STRENGTH OF BEAMS WITH
ECCENTRIC, REINFORCED, RECTANGULAR
WEB OPENINGS

by

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CHAPTER I INTRODUCTION

1.1 Problem Statement

In the design of high-rise steel building frames, the structural engineer is faced with the challenge of designing a structure which is safe as well as economical. When all other factors are approximately equal, the design which results in the lowest cost will usually be considered to be the best design. One method of achieving economy in a steel building frame is to reduce the total building height by locating wiring, piping and heating and air-conditioning ducts in the same space that is occupied by the floor beams and girders, thus reducing the height of each story compared with that of an alternate system where the utilities are located below the floor beams and girders. Thus it has become fairly common practice to cut openings in beams to permit the passage of utilities.

Cutting an opening in the web of a beam leads to a variety of problems. The strength of the beam in the vicinity of the opening can be considerably reduced, depending on the size of the member and the opening. This reduction in strength may or may not be critical, depending on the location of the opening on the span. It may be economical to use straight, horizontal duct work, and this results in openings which are not centered on the mid-depth of the floor members if floor beams and girders of different depth have been selected. In many cases, the beam will have to be reinforced in the vicinity of an opening to avoid using a heavier member. Reinforcement will probably be required if the opening is located at a section subjected to high shear forces, and almost certainly be required at sections where both high moments and high shear forces are present.

The objective of this thesis is to present and discuss the development of an ultimate strength analysis of steel beams with reinforced, eccentric web openings, which is the most general problem encountered when web openings are used. The results of the analysis are compared with the strength of uncut beams, and presented in the form of interaction diagrams relating shear strength and bending strength. Some numerical examples are also presented to illustrate the analysis, and to show the effects of the parameters involved in the problem.

1.2 Literature Review

Ultimate strength analyses of beams with web openings have been formulated by several investigators in recent years. In 1968, Bower published an ultimate strength analysis of concentric, unreinforced rectangular openings in the webs of beams (1). His analysis was based on the assumption that points of contraflexure are located in the tee-sections above and below the center of the openings, and that the stress distributions are the same at the high and low moment edges of the openings. Since the effect of strain hardening was neglected, experimental results showed that the ultimate loads were somewhat conservatively predicted by the lower-bound solution.

Redwood presented an ultimate strength analysis of the same problem in 1968 (2). He assumed that points of contra-flexure occur somewhere along the length of the opening, not necessarily at the center. In the analysis, a four-hinge mechanism was assumed in which bending, shear and direct force resultants were considered at each hinge. Only one tee-section was dealt with, and it was assumed that half of the total shear force is carried by each tee-section because of symmetry.

In 1969, Congdon presented in her M.S. thesis an ultimate strength analysis of beams with concentric, reinforced web openings (3). Basically, the assumptions used by Congdon were the same as Redwood's. An approximate method of analysis was also developed. Correlation with previous experimental work indicated that the theoretical results were conservative for large shear forces because the strain-hardening effect was neglected.

In 1971, Richard presented a M.S. thesis on the analysis of beams with eccentric, unreinforced rectangular web openings (4). His assumptions were primarily the same as Redwood's except that he assumed that points of contraflexure occur at the centers of the tee-sections above and below the opening.

From his analysis the variation of moment carrying capacity with different opening locations and dimensions was investigated. Shear forces were unequally distributed to the web areas above and below the eccentric openings, with the larger area carrying the larger shear force.

In 1973, Frost presented a method of predicting the ultimate strength of I-shaped beams with web holes that are not centered at the middepth of the beam and also not reinforced (5). The analysis was based on the assumption that the points of contraflexure occured somewhere along the length of the opening, not necessarily at the center. Comparison with experimental results showed that the ultimate loads were conservatively predicted by the approximate interaction formulas developed, and thus it was concluded that the formulas are satisfactory for design purpose.

In all the papers reviewed, no theoretical analysis was discovered which treats the subject of eccentric, reinforced rectangular openings.

1.3 Scope of the Investigation

The investigation presented in this thesis was limited to an analytical, ultimate strength analysis of steel W shape beams containing reinforced, eccentric rectangular openings in their webs. The general type of reinforcement used herein is pairs of steel bars of rectangular cross section welded on both sides of the web, parallel to the opening edge and with equal area for both top and bottom reinforcement. In the investigation the effects of variable opening lengths, opening eccentricities and reinforcing areas were considered.

CHAPTER II ULTIMATE STRENGTH ANALYSIS

2.1 Nomenclature

The symbols adopted in this thesis are defined where they first appear, and are summarized for convenient reference at the end of the thesis in the section " Nomenclature ".

2.2 Assumptions

The following assumptions were made to facilitate the solution of the problem :

a. The failure of the member will occur by the formation of a four-hinge mechanism with hinges located at the corners of the opening. The assumed failure mechanism of the beam is shown in Fig. 1.

b. Points of contraflexure occur somewhere within the sections above and below the opening, but not necessarily at the center of the opening. The location of the points of contraflexure is the same above and below the opening. This assumption permits the calculation of secondary bending due to shear (the so-called " Vierendeel action ").

c. Shear stresses are assumed to be carried only by the webs of the tee-sections and are uniformly distributed across those webs at hinge locations when hinges are fully formed.

d. Yielding in the flanges and reinforcing bars is in direct tension or compression.

e. Yielding in the webs at each of the four hinge locations under combined bending and shear must satisfy the Von-Mises yield criterion.

f. Reinforcing areas are assumed to be the same for the top and bottom tee-sections (see Fig. 2).

g. To simplify the derivation of formulas, the yield stresses of the material either in tension or compression for flanges, webs and reinforcing bars are assumed to be the same and equal to f_y. That is, $f_y = f_{yf} = f_{yw} = f_{yr}$.

h. For the purpose of this analysis, the opening will always be considered to be eccentric toward the top flange (or compression flange) of the beam. Eccentricity in that direction will be assumed positive and all derivations and examples will be based on positive eccentricity. If the eccentricity is toward the other direction (or bottom flange), it is taken as negative. However, the shape of the interaction curve will be exactly the same as that for positive eccentricity.

2.3 Equilibrium Equation

Free body diagrams of the tee-sections above and below the opening are shown in Fig. 3a. With an opening length of 2a, an opening depth of 2h and an eccentricity of e with respect to the center line of the beam, the opening is located a distance L=M/V from the near reaction. Horizontal equilibrium of the top tee requires that $Q_{1T}=Q_{2T}$. Similarly, for the bottom tee, $Q_{1B}=Q_{2B}$. Furthermore, horizontal equilibrium of sections one and two yields $Q_{1T}=Q_{1B}$ and $Q_{2T}=Q_{2B}$, respectively. Thus

$$Q = Q_{1T} = Q_{1B} = Q_{2T} = Q_{2B} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Referring to Fig. 3b, moment equilibrium at section one (the high-moment edge of the opening) yields

$$M = Q_{1T}(y_{1T}+y_{1B}+2h) - Va \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

and from Fig. 3c moment equilibrium at section two (the low-moment edge of the opening) gives

$$M = Q_{2T}(y_{2T}+y_{2B}+2h) + Va \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Moment equilibrium for the top tee-section yields

$$V_T = Q(y_{1T}-y_{2T})/2a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

while for the bottom tee-section,

$$V_B = Q(y_{1B}-y_{2B})/2a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Using Eqs. 4 and 5, and eliminating Q and a to find the shear distribution between the upper and lower tee-sections, the following relationship for V_T and V_B can be obtained :

$$\frac{V_T}{V_B} = \frac{y_{1T}-y_{2T}}{y_{1B}-y_{2B}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

2.4 Yield Criterion

When the four-hinge mechanism indicated in Fig. 1 is fully developed at ultimate load, the entire cross section of the top and bottom tee-sections at each of the hinge locations yields under combined bending and shear. It is assumed the combined stresses must satisfy the Von-Mises' yield criterion (6).

$$f_y^2 = f_b^2 + 3f_v^2 \quad \text{.............................(7)}$$

where f_y is the yield stress under uniaxial loading, and f_b and f_v are the concurrent bending (normal) and shear stresses.

In this thesis, it is assumed that the flanges and reinforcing bars take only bending stresses and that the stems of tee-sections take both bending and shear stresses. Thus, from Eq. 7, the bending stresses in the webs of the top and bottom tee-sections are

$$f_T^2 = f_y^2 - 3\left(\frac{V_T}{S_T w}\right)^2 \quad \text{..........................(8)}$$

$$f_B^2 = f_y^2 - 3\left(\frac{V_B}{S_B w}\right)^2 \quad \text{..........................(9)}$$

where S_T and S_B are the depths of the top and bottom web-parts, respectively, w is the thickness of the web and V_T and V_B are the total shear forces acting on the webs of the top and bottom tee-sections, respectively.

2.5 Stress Distributions

When the shear stresses are relatively low, the locations of the points of stress reversal at section one can be located in the stubs (web between reinforcing bars and opening edge), in the reinforcing bars or in the clear webs (between reinforcing bars and flange); therefore, there are nine possible combinations of the locations of stress reversal for the low shear case. The six stress distributions which are possible at section one for the low shear case and positive eccentricities are shown in Fig. 4. For low shear stress at section two, the points of stress reversal are assumed to be always in the flanges (see Fig. 5). The different locations of the points of stress reversal under low shear conditions at both sections one and two are shown in Table 1. The relationship between the low shear cases at section one is shown in Fig. 6 and will be discussed later.

Equations for the low shear Case SS are derived in the following section, and similar equations for other locations of stress reversal for low shear conditions are summarized in Appendix A .

2.6 Low Shear Case SS

Considering the first case of low shear in Fig. 4a, the assumed locations of stress reversal occur in the stubs at distances $k_{1T}S_T$ and $k_{1B}S_B$ from the opening edges for the top and bottom tee-sections at the high-moment edge of the opening (section one), respectively, and $k_{2T}t$ and $k_{2B}t$ from the outermost flange surfaces at the low-moment edge of the opening (section two) for the top and bottom tee-sections, respectively. The k's are coefficients between 0 and u/S_T for k_{1T}, 0 and u/S_B for k_{1B} and 0 and 1 for both k_{2T} and k_{2B}. The stress resultants at the hinge locations are ;

$$Q_{1T} = A_f f_y + S_T w(1-2k_{1T})f_T + A_r f_y \quad \text{.................} (10)$$

$$Q_{1B} = A_f f_y + S_B w(1-2k_{1B})f_B + A_r f_y \quad \text{.................} (11)$$

$$Q_{2T} = A_f f_y(1-2k_{2T}) + S_T w f_T + A_r f_y \quad \text{.................} (12)$$

$$Q_{2B} = A_f f_y(1-2k_{2B}) + S_B w f_B + A_r f_y \quad \text{.................} (13)$$

and

$$Q_{1T}y_{1T} = A_f f_y(S_T+0.5t) + 0.5S_T^2 w f_T(1-2k_{1T}^2) + A_r f_y(u+\tfrac{q}{2}) \quad \text{..........................} (14)$$

$$Q_{1B}y_{1B} = A_f f_y(S_B+0.5t) + 0.5S_B^2 w f_B(1-2k_{1B}^2) + A_r f_y(u+\tfrac{q}{2}) \quad \text{..........................} (15)$$

$$Q_{2T}y_{2T} = A_f f_y\left[(S_T+0.5t)-2k_{2T}(S_T+t)+tk_{2T}^2\right] + 0.5S_T^2 w f_T + A_r f_y(u+\tfrac{q}{2}) \quad \text{.................} (16)$$

$$Q_{2B}y_{2B} = A_f f_y\left[(S_B+0.5t)-2k_{2B}(S_B+t)+tk_{2B}^2\right] + 0.5S_B^2 w f_B + A_r f_y(u+\tfrac{q}{2}) \quad \text{.................} (17)$$

where A_f=b x t, the area of flange and A_r=q(c-w), the area of one pair of reinforcing bars.

Using Eqs. 1 and 10 through 13, all the k values can be represented in terms of k_{2T}. Thus, $Q_{1T}=Q_{2T}$ yields

$$k_{1T} = \frac{A_f f_y}{S_T w f_T} k_{2T} \quad \text{.................................(18)}$$

$Q_{2B}=Q_{2T}$ yields

$$k_{2B} = \frac{w(S_B f_B - S_T f_T)}{2A_f f_y} + k_{2T} \quad \text{.........................(19)}$$

and $Q_{1B}=Q_{2T}$ yields

$$k_{1B} = \frac{1}{2} - \frac{S_T f_T}{2S_B f_B} + \frac{A_f f_y}{S_B w f_B} k_{2T} \quad \text{......................(20)}$$

By combining Eqs. 1, 2 and 3 to eliminate M and h,

$$Q_{1T}y_{1T} - Q_{2T}y_{2T} + Q_{1B}y_{1B} - Q_{2B}y_{2B} - 2Va = 0 \quad \text{.....(21)}$$

Substituting Eqs. 14 through 17 into Eq. 21 to eliminate Q's and y's,

$$A_f f_y t(k_{2T}^2+k_{2B}^2) - 2A_f f_y\left[k_{2T}(S_T+t)+k_{2B}(S_B+t)\right]$$
$$+ w(S_T^2 f_T k_{1T}^2+S_B^2 f_B k_{1B}^2) + 2Va = 0 \quad \text{..................(22)}$$

Also, substituting all k - values from Eqs. 18 through 20 into Eq. 22 yields the following quadratic equation in k_{2T}:

$$A_{SS}k_{2T}^2 + B_{SS}k_{2T} + C_{SS} = 0 \quad \text{.......................(23)}$$

where

$$A_{SS} = \frac{A_f f_y}{w}\left(\frac{1}{f_T}+\frac{1}{f_B}\right) + 2t \quad \text{..........................(24)}$$

$$B_{SS} = -2(d-2h) + \frac{t}{A_f f_y}(S_B f_B - S_T f_T)\left(w+\frac{bf_y}{f_B}\right) \quad \text{.........(25)}$$

$$C_{SS} = \frac{2Va}{A_f f_y} + \frac{w(S_B f_B - S_T f_T)^2}{4A_f f_y}\left(\frac{1}{f_B} + \frac{w}{bf_y}\right) - \frac{w(S_B f_B - S_T f_T)}{A_f f_y}(S_B + t) \quad \text{......................(26)}$$

Only one solution of the quadratic equation will satisfy the requirement that k_{1T} and k_{1B} be less than u/S_T and u/S_B, respectively, and k_{2T} and k_{2B} be less than 1. The solution is discussed in the following section.

2.7 Method of Solution

A general approach for solving the problem will be described in this section. To assist in this description, a flow chart of the basic steps is shown in Fig. 7. Because of the unequal web areas above and below an eccentric opening, an indirect trial-and-error method is required to obtain a solution.

With the beam and opening properties known, the initial step is to assume that V is equal to zero (or any desired V value) and to assume a value of V_T/V_B. Then the corresponding values of V_T and V_B can be calculated as follows :

$$V_B = \frac{V}{1 + \frac{V_T}{V_B}} \quad \text{....................................(27)}$$

$$V_T = V - V_B \quad \text{....................................(28)}$$

and the corresponding values of f_T and f_B can be obtained from Eqs. 8 and 9. The coefficient k_{2T} can then be evaluated from the appropriate quadratic equation for each case; for Case SS Eqs. 23 through 26 are used. The desired root of the quadratic is positive and less than or equal to 1 .

After evaluating k_{2T}, other k values can also be calculated, and all the Q's and y's can be evaluated using the appropriate equations corresponding to each case, for Case SS these are Eqs. 10 through 17. Then a new value of V_T/V_B can be calculated from Eq. 6. If the calculated value of V_T/V_B equals, or is close to the assumed value, the solution is correct, and, after checking if k_{1T}, k_{1B} and k_{2B} are within the appropriate limits, the coordinates for one point on the interaction curve, M/M_P and V/V_P, can be obtained. M is the internal moment at the center of the cut section given by Eqs. 2 and 3, and M_P and V_P are the plastic moment capacity and plastic shear capacity for the uncut section, respectively, which are formulated as follows :

$$M_P = \left[A_f(d-t) + \frac{w}{4}(d-2t)^2\right]f_y \quad \ldots\ldots\ldots\ldots\ldots (29)$$

$$V_P = w(d-2t)f_y/\sqrt{3} \quad \ldots\ldots\ldots\ldots\ldots (30)$$

If the calculated value of V_T/V_B is not close to the assumed value, a new assumed V_T/V_B is obtained by taking the average of the calculated one and the assumed one and the procedures described above are repeated until the correct value of V_T/V_B is determined.

For the original assumed shear force, V, corresponding values of M/M_P and V/V_P can be evaluated as described above.

By repeating the whole procedure and incrementing V, more points on the interaction curve can be obtained for each case until V approaches V_{max}, where V_{max} is the maximum plastic shear capacity of the cut section and is formulated as follows :

$$V_{max} = w(S_T+S_B)f_y \,/\, \sqrt{3} \quad \text{.....................(31)}$$

CHAPTER III COMPUTER PROGRAM

3.1 Introduction

Using the procedure described previously, the computer programs shown in Appendix B were written to solve for the coordinates of points on the interaction diagram, V/V_P and M/M_P. Examples of the output from these programs are presented in Appendix C.

Programs were written separately for each case instead of one complete program involving all the cases, because the use of separate programs facilitated debugging, permitted monitoring the transition from one stress reversal case to the next, and helped to isolate special problems which are discussed later.

3.2 Initial Stress Reversal Locations

At the beginning of the program, it is necessary to determine the locations of the points of stress reversal when V = 0. From experience, the stress reversal in the top tee, Section 1, is always in the web stub initially, and stress reversals occur in the flanges for both the top and bottom tees at Section 2. In the bottom tee at Section 1, the initial location of the point of stress reversal is the plastic neutral axis of the cut, reinforced section. The plastic neutral axis can occur in the web stub, in the reinforcing bars or in the clear web. Therefore, it can be concluded that Case SS, SR or SW should be used to start with at Section 1 and Case FF at Section 2.

The eccentricity is the determining factor in selecting the initial case at Section 1, and the three possibilities can be summarized as follows :

a. For $e \leq u$, the plastic neutral axis is located in the web stub between the opening edge and reinforcing bars in the bottom tee. This limit was derived by equating the compression web area to the tension web area, since flange areas and reinforcing areas are the same for the top and bottom tee-sections. In this case, the problem is solved starting with Case SS, but from the viewpoint of practical design, it is not a common case since for such small values of e, the eccentricity could be neglected and the problem treated as a concentric opening.

b. For $u < e \leq u+q+\frac{A_r}{w}$, the plastic neutral axis is located within the reinforcing bars in the bottom tee. The upper limit for this case is derived by setting the plastic neutral axis at the lower edge of the reinforcing bars in the bottom tee-section and equating the total area including all the reinforcing area above the plastic neutral axis to the total area below it. Under such circumstances, the problem is solved starting with Case SR.

c. For $e > u+q+\frac{A_r}{w}$, the plastic neutral axis is located in the clear web between the reinforcing bars and the flange in the bottom tee, and the problem is solved starting with Case SW. This is a practical case only for small reinforcing area, otherwise such large eccentricities would leave no room for reinforcement in the top tee-section.

3.3 Roots of the Quadratic

An important step in the program involves solving for the roots of k_{2T} from the quadratic equation in order to evaluate the other k-values. Since there is only one root of the quadratic equation which will result in the other k-values being less than the limits (for example, for Case SS, k_{1T} and k_{1B} must be less than u/S_T and u/S_B, respectively, and k_{2T} and k_{2B} must be less than 1), the problem is to choose the right root. When the problem starts with a very small (or zero) value of V, one of the roots will be positive and less than 1 and the other root will be negative but close to zero. In this case the other k-values can not be evaluated within their limits from the former. Therefore, the technique adopted is to set the latter equal to zero in order to obtain a solution. This assumption can be checked numerically by comparing the M/M_P value obtained from the computer solution for V=0 with the ratio of plastic moment capacity of the cut, reinforced section to the plastic moment capacity of the gross cross section. Comparison of the two M/M_P values has shown in all cases that setting the small root equal to zero for small values of V yields correct results.

An interesting result from the numerical examples presented in Chapter IV is that in all cases the smaller root of k_{2T} from the quadratic equation is the root which yields the correction solution.

3.4 Transition between Stress Reversal Cases

For each value of V it is necessary to check the limits of the other k-values in order that the transition to the next case can be identified. For example, if the problem starts with Case SS, there will be only two possibilities : (1). k_{1T} hits the limit of u/S_T first, and the next case will be Case RS; and (2). k_{1B} hits the limit of u/S_B first, and the next case will be Case SR. By continuing to check the k-value limits, the interaction diagram can be completed by succesively selecting the right stress reversal cases.

3.5 Sensitivity when V Approaches Vmax

Since the program is based on assuming a value of V_T/V_B to match the calculated V_T/V_B from Eq. 6, the problem will become sensitive sometimes when V approaches V_{max}, given by Eq. 31. At that point the calculated V_T/V_B may be large enough to make either V_T or V_B in Eqs. 8 or 9 exceed V_{Tmax} or V_{Bmax}, where V_{Tmax} and V_{Bmax} are the maximum shear force possible on the top and bottom tee-sections, respectively, and are formulated as follows :

$$V_{Tmax} = wS_T f_y / \sqrt{3} \quad \text{.........................(32)}$$

$$V_{Bmax} = wS_Bf_y / \sqrt{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

When V_T or V_B exceeds the above limit given by Eq. 32 or 33, f_T or f_B becomes imaginary. Under such circumstances, a new suitable value of V_T/V_B may be obtained by a trial-and-error method. Also in this situation the permitted tolerance between the calculated V_T/V_B and the assumed V_T/V_B was increased arbitrarily from 0.01 to 0.025 to obtain a solution to the problem.

3.6 Maximum Shear Force

The interaction curve is limited by a maximum value of V/V_p, given by

$$\left(\frac{V_{max}}{V_p}\right) = \frac{d-2h-2t}{d-2t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

For some practical sizes of beams, opening and reinforcing area, this limiting value may not be reached, resulting in an imaginary solution to the quadratic in k_{2T}. In this case the stress reversal points at section one are still in the clear web for both top and bottom tee-sections, and at section two they are still in the flanges, as assumed. Consideration of other locations of stress reversal points did not give a real answer. Upon further investigation it was found that either the opening length must be less than a given maximum value or the reinforcing area greater than a minimum value to obtain a real solution to the quadratic in k_{2T}. From

Case WW in Appendix A, when V_{max} is reached f_T=0, yielding

$$k_{2T} = A_r / A_f \qquad (35)$$

Substituting Eqs. 31 and 35 into the quadratic equation for Case WW yields

$$a = \frac{A_r(d-2h-2u-q)\sqrt{3}}{w(S_T+S_B)} - \frac{A_r^2\sqrt{3}}{bw(S_T+S_B)} \qquad (36)$$

Eq. 36 is the same as Congdon's Eq. 30 (3), and indicates that eccentricity does not affect the maximum opening length required to reach V_{max}.

If the A_r^2 term is neglected and u, q and t are much smaller than d and h, Eq. 36 reduces to :

$$\text{Max.} \quad a = A_r\sqrt{3} / w \qquad (37)$$

or

$$\text{Min.} \quad A_r = aw / \sqrt{3} \qquad (38)$$

Eqs 37 and 38 are the same as Congdon's Eqs. 31 and 32 (3). If either value is satisfied, the maximum shear capacity of the section will be reached.

CHAPTER IV NUMERICAL EXAMPLES AND DISCUSSION OF RESULTS

4.1 Introduction

Some numerical examples are presented in this Chapter to explain how the computer program works, to show the effect of varying parameters on the interaction diagram and to check if the solution reduces to that of Congdon when e=0 (3) and to that of Frost when A_r=0 (5).

A W16x45 beam was used in all the numerical examples since this section had been tentatively selected for a proposed test program. Other properties and dimensions selected with the W16x45 section to serve as the basic case for the numerical examples are : $f_y=36^{ksi}$, a=4½", h=3", e=2", u=q=¼" and c-w=4".

Using these properties as the basic case, the effects of varying parameters a, e and A_r (actually, q was kept constant, varying c-w only) were investigated, while u, q, h and f_y were not varied. The values of the parameters investigated are shown in Table 2. An interaction diagram for the basic case is shown in Fig. 8, while the effects of varying parameters are shown in the interaction diagrams of Figs. 9 through 11.

4.2 Varying Eccentricity

The first parameter investigated was eccentricity. It is the most important one involved in the problem from the point of view that the eccentricity determines the correct stress reversal case to start the computations, as discussed in Section 3.2. Five different eccentricities were investigated, and they can be divided into three groups. The first group involves e=0", and the solution obtained is the same as that of Congdon (3). Case SS was the first case in the solution. The second group includes e=0.3", 1" and 2". Here, Case SR was the first case encountered in the solution. The third group involves e=3.5", and Case SW was selected to start the solution (see Table 2). The interaction diagrams for these examples are plotted in Fig. 9, from which it can be concluded:

a. With small eccentricity (say, $e \leq u$), the problem could be treated as a concentric opening, without any significant loss of accuracy.

b. As the eccentricity increases, the moment carrying capacity of the beam decreases for low shear forces and increases for high shear forces.

c. For the eccentricities included in group 2, the sensitivity problem was encountered when V approached V_{max}.

4.3 Varying Reinforcing Area

The second parameter investigated was c-w. With q kept constant, c-w was the term selected to vary the reinforcing area. There were five different c-w values investigated; the first case was c-w=0", which yielded results identical to Frost's solution (5). The remaining cases were c-w=1" through 4" in 1" increments. The results are plotted in Fig. 10, from which it can be concluded :

a. The moment carrying capacity is almost linearly proportional to the reinforcing area where the shear force is low, but this relationship becomes non-linear when the shear force is larger.

b. When c-w=0", the program terminated because it entered a region where low shear is " mixed " with high shear, i.e., Case FW was encountered at Section 1, and this case was not included in the present analysis.

c. When the reinforcing area provided was less than the minimum area given by Eq. 38 (min. A_r = 0.899 in^2), V_{max} was not reached (see Fig. 10; c-w=1", 2" and 3").

d. When c-w=4", the problem was sensitive when V approached V_{max}.

4.4 Varying Opening Length

The last parameter investigated was a. There were three different values investigated, a=3", 4½" and 6". The results are shown in Fig. 11, and conclusions can be formulated as follows :

a. When V=0, the moment carrying capacity does not vary with opening length.

b. As a increases, the moment carrying capacity decreases while V increases.

c. The curve for a=6" was terminated at $V < V_{max}$, because the opening length exceeds the maximum length given by Eq. 37 (max. a=5.006").

d. For the other two curves the sensitivity problem was encountered when V approached V_{max}.

4.5 Sequence of Stress Reversal Cases

The purpose of this section is to show by example how much of the interaction diagram is covered by each stress reversal case. The first example is the basic case, shown in Fig. 8, where there were only two stress reversal cases (Cases SR and RR) involved in the solution. In the second example there were five cases (Cases SW,SR,RR,RW and WW) included in the problem as shown in Fig. 12. The data for these two examples is presented in Appendix C. All of the sequences of stress reversal cases encountered in the numerical examples of this Chapter are shown in Fig. 6 by arrows between cases.

CHAPTER V CONCLUSIONS

An ultimate strength analysis of steel W shape beams containing eccentric, reinforced rectangular openings in their webs has been formulated for the low shear case. From a study of the parameters involved in the problem the following major conclusions can be made :

a. Eccentricity does not affect the maximum opening length and the minimum reinforcing area, hence Eqs. 36,37 and 38 are the same as obtained by Congdon (3).

b. With small eccentricity (say, $e \leq u$), the problem could be treated as a concentric opening without any significant loss of accuracy.

c. As the eccentricity increases, the moment carrying capacity of the beam decreases for low shear forces and increases for high shear forces.

d. The moment carrying capacity is almost linearly proportional to the reinforcing area when the shear force is low, but this relationship becomes non-linear when the shear force is larger.

e. As opening length increases, the moment carrying capacity decreases while shear force increases.

CHAPTER VI RECOMMENDATIONS FOR FURTHER STUDY

From the present analysis, the following recommendations are suggested for further investigation of the problem of eccentric, reinforced web openings :

a. All the computer programs shown in Appendix C should be combined into one program so that a complete interaction curve could be obtained in one calculation.

b. The same procedures described in this thesis could be used to analyze the high shear case at section one (i.e., Case FF) as well as those cases where low and high shear are " mixed ", i.e., Cases RF,FR,WF and FW (In the present analysis Case FW was found to be the next case for c-w=0").

c. An approximate solution should be formulated for design use.

d. An experimental study of beams with eccentric, reinforced rectangular openings in their webs would be helpful to check the analytical results presented here.

ACKNOWLEDGEMENTS

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Appreciation is extended to Dr. Robert R. Snell, the head of Civil Engineering Department, and my commitee members Dr. Hugh S. Walker and Dr. Stuart E. Swartz for their instruction in those courses which were related to the writing of this thesis.

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THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

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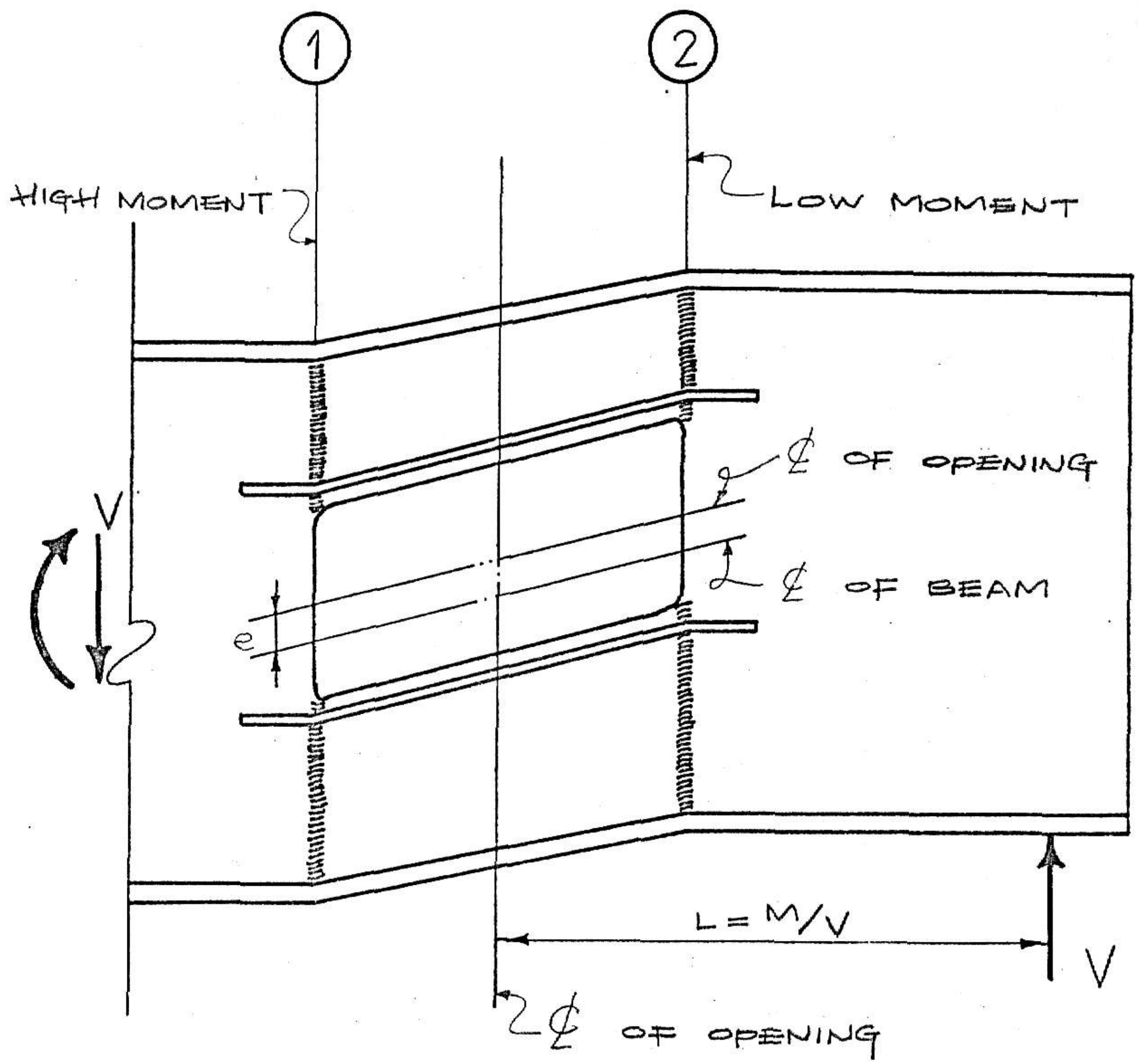


Fig 3 : Equilibrium of Tee-Sections

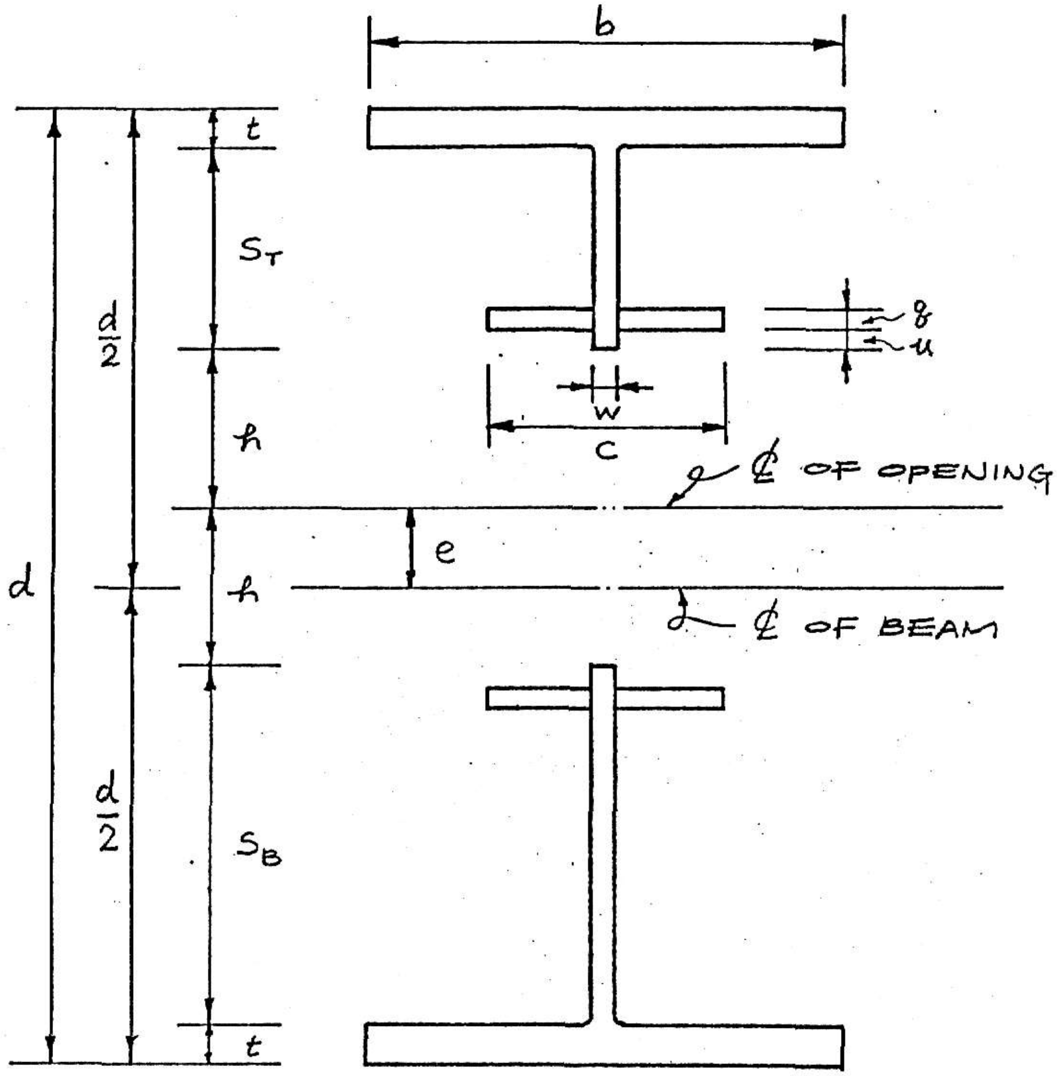


Fig. 2 : Beam Cross Section

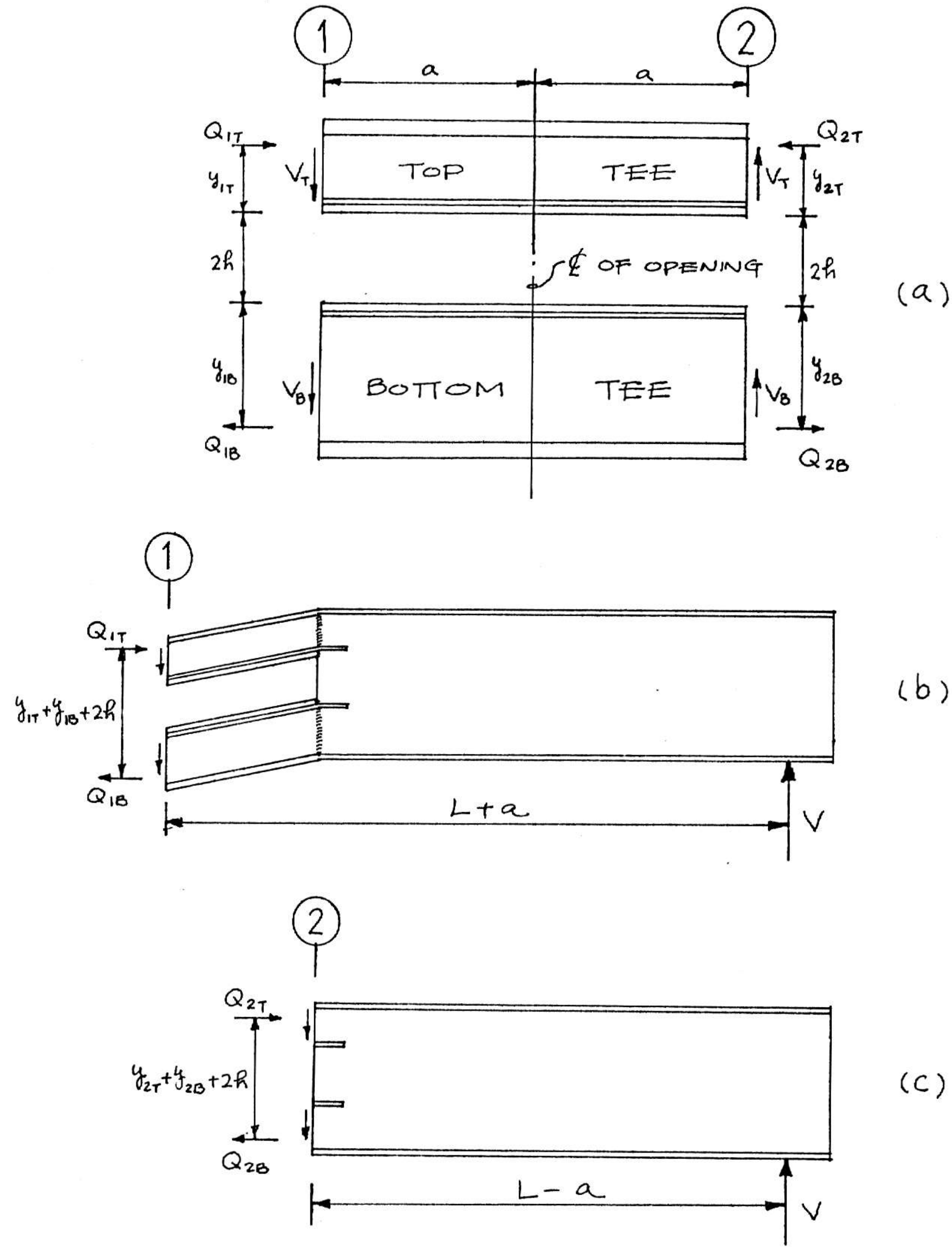


Fig. 3 : Free Body Diagrams

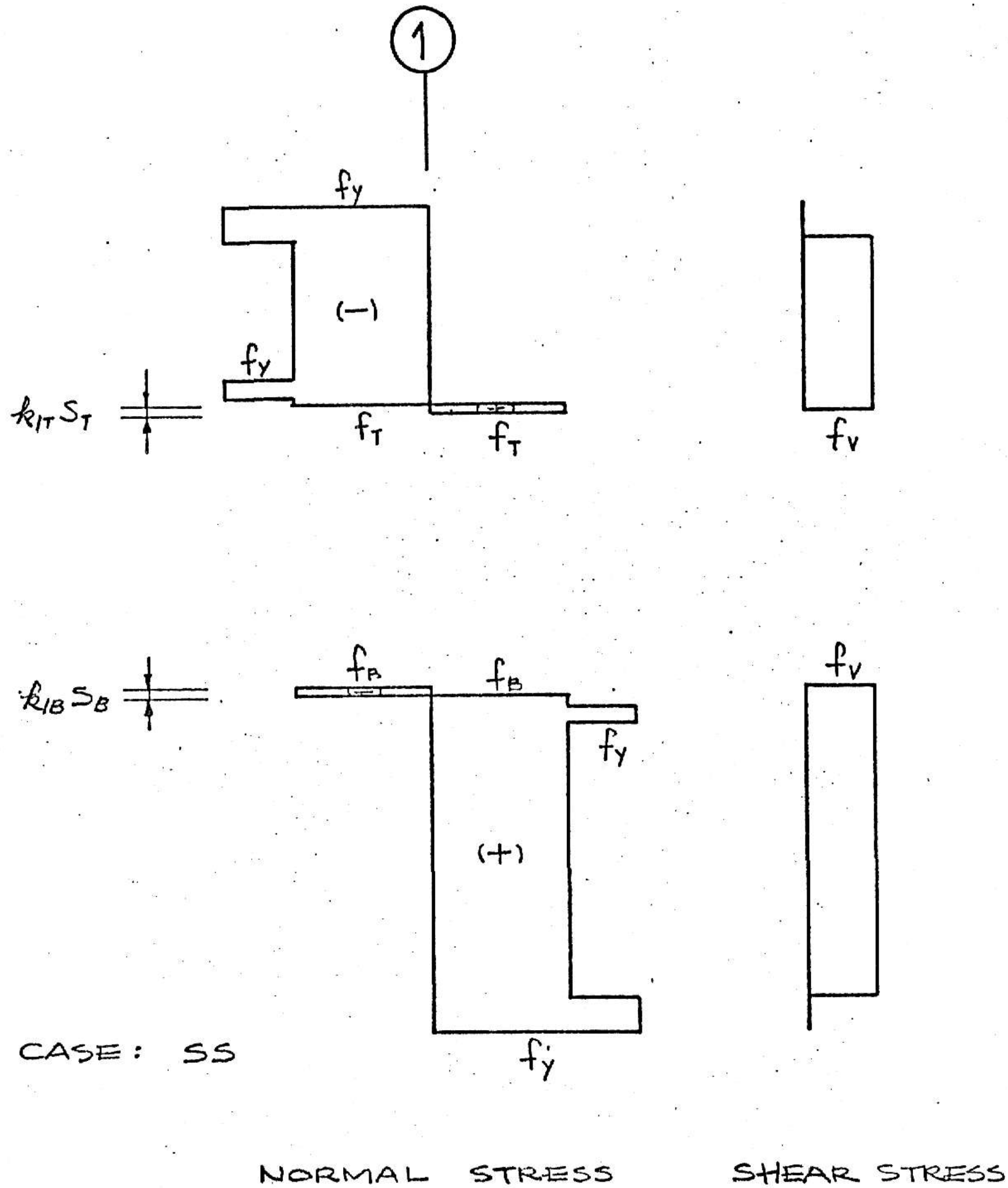


Fig. 4a : Stress Distribution at Section 1 for Low Shear Cases

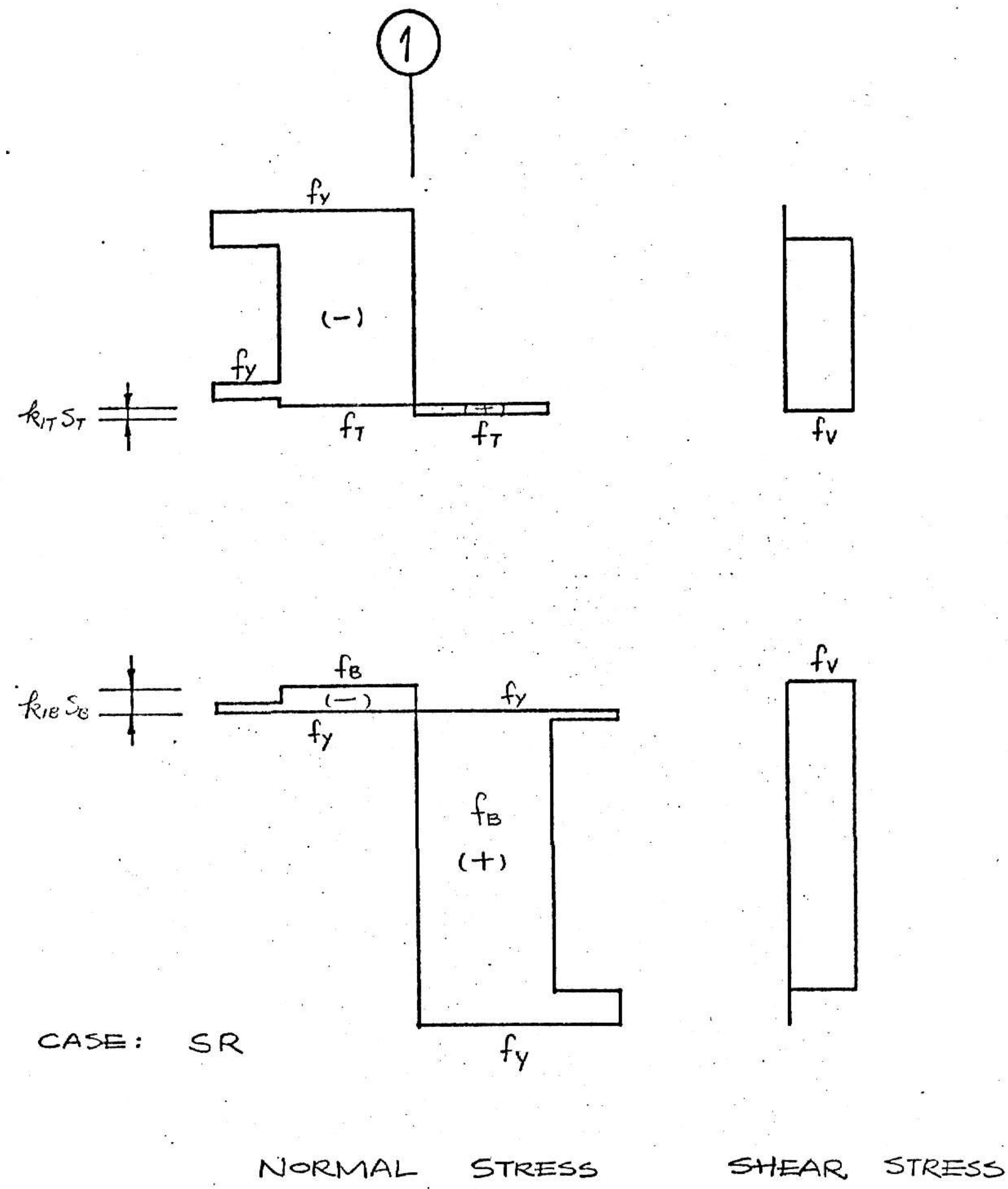


Fig. 4b : Stress Distribution at Section 1
for Low Shear Cases (cont'd)

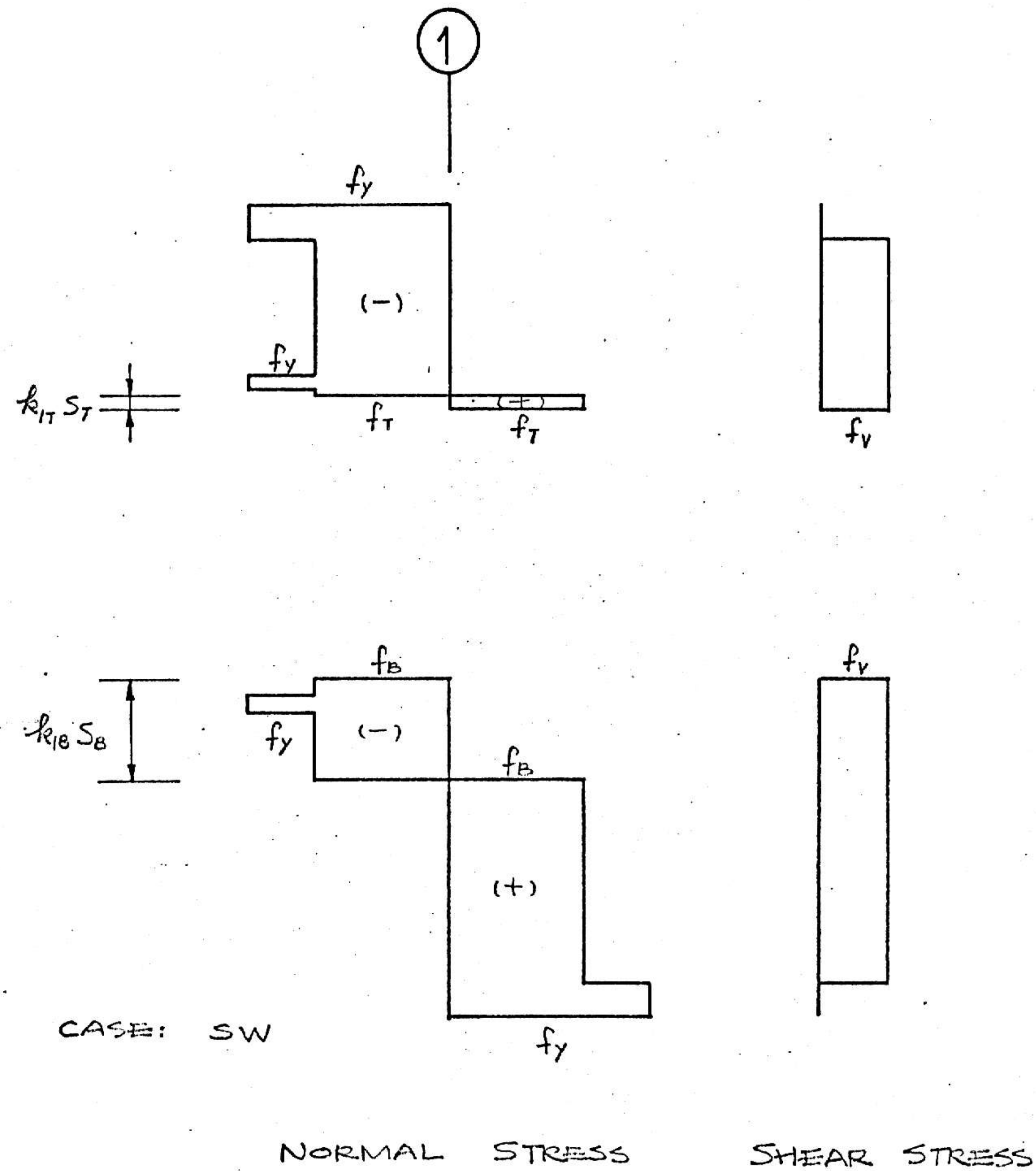


Fig. 4c : Stress Distribution at Section 1
for Low Shear Cases (cont'd)

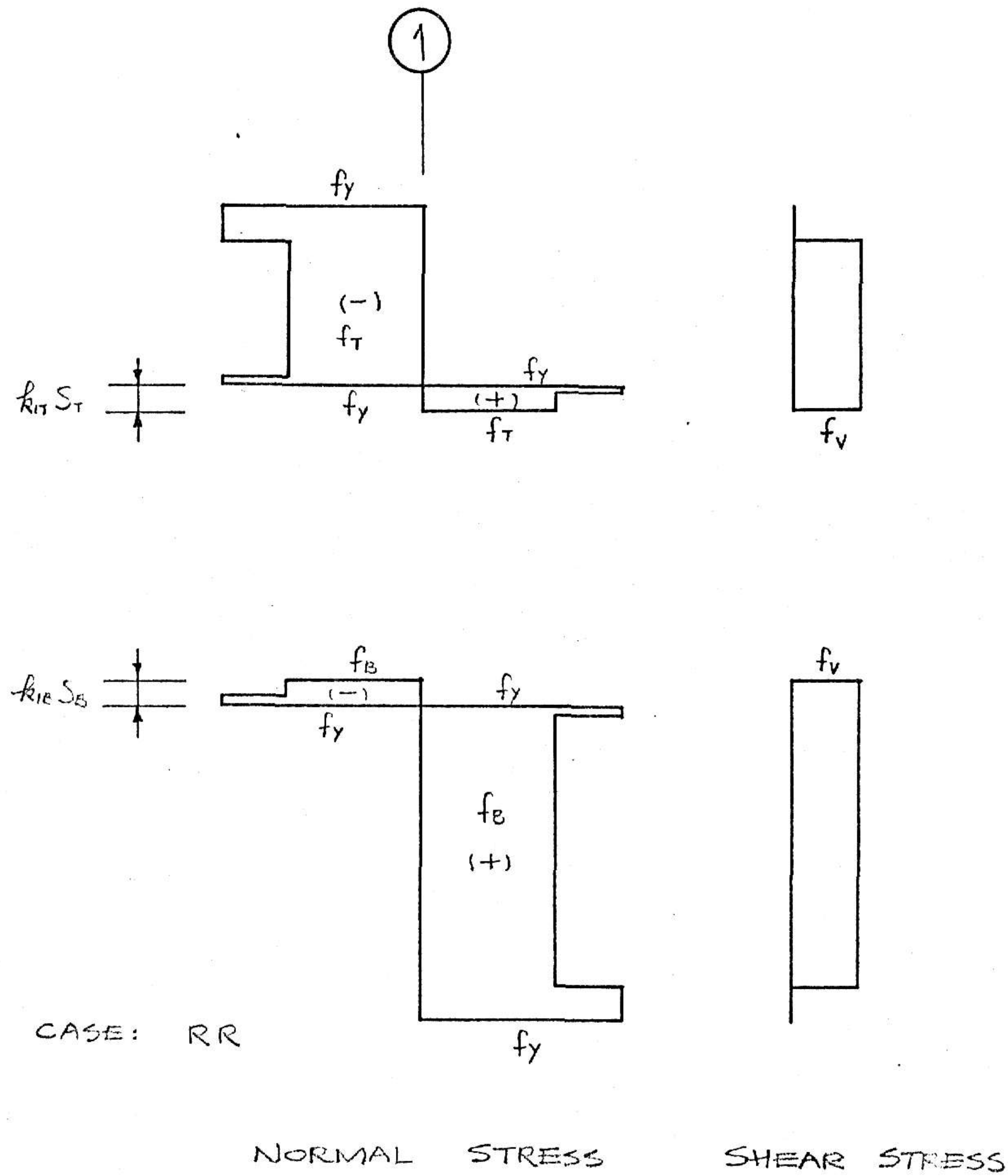


Fig. 4d : Stress Distribution at Section 1
for Low Shear Cases (cont'd)

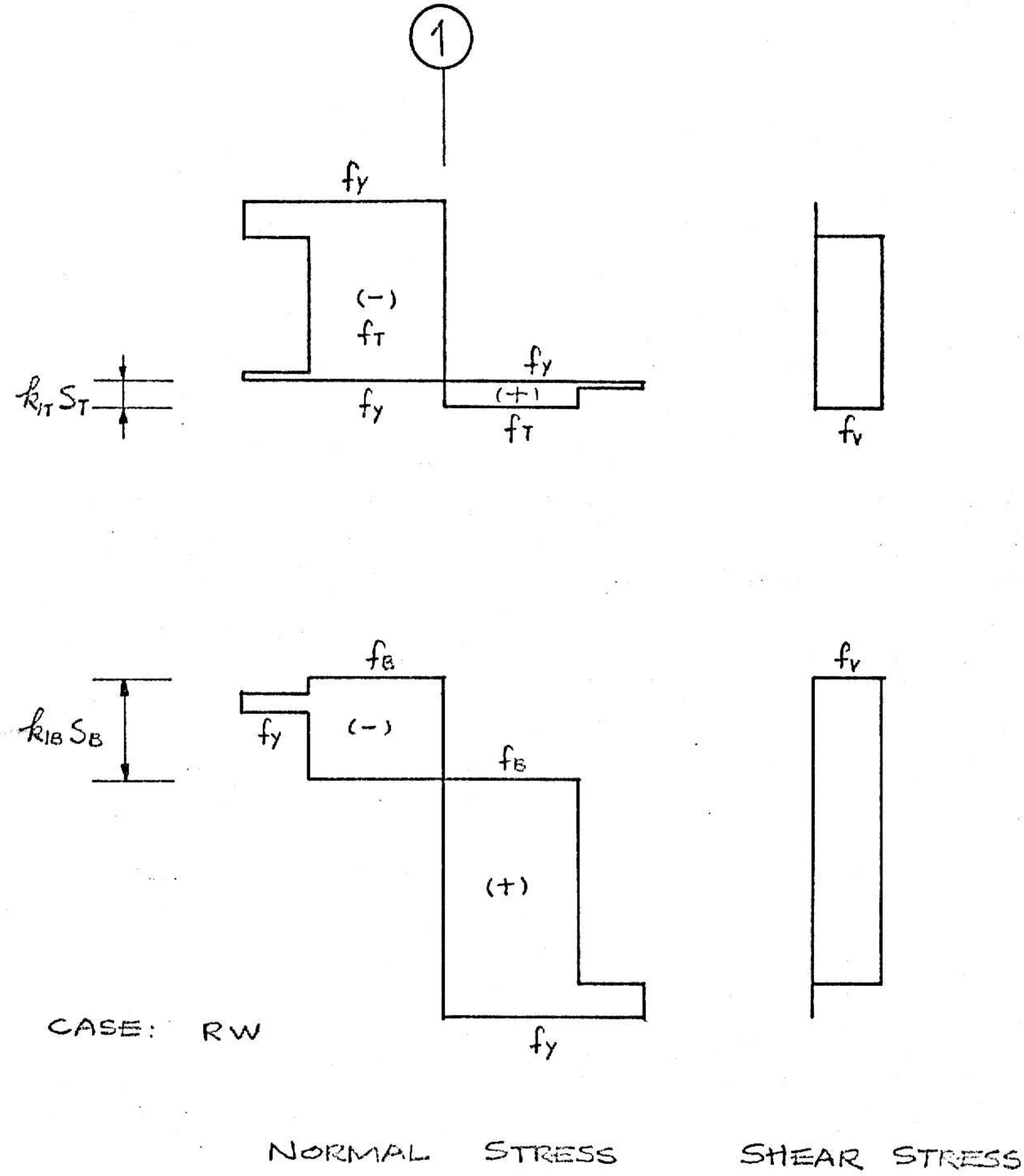


Fig. 4e : Stress Distribution at Section 1
for Low Shear Cases (cont'd)

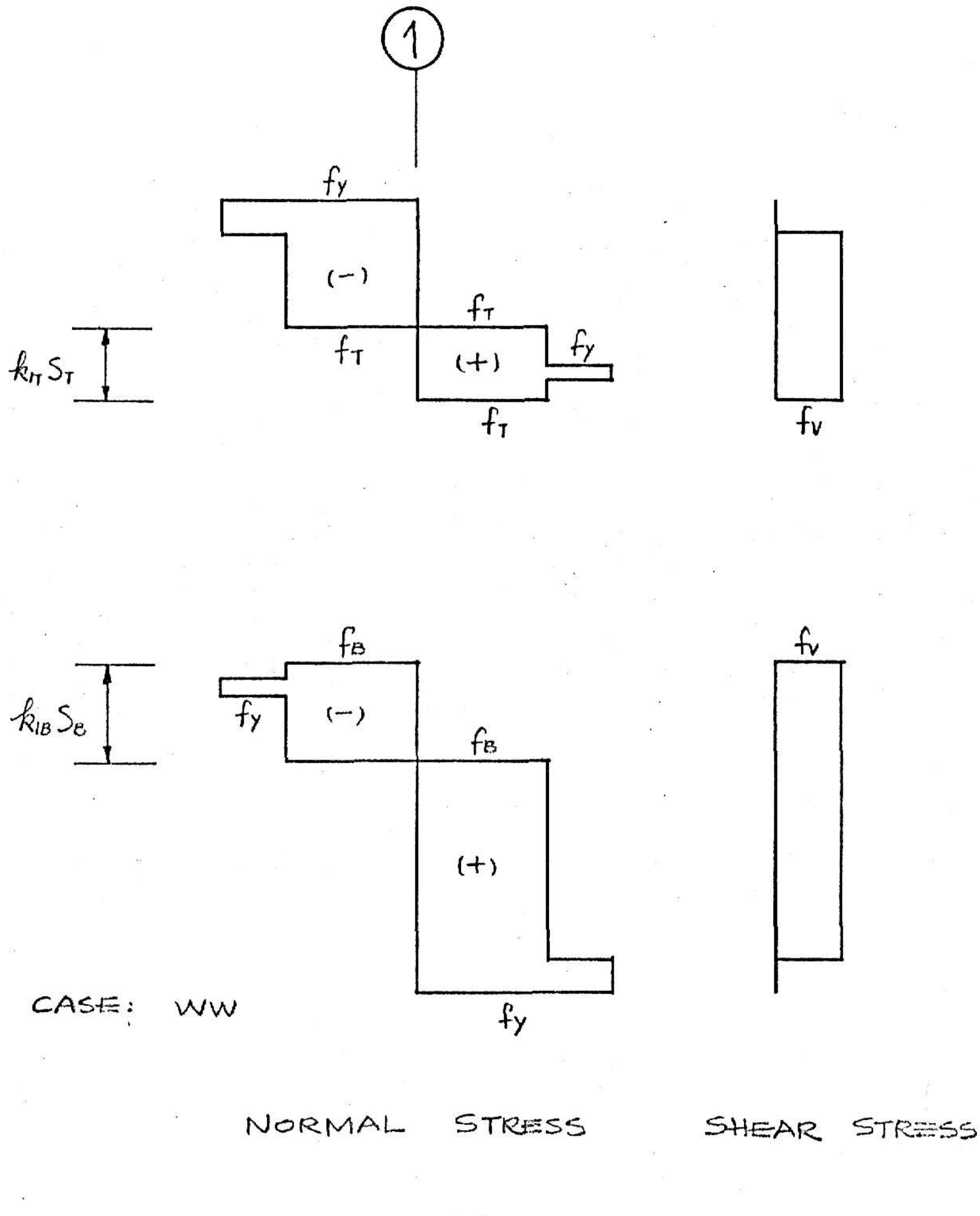


Fig. 4f : Stress Distribution at Section 1

for Low Shear Cases (cont'd)

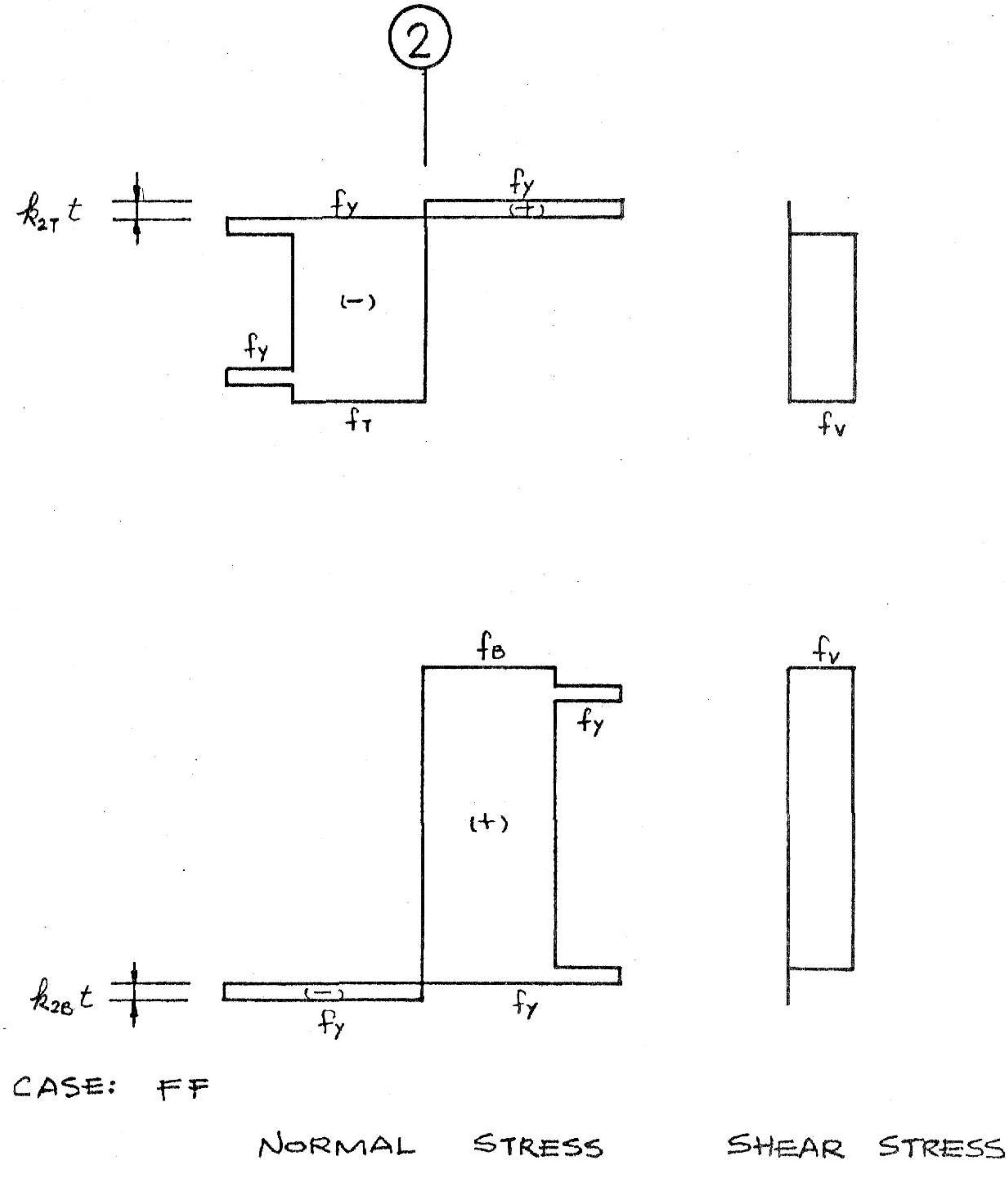
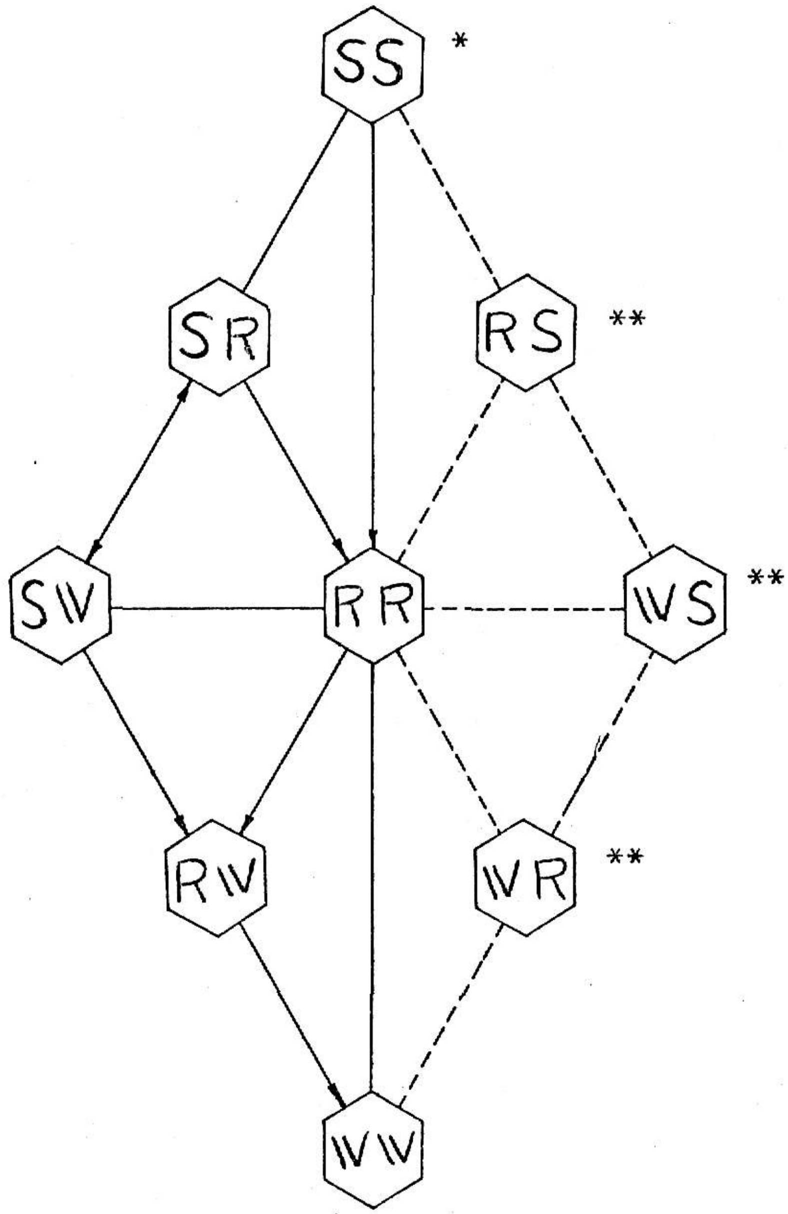


Fig. 5 : Stress Distribution at Section 2
for Low Shear Case



*. Notation defined in TABLE 1.

**. Cases not possible for positive eccentricity.

Fig. 6 : Possible Combinations for Low Shear Case at Section 1

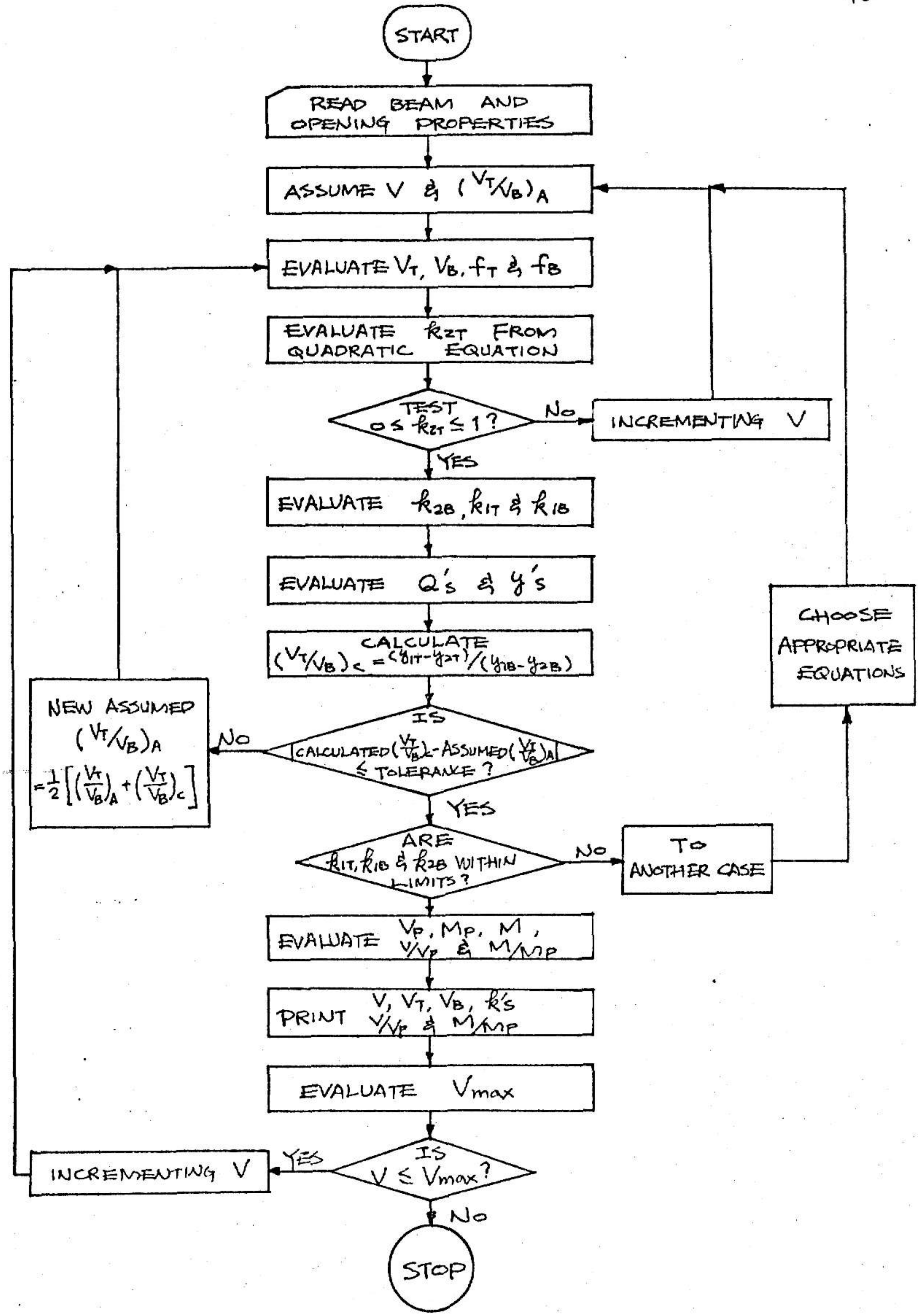


Fig. 7 : Flow Chart for Evaluating the Interaction Diagram

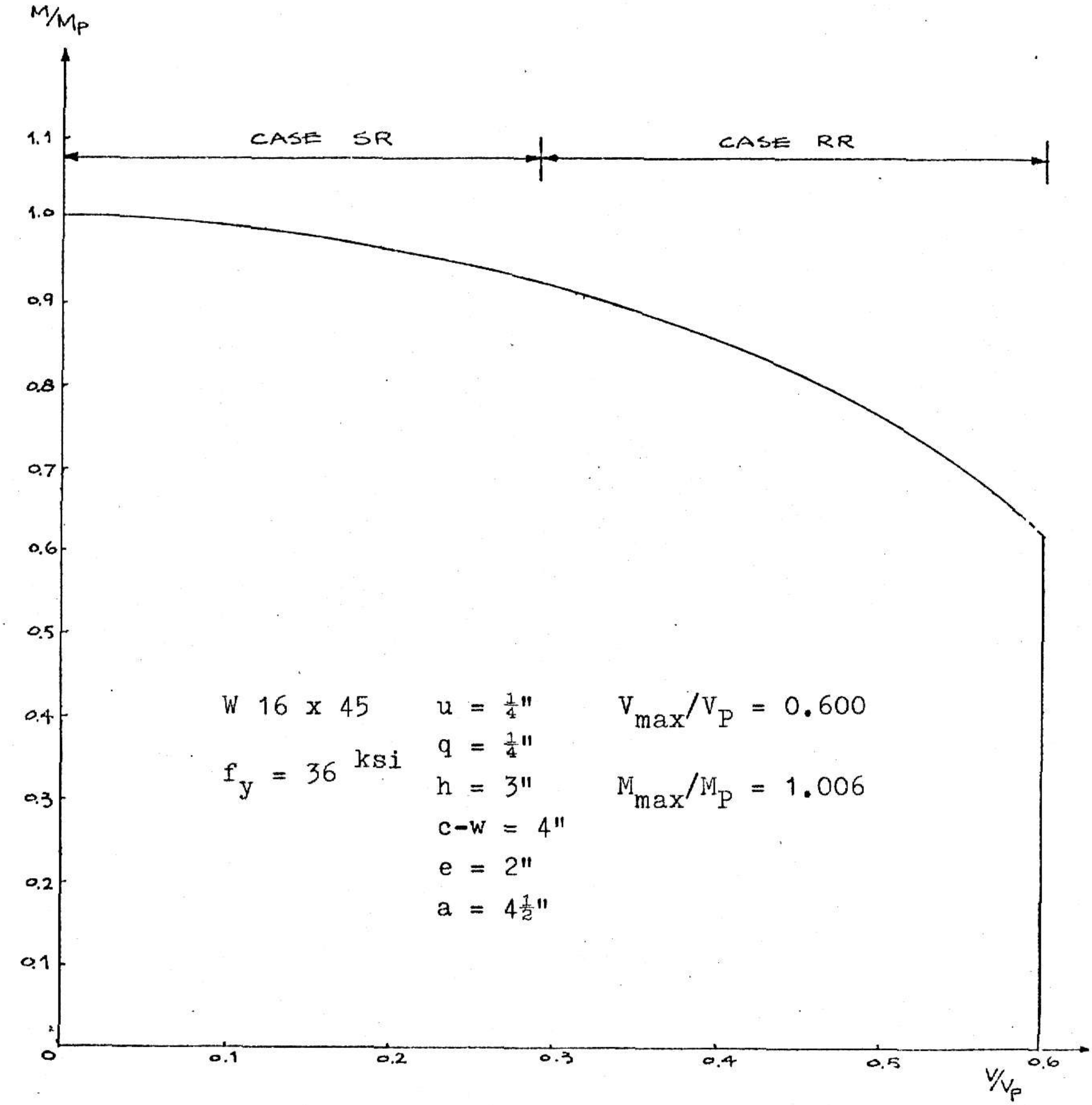


Fig. 8 : Interaction Curve for the Basic Case

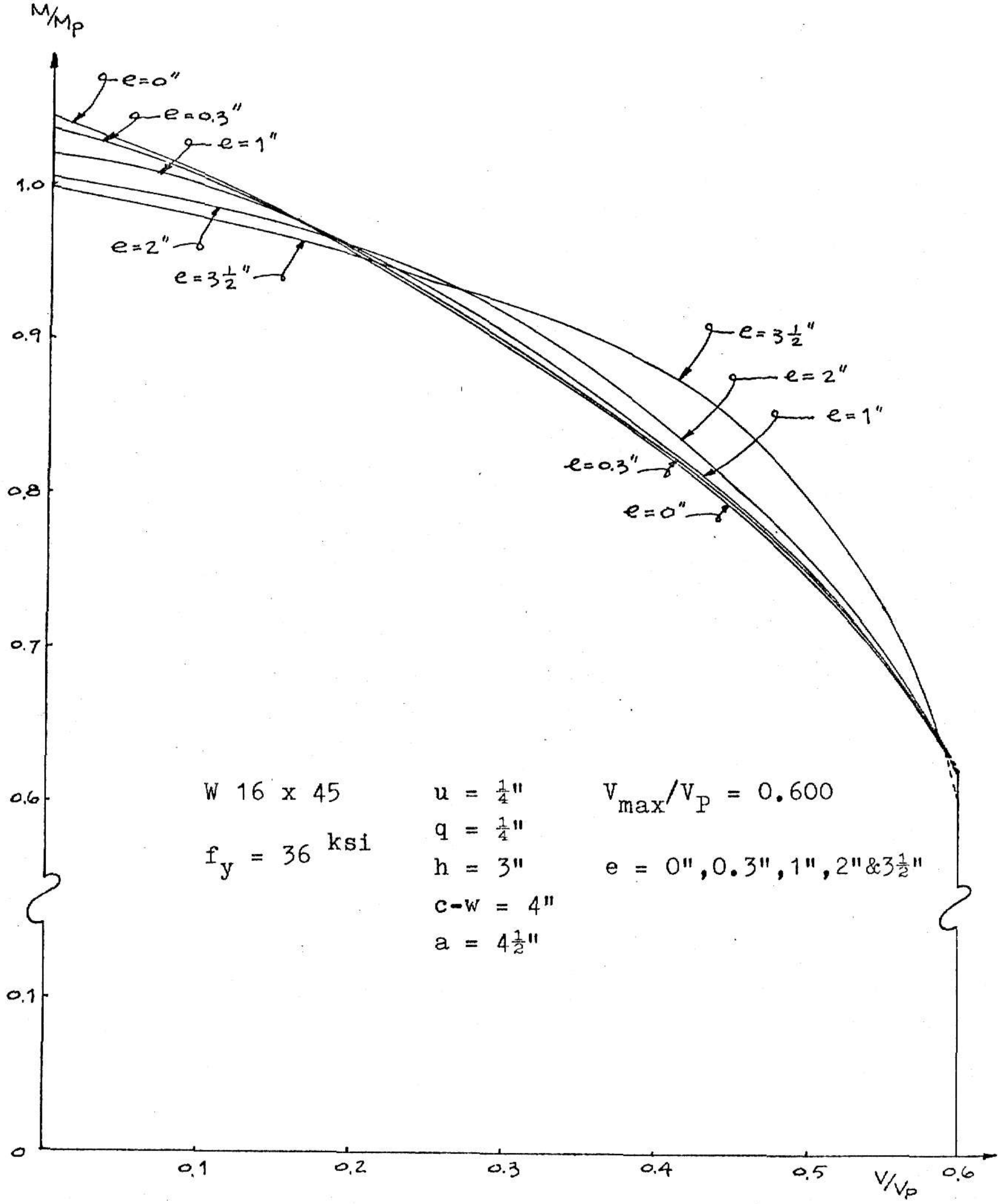


Fig. 9 : Interaction Curve with Varying Eccentricities

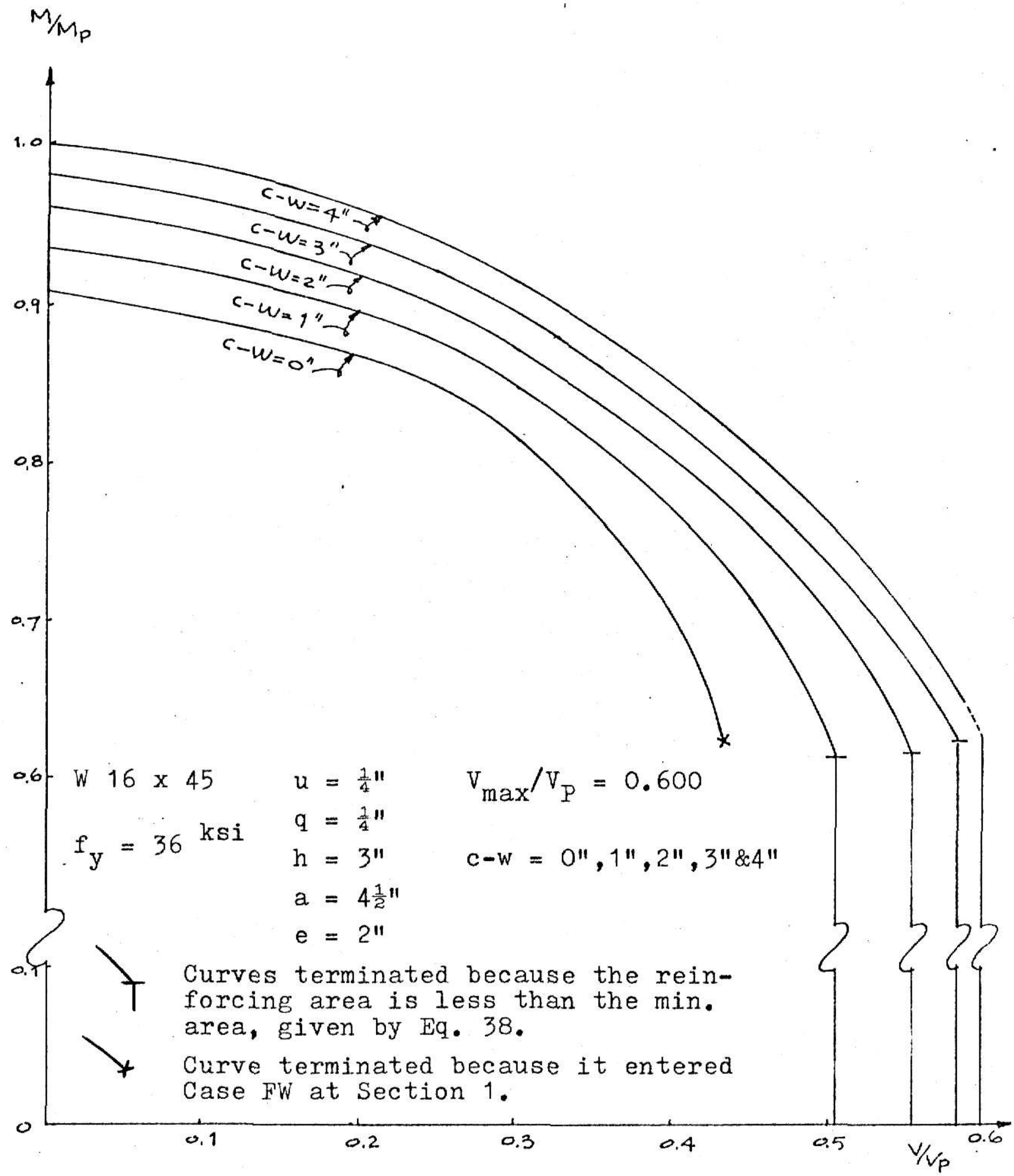


Fig. 10 : Interaction Curve with Varying Reinforcing Areas

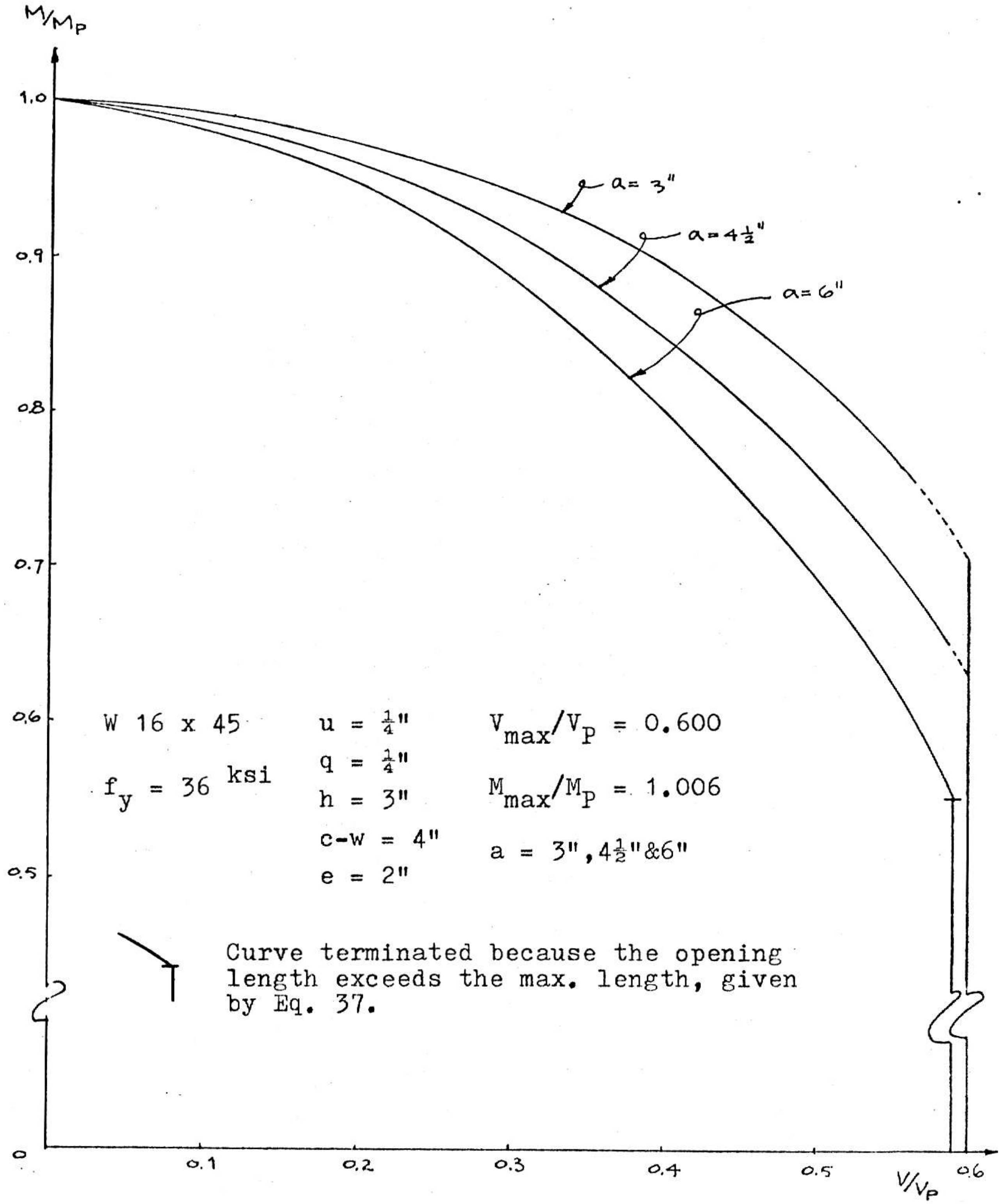


Fig. 11 : Interaction Curve with Varying Opening Lengths

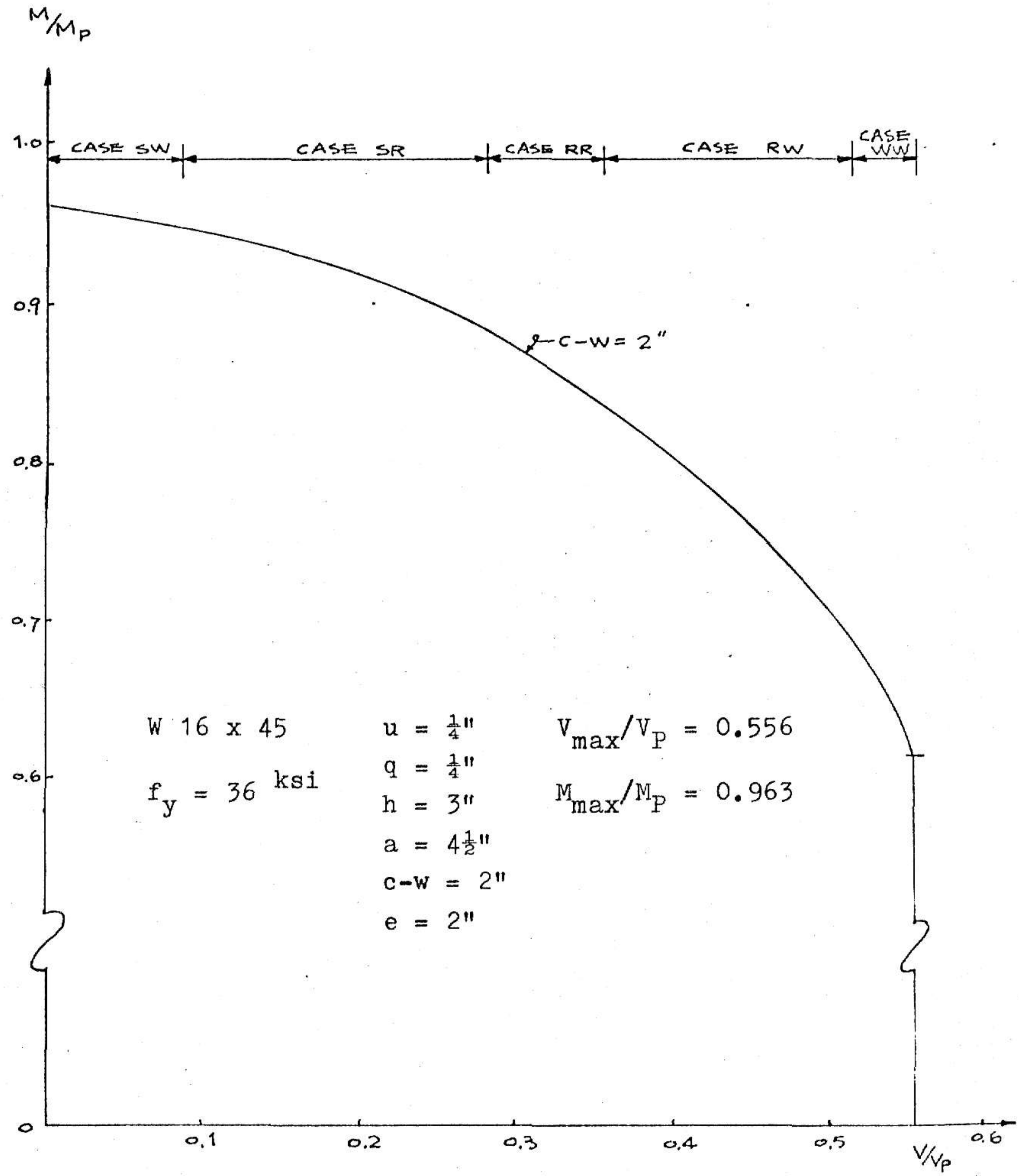


Fig. 12 : Interaction Curve for Illustrating Stress Reversal Cases

TABLE 1. Locations of Points of Stress Reversal

Section	Case	Stress reversal at top T-section in :	Stress reversal at bottom T-section in:
1	SS (L)[1]	Stub[2]	Stub
1	SR (L)	Stub	Reinforcing
1	SW (L)	Stub	Clear Web
1	RS* (L)	Reinforcing	Stub
1	RR (L)	Reinforcing	Reinforcing
1	RW (L)	Reinforcing	Clear Web
1	WS* (L)	Clear Web[3]	Stub
1	WR* (L)	Clear Web	Reinforcing
1	WW (L)	Clear Web	Clear Web
2	FF (L)	Flange	Flange

1. (L) - Low Shear Case.
2. Stub - Web between Reinforcing Bars and Opening Edge.
3. Clear Web - Web between Reinforcing Bars and Flange.

*. Cases not possible for Positive Eccentricity.

TABLE 2. Values of Parameters Investigated

a. Varying Parameter " e " :
($f_y=36^{ksi}$, $u=q=\frac{1}{4}$", h=3", $a=4\frac{1}{2}$" and c-w=4")

e(in)	Group	Complete Solution ?	Cases Involved
0	I[1]	Yes	(SS)[4],RR
0.3	II[2]	Yes*	(SR),RR
1	II	Yes*	(SR),RR
2	II	Yes*	(SR),RR
$3\frac{1}{2}$	III[3]	Yes	(SW),RR,RW

b. Varying Parameter " c-w " :
($f_y=36^{ksi}$, $u=q=\frac{1}{4}$", h=3", $a=4\frac{1}{2}$" and e=2")

c-w (in)	Group	Complete Solution ?	Cases Involved
0	III	$V=0-47^k$ [5]	(SW),RW,WW
1	III	$V=0-54.5^k$ [6]	(SW),RW,WW
2	III	$V=0-60.0^k$ [6]	(SW),SR,RR,RW,WW
3	II	$V=0-63.0^k$ [6]	(SR),RR,RW,WW
4	II	Yes*	(SR),RR

TABLE 2. (cont'd)

c. Varying Parameter " a " :
($f_y=36^{ksi}$, $u=q=\frac{1}{4}$", h=3", e=2" and c-w = 4")

a(in)	Group	Complete Solution ?	Cases Involved
3	II	Yes*	(SR),RR
4½	II	Yes*	(SR),RR
6	II	$V=0-63.7^k$ 7	(SR),RR,RW,WW

1. Group I - For $e \leq u$, Starting with Case SS.
2. Group II - For $u < e \leq u+q+A_r/w$, Starting with Case SR.
3. Group III - For $e > u+q+A_r/w$, Starting with Case SW.
4. The Case in parenthesis is the first case in the solution.
5. The program terminated because it entered Case FW at Section 1.
6. The program terminated because the reinforcing area is less than the minimum area, given by Eq. 38.
7. The program terminated because the opening length exceeds the maximum length, given by Eq. 37.

*. The program terminated because of the sensitivity problem when V approached V_{max}.

NOMENCLATURE

a	Half length of opening
A_f	Flange area (b x t)
A_r	Reinforcing area (c - w)q
b	Flange width
c	Total width of the reinforcing bars (including web thickness)
d	Depth of beam
e	Opening eccentricity
h	Half height of opening
f_b	Bending stress
f_B	Normal stress, bottom tee-section
f_T	Normal stress, top tee-section
f_v	Shear stress
f_y	Yield stress
f_{yf}	Yield stress for flange
f_{yr}	Yield stress for reinforcing area
f_{yw}	Yield stress for web
k_{1B}	Stress reversal coefficient, section one (bottom)
k_{1T}	Stress reversal coefficient, section one (top)
k_{2B}	Stress reversal coefficient, section two (bottom)
k_{2T}	Stress reversal coefficient, section two (top)
L	Horizontal distance from opening center line to closest support
q	Reinforcing bar thickness

Q_{1B}	Total normal force, section one (bottom)
Q_{1T}	Total normal force, section one (top)
Q_{2B}	Total normal force, section two (bottom)
Q_{2T}	Total normal force, section two (top)
M	Moment at center line of opening
M_P	Plastic moment capacity, uncut section
S_B	Web depth, bottom tee-section
S_T	Web depth, top tee-section
t	Flange thickness
u	Distance between opening and reinforcing bar
V	Total shear force ($V_T + V_B$)
V_B	Shear force, bottom tee-section
V_P	Plastic shear capacity, uncut section
V_T	Shear force, top tee-section
w	Web thickness
y_{1B}	Distance from opening edge to Q_{1B}
y_{1T}	Distance from opening edge to Q_{1T}
y_{2B}	Distance from opening edge to Q_{2B}
y_{2T}	Distance from opening edge to Q_{2T}
Case F	Stress reversal in the flange
Case R	Stress reversal in the reinforcing bars
Case S	Stress reversal in the web stub between reinforcing bars and opening edge
Case W	Stress reversal in the clear web between flange and reinforcing bars

REFERENCES

1. Bower, J.E., " Ultimate Strength of Beams with Rectangular Holes ", Journal of the Structural Division, A.S.C.E., Vol. 94, No. ST6, Proc. Paper 5982, June 1968.

2. Redwood, R.G., " The Strength of Steel Beams with Unreinforced Web Holes ", Civil Engineering and Public Works Review, Vol. 64, No. 755, London, June 1969.

3. Congdon, J.G., " Ultimate Strength of Beams with Reinforced Rectangular Openings ", Structural Mechanics Series - No. 14, Structures Laboratory of McGill University, Montreal, Canada. July 1969.

4. Richard, M.Y., " Ultimate Strength Analysis of Beams with Eccentric Rectangular Web Openings ", M.S. Thesis, Department of Civil Engineering, Kansas State University, Manhattan, Kansas, 1971.

5. Frost, R.W., " Behavior of Steel Beams with Eccentric Web Holes ", Technical Report 46.019-400(1), Applied Research Laboratory, United State Steel Corporation, February 1973.

6. Marin, J., Mechanical Behavior of Engineering Materials, Prentice-Hall, Inc. Englewood Cliffs, N.J., Second Printing, March 1963.

APPENDIX A

SUMMARY OF EQUATIONS FOR LOW AND HIGH SHEAR CASES

1. Low Shear Case SR :

a. Stress resultants :

$$Q_{1T} = A_f f_y + S_T w(1-2k_{1T})f_T + A_r f_y$$

$$Q_{1B} = A_f f_y + S_B w(1-2k_{1B})f_B + (c-w)(2u+q-2k_{1B}S_B)f_y$$

$$Q_{2T} = A_f f_y(1-2k_{2T}) + S_T w f_T + A_r f_y$$

$$Q_{2B} = A_f f_y(1-2k_{2B}) + S_B w f_B + A_r f_y$$

$$Q_{1T}y_{1T} = A_f f_y(S_T+0.5t) + 0.5S_T^2 w f_T(1-2k_{1T}^2) + A_r f_y(u+\frac{q}{2})$$

$$Q_{1B}y_{1B} = A_f f_y(S_B+0.5t) + 0.5S_B^2 w f_B(1-2k_{1B}^2) + (c-w)\left[u^2+uq+\frac{q^2}{2}-(k_{1B}S_B)^2\right]f_y$$

$$Q_{2T}y_{2T} = A_f f_y\left[(S_T+0.5t)-2k_{2T}(S_T+t)+tk_{2T}^2\right] + 0.5S_T^2 w f_T + A_r f_y(u+\frac{q}{2})$$

$$Q_{2B}y_{2B} = A_f f_y\left[(S_B+0.5t)-2k_{2B}(S_B+t)+tk_{2B}^2\right] + 0.5S_B^2 w f_B + A_r f_y(u+\frac{q}{2})$$

b. Stress reversal coefficients :

$$k_{1T} = \frac{A_f f_y}{S_T w f_T}\, k_{2T}$$

$$k_{1B} = \frac{1}{S_B w f_B + S_B (c-w) f_y} \left[A_f f_y k_{2T} + u(c-w) f_y + \frac{w}{2}(S_B f_B - S_T f_T)\right]$$

$$k_{2B} = \frac{w(S_B f_B - S_T f_T)}{2A_f f_y} + k_{2T}$$

c. Quadratic equation :

$$A_{SR} k_{2T}^2 + B_{SR} k_{2T} + C_{SR} = 0$$

$$A_{SR} = \frac{A_f f_y}{w f_T} + \frac{A_f f_y S_B}{S_B w f_B + S_B (c-w) f_y} + 2t$$

$$B_{SR} = -2(d-2h) + \frac{2u(c-w) f_y S_B}{S_B w f_B + S_B (c-w) f_y} + \frac{w(S_B f_B - S_T f_T) S_B}{S_B w f_B + S_B (c-w) f_y} + \frac{tw(S_B f_B - S_T f_T)}{A_f f_y}$$

$$C_{SR} = \frac{2Va}{A_f f_y} - \frac{u^2 (c-w)}{A_f} + \frac{u^2 (c-w)^2 f_y S_B}{A_f \left[S_B w f_B + S_B (c-w) f_y\right]} + \frac{w^2 (S_B f_B - S_T f_T)^2 S_B}{4A_f f_y \left[S_B w f_B + S_B (c-w) f_y\right]} + \frac{wu(c-w) S_B (S_B f_B - S_T f_T)}{A_f \left[S_B w f_B + S_B (c-w) f_y\right]} - \frac{w(S_B f_B - S_T f_T)(S_B + t)}{A_f f_y} + \frac{tw^2 (S_B f_B - S_T f_T)^2}{4A_f^2 f_y^2}$$

2. Low Shear Case SW :

a. Stress resultants :

$$Q_{1T} = A_f f_y + S_T w(1-2k_{1T}) f_T + A_r f_y$$

$$Q_{1B} = A_f f_y + S_B w(1-2k_{1B}) f_B - A_r f_y$$

$$Q_{2T} = A_f f_y (1-2k_{2T}) + S_T w f_T + A_r f_y$$

$$Q_{2B} = A_f f_y (1-2k_{2B}) + S_B w f_B + A_r f_y$$

$$Q_{1T}y_{1T} = A_f f_y(S_T+0.5t) + 0.5S_T^2 wf_T(1-2k_{1T}^2)$$

$$+ A_r f_y(u+\frac{g}{2})$$

$$Q_{1B}y_{1B} = A_f f_y(S_B+0.5t) + 0.5S_B^2 wf_B(1-2k_{1B}^2)$$

$$- A_r f_y(u+\frac{g}{2})$$

$$Q_{2T}y_{2T} = A_f f_y\left[(S_T+0.5t)-2k_{2T}(S_T+t)+tk_{2T}^2\right]$$

$$+ 0.5S_T^2 wf_T + A_r f_y(u+\frac{g}{2})$$

$$Q_{2B}y_{2B} = A_f f_y\left[(S_B+0.5t)-2k_{2B}(S_B+t)+tk_{2B}^2\right]$$

$$+ 0.5S_B^2 wf_B + A_r f_y(u+\frac{g}{2})$$

b. Stress reversal coefficients :

$$k_{1T} = \frac{A_f f_y}{S_T wf_T} k_{2T}$$

$$k_{1B} = \frac{1}{2} - \frac{S_T f_T}{2S_B f_B} - \frac{A_r f_y}{S_B wf_B} + \frac{A_f f_y}{S_B wf_B} k_{2T}$$

$$k_{2B} = \frac{w(S_B f_B - S_T f_T)}{2A_f f_y} + k_{2T}$$

c. Quadratic equation :

$$A_{SW}k_{2T}^2 + B_{SW}k_{2T} + C_{SW} = 0$$

$$A_{SW} = \frac{A_f f_y}{w}\left(\frac{1}{f_T} + \frac{1}{f_B}\right) + 2t$$

$$B_{SW} = -2(d-2h) + \frac{t}{A_f f_y}(S_B f_B - S_T f_T)(w+\frac{bf_y}{f_B}) - \frac{2A_r f_y}{wf_B}$$

$$C_{SW} = \frac{2Va}{A_f f_y} + \frac{w(S_B f_B - S_T f_T)^2}{4A_f f_y}\left(\frac{1}{f_B} + \frac{w}{bf_y}\right) + \frac{A_r}{A_f}(2u+q)$$

$$- \frac{w}{A_f f_y}(S_B f_B - S_T f_T)(S_B + t) + \frac{A_r^2 f_y}{A_f w f_B} - \frac{A_r}{A_f f_B}(S_B f_B - S_T f_T)$$

3. Low Shear Case RR :

a. Stress resultants :

$$Q_{1T} = A_f f_y + S_T w(1-2k_{1T})f_T + (c-w)(2u+q-2k_{1T}S_T)f_y$$
$$Q_{1B} = A_f f_y + S_B w(1-2k_{1B})f_B + (c-w)(2u+q-2k_{1B}S_B)f_y$$
$$Q_{2T} = A_f f_y(1-2k_{2T}) + S_T w f_T + A_r f_y$$
$$Q_{2B} = A_f f_y(1-2k_{2B}) + S_B w f_B + A_r f_y$$

$$Q_{1T}y_{1T} = A_f f_y(S_T+0.5t) + 0.5S_T^2 w f_T(1-2k_{1T}^2)$$
$$+ (c-w)\left[u^2+uq+\frac{q^2}{2}-(k_{1T}S_T)^2\right]f_y$$

$$Q_{1B}y_{1B} = A_f f_y(S_B+0.5t) + 0.5S_B^2 w f_B(1-2k_{1B}^2)$$
$$+ (c-w)\left[u^2+uq+\frac{q^2}{2}-(k_{1B}S_B)^2\right]f_y$$

$$Q_{2T}y_{2T} = A_f f_y\left[(S_T+0.5t)-2k_{2T}(S_T+t)+tk_{2T}^2\right]$$
$$+ 0.5S_T^2 w f_T + A_r f_y\left(u+\frac{q}{2}\right)$$

$$Q_{2B}y_{2B} = A_f f_y\left[(S_B+0.5t)-2k_{2B}(S_B+t)+tk_{2B}^2\right]$$
$$+ 0.5S_B^2 w f_B + A_r f_y\left(u+\frac{q}{2}\right)$$

b. Stress reversal coefficients :

$$k_{1T} = \frac{1}{S_T w f_T + S_T(c-w)f_y}\left[A_f f_y k_{2T} + u(c-w)f_y\right]$$

$$k_{1B} = \frac{1}{S_B w f_B + S_B(c-w)f_y}\left[A_f f_y k_{2T} + u(c-w)f_y + \frac{w}{2}(S_B f_B - S_T f_T)\right]$$

$$k_{2B} = \frac{w(S_B f_B - S_T f_T)}{2A_f f_y} + k_{2T}$$

c. Quadratic equation :

$$A_{RR}k_{2T}^2 + B_{RR}k_{2T} + C_{RR} = 0$$

$$A_{RR} = \frac{A_f f_y S_T}{S_T w f_T + S_T(c-w)f_y} + \frac{A_f f_y S_B}{S_B w f_B + S_B(c-w)f_y} + 2t$$

$$B_{RR} = -2(d-2h) - \frac{2u(c-w)f_y S_T}{S_T w f_T + S_T(c-w)f_y} + \frac{2u(c-w)f_y S_B}{S_B w f_B + S_B(c-w)f_y}$$

$$+ \frac{w(S_B f_B - S_T f_T)S_B}{S_B w f_B + S_B(c-w)f_y} + \frac{tw(S_B f_B - S_T f_T)}{A_f f_y}$$

$$C_{RR} = \frac{2Va}{A_f f_y} - \frac{2u^2(c-w)}{A_f} + \frac{u^2(c-w)^2 f_y S_T}{A_f\left[S_T w f_T + S_T(c-w)f_y\right]}$$

$$+ \frac{u^2(c-w)^2 f_y S_B}{A_f\left[S_B w f_B + S_B(c-w)f_y\right]} + \frac{w^2(S_B f_B - S_T f_T)^2 S_B}{4A_f f_y\left[S_B w f_B + S_B(c-w)f_y\right]}$$

$$+ \frac{wu(c-w)S_B(S_B f_B - S_T f_T)}{A_f\left[S_B w f_B + S_B(c-w)f_y\right]} - \frac{w(S_B + t)(S_B f_B - S_T f_T)}{A_f f_y}$$

$$+ \frac{tw^2(S_B f_B - S_T f_T)^2}{4A_f^2 f_y^2}$$

4. Low Shear Case RW :

a. Stress resultants :

$$Q_{1T} = A_f f_y + S_T w(1-2k_{1T})f_T + (c-w)(2u+q-2k_{1T}S_T)f_y$$

$$Q_{1B} = A_f f_y + S_B w(1-2k_{1B})f_B - A_r f_y$$

$$Q_{2T} = A_f f_y(1-2k_{2T}) + S_T w f_T + A_r f_y$$

$$Q_{2B} = A_f f_y(1-2k_{2B}) + S_B w f_B + A_r f_y$$

$$Q_{1T}y_{1T} = A_f f_y(S_T+0.5t) + 0.5S_T^2 w f_T(1-2k_{1T}^2)$$
$$+ (c-w)\left[u^2+uq+\frac{q^2}{2}-(k_{1T}S_T)^2\right]f_y$$

$$Q_{1B}y_{1B} = A_f f_y(S_B+0.5t) + 0.5S_B^2 w f_B(1-2k_{1B}^2)$$
$$- A_r f_y(u+\frac{q}{2})$$

$$Q_{2T}y_{2T} = A_f f_y\left[(S_T+0.5t)-2k_{2T}(S_T+t)+tk_{2T}^2\right]$$
$$+ 0.5S_T^2 w f_T + A_r f_y(u+\frac{q}{2})$$

$$Q_{2B}y_{2B} = A_f f_y\left[(S_B+0.5t)-2k_{2B}(S_B+t)+tk_{2B}^2\right]$$
$$+ 0.5S_B^2 w f_B + A_r f_y(u+\frac{q}{2})$$

b. Stress reversal coefficients :

$$k_{1T} = \frac{1}{S_T w f_T + S_T(c-w)f_y}\left[A_f f_y k_{2T} + u(c-w)f_y\right]$$

$$k_{1B} = \frac{1}{2} - \frac{S_T f_T}{2S_B f_B} - \frac{A_r f_y}{S_B w f_B} + \frac{A_f f_y}{S_B w f_B}\, k_{2T}$$

$$k_{2B} = \frac{w(S_B f_B - S_T f_T)}{2A_f f_y} + k_{2T}$$

c. Quadratic equation :

$$A_{RW}k_{2T}^2 + B_{RW}k_{2T} + C_{RW} = 0$$

$$A_{RW} = \frac{A_f f_y}{wf_B} + \frac{A_f f_y S_T}{S_T w f_T + S_T(c-w)f_y} + 2t$$

$$B_{RW} = -2(d-2h) + \frac{2u(c-w)f_y S_T}{S_T w f_T + S_T(c-w)f_y} - \frac{2A_r f_y}{wf_B}$$

$$+ \frac{t(S_B f_B - S_T f_T)}{A_f f_y}\left(\frac{bf_y}{f_B} + w\right)$$

$$C_{RW} = \frac{2Va}{A_f f_y} - \frac{(c-w)(u^2-2qu-q^2)}{A_f} + \frac{w(S_B f_B - S_T f_T)^2}{4\ A_f f_y f_B}$$

$$+ \frac{u^2(c-w)^2 f_y S_T}{A_f\left[S_T w f_T + S_T(c-w)f_y\right]} + \frac{A_r^2 f_y}{A_f w f_B} - \frac{A_r(S_B f_B - S_T f_T)}{A_f f_B}$$

$$- \frac{(S_B+t)w(S_B f_B - S_T f_T)}{A_f f_y} + \frac{tw^2(S_B f_B - S_T f_T)^2}{4A_f^2 f_y^2}$$

5. <u>Low Shear Case WW</u> :

a. Stress resultants :

$$Q_{1T} = A_f f_y + S_T w(1-2k_{1T})f_T - A_r f_y$$

$$Q_{1B} = A_f f_y + S_B w(1-2k_{1B})f_B - A_r f_y$$

$$Q_{2T} = A_f f_y(1-2k_{2T}) + S_T w f_T + A_r f_y$$

$$Q_{2B} = A_f f_y(1-2k_{2B}) + S_B w f_B + A_r f_y$$

$$Q_{1T}y_{1T} = A_f f_y(S_T+0.5t) + 0.5S_T^2 w f_T(1-2k_{1T}^2)$$

$$- A_r f_y\left(u+\frac{q}{2}\right)$$

$$Q_{1B}y_{1B} = A_f f_y(S_B+0.5t) + 0.5S_B^2 w f_B(1-2k_{1B}^2)$$

$$- A_r f_y(u+\frac{g}{2})$$

$$Q_{2T}y_{2T} = A_f f_y\left[(S_T+0.5t)-2k_{2T}(S_T+t)+tk_{2T}^2\right]$$

$$+ 0.5S_T^2 w f_T + A_r f_y(u+\frac{g}{2})$$

$$Q_{2B}y_{2B} = A_f f_y\left[(S_B+0.5t)-2k_{2B}(S_B+t)+tk_{2B}^2\right]$$

$$+ 0.5S_B^2 w f_B + A_r f_y(u+\frac{g}{2})$$

b. Stress reversal coefficients :

$$k_{1T} = \frac{1}{S_T w f_T}(A_f f_y k_{2T} - A_r f_y)$$

$$k_{1B} = \frac{1}{2} - \frac{S_T f_T}{2S_B f_B} - \frac{A_r f_y}{S_B w f_B} + \frac{A_f f_y}{S_B w f_B} k_{2T}$$

$$k_{2B} = \frac{w(S_B f_B - S_T f_T)}{2A_f f_y} + k_{2T}$$

c. Quadratic equation :

$$A_{WW}k_{2T}^2 + B_{WW}k_{2T} + C_{WW} = 0$$

$$A_{WW} = \frac{A_f f_y}{w}(\frac{1}{f_T} + \frac{1}{f_B}) + 2t$$

$$B_{WW} = -2(d-2h) + \frac{t}{A_f f_y}(S_B f_B - S_T f_T)(w+\frac{bf_y}{f_B})$$

$$- \frac{2A_r f_y}{w}(\frac{1}{f_T} + \frac{1}{f_B})$$

$$C_{WW} = \frac{2Va}{A_f f_y} + \frac{w(S_B f_B - S_T f_T)^2}{4A_f f_y}\left(\frac{1}{f_B} + \frac{w}{bf_y}\right)$$

$$- \frac{w}{A_f f_y}(S_B f_B - S_T f_T)(S_B + t) + \frac{A_r^2 f_y}{wA_f}\left(\frac{1}{f_T} + \frac{1}{f_B}\right)$$

$$- \frac{A_r(S_B f_B - S_T f_T)}{A_f f_B} + \frac{2A_r}{A_f}(2u+q)$$

APPENDIX B

COMPUTER PROGRAMS

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

```
C
C
C                                    APPENDIX  B
C
C
C
C
C
C     THIS IS THE FIRST PACKAGE OF THE COMPUTER PROGRAM TO BE PUT IN
C     FRONT OF EACH CASE.
C
C     ******************************************************************
C
C     THIS IS A PROGRAM FOR SOLVING THE INTERACTION DIAGRAM OF V/VP AND
C     M/MP FOR THE LOW SHEAR CASES :
C
C     ******************************************************************
C
      READ(5,20) B,D,T,W,U,Q,C,H,A,FY,ECCTRY,ROK2T,HIK2T,ROK2B,HIK2B
   20 FORMAT(8F10.3)
      ST=D/2.0-T-ECCTRY-H
      SB=D/2.0-T+ECCTRY-H
      WRITE(6,21) B,D,T,ST,SB
   21 FORMAT(//,'         B         D         T         ST        SB '//,
     1F10.3)
      WRITE(6,22) W,U,Q,C
   22 FORMAT(//,'         W         U         Q         C  '//,4F10.3)
      WRITE(6,23) H,A,FY,ECCTRY
   23 FORMAT(//,'         H         A         FY        ECCTRY  '//,4F10.3)
      AF=B*T
      AR=(C-W)*Q
      VP=W*(D-2.0*T)*FY/SQRT(3.)
      VMAX=W*(D-2.0*T-2.0*H)*FY/SQRT(3.)
      PM=FY*(AF*(D-T)+W*(D-2.0*T)**2/4.0)
      HIVVP=VMAX/VP
      WRITE(6,24) VP,VMAX,PM,HIVVP
   24 FORMAT(//,'         VP          VMAX          MP          VMAX/VP'//,2F10.5,F
     12.5,F10.5)
      WRITE(6,25) ROK2T,HIK2T,ROK2B,HIK2B
   25 FORMAT(//,'    LOK2T     HIK2T     LOK2B     HIK2B     FOR ALL THE F
     1OLLOWING CASES'/,4F10.5)
C
C     ******************************************************************
C
C     THIS SPACE IS RESERVED FOR THE APPROPRIATE CASE TO BE RUN.
C
C     ******************************************************************
C
C     THE FOLLOWING PACKAGE IS THE LAST ONE TO BE PUT AT THE END OF THE
C     PROGRAM.
C
C     ******************************************************************
C
 1990 WRITE(6,1991)
 1991 FORMAT(//,'   V IS GREATER THAN VMAX')
      GO TO 2000
 1992 WRITE(6,1993)
 1993 FORMAT(//,'   SET OR SB IS TOO SMALL')
 2000 STOP
      END
```

```
C
C
C
C     ****************************************************************************
C
C     CASE : SS  (THAT MEANS BOTH K1T AND K1B ARE LOCATED IN THE WEB BE
C                LOW THE REINFORCING BARS. )
C
C     ****************************************************************************
C
      ROK1T=0.0
      HIK1T=U/ST
      ROK1B=0.0
      HIK1B=U/SB
      WRITE(6,28) ROK1T,HIK1T,ROK1B,HIK1B
   28 FORMAT(/,'     LOK1T        HIK1T        LOK1B        HIK1B     FOR THE FOLLO
     1ING CASE SS'/,4F10.5)
      WRITE(6,29)
   29 FORMAT(//,'       V              VT              VB              V1              V2
     1K1T          K1B          K2T          K2B          V/VP          M/MP'//)
C
C     THIS SPACE IS RESERVED FOR ARBITRARY VALUE OF V AND V1 .
C
      VINCR=1.0
   30 VB=V/(1.0+V1)
      VT=V-VB
      FTEX=FY**2-3.0*VT**2/(ST**2*W**2)
      IF (FTEX .GT. 0.0) GO TO 35
      WRITE(6,32) FTEX
   32 FORMAT(/,'    FTEX =',F10.5)
      GO TO 2000
   35 FBEX=FY**2-3.0*VB**2/(SB**2*W**2)
      IF (FBEX .GT. 0.0) GO TO 40
      WRITE(6,37) FBEX
   37 FORMAT(/,'    FBEX =',F10.5)
      GO TO 2000
   40 FT=SQRT(FTEX)
      FB=SQRT(FBEX)
      IF (FT .LT. 0.1) GO TO 1992
      IF (FB .LT. 0.1) GO TO 1992
      ASS=AF*FY*(1.0/FT+1.0/FB)/W+2.0*T
      BSS=-2.0*(D-2.0*H)+T*(SB*FB-ST*FT)*(W+B*FY/FB)/(AF*FY)
      CSS=2.0*A*V/(AF*FY)+W*(SB*FB-ST*FT)**2*(1.0/FB+W/(B*FY))/(4.0*AF*F
     1Y)-W*(SB*FB-ST*FT)*(SB+T)/(AF*FY)
      DO=BSS**2-4.0*ASS*CSS
      IF (DO .GT. 0.0) GO TO 50
      WRITE(6,45) DO
   45 FORMAT(5H DO =F15.5)
      GO TO 2000
   50 R2T1=(-BSS+SQRT(DO))/(2.0*ASS)
      R2T2=(-BSS-SQRT(DO))/(2.0*ASS)
      IF (R2T1 .LT. 0.0) GO TO 52
      IF (R2T1 .LE. 1.0) GO TO 60
   52 IF (R2T2 .LT. 0.0) GO TO 57
      IF (R2T2 .LE. 1.0) GO TO 61
      WRITE(6,55) R2T1,R2T2
   55 FORMAT(/,'     R2T1         R2T2'/,2F10.5//)
      GO TO 2000
   57 R2T=0.0
      K=1
```

```
      GO TO 65
   60 R2T=R2T1
      K=0
      GO TO 65
   61 R2T=R2T2
      K=1
   65 R2B=W*(SB*FB-ST*FT)/(2.0*AF*FY)+R2T
      R1T=AF*FY*R2T/(ST*W*FT)
      R1B=AF*FY*R2B/(SB*W*FB)
      O1T=AF*FY+ST*W*FT*(1.0-2.0*R1T)+AR*FY
      O1B=AF*FY+SB*W*FB*(1.0-2.0*R1B)+AR*FY
      O2T=AF*FY*(1.0-2.0*R2T)+ST*W*FT+AR*FY
      O2B=AF*FY*(1.0-2.0*R2B)+SB*W*FB+AR*FY
      Y1T=(AF*FY*(ST+0.5*T)+0.5*ST**2*W*FT*(1.0-2.0*R1T**2)+AR*FY*(U+Q/
     1.0))/O1T
      Y1B=(AF*FY*(SB+0.5*T)+0.5*SB**2*W*FB*(1.0-2.0*R1B**2)+AR*FY*(U+Q/
     1.0))/O1B
      Y2T=(AF*FY*((ST+0.5*T)-2.0*R2T*(ST+T)+T*R2T**2)+0.5*ST**2*W*FT+A
     1FY*(U+Q/2.0))/O2T
      Y2B=(AF*FY*((SB+0.5*T)-2.0*R2B*(SB+T)+T*R2B**2)+0.5*SB**2*W*FB+AR
     1FY*(U+Q/2.0))/O2B
      V2=(Y1T-Y2T)/(Y1B-Y2B)
      IF (V2 .LT. 0.0) GO TO 85
      IF (V2 .GT. 1.0) GO TO 85
      R=V2-V1
      IF (ABS(R) .LE. 0.001) GO TO 66
      V1=(V1+V2)/2.0
      GO TO 30
   66 IF (R1T .GT. HIK1T) GO TO 86
      IF (R1B .GT. HIK1B) GO TO 88
      Z=O1T*(Y1T+Y1B+2.0*H)-V*A
      VVP=V/VP
      PMM=Z/PM
      IF (PMM .GT. 0.0) GO TO 80
      WRITE(6,70) PMM
   70 FORMAT(/,'    M/MP ='F10.5)
      GO TO 2000
   80 WRITE(6,81) V,VT,VB,V1,V2,R1T,R1B,R2T,R2B,VVP,PMM
   81 FORMAT(11F10.5)
      V=V+VINCR
      IF (V .GT. VMAX) GO TO 1990
      GO TO 30
   85 IF (K .EQ. 0) GO TO 52
      GO TO 2000
   86 IF (K .EQ. 0) GO TO 52
      WRITE(6,87)
   87 FORMAT(//,'    TO NEXT CASE KS')
      GO TO 2000
   88 IF (K .EQ. 0) GO TO 52
      WRITE(6,89)
   89 FORMAT(//,'    TO NEXT CASE SR')
      GO TO 2000
```

```
C
C
C
C     ****************************************************************
C
C     CASE : SR  (THAT MEANS K1T IS LOCATED IN THE WEB BELOW THE REIN-
C                 FORCING BARS AND K1B IS LOCATED IN THE REINFORCING
C                 BARS. )
C
C     ****************************************************************
C
      ROK1T=0.0
      HIK1T=U/ST
      ROK1B=U/SB
      HIK1B=(U+O)/SB
      WRITE(6,190) ROK1T,HIK1T,ROK1B,HIK1B
  190 FORMAT(/,'   LOK1T      HIK1T      LOK1B      HIK1B    FOR THE FOLLO
     1ING CASE SR'/.4F10.5)
      WRITE(6,200)
  200 FORMAT(//,'      V          VT          VB          V1          V2
     1K1T         K1B         K2T         K2B         V/VP        M/MP'//)
C
C     THIS SPACE IS RESERVED FOR ARBITRARY VALUE OF V AND V1 .
C
      VINCR=1.0
  210 VB=V/(1.0+V1)
      VT=V-VB
      FTEX=FY**2-3.0*VT**2/(ST**2*W**2)
      IF (FTEX .GT. 0.0) GO TO 214
      WRITE(6,213) FTEX
  213 FORMAT(/,'   FTEX =',F10.5)
      GO TO 2000
  214 FBEX=FY**2-3.0*VB**2/(SB**2*W**2)
      IF (FBEX .GT. 0.0) GO TO 217
      WRITE(6,216) FBEX
  216 FORMAT(/,'   FBEX =',F10.5)
      GO TO 2000
  217 FT=SORT(FTEX)
      FB=SORT(FBEX)
      IF (FT .LT. 0.1) GO TO 1992
      IF (FB .LT. 0.1) GO TO 1992
      PARA1=SB*W*FB+SB*(C-W)*FY
      PARA2=W*(SB*FB-ST*FT)
      PARA3=U*(C-W)*FY
      ASR=AF*FY/(W*FT)+AF*FY*SB/PARA1+2.0*T
      BSR=-2.0*(D-2.0*H)+2.0*PARA3*SB/PARA1+PARA2*SB/PARA1+T*PARA2/(AF*
     1Y)
      CSR=2.0*A*V/(AF*FY)-U*PARA3/(AF*FY)+SB*PARA3**2/(AF*FY*PARA1)+SB*
     1ARA2**2/(4.0*AF*FY*PARA1)+SB*PARA2*PARA3/(AF*FY*PARA1)-(SB+T)*PAR
     22/(AF*FY)+T*PARA2**2/(4.0*AF**2*FY**2)
      D1=BSR**2-4.0*ASR*CSR
      IF (D1 .GT. 0.0) GO TO 220
      WRITE(6,219) D1
  219 FORMAT(5H D1 =F15.5)
      GO TO 2000
  220 R2T1=(-BSR+SORT(D1))/(2.0*ASR)
      R2T2=(-BSR-SORT(D1))/(2.0*ASR)
      IF (R2T1 .LT. 0.0) GO TO 230
      IF (R2T1 .LE. 1.0) GO TO 250
  230 IF (R2T2 .LT. 0.0) GO TO 243
```

```
      IF (R2T2 .LE. 1.0) GO TO 251
      WRITE(6,240) R2T1,R2T2
  240 FORMAT(2F10.5)
      GO TO 2000
  243 R2T=0.0
      K=1
      GO TO 260
  250 R2T=R2T1
      K=0
      GO TO 260
  251 R2T=R2T2
      K=1
  260 R2B=PARA2/(2.0*AF*FY)+R2T
      R1T=AF*FY*R2T/(ST*W*FT)
      R1B=(AF*FY*R2T+PARA3+PARA2/2.0)/PARA1
      Q1T=AF*FY+ST*W*FT*(1.0-2.0*R1T)+AR*FY
      Q1B=AF*FY+SB*W*FB*(1.0-2.0*R1B)+(C-W)*(2.0*U+Q-2.0*R1B*SB)*FY
      Q2T=AF*FY*(1.0-2.0*R2T)+ST*W*FT+AR*FY
      Q2B=AF*FY*(1.0-2.0*R2B)+SB*W*FB+AR*FY
      Y1T=(AF*FY*(ST+0.5*T)+0.5*ST**2*W*FT*(1.0-2.0*R1T**2)+AR*FY*(U+Q/
     1.0))/Q1T
      Y1B=(AF*FY*(SB+0.5*T)+0.5*SB**2*W*FB*(1.0-2.0*R1B**2)+(C-W)*(U**2
     1U*Q+Q**2/2.0-(R1B*SB)**2)*FY)/Q1B
      Y2T=(AF*FY*((ST+0.5*T)-2.0*R2T*(ST+T)+T*R2T**2)+0.5*ST**2*W*FT+AR
     1FY*(U+Q/2.0))/Q2T
      Y2B=(AF*FY*((SB+0.5*T)-2.0*R2B*(SB+T)+T*R2B**2)+0.5*SB**2*W*FB+AR
     1FY*(U+Q/2.0))/Q2B
      V2=(Y1T-Y2T)/(Y1B-Y2B)
      R=V2-V1
      IF (V2 .LT. 0.0) GO TO 269
      IF (V2 .GT. 1.0) GO TO 269
      IF (ABS(R) .LE. 0.001) GO TO 261
      V1=(V1+V2)/2.0
      GO TO 210
  261 IF (R1T .GT. HIK1T) GO TO 270
      IF (R1B .LT. ROK1B) GO TO 272
      IF (R1B .GT. HIK1B) GO TO 274
      Z=Q1T*(Y1T+Y1B+2.0*H)-V*A
      VVP=V/VP
      PMM=Z/PM
      IF (PMM .GT. 0.0) GO TO 267
      WRITE(6,266) PMM
  266 FORMAT(/,'    M/MP ='F10.5)
      GO TO 2000
  267 WRITE(6,268) V,VT,VB,V1,V2,R1T,R1B,R2T,R2B,VVP,PMM
  268 FORMAT(11F10.5)
      V=V+VINCR
      IF (V .GT. VMAX) GO TO 1990
      GO TO 210
  269 IF (K .EQ. 0) GO TO 230
      GO TO 2000
  270 IF (K .EQ. 0) GO TO 230
      WRITE(6,271)
  271 FORMAT(//,'    TO NEXT CASE RR')
      GO TO 2000
  272 IF (K .EQ. 0) GO TO 230
      WRITE(6,273)
  273 FORMAT(//,'    TO NEXT CASE SS')
      GO TO 2000
  274 IF (K .EQ. 0) GO TO 230
```

```
      WRITE(6,275)
  275 FORMAT(//,'    TO NEXT CASE SW')
      GO TO 2000
```

```
C
C
C
C     *******************************************************************
C
C     CASE : SW  (THAT MEANS K1T IS LOCATED IN THE WEB BELOW THE REIN-
C                 FORCING BARS AND K1B IS LOCATED IN THE CLEAR WEB. )
C
C     *******************************************************************
C
      ROK1T=0.0
      HIK1T=U/ST
      ROK1B=(U+O)/SB
      HIK1B=1.0
      WRITE(6,401) ROK1T,HIK1T,ROK1B,HIK1B
  401 FORMAT(/,'    LOK1T       HIK1T       LOK1B       HIK1B      FOR THE FOLL
     1ING CASE SW'/,4F10.5)
      WRITE(6,402)
  402 FORMAT(//,'        V             VT            VB            V1           V2
     1K1T          K1B          K2T          K2B          V/VP          M/MP'//)
C
C     THIS SPACE IS RESERVED FOR ARBITRARY VALUE OF V AND V1 .
C
      VINCR=1.0
  410 VB=V/(1.0+V1)
      VT=V-VB
      FTEX=FY**2-3.0*VT**2/(ST**2*W**2)
      IF (FTEX .GT. 0.0) GO TO 413
      WRITE(6,412) FTEX
  412 FORMAT(/,'    FTEX =',F10.5)
      GO TO 2000
  413 FBEX=FY**2-3.0*VB**2/(SB**2*W**2)
      IF (FBEX .GT. 0.0) GO TO 420
      WRITE(6,415) FBEX
  415 FORMAT(/,'    FBEX =',F10.5)
      GO TO 2000
  420 FT=SQRT(FTEX)
      FB=SQRT(FBEX)
      IF (FT .LT. 0.1) GO TO 1992
      IF (FB .LT. 0.1) GO TO 1992
      PARA2=W*(SB*FB-ST*FT)
      ASW=AF*FY*(1.0/FT+1.0/FB)/W+2.0*T
      BSW=-2.0*(D-2.0*H)+T*(SB*FB-ST*FT)*(W+B*FY/FB)/(AF*FY)-2.0*AR*FY/
     1W*FB)
      CSW=2.0*V*A/(AF*FY)+(1.0/FB+W/(B*FY))*PARA2**2/(4.0*W*AF*FY)-PARA
     1*(SB+T)/(AF*FY)+AR*(2.0*U+O)/AF+AR**2*FY/(AF*W*FB)-AR*PARA2/(W*F
     2AF)
      D2=BSW**2-4.0*ASW*CSW
      IF (D2 .GT. 0.0) GO TO 432
      WRITE(6,430) D2
  430 FORMAT(5H D2 =F15.5)
      GO TO 2000
  432 R2T1=(-BSW+SQRT(D2))/(2.0*ASW)
      R2T2=(-BSW-SQRT(D2))/(2.0*ASW)
      IF (R2T1 .LT. 0.0) GO TO 434
      IF (R2T1 .LE. 1.0) GO TO 439
  434 IF (R2T2 .LT. 0.0) GO TO 438
      IF (R2T2 .LE. 1.0) GO TO 440
      WRITE(6,435) R2T1,R2T2
  435 FORMAT(2F10.5)
```

```
      GO TO 2000
  438 R2T=0.0
      K=1
      GO TO 441
  439 R2T=R2T1
      K=0
      GO TO 441
  440 R2T=R2T2
      K=1
  441 R2B=PARA2/(2.0*AF*FY)+R2T
      R1T=AF*FY*R2T/(ST*W*FT)
      R1B=(AF*FY*R2B-AR*FY)/(SB*W*FB)
      Q1T=AF*FY+ST*W*FT*(1.0-2.0*R1T)+AR*FY
      Q1B=AF*FY+SB*W*FB*(1.0-2.0*R1B)-AR*FY
      Q2T=AF*FY*(1.0-2.0*R2T)+ST*W*FT+AR*FY
      Q2B=AF*FY*(1.0-2.0*R2B)+SB*W*FB+AR*FY
      Y1T=(AF*FY*(ST+0.5*T)+0.5*ST**2*W*FT*(1.0-2.0*R1T**2)+AR*FY*(U+Q/
     1.0))/Q1T
      Y1B=(AF*FY*(SB+0.5*T)+0.5*SB**2*W*FB*(1.0-2.0*R1B**2)-AR*FY*(U+Q/
     1.0))/Q1B
      Y2T=(AF*FY*((ST+0.5*T)-2.0*R2T*(ST+T)+T*R2T**2)+0.5*ST**2*W*FT+AR
     1FY*(U+Q/2.0))/Q2T
      Y2B=(AF*FY*((SB+0.5*T)-2.0*R2B*(SB+T)+T*R2B**2)+0.5*SB**2*W*FB+AR
     1FY*(U+Q/2.0))/Q2B
      V2=(Y1T-Y2T)/(Y1B-Y2B)
      R=V2-V1
      IF (V2 .LT. 0.0) GO TO 470
      IF (V2 .GT. 1.0) GO TO 470
      IF (ABS(R) .LE. 0.001) GO TO 450
      V1=(V1+V2)/2.0
      GO TO 410
  450 IF (R1T .GT. HIK1T) GO TO 480
      IF (R1B .LT. ROK1B) GO TO 482
      IF (R1B .GT. HIK1B) GO TO 484
      Z=Q1T*(Y1T+Y1B+2.0*H)-V*A
      VVP=V/VP
      PMM=Z/PM
      IF (PMM .GT. 0.0) GO TO 460
      WRITE(6,459) PMM
  459 FORMAT(/,'    M/MP ='F10.5)
      GO TO 2000
  460 WRITE(6,461) V,VT,VB,V1,V2,R1T,R1B,R2T,R2B,VVP,PMM
  461 FORMAT(11F10.5)
      V=V+VINCR
      IF (V .GT. VMAX) GO TO 1990
      GO TO 410
  470 IF (K .EQ. 0) GO TO 434
      GO TO 2000
  480 IF (K .EQ. 0) GO TO 434
      WRITE(6,481)
  481 FORMAT(//,'    TO NEXT CASE RW')
      GO TO 2000
  482 IF (K .EQ. 0) GO TO 434
      WRITE(6,483)
  483 FORMAT(//,'    TO NEXT CASE SR')
      GO TO 2000
  484 IF (K .EQ. 0) GO TO 434
      WRITE(6,485)
  485 FORMAT(//,'    TO NEXT CASE SF')
      GO TO 2000
```

```
C
C
C
C     ****************************************************************
C
C     CASE : RR  (THAT MEANS BOTH K1T AND K1B ARE LOCATED IN THE REIN-
C                    FORCING BARS. )
C
C     ****************************************************************
C
      ROK1T=U/ST
      HIK1T=(U+Q)/ST
      ROK1B=U/SB
      HIK1B=(U+Q)/SB
      WRITE(6,278) ROK1T,HIK1T,ROK1B,HIK1B
  278 FORMAT(/,'    LOK1T       HIK1T       LOK1B       HIK1B      FOR THE FOLLO
     1ING CASE RR'/,4F10.5)
      WRITE(6,279)
  279 FORMAT(//,'        V             VT            VB            V1          V2
     1K1T          K1B          K2T          K2B          V/VP          M/MP'//)
C
C     THIS SPACE IS RESERVED FOR ARBITRARY VALUE OF V AND V1 .
C
      VINCR=1.0
      IND=0
  280 VB=V/(1.0+V1)
      VT=V-VB
      FTEX=FY**2-3.0*VT**2/(ST**2*W**2)
      IF (FTEX .GT. 0.0) GO TO 284
      IF (ABS(R) .LE. 0.0005) GO TO 282
      GO TO 286
  282 WRITE(6,283) FTEX
  283 FORMAT(/,'    FTEX =',F10.5)
      GO TO 2000
  284 FBEX=FY**2-3.0*VB**2/(SB**2*W**2)
      IF (FBEX .GT. 0.0) GO TO 291
      IF (ABS(R) .LE. 0.0005) GO TO 289
  286 IF (IND .EQ. 1) GO TO 287
      IND=1
      VVNCR=VINCR/2.0
      GO TO 288
  287 VVNCR=VVNCR/2.0
  288 V=VV+VVNCR
      V1=VV1
      GO TO 280
  289 WRITE(6,290) FBEX
  290 FORMAT(/,'    FBEX =',F10.5)
      GO TO 2000
  291 FT=SQRT(FTEX)
      FB=SQRT(FBEX)
      IF (FT .LT. 0.1) GO TO 1992
      IF (FB .LT. 0.1) GO TO 1992
      PARA1=SB*W*FB+SB*(C-W)*FY
      PARA2=W*(SB*FB-ST*FT)
      PARA3=U*(C-W)*FY
      PARA4=ST*W*FT+ST*FY*(C-W)
      ARR=AF*FY*ST/PARA4+AF*FY*SB/PARA1+2.0*T
      BRR=-2.0*(D-2.0*H)+2.0*PARA3*ST/PARA4+2.0*PARA3*SB/PARA1+PARA2*S
     1PARA1+T*PARA2/(AF*FY)
      CRR=2.0*V*A/(AF*FY)-2.0*U*PARA3/(AF*FY)+PARA3**2*ST/(AF*FY*PARA4)
```

```
     1PARA3**2*SB/(AF*FY*PARA1)+PARA2**2*SB/(4.0*AF*FY*PARA1)+PARA2*PAR
     23*SB/(AF*FY*PARA1)-PARA2*(SB+T)/(AF*FY)+T*PARA2**2/(4.0*AF**2*FY*
     32)
      D3=BRR**2-4.0*ARR*CRR
      IF (D3 .GT. 0.0) GO TO 294
      WRITE(6,293) D3
  293 FORMAT(5H D3 =F15.5)
      GO TO 2000
  294 R2T1=(-BRR+SQRT(D3))/(2.0*ARR)
      R2T2=(-BRR-SQRT(D3))/(2.0*ARR)
      IF (R2T1 .LT. 0.0) GO TO 300
      IF (R2T1 .LE. 1.0) GO TO 320
  300 IF (R2T2 .LT. 0.0) GO TO 310
      IF (R2T2 .LE. 1.0) GO TO 321
  310 WRITE(6,311) R2T1,R2T2
  311 FORMAT(2F10.5)
      GO TO 2000
  320 R2T=R2T1
      K=0
      GO TO 325
  321 R2T=R2T2
      K=1
  325 R2B=PARA2/(2.0*AF*FY)+R2T
      R1T=(AF*FY*R2T+PARA3)/PARA4
      R1B=(AF*FY*R2T+PARA3+PARA2/2.0)/PARA1
      Q1T=AF*FY+ST*W*FT*(1.0-2.0*R1T)+(C-W)*(2.0*U+Q-2.0*R1T*ST)*FY
      Q1B=AF*FY+SB*W*FB*(1.0-2.0*R1B)+(C-W)*(2.0*U+Q-2.0*R1B*SB)*FY
      Q2T=AF*FY*(1.0-2.0*R2T)+ST*W*FT+AR*FY
      Q2B=AF*FY*(1.0-2.0*R2B)+SB*W*FB+AR*FY
      Y1T=(AF*FY*(ST+0.5*T)+0.5*ST**2*W*FT*(1.0-2.0*R1T**2)+(C-W)*(U**2
     1U*Q+Q**2/2.0-(R1T*ST)**2)*FY)/Q1T
      Y1B=(AF*FY*(SB+0.5*T)+0.5*SB**2*W*FB*(1.0-2.0*R1B**2)+(C-W)*(U**2
     1U*Q+Q**2/2.0-(R1B*SB)**2)*FY)/Q1B
      Y2T=(AF*FY*((ST+0.5*T)-2.0*R2T*(ST+T)+T*R2T**2)+0.5*ST**2*W*FT+AR
     1FY*(U+Q/2.0))/Q2T
      Y2B=(AF*FY*((SB+0.5*T)-2.0*R2B*(SB+T)+T*R2B**2)+0.5*SB**2*W*FB+AR
     1FY*(U+Q/2.0))/Q2B
      V2=(Y1T-Y2T)/(Y1B-Y2B)
      IF (V2 .LT. 0.0) GO TO 349
      IF (V2 .GT. 1.0) GO TO 349
      R=V2-V1
      IF (IND .EQ. 1) GO TO 330
      IF (ABS(R) .LE. 0.001) GO TO 331
      V1=(V1+V2)/2.0
      GO TO 280
C
C     THE TOLERANCE FOR THE FOLLOWING STATEMENT MUST BE CHOSEN VERY CAR
C     FULLY WHEN IT APPROACHES VMAX IN THIS CASE.
C
  330 IF (ABS(R) .LE. 0.025) GO TO 331
      V1=(V1+V2)/2.0
      GO TO 280
  331 IF (R1T .LT. ROK1T) GO TO 350
      IF (R1T .GT. HIK1T) GO TO 352
      IF (R1B .LT. ROK1B) GO TO 354
      IF (R1B .GT. HIK1B) GO TO 356
      Z=Q1T*(Y1T+Y1B+2.0*H)-V*A
      VVP=V/VP
      PMM=Z/PM
      IF (PMM .GT. 0.0) GO TO 345
```

```
      WRITE(6,340) PMM
  340 FORMAT(/,'    M/MP ='F10.5)
      GO TO 2000
  345 WRITE(6,346) V,VT,VB,V1,V2,R1T,R1B,R2T,R2B,VVP,PMM
  346 FORMAT(11F10.5)
      VV=V
      VV1=V1
      V=V+VINCR
      IND=0
      IF (V .GT. VMAX) GO TO 1990
      GO TO 280
  349 IF (K .EQ. 0) GO TO 300
      GO TO 2000
  350 IF (K .EQ. 0) GO TO 300
      WRITE(6,351)
  351 FORMAT(//,'    TO NEXT CASE SR')
      GO TO 2000
  352 IF (K .EQ. 0) GO TO 300
      WRITE(6,353)
  353 FORMAT(//,'    TO NEXT CASE WR')
      GO TO 2000
  354 IF (K .EQ. 0) GO TO 300
      WRITE(6,355)
  355 FORMAT(//,'    TO NEXT CASE RS')
      GO TO 2000
  356 IF (K .EQ. 0) GO TO 300
      WRITE(6,357)
  357 FORMAT(//,'    TO NEXT CASE RW')
      GO TO 2000
```

```
C
C
C
C     ************************************************************************
C
C     CASE : RW  (THAT MEANS K1T IS LOCATED IN THE REINFORCING AND K1B
C                  IS LOCATED IN THE CLEAR WEB. )
C
C     ************************************************************************
C
      ROK1T=U/ST
      HIK1T=(U+O)/ST
      ROK1B=(U+O)/SB
      HIK1B=1.0
      WRITE(6,501) ROK1T,HIK1T,ROK1B,HIK1B
  501 FORMAT(/,'    LOK1T      HIK1T      LOK1B      HIK1B     FOR THE FOLLO
     1ING CASE RW'/,4F10.5)
      WRITE(6,502)
  502 FORMAT(//,'       V            VT           VB            V1          V2
     1K1T          K1B           K2T          K2B         V/VP         M/MP'//)
C
C     THIS SPACE IS RESERVED FOR ARBITRARY VALUE OF V AND V1 .
C
      VINCR=1.0
      IND=0
  510 VB=V/(1.0+V1)
      VT=V-VB
      FTEX=FY**2-3.0*VT**2/(ST**2*W**2)
      IF (FTEX .GT. 0.0) GO TO 514
      IF (ABS(R) .LE. 0.0005) GO TO 512
      GO TO 516
  512 WRITE(6,513) FTEX
  513 FORMAT(/,'    FTEX =',F10.5)
      GO TO 2000
  514 FBEX=FY**2-3.0*VB**2/(SB**2*W**2)
      IF (FBEX .GT. 0.0) GO TO 521
      IF (ABS(R) .LE. 0.0005) GO TO 519
  516 IF (IND .EQ. 1) GO TO 517
      IND=1
      VVNCR=VINCR/2.0
      GO TO 518
  517 VVNCR=VVNCR/2.0
  518 V=VV+VVNCR
      V1=VV1
      GO TO 510
  519 WRITE(6,520) FBEX
  520 FORMAT(/,'    FBEX =',F10.5)
      GO TO 2000
  521 FT=SQRT(FTEX)
      FB=SQRT(FBEX)
      IF (FT .LT. 0.1) GO TO 1992
      IF (FB .LT. 0.1) GO TO 1992
      PARA2=U*(SB*FB-ST*FT)
      PARA3=U*(C-W)*FY
      PARA4=ST*W*FT+ST*FY*(C-W)
      ARW=AF*FY*ST/PARA4+AF*FY/(W*FB)+2.0*T
      BRW=-2.0*(D-2.0*H)+2.0*PARA3*ST/PARA4+PARA2/(W*FB)-2.0*AR*FY/(W*FB
     1)+T*PARA2/(AF*FY)
      CRW=2.0*V*A/(AF*FY)-(C-W)*(U**2-2.0*O*U-O**2)/AF+PARA3**2*ST/(AF*F
     1Y*PARA4)+PARA2**2/(4.0*AF*FY*FB*W)+AR**2*FY/(AF*W*FB)-AR*PARA2/(A
```

```
      2*W*FB)-PARA2*(SB+T)/(AF*FY)+T*PARA2**2/(4.0*AF**2*FY**2)
      D4=BRW**2-4.0*ARW*CRW
      IF (D4 .GT. 0.0) GO TO 524
  522 WRITE(6,523) D4
  523 FORMAT(5H D4 =F15.5)
      GO TO 2000
  524 R2T1=(-BRW+SQRT(D4))/(2.0*ARW)
      R2T2=(-BRW-SQRT(D4))/(2.0*ARW)
      IF (R2T1 .LT. 0.0) GO TO 530
      IF (R2T1 .LE. 1.0) GO TO 550
  530 IF (R2T2 .LT. 0.0) GO TO 540
      IF (R2T2 .LE. 1.0) GO TO 551
  540 WRITE(6,541) R2T1,R2T2
  541 FORMAT(2F10.5)
      GO TO 2000
  550 R2T=R2T1
      K=0
      GO TO 555
  551 R2T=R2T2
      K=1
  555 R2B=PARA2/(2.0*AF*FY)+R2T
      R1T=(AF*FY*R2T+PARA3)/PARA4
      R1B=(AF*FY*R2B-AR*FY)/(SB*W*FB)
      Q1T=AF*FY+ST*W*FT*(1.0-2.0*R1T)+(C-W)*(2.0*U+Q-2.0*R1T*ST)*FY
      Q1B=AF*FY+SB*W*FB*(1.0-2.0*R1B)-AR*FY
      Q2T=AF*FY*(1.0-2.0*R2T)+ST*W*FT+AR*FY
      Q2B=AF*FY*(1.0-2.0*R2B)+SB*W*FB+AR*FY
      Y1T=(AF*FY*(ST+0.5*T)+0.5*ST**2*W*FT*(1.0-2.0*R1T**2)+(C-W)*(U**2+
     1U*Q+Q**2/2.0-(R1T*ST)**2)*FY)/Q1T
      Y1B=(AF*FY*(SB+0.5*T)+0.5*SB**2*W*FB*(1.0-2.0*R1B**2)-AR*FY*(U+Q/
     1.0))/Q1B
      Y2T=(AF*FY*((ST+0.5*T)-2.0*R2T*(ST+T)+T*R2T**2)+0.5*ST**2*W*FT+AR*
     1FY*(U+Q/2.0))/Q2T
      Y2B=(AF*FY*((SB+0.5*T)-2.0*R2B*(SB+T)+T*R2B**2)+0.5*SB**2*W*FB+AR*
     1FY*(U+Q/2.0))/Q2B
      V2=(Y1T-Y2T)/(Y1B-Y2B)
      IF (V2 .LT. 0.0) GO TO 579
      IF (V2 .GT. 1.0) GO TO 579
      R=V2-V1
      IF (IND .EQ. 1) GO TO 560
      IF (ABS(R) .LE. 0.001) GO TO 561
      V1=(V1+V2)/2.0
      GO TO 510
C
C     THE TOLERANCE FOR THE FOLLOWING STATEMENT MUST BE CHOSEN VERY CAR-
C     FULLY WHEN IT APPROACHES VMAX IN THIS CASE.
C
  560 IF (ABS(R) .LE. 0.015) GO TO 561
      V1=(V1+V2)/2.0
      GO TO 510
  561 IF (R1T .LT. ROK1T) GO TO 580
      IF (R1T .GT. HIK1T) GO TO 582
      IF (R1B .LT. ROK1B) GO TO 584
      IF (R1B .GT. HIK1B) GO TO 586
      Z=Q1T*(Y1T+Y1B+2.0*H)-V*A
      VVP=V/VP
      PMM=Z/PM
      IF (PMM .GT. 0.0) GO TO 575
      WRITE(6,570) PMM
  570 FORMAT(/,'    M/MP ='F10.5)
```

```
      GO TO 2000
  575 WRITE(6,576) V,VT,VB,V1,V2,R1T,R1B,R2T,R2B,VVP,PMM
  576 FORMAT(11F10.5)
      VV=V
      VV1=V1
      V=V+VINCR
      IND=0
      IF (V .GT. VMAX) GO TO 1990
      GO TO 510
  579 IF (K .EQ. 0) GO TO 530
      GO TO 2000
  580 IF (K .EQ. 0) GO TO 530
      WRITE(6,581)
  581 FORMAT(//,'    TO NEXT CASE SW')
      GO TO 2000
  582 IF (K .EQ. 0) GO TO 530
      WRITE(6,583)
  583 FORMAT(//,'    TO NEXT CASE WW')
      GO TO 2000
  584 IF (K .EQ. 0) GO TO 530
      WRITE(6,585)
  585 FORMAT(//,'    TO NEXT CASE RR')
      GO TO 2000
  586 IF (K .EQ. 0) GO TO 530
      WRITE(6,587)
  587 FORMAT(//,'    TO NEXT CASE RF')
      GO TO 2000
```

```
C
C
C
C     ******************************************************************
C
C     CASE : WW  (THAT MEANS BOTH K1T AND K1B ARE LOCATED IN THE CLEAR
C                WEBS. )
C
C     ******************************************************************
C
      ROK1T=(U+Q)/ST
      HIK1T=1.0
      ROK1B=(U+Q)/SB
      HIK1B=1.0
      WRITE(6,602) ROK1T,HIK1T,ROK1B,HIK1B
  602 FORMAT(/,'    LOK1T      HIK1T      LOK1B      HIK1B     FOR THE FOLL
     1ING CASE WW'/,4F10.5)
      WRITE(6,603)
  603 FORMAT(//,'       V              VT            VB           V1          V2
     1K1T        K1B        K2T        K2B       V/VP       M/MP'//)
C
C     THIS SPACE IS RESERVED FOR ARBITRARY VALUE OF V AND V1 .
C
      VINCR=1.0
      IND=0
  610 VB=V/(1.0+V1)
      VT=V-VB
      FTEX=FY**2-3.0*VT**2/(ST**2*W**2)
      IF (FTEX .GT. 0.0) GO TO 614
      IF (ABS(R) .LE. 0.0005) GO TO 612
      GO TO 616
  612 WRITE(6,613) FTEX
  613 FORMAT(/,'    FTEX =',F10.5)
      GO TO 2000
  614 FBEX=FY**2-3.0*VB**2/(SB**2*W**2)
      IF (FBEX .GT. 0.0) GO TO 621
      IF (ABS(R) .LE. 0.0005) GO TO 619
  616 IF (IND .EQ. 1) GO TO 617
      IND=1
      VVNCR=VINCR/2.0
      GO TO 618
  617 VVNCR=VVNCR/2.0
  618 V=VV+VVNCR
      V1=VV1
      GO TO 610
  619 WRITE(6,620) FBEX
  620 FORMAT(/,'    FBEX =',F10.5)
      GO TO 2000
  621 FT=SQRT(FTEX)
      FB=SQRT(FBEX)
      IF (FT .LT. 0.1) GO TO 1992
      IF (FB .LT. 0.1) GO TO 1992
      PARA2=W*(SB*FB-ST*FT)
      AWW=AF*FY*(1.0/FT+1.0/FB)/W+2.0*T
      BWW=-2.0*(D-2.0*H)+T*PARA2/(AF*FY)+PARA2/(W*FB)-2.0*AR*FY*(1.0/F
     11.0/FB)/W
      CWW=2.0*V*A/(AF*FY)+PARA2**2*(1.0/FB+W/(B*FY))/(4.0*W*AF*FY)-PARA
     1*(SB+T)/(AF*FY)+AR**2*FY*(1.0/FT+1.0/FB)/(W*AF)-AR*PARA2/(AF*FB*
     2+2.0*AR*(2.0*U+Q)/AF
      D5=BWW**2-4.0*AWW*CWW
```

```
      IF (D5 .GT. 0.0) GO TO 624
      WRITE(6,623) D5
  623 FORMAT(5H D5 =F15.5)
      GO TO 2000
  624 R2T1=(-BWW-SQRT(D5))/(2.0*AWW)
      R2T2=(-BWW+SQRT(D5))/(2.0*AWW)
      IF (R2T1 .LT. 0.0) GO TO 630
      IF (R2T1 .LE. 1.0) GO TO 650
  630 IF (R2T2 .LT. 0.0) GO TO 640
      IF (R2T2 .LE. 1.0) GO TO 651
  640 WRITE(6,641) R2T1,R2T2
  641 FORMAT(2F10.5)
      GO TO 2000
  650 R2T=R2T1
      K=0
      GO TO 655
  651 R2T=R2T2
      K=1
  655 R2B=PARA2/(2.0*AF*FY)+R2T
      R1T=(AF*FY*R2T-AR*FY)/(ST*W*FT)
      R1B=(AF*FY*R2B-AR*FY)/(SB*W*FB)
      Q1T=AF*FY+ST*W*FT*(1.0-2.0*R1T)-AR*FY
      Q1B=AF*FY+SB*W*FB*(1.0-2.0*R1B)-AR*FY
      Q2T=AF*FY*(1.0-2.0*R2T)+ST*W*FT+AR*FY
      Q2B=AF*FY*(1.0-2.0*R2B)+SB*W*FB+AR*FY
      Y1T=(AF*FY*(ST+0.5*T)+0.5*ST**2*W*FT*(1.0-2.0*R1T**2)-AR*FY*(U+Q/
     1.0))/Q1T
      Y1B=(AF*FY*(SB+0.5*T)+0.5*SB**2*W*FB*(1.0-2.0*R1B**2)-AR*FY*(U+Q/
     1.0))/Q1B
      Y2T=(AF*FY*((ST+0.5*T)-2.0*R2T*(ST+T)+T*R2T**2)+0.5*ST**2*W*FT+AR
     1FY*(U+Q/2.0))/Q2T
      Y2B=(AF*FY*((SB+0.5*T)-2.0*R2B*(SB+T)+T*R2B**2)+0.5*SB**2*W*FB+AR
     1FY*(U+Q/2.0))/Q2B
      V2=(Y1T-Y2T)/(Y1B-Y2B)
      IF (V2 .LT. 0.0) GO TO 679
      IF (V2 .GT. 1.0) GO TO 679
      R=V2-V1
      IF (IND .EQ. 1) GO TO 660
      IF (ABS(R) .LE. 0.001) GO TO 661
      V1=(V1+V2)/2.0
      GO TO 610
C
C     THE TOLERANCE FOR THE FOLLOWING STATEMENT MUST BE CHOSEN VERY CAR
C     FULLY WHEN IT APPROACHES VMAX IN THIS CASE.
C
  660 IF (ABS(R) .LE. 0.025) GO TO 661
      V1=(V1+V2)/2.0
      GO TO 610
  661 IF (R1T .LT. ROK1T) GO TO 680
      IF (R1T .GT. HIK1T) GO TO 682
      IF (R1B .LT. ROK1B) GO TO 684
      IF (R1B .GT. HIK1B) GO TO 686
      Z=Q1T*(Y1T+Y1B+2.0*H)-V*A
      VVP=V/VP
      PMM=Z/PM
      IF (PMM .GT. 0.0) GO TO 675
      WRITE(6,670) PMM
  670 FORMAT(/,'    M/MP ='F10.5)
      GO TO 2000
  675 WRITE(6,676) V,VT,VB,V1,V2,R1T,R1B,R2T,R2B,VVP,PMM
```

```
676 FORMAT(11F10.5)
    VV=V
    VV1=V1
    V=V+VINCR
    IND=0
    IF (V .GT. VMAX) GO TO 1990
    GO TO 610
679 IF (K .EQ. 0) GO TO 630
    GO TO 2000
680 IF (K .EQ. 0) GO TO 630
    WRITE(6,681)
681 FORMAT(//,'    TO NEXT CASE RW')
    GO TO 2000
682 IF (K .EQ. 0) GO TO 630
    WRITE(6,683)
683 FORMAT(//,'    TO NEXT CASE FW')
    GO TO 2000
684 IF (K .EQ. 0) GO TO 630
    WRITE(6,685)
685 FORMAT(//,'    TO NEXT CASE WR')
    GO TO 2000
686 IF (K .EQ. 0) GO TO 630
    WRITE(6,687)
687 FORMAT(//,'    TO NEXT CASE WF')
    GO TO 2000
```

APPENDIX C

SAMPLE COMPUTER PRINTOUT

(Sample 1)

H	D	T	ST	SB
7.039	16.120	0.563	2.497	6.497

W	U	D	C
0.346	0.250	0.250	4.346

H	A	FY	ECCTRY
3.000	4.500	36.000	2.000

VP	VMAX	MP	VMAX/VP
107.42890	64.44011	2419.54900	0.59984

LOK2T	HIK2T	LOK2B	HIK2B	
0.00000	1.00000	0.00000	1.00000	FOR ALL THE FOLLOWING CASES

LOK1T	HIK1T	LOK1B	HIK1B	
0.00000	0.10012	0.03848	0.07696	FOR THE FOLLOWING CASE SB

V	VT	VB	V1	V2	K1T	K1B	K2T	K2B	V/VP	M/MP
0.00000	0.00000	0.00000	0.00062	0.00000	0.00000	0.05992	0.00000	0.17462	0.00000	1.00566
1.00000	0.00062	0.99938	0.00062	0.00000	0.00000	0.05992	0.00000	0.17455	0.00927	1.00410
2.00000	0.00125	1.99875	0.00062	0.00000	0.00000	0.05989	0.00000	0.17436	0.01855	1.00251
3.00000	0.00187	2.99813	0.00062	0.00000	0.00000	0.05985	0.00000	0.17403	0.02782	1.00097
4.00000	0.00250	3.99750	0.00062	0.00000	0.00000	0.05980	0.00000	0.17358	0.03710	0.99920
5.00000	0.00312	4.99688	0.00062	0.00000	0.00000	0.05972	0.00000	0.17299	0.04637	0.99759
6.00000	0.00375	5.99625	0.00062	0.00000	0.00000	0.05963	0.00000	0.17227	0.05564	0.99575
7.00000	0.00437	6.99563	0.00062	0.00000	0.00000	0.05953	0.00000	0.17142	0.06492	0.99397
8.00000	0.00499	7.99501	0.00062	0.00000	0.00000	0.05941	0.00000	0.17043	0.07419	0.99215
9.00000	0.00562	8.99438	0.00062	0.00000	0.00000	0.05927	0.00000	0.16931	0.08347	0.99030
10.00000	0.00624	9.99376	0.00062	0.00000	0.00000	0.05911	0.00000	0.16805	0.09274	0.98840
11.00000	0.00687	10.99313	0.00062	0.00000	0.00000	0.05894	0.00000	0.16666	0.10201	0.98646
12.00000	0.00749	11.99251	0.00062	0.00000	0.00000	0.05875	0.00000	0.16512	0.11129	0.98449
13.00000	0.00812	12.99188	0.00062	0.00000	0.00000	0.05854	0.00000	0.16343	0.12056	0.98247
14.00000	0.00874	13.99126	0.00062	0.00000	0.00000	0.05831	0.00000	0.16160	0.12984	0.98041
15.00000	0.00936	14.99064	0.00062	0.00000	0.00000	0.05806	0.00000	0.15962	0.13911	0.97831
16.00000	0.00999	15.99001	0.00062	0.00000	0.00000	0.05780	0.00000	0.15749	0.14839	0.97617
17.00000	0.01062	16.98938	0.00062	0.00000	0.00000	0.05751	0.00000	0.15520	0.15766	0.97398
18.00000	0.01125	17.98875	0.00062	0.00000	0.00000	0.05721	0.00000	0.15275	0.16693	0.97174
19.00000	0.01187	18.98813	0.00062	0.00000	0.00000	0.05688	0.00000	0.15014	0.17620	0.96946
20.00000	0.01250	19.98750	0.00062	0.00000	0.00000	0.05653	0.00000	0.14736	0.18548	0.96713
21.00000	0.01312	20.98688	0.00062	0.00000	0.00000	0.05616	0.00000	0.14440	0.19475	0.96475
22.00000	0.01375	21.98625	0.00062	0.00000	0.00000	0.05576	0.00000	0.14125	0.20403	0.96232
23.00000	0.01436	22.98564	0.00062	0.00000	0.00000	0.05534	0.00000	0.13792	0.21330	0.95984
24.00000	0.01498	23.98502	0.00062	0.00000	0.00000	0.05490	0.00000	0.13439	0.22257	0.95728
25.00000	0.01561	24.98439	0.00062	0.00000	0.00000	0.05443	0.00000	0.13066	0.23185	0.95468
26.00000	0.01624	25.98376	0.00062	0.00000	0.00000	0.05393	0.00000	0.12671	0.24112	0.95201
27.00000	0.18370	26.81630	0.00685	0.00723	0.00920	0.05377	0.00201	0.12526	0.25040	0.94823
28.00000	0.63304	27.36696	0.02313	0.02407	0.03156	0.05417	0.00688	0.12782	0.25967	0.94292
29.00000	1.09056	27.90944	0.03908	0.03994	0.05401	0.05459	0.01175	0.13041	0.26894	0.93751
30.00000	1.54385	28.45615	0.05425	0.05504	0.07679	0.05499	0.01668	0.13303	0.27822	0.93197
31.00000	1.99249	29.00751	0.05864	0.05939	0.09998	0.05541	0.02166	0.13567	0.28749	0.92628

TO NEXT CASE RR

LOK1T	HIK1T	LOK1B	HIK1B	
0.10012	0.20024	0.03848	0.07696	FOR THE FOLLOWING CASE RR

V	VT	VB	V1	V2	K1T	K1B	K2T	K2B	V/VP	M/MP
32.00000	2.43909	29.56091	0.08251	0.08313	0.10197	0.05583	0.02669	0.13833	0.29677	0.92047
33.00000	2.88556	30.11444	0.09582	0.09637	0.10384	0.05626	0.03172	0.14098	0.30604	0.91458
34.00000	3.33224	30.66776	0.10866	0.10915	0.10573	0.05668	0.03678	0.14364	0.31531	0.90859
35.00000	3.77922	31.22078	0.12105	0.12148	0.10762	0.05711	0.04185	0.14630	0.32459	0.90251
36.00000	4.22661	31.77339	0.13302	0.13341	0.10954	0.05756	0.04695	0.14895	0.33386	0.89632
37.00000	4.67458	32.32542	0.14461	0.14494	0.11147	0.05793	0.05207	0.15161	0.34314	0.89004
38.00000	5.12302	32.87698	0.15582	0.15611	0.11342	0.05841	0.05721	0.15426	0.35241	0.88365
39.00000	5.55800	33.44200	0.16620	0.16720	0.11541	0.05885	0.06246	0.15688	0.36168	0.87710
40.00000	6.00874	33.99126	0.17677	0.17771	0.11739	0.05929	0.06766	0.15953	0.37095	0.87049
41.00000	6.46129	34.53871	0.18707	0.18790	0.11940	0.05973	0.07287	0.16218	0.38023	0.86377
42.00000	6.91493	35.08507	0.19709	0.19781	0.12142	0.06018	0.07812	0.16483	0.38951	0.85692
43.00000	7.36961	35.63039	0.20684	0.20745	0.12347	0.06062	0.08340	0.16747	0.39878	0.84995
44.00000	7.82555	36.17445	0.21633	0.21685	0.12554	0.06108	0.08871	0.17011	0.40805	0.84286
45.00000	8.28285	36.71715	0.22559	0.22602	0.12764	0.06153	0.09406	0.17274	0.41734	0.83564
46.00000	8.74164	37.25836	0.23462	0.23498	0.12977	0.06199	0.09945	0.17537	0.42660	0.82818
47.00000	9.20210	37.79790	0.24346	0.24373	0.13193	0.06245	0.10487	0.17799	0.43588	0.82062
48.00000	9.66444	38.33556	0.25210	0.25231	0.13412	0.06291	0.11034	0.18061	0.44515	0.81294
49.00000	10.12872	38.87128	0.26057	0.26071	0.13635	0.06338	0.11586	0.18322	0.45442	0.80497
50.00000	10.59511	39.40489	0.26888	0.26897	0.13862	0.06385	0.12143	0.18582	0.46370	0.79666
51.00000	11.05167	39.94833	0.27665	0.27743	0.14097	0.06432	0.12718	0.18836	0.47297	0.78845
52.00000	11.52878	40.47122	0.28486	0.28528	0.14131	0.06481	0.13281	0.19096	0.48225	0.77982
53.00000	12.01007	40.98993	0.29400	0.29296	0.14569	0.06529	0.13847	0.19356	0.49152	0.77115
54.00000	12.49513	41.50487	0.30105	0.30051	0.14813	0.06578	0.14419	0.19615	0.50079	0.76209
55.00000	12.98684	42.01316	0.30849	0.30856	0.15070	0.06627	0.15020	0.19866	0.51007	0.75249
56.00000	13.45065	42.54935	0.31611	0.31630	0.15331	0.06677	0.15620	0.20115	0.51934	0.74245
57.00000	13.93750	43.06250	0.32366	0.32407	0.15601	0.06727	0.16232	0.20363	0.52861	0.73256
58.00000	14.42769	43.57231	0.33112	0.33195	0.15883	0.06779	0.16858	0.20607	0.53789	0.72141
59.00000	14.94655	44.05345	0.33928	0.33866	0.16163	0.06833	0.17457	0.20855	0.54716	0.71068
60.00000	15.44539	44.55461	0.34666	0.34754	0.16474	0.06886	0.18119	0.21096	0.55644	0.69653
61.00000	15.97539	45.02461	0.35652	0.35416	0.16784	0.06963	0.18769	0.21369	0.56571	0.68848
62.00000	16.50377	45.49623	0.35775	0.36200	0.17133	0.07002	0.19424	0.21585	0.57499	0.67132
62.50000	16.63687	45.86313	0.36775	0.37679	0.17464	0.07015	0.20215	0.21529	0.57962	0.66069
62.75000	16.70341	46.04659	0.35275	0.35566	0.17453	0.07020	0.20671	0.21474	0.58194	0.65499
62.87500	17.18155	45.69345	0.37602	0.35198	0.17243	0.07085	0.19416	0.22069	0.58311	0.66066
62.93750	17.31314	46.05662	0.37602	0.36656	0.17803	0.07101	0.20117	0.22010	0.58776	0.66912
62.97500	17.45442	46.42018	0.37702	0.39747	0.18052	0.07110	0.21180	0.21953	0.56237	0.63642

V IS GREATER THAN VMAX

(Sample 2)

R	D	T	SI	SR
7.000	16.120	0.563	2.497	6.497

W	U	O	C
0.366	0.250	0.250	2.366

M	A	FY	FCCTRY
3.000	4.500	36.000	2.000

VP	VMAX	MP	VMAX/VP
107.82490	64.68011	2914.56000	0.59986

LOK2T	HIK2T	LOK2R	HIK2R	
0.00000	1.00000	0.00000	1.00000	FOR ALL THE FOLLOWING CASES.

LOK1T	HIK1T	LOK1R	HIK1R	
0.00000	0.19012	0.07696	1.00000	FOR THE FOLLOWING CASE SW

V	VT	VR	V1	V2	K1T	K1R	K2T	K2R	V/VP	M/MP
0.00000	0.00000	0.00000	0.00062	0.00000	0.00000	0.09561	0.00000	0.17462	0.00000	0.96315
1.00000	0.00062	0.99938	0.00062	0.00000	0.00000	0.09532	0.00000	0.17455	0.00927	0.96159
2.00000	0.00125	1.99875	0.00062	0.00000	0.00000	0.09503	0.00000	0.17436	0.01855	0.96030
3.00000	0.00187	2.99813	0.00062	0.00000	0.00000	0.08455	0.00000	0.17403	0.02782	0.95847
4.00000	0.00250	3.99750	0.00062	0.00000	0.00000	0.08309	0.00000	0.17358	0.03710	0.95670
5.00000	0.00312	4.99688	0.00062	0.00000	0.00000	0.08402	0.00000	0.17299	0.04637	0.95501
6.00000	0.00375	5.99625	0.00062	0.00000	0.00000	0.08195	0.00000	0.17227	0.05564	0.95327
7.00000	0.00437	6.99563	0.00062	0.00000	0.00000	0.08068	0.00000	0.17142	0.06492	0.95150
8.00000	0.00499	7.99501	0.00062	0.00000	0.00000	0.07920	0.00000	0.17043	0.07419	0.94968
9.00000	0.00562	8.99438	0.00062	0.00000	0.00000	0.07751	0.00000	0.16931	0.08347	0.94784

TO NEXT CASE SR

LOK1T	HIK1T	LOK1R	HIK1R	
0.00000	0.19012	0.03848	0.07696	FOR THE FOLLOWING CASE SR

V	VT	VR	V1	V2	K1T	K1R	K2T	K2R	V/VP	M/MP
10.00000	0.00620	9.99340	0.00062	0.00000	0.00000	0.07676	0.00000	0.16805	0.09274	0.94596
11.00000	0.00681	10.99319	0.00062	0.00000	0.00000	0.07645	0.00000	0.16665	0.10201	0.94404
12.00000	0.00743	11.99257	0.00062	0.00000	0.00000	0.07611	0.00000	0.16512	0.11129	0.94208
13.00000	0.00805	12.99195	0.00062	0.00000	0.00000	0.07576	0.00000	0.16345	0.12056	0.94008
14.00000	0.00867	13.99133	0.00062	0.00000	0.00000	0.07543	0.00000	0.16160	0.12984	0.93804
15.00000	0.00929	14.99071	0.00062	0.00000	0.00000	0.07409	0.00000	0.15962	0.13911	0.93595
16.00000	0.00991	15.99009	0.00062	0.00000	0.00000	0.07442	0.00000	0.15749	0.14838	0.93383
17.00000	0.01056	16.98944	0.00062	0.00000	0.00000	0.07390	0.00000	0.15520	0.15766	0.93166
18.00000	0.01115	17.98885	0.00062	0.00000	0.00000	0.07335	0.00000	0.15275	0.16693	0.92945
19.00000	0.01178	18.98822	0.00062	0.00000	0.00000	0.07277	0.00000	0.15014	0.17620	0.92719
20.00000	0.01239	19.98761	0.00062	0.00000	0.00000	0.07216	0.00000	0.14735	0.18548	0.92488
21.00000	0.01302	20.98698	0.00062	0.00000	0.00000	0.07147	0.00000	0.14439	0.19475	0.92253
22.00000	0.01364	21.98636	0.00062	0.00000	0.00000	0.07076	0.00000	0.14125	0.20403	0.92012
23.00000	0.01425	22.98575	0.00062	0.00000	0.00000	0.07000	0.00000	0.13792	0.21330	0.91766
24.00000	0.01488	23.98512	0.00062	0.00000	0.00000	0.06919	0.00000	0.13439	0.22257	0.91514
25.00000	0.01549	24.98451	0.00062	0.00000	0.00000	0.06834	0.00000	0.13066	0.23185	0.91257
26.00000	0.01611	25.98389	0.00062	0.00000	0.00000	0.06743	0.00000	0.12671	0.24112	0.90993
27.00000	0.21854	26.78146	0.00936	0.00962	0.01096	0.06730	0.00239	0.12580	0.25040	0.90596
28.00000	0.67539	27.32461	0.02453	0.02566	0.03339	0.06804	0.00727	0.12839	0.25967	0.90061
29.00000	1.12604	27.87396	0.04051	0.04138	0.05596	0.06887	0.01217	0.13102	0.26894	0.89516
30.00000	1.58342	28.41658	0.05572	0.05651	0.07882	0.06968	0.01712	0.13367	0.27822	0.88957

TO NEXT CASE RR

LOK1T	HIK1T	LOK1R	HIK1R	
0.19012	0.20026	0.03848	0.07696	FOR THE FOLLOWING CASE RR

V	VT	VR	V1	V2	K1T	K1R	K2T	K2R	V/VP	M/MP
31.00000	2.03317	28.96683	0.07019	0.07090	0.10061	0.07050	0.02212	0.13635	0.28749	0.88383
32.00000	2.48090	29.51910	0.08404	0.08467	0.10348	0.07132	0.02716	0.13905	0.29677	0.87794
33.00000	2.92836	30.07164	0.09733	0.09794	0.10737	0.07216	0.03222	0.14175	0.30604	0.87203
34.00000	3.37566	30.62434	0.11023	0.11073	0.11098	0.07303	0.03731	0.14445	0.31531	0.86599
35.00000	3.82295	31.17705	0.12262	0.12306	0.11463	0.07394	0.04243	0.14715	0.32459	0.85984
36.00000	4.27032	31.72968	0.13456	0.13497	0.11802	0.07470	0.04755	0.14986	0.33386	0.85359
37.00000	4.71788	32.28212	0.14615	0.14648	0.12164	0.07555	0.05271	0.15257	0.34314	0.84723
38.00000	5.16583	32.83417	0.15734	0.15762	0.12530	0.07643	0.05790	0.15528	0.35241	0.84076

TO NEXT CASE RW

LOK1T	HIK1T	LOK1R	HIK1R	
0.19012	0.20024	0.07696	1.00000	FOR THE FOLLOWING CASE RW

V	VT	VR	V1	V2	K1T	K1R	K2T	K2R	V/VP	M/MP
39.00000	5.61415	33.38585	0.16816	0.16841	0.12901	0.08021	0.06312	0.15800	0.36164	0.83417
40.00000	6.05067	33.94933	0.17823	0.17914	0.13282	0.08860	0.06847	0.16070	0.37096	0.82738
41.00000	6.50264	34.49736	0.18850	0.18935	0.13663	0.09746	0.07378	0.16345	0.38023	0.82056
42.00000	6.95714	35.04286	0.19853	0.19929	0.14051	0.10679	0.07913	0.16624	0.38951	0.81347
43.00000	7.41362	35.58638	0.20833	0.20899	0.14445	0.11665	0.08454	0.16905	0.39878	0.80627
44.00000	7.87225	36.12775	0.21790	0.21847	0.14846	0.12710	0.09001	0.17189	0.40805	0.79884
45.00000	8.33324	36.66676	0.22727	0.22776	0.15255	0.13822	0.09553	0.17476	0.41733	0.79131
46.00000	8.79578	37.20422	0.23665	0.23696	0.15673	0.15010	0.10112	0.17768	0.42660	0.78353
47.00000	9.26324	37.73676	0.24547	0.24580	0.16100	0.16265	0.10679	0.18064	0.43588	0.77553
48.00000	9.73282	38.26718	0.25434	0.25460	0.16537	0.17601	0.11252	0.18365	0.44515	0.76724
49.00000	10.20599	38.79401	0.26308	0.26322	0.16985	0.19155	0.11834	0.18672	0.45442	0.75879
50.00000	10.68306	39.31694	0.27172	0.27185	0.17446	0.20796	0.12425	0.18987	0.46371	0.74999
51.00000	11.16654	39.83346	0.28027	0.28035	0.17921	0.22581	0.13027	0.19311	0.47297	0.74087
52.00000	11.63971	40.36029	0.28840	0.28912	0.18419	0.24575	0.13651	0.19645	0.48225	0.73142
53.00000	12.13334	40.86166	0.29706	0.29736	0.18923	0.26806	0.14270	0.19991	0.49152	0.72146
54.00000	12.64516	41.35654	0.30577	0.30556	0.19447	0.29330	0.14906	0.20369	0.50079	0.71113
55.00000	13.16103	41.83897	0.31456	0.31374	0.19993	0.32220	0.15563	0.20752	0.51007	0.70025

TO NEXT CASE WW

LOK1T	HIK1T	LOK1R	HIK1R	
0.20026	1.00000	0.07696	1.00000	FOR THE FOLLOWING CASE WW

V	VT	VR	V1	V2	K1T	K1R	K2T	K2R	V/VP	M/MP
56.00000	13.65875	42.34125	0.32259	0.32266	0.25684	0.35651	0.16252	0.21167	0.51934	0.68836
57.00000	14.16838	42.83162	0.33079	0.33096	0.32804	0.39837	0.17011	0.21666	0.52861	0.67521
58.00000	14.67548	43.32452	0.33873	0.34005	0.41865	0.45184	0.17877	0.22214	0.53789	0.66011
59.00000	15.17601	43.82399	0.34629	0.34690	0.54249	0.52517	0.18939	0.22967	0.54716	0.64161
60.00000	15.67720	44.32241	0.35377	0.35562	0.71126	0.63015	0.20512	0.24971	0.55644	0.61747
[illegible]	-11.67760									

STRENGTH OF BEAMS WITH ECCENTRIC, REINFORCED, RECTANGULAR WEB OPENINGS

by

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ABSTRACT

An analytical method of determining the moment carrying capacity of steel W shape beams with eccentric, reinforced rectangular web openings is developed. Using this method, the effects of varying the opening eccentricities and lengths and the reinforcing areas were investigated. From this analysis, the following conclusions are drawn :

a. Eccentricity does not affect the maximum opening length and the minimum reinforcing area (see Eqs. 36, 37 and 38).

b. With small eccentricity (say, $e \leq u$), the problem could be treated as a concentric opening, without any significant loss of accuracy.

c. As the eccentricity increases, the moment carrying capacity of the beam decreases for low shear forces and increases for high shear forces.

d. The moment carrying capacity is almost linearly proportional to the reinforcing area when the shear force is low, but this relationship becomes non-linear when the shear force is larger.

e. As opening length increases, the moment carrying capacity decreases while shear force increases.