CONVECTION HEAT TRANSFER BETWEEN A FLAT PLATE AND A PARTIALLY IONIZED GAS

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NOMENCLATURE

A	Area
C	Specific heat
D	Diffusion coefficient
e	Electron charge
erfc	Complementary error function
$\overline{h}$	Average coefficient of transfer of heat or mass
k	Boltzmann constant
K	Thermal conductivity
L	Characteristic length
m	Mass
$\overline{Nu}$	Average Nusselt number
n	Number of particles per unit volume
Pr	Prandtl number
q	Heat transfer rate
r	Effective particle radius
R	Universal gas constant
Re	Reynolds number
Sc	Schmidt number
T	Absolute temperature
t	Static temperature
u	Component of velocity in the x direction
v	Component of velocity in the y direction
$\overline{v}$	Mean particle speed
$\overline{v^2}$	Mean square speed
w	Mass fraction

x	Local value of distance measured from the leading edge of the plate
x,y,z	Cartesian coordinates
α	Thermal diffusivity
β	Mobility
γ	Fraction of gas ionized
η	Variable
θ	Characteristic temperature of ionization
θ	Time
θ'	Dimensionless temperature
λ	Mean free path
μ	Absolute viscosity
ν	Kinematic viscosity
ρ	Density
σ	Effective hard sphere collision cross-section
$\emptyset'$	Dimensionless concentration of ions
ψ	Stream function

Subscripts

a	Ambipolar
a	Atom
c	Convection
c	Modified for Coulomb forces
d	Diffusion
e	Electrons
h	Heavy particles (ions and atoms)
i	Initial conditions
i	Ion

L	Based on length
R	Recombination
s	Quantities evaluated at the plate surface
∞	Quantities evaluated outside the boundary layer

INTRODUCTION

Recently, much effort has been expended in the field of high velocity, high temperature heat transfer study of ionized gases in thermodynamic equilibrium, i.e., thermally ionized gases in the temperature range of 10,000 degrees Fahrenheit to 15,000 degrees Fahrenheit and in the supersonic to hypersonic velocity range. The present uses for knowledge of such high temperature, high velocity heat transfer phenomena are in the areas of aerodynamic heating, ablation cooling, stagnation point heat transfer, and magnetohydrodynamic power generation upon which much emphasis has been placed due to the space age. Other uses include the study of material properties at high temperature, the plasma torch, and chemical synthesis.

Presently, little is known about heat transfer through the laminar boundary layer of a non-equilibrium, partially ionized gas at moderate pressures and low temperature. To the author's knowledge, only two references[5,6] exist on the study of heat transfer through laminar boundary layers in a partially ionized gas. Heat transfer in a completely ionized boundary layer is considered the simplest case; this was studied and the energy equation was solved by Jepson[8]. The region of investigation developed in this thesis is for a partially ionized gas. The ionization is produced by a high frequency electric field with a degree of ionization of 1 part in 10^5. The gas has a moderate pressure of 1 to 10 mm of mercury absolute and a macroscopic temperature in the range of 100 degrees Fahrenheit to 300 degrees Fahrenheit. Results of the study of heat transfer in this region are applicable to vacuum tube technology and to future low temperature propulsion systems. This thesis develops an analytical expression for a comparison of the heat transferred to a flat plate from an ionized gas to that which would be transferred if the gas were not ionized but at the same temperature, pressure, and velocity. Experimental results were obtained and compared with the developed theory. A research technique is investigated that can be used in the study of heat transfer in an ionized gas.

THEORY OF HEAT CONDUCTION IN A PARTIALLY IONIZED GAS

The kinetic theory of gases provides a method of estimating the transport coefficients of a partially ionized gas on a basis of the concept of mean free path. For a single component monatomic gas, the coefficients of viscosity, thermal conductivity, and self diffusion are given by[2]

$$\mu = \frac{5\pi}{32} nm\lambda\bar{v} \tag{1}$$

$$K = \frac{75\pi}{128} nk\lambda\bar{v} \tag{2}$$

$$D = \frac{3\pi}{16} \lambda\bar{v} \tag{3}$$

where

$$\lambda = [(2)^{\frac{1}{2}} n\,\sigma]^{-1} \tag{4}$$

$$\bar{v} = \left[\frac{8}{\pi}\frac{kT}{m}\right]^{\frac{1}{2}} \tag{5}$$

$$\sigma = 4\pi r^2 \tag{6}$$

These formulae assume smooth, rigid, elastic, spherical molecules with an effective radius r.

To extend these results to a partially ionized gas consisting of atoms, ions, and electrons, the following approximate formulae may be used as a starting point[1]:

$$\mu = \frac{5\pi}{32} \sum_k n_k m_k \lambda_k \bar{v}_k \tag{7}$$

$$K = \frac{75\pi}{128} k \sum_k n_k \lambda_k \bar{v}_k \tag{8}$$

$$\mathcal{D}_k = \frac{3\pi}{16} \lambda_k \bar{v}_k \tag{9}$$

where

$$\lambda_k = [(2)^{\frac{1}{2}} \sum_j n_j \sigma_{kj}]^{-1} \tag{10}$$

The subscript k denotes each kind of particle considered and σ_{kj} is the collision cross section of type k particles with type j particles. It is assumed that the transport of mass, momentum, and energy is the summation of the transport due to each individual kind of particle as given by equations (1), (2), and (3) but with the determination of the mean free path λ_k

modified to account for all types of collisions as in equation (10).

In applying these transport equations to a partially ionized gas, it must be noted that ions and atoms undergo appreciable momentum change only as a consequence of collisions among themselves. Electrons exchange momentum with each other and with ions and with atoms; however, there is no appreciable kinetic energy exchange between electrons and ions or atoms, but only between particles with a mass of the same order of magnitude. Therefore, in a few heavy particle collisions, the atoms and ions attain a Boltzmann distribution with a temperature T_a while within a few electron-electron collisions, the electrons approximate a Boltzmann distribution with a temperature T_e. The temperature T_e is not necessarily the same as T_a. The number of electron-heavy particle collisions necessary to equalize the two temperatures and bring the gas to an equilibrium state is approximately equal to the ratio of the mass of an atom to the mass of an electron.

The use of an electric field as a means of producing ionization at low temperatures also serves to accelerate the electrons between collisions, thereby keeping the average electron energy and the electron temperature T_e higher than in an equilibrium plasma. However, for purposes of this analysis, the electrons are assumed to have a Boltzmann distribution with temperature T_e.

From inspection of equation (7), it is noted that the electrons make no appreciable contribution to viscosity because of their extremely small mass. Therefore, the following equations apply to a monatomic ionized gas:

$$\mu = \frac{5\pi}{32} m_a \bar{v}_a (n_a \lambda_a + n_i \lambda_i) \tag{11}$$

where

$$\lambda_a = [(2)^{\frac{1}{2}}(n_i\sigma_{ai} + n_a\sigma_{aa})]^{-1} \tag{12}$$

and

$$\lambda_i = [(2)^{\frac{1}{2}}(n_i\sigma_{ii} + n_a\sigma_{ia})]^{-1} \tag{13}$$

The subscripts i and a refer to ion and atom respectively.

The thermal conductivity K of each constituent is considered separately because of the difference between electron and heavy particle temperature, hence a variation in properties. Considering the heavy particles first, the thermal conductivity is

$$K_h = \frac{15}{4}\frac{k\mu}{m_a} = \frac{3}{2}\frac{C_{pa}\mu}{m_a} \tag{14}$$

where C_{pa} is the specific heat per atom. Equation (14) was developed by substituting equation (7) into equation (8). The thermal conductivity K_e of the electrons is developed from equation (8) as

$$K_e = \frac{75\pi}{128} k\ n_e\overline{v}_e[n_a\sigma_{ae} + (2)^{\frac{1}{2}}\ \sigma_c]^{-1} \tag{15}$$

where σ_c is the Coulomb cross section section for charged electron interactions. The factor $(2)^{\frac{1}{2}}$ no longer appears in the first term of the brackets in equation (15)because the velocities of the electrons are much greater than those of heavy particles. The latter may be assumed to be at rest; a correction for relative velocities need not be made. The Coulomb cross section may be defined as follows[5]

$$\sigma_c = 1.8 \times 10^{-6} T^{-2} \ln \left[\frac{1.24 \times 10^4 \, T^{3/2}}{(n_e)^{\frac{1}{2}}} \right] \text{cm}^2 \tag{16}$$

This collision cross section takes into consideration the charge of an electron and its influence on the mean free path.

A partially ionized gas is a three component mixture; however, from the standpoint of particle diffusion, it consists of only two components, ion-electron pairs and atoms. This is a result of an interaction between ions and electrons when both are diffusing through neutral atoms. It is expected that the electrons, because of their smaller mass, will have a greater diffusion rate. However, this does not occur because the charge separation would result in a deceleration of the electron diffusion and an acceleration of the ion diffusion. This causes the electrons to be retarded in their motion by the slowly diffusing ions. Hence, the electrons and ions may be treated as if they were diffusing as ion-electron pairs. By denoting the electron diffusivity as $\mathcal{D}_e$ and the ion diffusivity as $\mathcal{D}_i$ with β_e and β_i being their respective mobilities in the presence of an electric field, the ambipolar diffusion coefficient for the ion-electron pairs is defined as[3]

$$\mathcal{D}_a = \frac{\beta_i \mathcal{D}_e + \beta_e \mathcal{D}_i}{\beta_i + \beta_e} \tag{17}$$

Because the mobility of charged particles is inversely proportional to their masses

$$\frac{\beta_i}{\beta_e} \ll 1 \tag{18}$$

and

$$\frac{\mathcal{D}_e}{\mathcal{D}_i} = \frac{\beta_e}{\beta_i} \tag{19}$$

Therefore, by substitution of equations (18) and (19) into (17)

$$\mathcal{D}_a \approx 2\mathcal{D}_i \tag{20}$$

The ambipolar diffusion coefficient is obtained from equations (9) and (20) as

$$\mathcal{D}_a = \frac{3\pi}{8} \bar{v}_a \left[(2)^{\frac{1}{2}}(n_i + n_a)\, \sigma_{ia}\right]^{-1} \tag{21}$$

PARTIALLY IONIZED LAMINAR BOUNDARY LAYER HEAT TRANSFER

There are many complexities, viz.; recombination and considerable variation in gas properties, involved in the exact solution of the energy equation which is used to determine the heat transfer between a flat plate and a partially ionized gas. As a result of the complexities, an approach, other than the exact solution to the energy equation, is made to determine the heat transfer. An expression for the ratio of the heat transferred from a partially ionized gas to a flat plate to that which is expected for a non-ionized gas will be developed.

From boundary layer theory[13], by consideration of a non-ionized fluid in laminar flow over a flat plate, it is shown that

$$\overline{Nu}_L = 0.664\, Re_L^{1/2}\, Pr^{1/3} \tag{22}$$

where $\overline{Nu}_L$ is the average Nusselt number for a flat plate of length L. By assuming a Prandtl number of unity, equation (22) can be written as

$$\overline{Nu}_L = \frac{\bar{h}_{cL}}{K} = 0.664\ (Re_L\ Pr)^{\frac{1}{2}} \tag{23}$$

Solving equation (23) for $\bar{h}_c$, the following is obtained

$$\bar{h}_c = 0.664 \left[\frac{C_p \rho V K}{L}\right]^{\frac{1}{2}} \tag{24}$$

By definition, the average convection coefficient of heat transfer is

$$\bar{h}_c = \frac{q}{A(t_\infty - t_s)} \tag{25}$$

Solving equation (25) for q

$$q = \bar{h}_c\ A(t_\infty - t_s) \tag{26}$$

From equations (24) and (26), the heat transfer q is

$$q = C\ (\rho C_p K)^{\frac{1}{2}}\ (t_\infty - t_s) \tag{27}$$

where C is a proportionality constant that includes V, L, 0.664, and A. The constant C is sufficient for a relative comparison of the heat transferred during ionized flow to that transferred during non-ionized flow. Upon consideration of a monatomic gas with three degrees of freedom in translation, and from elementary kinetic theory where $C_p = \frac{5}{2}R$ and $\rho = \frac{nk}{R}$, equation (27) may be written on a molecular basis as

$$q = C\left(\frac{5}{2}\,nk\,K\right)^{\frac{1}{2}}(t_{\infty} - t_s) \tag{28}$$

In actuality, the physical properties in equations (22) to (28) vary with temperature, but for the purpose of analysis it was assumed that the physical properties are constant.

Experimental data for heat transfer have been found to agree satisfactorily with the results predicted analytically for a non-ionized gas if the properties are evaluated at the mean temperature obtained from the surface of the body and the free-stream temperature[10]. For purposes of this derivation, the physical property variations through the boundary layer, other than electron temperature and concentration, are assumed to be linear with temperature for the ionized flow. These properties are evaluated at the mean temperature. The electron temperature and concentration used in the derivations are assumed to remain constant throughout the free stream and boundary layer.

The heavy particles and the electrons do not have the same temperature. This is justified by the fact that in an equilibrium plasma which is produced by thermal ionization, all particles in the plasma have the same energy on the basis of equipartition of energy[12]. The principle of equipartition of energy states that for a two component mixture consisting of electrons and heavy particles, the mean translational kinetic energy of the heavy molecules is equal to the mean translation kinetic energy of the electrons, or

$$\frac{1}{2}\,m_a\overline{v_a^{\,2}} = \frac{1}{2}\,m_e\overline{v_e^{\,2}} = \frac{3}{2}\,k\,T \tag{29}$$

This relationship is only valid for a monatomic gas with three degrees of freedom. Further, as a result of the electric field used to ionize the gas, the electron velocity is even greater than would be expected if thermal equilibrium existed; this is a consequence of the acceleration of the electrons by the electric field. Therefore, the effective "electron temperature" may be thousands of degrees; whereas, the heavy particle macroscopic temperature is in the range of 60 degrees to 300 degrees Fahrenheit. By assuming that the heavy particles and electrons have different temperatures and therefore different thermal boundary layer thicknesses, the equations for the heat transfer from the heavy particles and from the electrons become the following:

$$q_h = C \left[\frac{5}{2} (n_i + n_a) K_h k \right]^{\frac{1}{2}} (t_\infty - t_s) \tag{30}$$

$$q_e = C \left[\frac{5}{2} n_e K_e k \right]^{\frac{1}{2}} (t_\infty - t_s) \tag{31}$$

In addition to the transport of energy by molecular collisions with the surface, there is also the transport of energy to the surface by ambipolar diffusion of the ion-electron pairs; these may recombine at the surface and give up energy.

This heat transfer is given by

$$q_R = C \left[\mathcal{D}_a \right]^{\frac{1}{2}} \left(\frac{5}{2} n_e k\theta \right) \tag{32}$$

where $\frac{5}{2} k\theta$ is the ionization energy per atom[5]. Ionization energy is the amount of energy needed to remove one electron from the atom when it is in its normal state, leaving the ion in its normal state[4]. Subsequently, when a positive ion and an electron recombine at the surface of the plate, this same amount of energy is given up to the plate during the process of recombination. The θ is called the characteristic temperature of ionization.[4]

The characteristic temperature (θ) of argon which has an ionization energy of 15.755 electron-volts[4] is 73,000 degrees Kelvin.

Equation (31) was developed in the following manner. The energy equation and the diffusion equation are respectively

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 t}{\partial y^2} \tag{33}$$

$$u \frac{\partial w_i}{\partial x} + v \frac{\partial w_i}{\partial y} = \mathcal{D}_a \frac{\partial^2 w_i}{\partial y^2} \tag{34}$$

Introduction of the new variable

$$\eta = \frac{y}{2} \left(\frac{u_\infty}{\nu x} \right)^{\frac{1}{2}} \tag{35}$$

the stream function

$$\psi = f(\eta) \, (\nu u_\infty x)^{\frac{1}{2}} \tag{36}$$

and the dimensionless concentration ratio and the temperature ratio

$$\phi' = \frac{w_i - w_{is}}{w_{i\infty} - w_{is}} \qquad \theta' = \frac{t - t_s}{t_\infty - t_s} \tag{37}$$

into the partial differential equations (32) and (35) result in the following ordinary, nonlinear, second order differential equations

$$\frac{d^2\,\theta'}{d\,\eta^2} + Pr\;\; f(\eta)\,\frac{d\theta'}{d\eta} = 0 \tag{38}$$

$$\frac{d^2\,\phi'}{d\,\eta^2} + Sc\;\; f(\eta)\,\frac{d\phi'}{d\eta} = 0 \tag{39}$$

If the $S_c = P_r = 1$, then both equations have the same solution; by analogy, h_d is proportional to h_c where h_d is the coefficient of mass transfer. Therefore, from equation (23)

$$\frac{h_{dL}}{\mathcal{D}_a} = 0.664\;(Re_L\;Sc)^{\frac{1}{2}} \tag{40}$$

Solving equation (40) for h_d and including V, 0.644, and L in a constant C

$$h_d = C\;(\mathcal{D}_a)^{\frac{1}{2}} \tag{41}$$

The proportionally constants in equations (30), (31), and (32) are equal only for the case of zero gas viscosity. The effective viscosity for the electrons in equation (30) is zero, as compared with the heavy particles in equations (31) and (32); this is due to their extremely small mass which is $\frac{1}{73{,}000}$ as large as an argon atom. For a finite gas viscosity, the proportionality constants in (30) and (32) are less than that for equation (31) since the effect of viscosity on heat transfer is to reduce the heat transfer below its zero viscosity value.

If the Prandtl number $\frac{\mu C_p}{K}$ and Schmidt number $\frac{\mu}{\mathcal{D}_a \rho}$ are both about 1, then the heat transfer is about a factor of two less than that for the case of zero viscosity[5]. By introducing this factor of two, the total heat transfer q may be compared to that which would occur in a non-ionized gas q_a at the same temperature, velocity and particle density:

$$\frac{q}{q_a} = \frac{[2(\gamma K_e)^{\frac{1}{2}} + (K_h)^{\frac{1}{2}}][t_\infty - t_s] + [\frac{5}{2}(n_i + n_a)k \mathcal{D}_a]^{\frac{1}{2}} (\gamma\theta)}{[(1 - \gamma)K_a]^{\frac{1}{2}} [t_\infty - t_s]} \qquad (42)$$

Here γ is the fraction of the gas ionized.

EXPERIMENTAL WORK

A physical measurement of the ratio of the heat transfer film coefficient obtained in an ionized gas to that obtained from a non-ionized gas was made to provide a comparison with the theory developed in this thesis and to investigate a research technique that could be used in the study of heat transfer to an ionized gas.

Equipment and Materials

The equipment used to generate and contain the plasma consisted of a 12-inch by 12-inch by 24-inch-long wind tunnel, a vacuum pump, and an industrial radio frequency generator as shown on Plates I and II. An overall flow schematic is shown on Plate III. The generator operated at a frequency of 10.7 megacycles. The vacuum pump was a Kinney pump KDH 80 rated for 80 cubic feet per minute flow at 1800 rpm and atmospheric pressure with a guaranteed vacuum corresponding to a pressure of 10 microns of Hg, absolute. The wind tunnel was fabricated from ¼-inch steel, all welded construction, with 3½-inch pipe inlet and outlet, and 3/4-inch polished plate glass side plates for visual observation. The 3½-inch screw-on pipe flanges, which were used to make removable end caps, were modified to accept 6-inch diameter "O" rings. The tunnel also had two 6-inch square openings in the top which could be used to give access for instrumentation. Static pressure in the tunnel was controlled by a needle valve located between the pump and the tunnel.

All joints formed by removable components were sealed with rubber "O" rings. Leaks in the welds of the tunnel were detected with "Leak Tec" and sealed by applying a liberal amount of Apezon "Q" vacuum sealing compound at the location of the leak. The Apezon compound proved very successful at the pressure investigated.

The inner test chamber was constructed from 2-inch by 6-inch white pine, 3/4-inch plywood, formica, and double-strength window glass as shown by Plate IV. The inlet consisted of a 3-inch radius standard ASME, two-dimensional nozzle in which the radius of the nozzle inlet was six-tenths of the distance between the plates. This type of entrance straightened the velocity profile. The internal dimensions of the test chamber were 5¼ inches in the vertical direction by 7 inches in the axial direction. The electrodes used to generate the plasma consisted of two sections of 2-inch thick aluminum honeycomb material placed in series with the flow.[7] The resulting region of plasma appeared to be very uniform in color and density by visual observation. The electrode spacing was 7¼ inches.

A flat plate was chosen as the model upon which to make physical measurements in order to make a comparison of experimental results with theoretical predictions. A 4-inch by 6-inch flat plate made from ¼-inch thick polished plate glass with a 45-degree bevel on the leading edge was placed parallel to the flow at a zero angle of incidence. A 3-inch thickness of Kaylo insulation with a thermal conductivity of 0.0317 $\frac{\text{BTU}}{\text{hr ft }^{\circ}\text{F}}$ at 100 degrees Fahrenheit was used to insulate the bottom side of the plate.

The plate temperature at a depth of 1/32 inch was measured by using thermocouples made from number 36 AWG copper-constantan thermocouple wire, teflon insulated, with a fiberglass overall jacket. These wires were placed in 1/8-inch diameter holes drilled in the insulation side of the plate with a number 2600 Somaca carbide bit to within 1/32 inch from the top surface of the plate and bonded in place by 8201 epoxy resin. The glass plate and insulator are shown as mounted in the inner test chamber by Plate IV, and a test schematic is shown by Plate V.

Argon was used for the experimental procedure because of its relative ease of ionization, commercial availability, and because it is a monatomic gas. The argon, as supplied to the inlet of the wind tunnel, had a purity of 99.995 percent. A mercury manometer was used to measure the static pressure.

A dual-wire Langmuir probe was used to obtain data from which the electron temperature and the degree of ionization were calculated. The probe consisted of two tungsten wires 1.3 mm in diameter enclosed in a double bore, cylindrical, ceramic insulator which was 3.8 mm in diameter. The sensing end of the probe was cut in a plane perpendicular to the axis of the probe. The center to center spacing between the wires was 1.5 mm. A detail of the plasma sensing tip of the probe is shown on Plate IX. The probe was placed in the plasma perpendicular to the flow direction and a potential was applied to the probe circuit which is shown on Plate V. The potential of the probe was allowed to "float", i.e., not grounded, to make the probe data independent of the potential of the plasma.

Operations

During the first trial, an attempt was made to transfer heat from the plate to the ionized gas, holding the plate surface temperature constant with a heat source. Due to the temperature limitations of the epoxy resin used to bond the thermocouples in the plate, the maximum plate temperature that could be tolerated was 250 degrees Fahrenheit. During this trial, it was determined that the plasma temperature was much higher than originally anticipated. Macroscopic plasma temperatures of 250 degrees Fahrenheit to 300 degrees Fahrenheit were generated.

The test procedure was then revised to allow for heat transfer from the plasma to the cold plate. The plate was allowed to heat up from a uniform ambient temperature by the flow of ionized argon and then, from the same initial conditions, by heated non-ionized argon. The temperature of the plate at a depth of 1/32 inch was measured every five minutes for a period of 55 minutes for the ionized argon and 40 minutes for the heated non-ionized argon. A static pressure of 4 mm Hg, absolute, and a flow rate of 28 cubic feet per hour were controlled to maintain the same conditions for both the ionized argon and the heated argon. The velocity above the plate as calculated from flowmeter data was six feet per minute, which corresponds to a Reynolds number of 1.2 at the trailing edge of the plate. Curves of average plate temperature and gas temperature versus time for the ionized argon are shown on Plate VI and the corresponding curves for the heated non-ionized argon are shown on Plate VII.

The following equation[14] was used as a basis for the comparison between the average film coefficient $\bar{h}_c$ obtained during ionized flow and the average film coefficient $\bar{h}_c$ obtained during non-ionized flow.

$$\frac{t - t_i}{t_\infty - t_i} = \operatorname{erfc}\left[\frac{y}{2(\alpha\theta)^{\frac{1}{2}}}\right]$$

$$- \epsilon^{\frac{\bar{h}_c}{K}(\alpha\theta)^{\frac{1}{2}}\left[\frac{y}{(\alpha\theta)^{\frac{1}{2}}} + \frac{\bar{h}_c}{K}(\alpha\theta)^{\frac{1}{2}}\right]} \operatorname{erfc}\left[\frac{y}{2(\alpha\theta)^{\frac{1}{2}}} + \frac{\bar{h}_c}{K}(\alpha\theta)^{\frac{1}{2}}\right] \quad (43)$$

Equation (43) is the solution for the temperature distribution in a semi-infinite plate that is initially at a uniform temperature t_i throughout and suddenly at time $\theta = 0$ comes into contact with a fluid medium at temperature t_∞ with a film coefficient of heat transfer $\bar{h}_c$ present on the surface at $y = 0$. The temperature t in equation (43) is the temperature of the plate at some location y at time θ. The thermal conductivity K and the thermal diffusivity α pertain to the solid. Equation (43) is the solution to the one-dimensional transient heat conduction equation,

$$\frac{\partial^2 t}{\partial y^2} = \frac{1}{\alpha}\frac{\partial t}{\partial \theta} \quad (44)$$

and was obtained by use of the Laplace transformation[14] for the following initial and boundary conditions.

$$t = t_i \quad \text{at} \quad \theta = 0; \quad y \geqslant 0 \quad (45)$$

$$\frac{\partial t}{\partial y} = \frac{\bar{h}_c}{K}(t - t_\infty) \quad \text{at } y = 0;\ \theta > 0 \quad (46)$$

Due to the low thermal diffusivity of the glass plate and the relatively short time period during which data were recorded, equation (43) is the analytical solution for the temperature variation in the plate for the conditions of the experimental test. To make the comparison of the average film coefficient $\overline{h}_c$ obtained from the ionized argon with the film coefficient $\overline{h}_c$ obtained from the heated non-ionized argon, the dimensionless temperatures for equation (43) were evaluated from the curves on Plates VI and VII for both the ionized argon and the non-ionized argon at five minute intervals. Numerical values of $\overline{h}_c$ can be obtained for a given time θ by the solution of equation (43), using the appropriate experimental values of dimensionless temperatures.

By dividing equation (43) evaluated for the ionized gas by equation (43) evaluated for the non-ionized gas at the same time θ, it is seen from the result that the ratio obtained is a mathematical expression in which the film coefficients are the only unknowns. If the film coefficients are equal, then the ratio resulting from division of these dimensionless temperatures is unity. The results of the evaluation of the dimensionless temperatures for the ionized argon were divided by those obtained at corresponding times for the non-ionized argon. The ratios resulting from this comparison were essentially unity, within the limitations of the accuracy of the curves and measurements made with available equipment. Radiation effects between the gas and the plate and between the duct walls and the plate were assumed to be negligible.

From Langmuir probe data taken during the heating of the glass plate for ionized flow, a curve of probe current versus probe voltage is shown on Plate VIII. According to the theory developed by Johnson and Malter[9], the electron temperature corresponding to the curve on Plate VIII was 22,000 degrees Kelvin. The electron or ion concentration as developed from Langmuir's theory[11] was 3.5 x 10^{12} particles per cubic centimeter. These data were used in equation (42), developed from kinetic theory, to obtain a predicted value of the ratio of the heat transfer in the two cases. The result was essentially unity.

The problem of determining the macroscopic plasma temperature warrants some discussion. The plasma temperature as used in this thesis is the temperature of the positive ions and neutral atoms which are assumed to be equal in temperature. The plasma temperature was first measured by using a bare copper-constantan thermocouple junction placed in the center of the plasma. This did not prove satisfactory due to the recombination energy gained by the junction and a reading on the order of 1200 degrees Fahrenheit was obtained. This method of temperature measurement was revised by placing a glass capillary tube around the thermocouple junction, leaving the end of the tube open. This reduced the temperature indication approximately 900 degrees Fahrenheit to about 300 degrees Fahrenheit. However, by visual observation, it appeared that the shielded thermocouple was still attracting recombination energy. Next, two glass shielded thermocouples were placed in the ground potential electrode about ¼ inch from the inner edge. These thermocouples indicated a reading of about 50 degrees Fahrenheit lower than the glass shielded thermocouple in the center of the plasma. Finally, an unshielded thermocouple was placed approximately two inches downstream from the ground electrode; it gave readings 100 degrees Fahrenheit lower than had been indicated by the glass shielded thermocouple located in the center of the plasma. This reading was used as the macroscopic plasma temperature as previously defined.

CONCLUSIONS

At the degree of ionization investigated, the total heat transferred from an ionized gas to a flat plate was essentially the same as that which occurred in a non-ionized gas at the same velocity, temperature and pressure. This conclusion was based on both experimental and theoretical results at low temperatures and was restricted to laminar flow.

From the appearance of a bright and uniform plasma, as shown by the photographs on Plate X, it was expected that there would be a substantial degree of ionization present, hence an increase in heat transfer by recombination at the plate surface. However, from Langmuir probe data, it was determined that the gas was only very slightly ionized, i.e., one part in 10^5. Due to the nature of the ambipolar diffusion process, ion-electron pairs were very slowly transported to the surface of the plate upon which they gave up recombination energy. The glass plate used during the experimental test was essentially non-catalytic, i.e., did not speed up recombination. If a catalytic material had been used for a plate, more recombination energy would have been given up to the plate and a greater heat transfer would have resulted from the ionized gas.

Some specific areas of study pertaining to heat transfer in which there is a need for investigation are:

1. Development of more precise experimental techniques by means of which the effect of lower levels of ionization on heat transfer could be accurately determined.
2. Determination of the effect of different kinds of materials on recombination, i.e., catalytic effect.
3. Devise a method for obtaining a more accurate verification of

macroscopic plasma temperature as previously defined throughout the plasma.

4. Development of higher velocities of flow so that the effect of velocity on heat transfer in a plasma may be studied.

Considering the theory developed in the thesis, more rigorous kinetic theory could be applied to the transport equations developed in the section on Heat Conduction in a Partially Ionized Gas. The elementary kinetic theory used in development of these equations ignored the difference in charge between ions and neutral atoms and between electrons and atoms.

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APPENDIX

EXPLANATION OF PLATE I

Fig. 1. Plasma wind tunnel, radio frequency generator, and vacuum pump.

PLATE I

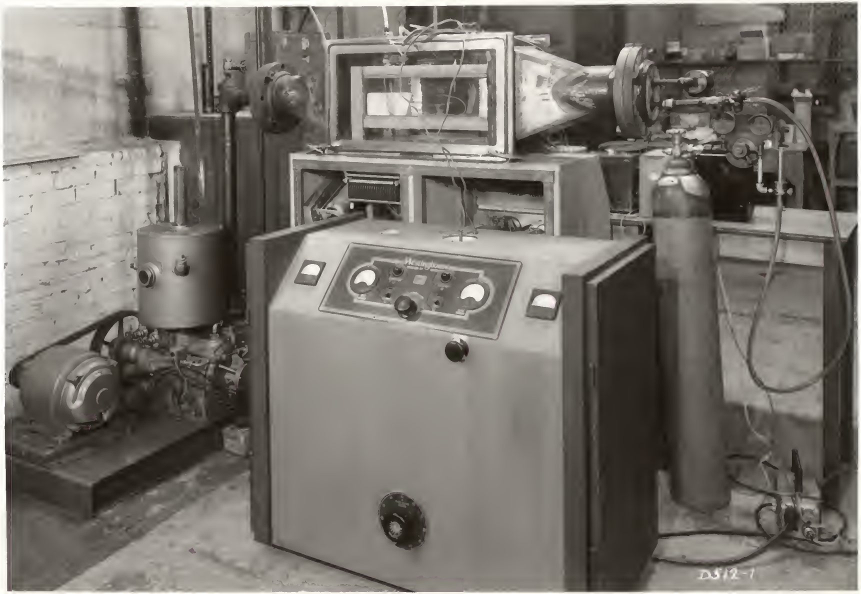

Fig. 1

EXPLANATION OF PLATE II

Fig. 2. Plasma wind tunnel and instrumentation.

PLATE II

Fig. 2

EXPLANATION OF PLATE III

Fig. 3. Test flow schematic.

PLATE III

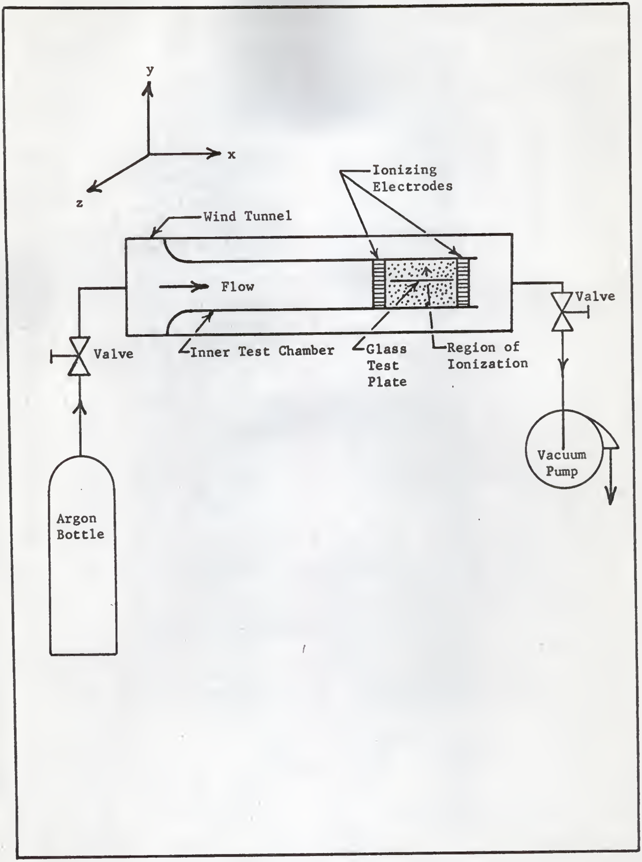

Fig. 3

EXPLANATION OF PLATE IV

Fig. 4. Inner test chamber.

PLATE IV

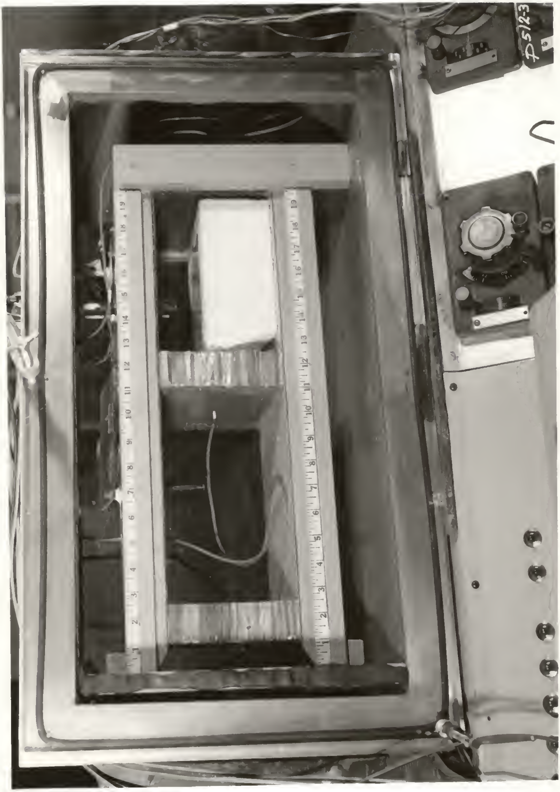

Fig. 4

EXPLANATION OF PLATE V

Fig. 5. Test schematic.

PLATE V

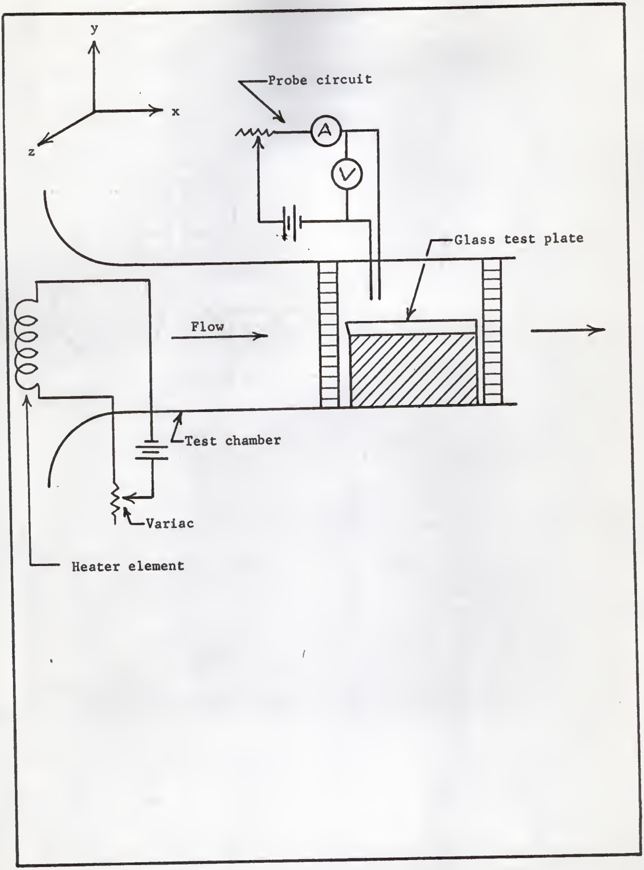
y
x
z
Probe circuit
A
V
Glass test plate
Flow
Test chamber
Variac
Heater element

Fig. 5

EXPLANATION OF PLATE VI

Fig. 6. Plot of plate temperature and gas temperature versus time for a 10.7 megacycle ionizing voltage, a pressure of 0.4 mm Hg, absolute, and a velocity of 6 feet per minute. The gas is argon.

PLATE VI

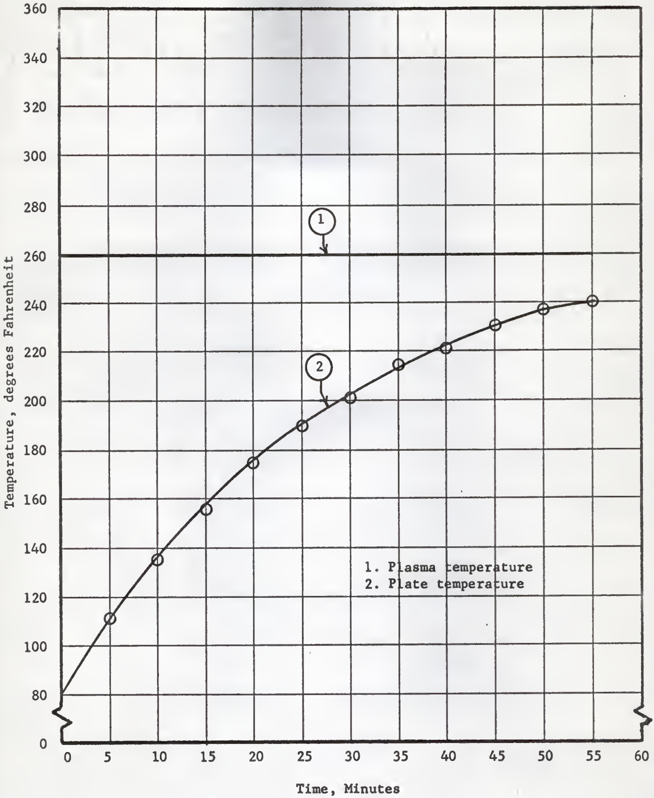

Fig. 6

EXPLANATION OF PLATE VII

Fig. 7. Plot of plate temperature and gas temperature versus time for a pressure of 0.4 mm Hg, absolute, and a velocity of 6 feet per minute. The gas is argon.

PLATE VII

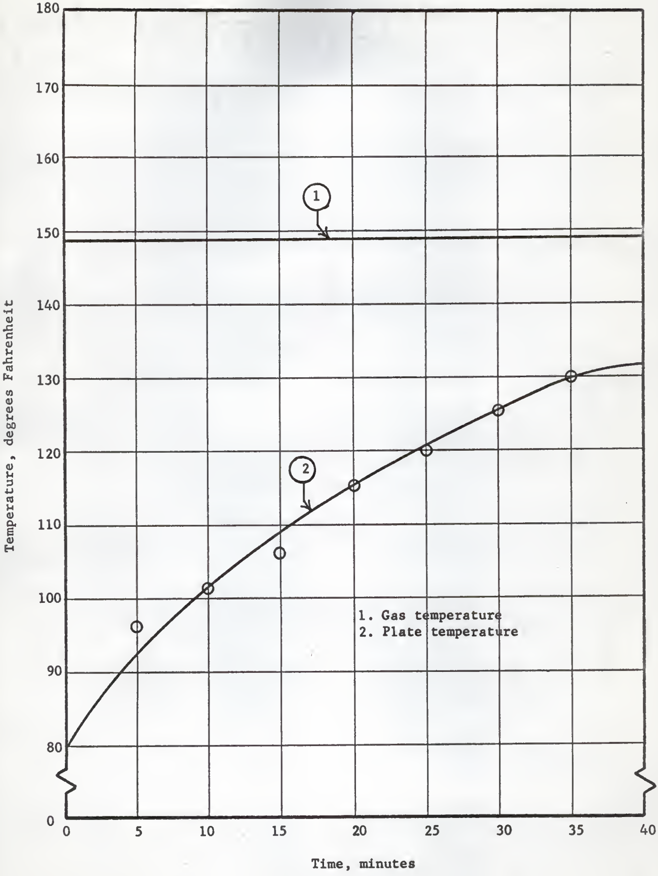

Fig. 7

EXPLANATION OF PLATE VIII

Fig. 8. Plot of probe current versus probe voltage for a 10.7 megacycle ionizing voltage, a pressure of 0.4 mm Hg, absolute, and a velocity of 6 feet per minute. The gas is argon.

PLATE VIII

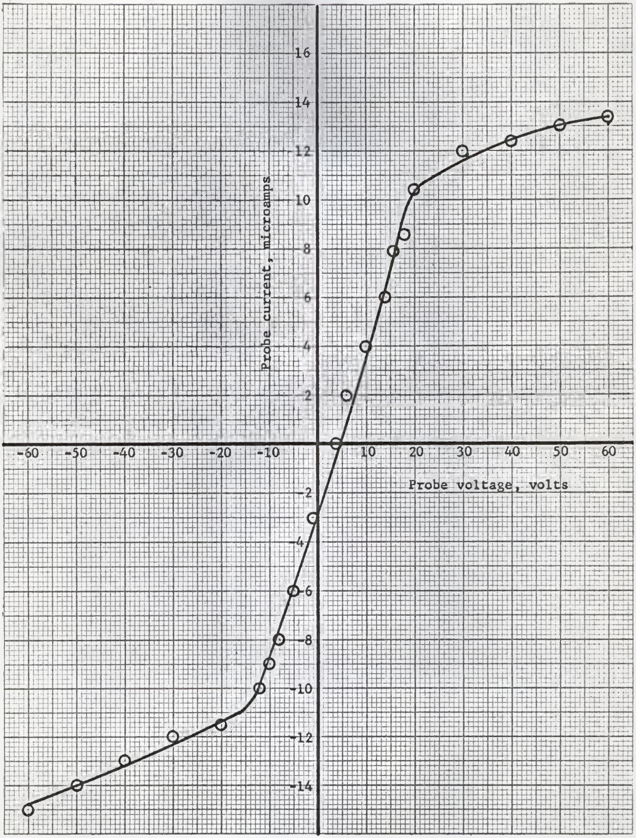

Fig. 8

EXPLANATION OF PLATE IX

Fig. 9. Detail of probe tip. Center to center spacing between tungsten wires = 1.5 mm.

PLATE IX

Fig. 9

EXPLANATION OF PLATE X

Fig. 10. Ionization produced by a 10.7 megacycle voltage at a pressure of .2 mm Hg, absolute. The gas is argon.

Fig. 11. Ionization produced by a 10.7 megacycle voltage at a pressure of 2 mm Hg, absolute. The gas is air.

PLATE X

Fig. 10

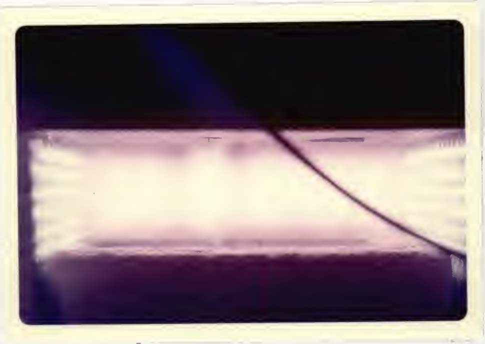

Fig. 11

CONVECTION HEAT TRANSFER BETWEEN A FLAT PLATE
AND A PARTIALLY IONIZED GAS

by

DON ELDEN CROY

B.S., Kansas State University, 1959

AN ABSTRACT OF A
MASTER'S THESIS

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MASTER OF SCIENCE

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1963

Low temperature heat transfer by convection between a flat plate and an ionized, monatomic gas was studied both experimentally and analytically. From kinetic theory, an expression was developed for the ratio of the heat transferred between an ionized gas and a flat plate to the heat transferred between a non-ionized gas and a flat plate at the same velocity, pressure and temperature. An experimental research technique was developed whereby from experimental data the heat transfer film coefficient obtained from the ionized gas was compared to that which occurred in the non-ionized gas. At the degree of ionization investigated, i.e., one part in 10^5, it was determined both from experimental results and from the theory that there was no appreciable difference between the heat transferred from the ionized gas to that which was transferred from the non-ionized gas.

A dual-wire Langmuir probe was used to take data from which the electron temperature and the degree of ionization were obtained. The problem of measuring the macroscopic plasma temperature was discussed. Suggestions for further research in factors relating to heat transfer utilizing existing plasma facilities have been included.