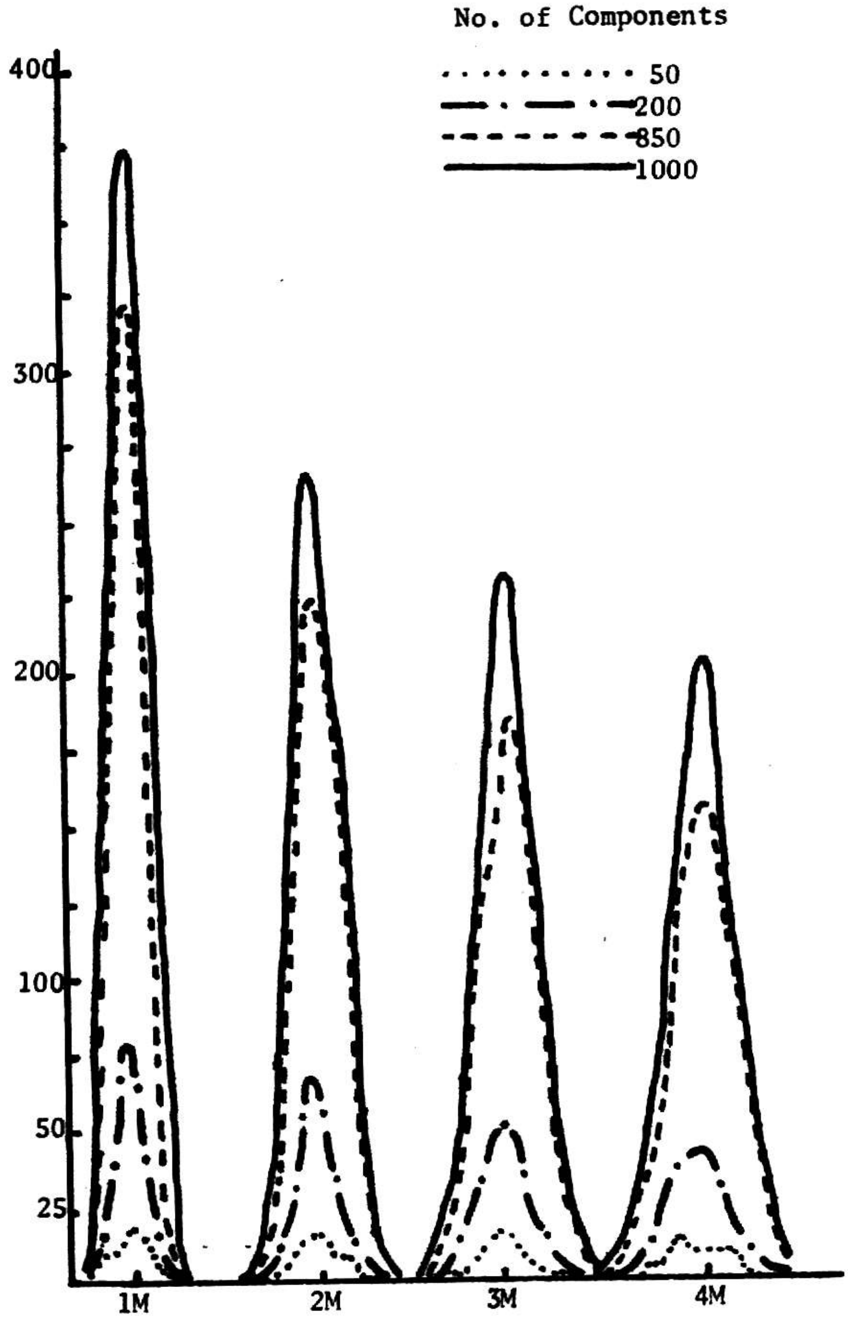


FIGURE 6. WEAROUT CURVES FOR FOUR GENERATIONS

BIBLIOGRAPHY

1. Bazovsky, Igor; Reliability Theory and Practice; 1961; Prentice-Hall, Inc.

2. Lloyd, David K. and Lipow, Myron; Reliability: Management, Methods and Mathematics; 1962; Prentice-Hall, Inc.

3. Meyer, Paul L.; Introductory Probability and Statistical Applications; 1965; Addison Wesley.

4. Molina, E. C.; Poisson's Exponential Binomial Limit; 1942; D. Van Nostrand Company, Inc.

5. Shooman, Martin L.; Probabilistic Reliability: An Engineering Approach; 1968; McGraw-Hill Book Company.

6. Wadsworth, George P., Bryan, Joseph G.; Introduction to Probability and Random Variables; 1960; McGraw-Hill Book Company, Inc.

APPENDIX 1

"Reliability of a System With Component Replacement"

by

David K. Lloyd and Myron Lipow

Lloyd and Lipow [2, p. 271] showed by using mathematical methods that the reliability of a complex system tends to approach that corresponding to the exponential failure law. They assumed a series system containing a large number of components operating simultaneously where a failed component is immediately replaced by a new one. The same failure distribution function F(t) and the probability density function f(t) are assumed for each component of the complex system. They defined the reliability (survival probability) of the system as a function of time t to be $R(t) = [1 - F(t)]^N$ where N is the number of components. Given the ages of N components, $X_1, X_2, \ldots, X_N$, the conditional probability of surviving from time t_0 (the beginning of the "mission") to time t is

$$R(t,t_0|X_1,X_2, \ldots, X_N) = \prod_{i=1}^{N} \frac{1-F(X_i+t)}{1-F(X_i)} .$$

Given the probability density function of the ages X_i as $g(X_i,t_0)$, the absolute reliability from t_0 to t is

$$R(t,t_0) = \left(\int_0^\infty \frac{1-F(X+t)}{1-F(X)} g(X,t_0)\ dX \right)^N$$

It is proved that $\lim_{t_0 \to \infty} g(X, t_0) = \frac{1-F(X)}{\mu}$ where μ is the mean time to failure and $\frac{1-F(X)}{\mu}$ is a probability density function of the system. Thus, $R(t,\infty) = \frac{1}{\mu^N} \left(\int_0^\infty [1-F(X+t)]\ dX \right)^N$. Now, consider the reliability $\tilde{R}$ for a single component $\tilde{R} = [R(t,\infty)]^{\frac{1}{N}} = \tilde{R} = \int_0^\infty \frac{1-F(X+t)}{\mu}\ dX$. Letting $y = X + t$ and $dy = dX$, $\tilde{R} = \int_t^\infty \frac{1-F(y)}{\mu}\ dy$.

or

$$\tilde{R} = \int_0^\infty \frac{1-F(y)}{\mu}\ dy - \int_0^t \frac{1-F(y)}{\mu}\ dy = 1 - \frac{t}{\mu} + \int_0^t \frac{F(y)}{\mu}\ dy$$

where

$$\int_0^\infty \frac{1-F(y)}{\mu}\ dy = 1.$$

Assuming the variance σ^2 of failure distribution exists, then for $y < \mu$,

$$F(y) \geq \frac{\sigma^2}{\sigma^2 + (\mu-y)^2}\ .$$

Now, considering only the term $\int_0^t \frac{F(y)}{\mu}\ dy$, we see that

$$\int_0^t \frac{F(y)}{\mu}\ dy \leq \frac{1}{\mu} \int_0^t \frac{\sigma^2 dy}{\sigma^2 + (\mu-y)^2}\ .$$

By letting

$$Z = \frac{(\mu-y)}{\sigma},$$

we get

$$\int_0^t \frac{F(y)}{\mu}\, dy = \frac{\sigma}{\mu} \int_{\frac{\mu-t}{\sigma}}^{\mu/\sigma} \frac{dZ}{1+Z^2}$$

$$= \frac{\sigma}{\mu} \left[\arctan\left(\frac{\mu}{\sigma}\right) - \arctan\left(\frac{\mu-t}{\sigma}\right)\right].$$

Using the identity $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$,

the previous expression becomes $\dfrac{\sigma}{\mu} \arctan \left(\dfrac{\frac{\mu}{\sigma} - \frac{\mu-t}{\sigma}}{1 + (\frac{\mu}{\sigma})(\frac{\mu-t}{\sigma})} \right)$

$$= \frac{\sigma}{\mu} \arctan \left(\frac{\frac{\sigma t}{\mu^2}}{[(\frac{\sigma}{\mu})^2 + 1 - \frac{t}{\mu}]} \right) \leq \frac{\sigma}{\mu} \arctan \left(\frac{\frac{\mu t}{\mu^2}}{1 - \frac{t}{\mu}} \right).$$

Since $\arctan Z \leq Z$, then $\displaystyle\int_0^t \frac{F(y)}{\mu}\, dy \leq \frac{\frac{\sigma^2}{\mu}}{\frac{\mu}{t} - 1}$.

Thus, $\tilde{R} = 1 - \dfrac{t}{\mu} + H$ where $H \leq \dfrac{(\frac{\sigma}{\mu})^2}{\frac{\mu}{t} - 1}$

For a fixed t, $\mu = \alpha N$ and $\alpha = O(N)$, which implies

$$H \simeq O\left(\frac{1}{N}\right)$$

Since $O(\frac{1}{N})$ is very small, then

$$R(t,\infty) \simeq \left[1 - \frac{t}{\alpha N} + O\left(\frac{1}{N}\right)\right]^N \simeq \left[1 - \frac{t}{\alpha N}\right]^N \to e^{-\frac{t}{\alpha}} \quad \text{as } N \to \infty.$$

Thus for large, but fixed N

$$R(t,\infty) \simeq e^{-\frac{tN}{\mu}} = e^{-\alpha t}$$

Therefore, the reliability of a complex system tends to approach the same form as that of an exponential distribution.

APPENDIX 2

The print-out of the computer program contained the following variables:

N = number of components in a generation

NG = number of generations

M = number of desired cells of histogram

MM = middle cell number of cell index to start calculating the the estimate of λ^*.

IX = seed value of Gauss and random generators

S = the standard deviation for the Gauss generator

AM = mean time to failure of the distribution for the Gauss generator

TI(J) = the failure time of a component occurring in the Jth string

L = cell index of the histogram which represents one day

NC(L) = the count of the number of failures in the Lth cell of the histogram

C = number of failures in a row of cells

CM = accumulation of failures in the system

R = number of rows of failures that have been added to calculate the estimator, lambda

λ^* = estimation of lambda

MAIN

```
C           SIMULATION OF BAZOVSKY'S EXAMPLE
            DIMENSION TI(10000),NC(16200)
C           INITIALIZE VARIABLES
C           N=NO. OF COMPONENTS,M=NO. OF CELLS OF HISTOGRAM
C           NG=NO. OF GENFRATIONS, MM=CELL NO. TO CALCULATING
C           OUTPUT,IX=SEED VALUE FOR GENERATOR, S=STANDARD
C           DEVIATION, AM=MEAN TIME TO FAILURE, A CELL REPRESENTS
C           24 HOURS OR 1 DAY.
            READ4,N,NG,M,MM
  4         FORMAT(4I5)
            TIME=0.
            IX=40001
            S=600.
            AM=7200.
            DO10I=1,N
            TI(I)=0.
         10 CONTINUE
            DO11 L=1,M
     11     NC(L)=0
C           FAILURE TIME FOR N COMPONENTS WILL BE NORMALLY
C           GENERATED FOR NG GENEPATIONS.
            DO 100 I=1,NG
            DO20 J=1,N
            CALL GAUSS(IX,S,AM,V)
C           FAILURE TIME OF J COMPONENT IS ADDED TO PREVIOUS
C           TIME AND CELL OF THE HISTOGRAM IS DETERMINED AND ONE
C                IS ADDED TO CELL.
            TI(J) = TI(J) + V
            TIME=((TI(J)/24.)-216.)
            L=TIME
            NC(L)=NC(L)+1
     20     CONTINUE
     100    CONTINUE
C           THE OUTPUT OF PROGRAM
C           THE NUMBER OF COMPONENTS AND GENERATIONS AND
C           STARTING PRINTOUT CELL ARE ECHO CHECKED
            PRINT7,N,NG,MM
  7         FORMAT(3I5)
C           HEADING OF OUTPUT
            PRINT 8
  8         FORMAT(6X,9HROW TOTAL,18H FAILURES CUM. SUM,3X,14Hλ*CURRENT EST.)
C           THE ESTIMATION OF LAMBDA OF THE DIST. IS CALCULATED
C           BY DIVIDING CM BY THE MULTIPLE OF T AND R WHERE-
C           CM=CUMMULATION OF FAILURES,T=TIME OF ONE ROW,
C           R=NUMBER OF ROWS
C           λ*=ESTIMATION OF LAMBDA
            I=0
            M=M-MM
            T=24.
```

```
      CM=0.
  2   M=M-24
      I=1+MM
      MM=MM+24
      C=0.
      DO 5 L=I,MM
      R=R+1.
  5   C=NC(L)+C
      CM=CM+C
      λ*=CM/(T*2)
      PRINT 6,C,CM,λ*
6     FORMAT(3F15.5)
      IF(M) 3,3,2
3     CONTINUE
      STOP
      END

C     NORMAL GENERATOR
      SUBROUTINE GAUSS(IX,S,AM,V)
      A=0.0
      DO 50 I=1,12
      CALL RANDU(IX,IY,Y)
      IX=IY
50    A=A+Y
      V=(A-6.0)*S+AM
      RETURN
      END

C     RANDOM NUMBER GENERATOR
      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY) 5,6,6
  5   IY=IY+2147483647+1
6     YFL=IY
      YFL=YFL*.4656613E-9
      RETURN
      END
```

APPENDIX 3

Print-out of the output for a system containing 50 components with 7 generations of replacement where the first 216 cells of the histogram were truncated.

50 7 0

ROW NO.	ROW TOTAL FAILURES	CUM. SUM	λ^* CURRENT EST.
1	1.00000	1.00000	0.00174
	8.00000	9.00000	0.00781
	10.00000	19.00000	0.01100
	17.00000	36.00000	0.01563
	11.00000	47.00000	0.01632
	1.00000	48.00000	0.01389
	2.00000	50.00000	0.01240
	0.0	50.00000	0.01085
	0.0	50.00000	0.00965
10	0.0	50.00000	0.00868
	0.0	50.00000	0.00789
	0.0	50.00000	0.00723
	1.00000	51.00000	0.00681
	4.00000	55.00000	0.00682
	12.00000	67.00000	0.00775
	16.00000	83.00000	0.00901
	8.00000	91.00000	0.00929
	7.00000	98.00000	0.00945
	1.00000	99.00000	0.00905
20	1.00000	100.00000	0.00868
	0.0	100.00000	0.00827
	0.0	100.00000	0.00789
	0.0	100.00000	0.00755
	1.00000	101.00000	0.00731
	2.00000	103.00000	0.00715
	1.00000	104.00000	0.00694
	8.00000	112.00000	0.00720
	15.00000	127.00000	0.00787
	13.00000	140.00000	0.00838
30	5.00000	145.00000	0.00839
	3.00000	148.00000	0.00829
	1.00000	149.00000	0.00808
	1.00000	150.00000	0.00789
	0.0	150.00000	0.00766
	0.0	150.00000	0.00744
	0.0	150.00000	0.00723
	0.0	150.00000	0.00704
	4.00000	154.00000	0.00704
	4.00000	158.00000	0.00703
40	13.00000	171.00000	0.00742
	9.00000	180.00000	0.00762
	9.00000	189.00000	0.00781
	9.00000	198.00000	0.00799
	1.00000	199.00000	0.00785
	1.00000	200.00000	0.00772
	0.0	200.00000	0.00755
	0.0	200.00000	0.00739
	0.0	200.00000	0.00723
	1.00000	201.00000	0.00712
50	2.00000	203.00000	0.00705
	7.00000	210.00000	0.00715
	5.00000	215.00000	0.00718
	10.00000	225.00000	0.00737
	13.00000	238.00000	0.00765
	7.00000	245.00000	0.00773
	3.00000	248.00000	0.00769
	1.00000	249.00000	0.00758

	1.00000	250.00000	0.00748
	0.0	250.00000	0.00736
60	0.0	250.00000	0.00723
	0.0	250.00000	0.00712
	0.0	250.00000	0.00700
	5.00000	255.00000	0.00703
	8.00000	263.00000	0.00713
	11.00000	274.00000	0.00732
	8.00000	282.00000	0.00742
	6.00000	288.00000	0.00746
	8.00000	296.00000	0.00756
	3.00000	299.00000	0.00752
70	1.00000	300.00000	0.00744
	0.0	300.00000	0.00734
	0.0	300.00000	0.00723
	0.0	300.00000	0.00713
	2.00000	302.00000	0.00709
	3.00000	305.00000	0.00706
	5.00000	310.00000	0.00708
	5.00000	315.00000	0.00710
	13.00000	328.00000	0.00730
	9.00000	337.00000	0.00741
80	8.00000	345.00000	0.00749
	3.00000	348.00000	0.00746
	2.00000	350.00000	0.00741
	0.0	350.00000	0.00732
	0.0	350.00000	0.00723
	0.0	350.00000	0.00715
	0.0	350.00000	0.00707
	0.0	350.00000	0.00698
	0.0	350.00000	0.00690
	0.0	350.00000	0.00683
	0.0	350.00000	0.00675
	0.0	350.00000	0.00668
	0.0	350.00000	0.00660
	0.0	350.00000	0.00653
	0.0	350.00000	0.00646
	0.0	350.00000	0.00640
	0.0	350.00000	0.00633
	0.0	350.00000	0.00626
	0.0	350.00000	0.00620
	0.0	350.00000	0.00614
	0.0	350.00000	0.00608
	0.0	350.00000	0.00602
	0.0	350.00000	0.00596
	0.0	350.00000	0.00590
	0.0	350.00000	0.00584
	0.0	350.00000	0.00579
	0.0	350.00000	0.00573
	0.0	350.00000	0.00568
	0.0	350.00000	0.00563
	0.0	350.00000	0.00557
	0.0	350.00000	0.00552
	0.0	350.00000	0.00547
	0.0	350.00000	0.00543
	0.0	350.00000	0.00538
	0.0	350.00000	0.00533
	0.0	350.00000	0.00528
	0.0	350.00000	0.00524
	0.0	350.00000	0.00519

APPENDIX 4

Print-out of the output for a system containing 850 components with 8 generations of replacement where the first 216 cells of the histogram were truncated.

850 8 0

ROW NO.	ROW TOTAL FAILURES	CUM. SUM	λ^* CURRENT EST.
1	6.00000	6.00000	0.01042
	82.00000	88.00000	0.07639
	215.00000	303.00000	0.17535
	323.00000	626.00000	0.27170
	187.00000	813.00000	0.28229
	31.00000	844.00000	0.24421
	6.00000	850.00000	0.21081
	0.0	850.00000	0.18446
	0.0	850.00000	0.16397
10	0.0	850.00000	0.14757
	0.0	850.00000	0.13415
	3.00000	853.00000	0.12341
	15.00000	868.00000	0.11592
	66.00000	934.00000	0.11582
	147.00000	1081.00000	0.12512
	224.00000	1305.00000	0.14160
	199.00000	1504.00000	0.15359
	142.00000	1646.00000	0.15876
	39.00000	1685.00000	0.15397
20	12.00000	1697.00000	0.14731
	3.00000	1700.00000	0.14054
	0.0	1700.00000	0.13415
	1.00000	1701.00000	0.12840
	5.00000	1706.00000	0.12341
	19.00000	1725.00000	0.11979
	52.00000	1777.00000	0.11866
	124.00000	1901.00000	0.12224
	148.00000	2049.00000	0.12705
	185.00000	2234.00000	0.13374
30	156.00000	2390.00000	0.13831
	97.00000	2487.00000	0.13928
	46.00000	2533.00000	0.13742
	15.00000	2548.00000	0.13405
	2.00000	2550.00000	0.13021
	0.0	2550.00000	0.12649
	5.00000	2555.00000	0.12322
	27.00000	2582.00000	0.12115
	39.00000	2621.00000	0.11975
	82.00000	2703.00000	0.12033
40	123.00000	2826.00000	0.12266
	151.00000	2977.00000	0.12606
	155.00000	3132.00000	0.12946
	122.00000	3254.00000	0.13138
	81.00000	3335.00000	0.13159
	46.00000	3381.00000	0.13044
	16.00000	3397.00000	0.12821
	4.00000	3401.00000	0.12563
	7.00000	3408.00000	0.12326
	16.00000	3424.00000	0.12132
50	33.00000	3457.00000	0.12003
	61.00000	3518.00000	0.11976
	104.00000	3622.00000	0.12093
	143.00000	3765.00000	0.12333
	124.00000	3889.00000	0.12503
	140.00000	4029.00000	0.12718
	101.00000	4130.00000	0.12804
	66.00000	4196.00000	0.12780

	32.00000	4228.00000	0.12656
	20.00000	4248.00000	0.12500
60	11.00000	4259.00000	0.12323
	12.00000	4271.00000	0.12156
	28.00000	4299.00000	0.12038
	59.00000	4358.00000	0.12009
	73.00000	4431.00000	0.12020
	116.00000	4547.00000	0.12145
	122.00000	4669.00000	0.12282
	140.00000	4809.00000	0.12461
	109.00000	4918.00000	0.12556
	82.00000	5000.00000	0.12581
70	47.00000	5047.00000	0.12517
	38.00000	5085.00000	0.12434
	17.00000	5102.00000	0.12302
	12.00000	5114.00000	0.12162
	23.00000	5137.00000	0.12052
	47.00000	5184.00000	0.12000
	76.00000	5260.00000	0.12016
	88.00000	5348.00000	0.12058
	115.00000	5463.00000	0.12159
	122.00000	5585.00000	0.12274
80	113.00000	5698.00000	0.12365
	88.00000	5786.00000	0.12401
	71.00000	5857.00000	0.12400
	57.00000	5914.00000	0.12370
	24.00000	5938.00000	0.12273
	28.00000	5966.00000	0.12185
	13.00000	5979.00000	0.12070
	37.00000	6016.00000	0.12005
	56.00000	6072.00000	0.11979
	79.00000	6151.00000	0.11999
90	108.00000	6259.00000	0.12074
	100.00000	6359.00000	0.12132
	114.00000	6473.00000	0.12215
	97.00000	6570.00000	0.12265
	93.00000	6663.00000	0.12306
	55.00000	6718.00000	0.12277
	40.00000	6758.00000	0.12221
	25.00000	6783.00000	0.12140
	14.00000	6797.00000	0.12041
	2.00000	6799.00000	0.11923
100	0.0	6799.00000	0.11804
	1.00000	6800.00000	0.11689
	0.0	6800.00000	0.11574
	0.0	6800.00000	0.11462
	0.0	6800.00000	0.11351
	0.0	6800.00000	0.11243
	0.0	6800.00000	0.11137
	0.0	6800.00000	0.11033
	0.0	6800.00000	0.10931
	0.0	6800.00000	0.10831
	0.0	6800.00000	0.10732
	0.0	6800.00000	0.10636
	0.0	6800.00000	0.10541
	0.0	6800.00000	0.10447
	0.0	6800.00000	0.10356
	0.0	6800.00000	0.10266
	0.0	6800.00000	0.10177
	0.0	6800.00000	0.10090

APPENDIX 5

Print-out of the output for a system containing 1000 components with 20 generations of replacement where the first 216 cells of the histogram were truncated.

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

1000 20 0

ROW NO.	ROW TOTAL FAILURES	CUM. SUM	λ^* CURRENT EST.
1	6.00000	6.00000	0.01042
	94.00000	100.00000	0.06661
	256.00000	356.00000	0.20662
	375.00000	731.00000	0.31727
	221.00000	952.00000	0.33056
	41.00000	993.00000	0.28733
	7.00000	1000.00000	0.24602
	0.0	1000.00000	0.21701
	0.0	1000.00000	0.19290
10	0.0	1000.00000	0.17361
	2.00000	1002.00000	0.15814
	3.00000	1005.00000	0.14540
	19.00000	1024.00000	0.13675
	65.00000	1089.00000	0.13504
	189.00000	1278.00000	0.14792
	264.00000	1542.00000	0.16732
	240.00000	1782.00000	0.18199
	140.00000	1922.00000	0.18558
	58.00000	1980.00000	0.18092
20	18.00000	1998.00000	0.17344
	2.00000	2000.00000	0.16534
	0.0	2000.00000	0.15783
	2.00000	2002.00000	0.15112
	8.00000	2010.00000	0.14540
	23.00000	2033.00000	0.14118
	52.00000	2085.00000	0.13922
	130.00000	2215.00000	0.14243
	195.00000	2410.00000	0.14943
	232.00000	2642.00000	0.15617
30	174.00000	2816.00000	0.16296
	109.00000	2925.00000	0.16381
	46.00000	2971.00000	0.16119
	28.00000	2999.00000	0.15778
	1.00000	3000.00000	0.15519
	3.00000	3003.00000	0.14896
	9.00000	3012.00000	0.14525
	16.00000	3028.00000	0.14208
	55.00000	3083.00000	0.14085
	90.00000	3173.00000	0.14125
40	146.00000	3319.00000	0.14405
	182.00000	3501.00000	0.14625

	41.00000	4975.00000	0.14692
	16.00000	4991.00000	0.14666
60	14.00000	5005.00000	0.14482
	17.00000	5022.00000	0.14293
	38.00000	5060.00000	0.14169
	57.00000	5117.00000	0.14101
	99.00000	5216.00000	0.14149
	111.00000	5327.00000	0.14228
	149.00000	5476.00000	0.14404
	177.00000	5653.00000	0.14648
	131.00000	5784.00000	0.14767
	99.00000	5883.00000	0.14802
70	63.00000	5946.00000	0.14747
	28.00000	5974.00000	0.14608
	27.00000	6001.00000	0.14470
	13.00000	6014.00000	0.14303
	30.00000	6044.00000	0.14180
	59.00000	6103.00000	0.14127
	74.00000	6177.00000	0.14110
	111.00000	6288.00000	0.14177
	138.00000	6426.00000	0.14303
	130.00000	6556.00000	0.14408
80	145.00000	6701.00000	0.14542
	115.00000	6816.00000	0.14609
	89.00000	6905.00000	0.14619
	51.00000	6956.00000	0.14550
	29.00000	6985.00000	0.14437
	25.00000	7010.00000	0.14318
	36.00000	7046.00000	0.14224
	40.00000	7086.00000	0.14140
	65.00000	7151.00000	0.14108
	88.00000	7239.00000	0.14121
90	120.00000	7359.00000	0.14196
	124.00000	7483.00000	0.14276
	133.00000	7616.00000	0.14372
	137.00000	7753.00000	0.14473
	90.00000	7843.00000	0.14485
	81.00000	7924.00000	0.14481
	39.00000	7963.00000	0.14401
	30.00000	7993.00000	0.14306
	36.00000	8029.00000	0.14224
	43.00000	8072.00000	0.14155
100	61.00000	8133.00000	0.14120
	73.00000	8206.00000	0.14105
	112.00000	8318.00000	0.14158
	115.00000	8433.00000	0.14214
	124.00000	8557.00000	0.14285
	124.00000	8681.00000	0.14354
	110.00000	8791.00000	0.14396
	90.00000	8881.00000	0.14410
	66.00000	8947.00000	0.14382
	34.00000	8981.00000	0.14305
110	31.00000	9012.00000	0.14223
	46.00000	9058.00000	0.14167
	38.00000	9096.00000	0.14100
	87.00000	9183.00000	0.14109
	84.00000	9267.00000	0.14113
	111.00000	9378.00000	0.14158
	130.00000	9513.00000	0.14230
	116.00000	9624.00000	0.14261

	109.00000	9732.00000	0.14320
	109.00000	9841.00000	0.14337
120	55.00000	9896.00000	0.14317
	74.00000	9970.00000	0.14305
	35.00000	10005.00000	0.14238
	39.00000	10044.00000	0.14177
	37.00000	10081.00000	0.14114
	61.00000	10142.00000	0.14086
	87.00000	10229.00000	0.14094
	104.00000	10333.00000	0.14125
	102.00000	10435.00000	0.14153
	123.00000	10558.00000	0.14209
130	112.00000	10670.00000	0.14249
	107.00000	10777.00000	0.14262
	99.00000	10876.00000	0.14305
	54.00000	10930.00000	0.14267
	47.00000	10977.00000	0.14222
	46.00000	11023.00000	0.14176
	45.00000	11068.00000	0.14129
	61.00000	11129.00000	0.14103
	60.00000	11189.00000	0.14076
	98.00000	11287.00000	0.14097
140	101.00000	11388.00000	0.14122
	114.00000	11502.00000	0.14162
	115.00000	11617.00000	0.14203
	102.00000	11719.00000	0.14228
	94.00000	11813.00000	0.14242
	74.00000	11887.00000	0.14233
	72.00000	11959.00000	0.14221
	48.00000	12007.00000	0.14181
	43.00000	12050.00000	0.14135
	55.00000	12105.00000	0.14104
150	72.00000	12177.00000	0.14094
	81.00000	12258.00000	0.14094
	99.00000	12357.00000	0.14114
	101.00000	12458.00000	0.14136
	116.00000	12574.00000	0.14175
	96.00000	12670.00000	0.14191
	105.00000	12775.00000	0.14217
	80.00000	12855.00000	0.14215
	76.00000	12931.00000	0.14209
	49.00000	12980.00000	0.14173
160	35.00000	13015.00000	0.14122
	54.00000	13069.00000	0.14093
	85.00000	13154.00000	0.14097
	84.00000	13238.00000	0.14100
	77.00000	13315.00000	0.14095
	90.00000	13405.00000	0.14105
	111.00000	13516.00000	0.14136
	100.00000	13616.00000	0.14155
	109.00000	13725.00000	0.14183
	77.00000	13802.00000	0.14179
170	82.00000	13884.00000	0.14179
	68.00000	13952.00000	0.14165
	53.00000	14005.00000	0.14136
	37.00000	14042.00000	0.14092
	69.00000	14111.00000	0.14079
	78.00000	14189.00000	0.14076
	93.00000	14282.00000	0.14058
	86.00000	14368.00000	0.14093

	95.00000	14463.00000	0.14106
	110.00000	14573.00000	0.14134
180	92.00000	14665.00000	0.14144
	95.00000	14760.00000	0.14157
	78.00000	14838.00000	0.14154
	76.00000	14914.00000	0.14149
	56.00000	14970.00000	0.14125
	58.00000	15028.00000	0.14103
	60.00000	15088.00000	0.14083
	83.00000	15171.00000	0.14085
	63.00000	15234.00000	0.14068
	92.00000	15326.00000	0.14078
190	87.00000	15413.00000	0.14084
	101.00000	15514.00000	0.14102
	102.00000	15616.00000	0.14120
	110.00000	15726.00000	0.14146
	69.00000	15795.00000	0.14135
	77.00000	15872.00000	0.14131
	63.00000	15935.00000	0.14115
	60.00000	15995.00000	0.14096
	65.00000	16060.00000	0.14082
	66.00000	16126.00000	0.14069
200	80.00000	16206.00000	0.14068
	96.00000	16302.00000	0.14061
	89.00000	16391.00000	0.14067
	77.00000	16468.00000	0.14084
	98.00000	16566.00000	0.14098
	88.00000	16654.00000	0.14104
	92.00000	16746.00000	0.14113
	73.00000	16819.00000	0.14105
	80.00000	16899.00000	0.14105
	71.00000	16970.00000	0.14097
210	71.00000	17041.00000	0.14088
	62.00000	17103.00000	0.14072
	72.00000	17175.00000	0.14065
	96.00000	17271.00000	0.14077
	68.00000	17339.00000	0.14067
	95.00000	17434.00000	0.14078
	85.00000	17519.00000	0.14081
	98.00000	17617.00000	0.14095
	88.00000	17705.00000	0.14100
	79.00000	17784.00000	0.14090
220	77.00000	17861.00000	0.14095
	61.00000	17922.00000	0.14079
	75.00000	17997.00000	0.14074
	74.00000	18071.00000	0.14069
	75.00000	18146.00000	0.14064
	85.00000	18231.00000	0.14067
	73.00000	18304.00000	0.14061
	78.00000	18382.00000	0.14059
	97.00000	18479.00000	0.14071
	91.00000	18570.00000	0.14078
230	92.00000	18662.00000	0.14087
	72.00000	18734.00000	0.14080
	89.00000	18823.00000	0.14086
	75.00000	18898.00000	0.14081
	73.00000	18971.00000	0.14075
	69.00000	19040.00000	0.14066
	72.00000	19112.00000	0.14060
	83.00000	19195.00000	0.14061

	78.00000	19273.00000	0.14059
	86.00000	19359.00000	0.14063
240	65.00000	19424.00000	0.14061
	92.00000	19516.00000	0.14059
	91.00000	19607.00000	0.14056
	72.00000	19679.00000	0.14060
	70.00000	19749.00000	0.14052
	65.00000	19814.00000	0.14041
	48.00000	19862.00000	0.14017
	44.00000	19906.00000	0.13992
	35.00000	19941.00000	0.13960
	17.00000	19958.00000	0.13915
	18.00000	19976.00000	0.13872
	11.00000	19987.00000	0.13825
	4.00000	19991.00000	0.13772
	3.00000	19994.00000	0.13720
	3.00000	19997.00000	0.13668
	2.00000	19999.00000	0.13616

APPENDIX 6

Print-out of the output for a system containing 200 components with 20 generations of replacement where the first 216 cells of the histogram were truncated.

200	20	0	
ROW NO.	ROW TOTAL FAILURES	CUM. SUM	λ^* CURRENT EST.
1	2.00^00	2.00000	0.00347
	18.00^00	20.00000	0.01736
	51.00^00	71.00000	0.04109
	80.00^00	151.00000	0.06554
	41.00^00	192.00000	0.06667
	5.00^00	197.00000	0.05700
	3.00^00	200.00000	0.04960
	0.0	200.00000	0.04340
	0.0	200.00000	0.03858
10	0.0	200.00000	0.03472
	0.0	200.00000	0.03157
	1.00^00	201.00000	0.02908
	3.00^00	204.00000	0.02724
	13.00^00	217.00000	0.02691
	35.00^00	252.00000	0.02917
	69.00^00	321.00000	0.03483
	44.00^00	365.00000	0.03728
	26.00^00	391.00000	0.03771
	7.00^00	398.00000	0.03637
20	2.00^00	400.00000	0.03472
	0.0	400.00000	0.03307
	0.0	400.00000	0.03157
	0.0	400.00000	0.03019
	0.0	400.00000	0.02894
	4.00^00	404.00000	0.02806
	13.00^00	417.00000	0.02784
	33.00^00	450.00000	0.02894
	47.00^00	497.00000	0.03082
	52.00^00	549.00000	0.03287
30	26.00^00	575.00000	0.03328
	18.00^00	593.00000	0.03321
	6.00^00	599.00000	0.03250
	1.00^00	600.00000	0.03157
	0.0	600.00000	0.03064
	0.0	600.00000	0.02976
	0.0	600.00000	0.02894
	3.00^00	603.00000	0.02829
	8.00^00	611.00000	0.02791
	25.00^00	636.00000	0.02831
40	39.00^00	675.00000	0.02930
	42.00^00	717.00000	0.03036
	33.00^00	750.00000	0.03100
	29.00^00	779.00000	0.03145
	12.00^00	791.00000	0.03121
	6.00^00	797.00000	0.03075
	3.00^00	800.00000	0.03019
	1.00^00	801.00000	0.02959
	1.00^00	802.00000	0.02901
	4.00^00	806.00000	0.02856
50	6.00^00	812.00000	0.02819
	21.00^00	833.00000	0.02836
	27.00^00	860.00000	0.02871
	43.00^00	903.00000	0.02958
	39.00^00	942.00000	0.03029
	24.00^00	966.00000	0.03049
	14.00^00	980.00000	0.03038
	11.00^00	991.00000	0.03018

	5.00^00	996.00000	0.02981
	5.00^00	1001.00000	0.02946
60	1.00^00	1002.00000	0.02899
	2.00^00	1004.00000	0.02857
	4.00^00	1008.00000	0.02823
	18.00^00	1026.00000	0.02827
	22.00^00	1048.00000	0.02843
	28.00^00	1076.00000	0.02874
	39.00^00	1115.00000	0.02933
	29.00^00	1144.00000	0.02964
	25.00^00	1169.00000	0.02985
	18.00^00	1187.00000	0.02987
70	6.00^00	1193.00000	0.02959
	7.00^00	1200.00000	0.02934
	1.00^00	1201.00000	0.02896
	5.00^00	1206.00000	0.02868
	4.00^00	1210.00000	0.02839
	9.00^00	1219.00000	0.02822
	18.00^00	1237.00000	0.02826
	33.00^00	1270.00000	0.02863
	30.00^00	1300.00000	0.02894
	31.00^00	1331.00000	0.02925
80	27.00^00	1358.00000	0.02947
	17.00^00	1375.00000	0.02947
	16.00^00	1391.00000	0.02945
	6.00^00	1397.00000	0.02922
	4.00^00	1401.00000	0.02896
	3.00^00	1404.00000	0.02868
	3.00^00	1407.00000	0.02840
	12.00^00	1419.00000	0.02832
	13.00^00	1432.00000	0.02825
	21.00^00	1453.00000	0.02834
90	32.00^00	1485.00000	0.02865
	31.00^00	1516.00000	0.02892
	21.00^00	1537.00000	0.02900
	29.00^00	1566.00000	0.02923
	17.00^00	1583.00000	0.02924
	12.00^00	1595.00000	0.02915
	3.00^00	1598.00000	0.02890
	6.00^00	1604.00000	0.02871
	5.00^00	1609.00000	0.02850
	7.00^00	1616.00000	0.02834
100	12.00^00	1628.00000	0.02826
	20.00^00	1648.00000	0.02833
	21.00^00	1669.00000	0.02841
	26.00^00	1695.00000	0.02857
	31.00^00	1726.00000	0.02881
	24.00^00	1750.00000	0.02894
	18.00^00	1768.00000	0.02896
	16.00^00	1784.00000	0.02895
	12.00^00	1796.00000	0.02887
	4.00^00	1800.00000	0.02867
110	5.00^00	1805.00000	0.02849
	7.00^00	1812.00000	0.02834
	12.00^00	1824.00000	0.02827
	13.00^00	1837.00000	0.02822
	19.00^00	1856.00000	0.02827
	33.00^00	1889.00000	0.02852
	22.00^00	1911.00000	0.02860
	25.00^00	1936.00000	0.02873

	21.00^00	1957.00000	0.02879
	16.00^00	1973.00000	0.02878
120	16.00^00	1989.00000	0.02878
	9.00^00	1998.00000	0.02867
	10.00^00	2008.00000	0.02857
	3.00^00	2011.00000	0.02838
	5.00^00	2016.00000	0.02823
	15.00^00	2031.00000	0.02821
	15.00^00	2046.00000	0.02819
	30.00^00	2076.00000	0.02838
	16.00^00	2092.00000	0.02837
	26.00^00	2118.00000	0.02850
130	27.00^00	2145.00000	0.02865
	19.00^00	2164.00000	0.02868
	16.00^00	2180.00000	0.02867
	13.00^00	2193.00000	0.02863
	8.00^00	2201.00000	0.02852
	8.00^00	2209.00000	0.02841
	6.00^00	2215.00000	0.02828
	9.00^00	2224.00000	0.02818
	13.00^00	2237.00000	0.02814
	21.00^00	2258.00000	0.02820
140	21.00^00	2279.00000	0.02826
	30.00^00	2309.00000	0.02843
	21.00^00	2330.00000	0.02849
	23.00^00	2353.00000	0.02857
	17.00^00	2370.00000	0.02857
	15.00^00	2385.00000	0.02856
	15.00^00	2400.00000	0.02854
	5.00^00	2405.00000	0.02840
	4.00^00	2409.00000	0.02826
	13.00^00	2422.00000	0.02822
150	10.00^00	2432.00000	0.02815
	15.00^00	2447.00000	0.02813
	27.00^00	2474.00000	0.02826
	24.00^00	2498.00000	0.02835
	23.00^00	2521.00000	0.02842
	15.00^00	2536.00000	0.02841
	26.00^00	2562.00000	0.02851
	18.00^00	2580.00000	0.02853
	15.00^00	2595.00000	0.02851
	8.00^00	2603.00000	0.02842
160	6.00^00	2609.00000	0.02831
	6.00^00	2615.00000	0.02820
	15.00^00	2630.00000	0.02819
	10.00^00	2640.00000	0.02812
	21.00^00	2661.00000	0.02817
	19.00^00	2680.00000	0.02820
	23.00^00	2703.00000	0.02827
	29.00^00	2732.00000	0.02840
	13.00^00	2745.00000	0.02837
	24.00^00	2769.00000	0.02845
170	17.00^00	2786.00000	0.02845
	14.00^00	2800.00000	0.02843
	3.00^00	2803.00000	0.02829
	10.00^00	2813.00000	0.02823
	10.00^00	2823.00000	0.02817
	12.00^00	2835.00000	0.02812
	18.00^00	2853.00000	0.02814
	16.00^00	2869.00000	0.02814

	29.00000	2898.00000	0.02827
	20.00000	2918.00000	0.02830
180	15.00000	2933.00000	0.02829
	20.00000	2953.00000	0.02832
	20.00000	2973.00000	0.02836
	17.00000	2990.00000	0.02837
	10.00000	3000.00000	0.02831
	12.00000	3012.00000	0.02827
	5.00000	3017.00000	0.02816
	13.00000	3030.00000	0.02813
	13.00000	3043.00000	0.02810
	21.00000	3064.00000	0.02815
190	23.00000	3087.00000	0.02821
	18.00000	3105.00000	0.02822
	18.00000	3123.00000	0.02824
	14.00000	3137.00000	0.02822
	21.00000	3158.00000	0.02826
	21.00000	3179.00000	0.02830
	15.00000	3194.00000	0.02829
	9.00000	3203.00000	0.02823
	11.00000	3214.00000	0.02818
	9.00000	3223.00000	0.02812
200	12.00000	3235.00000	0.02808
	17.00000	3252.00000	0.02809
	22.00000	3274.00000	0.02814
	16.00000	3290.00000	0.02814
	17.00000	3307.00000	0.02814
	19.00000	3326.00000	0.02817
	18.00000	3344.00000	0.02818
	21.00000	3365.00000	0.02822
	25.00000	3390.00000	0.02830
	9.00000	3399.00000	0.02823
210	5.00000	3404.00000	0.02814
	14.00000	3418.00000	0.02812
	9.00000	3427.00000	0.02806
	18.00000	3445.00000	0.02808
	20.00000	3465.00000	0.02811
	16.00000	3481.00000	0.02811
	21.00000	3502.00000	0.02815
	17.00000	3519.00000	0.02815
	23.00000	3542.00000	0.02821
	17.00000	3559.00000	0.02821
220	18.00000	3577.00000	0.02823
	16.00000	3593.00000	0.02823
	10.00000	3603.00000	0.02818
	12.00000	3615.00000	0.02814
	7.00000	3622.00000	0.02807
	12.00000	3634.00000	0.02804
	22.00000	3656.00000	0.02809
	17.00000	3673.00000	0.02809
	17.00000	3690.00000	0.02810
	17.00000	3707.00000	0.02810
230	21.00000	3728.00000	0.02814
	20.00000	3748.00000	0.02817
	15.00000	3763.00000	0.02816
	16.00000	3779.00000	0.02816
	13.00000	3792.00000	0.02813
	17.00000	3809.00000	0.02814
	12.00000	3821.00000	0.02811
	14.00000	3835.00000	0.02809

	14.00000	3849.00000	0.02808
	16.00000	3865.00000	0.02808
240	17.00000	3882.00000	0.02808
	12.00000	3894.00000	0.02805
	18.00000	3912.00000	0.02806
	19.00000	3931.00000	0.02808
	15.00000	3946.00000	0.02808
	14.00000	3960.00000	0.02806
	11.00000	3971.00000	0.02802
	14.00000	3985.00000	0.02801
	6.00000	3991.00000	0.02794
	3.00000	3994.00000	0.02785
250	3.00000	3997.00000	0.02776
	0.0	3997.00000	0.02765
	1.00000	3998.00000	0.02754
	1.00000	3999.00000	0.02744
	1.00000	4000.00000	0.02734
	0.0	4000.00000	0.02723

APPENDIX 7

Print-out of the output for a system containing 200 components with 50 generations of replacement where the first 216 cells of the histogram are truncated.

Fig. 3(a) shows the ROW TOTAL from row 1 to row 60

Fig. 3(b) shows the ROW TOTAL from row 238 to row 357

Fig. 3(c) shows that ROW TOTAL from row 400 to row 537

The data summarized as 0_1 in the χ^2 test of the Table 3 start at row 238 and go to row 417.

The data summarized as 0_2 in the χ^2 test of the Table 3 start at row 418 to row 597.

200 50 0

ROW NO.	ROW TOTAL FAILURES	CUM. SUM	λ^* CURRENT EST.	
1	2.00000	2.00000	0.00347	← FIG. 3(a) STARTS HERE
	18.00000	20.00000	0.01736	
	51.00000	71.00000	0.04109	
	80.00000	151.00000	0.06554	
	41.00000	192.00000	0.06667	
	5.00000	197.00000	0.05700	
	3.00000	200.00000	0.04960	
	0.0	200.00000	0.04340	
	0.0	200.00000	0.03858	
10	0.0	200.00000	0.03472	
	0.0	200.00000	0.03157	
	1.00000	201.00000	0.02908	
	3.00000	204.00000	0.02724	
	13.00000	217.00000	0.02691	
	35.00000	252.00000	0.02917	
	69.00000	321.00000	0.03483	
	44.00000	365.00000	0.03728	
	26.00000	391.00000	0.03771	
	7.00000	398.00000	0.03637	
20	2.00000	400.00000	0.03472	
	0.0	400.00000	0.03307	
	0.0	400.00000	0.03157	
	0.0	400.00000	0.03019	
	0.0	400.00000	0.02894	
	4.00000	404.00000	0.02806	
	13.00000	417.00000	0.02784	
	33.00000	450.00000	0.02894	
	47.00000	497.00000	0.03082	
	52.00000	549.00000	0.03287	
30	26.00000	575.00000	0.03328	
	18.00000	593.00000	0.03321	
	6.00000	599.00000	0.03250	
	1.00000	600.00000	0.03157	
	0.0	600.00000	0.03064	
	0.0	600.00000	0.02976	
	0.0	600.00000	0.02894	
	3.00000	603.00000	0.02829	
	8.00000	611.00000	0.02791	
	25.00000	636.00000	0.02831	
40	39.00000	675.00000	0.02930	
	42.00000	717.00000	0.03036	
	33.00000	750.00000	0.03100	
	29.00000	779.00000	0.03145	
	12.00000	791.00000	0.03121	
	6.00000	797.00000	0.03075	
	3.00000	800.00000	0.03019	
	1.00000	801.00000	0.02959	
	1.00000	802.00000	0.02901	
	4.00000	806.00000	0.02856	
50	6.00000	812.00000	0.02819	
	21.00000	833.00000	0.02836	
	27.00000	860.00000	0.02871	
	43.00000	903.00000	0.02958	
	39.00000	942.00000	0.03029	
	24.00000	966.00000	0.03049	
	14.00000	980.00000	0.03038	
	11.00000	991.00000	0.03018	

	5.00000	996.00000	0.02981
	5.00000	1001.00000	0.02946
60	1.00000	1002.00000	0.02899
	2.00000	1004.00000	0.02857
	4.00000	1008.00000	0.02823
	18.00000	1026.00000	0.02827
	22.00000	1048.00000	0.02843
	28.00000	1076.00000	0.02874
	39.00000	1115.00000	0.02933
	29.00000	1144.00000	0.02964
	25.00000	1169.00000	0.02985
	18.00000	1187.00000	0.02987
70	6.00000	1193.00000	0.02959
	7.00000	1200.00000	0.02934
	1.00000	1201.00000	0.02896
	5.00000	1206.00000	0.02868
	4.00000	1210.00000	0.02839
	9.00000	1219.00000	0.02822
	18.00000	1237.00000	0.02826
	33.00000	1270.00000	0.02863
	30.00000	1300.00000	0.02894
	31.00000	1331.00000	0.02925
80	27.00000	1358.00000	0.02947
	17.00000	1375.00000	0.02947
	16.00000	1391.00000	0.02945
	6.00000	1397.00000	0.02922
	4.00000	1401.00000	0.02896
	3.00000	1404.00000	0.02868
	3.00000	1407.00000	0.02840
	12.00000	1419.00000	0.02832
	13.00000	1432.00000	0.02825
	21.00000	1453.00000	0.02834
90	32.00000	1485.00000	0.02865
	31.00000	1516.00000	0.02892
	21.00000	1537.00000	0.02900
	29.00000	1566.00000	0.02923
	17.00000	1583.00000	0.02924
	12.00000	1595.00000	0.02915
	3.00000	1598.00000	0.02890
	6.00000	1604.00000	0.02871
	5.00000	1609.00000	0.02850
	7.00000	1616.00000	0.02834
100	12.00000	1628.00000	0.02826
	20.00000	1648.00000	0.02833
	21.00000	1669.00000	0.02841
	26.00000	1695.00000	0.02857
	31.00000	1726.00000	0.02881
	24.00000	1750.00000	0.02894
	18.00000	1768.00000	0.02896
	16.00000	1784.00000	0.02895
	12.00000	1796.00000	0.02887
	4.00000	1800.00000	0.02867
110	5.00000	1805.00000	0.02849
	7.00000	1812.00000	0.02834
	12.00000	1824.00000	0.02827
	13.00000	1837.00000	0.02822
	19.00000	1856.00000	0.02827
	33.00000	1889.00000	0.02852
	22.00000	1911.00000	0.02860
	25.00000	1936.00000	0.02873

	21.00000	1957.00000	0.02879	
	16.00000	1973.00000	0.02878	
120	16.00000	1989.00000	0.02878	← END OF FIG. 3(a) HERE
	9.00000	1998.00000	0.02867	
	10.00000	2008.00000	0.02857	
	3.00000	2011.00000	0.02838	
	5.00000	2016.00000	0.02823	
	15.00000	2031.00000	0.02821	
	15.00000	2046.00000	0.02819	
	30.00000	2076.00000	0.02838	
	16.00000	2092.00000	0.02837	
	26.00000	2118.00000	0.02850	
130	27.00000	2145.00000	0.02865	
	19.00000	2164.00000	0.02868	
	16.00000	2180.00000	0.02867	
	13.00000	2193.00000	0.02863	
	8.00000	2201.00000	0.02852	
	8.00000	2209.00000	0.02841	
	6.00000	2215.00000	0.02828	
	9.00000	2224.00000	0.02818	
	13.00000	2237.00000	0.02814	
	21.00000	2258.00000	0.02820	
140	21.00000	2279.00000	0.02826	
	30.00000	2309.00000	0.02843	
	21.00000	2330.00000	0.02849	
	23.00000	2353.00000	0.02857	
	17.00000	2370.00000	0.02857	
	15.00000	2385.00000	0.02856	
	15.00000	2400.00000	0.02854	
	5.00000	2405.00000	0.02840	
	4.00000	2409.00000	0.02826	
	13.00000	2422.00000	0.02822	
150	10.00000	2432.00000	0.02815	
	15.00000	2447.00000	0.02813	
	27.00000	2474.00000	0.02826	
	24.00000	2498.00000	0.02835	
	23.00000	2521.00000	0.02842	
	15.00000	2536.00000	0.02841	
	26.00000	2562.00000	0.02851	
	18.00000	2580.00000	0.02853	
	15.00000	2595.00000	0.02851	
	8.00000	2603.00000	0.02842	
160	6.00000	2609.00000	0.02831	
	6.00000	2615.00000	0.02820	
	15.00000	2630.00000	0.02819	
	10.00000	2640.00000	0.02812	
	21.00000	2661.00000	0.02817	
	19.00000	2680.00000	0.02820	
	23.00000	2703.00000	0.02827	
	29.00000	2732.00000	0.02840	
	13.00000	2745.00000	0.02837	
	24.00000	2769.00000	0.02845	
170	17.00000	2786.00000	0.02845	
	14.00000	2800.00000	0.02843	
	3.00000	2803.00000	0.02829	
	10.00000	2813.00000	0.02823	
	10.00000	2823.00000	0.02817	
	12.00000	2835.00000	0.02812	
	18.00000	2853.00000	0.02814	
	16.00000	2869.00000	0.02814	

	29.00000	2898.00000	0.02827
	20.00000	2918.00000	0.02830
180	15.00000	2933.00000	0.02829
	20.00000	2953.00000	0.02832
	20.00000	2973.00000	0.02836
	17.00000	2990.00000	0.02837
	10.00000	3000.00000	0.02831
	12.00000	3012.00000	0.02827
	5.00000	3017.00000	0.02816
	13.00000	3030.00000	0.02813
	13.00000	3043.00000	0.02810
	21.00000	3064.00000	0.02815
190	23.00000	3087.00000	0.02821
	18.00000	3105.00000	0.02822
	18.00000	3123.00000	0.02824
	14.00000	3137.00000	0.02822
	21.00000	3158.00000	0.02826
	21.00000	3179.00000	0.02830
	15.00000	3194.00000	0.02829
	9.00000	3203.00000	0.02823
	11.00000	3214.00000	0.02818
	9.00000	3223.00000	0.02812
200	12.00000	3235.00000	0.02808
	17.00000	3252.00000	0.02809
	22.00000	3274.00000	0.02814
	16.00000	3290.00000	0.02814
	17.00000	3307.00000	0.02814
	19.00000	3326.00000	0.02817
	18.00000	3344.00000	0.02818
	21.00000	3365.00000	0.02822
	25.00000	3390.00000	0.02830
	9.00000	3399.00000	0.02823
210	5.00000	3404.00000	0.02814
	14.00000	3418.00000	0.02812
	9.00000	3427.00000	0.02806
	18.00000	3445.00000	0.02808
	20.00000	3465.00000	0.02811
	16.00000	3481.00000	0.02811
	21.00000	3502.00000	0.02815
	17.00000	3519.00000	0.02815
	23.00000	3542.00000	0.02821
	17.00000	3559.00000	0.02821
220	18.00000	3577.00000	0.02823
	16.00000	3593.00000	0.02823
	10.00000	3603.00000	0.02818
	12.00000	3615.00000	0.02814
	7.00000	3622.00000	0.02807
	12.00000	3634.00000	0.02804
	22.00000	3656.00000	0.02809
	17.00000	3673.00000	0.02809
	17.00000	3690.00000	0.02810
	17.00000	3707.00000	0.02810
230	21.00000	3728.00000	0.02814
	20.00000	3748.00000	0.02817
	15.00000	3763.00000	0.02816
	16.00000	3779.00000	0.02816
	13.00000	3792.00000	0.02813
	17.00000	3809.00000	0.02814
	12.00000	3821.00000	0.02811
	14.00000	3835.00000	0.02809

	14.00000	3849.00000	0.02808	← FIG. 3(b) AND O_1 COUNTS START HERE
	16.00000	3865.00000	0.02808	
240	17.00000	3882.00000	0.02808	
	12.00000	3894.00000	0.02805	
	21.00000	3915.00000	0.02809	
	21.00000	3936.00000	0.02812	
	16.00000	3952.00000	0.02812	
	16.00000	3968.00000	0.02812	
	14.00000	3982.00000	0.02810	
	19.00000	4001.00000	0.02812	
	14.00000	4015.00000	0.02811	
	16.00000	4031.00000	0.02811	
250	15.00000	4046.00000	0.02810	
	6.00000	4052.00000	0.02803	
	15.00000	4067.00000	0.02802	
	15.00000	4082.00000	0.02801	
	23.00000	4105.00000	0.02806	
	18.00000	4123.00000	0.02807	
	21.00000	4144.00000	0.02810	
	12.00000	4156.00000	0.02808	
	15.00000	4171.00000	0.02807	
	17.00000	4188.00000	0.02807	
260	13.00000	4201.00000	0.02805	
	19.00000	4220.00000	0.02807	
	13.00000	4233.00000	0.02805	
	20.00000	4253.00000	0.02807	
	15.00000	4268.00000	0.02807	
	9.00000	4277.00000	0.02802	
	18.00000	4295.00000	0.02803	
	18.00000	4313.00000	0.02804	
	19.00000	4332.00000	0.02806	
	16.00000	4348.00000	0.02806	
270	18.00000	4366.00000	0.02807	
	18.00000	4384.00000	0.02809	
	16.00000	4400.00000	0.02808	
	9.00000	4409.00000	0.02804	
	24.00000	4433.00000	0.02809	
	19.00000	4452.00000	0.02811	
	13.00000	4465.00000	0.02809	
	6.00000	4471.00000	0.02802	
	17.00000	4488.00000	0.02803	
	16.00000	4504.00000	0.02803	
280	13.00000	4517.00000	0.02801	
	21.00000	4538.00000	0.02804	
	20.00000	4558.00000	0.02806	
	15.00000	4573.00000	0.02805	
	16.00000	4589.00000	0.02805	
	14.00000	4603.00000	0.02804	
	17.00000	4620.00000	0.02804	
	20.00000	4640.00000	0.02807	
	20.00000	4660.00000	0.02809	
	12.00000	4672.00000	0.02807	
290	8.00000	4680.00000	0.02802	
	16.00000	4696.00000	0.02802	
	12.00000	4708.00000	0.02799	
	23.00000	4731.00000	0.02803	
	19.00000	4750.00000	0.02805	
	14.00000	4764.00000	0.02804	
	12.00000	4776.00000	0.02801	
	15.00000	4791.00000	0.02801	

	19.00000	4810.00000	0.02802
	22.00000	4832.00000	0.02806
300	19.00000	4851.00000	0.02807
	16.00000	4867.00000	0.02807
	9.00000	4876.00000	0.02803
	15.00000	4891.00000	0.02802
	9.00000	4900.00000	0.02798
	22.00000	4922.00000	0.02802
	16.00000	4938.00000	0.02802
	20.00000	4958.00000	0.02804
	13.00000	4971.00000	0.02802
	16.00000	4987.00000	0.02802
310	16.00000	5003.00000	0.02802
	12.00000	5015.00000	0.02800
	25.00000	5040.00000	0.02804
	18.00000	5058.00000	0.02806
	10.00000	5068.00000	0.02802
	10.00000	5078.00000	0.02799
	17.00000	5095.00000	0.02799
	20.00000	5115.00000	0.02801
	15.00000	5130.00000	0.02801
	16.00000	5146.00000	0.02801
320	14.00000	5160.00000	0.02799
	15.00000	5175.00000	0.02799
	16.00000	5191.00000	0.02799
	23.00000	5214.00000	0.02803
	10.00000	5224.00000	0.02799
	23.00000	5247.00000	0.02803
	15.00000	5262.00000	0.02802
	7.00000	5269.00000	0.02797
	19.00000	5288.00000	0.02799
	13.00000	5301.00000	0.02797
330	20.00000	5321.00000	0.02799
	9.00000	5330.00000	0.02796
	27.00000	5357.00000	0.02801
	14.00000	5371.00000	0.02800
	19.00000	5390.00000	0.02802
	15.00000	5405.00000	0.02801
	21.00000	5426.00000	0.02804
	17.00000	5443.00000	0.02804
	6.00000	5449.00000	0.02799
	16.00000	5465.00000	0.02799
340	9.00000	5474.00000	0.02795
	19.00000	5493.00000	0.02797
	18.00000	5511.00000	0.02798
	12.00000	5523.00000	0.02795
	23.00000	5546.00000	0.02799
	18.00000	5564.00000	0.02800
	16.00000	5580.00000	0.02800
	20.00000	5600.00000	0.02802
	14.00000	5614.00000	0.02801
	20.00000	5634.00000	0.02803
350	7.00000	5641.00000	0.02798
	12.00000	5653.00000	0.02796
	15.00000	5668.00000	0.02796
	13.00000	5681.00000	0.02794
	17.00000	5698.00000	0.02794
	19.00000	5717.00000	0.02796
	18.00000	5735.00000	0.02797
	22.00000	5757.00000	0.02800 ← END OF FIG. 3(b)

	13.00000	5770.00000	0.02798	
	19.00000	5789.00000	0.02800	
360	14.00000	5803.00000	0.02799	
	17.00000	5820.00000	0.02799	
	16.00000	5836.00000	0.02799	
	10.00000	5846.00000	0.02796	
	14.00000	5860.00000	0.02795	
	13.00000	5873.00000	0.02793	
	19.00000	5892.00000	0.02795	
	10.00000	5902.00000	0.02792	
	16.00000	5918.00000	0.02792	
	24.00000	5942.00000	0.02796	
370	21.00000	5963.00000	0.02798	
	12.00000	5975.00000	0.02796	
	19.00000	5994.00000	0.02797	
	15.00000	6009.00000	0.02797	
	18.00000	6027.00000	0.02798	
	14.00000	6041.00000	0.02797	
	10.00000	6051.00000	0.02794	
	12.00000	6063.00000	0.02792	
	17.00000	6080.00000	0.02792	
	15.00000	6095.00000	0.02792	
380	16.00000	6111.00000	0.02792	
	24.00000	6135.00000	0.02796	
	20.00000	6155.00000	0.02797	
	14.00000	6169.00000	0.02796	
	15.00000	6184.00000	0.02796	
	14.00000	6198.00000	0.02795	
	19.00000	6217.00000	0.02796	
	11.00000	6228.00000	0.02794	
	23.00000	6251.00000	0.02797	
	15.00000	6266.00000	0.02797	
390	7.00000	6273.00000	0.02792	
	18.00000	6291.00000	0.02793	
	7.00000	6298.00000	0.02789	
	20.00000	6318.00000	0.02791	
	21.00000	6339.00000	0.02793	
	24.00000	6363.00000	0.02797	
	15.00000	6378.00000	0.02796	
	16.00000	6394.00000	0.02796	
	15.00000	6409.00000	0.02796	
	15.00000	6424.00000	0.02795	
400	8.00000	6432.00000	0.02792	
	24.00000	6456.00000	0.02795	
	16.00000	6472.00000	0.02795	
	10.00000	6482.00000	0.02792	
	15.00000	6497.00000	0.02792	
	10.00000	6507.00000	0.02789	
	22.00000	6529.00000	0.02792	
	18.00000	6547.00000	0.02793	
	21.00000	6568.00000	0.02795	
	19.00000	6587.00000	0.02796	
410	12.00000	6599.00000	0.02794	
	16.00000	6615.00000	0.02794	
	16.00000	6631.00000	0.02794	
	18.00000	6649.00000	0.02795	
	16.00000	6665.00000	0.02795	
	7.00000	6672.00000	0.02791	
	13.00000	6685.00000	0.02790	
	14.00000	6699.00000	0.02789	← 0_1 COUNT ENDS HERE

	14.00000	6713.00000	0.02788	← FIG. 3(c) AND O_2 COUNTS START HERE
	25.00000	6738.00000	0.02792	
420	22.00000	6760.00000	0.02794	
	18.00000	6778.00000	0.02795	
	12.00000	6790.00000	0.02793	
	19.00000	6809.00000	0.02795	
	23.00000	6832.00000	0.02797	
	14.00000	6846.00000	0.02797	
	8.00000	6854.00000	0.02793	
	15.00000	6869.00000	0.02793	
	17.00000	6886.00000	0.02793	
	7.00000	6893.00000	0.02790	
430	15.00000	6908.00000	0.02789	
	19.00000	6927.00000	0.02790	
	19.00000	6946.00000	0.02791	
	21.00000	6967.00000	0.02793	
	22.00000	6989.00000	0.02796	
	12.00000	7001.00000	0.02794	
	23.00000	7024.00000	0.02797	
	13.00000	7037.00000	0.02796	
	12.00000	7049.00000	0.02794	
	15.00000	7064.00000	0.02794	
440	14.00000	7078.00000	0.02793	
	11.00000	7089.00000	0.02791	
	19.00000	7108.00000	0.02792	
	14.00000	7122.00000	0.02791	
	19.00000	7141.00000	0.02792	
	18.00000	7159.00000	0.02793	
	17.00000	7176.00000	0.02793	
	21.00000	7197.00000	0.02795	
	16.00000	7213.00000	0.02795	
	16.00000	7229.00000	0.02795	
450	13.00000	7242.00000	0.02794	
	16.00000	7258.00000	0.02794	
	14.00000	7272.00000	0.02793	
	13.00000	7285.00000	0.02792	
	13.00000	7298.00000	0.02791	
	14.00000	7312.00000	0.02790	
	15.00000	7327.00000	0.02790	
	23.00000	7350.00000	0.02792	
	17.00000	7367.00000	0.02793	
	15.00000	7382.00000	0.02792	
460	19.00000	7401.00000	0.02793	
	19.00000	7420.00000	0.02794	
	17.00000	7437.00000	0.02795	
	13.00000	7450.00000	0.02794	
	15.00000	7465.00000	0.02793	
	15.00000	7480.00000	0.02793	
	14.00000	7494.00000	0.02792	
	13.00000	7507.00000	0.02791	
	17.00000	7524.00000	0.02791	
	15.00000	7539.00000	0.02791	
470	18.00000	7557.00000	0.02791	
	18.00000	7575.00000	0.02792	
	19.00000	7594.00000	0.02793	
	17.00000	7611.00000	0.02794	
	18.00000	7629.00000	0.02794	
	16.00000	7645.00000	0.02794	
	14.00000	7659.00000	0.02793	
	12.00000	7671.00000	0.02792	

	12.00000	7683.00000	0.02790
	22.00000	7705.00000	0.02793
480	14.00000	7719.00000	0.02792
	16.00000	7735.00000	0.02792
	11.00000	7746.00000	0.02790
	21.00000	7767.00000	0.02792
	20.00000	7787.00000	0.02793
	18.00000	7805.00000	0.02794
	19.00000	7824.00000	0.02795
	14.00000	7838.00000	0.02794
	13.00000	7851.00000	0.02793
	14.00000	7865.00000	0.02792
490	11.00000	7876.00000	0.02791
	20.00000	7896.00000	0.02792
	12.00000	7908.00000	0.02790
	18.00000	7926.00000	0.02791
	16.00000	7942.00000	0.02791
	14.00000	7956.00000	0.02790
	16.00000	7972.00000	0.02790
	20.00000	7992.00000	0.02792
	24.00000	8016.00000	0.02795
	13.00000	8029.00000	0.02793
500	13.00000	8042.00000	0.02792
	15.00000	8057.00000	0.02792
	11.00000	8068.00000	0.02790
	16.00000	8084.00000	0.02790
	17.00000	8101.00000	0.02791
	16.00000	8117.00000	0.02790
	13.00000	8130.00000	0.02789
	23.00000	8153.00000	0.02792
	14.00000	8167.00000	0.02791
	21.00000	8188.00000	0.02793
510	17.00000	8205.00000	0.02793
	17.00000	8222.00000	0.02793
	10.00000	8232.00000	0.02791
	18.00000	8250.00000	0.02792
	16.00000	8266.00000	0.02792
	10.00000	8276.00000	0.02790
	18.00000	8294.00000	0.02791
	13.00000	8307.00000	0.02790
	24.00000	8331.00000	0.02792
	15.00000	8346.00000	0.02792
520	12.00000	8358.00000	0.02790
	18.00000	8376.00000	0.02791
	12.00000	8388.00000	0.02790
	19.00000	8407.00000	0.02791
	22.00000	8429.00000	0.02793
	17.00000	8446.00000	0.02793
	14.00000	8460.00000	0.02792
	12.00000	8472.00000	0.02791
	14.00000	8486.00000	0.02790
	21.00000	8507.00000	0.02792
530	19.00000	8526.00000	0.02793
	10.00000	8536.00000	0.02791
	12.00000	8548.00000	0.02790
	17.00000	8565.00000	0.02790
	14.00000	8579.00000	0.02789
	20.00000	8599.00000	0.02790
	20.00000	8619.00000	0.02792
	15.00000	8634.00000	0.02791 ← END OF FIG. 3(c)

	20.00000	8654.00000	0.02793	
	16.00000	8670.00000	0.02793	
540	14.00000	8684.00000	0.02792	
	15.00000	8699.00000	0.02792	
	15.00000	8714.00000	0.02791	
	18.00000	8732.00000	0.02792	
	11.00000	8743.00000	0.02790	
	11.00000	8754.00000	0.02789	
	24.00000	8778.00000	0.02791	
	12.00000	8790.00000	0.02790	
	19.00000	8809.00000	0.02791	
	21.00000	8830.00000	0.02792	
550	15.00000	8845.00000	0.02792	
	12.00000	8857.00000	0.02791	
	20.00000	8877.00000	0.02792	
	16.00000	8893.00000	0.02792	
	12.00000	8905.00000	0.02791	
	16.00000	8921.00000	0.02791	
	14.00000	8935.00000	0.02790	
	15.00000	8950.00000	0.02790	
	19.00000	8969.00000	0.02791	
	19.00000	8988.00000	0.02791	
560	17.00000	9005.00000	0.02792	
	17.00000	9022.00000	0.02792	
	15.00000	9037.00000	0.02792	
	18.00000	9055.00000	0.02792	
	17.00000	9072.00000	0.02793	
	16.00000	9088.00000	0.02793	
	12.00000	9100.00000	0.02791	
	12.00000	9112.00000	0.02790	
	17.00000	9129.00000	0.02790	
	13.00000	9142.00000	0.02789	
570	21.00000	9163.00000	0.02791	
	20.00000	9183.00000	0.02792	
	17.00000	9200.00000	0.02792	
	13.00000	9213.00000	0.02791	
	15.00000	9228.00000	0.02791	
	14.00000	9242.00000	0.02790	
	20.00000	9262.00000	0.02792	
	16.00000	9278.00000	0.02792	
	15.00000	9293.00000	0.02791	
	11.00000	9304.00000	0.02790	
580	18.00000	9322.00000	0.02790	
	19.00000	9341.00000	0.02791	
	18.00000	9359.00000	0.02792	
	12.00000	9371.00000	0.02791	
	21.00000	9392.00000	0.02792	
	13.00000	9405.00000	0.02791	
	13.00000	9418.00000	0.02790	
	21.00000	9439.00000	0.02792	
	19.00000	9458.00000	0.02793	
	19.00000	9477.00000	0.02793	
590	9.00000	9486.00000	0.02791	
	8.00000	9494.00000	0.02789	
	25.00000	9519.00000	0.02792	
	14.00000	9533.00000	0.02791	
	18.00000	9551.00000	0.02792	
	16.00000	9567.00000	0.02791	
	13.00000	9580.00000	0.02791	
	12.00000	9592.00000	0.02789	← O_2 COUNT ENDS HERE

	19.00000	9611.00000	0.02790
	18.00000	9629.00000	0.02791
600	21.00000	9650.00000	0.02792
	15.00000	9665.00000	0.02792
	19.00000	9684.00000	0.02793
	7.00000	9691.00000	0.02790
	14.00000	9705.00000	0.02790
	21.00000	9726.00000	0.02791
	16.00000	9742.00000	0.02791
	14.00000	9756.00000	0.02790
	11.00000	9767.00000	0.02789
	17.00000	9784.00000	0.02789
610	16.00000	9800.00000	0.02789
	15.00000	9815.00000	0.02789
	18.00000	9833.00000	0.02789
	17.00000	9850.00000	0.02790
	15.00000	9865.00000	0.02789
	16.00000	9881.00000	0.02789
	7.00000	9888.00000	0.02787
	15.00000	9903.00000	0.02787
	13.00000	9916.00000	0.02786
	13.00000	9929.00000	0.02785
620	8.00000	9937.00000	0.02783
	11.00000	9948.00000	0.02781
	9.00000	9957.00000	0.02779
	9.00000	9966.00000	0.02777
	9.00000	9975.00000	0.02775
	4.00000	9979.00000	0.02772
	6.00000	9985.00000	0.02769
	1.00000	9986.00000	0.02765
	3.00000	9989.00000	0.02761
	3.00000	9992.00000	0.02758
630	2.00000	9994.00000	0.02754
	2.00000	9996.00000	0.02750
	2.00000	9998.00000	0.02746
	0.0	9998.00000	0.02742
	0.0	9998.00000	0.02738
	0.0	9998.00000	0.02733
	1.00000	9999.00000	0.02729
	0.0	9999.00000	0.02725
	0.0	9999.00000	0.02721
	1.00000	10000.00000	0.02717
640	0.0	10000.00000	0.02713
	0.0	10000.00000	0.02708
	0.0	10000.00000	0.02704
	0.0	10000.00000	0.02700
	0.0	10000.00000	0.02696
	0.0	10000.00000	0.02692
	0.0	10000.00000	0.02687
	0.0	10000.00000	0.02683
	0.0	10000.00000	0.02679
	0.0	10000.00000	0.02675
	0.0	10000.00000	0.02671
	0.0	10000.00000	0.02667
	0.0	10000.00000	0.02663
	0.0	10000.00000	0.02659
	0.0	10000.00000	0.02655
	0.0	10000.00000	0.02651
	0.0	10000.00000	0.02647
	0.0	10000.00000	0.02642

APPENDIX 8

The output of Bazovsky's Example where the first 216 cells of the histogram were truncated.

0000 7 0

ROW NO.	ROW TOTAL FAILURES	CUM. SUM	λ^* CURRENT EST.
1	71.00000	71.00000	0.12326
	735.00000	806.00000	0.69965
	2523.00000	3329.00000	1.92650
	3640.00000	6969.00000	3.02474
	2348.00000	9317.00000	3.23507
	613.00000	9930.00000	2.87326
	68.00000	9998.00000	2.47966
	0.0	9998.00000	2.16970
	0.0	9998.00000	1.92863
	0.0	9998.00000	1.73576
10	4.00000	10002.00000	1.57860
	26.00000	10028.00000	1.45081
	213.00000	10241.00000	1.36765
	725.00000	10966.00000	1.35987
	1639.00000	12605.00000	1.45891
	2558.00000	15163.00000	1.64529
	2500.00000	17663.00000	1.80382
	1484.00000	19147.00000	1.84674
	667.00000	19814.00000	1.81049
	154.00000	19968.00000	1.73333
20	30.00000	19998.00000	1.65327
	2.00000	20000.00000	1.57828
	16.00000	20016.00000	1.51087
	59.00000	20075.00000	1.45218
	204.00000	20279.00000	1.40826
	590.00000	20869.00000	1.39350
	1256.00000	22125.00000	1.42265
	1914.00000	24039.00000	1.49051
	2166.00000	26205.00000	1.56879
	1852.00000	28057.00000	1.62367
30	1167.00000	29224.00000	1.63665
	525.00000	29749.00000	1.61399
	192.00000	29941.00000	1.57518
	53.00000	29994.00000	1.53156
	23.00000	30017.00000	1.48894
	79.00000	30096.00000	1.45139
	196.00000	30292.00000	1.42136
	499.00000	30791.00000	1.40675
	932.00000	31723.00000	1.41217
	1530.00000	33253.00000	1.44327
40	1862.00000	35115.00000	1.48692
	1799.00000	36914.00000	1.52588
	1481.00000	38395.00000	1.55019
	891.00000	39286.00000	1.55011
	437.00000	39723.00000	1.53252
	200.00000	39923.00000	1.50675
	78.00000	40001.00000	1.47758
	97.00000	40098.00000	1.45030
	199.00000	40297.00000	1.42776
	419.00000	40716.00000	1.41375
50	748.00000	41464.00000	1.41149
	1222.00000	42686.00000	1.42515
	1593.00000	44279.00000	1.45044
	1638.00000	45917.00000	1.47624
	1566.00000	47483.00000	1.49883
	1184.00000	48667.00000	1.50877
	704.00000	49371.00000	1.50375
	397.00000	49768.00000	1.48970

	177.00000	49945.00000	1.46966
	143.00000	50088.00000	1.44931
60	194.00000	50282.00000	1.43107
	348.00000	50630.00000	1.41773
	646.00000	51276.00000	1.41303
	953.00000	52229.00000	1.41680
	1348.00000	53577.00000	1.43101
	1501.00000	55078.00000	1.44881
	1514.00000	56592.00000	1.46642
	1329.00000	57921.00000	1.47878
	946.00000	58867.00000	1.48115
	604.00000	59471.00000	1.47497
70	346.00000	59817.00000	1.46266
	203.00000	60020.00000	1.44724
	216.00000	60236.00000	1.43255
	296.00000	60532.00000	1.42014
	536.00000	61068.00000	1.41361
	842.00000	61910.00000	1.41424
	1149.00000	63059.00000	1.42178
	1314.00000	64373.00000	1.43280
	1420.00000	65793.00000	1.44587
	1264.00000	67057.00000	1.45523
80	1166.00000	68223.00000	1.46226
	824.00000	69047.00000	1.46187
	485.00000	69532.00000	1.45440
	251.00000	69783.00000	1.44227
	134.00000	69917.00000	1.42804
	53.00000	69970.00000	1.41251
	23.00000	69993.00000	1.39673
	5.00000	69998.00000	1.38096
	0.0	69998.00000	1.36544
	0.0	69998.00000	1.35027
	0.0	69998.00000	1.33543
	0.0	69998.00000	1.32092
	0.0	69998.00000	1.30671
	0.0	69998.00000	1.29281
	0.0	69998.00000	1.27920
	0.0	69998.00000	1.26588
	0.0	69998.00000	1.25283
	0.0	69998.00000	1.24004
	0.0	69998.00000	1.22752
	0.0	69998.00000	1.21524
	0.0	69998.00000	1.20321
	0.0	69998.00000	1.19141
	0.0	69998.00000	1.17985
	0.0	69998.00000	1.16850
	0.0	69998.00000	1.15737
	0.0	69998.00000	1.14645
	0.0	69998.00000	1.13574
	0.0	69998.00000	1.12522
	0.0	69998.00000	1.11490
	0.0	69998.00000	1.10477
	0.0	69998.00000	1.09481
	0.0	69998.00000	1.08504
	0.0	69998.00000	1.07544
	0.0	69998.00000	1.06600
	0.0	69998.00000	1.05673
	0.0	69998.00000	1.04762
	0.0	69998.00000	1.03867
	0.0	69998.00000	1.02987

THE LIMITING RELIABILITY OF A COMPLEX SYSTEM

OF COMPONENTS SUBJECT TO WEAROUT FAILURES

by

ROBERT LEE LYON

B.A., Southwestern College,

Winfield, Kansas, 1970

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1972

ABSTRACT

Igor Bazovsky, in his book, Reliability Theory and Practice, used an example to show graphically what happens in a system containing a large number of components operating simultaneously, when components are replaced immediately upon failure. He assumes that the components can fail only by wearout; early failures and chance failures are excluded. He assumes that the failure time of the individual components is normally distributed with a mean time to failure, M, and a standard deviation, σ. His problem was concerned with 10,000 lamps with M equal to 7,200 hours and σ equal to 600 hours. Bazovsky's graph implies that the number of failures becomes almost constant by the fourth generation. This constant failure rate implies that the system behaved exponentially.

The purpose of this report is to examine Bazovsky's assertion that the failure rate becomes constant and at what time or in what generation will this occur. A Fortran based computer program was used to simulate the complex system.

The computer program of the complex system was run varying the number of components and the number of generations. After a number of cases were run, it was observed that the number of components did not appear to affect the stabilization of the system. Using 200 components and running through 50 generations of replacement, graphs on the generated data showed the number of failures for a fixed time interval had started to stabilize. Thus, the system was assumed to be approaching exponentiality. The Chi-square goodness of fit test was used, and indicated that the system behaved exponentially after the 34th generation.

After simulating Bazovsky's example and then graphing the number of failures and the failure rate of his example against time, it appears that he confused the number of failures with the failure rate. Also, Bazovsky stated that this system would stabilize within the nth generation, where n is obtained from the formula $n = M/3\sigma$, giving $n = 7200/3(600) = 4$. Bazovsky does not give any reference or reasoning for this formula, and this report showed that it is defective.