

THE INFLUENCE OF MAGNETIC FIELDS
ON THE ANALYSIS OF SUNSPOT STRUCTURE

by 4589

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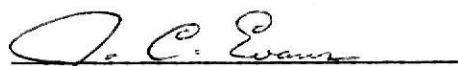
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TABLE OF CONTENTS

I. INTRODUCTION	1
II. THE UMBRAL MODEL	3
III. FURTHER CONFIRMATION OF THE MODEL.	15
IV. THE CALCULATION OF THE CURVES OF GROWTH.	19
V. THE ANALYSIS OF THE LINE DATA.	22
VI. CONCLUSIONS.	53
APPENDICES	
A. THE CALCULATION OF THE MODEL ATMOSPHERE.	54
B. THE CALCULATION OF INTENSITY LINE PROFILES	63
REFERENCES	72
ACKNOWLEDGMENTS.	75

**THIS BOOK
CONTAINS
NUMEROUS PAGES
WITH DIAGRAMS
THAT ARE CROOKED
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INFORMATION ON
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**THIS IS AS
RECEIVED FROM
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I. INTRODUCTION

In attempting to derive the physical conditions in sunspot umbrae by the analysis of equivalent widths or line profiles, the influence of the magnetic field on line formation, and consequently on the results obtained, has not been clearly shown. Several workers have considered the problem but are not in complete agreement as to the magnitude of this effect (see p. 138 in Bray and Loughhead 1964 and p. 85 in Zwaan 1965). The intent of this study was to quantitatively investigate with a somewhat more rigorous theoretical study the error that has been made by neglecting magnetic line formation. In order to accomplish this, I felt that a new semi-empirical umbral model was needed which achieved better agreement with the published continuum data than was possible with the predictions of existing models. By improving the reliability of the model using continuum measurements, one can be certain that the results obtained regarding the influence of the magnetic field are not made suspect by the physical conditions of the model.

Reviews of the physical conditions in sunspots have been made by Bray and Loughhead (1964) and Zwaan (1965) which roughly indicate hydrostatic pressures and a scaling down of the photospheric temperature distribution. Zwaan (1965) has derived one of the better umbral radiation models of the past few years, while more recently Mattig (1969) has obtained a nearly isothermal umbral model based primarily on a suspected limb brightening which only he has verified observationally. A different type of study of the physical conditions in sunspots in which no continuum radiation field is predicted are the magnetohydrostatic models by Deinzer (1965), Yun (1968), and Dicke (1970).

In outlining the major points of this study, Section II describes the

method by which the umbral model was obtained and the criterion used in establishing the validity of the model. Further confirmation for the validity of the model is provided in Section III. The method for calculating curves of growth and the method by which the Zeeman broadening was included in the computations is described in Section IV. The analysis and discussion of the line data is presented in Section V and the conclusions are presented in the final section.

II. THE UMBRAL MODEL

The model computations were made for a given temperature distribution, logarithm of the effective gravity, and chemical composition using a computational scheme due to Elste (1968). The digital computer program I used was written by Elste and Evans (1969). Hydrostatic equilibrium, neglecting magnetic and turbulent pressure, was assumed in the pressure model computation while the ionization equilibrium included contributions from hydrogen and nine element groups. Several elements with similar ionization potentials were considered together by adding their abundances and using the ionization potential and partition functions of the most abundant element in the group. Helium ionization, molecular formation, and the formation of negative ions were not included. Utilizing graphs given by Aller (1963), the assumption of no molecular hydrogen formation was found to be valid in the deeper layers of the umbral atmosphere, but deviated somewhat in the outer layers where the number of hydrogen molecules becomes a fairly large fraction of the total hydrogen content. However this increase was not believed to produce an important effect on the computations. The sources of continuous absorption included were bound-free and free-free transitions in H , H^- , H_2^+ , and the most abundant metals (Elste 1968). (For further details see Appendix A.)

In the derivation of the umbral model, temperature distributions of other authors (Zwaan 1965, Mattig 1969) were reviewed in order to examine the predicted continuum intensity ratios (spot to photosphere) as a function of wavelength for an LTE continuum formation mechanism. In checking the predicted values with available observational data, certain modifications seemed in order. In this investigation, the temperature distribution in the photosphere

due to Elste (1967) was scaled down until it resembled the one in Zwaan's model. It was then altered graphically in order to produce the best fit to the continuum and hydrogen line measurements. In Figure 1, the temperature distributions in the range $-4.0 \leq \log \tau_0 \leq +1.0$ (continuum optical depth at $5000 \text{ \AA}$) obtained by Zwaan (1965), Mattig (1969), and the one proposed in this investigation are shown. The proposed temperature stratification shows a smaller gradient than Zwaan's in the continuum formation region and larger than that proposed by Mattig. The temperature distribution agrees reasonably well with the one recently published by Henoux (1968). The model was derived for the logarithm of the effective gravity of 4.44 and the logarithm of the number ratio of helium nuclei to hydrogen nuclei of 0.125, and yielded an effective temperature of 4060° K . The details of the model are given in Table 1, where the first column is the logarithm of the continuum optical depth at $5000 \text{ \AA}$; the second column is $\theta = 5040/T$; the third, fourth, and fifth columns are the logarithms of the electron pressure, gas pressure, and density, respectively; the sixth column is the logarithm of the total absorption coefficient per hydrogen particle per unit electron pressure at $5000 \text{ \AA}$; and the seventh column is the geometrical depth in kilometers (see Appendix A).

Figure 2 shows the continuum intensity ratio of the umbra to the photosphere at the center of the disk as a function of wavelength for the three models as compared to observation. In the computations, the photospheric intensity from Elste's Solar Model 10 (see Table 2 and Elste 1967) was used since the temperature distribution recently proposed by Gingerich (1969) agrees well with Elste's determination. The observations of large spots are

EXPLANATION OF PLATE I

Fig. 1. - The temperature distribution ($\theta = 5040/T$) of the three models as a function of the continuum optical depth at 5000 Å.

PLATE I

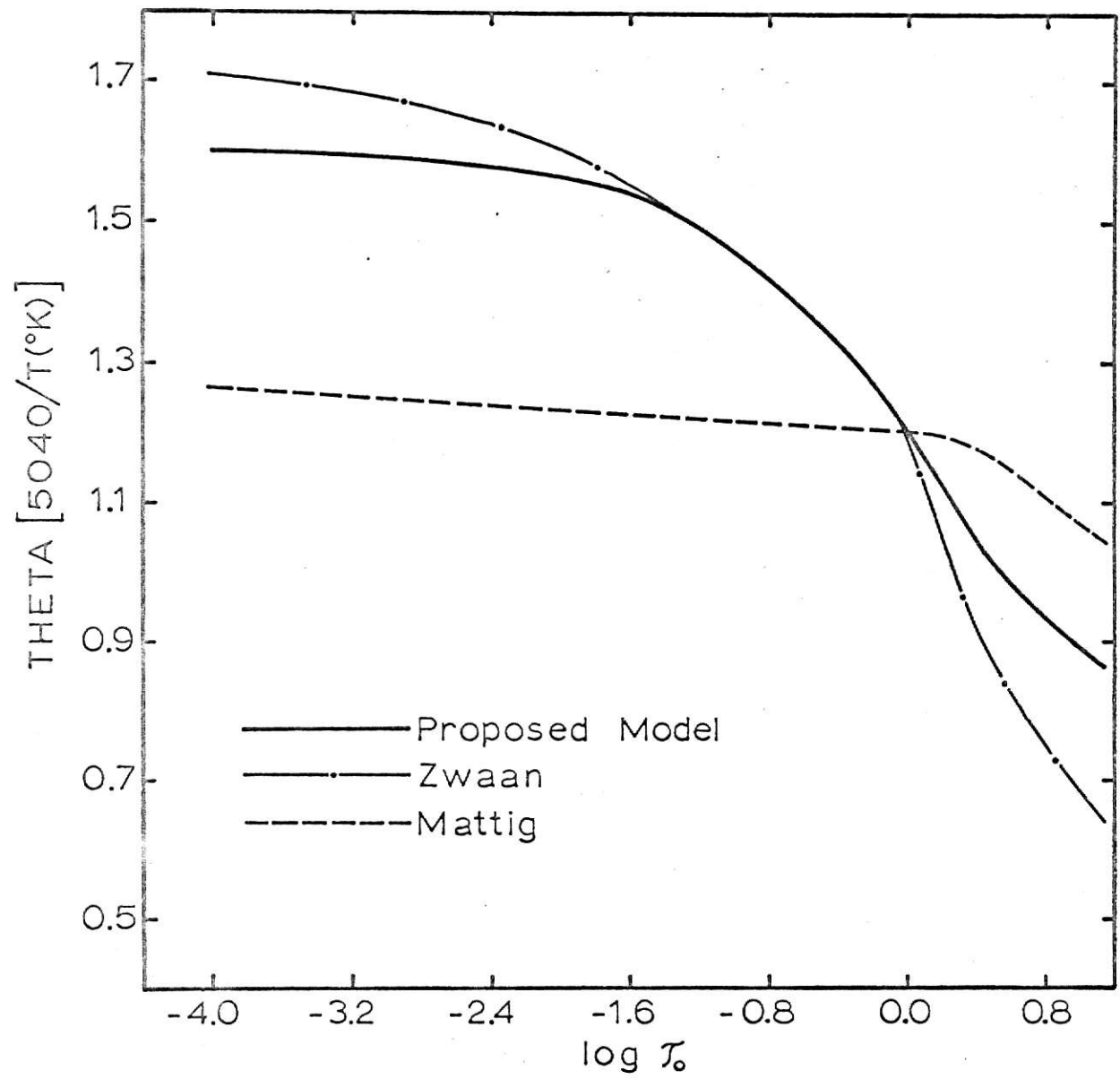


TABLE 1
UMBRAL MODEL

$\log \tau_0$	θ	$\log P_e$	$\log P_g$	$\log p$	$\log \kappa_0 / P_e$	h
-4.00	1.599	-2.04	3.40	-7.89	-24.14	-196
-3.80	1.599	-2.04	3.40	-7.89	-24.14	-174
-3.60	1.598	-1.94	3.53	-7.75	-24.15	-153
-3.40	1.597	-1.84	3.66	-7.63	-24.17	-132
-3.20	1.596	-1.74	3.78	-7.51	-24.18	-112
-3.00	1.594	-1.64	3.90	-7.39	-24.19	-93
-2.80	1.591	-1.54	4.02	-7.27	-24.20	-74
-2.60	1.587	-1.44	4.13	-7.16	-24.21	-55
-2.40	1.582	-1.33	4.25	-7.04	-24.22	-37
-2.20	1.576	-1.23	4.36	-6.93	-24.23	-18
-2.00	1.569	-1.13	4.47	-6.82	-24.25	0
-1.80	1.558	-1.02	4.58	-6.71	-24.27	18
-1.60	1.543	-0.91	4.70	-6.61	-24.29	37
-1.40	1.521	-0.79	4.81	-6.50	-24.32	56
-1.20	1.493	-0.66	4.92	-6.40	-24.36	75
-1.00	1.460	-0.52	5.03	-6.29	-24.41	94
-0.80	1.420	-0.36	5.14	-6.19	-24.47	114
-0.60	1.375	-0.19	5.25	-6.10	-24.54	134
-0.40	1.332	-0.01	5.36	-6.01	-24.61	154
-0.20	1.275	0.21	5.45	-5.93	-24.70	174
0.00	1.200	0.50	5.54	-5.87	-24.82	192
0.20	1.125	0.78	5.62	-5.81	-24.94	210
0.40	1.043	1.07	5.70	-5.77	-25.08	228
0.60	0.977	1.29	5.77	-5.73	-25.19	246
0.80	0.928	1.48	5.85	-5.67	-25.28	267
1.00	0.887	1.68	5.93	-5.61	-25.36	290
1.20	0.850	1.85	6.00	-5.56	-25.42	312

EXPLANATION OF PLATE II

Fig. 2. - The predicted continuum intensity ratio (spot to photosphere) as a function of wavelength for the three models as compared to observation.

PLATE II

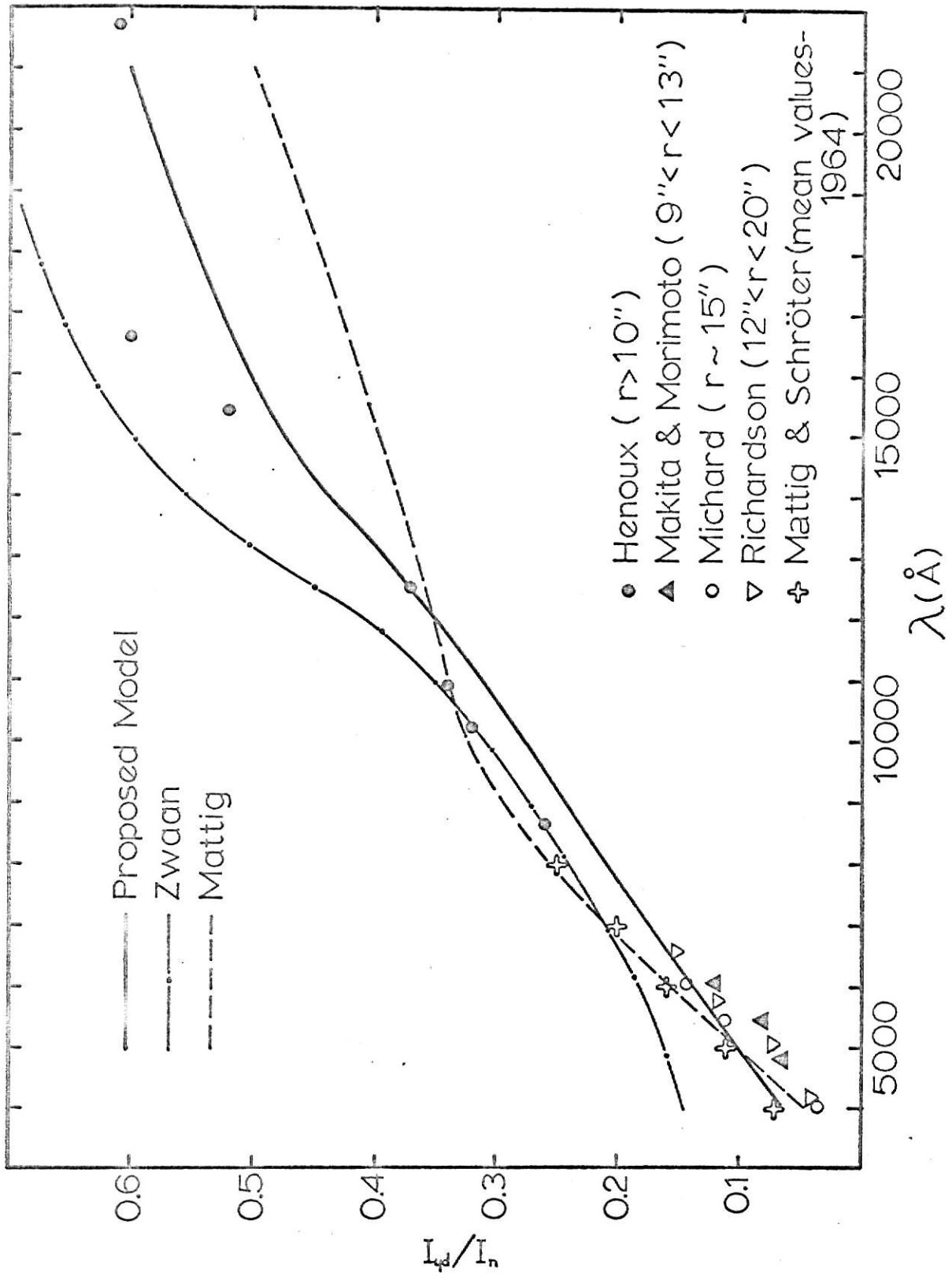


TABLE 2
ELSTE MODEL 10

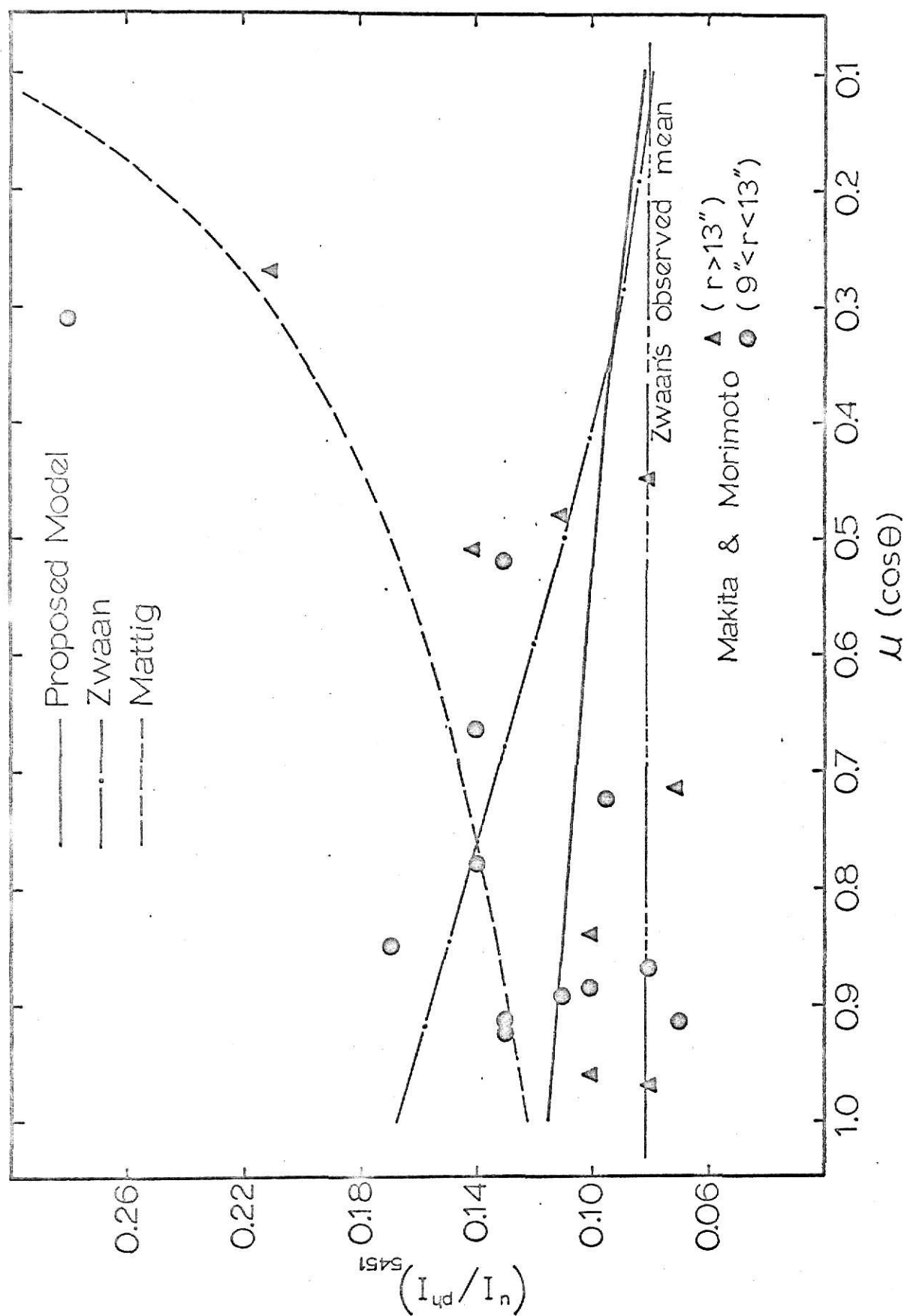
$\log \tau_0$	θ	$\log P_e$	$\log P_g$	$\log \rho$	$\log \kappa_0 / P_e$	h
-4.00	1.143	-1.14	3.16	-8.25	-24.83	-248
-3.80	1.143	-1.09	3.22	-8.18	-24.84	-222
-3.60	1.141	-0.98	3.34	-8.06	-24.85	-197
-3.40	1.140	-0.88	3.46	-7.95	-24.86	-172
-3.20	1.138	-0.79	3.58	-7.83	-24.87	-147
-3.00	1.135	-0.68	3.69	-7.72	-24.88	-123
-2.80	1.131	-0.58	3.80	-7.61	-24.89	-98
-2.60	1.125	-0.48	3.92	-7.50	-24.91	-74
-2.40	1.118	-0.37	4.03	-7.39	-24.92	-49
-2.20	1.107	-0.26	4.14	-7.28	-24.94	-25
-2.00	1.093	-0.15	4.25	-7.17	-24.97	0
-1.80	1.076	-0.02	4.36	-7.07	-25.00	25
-1.60	1.056	0.10	4.48	-6.97	-25.04	50
-1.40	1.034	0.24	4.59	-6.86	-25.08	76
-1.20	1.009	0.38	4.69	-6.77	-25.12	102
-1.00	0.982	0.53	4.80	-6.67	-25.17	128
-0.80	0.952	0.70	4.91	-6.58	-25.23	155
-0.60	0.918	0.89	5.00	-6.50	-25.29	180
-0.40	0.879	1.14	5.09	-6.43	-25.36	204
-0.20	0.834	1.45	5.17	-6.38	-25.44	225
0.00	0.784	1.81	5.22	-6.35	-25.54	242
0.20	0.730	2.22	5.26	-6.34	-25.65	255
0.40	0.675	2.64	5.29	-6.34	-25.75	266
0.60	0.633	2.97	5.32	-6.35	-25.83	274
0.80	0.603	3.20	5.34	-6.35	-25.87	282
1.00	0.584	3.36	5.36	-6.34	-25.90	291
1.20	0.572	3.46	5.37	-6.34	-25.91	300

taken from several sources and are summarized in Figure 2 (see Zwaan 1965), along with the umbral radius in seconds of arc (1 arc second = 725 km). The measurements by Michard (1953) and Richardson (1939), although initially corrected for scattered light (see Section V), were again corrected by Zwaan which brought them into fair agreement with the relative intensities measured by Makita and Morimoto (1960). The scattered light was undetectable in the measurements by Makita and Morimoto, but they do contain a correction for stray light due to blurring. Henoux's measurements also contain a correction for scattered light, while Mattig and Schröter (1964) have obtained mean values of other authors (Pettit and Nicholson 1930, Krat 1947, Michard 1953, Stepanov 1957, Makita and Morimoto 1960, Stumpff 1961, and Stankiewicz 1962) and normalized them to a value of $u_I/p^h_I = 0.16$ at $6000 \text{ \AA}$. Compared to our selected photospheric model, Zwaan's temperature distribution predicts too high a ratio in the visible, and Mattig's too low in the infrared. More consideration should be placed in obtaining a fit to the infrared data rather than the visible since the umbral spectrum is less vitiated by stray light with increasing wavelength. In addition the mean values of Mattig and Schröter include smaller spots in which the stray light would affect the intensity measurements greatly.

In Figure 3 the center-to-limb variation of the intensity ratios are compared for the three temperature distributions with observations at a continuum wavelength of $\lambda 5451$. The observed points are taken from Makita and Morimoto (1960) for which Zwaan suggested a mean value of 0.08 as shown, based on the assumption that systematic errors are chiefly caused by stray light. The measurements include only relatively large spots ($9'' < r < 13''$). If there is a rise in the observed ratio towards the limb then neither Zwaan's nor the

EXPLANATION OF PLATE III

Fig. 3. - The center-to-limb variation of the continuum intensity ratio (spot to photosphere) at a wavelength of $5451 \text{ \AA}$ for the three models as compared to observation.



proposed model agrees well. However the abrupt increase may be explained by the increase of stray light because of the foreshortening of the spots as they approach the limb. Furthermore, there is nothing to suggest that the data indicates a trend of brightening towards the limb. At least for $\cos\theta > 0.4$ the relative intensities show no clear center-to-limb effect.

III. FURTHER CONFIRMATION OF THE MODEL

As a further indication as to the validity of the model obtained in Section II, theoretical hydrogen line profiles for H_{α} and H_{β} for the three different models (see Section II) are contrasted with the observed profiles by Fricke and Elsasser (1965) in Figure 4. The theoretical line absorption coefficient is based on the work of Griem, Kolb, and Shen (1959) (see Griem 1964). As is evident from the graph the observed profiles, which have been corrected for photospheric scattered light, lie between the proposed model and Zwaan's while Mattig's temperature distribution predicts virtually no hydrogen lines. Assuming that the Balmer lines in the umbral spectrum show no wing formation as Fricke and Elsasser (1965) have claimed, the proposed model gives better agreement with the measurements. In contrast to the models of Zwaan and Mattig, the proposed umbral model clearly presents an improved picture of the excitation conditions in the umbra.

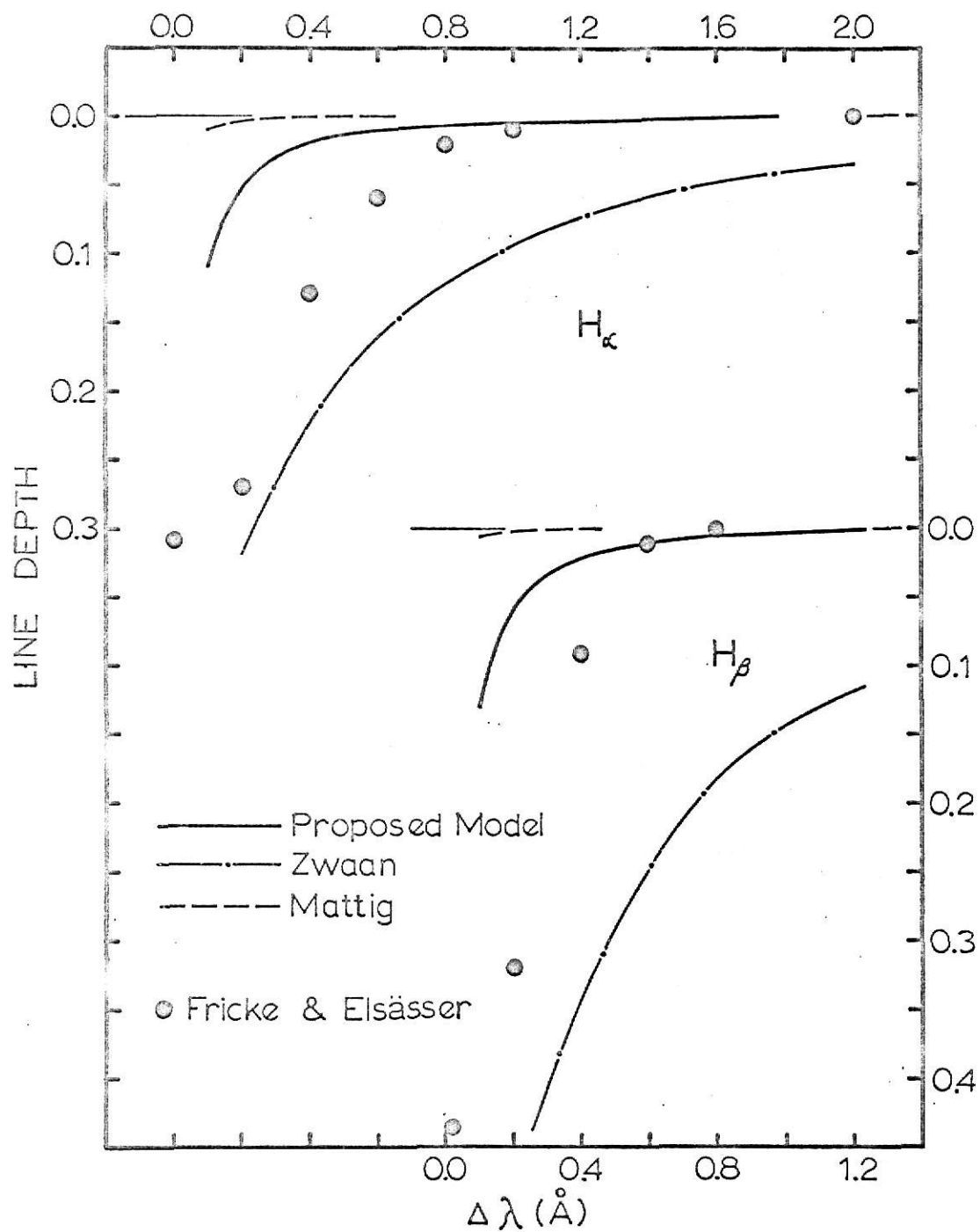
b) The Wilson Depression

As a sunspot approaches the limb side of the sun, the more rapid decrease in the disk side penumbral width as opposed to the slower decrease in the disk side penumbral width is known as the Wilson effect (Bray and Loughhead 1964). This has been interpreted as indicating that at any geometrical depth the umbra is more transparent than the photosphere. The difference in geometrical depth between photospheric and umbral layers at the same monochromatic optical depth is referred to as the Wilson depression (Zwaan 1965). In order to estimate this difference, I used the procedure suggested by Zwaan and dis-

EXPLANATION OF PLATE IV

Fig. 4. - The theoretical H_{α} and H_{β} line profiles for the three models of interest compared to the observed profiles by Fricke and Elsässer. The observed profiles have been corrected for stray light.

PLATE IV



cussed in the next paragraph. The large magnetic field of a sunspot, if not force-free (see p. 275 Bray and Loughhead 1964), contributes an additional pressure term, which when added to the umbral kinetic pressure produces a horizontal equilibrium with the photospheric kinetic pressure. If the spot is force-free, either some other mechanism provides for the horizontal pressure equilibrium, or the umbral kinetic pressures exceed hydrostatic pressures.

Assuming the spot to be cylindrical, with an axially symmetric magnetic field, orientated parallel to the solar radius, and ignoring any contribution from a curvature of the field lines in the outer layers of the umbra, a magnetic pressure ($H^2/8\pi$) was computed for reasonable values of the magnetic field (2000 to 5000 G). For umbral optical depths at $5000 \text{ \AA}$ of 0.01, 0.1, 1, and 10, the sum of the magnetic and kinetic pressure was equated to the photospheric pressure which was found to lie at a larger optical depth in the undisturbed photosphere. The Wilson depression was thus estimated as the difference in the geometrical depth between umbral and photospheric models for the correlation in optical depths obtained from the pressure equality. The expected Wilson depressions, given in Table 3, are in good agreement with the values of 400 to 800 km suggested by Zwaan (1968).

TABLE 3
THE PREDICTED WILSON DEPRESSION

Field Strength (Gauss)	Wilson Depression (km)
2000	280
2500	330
3000	390
3500	450
4000	520
4500	590
5000	670

IV. THE CALCULATION OF THE CURVES OF GROWTH

The curves of growth used in the analysis are similar to those defined by Aller, Elste, and Jugaku (1957) which is a plot of the equivalent width W against the equivalent width W^* assuming the line can not saturate (i.e., a weak line for any abundance of absorbing particles). For the ordinate of the theoretical curve, the equivalent width is obtained by integrating over a theoretical line profile while the common abscissa for both the theoretical and empirical curve is the unsaturated equivalent width given by

$$\log \left(\frac{W}{\lambda} \right)^* = \log \epsilon + \log C_\lambda \quad (1)$$

where

$$\log C_\lambda = \log gf\lambda + \log L_\lambda \quad (2)$$

where L_λ is a slightly modified version of the quantity defined by Aller, Elste, and Jugaku and depends on the atomic configuration, spectral range, intensity weight functions, and the excitation conditions in the atmosphere.

The line profile calculations follow a scheme similar to the one suggested by Gussman (1967) including the usual line broadening mechanisms of random thermal motions, radiation damping, quadratic Stark effect due to electrons and ions, and van der Waals interactions with neutral hydrogen and helium (see Unsöld 1955). The damping parameters are not critical to the analysis since the main concern of the analysis was with weak and medium strong lines. Therefore, a classical radiation damping constant and depth-dependent Stark and van der Waals values determined from a Lindholm relation were used in the

computations.

The inclusion of Zeeman broadening in the line computations uses a modification of the work by Stepanov (1958a, 1958b) due to Evans (1969). The intensity is divided equally between two beams of opposite rotary polarization, denoted by + and -, and these are distinguished by relative shifts in their absorption maxima. The line depth r in the total radiation at a distance $\Delta\lambda$ from the line center and at a limb distance μ is given by

$$r(\Delta\lambda, \mu) = \int_{-\infty}^{+\infty} D_{\lambda}(x, \mu) E_m(x, \Delta\lambda, \mu) dx, \quad (3)$$

$$\text{where } D_{\lambda} = \frac{1}{I_{\lambda}(0, \mu)} \left(\frac{dB_{\lambda}}{dx} \right) e^{-\tau_{\lambda}/\mu}, \quad (4)$$

$$E_m = 1 - \frac{1}{2} \left(e^{-\tau_1^+/\mu} + e^{-\tau_1^-/\mu} \right), \quad (5)$$

$\tau_1^{\pm}$ = the line optical depth for the beams of opposite rotary polarization,

τ_{λ} = the continuum optical depth,

where the integration variable x is the logarithm of the optical depth at 5000 Å. In order to calculate the line optical depth for a transition with i (violet V or red R) σ -components and j π -components, the line absorption coefficient per hydrogen particle is needed, which is given by the expression

$$K_1^{\pm} = \left(1 - e^{-hc/\lambda kT} \right) \left(\frac{N}{N_H} \right) \frac{\sqrt{\pi} e^2}{m c^2} \frac{\lambda^2}{\Delta\lambda_D} f \psi_{\pm}, \quad (6)$$

$$\text{where } \psi_{\pm} = \frac{1}{2} (H_V + H_R) + \frac{1}{4} (2H_{\pi} - H_V - H_R) \sin^2 \gamma \pm \frac{1}{2} \sqrt{\frac{1}{4} (2H_{\pi} - H_V - H_R)^2 \sin^2 \gamma + (H_V - H_R)^2 \cos^2 \gamma}, \quad (7)$$

$$H_V = \sum_i x H(a, v + v_m),$$

$$H_{\pi} = \sum_j y H(a, v + v_m),$$

$$H_R = \sum_i z H(a, v + v_m),$$

where each displaced Zeeman level will be broadened by the pressure and Doppler broadening mechanisms. Thus H is the Voigt function for each Zeeman component; x, y, z are the relative intensities of each component; v_m is the displacement due to the magnetic field in Doppler widths; and γ is the angle between the field lines and the line of sight. The other quantities have their usual meanings (see Appendix B).

V. THE ANALYSIS OF THE LINE DATA

In order to investigate the influence of the magnetic broadening on the analysis of umbral structure, ideally one would like to have line profiles. However, very little if any profile data for sunspots exists in the literature. Therefore in this study, I restricted the question to the influence of Zeeman splitting on the use of equivalent widths in deriving a radiation model for sunspots.

Of primary concern in obtaining umbral intensities for equivalent widths is the removal of stray light. This is light observed at one point of the image which does not originate from the corresponding point of the object. Processes causing the light to stray up to about twenty seconds of arc are a combined atmospheric and instrumental spread function referred to as blurring, while light is deviated over about one minute of arc or more by optical scattering processes in the atmosphere and observing instrument (Bray and Loughhead 1964, Zwaan 1965). Different procedures are required to correct for both components of the stray light which are discussed in the references cited.

Zwaan points out that it is very difficult to correct observed umbral intensities in small to medium size sunspots for the blurring effect. Thus only in large sunspots is the stray light correction made with some amount of confidence. By and large, however, observed equivalent widths are not corrected for stray light because of the uncertainties involved in the correction procedures (Zwaan 1968).

b) The Observed Equivalent Widths

The observed equivalent widths used in this investigation were taken from four sources: (i) ten Bruggencate and von Klüber (1939, 1944); (ii) Zwaan (1959); (iii) Kornilov (1961); (iv) Fricke and Elsässer (1965). Only the measured equivalent widths by Fricke and Elsässer have been corrected for stray light.

The equivalent widths of medium-strong and strong lines of Ti I, Ti II, and Fe I measured by ten Bruggencate and von Klüber (1939, 1944) were obtained from medium-large umbrae (radius ≈ 10 seconds of arc) and one large umbra ($r \approx 14''$). The authors tabulated mean values of W derived from the various spectra.

Zwaan (1959) measured equivalent widths of Ti I and Fe I which were obtained from one very large umbra ($r \approx 17''$). Very good agreement exists between Zwaan's measurements and those by ten Bruggencate and von Klüber. In neither case, however, were corrections applied for stray light.

Equivalent widths for Ti I, Ti II, Fe I, and Fe II were measured by Kornilov (1961) in the wavelength region $\lambda\lambda 4230-4580$. The umbral lines of Ti II and Fe II were not used because they appear to be stronger than the corresponding photospheric lines. Moreover the intensity of the umbral spectrum is low in this region of the spectrum, indicating Kornilov's results will be sensitive to the stray light (see Zwaan 1965). Only the Ti I lines were included in the curve of growth analysis although they do exhibit some scatter. The Fe I data was also discarded since it shows larger scatter than Zwaan's and ten Bruggencate and von Klüber's measurements.

Fricke and Elsässer (1965) measured equivalent widths of 148 lines of

Fe I, Fe II, Ti I, and Ti II which were obtained from a large umbra ($r \approx 13''$). The amount of scattered light was derived from the wings of the Balmer lines. The scattering was attributed to photospheric light in the wings of the lines since the authors' computations indicated no wings should be formed for the Balmer lines in the umbra.

The lines of Fe I, Fe II, Ti I, and Ti II used in this investigation from these sources are shown in Tables 4, 5, 6, and 7 where column (1) gives the multiplet number, column (2) the wavelength in Å, column (3) the excitation potential of the lower level in eV, column (4) the logarithm of the gf-value, column (5) the abscissa of the empirical curve of growth, and the remaining columns give the logarithm of the observed equivalent width for the various authors as indicated.

c) Neutral Iron

One means of estimating the influence of stray light and differences in umbral and atmospheric excitation conditions is to correlate with excitation potential the observed ratio of umbral to photospheric equivalent width (Zwaan 1965). In Figures 5 and 6, the observed intensifications in the form $\log(u_W/p_W)$ for the Fe I lines are plotted for two different sets of observations. The data by Zwaan (1959) and ten Bruggencate and von Klüber (1939) is plotted on one graph because of the good agreement between their measurements. Fricke and Elsässers' (1965) data is plotted separately since they made a correction for scattered light.

The photospheric equivalent widths were taken from, The Solar Spectrum 2935 Å to 8770 Å, by Moore, Minnaert, and Houtgast (1966). The line marked

TABLE 4
THE MEASURED EQUIVALENT WIDTHS OF Fe I LINES IN SUNSPOTS

Multiplet Number	λ	E.P.	$\log gf^*$	$\log C_\lambda$	$\log (W/\lambda)^+$			
	(Å)	(eV)		($\mu=1$)	(Z)	(BK)	(FE)	(Ph)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15	5269.54	0.86	-1.35	4.08	-3.80	-3.84	-3.80	-4.04
	5328.04	0.91	-1.46	3.97	-	-3.86	-3.76	-4.15
	5371.49	0.95	-1.60	3.83	-	-3.92	-3.76	-4.26
	5405.78	0.99	-1.75	3.68	-	-4.02	-3.81	-4.31
	5434.53	1.01	-1.97	3.47	-4.09	-4.13	-3.91	-4.47
	5397.13	0.91	-1.88	3.55	-	-4.04	-3.75	-4.35
	5429.70	0.95	-1.78	3.65	-3.91	-	-	-4.28
	5446.92	0.99	-1.90	3.54	-	-4.04	-3.86	-4.36
	5455.61	1.01	-2.03	3.41	-	-4.08	-4.04	-4.40
	5501.47	0.95	-2.66	2.78	-	-	-4.18	-4.68
	5506.78	0.99	-2.44	3.00	-	-	-4.12	-4.66
	5497.52	1.01	-2.49	2.95	-	-	-4.09	-4.63
553	5324.19	3.20	0.47	2.94	-	-4.17	-4.10	-4.20
	5283.63	3.23	0.20	2.67	-	-4.34	-4.15	-4.40
	5263.31	3.25	-0.19	2.28	-	-4.50	-4.24	-4.64
	5253.50	3.27	-0.80	1.67	-4.77	-4.68	-4.51	-4.85
	5217.40	3.20	-0.37	2.11	-4.60	-4.54	-4.33	-4.71
	5215.19	3.25	-0.17	2.31	-4.52	-	-4.26	-4.65
	5229.86	3.27	-0.19	2.29	-	-	-4.41	-4.63
	5393.17	3.23	-0.10	2.37	-	-4.46	-4.41	-4.55
	5339.94	3.25	-0.11	2.36	-	-4.44	-4.22	-4.52
	5302.31	3.27	-0.04	2.43	-4.48	-4.46	-4.33	-4.53
	5273.18	3.28	-0.27	2.20	-4.57	-4.56	-	-4.71
1087	5638.27	4.20	0.02	1.22	-	-	-4.70	-4.88
	5641.46	4.24	-0.13	1.07	-	-	-4.79	-4.95
	5662.52	4.16	0.39	1.59	-	-	-4.67	-4.79
	5731.77	4.24	-0.09	1.10	-	-	-4.67	-4.99
	5775.09	4.20	-0.16	1.03	-	-	-4.89	-5.08
	5873.21	4.24	-0.54	0.64	-	-	-5.21	-5.51

* Logarithm of f-value from Corliss and Tech (1968)

+ Logarithm of measured spot equivalent width column (6) by Zwaan (1959), column (7) by ten Bruggencate and von Klüber (1939), column (8) by Fricke and Elsässer (1965), column (9) photospheric value by Moore, Minnaert, and Houtgast (1966).

TABLE 5
THE MEASURED EQUIVALENT WIDTHS OF Fe II LINES IN SUNSPOTS

Multiplet Number	λ	E.P.	$\log gf^*$	$\log C_\lambda$	$\log (W/\lambda)^+$	
	(Å)	(eV)		($\mu = 1$)	(FE)	(Ph)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
27	4128.73	2.57	-2.76	-0.16	-5.57	-4.92
	4416.82	2.77	-2.09	0.25	-4.90	-4.76
28	4178.86	2.57	-2.00	0.60	-5.06	-4.72
37	4491.40	2.84	-2.09	0.13	-5.24	-4.83
	4515.34	2.83	-1.91	0.32	-5.04	-4.78
	4520.23	2.79	-1.87	0.41	-4.88	-4.82
	4555.89	2.82	-1.79	0.44	-4.92	-4.77
38	4508.28	2.84	-1.76	0.46	-4.99	-4.78
	4576.33	2.83	-2.22	0.00	-5.10	-4.91
	4583.83	2.79	-1.25	1.01	-4.52	-4.57
	4620.51	2.82	-2.63	-0.41	-5.55	-4.99
42	4923.92	2.88	-0.93	1.12	-4.63	-4.47
	5018.43	2.88	-0.86	1.20	-4.52	-4.38
43	4656.97	2.88	-2.53	-0.38	-5.82	-5.24
	4731.44	2.88	-2.26	-0.17	-4.79	-4.78

* Logarithm of f-value from Warner (1967).

+ Logarithm of measured spot equivalent width column (6) by Fricke and Elsässer (1965), column (7) photospheric value by Moore, Minnaert, and Houtgast (1966).

TABLE 6
THE MEASURED EQUIVALENT WIDTHS OF Ti I LINES IN SUNSPOTS

Multiplet Number	λ (Å)	E.P. (eV)	$\log gf^*$	$\log C_\lambda$ ($\mu = 1$)	$\log (W/\lambda)^+$			
					(Z)	(BK)	(K)	(Ph)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
37	5238.56	0.84	-1.54	3.92	-5.01	-4.86	-	-5.54
	5246.57	0.83	-1.79	3.67	-5.15	-	-	-6.12
	5250.95	0.82	-1.95	3.51	-5.22	-	-	-5.54
	5251.49	0.81	-2.18	3.28	-5.31	-	-	-6.32
	5248.40	0.81	-2.48	2.98	-5.88	-	-	-5.98
	5211.22	0.83	-2.45	3.01	-5.29	-	-	-5.81
	5224.14	0.82	-2.30	3.16	-5.42	-	-	-6.72
	5239.94	0.81	-2.68	2.78	-5.64	-	-	-6.24
42	4533.24	0.84	0.64	6.10	-	-	-4.44	-4.70
	4534.78	0.83	0.41	5.87	-	-	-4.48	-4.75
	4535.57	0.82	0.22	5.68	-	-	-4.39	-4.76
	4535.92	0.81	-0.09	5.37	-	-	-4.39	-4.49
	4555.49	0.84	-0.28	5.18	-	-4.54	-	-4.93
	4552.45	0.83	-0.17	5.29	-	-	-4.44	-4.62
	4548.76	0.82	-0.18	5.28	-	-4.51	-	-4.86
	4544.69	0.81	-0.31	5.21	-	-	-4.46	-4.67
	4512.73	0.83	-0.25	5.20	-	-4.54	-4.63	-4.91
	4518.02	0.82	-0.14	5.31	-	-4.43	-4.63	-4.86
	4522.80	0.81	-0.15	5.31	-	-4.41	-4.64	-4.84
	4527.31	0.81	-0.28	5.18	-	-	-4.52	-4.83
44	4305.91	0.84	0.72	6.17	-	-	-4.25	-4.44
	4301.09	0.83	0.54	5.99	-	-	-4.20	-4.38
	4300.57	0.82	0.38	5.83	-	-	-4.17	-4.53
	4298.66	0.81	0.22	5.67	-	-	-4.35	-4.75
	4295.75	0.81	-0.17	5.28	-	-	-4.47	-4.89
	4287.41	0.83	-0.14	5.31	-	-	-4.55	-4.88
	4286.01	0.82	-0.12	5.33	-	-	-4.37	-4.56
	4289.07	0.81	-0.10	5.35	-	-	-4.38	-4.77
	4290.93	0.81	-0.16	5.29	-	-	-4.20	-4.50
	4272.44	0.82	-1.14	4.31	-	-	-4.74	-5.63
	4281.37	0.81	-1.05	4.40	-	-	-4.88	-5.33

* Logarithm of f-value from Corliss and Bozman (1962).

+ Logarithm of measured spot equivalent width column (6) by Zwaan (1959), column (7) by ten Bruggencate and von Klüber (1944), column (8) by Kornilov (1961), column (9) photospheric value by Moore, Minnaert, and Houtgast (1966).

TABLE 7
THE MEASURED EQUIVALENT WIDTHS OF Ti II LINES IN SUNSPOTS

Multiplet Number	λ ($\text{\AA}$)	E.P. (eV)	$\log gf^*$	$\log C_\lambda$ ($\mu = 1$)	$\log (W/\lambda)^+$	
					(FE)	(Ph)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
18	4493.53	1.08	-1.92	2.89	-5.35	-5.24
19	4443.80	1.08	-0.74	4.07	-4.44	-4.55
20	4344.29	1.08	-1.67	3.16	-4.64	-4.76
30	4545.14	1.13	-1.61	3.13	-4.94	-5.03
31	4444.56	1.11	-1.86	2.92	-4.93	-4.95
	4468.49	1.13	-0.65	4.10	-4.59	-4.57
	4501.27	1.11	-0.79	3.98	-4.52	-4.61
40	4417.72	1.16	-1.18	3.53	-4.76	-4.66
	4464.46	1.16	-1.66	3.05	-4.71	-4.82
	4470.86	1.16	-1.80	2.91	-4.84	-4.92
41	4320.96	1.16	-2.27	2.46	-4.94	-4.84
50	4563.76	1.22	-0.86	3.76	-4.49	-4.58
51	4394.06	1.22	-1.47	3.17	-4.76	-4.79
	4399.77	1.23	-1.06	3.55	-4.65	-4.58
60	4544.01	1.24	-2.08	2.52	-5.07	-5.11
	4568.31	1.22	-1.93	2.69	-5.17	-5.26
61	4395.85	1.24	-1.53	3.08	-4.79	-4.86
	4409.52	1.23	-2.07	2.56	-4.94	-5.13
	4411.94	1.22	-2.11	2.53	-4.89	-4.90

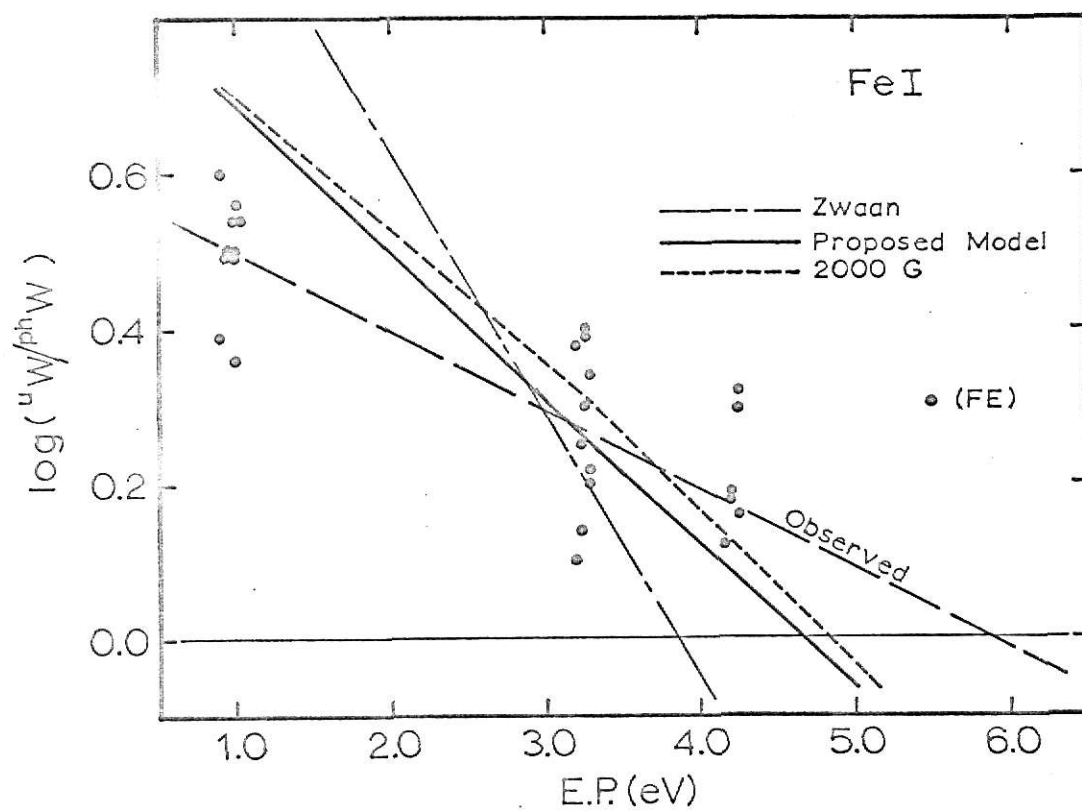
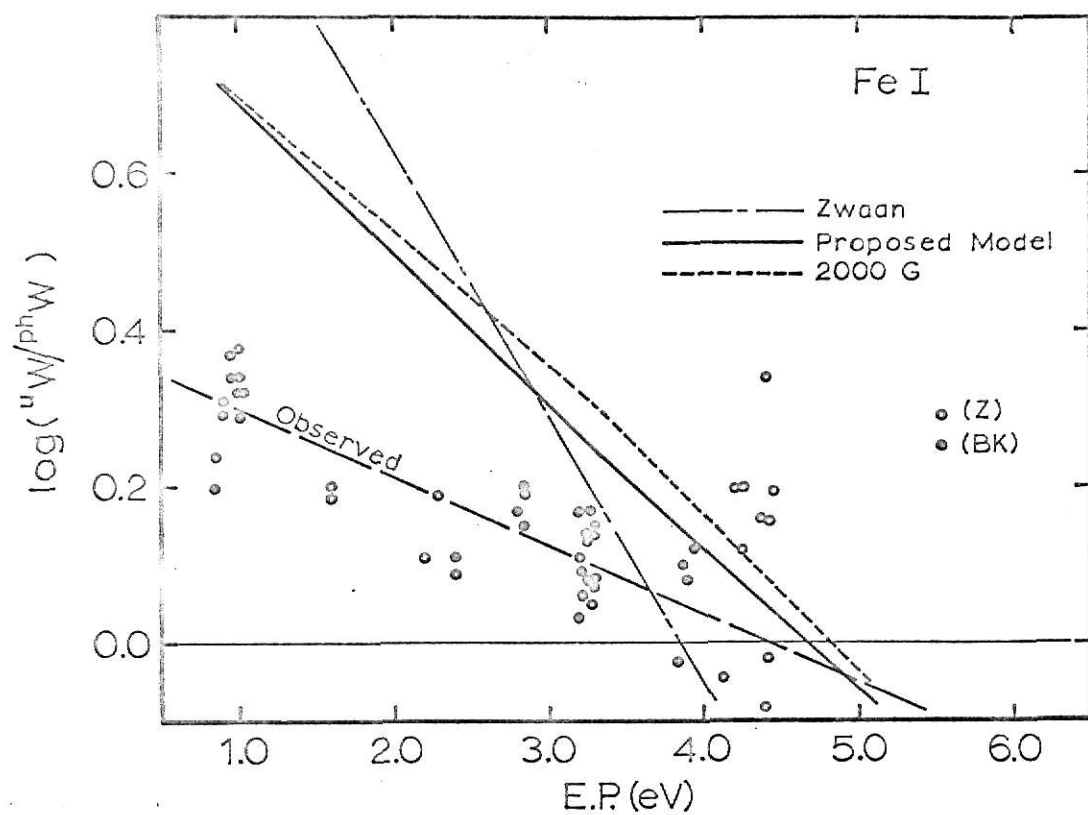
* Logarithm of f-value from Warner (1967).

+ Logarithm of measured spot equivalent width column (6) by Fricke and Elsasser (1965), column (7) photospheric value by Moore, Minnaert, and Houtgast (1966).

EXPLANATION OF PLATE V

Fig. 5. - In the graph at the top of Plate V are the theoretical curves showing the intensification of saturated lines of Fe I in the absence of stray light as a function of excitation potential. The curves are computed for thermal and pressure broadening (solid line) and for the addition of a 2000 G transverse magnetic field (dashed line). The curve marked observed is a visual estimate of the empirical relation and the long-short dashed line is the theoretical intensification for unsaturated lines computed by Zwaan for his model. The data, uncorrected for stray light is from Zwaan (Z) and ten Bruggencate and von Klüber (BK).

Fig. 6. - The same theoretical results as given in Fig. 5 (shown at the bottom of Plate V), except that the data is from the work of Fricke and Elsässer (FE) and has been corrected for scattered light.



observed represents my visual estimate of the empirical relation. The theoretical curves show the intensification of saturated umbral lines in the absence of stray light for thermal and pressure broadening (solid line) and the addition of a 2000 G transverse magnetic field (dashed line). As is evident from the graph, the effect of the magnetic broadening on the curves is less than the scatter in the observations.

Since nearly all neutral lines are intensified in the umbral spectrum (Zwaan 1968), a low-excitation umbral line would saturate sooner than the corresponding line in the normal photosphere, while high-excitation or ionized lines would saturate later. Thus for observed Fe I lines with excitation potential E.P. below about 4.8 eV, Figure 5 indicates that saturation will shift the decreasing ratio, moving the point closer to the axis $\log(^uW/^{ph}W) = 0$, while for E.P. above 4.8 eV the ratio increases which also moves the point closer to the axis. Stray photospheric light shifts the point still further towards the axis as the umbral spectrum will more closely resemble the photospheric spectrum. Hence the slope of the observed curve is expected to be less than the theoretical curve as is apparent in the figure. Also shown in Figures 5 and 6 is the theoretical curve for unsaturated lines computed by Zwaan for his umbral model. Although not shown on the graph, a similar curve was computed using our model which runs parallel to Zwaan's curve but intersects the axis at the same point as our theoretical curves for saturated lines.

Zwaan argues that the point of intersection with the horizontal axis $\log(^uW/^{ph}W) = 0$ is model dependent but unaffected by stray light. Therefore the use of such a correlation should provide a means of selecting a physically realistic model. One should expect then, for measured equivalent widths which

have been corrected for scattered light, that the slope should approach that of the theoretical curves while the intersection point remains the same. This does not appear to be the case in Figure 6 for Fricke and Elsässers' data where the intersection point has shifted upward; however the increased scatter in the corrected data masks the exact change. Our conclusion is that the model is, at least, not inconsistent with the observations, and represents it as well as other models.

Three multiplets of Fe I, RMT 15, 553, 1087, all in the wavelength region $\lambda\lambda 5000-5800$ were selected for the line analysis, in part because sufficient data exists for these multiplets. Secondly, multiplet 15 contains a non-splitting line $\lambda 5434.53$ while for multiplet 553 all lines are triplets, thus simplifying the interpretation.

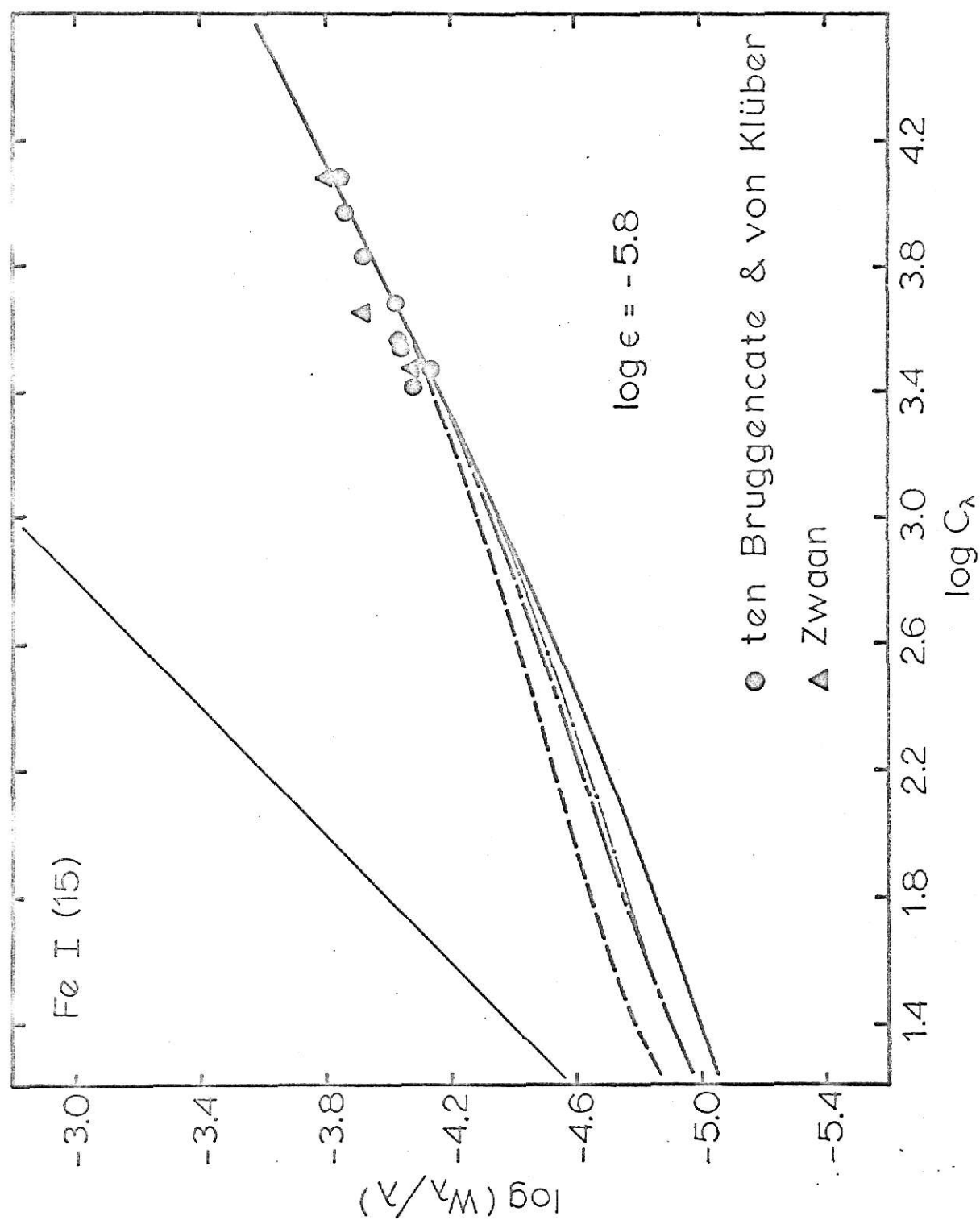
In Figure 7 is shown the empirical curve of growth using the data of ten Bruggencate and von Klüber and Zwaan. The data for this multiplet by Fricke and Elsässer is used to construct Figure 8. The f -values for all three multiplets were taken from Corliss and Tech (1968). The theoretical curves were obtained by varying the abundance for the line $\lambda 5455.61$ which has a Landé g factor of 1.5 and a triplet splitting pattern. The various computed curves are for thermal and pressure broadening only (solid line), the addition of 1.2 km/sec depth-independent microturbulence (dashed-dotted line), a 2.4 km/sec isotropic microturbulence (dashed line), and a 2000 G transverse magnetic field (long and short dashed line).

Although the field in sunspots is thought to be primarily longitudinal, a transverse component is not eliminated (Bray and Loughhead 1964). For a classical triplet, Zeeman broadening will produce no intensification for the

EXPLANATION OF PLATE VI

Fig. 7. - The umbral empirical and theoretical curve of growth for Fe I multiplet 15. The curves are computed for thermal and pressure broadening (solid line), the addition of a 1.2 km/sec microturbulence broadening (dashed-dotted line), a 2.4 km/sec microturbulence broadening (dashed line), and a 2000 G transverse magnetic field (long-short dashed line). The relative abundance $\log \epsilon$ derived from fitting the curves as shown is -5.8.

PLATE VI



longitudinal effect. However, a transverse component of the field, or the anomalous longitudinal effect will intensify the line with the intensification increasing with the number of Zeeman components.

In order to fit the theoretical curves to the observed points, the logarithm of the number density of Fe to H ($\log \epsilon$) adopted was -5.8 which is 0.4 smaller than the photospheric value determined by Goldberg, Kopp, and Dupree (1964). In Figure 8 the abundance adopted was -5.4 or the photospheric value. All the lines in this multiplet are too strong to be of any value in distinguishing magnetic effects. In addition, the absence of points in the weak portion of the curve of growth creates an uncertainty in fitting the observed and theoretical curves accurately to obtain the abundance.

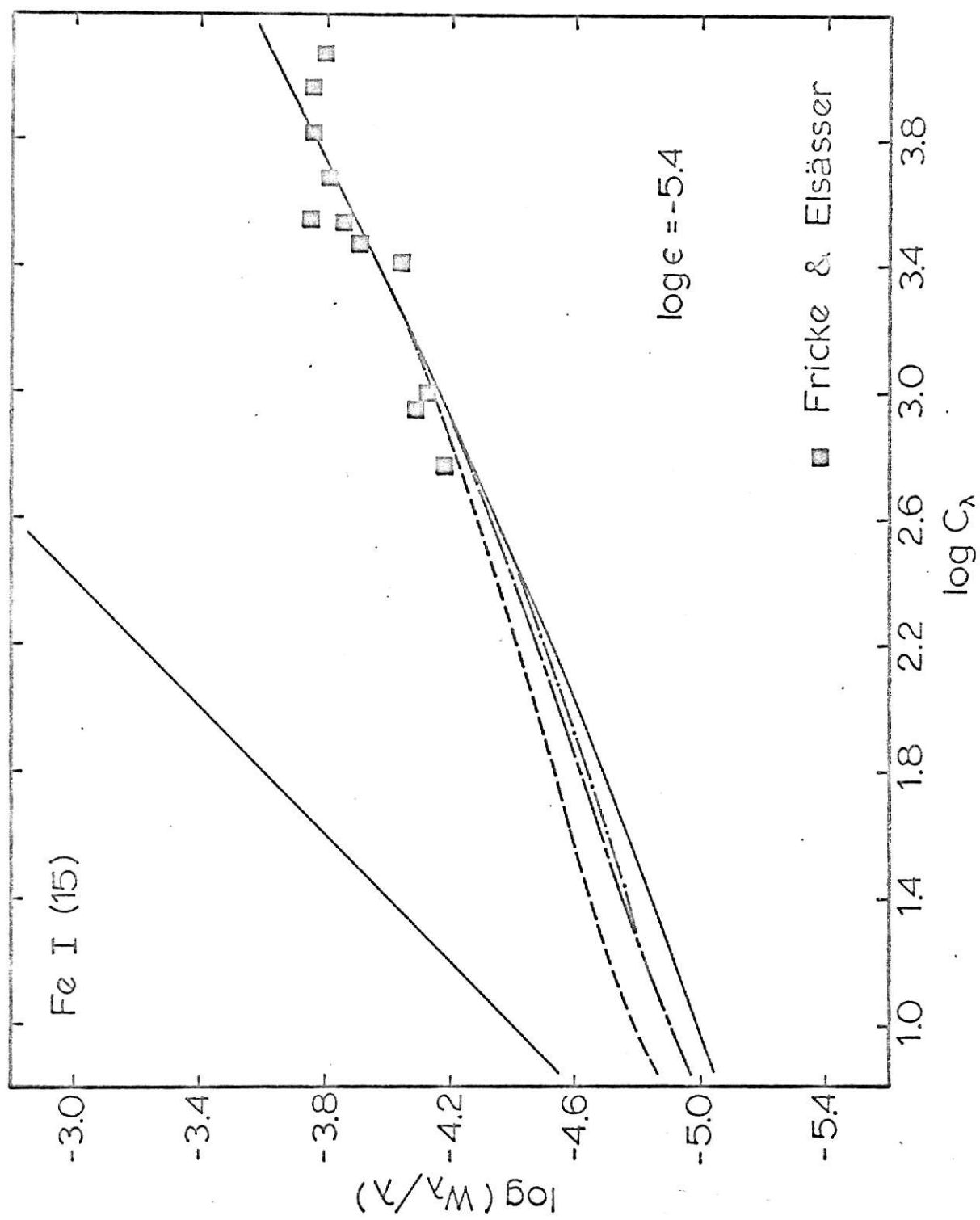
In Figures 9 and 10 are shown the results for multiplet 553 for which the adopted abundance is -5.6 and -5.4, respectively for the authors indicated. The theoretical curves were obtained with the line $\lambda 5253.50$, which is a triplet with a Landé splitting factor of 1.5, and the same broadening mechanisms as in Figures 7 and 8. If the low scatter in the observed points of Figure 9 is true, then the range of excitation conditions, turbulence, or magnetic effects in umbrae are close to the conditions used in the computations. Again, the abundance determinations are somewhat uncertain by the fact that lines are unavailable in the weak portion of the curve.

The results for multiplet 1087 are shown in Figure 11 with the data of Fricke and Elsässer. The broadening mechanisms for the theoretical curves are the same as those in Figure 7 and 8. The curves were obtained for the line $\lambda 5705.48$, which is a triplet with a Landé splitting factor of 1.5. A photospheric abundance was derived by fitting the curve as shown. The data

EXPLANATION OF PLATE VII

Fig. 8. - The same theoretical curves as are in Fig. 7, except that the observations are by Fricke and Elsasser. In contrast to Fig. 7 a relative abundance of -5.4 was derived which is the photospheric value.

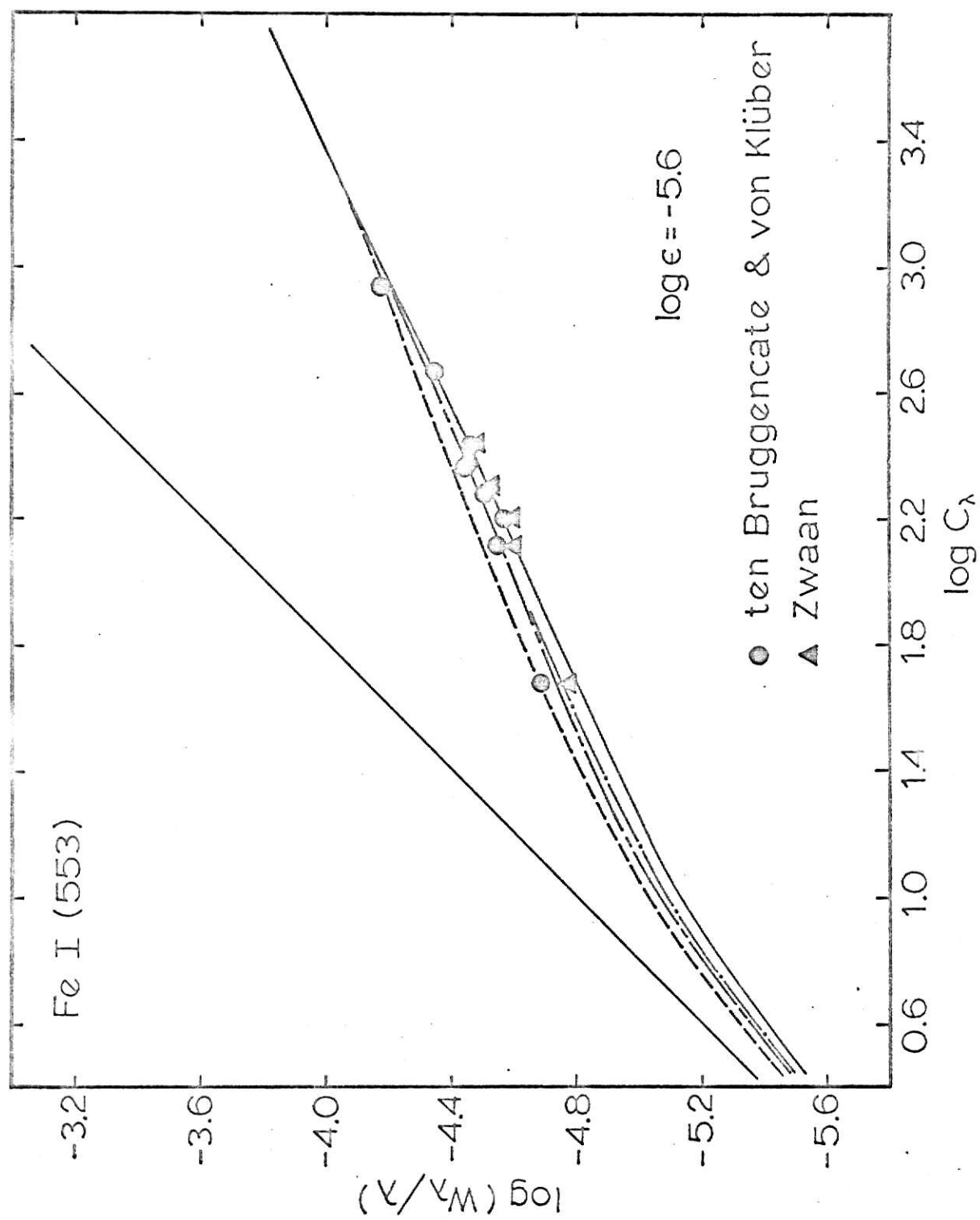
PLATE VII



EXPLANATION OF PLATE VIII

Fig. 9. - The umbral empirical and theoretical curve of growth for Fe I multiplet 553 where the broadening mechanisms are the same as in Fig. 7. The derived relative abundance is -5.6.

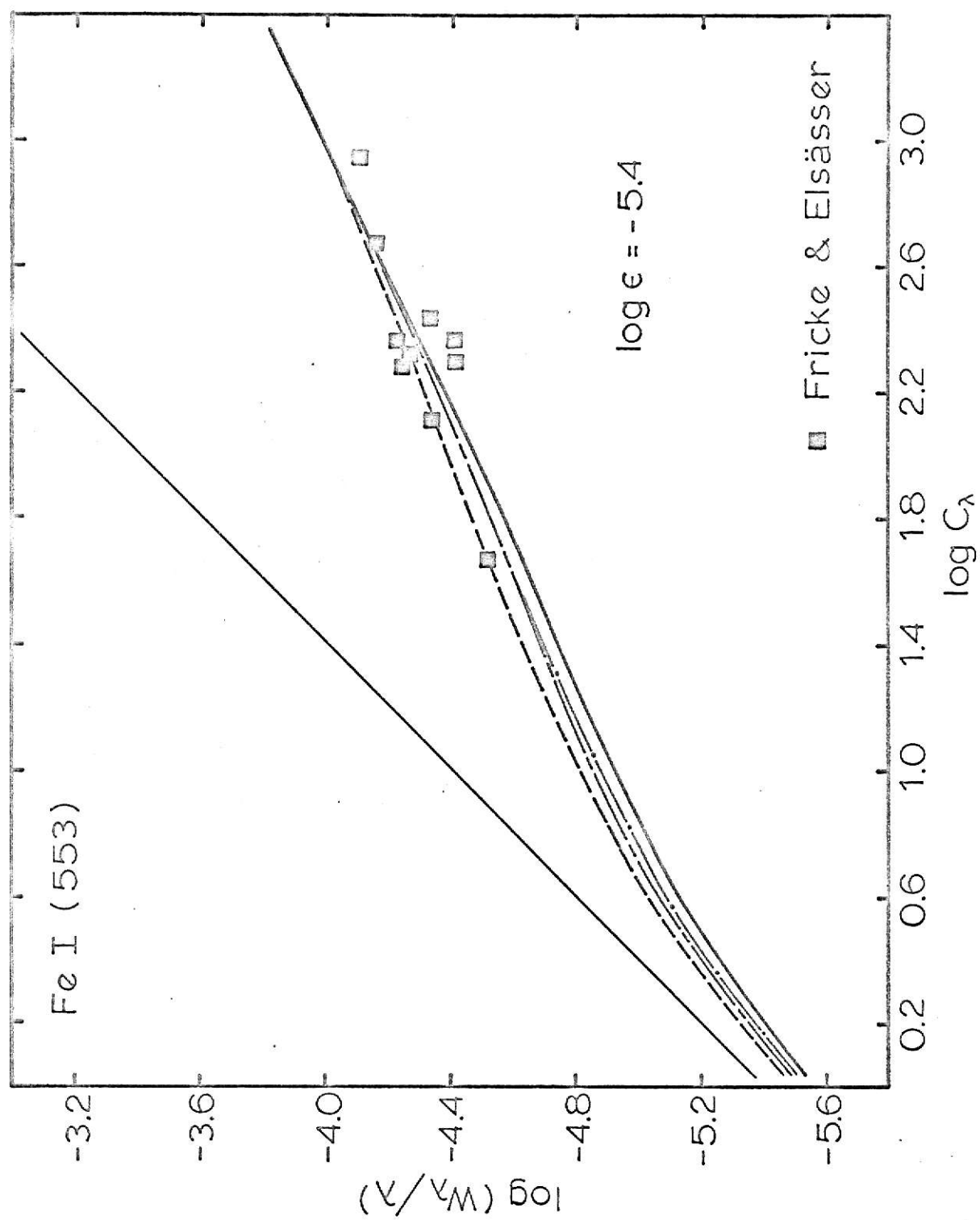
PLATE VIII



EXPLANATION OF PLATE IX

Fig. 10. - The same theoretical curves as are in Fig. 9, except that the observations are by Fricke and Elsässer. The derived relative abundance of -5.4 is the photospheric value.

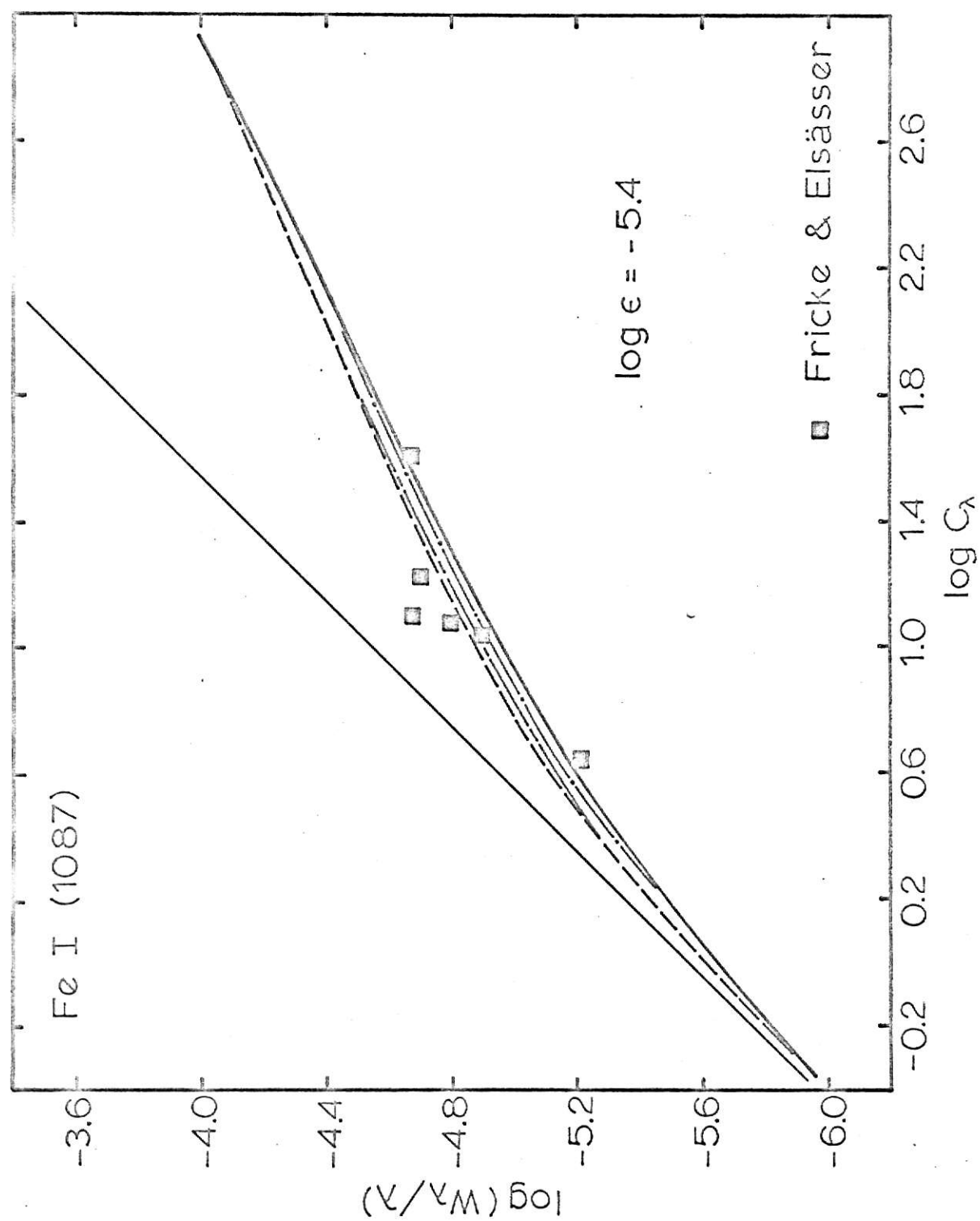
PLATE IX



EXPLANATION OF PLATE X

Fig. 11. - The umbral empirical and theoretical curve of growth for Fe I multiplet 1087 where the broadening mechanisms are the same as in Fig. 7. The derived relative abundance of -5.4 is the same as in Fig. 10 for the measurements of Fricke and Elsässer.

PLATE X



tends to outline the shape of the curve, but it is not good enough to distinguish between the broadening mechanisms.

The magnetic splitting delays the onset of optical saturation which produces an intensification effect similar to the effect produced by micro-turbulence. Thus it would seem that the arguments advanced in the literature (see Bray and Loughhead 1964 or Zwaan 1965) about a rise of the shoulder of the curve of growth over the photospheric curve thereby indicating a micro-turbulent velocity larger than in the undisturbed photosphere, have a more logical explanation in magnetic broadening (see Zwaan 1965). This same argument may well apply to stellar analyses which have obtained a small micro-turbulence from the curve of growth.

d) Singly Ionized Iron

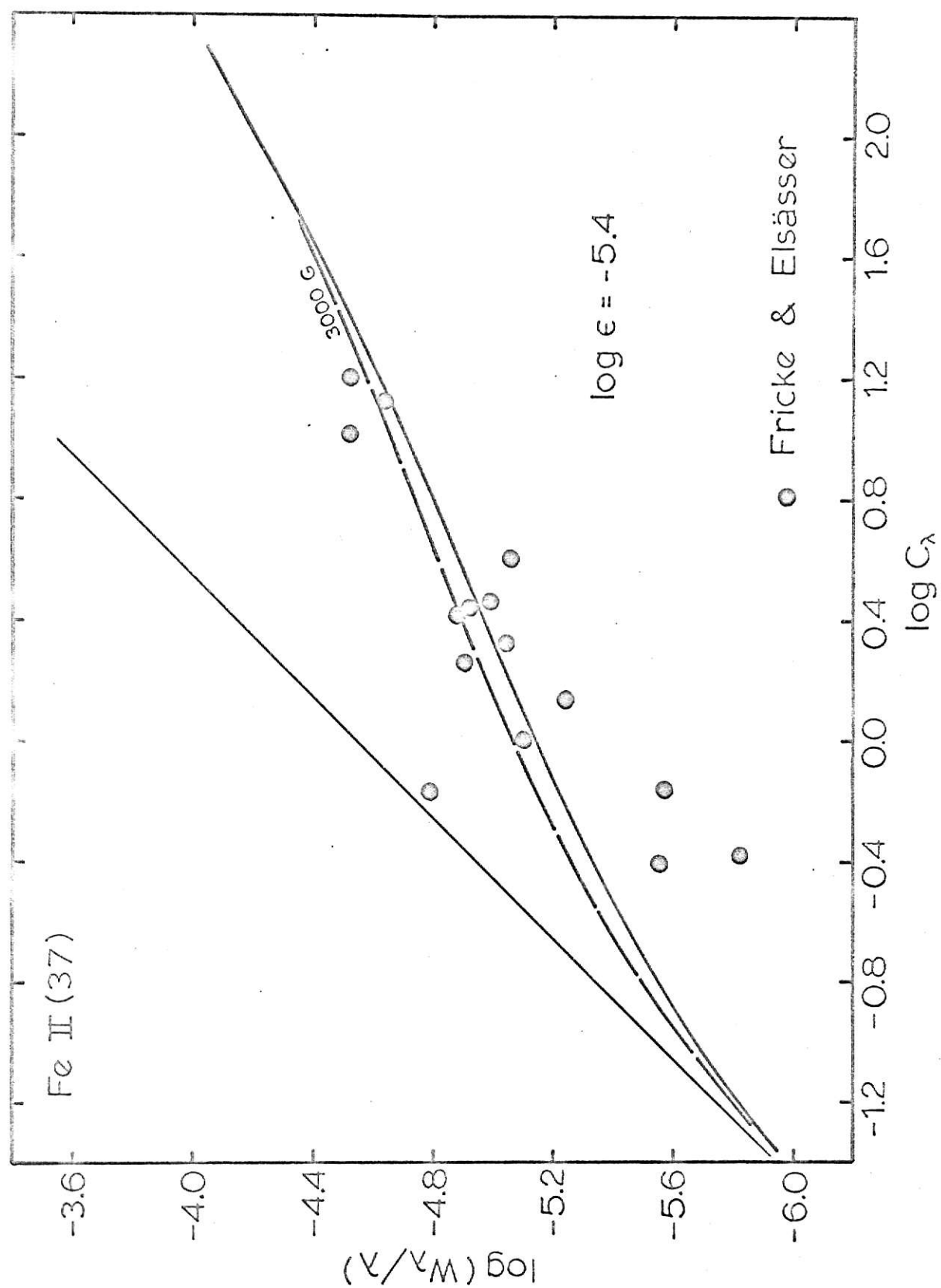
One multiplet, RMT 37, was used to compute the curve of growth for Fe II for the wavelength region $\lambda\lambda 4100-5000$ and range in excitation potential of 2.57 to 2.88 eV. The equivalent widths for the Fe II lines were taken from Fricke and Elsässer, since it is the most extensive and reasonably reliable data, and are shown in Figure 12. The theoretical curves were obtained with the line $\lambda 4629.34$, which is a triplet with a Landé splitting factor of 1.333. The solid line represents a thermal and pressure broadening and the dashed line represents the addition of a transverse magnetic field of 3000 G.

In order to place the f -values of Fe II, which were taken from Corliss and Bozman (1962), on the same absolute scale as the Fe I f -values, a value of -1.00 in the logarithm was added to the Corliss and Bozman f -values as suggested by Warner (1967). Thus the abundance I determined was -5.4, but

EXPLANATION OF PLATE XI

Fig. 12. - The umbral empirical and theoretical curve of growth for Fe II multiplets 27, 28, 37, 38, 42, and 43. The computed curves are for thermal and pressure broadening (solid line) and the addition of a 3000 G transverse magnetic field (dashed line). A relative abundance of -5.4 was derived for the fitting of the curves as shown.

PLATE XI



the uncertainty in this value is evident considering the large scatter in the data. Although the shape of the Fe I theoretical curves appears to fit its respective data very well, the same is not true of the Fe II curves. It seems that a steeper slope is needed to fit the Fe II data. The slope of the curves does depend on the excitation conditions of the model, but only to a small degree. Therefore significant changes in the excitation conditions of the model would be required to produce a steeper slope for the Fe II curves at the expense of the other predictions of the model. In addition, it should be noted that the umbral equivalent widths are measured at best to an accuracy of about twenty-five percent, and that the f -values are known at best to a factor of two.

As for the Fe I data, any real magnetic effects would be completely masked as is evident from the figure.

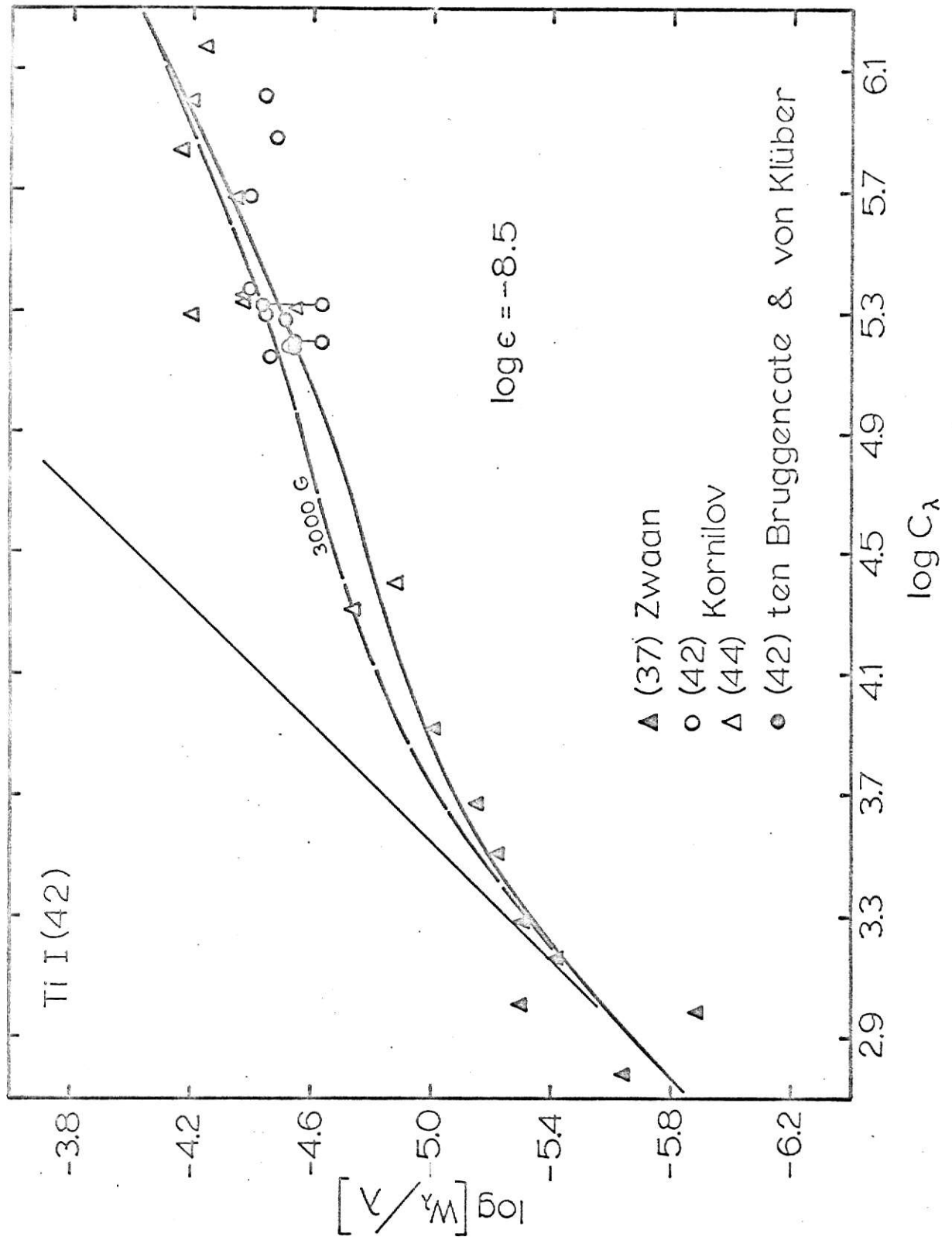
e) Neutral Titanium

In Figure 13 is shown the empirical curve of growth for Ti I using the data by Zwaan, ten Bruggencate and von Klüber, and Kornilov. Their measurements were used because they included weak lines which determined a fit to the linear portion of the curve of growth. The f -values are from Corliss and Bozman (1962). The theoretical curves were obtained for the wavelength region $\lambda\lambda 4300-5200$ and for an excitation potential of about 0.83 eV with the line $\lambda 4534.78$ (RMT 42) which is a triplet with a splitting factor of 1.350. The broadening mechanisms are the same as those used in Figure 12. The adopted abundance of -8.5 is an order of magnitude smaller than the photospheric value determined by Goldberg, Muller, and Aller (1960) of -7.30. The reason for this

EXPLANATION OF PLATE XII

Fig. 13. - The umbral empirical and theoretical curve of growth for Ti I multiplets 37, 42, and 44 where the theoretical curves are for the same broadening mechanisms as in Fig. 12. A relative abundance of -8.5 was derived by fitting the curves as shown.

PLATE XII



difference may lie partially in the fact that the data was uncorrected for stray light. These corrections would tend to intensify the lines and thus shift them upward which would then result in a larger abundance. The arguments made in the previous section on the excitation conditions of the model can also apply in this case. Again, any magnetic effects are indistinguishable in the figure.

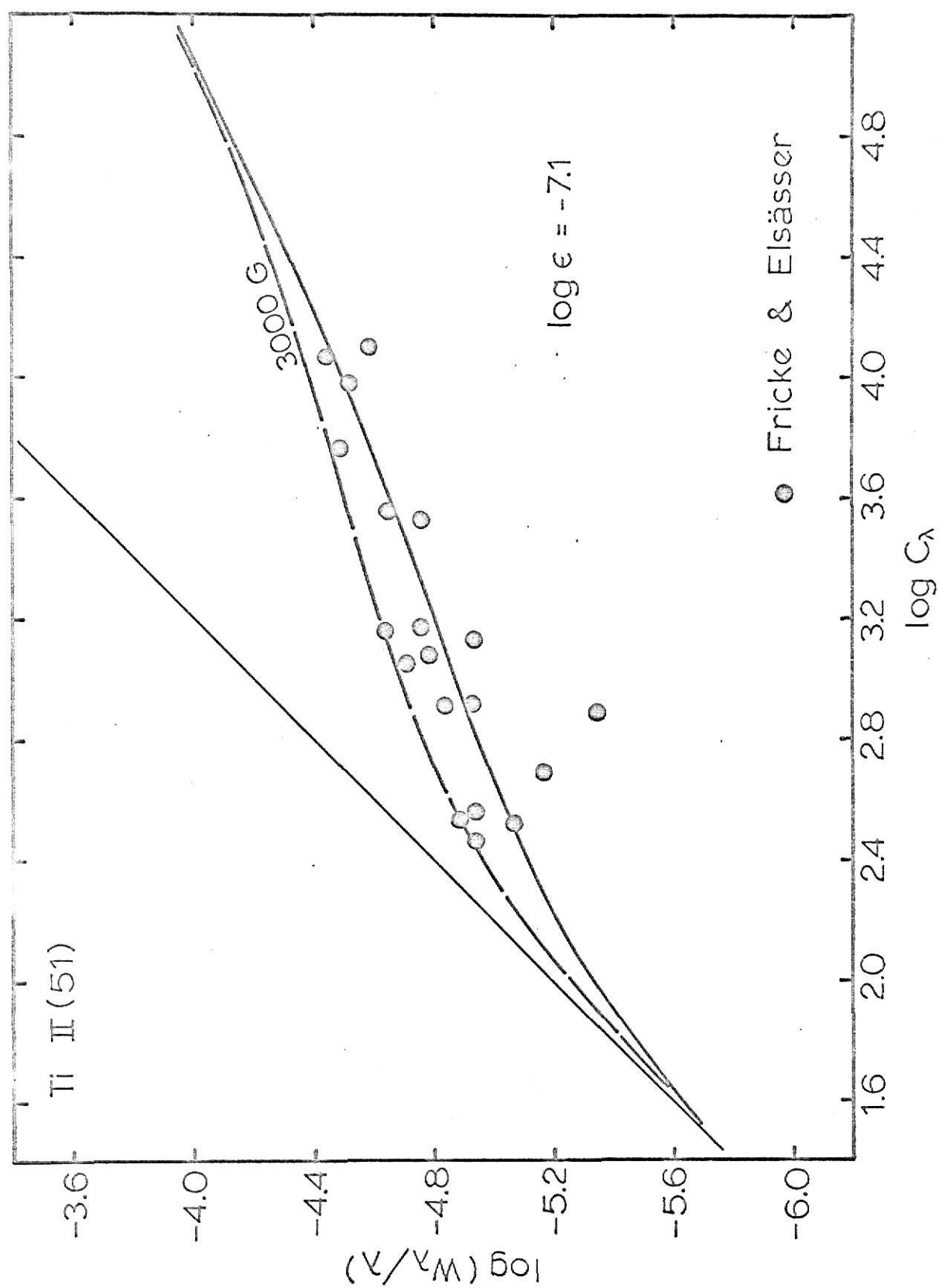
f) Singly Ionized Titanium

The empirical curve of growth for Ti II using the data by Fricke and Elsässer is shown in Figure 14. The curves were obtained for the wavelength region $\lambda\lambda 4300-4600$ and a range of excitation potential of about 1.08 to 1.24 eV using the line $\lambda 4432.09$ (RMT 51) which is an anomalous line of two π -components with a Landé splitting factor of 1.333. The curve representation is the same as in Figure 12. The f -values were taken from Warner (1967). Although the scatter in the data is substantial, an abundance of -7.1 was derived by fitting the curve as shown. The shape of the curves for Ti I and Ti II are nearly identical and differ only slightly from the computed photospheric curve. As was true for all the data used in this investigation, any magnetic effect is completely indistinguishable in Figure 14.

EXPLANATION OF PLATE XIII

Fig. 14. - The umbral empirical and theoretical curve of growth for Ti II multiplets 18, 20, 30, 31, 40, 41, 50, 51, 60, and 61 where the theoretical curves are for the same broadening mechanisms as in Fig. 12. The relative abundance derived was -7.1.

PLATE XIII



VI. CONCLUSIONS

It appears from considering the results of the line data analysis that the derived model and Zeeman broadening are unable to explain all the observed equivalent widths either with the correct slope for some of the curves of growth for the abundances. The inclusion of magnetic intensification does not greatly improve our understanding of the physical conditions in the umbrae of sunspots. A number of reasons along with magnetic intensification could account for all or some of the discrepancies. A non-LTE approach to the atmospheric conditions may be necessary for line and continuum formation. The problem of photospheric radiation entering the spot horizontally may have to be included in an analysis of the spot structure. Obviously this radiation would be most significant for smaller spots, decreasing in importance with increasing spot size. Besides these two reasons, it should be noted again that spectroscopic data available is subject to considerable uncertainty. This uncertainty arises from the method, which varies from observer to observer, in measuring the equivalent widths, from the stray light, and from consolidation of spectra from different sunspots. The model derived in this study however appears to represent the existing data in the most favorable compromise available.

APPENDIX A

THE CALCULATION OF THE MODEL ATMOSPHERE

a) Formulation of the Problem

The development of the model atmosphere concept originated in an attempt to explain the details of the emergent intensity from the solar surface and the emergent flux from stars. The atmosphere is defined as those layers of a star directly accessible to observation, in the sense that a detectable amount of radiation which is characteristic of those layers reaches the observer without being absorbed or scattered.

In order to derive a model atmosphere it is necessary to make certain simplifying assumptions concerning the geometry, equilibrium, and conservation equations which are consistent with observational evidence. These are discussed in greater detail in textbooks on stellar atmospheres such as Unsöld (1955) and Aller (1963).

The atmosphere is assumed to be stratified in homogeneous, steady-state, plane-parallel layers with the linear extent of the atmosphere small compared with the radius of the star. Each layer is thus defined by a single independent depth variable. The outer boundary is defined as the point at which no radiation flows back into the star across the boundary. In addition, the atmosphere is assumed to be in hydrostatic equilibrium under the action of a uniform gravitational field with no radiation, magnetic, or mechanical forces. This implies that the total gas pressure at each layer is just the kinetic pressure of the constituent particles.

The final assumptions concern the transfer of energy through the atmosphere.

Radiation and convection are known to be the primary modes of energy transport in the atmosphere which is assumed to possess no sources or sinks (Aller 1963). The state of the atmosphere is assumed to be one of local thermodynamic equilibrium (LTE). This means that at each layer the gas molecules obey Maxwell-Boltzmann statistics so that the ionization and excitation equilibrium is determined by the Saha and Boltzmann equations, respectively, for the local electron kinetic temperature. In LTE the source function, the ratio of the emission coefficient to the absorption coefficient, is the Planck function (Kourganoff 1963). In essence, scattering is neglected and pure absorption is assumed to be the only mechanism for the formation of the radiation field. The final assumption is the condition that the line and continuum formation can be separated, and hence line opacity does not dominate the atmospheric structure.

In atmospheres similar to the solar photosphere, such as sunspot umbra, the additional assumptions can be made that no molecular formation or helium ionization takes place, and that a scaled empirical solar temperature distribution can be used provided the opacity and ionization equilibrium are similar to those of the sun.

b) The Temperature Distribution

The structure of an atmosphere in LTE is determined by the radiation incident on the lower boundary of the atmosphere from the interior of the star, the mode of energy transport, and by the sources of continuous absorption. Generally the procedure used to find the run of temperature with depth is to require that the total energy flux is constant at each layer of the

atmosphere. An iterative computation on the temperature distribution is made until a unique temperature stratification is obtained which satisfies the condition of flux constancy at each depth. However, in the case of the solar photosphere it is possible to determine the variation of temperature with depth empirically from measurements of the limb darkening and the energy distribution at the center of the disk (see Aller 1963). In this investigation, a variation of this procedure was used as described in Section II.

The model atmosphere is the variation of the physical variables - temperature, pressure, density, opacity, and energy flux - as functions of a suitably defined depth variable. The computational scheme employed in this investigation for the pressure stratification is one due to Weidemann (1955) as modified by Elste (1968) and Evans (1969) to include the opacity and the emergent intensity. The independent depth variable was chosen as the logarithm of the continuum optical depth at $5000 \text{ \AA}$ (denoted by the subscript o). The relation to geometrical depth t is given by

$$x = \log \tau_o = \log \left(\int_0^{t'} \frac{\kappa_o(t)}{m_o \sum_i \epsilon_i \mu_i} \rho(t) dt \right), \quad (A1)$$

where κ_o = the continuous absorption coefficient per hydrogen particle at $5000 \text{ \AA}$,

m_o = the mass in grams of a unit of atomic weight,

ϵ_i = the number density of element i relative to hydrogen

μ_i = the atomic weight of species i ,

ρ = the density of stellar material.

The quantity $1/(m_o \sum_i \epsilon_i \mu_i)$ is the number of hydrogen particles per gram of material. As shown by Elste (1955), the logarithmic optical depth scale is approximately linearly proportional to geometrical depth, and is thus a

physically more desirable depth variable to use than optical depth (see Tables 1 and 2). The atmosphere was divided into twenty-seven layers extending from -4.0 to +1.2 in the logarithm of the optical depth. In order to obtain the corresponding geometrical depth at each layer of the atmosphere, equation (A1) was inverted and solved for the geometrical depth, and where a geometrical depth of zero was arbitrarily set at an optical depth of 0.01.

c) The Ionization Equilibrium

In determining the ionization equilibrium, the Saha equation can be used to calculate the number densities for two successive ionization stages assuming LTE. The equation for the ratio of singly ionized (denoted by the subscript 1) to neutral particles (denoted by the subscript 0) of atomic species i is

$$\frac{n_i}{n_o} = 10^{\log(u_i/u_o) + (9.0801 - 2.5 \log \theta - \log P_e) - \chi_o \theta} \equiv \Phi_i/P_e, \quad (A2)$$

where $u_r(\theta)$ = the partition function of the r th ionization stage,

θ = the reciprocal of the temperature expressed in electron volts
 $[=5040/T(^{\circ}\text{K})]$,

P_e = the electron pressure,

χ_r = the ionization potential between the r th and $(r + 1)$ st ionization stages in electron volts,

n_r = the number density of an atomic species in the r th ionization stage.

The ratio of the number density of ions to atoms and ions, or a quantity called the degree of ionization, is given by

$$\left(\frac{n_i}{n_o + n_i} \right)_i = \frac{\Phi_i / P_e}{1 + \Phi_i / P_e} \equiv X_i \quad . \quad (\text{A3})$$

Neglecting helium ionization, molecular dissociation, and negative ions, and assuming one ionization stage, the number density of free electrons due to the ionization of a particular species is equal to the number density of ions of that species. Thus the ratio of all atoms, ions, and electrons to electrons is

$$\frac{N}{N_e} = \frac{n_{\text{He}} + \sum_i (n_o + n_i + n_e)_i}{\sum_i (n_e)_i} \quad , \quad (\text{A4})$$

where n_{He} = the helium number density. Using the perfect gas law, then equation (A4) becomes

$$\frac{N}{N_e} = \frac{P_g}{P_e} = \frac{\sum_i \epsilon_i (1 + X_i)}{\sum_i \epsilon_i X_i} \quad , \quad (\text{A5})$$

where $X_{\text{Ne}} = 0$ and it has been incorporated in the summation. From equation (A5), the ratio P_g / P_e^2 , which is used in the calculation of the gas pressure, can now be written in terms of the degree of ionization and abundance as

$$\frac{P_g}{P_e^2} = \frac{\sum_i \epsilon_i (1 + X_i)}{P_e \sum_i \epsilon_i X_i} \quad . \quad (\text{A6})$$

d) The Sources of Continuous Absorption

The major assumptions involved in the calculations of the continuous absorption coefficient are: (a) the neglect of all molecular absorption

except H_2^+ ; (b) the neglect of all negative ion absorption except H^- ; and (c) the use of a hydrogenic approximation for metal absorption. The total absorption coefficient κ_λ includes, in order of importance, bound-free (bf) and free-free (ff) absorption due to H^- , H, H_2^+ , and a total absorption coefficient for all metals (Evans 1966, Elste 1968). The metal absorption coefficient was integrated over the ionization continuum rather than summing to obtain a smooth curve. In addition, the total absorption coefficient is multiplied by a correction term for stimulated emission of radiation and this product has added to it a scattering coefficient for Rayleigh scattering by neutral hydrogen and Thompson scattering by free electrons. The total monochromatic absorption coefficient per hydrogen particle per unit electron pressure at any depth in the atmosphere can be written in the form

$$\frac{\kappa_\lambda}{P_e} = \left[\frac{\kappa_\lambda(H_{bf,ff})}{P_e} + \frac{\kappa_\lambda(H_{bf})}{P_e} + \frac{\kappa_\lambda(H_{ff})}{P_e} + \frac{\kappa_\lambda(H_2^{+bf,ff})}{P_e} + \frac{\kappa_\lambda(Metals_{bf,ff})}{P_e} \right] \left(1 - e^{-hc/\lambda kT} \right) + \frac{\sigma(H, e^-)}{P_e} \quad (A7)$$

e) The Gas Pressure

The equation of hydrostatic equilibrium can be written in terms of the logarithmic optical depth scale as

$$dP_g = \frac{g m_o \sum_i \epsilon_i \mu_i}{\kappa_o} \left(\frac{\tau_o}{Mod} \right) dx, \quad (A8)$$

where g = the effective surface gravity,

$$Mod = \log_{10} e = 0.43429.$$

If this equation is multiplied by $P_g^{1/2}$ and integrated from the outer boundary

where $P_g = 0$ to the depth x , an expression for the gas pressure at x is obtained which in logarithmic form is

$$\log P_g = \frac{2}{3} \log \left(\frac{3}{2} \frac{K}{M_{\text{od}}} \right) + \frac{2}{3} \log \left[\int_{-\infty}^x \sqrt{\frac{P_g}{P_e^2}} \frac{\tau_0}{(\kappa_0/P_e)} dx' \right], \quad (\text{A9})$$

where $K = m_o g_i \sum_i \epsilon_i \mu_i$.

The function P_g/P_e^2 and κ_0/P_e are computationally advantageous for conditions found in the spot umbrae because they are much less sensitive to the value of the electron pressure than to the temperature as shown by Weidemann (1955). The electron pressure can be obtained by the relation

$$\log P_e = \frac{1}{2} \left[\log P_g - \log (P_g/P_e^2) \right]. \quad (\text{A10})$$

Using the perfect gas law written in logarithmic form, the density at each layer can be computed by the relation

$$\log \rho = \log P_g + \log \left(m_o \sum_i \epsilon_i \mu_i \right) - \log k - \log (5040/\theta), \quad (\text{A11})$$

where k = the Boltzmann constant.

f) The Emergent Intensity

The radiative transport equation for LTE and plane-parallel atmospheres is given by (Aller 1963)

$$\mu \frac{dI_{\lambda}(\tau_{\lambda}, \mu)}{d\tau_{\lambda}} = I_{\lambda}(\tau_{\lambda}, \mu) - S_{\lambda}(\tau_{\lambda}) \quad , \quad (A12)$$

where $\mu = \cos \theta$ is the angle between the emergent radiation and the outward normal to the surface of the atmosphere,

I_{λ} = the intensity of radiation at optical depth τ_{λ} ,

$B_{\lambda}(\tau_{\lambda})$ = the Planck function

An integral solution may be derived for the emergent intensity (Kourganoff 1963) where the optical depth at the wavelength λ other than 5000 Å is given by

$$\tau_{\lambda}(x) = \int_{-\infty}^x \frac{K_{\lambda}(x)}{K_0(x)} \left(\frac{\tau_0}{Mod} \right) dx \quad . \quad (A13)$$

The emergent monochromatic intensity can now be integrated by parts to obtain the relation

$$I_{\lambda}(0, \mu) = B_{\lambda}(-4.0) + \int_{-3.8}^{+1.2} \left(\frac{dB}{dx} \right) e^{-t/\mu} dx \quad , \quad (A14)$$

where $t = 10^x$.

g) The Computational Procedure

The digital computer program which performs the computation of the atmospheric model was developed primarily by Elste (1968) with modifications by Evans (1969). The geometrical depth routine was written specifically for this investigation by the author. For a given temperature distribution, effective

surface gravity, and chemical composition, an iteration is performed on the electron pressure from an initial estimate. This involves first the computation of the ionization equilibrium from equation (A6) and the continuous absorption coefficient at the wavelength of $5000 \text{ \AA}$ from equation (A7). These are then used to solve the hydrostatic equation [equation (A9)] for the depth-dependence of the gas pressure and a new estimate of the electron pressure from equation (A10). In practice the convergence does not depend strongly on the initial estimate of the electron pressure since P_g/P_e^2 and κ_o/P_e are stronger functions of temperature than electron pressure. The last step is the calculation of the density distribution from equation (A11) and the emergent intensity from equation (A14).

APPENDIX B

THE CALCULATION OF INTENSITY LINE PROFILES

a) The Definition of Line Depth

The assumption of the formation of the absorption lines in LTE yields the result that the source function is the Planck function. The LTE assumption is more valid for subordinate transitions, which tend to favor pure absorption, than for resonance transitions as a formation mechanism (Aller 1963).

The form of the equation from which the line depth in intensity is obtained is a modification of Gussman's (1967) gradient method by Evans (1969). In calculating the line profiles, I used the computer programs which were developed by Evans and which I assisted in writing and verifying. The line depth is defined as

$$r(\Delta\lambda, \mu) = \frac{I_{\lambda}(0, \mu) - I_{\Delta\lambda}(0, \mu)}{I_{\lambda}(0, \mu)}, \quad (\text{B1})$$

where $I_{\lambda}(0, \mu)$ = the emergent continuum intensity at a limb distance μ ,

$I_{\Delta\lambda}(0, \mu)$ = the emergent intensity in the line at the point $\Delta\lambda$ from the line center.

Throughout the visible region, except near absorption discontinuities, the continuum intensity is not a strong variable with wavelength. Therefore it is practical to use in the computation of the line depth a monochromatic continuum intensity for a wavelength region of 100 to 200 Å. If the equations for the intensity in the continuum and in the line are integrated by parts, the resulting equation for the line depth can be written in the form

$$r = \int_{-\infty}^{+\infty} D_{\lambda}(x, \mu) E(x, \Delta\lambda, \mu) dx \quad , \quad (B2)$$

where $D_{\lambda}(x, \mu) = \frac{1}{I_{\lambda}(0, \mu)} \left(\frac{dB_{\lambda}}{dx} \right) e^{-\tilde{\tau}_{\lambda}/\mu} \quad ,$

$$E(x, \Delta\lambda, \mu) = 1 - e^{-\tau_1/\mu} \quad ,$$

$$\tau_1(x, \Delta\lambda) = \text{the line optical depth.}$$

The optical depth $\tau_1(x, \Delta\lambda)$ in the line is given by

$$\tau_1(x, \Delta\lambda) = \int_{-\infty}^x \frac{\kappa_1}{\kappa_0} \left(\frac{\tau_0}{Mod} \right) dx \quad , \quad (B3)$$

where $\kappa_1(x, \Delta\lambda)$ is the line absorption coefficient per hydrogen particle, and κ_0 and τ_0 are obtained as functions of depth from the model atmosphere calculations. This relation follows from the definition of optical depth in equation (A1). The computation of the line depth is now reduced to the specification of the absorption coefficient in the line and its variation with wavelength.

The line absorption coefficient per hydrogen particle has the form (see Aller 1963)

$$\kappa_1 = (1 - e^{-hc/\lambda kT}) \left(\frac{N}{N_H} \right) \frac{e^2 \sqrt{\pi}}{m c^2} \frac{\lambda^2}{\Delta\lambda_D} f \phi(\Delta\lambda) \quad , \quad (B4)$$

where $(1 - e^{-hc/\lambda kT})$ = the correction for stimulated emission,

N = the number density of absorbing particles in
the lower level of the transition of interest,

N_H = the number density of hydrogen,

$$\begin{aligned}\Delta\lambda_D &= \text{the Doppler width,} \\ f &= \text{the oscillator strength,} \\ \phi(\Delta\lambda) &= \text{the broadening function,}\end{aligned}$$

and where the remaining quantities retain their usual meaning. The number of absorbing particles per unit volume, N , can be written in the form (see Evans 1966)

$$N = \left(\frac{n_{p,s}}{n_p} \right) \left(\frac{n_p}{\sum n_q} \right) N_i, \quad (\text{B5})$$

where p and s denote the ionization stage and excitation level, respectively. The quantity $\sum n_q$, which is just equal to the number abundance N_1 , represents the summation of particles in all ionization stages q . However in this investigation only three ionization stages were considered in the summation over q . Thus denoting the most abundant ionization stage for the temperature and pressure of interest by r , where $r-1$ and $r+1$ are the stages below and above r , respectively, then equation (B5) can be written in the form due to Evans (1966)

$$\left(\frac{n_{p,s}}{n_p} \right) \left(\frac{n_p}{\sum n_q} \right) N_i = \left(\frac{n_{p,s}}{n_p} \right) \left[\frac{n_p}{n_{r-1} + n_r + n_{r+1}} \right] N_i = \left(\frac{n_{p,s}}{n_r} \right) \left(\frac{1}{U} \right) N_i, \quad (\text{B6})$$

where u_r = the partition function of the r -th ionization stage,

$$U = u_r \left[\left(\frac{n_{r-1}}{n_r} \right) + 1 + \left(\frac{n_{r+1}}{n_r} \right) \right]$$

The first quantity on the right in equation (B6) depends on the ionization stage under consideration and is given in the form due to Evans (1966)

$$\log\left(\frac{n_{p,s} u_r}{n_r}\right) = \log g_{p,s} + \Delta\chi\theta + h(9.0801 - 2.5 \log \theta - \log P_e) , \quad (E7)$$

where $g_{p,s}$ = the statistical weight of the lower level,

$\chi_{r,s}$ = the excitation potential in electron volts,

and if $p = r-1$, then $h = -1$ and $\Delta\chi = \chi_{r-1} - \chi_{r-1,s}$

if $p = r$, then $h = 0$ and $\Delta\chi = -\chi_{r,s}$

if $p = r + 1$ then $h = +1$ and $\Delta\chi = -\chi_{r+1,s} - \chi_r$

The absorption coefficient per hydrogen particle, equation (B4), can now be written in the form

$$\kappa_1 = \epsilon_i \frac{\sqrt{\pi} e^2}{mc^2} \frac{\lambda^2}{\Delta\lambda_D} \frac{g_{p,s}}{U} \left[10^{\Delta\chi\theta + h(9.0801 - 2.5 \log \theta - \log P_e)} \right] \times (1 - e^{-hc/\lambda kT}) f \phi(\Delta\lambda) . \quad (B8)$$

The integrand for the line optical depth given in equation (B3) can now be expressed in the form (Evans 1966)

$$\left(\frac{\kappa_1}{\kappa_0}\right) \left(\frac{\tau_0}{Mod}\right) = \epsilon_i (gf\lambda)_i Z_i \phi(\Delta\lambda) , \quad (B9)$$

where Z_i defines a depth-dependent population factor and ϵ_i is the number density ratio of absorbing particles to hydrogen particles. The broadening function, $\phi(\Delta\lambda)$, will be discussed for metal lines and hydrogen lines in sections c and d, respectively.

b) Line Broadening Mechanisms

In recent years a number of reviews on spectral line broadening have

been written (see van Regemorter 1965). The broadening mechanisms of importance in stellar atmospheres may be divided into macroscopic and microscopic broadening agents:

(1) Macroscopic broadening agents

- (i) Rotational broadening due to contributions from different parts of a rotating stellar surface;
- (ii) Macroturbulence due to mass motions of turbulent elements whose linear extent is large compared to the mean free path of a photon;
- (iii) Zeeman broadening due to Zeeman splitting by a magnetic field.

(2) Microscopic broadening agents

- (i) Natural broadening due to the finite lifetime of excited states;
- (ii) Isotopic broadening, a pseudo-broadening due to isotopic differences;
- (iii) Thermal broadening due to the random thermal motions of the absorbing particles;
- (iv) Microturbulence due to the mass motions of turbulent elements whose linear extent is small compared to the mean free path of a photon;
- (v) Resonance broadening due to interactions with atoms of the same kind as the absorber;
- (vi) Stark broadening due to the interaction of the absorber with electrons and ions;
- (vii) van der Waals broadening due the interaction with neutral atoms or molecules of a different kind than the absorbing particle.

The line absorption coefficient is independent of the macroturbulence

and rotational broadening which, by their definition, take place after the line is formed. Therefore they affect only the line shape and not the equivalent width. Since the calculations are for sunspots, macroturbulence and rotational broadening agents are unimportant. The Zeeman splitting and all the microscopic agents which influence the line shape and equivalent width are included in the computation of the line absorption coefficient except for isotopic and resonance broadening which are negligible in most astrophysical cases.

c) The Metal Line Broadening Function

Although turbulence pressure was ignored in the computation of the atmospheric pressure model, microturbulence is included in the metal line broadening. Assuming an isotropic distribution of turbulent elements obeying a Maxwellian velocity distribution, microturbulence produces a Doppler broadening similar to thermal motions.

Doppler broadening produces a Gaussian line shape while natural and pressure broadening produce a dispersion profile. Assuming the statistical independence of each broadening mechanism, the convolution of the Gaussian and dispersion profiles results in a Voigt profile for the shape of the absorption coefficient per absorbing particle (Unsöld 1955, Aller 1963). The $1/e$ - width of the Gaussian profile is given by the Doppler width

$$\Delta\lambda_D = \frac{\lambda}{C} \sqrt{\xi_{th}^2 + \xi_t^2} \quad , \quad (B10)$$

where ξ_{th} = the most probable thermal speed of the Maxwellian distribution,

ξ_t = the most probable microturbulent speed,

and the half-width of the dispersion profile in circular frequency units is given by the sum of the half-widths for neutral, quadratic Stark, and van der Waals broadening,

$$\Gamma(x) = \Gamma_{Rad.} + \Gamma_{Stark} + \Gamma_{v.d. Waals} . \quad (B11)$$

The radiation damping constant is computed from the classical formula while the van der Waals broadening by hydrogen and helium atoms is obtained from a Lindholm relation. The Lindholm theory is also used to compute the contribution from quadratic Stark broadening by ions and electrons (Unsold 1955, Aller 1963). The half-width in units of the Doppler width can be written as

$$a(x) = \frac{\Gamma}{4\pi} \frac{\lambda}{\sqrt{\xi_{th}^2 + \xi_t^2}} . \quad (B12)$$

The metal line broadening function, which is the Voigt function can now be written as

$$\phi(\Delta\lambda) = H(a, v) = \frac{a}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{(v-y)^2 + a^2} dy . \quad (B13)$$

If the line is assumed to be formed in the presence of a homogeneous magnetic field, the broadening function becomes the $\psi_{\pm}$ discussed in Section IV.

d) The Hydrogen Line Broadening Function

Since the core of the Balmer lines are formed very high in the atmosphere

(above $\log \tau_0 = -3.0$) where deviations from LTE are probably important, only the wings are considered. In the wings, linear Stark broadening dominates Doppler broadening and radiation damping (see Aller 1963). In addition, Cayrel and Traving (1960) have shown that resonance broadening is important in the sun.

According to the Spitzer-Holstein criterion (Böhm 1960), the contribution of the slowly varying ion fields should be treated with a quasi-static approximation, while the fast moving electrons are treated with the impact approximation.

Investigations of the Balmer lines yielded several formulations for the treatment of the linear Stark broadening: David (1961), Griem (1964, 1967), and Edmonds, Schluter and Wells (1967). The form used in this investigation was the 1964 work of Griem which was an available computer code. The umbral hydrogen lines do not have extensive Stark broadened wings so that there was no necessity in utilizing the improved theories by Griem or Edmonds, et. al. for this study.

The hydrogen broadening function is given by the sum of two terms by Griem

$$f \phi(\Delta\lambda) = f_0 H(a, \nu) + \frac{\sqrt{\pi} \Delta\lambda_D}{F_0} f_{\pm} I[1 + R(\theta, P_e)], \quad (\text{B14})$$

where f_0 = the oscillator strength of the undisplaced component,

F_0 = the normal electric field strength,

$$= 0.1601 (P_e \theta)^{2/3},$$

$f_{\pm}$ = the oscillator strength of the displaced components (red denoted by + and violet denoted by -),

- I = the broadening function due to the quasi-static ion field,
 R = the ratio of the electron impact broadening to the ion
 broadening.

Further discussion and details are given by Evans (1966).

e) The Theoretical Curve of Growth

The theoretical curve of growth is a plot of the saturated equivalent width against the unsaturated equivalent width (Aller, Elste, and Jagaku 1957, Aller 1960, 1963). The unsaturated equivalent width is given by equations (1) and (2) in Section IV where the quantity L_λ is given by (Evans 1970)

$$\log L_\lambda = \log \left[\frac{\sqrt{\pi}}{c} \int_{-\infty}^{+\infty} \sqrt{\xi_{th}^2 + \xi_t^2} Z_i g_\lambda dx \right] , \quad (\text{B15})$$

and where g_λ is the intensity weighting function defined by Aller (1963).

The saturated equivalent width is defined by the integral of the line depth over wavelength

$$\log \left(\frac{W}{\lambda} \right) = \log \left[\frac{I}{\lambda} \int_0^\infty r(\Delta\lambda, \mu) d\lambda \right] \quad (\text{B16})$$

where the line depth is defined by equation (B1).

REFERENCES

- Aller, L. H. 1960, Stellar Atmospheres (Chicago: University of Chicago Press).
- _____. 1963, Astrophysics: The Atmospheres of the Sun and Stars (2d ed.; New York: The Ronald Press Co.).
- Aller, L. H, Elste, G., and Jugaku, J. 1957, Ap. J. Suppl., 3, 1.
- Bohm, K. 1960, Stellar Atmospheres (Chicago: University of Chicago Press).
- Bray, R. J., and Loughhead, R. E. 1964, Sunspots (Great Britain: Pitman Press).
- Cayrel, R., and Traving, G. 1960, Z. Astrophys., 50, 239.
- Corliss, C. H., and Bozman, W. R. 1962, NBS Monograph No. 53 (Washington D.C.: Department of Commerce).
- Corliss, C. H., and Tech, J. L. 1968, NBS Monograph No. 108 (Washington D.C.: Department of Commerce).
- David, K. 1961, Z. Astrophys., 53, 37.
- Deinzer, W. 1965, Ap. J., 141, 548.
- Dicke, R. H. 1970, Ap. J., 159, 25.
- Edmonds, Jr., F. N., Schlüter, H., and Wells III, D. C. 1967, Mem. R. Astr. Soc., 71, 271.
- Elste, G. 1955, Z. Astrophys., 37, 184.
- _____. 1967, Ap. J., 148, 857.
- _____. 1968, Solar Phys., 3, 106.
- Evans, J. C. 1966, Ph.D. thesis (Ann Arbor: University of Michigan).
- _____. 1969, Bull. Am. Astr. Soc., 1, 276.
- _____. 1970, unpublished research.
- Fricke, K., and Elsässer, H. 1965, Z. Astrophys., 63, 35.
- Gingerich, O. J. 1969, Bull. Am. Astr. Soc. 1, 277.
- Goldberg, L., Kopp, R. A., and Dupree, A. K. 1964, Ap. J., 140, 707.

- Goldberg, L. Müller, E. A., and Aller, L. H. 1960, Ap. J. Suppl., 45, 69.
- Griem, H. R., Kolb, A., and Shen, K. Y. 1959, Phys. Rev., 116, 4.
- Griem, H. R. 1964, Plasma Spectroscopy (New York: McGraw-Hill Book Co.).
- _____. 1968, Ap. J., 147, 1092.
- Gussman, E. A. 1967, Z. Astrophys., 65, 456.
- Henoux, J. C. 1968, Ann. Astrophys., 31, 511.
- Kornilov, A. I. 1961, Publ. Sternberg Astr. Inst., No. 117, 27.
- Kourganoff, V. 1963, Basic Methods in Transfer Problems (New York: Dover Publications, Inc.)
- Krat, T. W. 1947, Izv. glav. astr. Obx. Pulkove, 17, 56.
- Makita, M., and Morimoto, M. 1960, Publ. Astr. Soc. Japan, 12, 63.
- Mattig, W., and Schröter, E. H. 1964, Ap. J., 140, 804
- Mattig, W. 1969, Solar Phys., 6, 413.
- Michard, R. 1953, Ann. Astrophys., 16, 217.
- Moore, C. E., Minnaert, M. G. J., and Houtgast, J. 1966, NBS Monograph No. 61 (Washington D.C.: Department of Commerce).
- Pettit, E. A., and Nicholson, S. B. 1930, Ap. J., 71, 153.
- Richardson, R. S. 1939, Ap. J., 90, 230.
- Stankiewicz, A. 1962, Acta Astr., 12, 58.
- Stepanov, V. 1958a, Izv. Krym. Astrofiz. Obs., 18, 136.
- _____. 1958b, Izv. Krym. Astrofiz. Obs., 19, 20.
- Stepanov, W. E. 1957, Izv. Moscou, 100, 3.
- Stumpff, P. 1961, Z. Astrophys., 52, 73.
- ten Bruggencate, P., and von Klüber, H. 1939, Z. Astrophys., 18, 284.
- _____. 1944, Nachr. Akad. Wiss. Gött., Math-Phys., K1, 165.
- Unsold, A. 1955, Physik der Sternatmosphären Berlin: Springer-Verlag).

- van Regemorter, H. 1965, *Ann. Rev. Astr. Astrophys.*, Vol. 3, p. 71.
- Warner, B. 1967, *Mem. R. Astr. Soc.*, 70, 165.
- Weidemann, V. 1955, *Z. Astrophys.*, 36, 101.
- Yun, H. S. 1968, Ph.D. thesis (Bloomington: Indiana University).
- Zwaan, C. 1959, *Bull. Astr. Inst. Netherlands*, 14, 288.
- _____. 1965, *Rech. Astr. Obs. Utrecht*, 17, (4).
- _____. 1968, *Ann. Rev. Astr. Astrophys.*, Vol. 6, p. 135.

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THE INFLUENCE OF MAGNETIC FIELDS
ON THE ANALYSIS OF SUNSPOT STRUCTURE

by

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The question of the magnitude of the error made by neglecting magnetic line formation in the derivation of umbral models has been considered in this investigation. It was felt that a new semi-empirical umbral model was needed which achieved better agreement with the published continuum data than was possible with the predictions of existing models. Thus by improving the reliability of the model using continuum intensity measurements, one can be sure that the results obtained regarding the influence of the magnetic field are not made suspect by the physical conditions of the model. Assuming hydrostatic equilibrium with no magnetic pressure, an LTE source function, and no molecular hydrogen formation, the temperature distribution from Zwaan's model was altered graphically and used to predict the continuum intensity. The validation of the model consists primarily in comparing the predicted continuum intensity ratio of the umbral model to the model of the undisturbed photosphere by Elste to several sets of observations. The ratios have been obtained at several wavelengths in the range from 4000 - 22000 Å and for several limb distances at a wavelength of $\lambda 5451$. Additional confirmation comes from the agreement between observed hydrogen line profiles and the predicted ones of the model.

Line profiles were computed from the model using an LTE source function and then integrated to obtain theoretical curves of growth. Included in the calculations are the usual broadening mechanisms of thermal, pressure, and microturbulence along with the Zeeman broadening, which is the one of special interest. Empirical curves of growth published for Fe I, Fe II, Ti I, and Ti II from several different spots were compared to the theoretical curves. The scatter in the line data due to stray light and uncertainties in the

f-values considerably exceeds any magnetic intensification for typical spot field strengths. Thus the neglect of magnetic line formation does not appear to be the major weakness in our knowledge of sunspot structure.