

A LINEAR, THREE-DIMENSIONAL MODEL FOR THE
VIBRATIONS OF A SEMI-TRAILER TRUCK

by 7214

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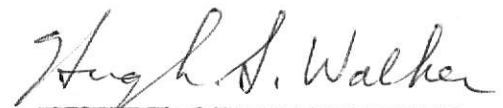
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NOMENCLATURE

Coordinates

- x_1 vertical motion of center of mass for front tractor axle
- x_2 vertical motion of center of mass for tractor
- x_3 vertical motion of center of mass for rear tractor axle
- x_4 vertical motion of center of mass for trailer
- x_5 vertical motion of center of mass for trailer axle
- x_6 longitudinal motion of center of mass for tractor
- x_7 transverse motion of center of mass for tractor
- x_8 longitudinal motion of center of mass for trailer
- x_9 transverse motion of center of mass for trailer
- z_1 transverse motion of center of mass for front tractor axle
- z_3 transverse motion of center of mass for rear tractor axle
- z_5 transverse motion of center of mass for trailer axle
- θ_1 rolling motion for front tractor axle
- θ_2 rolling motion for tractor
- θ_3 rolling motion for rear tractor axle
- θ_4 rolling motion for trailer
- θ_5 rolling motion for trailer axle
- ϕ_2 pitching motion for tractor
- ϕ_4 pitching motion for trailer
- ξ_2 yawing motion for tractor
- ξ_4 yawing motion for trailer

Dimensions

- a transverse offset of tractor center of mass from vehicle center line
- b longitudinal distance of tractor center of mass from kingpin
- c vertical distance of tractor center of mass from horizontal plane containing tractor spring attachment points
- d transverse offset of trailer center of mass from vehicle center line
- e longitudinal distance of trailer center of mass from kingpin
- f vertical distance of trailer center of mass from horizontal plane containing trailer spring attachment points
- g longitudinal distance from tractor center of mass to front spring attachment points of tractor
- h longitudinal distance from tractor center of mass to rear spring attachment points of tractor
- p vertical distance from tractor center of mass to kingpin
- q vertical distance from trailer center of mass to kingpin
- r longitudinal distance from trailer center of mass to spring attachment points of trailer
- s transverse distance between spring attachment points of tractor
- t transverse distance between spring attachment points of trailer
- u one-half the distance between axle suspension points
- v assumed rigid height for axle suspension
- w one-half the transverse distance between vehicle tires
- y assumed rigid height for vehicle tires

Inertia Parameters

m_1	mass of tractor front axle assembly
m_2	mass of tractor less suspension
m_3	mass of tractor rear axle assembly
m_4	mass of trailer less suspension
m_5	mass of trailer axle assembly
m_6	mass of tractor
m_7	mass of tractor less suspension
m_8	mass of trailer
m_9	mass of trailer less suspension
I_1	rolling inertia of tractor front axle
I_2	rolling inertia of tractor rigid body
I_3	rolling inertia of tractor rear axle
I_4	rolling inertia of trailer rigid body
I_5	rolling inertia of trailer axle
J_2	pitching inertia of tractor rigid body
J_4	pitching inertia of trailer rigid body
H_2	yawing inertia of tractor rigid body
H_4	yawing inertia of trailer rigid body
P_{62}	product of inertia for longitudinal-vertical plane of tractor
P_{67}	product of inertia for longitudinal-transverse plane of tractor
P_{27}	product of inertia for vertical-transverse plane of tractor
P_{84}	product of inertia for longitudinal-vertical plane of trailer
P_{89}	product of inertia for longitudinal-transverse plane of trailer
P_{49}	product of inertia for vertical-transverse plane of trailer

Springs and Dampers

- k_1, c_1 equivalent vertical spring and damping constant for left front suspension of tractor
- k_2, c_2 equivalent vertical spring and damping constant for right front suspension of tractor
- k_3, c_3 equivalent vertical spring and damping constant for left rear suspension of tractor
- k_4, c_4 equivalent vertical spring and damping constant for right rear suspension of trailer
- k_5, c_5 equivalent vertical spring and damping constant for left suspension of trailer
- k_6, c_6 equivalent vertical spring and damping constant for right suspension of trailer
- k_7, c_7 equivalent vertical spring and damping constant for left front tire of tractor
- k_8, c_8 equivalent vertical spring and damping constant for right front tire of tractor
- k_9, c_9 equivalent vertical spring and damping constant for left rear tire of tractor
- k_{10}, c_{10} equivalent vertical spring and damping constant for right rear tire of tractor
- k_{11}, c_{11} equivalent vertical spring and damping constant for left rear tire of trailer
- k_{12}, c_{12} equivalent vertical spring and damping constant for right rear tire of trailer

- k_{13}, c_{13} equivalent transverse spring and damping constant for left front tire of tractor
- k_{14}, c_{14} equivalent transverse spring and damping constant for right front tire of tractor
- k_{15}, c_{15} equivalent transverse spring and damping constant for left front suspension of tractor
- k_{16}, c_{16} equivalent transverse spring and damping constant for right front suspension of tractor
- k_{17}, c_{17} equivalent transverse spring and damping constant for left rear tire of tractor
- k_{18}, c_{18} equivalent transverse spring and damping constant for right rear tire of tractor
- k_{19}, c_{19} equivalent transverse spring and damping constant for left rear suspension of tractor
- k_{20}, c_{20} equivalent transverse spring and damping constant for right rear suspension of tractor
- k_{21}, c_{21} equivalent transverse spring and damping constant for left tire of trailer
- k_{22}, c_{22} equivalent transverse spring and damping constant for right tire of trailer
- k_{23}, c_{23} equivalent transverse spring and damping constant for left suspension of trailer
- k_{24}, c_{24} equivalent transverse spring and damping constant for right suspension of trailer

Chapter I

INTRODUCTION

Semi-trailer trucks are assuming an ever-increasing role in the transportation of consumer and capital goods. They provide the necessary link between the manufacturer and the consumer. As the advertisement states, "If you got it, a truck brought it." Although the semi-trailer truck has obtained increased importance, the understanding of its vibrational characteristics has lagged. Therefore the development of a three-dimensional model for the vibrations of a semi-trailer truck shall aid in a better understanding of the vehicle's motions.

A number of investigations have analyzed the vibrational characteristics of semi-trailer trucks, but have limited their studies to planar motions (8), (11), (16), (17), (18).^{*} The most comprehensive planar analysis was given by Potts (14), (15). The assumptions necessary for a planar analysis preclude certain physical realities. In a planar analysis the yawing and rolling motions of the vehicle are ignored. No distinction is made between the "bumps" encountered by the left and right side wheel sets. In the model to be presented, all these effects are accounted for.

After a number of assumptions were made to reduce the complexity of a "real" truck, energy methods and Lagrange's equation were used to develop the equations of motion for the vehicle. Twenty-one coordinates were initially chosen to describe the motions of the vehicle's components. Three kinematic

^{*}Numbers in parentheses refer to references given in the selected bibliography.

equations of constraint, due to the fifth wheel kingpin, and one ignorable coordinate were discovered. Thus the number of independent degrees of freedom was reduced to seventeen. Two digital computer programs were developed to solve various steady state problems. While these programs were used primarily as a check for the correctness of the mathematical model, they could be used in an extensive steady state analysis.

The three-dimensional model developed in this thesis provides a useful addition to the simulation of semi-trailer truck vibrations.

Chapter II

PHYSICAL DESCRIPTION OF MODEL

Physical Assumptions

The complexity of a real semi-trailer truck was reduced by making a number of simplifying assumptions. The tractor and trailer, less their suspensions, were assumed to be rigid bodies. Tandems and dual tire sets were lumped into an equivalent single axle-tire assembly; thus the vehicle was assumed to have three axles and six tires. Each of the equivalent suspension assemblies was assumed to be rigid in the longitudinal direction, thereby constraining axle movement to planar motion normal to the direction of travel. Each of the six tires was supposed equivalent to linear springs and viscous (linear) dampers possessing different properties in the vertical and transverse directions, as shown in figures 3, 4 and 5. The axle suspensions were also substituted by equivalent linear springs and dampers in both the vertical and transverse directions. These springs and dampers were assumed to be attached to the tractor and trailer directly above the axle.

Rigid bars, normal to the horizontal plane, were fixed to the rigid axles at the spring and tire attachment points (see Fig. 3). The transverse motion of these bar end points was used to describe the interaction between chassis, axle, and tire, due to the rolling motions of the vehicle components.

Oscillations about the equilibrium positions were supposed small, so that rigid body motions could be described by linear functions of the chosen coordinates.

At the tire-road interface, slippage was not permitted. Nonlinearities such as cargo movement, wheel-hop, and coulomb friction were ignored.

Describing Coordinates

The equilibrium positions of the sprung and unsprung centers of mass were used as the origins of the fixed coordinate systems. Figure 1 shows a general set of rectangular axes used. The longitudinal and transverse motions of the center of mass were described by the X and Z coordinates, respectively, where the X and Z axes lie in a horizontal plane parallel to the road surface, with the positive X axis coincident with the direction of travel. The positive Z axis points outward and left of the vehicle, as defined by the driver. The vertical component of motion was given along the Y axis. The rigid body rotations of roll, pitch, and yaw were given respectively by θ , ϕ and ξ . Positive angular rotations were defined by the right-hand rule.

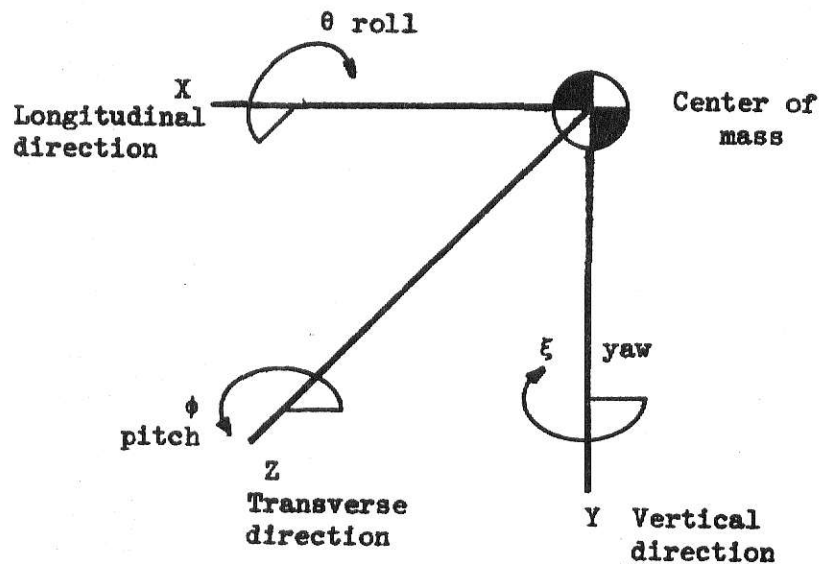


Figure 1: Rectangular axes.

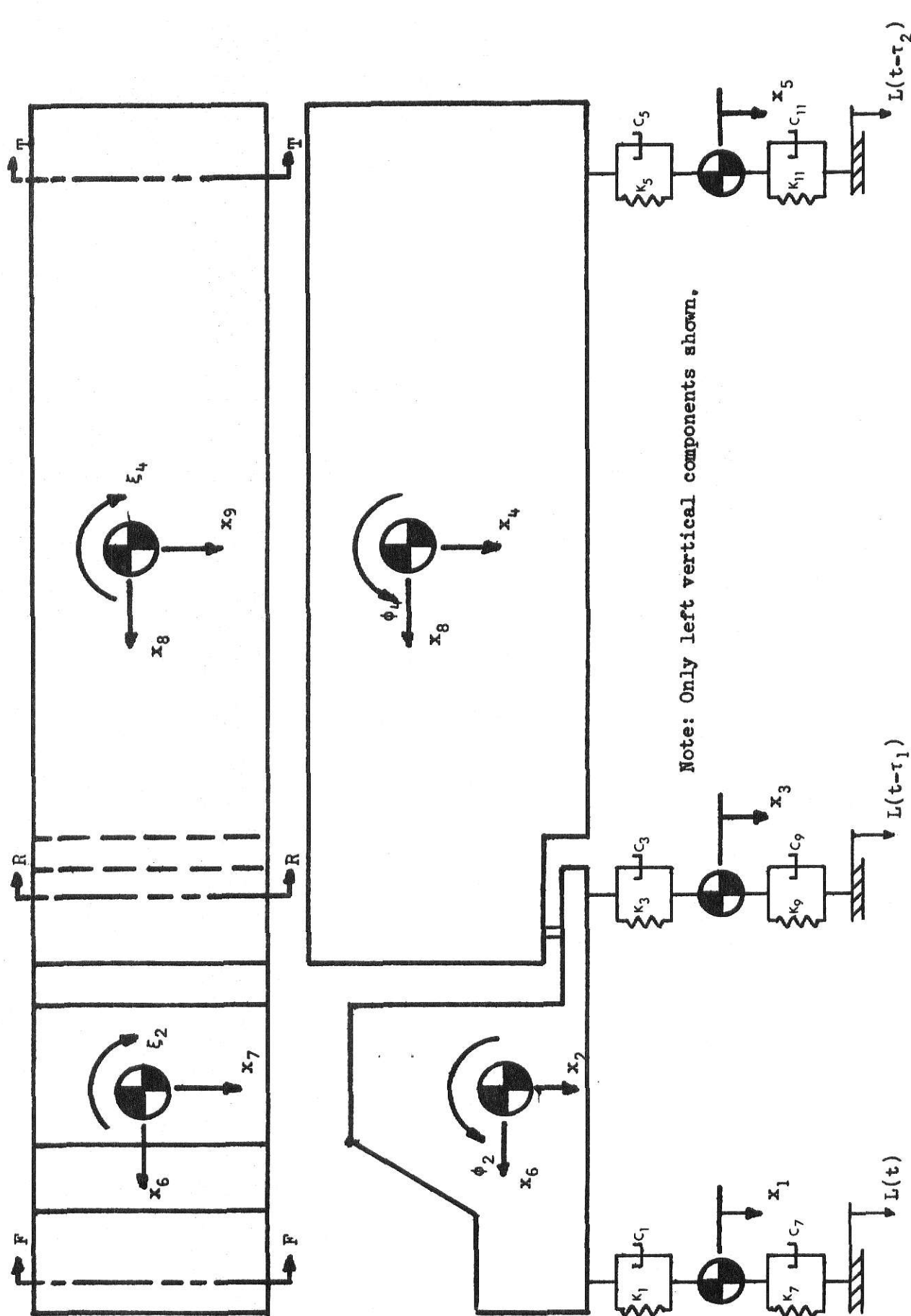


Figure 2: Top and left side view of semi-trailer truck.

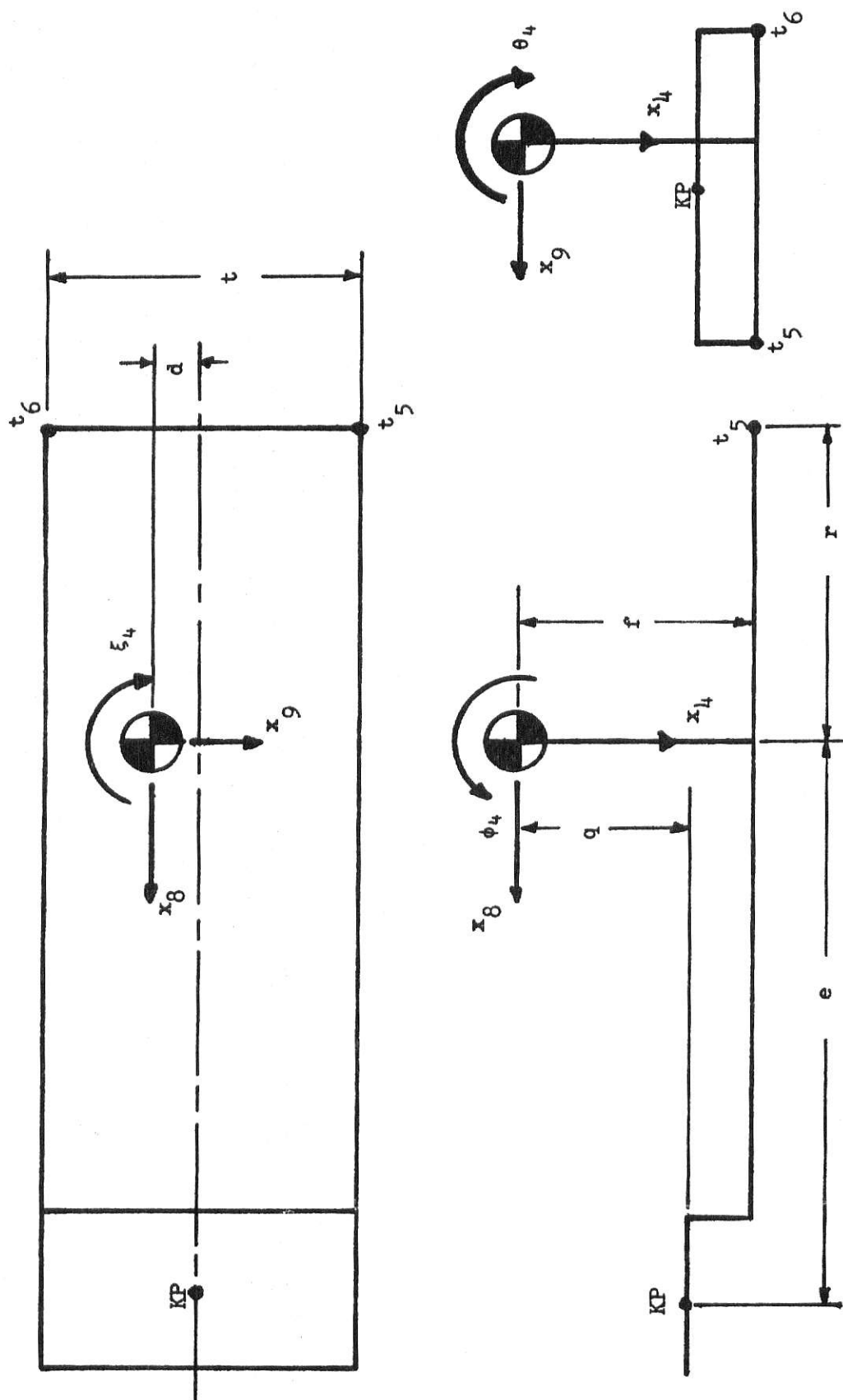


Figure 6. Dimensional schematic of trailer.

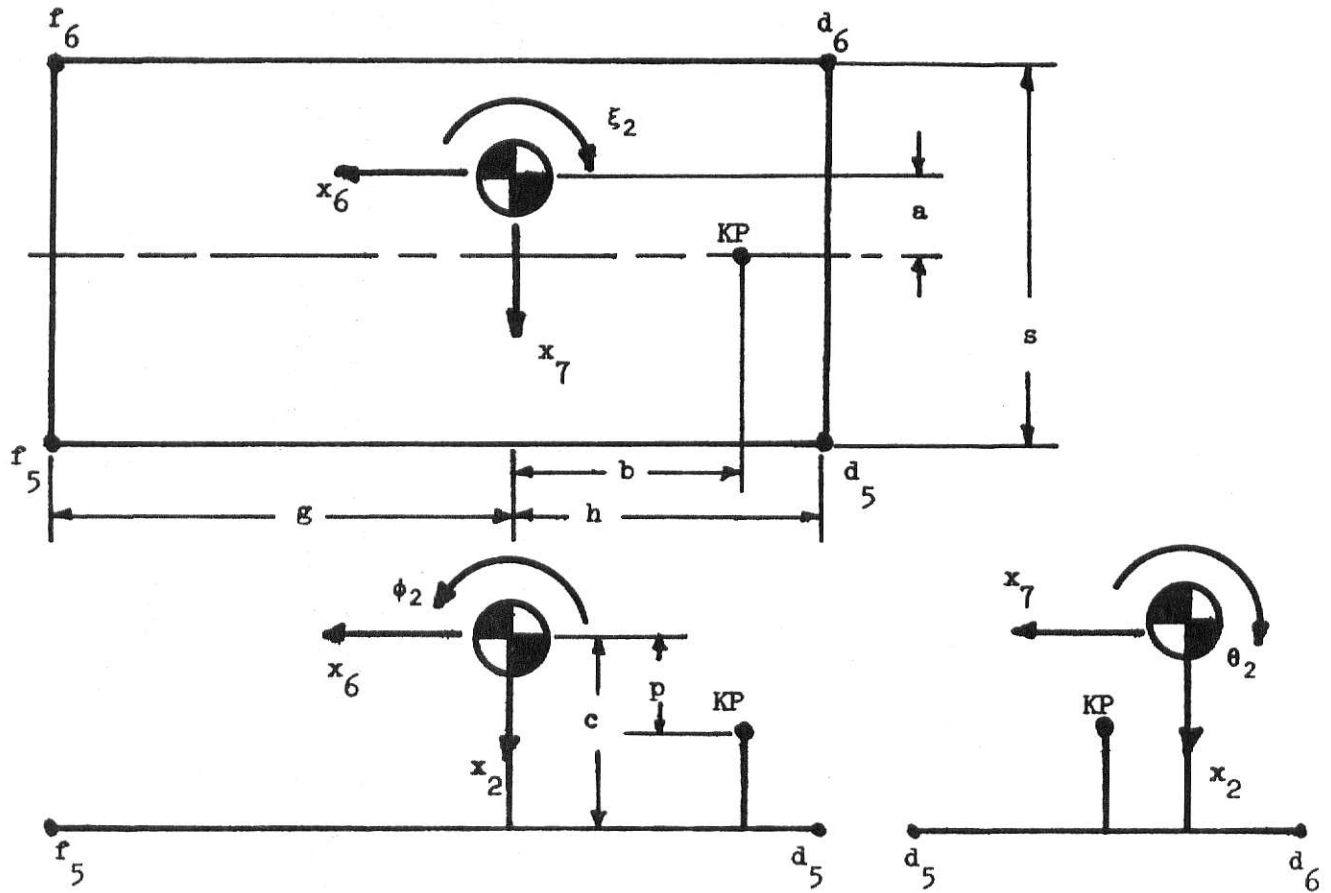


Figure 7: Dimensional schematic of tractor.

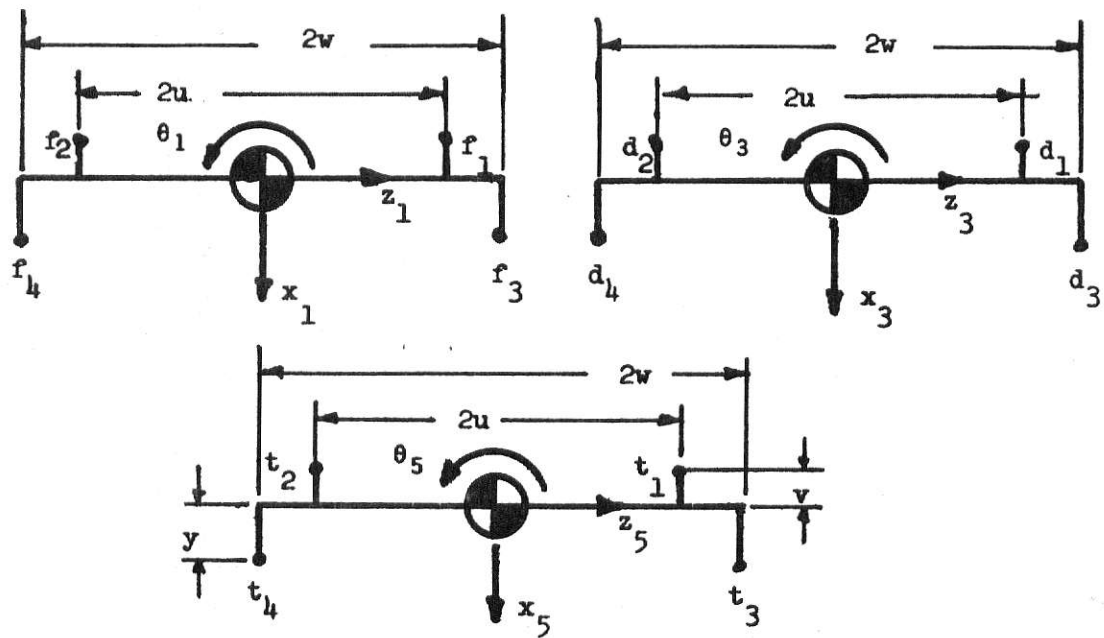


Figure 8: Dimensional schematic of vehicle axles.

Chapter III

MATHEMATICAL DESCRIPTION OF MODEL

Lagrange's Equation

The Lagrangian equations of motion are

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} + \frac{\partial D}{\partial \dot{x}_i} + \frac{\partial V}{\partial x_i} = q_i, \quad [1]$$

where

$i = 1, 2, \dots, n,$

n = number of degrees of freedom of the system,

T = kinetic energy function,

D = dissipation function,

V = potential energy function,

x = generalized coordinate,

q = generalized force.

When linear theory is used to analyze a vibrations problem, certain simplifications in the derivation of the equations of motion result. For linear theory, Langhaar(10) shows that the kinetic, potential, and dissipation functions can be expressed as homogeneous quadratic functions of the form

$$T = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n m_{ij} \dot{x}_i \dot{x}_j,$$

$$V = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n k_{ij} x_i x_j,$$

$$D = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n c_{ij} \dot{x}_i \dot{x}_j,$$

where the m_{ij} 's, k_{ij} 's, and c_{ij} 's are constants. Substituting the above into [1] yields an equivalent expression in summation form as

$$\sum_{j=1}^n (m_{ij} \ddot{x}_j + c_{ij} \dot{x}_j + k_{ij} x_j) = q_i \quad (i = 1, 2, \dots, n),$$

or in matrix form as

$$[m_{ij}]\{\ddot{X}\} + [c_{ij}]\{\dot{X}\} + [k_{ij}]\{X\} = \{Q\},$$

where

$$m_{ij} = \frac{\partial^2 T}{\partial \dot{x}_i \partial \dot{x}_j},$$

$$c_{ij} = \frac{\partial^2 D}{\partial \dot{x}_i \partial \dot{x}_j},$$

$$k_{ij} = \frac{\partial^2 V}{\partial x_i \partial x_j}.$$

[2]

Thus after the energy functions have been determined, the matrix elements are easily obtained by taking partial derivatives of the respective functions.

Energy Functions

Referring to figures 2, 3, 4, and 5, the kinetic energy can be written down immediately as

$$\begin{aligned}
 T = \frac{1}{2} \{ & m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2 + m_3 \dot{x}_3^2 + m_4 \dot{x}_4^2 + m_5 \dot{x}_5^2 + m_6 \dot{x}_6^2 + m_7 \dot{x}_7^2 + m_8 \dot{x}_8^2 + \\
 & m_9 \dot{x}_9^2 + m_1 \dot{z}_1^2 + m_3 \dot{z}_3^2 + m_5 \dot{z}_5^2 + I_1 \dot{\theta}_1^2 + I_2 \dot{\theta}_2^2 + I_3 \dot{\theta}_3^2 + I_4 \dot{\theta}_4^2 + \\
 & I_5 \dot{\theta}_5^2 + J_2 \dot{\phi}_2^2 + J_4 \dot{\phi}_4^2 + H_2 \dot{\xi}_2^2 + H_4 \dot{\xi}_4^2 - 2P_{62} \dot{\theta}_2 \dot{\xi}_2 - 2P_{67} \dot{\theta}_2 \dot{\phi}_2 - \\
 & 2P_{27} \dot{\xi}_2 \dot{\phi}_2 - 2P_{84} \dot{\theta}_4 \dot{\xi}_4 - 2P_{89} \dot{\theta}_4 \dot{\phi}_4 - 2P_{49} \dot{\xi}_4 \dot{\phi}_4 \}. \quad [3]
 \end{aligned}$$

The potential energy in terms of the rigid body attachment points (see figures 3, 4, and 5) is given by

$$\begin{aligned}
 V = \frac{1}{2} \{ & k_7 [f_{3v} - L(t)]^2 + k_{13} [f_{3h}]^2 + k_8 [f_{4v} - R(t)]^2 + k_{14} [f_{4h}]^2 + \\
 & k_{11} [t_{3v} - L(t - \tau_2)]^2 + k_{21} [t_{3h}]^2 + k_{12} [t_{4v} - R(t - \tau_2)]^2 + \\
 & k_{10} [d_{4v} - R(t - \tau_1)]^2 + k_{18} [d_{4h}]^2 + k_3 [d_{5v} - d_{1v}]^2 + k_{17} [d_{3h}]^2 + \\
 & k_{19} [d_{5h} - d_{1h}]^2 + k_4 [d_{6v} - d_{2v}]^2 + k_{20} [d_{6h} - d_{2h}]^2 + k_{22} [t_{4h}]^2 + \\
 & k_{16} [f_{6h} - f_{2h}]^2 + k_9 [d_{3v} - L(t - \tau_1)]^2 + k_1 [f_{5v} - f_{1v}]^2 + \\
 & k_{15} [f_{5h} - f_{1h}]^2 + k_5 [t_{5v} - t_{1v}]^2 + k_{23} [t_{5h} - t_{1h}]^2 + k_6 [t_{6v} - t_{2v}]^2 + \\
 & k_{24} [t_{6h} - t_{2h}]^2 + k_2 [f_{6v} - f_{2v}]^2 \}. \quad [4]
 \end{aligned}$$

where the second subscripts, v and h, represent respectively the vertical and

transverse displacements of the various attachment points.

Before the potential energy function can be written in terms of the chosen coordinates, the displacement of a general point in a rigid body due to small rotations is required. Consider the general point shown in figure 9 and defined by the column vector

$$\{R\} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}.$$

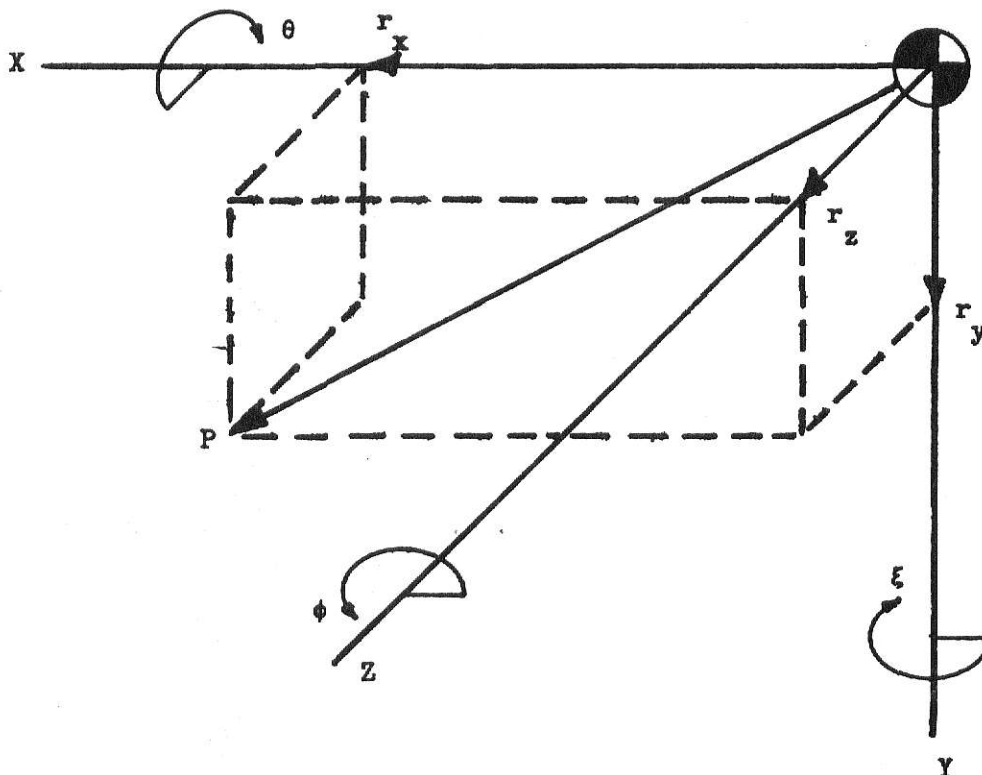


Figure 9: Definition of rigid body point.

The final position of point P after three successive rotations is

$$\{R\} = [E] [\Phi] [\Theta] \{R\},$$

where

$$[E] = \begin{vmatrix} \cos \xi & 0 & \sin \xi \\ 0 & 1 & 0 \\ -\sin \xi & 0 & \cos \xi \end{vmatrix},$$

$$[\Phi] = \begin{vmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{vmatrix},$$

$$[\Theta] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{vmatrix}.$$

From theory of matrices, it is known that general matrix products are noncommutative; thus the final position of the point P is dependent upon the order in which the rotations are assumed to occur. However, the transformation matrices become commutative if the magnitudes of their elements are sufficiently small. Since in linear theory we stipulate that the coordinates are limited to small oscillations, it may be assumed that $\cos \xi \approx 1$, $\cos \phi \approx 1$, $\cos \theta \approx 1$; and that $\sin \xi \approx \xi$, $\sin \phi \approx \phi$, $\sin \theta \approx \theta$. If in addition it is assumed that the products of the small quantities (ξ, ϕ, θ) are similar to zero, the final position of the point P is not dependent upon the

order of rotations.

With the above assumptions, the new position vector is given by

$$\{R'\} = \begin{vmatrix} 1 & -\phi & \xi \\ \phi & 1 & -\theta \\ -\xi & \phi & 1 \end{vmatrix} \{R\}.$$

The displacement of P is

$$\{\bar{R}\} = \{R'\} - \{R\},$$

or in scalar form,

$$\bar{r}_x = -r_y \phi + r_z \xi,$$

$$\bar{r}_y = r_x \phi - r_z \theta,$$

[5]

$$\bar{r}_z = -r_x \xi + r_y \theta.$$

The displacements of the attachment points used in the potential energy function [4] are determined by referring to the dimensional schematics for the vehicle (see figures 6, 7, and 8) and using the equations of [5]. Substituting the appropriate relationships for the point displacements gives the potential energy function in terms of the chosen coordinates as

$$\begin{aligned}
V = \frac{1}{2} \{ & k_7[x_1 - w\theta_1 - L(t)]^2 + k_{13}[z_1 + y\theta_1]^2 + k_8[x_1 + w\theta_1 - R(t)]^2 + k_{14}[z_1 + y\theta_1]^2 + \\
& k_{12}[x_5 + w\theta_5 - R(t - \tau_2)]^2 + k_{22}[z_5 + y\theta_5]^2 + k_5[x_4 - r\phi_4 - \frac{(t+2d)}{2}\theta_4 - x_5 + u\theta_5]^2 + \\
& k_{20}[x_7 + h\xi_2 + c\theta_2 - z_3 + v\theta_3]^2 + k_{11}[x_5 - w\theta_5 - L(t - \tau_2)]^2 + k_{21}[z_5 + y\theta_5]^2 + \\
& k_9[x_3 - w\theta_3 - L(t - \tau_1)]^2 + k_{17}[z_3 + y\theta_3]^2 + k_{10}[x_3 + w\theta_3 - R(t - \tau_1)]^2 + \\
& k_1[x_2 + g\phi_2 - \frac{(s+2a)}{2}\theta_2 - x_1 + u\theta_1]^2 + k_{15}[x_7 - g\xi_2 + c\theta_2 - z_1 + v\theta_1]^2 + \\
& k_2[x_2 + g\phi_2 + \frac{(s-2a)}{2}\theta_2 - x_1 - u\theta_1]^2 + k_{16}[x_7 - g\xi_2 + c\theta_2 - z_1 + v\theta_1]^2 + \\
& k_{19}[x_7 + h\xi_2 + c\theta_2 - z_3 + v\theta_3]^2 + k_4[x_2 - h\phi_2 + \frac{(s-2a)}{2}\theta_2 - x_3 - u\theta_3]^2 + \\
& k_{23}[x_9 + r\xi_4 + f\theta_4 - z_5 + v\theta_5]^2 + k_6[x_4 - r\phi_4 + \frac{(t-2d)}{2}\theta_4 - x_5 - u\theta_5]^2 + \\
& k_{18}[z_3 + y\theta_3]^2 + k_3[x_2 - h\phi_2 - \frac{(s+2a)}{2}\theta_2 - x_3 + u\theta_3]^2 + \\
& k_{24}[x_9 + r\xi_4 + f\theta_4 - z_5 + v\theta_5]^2 \}.
\end{aligned} \tag{6}$$

Since it was assumed that a viscous damper was in parallel with every spring, the dissipation function takes a form analogous to that of the potential energy function. By simply substituting the k 's by the c 's and the displacements by their time derivatives, the dissipation function could be determined. However before writing the dissipation function explicitly, it should be recognized that the chosen coordinates are not all independent.

After studying figure 2, it is seen that the kingpin is a point common to both the tractor and the trailer; thus the motion of this attachment point can be described in terms of either tractor or trailer coordinates. This common description of the kingpin motion results in three kinematic constraint equations. The motion of the kingpin in terms of the tractor and trailer coordinates is given respectively by the following vector equations.

$$\overline{KP} = \overline{S}_c + \overline{S}_{kp/c},$$

$$\overline{KP} = \overline{S}_{c'} + \overline{S}_{kp/c'}.$$

Equating the two expressions gives

$$\overline{S}_c + \overline{S}_{kp/c} = \overline{S}_{c'} + \overline{S}_{kp/c'}.$$

Resolving this vector equation into its rectangular components by using the equations of [5] yields the three constraint equations

$$x_8 - q\phi_4 + d\xi_4 = x_6 - p\phi_2 + a\xi_2,$$

$$x_4 + e\phi_4 - d\theta_4 = x_2 - b\phi_2 - a\theta_2,$$

$$x_9 - e\xi_4 + q\theta_4 = x_7 + b\xi_2 + p\theta_2.$$

Solving for x_8 , x_4 , x_9 in the above equations results in expressions for these dependent coordinates, given by

$$x_8 = x_6 - p\phi_2 + a\xi_2 + q\phi_4 - d\xi_4 ,$$

$$x_4 = x_2 - b\phi_2 - a\theta_2 - e\phi_4 + d\theta_4 , \quad [7]$$

$$x_9 = x_7 + b\xi_2 + p\theta_2 + e\xi_4 - q\theta_4 .$$

Thus the number of coordinates required to describe the motion of the vehicle has been reduced by three.

After eliminating the dependent variables from [6], using the equations of [7], it was observed that V was not a function of x_6 (note this also implies that D is not a function of $\dot{x}_6$). As described by Wells(19), the coordinate x_6 is recognized to be an ignorable coordinate. After the dependent velocities have been eliminated from [3] by the time derivative of the equations of [7], the Lagrangian equation of motion corresponding to the variable x_6 becomes

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_6}\right) = 0 = m_6 \ddot{x}_6 + m_8 (\ddot{x}_6 - p\ddot{\phi}_2 + a\ddot{\xi}_2 + q\ddot{\phi}_4 - d\ddot{\xi}_4) . \quad [8]$$

Solving [8] for $\ddot{x}_6$ gives

$$\ddot{x}_6 = \frac{m_8}{m_6 + m_8} (p\ddot{\phi}_2 - a\ddot{\xi}_2 - q\ddot{\phi}_4 + d\ddot{\xi}_4) . \quad [9]$$

Integrating [9] twice with respect to time yields

$$x_6 = \frac{m_8}{m_6+m_8} (p\phi_2 - a\xi_2 - q\phi_4 + d\xi_4) + C_1 t + C_2. \quad [10]$$

The constant C_1 is equal to zero because the coordinate x_6 describes oscillations about a fixed point and can not grow with time. Since it was assumed that $x_6(0) = 0$, C_2 must also be equal to zero. Having determined the constants C_1 and C_2 , the ignorable coordinate x_6 is discovered to be equal to

$$x_6 = \frac{m_8}{m_6+m_8} (p\phi_2 - a\xi_2 - q\phi_4 + d\xi_4). \quad [11]$$

Replacing the dependent variables by their equivalent functions, the kinetic, potential, and dissipation energy functions assume their final forms as given by [12], [13], and [14] respectively.

$$\begin{aligned} T = & \frac{1}{2} \{ m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2 + m_3 \dot{x}_3^2 + m_4 [\dot{x}_2 - b\dot{\phi}_2 - a\dot{\theta}_2 - e\dot{\phi}_4 + d\dot{\theta}_4]^2 + m_5 \dot{x}_5^2 + \\ & \frac{m_6 m_8}{m_6 + m_8} [p\dot{\phi}_2 - a\dot{\xi}_2 - q\dot{\phi}_4 + d\dot{\xi}_4]^2 + m_7 \dot{x}_7^2 + m_9 [\dot{x}_7 + b\dot{\xi}_2 + p\dot{\theta}_2 + e\dot{\xi}_4 - q\dot{\theta}_4]^2 + \\ & m_1 \dot{z}_1^2 + m_3 \dot{z}_3^2 + m_5 \dot{z}_5^2 + I_1 \dot{\theta}_1^2 + I_2 \dot{\theta}_2^2 + I_3 \dot{\theta}_3^2 + I_4 \dot{\theta}_4^2 + I_5 \dot{\theta}_5^2 + J_2 \dot{\phi}_2^2 + \\ & J_4 \dot{\phi}_4^2 + H_2 \dot{\xi}_2^2 + H_4 \dot{\xi}_4^2 - 2P_{62} \dot{\theta}_2 \dot{\xi}_2 - 2P_{67} \dot{\theta}_2 \dot{\phi}_2 - 2P_{27} \dot{\xi}_2 \dot{\phi}_2 - \\ & 2P_{84} \dot{\theta}_4 \dot{\xi}_4 - 2P_{89} \dot{\theta}_4 \dot{\phi}_4 - 2P_{49} \dot{\xi}_4 \dot{\phi}_4 \}, \end{aligned} \quad [12]$$

$$\begin{aligned}
V = & \frac{1}{2} \{ k_9 [x_3 - w\theta_3 - L(t - \tau_1)]^2 + k_{10} [x_3 + w\theta_3 - R(t - \tau_1)]^2 + k_{11} [x_5 - w\theta_5 - L(t - \tau_2)]^2 + \\
& k_{12} [x_5 + w\theta_5 - R(t - \tau_2)]^2 + (k_{13} + k_{14}) [z_1 + y\theta_1]^2 + (k_{17} + k_{18}) [z_3 + y\theta_3]^2 + \\
& (k_{15} + k_{16}) [x_7 - z_1 + v\theta_1 + c\theta_2 - g\xi_2]^2 + (k_{19} + k_{20}) [x_7 - z_3 + c\theta_2 + v\theta_3 + h\xi_2]^2 + \\
& k_1 [x_2 - x_1 + u\theta_1 - \theta_2 \frac{(s+2a)}{2} + g\phi_2]^2 + k_2 [x_2 - x_1 - u\theta_1 + \theta_2 \frac{(s-2a)}{2} + g\phi_2]^2 + \\
& k_3 [x_2 - x_3 - \theta_2 \frac{(s+2a)}{2} + u\theta_3 - h\phi_2]^2 + k_4 [x_2 - x_3 + \theta_2 \frac{(s-2a)}{2} - u\theta_3 - h\phi_2]^2 + \\
& k_5 [x_2 - x_5 - a\theta_2 - \frac{t\theta_4}{2} + u\theta_5 - b\phi_2 - (e+r)\phi_4]^2 + k_7 [x_1 - w\theta_1 - L(t)]^2 + \\
& k_6 [x_2 - x_5 - a\theta_2 + \frac{t\theta_4}{2} - u\theta_5 - b\phi_2 - (e+r)\phi_4]^2 + k_8 [x_1 + w\theta_1 - R(t)]^2 + \\
& (k_{23} + k_{24}) [x_7 - z_5 + p\theta_2 + (f-q)\theta_4 + v\theta_4 + b\xi_2 + (e+r)\xi_4]^2 + \\
& (k_{21} + k_{22}) [z_5 + y\theta_5]^2 \},
\end{aligned}$$

[13]

$$\begin{aligned}
D = \frac{1}{2} \{ & c_9 [\dot{x}_3 - w\dot{\theta}_3 - L(t-\tau_1)]^2 + c_{10} [\dot{x}_3 + w\dot{\theta}_3 - R(t-\tau_1)]^2 + c_{11} [\dot{x}_5 - w\dot{\theta}_5 - L(t-\tau_2)]^2 + \\
& c_{12} [\dot{x}_5 + w\dot{\theta}_5 - R(t-\tau_2)]^2 + (c_{13} + c_{14}) [\dot{z}_1 + y\dot{\theta}_1]^2 + (c_{17} + c_{18}) [\dot{z}_3 + y\dot{\theta}_3]^2 + \\
& (c_{15} + c_{16}) [\dot{x}_7 - \dot{z}_1 + v\dot{\theta}_1 + c\dot{\theta}_2 - g\dot{\xi}_2]^2 + (c_{19} + c_{20}) [\dot{x}_7 - \dot{z}_3 + c\dot{\theta}_2 + v\dot{\theta}_3 + h\dot{\xi}_2]^2 + \\
& c_1 [\dot{x}_2 - \dot{x}_1 + u\dot{\theta}_1 - \dot{\theta}_2 \frac{(s+2a)}{2} + g\dot{\phi}_2]^2 + c_2 [\dot{x}_2 - \dot{x}_1 - u\dot{\theta}_1 + \dot{\theta}_2 \frac{(s-2a)}{2} + g\dot{\phi}_2]^2 + \\
& c_3 [\dot{x}_2 - \dot{x}_3 - \dot{\theta}_2 \frac{(s+2a)}{2} + u\dot{\theta}_3 - h\dot{\phi}_2]^2 + c_4 [\dot{x}_2 - \dot{x}_3 + \dot{\theta}_2 \frac{(s-2a)}{2} - u\dot{\theta}_3 - h\dot{\phi}_2]^2 + \\
& c_5 [\dot{x}_2 - \dot{x}_5 - a\dot{\theta}_2 - \frac{t\dot{\theta}_4}{2} + u\dot{\theta}_5 - b\dot{\phi}_2 - (e+r)\dot{\phi}_4]^2 + c_7 [\dot{x}_1 - w\dot{\theta}_1 - L(t)]^2 + \\
& c_6 [\dot{x}_2 - \dot{x}_5 - a\dot{\theta}_2 - \frac{t\dot{\theta}_4}{2} - u\dot{\theta}_5 - b\dot{\phi}_2 - (e+r)\dot{\phi}_4]^2 + c_8 [\dot{x}_1 + w\dot{\theta}_1 - R(t)]^2 + \\
& (c_{23} + c_{24}) [\dot{x}_7 - \dot{z}_5 + p\dot{\theta}_2 + (f-q)\dot{\theta}_4 + v\dot{\theta}_4 + b\dot{\xi}_2 + (e+r)\dot{\xi}_4]^2 + \\
& (c_{21} + c_{22}) [\dot{z}_5 + y\dot{\theta}_5]^2 \}.
\end{aligned}$$

[14]

Generalized Forces

The generalized forces were determined by considering the virtual work done exclusively by the external forces when every coordinate was given an arbitrary virtual displacement. Since the virtual work done by the external forces is given as

$$\delta W_e = \sum_i^n \frac{\partial W_e}{\partial x_i} \delta x_i = \sum_i^n q_i \delta x_i ,$$

the generalized forces can be determined by the relation

$$q_i = \frac{\partial W_e}{\partial x_i} , \quad (i = 1, 2, \dots, n).$$

where

q_i = the generalized force associated with the i th coordinate

W_e = total work done by external forces.

When the external forces are due only to a "bumpy" road, the generalized force vector becomes

$$\begin{aligned}
\{Q\} = & \begin{array}{c}
k_7 L(t) + c_7 \dot{L}(t) + k_8 R(t) + c_8 \dot{R}(t) \\
0 \\
k_9 L(t-\tau_1) + c_9 \dot{L}(t-\tau_1) + k_{10} R(t-\tau_1) + c_{10} \dot{R}(t-\tau_1) \\
k_{11} L(t-\tau_2) + c_{11} \dot{L}(t-\tau_2) + k_{12} R(t-\tau_2) + c_{12} \dot{R}(t-\tau_2) \\
0 \\
0 \\
0 \\
0 \\
w[-k_7 L(t) - c_7 \dot{L}(t) + k_8 R(t) + c_8 \dot{R}(t)] \\
0 \\
w[-k_9 L(t-\tau_1) - c_9 \dot{L}(t-\tau_1) + k_{10} R(t-\tau_1) + c_{10} \dot{R}(t-\tau_1)] \\
0 \\
w[-k_{11} L(t-\tau_2) - c_{11} \dot{L}(t-\tau_2) + k_{12} R(t-\tau_2) + c_{12} \dot{R}(t-\tau_2)] \\
0 \\
0 \\
0 \\
0
\end{array}
\end{aligned}$$

After the generalized forces and the energy functions were determined, the seventeen equations of motion were written in the concise matrix form

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{Q\}, \quad [15]$$

where the mass, damping, and stiffness matrix elements were determined by applying the relationships of [2] to the energy functions [12], [13], and [14]. The respective non-zero matrix elements are given in subroutine DEFINE, found in appendix A.

Chapter IV

THEORY OF SOLUTION FOR MATHEMATICAL MODEL

Reduced Equation

The classical method of solving the equations of [15] in the absence of damping is to find the modal columns which satisfy the homogeneous matrix equation

$$[M]\{\ddot{X}\} + [K]\{X\} = \{0\} . \quad [16]$$

However when viscous damping is included, the modal columns for [16] do not in general uncouple the matrix equation

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{0\} .$$

Thus the primary objective for using normal coordinates is lost. This difficulty can be overcome by introducing an equivalent matrix equation called the reduced equation.

Employing the generalized velocities as auxiliary variables, equation [15] was written in the equivalent form

$$[A]\{\dot{Y}\} + [B]\{Y\} = \{F\} , \quad [17]$$

where

$$[A] = \begin{bmatrix} [C] & [M] \\ [M] & [0] \end{bmatrix} ,$$

$$[B] = \begin{vmatrix} [K] & [O] \\ [O] & -[M] \end{vmatrix} ,$$

$$\{F\} = \begin{vmatrix} \{Q\} \\ \{O\} \end{vmatrix} ,$$

$$\{Y\} = \begin{vmatrix} \{X\} \\ \{\dot{X}\} \end{vmatrix} .$$

Homogeneous Solution

The homogeneous solution of [17] is determined by premultiplying by $-[A]^{-1}$. Then homogeneous [17] becomes

$$-\{\dot{Y}\} + [D]\{Y\} = 0 , \quad [18]$$

where

$$[D] - [A]^{-1} [B] = \begin{vmatrix} [O] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{vmatrix} .$$

After substituting the assumed solution $\{Y\} = \{v\} e^{\lambda t}$ into [18], the result is

$$[D - \lambda I]\{v\} = 0 . \quad [19]$$

Equation [19] possesses a non-trivial solution if and only if

$$\det[D - \lambda I] = 0 . \quad [20]$$

This is recognized to be the characteristic equation of the matrix $[D]$. The $2n(34)$ roots of [20] are the eigenvalues of the matrix $[D]$. In general, the eigenvalues occur in complex conjugate pairs with corresponding conjugate eigenvectors or modal columns.

Orthogonality of Modes

Since each of the $2n$ eigenvalues with corresponding eigenvectors satisfies [19], the i th eigenvalue and associated eigenvector will also satisfy the equation resulting from premultiplying [19] by $-[A]$,

$$\lambda_i [A] \{u\}_i + [B] \{u\}_i = \{0\} . \quad [21]$$

Since $[A]$ and $[B]$ are known to be symmetric, the j th eigenvalue and corresponding eigenvector are a solution to the transpose of [21],

$$\lambda_j \{u\}_j^T [A] + \{u\}_j^T [B] = \{0\} . \quad [22]$$

Premultiplying [21] by $\{u\}_j^T$ and postmultiplying [22] by $\{u\}_i$ give respectively

$$\lambda_i \{u\}_j^T [A] \{u\}_i + \{u\}_j^T [B] \{u\}_i = 0 , \quad [23]$$

$$\lambda_j \{u\}_j^T [A] \{u\}_i + \{u\}_j^T [B] \{u\}_i = 0 . \quad [24]$$

Subtracting [24] from [23] yields

$$(\lambda_i - \lambda_j) \{u\}_j^T [A] \{u\}_i = 0 . \quad [25]$$

If the eigenvalues are distinct ($\lambda_i \neq \lambda_j$), we obtain the orthogonality relation

$$\{u\}_j^T [A] \{u\}_i = 0 . \quad [26]$$

Particular Solution

The particular solution of [17] can be determined by using the eigenvectors and the orthogonality condition. In order to solve [17], we expand $\{Y\}$ into a modal series of the form

$$\{Y\} = \sum_i^{2n} \{u\}_i z_i(t) . \quad [27]$$

After substituting [27] into [17], we obtain

$$[A] \sum_i^{2n} \{u\}_i \dot{z}_i(t) + [B] \sum_i^{2n} \{u\}_i z_i(t) = \{F\} . \quad [28]$$

Premultiplying [28] by $\{u\}_j^T$ and recalling the orthogonality condition, it is observed that the only term of the series that survives is for $i = j$; thus equation [28] becomes

$$\{u\}_j^T [A] \{u\}_j \dot{z}_j(t) + \{u\}_j^T [B] \{u\}_j z_j(t) = \{u\}_j^T \{F\} . \quad [29]$$

Referring back to equation [23], it is seen that by letting $i = j$,

$$\{u\}_j^T [B] \{u\}_j = -\lambda_j \{u\}_j^T [A] \{u\}_j .$$

After replacing this relation into [29], we obtain

$$\{u\}_j^T [A] \{u\}_j (\dot{z}_j(t) - \lambda_j z_j(t)) = \{u\}_j^T \{F\} . \quad [30]$$

If we require that $\{u\}_j^T [A] \{u\}_j = 1$, equation [30] is reduced to the simple form

$$\dot{z}_j(t) - \lambda_j z_j(t) = f_j, \quad [31]$$

where

$$f_j = \{u\}_j^T \{F\} .$$

Assuming the initial conditions are zero, the solution of [31] can be written in terms of the convolution integral

$$z_j(t) = \int_0^t f_j(\tau) e^{\lambda_j(t-\tau)} d\tau .$$

Recall that

$$\{Y(t)\} = \begin{bmatrix} \{X(t)\} \\ \{\dot{X}(t)\} \end{bmatrix} = [T] \{Z(t)\} = \begin{bmatrix} [\Psi] \\ [\lambda\Psi] \end{bmatrix} \{Z(t)\} , \quad [32]$$

where $[\Psi]$ is the upper $n \times 2n$ rectangular matrix of the square modal matrix $[T]$.

From [32] it is seen that the displacement vector is given by

$$\{X(t)\} = [\Psi] \{Z(t)\} ,$$

$$\{X(t)\} = \sum_{i=1}^{2n} \{\psi\}_i \int_0^t e^{\lambda_i(t-\tau)} f_i(\tau) d\tau, \quad [33]$$

Note that since

$$f_i(\tau) = \begin{vmatrix} \{\psi\} \\ \{\lambda\psi\} \end{vmatrix}^T \begin{vmatrix} \{Q(\tau)\} \\ \{0\} \end{vmatrix},$$

the expression for $f_i(\tau)$ in [33] may be replaced by $\{\psi\} \{Q(\tau)\}$ to give

$$\{X(t)\} = \sum_{i=1}^{2n} \{\psi\}_i \int_0^t e^{\lambda_i(t-\tau)} \{\psi\}_i^T \{Q(\tau)\} d\tau. \quad [34]$$

If it is assumed that $\{Q(\tau)\}$ is cosinusoidal (sinusoidal) and of the form

$$\{Q(\tau)\} = \text{Re} \left| \sum_k^S \{C\}_k e^{j w_k \tau} \right|,$$

where

$\{C\}_k = n \times 1$ column vector of complex constants giving phase information

$w_k =$ one of the S forcing frequencies

$j = \sqrt{-1}$

then [34] becomes

$$\begin{aligned}
X(t) &= \sum_i^{2n} \{\psi\}_i \int_0^t \{\psi\}_i^T \left(\sum_k^s \{C\}_k e^{j\omega_k \tau} \right) e^{\lambda_i(t-\tau)} d\tau \\
&= \sum_i^{2n} \{\psi\}_i \{\psi\}_i^T \left(\sum_k^s \{C\}_k \int_0^t e^{\tau(-\lambda_i + j\omega_k) + \lambda_i t} d\tau \right) . \quad [35]
\end{aligned}$$

The solution of [35] may be readily determined by taking the general r th factor of the sum for $\{Q\}$ and integrating the definite integral

$$\begin{aligned}
\{C\}_r \int_0^t e^{\tau(-\lambda_i + j\omega_r) + \lambda_i t} d\tau , \\
= \{C\}_r \frac{1}{-\lambda_i + j\omega_r} e^{\tau(-\lambda_i + j\omega_r) + \lambda_i t} \bigg|_0^t , \\
= \{C\}_r \frac{(e^{j\omega_r t} - e^{\lambda_i t})}{-\lambda_i + j\omega_r} . \quad [36]
\end{aligned}$$

From this result it is seen that the solution of [35] is

$$\{X(t)\} = \text{Re} \left| \sum_i^{2n} \{\psi\}_i \{\psi\}_i^T \sum_k^s \{C\}_k \frac{(e^{j\omega_k t} - e^{\lambda_i t})}{-\lambda_i + j\omega_k} \right| . \quad [37]$$

When only the steady state solution is desired, the second exponential term of [37] is recognized to be the transient response for the assumed zero initial conditions. Therefore, given sufficient time, the contribution of

$e^{\lambda_i t}$ in [37] may be ignored; thus the steady state response for sinusoidal inputs is given by

$$\{X(t)\} = \text{Re} \left| \sum_i^{2n} \{\psi\}_i \{\psi\}_i^T \sum_k^s \{C\}_k \frac{e^{j\omega_k t}}{-\lambda_i + j\omega_k} \right| . \quad [38]$$

Alternate Determination of Particular Solution

The particular solution for the equations of motion

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{Q\} , \quad [39]$$

may be solved by the method of undetermined coefficients. When $\{Q\} = \{q_1\}\cos \beta + \{q_2\}\sin \beta$ (where β is a linear function of time), assume that

$$\{X\} = \{E\} \cos \beta + \{F\} \sin \beta . \quad [40]$$

After substituting the respective time derivatives of [40] into [39] and equating the sine and cosine terms, the following equations result:

$$(-\beta^2 [M]\{E\} + \dot{\beta}[C]\{F\} + [K]\{E\}) \cos \beta = \{q_1\} \cos \beta ,$$

$$(-\beta^2 [M]\{F\} - \dot{\beta}[C]\{E\} + [K]\{F\}) \sin \beta = \{q_2\} \sin \beta .$$

After cancelling $\cos \beta$ and $\sin \beta$ from the above equations, the equations may be written in the equivalent partitioned matrix form as

$$\begin{vmatrix} -\ddot{\beta}^2 [M] + [K] & \dot{\beta} [C] \\ -\ddot{\beta} [C] & -\ddot{\beta}^2 [M] + [K] \end{vmatrix} \begin{vmatrix} \{E\} \\ \{F\} \end{vmatrix} = \begin{vmatrix} \{q_1\} \\ \{q_2\} \end{vmatrix} . \quad [41]$$

Since the tires on a given side experience the identical sinusoidal bump, the vehicle response for a single input at the respective tires may be determined simultaneously by altering [41] to the form

$$\begin{vmatrix} -\omega^2 [M] + [K] & \omega [C] \\ -\omega [C] & -\omega^2 [M] + [K] \end{vmatrix} \begin{vmatrix} E_1 & E_2 & E_3 \\ F_1 & F_2 & F_3 \end{vmatrix} = \begin{vmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \end{vmatrix} , \quad [42]$$

where ω is the common forcing frequency for a given side of the vehicle.

Assuming the equations of [42] represent forcing of the left side, the solution of the 2n algebraic equations gives the response of the vehicle as

$$\begin{aligned} \{X\} = & \{E_1\} \cos \omega t + \{F_1\} \sin \omega t + \{E_2\} \cos (\omega t - \tau_1) + \\ & \{F_2\} \sin (\omega t - \tau_1) + \{E_3\} \cos (\omega t - \tau_2) + \{F_3\} \sin (\omega t - \tau_2). \end{aligned} \quad [43]$$

Utilizing the trigonometric identities

$$\begin{aligned} \sin (y-z) &= \sin y \cos z - \cos y \sin z, \\ \cos (y-z) &= \cos y \cos z + \sin y \sin z, \end{aligned}$$

and defining

$$\{R\} = \{E_1\} + \{E_2\} \cos \tau_1 - \{F_2\} \sin \tau_1 + \{E_3\} \cos \tau_2 - \{F_3\} \sin \tau_2,$$

$$\{S\} = \{F_1\} + \{E_2\} \sin \tau_1 + \{F_2\} \cos \tau_1 + \{E_3\} \sin \tau_2 + \{F_3\} \cos \tau_2,$$

it can be shown that [43] is equivalent to

$$\{X\} = \{R\} \cos \omega t + \{S\} \sin \omega t. \quad [44]$$

Following the same procedure as given above, the response due to forcing of the right side can be determined. Adding these results gives the total response of the vehicle as

$$\{X\} = \{R\}_\ell \cos \omega t + \{S\}_\ell \sin \omega t + \{R\}_r \cos \omega t + \{S\}_r \sin \omega t, \quad [45]$$

where the subscripts indicate the respective sides of the vehicle.

Chapter V

DIGITAL COMPUTER SOLUTIONS

Two major digital computer programs were developed to obtain the particular solutions for the semi-trailer truck. The source listings given in appendix A and B are the programs corresponding to the reduced equation method and the undetermined coefficients method discussed in chapter IV. The following paragraphs include a discussion of the output for the two programs along with a brief description of the numerical methods employed.

Reduced Equations Program

The vehicle parameters (springs, dampers, masses and characteristic lengths) are read into the program under namelist format control and then printed out as shown in figure 10. After the mass, damping and stiffness matrix elements are computed in subroutine DEFINE, the undamped natural frequencies and damped eigenvalues are determined, using the method of Francis (5), and printed out as given in figures 11 and 12.

Next the complex conjugate eigenvectors are determined by inverse iteration, as explained by Wilkinson (20), and printed out as shown in figure 13 (note the eigenvectors are normalized as required in equation [31]).

Subroutine RIDE determines the sinusoidal response for the vehicle due to the sinusoidal forcing functions at the six tires, which were calculated in subroutine ROADIN. The sinusoidal road characteristics are explained on the next page, along with the response vectors as displayed in figure 14.

The final page of output, as indicated in figure 15, gives the maximum amplitudes of all the independent coordinates for the given road input.

FOR THIS SEMI-TRAILER TRUCK THE CHARACTERIZING
PARAMETERS ARE AS FOLLOWS.

DAMPERS (KIP-SEC/FT)	SPRINGS (KIP/FT)	DIMENSIONAL LENGTHS (FT)	MASS MOMENTS OF INERTIA (KIP-SEC**2-FT)	MASSES (KIP-SEC**2/FT)
C1= 0.780000E 00	K1= 0.198000E 02	A= 0.0	M11= 0.488000E 00	M1= 0.488000E-01
C2= 0.780000E 00	K2= 0.198000E 02	B= 0.648400E 01	M12= 0.238000E 01	M2= 0.258000E 00
C3= 0.240000E 00	K3= 0.150000E 03	CC= 0.250000E 01	M13= 0.149000E 01	M3= 0.149000E 00
C4= 0.240000E 00	K4= 0.150000E 03	D= 0.0	M14= 0.512000E 02	M4= 0.171000E 01
C5= 0.350000E 00	K5= 0.198000E 03	E= 0.170000E 02	M15= 0.127000E 01	M5= 0.127000E 00
C6= 0.350000E 00	K6= 0.198000E 03	F= 0.350000E 01	M12= 0.476000E 01	M6= 0.455800E 00
C7= 0.240000E-01	K7= 0.210000E 02	G= 0.376000E 01	M14= 0.149900E 03	M7= 0.258000E 00
C8= 0.240000E-01	K8= 0.210000E 02	H= 0.740000E 01	M12= 0.400000E 01	M8= 0.183700E 01
C9= 0.960000E-01	K9= 0.170000E 03	P= 0.108400E 01	M14= 0.151200E 03	M9= 0.171000E 01
C10= 0.960000E-01	K10= 0.170000E 03	Q= 0.300000E 01	P67= 0.0	
C11= 0.120000E 00	K11= 0.210000E 03	R= 0.116700E 02	P62= 0.0	
C12= 0.120000E 00	K12= 0.210000E 03	S= 0.350000E 01	P27= 0.0	
C13= 0.480000E-01	K13= 0.150000E 02	T= 0.350000E 01	P84= 0.0	
C14= 0.480000E-01	K14= 0.150000E 02	U= 0.175000E 01	P89= 0.0	
C15= 0.0	K15= 0.120000E 02	V= 0.750000E 00	P49= 0.0	
C16= 0.0	K16= 0.120000E 02	W= 0.295800E 01		
C17= 0.150000E 00	K17= 0.102000E 03	X= 0.0		
C18= 0.150000E 00	K18= 0.102000E 03	Y= 0.168500E 01		
C19= 0.0	K19= 0.187000E 02	Z= 0.0		
C20= 0.0	K20= 0.187000E 02			
C21= 0.200000E 00	K21= 0.126000E 03			
C22= 0.200000E 00	K22= 0.126000E 03			
C23= 0.0	K23= 0.250000E 02			
C24= 0.0	K24= 0.250000E 02			

Figure 10: Characterizing vehicle parameters.

THE N UNDAMPED CIRCULAR FREQUENCIES ARE

3.310115264267059D 00

6.726432012399262D 00

8.516004648279079D 00

1.000633374039780D 01

1.089404301032081D 01

1.467898336854767D 01

2.028969153925486D 01

2.113439555437766D 01

3.217689469462108D 01

3.702877707342676D 01

3.780928679435809D 01

4.258038166162345D 01

4.485904373484252D 01

5.797693591177322D 01

6.848518285529034D 01

6.942691803003756D 01

8.202576240139486D 01

THE ERROR IN THE EIGENVALUES IS COMPARABLE TO
1.048286182312220D-11

Figure 11: Undamped natural frequencies.

THE N DAMPED, COMPLEX CONJUGATE EIGENVALUE PAIRS ARE

REAL PART	IMAGINARY PART
-6.977365367288358D-03	3.310119819904249D 00
-6.977365367288358D-03	-3.310119819904249D 00
-2.433228024426425D-02	6.726678153693640D 00
-2.433228024426425D-02	-6.726678153693640D 00
-2.620388770238901D-02	8.516819042061824D 00
-2.620388770238901D-02	-8.516819042061824D 00
-8.605615222299350D-02	1.001290943392688D 01
-8.605615222299350D-02	-1.001290943392688D 01
-9.924787884981981D-01	1.110775877476442D 01
-9.924787884981981D-01	-1.110775877476442D 01
-1.865423224962665D-01	1.468319147763928D 01
-1.865423224962665D-01	-1.468319147763928D 01
-1.048229249213930D 00	2.044177285985040D 01
-1.048229249213930D 00	-2.044177285985040D 01
-2.475289146000497D-01	2.114228640837730D 01
-2.475289146000497D-01	-2.114228640837730D 01
-2.194299370101939D 00	3.311275532378840D 01
-2.194299370101939D 00	-3.311275532378840D 01
-4.360803660253532D 00	3.594218922000895D 01
-4.360803660253532D 00	-3.594218922000895D 01
-2.064598298907953D 01	3.608972855999870D 01
-2.064598298907953D 01	-3.608972855999870D 01
-7.228919689638735D-01	3.713737221178032D 01
-7.228919689638735D-01	-3.713737221178032D 01
-1.063615812073526D 00	4.485141758916870D 01
-1.063615812073526D 00	-4.485141758916870D 01
-1.769412451678008D 00	5.793561668086176D 01
-1.769412451678008D 00	-5.793561668086176D 01
-2.690821992762824D 00	6.840986622337274D 01
-2.690821992762824D 00	-6.840986622337274D 01
-2.623891127336520D 00	6.936841336596851D 01
-2.623891127336520D 00	-6.936841336596851D 01
-4.064845992317023D 00	8.191993220959171D 01
-4.064845992317023D 00	-8.191993220959171D 01

THE ERROR IN THE EIGENVALUES IS COMPARABLE TO
1.048288804625388D-13

Figure 12: Damped eigenvalues.

THE DAMPED EIGENVALUES AND THEIR ASSOCIATED EIGENVECTORS ARE		
1	2	3
-6.977365367288358D-03	3.310119819904249D 00	-6.977365367288358D-03
-3.857310191198520D-16	-3.857310191198520D-16	-2.889453777045809D-16
-2.635082338230157D-15	-2.635082338230157D-15	-3.603405245093768D-16
-1.618124432387662D-15	-1.618124432387662D-15	-1.618124432387662D-15
-3.690683411937736D-15	-2.792515701650395D-15	-3.690683411937736D-15
-2.728199854282866D-02	-2.742353756977890D-02	-2.728199854282866D-02
-6.639989073692518D-03	-6.752793441775530D-03	-6.639989073692518D-03
-6.963558947352552D-03	-6.963558947352552D-03	-6.963558947352552D-03
-1.982098435020087D-02	-1.987923675324778D-02	-1.982098435020087D-02
-1.348200883482611D-03	-1.439796670243658D-03	-1.348200883482611D-03
-2.971102975522255D-03	-2.931569626366941D-03	-2.971102975522255D-03
-1.308959528273037D-03	-1.305862643782849D-03	-1.308959528273037D-03
-2.809890927953280D-02	-2.806265752596438D-02	-2.809890927953280D-02
-7.804709946897892D-03	-7.840970224067032D-03	-7.804709946897892D-03
-4.467990146906618D-16	-1.756311260106459D-17	-4.467990146906618D-16
-1.859337963008272D-16	-8.758677502703067D-17	-1.859337963008272D-16
-1.844787895679693D-03	-1.844637369207446D-03	-1.844787895679693D-03
-1.276314321459505D-03	-1.283864536494062D-03	-1.276314321459505D-03
-1.420563311001877D-14	-1.026149675538217D-14	-1.420563311001877D-14
-6.759512576679273D-15	-1.004086481089147D-14	-6.759512576679273D-15
-1.137792768387593D-14	-5.867695762699617D-15	-1.137792768387593D-14
-6.215152260983815D-15	-9.818707177647042D-16	-6.215152260983815D-15
-9.096555171338160D-02	-9.011534006191690D-02	-9.096555171338160D-02
-2.239888504112708D-02	-2.193204272967295D-02	-2.239888504112708D-02
-2.309884235298370D-02	-2.302087358774764D-02	-2.309884235298370D-02
-6.594095383122970D-02	-6.547112844955040D-02	-6.594095383122970D-02
-4.775306384973719D-03	-4.452660478216533D-03	-4.775306384973719D-03
-9.724577194686047D-03	-9.814252213881110D-03	-9.724577194686047D-03
-4.331694908153410D-03	-4.323701397215145D-03	-4.331694908153410D-03
-9.308681523236090D-02	-9.281495310914840D-02	-9.308681523236090D-02
-2.600900725886404D-02	-2.577981576975443D-02	-2.600900725886404D-02
-1.542944655188194D-14	-1.269433000903279D-14	-1.542944655188194D-14
-1.577568512704007D-14	-1.183010121013263D-14	-1.577568512704007D-14
-6.118842475507425D-03	-6.093598268102474D-03	-6.118842475507425D-03
-4.258650759650020D-03	-4.215795339926313D-03	-4.258650759650020D-03

Figure 13: Damped eigenvalues with associated eigenvectors.

THE SPEED (FEET/SECOND) OF THIS VEHICLE IS EQUAL TO 8.80000 01

THE SINUSOIDAL FORCING FREQUENCIES (RAD/SEC) FOR THE LEFT AND FOR THE RIGHT SIDE WHEEL PATHS ARE RESPECTIVELY

WL= 5.000000000000000 01 WR= 5.000000000000000 01

THE AMPLITUDE (FEET) OF THE LEFT AND RIGHT SINUSOIDAL PATHS ARE RESPECTIVELY

8.35000-02 8.35000-02

THE WAVELENGTH OF THE LEFT AND RIGHT SINUSOIDAL DISPLACEMENT FUNCTIONS ARE RESPECTIVELY

1.10584061410 01 1.10584061410 01

OUTPUT(I) = A(I)*COS(WL*T) + B(I)*SIN(WL*T) + C(I)*COS(WR*T) + D(I)*SIN(WR*T)

A(I)	B(I)	C(I)	D(I)
-1.7440003657057630-02	-4.3390158270396110-03	-1.7440003657415960-02	-4.3390158268615280-03
2.38196223206841790-03	-5.3606556013424730-03	2.38196223208296230-03	-5.3606556017727390-03
-6.2659458814033260-03	3.0629268855155560-02	-6.2659458841121870-03	3.0629268873044420-02
5.6762041775309490-03	-2.4295097685164720-02	5.6762041779954540-03	-2.4295097687365240-02
1.3080150723528290-04	7.4989045240384150-05	-1.3080150707423430-04	-7.4989045027486410-05
2.3810329536165520-03	-2.1306837023809960-03	-2.3810329557375630-03	2.1306837015620140-03
5.4420372681433970-03	-5.9307274598775620-02	-5.4420372685178640-03	5.9307274600021200-02
1.026682145145840-02	1.0669203040090040-01	-1.0266821450558380-02	-1.0669203040778980-01
1.5558648358982500-03	6.4433809969376330-03	-1.5558648358477970-03	-6.4433809965164570-03
-9.8237947649124740-04	5.0313750467087420-03	9.8237947634043160-04	-5.0313750466841890-03
6.7430301945850870-03	-2.25928322268520940-02	-6.7430301946237880-03	2.2592832268555550-02
2.3021087192225610-05	-1.3266183676274120-04	-2.3021087205011490-05	1.3266183685784080-04
-5.4650347848213470-03	3.7600317744327170-03	5.4650347854126610-03	-3.7600317800453360-03
1.9507676159304200-04	1.8731472344702060-03	1.9507676157073380-04	1.8731472349726870-03
7.4542561926610150-05	-1.0555221459571440-03	7.4542561942561490-05	-1.0555221460525440-03
1.5021030524541320-04	-2.9787742577379580-04	-1.5021030526973280-04	2.9787742580888050-04
-1.1700480316721560-05	-2.7615199048349920-04	1.1700480311418300-05	2.7615199056013930-04

Figure 14: Vehicle response vectors.

THE MAXIMUM AMPLITUDES ARE

THE RELATIVE PHASE ANGLE IS

0.0

X1	=	3.590-02
X2	=	1.170-02
X3	=	6.250-02
X5	=	4.990-02
X7	=	2.670-13
Z1	=	2.270-12
Z3	=	1.300-12
Z5	=	6.950-12
THETA1	=	4.240-13
THETA2	=	1.530-13
THETA3	=	5.190-14
THETA4	=	9.600-14
THETA5	=	5.640-12
PHI2	=	3.770-03
PHI4	=	2.120-03
X12	=	4.270-14
X14	=	7.680-14

Figure 15: Maximum amplitudes for the coordinates.

Undetermined Coefficients Program

Except for the absence of the eigenvalues and eigenvectors, the output information and format for this program are identical to those of the previous program. However the numerical algorithms used are vastly different. Although the merits of the two programs will be discussed in the next chapter, it is of interest here to compare the solutions determined by this method, shown in figure 16, to those shown in figure 15. It is seen that the results are in close agreement.

THE MAXIMUM AMPLITUDES (FEET,RADIANS) ARE

THE RELATIVE PHASE ANGLE(RAD.) BETWEEN SIDES IS

0.0

X1 = 3.590-02

X2 = 1.170-02

X3 = 6.250-02

X5 = 4.990-02

X7 = 0.0

Z1 = 0.0

Z3 = 0.0

Z5 = 0.0

THETA1 = 0.0

THETA2 = 0.0

THETA3 = 0.0

THETA4 = 0.0

THETA5 = 0.0

PHI2 = 3.770-03

PHI4 = 2.120-03

XI2 = 0.0

XI4 = 0.0

Figure 16: Maximum amplitudes for the coordinates using the method of undetermined coefficients.

Chapter VI

RECOMMENDATIONS AND CONCLUSIONS

Recommendations

The scope of this thesis was to model a linear, three-dimensional semi-trailer truck. This object is a necessary first step for a better understanding of the vehicle's vibrations, but does not represent an end in itself. Many areas of analysis need to be initiated for a more complete understanding of the vehicle's vibrational characteristics. Using this model, a number of analyses could be effectively investigated. These are discussed in the following paragraphs.

The steady state solutions presented in this paper were used to verify the correctness of the mathematical model, not for a steady state analysis. However a comprehensive steady state analysis could be completed using this three-dimensional model. Bode plots similar to those presented by Potts (14) could be obtained for this three dimensional vehicle with few additions to the current programs.

The rolling motions experienced on asphalt highways due to phased sinusoidal wave forms could be investigated. The effect of different road frequencies experienced by the left and right sides of the vehicle could also be studied with small modifications to the present programs. Bode plots of the above phenomenon, considered separately or combined, would give a vivid display of the vehicle's response.

The consequences of crosswinds on the vehicle's motion could be explored by adding equivalent sinusoidal forcing functions to the existing programs. Dugoff (2) examined the lateral stability of highway vehicles due to aero-

dynamic forces, but did not include vehicular vibrations. With the inclusion of vehicular vibrations, a more comprehensive study would result.

For the particular solutions given in chapter V, it was assumed that the road bumps force the vehicle in the vertical direction only. However it is obvious that road bumps also force the tires of the vehicle in the longitudinal and transverse directions. After an appropriate "bump" model is determined, the forces on the vehicle's tires could be readily incorporated into this three dimensional vehicle by adding equivalent forcing functions.

The response of the vehicle to sudden irregularities in a road, such as chuck holes, could be determined with a transient analysis.

Since road profiles represent a random phenomenon, a random vibrational analysis might give the most practical information about the vehicle's motions. Using the set of eigenvalues and eigenvectors determined in the reduced equation program, Lancaster (19) indicates how the response of the system forced randomly may be determined by the simple relation

$$\{X\} = \sum_i^{2n} \frac{\{\psi\}_i^T \{Q\} \{\psi\}_i}{j\omega - \lambda_i}$$

where $\{Q\}$ is the power spectra for the generalized forces.

Conclusions

When deciding between the two programs, the number of frequencies investigated must be considered. The computing time required by the undetermined coefficients program is far less for up to twenty separate solutions. For investigations which exceed thirty solutions, the reduced equation program becomes superior.

Before using the undetermined coefficients program, the great flexibility of the reduced equation program should be considered. A given model is numerically characterized by its set of eigenvalues and eigenvectors determined in the reduced equation program. After they are determined, the set could be stored indefinitely and used repeatedly. Large numbers of particular solutions could be rapidly obtained by matrix multiplications, as opposed to solutions obtained by solving linear equations used in the undetermined coefficients program. As indicated in the recommendations, the reduced equation program can be used in transient and random vibrational analyses.

The three-dimensional model for the vibrations of a semi-trailer truck presented in this thesis is the most comprehensive model known to this author and will add to the understanding of the vibrations of semi-trailer trucks. However before concrete conclusions can be drawn from its use, the accuracy of the model must be determined by comparing it to "real" vehicles.

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Appendix A

EXPLANATION OF INPUT DATA FOR REDUCED EQUATION PROGRAM WITH SOURCE LISTING

All input data were read into the program under namelist format control. The values for the springs, dampers, masses, mass moments of inertia, and characteristic lengths were given in namelist data groups SPRING, DAMPER, MASSES, INERTA, and DIMEN respectively. The numerical suffixes of these REAL*4, fortran variables correspond to the subscripted parameters defined in the nomenclature (e.g. $K_{11} = k_{11}$). In namelist data group ROAD, the spring and damping constants for the vehicle tires were coded into REAL*8, fortran variables by using roman numerals for subscripts (e.g. $KVII = k_7$). The remaining REAL*8, input variables are explained below.

AL = amplitude of left side sinusoidal road (feet)

AR = amplitude of right side sinusoidal road (feet)

ZWIDE = (COMPLEX*16) distance from axle center of mass to equivalent
tire attachment point (feet)

BT = the lag angle of the right sinusoidal road relative to the left
sinusoidal road (radians)

GH1 = longitudinal distance from tractor front axle to tractor rear
axle (= $g+h$, feet)

GBER2 = longitudinal distance from tractor front axle to trailer axle
(= $g + b + e + r$, feet)

SPEED = vehicle speed (feet/second)

FWL = left sinusoidal frequency (radians/second)

FWR = right sinusoidal frequency (radians/second)

```

      IMPLICIT REAL*8 (A-H,O-Y),COMPLEX *16(Z)
      COMPLEX*16 DCMPLX,CDSQRT,DCONJG
      COMMON/COMX/ZVEC(35,34)
      COMMON/DYNAM/D(34,34),VALU(34,2)
      COMMON/MATMCK/AM(17,17),AC(17,17),AK(17,17)
      COMMON/A1/ DL(17,17),APA(17,17)
      COMMON/ZRIDE/ZRIDL(17),ZRIDR(17)
      COMMON/ZROAD/ZCL(6),ZCR(6)
      COMMON/ORDER/NMO,N,NPO,NTMO,NT,NTPO,NN,NTNT
      COMMON/INOUT/NREAD,NWRITE,NPUNCH
      COMMON/RIPPLE/WAVEL,WAVER,FAST,AMPL,AMPR,FAZE
      CALL DEFINE
      DO 2 I=1,N
      DO 2 J=1,N
2    AMA(J,I)=AM(J,I)
      CALL MINV(AM,N,NN)
      CALL GMPRD(AM,AK,DL,N,N,N,NN,NN,NN)
      DO 1 I=1,N
      LRR=I+N
      DO 1 J=1,N
1    D(LLR,J)=-DL(I,J)*1.D-2
      CALL FRANCI(DL,VALU,N,ANORM,N)
      CALL ARANGE(VALU,N,NMO,NPO,1)
      ERR=1.110223D-16*ANORM
      WRITE(NWRITE,7) ERR
C
C
      CALL GMPRD(AM,AC,DL,N,N,N,NN,NN,NN)
      DO 11 I=1,N
      LRR=I+N
      DO 11 J=1,N
      LRC=J+N
11   D(LRR,LRC)=-DL(I,J)*1.C-2
      DO 22 I=1,N
      IPNC=I+N
22   D(I,IPNC)=1.D-2
C
C
C    DETERMINE DAMPED EIGENVALUES AND EIGENVECTORS
C
      CALL FRANCI(D,VALU,NT,ANORM,NT)
      DO 300 I=1,NT
      VALU(I,1)=VALU(I,1)*1.D2
300  VALU(I,2)=VALU(I,2)*1.C2
      CALL ARANGE( VALU,NT,NTMO,NTPO,3)
      ERR=1.110223D-16*ANORM
      WRITE(NWRITE,7) ERR
      DO 43 J=1,NT,2
      DO 33 I=1,NT
33   ZVEC(I,J)=(1.00,1.00)
      ZVEC(35,J)=DCMPLX(VALU(J,1),VALU(J,2))
43   ZVEC(35,J+1)=DCONJG(ZVEC(35,J))
      CALL VECITR
      CALL BIORTH
      WRITE(NWRITE,155)
155  FORMAT('1',35X,'THE DAPPED EIGENVALUES AND THEIR ASSOCIATED'
1,X,'EIGENVECTORS ARE')
      DO 133 J=1,NT,2
      WRITE(NWRITE,144) ZVEC(NT+1,J),ZVEC(NT+1,J+1)

```

```

144 FORMAT('0',10X,1P2D25.15,10X,1P2D25.15)
WRITE(NWRITE,146)
JPO=J+1
WRITE(NWRITE,1122) J,JPO
1122 FORMAT('0',38X,12,58X,12//)
DO 134 M=1,NT
134 WRITE(NWRITE,145) ZVEC(M,J),ZVEC(M,J+1)
145 FORMAT(' ',10X,1P2D25.15,10X,1P2D25.15)
133 WRITE(NWRITE,147)
147 FORMAT('1')
146 FORMAT(////)
7 FORMAT('-',42X,'THE ERROR IN THE EIGENVALUES IS COMPARABLE TO'
1/' ',54X,1PD24.15)
NOREAD=1
CALL ROADIN(ZWL,ZWR,WL,WR,NOREAD)
CALL RIDE(ZWR,ZWL,NT,N)
WRITE(NWRITE,179)
179 FORMAT('1')
WRITE(NWRITE,210) FAST
WRITE(NWRITE,190)
WRITE(NWRITE,178) WL,WR
WRITE(NWRITE,3000) AMPL,AMPR
3000 FORMAT('-',25X,'THE AMPLITUDE (FEET) OF THE LEFT AND RIGHT'
1,1X,'SINUSOIDAL PATHS ARE RESPECTIVELY'/'0',47X,1P2D15.4)
190 FORMAT('-',10X,'THE SINUSOIDAL FORCING FREQUENCIES (RADIAN/SEC)'
1,1X,'FOR THE LEFT AND FOR THE RIGHT SIDE WHEEL PATHS ARE'
2,1X,'RESPECTIVELY')
210 FORMAT(' ',30X,'THE SPEED (FEET/SECOND) OF THIS VEHICLE'
1,1X,'IS EQUAL TO',1PD12.4)
178 FORMAT('0',35X,'WL=',1PD22.15,10X,'WR=',1PD22.15)
WRITE(NWRITE,4) WAVEL,WAVER
4 FORMAT('-',25X,'THE WAVELENGTH OF THE LEFT AND RIGHT SINUSOIDAL'
1,1X,'DISPLACEMENT FUNCTIONS ARE RESPECTIVELY'/'0',40X,1P2D25.10)
WRITE(NWRITE,177)
177 FORMAT('0',25X,'OUTPUT(I) = A(I)*COS(WL*T) + B(I)*SIN(WL*T) + '
1,1X,'C(I)*COS(WR*T) + C(I)*SIN(WR*T)')
180 FORMAT('0',1P4D30.15)
WRITE(NWRITE,181)
181 FORMAT('-',17X,'A(I)',26X,'B(I)',26X,'C(I)',26X,'D(I)')
DO 149 I=1,N
149 WRITE(NWRITE,180) ZRIDL(I),ZRIDR(I)
DIFFER=DABS(WL-WR)
IF(DIFFER.LE.1.D-7) CALL WEQULZ
STOP
END

```

```

SUBROUTINE DEFINE
REAL*8 M,C,K
REAL *4 M1,M2,M3,M4,M5,M6,M7,M8,M9
REAL *4 M11,M12,M13,M14,M15,MJ2,MJ4,MH2,MH4,P67,P62,P27,P84,P89,
UP49
REAL *4 K1,K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13,K14,K15,K16
REAL *4 K17,K18,K19,K20,K21,K22,K23,K24
COMMON/INOUT/NREAC,NWRITE,NPUNCH
COMMON/MATMCK/M(17,17),C(17,17),K(17,17)
NAMELIST/MASSES/M1,M2,M3,M4,M5,M6,M7,M8,M9
NAMELIST/INERTA/M11,M12,M13,M14,M15,MJ2,MJ4,MH2,MH4,
1P67,P62,P27,P84,P89,P49
NAMELIST/DAMPER/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15
1,C16,C17,C18,C19,C20,C21,C22,C23,C24
NAMELIST/SPRING/K1,K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13,K14,K15
1,K16,K17,K18,K19,K20,K21,K22,K23,K24
NAMELIST/DIMEN/A,B,CC,D,E,F,G,H,P,Q,R,S,T,U,V,W,X,Y,Z
READINREAD,MASSES)
READINREAD,INERTA)
READINREAD,DAMPER)
READINREAD,SPRING)
READINREAD,DIMEN)
30 FORMAT('1',T43,'FCR THIS SEMI-TRAILER TRUCK THE CHARACTERIZING'
1/' ',T53,'PARAMETERS ARE AS FOLLOWS.'////)
40 FORMAT(' ',T14,'DAMPERS',T40,'SPRINGS',T60,'DIMENSIONAL LENGTHS'
1,T87,'MASS MOMENTS OF INERTIA',T120,'MASSES')
60 FORMAT(' ',T12,'(KIP-SEC/FT)',T40,'(KIP/FT)',T68,'(FT)',T91,
1'(KIP-SEC**2-FT)',T115,'(KIP-SEC**2/FT)')
1 FORMAT('0',T11,'C1=',E13.6,T37,'K1=',E13.6,T64,'A=',E13.6,T90,
1'M11=',E13.6,T116,'M1=',E13.6)
2 FORMAT('0',T11,'C2=',E13.6,T37,'K2=',E13.6,T64,'B=',E13.6,T90,
2'M12=',E13.6,T116,'M2=',E13.6)
3 FORMAT('0',T11,'C3=',E13.6,T37,'K3=',E13.6,T63,'CC=',E13.6,T90,
3'M13=',E13.6,T116,'M3=',E13.6)
4 FORMAT('0',T11,'C4=',E13.6,T37,'K4=',E13.6,T64,'D=',E13.6,T90,
4'M14=',E13.6,T116,'M4=',E13.6)
5 FORMAT('0',T11,'C5=',E13.6,T37,'K5=',E13.6,T64,'E=',E13.6,T90,
5'M15=',E13.6,T116,'M5=',E13.6)
6 FORMAT('0',T11,'C6=',E13.6,T37,'K6=',E13.6,T64,'F=',E13.6,T90,
6'MJ2=',E13.6,T116,'M6=',E13.6)
7 FORMAT('0',T11,'C7=',E13.6,T37,'K7=',E13.6,T64,'G=',E13.6,T90,
7'MJ4=',E13.6,T116,'M7=',E13.6)
8 FORMAT('0',T11,'C8=',E13.6,T37,'K8=',E13.6,T64,'H=',E13.6,T90,
8'MH2=',E13.6,T116,'M8=',E13.6)
9 FORMAT('0',T11,'C9=',E13.6,T37,'K9=',E13.6,T64,'P=',E13.6,T90,
9'MH4=',E13.6,T116,'M9=',E13.6)
10 FORMAT('0',T10,'C10=',E13.6,T36,'K10=',E13.6,T64,'Q=',E13.6,T90,
2'P67=',E13.6)
11 FORMAT('0',T10,'C11=',E13.6,T36,'K11=',E13.6,T64,'R=',E13.6,T90,
2'P62=',E13.6)
12 FORMAT('0',T10,'C12=',E13.6,T36,'K12=',E13.6,T64,'S=',E13.6,T90,
2'P27=',E13.6)
13 FORMAT('0',T10,'C13=',E13.6,T36,'K13=',E13.6,T64,'T=',E13.6,T90,
2'P84=',E13.6)
14 FORMAT('0',T10,'C14=',E13.6,T36,'K14=',E13.6,T64,'U=',E13.6,T90,
2'P89=',E13.6)
15 FORMAT('0',T10,'C15=',E13.6,T36,'K15=',E13.6,T64,'V=',E13.6,T90,
2'P49=',E13.6)
16 FORMAT('0',T10,'C16=',E13.6,T36,'K16=',E13.6,T64,'W=',E13.6)
17 FORMAT('0',T10,'C17=',E13.6,T36,'K17=',E13.6,T64,'X=',E13.6)

```

```

18 FORMAT('0',T10,'C18=',E13.6,T36,'K18=',E13.6,T64,'Y=',E13.6)
19 FORMAT('0',T10,'C19=',E13.6,T36,'K19=',E13.6,T64,'Z=',E13.6)
20 FORMAT('0',T10,'C20=',E13.6,T36,'K20=',E13.6)
21 FORMAT('0',T10,'C21=',E13.6,T36,'K21=',E13.6)
22 FORMAT('0',T10,'C22=',E13.6,T36,'K22=',E13.6)
23 FORMAT('0',T10,'C23=',E13.6,T36,'K23=',E13.6)
24 FORMAT('0',T10,'C24=',E13.6,T36,'K24=',E13.6)
WRITE(NWRITE,30)
WRITE(NWRITE,40)
WRITE(NWRITE,60)
WRITE(NWRITE, 1) C1 ,K1 ,A ,M11,M1
WRITE(NWRITE, 2) C2 ,K2 ,B ,M12,M2
WRITE(NWRITE, 3) C3 ,K3 ,CC,M13,M3
WRITE(NWRITE, 4) C4 ,K4 ,D ,M14,M4
WRITE(NWRITE, 5) C5 ,K5 ,E ,M15,M5
WRITE(NWRITE, 6) C6 ,K6 ,F ,MJ2,M6
WRITE(NWRITE, 7) C7 ,K7 ,G ,MJ4,M7
WRITE(NWRITE, 8) C8 ,K8 ,H ,MH2,M8
WRITE(NWRITE, 9) C9 ,K9 ,P ,MH4,M9
WRITE(NWRITE,10) C10,K10,Q,P67
WRITE(NWRITE,11) C11,K11,R,P62
WRITE(NWRITE,12) C12,K12,S,P27
WRITE(NWRITE,13) C13,K13,T,P84
WRITE(NWRITE,14) C14,K14,U,P89
WRITE(NWRITE,15) C15,K15,V,P49
WRITE(NWRITE,16) C16,K16,W
WRITE(NWRITE,17) C17,K17,X
WRITE(NWRITE,18) C18,K18,Y
WRITE(NWRITE,19) C19,K19,Z
WRITE(NWRITE,20) C20,K20
WRITE(NWRITE,21) C21,K21
WRITE(NWRITE,22) C22,K22
WRITE(NWRITE,23) C23,K23
WRITE(NWRITE,24) C24,K24
C
C*****
C
MR=M6*M8/(M6+M8)
SPA=S*.5+A
SMA=S*.5-A
EPR=E+R
TT=T*.5
FMQ=F-Q
M(1,1)=M1
M(2,2)=M2+M4
M(2,10)=-M4*A
M(2,12)=M4*D
M(2,14)=-M4*B
M(2,15)=-M4*E
M(3,3)=M3
M(4,4)=M5
M(5,5)=M7+M9
M(5,10)=M9*P
M(5,12)=-M9*Q
M(5,16)=M9*B
M(5,17)=M9*E
M(6,6)=M1
M(7,7)=M3
M(8,8)=M5
M(9,9)=M11

```


[illegible]

```

K(2,9)=U*(K1-K2)
K(2,10)=-SPA*(K1+K3)+SMA*(K2+K4)-A*(K5+K6)
K(2,11)=U*(K3-K4)
K(2,12)=TT*(K6-K5)
K(2,13)=U*(K5-K6)
K(2,14)=K1*G+K2*G-K3*H-K4*H-K5*B-K6*B
K(2,15)=-EPR*(K5+K6)
K(3,3)=K3+K4+K9+K10
K(3,10)=SPA*K3-SMA*K4
K(3,11)=W*(K10-K9)+U*(K4-K3)
K(3,14)=K3*H+K4*H
K(4,4)=K5+K6+K11+K12
K(4,10)=A*(K5+K6)
K(4,12)=TT*(K5-K6)
K(4,13)=W*(K12-K11)+U*(K6-K5)
K(4,14)=K5*B+K6*B
K(4,15)=EPR*(K5+K6)
K(5,5)=K15+K16+K19+K20+K23+K24
K(5,6)=-K15-K16
K(5,7)=-K19-K20
K(5,8)=-K23-K24
K(5,9)=V*(K15+K16)
K(5,10)=CC*(K15+K16+K19+K20)+K23*P+K24*P
K(5,11)=V*(K19+K20)
K(5,12)=(K23+K24)*FMQ
K(5,13)=V*(K23+K24)
K(5,16)=-K15*G-K16*G+K19*H+K20*H+K23*B+K24*B
K(5,17)=(K23+K24)*EPR
K(6,6)=K13+K14+K15+K16
K(6,9)=Y*(K13+K14)-V*(K15+K16)
K(6,10)=-K15*CC-K16*CC
K(6,16)=K15*G+K16*G
K(7,7)=K17+K18+K19+K20
K(7,10)=-K19*CC-K20*CC
K(7,11)=Y*(K17+K18)-V*(K19+K20)
K(7,16)=-K19*H-K20*H
K(8,8)=K21+K22+K23+K24
K(8,10)=-K23*P-K24*P
K(8,12)=-K23+K24)*FMQ
K(8,13)=Y*(K21+K22)-V*(K23+K24)
K(8,16)=-B*(K23+K24)
K(8,17)=-K23+K24)*EPR
K(9,9)=W*W*(K7+K8)+Y*Y*(K13+K14)+U*U*(K1+K2)+V*V*(K15+K16)
K(9,10)=-U*(SPA*K1+SMA*K2)+V*CC*(K15+K16)
K(9,14)=U*G*(K1-K2)
K(9,16)=-V*G*(K15+K16)
K(10,10)=SPA*SPA*(K1+K3)+SMA*SMA*(K2+K4)+CC*CC*(K15+K16+K19+K20)
1+A*A*(K5+K6)+P*P*(K23+K24)
K(10,11)=-U*(SPA*K3+SMA*K4)+CC*V*(K19+K20)
K(10,12)=A*TT*(K5-K6)+P*FMQ*(K23+K24)
K(10,13)=A*U*(K6-K5)+P*V*(K23+K24)
K(10,14)=-G*(SPA*K1-SMA*K2)+H*(SPA*K3-SMA*K4)+A*B*(K5+K6)
K(10,15)=A*EPR*(K5+K6)
K(10,16)=-CC*G*(K15+K16)+CC*H*(K19+K20)+P*B*(K23+K24)
K(10,17)=P*EPR*(K23+K24)
K(11,11)=W*W*(K9+K10)+Y*Y*(K17+K18)+U*U*(K3+K4)+V*V*(K19+K20)
K(11,14)=U*H*(K4-K3)
K(11,16)=V*H*(K19+K20)
K(12,12)=TT*TT*(K5+K6)+FMQ*FMQ*(K23+K24)
K(12,13)=-TT*U*(K5+K6)+FPQ*V*(K23+K24)

```

```

K(12,14)=TT*B*(K5-K6)
K(12,15)=TT*EPR*(K5-K6)
K(12,16)=B*FMQ*(K23+K24)
K(12,17)=EPR*FMQ*(K23+K24)
K(13,13)=W*W*(K11+K12)+Y*Y*(K21+K22)+U*U*(K5+K6)+V*V*(K23+K24)
K(13,14)=U*B*(K6-K5)
K(13,15)=U*EPR*(K6-K5)
K(13,16)=V*B*(K23+K24)
K(13,17)=V*EPR*(K23+K24)
K(14,14)=G*G*(K1+K2)+H*H*(K3+K4)+B*B*(K5+K6)
K(14,15)=B*EPR*(K5+K6)
K(15,15)=(K5+K6)*EPR*EPR
K(16,16)=G*G*(K15+K16)+H*H*(K19+K20)+B*B*(K23+K24)
K(16,17)=B*EPR*(K23+K24)
K(17,17)=(K23+K24)*EPR*EPR
DO 70 I=1,16
N=I+1
DO 80 J=N,17
M(J,I)=M(I,J)
C(J,I)=C(I,J)
80 K(J,I)=K(I,J)
70 CONTINUE
RETURN
END

```

```

C
C ..... MINV 10
C ..... MINV 20
C ..... MINV 30
C SUBROUTINE MINV MINV 40
C ..... MINV 50
C PURPOSE MINV 60
C   INVERT A MATRIX MINV 70
C ..... MINV 80
C USAGE MINV 90
C   CALL MINV(A,N,D,L,M) MINV 100
C ..... MINV 110
C DESCRIPTION OF PARAMETERS MINV 120
C   A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY MINV 130
C     RESULTANT INVERSE. MINV 140
C   N - ORDER OF MATRIX A MINV 150
C   D - RESULTANT DETERMINANT MINV 160
C   L - WORK VECTOR OF LENGTH N MINV 170
C   M - WORK VECTOR OF LENGTH N MINV 180
C ..... MINV 190
C REMARKS MINV 200
C   MATRIX A MUST BE A GENERAL MATRIX MINV 210
C ..... MINV 220
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED MINV 230
C   NONE MINV 240
C ..... MINV 250
C METHOD MINV 260
C   THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT MINV 270
C   IS ALSO CALCULATED. A DETERMINANT OF ZERO INDICATES THAT MINV 280
C   THE MATRIX IS SINGULAR. MINV 290
C ..... MINV 300
C ..... MINV 310
C ..... MINV 320
C
C SUBROUTINE MINV(A,N,NN)
C REAL *8 A,D,BIGA,HOLD,DABS
C DIMENSION A(NN),L(N),M(N)
C DIMENSION L(34),M(34)
C DIMENSION A(NN)
C
C ..... MINV 350
C ..... MINV 360
C ..... MINV 370
C IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE MINV 380
C C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION MINV 390
C STATEMENT WHICH FOLLOWS. MINV 400
C ..... MINV 410
C DOUBLE PRECISION A,D,BIGA,HOLD MINV 420
C ..... MINV 430
C THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS MINV 440
C APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS MINV 450
C ROUTINE. MINV 460
C ..... MINV 470
C THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO MINV 480
C CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS. ABS IN STATEMENT MINV 490
C 10 MUST BE CHANGED TO DABS. MINV 500
C ..... MINV 510
C ..... MINV 520
C ..... MINV 530
C SEARCH FOR LARGEST ELEMENT MINV 540
C ..... MINV 550
C D=1.0 MINV 560
C NK=-N MINV 570

```

```

DO 80 K=1,N
NK=NK+N
L(K)=K
M(K)=K
KK=NK+K
BIGA=A(KK)
DO 20 J=K,N
IZ=N*(J-1)
DO 20 I=K,N
IJ=IZ+I
10 IF(DABS(BIGA)-DABS(A(IJ))) 15,20,20
15 BIGA=A(IJ)
L(K)=I
M(K)=J
20 CONTINUE
C
C      INTERCHANGE ROWS
C
J=L(K)
IF(J-K) 35,35,25
25 KI=K-N
DO 30 I=1,N
KI=KI+N
HOLD=-A(KI)
JI=KI-K+J
A(KI)=A(JI)
30 A(JI)=HOLD
C
C      INTERCHANGE COLUMNS
C
35 I=M(K)
IF(I-K) 45,45,38
38 JP=N*(I-1)
DO 40 J=1,N
JK=NK+J
JI=JP+J
HOLD=-A(JK)
A(JK)=A(JI)
40 A(JI)=HOLD
C
C      DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C      CONTAINED IN BIGA)
C
45 IF(BIGA) 48,46,48
46 D=0.0
RETURN
48 DO 55 I=1,N
IF(I-K) 50,55,50
50 IK=NK+I
A(IK)=A(IK)/(-BIGA)
55 CONTINUE
C
C      REDUCE MATRIX
C
DO 65 I=1,N
IK=NK+I
HOLD=A(IK)
IJ=I-N
DO 65 J=1,N
IJ=IJ+N

```

```

MINV 580
MINV 590
MINV 600
MINV 610
MINV 620
MINV 630
MINV 640
MINV 650
MINV 660
MINV 670
MINV 690
MINV 700
MINV 710
MINV 720
MINV 730
MINV 740
MINV 750
MINV 760
MINV 770
MINV 780
MINV 790
MINV 800
MINV 810
MINV 820
MINV 830
MINV 840
MINV 850
MINV 860
MINV 870
MINV 880
MINV 890
MINV 900
MINV 910
MINV 920
MINV 930
MINV 940
MINV 950
MINV 960
MINV 970
MINV 980
MINV 990
MINV1000
MINV1010
MINV1020
MINV1030
MINV1040
MINV1050
MINV1060
MINV1070
MINV1080
MINV1090
MINV1100
MINV1110
MINV1120
MINV1130
MINV1140
MINV1150
MINV1160
MINV1170

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```

      IF(I-K) 60,65,60
60  IF(J-K) 62,65,62
62  KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
65  CONTINUE
C
C      DIVIDE ROW BY PIVOT
C
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
70  A(KJ)=A(KJ)/BIGA
75  CONTINUE
C
C      PRODUCT OF PIVCTS
C
      D=D*BIGA
C
C      REPLACE PIVOT BY RECIPROCAL
C
      A(KK)=1.0/BIGA
80  CONTINUE
C
C      FINAL ROW AND COLUMN INTERCHANGE
C
      K=N
100 K=(K-1)
      IF(K) 150,150,105
105 I=L(K)
      IF(I-K) 120,120,108
108 JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
110 A(JI)=HOLD
120 J=M(K)
      IF(J-K) 100,100,125
125 KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
130 A(JI)=HOLD
      GO TO 100
150 RETURN
      END

```

```

MINV1180
MINV1190
MINV1200
MINV1210
MINV1220
MINV1230
MINV1240
MINV1250
MINV1260
MINV1270
MINV1280
MINV1290
MINV1300
MINV1310
MINV1320
MINV1330
MINV1340
MINV1350
MINV1360
MINV1370
MINV1380
MINV1390
MINV1400
MINV1410
MINV1420
MINV1430
MINV1440
MINV1450
MINV1460
MINV1470
MINV1480
MINV1490
MINV1500
MINV1510
MINV1520
MINV1530
MINV1540
MINV1550
MINV1560
MINV1570
MINV1580
MINV1590
MINV1600
MINV1610
MINV1620
MINV1630
MINV1640
MINV1650
MINV1660
MINV1670

```

```

C
C ..... GMPR 10
C SUBROUTINE GMPRD ..... GMPR 20
C ..... GMPR 30
C SUBROUTINE GMPRD ..... GMPR 40
C ..... GMPR 50
C PURPOSE ..... GMPR 60
C   MULTIPLY TWO GENERAL MATRICES TO FORM A RESULTANT GENERAL ..... GMPR 70
C   MATRIX ..... GMPR 80
C ..... GMPR 90
C USAGE ..... GMPR 100
C   CALL GMPRD(A,B,R,N,M,L) ..... GMPR 110
C ..... GMPR 120
C DESCRIPTION OF PARAMETERS ..... GMPR 130
C   A - NAME OF FIRST INPUT MATRIX ..... GMPR 140
C   B - NAME OF SECOND INPUT MATRIX ..... GMPR 150
C   R - NAME OF OUTPUT MATRIX ..... GMPR 160
C   N - NUMBER OF ROWS IN A ..... GMPR 170
C   M - NUMBER OF COLUMNS IN A AND ROWS IN B ..... GMPR 180
C   L - NUMBER OF COLUMNS IN B ..... GMPR 190
C ..... GMPR 200
C REMARKS ..... GMPR 210
C   ALL MATRICES MUST BE STORED AS GENERAL MATRICES ..... GMPR 220
C   MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A ..... GMPR 230
C   MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX B ..... GMPR 240
C   NUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROWS ..... GMPR 250
C   OF MATRIX B ..... GMPR 260
C ..... GMPR 270
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED ..... GMPR 280
C   NONE ..... GMPR 290
C ..... GMPR 300
C METHOD ..... GMPR 310
C   THE M BY L MATRIX B IS PREMULIPLIED BY THE N BY M MATRIX A ..... GMPR 320
C   AND THE RESULT IS STORED IN THE N BY L MATRIX R. ..... GMPR 330
C ..... GMPR 340
C ..... GMPR 350
C ..... GMPR 360
C
C SUBROUTINE GMPRD(A,B,R,N,M,L,NH,ML,NL)
C REAL *8 A,B,R
C DIMENSION A(NH),B(ML),R(NL)
C
C   IR=0 ..... GMPR 390
C   IK=-M ..... GMPR 400
C   DO 10 K=1,L ..... GMPR 410
C   IK=IK+M ..... GMPR 420
C   DO 10 J=1,N ..... GMPR 430
C   IR=IR+1 ..... GMPR 440
C   JI=J-N ..... GMPR 450
C   IB=IK ..... GMPR 460
C   R(IR)=0 ..... GMPR 470
C   DO 10 I=1,H ..... GMPR 480
C   JI=JI+N ..... GMPR 490
C   IB=IB+1 ..... GMPR 500
C 10 R(IR)=R(IR)+A(JI)*B(IB) ..... GMPR 510
C   RETURN ..... GMPR 520
C   END ..... GMPR 530
C ..... GMPR 540

```

```

      SUBROUTINE FRANCI (A,VALU,NSUB,ANORM,NMAX)
C
C      EIGENVALUES OF REAL MATRICES
C
C**** REMOVE OR MODIFY NEXT FOUR STATEMENTS IN SINGLE PRECISION VERSION
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 DEL
      COMMON /QR/ ITER(200), DUMMY(100)
      DATA EPS/Z33800000C00000000/
      DIMENSION A(NMAX,NSUB), VALU(NMAX,2)
C
      N=NSUB
C
C      REDUCE MATRIX TO UPPER HESSENBERG FORM
C
      CALL SUBDIA(A,N,NMAX)
C
C      COMPUTE MATRIX NORM
C
      L=1
10  ANORM2=0.0
      DO 40 I=1,N
        I1=I-1+1/I
        DO 40 J=I1,N
          ANORM2=ANORM2+A(I,J)**2
40  CONTINUE
C**** CHANGE FUNCTION NAME IN NEXT STATEMENT IN SINGLE PRECISION
      ANORM2=DSQRT(ANORM2)
      IF (L.EQ.2) GO TO 190
      L=2
      ANORM=      ANORM2
C
      DEL=ANORM2*EPS
C
C      BEGINNING OF LOOP FOR ITERATIVE DETERMINATION OF EIGENVALUES
C      (ARRAY ITER HAS EFFECTIVELY BEEN CLEARED TO ZERO BY SUBDIA)
C
50  K=NSUB+1-N
C
C      FIND ROOTS OF LOWER 2X2 MINOR
C
      IF(N.EQ.0) GO TO 250
60  ANN=A(N,N)
      IF (N-1) 250, 220, 70
      CHANGE DABS TO ABS FOR SINGLE PRECISION
70  IF (DABS(A(N,N-1)).LE.DEL) GO TO 220
      DIF=(A(N-1,N-1)-ANN)*0.5
      DISCSQ=DIF**2+A(N-1,N)*A(N,N-1)
      IF (DISCSQ.LE.0.0) GO TO 100
C**** CHANGE FUNCTION NAMES IN NEXT STATEMENT IN SINGLE PRECISION
      DISC=DSIGN(DSQRT(DISCSQ),DIF)
      E=DISC+DIF+ANN
      G=E-DISC-DISC
      F=0.0
      GO TO 110
100 E=DIF+ANN
      G=E
C**** CHANGE FUNCTION NAME IN NEXT STATEMENT IN SINGLE PRECISION
      F=DSQRT(-DISCSQ)
110 IF (N.EQ.2) GO TO 230

```

CC ADDED

```

C      CHANGE DABS TO ABS FOR SINGLE PRECISION
      IF (DABS(A(N-1,N-2)).LE.DEL) GO TO 230
C
C      CHOOSE SHIFT TOWARD ACCELERATING CONVERGENCE
C
      SIG=E+G
      RHO=E*G+F**2
      GO TO 200
C
C      SHIFT UNSUCCESSFUL-- CHOOSE DIFFERENTLY TO BREAK UP CLUSTERS
C
190  SIG=0.125*      AADRM2
C
C      PERFORM QR ITERATION
C
200  CALL QR2(A,N,DEL,SIG,RHO,NMAX)
C
      ITER(K)=ITER(K)+1
      IF (MOD(ITER(K),50).NE.0) GO TO 60
      GO TO 10
C
C      SINGLE EIGENVALUE CONVERGED
C
220  VALU(K,1)=ANN
      VALU(K,2)=0.0
      N=N-1
      GO TO 50
C
C      PAIR OF EIGENVALUES CONVERGED
C
230  VALU(K,1)=E
      VALU(K,2)=F
      VALU(K+1,1)=G
      VALU(K+1,2)=-F
      N=N-2
      GO TO 50
C
250  RETURN
      END

```

```

      SUBROUTINE SUBDIA(A,N,NMAX)
C
C      HOUSEHOLDER REDUCTION OF REAL MATRIX TO UPPER HESSENBERG FORM
C
C**** REMOVE OR MODIFY NEXT      STATEMENT IN SINGLE PRECISION VERSION
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(NMAX,N), WVEC(100), PVEC(100), QVEC(100)
      COMMON /QR/ WVEC, PVEC
      EQUIVALENCE (PVEC,QVEC)
C
      DO 200 I=1,N
C
C      REDUCE COLUMN OF MATRIX
C
      WVEC(I)=0.0
      I1=I+1
      I2=I1+1
      IF (I2.GT.N) GO TO 200
      SUM=0.0
      DO 70 J=I2,N
70  SUM=SUM+A(J,I)**2
      IF (SUM.EQ.0.0) GO TO 200
75  J=I1
      TEMP=A(J,I)
C**** CHANGE FUNCTION NAMES IN NEXT FOUR STATEMENTS IN SINGLE PRECISION
      SUM=DSQRT(SUM+ TEMP **2)
      A(J,I)=-DSIGN(SUM, TEMP )
      WVEC(J)=DSQRT( 1.0+DABS( TEMP )/SUM)
      DIV=DSIGN( WVEC(J)*SUM, TEMP )
      DO 85 J=I2,N
85  WVEC(J)=A(J,I)/DIV
      SCALAR=0.0
      DO 95 J=I1,N
      PVEC(J)=0.0
      DO 90 K=I1,N
90  PVEC(J)=PVEC(J)+A(K,J)*WVEC(K)
      SCALAR=SCALAR+PVEC(J)*WVEC(J)
95  CONTINUE
      SCALAR=SCALAR/2.0
      DO 120 J=I1,N
      QVEC(J)=PVEC(J)-SCALAR*WVEC(J)
      DO 120 K=I1,N
      A(K,J)=A(K,J)-WVEC(K)*QVEC(J)
120 CONTINUE
      DO 180 K=I1,N
      QVEC(K)=-SCALAR*WVEC(K)
      DO 170 J=I1,N
170 QVEC(K)=QVEC(K)+A(K,J)*WVEC(J)
      DO 180 J=I1,N
      A(K,J)=A(K,J)-QVEC(K)*WVEC(J)
180 CONTINUE
C
200 CONTINUE
C
      RETURN
      END

```

```

      SUBROUTINE QR2(A,N,DEL,SIG,RHO,NMAX)
C
C      DOUBLE QR ITERATION
C
C**** REMOVE OR MODIFY NEXT THREE STATEMENTS IN SINGLE PRECISION VERSION
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 DEL
      REAL*8 KAP
      DIMENSION A(NMAX,N), GAM(3), PSI(2)
      INTEGER P,Q
C
      N1=N-1
      N2=N1-1
C
C      FIND P
C
      DO 50 I1=1,N2
      I=N1-I1
      IF (I1.EQ.1) GO TO 40
      A10=A(I+1,I)
C      CHANGE DABS TO ABS FOR SINGLE PRECISION
      IF (DABS(A10).LE.DEL) GO TO 60
      A12=A(I+1,I+2)
      A21=A(I+2,I+1)
      A11=A(I+1,I+1)
      DIF=A11-SIG
      DENOM=DIF*A11+A12*A21+RHO
      IF (DENOM.EQ.0.0) GO TO 40
C**** CHANGE FUNCTION NAME IN NEXT STATEMENT IN SINGLE PRECISION
      A32=DABS(A(I+3,I+2))
      A22=A(I+2,I+2)
C**** CHANGE FUNCTION NAME IN NEXT STATEMENT IN SINGLE PRECISION
C      CHANGE DABS TO ABS FOR SINGLE PRECISION
      IF (DABS(A10*A21*(DABS(A22+DIF)+A32)/DENOM).LE.DEL) GO TO 60
      40 P=I
      50 CONTINUE
C
C      FIND Q
C
      60 DO 100 I1=1,P
      I=P+1-I1
      C      CHANGE DABS TO ABS FOR SINGLE PRECISION
      IF (DABS(A(I+1,I)).LE.DEL) GO TO 110
      Q=I
      100 CONTINUE
C
C      ITERATE
C
      110 DO 300 I=P,N1
      IF (I.NE.P) GO TO 130
      DIF=A(I,I)-SIG
      GAM(1)=DIF*A(I,I)+A(I,I+1)*A(I+1,I)+RHO
      GAM(2)=A(I+1,I)*(DIF+A(I+1,I+1))
      GAM(3)=A(I+1,I)*A(I+2,I+1)
      A(I+2,I)=0.0
      GO TO 150
      130 GAM(1)=A(I,I-1)
      GAM(2)=A(I+1,I-1)
C
C      THE STATEMENTS WITH CC ADDED WERE ADDED SO THAT THIS PROGRAM WOULD C ADDED
      CC ADDED

```

```

C      NOT TERMINATE WHILE USING THE WATFIV COMPLIER   3/19/1971      CC ADDED
C
C      IF(I.EQ.N1) GO TO 140      CC ADDED
C      GAM(3)=A(I+2,I-1)      CC ADDED
C      IF (I.EQ.N1) GAM(3)=0.0      CC ADDED
C      GO TO 150      CC ADDED
140    GAM(3)=0.00      CC ADDED
C**** CHANGE FUNCTION NAMES IN NEXT STATEMENT IN SINGLE PRECISION
150    KAP=DSIGN(DSQRT(GAM(1)**2+GAM(2)**2+GAM(3)**2),GAM(1))
      GK=GAM(1)+KAP
      IF (GK.NE.0.0) GO TO 170
      ALF=-2.0
      GK=ALF
      GO TO 180
170    ALF=-GK/KAP
180    PSI(1)=GAM(2)/GK
      PSI(2)=GAM(3)/GK
      IF (I.EQ.Q) GO TO 220
      IF (I.EQ.P) KAP=A(I,I-1)
      A(I,I-1)=-KAP
C
C      ROW OPERATION
C
220    DO 250 J=I,N
      IF(I.EQ.N1) GO TO 230
      ETA=ALF*(A(I,J)+PSI(1)*A(I+1,J)+PSI(2)*A(I+2,J))
      GO TO 221
      CC ADDED
230    ETA=ALF*(A(I,J)+PSI(1)*A(I+1,J))
      CC ADDED
221    A(I,J)=ETA+A(I,J)
      CC ADDED
      A(I+1,J)=A(I+1,J)+PSI(1)*ETA
      IF (I.NE.N1) A(I+2,J)=A(I+2,J)+PSI(2)*ETA
250    CONTINUE
C
C      COLUMN OPERATION
C
      L=I+2
      IF (L.GT.N) L=N
      DO 280 J=Q,L
      IF(I.EQ.N1) GO TO 260
      ETA=ALF*(A(J,I)+PSI(1)*A(J,I+1)+PSI(2)*A(J,I+2))
      GO TO 281
      CC ADDED
260    ETA=ALF*(A(J,I)+PSI(1)*A(J,I+1))
      CC ADDED
281    A(J,I)=ETA+A(J,I)
      CC ADDED
      A(J,I+1)=A(J,I+1)+PSI(1)*ETA
      IF (I.NE.N1) A(J,I+2)=A(J,I+2)+PSI(2)*ETA
280    CONTINUE
      IF (I.GE.N2) GO TO 300
      ETA=ALF*PSI(2)*A(I+3,I+2)
      A(I+3,I)=ETA
      A(I+3,I+1)=A(I+3,I)*PSI(1)
      A(I+3,I+2)=A(I+3,I+2)+PSI(2)*ETA
300    CONTINUE
C
      RETURN
      END

```

```

SUBROUTINE ARANGE(VALU,NT,NTMO,NTPD,NUMBER)
IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
COMMON/INOUT/NREAC,NWRITE,NPUNCH
DIMENSION VALU(NT,2),VALRE(34),VALIM(34)

C
C IF THE EIGENVALUES ARE KNOWN TO BE REAL SET NUMBER.EQ.1.
C
C
C IF THE EIGENVALUES ARE KNOWN TO BE COMPLEX SET NUMBER.NE.1.
C
C IF(NUMBER.EQ.1) GC TO 100
C
C COMPLEX EIGENVALUES ARE SORTED INTO ASCENDING ORDER IN THE FOLLOWING
C STATEMENTS ENDING ON STATEMENT 99.
C
DO 1 I=1,NT
VALRE(I)=VALU(I,1)
1 VALIM(I)=VALU(I,2)
BIG=0.DO
DO 99 I=1,NTMO
JQUIT=NTPD-I
JQUTMO=JQUIT-1
DO 98 J=1,JQUTMO
IF(DABS(VALIM(J)).LE.DABS(BIG)) GO TO 98
JPO=J+1
IF(DABS(VALIM(JPO)).GE.DABS(VALIM(J))) GO TO 98
BIG=VALIM(J)
REALPT=VALRE(J)
DO 97 K=J,JQUTMO
KPO=K+1
VALIM(K)=VALIM(KPO)
97 VALRE(K)=VALRE(KPO)
VALIM(JQUIT)=BIG
VALRE(JQUIT)=REALPT
98 CONTINUE
BIG=VALIM(JQUTMO)
99 CONTINUE

C
C THE DAMPED,COMPLEX EIGENVALUES HAVE BEEN ORDERED FROM SMALLEST TO LARGEST
C IN ABSOLUTE VALUE WITH RESPECT TO THEIR IMAGINARY PARTS.(WHICH ARE THE
C CIRCULAR FREQUENCIES)
C
C WRITE OUT THESE ORDERED EIGENVALUES,PLACE THEM IN VALU(I,2) AND RETURN
C TO MAIN.
C
WRITE(NWRITE,7)
7 FORMAT('1',45X,'THE N DAMPED,COMPLEX CONJUGATE EIGENVALUE PAIRS
1 ARE','-',54X,'REAL PART',16X,'IMAGINARY PART')
WRITE(NWRITE,8) (VALRE(I),VALIM(I),I=1,NT)
8 FORMAT('0',40X,1P2D28.15/' ',40X,1P2D28.15)
DO 2 I=1,NT
VALU(I,1)=VALRE(I)
2 VALU(I,2)=VALIM(I)
RETURN

C
C REAL EIGENVALUES ARE SORTED IN ASCENDING ORDER IN THE FOLLOWING STATEMENT
C ENDING ON 78.
C
100 CONTINUE
DO 75 I=1,NT

```

```

75 VALRE(I)=VALU(I,1)
   BIG=0.00
   DO 78 I=1,NTMO
     JQUIT=NTPO-I
     JQUTMO=JQUIT-1
     DO 77 J=1,JQUTMO
       IF(VALRE(J).LE.BIG) GO TO 77
       JPO=J+1
       IF(VALRE(JPO).GE.VALRE(J)) GO TO 77
       BIG=VALRE(J)
       DO 76 K=J,JQUTMO
         KPO=K+1
76 VALRE(K)=VALRE(KPC)
       VALRE(JQUIT)=BIG
77 CONTINUE
       BIG=VALRE(JQUTMO)
78 CONTINUE
C
C   THE UNDAMPED,REAL EIGENVALUES HAVE BEEN ORDERED. PLACE THESE ORDERED
C   EIGENVALUES IN VALU(I,1). DETERMINE AND WRITE OUT THE UNDAMPED CIRCULAR
C   FREQUENCIES.
C
   DO 79 I=1,NT
79 VALU(I,1)=VALRE(I)
   DO 81 I=1,NT
81 VALRE(I)=DSQRT(DABS(VALRE(I)))
     WRITE(NWRITE,83)
83 FORMAT('1',45X,'THE N UNDAMPED CIRCULAR FREQUENCIES ARE')
     WRITE(NWRITE,85) (VALRE(I),I=1,NT)
85 FORMAT('-',48X,1P1028.15)
     RETURN
   END

```

```

SUBROUTINE VECITR
  IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
  COMPLEX*16 DCMLX,DCONJG
  COMMON/COMX/ZVEC(35,34)
  COMMON/DYNAM/D(34,34),VALU(34,2)
  COMMON/MATMCK/AM(17,17),AC(17,17),AK(17,17)
  COMMON/A1/DL(17,17),AMA(17,17)
  COMMON/ORDER/NMO,N,NPO,NTMO,NT,NTPO,NN,NTNT
  COMMON/INOUT/NREAD,NWRITE,NPUNCH
  DIMENSION ZHOLD(34),ZRIGHT(17)
  DIMENSION OVERZR(34)
  EQUIVALENCE(OVERZR,ZRIGHT)
  DIMENSION RIGHT(34)
  DO 1 L=1,NT,2
    ZLAMB=ZVEC(35,L)
    ROOTR=VALU(L,1)
    ROOTI=VALU(L,2)
    RRMII=ROOTR*ROOTR-ROOTI*ROOTI
    TRI=2.00*ROOTR*ROOTI
    DO 11 I=1,N
      IPN=I+N
      DO 11 J=1,N
        JPN=J+N
        R=RRMII*AMA(I,J)+ROOTR*AC(I,J)+AK(I,J)
        AI=TRI*AMA(I,J)+RCOTI*AC(I,J)
        O(I,J)=R
        D(IPN,JPN)=R
        D(I,JPN)=-AI
11      D(IPN,J)=AI
    DO 22 I=1,NT
22      ZHOLD(I)=ZVEC(I,L)
    DO 33 I=1,N
      ZKX=(0.00,0.00)
      ZMY=ZKX
      DO 44 J=1,N
        ZKX=ZKX+DCMLX(AK(I,J),0.00)*ZHOLD(J)
44      ZMY=ZMY+DCMLX(AMA(I,J),0.00)*ZHOLD(J+N)
33      ZRIGHT(I)=ZKX-ZLAMB*ZMY
    DO 55 I=1,N
      IR=2*I-1
      II=2*I
      RIGHT(I)=OVERZR(IR)
55      RIGHT(I+N)=OVERZR(II)
      CALL DGELG(RIGHT,C,NT,1,1.E-12,IER)
    DO 66 I=1,N
      IPN=I+N
66      ZHOLD(IPN)=DCMLX(RIGHT(I),RIGHT(IPN))
    DO 77 I=1,N
77      ZHOLD(I)=(ZHOLD(I+N)-ZHOLD(I))/ZLAMB
    DO 88 I=1,NT
      ZVEC(I,L)=ZHOLD(I)
88      ZVEC(I,L+1)=DCONJG(ZHOLD(I))
1  CONTINUE
  RETURN
  END

```

```

C
C
C .....DGELG002
C                                     DGELG003
C
C SUBROUTINE DGELG (THIS IS ONE OF THE SUBROUTINES IN THE IBM
C SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE).
C
C PURPOSE                                     DGELG005
C   TO SOLVE A GENERAL SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS. DGELG006
C                                     DGELG007
C USAGE                                     DGELG008
C   CALL DGELG(R,A,M,N,EPS,IER)             DGELG009
C                                     DGELG010
C DESCRIPTION OF PARAMETERS                 DGELG011
C   R   - DOUBLE PRECISION M BY N RIGHT HAND SIDE MATRIX DGELG012
C         (DESTROYED). ON RETURN R CONTAINS THE SOLUTIONS DGELG013
C         OF THE EQUATIONS.                      DGELG014
C   A   - DOUBLE PRECISION M BY M COEFFICIENT MATRIX DGELG015
C         (DESTROYED).                      DGELG016
C   M   - THE NUMBER OF EQUATIONS IN THE SYSTEM. DGELG017
C   N   - THE NUMBER OF RIGHT HAND SIDE VECTORS. DGELG018
C   EPS - SINGLE PRECISION INPUT CONSTANT WHICH IS USED AS DGELG019
C         RELATIVE TOLERANCE FOR TEST ON LOSS OF DGELG020
C         SIGNIFICANCE.                      DGELG021
C   IER - RESULTING ERROR PARAMETER CODED AS FOLLOWS DGELG022
C         IER=0 - NO ERROR,                  DGELG023
C         IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR DGELG024
C                 PIVOT ELEMENT AT ANY ELIMINATION STEP DGELG025
C                 EQUAL TO 0,                DGELG026
C         IER=K - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI- DGELG027
C                 CANCE INDICATED AT ELIMINATION STEP K+1, DGELG028
C                 WHERE PIVOT ELEMENT WAS LESS THAN OR DGELG029
C                 EQUAL TO THE INTERNAL TOLERANCE EPS TIMES DGELG030
C                 ABSOLUTELY GREATEST ELEMENT OF MATRIX A. DGELG031
C                                     DGELG032
C REMARKS                                     DGELG033
C   INPUT MATRICES R AND A ARE ASSUMED TO BE STORED COLUMNWISE DGELG034
C   IN M*N RESP. M*M SUCCESSIVE STORAGE LOCATIONS. ON RETURN DGELG035
C   SOLUTION MATRIX R IS STORED COLUMNWISE TOO. DGELG036
C   THE PROCEDURE GIVES RESULTS IF THE NUMBER OF EQUATIONS M IS DGELG037
C   GREATER THAN 0 AND PIVOT ELEMENTS AT ALL ELIMINATION STEPS DGELG038
C   ARE DIFFERENT FROM 0. HOWEVER WARNING IER=K - IF GIVEN - DGELG039
C   INDICATES POSSIBLE LOSS OF SIGNIFICANCE. IN CASE OF A WELL DGELG040
C   SCALED MATRIX A AND APPROPRIATE TOLERANCE EPS, IER=K MAY BE DGELG041
C   INTERPRETED THAT MATRIX A HAS THE RANK K. NO WARNING IS DGELG042
C   GIVEN IN CASE M=1. DGELG043
C                                     DGELG044
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED DGELG045
C   NONE DGELG046
C                                     DGELG047
C METHOD DGELG048
C   SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION WITH DGELG049
C   COMPLETE PIVOTING. DGELG050
C                                     DGELG051
C .....DGELG053
C                                     DGELG054
C SUBROUTINE DGELG(R,A,M,N,EPS,IER) DGELG055
C                                     DGELG056
C                                     DGELG057
C DIMENSION A(1),R(1) DGELG058
C DOUBLE PRECISION R,A,PIV,T0,TOL,PIVI DGELG059

```

	IF(M)23,23,1	DGELG060
C		DGELG061
C	SEARCH FOR GREATEST ELEMENT IN MATRIX A	DGELG062
1	IER=0	DGELG063
	PIV=0.00	DGELG064
	MM=M*M	DGELG065
	NM=N*M	DGELG066
	DO 3 L=1,MM	DGELG067
	TB=DABS(A(L))	DGELG068
	IF(TB-PIV)3,3,2	DGELG069
2	PIV=TB	DGELG070
	I=L	DGELG071
3	CONTINUE	DGELG072
	TOL=EPS*PIV	DGELG073
C	A(I) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I).	DGELG074
C		DGELG075
C		DGELG076
C	START ELIMINATION LOOP	DGELG077
	LST=1	DGELG078
	DO 17 K=1,M	DGELG079
C		DGELG080
C	TEST ON SINGULARITY	DGELG081
	IF(PIV)23,23,4	DGELG082
4	IF(IER)7,5,7	DGELG083
5	IF(PIV-TOL)6,6,7	DGELG084
6	IER=K-1	DGELG085
7	PIV=1.00/A(I)	DGELG086
	J=(I-1)/M	DGELG087
	I=I-J*M-K	DGELG088
	J=J+1-K	DGELG089
C	I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT	DGELG090
C		DGELG091
C	PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R	DGELG092
	DO 8 L=K,NM,M	DGELG093
	LL=L+I	DGELG094
	TB=PIV*I*R(LL)	DGELG095
	R(LL)=R(L)	DGELG096
8	R(L)=TB	DGELG097
C		DGELG098
C	IS ELIMINATION TERMINATED	DGELG099
	IF(K-M)9,18,18	DGELG100
C		DGELG101
C	COLUMN INTERCHANGE IN MATRIX A	DGELG102
9	LEND=LST+M-K	DGELG103
	IF(J)12,12,10	DGELG104
10	II=J*M	DGELG105
	DO 11 L=LST,LEND	DGELG106
	TB=A(L)	DGELG107
	LL=L+II	DGELG108
	A(L)=A(LL)	DGELG109
11	A(LL)=TB	DGELG110
C		DGELG111
C	ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A	DGELG112
12	DO 13 L=LST,MM,M	DGELG113
	LL=L+I	DGELG114
	TB=PIV*I*A(LL)	DGELG115
	A(LL)=A(L)	DGELG116
13	A(L)=TB	DGELG117
C		DGELG118
C	SAVE COLUMN INTERCHANGE INFORMATION	DGELG119

	A(LST)=J	DGELG120
C		DGELG121
C	ELEMENT REDUCTION AND NEXT PIVOT SEARCH	DGELG122
	PIV=0.00	DGELG123
	LST=LST+1	DGELG124
	J=0	DGELG125
	DO 16 II=LST,LEND	DGELG126
	PIVI=-A(II)	DGELG127
	IST=II+M	DGELG128
	J=J+1	DGELG129
	DO 15 L=IST,MM,M	DGELG130
	LL=L-J	DGELG131
	A(L)=A(L)+PIVI*A(LL)	DGELG132
	TB=DABS(A(L))	DGELG133
	IF(TB-PIV)15,15,14	DGELG134
14	PIV=TB	DGELG135
	I=L	DGELG136
15	CONTINUE	DGELG137
	DO 16 L=K,NM,M	DGELG138
	LL=L+J	DGELG139
16	R(LL)=R(LL)+PIVI*R(L)	DGELG140
17	LST=LST+M	DGELG141
C	END OF ELIMINATION LOOP	DGELG142
C		DGELG143
C		DGELG144
C	BACK SUBSTITUTION AND BACK INTERCHANGE	DGELG145
18	IF(M-1)23,22,19	DGELG146
19	IST=MM+M	DGELG147
	LST=M+1	DGELG148
	DO 21 I=2,M	DGELG149
	II=LST-I	DGELG150
	IST=IST-LST	DGELG151
	L=IST-M	DGELG152
	L=A(L)+.500	DGELG153
	DO 21 J=II,NM,M	DGELG154
	TB=R(J)	DGELG155
	LL=J	DGELG156
	DO 20 K=IST,MM,M	DGELG157
	LL=LL+1	DGELG158
20	TB=TB-A(K)*R(LL)	DGELG159
	K=J+L	DGELG160
	R(J)=R(K)	DGELG161
21	R(K)=TB	DGELG162
22	RETURN	DGELG163
C		DGELG164
C		DGELG165
C	ERROR RETURN	DGELG166
23	IER=-1	DGELG167
	RETURN	DGELG168
	END	DGELG169

```

BLOCK DATA
IMPLICIT REAL*8 (A-H,O-Y),COMPLEX *16(Z)
COMMON/COMX/ZVEC(35,34)
COMMON/DYNAM/D(34,34),VALU(34,2)
COMMON/MATMCK/AM(17,17),AC(17,17),AK(17,17)
COMMON/A1/ DL(17,17),AMA(17,17)
COMMON/ORDER/NMO,N,NPO,NTMO,NT,NTPO,NN,NTNT
COMMON/INOUT/NREAD,NWRITE,NPUNCH
DATA D,VALU/1224*0.00/
DATA DL,AMA/578*0.00/
DATA NMO,N,NPO,NTPO,NT,NTPO,NN,NTNT/16,17,18,33,34,35,289,1156/
DATA NREAD,NWRITE,NPUNCH/5,6,7/
DATA AM,AC,AK/867*0.00/
END

SUBROUTINE BIORTH
IMPLICIT REAL*8 (A-H,O-Y),COMPLEX *16(Z)
COMPLEX*16 CDSQRT,DCMPLX,DCONJG
COMMON/COMX/ZVEC(35,34)
COMMON/DYNAM/D(34,34),VALU(34,2)
COMMON/MATMCK/AM(17,17),AC(17,17),AK(17,17)
COMMON/A1/ DL(17,17),AMA(17,17)
COMMON/ORDER/NMO,N,NPO,NTMO,NT,NTPO,NN,NTNT
COMMON/INOUT/NREAD,NWRITE,NPUNCH
DIMENSION ZHOLD(17)
DO 1 K=1,NT,2
  ROOTR=VALU(K,1)
  ROOTI=VALU(K,2)
  DO 11 I=1,N
11  ZHOLD(I)=ZVEC(I,K)
    ZSUMT=(0.00,0.00)
    DO 22 J=1,N
      ZSUMO=(0.00,0.00)
      DO 33 I=1,N
33  ZSUMO=ZSUMO+
      DCMPLX(2.00*ROOTR*AMA(I,J)+AC(I,J),2.00*ROOTI*AMA(I,J))*ZHOLD(I)
22  ZSUMT=ZSUMT+ZSUMO*ZHOLD(J)
      ZSQ=CDSQRT((1.00,0.00)/ZSUMT)
      ZSQCON=DCONJG(ZSQ)
      DO 44 I=1,NT
        ZVEC(I,K+1)=ZVEC(I,K+1)*ZSQCON
44  ZVEC(I,K)=ZVEC(I,K)*ZSQ
1  CONTINUE
  RETURN
END

```

```

SUBROUTINE ROADIN(ZWL,ZWR,WL,WR,NOREAD)
IMPLICIT REAL*8(A-H,K,O-Y),COMPLEX*16(Z)
COMPLEX*16 DCMLPX
REAL *8 DSIN,DCOS
NAMELIST/ROAD/ CVII,CVIII,CIX,CX,CXI,CXII,KVII,KVIII,KIX,KX,KXI,
1 KXII,AL,AR,ZWIDE,BT,GH1,GBER2,SPEED,FWL,FWR
DIMENSION ZTIME(5)
COMMON/ZROAD/ZCL(6),ZCR(6)
COMMON/INOUT/NREAD,NWRITE,NPUNCH
COMMON/RIPPLE/WAVEL,WAVER,FAST,AMPL,AMPR,FAZE
IF(NOREAD,NE.1) GO TO 10
READ(NREAD,ROAD)
WL=FWL
WR=FWR
FAST=SPEED
AMPL=AL
AMPR=AR
FAZE=BT
10 CONTINUE
T1=WL*GH1/SPEED
T2=WL*GBER2/SPEED
ZTIME(1)=DCMLPX(DCOS(T1),-DSIN(T1))
ZTIME(2)=DCMLPX(DCOS(T2),-DSIN(T2))
ZTIME(3)=DCMLPX(DCOS(BT),-DSIN(BT))
T1=WR*T1/WL
T2=WR*T2/WL
ZTIME(4)=DCMLPX(DCOS(T1+BT),-DSIN(T1+BT))
ZTIME(5)=DCMLPX(DCOS(T2+BT),-DSIN(T2+BT))
ZCL(1)=DCMLPX(CVII*WL*AL,-KVII*AL)
ZCL(2)=DCMLPX(CIX*WL*AL,-KIX*AL)*ZTIME(1)
ZCL(3)=DCMLPX(CXI*WL*AL,-KXI*AL)*ZTIME(2)
ZCL(4)=-ZWIDE*ZCL(1)
ZCL(5)=-ZWIDE*ZCL(2)
ZCL(6)=-ZWIDE*ZCL(3)
ZCR(1)=DCMLPX(CVIII*WR*AR,-KVIII*AR)*ZTIME(3)
ZCR(2)=DCMLPX(CX*WR*AR,-KX*AR)*ZTIME(4)
ZCR(3)=DCMLPX(CXII*WR*AR,-KXII*AR)*ZTIME(5)
ZCR(4)=ZWIDE*ZCR(1)
ZCR(5)=ZWIDE*ZCR(2)
ZCR(6)=ZWIDE*ZCR(3)
ZWL=DCMLPX(0.00,WL)
ZWR=DCMLPX(0.00,WR)
WAVEL=6.283185307179586*SPEED/WL
WAVER=6.283185307179586*SPEED/WR
RETURN
END

```

```

SUBROUTINE RIDE(ZWR,ZWL,NT,N)
IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
COMPLEX*16 DCONJG
COMMON/COMX/ZVEC(35,34)
COMMON/ZRIDE/ZRIDL(17),ZRIDR(17)
COMMON/ZROAD/ZCL(6),ZCR(6)
COMMON/INOUT/NREAD,NWRITE,NPUNCH
DIMENSION ZSCALL(34),ZSCALR(34)
DIMENSION ZHOLD(6)
DO 1 K=1,NT
  ZLAMB=ZVEC(NT+1,K)
  ZHOLD(1)=ZVEC(1,K)
  ZHOLD(2)=ZVEC(3,K)
  ZHOLD(3)=ZVEC(4,K)
  ZHOLD(4)=ZVEC(9,K)
  ZHOLD(5)=ZVEC(11,K)
  ZHOLD(6)=ZVEC(13,K)
  ZSUM=(0.00,0.00)
  DO 11 I=1,6
11  ZSUM=ZSUM+ZHOLD(I)*ZCL(I)
    ZSCALL(K)=ZSUM/(ZWL-ZLAMB)
    ZSUM=(0.00,0.00)
    DO 22 I=1,6
22  ZSUM=ZSUM+ZHOLD(I)*ZCR(I)
      1 ZSCALR(K)=ZSUM/(ZWR-ZLAMB)
55  FORMAT('1',34(' ',1P4D30.15/))
    DO 33 I=1,N
33  ZRIDL(I)=(0.00,0.00)
      DO 44 K=1,NT
44  ZRIDR(I)=ZRIDR(I)+ZVEC(I,K)*ZSCALR(K)
      DO 66 I=1,N
66  ZRIDL(I)=DCONJG(ZRIDL(I))
      ZRIDR(I)=DCONJG(ZRIDR(I))
    RETURN
  END

```

```

SUBROUTINE WEQULZ
  IMPLICIT REAL *8(A-H,O-Y), COMPLEX *16(Z)
  COMPLEX*16 C^ABS
  COMMON/ZRIDE/ZRIDL(17),ZRIDR(17)
  COMMON/URDER/NMO,N,NPO,NTMO,NT,NTPD,NN,NTNT
  COMMON/RIPPLE/WAVEL,WAVER,FAST,AMPL,AMPR,FAZE
  COMMON/INOUT/NREAD,NWRITE,NPUNCH
  DIMENSION AMPMAX(17)
  DO 1000 I=1,N
1000 AMPMAX(I)=CDABS(ZRIDL(I)+ZRIDR(I))
  WRITE(NWRITE,111)
  WRITE(NWRITE,222) FAZE
  WRITE(NWRITE, 1) AMPMAX( 1)
  WRITE(NWRITE, 2) AMPMAX( 2)
  WRITE(NWRITE, 3) AMPMAX( 3)
  WRITE(NWRITE, 4) AMPMAX( 4)
  WRITE(NWRITE, 5) AMPMAX( 5)
  WRITE(NWRITE, 6) AMPMAX( 6)
  WRITE(NWRITE, 7) AMPMAX( 7)
  WRITE(NWRITE, 8) AMPMAX( 8)
  WRITE(NWRITE, 9) AMPMAX( 9)
  WRITE(NWRITE,10) AMPMAX(10)
  WRITE(NWRITE,11) AMPMAX(11)
  WRITE(NWRITE,12) AMPMAX(12)
  WRITE(NWRITE,13) AMPMAX(13)
  WRITE(NWRITE,14) AMPMAX(14)
  WRITE(NWRITE,15) AMPMAX(15)
  WRITE(NWRITE,16) AMPMAX(16)
  WRITE(NWRITE,17) AMPMAX(17)
  1 FORMAT('0',56X,' X1 = ',1P109.2)
  2 FORMAT('0',56X,' X2 = ',1P109.2)
  3 FORMAT('0',56X,' X3 = ',1P109.2)
  4 FORMAT('0',56X,' X5 = ',1P109.2)
  5 FORMAT('0',56X,' X7 = ',1P109.2)
  6 FORMAT('0',56X,' Z1 = ',1P109.2)
  7 FORMAT('0',56X,' Z3 = ',1P109.2)
  8 FORMAT('0',56X,' Z5 = ',1P109.2)
  9 FORMAT('0',56X,' THETA1 = ',1P109.2)
 10 FORMAT('0',56X,' THETA2 = ',1P109.2)
 11 FORMAT('0',56X,' THETA3 = ',1P109.2)
 12 FORMAT('0',56X,' THETA4 = ',1P109.2)
 13 FORMAT('0',56X,' THETA5 = ',1P109.2)
 14 FORMAT('0',56X,' PHI2 = ',1P109.2)
 15 FORMAT('0',56X,' PHI4 = ',1P109.2)
 16 FORMAT('0',56X,' XI2 = ',1P109.2)
 17 FORMAT('0',56X,' XI4 = ',1P109.2)
 111 FORMAT('1',52X,' THE MAXIMUM AMPLITUDES ARE'//)
 222 FORMAT('1',35X,' THE RELATIVE PHASE ANGLE IS'/'0',60X,1P1010.3//)
  RETURN
  END

```

Appendix B

EXPLANATION OF INPUT DATA FOR UNDETERMINED COEFFICIENTS PROGRAM WITH SOURCE LISTING

The required input for this program is identical to that for the program explained in appendix A except for the namelist data group FORCE. Data group FORCE contains the same variables as data group ROAD, except ZWIDE becomes the REAL*8 variable, WIDE, and possesses the additional variables defined below. If a Bode plot is desired for a given vehicle, these variables may be employed.

NUMB = the number of discrete steps desired (integer)

RUNGL = the step size for the left side (radians/second)

RUNGR = the step size for the right side (radians/second)

```

      IMPLICIT REAL*8(A-H,O-Y)
      COMMON/DYNAM/D(34,34),VALU(34,2)
      COMMON/MATHCK/AM(17,17),AC(17,17),AK(17,17)
      COMMON/ORDER/NMO,N,NPO,NTMO,NT,NTPO,NN,NTNT
      COMMON/A1/DL(17,17),AMA(17,17)
      COMMON/INDUT/NREAC,NWRITE,NPUNCH
      COMMON/R1/TIME(10)
      COMMON/MISC/WL,WR,FAST,STEPL,STEPR,AMPL,AMPR,FAZE,NSTEPS
      DIMENSION      CI(17),CII(17),CIII(17),SI(17),SII(17),SIII(17)
      COMMON/DOUT/XCL(17),XSL(17),XCR(17),XSR(17)
      COMMON/COSSIN/SAC(34,3)
      EQUIVALENCE(SAC(1,1),CI),(SAC(1,2),CII),(SAC(1,3),CIII)
      EQUIVALENCE(SAC(18,1),SI),(SAC(18,2),SII),(SAC(18,3),SIII)
      CALL DEFINE
      DO 2 I=1,N
      DO 2 J=1,N
2    AMA(I,J)=AM(I,J)
      NOREAD=0
      ERO=0.00
200  NSIDE=1
      DO 3 J=1,3
      DO 3 I=1,NT
3    SAC(I,J)=0.00
      CALL FORCIN(NOREAC,NSIDE)
      DO 100 I=1,N
      IPN=I+N
      DO 100 J=1,N
      JPN=J+N
      WMK=-WL*WL*AMA(I,J)+AK(I,J)
      WC=WL*AC(I,J)
      D(I,J)=WMK
      D(IPN,JPN)=WMK
      D(I,JPN)=WC
100  D(IPN,J)=-WC
      CALL DGELG(SAC,D,NT,3,1.E-14,IER)
      IF(IER.NE.0) WRITE(NWRITE,333)
333  FORMAT(' ',50X,IIC)
      DO 110 I=1,N
      XCL(I)=ERO
      XSL(I)=ERO
      XCR(I)=ERO
110  XSR(I)=ERO
      DO 120 I=1,N
      XCL(I)=CI(I)+CII(I)*TIME(2)-SII(I)*TIME(1)+CIII(I)*TIME(4)-SIII(I)
      1*TIME(3)
120  XSL(I)=SI(I)+CII(I)*TIME(1)+SII(I)*TIME(2)+CIII(I)*TIME(3)+SIII(I)
      1*TIME(4)
      DO 130 J=1,3
      DO 130 I=1,NT
130  SAC(I,J)=ERO
      NOREAD=1+NOREAD
      NSIDE=2
      CALL FORCIN(NOREAC,NSIDE)
      DO 140 I=1,N
      IPN=I+N
      DO 140 J=1,N
      JPN=J+N
      WMK=-WR*WR*AMA(I,J)+AK(I,J)
      WC=WR*AC(I,J)
      D(I,J)=WMK

```

```

      D(IPN,JPN)=WMK
      D(I,JPN)=WC
140  D(IPN,J)=-WC
      CALL DGELG(SAC,D,NT,3,1.E-14,IER)
      IF(ITER.NE.0) WRITE(NWRITE,333)
      DO 160 I=1,N
        XCR(I)=C(I)*TIME(6)-SI(I)*TIME(5)+CII(I)*TIME(8)-SII(I)*TIME(7)
        1+CIII(I)*TIME(10)-SIII(I)*TIME(9)
160  XSR(I)=C(I)*TIME(5)+SI(I)*TIME(6)+CII(I)*TIME(7)+SII(I)*TIME(8)
        1+CIII(I)*TIME(9)+SIII(I)*TIME(10)
      WRITE(NWRITE,179)
179  FORMAT('1')
      WRITE(NWRITE,210) FAST
      WRITE(NWRITE,190)
      WRITE(NWRITE,178) WL,WR
      WRITE(NWRITE,3000) AMPL,AMPR
3000 FORMAT('-',25X,'THE AMPLITUDE (FEET) OF THE LEFT AND RIGHT'
      1,1X,'SINUSOIDAL PATHS ARE RESPECTIVELY'/'0',47X,1P2D15.4)
190  FORMAT('-',10X,'THE SINUSOIDAL FORCING FREQUENCIES (RADIAN/SEC)'
      1,1X,'FOR THE LEFT AND FOR THE RIGHT SIDE WHEEL PATHS ARE'
      2,1X,'RESPECTIVELY')
210  FORMAT(' ',30X,'THE SPEED (FEET/SECOND) OF THIS VEHICLE'
      1,1X,'IS EQUAL TO',1P1D12.4)
178  FORMAT('0',35X,'WL=',1P1D22.15,10X,'WR=',1P1D22.15)
      WAVEL=6.283185307179586*FAST /WL
      WAVER=6.283185307179586*FAST /WR
      WRITE(NWRITE,4) WAVEL,WAVER
      4 FORMAT('-',25X,'THE WAVELENGTH OF THE LEFT AND RIGHT SINUSOIDAL'
      1,1X,'DISPLACEMENT FUNCTIONS ARE RESPECTIVELY'/'0',40X,1P2D25.10)
      WRITE(NWRITE,177)
177  FORMAT('0',25X,'OUTPUT(I) = A(I)*COS(WL*T) + B(I)*SIN(WL*T) + '
      1,1X,'C(I)*COS(WR*T) + D(I)*SIN(WR*T)')
      WRITE(NWRITE,181)
181  FORMAT('-',17X,'A(I)',26X,'B(I)',26X,'C(I)',26X,'D(I)')
      DO 170 I=1,N
170  WRITE(NWRITE,180) XCL(I),XSL(I),XCR(I),XSR(I)
180  FORMAT('0',1P4D30.15)
      DIFFER=DABS(WL-WR)
      IF(DIFFER.LT.1.D-7) CALL WEQUAL
      WL=WL+STEPL
      WR=WR+STEPR
      IF(NSTEPS.GE.NOREAD) GO TO 200
      STOP
      END

```

```

SUBROUTINE DEFINE
REAL *8 M,C,K
REAL *4 M1,M2,M3,M4,M5,M6,M7,M8,M9
REAL *4 M11,M12,M13,M14,M15,MJ2,MJ4,MH2,MH4,P67,P62,P27,P84,P89,
UP49
REAL *4 K1,K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13,K14,K15,K16
REAL *4 K17,K18,K19,K20,K21,K22,K23,K24
COMMON/INOUT/NREAD,NWRITE,NPUNCH
COMMON/MATHCK/M(17,17),C(17,17),K(17,17)
NAMELIST/MASSES/M1,M2,M3,M4,M5,M6,M7,M8,M9
NAMELIST/INERTA/M11,M12,M13,M14,M15,MJ2,MJ4,MH2,MH4,
1P67,P62,P27,P84,P89,P49
NAMELIST/DAMPER/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15
1,C16,C17,C18,C19,C20,C21,C22,C23,C24
NAMELIST/SPRING/K1,K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13,K14,K15
1,K16,K17,K18,K19,K20,K21,K22,K23,K24
NAMELIST/DIMEN/A,B,CC,D,E,F,G,H,P,Q,R,S,T,U,V,W,X,Y,Z
READ(NREAD,MASSES)
READ(NREAD,INERTA)
READ(NREAD,DAMPER)
READ(NREAD,SPRING)
READ(NREAD,DIMEN)
30 FORMAT('1',T43,'FOR THIS SEMI-TRAILER TRUCK THE CHARACTERIZING'
1/' ',T53,'PARAMETERS ARE AS FOLLOWS.'////)
40 FORMAT(' ',T14,'DAMPERS',T40,'SPRINGS',T60,'DIMENSIONAL LENGTHS'
1,T87,'MASS MOMENTS OF INERTIA',T120,'MASSES')
60 FORMAT(' ',T12,'(KIP-SEC/FT)',T40,'(KIP/FT)',T68,'(FT)',T91,
1'(KIP-SEC**2-FT)',T115,'(KIP-SEC**2/FT)')
1 FORMAT('0',T11,'C1=',E13.6,T37,'K1=',E13.6,T64,'A=',E13.6,T90,
1'M11=',E13.6,T116,'M1=',E13.6)
2 FORMAT('0',T11,'C2=',E13.6,T37,'K2=',E13.6,T64,'B=',E13.6,T90,
2'M12=',E13.6,T116,'M2=',E13.6)
3 FORMAT('0',T11,'C3=',E13.6,T37,'K3=',E13.6,T63,'CC=',E13.6,T90,
3'M13=',E13.6,T116,'M3=',E13.6)
4 FORMAT('0',T11,'C4=',E13.6,T37,'K4=',E13.6,T64,'D=',E13.6,T90,
4'M14=',E13.6,T116,'M4=',E13.6)
5 FORMAT('0',T11,'C5=',E13.6,T37,'K5=',E13.6,T64,'E=',E13.6,T90,
5'M15=',E13.6,T116,'M5=',E13.6)
6 FORMAT('0',T11,'C6=',E13.6,T37,'K6=',E13.6,T64,'F=',E13.6,T90,
6'MJ2=',E13.6,T116,'M6=',E13.6)
7 FORMAT('0',T11,'C7=',E13.6,T37,'K7=',E13.6,T64,'G=',E13.6,T90,
7'MJ4=',E13.6,T116,'M7=',E13.6)
8 FORMAT('0',T11,'C8=',E13.6,T37,'K8=',E13.6,T64,'H=',E13.6,T90,
8'MH2=',E13.6,T116,'M8=',E13.6)
9 FORMAT('0',T11,'C9=',E13.6,T37,'K9=',E13.6,T64,'P=',E13.6,T90,
9'MH4=',E13.6,T116,'M9=',E13.6)
10 FORMAT('0',T10,'C10=',E13.6,T36,'K10=',E13.6,T64,'Q=',E13.6,T90,
2'P67=',E13.6)
11 FORMAT('0',T10,'C11=',E13.6,T36,'K11=',E13.6,T64,'R=',E13.6,T90,
2'P62=',E13.6)
12 FORMAT('0',T10,'C12=',E13.6,T36,'K12=',E13.6,T64,'S=',E13.6,T90,
2'P27=',E13.6)
13 FORMAT('0',T10,'C13=',E13.6,T36,'K13=',E13.6,T64,'T=',E13.6,T90,
2'P84=',E13.6)
14 FORMAT('0',T10,'C14=',E13.6,T36,'K14=',E13.6,T64,'U=',E13.6,T90,
2'P89=',E13.6)
15 FORMAT('0',T10,'C15=',E13.6,T36,'K15=',E13.6,T64,'V=',E13.6,T90,
2'P49=',E13.6)
16 FORMAT('0',T10,'C16=',E13.6,T36,'K16=',E13.6,T64,'W=',E13.6)
17 FORMAT('0',T10,'C17=',E13.6,T36,'K17=',E13.6,T64,'X=',E13.6)

```

```

18 FORMAT('O',T10,'C18=',E13.6,T36,'K18=',E13.6,T64,'Y=',E13.6)
19 FORMAT('O',T10,'C19=',E13.6,T36,'K19=',E13.6,T64,'Z=',E13.6)
20 FORMAT('O',T10,'C20=',E13.6,T36,'K20=',E13.6)
21 FORMAT('O',T10,'C21=',E13.6,T36,'K21=',E13.6)
22 FORMAT('O',T10,'C22=',E13.6,T36,'K22=',E13.6)
23 FORMAT('O',T10,'C23=',E13.6,T36,'K23=',E13.6)
24 FORMAT('O',T10,'C24=',E13.6,T36,'K24=',E13.6)
WRITE(NWRITE,30)
WRITE(NWRITE,40)
WRITE(NWRITE,60)
WRITE(NWRITE, 1) C1 ,K1 ,A ,M11,M1
WRITE(NWRITE, 2) C2 ,K2 ,B ,M12,M2
WRITE(NWRITE, 3) C3 ,K3 ,CC,M13,M3
WRITE(NWRITE, 4) C4 ,K4 ,D ,M14,M4
WRITE(NWRITE, 5) C5 ,K5 ,E ,M15,M5
WRITE(NWRITE, 6) C6 ,K6 ,F ,MJ2,M6
WRITE(NWRITE, 7) C7 ,K7 ,G ,MJ4,M7
WRITE(NWRITE, 8) C8 ,K8 ,H ,MH2,M8
WRITE(NWRITE, 9) C9 ,K9 ,P ,MH4,M9
WRITE(NWRITE,10) C10,K10,Q,P67
WRITE(NWRITE,11) C11,K11,R,P62
WRITE(NWRITE,12) C12,K12,S,P27
WRITE(NWRITE,13) C13,K13,T,P84
WRITE(NWRITE,14) C14,K14,U,P89
WRITE(NWRITE,15) C15,K15,V,P49
WRITE(NWRITE,16) C16,K16,W
WRITE(NWRITE,17) C17,K17,X
WRITE(NWRITE,18) C18,K18,Y
WRITE(NWRITE,19) C19,K19,Z
WRITE(NWRITE,20) C20,K20
WRITE(NWRITE,21) C21,K21
WRITE(NWRITE,22) C22,K22
WRITE(NWRITE,23) C23,K23
WRITE(NWRITE,24) C24,K24
C
C*****
C
MR=M6*M8/(M6+M8)
SPA=S*.5+A
SMA=S*.5-A
EPR=E+R
TT=T*.5
FMQ=F-Q
M(1,1)=M1
M(2,2)=M2+M4
M(2,10)=-M4*A
M(2,12)=M4*D
M(2,14)=-M4*B
M(2,15)=-M4*E
M(3,3)=M3
M(4,4)=M5
M(5,5)=M7+M9
M(5,10)=M9*P
M(5,12)=-M9*Q
M(5,16)=M9*B
M(5,17)=M9*E
M(6,6)=M1
M(7,7)=M3
M(8,8)=M5
M(9,9)=M11

```

```

M(10,10)=M4*P+A*M5*P*P+M12
M(10,12)=-A*D*M4-F*Q*M9
M(10,14)=M4*A*B-P67
M(10,15)=M4*A*E
M(10,16)=M9*P*B-P62
M(10,17)=M9*P*E
M(11,11)=M13
M(12,12)=M4*D*D+M5*Q*Q+M14
M(12,14)=-M4*D*B
M(12,15)=-M4*D*E-P89
M(12,16)=-M9*Q*B
M(12,17)=-M9*Q*E-P84
M(13,13)=M15
M(14,14)=M4*B*B+MR*P*P+M12
M(14,15)=M4*B*E-MR*P*Q
M(14,16)=-P*A*MR-P27
M(14,17)=MR*P*D
M(15,15)=M4*B*E+MR*Q*Q+M14
M(15,16)=MR*Q*A
M(15,17)=-MR*Q*D-P49
M(16,16)=MR*A*A+M5*B*B+M12
M(16,17)=-A*D*MR+B*E*M9
M(17,17)=MR*D*D+M5*E*E+M14

```

[illegible]

```

C(1,1)=C1+C2+C7+C8
C(1,2)=-C1-C2
C(1,9)= W*(C8-C7)+U*(C2-C1)
C(1,10)= SPA*C1-SMA*C2
C(1,14)=-G*(C1+C2)
C(2,2)=C1+C2+C3+C4+C5+C6
C(2,3)=-C3-C4
C(2,4)=-C5-C6
C(2,9)=U*(C1-C2)
C(2,10)=-SPA*(C1+C3)+SMA*(C2+C4)-A*(C5+C6)
C(2,11)=U*(C3-C4)
C(2,12)=TT*(C6-C5)
C(2,13)=U*(C5-C6)
C(2,14)=C1*G+C2*G-C3*H-C4*H-C5*B-C6*B
C(2,15)=-EPR*(C5+C6)
C(3,3)=C3+C4+C9+C10
C(3,10)=SPA*C3-SMA*C4
C(3,11)=W*(C10-C9)+U*(C4-C3)
C(3,14)=C3*H+C4*H
C(4,4)=C5+C6+C11+C12
C(4,10)=A*(C5+C6)
C(4,12)=TT*(C5-C6)
C(4,13)=W*(C12-C11)+U*(C6-C5)
C(4,14)=C5*B+C6*B
C(4,15)=EPR*(C5+C6)
C(5,5)=C15+C16+C15+C20+C23+C24
C(5,6)=-C15-C16
C(5,7)=-C19-C20
C(5,8)=-C23-C24
C(5,9)=V*(C15+C16)
C(5,10)=CC*(C15+C16+C19+C20)+C23*P+C24*P
C(5,11)=V*(C19+C20)
C(5,12)=(C23+C24)*FMQ
C(5,13)=V*(C23+C24)

```

```
C(5,16)=-C15*G-C16*G+C19*H+C20*H+C23*B+C24*B
C(5,17)=(C23+C24)*EPR
C(6,6)=C13+C14+C15+C16
C(6,9)=Y*(C13+C14)-V*(C15+C16)
C(6,10)=-C15*CC-C16*CC
C(6,16)=C15*G+C16*G
C(7,7)=C17+C18+C19+C20
C(7,10)=-C19*CC-C20*CC
C(7,11)=Y*(C17+C18)-V*(C19+C20)
C(7,16)=-C19*H-C20*H
C(8,8)=C21+C22+C23+C24
C(8,10)=-C23*P-C24*P
C(8,12)=-FMQ*(C23+C24)
C(8,13)=Y*(C21+C22)-V*(C23+C24)
C(8,16)=-B*(C23+C24)
C(8,17)=-C23+C24)*EPR
C(9,9)=W*W*(C7+C8)+Y*Y*(C13+C14)+U*U*(C1+C2)+V*V*(C15+C16)
C(9,10)=-U*(SPA*C1+SMA*C2)+V*CC*(C15+C16)
C(9,14)=U*G*(C1-C2)
C(9,16)=-V*G*(C15+C16)
C(10,10)=SPA*SPA*(C1+C3)+SMA*SMA*(C2+C4)+CC*CC*(C15+C16+C19+C20)+
1A*A*(C5+C6)+P*P*(C23+C24)
C(10,11)=-U*(SPA*C3+SMA*C4)+CC*V*(C19+C20)
C(10,12)=A*TT*(C5-C6)+P*FMQ*(C23+C24)
C(10,13)=A*U*(C6-C5)+P* V*(C23+C24)
C(10,14)=-G*(SPA*C1-SMA*C2)+H*(SPA*C3-SMA*C4)+A*B*(C5+C6)
C(10,15)=A* EPR *(C5+C6)
C(10,16)=-CC*G*(C15+C16)+CC*H*(C19+C20)+P*B*(C23+C24)
C(10,17)=P*EPR *(C23+C24)
C(11,11)=W*W*(C9+C10)+Y*Y*(C17+C18)+U*U*(C3+C4)+V*V*(C19+C20)
C(11,14)=U*H*(C4-C3)
C(11,16)=V*H*(C19+C20)
C(12,12)=TT*TT*(C5+C6)+FMQ*FMQ*(C23+C24)
C(12,13)=-TT*U*(C5+C6)+FMQ*V*(C23+C24)
C(12,14)=TT*B*(C5-C6)
C(12,15)=JT*EPR*(C5-C6)
C(12,16)=B*FMQ*(C23+C24)
C(12,17)=EPR*FMQ*(C23+C24)
C(13,13)=W*W*(C11+C12)+Y*Y*(C21+C22)+U*U*(C5+C6)+V*V*(C23+C24)
C(13,14)=U*B*(C6-C5)
C(13,15)=U*EPR*(C6-C5)
C(13,16)=V*B*(C23+C24)
C(13,17)=V*EPR*(C23+C24)
C(14,14)=G*G*(C1+C2)+H*H*(C3+C4)+B*B*(C5+C6)
C(14,15)=B*EPR*(C5+C6)
C(15,15)=(C5+C6)*EPR*EPR
C(16,16)=G*G*(C15+C16)+H*H*(C19+C20)+B*B*(C23+C24)
C(16,17)=B*EPR*(C23+C24)
C(17,17)=(C23+C24)*EPR*EPR

C
C
C
```

```
K(1,1)=K1+K2+K7+K8
K(1,2)=-K1-K2
K(1,9)=W*(K8-K7)+L*(K2-K1)
K(1,10)=SPA*K1-SMA*K2
K(1,14)=-G*(K1+K2)
K(2,2)=K1+K2+K3+K4+K5+K6
K(2,3)=-K3-K4
K(2,4)=-K5-K6
```

```

K(2,9)=U*(K1-K2)
K(2,10)=-SPA*(K1+K3)+SMA*(K2+K4)-A*(K5+K6)
K(2,11)=U*(K3-K4)
K(2,12)=TT*(K6-K5)
K(2,13)=U*(K5-K6)
K(2,14)=K1*G+K2*G-K3*H-K4*H-K5*B-K6*B
K(2,15)=-EPR*(K5+K6)
K(3,3)=K3+K4+K9+K10
K(3,10)=SPA*K3-SMA*K4
K(3,11)=W*(K10-K9)+U*(K4-K3)
K(3,14)=K3*H+K4*H
K(4,4)=K5+K6+K11+K12
K(4,10)=A*(K5+K6)
K(4,12)=TT*(K5-K6)
K(4,13)=W*(K12-K11)+U*(K6-K5)
K(4,14)=K5*B+K6*B
K(4,15)=EPR*(K5+K6)
K(5,5)=K15+K16+K19+K20+K23+K24
K(5,6)=-K15-K16
K(5,7)=-K19-K20
K(5,8)=-K23-K24
K(5,9)=V*(K15+K16)
K(5,10)=CC*(K15+K16+K19+K20)+K23*P+K24*P
K(5,11)=V*(K19+K20)
K(5,12)=(K23+K24)*FMQ
K(5,13)=V*(K23+K24)
K(5,16)=-K15*G-K16*G+K19*H+K20*H+K23*B+K24*B
K(5,17)=(K23+K24)*EPR
K(6,6)=K13+K14+K15+K16
K(6,9)=Y*(K13+K14)-V*(K15+K16)
K(6,10)=-K15*CC-K16*CC
K(6,16)=K15*G+K16*G
K(7,7)=K17+K18+K19+K20
K(7,10)=-K19*CC-K20*CC
K(7,11)=Y*(K17+K18)-V*(K19+K20)
K(7,16)=-K19*H-K20*H
K(8,8)=K21+K22+K23+K24
K(8,10)=-K23*P-K24*P
K(8,12)=-K23+K24)*FMQ
K(8,13)=Y*(K21+K22)-V*(K23+K24)
K(8,16)=-B*(K23+K24)
K(8,17)=-K23+K24)*EPR
K(9,9)=W*W*(K7+K8)+Y*Y*(K13+K14)+U*U*(K1+K2)+V*V*(K15+K16)
K(9,10)=-U*(SPA*K1+SMA*K2)+V*CC*(K15+K16)
K(9,14)=U*G*(K1-K2)
K(9,16)=-V*G*(K15+K16)
K(10,10)=SPA*SPA*(K1+K3)+SMA*SMA*(K2+K4)+CC*CC*(K15+K16+K19+K20)
1+A*A*(K5+K6)+P*P*(K23+K24)
K(10,11)=-U*(SPA*K3+SMA*K4)+CC*V*(K19+K20)
K(10,12)=A*TT*(K5-K6)+P*FMQ*(K23+K24)
K(10,13)=A*U*(K6-K5)+P*V*(K23+K24)
K(10,14)=-G*(SPA*K1-SMA*K2)+H*(SPA*K3-SMA*K4)+A*B*(K5+K6)
K(10,15)=A*EPR*(K5+K6)
K(10,16)=-CC*G*(K15+K16)+CC*H*(K19+K20)+P*B*(K23+K24)
K(10,17)=P*EPR*(K23+K24)
K(11,11)=W*W*(K9+K10)+Y*Y*(K17+K18)+U*U*(K3+K4)+V*V*(K19+K20)
K(11,14)=U*H*(K4-K3)
K(11,16)=V*H*(K19+K20)
K(12,12)=TT*TT*(K5+K6)+FMQ*FMQ*(K23+K24)
K(12,13)=-TT*U*(K5+K6)+FMQ*V*(K23+K24)

```

```

K(12,14)=TT*B*(K5-K6)
K(12,15)=TT*EPR*(K5-K6)
K(12,16)=B*FMQ *(K23+K24)
K(12,17)=EPR*FMQ*(K23+K24)
K(13,13)=W*W*(K11+K12)+Y*Y*(K21+K22)+U*U*(K5+K6)+V*V*(K23+K24)
K(13,14)=U*B*(K6-K5)
K(13,15)=U*EPR *(K6-K5)
K(13,16)=V*B*(K23+K24)
K(13,17)=V*EPR*(K23+K24)
K(14,14)=G*G*(K1+K2)+H*H*(K3+K4)+B*B*(K5+K6)
K(14,15)=B* EPR* (K5+K6)
K(15,15)=(K5+K6)*EPR*EPR
K(16,16)=G*G*(K15+K16)+H*H*(K19+K20)+B*B*(K23+K24)
K(16,17)=B*EPR *(K23+K24)
K(17,17)=(K23+K24)*EPR*EPR
DO 70 I=1,16
N=N+1
DO 80 J=N,17
M(J,I)=M(I,J)
C(J,I)=C(I,J)
80 K(J,I)=K(I,J)
70 CONTINUE
RETURN
END

```

```
BLOCK DATA
IMPLICIT REAL*8(A-H,O-Y)
COMMON/DYNAM/D(34,34),VALU(34,2)
COMMON/HATMCK/AM(17,17),AC(17,17),AK(17,17)
COMMON/A1/DL(17,17),AMA(17,17)
COMMON/ORDER/NMO,N,NPO,NTMO,NT,NTPD,NN,NTNT
COMMON/INOUT/NREAC,NWRITE,NPUNCH
DATA D,VALU/1224*0.00/
DATA NMO,N,NPO,NTPD,NT,NTPD,NN,NTNT/16,17,18,33,34,35,289,1156/
DATA NREAD,NWRITE,NPUNCH/5,6,7/
DATA AM,AC,AK/867*0.00/
END
```

```

SUBROUTINE FORCIN(NOREAD,NSIDE)
  IMPLICIT REAL*8(A-H,K,O-Y)
  REAL*8 DCOS,DSIN
  NAMELIST/FORCE/CVII,CVIII,CIX,CX,CXI,CXII,KVII,KVIII,KIX,KX,KXI,
  IKXII,AL,AR,WIDE,BT,GH1,GBER2,SPEED,FWL,FWR,RUNGL,RUNGR,NUMB
  COMMON/R1/TIME(10)
  COMMON/COSSIN/SAC(34,3)
  COMMON/MISC/WL,WR,FAST,STEPL,STEPR,AMPL,AMPR,FAZE,NSTEPS
  COMMON/INDUT/NREAD,NWRITE,NPUNCH
  DIMENSION CI(17),CII(17),CIII(17),SI(17),SII(17),SIII(17)
  EQUIVALENCE(SAC(1,1),CI),(SAC(1,2),CII),(SAC(1,3),CIII)
  EQUIVALENCE(SAC(18,1),SI),(SAC(18,2),SII),(SAC(18,3),SIII)
  IF(NOREAD.NE.0) GO TO 10
  READ(NREAD,FORCE)
  WL=FWL
  WR=FWR
  FAST=SPEED
  AMPL=AL
  AMPR=AR
  STEPL=RUNGL
  STEPR=RUNGR
  FAZE=BT
  NSTEPS=NUMB
10 CONTINUE
  IF(NSIDE.EQ.2) GO TO 20
  T1=WL*GH1/FAST
  T2=WL*GBER2/FAST
  TIME(1)=DSIN(T1)
  TIME(2)=DCOS(T1)
  TIME(3)=DSIN(T2)
  TIME(4)=DCOS(T2)
  T1=WR*GH1/FAST
  T2=WR*GBER2/FAST
  TIME(5)=DSIN(BT)
  TIME(6)=DCOS(BT)
  TIME(7)=DSIN(BT+T1)
  TIME(8)=DCOS(BT+T1)
  TIME(9)=DSIN(BT+T2)
  TIME(10)=DCOS(BT+T2)
  AW=AL*WL
  CI(1)=AW*CVII
  CI(9)=-WIDE*CI(1)
  SI(1)=AL*KVII
  SI(9)=-WIDE*SI(1)
  CII(3)=AW*CIX
  CII(11)=-WIDE*CII(3)
  SII(3)=KIX*AL
  SII(11)=-WIDE*SII(3)
  CIII(4)=CXI*AW
  CIII(13)=-WIDE*CIII(4)
  SIII(4)=AL*KXI
  SIII(13)=-WIDE*SIII(4)
  RETURN
20 AW=AR*WR
  CI(1)=AW*CVIII
  CI(9)=WIDE*CI(1)
  SI(1)=AR*KVIII
  SI(9)=WIDE*SI(1)
  CII(3)=AW*CX
  CII(11)=WIDE*CII(3)

```

```
SII(3)=AR*KX  
SII(11)=WIDE*SII(3)  
CIII(4)=AW*CXII  
CIII(13)=WIDE*CIII(4)  
SIII(4)=AR*KXII  
SIII(13)=WIDE*SIII(4)  
RETURN  
END
```

```

SUBROUTINE DGELG (THIS IS ONE OF THE SUBROUTINES IN THE IBM
SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE).

PURPOSE
  TO SOLVE A GENERAL SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS.

USAGE
  CALL DGELG(R,A,M,N,EPS,IER)

DESCRIPTION OF PARAMETERS
  R      - DOUBLE PRECISION M BY N RIGHT HAND SIDE MATRIX
           (DESTROYED). ON RETURN R CONTAINS THE SOLUTIONS
           OF THE EQUATIONS.
  A      - DOUBLE PRECISION M BY M COEFFICIENT MATRIX
           (DESTROYED).
  M      - THE NUMBER OF EQUATIONS IN THE SYSTEM.
  N      - THE NUMBER OF RIGHT HAND SIDE VECTORS.
  EPS    - SINGLE PRECISION INPUT CONSTANT WHICH IS USED AS
           RELATIVE TOLERANCE FOR TEST ON LOSS OF
           SIGNIFICANCE.
  IER    - RESULTING ERROR PARAMETER CODED AS FOLLOWS
           IER=0 - NO ERROR,
           IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR
                   PIVOT ELEMENT AT ANY ELIMINATION STEP
                   EQUAL TO 0,
           IER=K - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-
                   CANCE INDICATED AT ELIMINATION STEP K+1,
                   WHERE PIVOT ELEMENT WAS LESS THAN OR
                   EQUAL TO THE INTERNAL TOLERANCE EPS TIMES
                   ABSOLUTELY GREATEST ELEMENT OF MATRIX A.

REMARKS
  INPUT MATRICES R AND A ARE ASSUMED TO BE STORED COLUMNWISE
  IN M*N RESP. M*M SUCCESSIVE STORAGE LOCATIONS. ON RETURN
  SOLUTION MATRIX R IS STORED COLUMNWISE TOO.
  THE PROCEDURE GIVES RESULTS IF THE NUMBER OF EQUATIONS M IS
  GREATER THAN 0 AND PIVOT ELEMENTS AT ALL ELIMINATION STEPS
  ARE DIFFERENT FROM 0. HOWEVER WARNING IER=K - IF GIVEN -
  INDICATES POSSIBLE LOSS OF SIGNIFICANCE. IN CASE OF A WELL
  SCALED MATRIX A AND APPROPRIATE TOLERANCE EPS, IER=K MAY BE
  INTERPRETED THAT MATRIX A HAS THE RANK K. NO WARNING IS
  GIVEN IN CASE M=1.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
  NONE

METHOD
  SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION WITH
  COMPLETE PIVOTING.

```

C	IF(M)23,23,1	DGELG060
C	SEARCH FOR GREATEST ELEMENT IN MATRIX A	DGELG061
1	IER=0	DGELG062
	PIV=0.00	DGELG063
	MM=M*M	DGELG064
	NN=N*M	DGELG065
	DO 3 L=1,MM	DGELG066
	TB=DABS(A(L))	DGELG067
	IF(TB-PIV)3,3,2	DGELG068
2	PIV=TB	DGELG069
	I=L	DGELG070
3	CONTINUE	DGELG071
	TOL=EPS*PIV	DGELG072
C	A(I) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I).	DGELG073
C		DGELG074
C		DGELG075
C	START ELIMINATION LOOP	DGELG076
	LST=1	DGELG077
	DO 17 K=1,M	DGELG078
C		DGELG079
C	TEST ON SINGULARITY	DGELG080
	IF(PIV)23,23,4	DGELG081
4	IF(IER)7,5,7	DGELG082
5	IF(PIV-TOL)6,6,7	DGELG083
6	IER=K-1	DGELG084
7	PIV=1.00/A(I)	DGELG085
	J=(I-1)/M	DGELG086
	I=I-J*M-K	DGELG087
	J=J+1-K	DGELG088
C	I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT	DGELG089
C		DGELG090
C	PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R	DGELG091
	DO 8 L=K,MM,M	DGELG092
	LL=L+I	DGELG093
	TB=PIV*A(LL)	DGELG094
	R(LL)=R(L)	DGELG095
8	R(L)=TB	DGELG096
C		DGELG097
C	IS ELIMINATION TERMINATED	DGELG098
	IF(K-M)9,18,18	DGELG099
C		DGELG100
C	COLUMN INTERCHANGE IN MATRIX A	DGELG101
9	LEND=LST+M-K	DGELG102
	IF(J)12,12,10	DGELG103
10	II=J*M	DGELG104
	DO 11 L=LST,LEND	DGELG105
	TB=A(L)	DGELG106
	LL=L+II	DGELG107
	A(L)=A(LL)	DGELG108
11	A(LL)=TB	DGELG109
C		DGELG110
C	ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A	DGELG111
12	DO 13 L=LST,MM,M	DGELG112
	LL=L+I	DGELG113
	TB=PIV*A(LL)	DGELG114
	A(LL)=A(L)	DGELG115
13	A(L)=TB	DGELG116
C		DGELG117
C	SAVE COLUMN INTERCHANGE INFORMATION	DGELG118
		DGELG119

C	A(LST)=J	DGELG120
C	ELEMENT REDUCTION AND NEXT PIVOT SEARCH	DGELG121
	PIV=0.00	DGELG122
	LST=LST+1	DGELG123
	J=0	DGELG124
	DO 16 II=LST,LEND	DGELG125
	PIV=-A(II)	DGELG126
	IST=II+M	DGELG127
	J=J+1	DGELG128
	DO 15 L=IST,MM,M	DGELG129
	LL=L-J	DGELG130
	A(L)=A(L)+PIV*A(LL)	DGELG131
	TB=DABS(A(L))	DGELG132
	IF(TB-PIV)15,15,14	DGELG133
14	PIV=TB	DGELG134
	I=L	DGELG135
15	CONTINUE	DGELG136
	DO 16 L=K,NM,M	DGELG137
	LL=L+J	DGELG138
16	R(LL)=R(LL)+PIV*R(L)	DGELG139
17	LST=LST+M	DGELG140
C	END OF ELIMINATION LOOP	DGELG141
C		DGELG142
C		DGELG143
C	BACK SUBSTITUTION AND BACK INTERCHANGE	DGELG144
18	IF(M-1)23,22,19	DGELG145
19	IST=MM+M	DGELG146
	LST=M+1	DGELG147
	DO 21 I=2,M	DGELG148
	II=LST-I	DGELG149
	IST=IST-LST	DGELG150
	L=IST-M	DGELG151
	L=A(L)+.500	DGELG152
	DO 21 J=II,NM,M	DGELG153
	TB=R(J)	DGELG154
	LL=J	DGELG155
	DO 20 K=IST,MM,M	DGELG156
	LL=LL+1	DGELG157
20	TB=TB-A(K)*R(LL)	DGELG158
	K=J+L	DGELG159
	R(J)=R(K)	DGELG160
21	R(K)=TB	DGELG161
22	RETURN	DGELG162
C		DGELG163
C		DGELG164
C	ERROR RETURN	DGELG165
23	IER=-1	DGELG166
	RETURN	DGELG167
	END	DGELG168
		DGELG169

```

SUBROUTINE WEQUAL
  IMPLICIT REAL*8(A-H,O-Y)
  REAL*8 DSQRT
  COMMON/DOOUT/XCL(17),XSL(17),XCR(17),XSR(17)
  COMMON/MISC/WL,WR,FAST,STEPL,STEPR,AMPL,AMPR,FAZE,NSTEPS
  COMMON/ORDER/NMO,N,NPO,NTMO,NT,NTPO,NN,NTNT
  COMMON/INDOUT/NREAC,NWRITE,NPUNCH
  DIMENSION AMPMAX(17)
  DO 21 I=1,N
    XSIN=XSL(I)+ XSR(I)
    XCOS=XCL(I)+XCR(I)
21  AMPMAX(I)=DSQRT(XSIN*XSIN+XCOS*XCOS)
    WRITE(NWRITE,111)
    WRITE(NWRITE,222) FAZE
    WRITE(NWRITE, 1) AMPMAX( 1)
    WRITE(NWRITE, 2) AMPMAX( 2)
    WRITE(NWRITE, 3) AMPMAX( 3)
    WRITE(NWRITE, 4) AMPMAX( 4)
    WRITE(NWRITE, 5) AMPMAX( 5)
    WRITE(NWRITE, 6) AMPMAX( 6)
    WRITE(NWRITE, 7) AMPMAX( 7)
    WRITE(NWRITE, 8) AMPMAX( 8)
    WRITE(NWRITE, 9) AMPMAX( 9)
    WRITE(NWRITE,10) AMPMAX(10)
    WRITE(NWRITE,11) AMPMAX(11)
    WRITE(NWRITE,12) AMPMAX(12)
    WRITE(NWRITE,13) AMPMAX(13)
    WRITE(NWRITE,14) AMPMAX(14)
    WRITE(NWRITE,15) AMPMAX(15)
    WRITE(NWRITE,16) AMPMAX(16)
    WRITE(NWRITE,17) AMPMAX(17)
1  FORMAT('O',56X,' X1 = ',1P109.2)
2  FORMAT('O',56X,' X2 = ',1P109.2)
3  FORMAT('O',56X,' X3 = ',1P109.2)
4  FORMAT('O',56X,' X5 = ',1P109.2)
5  FORMAT('O',56X,' X7 = ',1P109.2)
6  FORMAT('O',56X,' Z1 = ',1P109.2)
7  FORMAT('O',56X,' Z3 = ',1P109.2)
8  FORMAT('O',56X,' Z5 = ',1P109.2)
9  FORMAT('O',56X,' THETA1 = ',1P109.2)
10 FORMAT('O',56X,' THETA2 = ',1P109.2)
11 FORMAT('O',56X,' THETA3 = ',1P109.2)
12 FORMAT('O',56X,' THETA4 = ',1P109.2)
13 FORMAT('O',56X,' THETA5 = ',1P109.2)
14 FORMAT('O',56X,' PH12 = ',1P109.2)
15 FORMAT('O',56X,' PH14 = ',1P109.2)
16 FORMAT('O',56X,' XI2 = ',1P109.2)
17 FORMAT('O',56X,' XI4 = ',1P109.2)
111 FORMAT('I',46X,' THE MAXIMUM AMPLITUDES (FEET,RADIANS) ARE'//)
222 FORMAT('I',42X,' THE RELATIVE PHASE ANGLE(RAD.) BETWEEN SIDES IS'
1/'O',60X,1P1010.3//)
  RETURN
  END

```

```
BLOCK DATA
IMPLICIT REAL*8(A-H,O-Y)
COMMON/DYNAM/D(34,34),VALU(34,2)
COMMON/HATMCK/AM(17,17),AC(17,17),AK(17,17)
COMMON/AL/DL(17,17),AMA(17,17)
COMMON/ORDER/NMO,N,NPO,NTMO,NT,NTPD,NN,NTNT
COMMON/INOUT/NREAC,NWRITE,NPUNCH
DATA D,VALU/1224*0.00/
DATA NMO,N,NPO,NTMO,NT,NTPD,NN,NTNT/16,17,18,33,34,35,289,1156/
DATA NREAD,NWRITE,NPUNCH/5,6,7/
DATA AM,AC,AK/867*0.00/
END
```

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A LINEAR, THREE-DIMENSIONAL MODEL FOR THE
VIBRATIONS OF A SEMI-TRAILER TRUCK

by

CHRISTOPHER CHARLES CHAPMAN

B. S., Kansas State University, 1970

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1971

Name: Christopher Charles Chapman

Date of Degree: August, 1971

Institution: Kansas State University

Location: Manhattan, Kansas

Title of Study: A LINEAR, THREE DIMENSIONAL
MODEL FOR THE VIBRATIONS OF
A SEMI-TRAILER TRUCK

Pages in Study: 110

Candidate for Degree of Master of Science

Major Field: Mechanical Engineering

Scope and Method of Study: After a number of simplifying assumptions had been made, a mathematical model was developed for simulation of a three dimensional semi-trailer truck. The linear, differential equations of motion were determined using the energy method and Lagrange's Equations.

Two methods of solution were described. The first was based on the reduced equation, which determines the transient solution and the steady state solution. The second was based on the undetermined coefficients method, which determines the steady state solution only.

Two digital computer programs, based on the two methods, were developed for determining the steady state responses of the vehicle due to sinusoidal inputs. The utility of the two programs was compared.

Findings and Conclusions: It was concluded that the three dimensional model provided a more comprehensive addition to simulation of the vibrations of a semi-trailer truck. However it is recommended that the accuracy of the model be determined by comparing it to "real" vehicles. This could be a subject for further study.

MAJOR PROFESSOR'S APPROVAL

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