NUMERICAL OPTIMIZATION OF A STOCHASTIC INVENTORY SYSTEM UNDER CONSTRAINT

by

SURESH V. GADAD

B. Tech. (Metallurgical), Indian Inst. of Tech., Bombay, India, 1965

Diploma I. I. T. (I.E.), Indian Inst. of Tech., Madras, India, 1969

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Approved by

Major Professor

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CHAPTER 1

INTRODUCTION

1.1 INVENTORY MANAGEMENT:

Inventory has been defined by Hanssman (20) as "An idle resource of any kind, provided that such resource has economic value". Inventory management deals with determination of the optimal procedures for aquiring (through purchasing, manufacturing,etc.) these resources that will be demanded in the future. The field of inventory management has commanded considerable interest in the past; the last few years in particular have witnessed a good deal of work in this field, both with respect to the quality and quantity. This can be attributed to the recent parallel development in research and business. Because of the important role of inventory in business, the management has been willing to cooperate in experimenting with the application of new theoretical techniques. Also, the nature of the problems in inventory management has been such as to engross the attention of the scientists in the field of operations research, statistics and economics. One tends to agree with Churchman's (10, p. 14) statement that "More operations research has been directed towards inventory control than any other problem area in business and industry." Inventory management is a very vast field covering a diversity of problems. A brief description of the properties of an inventory system follows.

1.2 PROPERTIES OF AN INVENTORY SYSTEM:

To prepare an exhaustive list of the properties of an inventory system would be an impossible task. Even a study of the more significant

properties would reveal the existance of a large number of inventory problems. Following Naddor's (29) classification, inventory system can be divided into four components: demands, replenishments, costs and constraints. Demands are what is taken out of inventory. Replenishments are what is put into inventory. Costs are the pertinent measures associated with positive (holding cost) and negative inventory (shortage cost) and with raising the level of inventories (the replenishment cost or order cost). Constraints are various administrative, physical, financial and other factors that place limitations on demands, replenishments and costs.

Table 1, prepared on the lines of one given by Churchman et al (10, p. 200), lists a number of important characteristics of an inventory system.

1.3 INVENTORY PROBLEM UNDER CONSTRAINT:

The inventory systems encountered in practice involve constraints of some sort, more often than not. These constraints affect formulation of the inventory models. The solution of inventory systems under constraints, to obtain optimal inventory policy, is quite complicated. A few of the constraints are:

1.3.1 Unit Constraint: When the units involved are discrete (e.g. automobiles, no. of spares) the mathematical analysis necessary for optimization is quite different from the case when the units are continuous.

1.3.2 Inventory Levels: The management may place certain restriction on the aggregate amount of capital to be invested in inventory.

1.3.3 Number of Annual Orders: The management may be interested in finding the optimal inventory policy, without altering the annual total number of orders placed.

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TABLE 1

CHARACTERISTICS OF AN INVENTORY SYSTEM

	I. DEMAND		
1)	Demand size and rate...	A. Known.....	a. Constant b. Variable
		B. Estimated	
2)	Quantities required....	A. Descrete units	
		B. Continuous units	
3)	Demand pattern.....	A. Continuous	a. Constant rate b. Variable rate
		B. Discontinuous	
	II. REPLENISHMENT		
4)	Replenishment cycle time....	A. Known	a. Constant b. Variable
		B. Estimated	
5)	Lead time.....	A. Virtually zero	a. Constant b. Variable
		B. Positive	
6)	Replenishment size....	A. Descrete	a. Constant b. Variable
		B. Continuous	
7)	Replenishment pattern.	A. Continuous	a. Constant rate b. Variable rate
		B. Discontinuous	
	III. COST		
8)	Purchase or manufacturing cost		
9)	Stock holding cost/unit/unit time	.....	a. Constant b. Variable
10)	Replenishment cost		
11)	Out of stock cost....	A. Fixed	
		B. Function of duration and amount of stockout	

TABLE 1

CHARACTERISTICS OF AN INVENTORY SYSTEM (continued)

IV. CONSTRAINT

12) Administrative
13) Physical

A. Expected value of the constraint

B. Maximum or certain percentile of the constraint

1.3.4 Space Constraint: The amount of space for storing inventories may be limited.

The last constraint plays an important role in designing optimal inventory policy for the items to be stored in an automated warehouse. The treatment of the inventory system under the constraint of warehouse space forms the topic for this thesis.

1.4 INVENTORY CONTROL IN AN AUTOMATED WAREHOUSE:

As of today, not many automated warehouses are in existance. However, due to the increased availability of computers, a rapid growth in the field of automated warehousing is predicted. Dallimonti (11) defines an automated warehouse as 'a system with a central controller, usually a computer, which handles both the information and data processing and the physical handling of the materials according to the operating business decisions'.

Two features of the inventory problem in an automated warehouse that distinguish it from the inventory problems in any other warehouse or business are:

a) High cost of the space in automated warehouse, which makes it imperative to use warehouse space efficiently.

b) Availability of the computer.

1.4.1 Efficient Use of Warehouse Space: The importance of using space in automated warehouse cannot be over emphasized. The capital cost of automated warehouse space is very high. As Kay (26) puts it, "the real difficulties of warehouse automation are the design and COST of automated handling equipment". As a number of items are to be stored interchangably,

in an automated warehouse, there is competition for the same space. The result is a multi-item inventory problem under the constraint of maximum available space.

The problem of obtaining a constrained optimal inventory policy, has been more often tackled in the literature using an average constraint. Such a treatment of average constraint is sometimes satisfactory; e.g. 1) average amount of the capital investment in inventory should not exceed certain figure, or 2) average number of total annual orders should not exceed a given figure. This approach, however, is not valid while considering the space constraint of a warehouse. Designing the inventory policy for the average space used to be equal to (or less than) the available space, would mean that 50% of the time the management would run out of capacity. It is hardly practical to expect management to arrange alternative storage 50% of the time.

An alternative would be to consider the absolute constraint; i.e. the maximum space used under the inventory policy should be equal to the available space. This has been the approach taken by Kalinhoff (25). This is a realistic constraint in some practical problems, where making alternative arrangement for storage is impossible. However, in most of the real-world problems, alternative arrangements for storage (at additional cost or risk of spoilage) are possible. Obviously then, the realistic way of designing the optimal inventory policy, for an automated warehouse, would be to consider the available space equal to some percentage (less than 100) of the maximum required space. A detailed analysis of this aspect of the problem is given in Chapter 3. Hadley and Whitin (19) stress

this point*: "When absolute constraint is specified the problem takes a fundamentally different nature. The task of developing models which include constraints in a realistic way is difficult. The problem of how to deal with the realistic constraint appears to be extremely difficult and it seems that no attempts have been made to solve this problem."

1.4.2 Availability of the Computer: A major obstacle in the application of rigorous mathematical techniques to the real-world inventory problems is the difficulty in the implementation of the solution thus obtained. The calculations involved would be too complicated to be handled manually, without increasing the systemic costs to prohibitively high level. The use of approximate inventory models and tools like graphs, nomograms, stoculators etc. to obtain approximately optimal inventory policies, in practice can be thus explained. The approximate answers are justified because the total inventory cost function is often flat in the vicinity of the optimal point. Eilon (14) has shown the insensitivity of the optimal solution (even 1% increase in cost allows a range of 15.2% for optimal lot size, for a simple model that he has described), and suggested that the concept of an optimal range could be useful. However, the use of unrealistically approximate inventory models to obtain the optimal inventory policy can significantly affect the inventory cost. The availability of computer facility makes the exact treatment of the complex real world problem possible, feasible and hopefully profitable.

1.4.3 Postulates of the Inventory System:

The complex inventory problem of an automated warehouse, the solution for which is attempted in this thesis, is based on the following postulates.

*'constraint' has been used in place of 'budget', in the quotation.

a) The inventory problem under consideration is dynamic; in other words a number of orders are possible, because an optimal inventory policy is sought for an infinite time horizon.

b) The demand per unit of time is stochastic and follows a distribution which is stationary over all time periods. The assumption regarding stationary property is quite restricting; however, the analysis otherwise becomes very complex.

c) The lead time is positive and follows a stationary distribution.

d) The inventory system includes a number of items. The only interaction between the various items in the system is that they are competing for the same warehouse space. So, the problem is one of constrained optimization.

e) No assumptions regarding the type of distribution for demand rate or lead time are to be made. The approach should be such that it can use historical data for these variables (or for the lead time demand, if available).

1.5 DEFINITIONS OF THE TERMS USED:

The symbols used in the thesis are defined below:

n = Number of items to be stored in the warehouse.

TC = Total expected inventory cost rate for 'n' items.

CAP = Available space of the warehouse.

λ = Lagrangian multiplier.

L = Lagrangian

S_i = Demand in units/unit of time.

$\bar{S}_i$ = Average demand in units per unit of time.

T_{L_i} = Lead time in time units, the interval of time elapsing between the point of time at which a replenishment is triggered and the point of time at which the replenishment occurs.

$\bar{T}_{L_i}$ = Average lead time, in units of time.

S_{L_i} = Demand during lead time in units of product. It is a random variable with p.d.f. $f(S_{L_i})$. It is a function of the two variables S_i and T_{L_i}. The average demand during lead time is $\bar{S}_{L_i} = \bar{S}_i \bar{T}_{L_i}$.

Q_i = Order size in units of the commodity.

I_{T_i} = Reorder point in units of the commodity.

C_i = Purchasing cost in dollars per unit.

C_{S_i} = Total expected inventory cost rate.

C_{I_i} = Inventory holding cost, in dollars per unit per unit of time.

T_{Q_i} = Order period.

C_{F_i} = Order cost in dollars per lot.

C_{D_i} = Out of stock cost in

a) dollars per stockout occurance, when stockout penalty is proportional to stockout occasions.

b) dollars per unit out of stock, when stockout penalty is proportional to the units out of stock.

c) dollars per unit per unit of time, when the stockout cost is proportional to the product of the size and duration of the stockout.

d) dollars per unit of time, when stockout penalty is proportional to the duration of stockout.

$f(S_{L_i})$ = Probability density function of the demand during lead time.

For the demand during lead time for any item,

N = The number of classes of equal class intervals into which demand during lead time is divided.

I_{T_1} = Minimum lead time demand.

$I_{T_{N+1}}$ = Maximum lead time demand.

For any class j, j = 1, 2, ..., N,

f_j = Frequency of lead time demand being in the jth class.

p_j = Probability of lead time demand being in the jth class.

S_{L_j} = Mid-value of the jth class.

1.5 PLAN OF THE THESIS:

The first chapter introduces the problem of finding an optimal inventory policy for a multi-item case with a space constraint, and defines the nomenclature to be used in the thesis. In the second chapter the work done in the past, relevant to the problem under investigation, is reviewed. In the third chapter the mathematical formulation of the problem is presented. The detailed procedure for the solution of the constrained optimization is described. The computer programs developed for solving the

problem are given along with the flow charts. The fourth chapter describes the three problems solved and discusses the results obtained. In the fifth and final chapter, conclusions are drawn and the scope for further work is outlined.

CHAPTER 2

LITERATURE REVIEW

2.1. INTRODUCTION:

The earliest attempt to employ analytical technique to study inventory problem dates as far back as 1915, when Ford Harris (22), obtained the well known 'simple lot size formula', also known as Wilson's square root formula. The stochastic version of the simple lot size formula was developed 38 years later by Whitin (37), in 1953.

Although papers in the field of inventory abound in the literature, the case of multi-item stochastic inventory system under constraint has not received much attention. As Hanssman (21) says, the final goal of every inventory study must be to make statements about the total system consisting of interacting items, rather than an individual item. It can be said at the outset that the exact analytical solution to the problem has still not been obtained. However, some approximate solutions or solutions to (simplified) particular cases are available.

2.2. WAREHOUSING PROBLEM:

In the well known 'warehousing problem', a space constraint is involved. It was formulated by A. Cahn as: 'given a warehouse with fixed capacity and an initial stock of a certain product which is subject to known seasonal price and cost variations, what is the optimal pattern of purchasing, storage and sales?'. Charnes and Cooper (8) have solved this problem by using linear programming. They have discussed the generalizations of the warehousing problem such as multi-product system,

effect of increasing the warehouse capacity. Also, they have presented special methods (which rest on the dual theorem and the regrouping principle), which take advantage of the structure of such problems, in order to evolve efficient computational devices. Eastman (13) has given a special matrix formulation to yield a simple solution to the multi-product warehousing problem. Bellman (1) has used dynamic programming to solve this problem.

2.3 QUEUEING THEORY:

All inventory situations, since they deal with flows and rates of flows, can be considered as queues. But the inventory models which allow solutions in closed form or for which good approximate solutions can be obtained in closed form, are based on unrealistic restrictive assumptions such as:

1) distribution of demand and replenishment time are poisson and exponential respectively,
2) queue descipline is FIFO.

An inventory model which is solved by queueing theory and which is closest to the real world problem has been described by Buchan and Koenigsberg (4, pp. 410-12). It is based on the following assumptions:

1) 'q' units are ordered when the stock level falls to 'p',
2) no back orders are allowed,
3) demand follows poisson distribution and replenishment time follows exponential distribution.

2.4 STOCHASTIC, UNCONSTRAINED INVENTORY SYSTEM:

The knowledge of the distribution of lead time demand is essential, for obtaining the optimal inventory policy, for any stochastic inventory

system. Generally, the available inventory records can be used to obtain information regarding lead time and demand rate distributions.

Clark (9) has obtained the mean, standard deviation and third central moment of the demand rate and lead time, from inventory records, using a computer. These were used to calculate the mean (μ), standard deviation (σ) and third central moment of the distribution of lead time demand. A factor 'r' ($r>1$) was found experimentally such that a normal distribution with mean μ and standard deviation $r\sigma$, fitted the actual distribution of lead time demand (which was skewed to the right) in the right hand tail of the distribution.

Plossl and Wight (31) use the average demand per order and the average number of orders during lead time (the latter assumed to follow a poisson distribution), to calculate the reorder points for specific service level.

In obtaining the distribution of lead time demand, many times, the variability of the lead time is neglected. Bramson (2) has stressed the importance of considering this variability of lead time and demonstrated that even a modest degree of lead time variability can influence the distribution of lead time demand, considerably.

Lu et al (27) have obtained an expression for the standard deviation of the lead time demand assuming that, 1) the demand rate follows a normal distribution and 2) the lead time is small. Using this standard deviation, nomographs have been prepared for determining the order size and reorder points, independently.

Carlson (6) has used the cumulants of the lead time and demand rate to obtain cumulants of the lead time demand. For poisson and geometric

distributed lead time and a demand rate having all finite cumulants, the asymptotic distributions of the lead time demand are respectively normal and exponential. He has shown that even for fairly low values of average lead time, the asymptotic value of the reorder point is often sufficiently close to the exact value of the reorder point.

Parker (30) has used an approximate expression for the right tail of the lead time demand distribution (assumed normal) to analytically calculate the optimal order quantity and reorder point. The use of these values, he has shown by simulation, will lead to 5-7% reduction in total cost, when compared with the use of Wilson's economic order quantities.

As Smith (34) points out, the cost of a stockout is seldom predictable. It often has an intangible (such as loss of good will) as well as measurable value. Schwartz (33) has introduced the concept of 'perturbed demand'. A stockout changes the subsequent demand pattern, due to loss of good will. He claims that perturbed demand approach to loss of good will is both substantially more valid and more practical than any previously considered approach in the literature of inventory theory.

Burgin and Wild (5) have used the first two moments of the lead time (L_1 and L_2) and demand rate (S_1 and S_2), to obtain the two moments for the lead time demand (D_1 and D_2). A Gamma distribution is fitted to lead time demand, which is used to decide service levels, using tables. They point out the difficulty in obtaining enough data on lead time demand to test high protective levels properly, and suggest the use of simulation using lead time and demand data separately.

Herron (23) has used a normal distribution for the lead time demand and provided graphical solutions for the cases where a dollar stockout

penalty is proportional to 1) the number of stockout occurances and 2) the number of units out of stock. For the multi-item case, he has shown how, under a composite service level (service level for all items together must be equal to or greater than a specific value), the problem can be treated as the constrained optimization problem, using the Lagrange multiplier method.

Pressutti (32) has used psedo exponential distributions for lead time demands and obtained explicit expressions for optimal order quantities and reorder points, for multi-item inventory system subject to specific stockout constraints.

2.5 CONSTRAINTS:

The task of developing models with realistic constraints is very difficult. For simplifying the analytical treatment, expected values of the constraints are used; e.g. expected number of annual orders, expected aggregate investment in inventory (24), (10), (35). This approach is not satisfactory in many real world problems, (19), (28). For realistic treatment of the space constraint of a warehouse, wherein the same space can be used for all the items, a certain fraction of the maximum required capacity ought to be used. This fraction will depend on the number of items in the system, among other things. A good idea of this fraction can be got by simulating the utilization of the warehouse space.

2.6 CONSTRAINED OPTIMIZATION FOR DETERMINISTIC INVENTORY SYSTEM:

Dano and Jensen (12) have used linear programming to solve the problem of production and inventory planning, given the sales forecast for each month, under capacity limitation, all products sharing the same capacity.

Chambers, Bond and Leake (7) have shown how optimal lot sizes for parts manufactured in 'N' periods, are determined, subject to constraints such as the company's policy of keeping labour force constant. N-dimensional search and dynamic programming have been used to determine the unconstrained optimal solution. If any constraint is violated, the solution is adjusted. Management thus provided with a quantitative cost incurred by the constraint, can decide if consideration of the constraint is worth the additional cost.

A packaged program called 'LIMIT' (lot-size inventory management interpolation technique) has been prepared (31), to give optimal order quantities under constraints such as the total setup hours, the total annual orders.

Hadley and Whitin (19) and many others (24), (10), (35) have obtained solutions for specific problems in this category.

2.7 CONSTRAINED OPTIMIZATION FOR STOCHASTIC INVENTORY SYSTEM:

Gourary (17) has considered the problem of stocking a vessel with the necessary commodities, subject to the limitation of available space. A rather idealized model wherein the average number of demands fulfilled is maximized subject to the space constraint, is formulated. The Lagrange multiplier method is used; optimal λ corresponding to the available space is determined by an iterative procedure. The iterative procedure has not been described.

Holt et al (24) have formulated the general constrained stochastic inventory problem. Numerical evaluation of the optimal inventory policies using Lagrange multipliers has been suggested. They have obtained explicit solutions for some special cases.

Buchan and Koenigsberg (4, pp. 331-58) have obtained expressions for service levels in terms of coefficient of variation and reorder points, using three distributions for lead time demand, namely normal, uniform and exponential. The determination of the optimal order quantity and reorder point has been suggested, using Wilson's economic order size formula and specified service levels, respectively. Alternatively, a total cost equation is optimized to obtain explicit expressions for optimal order sizes and reorder points, for the above three distributions of lead time demand. It is easy to incorporate the space constraint in this model.

2.8 LAGRANGE MULTIPLIER METHOD:

To obtain a constrained optimal solution to the stochastic inventory problem, the use of Lagrange multiplier method seems to be the obvious choice. It is not possible, however, to get an explicit expression for an optimal λ corresponding to the given constraint on resource level. Everett (15) has dealt with the problem of 'generalized Lagrange multiplier method for solving problems of optimal allocation of resources'. He has pointed out that the conversion of constrained optimization problem to unconstrained optimization problem is especially crucial for the cell problem*. Because, the problem which was essentially of combinatorial nature is uncoupled. (The inventory problem under space constraint is a cell or separable problem). He has given theoretical proof for the validity of searching λ^S corresponding to given resource levels; this procedure gives constrained optimal solutions to any desired accuracy.

---

*A cell problem is one in which there are 'm' independent areas into which the resources may be committed and for which the overall payoff that accrues is simply the sum of the payoffs that accrue from each independent venture.

Brooks and Geoffrion (3) have indicated how one might approximate the desired Lagrange multipliers (for given set of resources) by means of linear programming.

Fox and Lundi (16) have observed that minimax sequential search for obtaining Lagrange multiplier in one constraint optimization problem (such as the one attempted in this thesis) is bisection. However, they caution that for certain applications, it pays to design search procedures that take advantage of the special problem structure.

CHAPTER 3

NUMERICAL OPTIMIZATION OF THE STOCHASTIC INVENTORY PROBLEM UNDER CONSTRAINT

3.1 FORMULATION OF THE MATHEMATICAL MODEL:

Formulation of the mathematical model for an inventory system consists of expressing the expected cost rate (i.e. the cost per an arbitrarily selected unit of time) as a function of the independent or decision variables. The decision variables will depend on the subgroup of the inventory policy to be used. The two subgroups commonly in use are:

a) Q-system, wherein a quantity 'Q' (order size) is ordered when the stock on hand plus on order reaches a level 'I_T' (reorder point) and

b) P-system, wherein the order period is fixed and a variable quantity, depending on the fluctuations in demand, is ordered.

A number of other subgroups of inventory policies are possible.

3.1.1 Selection of the Subgroup of Inventory Policy:

The problem of finding an overall optimal inventory policy is an exceedingly difficult task. Generally, the optimal inventory policy for a specific subgroup is obtained. For a complex inventory system that includes many items, such as the one in consideration, it is not possible to make a comparison between the optimal inventory policies of two different subgroups. The literature is replete with examples of the proofs of optimality of one subgroup or the other. But these proofs of

optimality are generally developed under restrictive assumptions, e.g. zero lead time. (34, p. 116). No attempt is made in this thesis to obtain an overall optimal inventory policy. Starr and Miller (35, p. 124) have shown that the optimal Q-system is generally less expensive than the optimal P-system for the same item, if the systemic cost of perpetual inventory surveillance and the saving in cost due to joint ordering are neglected. Assuming that, when a computer is available for keeping track of the inventories, the systemic cost for perpetual inventory auditing is insignificant and neglecting the advantages of joint ordering, the Q-system is selected for the inventory control of the stochastic inventory system of the automated warehouse subject to the space constraint.

3.1.2 Postulates Underlying the Inventory Model:

The three important relevant costs in an inventory system are:

1) cost of ordering or cost of replenishment,
2) cost of holding inventory or carrying cost and
3) out of stock cost.

Expressions for the above costs, for a single item, are obtained using the following postulates:

1) The lead time is shorter than the order period or lot time. If lead time is greater than lot time the decision rule must be modified - an order should be placed when the inventory on hand plus that on order reaches the reorder point. However, Holt et al (24, p. 239) have shown that the expression for cost rate remains unchanged when lead time is greater than lot time.

2) The reorder point is positive.

3) There is no serial correlation of demand rates between the periods considered.

4) When out of stock, the orders are backlogged for future delivery.

5) Lead time is constant and known.

Cost rate functions in the absence of assumptions 2, 3 and 4 are obtained by Holt et al (24, p. 240). However, for practical problems, an inventory model with the above assumptions is good enough.

The effect of variable lead time is considered in 3.1.6.

3.1.3 Cost of Replenishment:

C_F = Cost of replenishment per order.

$\bar{S}$ = Expected demand rate.

Q = Order quantity.

$\frac{\bar{S}}{Q}$ = Expected number of orders per unit time.

Therefore, expected replenishment cost rate = $\frac{C_F \bar{S}}{Q}$ (3.1)

3.1.4 Cost of Holding Inventory:

Figure 1 shows the relationship between time and inventory levels. If the inventory position during lead time is observed, two things can happen before an order arrives.

1) When an order arrives, there is still some inventory on hand, as in Lot 1.

2) When an order arrives, there is no inventory on hand; in other words, a stockout has occurred, as in Lot 2.

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

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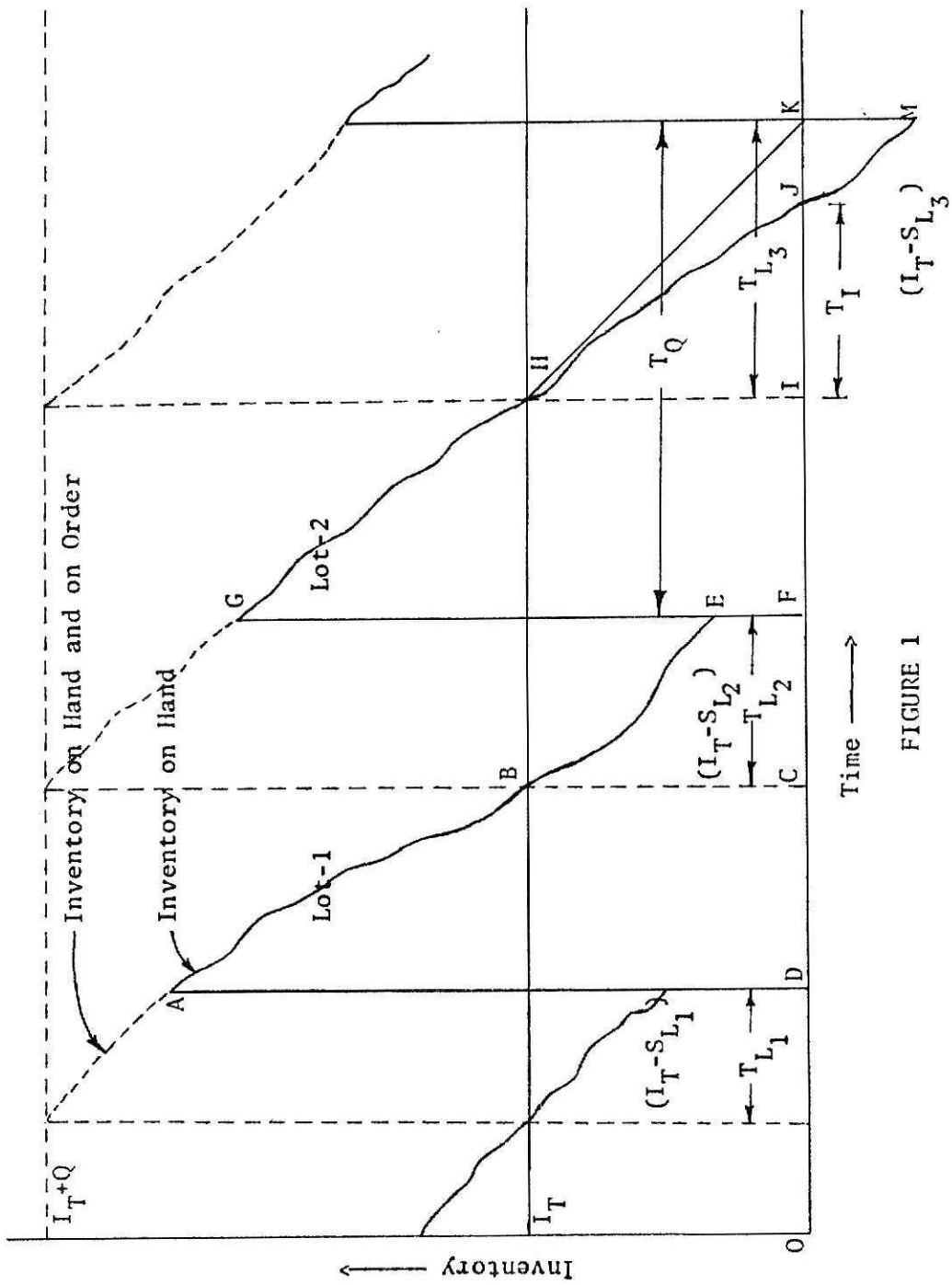


FIGURE 1

INVENTORY LEVELS WITH TIME

The cost of inventory, for the two lots is proportional to the area ADEF and GFJH, respectively. The factor of proportionality is C_I, the cost of storing one unit for one unit of time. Holding cost for the two lots until reorder point is reached, will have the same expression. The area ABCD is approximately:

$$ABCD = \frac{1}{2}(T_Q - T_L)(Q + (I_T - S_{L_1}) + I_T) \tag{3.2}$$

$$= \frac{1}{2S}(Q - T_L S)(Q + 2I_T - S_{L_1}) \tag{3.3}$$

$\therefore T_Q$= Order period = Q/S

The area GFIH is obtained if S_{L_1} is replaced by S_{L_2}.

The cost of holding inventory during a lead time is given by the following:

A) Demand during lead time is less than reorder level:

As shown in Lot 1, this cost is proportional to the area BCFE, which is approximately:

$$BCEF = \frac{1}{2} T_L (I_T + I_T - S_{L_2}) \tag{3.4}$$

$$= \frac{1}{2} T_L (2I_T - S_{L_2}) \tag{3.5}$$

B) Demand during lead time is more than reorder level:

As shown in Lot 2, this cost is proportional to the area HIJ, which is approximately:

$$HIJ = \frac{1}{2} I_T \cdot T_I \tag{3.6}$$

By similar triangles, $\frac{T_I}{T_L} = \frac{I_T}{S_{L_3}}$

$$HIJ = \frac{1}{2} I_T \left(\frac{I_T \cdot T_L}{S_{L_3}}\right) \tag{3.7}$$

$$= \frac{I_T^2 \cdot T_L}{S_{L_3}} \tag{3.8}$$

Assuming that

1) S_{L_1}, S_{L_2} and S_{L_3} are distributed independently with the same probability density function and

2) S is independent of S_{L_1}, S_{L_2} and S_{L_3}, the holding cost per lot is

$$HCPL = \frac{C_I}{2S} (Q - T_L S)(Q + 2I_T - S_L) + \frac{C_I T_L}{2} \begin{cases} (2I_T - S_L) & \text{if } S_L \leq I_T \\ \frac{I_T^2}{S_L} & \text{if } S_L > I_T \end{cases} \tag{3.9}$$

The holding cost rate, therefore, is obtained by multiplying (3.9) by $\frac{1}{T_Q}$ or $\frac{S}{Q}$:

$$HCR = \frac{C_I}{2Q} (Q - T_L S)(Q + 2I_T - S_L) + \frac{C_I S T_L}{2Q} \begin{cases} (2I_T - S_L) & \text{if } S_L \leq I_T \\ \frac{I_T^2}{S_L} & S_L > I_T \end{cases} \tag{3.10}$$

Expected holding cost rate, EHCR, therefore is

$$EHCR = \frac{C_I}{2Q}(Q - \bar{S}_L)(Q + 2I_T - \bar{S}_L) + \frac{C_I\bar{S}_L}{2Q}\int_0^{I_T}(2I_T - S_L)\, f(S_L)\,dS_L$$

$$+ \frac{C_I\bar{S}_L}{2Q}\int_{I_T}^{\infty}\frac{I_T^2}{S_L} f(S_L)\,dS_L \qquad (3.11)$$

Now,

$$\int_0^{I_T}(2I_T-S_L)\, f(S_L)\,dS_L = \int_0^{\infty}(2I_T-S_L)\, f(S_L)\,dS_L - \int_{I_T}^{\infty}(2I_T-S_L)\, f(S_L)\,dS_L \qquad (3.12)$$

$$= (2I_T-\bar{S}_L) - \int_{I_T}^{\infty}(2I_T-S_L)\, f(S_L)\,dS_L \qquad (3.13)$$

Substituting (3.13) in (3.11), the expected holding cost rate is

$$EHCR = \frac{C_I}{2Q}(Q - \bar{S}_L)(Q + 2I_T - \bar{S}_L) + \frac{C_I\bar{S}_L}{2Q}(2I_T-\bar{S}_L)$$

$$+ \frac{C_I\bar{S}_L}{2Q}\int_{I_T}^{\infty}\frac{(S_L-I_T)^2}{S_L} f(S_L)\,dS_L \qquad (3.14)$$

simplifying,

$$EHCR = C_I\left(\frac{Q}{2} + I_T - \bar{S}_L\right) + \frac{C_I\bar{S}_L}{2Q}\int_{I_T}^{\infty}\frac{(S_L-I_T)^2}{S_L} f(S_L)\,dS_L \qquad (3.15)$$

The above expression can be considerably simplified if the holding cost during those lead times in which stockouts occur, is approximated. Instead of the triangle HIJ, triangle HIK is used to calculate the

holding cost. Using this area HIK, the approximate holding cost rate is:

$$EHCR = C_I\,(Q/2 + I_T - \bar{S}_L) + \frac{C_I \bar{S}_L}{2Q} \int_{I_T}^{\infty} (S_L - I_T)\, f(S_L)\, dS_L \qquad (3.16)$$

Thus, (3.15) gives an exact holding cost rate, whereas (3.16) gives an approximate holding cost rate.

3.1.5 Cost of Stockout:

The stockout cost is very difficult to estimate and quantify. It would differ largely in various inventory problems. It would depend on the product, industry and market involved in the inventory problem. A few simple formulations are given below:

A) Stockout Cost Proportional to the Number of Stockouts:

Whenever a stockout occurs a fixed cost C_{D1} (in dollars) is incurred, irrespective of the out of stock quantity or the time for which the item was out of stock.

$$\text{Expected stockout cost rate} = \frac{C_{D1} \bar{S}}{Q} \int_{I_T}^{\infty} f(S_L)\, dS_L \qquad (3.17)$$

B) Stockout Cost Proportional to the Number of Units out of Stock. C_{D2} in dollars per unit of the item out of stock.

$$\text{Expected stockout cost rate} = \frac{C_{D2} \bar{S}}{Q} \int_{I_T}^{\infty} (S_L - I_T)\, f(S_L)\, dS_L \qquad (3.18)$$

C) Stock Cost Proportional to the Product of the Number of Units Out of Stock Times the Duration of the Stockout. C_{D3} in dollars per unit

time per unit out of stock.

$$\text{Stockout Cost} = C_{D3}\,(T_L - T_I)(S_L - I_T), \quad \text{if } S_L > I_T \tag{3.19}$$

From Fig. 1, by similar triangles,

$$\frac{T_L - T_I}{T_L} = \frac{S_L - I_T}{S_L} \tag{3.20}$$

$$\therefore \text{ Stockout cost} = \frac{(S_L - I_T)^2}{S_L}\, T_L, \quad \text{if } S_L > I_T \tag{3.21}$$

Since T_L is constant, $\bar{S}T_L = \bar{S}_L$. Therefore,

$$\text{Expected stockout cost rate} = \frac{C_{D3}\bar{S}_L}{Q} \int_{I_T}^{\infty} \frac{(S_L - I_T)^2}{S_L}\, f(S_L)\, dS_L \tag{3.22}$$

D) Stockout Cost Proportional to the Maximum Duration of the Stockout.

C_{D4} in dollars per time unit.

$$\text{Stockout cost} = C_{D4}\,(T_L - T_I), \quad \text{if } S_L > I_T \tag{3.23}$$

$$= C_{D4}\,\left(T_L - \frac{T_L I_T}{S_L}\right), \quad \text{if } S_L > I_T \tag{3.24}$$

$$= C_{D4}\, T_L\, \left(1 - \frac{I_T}{S_L}\right), \quad \text{if } S_L > I_T \tag{3.25}$$

$$\therefore \text{ Expected stockout cost rate} = \frac{C_{D4}\bar{S}_L}{Q} \int_{I_T}^{\infty} \left(\frac{S_L - I_T}{S_L}\right) f(S_L)\, dS_L \tag{3.26}$$

3.1.6 Effect of Variable Lead Time:

Variability of lead time brings in considerable complexities in the formulation of an inventory model. Let T_L and S_L be jointly distributed, independent of S, T_{L_1}, T_{L_2}, T_{L_3}, S_{L_1}, S_{L_2}, S_{L_3}, with the p.d.f. $f_1(T_L,S_L)$. All the costs then, where S_L and T_L are present, will be altered. For example, inventory holding cost rate given by (3.15) will be changed to

$$C_I \left(\frac{Q}{2} + I_T - \bar{S}_L\right) + \frac{C_I \bar{S}}{Q} \int_{I_T}^{\infty} \int_{0}^{\infty} T_L \frac{(S_L - I_T)^2}{S_L} f_1 (T_L,S_L)\, dS_L\, dT_L \qquad (3.27)$$

Let $g(T_L|S_L)$ be the conditional p.d.f. of T_L given S_L, and $f(S_L)$ be the marginal p.d.f. of S_L.

Then, $f_1(T_L, S_L) = g(T_L|S_L)\ f(S_L)$ (3.28)

Therefore,

$$\int_0^{\infty} T_L\, f_1(T_L,S_L)\, dT_L = \int_0^{\infty} T_L\, f(S_L)\, g(T_L|S_L)\, dT_L \qquad (3.29)$$

$$= T_L(S_L)\ f(S_L) \qquad (3.30)$$

Where, $T_L(S_L)$ is the conditional mean of T_L given S_L, which is not easy to obtain. One realistic approximation that can be used is

$$T_L(S_L) = \frac{S_L}{\bar{S}} \qquad (3.31)$$

Using this approximation, various cost functions would be modified as shown below:

Equation		is modified to	
(3.15)	$C_I(\frac{Q}{2} + I_T - \bar{S}_L) + \frac{C_I}{2Q}\int_{I_T}^{\infty} (S_L - I_T)^2 f(S_L)\,dS_L$		(3.32)
(3.16)	$C_I(\frac{Q}{2} + I_T - \bar{S}_L) + \frac{C_I}{2Q}\int_{I_T}^{\infty} S_L(S_L - I_T) f(S_L)\,dS_L$		(3.33)
(3.22)	$\frac{C_{D3}}{Q}\int_{I_T}^{\infty} (S_L - I_T)^2 f(S_L)\,dS_L$		(3.34)
(3.26)	$\frac{C_{D4}}{Q}\int_{I_T}^{\infty} (S_L - I_T) f(S_L)\,dS_L$		(3.35)

Equations (3.17) and (3.18) remain unchanged.

3.1.7 Various Models

A large number of inventory models can be formulated, taking various combinations of the forms for holding and stockout costs, with and without considering variability of lead time. For example,

1) Approximate inventory holding cost, stockout cost proportional to the number of stockouts, constant and known lead time.
2) Same as 1), except that lead time is variable.
3) Approximate inventory holding cost, stockout cost proportional to the number of units out of stock, constant and known lead time.
4) Exact inventory holding cost, stockout cost proportional to the product of the number of units times the duration of the stockout, constant and known lead time.

In this chapter, the model mentioned last will be used, from now on. The total expected inventory cost rate, for a single item is obtained by summing the three costs given by (3.1), (3.15) and (3.22).

$$CS = C_F \frac{\bar{S}}{Q} + C_I \left(\frac{Q}{2} + I_T - \bar{S}_L\right) + \frac{\bar{S}_L}{Q}\left(\frac{C_I}{2} + C_{D3}\right) \int_{I_T}^{\infty} \frac{(S_L - I_T)^2}{S_L} f(S_L)\, dS_L \tag{3.36}$$

When 'n' items in the system are considered the total cost is the sum of the individual costs given by (3.36).

$$TC = \sum_{i=1}^{n} CS_i$$

$$= \sum_{i=1}^{n} \left(C_{F_i} \frac{\bar{S}_i}{Q_i} + C_{I_i} \left(\frac{Q_i}{2} + I_{T_i} - \bar{S}_{L_i}\right) \right.$$

$$\left. + \frac{\bar{S}_{L_i}}{Q_i}\left(\frac{C_{I_i}}{2} + C_{D_i}\right) \int_{I_{T_i}}^{\infty} \frac{(S_{L_i} - I_{T_i})^2}{S_{L_i}} f(S_{L_i})\, dS_{L_i} \right) \tag{3.37}$$

(C_D will be used in place of C_{D3}).

3.1.8 Warehouse Space Constraint:

A warehouse space constraint can be used in a number of ways. In a warehouse storing 'n' items, the maximum space that would be required is when

1) replenishment of all the items occurs at the same time and

2) there is no demand for any of the items during their respective

lead times, immediately preceding the event of the simultaneous replenishment of all the items.

Considering the constraint of the maximum required capacity,

$$CAP = \sum_{i=1}^{n} w_i \, (Q_i + I_{T_i}) \tag{3.38}$$

Designing an inventory policy for such a constraint is not a practical solution. The utilization of space under such a policy would be very poor. The probability of this maximum space requirement is indeed small. To get a more realistic solution, the assumption that there is no demand for any item during the respective lead times, immediately preceding the simultaneous replenishment can be discarded. Instead, average demand during lead time is assumed, whereby, the constraint takes the form

$$CAP = \sum_{i=1}^{n} w_i \, (Q_i + I_{T_i} - \bar{S}_{L_i}) \tag{3.39}$$

Another form of constraint, commonly used in the literature is the average constraint, which can be represented by

$$CAP = \sum_{i=1}^{n} w_i \, (\frac{Q_i}{2} + I_{T_i} - \bar{S}_{L_i}) \tag{3.40}$$

Clearly, this is not a realistic constraint, because the management would run out of capacity 50% of the time. A rational criterion would be: 'warehouse space should be sufficient on 'X'% days and on '(100-X)'% days, an alternative arrangement for storage need be made'. The figure

'X' will depend on the cost of running out of capacity (the cost of hiring extra capacity at short notice, the loss due to spoilage etc.). This cost may not be explicitly available. The management, however, should be able to specify 'X', which implies this cost.

The procedure followed in this thesis is to use constraint (3.39) and find out optimal inventory policy. The utilization of warehouse space is simulated using the optimal Q_i^S and $I_{T_i}^S$. Knowing the figure 'X', a factor 'K' (less than 1, normally) is obtained to alter the constraint as below:

$$CAP = K \sum_{i=1}^{n} w_i \, (Q_i + I_{T_i} - \bar{S}_{L_i}) \tag{3.41}$$

The revised constraint can be used to obtain the optimal inventory policy.

3.2 OPTIMIZATION OF THE INVENTORY MODEL UNDER SPACE CONSTRAINT:

The problem is to minimize the total cost function given by (3.37) subject to the constraint (3.39).

Minimize

$$\sum_{i=1}^{n} \left(C_{F_i} \frac{\bar{S}_i}{Q_i} + C_{I_i} \left(\frac{Q_i}{2} + I_{T_i} - \bar{S}_{L_i} \right) + \frac{\bar{S}_{I_i}}{Q_i} \left\{ \frac{C_{I_i}}{2} + C_{D_i} \right\} \int_{I_{T_i}}^{\infty} \frac{(S_{L_i} - I_{T_i})^2}{S_{L_i}} f(S_{L_i}) \, dS_{L_i} \right)$$

Subject to

$$CAP = \sum_{i=1}^{n} w_i \, (Q_i + I_{T_i} - \bar{S}_{L_i})$$

This can be done by using the classical Lagrange multiplier method, whereby constrained optimization problem is converted to unconstrained

optimization problem. The Lagrangian would be

$$L = \sum_{i=1}^{n} \left\{ C_{F_i} \frac{\bar{S}_i}{Q_i} + C_{I_i} (Q_i/2 + I_{T_i} - \bar{S}_{L_i}) + \frac{\bar{S}_{L_i}}{Q_i} \left\{ \frac{C_{I_i}}{2} + C_{D_i} \right\} \int_{I_{T_i}}^{\infty} \frac{(S_{L_i} - I_{T_i})^2}{S_{L_i}} \right.$$

$$\left. f(S_{L_i})\, dS_{L_i} \right\} + \lambda \left\{ CAP - \sum_{i=1}^{n} w_i (Q_i + I_{T_i} - \bar{S}_{L_i}) \right\} \qquad (3.42)$$

In order to minimize L with respect to Q_i, I_{T_i} for i = 1, 2, ..., n and λ, partial derivatives of L with respect to these are set equal to zero. (2n+1) equations thus obtained can be solved to obtain (2n+1) unknowns. These (2n+1) equations, hereafter called Basic equations, are

$$\frac{\partial L}{\partial Q_i} = - \frac{\bar{S}_i}{Q_i^2} \left(C_{F_i} + T_{L_i} \left\{ \frac{C_{I_i}}{2} + C_{D_i} \right\} \int_{I_{T_i}}^{\infty} \frac{(S_{L_i} - I_{T_i})^2}{S_{L_i}} f(S_{L_i})\, dS_{L_i} \right)$$

I)

$$+ \left(\frac{C_{I_i}}{2} - w_i \lambda \right) = 0 \qquad i = 1, 2, \ldots, n \qquad (3.43)$$

$$\frac{\partial L}{\partial I_{T_i}} = - \frac{2\bar{S}_{L_i}}{Q_i} \left\{ \frac{C_{I_i}}{2} + C_{D_i} \right\} \int_{I_{T_i}}^{\infty} \frac{S_{L_i} - I_{T_i}}{S_{L_i}} f(S_{L_i})\, dS_{L_i}$$

II)

$$+ (C_{I_i} - w_i \lambda) = 0 \qquad i = 1, 2, \ldots, n. \qquad (3.44)$$

III) $$\frac{\partial L}{\partial \lambda} = CAP - \sum_{i=1}^{n} w_i (Q_i + I_{T_i} - \bar{S}_{L_i}) = 0 \qquad (3.45)$$

The value of λ, must be such that equation (3.45) is satisfied. In general, an explicit analytical solution is not available, because it is not possible to eliminate λ from (3.43) and (3.44). Even if it were possible, the presence of the particular integrals, forbids explicit solution. These integrals cannot always be evaluated, even in the case of standard distributions for demand during lead time. For the gamma distribution, some particular integrals can be obtained using a computer (24), but the computation procedure is quite involved. Besides, demand during lead time may not follow any known standard distribution. It is possible that lead time follows certain known distribution, and demand rate follows certain other known distribution. The product of the two distributions (which is the distribution for demand during lead time), however, might not be tractable.

The conclusion, therefore, is that the optimal solution, for a general inventory model, for a general real world inventory problem, can not be obtained explicitly and analytically, from the Basic Equations. To deal with the general inventory problem, recourse to numerical method and numerical integration needs to be taken. The knowledge of the distribution of demand during lead time is essential. This variable is discussed below briefly.

3.3 LEAD TIME DEMAND:

The information about lead time demand can be obtained from following three sources.

3.3.1 Historical Records:

It is not possible to get lead time demands directly from the usual

historical records. The records give information about demand rate and lead time. It is possible to use this information to get the lead time demand distribution (19), (31), (2), (27), (6). In an automated warehouse, it should not be difficult to obtain the lead time demand, directly. The information obtained from past data, modified suitably to be used for the future, would be reasonably good data to be used for the optimization problem.

3.3.2 Simulated Distribution:

If the distributions for the demand rate and lead time (Poisson and Erlang respectively, for instance) are known, it is possible to simulate the demand during lead time.

3.3.3 Known Distribution for Lead Time Demand:

It is possible to obtain the simulated lead time demands knowing lead time demand distribution, e.g. normal (30), (23), pseudo-exponential (32).

3.4 FREQUENCY DISTRIBUTION FOR LEAD TIME DEMAND:

For numerical integration, it is necessary to split up the lead time demand distribution, into a frequency distribution, with a predetermined number of classes. When demand is given (either from historical records or simulated) in integer quantities, special precaution must be taken, in obtaining frequency distribution. If the class interval is 0.5, alternate classes will have zero probabilities, if an integer demand variable is tabulated. Hence, even if the actual demand (e.g. simulated from a Poisson distribution) is monotonically increasing or decreasing,

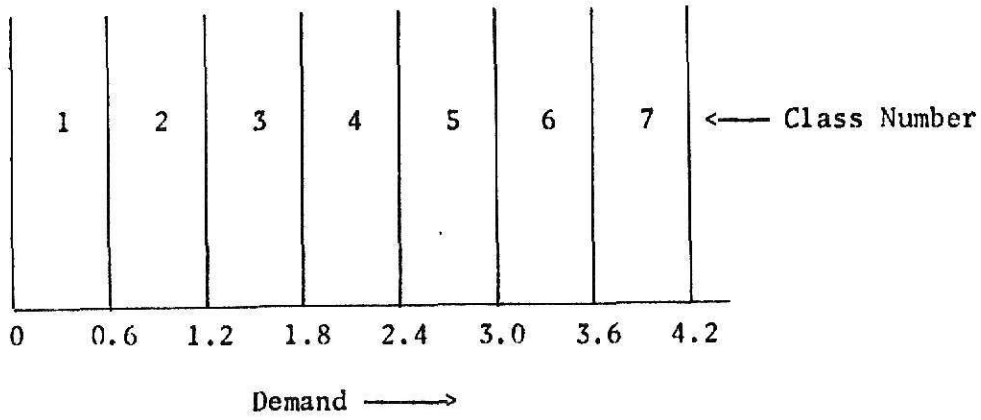


FIGURE 2

FREQUENCY DISTRIBUTION FOR INTEGER DEMAND

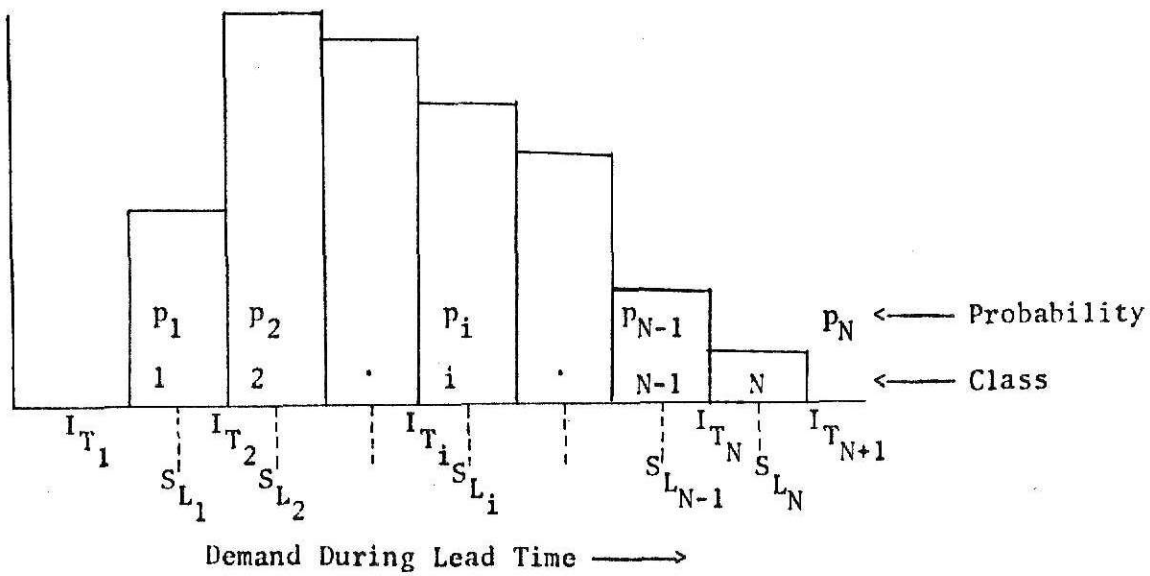


FIGURE 3

FREQUENCY DISTRIBUTION FOR DEMAND DURING LEAD TIME

the frequency distribution will not reporduce it, there will be inversions, which will affect the integrals in the Basic Equations such that an optimal solution will not be reached. In order to avoid this, any integer demand should be assumed to have equal probability between the interval $\pm$ 0.5 about the integer value. To illustrate this by example, let the class interval be 0.6, as shown in Figure 2 .

A demand of 3 units will add to the classes 5 and 6 the frequencies 0.5 and 0.5. A demand of 2 units will add to the classes 3, 4 and 5 frequencies of 0.3, 0.5 and 0.1 respectively.

3.5 NUMERICAL INTEGRATION:

Let the frequency distribution, of lead time demand for an item obtained as described in 3.4, be as shown in Figure 3.

Demand during lead time is distributed in 'N' classes of equal class intervals.

f_i = frequency of the i^{th} class.

p_i = probability of the lead time demand being in the i^{th} class.

S_{L_i} = mid-value of the lead time demand for the i^{th} class.

I_{T_1} = minimum lead-time-demand.

$I_{T_{N+1}}$ = maximum lead-time-demand.

Let the integral to be obtained be:

$$\int_{I_T}^{\infty} g(S_L, I_T)\ f(S_L)\, dS_L \qquad (3.46)$$

where, $g(S_L, I_T)$ is any function of S_L and I_T.

This is approximated as:

$$\sum_{S_L=I_T}^{\infty} + \; g(S_L, I_T) \text{ Prob } (S_L) \tag{3.47}$$

It is assumed that the probability of a class is associated with the demand corresponding to the central value of the class. '(N+1)' integrals, corresponding to '(N+1)' end points of 'N' classes are thus obtained and stored in the computer for further use.

For example, when $I_T = I_{T_i}$

$$\int_{I_{T_i}}^{\infty} g(S_L, I_T)\, f(S_L)\, dS_L = \sum_{j=i}^{N} g(S_{L_j}, I_{T_i}) \text{ Prob } (S_{L_j}) \tag{3.48}$$

3.6 ITERATIVE PROCEDURE FOR OBTAINING OPTIMAL Q AND I_T:

As discussed earlier, the (2n+1) equations given by the set of Basic Equations, can not be solved directly. The following iterative procedure is used.

Step 1: Assume a value of λ.

Step 2: Obtain Q_i and I_{T_i} from the two available equations for i^{th} item, namely (3.43) and (3.44).

Using these two equations Q_i can be expressed as two functions of I_{T_i}.

From (3.43),

$$Q_{1_i} = \left\{ \frac{S_i}{\frac{C_{I_i}}{2} - w_i\lambda} \left\{ C_{F_i} + T_{L_i} \left(\frac{C_{I_i}}{2} + C_{D_i} \right) \int_{I_{T_i}}^{\infty} \frac{(S_{L_i} - I_{T_i})^2}{S_{L_i}} f(S_{L_i}) dS_{L_i} \right\} \right\}^{1/2} \tag{3.49}$$

From (3.44)

$$Q_{2_i} = \frac{2S_{L_i}}{C_{I_i} - w_i\lambda} \left\{ \frac{C_{I_i}}{2} + C_{D_i} \right\} \int_{I_{T_i}}^{\infty} \frac{S_{L_i} - I_{T_i}}{S_{L_i}} f(S_{L_i}) dS_{L_i} \tag{3.50}$$

The value of $I_{T_i} = I_{T_i}^*$ is to be found which gives $Q_{1_i} = Q_{2_i}$. Then, for the assumed λ,

Optimal order size = $Q_i^* = Q_{1_i} = Q_{2_i}$

Optimal reorder point = $I_{T_i}^*$

This is done by starting from $I_{T_i} = I_{T_{i,N+1}}$, the maximum lead time demand for the i^{th} item. I_{T_i}, then is reduced by the class interval, in each subsequent computation. Q_{1_i} and Q_{2_i} are monotonically increasing as I_{T_i} decreases as shown in Figure 4.

The monotonic behaviour of the two functions, Q_{1_i} and Q_{2_i}, offers the facility of the following procedure.

a) Compute Q_{1_i} and Q_{2_i} for $I_{T_i} = I_{T_{i,N+1}}$, the maximum demand during lead time.

b) Decrease I_{T_i} by the class interval. Compute Q_{1_i} and Q_{2_i}.

c) If $Q_{1_i} > Q_{2_i}$, go to step 2.

If $Q_{1_i} < Q_{2_i}$, the desired $I_{T_i}^*$ is between the current I_{T_i} and (I_{T_i} + class interval).

d) Use linear interpolation to find Q_i^* and $I_{T_i}^*$.

Step 3: Obtain optimal order size and reorder point, for all 'n' items, for the value of λ assumed in step 1.

Step 4: Obtain the capacity required for this λ.

$$\text{CAPACITY } (\lambda) = \sum_{i=1}^{n} w_i (Q_i^*(\lambda) + I_{T_i}^*(\lambda) - \bar{S}_{L_i})$$

If this capacity is less than available capacity (CAP), check λ. If λ is zero, then this problem is unconstrained problem. That is, the available capacity is larger than that required by unconstrained problem. Therefore, an optimal solution for CAP, is the same as that for an unconstrained problem. If λ is negative, increase λ, and repeat steps (2), (3) and (4). If this calculated required capacity is more than the available capacity (CAP) reduce λ and repeat steps (2), (3) and (4).

The essence of the problem, thus, is to find an optimal λ^*, for which the calculated capacity - CAPACITY (λ) - is equal to the available capacity, within predetermined permissible error (e.g. $\pm$ 0.1%).

To search for λ^*, for a given available capacity (CAP), two alternative procedures have been used.

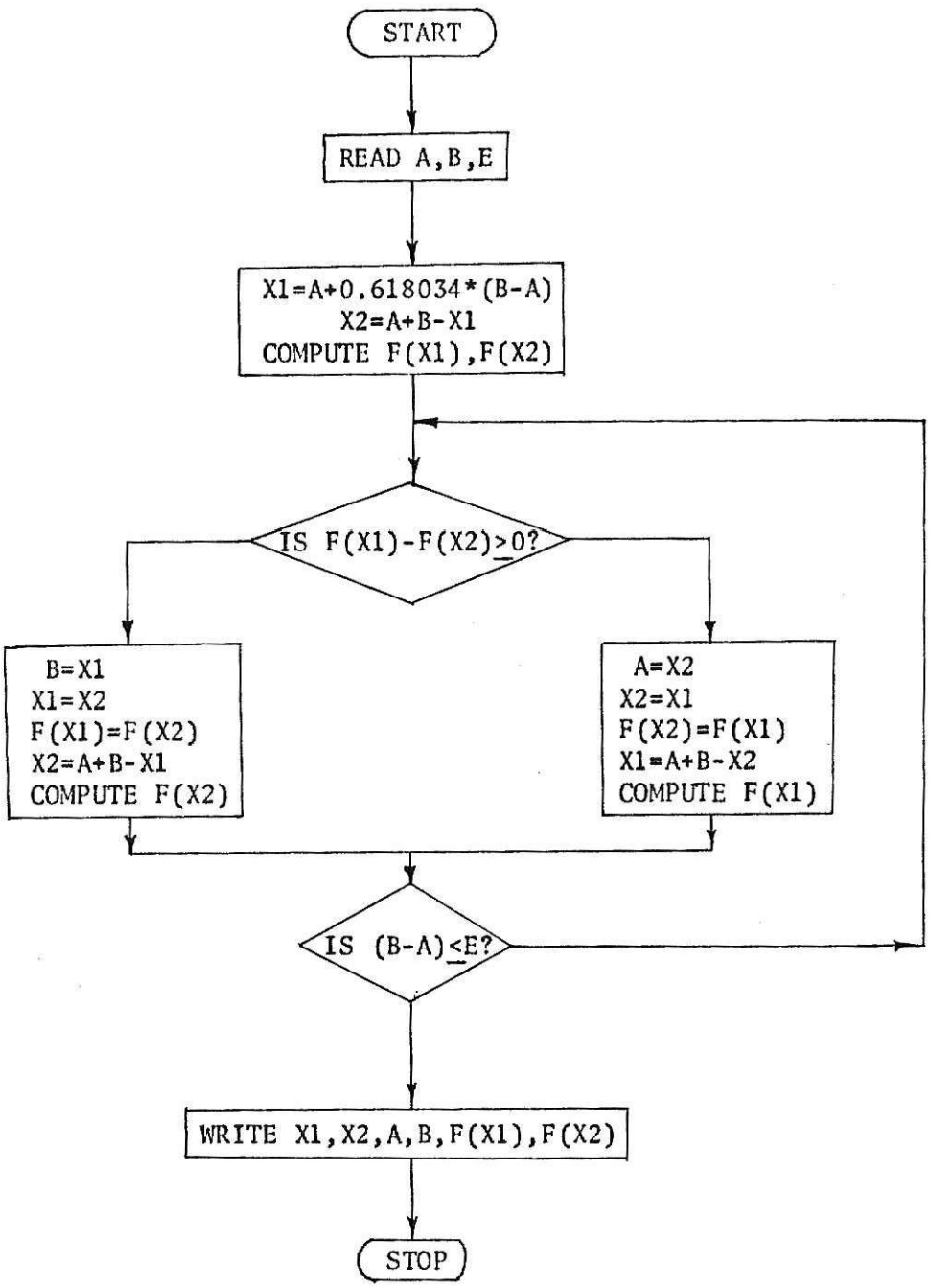


FIGURE 5

FLOW CHART OF GOLDEN SECTION SEARCH
TO MAXIMIZE S=F(X), A X B.

3.6.1 Golden Section Search:

An upper limit of $\lambda_1 = 0$, and a suitable lower limit equal to λ_2 are taken. If CAPACITY (λ_2) is more than available capacity, λ_2 is reduced till CAPACITY (λ_2) becomes smaller than available capacity. Using this range of 0 and negative λ_2, the golden section search is used to find λ^*. A flow diagram for general golden section search is given in Figure 5.

3.6.2 Progressive Linear Interpolation/Extrapolation:

Range of λ ($\lambda_1 = 0$, $\lambda_2 = -1.0$ say) is selected. By linear interpolation/extrapolation, λ corresponding to the available capacity is obtained. If CAPACITY (λ) is equal to CAP, within permissible error the search is stopped. Otherwise, this new value of λ is called λ_2; earlier λ_2 is redefined as λ_1. In this manner, using the latest two values of λ, interpolation/extrapolation is continued, till the optimal solution is obtained.

3.7 DIFFICULTIES IN OBTAINING OPTIMAL SOLUTION:

The integrals appearing in the Basic Equations are available only at a limited number of points - the end points of classes. In the iterative procedure for finding Q_i^* and $I_{T_i}^*$, for an assumed value of λ, linear interpolation has been used. This brings in an error. As λ decreases, CAPACITY (λ) should decrease. However, due to this error, sometimes, CAPACITY (λ) increases as λ decreases. When this happens, the solution procedure adopted in the golden section and progressive linear interpolation/extrapolation methods, to find λ^*, fails and the optimal solution is not obtained. To overcome this difficulty, the

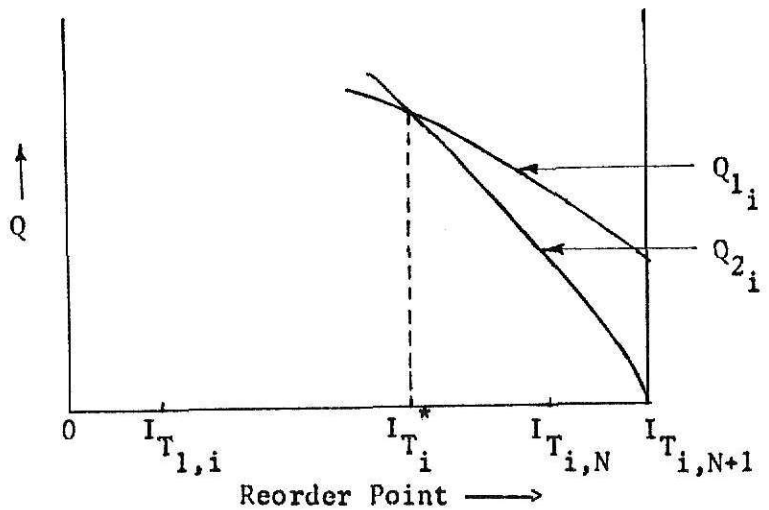


FIGURE 4

ITERATIVE PROCEDURE FOR OBTAINING OPTIMAL SOLUTION

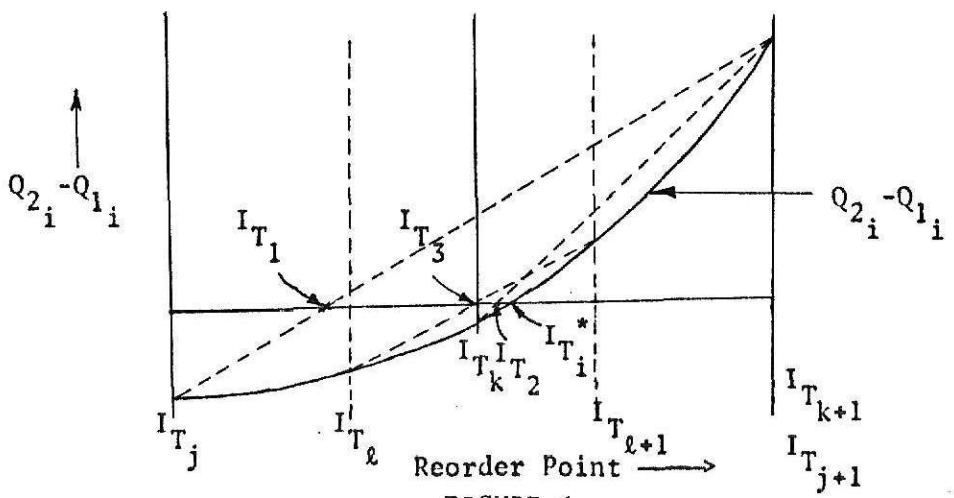


FIGURE 6

EFFECT OF CLASS INTERVAL ON ACCURACY OF OPTIMAL SOLUTION

following steps can be tried.

1) In place of linear interpolation in finding Q^* and I_T^*, use more accurate interpolation - e.g. cubical interpolation.

2) Obtain a graph of λ v/s required capacity for the inventory problem. Use λ corresponding to available capacity and obtain an optimal solution.

3) Increase the number of classes into which demand during lead time has been distributed. This in general should improve the solution; but the error due to interpolation still remains. This error can be sometimes, paradoxically, more in the case of larger number of classes. However, if the number of classes is an integral multiple, then certainly a better solution can be ensured for larger number of classes. This is illustrated in Figure 6.

Three class intervals are considered.

1) $I_{T_j} - I_{T_{j-1}}$ 2) $I_{T_k} - I_{T_{k-1}}$ 3) $I_{T_\ell} - I_{T_{\ell-1}}$

I_{T_1}, I_{T_2}, I_{T_3} and I_T^* are the optimum solutions obtained for the three class intervals and true optimal reorder point, respectively. The class interval for (2) is half that of (1). I_{T_2} is found to be better than I_{T_1}. However, although the class interval for (3) is smaller than that for (2), I_{T_3} is worse than I_{T_2}. This clearly indicates that the accuracy of (optimal) solution obtained depends on

A) class interval and

B) the relative position of the optimal solution from the end values of the class.

4) Instead of using integrals evaluated at finite points, obtain a suitable continuous function of I_{T_i} to describe the dependence of integrals

on I_{T_i}. Use of such a regression function would improve the accuracy of an optimal solution.

3.8 COMPUTER PROGRAMS:

The problem of finding the optimal solution to a constrained inventory problem has been divided into three subproblems:

1) Generating lead time demand distribution and tabulating it in 'N' classes.
2) Obtaining optimal order sizes and reorder points.
3) Simulation of warehouse space utilization.

3.8.1 Lead Time Demand Distribution:

In order to use the numerical approach for finding the optimal order sizes and reorder points, it is necessary to obtain the lead time demand distribution divided in 'N' classes. The objective is to divide the lead time demand, obtained from one of the three sources mentioned in 3.3, into the desired number of classes, and obtain the minimum, maximum class interval and probabilities of the demand for all the classes. A computer program, shown in Appendix 3, has been prepared to accomplish this, when the distributions for demand rate and lead time are given. With minor modifications, shown in the flowchart (Appendix 4), information as given by the other two sources, can be used. In addition to the printed output, punched output suitable to be used in subproblem 2, described in 3.8.2, is obtained.

3.8.2 Obtaining Optimal Order Sizes and Reorder Points:

Knowing the inventory model, the Basic Equations are obtained. For

each item, the nature of the two equations, irrespective of the inventory model, is as below:

$$Q_{1_i} = \left(\frac{C_{I_i}}{2} - w_i\lambda\right)^{-1/2} f_1(I_T) \qquad (3.51)$$

$$Q_{2_i} = (C_{I_i} - w_i\lambda)^{-1} f_2(I_T) \qquad (3.52)$$

$f_1(I_T)$ and $f_2(I_T)$ depend on the inventory model. In subroutine ITGRLS (to be supplied by the user, for his inventory model), these functions are calculated and stored for the (N+1) values of I_T, where N is the number of classes into which the lead time demand for i^{th} item has been divided. The computing procedure obtains the optimal order sizes and reorder points, for any available capacity CAP, such that

$$CAP = \sum_{i=1}^{n} w_i \left(Q_i + I_{T_i} - \bar{S}_{L_i}\right).$$

Flow chart and computer program are given in Appendix 5 and Appendix 6 respectively.

3.8.3 Utilization of Warehouse Space:

Using the optimal order sizes and reorder points obtained in 3.8.2, the warehouse space utilization is simulated. The cumulative probability distribution of the warehouse space used is obtained. If the management can provide the protection level ('X' in 3.1.8) the space constraint can be suitably altered and the optimal order sizes and reorder points can be recalculated.

The effect of the initial inventories of all the items on the space utilization is also studied. The initial inventories are obtained randomly. It is not possible to find the best set of initial inventories; however, a 'good' set can be obtained.

Flow chart and computer program are given in Appendix 7 and Appendix 8 respectively.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 PROBLEM 1:

This problem is constructed for the purpose of solving the constrained optimization problem of an inventory system in all phases: generation of lead time demand distribution, finding optimum order sizes and reorder points and simulation of the warehouse space utilization.

4.1.1 Characteristics of the Inventory System:

1) Number of items in the system is six.
2) Demand rates for all items follow the Poisson distribution.
3) Lead times follow the Erlang distribution with parameter = 4.
4) The lead time variability is used only for generating the lead time demand distributions. For formulation of the model, however, a constant and known lead time is used. This constant and known lead time is equal to the mean of the lead time distribution used for generating the lead time demand distribution.
5) Inventory holding cost is approximated by the equation (3.16).
6) Out of stock cost is proportional to the maximum number of units out of stock - equation (3.18).
7) The constrained optimal solutions are to be obtained for the following five available capacities.
   a) 270 $ft.^3$
   b) 500 $ft.^3$
   c) 1000 $ft.^3$
   d) 1500 $ft.^3$
   e) 2000 $ft.^3$

TABLE 2

DATA FOR THE SIX ITEMS

Item i	Mean Demand Rate units/day	Mean Lead Time days	C_i in \$/unit	C_{F_i} in \$/order	C_{I_i} in \$/unit/day	C_{D_i} in \$/unit	W_i in $ft.^3$/unit
1	4.000	11.000	1.500	20.00	0.00095	4.50	1.30
2	8.000	6.000	2.000	25.00	0.00120	6.70	0.90
3	5.000	3.000	1.000	20.00	0.00061	3.00	1.10
4	5.600	7.300	1.600	20.00	0.00096	5.20	0.80
5	2.000	10.300	1.500	30.00	0.00089	4.80	1.20
6	8.300	3.900	1.800	20.00	0.00116	6.30	0.80

9) The following lead time demand distributions are used to obtain the constrained optimal solutions:

	Lead Time Demands	Classes into which lead time demands are divided
a	1000	25
b	1000	30
c	1000	40
d	1000	50
e	2000	25
f	2000	30
g	2000	40
h	2000	50

4.1.2 Mathematical Model of the Inventory System:

The Lagragian is

$$L = \sum_{i=1}^{6} \left(C_{F_i} \frac{\bar{S}_i}{Q_i} + C_{I_i} \left(\frac{Q_i}{2} + I_{T_i} - \bar{S}_{L_i}\right) + \frac{\bar{S}_i}{Q_i} \left(\frac{C_{I_i} T_{L_i}}{2} + C_{D_i}\right) \int_{I_{T_i}}^{\infty} (S_{L_i} - I_{T_i}) \right.$$

$$\left. f(S_{L_i})\, dS_{L_i} \right) + \lambda \left(CAP - \sum_{i=1}^{6} w_i \left(Q_i + I_{T_i} - \bar{S}_{L_i}\right) \right) \qquad (4.1)$$

The Basic Equations are:

$$\frac{\partial L}{\partial Q_i} = -\frac{\bar{S}_i}{Q_i^2} \left(C_{F_i} + \left(\frac{C_{I_i} T_{L_i}}{2} + C_{D_i}\right) \int_{I_{T_i}}^{\infty} (S_{L_i} - I_{T_i})\, f(S_{L_i})\, dS_{L_i} \right)$$

$$+ \left(\frac{C_{I_i}}{2} - \lambda w_i\right) = 0; \qquad i = 1, 2, \ldots, 6. \qquad (4.2)$$

$$\frac{\partial L}{\partial I_{T_i}} = -\frac{\bar{S}_i}{Q_i}\left(\left(\frac{C_{I_i} T_{L_i}}{2} + C_{D_i}\right)\int_{I_{T_i}}^{\infty} f(S_{L_i})\,dS_{L_i}\right) + (C_{I_i} - \lambda w_i) = 0$$

$$i = 1, 2, \ldots, 6 \qquad (4.3)$$

$$\frac{\partial L}{\partial \lambda} = CAP - \sum_{i=1}^{6} w_i\,(Q_i + I_{T_i} - \bar{S}_{L_i}) = 0 \qquad (4.4)$$

(4.2) and (4.3) yield following two equations for each item:

$$Q_{1_i} = \left\{\frac{1}{C_{I_i}/2 - \lambda w_i}\left(\bar{S}_i\left\{C_{F_i} + \left(\frac{C_{I_i} T_{L_i}}{2} + C_{D_i}\right)\int_{I_{T_i}}^{\infty}(S_{L_i} - I_{T_i}) f(S_{L_i})\,dS_{L_i}\right\}\right)\right\}^{1/2} \qquad (4.5)$$

$$Q_{2_i} = \frac{1}{C_{I_i} - \lambda w_i}\left(\bar{S}_i\left(\frac{C_{I_i} T_{L_i}}{2} + C_{D_i}\right)\int_{I_{T_i}}^{\infty} f(S_{L_i})\,dS_{L_i}\right) \qquad (4.6)$$

The functions to be evaluated in SUBROUTINE ITGRLS at (N+1) - N is the number of classes into which the lead time demand distribution is divided-values of the reorder points therefore are:

$$f_1(I_{T_i}) = \left(\bar{S}_i\left\{C_{F_i} + \left(\frac{C_{I_i} T_{L_i}}{2} + C_{D_i}\right)\int_{I_{T_i}}^{\infty}(S_{L_i} - I_{T_i})\, f(S_{L_i})\,dS_{L_i}\right\}\right)^{1/2} \qquad (4.7)$$

$$f_2(I_{T_i}) = \bar{S}_i\left(\frac{C_{I_i} T_{L_i}}{2} + C_{D_i}\right)\int_{I_{T_i}}^{\infty} f(S_{L_i})\,dS_{L_i} \qquad (4.8)$$

4.1.3 Discussion of the Results:

A. Lead time demand distribution:

For each item the above distributions were obtained as printed and punched card output. A sample printed output for an item is given in Appendix 1. The precaution taken in using integer lead time demands - 3.4 - has considerably reduced the inversions. The corresponding distributions, for the same item, when this precaution was not taken, are shown in Appendix 2.

B. Optimum order sizes and reorder points:

The eight sets of distributions were used in obtaining constrained optimal solution, for each of the five available capacities. The search for λ was made using:

1) Golden section search and

2) Progressive linear interpolation/extrapolation.

Hence, for a given available capacity, there were 16 ways of obtaining an optimal solution. An optimal solution was said to be reached if the search procedure for λ arrived at a λ for which the calculated capacity is equal to the available capacity within $\pm$ 0.1% of the available capacity. In a number of cases the optimal solution was not reached. Table 3 shows the number of iterations (the number of values of λ tried) needed to obtain an optimal solution; it also shows the cases wherein an optimal solution could not be reached. In 29 out of 80 cases (about 36%) the optimal solution was not reached, because of the difficulty in using integrals at a finite number of points, as explained in 3.7. One way of overcoming this difficulty, is to find a relationship between λ and the required capacity for the optimal solution, plot the relationship

TABLE 3

NUMBER OF ITERATIONS REQUIRED TO OBTAIN OPTIMAL SOLUTION

(* indicates that an optimal solution was not reached)

Available Capacity in $ft.^3$	Progressive Linear Interpolation/Extrapolation							
	Classes for 1000 Lead-time-demands				Classes for 2000 Lead-time-demands			
	25	30	40	50	25	30	40	50
270	6	6	6	*	7	5	5	5
500	18	20	18	18	21	22	17	*
1000	22	*	19	*	*	*	24	20
1500	20	19	17	17	20	17	*	18
2000	15	15	14	*	15	14	14	15
	Golden Section Search							
	Classes for 1000 Lead-time-demands				Classes for 2000 Lead-time-demands			
	25	30	40	50	25	30	40	50
270	*	13	13	14	13	13	12	12
500	13	16	17	15	16	8	15	15
1000	*	*	18	*	16	18	18	*
1500	*	*	*	*	*	*	*	*
2000	*	*	*	*	*	*	*	*

and use the graph. For all the eight distributions, this relationship was computed, for a number of values of λ. The graphs are plotted for 1000 and 2000 lead-time demands divided into 25 and 50 classes, in Figure 7 and Figure 8. The smoothing of the curve when classes are doubled is evident. The graph for a different range of λ is shown in Figure 9.

The optimal solution for a given available capacity is considerably influenced by the distribution of lead time demand used in the analysis. For an available capacity of 500, the optimal solutions obtained for the various distributions are shown in Table 4.

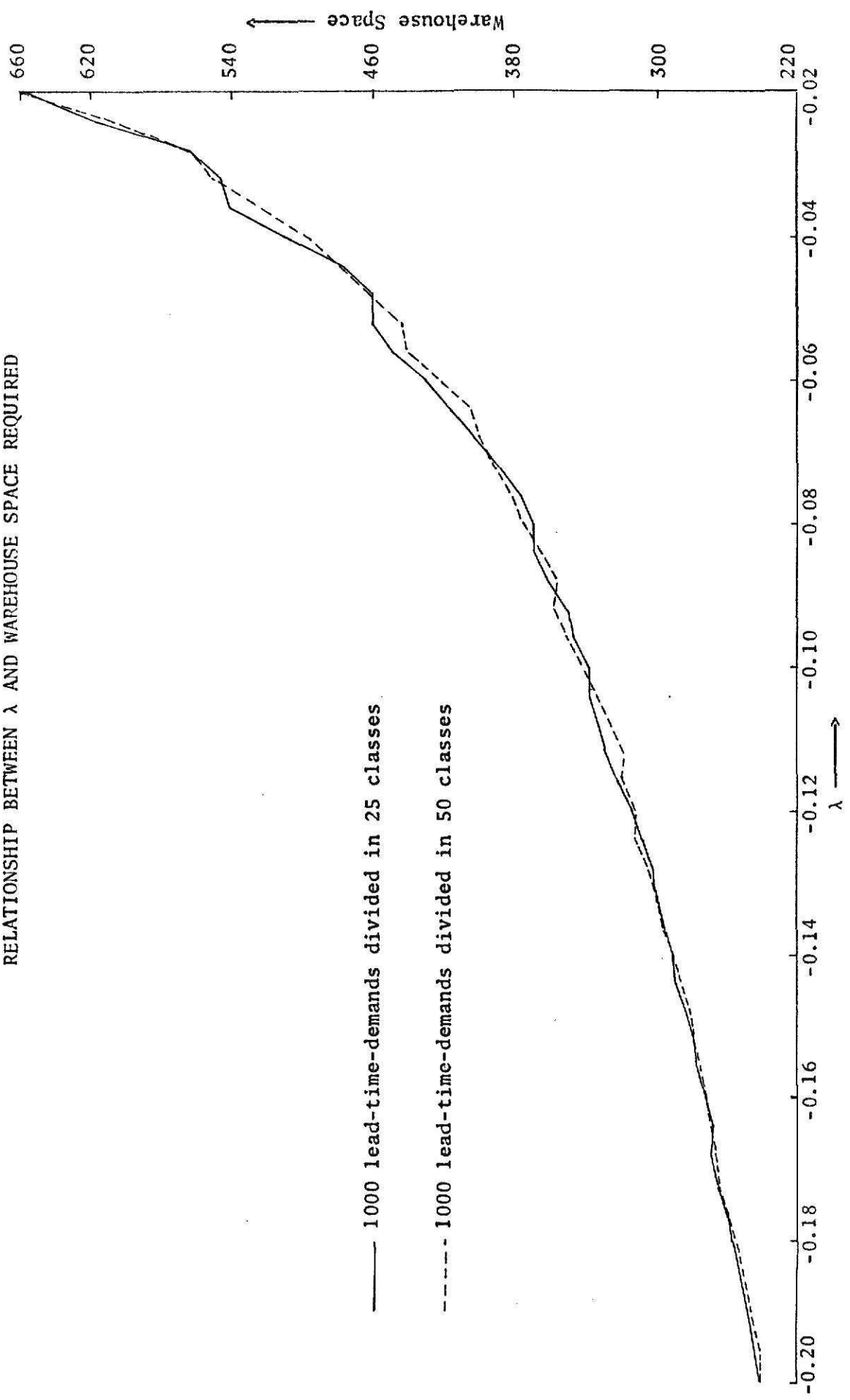
It has been observed that the optimal solution obtained from the distribution divided in 50 classes, is better than the optimal solutions using 25, 30 or 40 classes. The optimal solutions for the five available capacities tried, using the distributions obtained from 1000 and 2000 lead time demands divided into 50 classes are given in Table 5 and Table 6. No meaningful comparison of the two search methods, used for determining an optimal λ for a given capacity, could be made.

c) Utilization of Warehouse Capacity:

Using the optimal solution (optimal order sizes and optimal reorder points) obtained for an available capacity of 1500, using 1000/50 distribution, utilization of capacity was simulated. Six sets of initial inventories were used. Utilization of the capacity was printed out at every 500 time units, upto 3500 time units. The region of interest is that of the higher values for capacities used. e.g. capacities that are sufficient with probability 0.9 and more. The effect of the length of simulation on stabilization of capacity utilization is shown in Figure 10. By visual

FIGURE 7

RELATIONSHIP BETWEEN λ AND WAREHOUSE SPACE REQUIRED



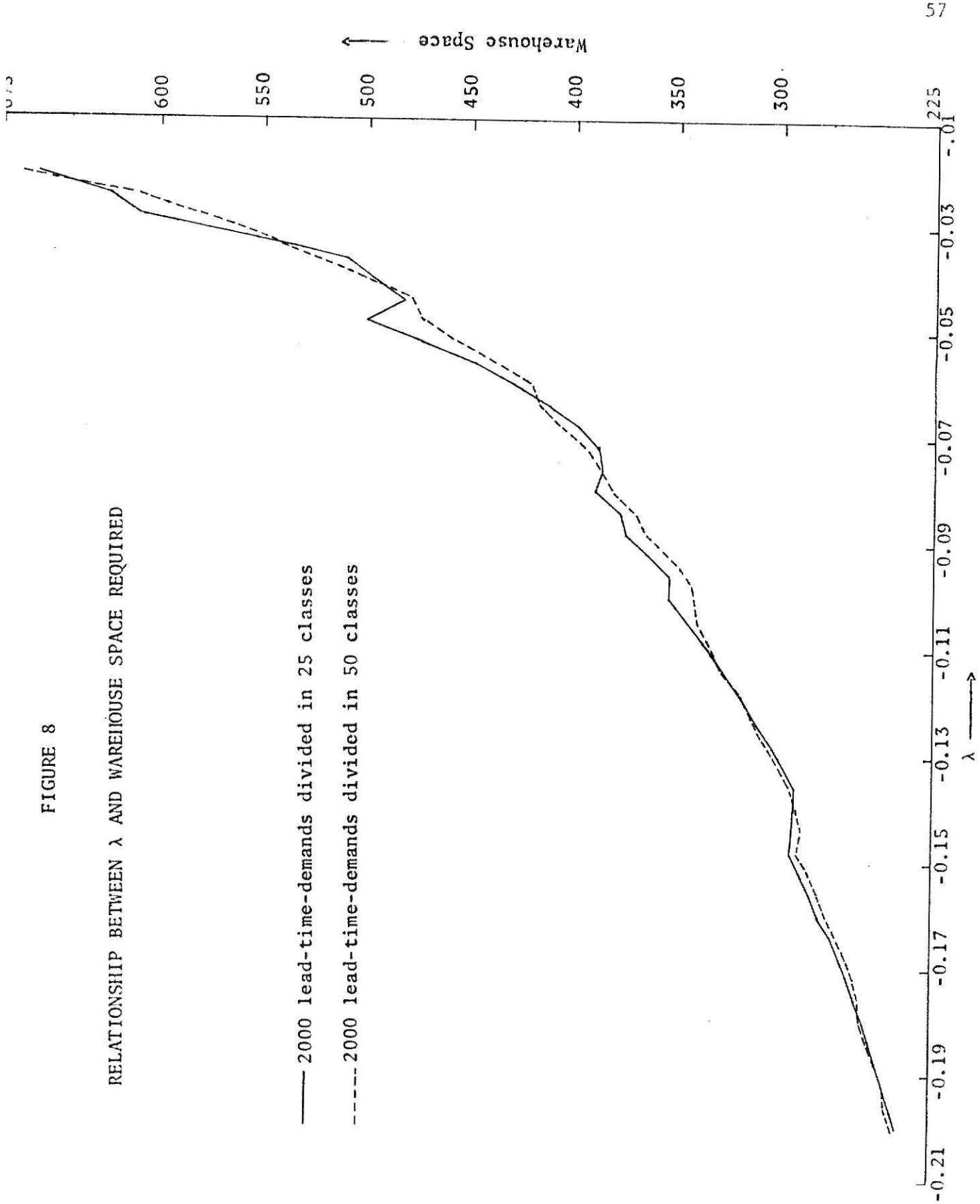


FIGURE 8

RELATIONSHIP BETWEEN λ AND WAREHOUSE SPACE REQUIRED

FIGURE 9

RELATIONSHIP BETWEEN λ AND WAREHOUSE SPACE REQUIRED

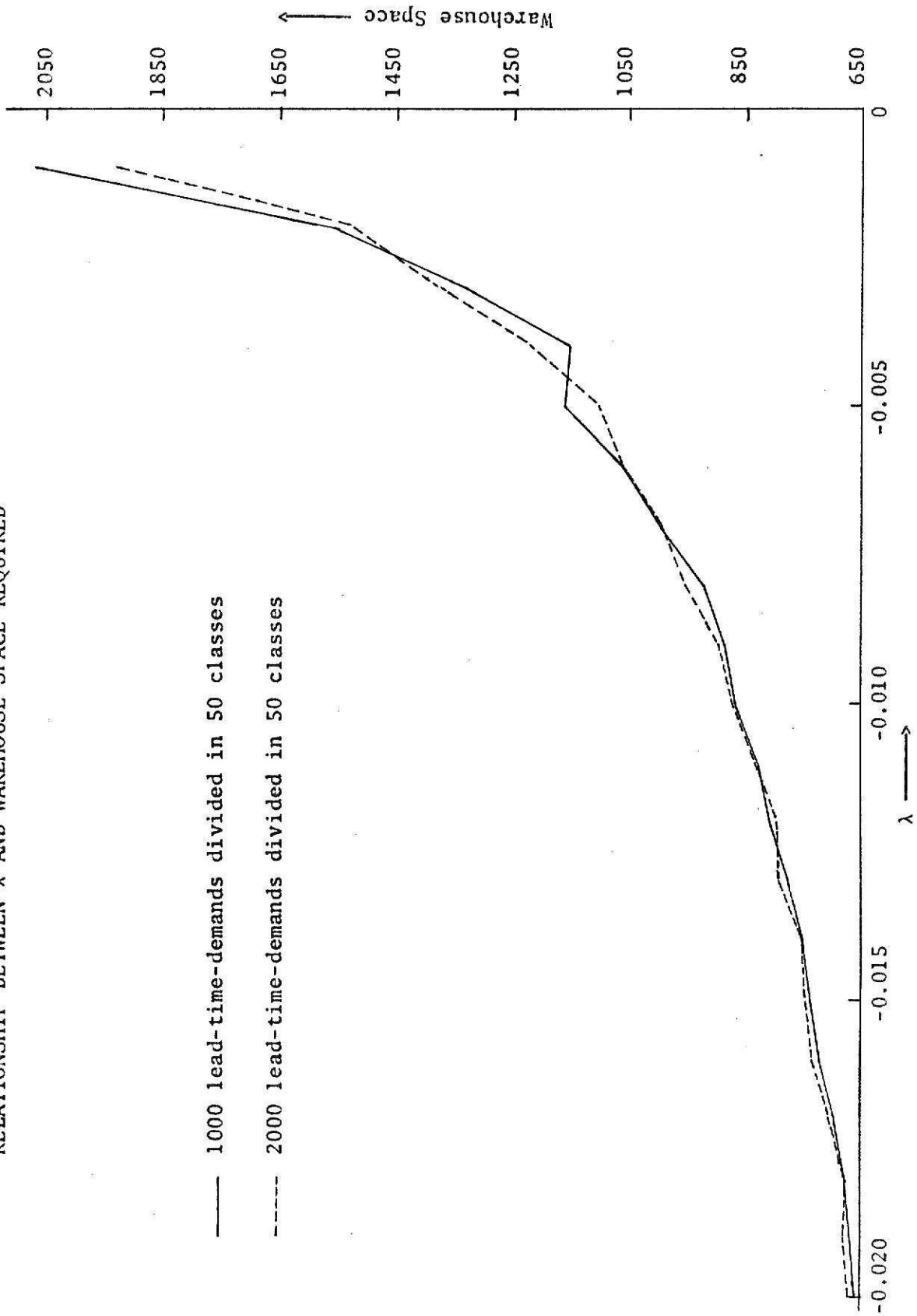


TABLE 4

BEHAVIOUR OF OPTIMAL SOLUTION WITH DISTRIBUTION FOR AVAILABLE CAPACITY OF 1500 FT.^{3}

(Progressive linear interpolation/extrapolation was used in the search for λ)

Distribution Lead Time Demands	Classes	Optimal Order Size for Item 1	2	3	4	5	6	Optimal Reorder Point for Item 1	2	3	4	5	6	Required Capacity in ft.^{3}	Total Cost Rate $/Day	Number of Iterns.	λ
1000	25	162	244	195	246	143	308	93	115	30	91	41	77	1499.84	4.374	20	-0.00240
1000	30	156	319	189	190	141	302	92	108	29	98	41	77	1499.97	4.379	19	-0.00258
1000	40	162	265	235	213	138	249	97	114	29	98	43	81	1498.57	4.374	17	-0.00208
1000	50	169	298	184	207	143	275	94	112	31	99	42	79	1499.68	4.349	17	-0.00215
2000	25	172	262	174	271	123	314	90	113	31	89	43	76	1499.93	4.432	20	-0.00265
2000	30	145	270	174	216	136	355	100	115	32	98	43	76	1499.67	4.443	17	-0.00225
2000	40	OPTIMUM SOLUTION COULD NOT BE OBTAINED DUE TO INVERSION															
2000	50	149	245	207	249	148	291	98	115	30	93	41	79	1499.98	4.405	18	-0.00237

TABLE 5

OPTIMAL SOLUTION FOR VARIOUS AVAILABLE CAPACITIES USING 1000 LEAD-TIME-DEMANDS DIVIDED IN 50 CLASSES

(G: Golden Section Search; L = Progressive Linear Interpolation/Extrapolation)

Available Capacity in $ft.^3$		Optimal Order Size for item						Optimal Reorder Point for item						Required Capacity in $ft.^3$	Total Cost Rate $/Day	No. of Itrns.	λ
		1	2	3	4	5	6	1	2	3	4	5	6				
270	G	33	46	26	35	25	46	45	78	18	62	19	54	269.97	34.046	14	-0.16193
	L	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															
500	G	47	87	48	72	40	81	67	91	24	71	30	64	499.58	14.801	15	-0.03995
	L	47	87	48	72	40	81	67	91	24	71	30	64	499.98	14.786	18	-0.03990
1000	G	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															
	L	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															
1500	G	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															
	L	169	298	184	207	145	275	94	112	31	99	42	79	1499.68	4.349	17	-0.00215
2000	G	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															
	L	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															

TABLE 6

OPTIMAL SOLUTION FOR VARIOUS AVAILABLE CAPACITIES USING 2000 LEAD-TIME-DEMANDS DIVIDED IN 50 CLASSES

(G: Golden Section Search; L = Progressive Linear Interpolation/Extrapolation)

Available Capacity in ft.^{3}		Optimal Order Size for item						Optimal Reorder Point for item						Required Capacity in ft.^{3}	Total Cost Rate $/Day	No. of Itrns.	λ
		1	2	3	4	5	6	1	2	3	4	5	6				
270	G	35	45	27	42	25	44	44	78	18	57	18	54	270.03	34.616	12	-0.16493
	L	35	45	27	42	25	43	44	78	18	57	18	54	269.84	34.647	5	-0.16511
500	G	50	83	48	67	39	82	66	93	24	74	30	64	500.45	14.991	15	-0.04007
	L	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															
1000	G	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															
	L	102	175	125	140	85	175	84	108	28	91	39	76	999.85	6.401	20	-0.00693
1500	G	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															
	L	149	245	207	249	148	291	98	115	30	93	41	79	1499.98	4.405	18	-0.00237
2000	G	NO SOLUTION DUE TO THE NONMONOTONICALLY DECREASING INTEGRALS															
	L	231	402	268	318	189	373	100	114	33	98	45	81	2000.09	3.635	15	-0.00093

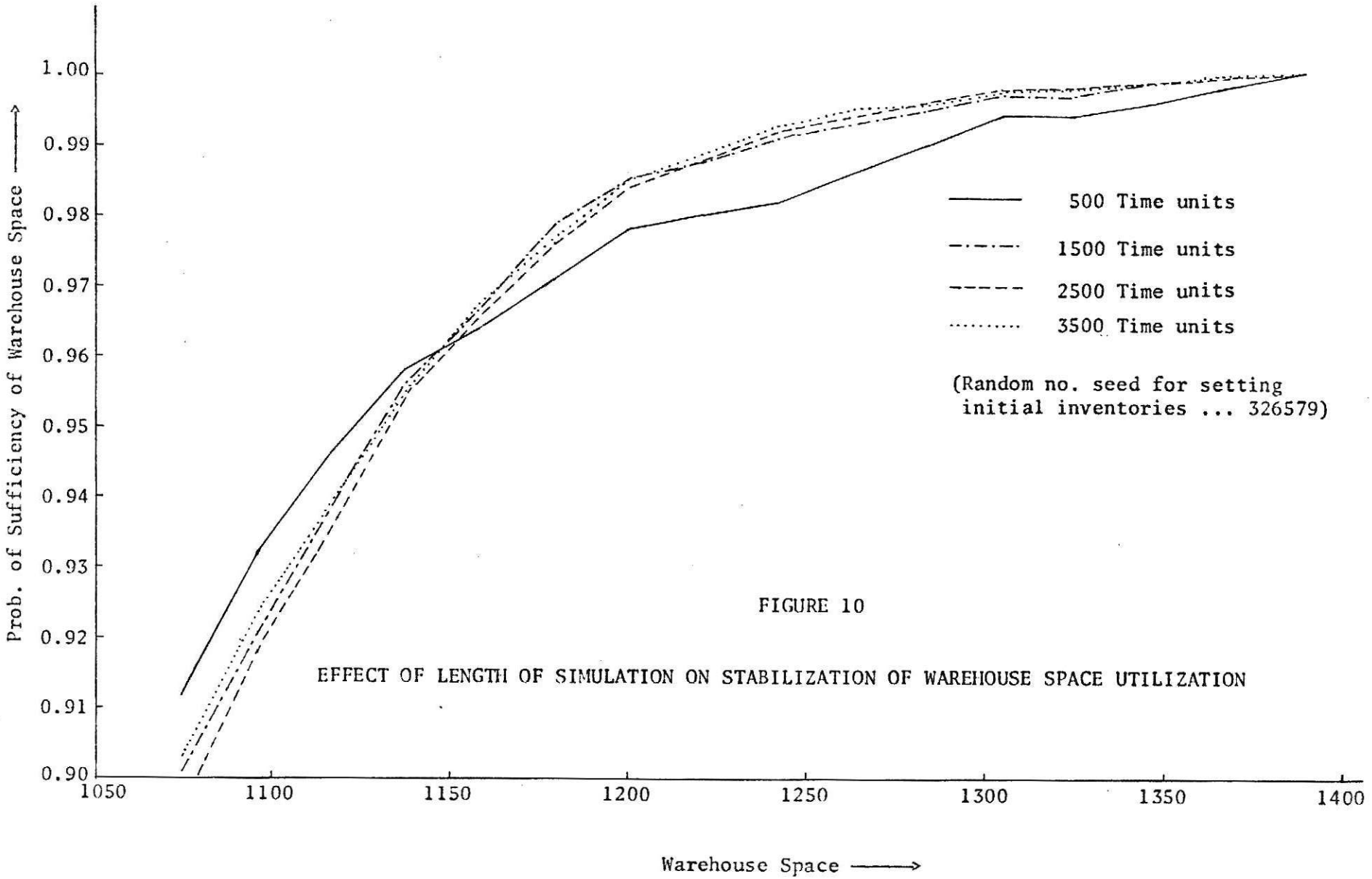


FIGURE 10

EFFECT OF LENGTH OF SIMULATION ON STABILIZATION OF WAREHOUSE SPACE UTILIZATION

inspection it is seen that 3500 time units is a sufficiently long simulation period. The effect of the initial inventories on the capacity utilization is shown in Figure 11. At 1% and 5% risk of running out of capacity, the actual required capacities, for the six sets of initial inventories, are shown in Table 7.

At 1% risk of running out of capacity, the maximum capacity that has been used is 1230 ft^3. Using this information the capacity constraint can be modified.

$$CAP = k \sum_{i=1}^{6} w_i \, (Q_i + I_{T_i} - \bar{S}_{L_i}).$$

There is no way to determine 'k' exactly. A value somewhat larger than the ratio of the actual capacity used to the available capacity, would be a good trial value.

$$k \simeq \frac{1230}{1500} = 0.85.$$

The optimal order sizes and reorder points are not obtained using this modified constraint, because

1) The risk of running out of space would depend on the specific problem.

2) k would have to be obtained by trial and error.

4.2 PROBLEM 2:

This problem is identical to the earlier problem, except for the variability of the lead time, which is used both for generating the lead time demand distribution and for the formulation of the inventory model.

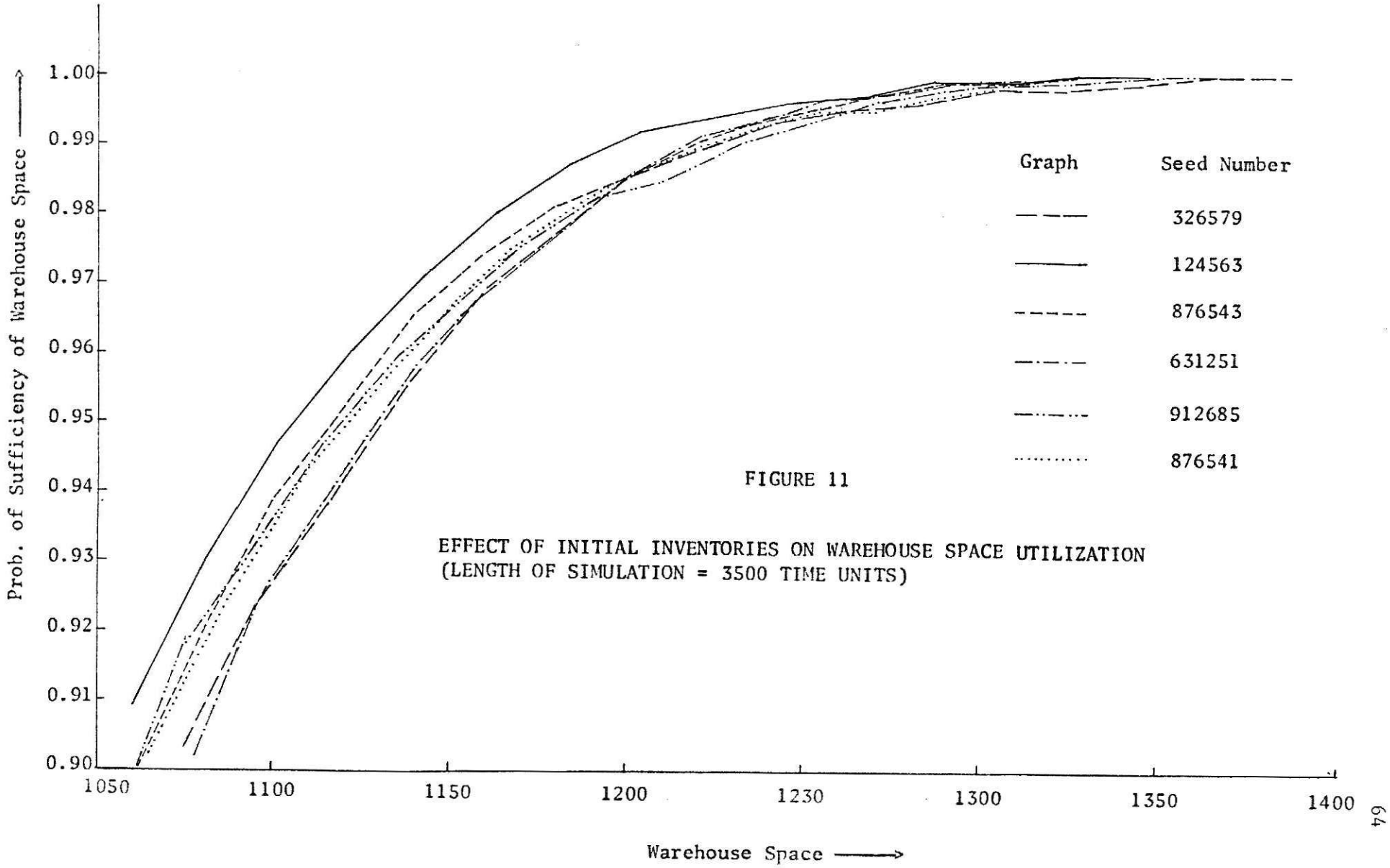


FIGURE 11

EFFECT OF INITIAL INVENTORIES ON WAREHOUSE SPACE UTILIZATION
(LENGTH OF SIMULATION = 3500 TIME UNITS)

TABLE 7

SIMULATION OF CAPACITY UTILIZATION USING DIFFERENT SETS OF INITIAL INVENTORIES

(Available Capacity = 1500 ft.^{3})

Seed for Setting Initial Inventories	Capacity Required in ft.^{3} at a risk of			Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
	1%	5%							
326579	1224	1133	Initial Inventory	156	195	18	33	5	299
			Final Inventory	210	218	164	311	44	54
			Stockout Prob.	0.0080	0.0029	0.0017	0.0026	0.0097	0.0023
124563	1196	1106	Initial Inventory	129	197	124	135	80	131
			Final Inventory	183	193	99	140	149	93
			Stockout Prob.	0.0020	0.0014	0.0026	0.0026	0.0034	0.0014
876543	1219	1117	Initial Inventory	121	122	50	7	140	183
			Final Inventory	58	99	69	150	75	249
			Stockout Prob.	0.0029	0.0020	0.0011	0.0017	0.0009	0.0003
631251	1216	1130	Initial Inventory	42	120	23	242	21	248
			Final Inventory	111	297	189	121	46	374
			Stockout Prob.	0.0091	0.0009	0.0023	0.0023	0.0046	0.0009
912685	1230	1120	Initial Inventory	138	29	89	185	53	137
			Final Inventory	191	110	64	218	63	138
			Stockout Prob.	0.0071	0.0026	0.0014	0.0017	0.0040	0.0020
876541	1226	1122	Initial Inventory	121	122	50	5	54	179
			Final Inventory	162	250	9	98	46	223
			Stockout Prob.	0.0114	0.0011	0.0020	0.0051	0.0034	0.0006

As explained in 3.1.6 the knowledge of the conditional mean of lead time, given the lead time demand, is necessary. An approximation of this conditional mean is used.

$$\text{Conditional mean of } T_L = T_L\,(S_L) = \frac{S_L}{\bar{S}}$$

With this approximation, the expected approximate inventory holding cost and stockout cost rates would be given by equations (3.33) and (3.18) respectively.

Also, since this problem is solved just to see the effect of lead time variability, only two available capacities are used:

a) 500 ft.^{3} and b) 1000 ft.^{3}

The two lead time demand distributions used are:

a) 1000 lead time demands divided in 50 classes and

b) 2000 lead time demands divided in 50 classes.

4.2.1 Mathematical Model of the Inventory System:

$$L = \sum_{i=1}^{6} \left(C_{F_i} \frac{\bar{S}_i}{Q_i} + C_{I_i} \left(\frac{Q_i}{2} + I_{T_i} - \bar{S}_{L_i}\right) + \frac{C_{D_i} \bar{S}_i}{Q_i} \int_{I_{T_i}}^{\infty} (S_{L_i} - I_{T_i}) f(S_{L_i})\, dS_{L_i} \right.$$

$$\left. + \frac{C_{I_i}}{2Q_i} \int_{I_{T_i}}^{\infty} S_{L_i} (S_{L_i} - I_{T_i})\, f(S_{L_i})\, dS_{L_i} \right)$$

$$+ \lambda \left(CAP - \sum_{i=1}^{6} w_i (Q_i + I_{T_i} - \bar{S}_{L_i}) \right) \qquad (4.9)$$

The Basic Equations are:

$$\frac{\partial L}{\partial Q_i} = -\frac{1}{Q_i^2}\left[\bar{S}_i\left\{C_{F_i} + C_{D_i}\int_{I_{T_i}}^{\infty}(S_{L_i} - I_{T_i})\, f(S_{L_i})dS_{L_i}\right\}\right.$$

$$\left. + \frac{C_{I_i}}{2}\int_{I_{T_i}}^{\infty} S_{L_i}(S_{L_i} - I_{T_i})\, f(S_{L_i})dS_{L_i}\right] + \left(\frac{C_{I_i}}{2} - \lambda w_i\right) = 0;$$

$$i = 1, \ldots, 6 \qquad (4.10)$$

$$\frac{\partial L}{\partial I_{T_i}} = -\frac{1}{Q_i}\left[\bar{S}_i C_{D_i}\int_{I_{T_i}}^{\infty} f(S_{L_i})dS_{L_i} + \frac{C_{I_i}}{2}\int_{I_{T_i}}^{\infty} S_{L_i} f(S_{L_i})dS_{L_i}\right] = 0$$

$$i = 1, \ldots, 6 \qquad (4.11)$$

$$\frac{\partial L}{\partial \lambda} = CAP - \sum_{i=1}^{6} w_i\,(Q_i + I_{T_i} - \bar{S}_{L_i}) = 0 \qquad (4.12)$$

From (5.10) and (5.11), for $i = 1, \ldots, 6$

$$Q_{1_i} = \left\{\frac{1}{C_{I_i}/2 - \lambda w_i}\left[\bar{S}_i\left\{C_{F_i} + C_{D_i}\int_{I_{T_i}}^{\infty}(S_{L_i} - I_{T_i}) f(S_{L_i})dS_{L_i}\right\}\right.\right.$$

$$\left.\left. + \frac{C_{I_i}}{2}\int_{I_{T_i}}^{\infty} S_{L_i}(S_{L_i} - I_{T_i}) f(S_{L_i})dS_{L_i}\right]\right\}^{1/2} \qquad (4.13)$$

$$Q_{2_i} = \frac{1}{C_{I_i} - \lambda w_i}\left[\bar{S}_i C_{D_i}\int_{I_{T_i}}^{\infty} f(S_{L_i})dS_{L_i} + \frac{C_{I_i}}{2}\int_{I_{T_i}}^{\infty} S_{L_i} f(S_{L_i})dS_{L_i}\right] \qquad (4.14)$$

The functions to be evaluated in SUBROUTINE ITGRLS at (N+1) - N is the number of classes in which lead-time-demand is divided-values of the reorder points are:

$$f_1(I_{T_i}) = \left(\bar{S}_i\left\{C_{F_i} + C_{D_i}\int_{I_{T_i}}^{\infty}(S_{L_i} - I_{T_i})f(S_{L_i})dS_{L_i}\right\} + \frac{C_{I_i}}{2}\int_{I_{T_i}}^{\infty} S_{L_i}(S_{L_i} - I_{T_i})\right.$$

$$\left. f(S_{L_i})dS_{L_i}\right)^{1/2} \qquad (4.15)$$

$$f_2(I_{T_i}) = \bar{S}_i C_{D_i}\int_{I_{T_i}}^{\infty} f(S_{L_i})dS_{L_i} + \frac{C_{I_i}}{2}\int_{I_{T_i}}^{\infty} S_{L_i}\, f(S_{L_i})dS_{L_i} \qquad (4.16)$$

4.2.2 Discussion of the Results:

The optimal solutions obtained in this problem (variable lead time) are compared with those obtained in problem 1 (constant lead time), in Table 8. It is observed that the solutions are not significantly different.

4.3 PROBLEM 3:

Kalinhoff (25) has solved a problem of inventory control for finished feeds under constraint. He has used Cramer's inequality for the upper limit of the probability of deviations from the mean in one direction, for the lead time demand distribution. In order to compare his results with the results that can be obtained by the numerical approach, the same problem is taken. He has used constant lead time and the mean and standard deviation for the demand rate. Instead, an Erlang distribution for the demand rate is fitted, knowing the mean and standard deviation, in this problem.

TABLE 8

COMPARISON OF OPTIMAL SOLUTIONS OBTAINED WITH VARIABLE AND CONSTANT LEAD TIMES

Available Capacity in ft.^{3}	Distribution	Lead Time	Optimal Order Size for Item 1	2	3	4	5	6	Optimal Reorder Point for Item 1	2	3	4	5	6	Required Capacity	Total Cost Rate
500	2000/50	Constant	50	83	48	67	39	82	66	93	24	74	30	64	500.45	14.99
		Variable	No solution due to the nonmonotonically decreasing integrals													
	1000/50	Constant	47	87	48	72	40	81	67	91	24	71	30	64	499.58	14.80
		Variable	47	87	48	72	40	81	67	91	24	71	30	64	499.94	14.79
1000	2000/50	Constant	102	175	125	140	85	175	84	108	28	91	39	76	999.85	6.401
		Variable	102	175	125	140	85	175	85	108	28	91	39	76	999.43	6.404
	1000/50	Constant	No solution due to the nonmonotonically decreasing integrals													
		Variable	106	171	117	138	89	181	83	110	28	90	38	76	1000.08	6.33

4.3.1 Characteristics of the Inventory System:

1) Number of items in the system is sixteen.

2) Mean and standard deviation of the demand rate for each item are given. It is assumed that the demand rate follows an Erlang distribution. The parameter of the Erlang distribution is obtained by rounding off $[\frac{\text{mean demand rate}}{\text{std. deviation of demand rate}}]^2$. If the calculated parameter for an item becomes 0 or 1, a poisson distribution with the given mean is assumed.

3) For generating the lead time demand distribution, it is assumed that the lead time follows the Erlang distribution with a parameter value of four, and the mean equal to the stated constant lead time. For the formulation of the inventory model, a constant lead time is used.

4) Inventory holding cost is approximated by equation (3.16).

5) Out of stock cost is proportional to the number of stockouts, equation (3.17).

6) The data for the sixteen items is given in Table 9.

7) The constrained optimal solutions are to be obtained for the following available capacities: a) 383 T. b) 384 T. and c) 500 T.

8) The lead time demand distributions used are similar to those used in Problem 1.

4.3.2 Mathematical Model of the Inventory System:

The Lagrange is:

TABLE 9

DATA FOR 16 ITEMS

Item i	Mean Demand Rate T/Month	Std. Dev. of Demand Rate T/Month	Mean Lead Time in Months	C_i \$/Ton	C_{F_i} \$/Order	C_{I_i} \$/Month /Ton	C_{D_i} \$/Stock-out	W_i Ton/Ton
1	40.000	17.750	0.500	50.00	15.00	0.028	30.00	1.00
2	38.790	15.100	0.500	50.00	15.00	0.028	30.00	1.00
3	24.330	10.130	0.500	50.00	10.00	0.028	30.00	1.00
4	24.110	8.770	0.500	50.00	15.00	0.028	30.00	1.00
5	24.080	10.250	0.500	50.00	15.00	0.028	30.00	1.00
6	20.970	12.570	0.500	50.00	15.00	0.028	30.00	1.00
7	16.000	14.570	0.500	50.00	15.00	0.028	30.00	1.00
8	15.450	8.120	0.500	50.00	10.00	0.028	30.00	1.00
9	13.880	5.670	0.500	50.00	15.00	0.028	30.00	1.00
10	13.590	6.420	0.500	50.00	10.00	0.028	30.00	1.00
11	13.500	14.120	0.500	50.00	15.00	0.028	30.00	1.00
12	13.210	10.570	0.500	50.00	15.00	0.028	30.00	1.00
13	11.770	7.600	0.500	50.00	10.00	0.028	30.00	1.00
14	9.570	3.940	0.500	50.00	15.00	0.028	30.00	1.00
15	7.700	6.310	0.500	50.00	15.00	0.028	30.00	1.00
16	7.140	4.300	0.500	50.00	10.00	0.028	30.00	1.00

$$L = \sum_{i=1}^{16} \left(C_{F_i} \frac{\bar{S}_i}{Q_i} + C_{I_i} \left\{ \frac{Q_i}{2} + I_{T_i} - \bar{S}_{L_i} \right\} + \frac{C_{I_i} T_{L_i} \bar{S}_i}{2Q_i} \int_{I_{T_i}}^{\infty} (S_{L_i} - I_{T_i}) \, f(S_{L_i}) dS_{L_i} \right.$$

$$\left. + \frac{C_{D_i} \bar{S}_i}{Q_i} \int_{I_{T_i}}^{\infty} f(S_{L_i}) dS_{L_i} \right) + \lambda \left(CAP - \sum_{i=1}^{16} w_i (Q_i + I_{T_i} - \bar{S}_{L_i}) \right) \tag{4.17}$$

The Basic Equations are

$$\frac{\partial L}{\partial Q_i} = - \frac{\bar{S}_i}{Q_i^2} \left(C_{F_i} + \frac{C_{I_i} T_{L_i}}{2} \int_{I_{T_i}}^{\infty} (S_{L_i} - I_{T_i}) \, f(S_{L_i}) dS_{L_i} + C_{D_i} \int_{I_{T_i}}^{\infty} f(S_{L_i}) dS_{L_i} \right)$$

$$+ \left(\frac{C_{I_i}}{2} - \lambda w_i \right) = 0 \qquad i = 1, \ldots, 6 \tag{4.18}$$

$$\frac{L}{I_{T_i}} = - \frac{\bar{S}_i}{Q_i} \left(\frac{C_{I_i} T_{L_i}}{2} \int_{I_{T_i}}^{\infty} f(S_{L_i}) dS_{L_i} + C_{D_i} f(I_{T_i}) \right) + C_{I_i} - \lambda w_i = 0$$

$$i = 1, 2, \ldots, 6 \tag{4.19}$$

$$\frac{\partial L}{\partial \lambda} = CAP - \sum_{i=1}^{16} w_i \, (Q_i + I_{T_i} - \bar{S}_{L_i}) = 0 \tag{4.20}$$

(4.18) and (4.19) yield following two equations for each item:

$$Q_{1_i} = \left\{ \frac{1}{C_{I_i}/2 - \lambda w_i} \left(\bar{S}_i \left\{ C_{F_i} + \frac{C_{I_i} T_{L_i}}{2} \int_{I_{T_i}}^{\infty} (S_{L_i} - I_{T_i}) f(S_{L_i}) dS_{L_i} \right. \right. \right.$$

$$\left. \left. \left. + C_{D_i} \int_{I_{T_i}}^{\infty} f(S_{L_i}) dS_{L_i} \right\} \right) \right\}^{1/2} \tag{4.22}$$

$$Q_{2_i} = \frac{1}{C_{I_i} - \lambda w_i} \left(\bar{S}_i \left\{ \frac{C_{I_i} T_{L_i}}{2} \int_{I_{T_i}}^{\infty} f(S_{L_i}) dS_{L_i} + C_{D_i} f(I_{T_i}) \right\} \right) \quad (4.22)$$

The functions to be evaluated in SUBROUTING ITGRLS at (N+1) - N is the number of classes in which lead-time-demand is divided - values of the reorder points, are:

$$f_1(I_{T_i}) = \left(\bar{S}_i \left\{ C_{F_i} + \frac{C_{I_i} T_{L_i}}{2} \int_{I_{T_i}}^{\infty} (S_{L_i} - I_{T_i}) f(S_{L_i}) dS_{L_i} + C_{D_i} \int_{I_{T_i}}^{\infty} f(S_{L_i}) dS_{L_i} \right) \right)^{1/2} \quad (4.23)$$

$$f_2(I_{T_i}) = \left(\bar{S}_i \left\{ \frac{C_{I_i} T_{L_i}}{2} \int_{I_{T_i}}^{\infty} f(S_{L_i}) dS_{L_i} + C_{D_i} f(I_{T_i}) \right\} \right) \quad (4.24)$$

4.3.3 Discussion of the Results:

Lead time demand distributions, eight for each item, were obtained by simulation, both in printed and punched form. The constrained optimal order sizes and reorder points for each available capacity were calculated using the eight distributions and the two methods of searching for optimal λ. Table 10 shows the number of iterations (the number of values of λ tried) needed to obtain the optimal solution, when one is available. It is observed that the number of iterations required for the progressive linear interpolation/extrapolation method is smaller than the number of iterations required by the golden section search method. This can be attributed to the near-linear relationship between λ and the required capacity in the region of capacities considered in this problem.

TABLE 10

NUMBER OF ITERATIONS REQUIRED TO OBTAIN OPTIMAL SOLUTION

(* indicates that optimal solution was not reached)

Available Capacity	Progressive Linear Interpolation/Extrapolation							
	Classes for 1000 Lead-time-demands				Classes for 2000 Lead-time-demands			
	25	30	40	50	25	30	40	50
383 T	9	6	8	10	7	8	9	8
384 T	8	6	8	10	7	9	9	8
500 T	10	*	10	10	8	10	11	12

	Golden Section Search							
	Classes for 1000 Lead-time-demands				Classes for 2000 Lead-time-demands			
	25	30	40	50	25	30	40	50
383 T	16	14	8	16	15	13	8	15
384 T	15	16	16	14	13	15	15	13
500 T	18	18	17	14	16	16	16	16

The relationship between λ and the required warehouse capacity is shown in Figure 12.

Kalinhoff (25) has obtained the optimal inventory policy for an available capacity of 383 T., using Cramer's inequality for the lead time demand distribution. The optimal inventory policies for the same available capacity, using progressive linear interpolation/extrapolation in conjunction with the distributions,

a) 1000 lead time demands divided in 50 classes and

b) 2000 lead time demands divided in 50 classes,

have been compared with Kalinhoff's results in Table 11.

The total cost per month of the optimal inventory policy, when Cramer's inequality is used is about 40% higher than the cost for the optimal inventory policy when the numerical approach developed in this paper is used. This is not surprising. Cramer's inequality gives an 'upper limit' of the probability of the deviation from the mean in one direction. The inequality gives very conservative estimate for the spread of the lead time demand. The demand rates may not follow Erlang distributions as assumed in this analysis. However, the optimal inventory policy can be obtained by using the actual data, for the numerical approach.

FIGURE 12

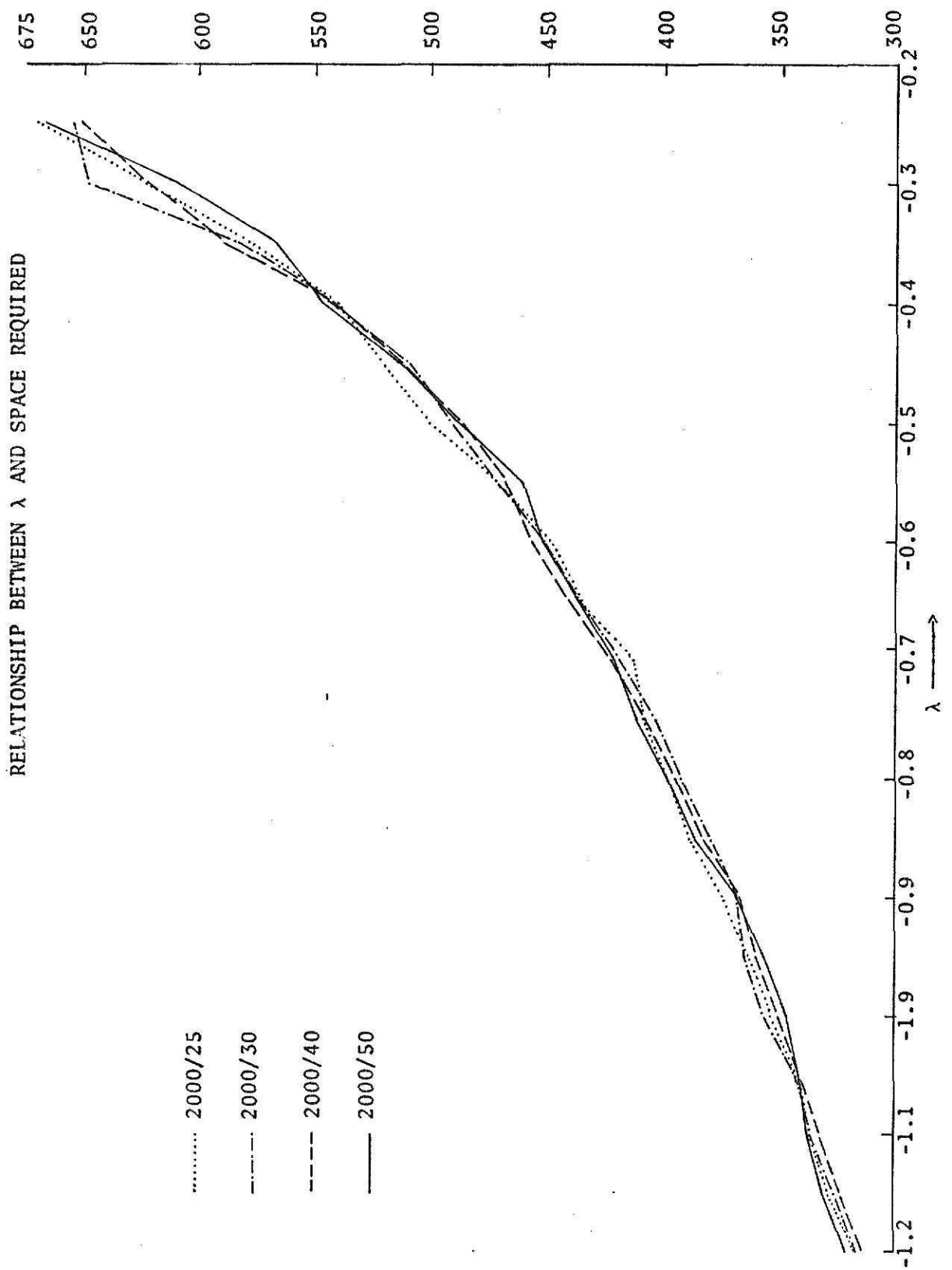

RELATIONSHIP BETWEEN λ AND SPACE REQUIRED
675
650
600
550
500
450
400
350
300
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.9
-1.1
-1.2
λ ⟶
2000/25
2000/30
2000/40
2000/50

TABLE 11

COMPARISON OF OPTIMAL INVENTORY POLICIES FOR AN AVAILABLE CAPACITY OF 383 T

	Kalinhoff's Method		1000/50 Distribution		2000/50 Distribution	
	Batch Size in Tons	Reorder Point in Tons	Batch Size in Tons	Reorder Point in Tons	Batch Size in Tons	Reorder Point in Tons
Item - 1	23.42	36.73	33.29	23.74	37.89	18.77
Item - 2	22.63	34.44	31.86	24.25	30.30	24.88
Item - 3	14.81	22.48	19.24	19.89	24.67	15.44
Item - 4	17.22	21.84	27.29	14.33	27.15	15.14
Item - 5	17.51	22.83	23.78	15.70	24.39	15.84
Item - 6	16.91	22.43	22.31	12.62	25.17	11.34
Item - 7	15.39	20.43	19.02	13.79	19.08	13.16
Item - 8	11.72	15.68	18.71	11.06	18.79	10.93
Item - 9	12.86	13.65	15.52	12.06	17.91	11.32
Item -10	10.77	13.50	15.56	11.16	15.05	11.14
Item -11	14.26	18.55	20.06	9.68	17.25	10.78
Item -12	13.53	16.44	17.28	11.40	17.87	10.61
Item -13	10.33	13.14	14.77	9.96	14.37	10.30
Item -14	10.50	9.76	15.71	7.99	14.36	8.36
Item -15	10.00	10.29	13.03	7.29	12.92	7.32
Item -16	7.73	8.20	10.95	6.72	11.35	6.78
Total cost/ month in dollars	399.85		282.913		281.342	

CHAPTER 5

CONCLUSIONS

An immense amount of research work has been done in the field of inventory management. A number of inventory models have been formulated to find the optimal inventory policy, analytically. These models are based on specific assumptions about the costs, distributions of demand rate and lead time, constraints etc. In many real world problems, these assumptions do not hold good. Such problems, where analytical methods can not be applied for the optimization, have escaped the attention of the research workers. The practicing inventory controllers are interested in just these problems.

In this paper, a hypothetical automated warehouse is considered for determining the optimal inventory policy. The inventory system used is a Q-system, wherein a constant quantity 'Q' is ordered, whenever the stock on hand (plus on order) reaches a level 'I_T' or reorder point. The warehouse space is such that it can be used by all the items. A special case of this would be a warehouse which provides distinctly different space for each item. The procedure developed to determine the optimal inventory policy, under space constraint, should be capable of using any distribution for lead time demand (obtained from historical records or generated from the distributions for the lead time and demand rate), standard or otherwise. It should also allow the use of approximate and exact inventory holding cost and any kind of dollar stockout penalty in the mathematical model of the inventory system.

The constrained optimization problem is converted to the unconstrained optimization problem using the Lagrange multiplier method. Since the multi-item inventory problem under space constraint is a separable problem, introduction of Lagrange multiplier (λ) simplifies the optimization problem considerably. If the value of λ corresponding to the available space used in the constraint is known, only two equations have to be solved simultaneously, to find the optimal order size and reorder point for an item. It is not possible, however, to obtain an explicit expression for λ, corresponding to the available warehouse space. The property of monotonic variation of the required space with λ - Everett (15) - is used in obtaining the constrained optimal solution. The mathematical model and the two conditions per item for an optimal solution (these conditions are called 'Basic Equations' in the text) involve particular integrals. Analytical expressions for these integrals cannot be obtained for a general problem. Therefore, numerical integration is used. The lead time demand is divided into a predetermined number of classes; the probability for each class is obtained; and the integrals are evaluated at the end points of the classes. For any assumed value of λ (which corresponds to some required warehouse space), the optimal order sizes and reorder points are calculated, employing an iterative procedure (starting from the reorder point equal to the maximum demand during lead time) and the above integrals. A search method (either golden section or progressive linear interpolation/extrapolation) is used to find λ^* corresponding to the available space within permissible error. The optimal inventory policy corresponding to λ^* is the desired constrained optimal solution. Alternatively,

the relationship between λ and required space is plotted. This graph is used in finding λ corresponding to the available warehouse space; this λ is, in turn, used in determining the optimal inventory policy.

The numerical approach developed is quite inexpensive. If the lead time demand distribution is given the calculation of constrained optimal inventory policy for a 16 - item problem requires less than two dollars on 360/50 computer. If the range of λ corresponding to the available space is approximately known, the best method for searching optimal λ^* is linear interpolation. This has been exhibited in Problem 3. If the range of λ is not known, the relationship between λ and the required capacity is quite nonlinear as the range of λ involved is wide. In such case, a golden section search would be better.

Suggestions for Further Work:

1. Use actual data for lead times and demand rates to obtain the lead time demand distributions, using simulation. Compare the optimal inventory policy obtained using these distributions in the numerical approach, developed in this paper, with the optimal inventory policy obtained by using Cramer's inequality. It may be pointed out here, that the method of using Cramer's inequality as used by Kalinhoff (25) is applicable only when the out of stock is proportional to the number of stockouts.

2. Use a combination of the bisection search and progressive linear interpolation/extrapolation for finding λ^*. The first few iterations should use the bisection search.

3. It has been observed that the iterative procedure of finding λ^*, fails sometimes, due to the coarseness of the integrals. The integrals

evaluated on a finite set of points can be used to obtain a regression function which gives 'smooth interpolation' relating the integrals with the reorder points. Use of such a regression function will improve the accuracy of the solution and also the speed of obtaining the constrained optimal solution.

4. In case of variable lead time, use the actual conditional mean of the lead time (given lead time demand) instead of the approximation used in this paper.

5. Study the effect of the number of items on the smoothing of warehouse space utilization.

6. The optimal allocation of the limited warehouse space using a constraint of the form (3.39) is quite pessimistic. As observed in Table 7, for a six-item case 99% of the time only 1230 ft.^{3}, of the 1500 ft.^{3} available warehouse space, is used. For an inventory system comprising a greater number of items, the problem of poor utilization would be aggrevated. As the number of items increases, the variability of the warehouse space utilization decreases and the actual space required tends to cluster closely about the average. Thus if management is willing to choose a risk level for exceeding warehouse capacity, it is possible to obtain an optimal inventory policy which more or less corresponds to this risk using a constraint of the form (3.41).

Solution of the problem (finding optimal Q and I_T and simulating warehouse space utilization) more than once is necessary to achieve satisfactory utilization. The high cost of warehouse space may justify the additional computational cost. The effect of modifying the space constraint on improvement in utilization needs to be examined.

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APPENDIX 1

INTEGER LEAD TIME DEMAND DISTRIBUTION

AVERAGE LEAD TIME	11.000
AVERAGE DAILY DEMAND	4.000
STANDARD DEVIATION OF DAILY DEMAND	4.000
ERLANG PARAMETER FOR LEAD TIME	4
ERLANG PARAMETER FOR DAILY DEMAND	1
NUMBER OF SIMULATED DEMANDS DURING LEAD TIME	1000

(NO. OF CELLS = 25)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
6.880	27.1	0.027
12.640	61.0	0.061
18.400	65.7	0.066
24.160	99.9	0.100
29.920	124.0	0.124
35.680	103.4	0.103
41.440	112.6	0.113
47.200	79.5	0.080
52.960	77.5	0.077
58.720	71.3	0.071
64.480	50.6	0.051
70.240	35.9	0.036
76.000	19.4	0.019
81.760	21.3	0.021
87.520	10.5	0.011
93.280	7.5	0.008
99.040	9.7	0.010
104.800	7.2	0.007
110.560	4.8	0.005
116.320	3.0	0.003
122.080	1.5	0.001
127.840	1.5	0.002
133.600	0.0	0.0
139.360	3.5	0.003
145.120	1.5	0.002

MEAN	STD. DEV.	MINIMUM	MAXIMUM
42.990	22.385	4.000	148.000

(NO. OF CELLS = 30)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
6.400	17.3	0.017
11.200	42.1	0.042
16.000	50.6	0.051
20.800	73.5	0.073
25.600	90.0	0.090
30.400	104.2	0.104
35.200	89.7	0.090
40.000	90.8	0.091
44.800	83.8	0.084
49.600	59.5	0.059
54.400	64.7	0.065
59.200	55.9	0.056
64.000	41.7	0.042
68.800	33.5	0.034
73.600	21.7	0.022
78.400	14.8	0.015
83.200	17.4	0.017
88.000	8.6	0.009
92.800	5.9	0.006
97.600	9.3	0.009
102.400	6.3	0.006
107.200	6.8	0.007
112.000	1.8	0.002
116.800	2.1	0.002
121.600	1.0	0.001
126.400	2.0	0.002
131.200	0.0	0.0
136.000	0.9	0.001
140.800	3.1	0.003
145.600	1.0	0.001

MEAN	STD. DEV.	MINIMUM	MAXIMUM
42.990	22.885	4.000	148.000

(NO. OF CELLS = 40)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
5.800	9.5	0.009
9.400	29.1	0.029
13.000	38.2	0.038
16.600	33.2	0.033
20.200	55.5	0.055
23.800	65.0	0.065
27.400	69.9	0.070
31.000	77.3	0.077
34.600	68.9	0.069
38.200	64.4	0.064
41.800	69.8	0.070
45.400	61.2	0.061
49.000	42.2	0.042
52.600	45.9	0.046
56.200	50.9	0.051
59.800	41.1	0.041
63.400	29.4	0.029
67.000	26.9	0.027
70.600	28.8	0.029
74.200	11.8	0.012
77.800	9.4	0.009
81.400	17.4	0.017
85.000	7.8	0.008
88.600	6.2	0.006
92.200	4.2	0.004
95.800	6.2	0.006
99.400	6.5	0.007
103.000	4.6	0.005
106.600	3.6	0.004
110.200	4.1	0.004
113.800	1.1	0.001
117.400	1.9	0.002
121.000	1.0	0.001
124.600	1.0	0.001
128.200	1.0	0.001
131.800	0.0	0.0
135.400	0.0	0.0
139.000	2.0	0.002
142.600	2.0	0.002
146.200	1.0	0.001

MEAN	STD. DEV.	MINIMUM	MAXIMUM
42.999	22.885	4.000	148.000

(NO. OF CELLS = 50)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
5.440	7.8	0.008
8.320	19.3	0.019
11.200	20.6	0.021
14.080	40.4	0.040
16.960	21.9	0.022
19.840	43.8	0.044
22.720	48.0	0.048
25.600	51.9	0.052
28.480	62.9	0.063
31.360	61.0	0.061
34.240	58.9	0.059
37.120	44.6	0.045
40.000	59.7	0.060
42.880	52.9	0.053
45.760	48.2	0.048
48.640	31.3	0.031
51.520	41.7	0.042
54.400	35.8	0.036
57.280	38.5	0.039
60.160	32.8	0.033
63.040	25.8	0.026
65.920	24.8	0.025
68.800	16.4	0.016
71.680	19.4	0.019
74.560	10.4	0.010
77.440	9.0	0.009
80.320	11.3	0.011
83.200	10.0	0.010
86.080	5.8	0.006
88.960	4.8	0.005
91.840	2.8	0.003
94.720	4.8	0.005
97.600	5.8	0.006
100.480	3.9	0.004
103.360	4.3	0.004
106.240	2.9	0.003
109.120	4.8	0.005
112.000	0.0	0.0
114.880	1.8	0.002
117.760	1.2	0.001
120.640	1.0	0.001
123.520	0.5	0.000
126.400	0.9	0.001
129.280	0.7	0.001
132.160	0.0	0.0
135.040	0.0	0.0
137.920	1.0	0.001
140.800	2.5	0.002
143.680	0.5	0.001
146.560	1.0	0.001

MEAN	STD. DEV.	MINIMUM	MAXIMUM
43.990	22.885	4.000	148.000

AVERAGE LEAD TIME.....	11.000
AVERAGE DAILY DEMAND.....	4.000
STANDARD DEVIATION OF DAILY DEMAND.....	4.000
ERLANG PARAMETER FOR LEAD TIME.....	4
ERLANG PARAMETER FOR DAILY DEMAND.....	1
NUMBER OF SIMULATED DEMANDS DURING LEAD TIME.....	2000

(NO. OF CELLS = 25)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
7.040	56.0	0.028
13.120	123.3	0.062
19.200	163.2	0.082
25.280	210.2	0.105
31.360	238.6	0.119
37.440	245.0	0.123
43.520	201.1	0.101
49.600	151.7	0.076
55.680	160.9	0.080
61.760	127.9	0.064
67.840	95.0	0.048
73.920	59.9	0.030
80.000	48.6	0.024
86.080	26.2	0.013
92.160	19.3	0.010
98.240	22.8	0.011
104.320	14.3	0.007
110.400	8.1	0.004
116.480	10.0	0.005
122.560	5.0	0.002
128.640	3.2	0.002
134.720	1.1	0.001
140.800	6.1	0.003
146.880	1.7	0.001
152.960	1.0	0.000

MEAN	STD. DEV.	MINIMUM	MAXIMUM
43.614	23.411	4.000	156.000

(NO. OF CELLS = 30)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
6.533	40.3	0.020
11.600	88.0	0.044
16.667	123.0	0.062
21.733	159.2	0.080
26.800	187.7	0.094
31.867	192.9	0.096
36.933	206.5	0.103
42.000	173.5	0.087
47.067	130.9	0.065
52.133	142.2	0.071
57.200	132.5	0.066
62.267	101.0	0.050
67.333	80.6	0.040
72.400	57.9	0.029
77.467	40.7	0.020
82.533	31.7	0.016
87.600	22.8	0.011
92.667	15.3	0.008
97.733	20.7	0.010
102.800	12.8	0.006
107.867	11.4	0.006
112.933	3.3	0.002
118.000	8.0	0.004
123.067	4.0	0.002
128.133	2.2	0.001
133.200	1.8	0.001
138.267	2.3	0.001
143.333	4.7	0.002
148.400	1.0	0.000
153.467	1.0	0.000

MEAN	STD. DEV.	MINIMUM	MAXIMUM
43.624	23.411	4.000	156.000

(NO. OF CELLS = 40)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
5.900	22.7	0.011
9.700	56.6	0.028
13.500	85.8	0.043
17.300	86.3	0.043
21.100	115.1	0.058
24.900	131.5	0.066
28.700	153.3	0.077
32.500	139.9	0.070
36.300	157.6	0.079
40.100	139.7	0.070
43.900	126.5	0.063
47.700	87.1	0.044
51.500	109.0	0.054
55.300	101.8	0.051
59.100	90.6	0.045
62.900	74.3	0.037
66.700	64.5	0.032
70.500	58.8	0.029
74.300	26.5	0.013
78.100	29.4	0.015
81.900	28.6	0.014
85.700	14.9	0.007
89.500	14.3	0.007
93.300	12.0	0.006
97.100	16.2	0.008
100.900	9.9	0.005
104.700	10.2	0.005
108.500	8.6	0.004
112.300	2.4	0.001
116.100	4.9	0.002
119.900	5.3	0.003
123.700	2.7	0.001
127.500	2.0	0.001
131.300	2.0	0.001
135.100	0.0	0.0
138.900	2.3	0.001
142.700	4.7	0.002
146.500	0.9	0.000
150.300	0.1	0.000
154.100	1.0	0.000

MEAN	STD. DEV.	MINIMUM	MAXIMUM
43.624	23.411	4.000	156.000

(NO. OF CELLS = 50)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
5.520	18.2	0.009
8.560	37.8	0.019
11.600	46.8	0.023
14.640	76.4	0.038
17.680	72.2	0.036
20.720	91.1	0.046
23.760	107.0	0.053
26.800	103.2	0.052
29.840	123.8	0.062
32.880	114.8	0.057
35.920	130.9	0.065
38.960	114.1	0.057
42.000	102.4	0.051
45.040	98.7	0.049
48.080	64.8	0.032
51.120	87.0	0.043
54.160	82.0	0.041
57.200	78.9	0.039
60.240	67.8	0.034
63.280	60.0	0.030
66.320	54.6	0.027
69.360	40.4	0.020
72.400	35.7	0.018
75.440	24.2	0.012
78.480	24.2	0.012
81.520	24.3	0.012
84.560	11.6	0.006
87.600	14.6	0.007
90.640	8.8	0.004
93.680	10.5	0.005
96.720	12.7	0.006
99.760	10.1	0.005
102.800	7.1	0.004
105.840	7.2	0.004
108.880	7.8	0.004
111.920	0.3	0.000
114.960	4.0	0.002
118.000	6.0	0.003
121.040	3.0	0.001
124.080	2.0	0.001
127.120	2.0	0.001
130.160	1.2	0.001
133.200	0.8	0.000
136.240	0.3	0.000
139.280	2.0	0.001
142.320	4.0	0.002
145.360	0.7	0.000
148.400	1.0	0.000
151.440	0.0	0.0
154.480	1.0	0.000

MEAN	STD. DEV.	MINIMUM	MAXIMUM
43.624	23.411	4.000	156.000

APPENDIX 2

INTEGER LEAD TIME DEMAND DISTRIBUTION

AVERAGE LEAD TIME	11.000
AVERAGE DAILY DEMAND	4.000
STANDARD DEVIATION OF DAILY DEMAND	4.000
ERLANG PARAMETER FOR LEAD TIME	4
ERLANG PARAMETER FOR DAILY DEMAND	1
NUMBER OF SIMULATED DEMANDS DURING LEAD TIME	1000

(NO. OF CELLS = 25)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
6.880	25.0	0.025
12.640	63.0	0.063
18.400	70.0	0.070
24.160	104.0	0.104
29.920	110.0	0.110
35.680	108.0	0.108
41.440	117.0	0.117
47.200	83.0	0.083
52.960	65.0	0.065
58.720	76.0	0.076
64.480	53.0	0.053
70.240	35.0	0.035
76.000	19.0	0.019
81.760	21.0	0.021
87.520	11.0	0.011
93.280	8.0	0.008
99.040	9.0	0.009
104.800	7.0	0.007
110.560	5.0	0.005
116.320	3.0	0.003
122.080	1.0	0.001
127.840	2.0	0.002
133.600	0.0	0.0
139.360	4.0	0.004
145.120	1.0	0.001

MEAN	STD. DEV.	MINIMUM	MAXIMUM
42.990	22.885	4.000	148.000

(NO. OF CELLS = 30)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
6.400	14.0	0.014
11.200	44.0	0.044
16.000	53.0	0.053
20.800	77.0	0.077
25.600	74.0	0.074
30.400	110.0	0.110
35.200	94.0	0.094
40.000	94.0	0.094
44.800	85.0	0.085
49.600	49.0	0.049
54.400	68.0	0.068
59.200	59.0	0.059
64.000	44.0	0.044
68.800	35.0	0.035
73.600	17.0	0.017
78.400	15.0	0.015
83.200	19.0	0.019
88.000	9.0	0.009
92.800	6.0	0.006
97.600	8.0	0.008
102.400	7.0	0.007
107.200	7.0	0.007
112.000	2.0	0.002
116.800	2.0	0.002
121.600	1.0	0.001
126.400	2.0	0.002
131.200	0.0	0.0
136.000	1.0	0.001
140.800	3.0	0.003
145.600	1.0	0.001

MEAN	STD. DEV.	MINIMUM	MAXIMUM
42.990	22.885	4.000	148.000

(NO. OF CELLS = 40)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
5.800	9.0	0.009
9.400	32.0	0.032
13.000	31.0	0.031
16.600	39.0	0.039
20.200	47.0	0.047
23.800	71.0	0.071
27.400	78.0	0.078
31.000	65.0	0.065
34.600	76.0	0.076
38.200	51.0	0.051
41.800	80.0	0.080
45.400	66.0	0.066
49.000	35.0	0.035
52.600	51.0	0.051
56.200	45.0	0.045
59.800	45.0	0.045
63.400	32.0	0.032
67.000	23.0	0.023
70.600	32.0	0.032
74.200	9.0	0.009
77.800	11.0	0.011
81.400	19.0	0.019
85.000	6.0	0.006
88.600	7.0	0.007
92.200	3.0	0.003
95.800	7.0	0.007
99.400	7.0	0.007
103.000	4.0	0.004
106.600	4.0	0.004
110.200	4.0	0.004
113.800	1.0	0.001
117.400	2.0	0.002
121.000	1.0	0.001
124.600	1.0	0.001
128.200	1.0	0.001
131.800	0.0	0.0
135.400	0.0	0.0
139.000	2.0	0.002
142.600	2.0	0.002
146.200	1.0	0.001

MEAN	STD. DEV.	MINIMUM	MAXIMUM
42.990	22.885	4.000	148.000

(NO. OF CELLS = 50)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
5.440	7.0	0.007
8.320	18.0	0.018
11.200	21.0	0.021
14.080	42.0	0.042
16.960	23.0	0.023
19.840	47.0	0.047
22.720	51.0	0.051
25.600	53.0	0.053
28.480	45.0	0.045
31.360	65.0	0.065
34.240	62.0	0.062
37.120	46.0	0.046
40.000	62.0	0.062
42.880	55.0	0.055
45.760	48.0	0.048
48.640	35.0	0.035
51.520	29.0	0.029
54.400	36.0	0.036
57.280	41.0	0.041
60.160	35.0	0.035
63.040	27.0	0.027
65.920	26.0	0.026
68.800	17.0	0.017
71.680	18.0	0.018
74.560	8.0	0.008
77.440	11.0	0.011
80.320	10.0	0.010
83.200	11.0	0.011
86.080	6.0	0.006
88.960	5.0	0.005
91.840	3.0	0.003
94.720	5.0	0.005
97.600	6.0	0.006
100.480	3.0	0.003
103.360	4.0	0.004
106.240	3.0	0.003
109.120	5.0	0.005
112.000	0.0	0.0
114.880	2.0	0.002
117.760	1.0	0.001
120.640	1.0	0.001
123.520	0.0	0.0
126.400	1.0	0.001
129.280	1.0	0.001
132.160	0.0	0.0
135.040	0.0	0.0
137.920	1.0	0.001
140.800	3.0	0.003
143.680	0.0	0.0
146.560	1.0	0.001

MEAN	STD. DEV.	MINIMUM	MAXIMUM
42.990	22.885	4.000	148.000

AVERAGE LEAD TIME	11.000
AVERAGE DAILY DEMAND	4.000
STANDARD DEVIATION OF DAILY DEMAND	4.000
ERLANG PARAMETER FOR LEAD TIME	4
ERLANG PARAMETER FOR DAILY DEMAND	1
NUMBER OF SIMULATED DEMANDS DURING LEAD TIME	2000

(NO. OF CELLS = 25)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
7.040	61.0	0.030
13.120	124.0	0.062
19.200	165.0	0.082
25.280	210.0	0.105
31.360	235.0	0.117
37.440	242.0	0.121
43.520	199.0	0.099
49.600	149.0	0.074
55.680	159.0	0.079
61.760	129.0	0.064
67.840	93.0	0.046
73.920	64.0	0.032
80.000	55.0	0.027
86.080	24.0	0.012
92.160	19.0	0.009
98.240	22.0	0.011
104.320	14.0	0.007
110.400	8.0	0.004
116.480	10.0	0.005
122.560	5.0	0.002
128.640	3.0	0.001
134.720	1.0	0.000
140.800	6.0	0.003
146.880	2.0	0.001
152.960	1.0	0.000

MEAN	STD. DEV.	MINIMUM	MAXIMUM
43.624	23.411	4.000	156.000

(NO. OF CELLS = 30)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
6.533	49.0	0.024
11.600	90.0	0.045
16.667	122.0	0.061
21.733	158.0	0.079
26.800	187.0	0.093
31.867	189.0	0.094
36.933	204.0	0.102
42.000	171.0	0.085
47.067	129.0	0.064
52.133	141.0	0.070
57.200	131.0	0.065
62.267	102.0	0.051
67.333	81.0	0.040
72.400	58.0	0.029
77.467	41.0	0.020
82.533	37.0	0.018
87.600	23.0	0.011
92.667	15.0	0.007
97.733	20.0	0.010
102.800	13.0	0.006
107.867	11.0	0.005
112.933	3.0	0.001
118.000	8.0	0.004
123.067	4.0	0.002
128.133	2.0	0.001
133.200	2.0	0.001
138.267	2.0	0.001
143.333	5.0	0.002
148.400	1.0	0.000
153.467	1.0	0.000

MEAN	STD. DEV.	MINIMUM	MAXIMUM
43.624	23.411	4.000	156.000

(NO. OF CELLS = 40)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
5.900	20.0	0.010
9.700	58.0	0.029
13.500	90.0	0.045
17.300	93.0	0.046
21.100	89.0	0.044
24.900	139.0	0.069
28.700	159.0	0.079
32.500	147.0	0.073
36.300	164.0	0.082
40.100	112.0	0.056
43.900	135.0	0.067
47.700	93.0	0.046
51.500	115.0	0.057
55.300	107.0	0.053
59.100	75.0	0.037
62.900	77.0	0.038
66.700	68.0	0.034
70.500	62.0	0.031
74.300	27.0	0.013
78.100	23.0	0.011
81.900	32.0	0.016
85.700	15.0	0.007
89.500	15.0	0.007
93.300	13.0	0.006
97.100	12.0	0.006
100.900	12.0	0.006
104.700	11.0	0.005
108.500	9.0	0.004
112.300	3.0	0.001
116.100	3.0	0.001
119.900	6.0	0.003
123.700	3.0	0.001
127.500	2.0	0.001
131.300	2.0	0.001
135.100	0.0	0.0
138.900	2.0	0.001
142.700	5.0	0.002
146.500	1.0	0.000
150.300	0.0	0.0
154.100	1.0	0.000

MEAN	STD. DEV.	MINIMUM	MAXIMUM
43.624	23.411	4.000	156.000

(NO. OF CELLS = 50)

DISTRIBUTION OF DEMAND OVER LEAD TIME

MID-VALUE OF CELL	FREQUENCY	PROBABILITY
5.520	20.0	0.010
8.560	41.0	0.020
11.600	49.0	0.024
14.640	75.0	0.037
17.680	76.0	0.038
20.720	89.0	0.044
23.760	108.0	0.054
26.800	102.0	0.051
29.840	121.0	0.060
32.880	114.0	0.057
35.920	130.0	0.065
38.960	112.0	0.056
42.000	101.0	0.050
45.040	98.0	0.049
48.080	63.0	0.031
51.120	86.0	0.043
54.160	81.0	0.040
57.200	78.0	0.039
60.240	67.0	0.033
63.280	62.0	0.031
66.320	55.0	0.027
69.360	38.0	0.019
72.400	40.0	0.020
75.440	24.0	0.012
78.480	23.0	0.011
81.520	32.0	0.016
84.560	10.0	0.005
87.600	14.0	0.007
90.640	8.0	0.004
93.680	11.0	0.005
96.720	12.0	0.006
99.760	10.0	0.005
102.800	7.0	0.003
105.840	7.0	0.003
108.880	8.0	0.004
111.920	0.0	0.0
114.960	4.0	0.002
118.000	6.0	0.003
121.040	3.0	0.001
124.080	2.0	0.001
127.120	2.0	0.001
130.160	1.0	0.000
133.200	1.0	0.000
136.240	0.0	0.0
139.280	2.0	0.001
142.320	4.0	0.002
145.360	1.0	0.000
148.400	1.0	0.000
151.440	0.0	0.0
154.480	1.0	0.000

MEAN	STD. DEV.	MINIMUM	MAXIMUM
43.624	23.411	4.000	156.000

APPENDIX 3

COMPUTER PROGRAM

GENERATION OF LEAD TIME DEMAND DISTRIBUTION

```
$JOB            ANP,RUN=CHECK,KP=29,TIME=5,PAGES=25
 1          COMMON IX
 2          DIMENSION TIMELD(2000),DEMDLT(2000),LTDEMD(2000)
 3        999 FORMAT('1',////////////////////,10X,'AVERAGE LEAD TIME.....
           1.....',F10.3)
 4        998 FORMAT(10X,'AVERAGE DAILY DEMAND.....',
           1F10.3)
 5        997 FORMAT(10X,'STANDARD DEVIATION OF DAILY DEMAND.....',
           1F10.3)
 6        996 FORMAT(10X,'ERLANG PARAMETER FOR LEAD TIME.....',
           1I10)
 7        995 FORMAT(10X,'ERLANG PARAMETER FOR DAILY DEMAND.....',
           1I10)
 8        994 FORMAT(10X,'NUMBER OF SIMULATED DEMANDS DURING LEAD TIME.....',
           1I10)
 9        993 FORMAT(33X,'(NO. OF CELLS =',I3,')')
10        992 FORMAT(/,22X,'DISTRIBUTION OF DEMAND OVER LEAD TIME')
11        991 FORMAT(2F10.3,3I5,F10.3)
12        990 FORMAT('1')
       C
       C.........AVTL    = AVERAGE LEAD TIME IN TIME UNITS.
       C.........AVDEMD  = AVERAGE DEMAND PER TIME UNIT.
       C.........STDEV   = STANDARD DEVIATION OF DEMAND.
       C.........KERLGS  = PARAMETER OF ERLANG DISTRIBUTION FOR LEAD TIME.
       C.........KERLNG  = PARAMETER OF ERLANG DISTN. FOR DEMAND PER TIME UNIT.
       C.........NLDTMS  = NUMBER OF LEAD TIME DEMANDS TO BE SIMULATED.
       C.........NCELLS  = NUMBER OF CLASSES INTO WHICH LEAD TIME DEMANDS ARE
       C                   TO BE DIVIDED.
       C
13          N=0
14        91 N=N+1
       C
       C.........READ THE DATA. SET SEED FOR RANDOM NUMBER GENERATOR. CALCULATE
       C           THE PARAMETER 'KERLNG'. IF KERLNG IS LESS THAN OR EQUAL TO 1,
       C           THE DISTRIBUTION FOR DEMAND RATE IS ASSUMED TO BE POISSON.
       C           OTHERWISE ERLANG.
       C
15          READ 991,AVTL,AVDEMD,KERLGS,NLDTMS,NCELLS,STDEV
16          K1=KERLGS
17          KERLNG=(AVDEMD/STDEV)**2
18          IX=181043
19          MM=1
20        12 IF(KERLNG-1)9,9,10
       C
       C.........GENERATE 'NLDTMS' LEAD TIME DEMANDS FOR POISSON DISTRIBUTED
       C           DEMAND RATE AND ERLANG DISTRIBUTED LEAD TIME.
       C
21         9 DO 23 K2=MM,NLDTMS
       C
       C.........GENERATE LEAD TIME.
       C
22          R=1.
23          DO 24 K3=1,KERLGS
24        24 R=R*RA(X)
25          TIMELD(K2)=-(AVTL/K1)*ALOG(R)
       C
       C.........GENERATE LEAD TIME DEMAND.
       C
26          M=TIMELD(K2)
27          X=TIMELD(K2)-M
```

```
28          LTDEMD(K2)=0
29          IF(M)21,21,22
30       22 DO 20 I=1,M
31          CALL PCISN(AVDEMD,NPOISN)
32       20 LTDEMD(K2)=LTDEMD(K2)+NPOISN
33       21 X=X*AVDEMD
34          CALL PCISN(X,NPOISN)
35          LTDEMD(K2)=LTDEMD(K2)+NPOISN
36       23 CONTINUE
37          GO TO 25
      C
      C.........GENERATE 'NLDTMS' LEAD TIME DEMANDS FOR ERLANG DISTRIBUTED
      C           LEAD TIME AND DEMAND RATE.
      C
38       10 DO 2  K2=MM,NLDTMS
      C
      C.........GENERATE LEAD TIME.
      C
39          R=1.
40          DO 3 K3=1,K1
41        3 R=R*RA(X)
42          TIMELD(K2)=-(AVTL/K1)*ALOG(R)
      C
      C.........GENERATE LEAD TIME DEMAND.
      C
43          M=TIMELD(K2)
44          X=TIMELD(K2)-M
45          DEMDLT(K2)=0.
46          IF(M)5,5,6
47        6 DO 4 K4=1,M
48          R=1.
49          DO 7 I=1,KERLNG
50        7 R=R*RA(X)
51        4 DEMDLT(K2)=DEMDLT(K2)-(AVDEMD/KERLNG)*ALOG(R)
52        5 X=X*AVDEMD
53          R=1.
54          DO 8 I=1,KERLNG
55        8 R=R*RA(X)
56          DEMDLT(K2)=DEMDLT(K2)-(X/KERLNG)*ALOG(R)
57        2 CONTINUE
58       25 NN=0
      C
      C.........PRINT THE DETAILS FOR LEAD TIME DEMAND DISTRIBUTION.
      C
59          PRINT 999,AVTL
60          PRINT 998,AVDEMD
61          PRINT 997,STDEV
62          PRINT 996,KERLGS
63          PRINT 995,KERLNG
64          PRINT 994,NLDTMS
65       32 PRINT 990
66          PRINT 993,NCELLS
67          PRINT 992
68          NN=NN+1
      C
      C.........IF THE LEAD TIME DEMAND IS IN INTEGERS, CALL DIST TO TABULATE
      C           INTEGER LEAD TIME DEMANDS. OTHERWISE, CALL DIST1 TO TABULATE
      C           CONTINUOUS LEAD TIME DEMAND.
      C
69          IF(KERLNG-1)40,40,41
```

```
70        40 CALL DIST (LTDEMD,NLDTMS,NCELLS)                          110
71           GO TO 42
72        41 CALL DIST1(DEMDLT,NLDTMS,NCELLS)
      C
      C.....IF 'NLDTMS' DEMANDS ARE TO BE DIVIDED IN SOME OTHER NUMBER OF
      C        CLASSES, CALL DIST OR DIST1 AGAIN.
      C
73        42 GO TO(26,27,28,29),NN
74        26 NCELLS=30
75           GO TO 32
76        27 NCELLS=40
77           GO TO 32
78        28 NCELLS=50
79           GO TO 32
80        29 IF(NLDTMS-1000)30,30,31
81        30 MM=NLDTMS+MM
82           NLDTMS=NLDTMS+1000
83           NCELLS=25
84           GO TO 12
      C
      C.....IF LEAD TIME DEMANDS FOR ANY MORE ITEM ARE TO BE GENERATED,
      C        REPEAT THE PROCEDURE ALL OVER AGAIN.
      C
85        31 IF(N-1)91,92,92
86        92 STOP
87           END

88           SUBROUTINE POISN (AV,NPOSN)
89           COMMON IX
      C
      C.....THIS SUBROUTINE GENERATES A POISSON DISTRIBUTED RANDOM NUMBER
      C        USING RA(X) - A FUNCTION SUBPROGRAM TO GENARATE UNIFORMLY
      C        DISTRIBUTED RANDOM NUMBER.
      C
90           NPOSN=0
91         1 Y=EXP(-AV)
92           X=1.0
93         5 X=X*RA( X)
94           IF(X-Y)3,4,4
95         4 NPOSN=NPOSN+1
96           GO TO 5
97         3 RETURN
98           END

99           FUNCTION RA( X)
100          COMMON IX
      C
      C.....THIS FUNCTION SUBPROGRAM GENERATES UNIFORMLY DISTRIBUTED RANDOM
      C        NUMBER.
      C
101          IX=IX*65539
102          IF(IX)1,2,2
103        1 IX=IX+2147483647+1
104        2 YFL=IX
105          RA    =YFL*0.4656613D-9
106          RETURN
107          END

108          SUBROUTINE DIST(L,N,NCELLS)
109          COMMON IX
```

```
110         DIMENSION      FREQ(50),PROB(50),CUMPRC(50),L(2000)        111
111      599 FORMAT(15X,F10.3,2X,F10.3,2X,F10.3,3X,F10.3)
112      598 FORMAT(/,20X,'MEAN      STD. DEV.      MINIMUM      MAXIMUM')
113      597 FORMAT(/,19X,'MID-VALUE OF CELL    FREQUENCY    PROBABILITY',/)
114      596 FORMAT(21X,F10.3,5X,F10.1,4X,F10.3)
115      595 FORMAT(8F10.5)
         C
         C.........THIS SUBROUTINE TABULATES 'N' VALUES OF AN INTEGER VARIABLE 'L'
         C           INTO NCELLS CLASSES.
         C.........FIND MINIMUM AND MAXIMUM.
         C
116           AMIN=1.0E75
117           AMAX=-1.0E75
118           DO 2 I=1,N
119           IF(L(I)-AMIN)4,5,5
120         4 AMIN=L(I)
121         5 IF(L(I)-AMAX)2,2,6
122         6 AMAX=L(I)
123         2 CONTINUE
         C.........INITILISE FREQUENCIES.
124           DO 7 I=1,NCELLS
125         7 FREQ(I)=0.
         C.........DETERMINE INTERVAL SIZE.
126           SINT=(AMAX-AMIN)/NCELLS
127           AV=0.
128           VAR=0.
         C
         C.........DETERMINE THE CLASS OR CLASSES TO WHICH OBSERVED VARIABLE
         C           BELONGS. UPDATE THE FREQUENCIES OF THE CLASSES. OBTAIN SUM AND
         C           SUM OF SQUARES OF 'L'.
         C
129           DO 10 I=1,N
130           AV=AV+L(I)
131           VAR=VAR+L(I)*L(I)
132           K=(L(I)-AMIN)/SINT+1
133           IF(K.GT.NCELLS)K=NCELLS
134        10 FREQ(K)=FREQ(K)+1.
         C
         C.........COMPUTE MEAN AND STANDARD DEVIATION OF 'L'.
         C
135        43 STDEV=SQRT(ABS((VAR-AV*AV/N)/(N-1.)))
136           AV=AV/N
         C
         C.........COMPUTE PROBABILITY THAT 'L' BELONGS TO A CLASS. GET THESE
         C           PROBABILITIESAND MEAN, STD. DEVIATION, MINIMUM AND MAXIMUM OF
         C           'L' IN PRINTED AND PUNCHED OUTPUT.
         C
137           DO 12 I=1,NCELLS
138        12 PROB(I)=FREQ(I)/N
139           PRINT 597
140           B=AMIN-SINT/2.
141           DO 14 I=1,NCELLS
142           B=B+SINT
143        14 PRINT 596,B,FREQ(I),PROB(I)
144           PRINT 598
145           PRINT 599,AV,STDEV,AMIN,AMAX
146           PUNCH 595,AV,STDEV,SINT,AMIN,AMAX
147           PUNCH 595,(PROB(I),I=1,NCELLS)
148           RETURN
149           END
```

```
150         SUBROUTINE DIST1(A,N,NCELLS)
151         COMMON IX
152         DIMENSION A(1),FREQ(50),PROB(50),CUMPRO(50)
153       999 FORMAT(15X,F10.3,2X,F10.3,2X,F10.3,3X,F10.3)
154       998 FORMAT(/,20X,'MEAN      STD. DEV.      MINIMUM      MAXIMUM')
155       997 FORMAT(/,19X,'MID-VALUE OF CELL    FREQUENCY    PROBABILITY',/)
156       996 FORMAT(21X,F10.3,5X,F10.1,4X,F10.3)
157       995 FORMAT(8F10.5)
          C
          C.........THIS SUBROUTINE TABULATES 'N' VALUES OF A CONTINUOUS VARIABLE
          C         'A' INTO 'NCELLS' CLASSES.
          C.........FIND MINIMUM AND MAXIMUM OF 'A'.
          C
158           AMIN=1.0E75
159           AMAX=-1.0E75
160           DO 2 I=1,N
161           IF(A(I)-AMIN)4,5,5
162         4 AMIN=A(I)
163         5 IF(A(I)-AMAX)2,2,6
164         6 AMAX=A(I)
165         2 CONTINUE
          C.........INITILISE FREQUENCIES.
166           DO 7 I=1,NCELLS
167         7 FREQ(I)=0.
          C.........DETERMINE INTERVAL SIZE.
168           SINT=(AMAX-AMIN)/NCELLS
169           AV=0.
170           VAR=0.
          C
          C.........DETERMINE THE CLASS TO WHICH THE VALUE OF 'A' BELONGS, UPDATE
          C         THE FREQUENCY OF THAT CLASS. OBTAIN THE SUM AND SUM OF SQUARES
          C         OF 'A'.
          C
171           DO 10 I=1,N
172           AV=AV+A(I)
173           VAR=VAR+A(I)*A(I)
174           K=(A(I)-AMIN)/SINT+1
175           IF(K.GT.NCELLS)K=NCELLS
176        10 FREQ(K)=FREQ(K)+1.
          C
          C.........COMPUTE MEAN AND STD. DEVIATION OF 'A'. DETERMINE PROBABILITY
          C.........THAT 'A' BELONGS TO A CLASS. GET THE PROBABILITIES,MEAN, STD.
          C.........DEVIATION, MINIMUM AND MAXIMUMOF 'A' IN PRINTED AND PUNCHED FORM.
          C
177           STDEV=SQRT(ABS((VAR-AV*AV/N)/(N-1.)))
178           AV=AV/N
179           DO 12 I=1,NCELLS
180        12 PROB(I)=FREQ(I)/N
181           PRINT 997
182           B=AMIN-SINT/2.
183           DO 14 I=1,NCELLS
184           B=B+SINT
185        14 PRINT 996,B,FREQ(I),PROB(I)
186           PRINT 998
187           PRINT 999,AV,STDEV,AMIN,AMAX
188           PUNCH 995,AV,STDEV,SINT,AMIN,AMAX
189           PUNCH 995,(PROB(I),I=1,NCELLS)
190           RETURN
191           END
```

APPENDIX 4

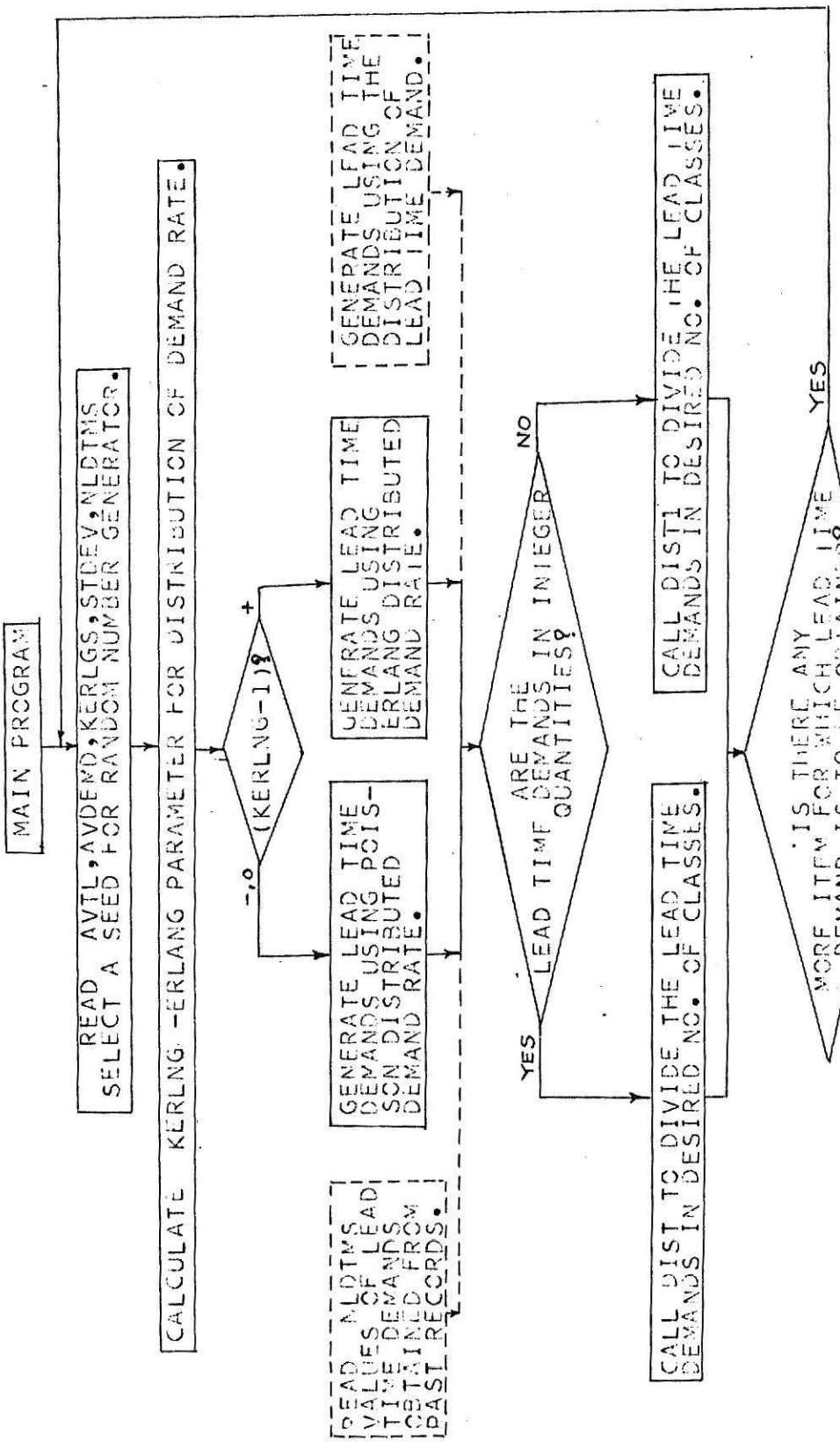
FLOW CHARTS

GENERATION OF LEAD TIME DEMAND DISTRIBUTION

Flow charts for subroutine DIST and subroutine DIST 1 are similar to the one given on p. 145.

Flow charts for subroutine POISN is given on p. 144.

Flow chart for function RA(X) is given on p. 144.



FLOW CHART OF MAIN PROGRAM

APPENDIX 5

COMPUTER PROGRAM

OBTAINING OPTIMAL ORDER SIZES AND REORDER POINTS

```
      $JOB            ANP,RUN=CHECK,KP=29,TIME=5,PAGES=25
 1          COMMCN PROB(50,20),STAT( 3,2C),AREA1(51,20),AREA2(51,20)
 2          CCMMCN NTYPE(20),NCELL(20),S(20),ARE1(51,20),ARE2(51,20)
 3          COMMCN    C(20),TL(20),CI(20),CF(20),CD(20),W(20),BOU(20),BOL(20)
 4          COMMCN SINT(20),REORD(20),Q(20),NITMS,CAP
 5        599 FORMAT(2I5)
 6        598 FORMAT(10X,'NO. OF ITEMS =',I3,'.  NO. OF PROBLEMS ='I3,/)
 7        597 FORMAT(2I5,3F10.3,F10.6,3F10.3)
 8        596 FORMAT(10X,'ITEM NO.  CODE NC.  NO. OF CELLS  DEMAND RATE  LEAD TI
            1ME  CCST/UNIT  INV. COST  FIX. COST  STOUT COST  SPACE/UNIT',/)
 9        595 FORMAT(10X,I5,5X,I5,5X,I8,5X,F10.3,1X,F10.3,F10.3,4(1X,F10.3),2(2X
            1,F10.3))
10        594 FORMAT(8F10.5)
11        593 FORMAT(/,10X,'ITEM    MEAN      STC. DEV.   CELL WIDTH    MINIMUM
            1  MAXIMUM',/)
12        592 FORMAT(8X,I5,5(4X,F8.3) )
13        591 FORMAT(10X,'ITEM NO. =',I5,'.   PROBABILITY DISTRIBUTION OVER',I5,
            1' CELLS',/)
          C
          C......NITMS...NUMBER OF ITEMS TO BE STORED IN THE WAREHOUSE.
          C......CAP.....AVAILABLE CAPACITY.
          C......NO......NUMBER OF AVAILABLE CAPACITIES FOR WHICH CCNSTRAINED OPT-
          C               IMUM SOLUTIONS ARE TO BE OBTAINED.
          C......N.......NUMBER OF ITERATICNS REQUIRED TC SCLVE THE PROBLEM.
          C......FOR I'TH ITEM,
          C......S(I).....DEMANC PER UNIT TIME.
          C......C(I).....CCST PER UNIT.
          C......W(I).....WAREHOUSE SPACE REQUIREC PER UNIT.
          C......TL(I).....LEAD TIME.
          C......CI(I).....INVENTORY HCLDING CCST PER UNIT PER UNIT TIME.
          C......CF(I).....ORDERING OR REPLENISHMENT CCST PER LOT.
          C......CD(I).....OUT OF STOCK CCST.
          C......NTYPE(I)..CODE NUMBER.
          C......NCELL(I)..NUMBER OF CLASSES IN WHICH LEAC TIME CEMANC IS CIVIDED.
          C......BOL(I)....MINIMUM LEAD TIME DEMAND.
          C......BOU(I)....MAXIMUM LEAD TIME DEMAND.
          C......SINT(I)...CLASS INTERVAL FOR LEAD TIME CEMAND.
          C......PROB(J,I).PROBABILITY THAT LEAD TIME DEMANC IS IN J'TH CLASS.
          C......STAT(2,J).MEAN OF THE LEAD TIME DEMAND.
          C......STAT(3,J).STD. DEV. OF THE LEAD TIME CEMANC.
          C
14          READ 599,NITMS,NO
15          PRINT 598,NITMS,NO
16          PRINT 596
          C
          C......READ AND PRINT THE DATA FOR ALL THE ITEMS.
          C
17          DO 1 J=1,NITMS
18          READ 597,NTYPE(J),NCELL(J),S(J),TL(J),C(J),CI(J),CF(J),CD(J),W(J)
19        1 PRINT 595,J,NTYPE(J),NCELL(J),S(J),TL(J),C(J),CI(J),CF(J),CD(J),W(
            1J)
20          KLK=0
21       91 KLK=KLK+1
22          READ 599,NCLL
          C
          C......READ AND PRINT THE DISTRIBUTION OF LEAD-TIME-DEMANDS.
          C
23          DO 81 J=1,NITMS
24          NCELL(J)=NCLL
25          K=NCELL(J)
```

```
26          READ 994,STAT(2,J),STAT(3,J),SINT(J),BOL(J),BOU(J)
27       81 READ  994,(PROB(I,J),I=1,K)
28          PRINT 993
29          DO 82 J=1,NITMS
30       82 PRINT 992,J,STAT(2,J),STAT(3,J),SINT(J),BOL(J),BOU(J)
31          DO 83 J=1,NITMS
32          K=NCELL(J)
33          PRINT 991,J,NCELL(J)
34       83 PRINT 994,(PROB(I,J),I=1,K)
35          CALL ITGRLS
36          LLL=0
37       19 READ  994,CAP
38          N=0
39          LLL=LLL+1
      C
      C......SELECT LOWER AND UPPER BOUNDS FOR LYAMBDA.
      C
40          AMD1=0.
41          AMD2=-0.2
42          CALL KALPKM(AMD1,CAP1)
43          N=N+1
      C
      C.....EXAMINE WHETHER AVAILABLE WAREHOUSE CAPACITY IS MORE THAN THAT RE-
      C        QUIRED BY UNCONSTRAINED OPTIMAL SOLUTION.
      C
44          IF (CAP1-CAP)2,2,3
45        3 CALL KALPKM(AMD2,CAP2)
46          N=N+1
      C
      C.....EXAMINE WHETHER 'AMD2' IS SUFFICIENTLY SMALL. IF NOT, IT IS REDUC-
      C        ED SO THAT IT BECOMES A LOWER BOUND FOR GOLDEN SECTION SEARCH.
47          IF(CAP2-CAP)4,4,5
      C
48        5 AMD2=AMD2-1.0
49          GO TO 3
      C
      C......SEARCH BY LINEAR INTER AND EXTRAPOLATION STARTS.
      C......ALTERNATIVELY, GOLDEN SECTION SEARCH COULD BE USED.
      C
50        4 AMAX=CAP1
51          AMIN=CAP2
52          YY1=AMD1
53          YY2=AMD2
54        9 N=N+1
55       13 X=AMD1-(AMD1-AMD2)*(CAP1-CAP)/(CAP1-CAP2)
56          IF(X-YY2)14,15,15
57       14 CAP1=CAP2
58          AMD1=AMD2
59          CAP2=AMIN
60          AMD2=YY2
61          GO TO 13
62       15 IF(X-YY1)16,17,17
63       17 CAP1=CAP2
64          AMD1=AMD2
65          CAP2=AMAX
66          AMD2=YY1
67          GO TO 13
68       16 CALL KALPKM(X,CAPX)
69          IF(AMD1-AMD2)60,60,61
70       60 IF(X-AMD2)62,62,63
```

```
      62 IF(X-AMD1)67,67,66
      66 IF(CAPX-CAP1)10,64,64
      64 IF(CAPX-CAP2)65,65,10
      67 IF(CAPX-CAP1)65,65,10
      63 IF(CAPX-CAP2)10,65,65
      61 IF(X-AMD2)68,68,69
      68 IF(CAPX-CAP2)65,65,10
      69 IF(X-AMD1)70,70,71
      71 IF(CAPX-CAP1)10,65,65
      70 IF(CAPX-CAP2)10,72,72
      72 IF(CAPX-CAP1)65,65,10
      65 AMD1=AMD2
         CAP1=CAP2
         AMD2=X
         CAP2=CAPX
         IF(CAP2-CAP)50,51,51
      51 YY1=AMD2
         AMAX=CAP2
         GO TO 52
      50 YY2=AMD2
         AMIN=CAP2
      52 Y=ABS(CAP2-CAP)
         IF(Y-0.001*CAP)2,2,9
C
C.....SEARCH BY LINEAR INTER AND EXTRAPOLATION ENDS.
C
       2 PRINT 980,N
     980 FORMAT(10X,'NUMBER OF ITERATIONS REQUIRED =',I4)
         GO TO 18
      10 PRINT 992
         GO TO 18
C
C.....IF ANY MORE CONSTRAINED CAPACITY OPTIMUM SOLUTION IS TO BE OBTAI-
C.....NED, THE COMPUTING PROCEDURE IS REPEATED.
C
      18 IF(LLL-NO)19,20,20
C
C.....IF ANY OTHER DISTRIBUTION IS TO BE USED TO SOLVE THIS CONSTRAINED
C.....CAPACITY PROBLEM THE COMPUTING PROCEDURE IS REPEATED.
C
      20 IF(KLK-8)91,92,92
      92 STOP
         END

         SUBROUTINE KALPKM (AMDA,CAP1)
         COMMON PROB(50,20),STAT( 3,20),AREA1(51,20),AREA2(51,20)
         COMMON NTYPE(20),NCELL(20),S(20),ARE1(51,20),ARE2(51,20)
         COMMON    C(20),TL(20),CI(20),CF(20),CD(20),W(20),BOU(20),BOL(20)
         COMMON SINT(20),REORD(20),Q(20),NITMS,CAP
         DIMENSION IQ(20),IREORD(20)
     999 FORMAT(10X,'INTERMEDIATE RESULT AT LAMBDA =',F8.5,/)
     998 FORMAT(10X,'ITEM NO.   ORDER QUANTITY   REORD. PT.   ORDER COST
        1INVT. COST   SAFTY COST   STOUT COST   TOTAL COST',/)
     997 FORMAT(10X,I5,11X,I3,12X,I3,2X,5F13.3)
     996 FORMAT(/,10X,'TOTAL COST'  ,15('.'),F8.3,/,10X,'AVAILABLE CAPACITY
        1',7('.'),F8.3,/,10X,'REQUIRED CAPACITY',8('.'),F8.3,///)
C
C.....THIS SUBROUTINE FINDS OPTIMUM SOLUTION FOR A GIVEN VALUE OF LYAM-
C      BDA - 'AMDA'. CORRESPONDING CAPACITY REQUIRED, THE VARIOUS INVEN-
C      TORY COSTS FOR INDIVIDUAL ITEMS AND THE TOTAL COST OF THE INVENT-
```

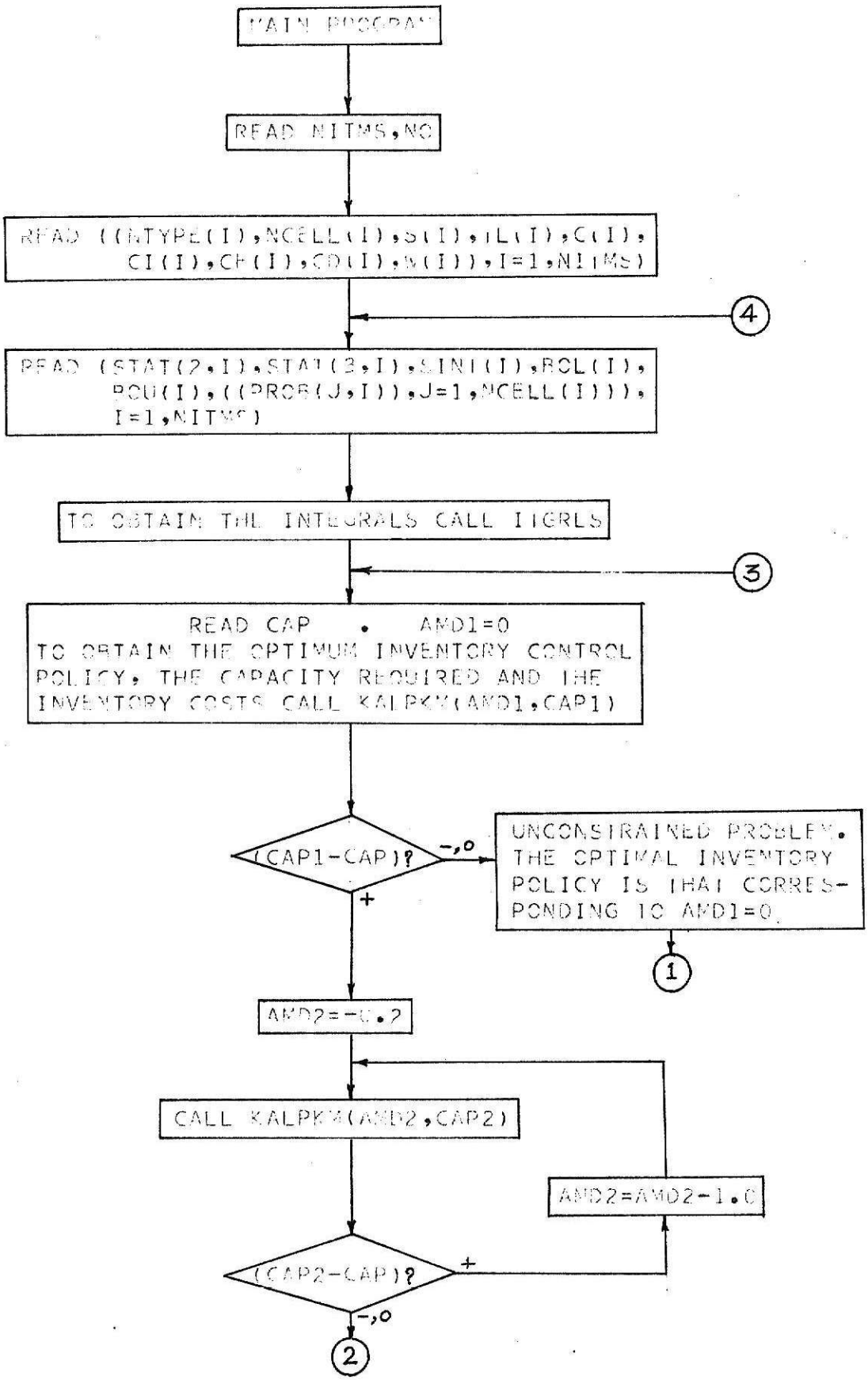
```
      C       DRY POLICY ARE COMPUTED AND PRINTED OUT.
      C......CAP1....WAREHOUSE CAPACITY REQUIRED FOR THE OPTIMUM INVENTORY
      C......          POLICY FOR A GIVEN LYAMBDA.
      C......TC......TOTAL INVENTORY COST RATE FOR THE INVENTORY POLICY.
      C......FOR ANY ITEM,
      C......C1......REPLENISHMENT COST RATE.
      C......C2......INVENTORY HOLDING COST RATEFOR AVERAGE INVENTORY.
      C......C3......INVENTORY HOLDING COST RATE FOR SAFETY STOCK.
      C......C4......OUT OF STOCK COST RATE.
      C......IQ......OPTIMUM ORDER SIZE.
      C......IRECRD..OPTIMUM REORDER POINT.
      C
  113         PRINT 599,AMDA
  114         PRINT 598
  115         CAP1=0.
  116         TC=0.
  117         DO 1 I=1,NITMS
  118         E=CI(I)-W(I)*AMDA
  119         B=SQRT(1.0/((CI(I)/2.0)-W(I)*AMDA))
  120         K=NCELL(I)+1
  121         DO 2 J=1,K
  122         P1=AREA1(J,I)/E
  123         P2=B*AREA2(J,I)
  124         P3=P2-P1
  125         P=(P1+P2)/2.0
  126         IF(P3)3,3,4
  127       4 PQ=P
  128         P4=P3
  129       2 CONTINUE
      C
      C......REORDER POINT WOULD BE LESS THAN THE MINIMUM DEMAND DURING LEAD
      C......TIME, FOR 'AMDA'.
      C
  130       3 X=P4*SINT(I)/(P4-P3)
  131         TEGRL1=ARE1(J,I)-X*(ARE1(J,I)-ARE1(J-1,I))/SINT(I)
  132         TEGRL2=ARE2(J,I)-X*(ARE2(J,I)-ARE2(J-1,I))/SINT(I)
  133         Q(I)=P-(P-PQ)*X/SINT(I)
  134         REORD(I)=BOU(I)-(J-1)*SINT(I)+X
      C
      C......MAKE ORDER SIZE AND REORDER POINT INTEGER NUMBERS BY ROUNDING OFF
      C......IF NECESSARY.
      C
  135         C1=CF(I)*S(I)/Q(I)
  136         C2=CI(I)*Q(I)/2.0
  137         C3=CI(I)*((REORD(I)-S(I)*TL(I))+TEGRL2/(2.0*Q(I)))
  138         C4=CD(I)*S(I)*TEGRL1/Q(I)
  139         TC1=C1+C2+C3+C4
  140         TC=TC+TC1
  141         CAP1=CAP1+W(I)*(Q(I)+REORD(I)-S(I)*TL(I))
  142         IQ(I)=Q(I)
  143         IF(Q(I)-IQ(I)-0.5)5,6,6
  144       6 IQ(I)=IQ(I)+1
  145       5 IRECRD(I)=REORD(I)
  146         IF(RECRD(I)-IREORD(I)-0.5)1,7,7
  147       7 IREORD(I)=IREORD(I)+1
  148       1 PRINT 597,I,IQ(I),IREORD(I),C1,C2,C3,C4,TC1
  149         PRINT 596,TC,CAP,CAP1
  150         RETURN
  151         END
```

```
      SUBROUTINE ITGRLS
      COMMON PROB(50,20),STAT( 3,20),AREA1(51,20),AREA2(51,20)
      COMMON NTYPE(20),NCELL(20),S(20),ARE1(51,20),ARE2(51,20)
      COMMON    C(20),TL(20),CI(20),CF(20),CD(20),W(20),BOU(20),BOL(20)
      COMMON SINT(20),REORD(20),Q(20),NITMS,CAP
      DIMENSION  PROBDE(51,20)
  999 FORMAT(10F10.4)
C
C.....THIS SUBROUTINE IS USED TO FIND THE INTEGRALS AND THE TWO FUNCTI-
C.....ONS OF THE INTEGRALS IN THE BASIC EQUATIONS.
C.....CHARACTERISTICS OF THE INVENTORY MODEL ARE,
C.......1) APPROXIMATE INVENTORY HOLDING COST.
C.......2)STOCK-OUT-COST PROPORTIONAL TO THE MAXIMUM STOCK-OUT.
C.......3) CONSTANT AND KNOWN LEAD TIME.
C.....FOR J'TH ITEM AND I'TH REORDER POINT COUNTING DOWN FROM THE MAXI-
C.....MUM REORDER POINT,
C..... AREA1(I,J)..FUNCTION OF INTEGRALS IN THE FIRST EQUATION.
C..... AREA2(I,J)..FUNCTION OF INTEGRALS IN THE SECOND EQUATION.
C
      DO 1 J=1,NITMS
      ARE1(1,J)=0.
      ARE2(1,J)=0.
      AREA1(1,J)=0.
      AREA2(1,J)=SQRT(S(J)*CF(J))
      K=NCELL(J)+1
      E=CD(J)+CI(J)*TL(J)/2.0
      B=S(J)*E
      DO 2 I=2,K
      ARE1(I,J)=0.
      ARE2(I,J)=0.
      ARE3=0.
      ARE4=0.
      MM=I-1
      N=K-I
      DO 3 M=1,MM
      L=N+M
      ABC=BOL(J)+(L-0.5)*SINT(J)
      ARE3=ARE3+PROB(L,J)
      ARE4=ARE4+ABC*PROB(L,J)
      ARE1(I,J)=ARE1(I,J)+(M-0.5)*PROB(L,J)*SINT(J)
    3 ARE2(I,J)=ARE2(I,J)+ABC*(M-0.5)*PROB(L,J)*SINT(J)
      AREA1(I,J)=S(J)*CD(J)*ARE3+CI(J)*ARE4/2.0
      AREA2(I,J)=SQRT(S(J)*(CF(J)+CD(J)*ARE1(I,J))+CI(J)*ARE2(I,J)/2.0)
    2 CONTINUE
    1 CONTINUE
C
C.....PRINT OUT THE TWO FUNCTIONS OF THE INTEGRALS AT (NCELLS+1) VALUES
C.....OF REORDER POINTS, FOR ALL THE ITEMS.
C
      DO 4 J=1,NITMS
      K=NCELL(J)+1
    4 PRINT 999,(AREA1(I,J),I=1,K),(AREA2(I,J),I=1,K)
      RETURN
      END

$ENTRY
```

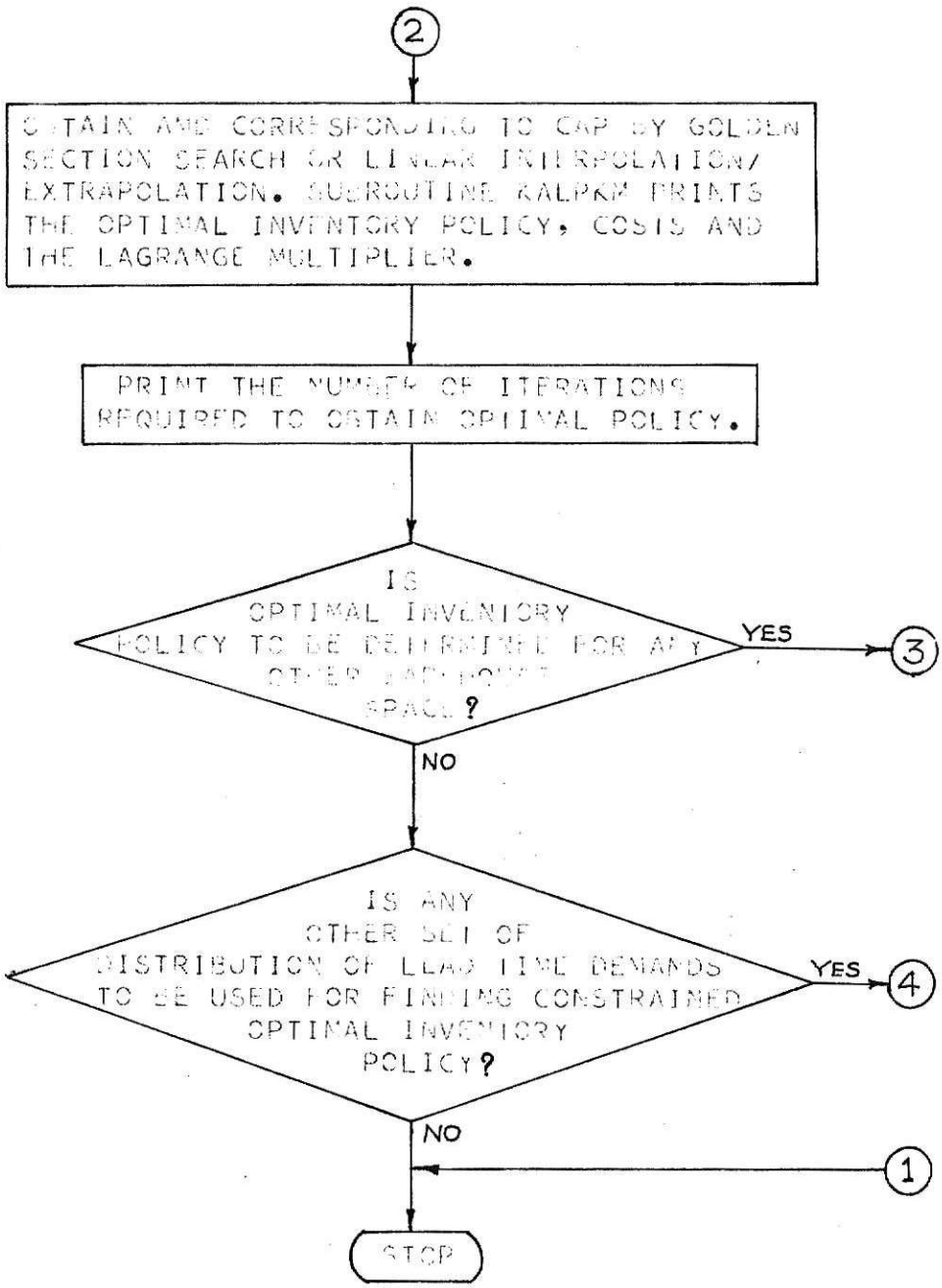
APPENDIX 6

FLOW CHARTS

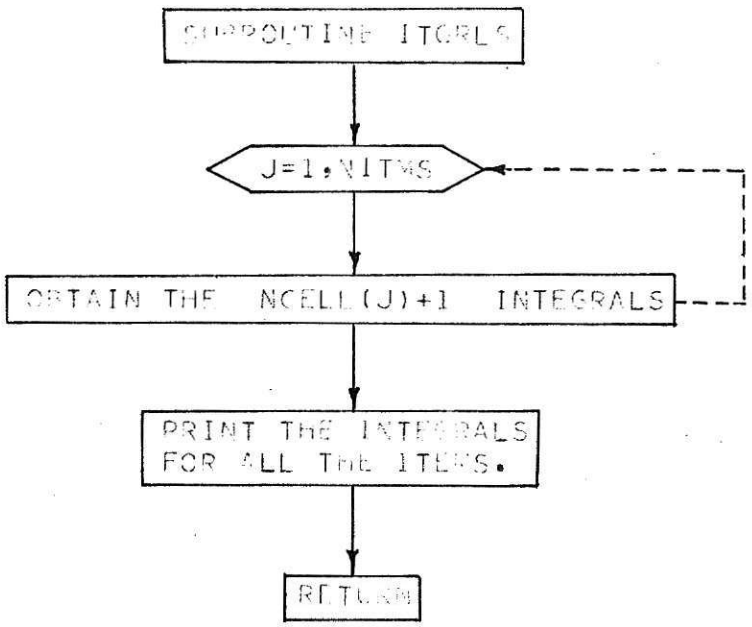
OBTAINING OPTIMAL ORDER SIZES AND REORDER POINTS



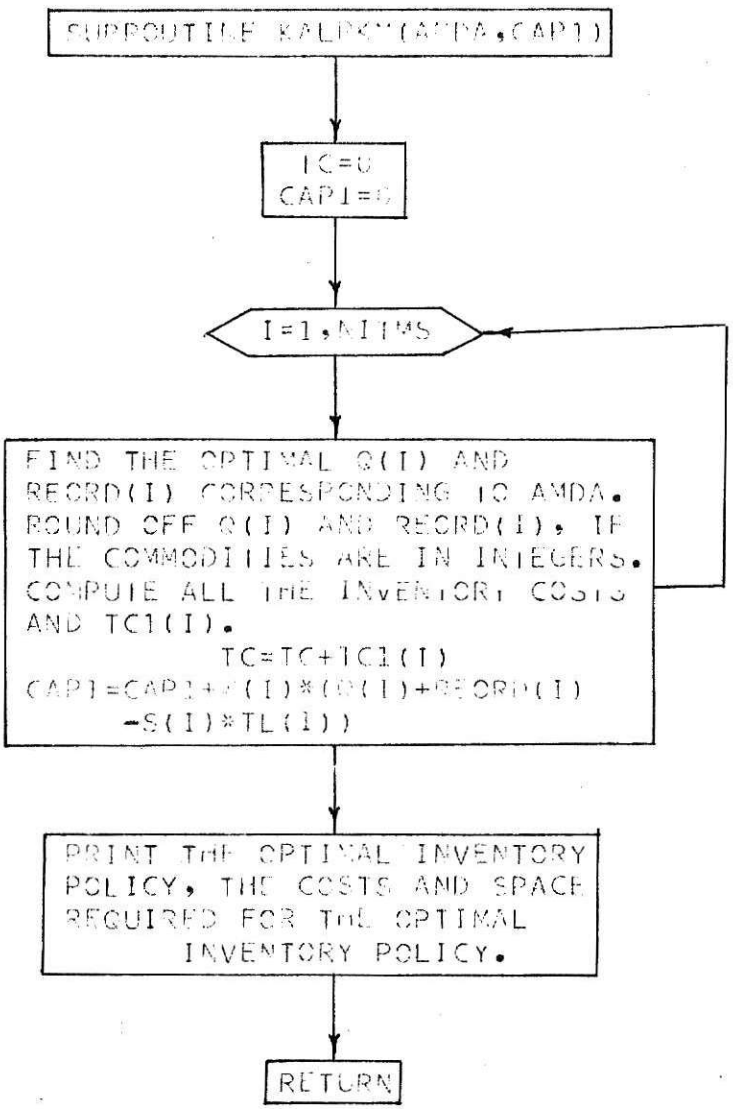
FLOW CHART OF MAIN PROGRAM



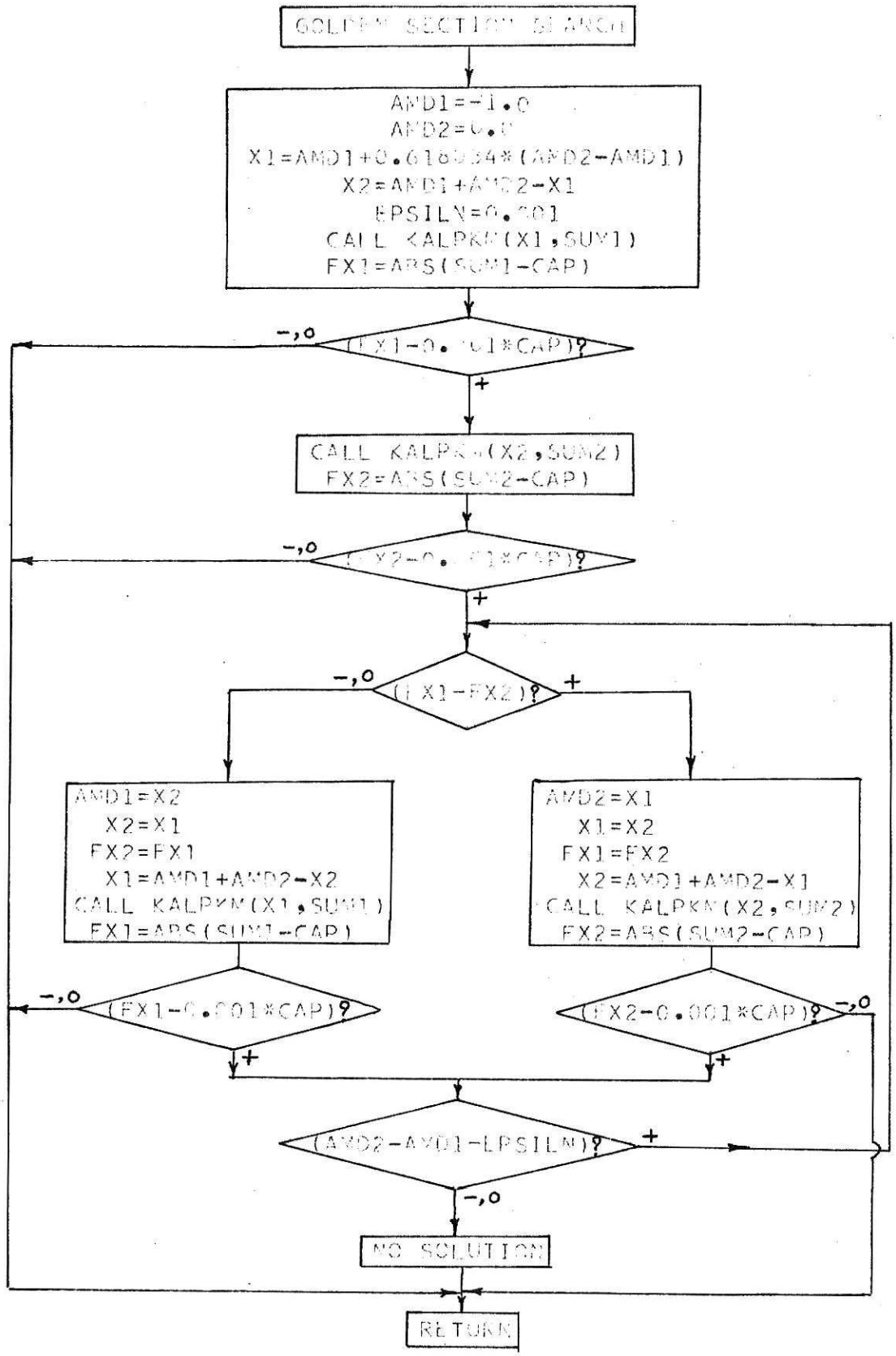
FLOW CHART OF MAIN PROGRAM (CONTD.)



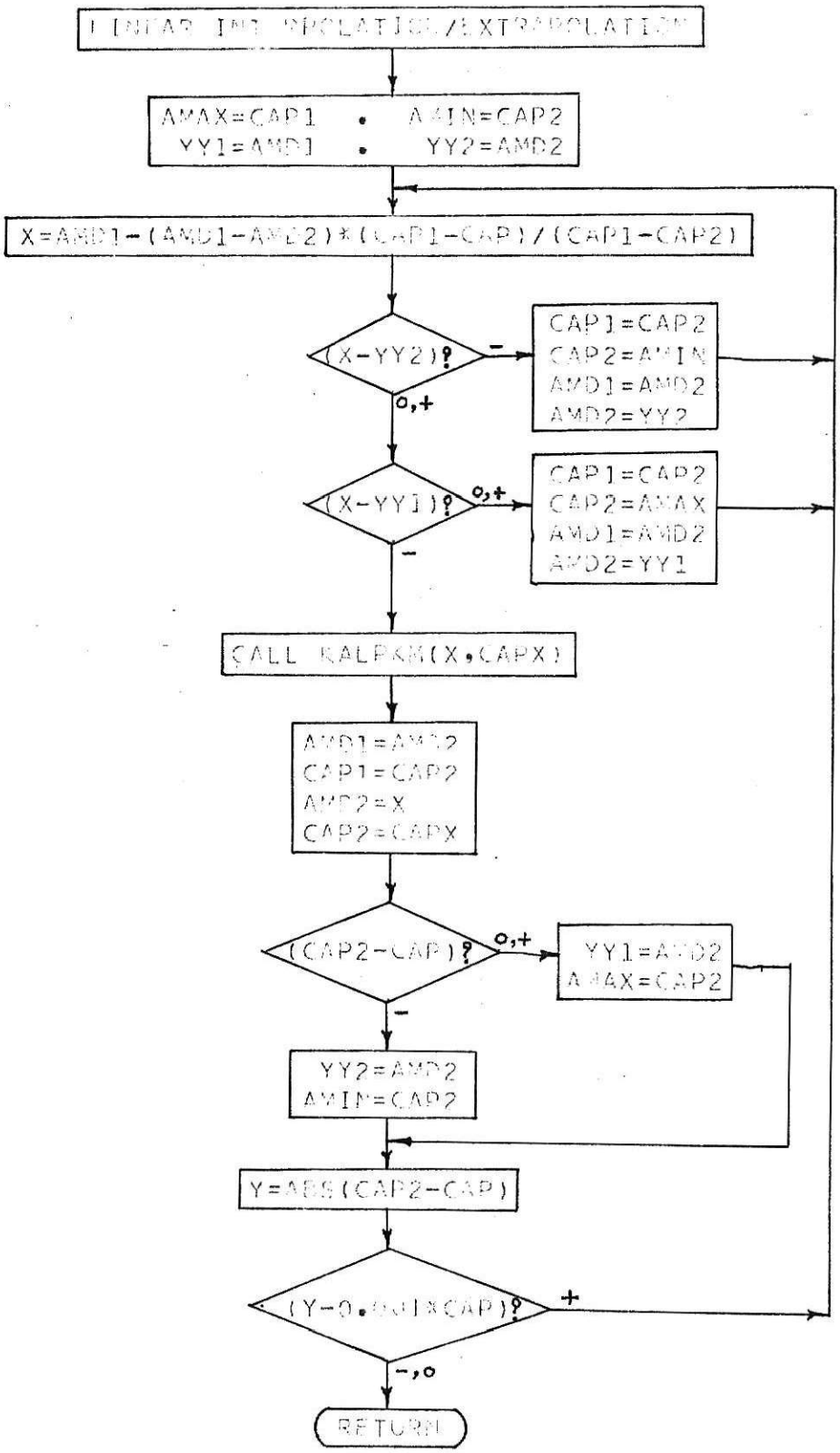
FLOW CHART OF SUBROUTINE ITGRLS



FLOW CHART OF SUBROUTINE KALPKM



FLOW CHART OF GOLDEN SECTION SEARCH



FLOW CHART OF LINEAR INTERPOLATION/EXTRAPOLATION

APPENDIX 7

COMPUTER PROGRAM

WAREHOUSE SPACE UTILIZATION

```
      $JOB            ANP,RUN=CHECK,KP=29,TIME=5,PAGES=25
     1      COMMON IM,IMM,MX,MXX,MAXQS,MAXNS,NQ,MLC,MFE,MLE,ID,MAXNQ,ENQ,VNQ,Q
            1TIME,MFA,TIME,ITEM,IX ,TNOW
     2      COMMON NSET(300),QSET(100),ATRIB(1),JTRIB(3)
     3      DIMENSION CAP(4000),STCKOT(100),NOUT(100),INVTRY(100),KODE(100)
     4      DIMENSION IQ(100),IRECRD(100),W(100),S(100),AVTL(100),KERLNG(100)
     5      DIMENSION FREQ(100),PROB(100),CUMPRC(100),NIVTRY(100)
      C
      C.........COMPUTTR PROGRAMME TO SIMULATE WAREHOUSE CAPACITY UTILISATION.
      C.........NITMS  = NUMBER OF ITEMS STORED IN THE WAREHOUSE.
      C.........NREPOT = PERIOD IN TIME UNITS BETWEEN TWO SIMULATION REPORTS.
      C.........NODAYS = LENGTH OF SIMULATION IN TIME UNITS.
      C.........NCELLS = NUMBER OF CELLS IN WHICH CAPACITY UTILISATION IS DIVIDED.
      C.........ISDSIM = RANDOM NO. SEED FOR SIMULATING CAPACITY UTILISATION.
      C.........NSEEDS = NO. OF RANDOM NO. SEEDS USED TO SET INITIAL INVENTORY.
      C.........AN EVENT FILE-ARRIVAL OF ORDERS-IS USED. FOR THIS FILE,
      C.........TNOW      = CURRENT TIME.
      C.........IM        = NUMBER OF FIXED POINT ATTRIBUTES.
      C.........IMM       = NUMBER OF FLOATING POINT ATTRIBUTES.
      C.........ID        = MAXIMUM EXPECTED NUMBER OF ENTRIES IN THE FILE.
      C.........ATRIB(1) = TIME AT WHICH ORDER IS ARRIVING.
      C.........JTRIB(1) = ITEM, ORDER FOR WHICH IS ARRIVING.
      C.........JTRIB(2) JTRIB(3) ARE POINTERS.
      C..........FOR I'TH ITEM,
      C.........AVTL(I)    = AVERAGE LEAD TIME IN TIME UNITS.
      C.........S(I)       = AVERAGE DEMAND PER TIME UNIT.
      C.........W(I)       = WAREHOUSE SPACE REQUIRED PER UNIT.
      C.........IQ(I)      = ORDER QUANTITY.
      C.........IRECRD(I) = REORDER POINT.
      C.........KERLNG(I) = ERLANG PARAMETER FOR LEAD TIME.
      C.........NIVTRY(I) = ERLANG PARAMETER FOR LEAD TIME.
      C.........INVTRY(I) = INVENTORY AT THE END OF DAY.
      C.........KODE(I)    = 2,IF OREDR HAS ALREADY BEEN PLACED.1, OTHERWISE.
      C.........NOUT(I)    = NUMBER OF TIME UNITS, THE ITEM IS OUT OF STOCK.
      C
     6      599 FORMAT(8I10)
     7      598 FORMAT(3F10.3,3I10)
     8      597 FORMAT('1')
     9      596 FORMAT(18X,'SIMULATION OF WAREHOUSE CAPACITY UTILISATION',/)
    10      595 FORMAT(10X,'NUMBER OF ITEMS STORED IN WAREHOUSE',25('.'),I10)
    11      594 FORMAT(10X,'LENGTH OF SIMULATION RUN IN TIME-UNITS',22('.'),I10)
    12      593 FORMAT(10X,'NUMBER OF CELLS IN WHICH CAPACITY UTILISATION IS DIVID
               1ED',4('.'),I10)
    13      592 FORMAT(10X,'RANDOM NUMBER SEED FOR SETTING INITIAL INVENTORY',12('
               1.'),I10)
    14      591 FORMAT(10X,'RANDOM NUMBER SEED FOR SIMULATING CAPACITY UTILISATION
               1',6('.'),I10)
    15      590 FORMAT(/,25X,'DECRIPTION                    ITEM 1    ITEM 2    ITEM
               1 3    ITEM 4    ITEM 5    ITEM 6',/)
    16      589 FORMAT(10X,'ORDER QUANTITY',26('.'),3X,6(I4,6X))
    17      588 FORMAT(10X,'REORDER POINTY',26('.'),3X,6(I4,6X))
    18      587 FORMAT(10X,'MEAN LEADTIME',27('.'),1X,6(F8.3,2X))
    19      586 FORMAT(10X,'ERLANG PARAMETER FOR LEAD TIME',10('.'),3X,6(I4,6X))
    20      585 FORMAT(10X,'MEAN DEMAND RATE (POISSON DISTRIBUTED)..',1X,6(F8.3,2X
               1))
    21      584 FORMAT(10X,'SPACE OCCUPIED PER UNIT',17('.'),1X,6(F8.3,2X))
    22      583 FORMAT(10X,'INITIAL INVENTORY'23('.'),3X,6(I4,6X))
    23      582 FORMAT(10X,'PROBABILITY OF GOING OUT OF STOCK'7('.'),1X,6(F8.5,2X)
               1)
    24      581 FORMAT(10X,'FINAL INVENTORYRY'23('.'),3X,6(I4,6X))
```

```
25          READ 999,NITMS,NREPOT,NODAYS,NCELLS,ISDSIM,NSEEDS
26          DO 1 I=1,NITMS
27        1 READ 998,AVTL(I),S(I),W(I),IQ(I),IREORD(I),KERLNG(I)
28          KLM=0
29       40 KLM=KLM+1
      C
      C.........INITIALISE EVENT FILE, FOR ARRIVAL OF ORDERS.
      C
30          TNOW=0.
31          IM=1
32          IMM=1
33          ID=100
34          CALL INIT
      C
      C.........READ RANDOM NUMBER SEED FOR SETTING INITIAL INVENTORIES.
      C           SET INITIAL INVENTORIES BETWEEN ZERO AND ORDER SIZE. IF IT IS
      C           BELOW REORDER POINT, SCHECULE AN ARRIVAL OF ORDER FOR THIS ITEM.
      C           SET KODE=2. ENTER THIS EVENT IN THE FILE.
      C
35          READ 999,IX
36          IZ=IX
37          DO 31 I=1,NITMS
38          NOUT(I)=0
39          KODE(I)=1
40          NIVTRY(I)=RA(X)*IQ(I)
41          IF(NIVTRY(I)-IRECRD(I))32,32,31
42       32 Y=1.0
43          KERL=KERLNG(I)
44          DO 34 K=1,KERL
45       34 Y=Y*RA(X)
46          TL=(-AVTL(I)/KERL)*ALOG(Y)
47          LEADTM=TL
48          IF(TL-LEADTM-0.5)36,36,35
49       35 TL=TL+1.
50       36 A=INVTRY(I)
51          B=IRECRD(I)
52          TL=TL*A/B
53          ATRIB(1)=TNOW+TL
54          JTRIB(1)=I
55          KODE(I)=2
56          CALL ENTER(&18)
57       31 CONTINUE
58          ND=NREPOT
59          IX=ISDSIM
60          IY=IX
61          M=1
      C
      C.........START SIMULATION OF CAPACITY UTILISATION.
      C
62       39 DO 2 J=M,ND
63          TNOW=TNOW+1.
      C
      C.........INITILISE CAPACITY USED AT THIS TIME UNIT.
      C
64          CAP(J)=0.
      C
      C.........TEST IF ANY ORDER IS ARRIVING AT THIS TIME UNIT. IF AN ORDER IS
      C           ARRIVING BRING UP THE INVENTORY OF THE ITEM BY ITS ORDER SIZE.
      C           REMOVE THIS ENTRY FROM THE FILE. GO BACK TO CHECK IF ANY OTHER
      C           ORDER IS ARRIVING.
```

```
      C
   65       3 IF(TNOW-TIME)4,5,5
   66       5 INVTRY(ITEM)=INVTRY(ITEM)+IQ(ITEM)
   67         KODE(ITEM)=1
   68         CALL REMOVE
   69         GO TO 3
   70       4 DO 2 I=1,NITMS
      C
      C.........FIND THE DEMAND FOR THE ITEM DURING THIS TIME UNIT. BRING DOWN
      C           THE INVENTORY BY THIS DEMAND TO GET THE INVENTORY AT THE END OF
      C           THIS TIME UNIT.
      C
   71         CALL DEMAND(S(I),KK)
   72         INVTRY(I)=INVTRY(I)-KK
      C
      C.........IF INVENTORY AT THE END OF  THE TIME UNIT IS ZERO OR NEGATIVE,
      C           INCREASE NOUT(I) BY ONE. OTHERWISE, ADD CAPACITY REQUIRED BY
      C           THIS ITEM TO CAP(J).
      C
   73         IF(INVTRY(I))8,9,9
   74       8 NOUT(I)=NOUT(I)+1
   75         GO TO 7
   76       9 CAP(J)=CAP(J)+W(I)*INVTRY(I)
      C
      C.........IF INVENTORY AT THE END OF THE DAY IS LESS THAN OR EQUAL TO THE
      C           RECRDER POIMT SCHEDULE AN ORDER. SET KODE(I)=2. FILE THE EVENT.
      C           RECRDER POINT SCHEDULE AN ORDER. SET KODE(I)=2. FILE THE EVENT.
      C
   77       7 IF (INVTRY(I)-IREORD(I))10,10,2
   78      10 MMM=KODE(I)
   79         GO TO (12,2 ),MMM
   80      12 Y=1.0
   81         KERL=KERLNG(I)
   82         DO 13 K=1,KERL
   83      13 Y=Y*RA(X)
   84         TL=(-AVTL(I)/KERLNG(I))*ALOG(Y)
   85         LEADTM=TL
   86         IF(TL-LEADTM-0.5)14,14,15
   87      15 LEADTM=LEADTM+1
   88      14 TL=LEADTM
   89         ATRIB(1)=TNOW+TL
   90         JTRIB(1)=I
   91         KODE(I)=2
   92         CALL ENTER(&18)
   93       2 CONTINUE
      C
      C.........AFTER SIMULATING FOR A PERIOD EQUAL TO NREPOT, CALL SUBROUTINE
      C           DIST WHICH PREPARES AND PRINTS OUT THE DISTRIBUTION OF CAPACITY
      C           UTILISATION.
      C
   94         CALL DIST(CAP,ND,NCELLS)
      C
      C.........CHECK IF SIMULATON HAS BEEN DONE FOR THE DESIRED NUMBER OF TIME
      C           UNITS. IF YES, CALCULATE THE PROBABILITY THAT AN ITEM WAS OUT
      C           OF STOCK. PRINT OUT THE DATA USED, INITIALAND FINAL INVENTORY.
      C
   95         IF(ND-NODAYS)37,38,38
   96      37 ND=ND+NREPOT
   97         M=M+NREPOT
   98         GO TO 39
```

```
 99       38 XY=NODAYS
100          DO 16 I=1,NITMS
101          XYZ=NCLT(I)
102       16 STCKOT(I)=XYZ/XY
103          PRINT 997
104          PRINT 996
105          PRINT 995,NITMS
106          PRINT 994,NODAYS
107          PRINT 993,NCELLS
108          PRINT 992,IZ
109          PRINT 991,IY
110          PRINT 990
111          PRINT 989,(IQ     (I),I=1,NITMS)
112          PRINT 988,(IREORD(I),I=1,NITMS)
113          PRINT 987,(AVTL   (I),I=1,NITMS)
114          PRINT 986,(KERLNG(I),I=1,NITMS)
115          PRINT 985,(S      (I),I=1,NITMS)
116          PRINT 984,(W      (I),I=1,NITMS)
117          PRINT 983,(NIVTRY(I),I=1,NITMS)
118          PRINT 982,(STCKOT(I),I=1,NITMS)
119          PRINT 981,(INVTRY(I),I=1,NITMS)
         C
         C.........IF ANY MORE SEED TO SET THE INITIAL INVENTORIES IS REMAINING, GO
         C           BACK AND START THE SIMULATION ALL OVER AGAIN.
         C
120          IF(KLM-NSEEDS)40,18,18
121       18 STOP
122          END

123          SUBROUTINE DIST(A,N,NCELLS)
124          COMMON IM,IMM,MX,MXX,MAXQS,MAXNS,NQ,MLC,MFE,MLE,ID,MAXNQ,ENQ,VNQ,Q
            1TIME,MFA,TIME,ITEM,IX ,TNOW
125          COMMON NSET(300),QSET(100),ATRIB(1),JTRIB(3)
126          DIMENSION      FREQ(50),PROB(50),CUMPRO(50) ,A(4000)
127
         C.........THIS SUBROUTINE DIVIDES THE 'N' VALUES OF THE VARIABLE 'CAPACITY
         C           USED' INTO 'NCELL' CELLS. PRINTS OUT THE DISTRIBUTION,MEAN,STD.
         C           DEVIATION,MINIMUM AND MAXIMUM CAPACITY USED AND INTERVAL SIZE.
         C
**WARNING**  BLANK CARD ENCOUNTERED
128        999 FORMAT(22X,'DISTRIBUTION OF CAPACITY UTILISATION AT ',I4,' TIME UN
              1ITS',/)
129        998 FORMAT(10X,'MID-VALUE OF CELL     FREQUNCY     PROBABILITY     END VA
              1LUE OF CELL     CUM. PROB.',/)
130        997 FORMAT(13X,F10.3,8X,F7.3,6X,F8.5,9X,F10.3,11X,F10.3)
131        996 FORMAT(25X,F8.3,3X,F8.3,5X,F8.3,4X,F8.3)
132        995 FORMAT(28X,'MEAN      STD. DEV.       MINIMUM       MAXIMUM',/)
133        994 FORMAT('1')
         C
         C.........DECIDE MINIMUM AND MAXIMUM CAPACITY USED.
         C
134          AMIN=1.0E75
135          AMAX=-1.0E75
136          DO 2 I=1,N
137          IF(A(I)-AMIN)4,5,5
138        4 AMIN=A(I)
139        5 IF(A(I)-AMAX)2,2,6
140        6 AMAX=A(I)
141        2 CONTINUE
         C
```

```
      C.....INITILISE FREQUENCIES AND CUMULATIVE PROBABILITIES.
      C
142           DO 7 I=1,NCELLS
143           CUMPRC(I)=0.
144         7 FREQ(I)=0.
      C
      C.....DETERMINE INTERVAL SIZE. INITILISE SUM AND SUM OF SQUARES.
      C
145           SINT=(AMAX-AMIN)/NCELLS
146           AV=0.
147           VAR=0.
      C
      C.....BRING UP SUM AND SUM OF SQUARES. PLACE THE VARIABLE IN ITS CELL.
      C        INCREASE THE FREQUENCY OF THE CELL.
      C
148           DO 10 I=1,N
149           AV=AV+A(I)
150           VAR=VAR+A(I)*A(I)
151           K=(A(I)-AMIN)/SINT+1
152           IF(K.GT.NCELLS)K=NCELLS
153        10 FREQ(K)=FREQ(K)+1.
      C
      C.....CALCULATE MEAN,STANDARD DEVIATION,PROBABILITY OF THE CELL,
      C        CUMLLATIVE PROBABILITY UPTO THE CEL AND PRINT THE RESULTS.
      C
154           STDEV=SQRT(ABS((VAR-AV*AV/N)/(N-1.)))
155           AV=AV/N
156           DO 12 I=1,NCELLS
157        12 PROB(I)=FREQ(I)/N
158           DO 13 I=1,NCELLS
159           DO 13 J=1,I
160        13 CUMPRC(I)=CUMPRO(I)+PROB(J)
161           PRINT 994
162           PRINT 999,N
163           PRINT 998
164           B=AMIN
165           C=AMIN-SINT/2.0
166           DO 14 I=1,NCELLS
167           B=B+SINT
168           C=C+SINT
169        14 PRINT 997,C,FREQ(I),PROB(I),B,CUMPRC(I)
170           PRINT 995
171           PRINT 996,AV,STDEV,AMIN,AMAX
172           RETURN
173           END

174           SUBROUTINE INIT
175           COMMON IM,IMM,MX,MXX,MAXQS,MAXNS,NQ,MLC,MFE,MLE,ID,MAXNQ,ENQ,VNQ,Q
             1TIME,MFA,TIME,ITEM,IX ,TNOW
176           COMMON NSET(300),QSET(100),ATRIB(1),JTRIB(3)
177           MXX=IM+2
178           MX=IM+1
179           MAXQS=ID*IMM
180           MAXNS=ID*MXX
      C        INITILISE FILE TO ZERO.
181           DO 1 I=1,MAXQS
182         1 QSET(I)=0.0
183           DO 2 I=1,MAXNS
184         2 NSET(I)=0
      C        INITILISE POINTERS IN NSET.
```

```
185           DO 3 I=1,ID
186           INDX=I*MXX
187           NSET(INDX-1)=I+1
188         3 NSET(INDX)=I-1
189           NSET(MAXNS-1)=8888
      C
      C          FIRST AVAILABLE COLUMN IS 1. FIRST AND LAST ENTRIES ARE ZERO.
      C
190           MFE=0
191           MLE=0
192           MFA=1
      C
      C          SINCE THE FILE IS EMPTY, TIME IS SET TO A LARGE QUANTITY.
      C
193           TIME=10000.
194           RETURN
195           END

196           SUBROUTINE REMOVE
197           COMMON IM,IMM,MX,MXX,MAXQS,MAXNS,NQ,MLC,MFE,MLE,ID,MAXNQ,ENQ,VNQ,Q
             1TIME,MFA,TIME,ITEM,IX ,INCW
198           COMMON NSET(300),QSET(100),ATRIB(1),JTRIB(3)
      C
      C.........THIS SUBROUTINE REMOVES FIRST ENTRY FROM THE FILE,
      C          CHOOSE FIRST ENTRY
      C
199           N=MFE
200           IF(MFE)1,1,2
201         1 TIME=10000.
202           GO TO 13
      C          TAKE OUT DESIRED QUANTITIES.
203         2 INDX=(MFE-1)*IMM
204           DO 3 I=1,IMM
205           INDX=INDX+1
206         3 QSET(INDX)=0.0
207           INDX=(MFE-1)*MXX
208           DO 4 I=1,IM
209           INDX=INDX+1
210         4 NSET(INDX)=0
211           INDX=MFE*MXX-1
      C          FIND NEXT ENTRY.
212           JL=NSET(INDX)
213           IF(JL-7777)5,6,6
      C          THIS IS THE ONLY ENTRY
214         6 MFE=0
215           MLE=0
216           TIME=10000.
      C
      C.........COLUMN FROM WHICH THE ENTRY WAS REMOVED BECOMES MFA. POINTERS
      C          ARE READJUSTED.
      C
217        10 INDX=N*MXX-1
218           NSET(INDX)=MFA
219           NSET(INDX+1)=9999
220           IF(MFA-8888)11,12,12
221        11 INDX=MFA*MXX
222           NSET(INDX)=N
223        12 MFA=N
224           RETURN
      C
```

```
      C.....ADJUST THE POINTERS. REDEFINE TIME AND ITEM.
      C
  225      5 INDX=JL*MXX
  226        NSET(INDX)=9999
  227        INDX=(JL-1)*IMM+1
  228        TIME=QSET(INDX)
  229        INDX=(JL-1)*MXX+1
  230        ITEM=NSET(INDX)
  231        MFE=JL
  232        GO TO 10
  233       13 RETURN
  234        END

  235        SUBROUTINE ENTER(*)
  236        COMMON IM,IMM,MX,MXX,MAXQS,MAXNS,NQ,MLC,MFE,MLE,ID,MAXNQ,ENQ,VNQ,Q
           1TIME,MFA,TIME,ITEM,IX ,TNOW
  237        COMMON NSET(300),QSET(100),ATRIB(1),JTRIB(3)
      C
      C.....THIS SUBROUTINE FILES AN ENTRY INTO THE FILE IN SUCH A MANNER
      C        THAT THE LOWEST VALUE OF TIME OF ARRIVAL OF ORDER GETS THE 1ST
      C        RANK AND SO ON.
      C        TEST AVAILABILITY OF COLUMN FOR STORAGE.
      C
  238        IF(MFA-ID)1,1,2
  239      2 PRINT 3
  240      3 FORMAT(//,' THE FILE IS FULL')
  241        RETURN1
      C        PUT ATTRIBUTE VALUES IN FILE. ITEM TO BE INSERTED WILL BEPUT IN
      C        COLUMN MFA.
  242      1 INDX=(MFA-1)*IMM
  243        DO 4 I=1,IMM
  244        INDX=INDX+1
  245      4 QSET(INDX)=ATRIB(I)
  246        INDX=(MFA-1)*MXX
  247        DO 5 I=1,IM
  248        INDX=INDX+1
  249      5 NSET(INDX)=JTRIB(I)
      C        MFEX IS FIRST ENTRY WHICH HAS NOT BEEN COMPARED WITH ITEM TO BE
      C        INSERTED. KNT IS A CHECK CODE. EQUALS 2 IF NO COMPARISON HAS
      C        BEEN MADE. EQUALS 1 OTHERWISE.
  250        MFEX=MFE
  251        KNT=2
      C        THE ROW ON WHICH RANKING IS TO BE DONE IS FIRST ROW IN QSET.THE
      C        SCHEDULED TIME OF ARRIVAL OF ORDER. FIND SUCCESSOR OF MFA.
  252        INDX=MFA*MXX-1
  253        NXFA=NSET(INDX)
      C        DESCIPLINE OF THE FILE IS FIFO. TRY INSERTING THIS ENTRY AT END
      C        OF FILE. LAST ENTRY IS MLEX. IF MLEX IS ZERO, FILE IS EMPTY.
  254        MLEX=MLE
  255        IF(MLEX)100,10,11
  256       10 INDX=MFA*MXX
  257        NSET(INDX)=9999
  258        ITEM=JTRIB(1)
  259        TIME=ATRIB(1)
  260        MFE=MFA
      C        THERE IS NO SUCCESSOR TO THIS ITEM INSERTED.
  261       17 INDX=MFA*MXX-1
  262        NSET(INDX)=7777
  263        MLE=MFA
      C        SET NEW MFA EQUAL TO SUCCESSOR OF OLD MFA. THIS NEW MFA HAS NO
```

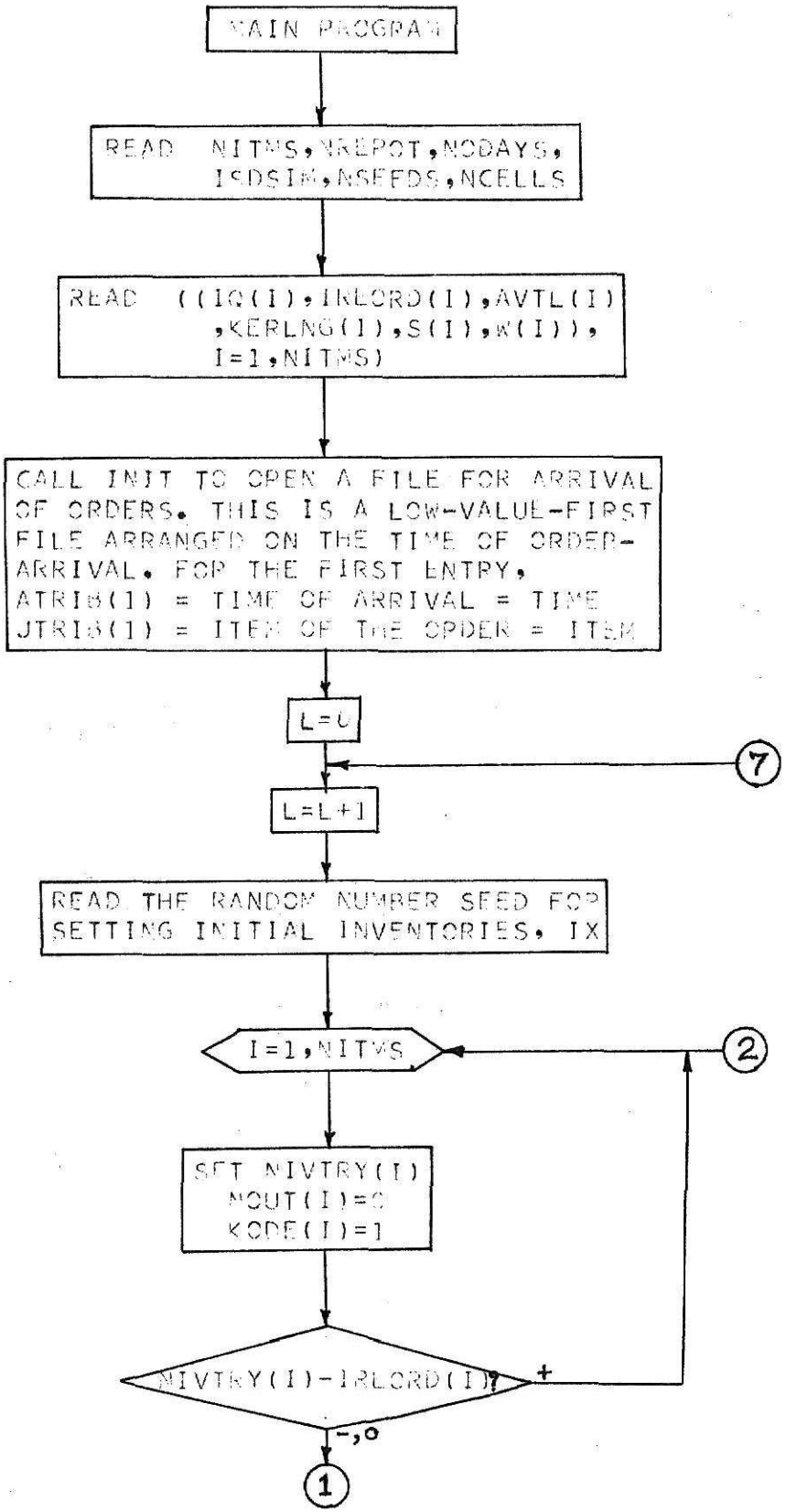
```
      C         PREDECESSOR.
264       14 MFA=NXFA
265          IF(MFA-8888)237,238,238
266       237 INDX=MFA*MXX
267          NSET(INDX)=9999
268       238 RETURN
      C         THE RANKING ROW IS FIRST IN QSET.
269       11 INDX1=(MFA- 1)*IMM+1
270          INDX2=(MLEX-1)*IMM+1
      C         TEST RANKING VALUE OF NEW ITEM AGAINST THAT IN MLEX.
271          IF (QSET(INDX1)-QSET(INDX2))12,13,13
      C         INSERT ITEM AFTER COLUMN MLEX. LET SUCCESSOR OF MLEX BE MSU.
272       13 INDX=MLEX*MXX-1
273          MSU=NSET(INDX)
274          NSET(INDX)=MFA
275          INDX=MFA*MXX
276          NSET(INDX)=MLEX
277          GO TO (18,17),KNT
      C         SINCE KNT=1, A COMPARISON HAS BEEN MADE, AND THERE IS A
      C         SUCCESSOR TO MLEX.THAT IS, MSU IS NOT EQUAL TO 7777. POINT
      C         COLUMN MFA TO MSU AND VICE VERSA.
278       18 INDX=MFA*MXX-1
279          NSET(INDX)=MSU
280          INDX=MSU*MXX
281          NSET(INDX)=MFA
282          GO TO 14
      C         SET KNT=1. TEST MFA AGAINST PREDECESSOR OF MLEX.
283       12 KNT=1
284          INDX=MLEX*MXX
285          MLEX=NSET(INDX)
286          IF(MLEX-9999)11,16,16
287       16 INDX=MFA*MXX
288          NSET(INDX)=9999
289          MFE=MFA
      C         SUCCESSOR OF MFA IS MFEX. PREDECESSOR OF MFEX IS MFA.
290          INDX=MFA*MXX-1
291          NSET(INDX)=MFEX
292          INDX=MFEX*MXX
293          NSET(INDX)=MFA
294          TIME=ATRIB(1)
295          ITEM=JTRIB(1)
296          GO TO 14
297       100 RETURN1
298          END

299          SUBROUTINE DEMAND(AV,NPOSN)
300          COMMON IM,IMM,MX,MXX,MAXQS,MAXNS,NQ,MLC,MFE,MLE,ID,MAXNQ,ENQ,VNQ,C
            1TIME,MFA,TIME,ITEM,IX ,TNOW
301          COMMON NSET(300),QSET(100),ATRIB(1),JTRIB(3)
      C
      C.....THIS SUBROUTINE GENERATES POISSON DISTRIBUTED DEMAND.
      C
302          NPOSN=0
303        1 Y=EXP(-AV)
304          X=1.0
305        5 X=X*RA( X)
306          IF(X-Y)3,4,4
307        4 NPOSN=NPOSN+1
308          GO TO 5
309        3 RETURN
```

```
310           END

311           FUNCTION RA( X)
312           COMMON IM,IMM,MX,MXX,MAXQS,MAXNS,NQ,MLC,MFE,MLE,ID,MAXNQ,ENQ,VNQ,Q
             1TIME,MFA,TIME,ITEM,IX ,TNOW
313           COMMON NSET(300),QSET(100),ATRIB(1),JTRIB(3)
      C
      C.........THE FUNCTION SUBPROGRAMME GENERATES A RANDOM NUMBER, UNIFORMALLY
      C           DISTRIBUTED BETWEEN ZERO AND ONE.
      C
314           IX=IX*65539
315           IF(IX)1,2,2
316         1 IX=IX+2147483647+1
317         2 YFL=IX
318           RA     =YFL*0.4656613D-9
319           RETURN
320           END

      $ENTRY
```

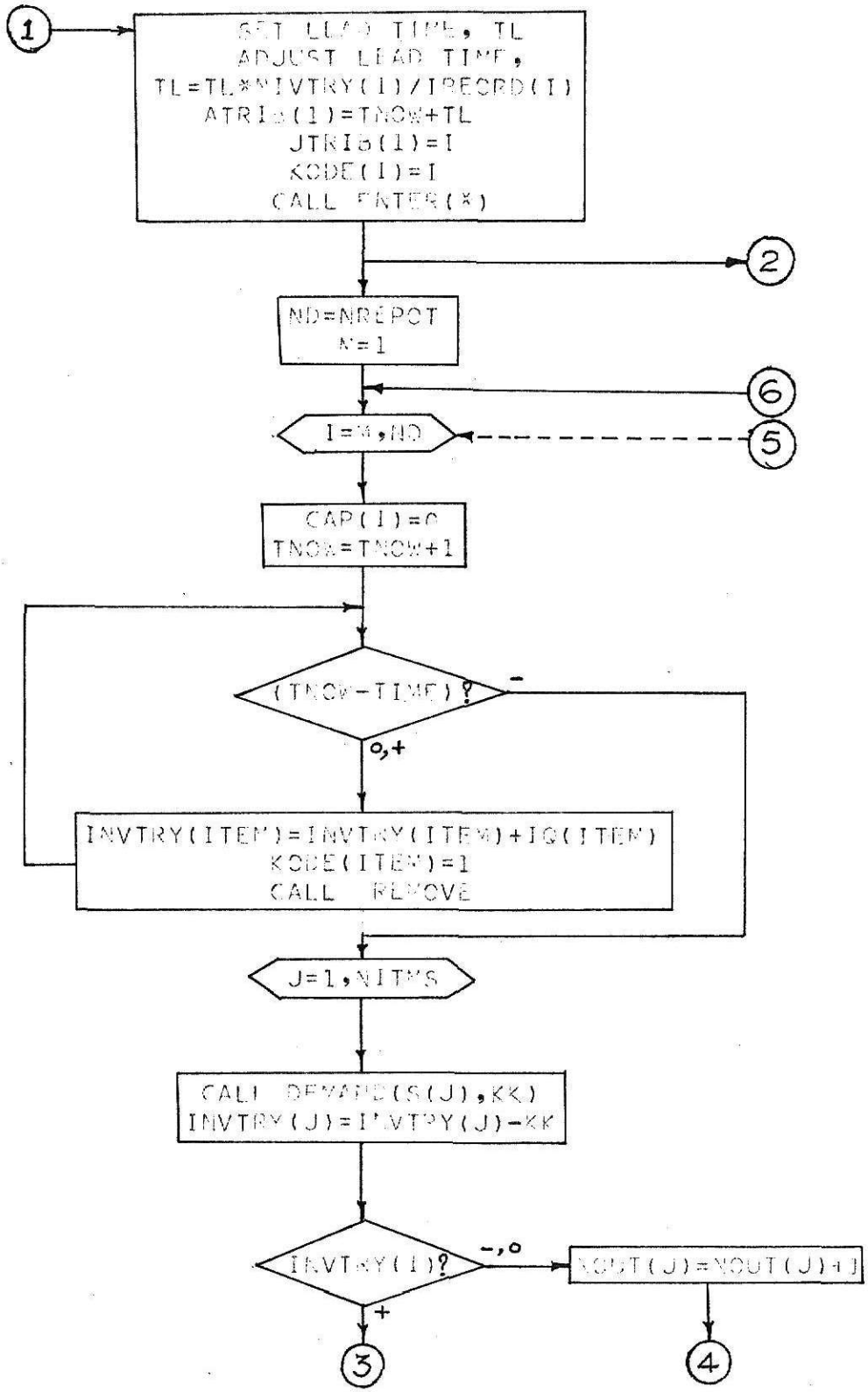
APPENDIX 8

FLOW CHARTS

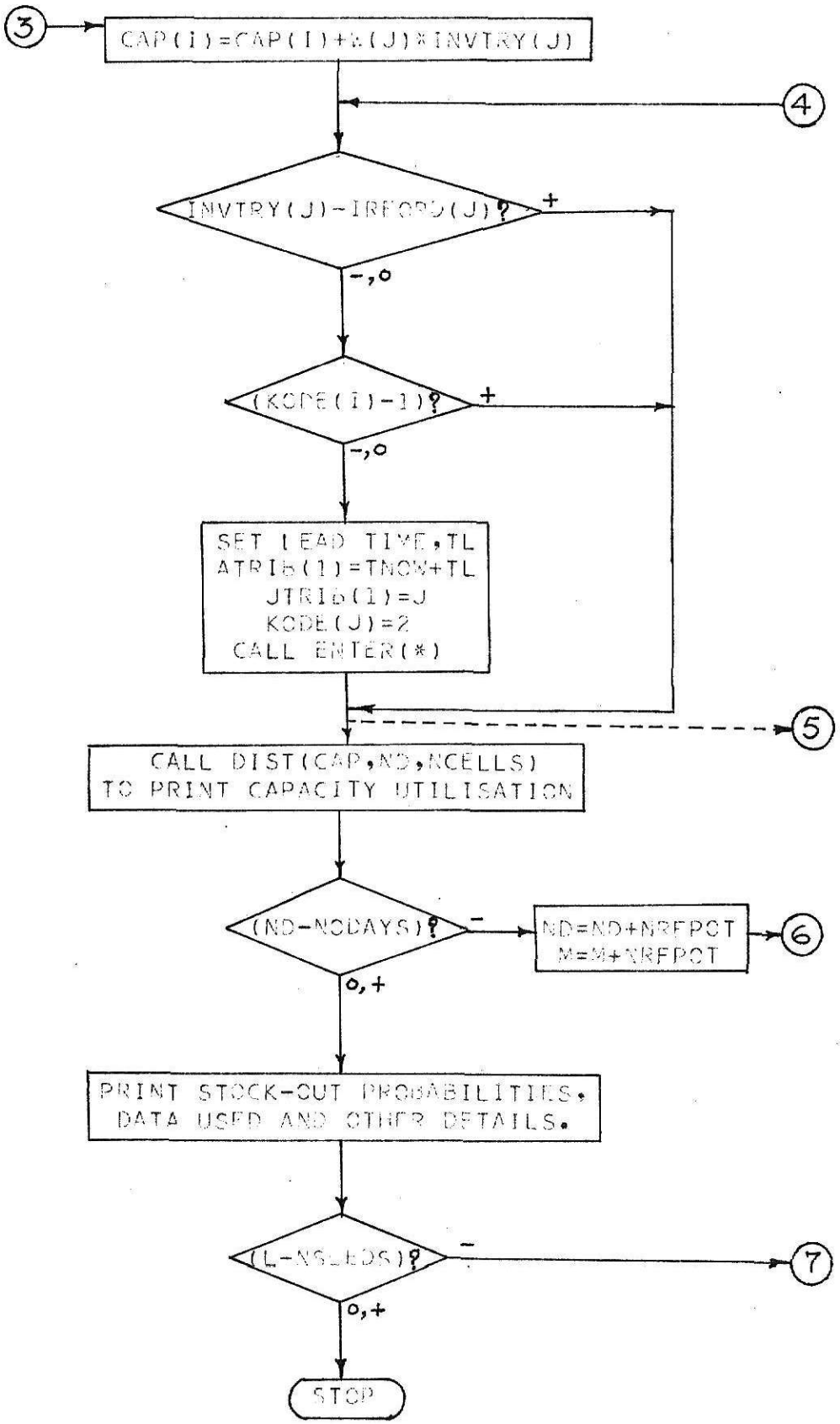
WAREHOUSE SPACE UTILIZATION



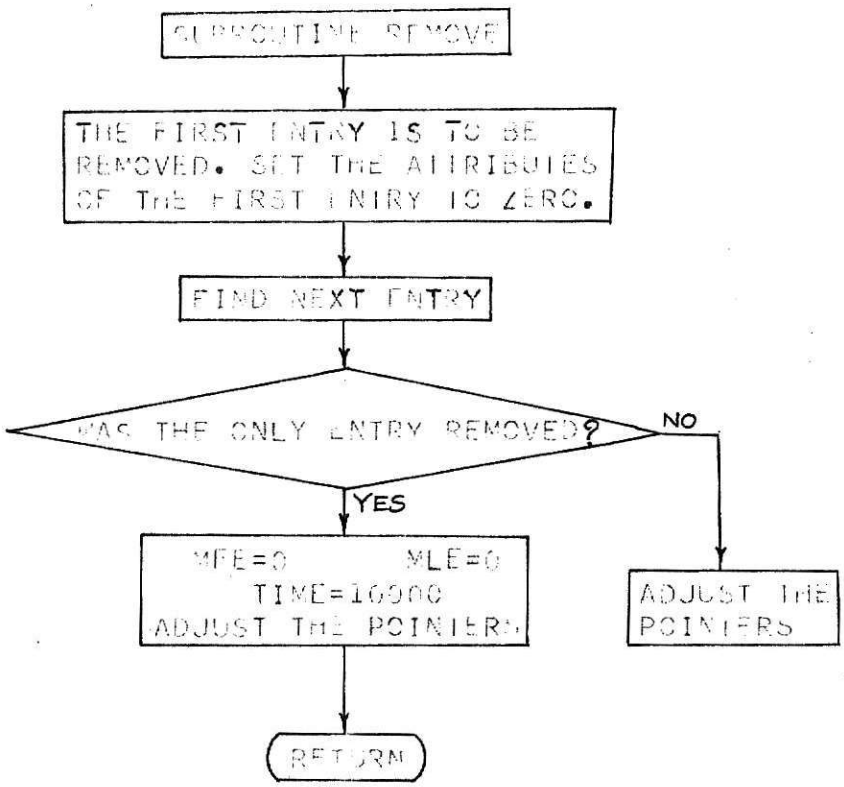
FLOW CHART OF MAIN PROGRAM



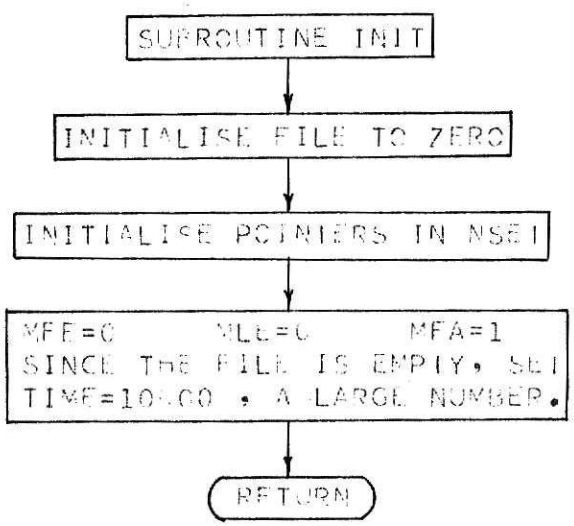
FLOW CHART OF MAIN PROGRAM (CONT'D.)



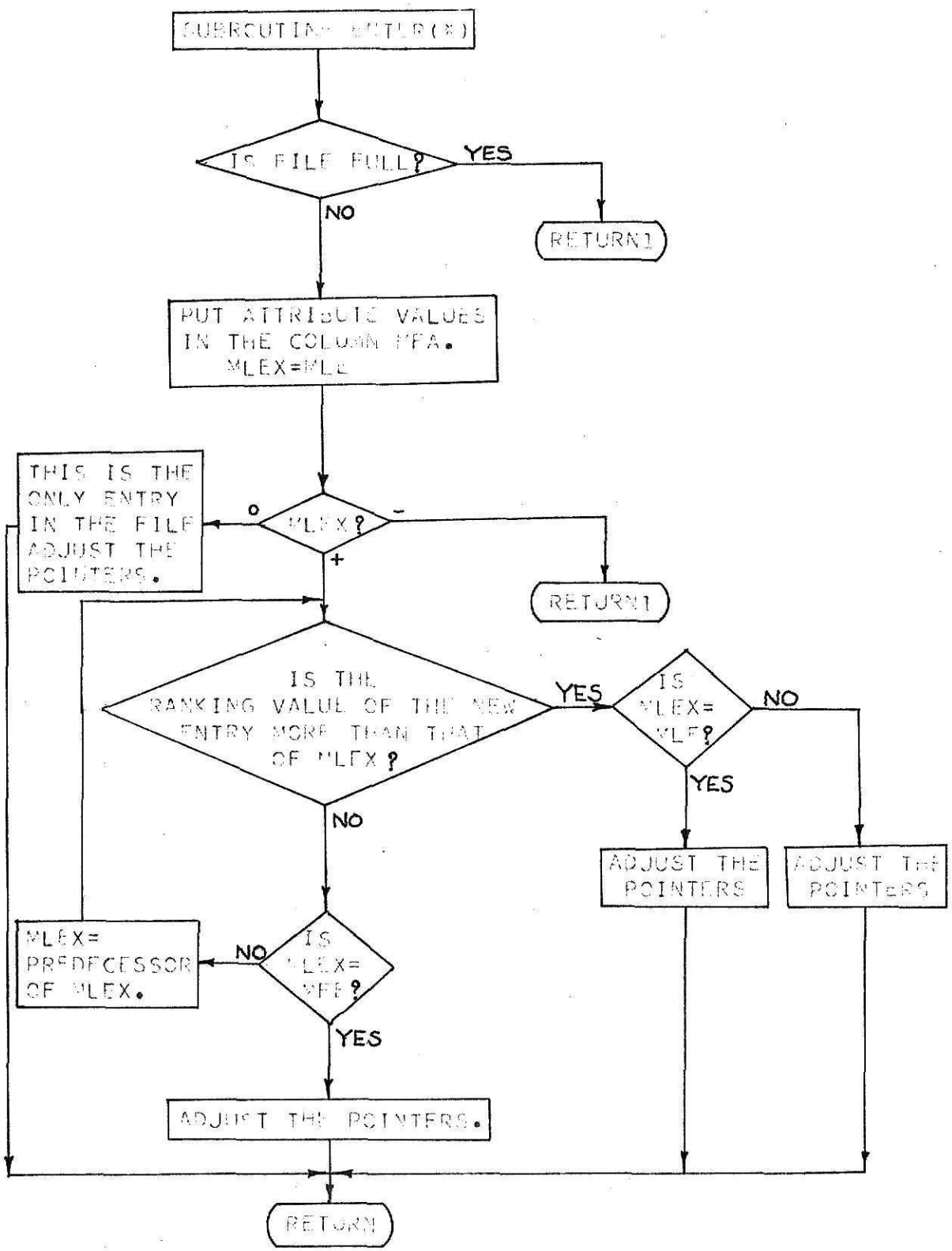
FLOW CHART OF MAIN PROGRAM (CONTD.)



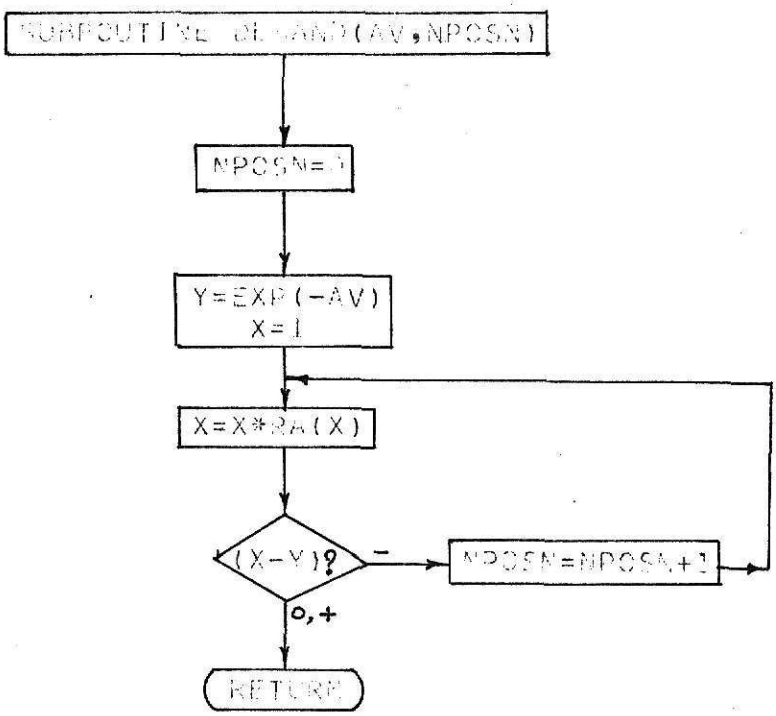
FLOW CHART OF SUBROUTINE REMOVE



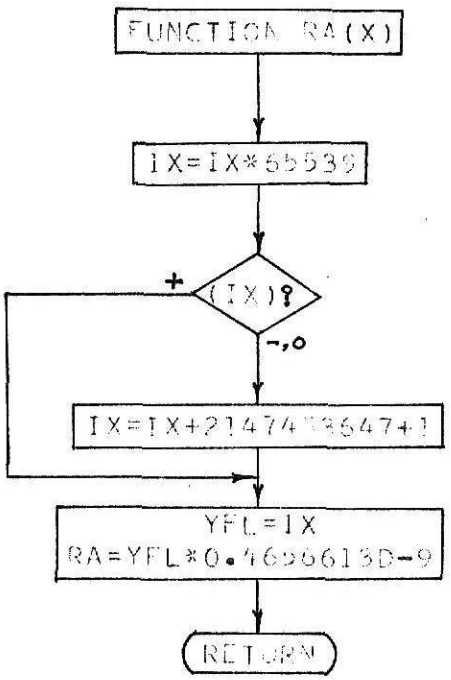
FLOW CHART OF SUBROUTINE INIT



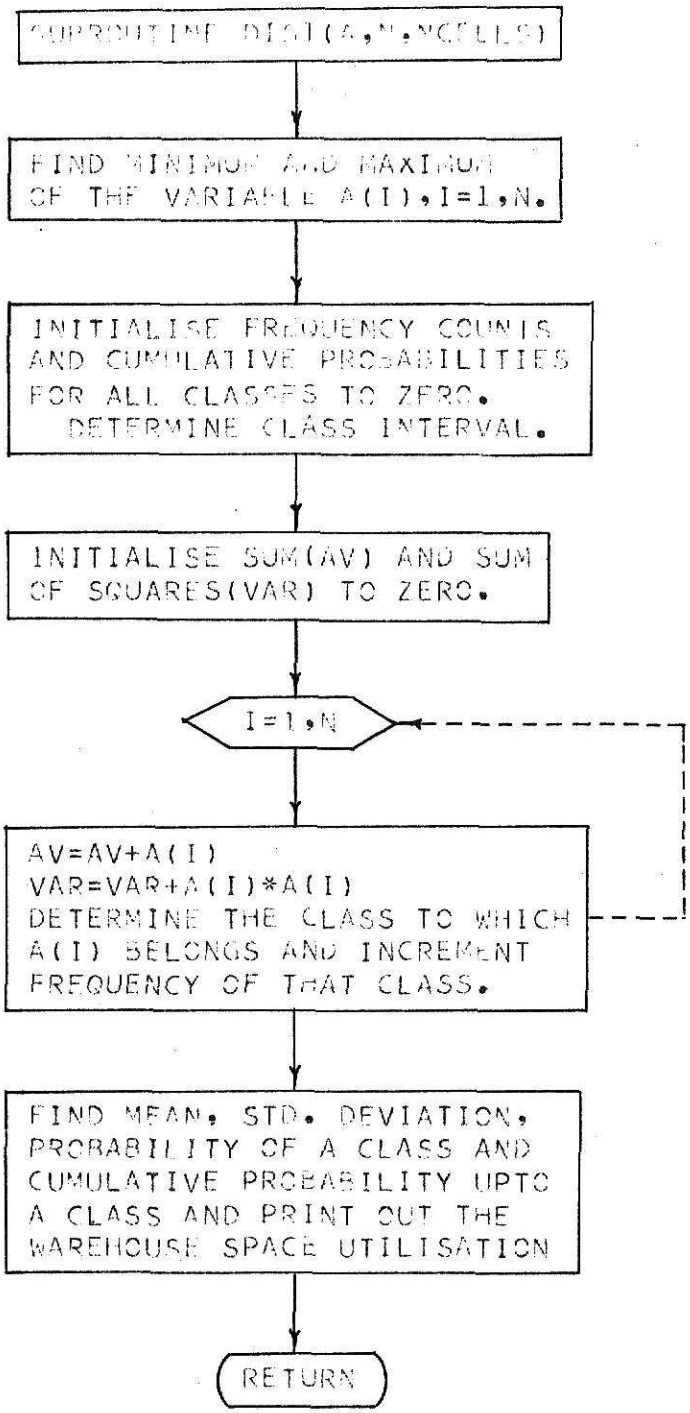
FLOW CHART OF SUBROUTINE ENTER(*)



FLOW CHART OF SUBROUTINE DEXAM



FLOW CHART OF FUNCTION RA(X)



FLOW CHART OF SUBROUTINE DIST

NUMERICAL OPTIMIZATION OF A STOCHASTIC INVENTORY SYSTEM UNDER CONSTRAINT

by

SURESH V. GADAD

B. Tech. (Metallurgical), Indian Inst. of Tech., Bombay, India, 1965

Diploma I. I. T. (I.E.), Indian Inst. of Tech., Madras, India, 1969

AN ABSTRACT OF A MASTER'S THESIS

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MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

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The inventory models discussed in the literature have been developed for simple inventory systems, in order to obtain the optimal inventory policies analytically. The real life inventory systems differ widely from these 'classical' inventory systems. The classical models are restricted to only 1) a few standard distributions of the demand rate, lead time and lead time demand, 2) limited forms of inventory holding and stockout costs and 3) simple and quite unrealistic constraints.

In this paper, a multi-item inventory problem of an automated warehouse with the available warehouse space as constraint has been dealt with. For this inventory problem, the lead time demand may follow any standard or nonstandard distribution. Even if this distribution is not available, it may be obtained by simulation, using the information about the lead time and demand rate. The inventory holding cost and stockout cost may be expressed in a number of alternative ways.

The absolute constraint, i.e., the maximum required space should be less than or equal to the available space, is incorporated in the inventory model by means of the Lagrange multiplier moethod. This constraint is more realistic than the average constraint, wherein the expected value of the resource used should not be more than a specified quantity, too often used in the literature. A numerical method based on numerical integration is used for finding the optimal inventory policy. For an assumed value of λ, the Lagrange multiplier, the inventory policy is obtained by an iterative method; also, the warehouse space required for this inventory policy is computed. For a given available warehouse space, the optimal λ^* and the corresponding solution, which is also the desired

constrained optimal inventory policy, are obtained by golden section or progressive linear interpolation/extrapolation method.

A constant-time-advance simulation, creating a file with pointer system to keep track of the arrival of orders, is carried out to study the warehouse space utilization. Knowing the actual space utilization, a more realistic space constraint, based on the risk of running out of capacity that the management is willing to take, can be developed.

The drawbacks of the numerical method developed in the paper have been brought out and the scope for further work is outlined.