CONSTRAINED AGGREGATE DYNAMIC INVENTORIES IN A BOTTLING PLANT - A CASE STUDY

by

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CHAPTER I

INTRODUCTION

1.1 Introduction

This report is a case study of prevailing production and inventory control practices for a bottling plant engaged in the production and retail sale of various soft drink flavours. Various case studies are available in the literature for particular problems in inventory control (1). This report narrates an overall view of the existing and proposed operations, practices, and policies in the bottling plant. The existing system was studied in relation to various problems such as identifying the constraints, determining the related penalty costs and suggesting modifications to lower the operating costs by reducing inventory costs and the cooling costs.

In certain situations the existance of constraints make the implementation of the optimum policies impracticable. Violation of the constraints, in general, increases the operating costs of the firm. This study discusses the effects of warehousing constraints on the inventory costs in the existing system. The cost of slackening the storage constraint has been economically justified through the resulting savings in inventory costs.

In the absence of reliable parameters for the inventory problem formulations, operations research techniques for optimization can be viewed as guidelines for achieving efficient production and inventory control. The results of the study are based on numerous simplifying assumptions and approximate parameter values. The results of this study should, therefore, be viewed as recommendations rather than the exact solution to the problems in the existing system.

Production and inventory control is among the widely discussed topics in operations research. Methods for solving constrained inventory problems (3), (6), (9), (10) and the evaluation of inventory costs due to constraints (3) are frequently discussed in the literature. Most of the books on this subject include discussion of the methods for solving inventory problems with well behaved stochastic models for demand distribution. The methods for solving the dynamic inventory problems with uncertain demand distribution are presented by Starr and Miller (10).

1.2 Objective

This report is a preliminary study of production and inventory control practices in a bottling plant. This report studies the application of operations research principles to modify the existing practices in the plant.

This study serves many purposes, namely

1) To recommend improvements in the existing system in order to reduce the operating costs of the firm,
2) To give an insight into the importance of analytical techniques in daily decisions,
3) It would lead the firm to undertake similar studies concerning other problem areas in the system.

The principal aspects discussed in this report are;

1) The study of existing operations, processes, and constraints and the bottlenecks therein,
2) The determination of demand distribution, lead time, and the relevant inventory costs in the inventory system,

3) The recommendation, based on the principles of operations research, for modifications in the production and inventory control practices for achieving savings in the operating costs,

4) The determination of a penalty cost due to the presence of constraints on the inventory system,

5) To avoid freezing the bottle contents by raising the cooler temperatures and compensate for the reduction of cooling rates by introducing a forced draft in the coolers.

The data necessary for this study is adapted from the B. S. Factory, Amravati, India. The author hopes that this study would help the firm in the lowering of operating costs.

CHAPTER II

PROCESS DESCRIPTION

2.1 Existing Processes

2.1.1 General

The B. S. Factory is engaged in the production and retail sale of nine different soft drink flavours. The firm owns a factory where all soft drinks are bottled and warehoused. The firm also owns four retail shops in the city where the drinks are sold to the customers.

The sale of all flavours is affected by seasonal fluctuations. For all products the sale patterns exhibit a significant high during the summer season, March through June. In 1971 the daily total sale during the summer season averaged slightly more than 2.5 times the daily total sale during the winter and rainy seasons (Table 1). For achieving a better control over the operations the management of the firm therefore divides the fiscal year into two periods, summer and nonsummer seasons. Talbe 1 displays the statistics concerning the seasonal sale patterns during year 1971. The process chart and Flow Diagram shown in Figures 1 and 2 respectively represent the processes and the operations undertaken in the bottling plant.

2.1.2 Products

All drinks are bottled in various sizes, shapes and brands of bottles. Table 1 lists the types of bottle used for bottling the flavour. The drinks carbonated at high pressures are bottled in type A bottles while the other drinks filled at little or no pressure are bottled in type B, C or D

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

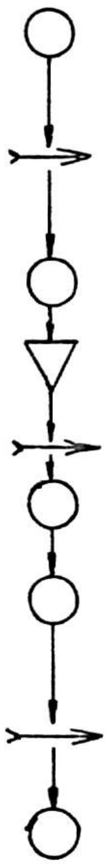
THIS IS AS RECEIVED FROM CUSTOMER.

Table 1

Seasonal Sale Patterns

Product	Bottle Type	Daily Sale in Cases					
		Summer Season			Winter and Rainy Seasons		
		Mean	Std. Dev.	Range	Mean	Std. Dev.	Range
Soda	A	297.20	102.64	440	132.70	49.79	250
ICS	B	21.58	12.53	49	3.22	2.34	12
Lemon	B	15.79	10.27	43	1.90	1.72	9
Orange	C	13.20	9.35	103	2.05	1.66	35
LO	D	6.06	3.68	15	1.57	1.21	7
Mangola	B	2.56	2.43	21	*	*	*
GF	B	1.49	1.07	7	*	*	*
Ginger	B	1.28	1.55	4	0.64	3.00	3
Cola	B	0.14	0.41	2	*	*	*

* Indicates the product not sold during winter and rainy seasons.

SYMBOL	DESCRIPTION
	Empty (washed) bottles in cases are stored in the factory warehouse. Cases of empty bottles are transferred to bottling machines for filling. Bottles are filled with drinks and sealed. Filled bottles of different flavours are stored in the warehouse. Cases are delivered to shops as per requisition . Bottles in cases are cooled in the cooler at the shops. Cold drinks are served to the customers. Unsatisfactorily cooled drinks are served with crushed ice. Empty bottles are returned to the warehouse whenever orders are delivered to shops. Empty bottles are washed and transferred to warehouse .

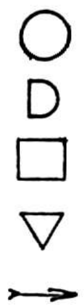
SYMBOL	
	Operation Delay, Temporary Storage Inspection Storage Transportation

Fig. 1. Process Chart of Operations in Existing System.

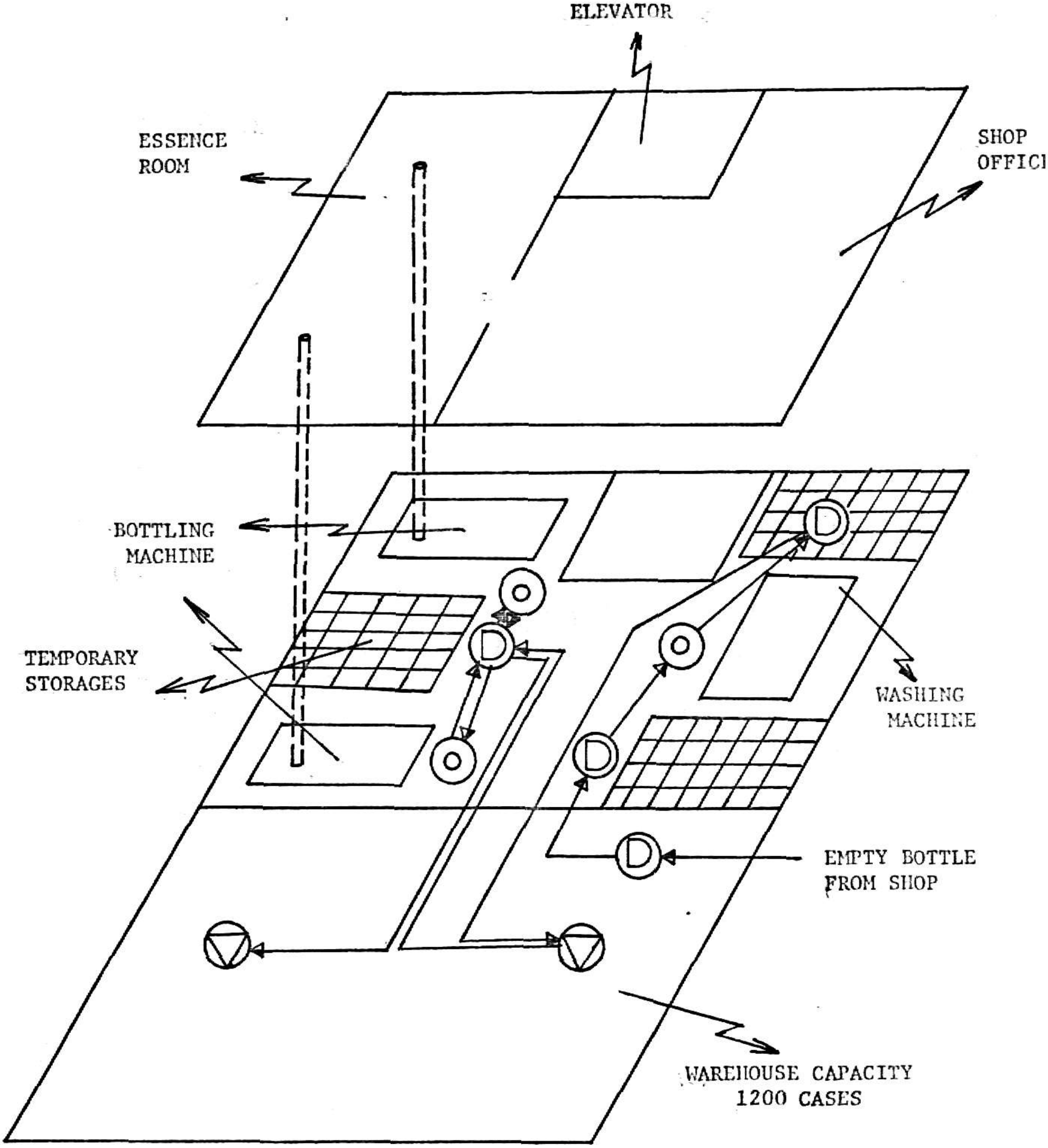


Fig. 2. Flow Diagram for Manufacturing Processes

bottles. All drinks are sold to customers in the shops and are served in drinking glasses.

Because of the low sales experienced during winter and rainy seasons in the past years, the flavours G.F. and C. Cola are manufactured only during the summer season. Another flavour, Mangola, has a shelf life of one week and is not bottled during the winter and rainy seasons due to nonavailability of its raw materials.

2.1.3 Ordering System

Delivery orders are placed by the shop managers on the factory warehouse. In the existing system only one daily delivery order is allowed during the winter and rainy seasons. Multiple daily orders can be placed during the summer season. Multiple daily deliveries are possible. Lead time for the delivery orders averages three hours.

"Emergency delivery orders" are placed by the shops upon a recognition of a stockout of one or more flavours. These orders are given special considerations at the factory. If enough stock is available at the warehouse, a special delivery is arranged, otherwise an emergency production order is placed by the production manager. The emergency production order, in general, involves 30 minutes of machine downtime due to the new machine set up required. However, insufficient warehouse inventories required to meet an ordinary delivery order do not trigger an emergency production order. "Back orders" are not processed. The undelivered portion of an order is added to the next delivery order placed by that shop.

The firm's shipping clerk keeps a record of the warehouse inventories and notifies the production manager when the stock levels are low. The production manager then schedules the production order. In the existing system no qualitative calculations are made towards the reduction of the firm's operational costs through an optimum production planning. The current production policy tends to produce smaller batches and results in frequent set up.

2.1.4 Manufacturing Processes

All soft drinks are bottled at the factory working a twelve hour shift. The shops also work on the twelve hour shift basis. The entire production process can be classified broadly into five major operations.

2.1.4.1 Essence Preparation

An advanced order is placed for the preparation of the essences. This operation involves mixing the extracts with a sugar solution and other additives in the prespecified proportions. In the existing system the essences are prepared during the set up time to meet the requirements of the production batch. The amount of essence prepared at one time is limited by its shelf life of 3 days. Since only one person is engaged for the task, the essences for more than one flavour cannot be prepared simultaneously.

The essence preparation operation does not require a finite lead time and averages 20 minutes for each flavour.

2.1.4.2 Machine Set Up

Any production change on either of two bottling machines requires 30 minutes for a machine set up, which normally includes water flow regulation from softeners, essence fill up, periodic maintenance and five minutes of early production wastage. All machine set ups are supervised by a set up man. Since 30 minutes of the production loss is costly the number of set ups should be minimized.

2.1.4.3 Bottling Operation

The firm owns two semiautomatic bottling machines, each with a production capacity of 30 two dozen cases per hour. Each machine is attended by an operator and his helper. The bottling operation includes filling the bottles with the essence and water at a carbonation pressure and sealing the bottles. Filled bottles are transferred in cases to the factory warehouse.

2.1.4.4 Bottle Washing

Cases of empty bottles are collected whenever the deliveries are made to the shops. The empty bottles are washed in a semiautomatic washing machine with a capacity of 150 cases per hour. The bottles are washed by a pressurized soap solution, cleaned against revolving brushes and then rinsed in clean water tanks. The clean bottles are stored in cases in the factory warehouse till requisitioned for rebottling. Proper arrangements are made to ensure cleanliness and sterility of the stored washed bottles for at least two weeks. For sanitary reasons the unwashed bottles are not stored in the warehouse although this would facilitate feeding freshly cleaned bottles to the bottling machines directly from the washer.

2.1.4.5 Factory Warehouse

The filled and empty bottles are stored in cases in a factory warehouse with a storage capacity of 1200 cases. The firm's shipping clerk keeps a record of the warehouse inventories and is responsible for arranging the deliveries to the shops.

2.1.5 Cooling System

Natural draft air cooled chambers are used in each shop to cool the soft drinks for sale. The coolers are currently maintained at 20°F instead of a desired range of 40° to 45°F. The low cooler temperatures provide higher cooling rates which allow a higher turnover in the coolers during the summer season. The firm has an option of serving the unsatisfactorily cooled drinks with crushed ice. Using crushed ice is more expensive and undesirable bacause of the customer dissatisfaction involved.

Table 2 represents the existing cooling capacities in the shops.

2.1.6 Storages in Shops

Each shop has provisions for storing the cases of filled and empty bottles. The delivered cases are unloaded in this storage space before loading them into coolers. Table 2 lists the existing storage capacities in the shops.

The preceeding section briefly outlines the major operations and policies in the existing system. Even though the description is incomplete, the major problem areas and bottlenecks in the existing system can be identified after a careful study.

2.2 Problems in Existing System

2.2.1 Emergency Production Orders

The emergency production orders involve a high set up cost due to the production downtime. The firm's operational costs can be reduced by minimizing the frequency of such machine set ups. In the absence of a known demand distribution, the desired optimum can be achieved by fixing a service policy, i.e., by choosing a low allowable probability of the stockout at the warehouse. This policy should practically eliminate high additional costs of the emergency production orders.

2.2.2 Cooling System

All coolers are currently maintained at a very low temperature during the summer season to cool the high selling flavours satisfactorily. This

Table 2

Cooling and Storage Capacities in Shops

Shop	Cooler Capacity in cases	Storage Capacity in cases
1	140	90
2	80	144
3	90	108
4	80	50
Total	390	392

practice freezes some low selling soft drinks. The frozen drinks are difficult to pour into glasses. Prolonged freezing may result in bottle breakage inside the coolers. Maintaining low temperatures in the cooler is costly. The low cooler temperatures are therefore undesirable since they introduce serious constraints such as shelf time in the coolers. To decrease the cost of cooling, the management of the firm suggests increasing the ambient cooler temperature to 30° - 35°F which is above the average freezing temperature of the soft drinks, 28.5°F. The corresponding reduction in the cooling rate can be compensated by introducing a forced draft system in the coolers. A detailed heat transfer investigation should be conducted to justify the increase in the cooler temperatures.

2.2.3 Some Constraints

2.2.3.1 Number of Bottles

Various sizes, shapes and brands of bottles are currently used to bottle different soft drinks (Table 3). The brand on the bottles identifies the flavour in the bottle. Bottle type B, C and D are similar in shape and size but possess different brands. These bottles are cheaper than type A bottles and are designed to stand only light pressures. Type A bottles are expensive and are designed to stand high pressures. Since the existing system is seriously constrained due to the number of each bottle type, the firm suggests using only type A and type B bottles. This involves reclassifying type C and type D bottles as type B, i.e., ignoring the brands on the bottles. Since the customers never handle the bottles, the problems such as developing the customer's attitude towards reclassification of bottles do not exist. This practice should reduce the number of constraints. Table 3 represents the existing and proposed number of bottles in the system.

Table 3

Number of Bottles

Product	Bottle Type	Cases of Bottles	
		Existing	Proposed
Soda	A	1250	1250
ICS	B	315	485
Lemon			
Ginger			
GF			
Mangola			
C Cola			
Orange	C	85	
LO	D	85	
Total		1735	1735

2.2.3.2 Storage Constraints

The existing production policy of the firm hints at the presence of severe storage constraints in the warehouse and the shops. These constraints should be considered in the formulation of production and inventory control policies. Table 2 lists the storage capacities in the shops.

Acquisition of additional warehouse space by building and/or renting a warehouse is recommended. Based on the floor area available, an additional warehouse capacity of 1800 cases can be built within the factory premises. Any additional storage capacity must be rented at a higher cost and at a distance from the factory. The effects of the increase in the storage costs on the inventory problem formulation are discussed in Chapter 4.

2.2.3.3 Production Rate on Bottling Machines

Both bottling machines have a production capacity of 30 cases per hour. Because of its low production capacity one bottling machine is engaged 82.5% of a 12 hour working day to meet the average daily summer demand of the highest selling flavour, Soda (Table 4). A finite replenishment rate inventory model should be considered for this product. Table 4 represents the percent of a working day during which both machines would be busy satisfying the average daily summer demand of all flavours. Figure 3 is a graphical representation of the data in Table 4.

The graph in Figure 3 indicates that one bottling machine can be utilised 82.5% on a twelve hour working day basis to produce the highest selling flavour, Soda. The second machine would then be utilised 77.5% on a eight hour working day basis for bottling the other eight flavours.

Table 4

Percent Working Day Busy

Product	Percent*	Product	Percent*
Soda	41.250	Mangola	0.356
ICS	2.995	GF	0.178
Lemon	2.190	Ginger	0.207
Orange	1.842	C Cola	0.019
LO	0.842		
		Idle	48.88

* Excluding the set up time involved.

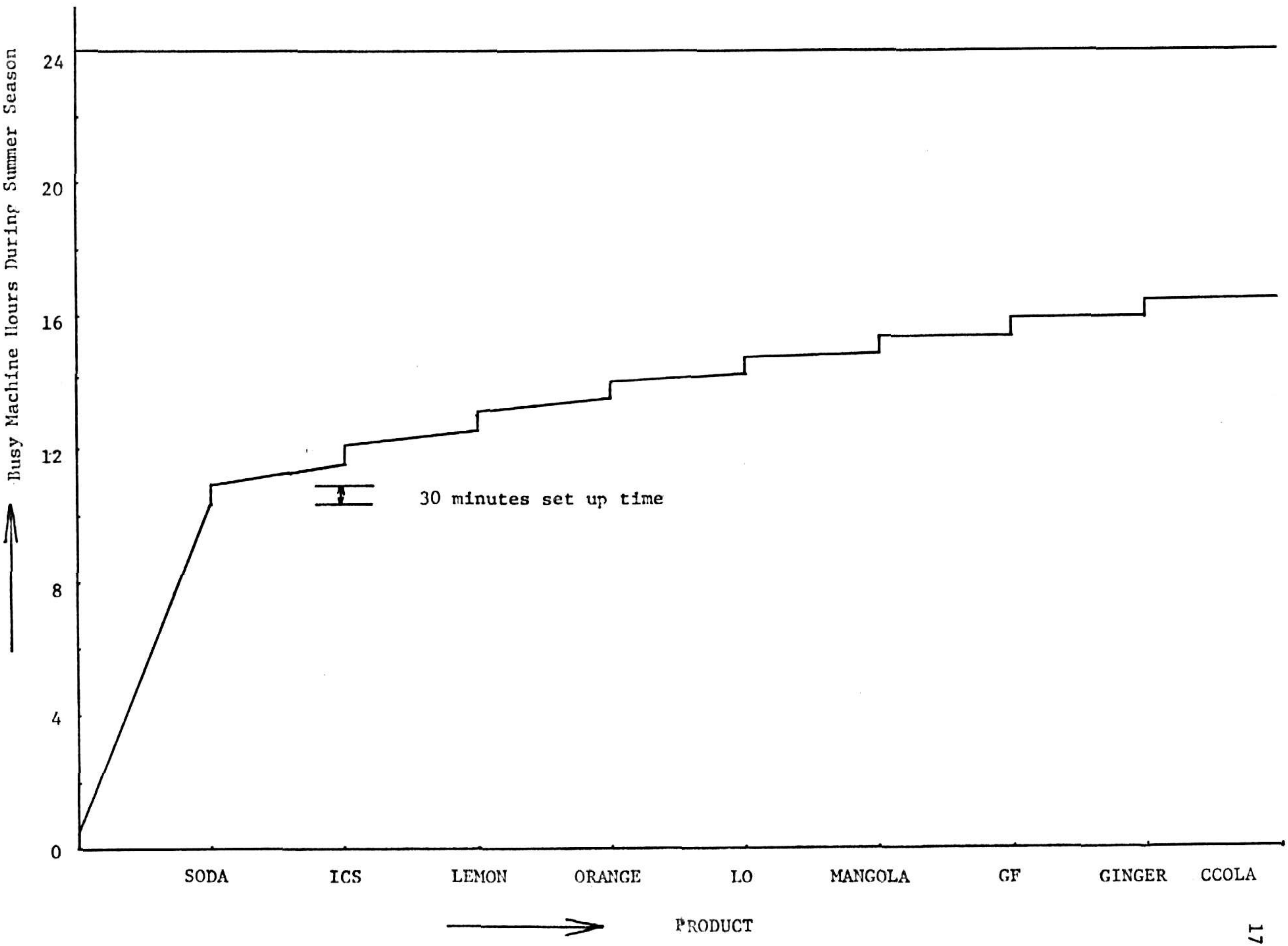


Fig. 3. Busy Machine Hours

2.2.4 Acquisition of Raw Material

Essences form an important portion of raw materials, partly because of their high acquisition costs. The consumption of the essences is the highest during the summer season, but they are cheaper and available with discounts during the winter and rainy seasons. An efficient acquisition policy would lower the firm's operating costs. The design of an optimum policy, though intresting, shall not be dealt here due to the lack of adequate data.

2.2.5 Working Hours

Reduction of factory hours from 12 hours to 8 hours per day is a long term goal of the firm. This change would require relaxation of many constraints and further capital investments. A series of studies would be required for an efficient conversion to an eight hour working day system.

The above discussion describes many phases of operations where modifications are desirable. The following chapters discuss a few of the modifications explicitly.

CHAPTER III

TIME FACTOR IN COOLING

3.1 Introduction

The soft drinks are cooled in natural draft air cooled chambers before serving to the customers. To cool the high selling flavours satisfactorily all coolers are currently maintained at 20°F during the summer season. The low cooler temperatures tend to freeze the low selling flavours. Maintaining low temperatures in the coolers is expensive. The firm has, therefore, suggested an increase in the cooler temperature to 34°F to reduce the cost of cooling and to avoid undesired freezing. The corresponding decrease in the cooling rate can be compensated by introducing a forced draft in the coolers. This chapter summarizes the heat transfer investigation conducted to justify the temperature raise in the coolers by comparing the cooling time at the two temperatures with and without introducing the forced draft in the coolers. All formulae, empirical relations and the value of physical properties are adopted from the books "Principles of Heat Transfer" by Frank Kreith (7) and "Heat Transmission" by W. H. McAdams (8).

3.2 Assumptions

The following simplifying assumptions are made to determine the cooling time conveniently.

1) The air temperature in the coolers is uniform and constant at 34°F.
2) All drinks are at a constant initial temperature of 95°F when loaded into the coolers.

3) Drinks are served to the customers at 38°F.

4) Freezing temperature for all soft drinks is constant at 28.5°F.

5) The temperature difference between the surface of the bottles and the surrounding air is not large enough to cause changes in the local properties of air.

6) All type A and type B bottles are identical and have the following physical properties.

Type A bottles

(a) External diameter = 0.1849'.

(b) Thickness of glass wall = 0.020'.

(c) Volumetric capacity = 10 fluid ounces.

(d) Mass of an empty bottle = 1.30 lbm.

(e) Equivalent cylindical height of the bottle = 0.51'.

Type B bottles differ in the following respects.

(a) External diameter of bottles = 0.1762'.

(b) Thickness of glass wall = 0.01562'.

(c) Mass of an empty bottle = 1 lbm.

7) The physical properties of all soft drinks are as follows.

(a) Specific gravity = 1 lbm./cu. ft.

(b) Specific heat = 1 Btu /lbm.°F.

(c) Thermal conductivity = 0.332 Btu /hr. ft. °F.

8) Constant volumes equal to 10 fluid oz. are filled in each bottle.

3.3 Heat Transfer Coefficient

The convective heat transfer coefficient between a surface and a fluid is computed by the following relation.

$$q = \bar{h}_c . A_s . \Delta T$$

where q = Rate of convective heat transfer, Btu /hour;

A_s = Surface area across which heat flows, sq. ft.;

ΔT = Temperature difference between the surface and the fluid, °F;

$\bar{h}_c$ = Average unit thermal convective conductance or convective heat transfer coefficient, Btu /hr. sq. ft. °F.

The rate of heat transfer and hence the time required to cool the drinks depends on the value of convective heat transfer coefficient between the surface of the bottle and the air in the cooler. For a free convection, the value of the heat transfer coefficient depends on the type of air flow over the bottle surface. The air flow can be laminar or turbulent depending on the value of Grashof number. For a laminar flow the Grashof number should not exceed 10^8. Grashof number, Gr, is computed by the following relationship.

$$Gr = \frac{\ell^2 g \beta (T_0 - T_\infty) L^3}{\mu^2} \qquad (7, \text{ p. } 335)$$

where ℓ = Density of air, lbm./cu. ft.;

g = Gravity force, ft./sec^2;

β = Coefficient of thermal expansion of air, 1/°F;

μ = Viscosity of air, lbm./ft. sec.;

L = Length of the cylindrical body, ft.;

$T_0 - T_\infty$ = Temperature difference between the surface of the cylindrical bottle and air, °F;

= 95° - 34°

= 61°F.

From the tables of physical properties (7, p. 595-599),

$$\frac{g\beta\ell^2}{\mu^2} = 3.16 \times 10^6 \text{ } 1/°\text{F cu. ft. at } 34°\text{F.}$$

Substitution of the parameter values yields

$$Gr = 0.256 \times 10^8.$$

Since $Gr<10^8$, the air flow over the bottle surface is laminar. For a laminar flow around a vertical cylinder such as a bottle the average value of the Nusselt number $\overline{Nu}_L$ is computed by the following empirical relation.

$$\overline{Nu}_L = \frac{\overline{h}_c . L}{k} = 0.48 \, (Gr_L)^{1/4}, \quad (7, \text{ eq. 7-19, p. 337})$$

where k = Coefficient of thermal conductivity of air at 34°F;

= 0.014 Btu /hr. ft. °F;

L = Significant cylindrical length of the bottle;

= 0.51'.

The empirical relation gives

$\overline{Nu}_L = 34.1$

$\overline{h}_c = 0.935$ Btu /hr. sq. ft. °F.

That is, the convective heat transfer coefficient between the surface of the bottle and the air in the cooler = 0.935 Btu /hr. sq. ft. °F.

3.4 Cooling Time in Proposed System

3.4.1 For Type B Bottles

The calculations for the determination of the cooling time is greatly simplified if the internal resistance of the filled bottles is negligible. A temperature gradient exists across the body with high internal resistance. A measure of the relative importance of thermal resistance within a solid body is the ratio of internal to external resistance of the body. A corresponding dimensionless ratio is the Biot number. The Biot number, Bi, is defined by the equation,

$$Bi = \frac{\bar{h}_c L_e}{k_s} \qquad (7, p. 128)$$

where k_s = Thermal conductivity of the filled bottle, Btu/hr. ft. °F;

L_e = Characteristic length dimension of the glass bottle,

= Volume/Surface area of the bottle,

$= \frac{\pi}{4} D^2 L \; / (2 \times \frac{\pi}{4} D^2 + \pi DL)$ ft.

where D = External diameter of the bottle, ft.;

= 0.1762'.

L = Equivalent length of the bottle, ft.;

= 0.51'.

or L_e = 0.0405'.

For perfectly cylindrical bodies the error introduced by the assumption that the temperature at any instant is uniform across the cross section of the body will be less than 5% when Bi < 0.1. For a composite body such as a filled bottle, k_s is unknown and difficult to compute. However, independent Biot tests can be easily conducted for two imaginary bodies made of glass and soft drink respectively. If Bi < 0.1 for both bodies then the internal resistance of the composite body can be neglected.

Case A : For a glass body -

k_s = 0.45 Btu/hr. ft. °F;

or, Bi = 0.0842, upon substitution.

< 0.1.

Case B : For a liquid body -

k_s = 0.340 Btu/hr. ft. °F at the average film temperature equal to 50°F.

or, Bi ≃ 0.1, upon substitution.

Since Biot number criterion is satisfied in both cases, the internal resistance or the temperature gradients inside glass bottle can be safely neglected. Equation (a) hence dictates the time required to cool the drinks to a temperature T °F (7, p. 128-9).

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-(\bar{h}_c A_s / c \ell V).\theta} \qquad \text{(a)}$$

where T_∞ = Temperature of the surrounding media, air, °F;

T_o = Initial temperature of soft drinks, °F;

θ = Time required to cool the soft drink bottle to a temperature T °F;

c = Specific heat of the composite body, Btu/lbm. °F;

ℓ = Density of the composite body, lbm./cu. ft.;

V = Volume of the bottle, cu. ft.;

A_s = Surface area of the bottle, sq. ft.

The expression ($c\ell V$) represents the thermal capacity of the composite body and can be calculated by using the following relation.

$$(c\ell V)_{\text{composite body}} = (c\ell V)_{\text{glass}} + (c\ell V)_{\text{drink}}.$$

$$= m_{\text{glass}}c_{\text{glass}} + m_{\text{drink}}c_{\text{drink}}.$$

m_{glass} and m_{drink} represent the mass of empty glass bottle and the soft drink respectively.

Substitution yields, $(c\ell V)_{\text{composite body}}$ = 0.8505 Btu. Equation (a) can be used now to calculate the time required to cool the drinks to 38°F.

$$\theta_{38°} = 2.76 \ln((95 - 34)/(38 - 34))$$

$$= 7.52 \text{ hours.}$$

That is, all soft drink bottles should be left at least for 7.52 hours before serving them to the customers.

3.4.2 For Type A Bottles

Bottles with thicker walls are used for bottling the high pressure soft drinks. The physical properties of these bottles are described under Section 3.2. Similar heat transfer analysis shows an increase in the cooling time from 7.52 hours to 7.58 hours. The two cooling times, therefore, can be assumed equal.

3.5 Cooling Time in Existing System

In the existing system all coolers are maintained at 20°F during the summer season. A similar heat transfer analysis yields the time to cool the drinks to 38°F as 3.70 hours when the cooler temperatures are 20°F. The relevant calculations are summarized in the following steps.

1) Grashof Number, $Gr = 0.352 \times 10^8$.

$< 10^8$.

The air flow over the bottle surface is laminar.

2) Nusselt Number, $\bar{Nu} = 37$

3) Coefficient of convective heat transfer, $\bar{h}_c = 0.995$ Btu/hr.sq.ft.°F.

4) Case A : for glass body -

$Bi = 0.0895$.

< 0.1.

Case B : for liquid body -

$Bi = 0.1113$.

$\simeq 0.10$.

The internal resistance of the filled bottles can be safely neglected.

5) Equation (a) yields,

$$\Theta_{38} = 2.585 \times \ln (.75/18).$$
$$= 3.7 \text{ hours}.$$

i.e., all drinks should be cooled at least for 3.7 hours before serving them to the customers.

3.6 Forced Draft System

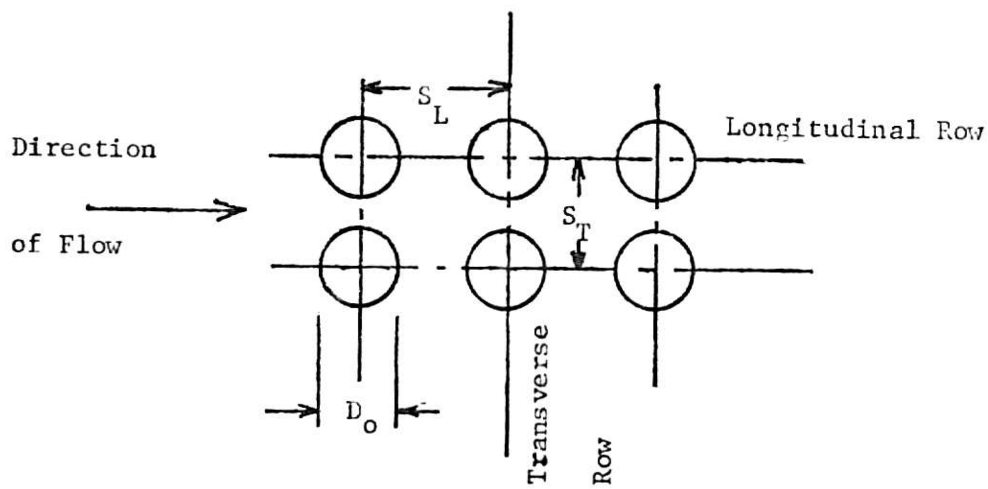
3.6.1 Introduction

Raising the cooler temperature from 20°F to 34°F increases the cooling time from 3.7 hours to 7.52 hours. The cooling time of 3.7 hours when the cooler temperature is 34°F can be achieved by introducing a forced air draft inside the cooler. Fans can be installed inside coolers to maintain a satisfactory air velocity. The properties of an air current are computed in the following heat transfer analysis.

3.6.2 Assumption

The following assumptions are made in addition to those mentioned in the preceeding sections.

1) The stacked bottles can be viewed as an inline tube bank.



Air flowing normal to a bank of pipes in line.

S_L = Longitudinal pitch = 0.3262'.

S_T = Transverse pitch = 0.3262'.

Do = External diameter of bottles = 0.1762'.

2) A cross air flow at a constant velocity surrounds the bottles.

3) The bank of bottles is more than ten rows deep.

3.6.3 Air Velocity Computations

The heat transfer in a flow over the tube bundles depends largely on the flow pattern and the degree of turbulence, which in turn are functions of the air velocity and the size and arrangement of the tubes. Knowing the cooling time, the effective heat transfer coefficient can be calculated by using the equation (a).

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-(\bar{h}_c A_s/(c\ell V))\theta}.$$

or, $$\frac{38 - 34}{95 - 34} = e^{-(\bar{h}_c \times 0.33/(0.8505)) \times 3.7}.$$

or, $$\bar{h}_c = 1.9 \text{ Btu/hr. sq. ft. °F.}$$

The following emperical relationship is suggested by W. H. McAdams (8) for air flowing normal to the bank of tubes in line for $\left(\frac{G_{max}\,D_o}{\mu_f}\right)$ from 2000 to 32000 (8, eq 10-11a, p. 272).

$$\left(\frac{\bar{h}_c D_o}{k}\right)_f = 0.26\left(\frac{C_p\,\mu}{k}\right)_f^{1/3} \cdot \left(\frac{D_o\,G_{max}}{\mu_f}\right)^{0.6}.$$

$$= 0.26\,(Pr)_f^{1/3} \cdot (Re)_f^{0.6} \qquad \text{(b)}$$

where k = Thermal conductivity of air, Btu/hr. ft. °F;

$= 0.0147$ at the film temperature $t_f = 1/2(\frac{95 + 38}{2} + 34)$

$= 66°F;$

Pr = Prandlt Number,

$= 0.72$;

Re = Reynolds Number,

$$= \left(\frac{Gmax\ Do}{\mu}\right)_f \ ;$$

μ = Viscosity of air,

$= 1.225 \times 10^{-5}$ lbm/ft. sec.;

Gmax = Mass flow rate of air per square foot across the tubes,

$$= \ell_f \cdot V_\infty \cdot \frac{A\ total}{A\ open} \ ;$$

$$= \ell_f \cdot V_\infty \cdot \frac{S_T}{S_T - Do} \ ;$$

$$= 0.0785 \times \frac{0.3262}{0.3262 - 0.1762} \ V_\infty \ ;$$

$= 0.171\ V_\infty$;

ℓ_f = Density of air,

$= 0.0785$ lb./cu. ft.;

V_∞ = Velocity of air in the cooler, ft./hour;

The subscript f denotes the film temperature.

$$\therefore \quad Re = \frac{0.171 \times .1762 \times V_\infty}{3600 \times 1.225 \times 10^{-5}} \ ;$$

$$= 0.6845\ V_\infty \ .$$

From equation (b)

$V_\infty =$ 3070 ft./hour.

A check on the value of Reynolds Number should be made for validity of equation (b).

$Re = 0.6845 \times 3070$

$= 2100.$

i.e., $2000 < Re < 32000$.

Hence, equation (b) is applicable.

The fan specifications can be expressed as its volumetric capacity. The volume of air passing accross a fan in the cooler is

$$Q = A \times V_{\infty} .$$

where A = half cross section area of the cooler,

$= 24$ sq. ft.

$$Q = \frac{24 \times 3070}{60} ;$$

$= 1230$ cfm.

The fan should have the volumetric capacity of 1230 cfm or higher.

3.7 Summary

The suggested raise in the cooler temperature increases the cooling time by 3.82 hours. However, the estimates of cooling time are only approximate because of the assumptions involved in their computations. Higher cooler temperatures are less expensive to maintain and do not freeze the bottle contents, but reduce the turnover of the bottles in the coolers. Low turnover could be detrimental during the summer season due to the loss of sale and customer dissatisfaction. On the average 3840 cooled bottles are sold daily in a shop at the sale peak during May. The rate of turnover of bottles then averages 2 per day. The mean permissible cooling time should not, therefore, exceed six hours to allow this turnover in the coolers. Allowing for the time for loading and unloading the coolers, the existing cooling time is within the desired cooling time range. Before raising the temperature to 34°F, fans should be installed in the cooler to effect an air draft of 51.2 ft. per minute to achieve the satisfactory cooling rate.

CHAPTER IV

INVENTORY COSTS

4.1 Introduction

All inventory problems manifest the existance of two or more opposing inventory costs. A specific inventory problem may oppose the procurement costs against the cost of carrying large inventories and the penalty cost of stockouts. An optimum lot or batch size can be computed which minimizes the total inventory costs. This chapter focuses on the recognition and the measurement of all costs relevant to the bottling plant inventory.

All inventories in the warehouse and shops are termed dynamic inventories due to the possibility of multiple production and delivery orders during the planning horizon. The following inventory costs are usually attached to the dynamic inventory problems.

1. Procurement cost,
2. Holding costs for the average inventories and the safety stock,
3. Shortage cost.

These costs are defined in the following sections. The costs differ in values for the production and delivery orders.

Delivery Order: A delivery order is placed by the shops and may include the requisition of one or more flavours. A typical delivery order is a combination of four individual orders placed by the shops.

Production Order: The production order is placed by the production manager. A request for the production of a single flavour constitutes a production order.

4.2 Acquisiton Cost

Acquisition cost represents the average cost per case of the inventory. The determination of the acquisition cost is important for calculating the inventory holding cost for the firm. The cost of acquisition of a case of the drink is different for the production and the delivery orders. The production order involves warehouse inventories and the corresponding acquisition cost can be calculated by substracting the average selling costs and the profit from the average selling price of the drinks. The average value of before tax profit and the selling price can be inferred from firm's past accounting data. The average before tax profit is estimated at $1.128 per case of the drinks. The average selling price can be calculated by using the following relationship.

$$\text{Average Selling Price} = \frac{\sum_{i=1}^{9} (\text{Sale})_i \times (\text{Selling Price})_i}{\sum_{i=1}^{9} (\text{Sale})_i}$$

where i denotes a flavour sold by the bottling plant. Based on the sales during the year 1971 the average selling price is approximated as $3.864 per case of drinks.

<u>Delivery Order:</u> The average acquisition cost of the inventories in the shops = Average selling price - Average before tax profit per case = $2.736 per case.

<u>Production Order</u>: The factory cost of the drinks can be computed from the direct material, direct labor and the overhead costs etc. In the absence of enough adequate accounting data the factory cost can be approximated if the selling costs and the overheads in the shops are known. Table 5 lists the elemental breakdown of the selling costs and the shop overheads. From

Table 5

Selling Cost

Description of Cost	$ per case
Cooling Costs: electricity consumption, depreciation on coolers and equipments etc.	0.2520
Repair: coolers, equipments and other maintenance expenses	0.0374
Rent: shops	0.1650
Salaries: personnel in shops	0.8350
Transportation: fuel, depriciation on the truck, salary to truck driver etc.	0.0352
Shop overheads: assumed 20% of the labor cost	0.1670
Insurance, Obsolesence and other systemic costs are assumed negligible.	
Total selling cost + overheads:	$ per case= 1.492

Table 5, the average acquisition cost for the inventories in the warehouse = 2.736 - 1.492 ≈ 1.25 $ per case.

4.3 Procurement Cost

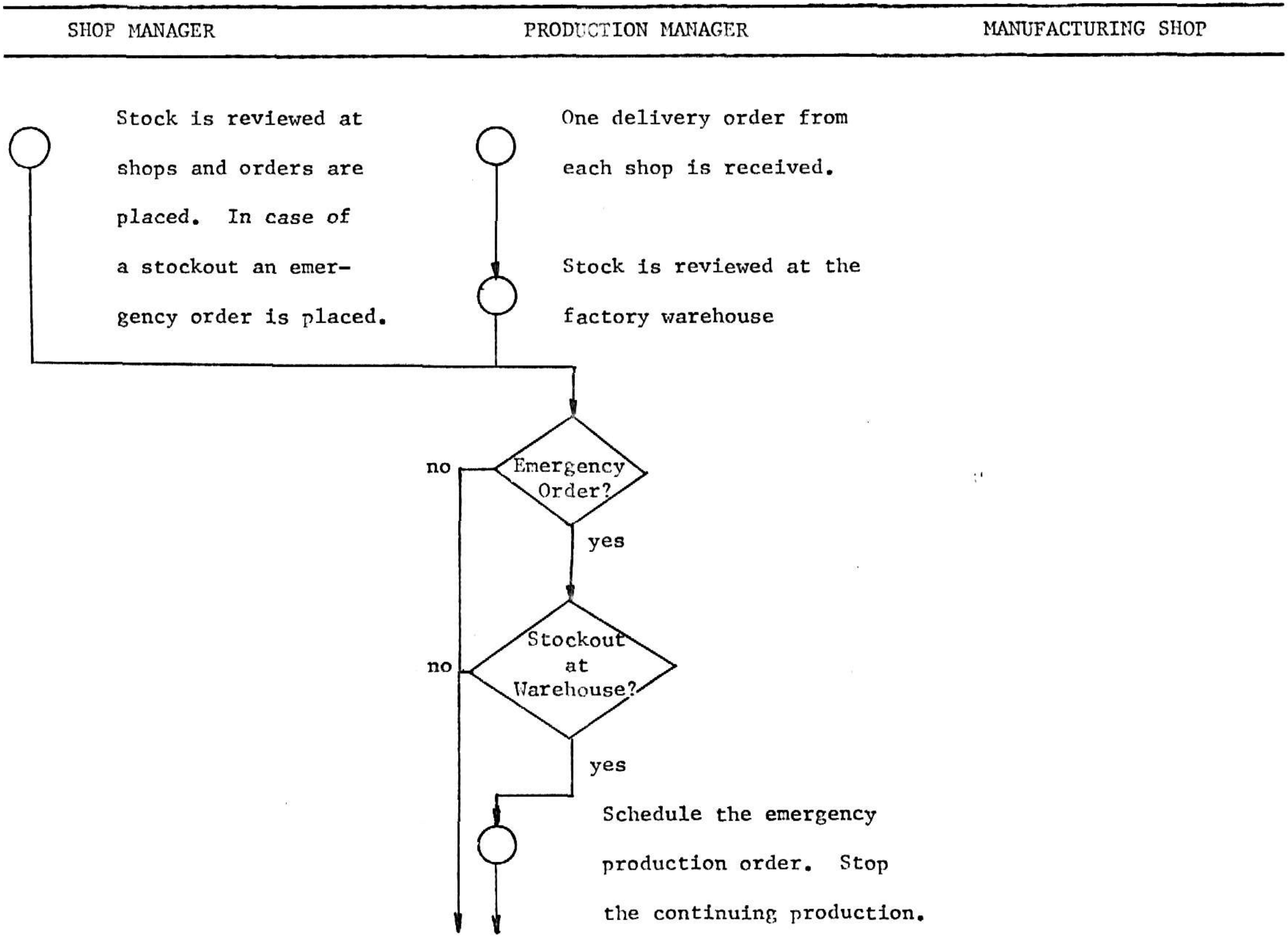
The procurement cost is the unit cost of replenishing inventories. The procurement cost is the set up cost or the ordering cost when the commodities are self supplied or supplied through a vendor respectively. The production order therefore involves the set up cost while the delivery order involves the ordering cost. Both types of procurement costs play the same role in the analytical formulation of the inventory problems. The major constituents of the set up cost and the ordering cost are identified and determined in the following subsections.

4.3.1 Ordering Cost

The ordering cost is related to the delivery order and is composed of the following major cost factors.

1. The cost of handling the issue transaction,
2. The cost of handling the receipt transaction,
3. The cost of making and sending a purchase order,
4. The cost of expediting,
5. The cost of updating,
6. The administrative and overhead costs,
7. Other costs.

The process chart (Figure 4) represents the various modes of operation in processing a delivery order. Table 6 lists the elemental costs of each of the cost factors mentioned above. From Table 6 the ordering cost for delivery orders placed by the shops equals $12 per order.



SHOP MANAGER
PRODUCTION MANAGER
MANUFACTURING SHOP
Stock is reviewed at shops and orders are placed. In case of a stockout an emergency order is placed.
One delivery order from each shop is received.
Stock is reviewed at the factory warehouse
Emergency Order?
no
yes
Stockout at Warehouse?
no
yes
Schedule the emergency production order. Stop the continuing production.

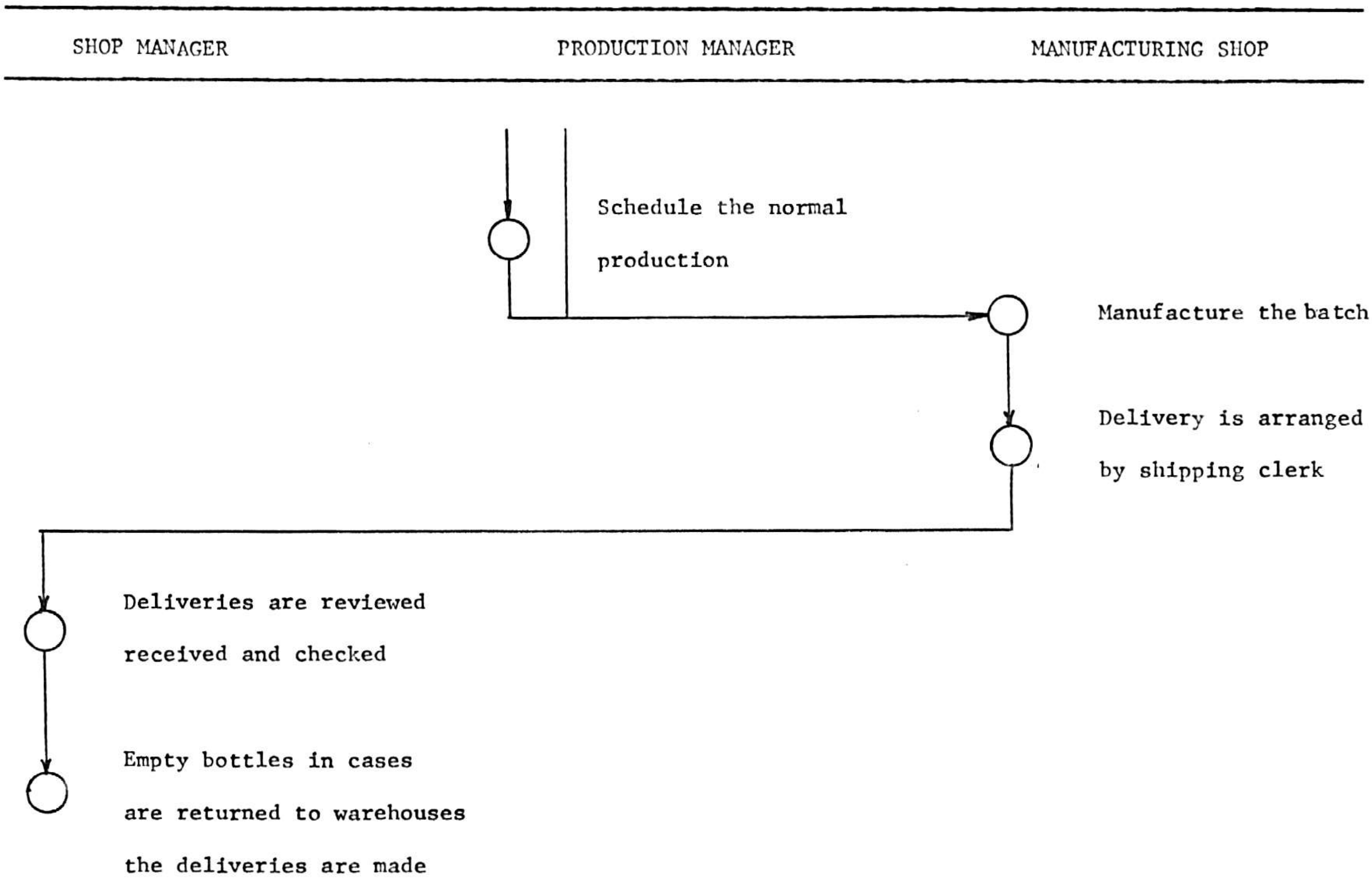


Fig. 4. Process Chart for Ordering System

Table 6

Ordering Cost

Description of cost		$ per order
Cost of making and sending a purchase order		
Stock review at shops before placing order	: 10 minutes per shop	0.970
Placing a delivery order	: 10 minutes per shop	0.970
Cost of handling an issue transaction		
Receiving and recording the delivery order at the factory	: 40 minutes	0.696
Stock review at the warehouse	: 20 minutes	0.348
Loading the order in the truck,		
- loader and unloader	: 30 minutes	0.375
(It is assumed that the loading and undloading time is independent of the quantity involved.)		
- supervising loading	: 30 minutes	0.520
- supervising and recording unloaded empty bottles	: 10 minutes	0.17
- unloading empty bottles at washers	: 20 minutes	0.25
Cost of expediting		
Transportation to and from the shops, fuel + driver's salary	: 150 minutes	4.615

Depreciation on truck and the equipments is assumed negligible.

Table 6 (cont)

Description of cost		$ per order
Cost of handling reciept transaction		
Unloading the filled cases at four shops		
loader and unloader :	60 minutes	0.750
Loading cases of empty bottles in		
the truck :	20 minutes	0.250
Supervise and record the quantity		
loaded and delivered :	40 minutes	0.970
Administrative and overhead costs are		
assumed to be 10% of ordering cost.		1.088
Ordering Cost :	$ per order	12.000

4.3.2 Set Up Cost

A self supplier's procurement costs are called set up costs. Set up costs include the cost of changing over the production process to produce the required commodity. The set up cost is composed of the following cost factors.

1) The cost of making and sending the production order,

2) The cost of labor in setting up,

3) Other expenses in initiating the production,

4) The cost of set up time due to the loss of production.

The cost of set up time is not considered important during the winter and rainy seasons. Because of the high demand rate during the summer season this loss is viewed as a penalty against changing the production on either of the two bottling machines. Depending upon the situation the set up cost may depend on the production batch size but in the present case it is independent of the production order size. Table 7 lists the components of the set up cost.

The set up cost, therefore, depends on the season. The set up cost for winter and rainy season equals \$8.00 per set up, and for summer season equals \$24.6 per set up (Table 7).

4.4 Holding Cost

Holding costs are the cost of carrying the inventories and are composed of the following cost factors:

1) The cost of money tied up in inventory,

2) Storage costs,

3) Deterioration cost,

4) Insurance costs,

5) Taxes on inventories.

Table 7

Set Up Cost

Description of Cost		$ per set up
Cost of making and placing a production order		
Warehouse stock review before placing an order	: 90 minutes	2.190
Other expenses in initiating the production		
Essence preparation	: 20 minutes	0.416
Early production wastage on bottling machine capacity 30 cases per hour, cost of goods is $1.25 per case	: 5 minutes	3.125
Cost of labor in setting up the machine		
Set up man sets up machines	: 30 minutes	0.650
Machine operators and helpers are idle	: 30 minutes	0.895
Administrative and overhead expenses		0.725
Set up cost during winter and rainy seasons	: $ per set up	≈ 8.00
Production loss during summer season, bottling capacity = 30 cases per hour Expected profit on each drink = 1.128 per case	: 30 minutes	16.192
Set up cost during summer season:	$ per set up	≈ 24.6

Each of the cost components mentioned above can be determined from the records of the firm. Holding costs are assumed to be the same for the production and delivery orders.

The addition of individual cost components yields the firm's holding costs as \$0.3582 per year per dollar invested in the inventory.

To meet the requirement of additional warehousing capacity the firm intends to build an additional warehouse inside the factory premises. Based on the floor area available, the firm proposes to build an additional 1800 case capacity warehouse. To maintain a sufficient level of the inventory of bottles inside the new warehouse, the firm roughly estimates to buy 1800 cases of type B bottles at the rate of \$5.00 per case. The inventory storage cost in this situation can be conveniently computed by combining the first costs and the yearly operating costs for maintaining the new warehouse and the proposed levels of bottles inventory. Assuming 5 - 7% breakage of bottles per year the revised estimate of inventory holding costs for 3000 case warehouse is \$0.4510 per year per dollar invested in inventory.

4.5 Shortage Cost

Shortage costs arise due to not carrying sufficient inventories, and differ for the production and the delivery orders. There are two varients of this cost depending on the reaction of the prospective customers.

1) Back order cost,

2) Loss of sale and goodwill.

Back order costs arise when the emergency expediting procedures are adopted to get the additional stock. In the existing system the shops place emergency delivery orders upon the recognition of a stockout of any flavour.

Another mode of incurring shortage costs is through the loss of sale and the goodwill when the customers are not satisfied. However, most customers do make a choice of another soft drink flavour if their demands are not met. It can, therefore, be assumed that the firm faces only a constant shortage cost equal to the delivery order cost in response to a stockout. The goodwill loses are difficult to determine and are neglected in this discussion. The constant shortage cost for the delivery orders is, therefore, equal to $12.00.

In the existing system an emergency production order is placed whenever enough stock is not available at the factory warehouse in response to an emergency delivery order. The emergency production order, in general involves a change of set up on the bottling machine. It could also involve two changes in set up if an ongoing production order is interrupted and reinstalled at the end of emergency production. However, since the firm does not adopt such a policy the cost of a double set up is avoided. Neglecting the effects of loss of goodwill the shortage cost for the production order:

During the winter and rainy seasons = $ 8.00,

During the summer season = $24.60.

CHAPTER V

METHODOLOGY

5.1 Introduction

This chapter discusses Operations Research technique for obtaining the optimum production and inventory policies. In the absence of reliable data these techniques can be viewed as the general principles for obtaining efficient production and inventory control. Its results would yield a better insight of the shortcomings in the existing system and the necessary improvements for reducing the firm's operating costs. An unconstrained optimisation of the inventory system would indicate the possible inventory cost savings over the constrainted situation. The existing inventory system has serious constraints due to the limited warehousing space and the number of bottles. The economic comparison of the cost of removing constraints against the cost savings should be undertaken before implementing the proposed changes in the system.

5.2 Important Features of Inventory Problems

The inventory problem under consideration deals with the aggregate inventories. It has a dynamic nature because more than one order can be placed. The relevant inventory costs are discussed in Chapter 4. The other important features of this class of inventory problems are its lead time, demand distributions and the scheduling period.

5.2.1 Lead Time

Lead time is the time interval between placing an order and its addition to inventory. Lead time for delivery order is three hours. Its value for production order depends upon the amount produced. In this inventory problem formulation the effects of lead time are considered negligible because of its small magnitude.

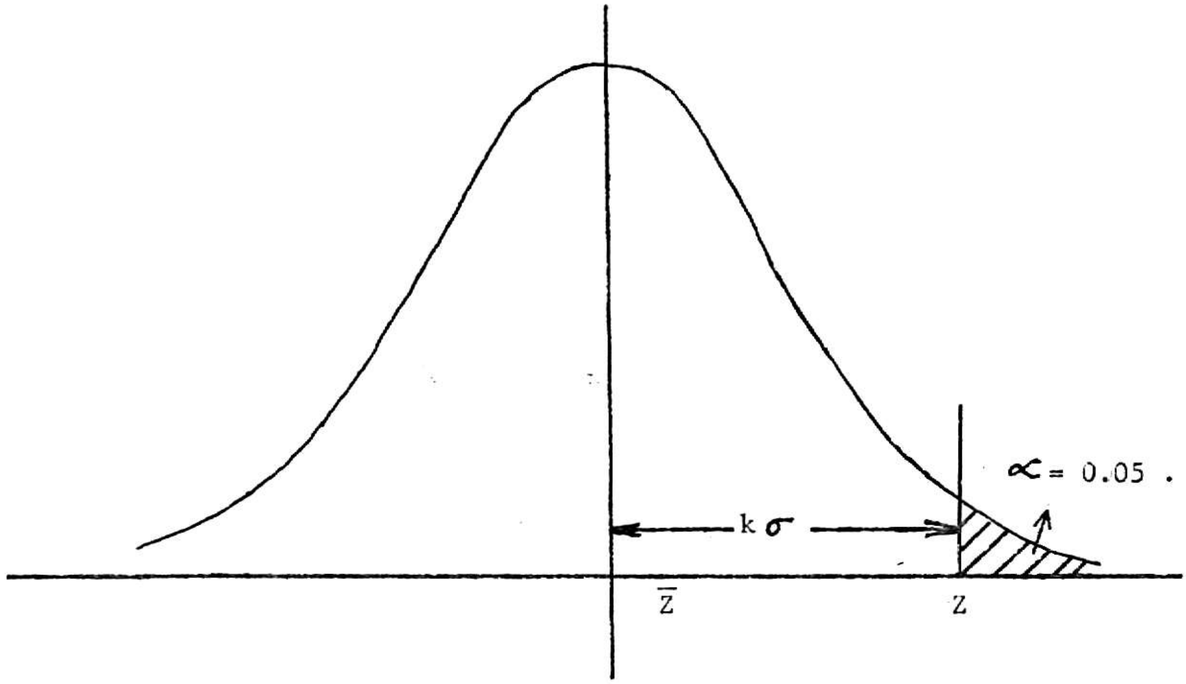
5.2.2 Demand Distribution

Attempts were made to fit standard probability distributions such as Poisson and Gamma functions to the frequency polygons for summer and nonsummertime sales of different products. Chi-square tests were conducted to test the fit. However, none of the attempted distributions could be statistically justified to represent 1971 sale fluctuations. Because of these difficulties in establishing the exact demand distributions, they were assumed uncertain and their first two moments were computed (Table 1).

The demand distribution gives the expected probability of stockouts. The allowable probability of stockout is decided from the firm's service policy. The service level is assumed at 95% for the production orders. The central limit theorm can be used to determine the amount of reserve stock needed. According to the central limit theorm, for a random sample of size n ($n > 5$) from any distribution having positive variance σ^2 and finite mean μ the random variable $\sqrt{n}(\bar{Z} - \mu)/\sigma$ has a limiting normal distribution with zero mean and unit variance (5, p. 182). The mean and standard deviations of the sale of each product (Table 1) are based on a large sample size ($n = 122$ days during the summer season). For $\alpha = 0.05$ (Figure 5), the value of number of standard deviations, k, can be determined by applying the central limit theorm.

5.2.3 Scheduling Period

The time interval between two successive orders is called the scheduling period. The optimum value of a scheduling period minimizes the inventory total cost. In certain situations the scheduling period is



From the Table of Normal Distribution, for $\alpha = 0.05$, $k = 1.645$.

Figure 5. Normal Demand Distribution

constant and prescribed, depending upon factors like the number of orders that can be treated in a fiscal year, etc. The effects of prescribed scheduling period are discussed by Naddor (9) in the book "Inventory Systems".

5.3 Total Cost Equation

The inventory cost based on 365 days per year is expressed as follows (10, p. 127-30).

$$TCi \leq \underbrace{\frac{365 C_r}{t}}_{\text{Procurement cost}} + \underbrace{\frac{\bar{Z} t c C_c}{2}}_{\text{Holding costs for average inventory}} + \underbrace{k\sigma\sqrt{tc}\,C_c}_{\text{Holding cost for reserve stock}} + \underbrace{\frac{365\ K}{t}\int_{\bar{Z}+r}^{\infty} f^t(y)\,dy}_{\text{Shortage cost}} \qquad (A)$$

where TCi = Total inventory cost for i^{th} product, \$/year;

Cr = Procurement cost, \$/order;

t = Scheduling period,

$= \frac{x}{\bar{Z}}$ days;

x = Batch size, units/order;

$\bar{Z}$ = Average daily demand, units/day;

C_c = Inventory holding cost, \$/\$ year;

C = Unit acquisition cost, \$/unit;

$\sigma\sqrt{t}$ = t^{th} convolution of the standard deviation, σ, units;

r = Reserve stock against the demand fluctuations,

$= k\sigma\sqrt{t}$ units;

k = Constant (Figure 5) associated with service level

= 1.645 for service level equal .95

K = Constant shortage cost independent of the number of units short, \$/order;

$f^t(y)$ = t^{th} convolution of the demand distribution $f(y)$;

$\int_{\bar{Z}+r}^{\infty} f^t(y)dy$ = Probability of stockout,

= 0.05.

The total inventory cost can be minimized by partially differentiating equation (A) with respect to t. The constraints are ignored in equation (A).

The assumption of a finite replenishment rate for the highest selling flavour, Soda (Section 2.2.3.3). It alters the average inventory carried. The inventory total cost formulation changes slightly for the finite replenishment rate case.

$$TC_{Soda} = \frac{365\ Cr}{t} + \frac{t\bar{Z}cCc}{2} \cdot (1 - \bar{Z}/p) + k\sigma\sqrt{t}cCc$$

$$+ \frac{365}{t} \cdot \int_{\bar{Z}+r}^{\infty} f^t(y)dy \qquad (B)$$

where p = Production rate on a bottling machine,

= 360 units/day.

5.4 Unconstrained Optimization

Differentiating equation (A) with respect to t and equating to zero,

$$\frac{\partial TC}{\partial t} = o = -\frac{365\ Cr}{t^2} + \frac{\bar{Z}cCc}{2} + \frac{k\sigma cCc}{2\sqrt{t}} - \frac{365\ K}{t^2} \times 0.05 = 0$$

or, $$\frac{\bar{Z}cCc}{2}\ t^2 + \frac{k\sigma cCc}{2}\ t^{3/2} - 365(Cr + 0.05\ K) = 0, \qquad (C)$$

or, for the finite replenishment rate case,

$$\frac{\bar{Z}cCc(1-\bar{Z}/P)}{2} \cdot t^2 + \frac{k\sigma cCc}{2}\ t^{3/2} - 365(Cr + 0.05\ K) = 0 \qquad (D)$$

Equations (C) and (D) can be solved for the optimum value of t by using a one dimensional search technique such as Newton's method.

The optimum batch size, x_i and reserve stock, r_i for product i can be determined as follows.

$$X_{i_{optimum}} = t_{i_{optimum}} \times \bar{Z}_i,$$

$$r_i = 1.645 \times \sigma_i \times \sqrt{t_{i_{optimum}}}\ .$$

Equations (A) and (B) can be used directly to determine the inventory total cost, batch size and reserve stock when the scheduling period is prescribed at $t = t_p$.

5.5 Constrained System

The solution to the constrained inventory problem can be interpreted from the results of the unconstrained inventory problem. The average warehouse inventory level of product i is $(\frac{x}{2} + r)_i$. The values of the average inventory and the inventory total costs can be calculated for all products for different values of scheduling period t in equations (A) and (B). The inventory total cost decreases while the average inventory of a product increases with increase in t until the optimum is reached. The shadow price λ is defined as the savings in inventory costs per additional case of average inventory.

Or,

$$\lambda_{i,t} = \frac{\Delta(\text{Inventory total cost})\ i,\ t}{\Delta(\text{Average inventory})\ i,\ t},$$

$$\text{for } t = 1, 2, 3 - - - - t_{\text{optimum}}$$

Subscripts i and t designate the product and the scheduling period respectively. For all products the shadow price decreases with increasing scheduling period. A minimum value of shadow price is desired so that the storage constraint is not violated, i.e., choose a minimum λ, equal for all products such that,

$$\sum_{i=1}^{i} \left(\frac{x}{2} + r\right)_i \leq \text{storage capacity available for all products.}$$

The shadow price $\lambda_{i,\ t}$ is plotted against average inventory (Figure 6) to determine the optimum inventory policy. Figure 7 is a plot of shadow price versus total average warehouse capacity needed for all i products. However, in the above mentioned analysis concessions should be made for the storage of empty bottles.

The inventory analysis for delivery orders is not undertaken in this study because the demand distribution is not known for individual shops. Since the demand rate is not critical during the rainy and the winter seasons, the inventory problem is analysed for the production orders during the summer season only.

The inventory problem formulation for the existing system is based on the following parameters (defined on page 46):

$$Cr = \$24.6/\text{order},$$

$$C = \$1.25/\text{case},$$

$$C_c = \$0.3582/\$ \text{ year},$$

$$K = \$24.6/\text{order},$$

$$p = 360 \text{ cases/day for soda},$$

$$t_p = 1 \text{ day for daily production for all products}.$$

Because the exact inventory and production policy of the firm is not known, a hypothetical inventory system is assumed where all products are produced daily.

In the proposed system one bottling machine is used exclusively to produce Soda. This practice would lower the set up costs because certain steps of set up are eliminated. The corresponding parameters are

$$Cr = \$5.5/\text{order},$$

$$C = \$1.25/\text{case},$$

$$C_c = \$0.4510/\$ \text{ year},$$

$$K = \$5.5/\text{order}.$$

For the other eight products the parameters of the inventory problem in the proposed system are

$$Cr = \$24.6/\text{order},$$

$$c = \$1.25/\text{case},$$

$$C_c = \$0.4510/\$\ \text{year},$$

$$K = \$24.6/\text{order}.$$

The parameter values are substituted in the equations (A) and (C) for obtaining the optimum solutions for these products.

The results obtained on the basis of the methods discussed here are summarized and discussed in the following chapter.

CHAPTER VI

RESULTS AND DISCUSSION

6.1 General

This chapter summarizes and discusses the analysis results based on the methods discussed in the prior chapters. The following modifications to the existing system have been proposed.

1) The cooler temperature should be raised from 20°F to 34°F and a circulating fan installed in each cooler to maintain the rate of cooling.

2) One bottling machine should be used on a twelve hour work day basis for bottling the highest selling flavour, Soda. The other bottling machine should be used eight hours per day for bottling the remaining eight products. One of the advantages of using one machine for manufacturing Soda exclusively is the saving in the number of set ups. Only the early production wastage and essence preparation constitute the set up. This reduces the set up and the shortage cost to $5.5 per order. A finite replenishment rate of 360 cases per day is considered for Soda. For the remaining eight products, the replenishment rate is assumed infinite.

3) The acquisition of an additional 1800 cases of type B bottles and 1800 case capacity warehouse is proposed. The filled and the empty bottles in cases are stored in the warehouse. The existing warehouse capacity is 1200 cases and the available bottle inventory is 1735 cases.

4) An efficient production and inventory control policy should be designed for lowering of the firm's operating costs. The proposed system has the constraints due to the warehouse capacity of 3000 cases and the bottle inventory of 3535 cases. The results of the analysis conducted for each modification are discussed in the following sections.

6.2 Heat Transfer Investigation

The proposed increase in the cooler temperature from 20°F to 34°F increases the cooling time (to 38°F) from 3.7 hours to 7.52 hours. The higher cooling temperature reduces the daily turnover of the bottles in the coolers from 3.24 to 1.6 (Chapter 3), but completely eliminates the possibility of freezing the bottle contents. The cooling rate is increased when a forced air draft is introduced in the coolers across the bottles. For achieving the desired cooling rate, the velocity of the air current should be maintained at 51.2 fpm. The corresponding fan capacity is approximated at 1230 cfm.

6.3 Production and Inventory Control

The proposed inventory system for the production orders is optimized without considering the constraints, to detect the tightness of the constraints. The unconstrained optimization also gives an idea of the penalty paid due to the presence of the constraints in the existing system.

6.3.1 Unconstrained Optimization

The unconstrained optimization for the proposed system is done with the following assumptions:

1) The scheduling period for Mangola (whose shelf life is one week) is assumed at 6 days.

2) A finite replenishment rate model is considered for Soda.

3) The cost of a set up on the bottling machine producing Soda exclusively is \$5.5 per order. An infinite replenishment rate is assumed for all products except Soda.

The results of the unconstrained optimisation are represented in Table 8. The average inventory of a product is calculated as $(\frac{x}{2} \cdot (1-\bar{Z}/P)+r)$

where x = Production batch size, cases;

r = Reserve stock, cases;

$\bar{Z}$ = Average daily demand, cases/day;

P = Replenishment rate,

= 360 cases/day for Soda.

As indicated in Table 8, the net average inventory of the filled bottles is 2627.5 cases. Before commenting upon the sufficiency of the proposed storage capacity, an important assumption is made here regarding the amount of storage allocated for storing the empty bottles. A standby supply of 1000 cases of empty bottles is assumed. This is a sufficient amount to maintain full production on both machines for two days, assuming 85% machine utilization. The average storage required for implementing the unconstrained optimum policy is thus, 3627.5 cases. The unconstrained optimal policy is also restricted by the number of the bottles in the system.

A large warehouse space and bottle inventory is required for Soda in the unconstrained optimum policy. An alternate solution to this problem would be to undertake a continuous production for Soda. This means that one machine would bottle Soda for an average of 9.9 hours per twelve hour working day. The corresponding batch size, reserve stock, and average inventory are 297, 168 and 220 cases respectively. The saving in the average

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Table 9

Inventory Costs In Existing System

Product	Scheduling Period, Days	Batch Size, Cases	Reserve Stock, Cases	Total Inventory Cost During Summer, Dollars
Soda	1	297	169	3179.25
ICS	1	22	20	3148.55
Lemon	1	16	17	3147.30
Orange	1	13	15	3146.75
LO	1	6	6	3144.35
Mangola	1	3	4	3143.30
GF	1	1	2	3143.12
Ginger	1	1	2	3143.24
C Cola	1	1	1	3142.79
Total Inventory Cost				28338.45

inventory requirement is coupled with an increase in the inventory costs by (\$754.00 - \$216.48) = \$537.02 (Table 8 and Table 10). This system also possesses a reserve machine capacity of 2.1 hours per day to build the reserve stock against the fluctuations in the daily demand.

In the unconstrained optimum policy the scheduling periods for GF, Ginger and C Cola are too large. Each product has scheduling periods longer than the duration of the summer season. The longer scheduling period results in higher average inventory levels. A scheduling period of 120 days is, therefore, suggested for these products. The three products would be manufactured during the presummer season in large enough quantities to satisfy the summertime demand. This policy would transfer the related man and machine hours to the less critical winter season. The parameter values for the corresponding inventory problem formulation for these products are

$$Cr = \$8.00/\text{order};$$

$$c = \$1.25/\text{case};$$

$$Cc = \$0.4510/\$\ \text{year};$$

$$K = \$24.6/\text{order};$$

$$t_p = 120\ \text{days}.$$

Table 10 represents the batch size, reserve stock, average inventory and the inventory cost corresponding to t_p = 120 days.

6.3.2 Constrained System

After proposing the inventory policy for the five products discussed above, the inventory policy for the last four products is determined so that the storage constraint is not violated. The average storage available for the four products is calculated as follows (Table 10).

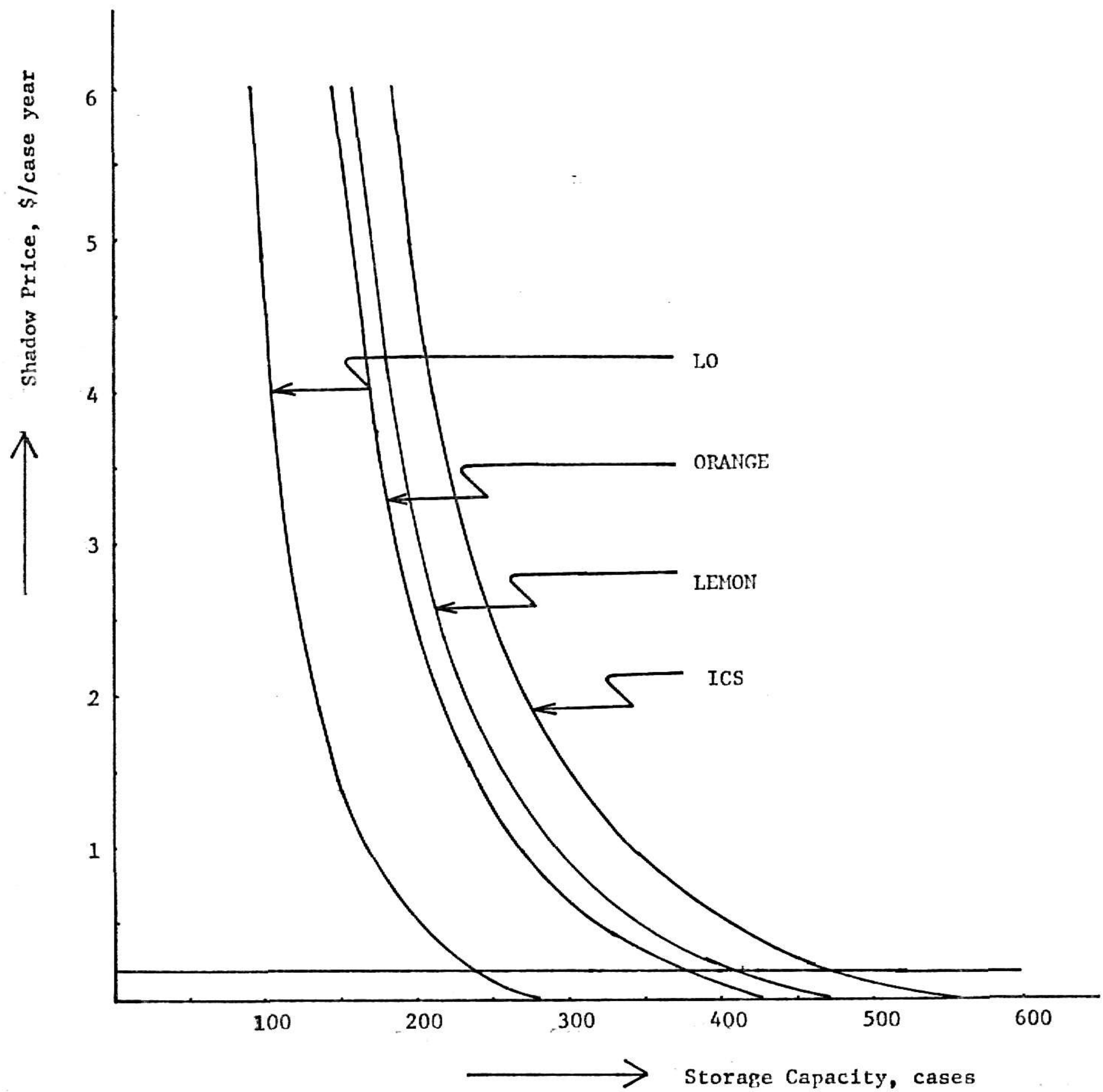

Fig. 6. Shadow Price Versus Storage Capacity

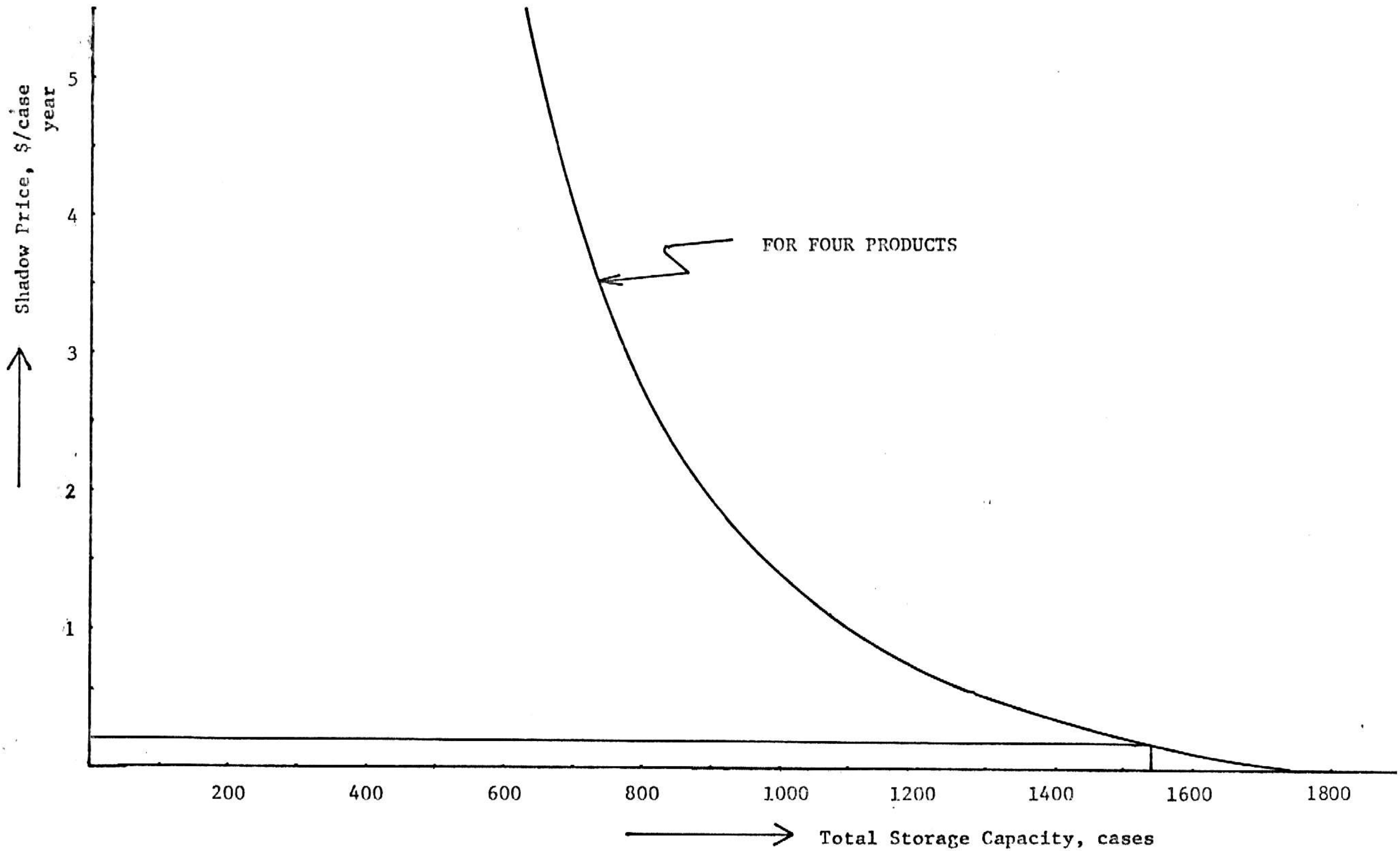

Fig. 7. Shadow Price Versus Total Storage Capacity

Table 10

Recommended Inventory System

Product	Scheduling Period, Days	Batch Size, Cases	Reserve Stock, Cases	Average Inventory, Cases	Total Inventory Cost During Summer, Dollars
Soda	1	297	168	220.0	754.00
ICS	34	734	120	487.0	183.95
Lemon	41	647	108	431.5	157.81
Orange	44	584	102	394.0	145.49
LO	60	364	47	229.0	95.35
Mangola	6	15	10	17.5	527.06
GF	120	178	19	108.0	45.77
Ginger	120	153	28	104.5	45.03
C Cola	120	17	7	15.5	28.31
Total				2007.0	1982.77

Available storage capacity	=	3000 cases
Average inventory of empty bottles	=	1000 cases
Average inventory		
- for Soda, tp = 1 day	=	220.0 cases
- for Mangola, tp = 6 days	=	17.5 cases
- for Gf, tp = 120 days	=	108.0 cases
- for Ginger, tp = 120 days	=	104.5 cases
- for C Cola, tp = 120 days	=	15.5 cases
Total committed storage capacity	=	1465.5 cases

Therefore, available net storage capacity

for remaining four products = 1535 cases

The unconstrained optimum policy for the four products (Table 8) cannot be implemented because of the storage constraints. Shadow prices are therefore calculated to facilitate the determination of the constrained optimum policy for these products. Figure 5 and 6 are the plots of shadow price versus the average inventory and the total average inventory of the four products. Corresponding to the available 1535 case capacity for the four products, the shadow price is $0.15 per case per year (Figure 6) and the individual average storages for ICS, Lemon, Orange and Lo are 487.0, 431.5, 394.0 and 229.0 cases respectively (Figure 5). Table 10 summarizes the recommended inventory policy for the production orders during the summer season. The effects of the proposed modifications can be illustrated by comparing the total costs in the existing and the proposed system. Based on the assumption of the daily production of all the products, the inventory total cost in the existing system is $28338.45 during the summer season (Table 9). The savings achieved in the proposed system is $26355.68, i.e., approximately 93% of the inventory cost in the existing system. Though the results of this analysis are not accurate because of the assumptions involved, they are illustrative. However, the expected inventory cost savings indicate that the proposed 3000 case warehouse capacity is adequate. The acquisition of an additional 1800 case warehouse capacity is, therefore desirable.

6.4 Machine Utilization

Machine utilization is one of the criteria for comparing the existing and the proposed system. In the existing system both bottling machine work on a twelve hour shift basis. The proposed system recommends using one machine for twelve hours per day and the other for eight hours per day. The corresponding saving in the labor can be utilized productively for alternate purposes.

In the existing system the combined utilization of both machines is 50.5% with 4.5 hours required daily for the set ups. The proposed systen has the combined machine utilization 59.85% with 0.142 hours daily for set ups. The proposed system has higher machine utilization due to lower daily machine hours recommended. However, there is a remarkable decrease in the average daily set up time.

6.5 Economic Feasibility of Proposals

Modifications are suggested in many problem areas in the existing system. A considerable capital investment is necessary for constructing the warehouse, purchasing the bottles and installing the forced draft fans in the coolers. The firm estimates the necessary investment at

\$60000 to \$70000. This amount is not large enough to forbid the possibility of financing the project.

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CONSTRAINED AGGREGATE DYNAMIC INVENTORIES

IN A BOTTLING PLANT - A CASE STUDY

by

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ABSTRACT

This report is a case study of prevailing inventory and production practices in a bottling plant engaged in the manufacture and retail sale of various soft drinks. This study summarizes the overall existing system in many segments.

a) The description of existing operations, practices and policies,

b) Identify the constraints therein and

c) Propose some modifications and recommends an efficient production and inventory control system.

Improvements are suggested in the bottle cooling system. A saving in the cooling cost is expected by maintaining high cooler temperatures coupled with the introduction of forced draft fans in the coolers to improve the cooling rate. Acquisition of an additional 1800 cases of bottles and 1800 case storage capacity is suggested for an efficient inventory system. Based on general principles of operations research, an inventory control system with lower inventory costs and better machine utilization is recommended. The expected total capital investment for fans, bottles and warehouse is $60000 to $70000.

In the absence of reliable data many approximations are made in the problem analysis. More refined data regarding inventory cost, demand distributions for retail shops and the existing production system should be collected for obtaining a more accurate and reliable solution to the inventory and production problems.