

Thermomechanical soil structure interaction in energy piles exhibiting reversible and irreversible interface behavior

by

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B.S., University of Kerman, 2013

M.S., Tarbiat Modares University, 2016

AN ABSTRACT OF A DISSERTATION

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Department of Civil Engineering
Carl R. Ice College of Engineering

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Abstract

Energy piles are sustainable, eco-friendly, deep thermo-active foundations that combine the roles of a structural support and thermal energy carrier, thus facilitating the exchange of renewable thermal energy between the subsurface and superstructure. Although these earth-contact structures present advantageous technology based on the use of renewable thermal energy for space heating and cooling the unprecedented effects of a simultaneous action of mechanical and thermal loads require development of novel theoretical models and analyses that will ultimately enable their wider use.

To this end, in this study, multiple series of mechanics based analytical solutions were derived to predict the thermomechanical interaction between the soil and energy piles. Specifically, equations that predict displacements, axial strain and stress, and interface shear stress that develops in energy piles exhibiting reversible and irreversible interface behavior were derived starting with a single energy pile embedded in the homogeneous soil profile pile exhibiting reversible interface behavior. A subsequent validation of these solutions against the results of a geotechnical centrifuge tests was successful. Additionally, upon the development of analytical solutions for a multi-layered soil profile a validation against the full scale pile test at the site comprising four different soil layers underlain by the bedrock was also successful.

Furthermore, for the first time, a series of analytical solutions incorporating yielding of the soil pile interface, which is referred to as irreversible soil pile interface behavior, were derived and validated against the full-scale pile test. This series of solutions elucidated important features of the irreversible interface behavior including generation of a residual displacement, strain, axial stress and interface shear stress in a thermo-elastic energy pile, upon the inception of the interface yielding. In addition, the inception of the interface yielding also produced a change in the location

of the null point. Consequently, this research contributes to filling the long-lasting gap of lack of analytical solutions that quantitatively and qualitatively describe the response of a single energy pile under thermal and combined thermomechanical load.

A subsequent implementation of the analytical solutions derived in this research resulted in creation of mechanics-based load transfer diagrams that delineate a thermomechanical behavior of energy piles. Previously provided load transfer diagrams were largely speculative in that they were mostly based on the trends extracted from the experimental data. While they addressed the effects of head and tip restraints, they failed to provide quantitative assessment of displacement, strain, axial and interface shear stresses. Furthermore, the load transfer diagrams generated in this study also delineated the progression of interface yielding that culminates in the development of a constant shear stress along the pile shaft.

Finally, fully coupled thermo-hydro-mechanical finite element analyses and analytical solutions were combined with statistical analyses to develop a correlation between the parameters of a finite element model and analytical solutions for isotropic and cross-anisotropic soils. Multiple linear regression models were developed and used to equalize maximum thermal axial stresses in the energy piles obtained from the finite element analyses and analytical solutions, thus leading to a correlation between the soil parameters used by analytical and finite element solutions.

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Table of Contents

List of Figures	xi
List of Tables	xvi
Acknowledgements	xvii
Chapter 1 - Introduction.....	1
Background.....	1
Literature review	3
Physical modeling	4
Numerical and computational modeling	7
Load transfer diagrams.....	9
Analytical solutions	10
Research objectives.....	11
Governing equations and analytical solutions	12
Assumptions of this study	12
Chapter 2 - Thermomechanical Soil Structure Interaction in Single Energy Piles Exhibiting	
Reversible Interface Behavior	20
Abstract.....	20
Introduction.....	21
Analytical solutions for semi- floating energy pile	28
Thermal Load.....	28
Mechanical load	32
Special cases	33
Mechanical Load.....	33
Thermal Load.....	35
Validation of the analytical model.....	36
Case 1 (Stewart and McCartney, 2013)	36
Case 2 (Goode and McCartney, 2015).....	38
Load transfer diagram	39
Mechanical Load.....	41
Thermal Load.....	43

Thermal, mechanical and combined thermomechanical load	44
Thermal null point and development of tensile zone.....	53
Head restraint effects	59
Conclusions.....	61
Chapter 3 - Analytical Solutions for Thermo-Mechanical Soil Structure Interaction in Single	
Semi Floating Energy Piles Embedded in a Layered Soil Profile	64
Abstract:.....	64
Introduction.....	65
Assumptions and governing laws	69
Analytical solutions	70
Thermal Load.....	71
Mechanical Load.....	80
Validation of analytical solutions	85
Conclusions.....	97
Chapter 4 - Analytical Solutions for Thermomechanical Soil Structure Interaction in Single	
Energy Piles Exhibiting Irreversible Interface Behavior.....	99
Abstract:.....	99
Introduction.....	100
Governing laws and assumptions	106
Analytical solutions for irreversible interface behavior	109
Thermomechanical Loading	109
Semi Floating Energy Pile	110
Special Cases	116
Mechanical Loading.....	118
Semi Floating Pile.....	119
Special Cases	120
Validation of the analytical solutions	121
Load transfer diagrams	125
Irreversible Response Mechanism	126
Thermal and Mechanical Interface Yielding	129
Progression of Yielding under Thermomechanical Loading.....	132

Conclusions.....	137
Chapter 5 - Computational, Analytical and Statistical Analyses for Assessing Effects of Soil	
Anisotropy on Response of Energy Piles	139
Abstract	139
Introduction.....	140
Methodology	142
Computational Modeling	143
Isotropic and anisotropic elastic soils	145
Statistical Analysis.....	148
Multiple Linear Regression Analysis.....	149
Predictions of full-scale experimental data.....	154
Cross-validation of FEM and analytical predictions	157
Conclusions.....	160
Chapter 6 - Conclusions and Recommendations	162
References	164

List of Figures

Figure 1.1 Thermal behavior of energy piles under (a) cooling, and (b) heating	2
Figure 1.2 Schematic of the physical model of energy pile (after Saeidi Rashk Olia and Perić 2021c)	14
Figure 1.3 FBDs of a semifloating energy pile subjected to $\Delta T > 0$, a) the entire pile, b) an infinitesimal segment of the pile above the null point, c) an infinitesimal segment of the pile below the null point (after Saeidi Rashk Olia and Perić 2021a).....	16
Figure 2.1. Proposed model of energy pile	27
Figure 2.2 FBDs of a semi-floating energy pile subjected to $\Delta T > 0$, a) the entire pile, b) an infinitesimal segment of the pile above the null point, c) an infinitesimal segment of the pile below the null point	28
Figure 2.3. FBDs of a semi-floating energy pile subjected to $F < 0$, a) the entire pile, b) an infinitesimal segment of the pile	32
Figure 2.4. Comparison of analytical predictions with centrifuge test results (Stewart and McCartney, 2013): (a) thermal displacement, (b) thermal strain, (c) thermal stress	37
Figure 2.5. Comparison of analytical predictions with centrifuge test results (Goode and McCartney, 2015): (a) thermal displacement, (b) thermal strain, (c) thermal stress	39
Figure 2.6. Comparison of: (a) displacement, (b) strain, and (c) stress for mechanical load ($F < 0$) in ideally end bearing and fully floating energy piles, $k_h = 0$, soil A2.....	42
Figure 2.7. Comparison of: (a) displacement, (b) strain, and (c) stress for thermal load ($\Delta T > 0$) in ideally end bearing and fully floating energy piles, $k_h = 0$, soil A2.....	43
Figure 2.8. (a) Displacement, (b) strain, and (c) stress in ideally end bearing energy pile subjected to a combined mechanical and equivalent thermal load, scenario (ii), $k_h = 0$, soil A2.....	46
Figure 2.9. (a) Displacement, (b) strain, and (c) stress in ideally end bearing energy pile subjected to a combined mechanical and equivalent thermal load, scenario (i), $k_h = 0$, soil A2.....	48
Figure 2.10. (a) Displacement, (b) strain, and (c) stress in fully floating energy pile subjected to a combined mechanical and equivalent thermal load, scenario (ii), $k_h = 0$, soil A2.....	50

Figure 2.11. (a) Displacement, (b) strain, and (c) stress in fully floating energy pile subjected to a combined mechanical and equivalent thermal load, scenario (i), $k_h = 0$, soil A2	51
Figure 2.12. (a) Displacement, (b) strain, and (c) stress in semi-floating energy pile subjected to a combined mechanical and equivalent thermal load, scenario (ii), $k_h = 0$, soil A2	52
Figure 2.13. (a) Displacement, (b) strain, and (c) stress in semi-floating energy pile subjected to a combined mechanical and equivalent thermal load, scenario (i), $k_h = 0$, soil A2	53
Figure 2.14. Normalized location of thermal null point with respect to normalized k_b , $k_h = 0$, soils A2 and C	54
Figure 2.15. Mechanical and combined stresses versus depth for ideally end bearing energy pile subjected to loading scenario (i) and $k_h = 0$, a) $\psi=0.0384/\text{m}$ (soil A2), b) $\psi=0.1289/\text{m}$ (soil C).....	55
Figure 2.16. Normalized location of zero combined stress versus η for ideally end bearing energy pile, scenarios (i) and (iv), $k_h = 0$, soils A2 and C.....	56
Figure 2.17. Mechanical and combined stresses versus depth in nearly floating energy pile subjected to loading scenario (i) and $k_h = 0$, a) $\psi=0.0384/\text{m}$ (soil A2), (b) $\psi=0.1289/\text{m}$ (soil C).....	57
Figure 2.18. Normalized location of zero combined stress versus η for fully floating energy pile, scenarios (i) and (iv), $k_h = 0$, soils A2 and C.....	58
Figure 2.19. Mechanical and combined stresses versus depth in ideally end bearing energy pile for loading scenario (ii) and $k_h = 0$, (a) $\psi=0.0384/\text{m}$ (soil A2), (b) $\psi=0.1289/\text{m}$ (soil C) ...	59
Figure 2.20. Effect of head restraint (k_h) on normalized thermal stress ($\Delta T > 0$) in: (a) ideally end bearing energy pile (soil A2), (b) fully floating energy pile (soil A2).....	60
Figure 2.21. Effect of head restraint (k_h) on the normalized location of null point in fully floating and semi-floating energy piles, soils A2 and C	61
Figure 3.1. FBDs of the energy pile in layered soil profile subjected to thermal load ($\Delta T > 0$), a) the entire energy pile, b) an infinitesimal segment of energy pile above the null point	71
Figure 3.2 Flowchart depicting the analytical solution procedure for a single energy pile embedded in n soil layers, and subjected to thermal load.....	79
Figure 3.3 FBDs of the energy pile in layered soil subjected to mechanical load ($F < 0$), a) the entire energy pile, b) an infinitesimal segment of the energy pile	80

Figure 3.4. Flowchart depicting the analytical solution procedure for a single energy pile embedded in n soil layers, and subjected to mechanical load.....	85
Figure 3.5. Layered soil profile at Swiss Federal Institute of Technology, Lausanne	87
Figure 3.6. Analytical prediction of vertical pile head displacements versus time for test T1.....	88
Figure 3.7. Analytical prediction of axial strain versus depth for test T1 ($\Delta T = 13.4^\circ\text{C}$).....	89
Figure 3.8. Analytical prediction of axial stress versus depth for test T1 ($\Delta T = 13.4^\circ\text{C}$).....	89
Figure 3.9. Analytical prediction of axial stress versus depth for thermal load ($\Delta T = 14^\circ\text{C}$) and mechanical load ($F = -1000\text{ kN}$), test T7	90
Figure 3.10. Analytical prediction of axial strain versus depth for test T7 at $\Delta T = 14^\circ\text{C}$, $k_h =$ 0.0625 GPa/m	92
Figure 3.11. Analytical prediction of axial stress versus depth for test T7 at $\Delta T = 14^\circ\text{C}$, $k_h =$ 0.0625 GPa/m	92
Figure 3.12. Analytical prediction of axial strain versus depth for test T7 at $\Delta T = 14^\circ\text{C}$, no head restraint	93
Figure 3.13. Analytical prediction of axial stress versus depth for test T7 at $\Delta T = 14^\circ\text{C}$, no head	93
Figure 3.14. Location of the thermal null point for different values of spring tip stiffness	94
Figure 4.1 Load transfer functions: a) along the pile shaft, b) at the tip.....	107
Figure 4.2. FBDs of a semi floating energy pile subjected to a thermo-mechanical load, $F < 0$ and $\Delta T > 0$: (a) the entire pile, (b) an infinitesimal segment of the pile above the null point exhibiting reversible interface behavior, and (c) an infinitesimal segment of the pile above the null point exhibiting irreversible interface behavior	107
Figure 4.3. Flowchart depicting the procedure of derivation of analytical solutions for a single semi- floating energy pile exhibiting soil-pile interface yielding under thermal load.....	118
Figure 4.4. Validation of analytical solutions against experimental data for the heat sink pile at Lambeth College under heating ($\Delta T = 29.4^\circ\text{C}$): (a) thermally induced axial strain, and (b) thermally induced axial load	123
Figure 4.5 Comparison between analytical predictions and experimental data for the main test pile at Lambeth college under mechanical loading ($F < 0$)	124
Figure 4.6 Mechanism of development of mechanical residual: (a) displacement, (b) axial stress, and (c) shear stress in Lambeth College main pile subjected to mechanical loading.	125

Figure 4.7. Mechanism of development of thermal irreversible: (a) strain, (b) axial stress, and (c) shear stress in an end bearing energy pile subjected to heating (10°C beyond $\Delta T_{y,head}$)	127
Figure 4.8. Mechanism of development of thermal irreversible: (a) strain, (b) axial stress, and (c) shear stress in a fully floating energy pile subjected to heating (10°C beyond ΔT_y)	128
Figure 4.9. Comparison of thermal (a) displacement, (b) shear stress, and (c) axial stress under heating in ideally end bearing and fully floating energy piles ($\Delta T = 30^{\circ}\text{C}$)	130
Figure 4.10. Comparison of mechanical (a) displacement; (b) shear stress, and (c) axial stress under compressive mechanical load in an ideally end bearing fully floating energy pile ..	131
Figure 4.11. Progression of thermal soil pile interface yielding: (a) displacement, (b) shear stress, and (c) axial stress for an end bearing energy pile subjected to mechanical compression and heating	132
Figure 4.12. Progression of thermal soil pile interface yielding: (a) displacement, (b) shear stress, and (c) axial stress for a fully floating energy pile subjected to mechanical compression and heating	134
Figure 4.13. Progression of thermal soil pile interface yielding; (a) displacement, (b) shear stress, and (c) axial stress for a semi floating energy pile subjected to mechanical compression and heating	134
Figure 4.14. Progression of thermal soil pile interface yielding: (a) displacement, (b) shear stress, and (c) axial stress for a fully floating energy pile subjected to mechanical compression and cooling	136
Figure 5.2. Temperature distribution in for a) $\Delta T = 0^{\circ}\text{C}$ b) $\Delta T = +10^{\circ}\text{C}$ and c) $\Delta T = +15^{\circ}\text{C}$ with a deformation scale factor of 1500 (NT stands for nodal temperatures in $^{\circ}\text{C}$)	144
Figure 5.3. The effect of soil anisotropy, based on the computational model, on (a) displacement; (b) strain; and (c) stress for the thermal load ($\Delta T = +15^{\circ}\text{C}$)	148
Figure 5.4. a) Residuals vs predicted values and b) distribution of residuals of the regression analysis of computational data (5.16)	152
Figure 5.5. a) Residuals vs predicted values and b) distribution of residuals of the regression analysis of analytical data (Eq. (5.17))	152
Figure 5.6. Predicted versus actual (observed) values of maximum axial stress along the pile a) computational regression model Eq. ((5.16)), b) analytical regression model Eq. ((5.17)) 153	

Figure 5.7. The effect of soil anisotropy on the magnitude of maximum axial stress based on Eq. (5.17)	153
Figure 5.8. Predictions of axial strain in the full-scale Lambeth College main energy pile under 1200kN of compressive mechanical load	156
Figure 5.9. The effect of soil anisotropy reflected in analytical solutions on (a) displacement; (b) strain; and (c) stress for the thermal load ($\Delta T = +15^{\circ}\text{C}$)	157
Figure 5.10. Comparison of computational and analytical predictions for energy pile subjected to $\Delta T = +15^{\circ}\text{C}$ embedded into isotropic soil (A1) a) displacement b) strain and c) stress.....	159
Figure 5.11. Comparison of computational and analytical predictions for energy pile subjected to $\Delta T = +15^{\circ}\text{C}$ embedded in anisotropic soil (B) of a) displacement b) strain and c) stress..	160

List of Tables

Table 2.1. Displacement, strain and stress at the pile tip and pile head for mechanical and thermal loading, $k_h=0$	40
Table 2.2. Directions and signs of displacement, strain and stress in ideally end bearing, fully floating and semi-floating energy piles subjected to different loading scenarios ($k_h=0$).....	47
Table 3.1. Coefficients for $C_{iT,1}$	76
Table 3.2 Coefficients for x'_{oi}	78
Table 3.3 Temperature histories for tests T1 and T7	86
Table 3.4. Yielding check at the pile soil interface along the pile depth, Test T1.....	96
Table 3.5. Yielding check at the pile soil interface along the pile depth, Test T7.....	96
Table 3.6 Yielding check at the pile soil interface at the pile tip, Tests T1 and T7	97
Table 4.1. Summary of load transfer diagrams of energy piles presented by Bourne-Webb et al. (2009), Bourne-Webb et al. (2013), and Amatya et al. (2012)	102
Table 4.2. Summary of load transfer diagrams of energy piles presented by Rotta Loria and Laloui (2019)	103
Table 4.3. Summary of load transfer diagrams of energy piles presented by Saeidi Rashk Olia and Perić(2021a)	104
Table 5.1. Properties of soil and bedrock (Perić et al. 2017).....	158
Table 5.2. Properties of reinforced concrete (Laloui et al. 2006)	158

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Chapter 1 - Introduction

Background

Since the 1800s, energy consumption and supply have been one of the main contributors to environmental pollution. National and international policies and regulations are increasingly requiring the use of "environmentally friendly" technologies that reduce negative environmental impacts. Earth-contact structures such as thermo-active foundations can harvest renewable thermal energy from the ground and transport it to the surface where it can be used for space heating and cooling as well as for production of hot water. This advantageous technology that relies on use of naturally occurring renewable thermal energy available worldwide directly contributed to development of carbon-neutral buildings and infrastructure, thus ultimately meeting international standards for built environment.

Geothermal energy represents the second most abundant source of primary energy on Earth, after solar energy (Lee et al., 2007). It is classified as both renewable and sustainable (Lund, 2009), and represents one of the energy sources that can be used for development of low-carbon buildings. Energy piles, which belong to the group of thermo-active earth-contact structures are innovative, multifunctional foundations that can be used for energy transfer applications as well as for providing the structural support to superstructures. Similarly, to other shallow geothermal systems, energy piles involve low enthalpy and take advantage of the relatively constant temperature field in the shallow subsurface throughout the year for their heat exchanger role (Batini et al., 2015). Thus, the additional function of the energy piles, in addition to of their structural support involves transferring ground heat to the building in the winter and out of the building in the summer. A fluid circulation within the geothermal loop that is embedded within

the energy piles accomplishes this task. Other heating/cooling systems can supplement energy piles as needed.

The multifunctional role of energy piles implies simultaneous application of thermal and mechanical loads, whereby the former is expected to have a cyclic nature and vary significantly compared to the mechanical load that is expected to be constant after completion of the superstructure construction. This causes the pile shaft to undergo a relatively wide range of temperature changes throughout each cycle. Considering the restraints imposed on the pile by the surrounding soil, bedrock, or underlying soil layers, the free thermal expansion and contraction of the pile is restrained, thus leading to generation of compressive and tensile stresses in the pile respectively.

Thermal behavior of an energy pile under heating and cooling is shown in Figure 1.1. As shown the ability to contract under cooling and expand under heating is limited by the presence of the soil surrounding the energy pile, and the underlying soil layer.

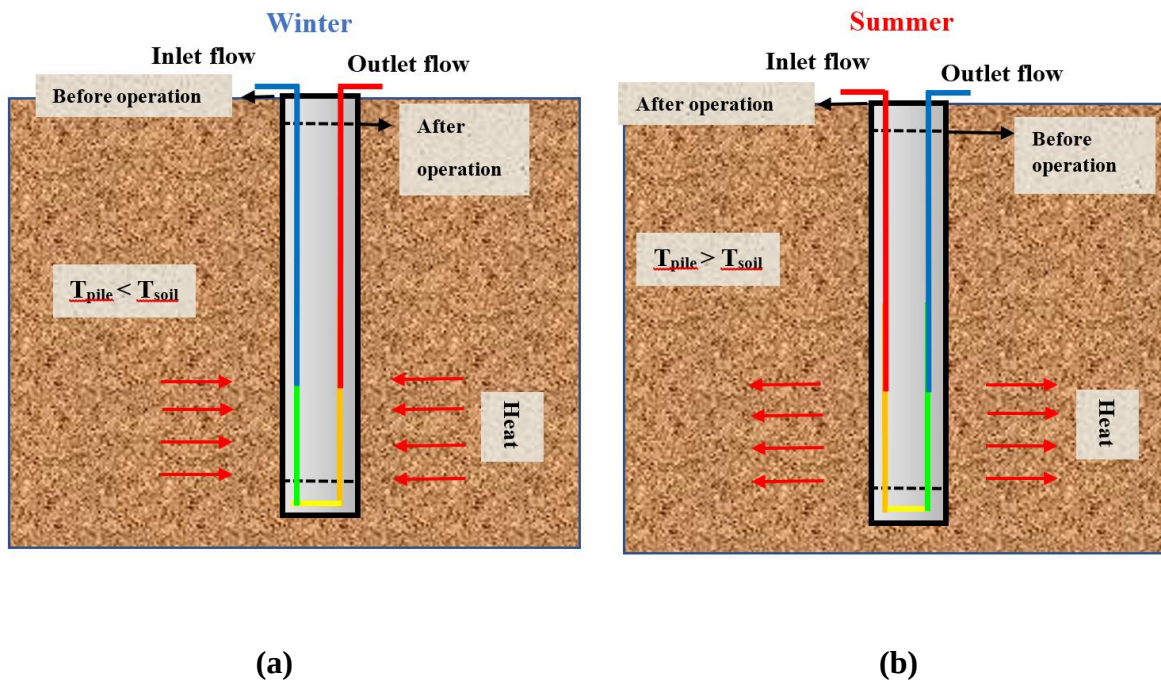


Figure 1.1 Thermal behavior of energy piles under (a) cooling, and (b) heating

The multifunctional nature of energy piles poses challenges linked to understanding their behavior and developing corresponding analysis and design methods. The unprecedented effects of combined application of mechanical and thermal loads require development of a novel theory capable of providing the solutions for load transfer that need to be validated against the experimental data. A qualitative and quantitative understanding of the thermomechanical displacement, strain and stress, and interface shear stress that develop in energy piles is crucial for their analysis, design and wider implementation in the engineering practice that will in turn provide increased reliance on clean energy while increasing the energy efficiency of the civil infrastructure.

To this end, the main focus of this study is on advancing the knowledge of the soil structure interaction in energy piles by employing analytical and computational modeling and including the model validation against the experimental data from both, full scale pile tests and geotechnical centrifuge tests.

Literature review

Three approaches have been employed to provide understanding of the thermal and thermomechanical soil structure interaction in energy piles, including physical, computational, and, more recently, analytical modeling. To this end, various researchers used different approaches in their attempts to provide what Rotta Loria and Laloui (2018) referred to as “thermomechanical schemes”, i.e. diagrams of displacement, strain and axial stress and shear stress evolutions versus depth. After these schemes, referred to herein as load transfer diagrams, are rigorously developed and validated they can provide a general framework for qualitative and quantitative understanding of the thermomechanical response of energy piles.

Physical modeling

A full-scale pile test was conducted at the Swiss Federal Institute of Technology (Laloui et al. 2006; Laloui. et al. 1999) at Lausanne, Switzerland. Only one pile out of 97 constructed on that site, which has an average diameter of 1m and approximate length of 26 m, was constructed as an energy pile. The bedrock at this site was encountered at a depth of 25.5 m, thus providing a restraint at the pile tip. Among several tests conducted on this pile, test T1 was performed prior to the construction of the building, while test T7 was conducted upon the completion of the construction. Thus, the pile in the former test experienced only thermal load, while in the latter case the pile experienced a combined thermomechanical load. Both tests included a heating-cooling cycle during which the pile remained in a net heated state. The axial compressive force imposed by the building during the test T7 was reported to be of 1000 kN (Knellwolf et al. 2011). The energy pile was embedded in four layers that Perić et al. (2017) classified as low plasticity clays (CL) based on the laboratory test results reported by Laloui et al. (1999).

Another full-scale pile test, which was conducted at the Lambeth College in London, incorporated temperature cycles while under an extended period of maintained loading. Two piles were used in this test, a main pile having a length of 23 m, and a heat sink pile with a length of 30 m. For the most part these piles were embedded in the London clay that produced negligible restraints at the pile tips, thus rendering them floating piles. The recorded thermal strains in the main pile and the heat sink pile provided valuable data for model validations.

Murphy et al. (2015) conducted a full-scale energy pile test on an end bearing energy pile with a length of 15.2 m and a diameter of 0.61 m at the shower-shave building constructed at the U.S. Air Force Academy (USAFA). An axial mechanical load of 833 kN was applied to the pile before heating in the amount of 6, 12, and 19°C were applied to the pile.

Geotechnical centrifuge modeling has also been employed to explore the thermally induced response of energy piles. Centrifuge modeling relies on the concept of a geometric similitude. The concept assumes that a full-scale prototype soil layer experiences the same stress state as a model scale soil layer that is spinning in a geotechnical centrifuge at a centripetal acceleration that is N times larger than that of Earth's gravity (Ko 1988; Taylor 1995). The model is N times smaller than the prototype whereby N corresponds to the g-level of the centrifuge test. Geometric similitude can be employed to extrapolate the load-settlement behavior and thermal soil-structure interaction phenomena of model-scale energy foundations to full-scale prototype foundations. To this end, Stewart and McCartney (2013), Goode and McCartney (2015), and Ng et al. (2015) conducted thermal experiments on model-scale energy piles in the geotechnical centrifuges and reported the axial strain stress and displacements under different thermal loads.

Stewart and McCartney (2013) tested the concrete model of a single end bearing energy pile having a diameter of 50.8 mm and length of 533.4 mm in the geotechnical centrifuge at the 24 g-level, thus corresponding to the prototype pile with a diameter of 1.22 m and 12.8 m length. The pile was embedded in a layer of unsaturated Bonny silt and was heated to 5°C, 10°C, 15°C, and 19°C. The energy pile exhibited the minimum magnitude of a displacement of almost zero at its tip, and the maximum stress magnitude near the pile tip. In addition, Goode and McCartney (2015) tested the semi floating concrete energy pile having a diameter of 63.5 mm and length of 342.9 mm in the geotechnical centrifuge at a g-level of 24. The corresponding prototype energy pile, thus, had a length of 8.2 m and a diameter of 1.5 m. The pile was embedded in Nevada sand and heated to 7°C, 12°C, and 18°C. In this case, the null point was observed to be located near the mid-depth of the pile, where the energy pile also experienced the maximum stress. Also, Ng et al. (2015) tested instrumented, semi floating aluminum alloy model pipe piles with an external

diameter of 22mm and length of 600mm that were embedded in saturated sand at a centrifuge g-level of 40. Thus, the corresponding prototype-scale energy pile has a length of 19.6 m and a diameter of 0.88 m and was heated to 15°C and 30°C. They observed the movement of the null point from the mid-depth of the energy pile towards the pile tip as the temperature in the pile increased.

Small-scale experimental programs also provided information about thermomechanical behavior of energy piles. To this end, Kalantidou et al. (2012) conducted a series of preliminary small-scale tests on an energy pile subjected to different mechanical loads while undergoing several thermal loading cycles. Their findings showed that the pile response remains reversible once the global factor of safety of the pile under mechanical load is above 2.5, and they observed an irreversible displacement in the pile shaft once the applied mechanical load exceeded 40% of the pile axial mechanical capacity. They did not explain the nature of the irreversible displacement, and did not link it to the behavior of the soil-pile interface. In another study, Elzeiny and Suleiman (2021) investigated the impact of the number of thermal loading cycles on the pull-out capacity of a small-scale energy pile. Their findings indicate that although thermal cycles lead to a decrease in the pull-out capacity of the energy pile, the impact of 100 cycles was not significantly larger than the impact of 5 cycles of cooling with the same thermal load. Additional experimental studies were conducted worldwide in recent years to further understand the behavior of energy piles (Brandl 2006, Bourne-Webb et al. 2009, Adam, and Markiewicz 2009, Wood et al. 2009, Amatya et al. 2012, Akrouch et al. 2014, Sung et al. 2018, Fang et al. 2020, Moradshahi et al. 2021, and Yang et al. 2022).

Numerical and computational modeling

Different numerical and computational models have been used for decades to analyze and design foundations subjected to mechanical loads. Since the introduction of energy piles, many researchers have worked on advancing the understanding of thermal, mechanical, and thermomechanical responses of these foundations by employing different numerical approaches.

Laloui et al. (2006) developed a fully coupled thermo-hydro-mechanical Finite Element (FE) code that models thermomechanical response of energy piles. They used the code to obtain thermally induced stress along the energy pile tested at the Lausanne site. Furthermore, they concluded that 1°C of temperature increment could induce 100 kN of axial force to the pile shaft. While this information may be useful, the particular numerical value is applicable only to that site and that energy pile, and it cannot be generalized. Di Donna et al. (2016) and Perić et al. (2017) later corrected the values of the isotropic elastic parameters for the Lausanne site that were initially reported by Laloui et al. (2006). In addition, during the model calibration procedure, Perić et al. (2017) discovered the anisotropic nature of the two bottom layers at the Lausanne site. Upon a subsequent calibration of a selected anisotropic elastic model and use of the corresponding computational model they obtained improved agreement with the experimental data at the Lausanne site.

Knellwolf et al. (2011) used Finite Difference Method (FDM), thus enabling them to consider several soil layers. In their FDM model, the surrounding soil was replaced by a series of discrete shear springs and a normal spring at the pile tip. Knellwolf et al. (2011) also modeled the restraint to the free pile head movement that is induced by the presence of a superstructure during heating.. They used this model to predict the response of the energy piles at the Lausanne site

whereby the stiffness of shear springs representing the action of the soil on the pile was determined from the results of the full-scale tests (Laloui et al. 1999).

Chen and McCartney (2016) and Moradshahi et al. (2020) developed nonlinear T-Z and the Q-Z response curves and included them in their FD models. Furthermore, they pointed out that capturing the accurate value of the zero-displacement location is important for successful analysis of energy piles.

Recently, Arzanfudi et al. (2020) created a new thermo-hydro-mechanical finite element model for energy piles under cyclic thermal loading, incorporating features such as three-dimensionality, embedded heat exchangers, soil constitutive modeling, and pile-soil interface. In another study, Iodice et al. (2023) explored how different constitutive models impact the axial force development of energy piles and developed an efficient design method for energy piles under thermal loading.

Numerical and computational models of energy piles with different levels of complexity including constitutive relationships, and pile-soil interaction models continue to be developed and used to better capture thermomechanical displacement, strain and stress in energy piles. While these methods provide a valuable insight into the response of energy piles, a significant time and expertise, which are required to address numerical simulations coupled with calibration of advanced soil constitutive models that requires a comprehensive laboratory investigation program makes these highly advanced models daunting even for the scientific community, let alone the engineering practice. On the contrary, analytical solutions are much simpler and typically require calibration of the smaller number of parameters. Furthermore, they provide in depth qualitative and quantitative understanding of the load transfer diagrams. In addition, the analytical solutions

employ a continuous shear spring along the pile shaft unlike the discrete shear that is used in FD models.

Load transfer diagrams

Distribution of thermal, mechanical, and thermomechanical axial displacement, strain, stress as well as shear stress along the energy pile are referred to as load transfer diagrams. These diagrams can provide valuable qualitative and quantitative insights into the thermomechanical behavior of energy piles and serve as a basis for validation of more advanced numerical and computational models. Prior to this study, several authors (Bourne-Webb et al., 2009; Bourne-Webb et al., 2013; Amatya et al., 2012, Rotta Loria and Laloui, 2018) presented load transfer diagrams for energy piles that are embedded in a single soil layer. These authors assumed that temperature was uniform throughout the pile. Bourne-Webb et al. (2009), Bourne-Webb et al. (2013), and Amatya et al. (2012) simply postulated their load transfer diagrams based on the assumption of the uniform shaft resistance (q_s) along the pile length. These diagrams, which they referred to as "a simple set of mechanisms," exhibit linear evolutions of axial force, strain, and restrained thermal strain versus depth in the case of mechanical loading, while these evolutions are bilinear in the case of a combined thermomechanical loading. Nevertheless, Bourne-Webb et al. (2009), Bourne-Webb et al. (2013), and Amatya et al. (2012) later neglected their own assumption of a constant magnitude of a shaft resistance versus depth in the case of a combined thermomechanical loading while Amatya et al. (2012) neglected this assumption also in the case of thermal loading without any clarification or justification. Furthermore, it is noted that Bourne-Webb et al. (2013) used the restrained portion of thermal strain (ϵ_{T-Rstr}) to denote both the truly restrained portion of thermal strain, which does not materialize due to the presence of the soil around the pile, and mechanically induced strain that materializes. Amatya et al. (2012) presented

both ε_{T-Rstr} and ε_{T-Obs} , whereby the latter denotes the actual, observed strain. Finally, Bourne-Webb et al. (2013) also stated that the pile has a constant cross-sectional area and that it is made from a linear elastic material. Similarly, Amatya et al. (2012) also stated that the pile has a constant cross-sectional area and that it is installed in a rigid plastic soil.

Rotta Loria and Laloui (2018) stated that the number of conditions of validity of load transfer diagrams described by the abovementioned authors has never been explicated. They proposed "thermomechanical schemes" for the response of single energy piles that, according to them, are based on the thermo-elasticity theory. Rotta Loria and Laloui (2018) presented an approximately linear evolution of axial stress in the pile and approximately constant interface shear stress in the case of mechanical loading, similarly to the corresponding evolutions presented by Bourne-Webb et al. 2009; Bourne-Webb et al. 2013; Amatya et al. 2012. Nevertheless, in the case of thermal loading and combined thermomechanical loading, the stress evolution with depth is non-linear, while evolutions of shear stress and vertical displacements are linear, according to Rotta Loria and Laloui (2018). None of the mentioned authors, however, provided a quantitative mechanics-based framework for their presented load transfer diagrams.

Analytical solutions

Analytical solutions are based on a rational mechanical approach and a corresponding set of assumptions. Analytical solutions that have been successfully validated against the experimental data, represent an effective approach for addressing the load-transfer diagrams of energy piles. Despite the accuracy and applicability of these solutions, the solutions that account for irreversible soil pile interface behavior became available only recently (Cossel 2019, Perić et al. 2020, and Iodice et al. 2020) for Furthermore, these solutions alone can also provide load transfer diagrams they also need to be extended by accounting for the irreversible soil pile interface response. Unlike

previously provided load transfer diagrams by (Bourne-Webb et al. 2009, Bourne-Webb et al. 2013, Amatya et al. 2012, and Rotta Loria and Laloui 2018) that were based solely on certain postulates and experimental data trends, the mechanics-based analytical solutions are based on rational, mechanics-based approach.

Research objectives

After conducting a detailed literature review on the state of knowledge on thermally induced stress in the energy pile, it was understood that very few studies provided analytical solutions for predicting the energy pile response to thermal and mechanical loads. Additionally, these studies did not comprehensively address the response for different end restraint conditions, including fully floating, semi floating, and end bearing energy piles with and without the head restraint imposed by a superstructure. Furthermore, use of physical and numerical models inherently limits the corresponding experiments and analyses in number while impeding the development of a qualitative understanding.

Consequently, the main aim of this research is to develop and use analytical solutions to provide a fundamental, rational, qualitative and quantitative, mechanics-based understanding of the thermomechanical load transfer mechanism in fully floating, semifloating, and end bearing piles. To this end, one set of solutions was derived for single energy piles embedded a homogenous soil profile with a reversible soil pile interface. Another set of solutions is developed for energy piles exhibiting irreversible soil-pile interface behavior. Another set of solutions was derived for a multi-layered soil profile whereby the soil pile behavior is reversible. The capability of all aforementioned solutions for predicting a response of a single energy pile is evaluated through validation against the experimental data.

Furthermore, fully coupled thermo-hydro-mechanical finite element analyses and analytical modeling were combined with advanced statistical modeling to provide deeper understanding of the key parameters affecting the development of axial stress in the energy piles. In addition, these analyses enabled development of statistical models that correlate parameters of a continuum employed in FE modeling and elastic parameters of a discrete approach used in the analytical solutions. Correlations were developed for isotropic and cross-anisotropic soils. This research has resulted in several peer-reviewed journal and international conference publications that have garnered interest and attention. The journal publications construct different chapters of this dissertation. Additional papers published as conference proceedings are provided in the list of references.

Governing equations and analytical solutions

In this section, a comprehensive extended derivation of solutions for thermal displacement, strain and stress of a semi floating energy pile with reversible interface behavior is provided. This derivation will help in delineating all the steps, thus providing the context upon which the analytical solutions are derived.

Assumptions of this study

The main assumptions and laws of mechanics, which provide basis of the analytical solutions found herein, are summarized as follows:

- The 1D formulation is employed whereby the single coordinate is oriented in the vertical direction.
- Temperature change is uniform throughout the entire pile at any given time.
- Higher order terms in the kinematic relationship are neglected.
- The energy pile is assumed to follow a thermo-elastic constitutive law.

- For the solutions employing reversible soil pile interface, the interface is assumed to remain in the elastic range, whereby the shear stress at the interface is a linear function of the vertical shaft displacement, and normal stress at the pile tip is a linear function of the vertical tip displacement.
- For the solutions employing irreversible soil pile interface, the interface is assumed to follow an elastic-perfectly plastic shear stress-axial displacement relationship.
- Temperature change does not affect material properties of the soil and pile.

It is noted that Laloui et al. (2003) stated that the vertical displacements of energy piles are dominant in comparison with the horizontal displacements.

The physical model of energy pile, which is employed herein, is presented in Figure 1.2. As depicted, the continuous shear spring having stiffness k_s represents the restraining effect of the surrounding soil, and normal springs at the pile head and pile tip having stiffness of k_h and k_b represent restraining effects from the superstructure and bedrock, respectively. The coordinate system for analytical solutions is set to have $x = 0$ at the pile tip with the positive upward direction.

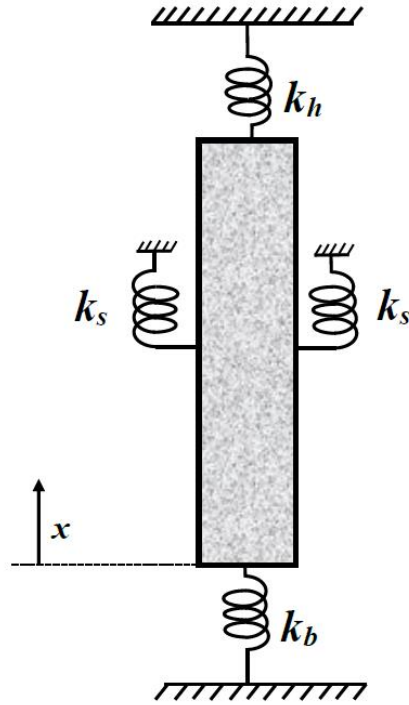


Figure 1.2 Schematic of the physical model of energy pile (after Saeidi Rashk Olia and Perić 2021c)

Similarly, to Knellwolf et al. (2011) and Perić et al. (2020), the self-weight of the pile is neglected in this study.

Positive displacement is directed upward, while compressive axial strain and stress are both negative. Furthermore, heating of the pile induces a positive temperature change while compressive axial force is negative and vice versa, cooling induces negative temperature change while tensile axial force is positive.

The kinematic relationship for the pile corresponding to the infinitesimal strain is given by

$$\varepsilon = \frac{du}{dx} \quad (1.1)$$

where $u = u(x)$ is the axial (or vertical) displacement, and $\varepsilon = \varepsilon(x)$ is the axial strain. The thermo-elastic constitutive law is given by

$$\sigma = E(\varepsilon - \alpha\Delta T) \quad (1.2)$$

where $\sigma = \sigma(x)$ is the axial stress. In the 1D formulation used herein u , ε and σ are simply referred to as displacement, strain, and stress. The elastic modulus and the coefficient of thermal expansion of the pile are denoted by E and α , respectively, while ΔT is a temperature difference of the pile relative to the surrounding soil.

In the case of a reversible soil pile interface, the interface along the pile shaft is modeled with a continuous linear elastic shear spring whereby the magnitude of the interface shear stress, $\tau = \tau(x)$ is given by

$$|\tau| = k_s |u| \quad (1.3)$$

and k_s is the elastic stiffness of a continuous shear spring, which is constant with depth for a given soil layer. Load transfer at the pile tip is modeled by the normal elastic spring having stiffness k_b , thus resulting in

$$\sigma(0) = k_b u(0) \quad (1.4)$$

Eq. (1.4) represents the boundary condition for a semifloating pile, whereby neither the stress nor the displacement at the pile tip is equal to zero. For comparison, tip displacement is equal to zero in an ideally end bearing pile, while in the case of an ideally fully floating pile, the tip stress is equal to zero. Such piles are referred to as an end bearing and fully floating piles in this study.

The Free Body Diagrams (FBDs) of an entire semi-floating energy pile and its infinitesimal segments representing the portions above and below the thermal null point are shown in Figure 1.3 for $\Delta T > 0$.

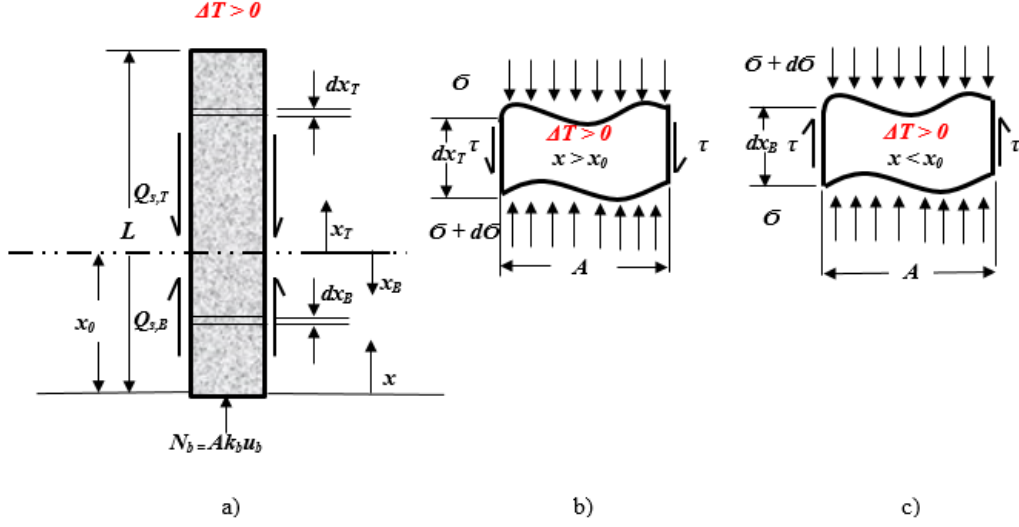


Figure 1.3 FBDs of a semi floating energy pile subjected to $\Delta T > 0$, a) the entire pile, b) an infinitesimal segment of the pile above the null point, c) an infinitesimal segment of the pile below the null point (after Saeidi Rashk Olia and Perić 2021a)

In the case of a semi floating pile, both stress and displacement develop at the pile tip. Thus, the null point is located somewhere in the interior of the pile length. As shown in Figure 1.3, the thermal null point (x_0) is also the origin of both local coordinate systems, x_B , and x_T .

By considering the infinitesimal pile segments (Figure 1.3 b and c), vertical equilibrium is enforced by

$$A(\sigma_i + d\sigma_i) - \tau_i p dx_i - A\sigma = 0 \quad \Rightarrow \quad Ad\sigma - \tau p dx_i = 0 \quad (1.5)$$

where p and A are the perimeter and cross-sectional area of the pile, respectively. Then by replacing τ with its definition in Eq. (1.3), and considering the local coordinate system, where $u_i = u_i(x_i)$, and $\sigma_i = \sigma_i(x_i)$ the equilibrium in Eq. (1.5) takes the following form

$$Ad\sigma_i(x_i) - k_s u_i(x_i) p dx_i = 0 \quad i = T \text{ and/or } B \quad (1.6)$$

where B and T denote the regions of the pile located below and above the thermal null point, respectively. The following holds (Figure 1.3) .

$$-x_0 \leq x_B \leq 0 \quad \text{and} \quad 0 \leq x_T \leq L - x_0 \quad (1.7)$$

Combining Eqs. (1.1) and (1.2) gives

$$\sigma_i(x_i) = E \left[\frac{du_i(x_i)}{dx_i} - \alpha \Delta T \right] \quad i = T \text{ and/or } B \quad (1.8)$$

Substituting Eq. (1.8) into Eq. (1.6) results in

$$AE d \left[\frac{du_i(x_i)}{dx_i} - \alpha \Delta T \right] - k_s u_i(x_i) p dx_i = 0 \quad i = T \text{ and/or } B \quad (1.9)$$

and subsequently

$$AE \frac{d^2 u_i(x_i)}{dx_i^2} - k_s p u_i(x_i) dx_i = 0 \quad i = T \text{ and/or } B \quad (1.10)$$

Dividing Eq. (1.10) by AE results in

$$\frac{d^2 u_i(x_i)}{dx_i^2} - \frac{k_s p}{AE} u_i(x_i) dx_i = 0 \quad i = T \text{ and/or } B \quad (1.11)$$

Next, parameter ψ , which contains the pile geometry in the form of ξ_g and the ratio of the stiffness of the soil and the pile in the term ξ_s , is introduced as

$$\psi^2 = \left(\frac{p}{A} \right) \left(\frac{k_s}{E} \right) = \xi_g \xi_s \quad (1.12)$$

and substituting Eq. (1.12) into Eq. (1.11) results in

$$\frac{d^2 u_i(x_i)}{dx_i^2} - \psi^2 u_i(x_i) = 0 \quad i = B \text{ and/or } T \quad (1.13)$$

Solving for the unknown function $u_i(x_i)$ from Eq. (1.13) gives

$$u_i(x_i) = C_{1i} e^{\psi x} + C_{2i} e^{-\psi x} \quad i = B \text{ and/or } T \quad (1.14)$$

Continuity of stresses and displacement at the null point, in terms of local coordinates is given by

$$u_T(0) = u_B(0) = 0 \quad \text{and} \quad \sigma_T(0) = \sigma_B(0) \quad (1.15)$$

Enforcing the continuity of displacements at the null point given in Eq. (1.15) results in the following

$$C_{1i} = -C_{2i} \quad i = B \text{ and/or } T \quad (1.16)$$

and subsequently

$$u_i(x_i) = 2C_{1i} \sinh(\psi x_i) \quad i = B \text{ and/or } T \quad (1.17)$$

The boundary condition at the pile head in terms of a global coordinate x is given by

$$\sigma(L) = -k_h u(L) \quad (1.18)$$

Enforcing the continuity of stress at the null point results in

$$C_{1T} = C_{1B} \quad (1.19)$$

The boundary condition at the pile head in terms of a local coordinate is given by

$$\sigma_T(L - x_0) = -k_h u_T(L - x_0) \quad (1.20)$$

Combining Eqs. (1.20) (1.17), (1.1) and (1.2) leads to

$$2E\psi C_{1T} \cosh[\psi(L - x_0)] - E\alpha\Delta T = -2k_h C_{1T} \sinh[\psi(L - x_0)] \quad (1.21)$$

from which C_{1T} is obtained as

$$C_{1T} = \frac{\alpha\Delta T}{2\left[\psi \cosh[\psi(L - x_0)] + \frac{k_h}{E} \sinh[\psi(L - x_0)]\right]} \quad (1.22)$$

Substituting Eq. (1.22) into Eq. (1.17) results in $u_i(x_i)$ that is given by

$$u_i(x_i) = \frac{\alpha\Delta T \sinh(\psi x_i)}{\psi \cosh[\psi(L - x_0)] + \frac{k_h}{E} \sinh[\psi(L - x_0)]} \quad i = B \text{ and/or } T \quad (1.23)$$

The relationship between the local and global coordinates given by

$$x_i = x - x_0 \quad i = B \text{ and/or } T \quad (1.24)$$

By incorporating Eq. (1.24) along with the kinematic relationship and thermo-elastic constitutive law solution for displacement, strain and stress in the entire pile is given by

$$u(x) = \frac{\alpha\Delta T \sinh[\psi(x - x_0)]}{\psi \cosh[\psi(L - x_0)] + \frac{k_h}{E} \sinh[\psi(L - x_0)]} \quad (1.25)$$

$$\varepsilon(x) = \frac{\alpha\Delta T \cosh[\psi(x - x_0)]}{\cosh[\psi(L - x_0)] + \frac{k_h}{E\psi} \sinh[\psi(L - x_0)]} \quad (1.26)$$

$$\sigma(x) = E\alpha\Delta T \left[\frac{\cosh[\psi(x-x_0)]}{\cosh[\psi(L-x_0)] + \frac{k_h}{E\psi} \sinh[\psi(L-x_0)]} - 1 \right] \quad (1.27)$$

The solution for x_0 is obtained by enforcing the boundary condition at the pile tip given in Eq.(1.4), thus resulting in

$$x_0 = \frac{1}{\psi} \tanh^{-1} \left[\frac{\cosh(\psi L) - 1 + \frac{k_h}{E\psi} \sinh(\psi L)}{\sinh(\psi L) + \frac{k_b}{E\psi} + \frac{k_h}{E\psi} \cosh(\psi L)} \right] \quad (1.28)$$

Details of this derivation were provided in this chapter so that the reader can refer to them if needed while reading the remaining chapters.

Chapter 2 - Thermomechanical Soil Structure Interaction in Single Energy Piles Exhibiting Reversible Interface Behavior

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Abstract

Analytical solutions for displacement, strain and stress in a single energy pile provide rational, mechanics based qualitative and quantitative understanding of thermomechanical load transfer mechanism in energy piles. To this end, thermomechanically induced axial displacement, strain and stress evolutions are derived for different tip restraints and for several different loading scenarios including net heating, net cooling, and compressive and tensile mechanical loads. In addition, the analytical solutions are successfully validated against centrifuge test results.

The analytical solution for the location of a thermal null point indicates that it depends on the geometry and stiffness of the pile, stiffness of the soil, and amount of head and tip restraints. In addition, the thermal null point moves upward into the top half of the pile with increase in the ratio between the amounts of head and tip restraints. It is also found that formation of a tension zone in energy piles is likely for several different load scenarios. At a constant mechanical load, the length of a tension zone depends on the geometry of the pile, ratio of the soil and pile stiffness, and on the magnitude of thermal load. An increased magnitude of a net cooling combined with a constant compressive axial force increases the length of a tension zone and magnitude of tensile stress in the pile. On the contrary, the length of a tension zone and magnitude of tensile stress decrease with increase in magnitude of heating at constant tensile axial force. In summary, the analytical

solutions and their features presented herein provide a fundamental, rational, mechanics based framework for advancing the understanding of a load transfer mechanism and soil structure interaction in energy piles, thus contributing directly towards their safer design and wider use, and increased sustainability of civil engineering infrastructure.

Key words: Analytical solution, Energy pile, Thermo-elasticity, Thermomechanical, Soil structure interaction

Introduction

In addition to enabling safe transfer of structural loads to the underlying soils and rocks, piles by virtue of their length also provide a direct access to renewable geothermal energy in the shallow subsurface. To enable transfer of thermal energy between the subsurface and superstructures a closed loop tubing, which serves as a fluid conduit for the heat exchange fluid, is inserted in reinforced concrete piles. Consequently, these multifunctional foundations known as energy piles, are sustainable foundations that use a low enthalpy geothermal energy for a supplemental space heating and cooling, thus improving the energy efficiency of buildings while decreasing the emission of carbon dioxide into the atmosphere.

A multifunctional nature of energy piles implies a simultaneous application of thermal and mechanical loads whereby the former is expected to vary more significantly than the latter. It is because a free thermally induced deformation of energy pile is inhibited by the surrounding soil that thermal load induces additional displacement, strain and stress in energy piles, thus necessitating the in depth understanding of the load transfer mechanism for thermal and mechanical loads. To this end, three different approaches have been employed including physical and numerical modeling, and more recently analytical modeling.

The load transfer diagrams presented by Bourne-Webb et al. (2009), Amatya et al. (2012) and Bourne-Webb et al. (2013) are based on the response observed during full scale in situ tests. Furthermore, all their diagrams employ the assumption of a constant shaft resistance versus depth. Depicted axial force and strain are linear functions of depth while pile displacements were not included.

Bourne-Webb et al. (2009) presented schematic diagrams for a fully floating energy pile without a head restraint subjected to a thermal, mechanical and combined thermomechanical loads. The shapes of their thermally induced load and strain distributions are equal to each other, thus indicating that the depicted strain is a so-called restrained strain or a suppressed portion of the strain.

Amatya et al. (2012) presented load transfer diagrams for a fully floating pile with and without head restraint subjected to thermal, mechanical and combined loads. They also showed load transfer diagrams for a semifloating pile with and without head restraint. The former was subjected to a thermal loading only while the latter was subjected to thermal, mechanical and combined loading. Actual strain, which they referred to as an observed strain, was shown only for thermal loading in the form of heating. Otherwise, for combined loading only a thermal restrained strain was depicted.

Bourne-Webb et al. (2013) adopted a simplified approach similar to the one presented by Bourne-Webb et al. (2009). They presented diagrams for a thermal, mechanical and combined loading in the absence and in the presence of a head restraint for fully floating and semifloating piles.

Rotta Loria and Laloui (2019) stated that a number of conditions of validity of the schematic load transfer diagrams provided by Bourne-Webb et al. (2009) and Bourne-Webb et al.

(2013) have never been explained. Furthermore, they pointed out that the new load transfer diagrams, which they presented, were based on thermo-elasticity. These schematics differ from those previously proposed by Bourne-Webb et al. (2009), Amatya et al. (2012) and Bourne-Webb et al. (2013). Rotta Loria and Laloui (2018) presented load transfer diagrams for a fully floating pile with and without head restraint subjected to mechanical, thermal and combined loading. In addition, they depicted load transfer diagrams for a semi floating pile with and without head restraint subjected to mechanical, thermal and combined loading. No analytical expressions were given for any of these functions. The presented distribution of mechanically induced pile stress is linear with depth while the distribution of thermally induced pile stress is non-linear. Furthermore, their distribution of mechanically induced pile displacement is constant versus depth while distribution of thermally induced displacement is linear. Finally, distribution of mechanically induced interface shear stress is constant with depth while the distribution of thermally induced interface shear stress is linear versus depth.

To our best knowledge Cossel (2019) was the first to derive analytical solutions for axial displacement, strain and stress in ideally end bearing and semifloating energy piles with and without head restraints. The pile is embedded in homogeneous and stratified soil profiles, and subjected to thermal and mechanical loads. The solutions assumed that soil pile interface remains in the elastic regime while the pile obeys thermo-elastic constitutive law.

Iodice et al. (2020) derived analytical solution for axial displacement, force and interface shear stress in a single energy pile embedded into homogeneous and two-layered soils, and for soil whose stiffness increases linearly with depth. They also assumed that the pile is thermo-elastic material while the soil pile interface response remains in the elastic regime.

In summary, very few studies provided analytical solutions for predicting the energy pile response to thermal and mechanical loads. The studies did not comprehensively address the response for different end restraint conditions including floating, semifloating and end bearing energy piles with and without the head restraint imposed by a superstructure. Furthermore, previous use of physical and numerical models inherently limits the corresponding experiments and analyses in a number while impeding development of a qualitative understanding. To this end, the main goal of this study was to develop and use analytical solutions to provide fundamental, rational, qualitative and quantitative, mechanics-based understanding of the thermomechanical load transfer mechanism in fully floating, semifloating, and end bearing piles. Different loading scenarios are considered including compressive and tensile axial forces, and cooling and heating. For graphical presentations magnitudes of thermal and mechanical loads were carefully selected by using the concept of equivalent thermal load (Perić et al. 2020) that enables a direct conversion between thermal and mechanical loads. In addition, solutions for the location of a thermal null point and length of a tension zone are also derived and illustrated graphically. The latter brings attention to ensuring the structural integrity of reinforced concrete energy piles subjected not only to compressive but also to tensile stresses, whereby several loading scenarios that lead to generation of tensile stresses are identified.

Governing laws and assumptions

Traditional load transfer analyses for mechanical loads were proposed by Seed and Reese (1957) and Coyle and Reese (1966). Presently, various load transfer functions are available in the literature (Randolph and Wroth 1978; Frank and Zhao 1982; Armaleh and Desai 1987; Frank et al. 1991, Plaseied 2012; Suryatriyastuti et al. 2014).

Laloui et al. (2003) and Bourne-Webb et al. (2009) found from the full scale in situ tests, and Laloui et al. (2006), Knellwolf et al. (2011), Ozudogru et al. (2015), Perić et al. (2017), Iodice et al. (2020) found from the numerical and analytical models that energy piles exhibited fully reversible behavior, thus remaining in the elastic state during the full scale field tests. Consequently, it is assumed herein that the pile obeys thermo-elastic constitutive law while the soil pile interface remains in the elastic regime.

Assumptions and laws of mechanics upon which the analytical solutions found herein are based are summarized as follows:

- 1D formulation is used whereby the single coordinate is oriented in the vertical direction.
- Temperature change is uniform throughout the entire pile at any given time.
- Equilibrium in vertical direction is enforced.
- Higher order terms in kinematic relationship are neglected.
- The energy pile is assumed to follow a thermo-elastic constitutive law.
- The soil pile interface is assumed to remain in elastic range whereby the shear stress at the interface is a linear function of the vertical shaft displacement and normal stress at the pile tip is a linear function of the vertical tip displacement.
- Material properties of the soil and pile are not affected by temperature.

It is noted that Laloui et al. (2003) stated that the vertical displacements of energy piles are dominant in comparison with the horizontal displacements. Furthermore, the successful validation of the analytical solutions presented herein against the two different centrifuge tests performed on nearly ideally end bearing and semifloating energy piles demonstrates that these assumptions are reasonable.

Both, the positive x -axis and positive displacement are directed upward while compressive axial strain and stress are both negative. Furthermore, heating of the pile induces a positive temperature change while compressive axial force is negative.

The kinematic relationship for the pile and infinitesimal strain is given by

$$\varepsilon = \frac{du}{dx} \quad (2.1)$$

where $u = u(x)$ is the axial (or vertical) displacement, and $\varepsilon = \varepsilon(x)$ is the axial strain. The thermo-elastic constitutive law is given by

$$\sigma = E(\varepsilon - \alpha \Delta T) \quad (2.2)$$

where $\sigma = \sigma(x)$ is the axial stress. In 1D formulation used herein u , ε and σ are simply referred to as displacement, strain and stress. The elastic modulus and the coefficient of thermal expansion of the pile are denoted by E and α respectively while ΔT is a temperature difference of the pile relative to the surrounding soil.

The soil pile interface along the pile shaft is modeled with a continuous linear elastic shear spring whereby the magnitude of the interface shear stress, $\tau = \tau(x)$ is given by

$$|\tau| = k_s |u| \quad (2.3)$$

and k_s is the elastic stiffness of a continuous shear spring, which is constant with depth for a given soil layer. Load transfer at the pile tip is modeled by the normal elastic spring having stiffness k_b , thus resulting in

$$\sigma(0) = k_b u(0) \quad (2.4)$$

As depicted in Figure 2.1 the restraining effects of the surrounding soil and the superstructure on the pile are represented by the continuous shear spring having stiffness k_s , and normal springs at the pile head and pile tip having stiffness k_h and k_b respectively, with $x=0$ at the pile tip. The normal spring at the pile head represents the restraint to the free movement of the pile

head imposed by the superstructure. As explained by Knellwolf et al. (2011) it is activated solely by thermal loading. Similarly, to Knellwolf et al. (2011) and Perić et al. (2020) the self-weight of the pile is not taken into account in this study.

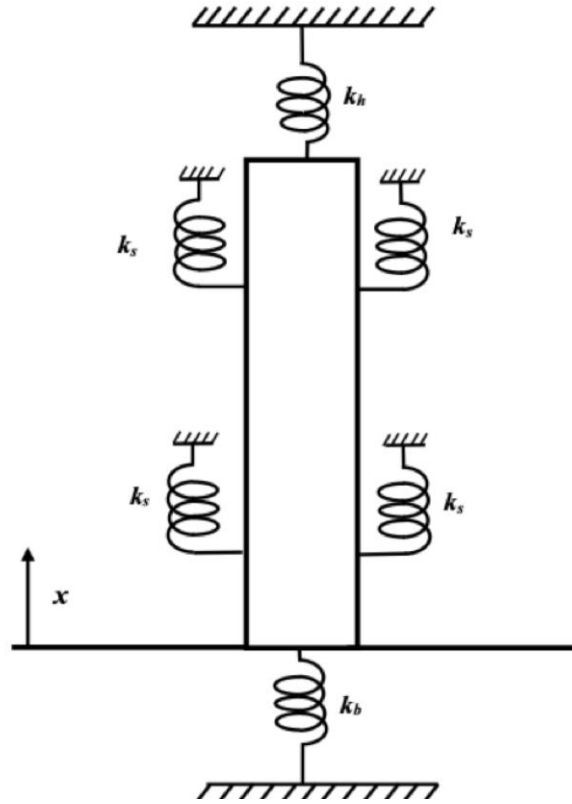


Figure 2.1. Proposed model of energy pile

Eq. (2.4) represents the boundary condition for a semifloating pile, as the most general case, whereby neither the stress nor the displacement at the pile tip is equal to zero. In the case of an ideally end bearing pile, the tip displacement is equal to zero with the corresponding non-zero stress. In the case of an ideally fully floating pile the stress at the pile tip is zero with the corresponding non-zero displacement. Thus, the ideally fully floating pile has infinite length. Nevertheless, the thermal null point can be located arbitrarily close to the mid-length of the pile, which has a finite length, thus resulting in a small amount of stress being transferred at the pile tip.

Hereafter in this study this type of pile is referred to as a fully floating pile while an energy pile with absolutely zero displacement at the tip is referred to as an ideally end bearing energy pile.

Analytical solutions for semi- floating energy pile

A soil profile that consists of a single soil layer underlain by a bedrock or another soil stratum whose top surface is flush with the pile tip is referred to as a homogeneous soil profile herein. The solution for a semifloating energy pile is presented first because it is the most general case. It is subsequently shown that the special cases of ideally end bearing and fully floating piles can be derived from the solution for the semifloating pile.

Thermal Load

Only thermal load (ΔT) is applied in this case. The Free Body Diagrams (FBDs) of the entire semifloating energy pile and its infinitesimal segments representing the portions above and below the thermal null point are shown in for $\Delta T > 0$.

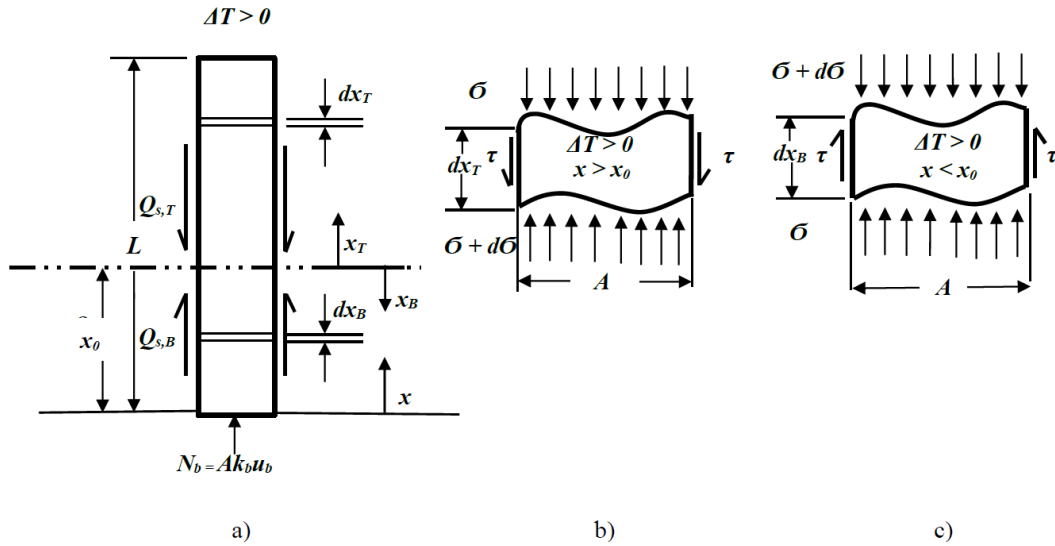


Figure 2.2 FBDs of a semi-floating energy pile subjected to $\Delta T > 0$, a) the entire pile, b) an infinitesimal segment of the pile above the null point, c) an infinitesimal segment of the pile below the null point

In the case of a semifloating pile both, stress and displacement develop at the pile tip. Thus, the location of zero displacement known as a thermal null point is somewhere in the interior of the pile. As shown in Figure 2.2, the thermal null point (x_0) is also the origin of both local coordinate systems, x_B and x_T . Imposing equilibrium of the infinitesimal pile segment (Figure 2.2) in the vertical direction and using Eqs. (2.1), (2.2) and (2.3) results in the following homogeneous second order differential equation

$$\frac{d^2 u_i}{dx_i^2} - \psi^2 u_i = 0 \quad i = B \text{ and/or } T \quad (2.5)$$

where B and T denote the regions of the pile located below and above the thermal null point respectively. The local coordinates (x_i) are given by

$$-x_0 \leq x_B \leq 0 \quad \text{and} \quad 0 \leq x_T \leq L - x_0 \quad (2.6)$$

The parameter ψ is a constant given by

$$\psi^2 = \left(\frac{p}{A}\right) \left(\frac{k_s}{E}\right) = \xi_g \xi_s \quad (2.7)$$

where p and A are the perimeter and cross-sectional area of the pile respectively. Thus, the parameter ψ contains both, the pile geometry in the form of ξ_g and the ratio of the stiffness of the soil and the pile in the term ξ_s .

Solving for the unknown function $u_i(x_i)$ from Eq. (2.5) results in

$$u_i(x_i) = C_{1i} e^{\psi x} + C_{2i} e^{-\psi x} \quad i = B \text{ and/or } T \quad (2.8)$$

Enforcing the continuity of displacements at the thermal null point while setting them equal to zero results in the following

$$u_i(x_i) = 2C_{1i} \sinh(\psi x_i) \quad i = B \text{ and/or } T \quad (2.9)$$

The boundary condition at the pile head in terms of global coordinate x is given by

$$\sigma(L - x_0) = 0 \quad (2.10)$$

where L is the length of the pile. Combining Eqs. (2.1), (2.2), (2.9) and (2.10) results in

$$C_{1T} = \frac{\alpha \Delta T}{2\psi \cosh[\psi(L - x_0)]} \quad (2.11)$$

After substituting Eq. (2.11) into Eq. (2.9) and updating the expressions for strain and stress the following solution is obtained

$$u_T(x) = \frac{\alpha \Delta T \sinh[\psi(x - x_0)]}{\psi \cosh[\psi(L - x_0)]} \quad (2.12)$$

$$\varepsilon_T(x) = \frac{\alpha \Delta T \cosh[\psi(x - x_0)]}{\cosh[\psi(L - x_0)]} \quad (2.13)$$

$$\sigma_T(x) = E\alpha \Delta T \left\{ \frac{\cosh[\psi(x - x_0)]}{\cosh[\psi(L - x_0)]} - 1 \right\} \quad (2.14)$$

It is

noted that Eqs. (2.12) through (2.14) are expressed in terms of the global coordinate (x). The relationship between the local and global coordinates (Figure 2.2a) is given by

$$x_i = x - x_0 \quad i = B \text{ and/or } T \quad (2.15)$$

To find the solution for the bottom region of the pile the continuity of stress at the thermal null point is enforced, thus resulting in

$$C_{1T} = C_{1B} \quad (2.16)$$

which gives the following solution for displacement, strain and stress in the entire pile

$$u(x) = \frac{\alpha \Delta T \sinh[\psi(x - x_0)]}{\psi \cosh[\psi(L - x_0)]} \quad (2.17)$$

$$\varepsilon(x) = \frac{\alpha \Delta T \cosh[\psi(x - x_0)]}{\cosh[\psi(L - x_0)]} \quad (2.18)$$

$$\sigma(x) = E\alpha \Delta T \left\{ \frac{\cosh[\psi(x - x_0)]}{\cosh[\psi(L - x_0)]} - 1 \right\} \quad (2.19)$$

The location of the thermal null point is obtained by imposing the constraint given in Eq. (2.4), which results in

$$x_0 = \frac{1}{\psi} \tanh^{-1} \left[\frac{\cosh(\psi L) - 1}{\sinh(\psi L) + \frac{k_b}{E\psi}} \right] \leq \frac{L}{2} \quad (2.20)$$

Knellwolf et al. (2011) used a finite difference method to model the soil structure interaction in energy piles. In their model they attached a normal spring at the pile head to simulate the restraint imposed by the superstructure during the application of a thermal load to the pile. To this end, the present solution for thermal loading is further modified by adding the normal elastic spring having stiffness k_h at the pile head. The solution is obtained by following the previously described procedure for thermal loading in the absence of a head spring with exception of Eq. (2.10), which is replaced by Eq. (2.21) given by

$$\sigma(L - x_0) = -k_h u(L - x_0) \quad (2.21)$$

thus, leading to the following solution

$$u(x) = \frac{\alpha \Delta T \sinh[\psi(x - x_0)]}{\psi \cosh[\psi(L - x_0)] + \frac{k_h}{E} \sinh[\psi(L - x_0)]} \quad (2.22)$$

$$\varepsilon(x) = \frac{\alpha \Delta T \cosh[\psi(x - x_0)]}{\cosh[\psi(L - x_0)] + \frac{k_h}{E\psi} \sinh[\psi(L - x_0)]} \quad (2.23)$$

$$\sigma(x) = E \alpha \Delta T \left[\frac{\cosh[\psi(x - x_0)]}{\cosh[\psi(L - x_0)] + \frac{k_h}{E\psi} \sinh[\psi(L - x_0)]} - 1 \right] \quad (2.24)$$

The solution for x_0 is obtained by enforcing the boundary condition at the pile tip given by Eq. (2.4), which results in

$$x_0 = \frac{1}{\psi} \tanh^{-1} \left[\frac{\cosh(\psi L) - 1 + \frac{k_h}{E\psi} \sinh(\psi L)}{\sinh(\psi L) + \frac{k_b}{E\psi} + \frac{k_h}{E\psi} \cosh(\psi L)} \right] \quad (2.25)$$

It can be shown by a simple rearrangement of terms that

$$\frac{\cosh(\psi L) - 1 + \frac{k_b}{E\psi}}{\sinh(\psi L) + \frac{k_b}{E\psi}} < 1 \quad (2.26)$$

and

$$\frac{\cosh(\psi L) - 1 + \frac{k_h}{E\psi} \sinh(\psi L)}{\sinh(\psi L) + \frac{k_b}{E\psi} + \frac{k_h}{E\psi} \cosh(\psi L)} < 1 \quad (2.27)$$

Thus, the location of the thermal null point can always be found based on Eqs. (2.20) and (2.25).

Mechanical load

Next, the solution is derived for mechanical load in the form of an axial force applied at the pile head. The free body diagrams of the entire pile and its infinitesimal segment are shown in Figure 2.3

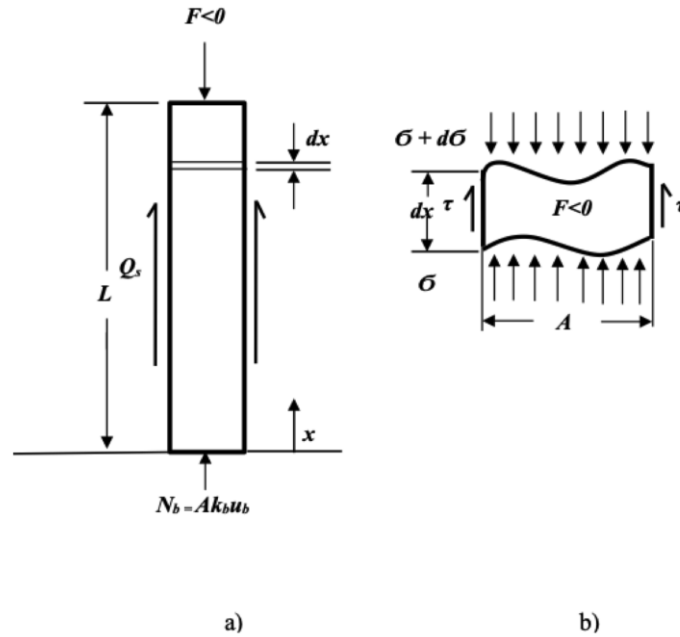


Figure 2.3. FBDs of a semi-floating energy pile subjected to $F < 0$, a) the entire pile, b) an infinitesimal segment of the pile

By enforcing the equilibrium of the infinitesimal pile segment and combining it with Eqs. (2.1) through (2.3) the following governing equation is obtained

$$\frac{d^2u}{dx^2} - \psi^2 u = 0 \quad (2.28)$$

The solution is given by

$$u(x) = C_1 e^{\psi x} + C_2 e^{-\psi x} \quad (2.29)$$

The relationship between constants C_1 and C_2 is obtained by combining Eq. (29) with Eqs. (2.1), (2.2) and (2.4). It is given by

$$C_2 = \frac{E\psi - k_b}{E\psi + k_b} C_1 \quad (2.30)$$

The boundary condition at the pile head is given by

$$\sigma(L) = \frac{F}{A} \quad (2.31)$$

After substituting Eq. (2.30) into Eq. (2.29), combining the result with Eqs. (2.1), (2.2) and (2.31), and rearranging the following solution for displacement, strain and stress is obtained

$$u(x) = \frac{F[E\psi \cosh(\psi x) + k_b \sinh(\psi x)]}{AE\psi[E\psi \sinh(\psi L) + k_b \cosh(\psi L)]} \quad (2.32)$$

$$\varepsilon(x) = \frac{F[E\psi \sinh(\psi x) + k_b \cosh(\psi x)]}{AE[E\psi \sinh(\psi L) + k_b \cosh(\psi L)]} \quad (2.33)$$

$$\sigma(x) = \frac{F[E\psi \sinh(\psi x) + k_b \cosh(\psi x)]}{A[E\psi \sinh(\psi L) + k_b \cosh(\psi L)]} \quad (2.34)$$

Special cases

Special cases of the solutions presented above that represent an ideally end bearing and fully floating energy piles are presented next.

Mechanical Load

For an ideally end bearing pile solutions for displacement, strain and stress due to an axial force applied at the pile head are obtained by taking the limit as $k_b \rightarrow \infty$ from Eqs. (2.32) through (2.34), which results in

$$u(x) = \frac{F \sinh(\psi x)}{AE \psi \cosh(\psi L)} \quad (2.35)$$

$$\varepsilon(x) = \frac{F \cosh(\psi x)}{AE \cosh(\psi L)} \quad (2.36)$$

$$\sigma(x) = \frac{F \cosh(\psi x)}{A \cosh(\psi L)} \quad (2.37)$$

As expected Eq. (2.35) indicates that the displacement at the tip of the ideally end bearing pile is equal to zero.

The stiffness of the spring attached to the tip of the semifloating pile (k_b) is larger than zero, but smaller than infinity. As the stiffness decreases so does the amount of axial load transferred to the surrounding soil or rock at the pile tip. In case of ideally floating pile ($k_b \rightarrow 0$) there would be no axial load transfer at the pile tip. This scenario corresponds to an infinitely long pile.

For a fully floating pile of a finite length the stiffness of the spring attached to the pile tip can be obtained by selecting a desired value of a ratio (β) between the mechanical stress at the pile tip and pile head and substituting it into Eq. (2.34), thus resulting in

$$k_b = \frac{\beta E \psi \sinh(\psi L)}{1 - \beta \cosh(\psi L)} \quad (2.38)$$

Finally, the solution for the displacement, strain and stress in a fully floating pile subjected to the mechanical load can be obtained by substituting the value of k_b from Eq. (2.38) into Eqs. (2.32) through (2.34), thus resulting in

$$u(x) = \frac{F [\cosh(\psi x) - \beta \cosh(\psi(L - x))]}{AE \psi \sinh(\psi L)} \quad (2.39)$$

$$\varepsilon(x) = \frac{F [\sinh(\psi x) + \beta \sinh(\psi(L - x))]}{AE \sinh(\psi L)} \quad (2.40)$$

$$\sigma(x) = \frac{F[\sinh(\psi x) + \beta \sinh(\psi(L-x))]}{A \sinh(\psi L)} \quad (2.41)$$

As β decreases the second term in the numerator becomes less significant and the solution is mainly affected by the first term in the numerator of Eqs. (2.39) through (2.41). As $\beta \rightarrow 0$ the second term in the numerator of Eqs. (2.39) through (2.41) vanishes, thus representing the infinitely long ideally fully floating pile.

Thermal Load

For an ideally end bearing pile ($k_b \rightarrow \infty$) subjected to thermal load and $k_h = 0$ the solution is given by Eqs. (2.17) through (19) whereby. Eq. (2.20) indicates that the thermal null point in this case is located at the pile tip. Alternatively, for $k_h \neq 0$, the solution is given by Eqs. (2.22) through (2.24) while Eq. (2.25) indicates that the thermal null point in this case is also located at the pile tip.

For a fully floating pile ($k_b \rightarrow 0$) subjected to thermal load and $k_h = 0$, the solution is also given by Eqs. (2.17) through (2.19). Alternatively, for $k_h \neq 0$ the solution is given by Eqs. (2.22) through (2.24). The location of the thermal null point for $k_h = 0$ can be obtained by substituting Eq. (2.38) into Eq. (2.20), thus resulting in

$$x_0 = \frac{1}{\psi} \tanh^{-1} \left[\frac{\cosh(\psi L) - 1}{\sinh(\psi L) + \frac{\beta \sinh(\psi L)}{1 - \beta \cosh(\psi L)}} \right] \quad (2.42)$$

The location of the thermal null point for $k_h \neq 0$ is obtained by substituting Eq. (2.38) into Eq. (2.25), thus resulting in

$$x_0 = \frac{1}{\psi} \tanh^{-1} \left[\frac{\cosh(\psi L) - 1 + \frac{k_h}{E\psi} \sinh(\psi L)}{\sinh(\psi L) + \frac{k_h}{E\psi} \cosh(\psi L) + \frac{\beta \sinh(\psi L)}{1 - \beta \cosh(\psi L)}} \right] \quad (2.43)$$

Validation of the analytical model

The analytical solutions presented herein were validated against two centrifuge tests (Stewart and McCartney 2013; Goode and McCartney 2015). While the energy pile in the former study represents a nearly ideally end bearing pile, the energy pile in the latter study effectively represents a semifloating energy pile that is very close to a fully floating pile.

Case 1 (Stewart and McCartney, 2013)

A concrete model of a single end bearing energy pile having diameter of 50.8 mm and length of 533.4 mm was tested in a geotechnical centrifuge at g-level of 24. Thus, the corresponding dimensions of the prototype energy pile were 12.8 m in length and 1.22 m in diameter. The pile was embedded in a layer of unsaturated Bonny silt and heated during the centrifuge tests. Stewart and McCartney (2013) reported that the pile had no head restraint, and that its coefficient of thermal expansion was $7.5 \times 10^{-6} \text{ } 1/^{\circ}\text{C}$ and its Young's modulus was 7.17 GPa. The best fit between predictions of the analytical solutions and the centrifuge test results was obtained for $k_s = 55 \text{ MPa/m}$ herein.

The thermal axial displacement, strain and stress measured in the centrifuge tests for the pile subjected to the temperature increases of 5°, 10°, 15° and 19°C are shown in Figure 2.4(a) through 4(c) respectively along with the responses predicted by the analytical solutions derived in this study. Both, the experimental data and model predictions show that the energy pile experienced almost no thermal axial displacement at the pile tip. Consequently, the minimum thermal strain and stress are predicted at the pile tip, which fits closely with the available experimental data. This clearly indicates that the physical model of the energy pile in Stewart and McCartney (2013) is almost the ideally end bearing pile. Overall, the analytical solutions for ideally end bearing pile successfully predicted the energy pile response along the entire length of

the pile. Specifically, the experimental and analytically predicted responses show an excellent agreement for the temperature increases of 15°C and 19°C. There is slightly less agreement for the temperature increases of 15°C and 19°C. There is slightly less agreement between predicted and experimentally observed responses for lower temperature increases (5°C and 10°C) most likely because the self-weight, whose effect is more pronounced at lower magnitude of thermal loads, is not accounted for in the analytical solutions.

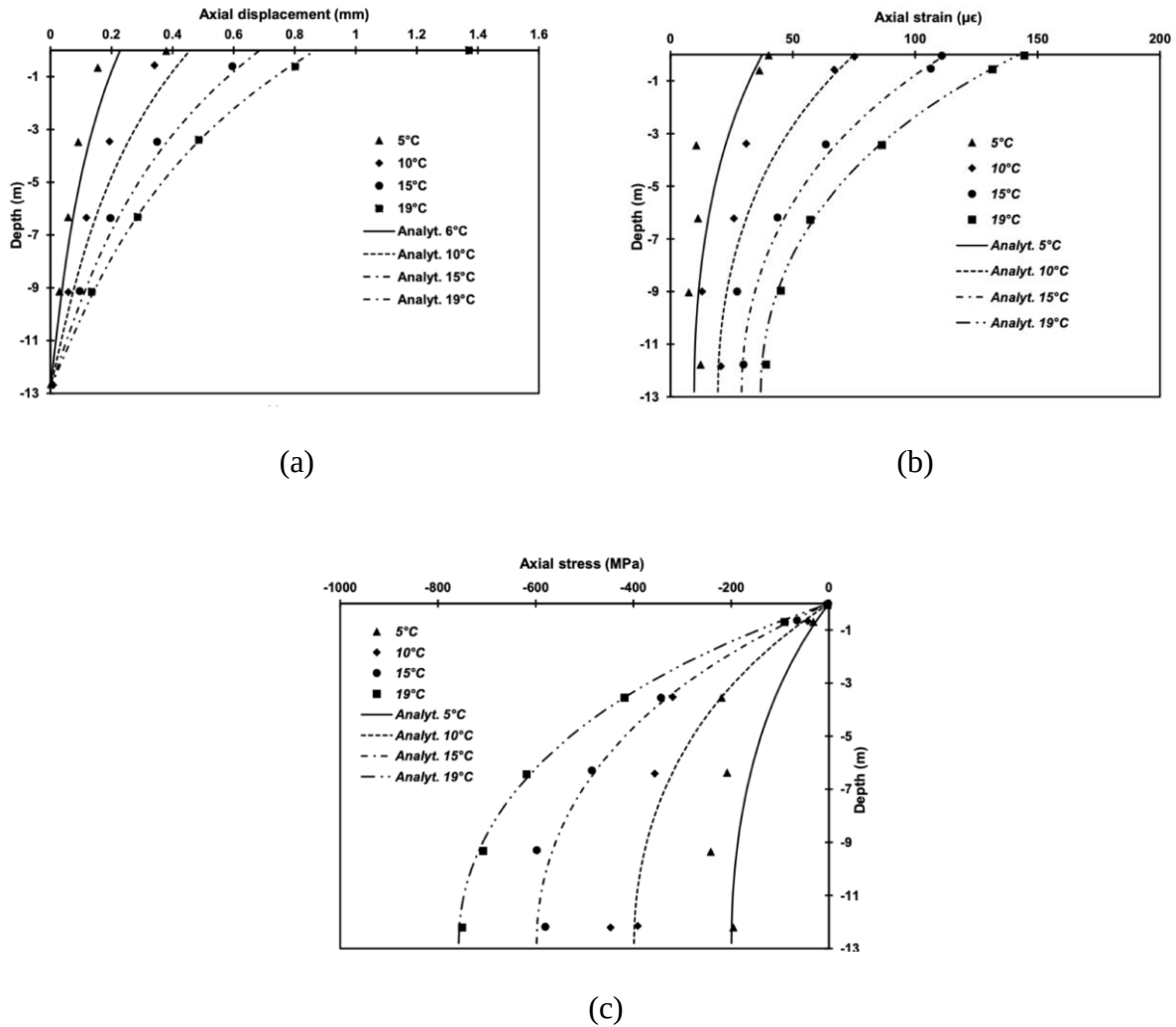
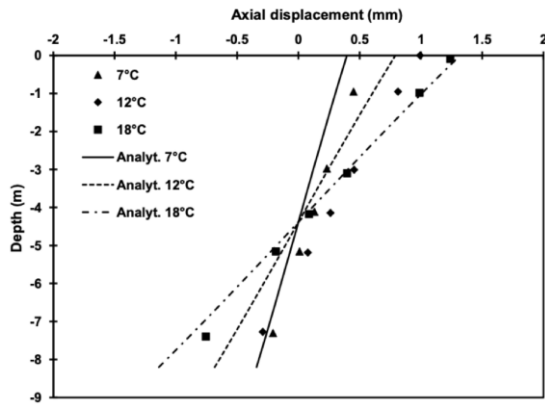


Figure 2.4. Comparison of analytical predictions with centrifuge test results (Stewart and McCartney, 2013): (a) thermal displacement, (b) thermal strain, (c) thermal stress

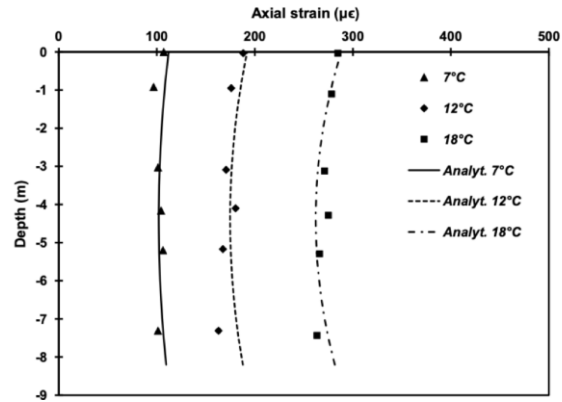
Case 2 (Goode and McCartney, 2015)

In this study, the concrete semifloating energy pile having diameter of 63.5 mm and length of 342.9 mm was tested in a geotechnical centrifuge at g-level of 24. Thus, the corresponding prototype energy pile had length of 8.2 m and diameter of 1.5m. The pile was embedded in Nevada sand and heated. Goode and McCartney (2015) reported that the pile had the coefficient of thermal expansion of $16 \times 10^{-6} \text{ 1/}^\circ\text{C}$ and Young's modulus of 33 GPa, and no head restraint. The best fit between predictions of analytical solutions and experimental results was obtained for $k_s = 125 \text{ MPa/m}$ and $k_b = 205 \text{ MPa/m}$ herein.

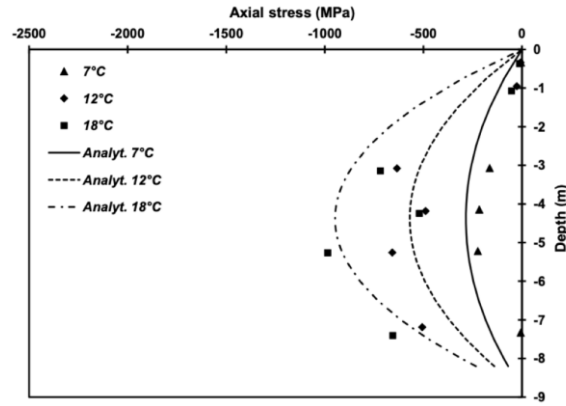
The model predictions depicted in Figure 2.5 along with the experimental data indicate that the analytical solutions successfully captured the overall trend in change of thermal axial displacement, strain and stress versus depth. Furthermore, analytically predicted and experimentally observed locations of the null point match closely. In addition, the variation of axial displacement, strain and stress versus depth along with the location of the null point indicate that this semifloating pile is very close to being fully floating pile.



(a)



(b)



(c)

Figure 2.5. Comparison of analytical predictions with centrifuge test results (Goode and McCartney, 2015): (a) thermal displacement, (b) thermal strain, (c) thermal stress

Based on the results of the two centrifuge tests it can be concluded that the analytical solutions have successfully captured the responses of ideally end bearing, semifloating and fully floating energy piles, thus fully validating the proposed analytical model and its solutions.

Load transfer diagram

Axial displacement, strain and stress, as well as shear stress at the pile soil interface for thermal, mechanical, and combined thermal and mechanical loads are referred to as load transfer diagrams herein. To facilitate understanding of load transfer diagrams values of displacement, strain and stress at the pile tip and pile head for thermal and mechanical loads are shown in Table 2.1. Load transfer diagrams are provided for a hypothetical homogeneous soil profile comprising a single soil layer underlain by a bedrock or another soil stratum, the top surface of which, is flush with the pile tip. Dimensions and properties of the pile correspond to those encountered in the single energy pile constructed at Swiss Federal Institute of Technology at Lausanne (Laloui et al. 2006). Thus, the length and the diameter of the pile are equal to 26 m and 1 m respectively while its elastic modulus and coefficient of thermal expansion are equal to 29.2 GPa and $1 \times 10^{-5}/^{\circ}\text{C}$ respectively. It is noted that the actual soil profile at Swiss Federal Institute of Technology consists

of four different soil layers located above the bedrock that were referred to as soils A1, A2, B and C by Laloui et al. (2006). Thus, the soil profile used herein is hypothetical in that only a single soil is located above the bedrock.

Table 2.1. Displacement, strain and stress at the pile tip and pile head for mechanical and thermal loading, $k_h=0$

			Ideally End bearing	Ideally Fully floating
Mechanical	u	$u(0)$	0	$\frac{F}{AE\psi\sinh(\psi L)}$
		$u(L)$	$\frac{F}{AE\psi}\tanh(\psi L)$	$\frac{F}{AE\psi\tanh(\psi L)}$
	ε	$\varepsilon(0)$	$\frac{F}{AE\cosh(\psi L)}$	0
		$\varepsilon(L)$	$\frac{F}{AE}$	$\frac{F}{AE}$
	σ	$\sigma(0)$	$\frac{F}{A\cosh(\psi L)}$	0
		$\sigma(L)$	$\frac{F}{A}$	$\frac{F}{A}$
Thermal	u	$u(0)$	0	$-\frac{\alpha\Delta T}{\psi}\tanh\frac{\psi L}{2}$
		$u(L)$	$\frac{\alpha\Delta T}{\psi}\tanh(\psi L)$	$\frac{\alpha\Delta T}{\psi}\tanh\frac{\psi L}{2}$
	ε	$\varepsilon(0)$	$\frac{\alpha\Delta T}{\cosh(\psi L)}$	$\alpha\Delta T$
		$\varepsilon(L)$	$\alpha\Delta T$	$\alpha\Delta T$
	σ	$\sigma(0)$	$E\alpha\Delta T[\frac{1}{\cosh(\psi L)} - 1]$	0
		$\sigma(L)$	0	0

Perić et al. (2017) classified all four soils as low plasticity clays. While most of figures herein were developed for soil A2 some figures also include soil C. Knellwolf et al. (2011) used

finite difference method to model a response of the energy pile located at Swiss Federal Institute of Technology. They reported the values of elastic shear spring stiffness substituting soils A2 and C, which were determined from full scale field tests, to be equal to 10.8 MPa/m and 121.4 MPa/m respectively. These values resulted in the values of parameter ψ determined from Eq. (2.7) being equal to 0.0384/m and 0.1289/m for soils A2 and C respectively. They also reported that the stiffness of the normal spring attached to the pile tip ranged between 667.7 MPa/m and 1335.54 MPa/m. In the present study the mean value of $k_b = 1001.55$ MPa/m was used for the semifloating pile. For $\beta = 0.01$ and fully floating pile embedded in soils A2 and C Eq. (2.38) results in $k_b = 13.41$ MPa/m and 627.32 MPa/m respectively.

Mechanical Load

The normalized displacement, strain and stress are shown for both, ideally end bearing and fully floating piles. Figure 2.6a shows that the ideally end bearing pile accommodates mechanical load by developing the displacement of a smaller magnitude than the fully floating pile. Nevertheless, the smaller magnitude of displacement is compensated by the larger magnitude of strain and stress that develop in the ideally end bearing pile as compared to the fully floating pile. Although values provided in Table 2.1 indicate that for $\tanh(\psi L) \rightarrow 1$ the head displacements of ideally end bearing and fully floating piles would be equal the latter would still exhibit larger magnitude of displacements over the entire length of the pile, and thus smaller magnitudes of stress and strain.

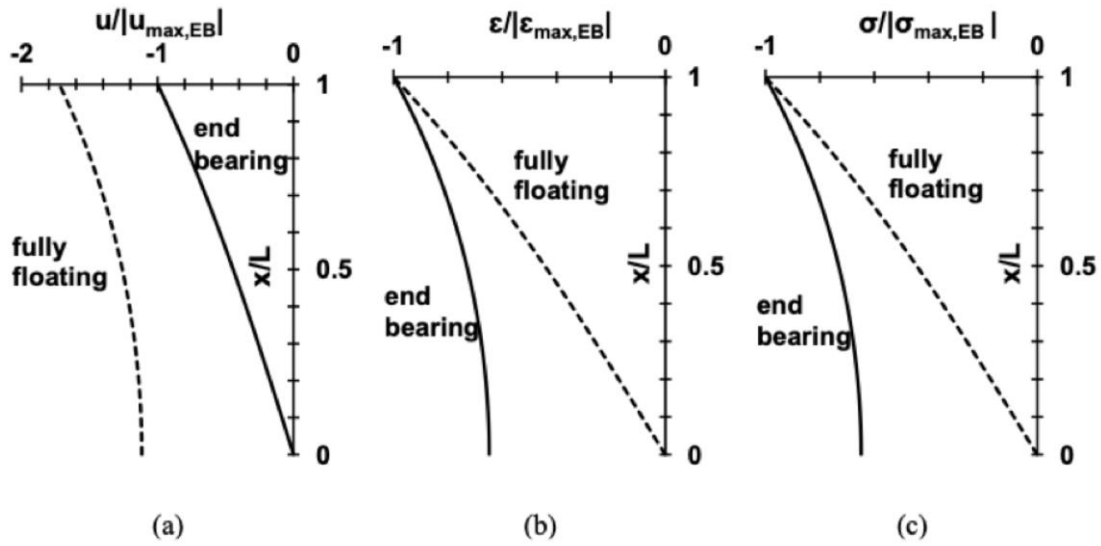


Figure 2.6. Comparison of: (a) displacement, (b) strain, and (c) stress for mechanical load ($F < 0$) in ideally end bearing and fully floating energy piles, $k_h = 0$, soil A2

Displacements of ideally end bearing and fully floating piles provide the lower and upper bounds respectively for the magnitude of displacement of a semifloating pile. On the contrary, stress and strain responses of ideally end bearing and fully floating piles provide the upper and lower bounds for the magnitude of stress and strain in a semifloating pile. The stiffer the bedrock or the soil at the pile tip the closer the response of an energy pile will be to that of the ideally end bearing pile.

Pile foundations are most often used to transfer compressive mechanical loads. Nevertheless, they could also be subjected to tensile mechanical loads, for example in case of uplift forces due to hydrostatic pressure, seismic activities or overturning moments. It is noted that in the case of a tensile force applied at the pile head the normalized load transfer diagrams would have the same magnitude as shown in Figure 2.6 while their sign would be the opposite, thus positive.

Thermal Load

Figure 2.7 shows the normalized load transfer diagrams for a thermal load in the form of heating. In the case of ideally end bearing pile the thermal load is accommodated by a larger magnitude of displacement at the pile head than in the case of fully floating pile. On the contrary, at the pile tip the fully floating pile exhibits a larger magnitude of displacement than the ideally end bearing pile.

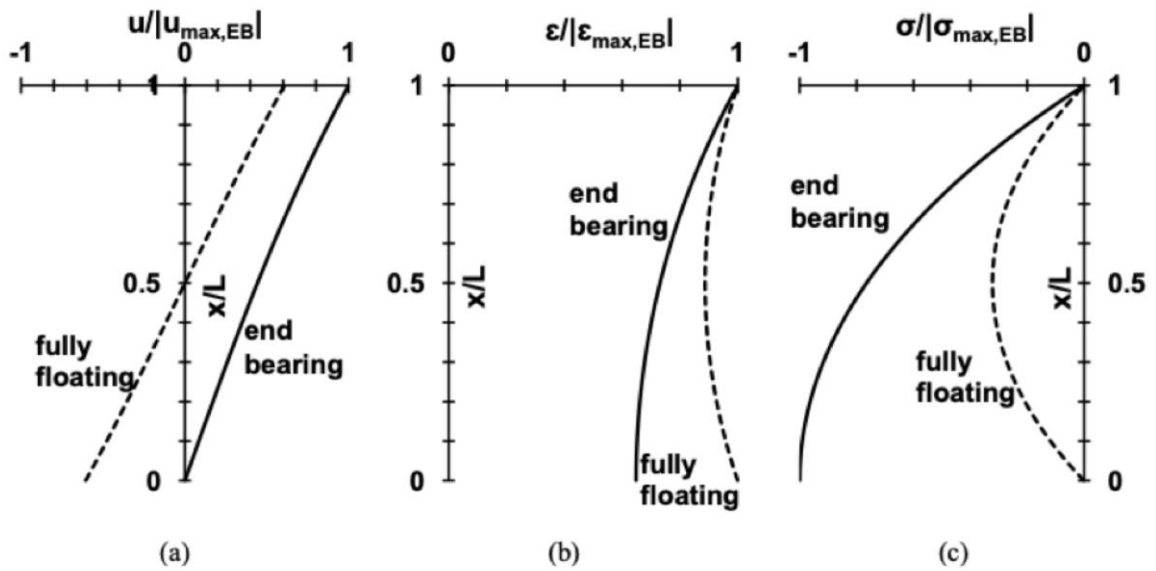


Figure 2.7. Comparison of: (a) displacement, (b) strain, and (c) stress for thermal load ($\Delta T > 0$) in ideally end bearing and fully floating energy piles, $k_h = 0$, soil A2

Furthermore, Figure 2.7 shows that the thermal null point is located in the vicinity of the middle of the pile for the selected value of β in accordance with Eq. (2.43). In addition, the magnitude of strain is larger in the case of the fully floating pile throughout its entire length with the exception of the pile head where the both magnitudes are equal. On the contrary, the magnitude of stress is larger in the ideally end bearing pile than in the fully floating pile throughout the pile length with the exception of the pile head where they are both equal to zero. The load transfer diagrams for cooling can be obtained by preserving the magnitude while changing the sign of

diagrams depicted in Figure 2.7, thus resulting in the compressive strain and tensile stress. Finally, the magnitudes of stress and strain at the pile head in the ideally end bearing and fully floating piles are equal to each other for both, thermal and mechanical loads. Figure 2.7b also depicts a degree of freedom (n), which was defined by Knellwolf et al. (2011) as

$$n = \frac{\varepsilon}{\varepsilon_f} \quad \text{where} \quad \varepsilon_f = \alpha \Delta T \quad (2.44)$$

Thermal, mechanical and combined thermomechanical load

The next series of figures illustrates responses of ideally end bearing, fully floating and semifloating energy piles to a combined thermal and mechanical load. To this end, several load scenarios are considered as follows:

- Case (i): $F < 0$ (compressive force) and $\Delta T < 0$ (cooling),
- Case (ii): $F < 0$ (compressive force) and $\Delta T > 0$ (heating),
- Case (iii): $F > 0$ (tensile force) and $\Delta T < 0$ (cooling)
- Case (iv): $F > 0$ (tensile force) and $\Delta T > 0$ (heating)

A comparison of Eqs. (2.17) and (2.18) for $k_b \rightarrow \infty$ with Eqs. (2.35) and (2.36) indicates that the magnitudes of displacements and strains in an ideally end bearing pile that are induced by thermal and mechanical load are equal to each other if

$$\alpha |\Delta T| = \frac{|F|}{AE} \quad (2.45)$$

It is noted that thermal load is inherently changing with time while mechanical load, which is imposed by the superstructure, is approximately constant. To this end, the equivalent thermal load (ΔT_{eq}) was defined by Perić et al. (2020) as

$$\Delta T_{eq} = \pm \frac{F}{AE\alpha} \quad (2.46)$$

whereby plus sign applies to the cases (i) and (iv) while the minus sign applies to the cases (ii) and (iii). For example, in the case of Swiss Federal Institute of Technology the magnitude of mechanical load applied to the single energy pile installed was equal to 1 MN (Knellwolf et al. 2011). Thus, according to Eq. (2.46) the magnitude of the corresponding equivalent thermal load would be 4.36°C. Based on Eq. (2.46) thermal load can be classified into three groups as follows

$$|\Delta T| \begin{cases} < |\Delta T_{eq}| \\ = |\Delta T_{eq}| \\ > |\Delta T_{eq}| \end{cases} \quad (2.47)$$

For a more balanced comparison of effects of thermal and mechanical loads the magnitude of thermal load is selected to be equal to the magnitude of the equivalent thermal load in the next series of figures. Although the equivalent thermal load is defined based on the response of the ideally end bearing pile it is used herein to illustrate responses of nearly fully floating and semifloating piles as well, thus providing a common reference response for various comparisons.

Figures. 2.8 through 2.13 depict various load transfer diagrams for $\Delta T = \Delta T_{eq}$ and $F = -1 \text{ MN}$. Figure 2.8 shows displacement, stress and strain for ideally end bearing pile and load scenario (ii). In this case the thermally and mechanically induced displacement and strain cancel each other, thus resulting in the net zero displacement and strain throughout the entire length of the pile. Nevertheless, mechanically and thermally induced stresses are both compressive and they add up to produce the constant axial stress along the pile length that is equal to F/A .

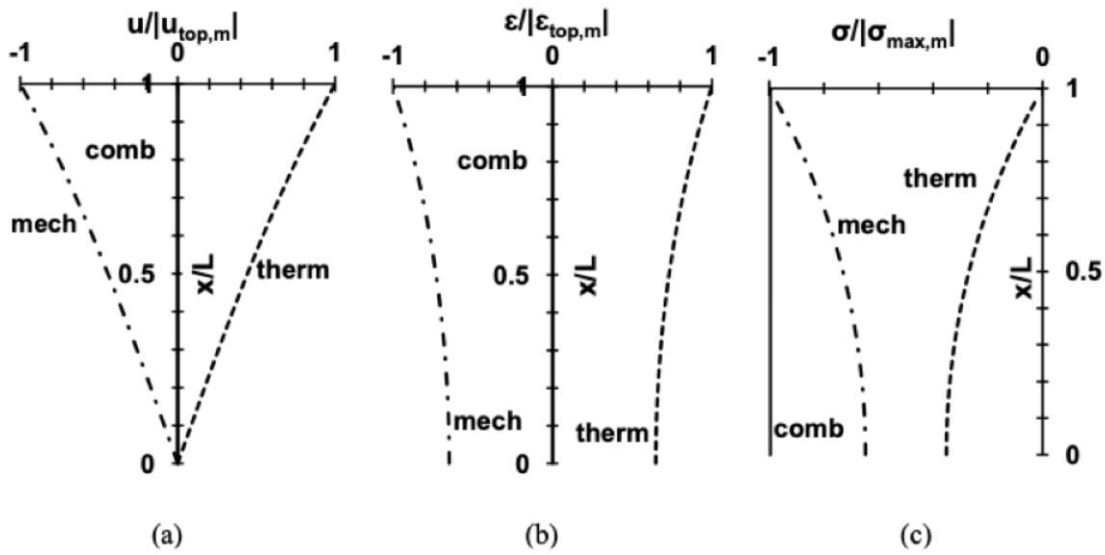


Figure 2.8. (a) Displacement, (b) strain, and (c) stress in ideally end bearing energy pile subjected to a combined mechanical and equivalent thermal load, scenario (ii), $k_h = 0$, soil A2

Table 2.2 shows signs and directions of displacement, strain and stress in energy piles subjected to mechanical and thermal loads. In case of displacements the depicted signs apply only to the region above the thermal null point. The symbols c_m and t_m denote compressive and tensile strain or stress respectively that are induced by a mechanical load while c_t and t_t denote compressive and tensile strain or stress respectively that are induced by a thermal load. The last column shows the possible signs of a combined thermally and mechanically induced stress. It is noted that in the cases (ii) and (iii), each corresponding to different signs of thermal and mechanical loads, the resultant combined stress is always compressive (ii) and tensile (iii) respectively throughout the length of the pile. Case (iii) corresponds to a tensile force applied at the pile head combined with cooling. The corresponding response can be obtained by flipping the diagrams shown in Figure 2.8 whereby the resulting mechanically induced displacement and strain become positive. Likewise, the resulting combined stress is tensile and equal to F/A . While tensile stresses

always develop in the pile in cases (iii) and (iv) they might also develop in case (i) provided that the magnitude of thermal load is sufficiently large.

Table 2.2. Directions and signs of displacement, strain and stress in ideally end bearing, fully floating and semi-floating energy piles subjected to different loading scenarios ($k_h=0$)

Case		Thermal			Mechanical			Combined stress
		u	ε	σ	u	ε	σ	
(i)	$F < 0 \ \& \ \Delta T < 0$	< 0 ↓	< 0 (c_t)	> 0 (t_t)	< 0 ↓	< 0 (c_m)	< 0 (c_m)	1-compression 2-tension at bottom and compression at top of the pile
(ii)	$F < 0 \ \& \ \Delta T > 0$	> 0 ↑	> 0 (t_t)	< 0 (c_t)	< 0 ↓	< 0 (c_m)	< 0 (c_m)	compression
(iii)	$F > 0 \ \& \ \Delta T < 0$	< 0 ↓	< 0 (c_t)	> 0 (t_t)	> 0 ↑	> 0 (t_m)	> 0 (t_m)	tension
(iv)	$F > 0 \ \& \ \Delta T > 0$	> 0 ↑	> 0 (t_t)	< 0 (c_t)	> 0 ↑	> 0 (t_m)	> 0 (t_m)	1-tension 2-compression at bottom and tension at top of the pile

Figure 2.9 shows load transfer diagrams for load scenario (i). In this case the application of the equivalent thermal load in addition to the mechanical load simply doubles the downward displacement and compressive strain. Although the stress due to the combined equivalent thermal and mechanical loads remains compressive its magnitude is smaller than that of the stress induced

by mechanical load. Table 2.2 indicates that the combined stress in this case could be compressive along the entire pile length or it could be compressive in the upper part of the pile and tensile in the bottom part of the pile. The latter scenario requires the magnitude of thermal load to be larger than the magnitude of the equivalent thermal load.

Similarly, load transfer diagrams for case (iv) can be obtained by flipping the diagrams depicted in Figure 2.9. Thus, the combined equivalent thermal and mechanical load in this case will double the upward displacement and tensile strain. As indicated in Table 2.2 the combined stress will remain tensile in the case of equivalent thermal load. Nevertheless, in case that the magnitude of thermal load is larger than that of equivalent thermal load the combined stress will be compressive in the bottom portion of the pile and tensile in the top portion of the pile.

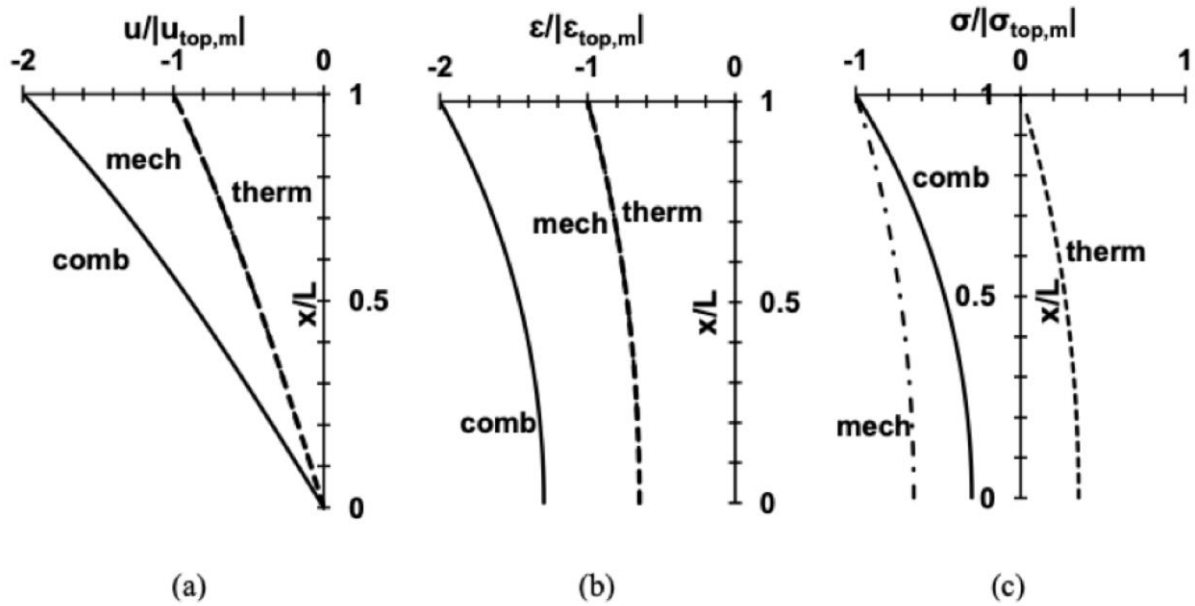


Figure 2.9. (a) Displacement, (b) strain, and (c) stress in ideally end bearing energy pile subjected to a combined mechanical and equivalent thermal load, scenario (i), $k_h = 0$, soil A2

Load transfer diagrams for a fully floating pile are depicted in Figures. 2.10 and 2.11 for load scenarios (ii) and (i) respectively. As shown in Figure 2.10a heating causes the upper half of

the fully floating pile to move upward, while its lower half moves downward. Thus, the thermally induced null point is located in the mid-depth of the pile. The mechanical load causes a downward displacement throughout the pile length whereby the maximum magnitude occurs at the pile head. It is noted that the magnitude of mechanical displacement is considerably larger than that induced by the equivalent thermal load, thus causing the entire pile to move downward when subjected to the combined load. Figure 2.10b shows that while heating induces tensile strain, compressive force causes compressive strain throughout the pile length. Unlike in the case of the ideally end bearing pile shown in Figure 2.8b thermal and mechanical strain in the floating pile do not cancel each other throughout the entire length of the pile. The magnitudes of thermal and mechanical strains are equal to each other only at the pile head. Furthermore, the combined thermal and mechanical strain is tensile due to the larger magnitude of thermal strain throughout the entire length of the pile. Both, thermal and mechanical loads induce compressive stress in the case of the floating pile just like in the case of the ideally end bearing pile. While the resulting combined stress is compressive in the case of equivalent thermal load as shown in Figure 2.10c it is not constant versus depth as in the case of the ideally end bearing pile depicted in Figure 2.8c. It is also noted that in the case of the fully floating pile the maximum magnitude of thermal stress occurs at the mid-length of the pile, thus corresponding to the location of the thermal null point.

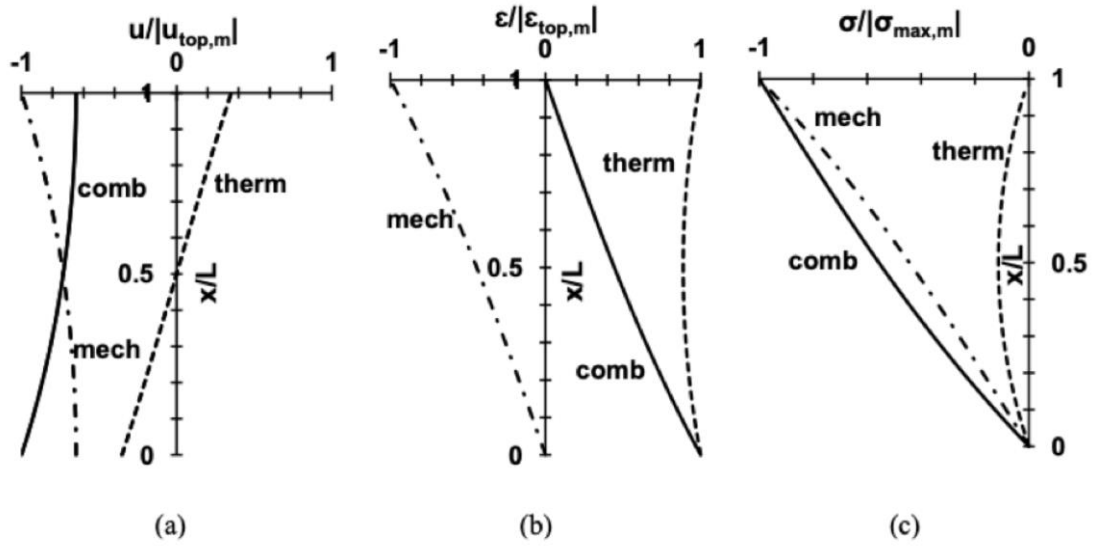


Figure 2.10. (a) Displacement, (b) strain, and (c) stress in fully floating energy pile subjected to a combined mechanical and equivalent thermal load, scenario (ii), $k_h = 0$, soil A2

Figure 2.11 depicts displacement, strain and stress diagrams for fully floating pile subjected to load scenario (i) and equivalent thermal load. As expected, Figure 2.11a shows that the thermal null point is located at the mid-depth of the pile, but in this case the top part of the pile moves downward due to cooling. Furthermore, a larger magnitude of mechanical displacement results in the downward directed combined displacement throughout the length of the pile. Figure 2.11b shows that the combined strain is compressive throughout the entire length of the pile. As indicated in Table 2.2 and Figure 2.11c, the combined stress is also compressive throughout the entire length of the pile. It is because the magnitude of the mechanical stress in the case of the equivalent thermal load is noticeably larger than that of thermal stress. Thus, the sign of the combined stress is the same as the sign of mechanical stress. In other words, both of these stresses are compressive.

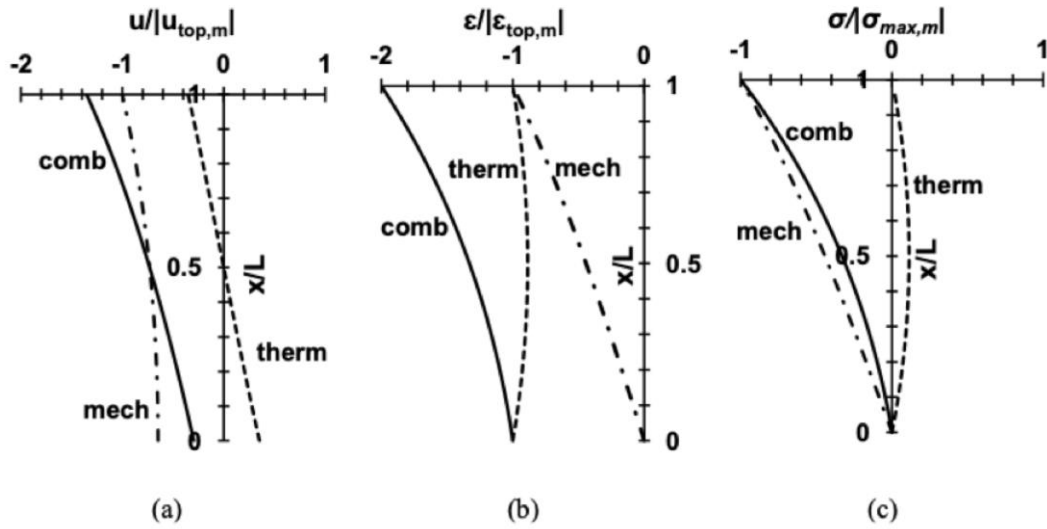


Figure 2.11. (a) Displacement, (b) strain, and (c) stress in fully floating energy pile subjected to a combined mechanical and equivalent thermal load, scenario (i), $k_h = 0$, soil A2

Figure 2.12 depicts load transfer diagrams for a semifloating pile subjected to the load scenario (ii) with equivalent thermal load. It can be seen from Figure 2.12a that the thermal null point in this case is located within the bottom half of the pile length. It is noted that none of the figures so far include effects of a head spring. As expected, heating causes upward displacement above the thermal null point and downward displacement below the thermal null point. Adding the equivalent thermal load to the mechanical load decreases the downward displacement above the thermal null point while increasing the downward displacement below the thermal null point. Figure 2.12b indicates that the combined strain is tensile because magnitude of thermally induced strain is larger than that of mechanical strain throughout the length of the pile. It is noted that both strains, thermal and mechanical have equal magnitudes at the pile head. Figure 2.12c shows that the combined stress is compressive throughout the length of the pile as indicated in Table 2.2. Nevertheless, the maximum magnitude of thermal stress is located within the bottom half of the pile length, thus corresponding to the location of the thermal null point.

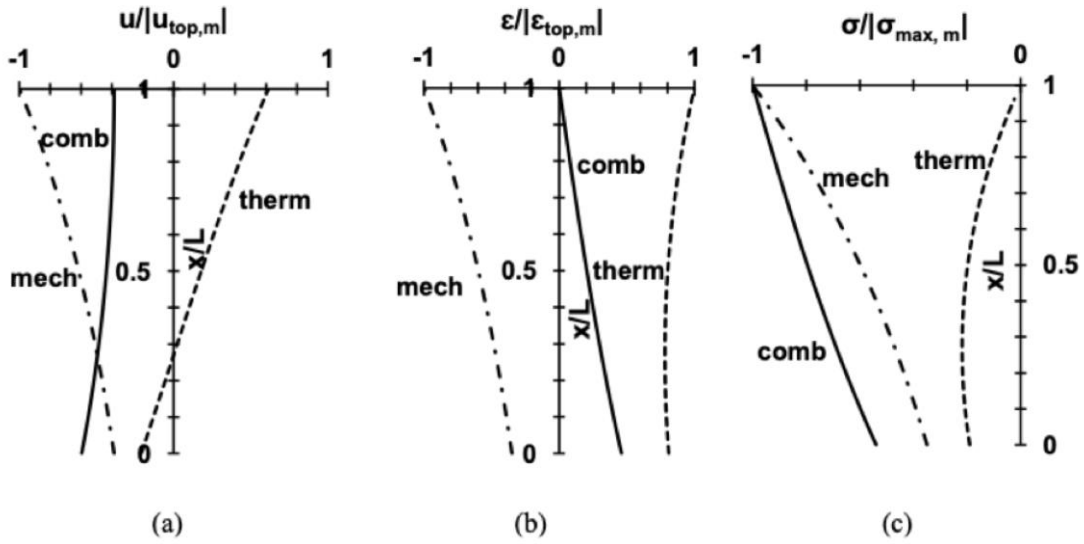


Figure 2.12. (a) Displacement, (b) strain, and (c) stress in semi-floating energy pile subjected to a combined mechanical and equivalent thermal load, scenario (ii), $k_h = 0$, soil A2

Figure 2.13 illustrates the response of the semifloating energy pile subjected to the loading scenario (i). As expected, the thermal null point is located within the bottom half of the pile. Similarly, to the fully floating pile, magnitude of mechanical displacement is larger than that of thermal displacement thus resulting in the overall downward displacement. Specifically, adding thermal load further increases the magnitude of the downward mechanical displacement above the thermal null point and decreases the downward displacement below the thermal null point. Figure 2.13b shows that the resulting combined mechanical strain is compressive throughout the length of the pile. Figure 2.13c shows that the combined stress remains compressive in accordance with Table 2.2 in the case of the equivalent thermal load.

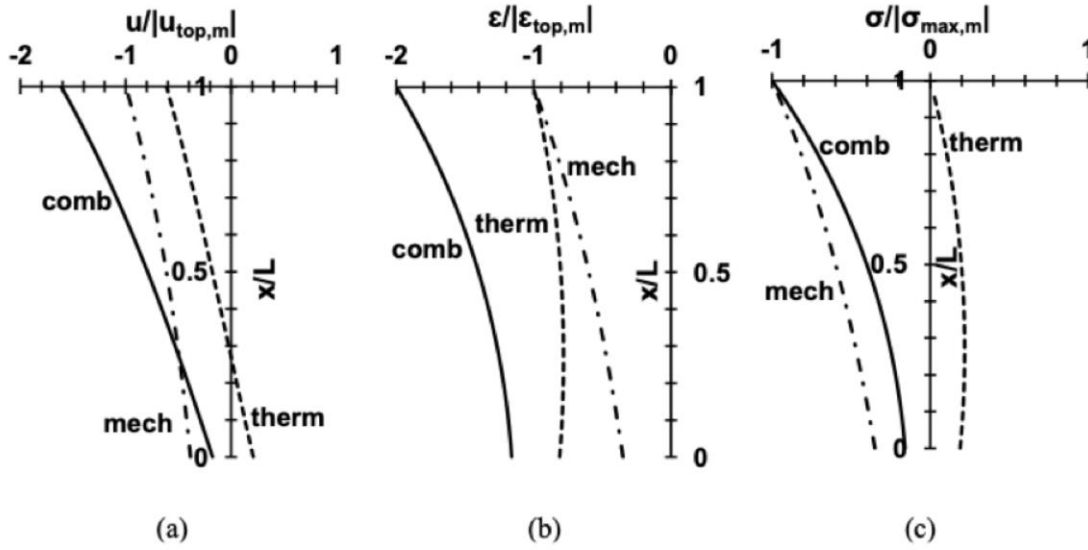


Figure 2.13. (a) Displacement, (b) strain, and (c) stress in semi-floating energy pile subjected to a combined mechanical and equivalent thermal load, scenario (i), $k_h = 0$, soil A2

As seen in Figures 2.8 to 2.13 and in accordance with Eq. (2.20), in the absence of a head spring thermal null point can be located anywhere between the pile tip and its mid-length. In the case of an ideally end bearing pile ($k_b \rightarrow \infty$) no displacement occurs at the pile tip, thus resulting in the thermal null point being located at the pile tip while in the case of a fully floating pile ($k_b \approx 0$) the thermal null point is located arbitrarily close to the mid-length of the pile.

Thermal null point and development of tensile zone

Due to different directions of displacements induced in the top and bottom portions of the semifloating and fully floating energy piles subjected to thermal load, development of the zero-displacement point or thermal null point is inevitable in the absence of mechanical load. Nevertheless, in case of a thermomechanical loading a combined null point development depends on the loading scenario, and magnitudes of thermal and mechanical loads (Saeidi Rashk Olia and Perić 2021b). Figure 2.14 illustrates Eq. (2.20) in the normalized form whereby ψ values of

0.1289/m and 0.0384/m correspond to the soils C and A2 respectively. Knellwolf et al. (2011) reported k_s value of 121.4 MPa/m for soil C.

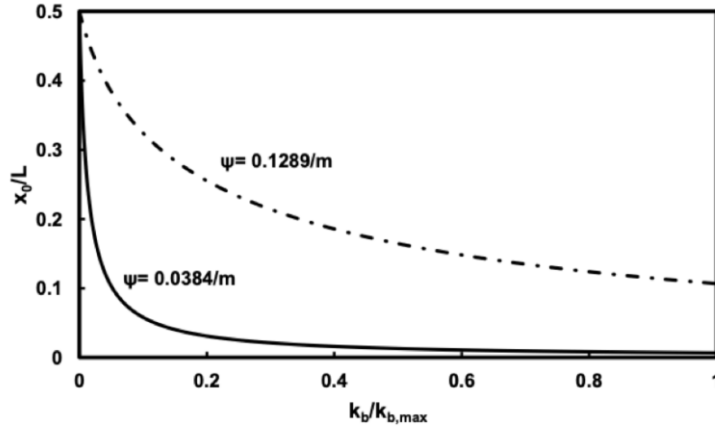


Figure 2.14. Normalized location of thermal null point with respect to normalized k_b , $k_h = 0$, soils A2 and C

In Figure 2.14 stiffness of the normal spring (k_b) is normalized with the realistic maximum value for rock, which was obtained by using Eq. (2.48) proposed by Scott (1981) that is given by

$$k_b = \frac{2G_b}{(1 - \nu_b)d} \quad (2.48)$$

where G_b and ν_b are shear modulus and Poisson's ratio of the bedrock respectively, and d is the diameter of the pile. Realistic maximum values of G_b and ν_b corresponding to 30 GPa and 0.35 were obtained from Palmström and Singh (2001), thus resulting in $k_{b,max} = 92.31$ GPa/m. Figure 2.14 illustrates that in the case of the softer soil (A2) the location of the thermal null point rapidly moves to the vicinity of the pile tip with increase in the tip restraint. Nevertheless, in the case of the stiffer soil (C) the thermal null point is still located at $0.12L$ above the pile tip at $k_b = k_{b,max}$.

As shown in Table 2.2 for loading scenarios (i) and (iv) where both, thermal and mechanical loads have the same sign, there is a possibility of development of a tensile stress zone in the energy pile. The tension zone will develop within the bottom or top part of the pile in cases (i) or (iv) respectively for sufficiently large magnitude of thermal load. It is noted that in case (iv)

the entire pile will experience tensile stress provided that the magnitude of thermal load is sufficiently small. To this end, the ratio of the actual and equivalent thermal load (η) is defined as

$$\eta = \frac{\Delta T}{\Delta T_{eq}} > 0 \quad (2.49)$$

Figure 2.15 depicts normalized mechanical and combined stresses in the ideally end bearing energy pile for scenario (i) and several values of η . It can be seen from Figure 2.15 that the length of tension zone increases with increasing magnitude of thermal load.

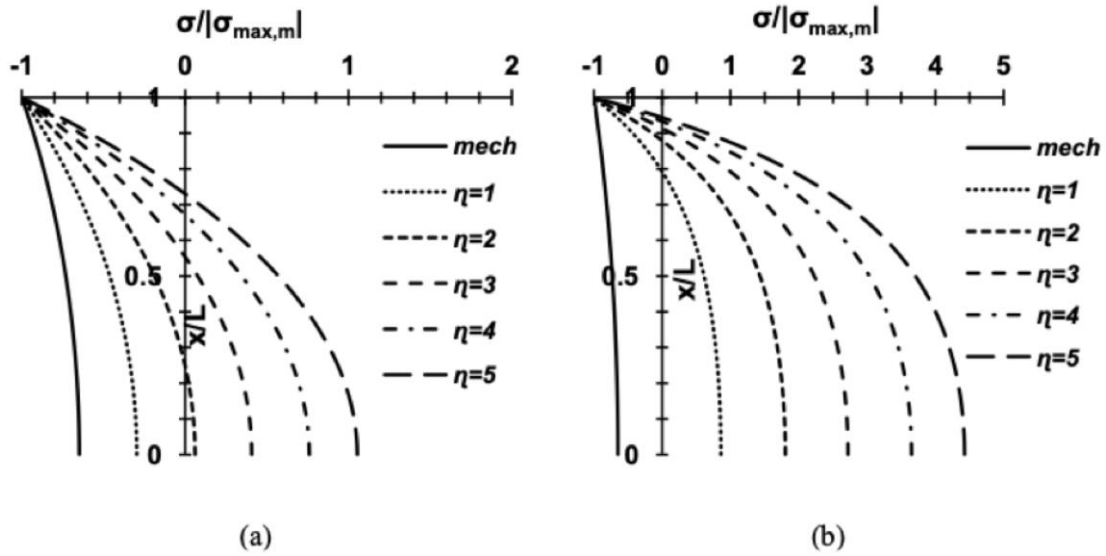


Figure 2.15. Mechanical and combined stresses versus depth for ideally end bearing energy pile subjected to loading scenario (i) and $k_h = 0$, a) $\psi=0.0384/\text{m}$ (soil A2), b) $\psi=0.1289/\text{m}$ (soil C)

The x -coordinate corresponding to the zero combined stress (x_{ten}) in ideally end bearing pile is obtained by combining Eqs. (2.19) for $x_0 = 0$, (2.37) and (2.49). It is given by

$$x_{ten} = \frac{1}{\psi} \cosh^{-1} \left[\frac{\cosh(\psi L)}{1 + \frac{1}{\eta}} \right] \quad (2.50)$$

thus, requiring that

$$\eta > \frac{1}{\cosh(\psi L) - 1} \quad (2.51)$$

Figure 2.16 illustrates Eq. (2.50) in a normalized form. It is noted that the length of the tension zone for the loading scenario (i) is given by x_{ten} while for the loading scenario (iv) it is given by $(L-x_{ten})$. For the scenario (i) the normalized length of the tension zone increases with increasing magnitude of thermal load. For the scenario (iv) the normalized length of the tension zone decreases with the increasing magnitude of thermal load. Furthermore, the stiffer the soil, the longer the tension zone for a given magnitude of thermal load for the loading scenario (i) and vice versa for the loading scenario (iv). For example, for $\eta=1$ energy pile in soil A2 does not develop tensile zone while in soil C it develops tensile zone along 80% of its length for load scenario (i) and along 20% of its length for load scenario (iv).

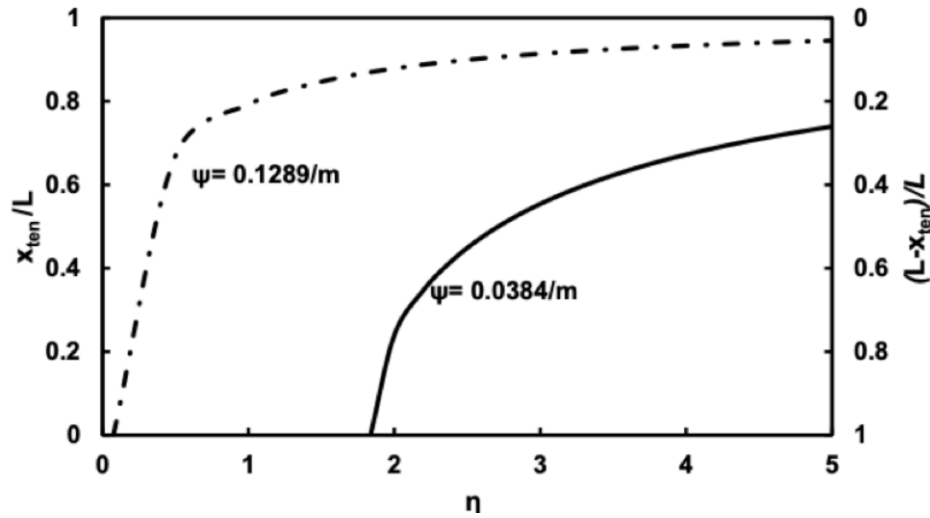


Figure 2.16. Normalized location of zero combined stress versus η for ideally end bearing energy pile, scenarios (i) and (iv), $k_h = 0$, soils A2 and C

Figure 2.17 depicts mechanical and combined stresses in the fully floating pile for the loading scenario (i) and several values of η . Similarly, to the behavior of ideally end bearing pile shown in Figure 2.15, it can be observed that the length of tension zone increases with increasing

magnitude of thermal load and that for given value of η the length of the tension zone is larger in a stiffer soil.

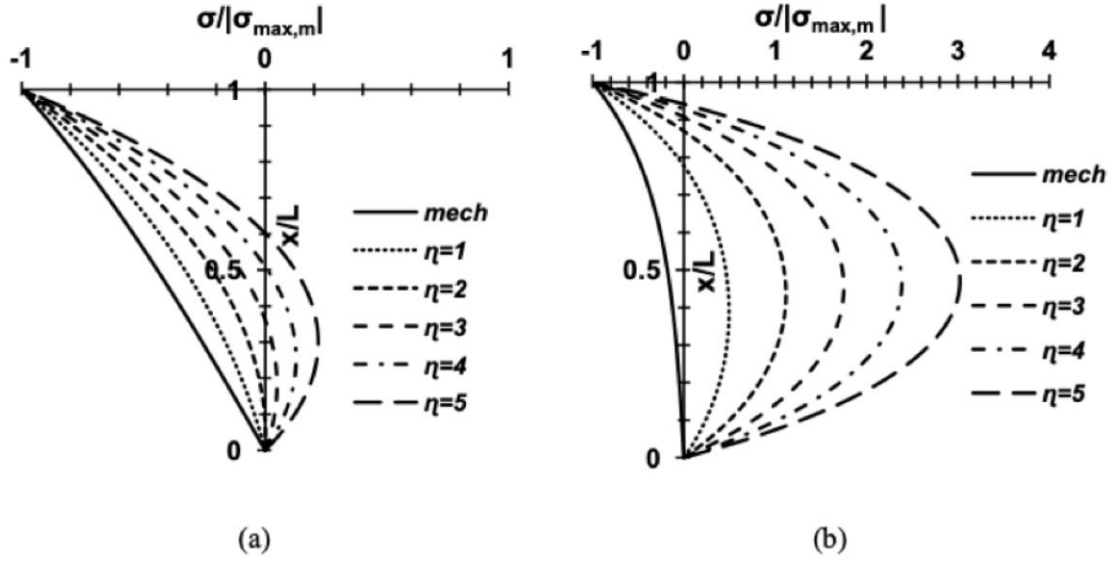


Figure 2.17. Mechanical and combined stresses versus depth in nearly floating energy pile subjected to loading scenario (i) and $k_h = 0$, a) $\psi=0.0384/\text{m}$ (soil A2), (b) $\psi=0.1289/\text{m}$ (soil C)

The x -coordinate corresponding to zero combined stress (x_{ten}) in a fully floating pile is obtained by combining Eqs. (2.19) for $x_0=L/2$, (2.41) for $\beta \approx 0$, and (2.49). It is given by

$$x_{ten} = \frac{2}{\psi} \tanh^{-1} \left[\tanh\left(\frac{\psi L}{2}\right) - \frac{1}{\eta \sinh(\psi L)} \right] \quad (2.52)$$

thus requiring that

$$\eta > \frac{1}{\sinh(\psi L) \left[\tanh\left(\frac{\psi L}{2}\right) - 1 \right]} \quad (2.53)$$

Figure 2.18 illustrates Eq. (2.52) in a normalized form. For the scenario (i) the length of the tension zone increases with increasing magnitude of thermal load, while for the scenario (iv) the length of tension zone decreases with increasing magnitude of thermal load. Furthermore, it is noted that development of tension zone in the fully floating energy pile requires larger magnitude of thermal load than in the ideally end bearing pile for the load scenario (i) and vice versa for the

load scenario (iv). This trend is far more pronounced in soil A2 than in soil C. Specifically, at $\eta = 2$ the length of tension zone in the fully floating pile embedded in soil A2 is equal to about 7% of the entire pile length (Figure 2.18), while it is equal to about 24% of the pile length in the ideally end bearing pile (Figure 2.16) for the load scenario (i).

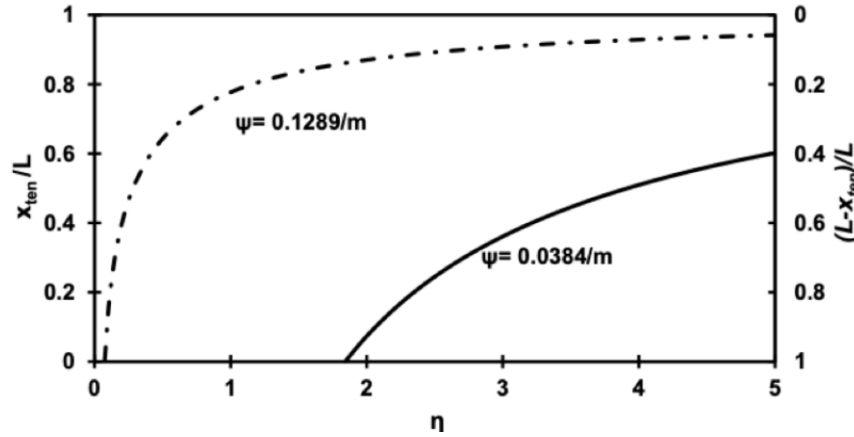


Figure 2.18. Normalized location of zero combined stress versus η for fully floating energy pile, scenarios (i) and (iv), $k_h = 0$, soils A2 and C

Figure 2.19 depicts distribution of mechanical and combined stresses in the ideally end bearing pile for the loading scenario (ii) and different values of η . It is seen that the combined stress is constant with depth for equivalent thermal load and its magnitude increases beyond that for thermal loads of a larger magnitude. In the latter case the maximum magnitude of the combined stress is located at the pile tip. Fig. 19 also shows that the magnitude of a combined stress can be significantly affected by soil shear stiffness of soil. A distribution of a combined stress for loading scenario (iii) can be obtained by changing the sign of stress depicted in Figure 2.19, thus indicating development of a tension zone throughout the entire length of the pile.

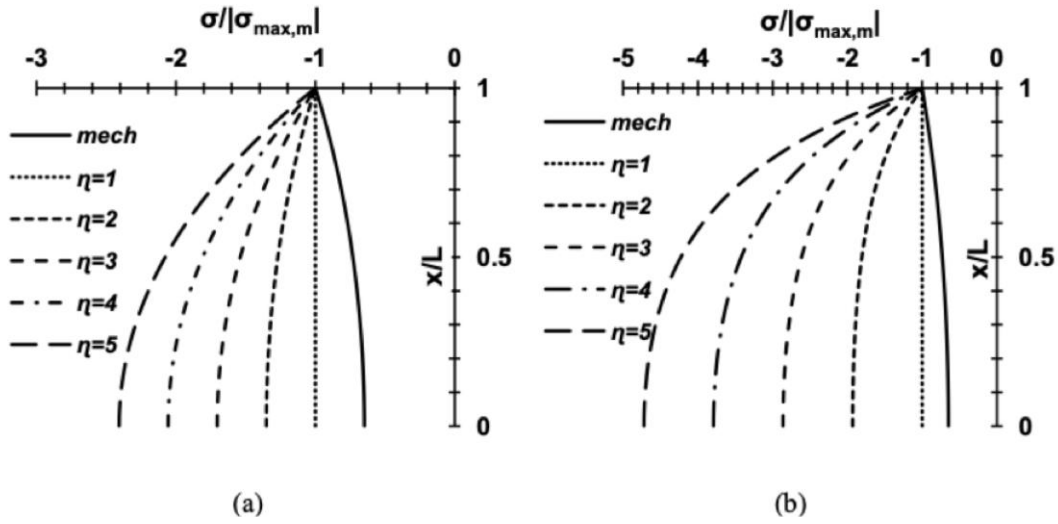


Figure 2.19. Mechanical and combined stresses versus depth in ideally end bearing energy pile for loading scenario (ii) and $k_h = 0$, (a) $\psi=0.0384/\text{m}$ (soil A2), (b) $\psi=0.1289/\text{m}$ (soil C)

Head restraint effects

Head restraints have significant impacts on thermomechanical response of energy piles. The stress in the pile varies drastically depending on restraints at the pile tip and the pile head (Saeidi Rashk Olia and Perić 2021c). To this end, Figure 2.20 depicts distribution of thermal stress versus depth for different magnitudes of head restraint in ideally end bearing and fully floating piles subjected to heating. It is seen that presence of the head restraint increases magnitude of thermal stress more significantly in the ideally end bearing pile. Specifically, the magnitude of thermal stress at the mid-length of the ideally end bearing pile increases 2.1 times as k_h/k_b increases from 0 to 1. In fully floating pile the magnitude of thermal stress at the mid-length increases 1.75 times for the same increase in the stiffness of the head spring. Furthermore, the increase in k_h/k_b from 0 to 1 induces larger increase in the magnitude of thermal stress at the pile head than at the pile tip for both, ideally end bearing and fully floating piles.

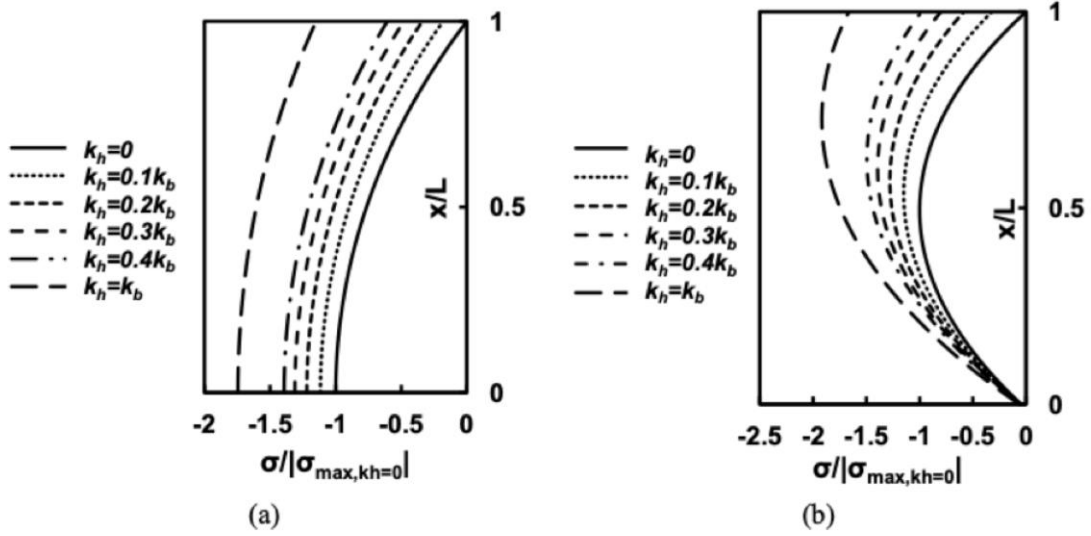


Figure 2.20. Effect of head restraint (k_h) on normalized thermal stress ($\Delta T > 0$) in: (a) ideally end bearing energy pile (soil A2), (b) fully floating energy pile (soil A2)

Figure 2.21 illustrates effects of head and tip restraints on the location of a thermal null point in semifloating and nearly fully floating energy piles. The normalized location of the thermal null point is depicted for soils A2 and C based on Eq. (2.25). Clearly, location of the thermal null point in the absence of any head and tip restraints is located at the mid-length of the pile. If tip restraint is present, but head restraint is absent Eq. (2.25) indicates that the thermal null point will be located within the bottom half of the pile. As the ratio of head and tip restraint increases the location of the null point moves into the top half of the pile. It is also noted that the effect of end restraints on the location of the thermal null point is highly affected by the shear stiffness of the soil. Specifically, the location of the null point is much less affected in the stiffer soil (C), than in the softer soil (A2).

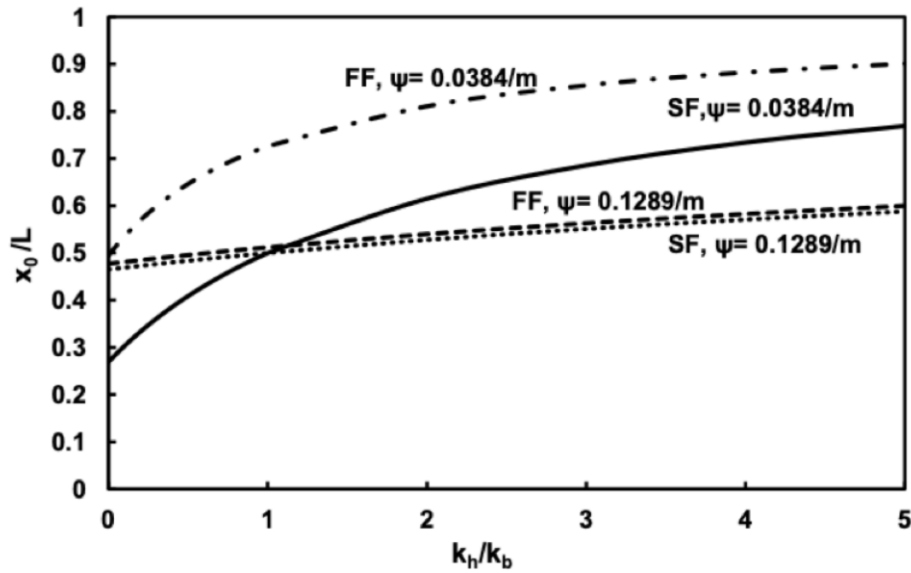


Figure 2.21. Effect of head restraint (k_h) on the normalized location of null point in fully floating and semi-floating energy piles, soils A2 and C

Conclusions

Analytical solutions for axial displacement, stress and strain in a single energy pile exhibiting reversible interface behavior subjected to different end restrains and combined thermal and mechanical loads have been derived. The solutions provide a fundamental, rational, mechanics based qualitative and quantitative understanding of the load transfer mechanism. Resulting evolutions of axial displacement, strain and stress in the pile are presented.

Analytical model was successfully validated against the results of two centrifuge tests involving a semifloating pile and a nearly ideally end bearing pile. It is because the thermal null point in a semifloating pile was located close to its mid-length that this pile also represents fully floating pile as defined in this study. Thus, the validation can be considered to have been successfully completed for nearly ideally end bearing, fully floating and semifloating piles.

The load transfer diagrams indicate thermal load is accommodated by a larger magnitude of normalized displacement in an ideally end bearing pile for the most part than in the fully floating pile, thus resulting in a smaller magnitude of a normalized strain in the ideally end bearing pile than in the fully floating pile. Nevertheless, thermal load is also accommodated by a larger magnitude of normalized stress in the ideally end bearing pile than in the fully floating pile.

Furthermore, it was found that mechanical load is dominant when combined with an equivalent thermal load regardless of a loading scenario. Nevertheless, thermal load eventually becomes dominant as its magnitude increases beyond that of an equivalent thermal load. This process leads to a formation of increasingly larger tensile stress and longer tension zone in energy piles in the case of a combined compressive mechanical load and net cooling. On the contrary, in the case of a combined tensile mechanical load and heating the length of a tension zone becomes smaller with increasing magnitude of thermal load. It is also found that at a constant mechanical load, the length of a tension zone depends on the geometry of the pile, ratio of the soil and pile stiffness, and on the magnitude of thermal load. Otherwise, in the case of a combined compressive axial force and heating the stress in the pile is always compressive throughout the entire pile, and in the case of a combined tensile axial force and cooling the stress is always tensile throughout the entire pile. Clearly, the structural integrity of reinforced concrete energy piles needs to be ensured not only in case that compressive stress develops but also in the case that tensile stress evolves. Addition of thermal load can also increase the interface shear stress as well as the normal stress at the pile tip. Furthermore, in the case that the tensile stress develops at the pile tip the pile has to be safely anchored into the bedrock to ensure the transfer of a tensile stress.

In ideally end bearing energy piles a thermal null point remains at the pile tip regardless of a presence of a head restraint. Otherwise, it is found that location of the thermal null point depends

on the geometry and stiffness of the pile, stiffness of the soil, and amount of head and tip restraints. The highest location of the thermal null point in the absence of a head restraint occurs in an ideally fully floating pile and it coincides with the mid-depth of the pile. For a semifloating pile in the absence of a head restraint the thermal null point is located in the bottom half of the pile. Finally, the thermal null point moves upward with increasing stiffness of a head restraint, thus implying that the thermal null point ends up being located within the top half of the pile even in a semifloating pile for a sufficiently large head restraint.

Chapter 3 - Analytical Solutions for Thermo-Mechanical Soil

Structure Interaction in Single Semi Floating Energy Piles

Embedded in a Layered Soil Profile

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Abstract:

Analytical solutions for axial displacement, strain, and stress in a single semi-floating energy pile embedded in a layered soil profile subjected to thermal and mechanical loads have been derived. Furthermore, analytical solution for the location of zero-displacement, also known as a null point, was derived for a pile subjected to thermal load. It is shown that the location of the thermal null point corresponds to the location of the maximum magnitude of thermal axial stress. It is also shown that the location of the null point moves towards the pile tip as the stiffness of the bedrock below the pile tip increases. The analytical solutions were validated against in situ full scale energy pile tests. The solutions along with the validation process delineate the load transfer mechanism in energy piles subjected to thermal, mechanical and combined thermo-mechanical loads embedded in a four-layered soil profile. Furthermore, the flowcharts delineating the procedures for obtaining the analytical solutions for a single energy pile embedded in an arbitrary number of layers, and subjected to thermal and mechanical loads are also provided. While continuity of stresses and displacements at the interface of different soil layers is

maintained, displacement, strain and stress diagrams exhibit a lack of smoothness, the amount of which depends on the difference in the stiffness of these layers. In summary, the presented solutions provide a rational, mechanics-based framework for advancing the understanding of thermo-mechanical response of energy piles that is not only essential for analysis and design, but it ultimately contributes to a wider use of energy piles and increased sustainability of civil engineering infrastructure.

Key words: analytical solutions, energy pile, null point, semi-floating, soil structure interaction

Introduction

Burger et al. (1985) pointed out that the ground temperature at depths of below 3 to 5 m is constant and approximately equal to the mean air temperature. Brandl (2006) noted that the constant temperature provides a stable source for an efficient heat exchange between the shallow subsurface and superstructure. Pile foundations provide access to these depths, thus enabling harvesting of the renewable thermal energy in addition to transferring the superstructure loads to the subsurface. To this end, reinforced concrete energy piles are equipped with a closed loop tubing that serves as a fluid conduit. The heat exchange fluid, which circulates inside the tubing, is heated in the subsurface during winters and cooled during summers, thus providing a supplemental or a sole source of energy for space heating and cooling. Furthermore, the use of a subsurface renewable thermal energy for space heating and cooling decreases the emission of carbon dioxide into the atmosphere as compared to the sole use of fossil fuels.

The dual role of energy piles implies the dual nature of their loads, mechanical and thermal. The latter are induced due to the presence of soil and rock that surround the piles, thus inhibiting their free thermal deformation. A rational, mechanics-based understanding of the underlying load

transfer mechanisms including thermo-mechanically induced displacement, strain and stress is vitally important for analysis, design and wider use of energy piles, through which sustainability of civil infrastructure can be enhanced.

Physical and numerical modeling have been used to advance the knowledge of load transfer mechanisms in energy piles. The former typically employs full scale field tests, geotechnical centrifuge tests and small-scale pile tests. The latter usually relies on the use of finite element and finite difference methods for capturing the response of energy piles. The finite difference method uses load transfer functions that characterize the load transfer between the pile and the soil.

Traditional load transfer analyses for mechanical loads were proposed by Seed and Reese (1957) and Coyle and Reese (1966). Many load transfer functions are available in the literature (Randolph and Wroth 1978; Frank and Zhao 1982; Armaleh and Desai 1987; Frank et al. 1991). Knellwolf et al. (2011) were probably the first to use the load transfer concept within the framework of energy piles. They adopted the piecewise linear T-z and Q-z functions that characterize the load transfer along the pile shaft and at the pile tip respectively. These load transfer functions were previously proposed by Frank and Zhao (1982) for the transfer of mechanical loads. In addition, Knellwolf et al. (2011) added the unloading branch that describes the irreversible behavior. Plaseied (2012) proposed a smooth non-linear T-z curve with relatively large initial linear portion. To understand the effects and the range of parameters of load transfer function proposed by Plaseied (2012) Chen and McCartney (2017) calibrated the model parameters against the results of full scale and geotechnical centrifuge tests.

Suryatriyastuti et al. (2014) proposed a cyclic T-z function that comprises a cyclic hardening/softening mechanism to address cyclic thermal loads. Their load transfer function is able to account for degradation of the soil –pile capacity during two-way cyclic thermal loading in

energy piles. Ouyang et al. (2011) investigated the applicability of a hybrid load transfer approach that uses the elastic continuum solution to model the interaction effects among piles, while adopting the load transfer approach for the soil-pile interaction. Pasten and Santamarina (2014) investigated the long term response of a single pile subjected to thermal cycles by using one dimensional load transfer formulation modified to consider thermoplastic effects.

Bourne-Webb et al. (2009), Amatya et al. (2012), and Bourne-Webb et al. (2013) presented schematic diagrams of axial strain, axial load, and interface shear stress in energy piles, to which they referred as “simplified mechanism” and “descriptive framework”. They assumed that the pile is installed in a soil that exhibits constant shaft resistance with depth and inferred the forces mobilized in the pile and at the pile soil interface based on the experimentally observed response. Rotta Loria and Laloui (2018) stated that the load transfer diagrams, which they presented, are based on theory of thermo-elasticity. While their schematics differ from those previously proposed by Bourne-Webb et al. (2009), Amatya et al. (2012), and Bourne-Webb et al. (2013) none of these authors provided mathematical expressions for vertical displacement, shear stress, axial strain and stress. Furthermore, Rotta Loria and Laloui (2018) pointed out that a number of conditions of validity of the schematics provided by Bourne-Webb et al. (2009) and Bourne-Webb et al. (2013) have never been explained.

Analytical solutions (Cossel 2019, Perić et al. 2020, Iodice et al. 2020, Saeidi Rashk Olia and Perić 2021a) for soil structure interaction in energy piles have recently been derived. Cossel (2019) provided analytical solutions for displacement, strain and stress in a single energy pile embedded into a homogeneous and four-layered soil profiles having constant stiffness with depth. Perić et al. (2020) presented analytical solutions for displacement, strain and stress in a single ideally end-bearing energy pile embedded into a four layered soil whereby the stiffness of each

soil layer was constant with depth. The solutions were validated against the in situ full scale energy pile tests. Iodice et al. (2020) presented analytical solutions for the energy pile embedded into a single soil layer whereby the stiffness of a shear spring was both, constant and a linear function of depth. They also presented the analytical solution for a two layered soil profile. They provided explicit expressions for displacement and stress in the pile as well as for the interface shear stress. Furthermore, Iodice et al. (2020) derived the solutions for a single layered and multi layered soil profiles based on the approximate energy method. These latter solutions were validated against the full-scale energy pile tests. Saeidi Rashk Olia and Perić (2021a) provided analytical solutions for displacement, strain and stress in an energy pile embedded into a single soil layer having constant stiffness with depth and subjected to thermal and mechanical loads as well as to different end restraints. They validated these solutions against the centrifuge tests.

This study provides explicit analytical solutions for axial displacement, strain and stress in a single semi-floating energy pile embedded into a four layered soil profile whereby the stiffness of each layer is constant with depth. In addition, the analytical solution procedures for a single energy pile embedded into an arbitrary number of soil layers are provided for both, thermal and mechanical loads. The solutions contribute to advancing the qualitative and quantitative understanding of load transfer mechanism in energy piles subjected to thermal and thermo-mechanical loads. Namely, the solutions provide an aid in interpretation of results obtained from numerical and physical models while contributing towards development of improved design procedures.

Assumptions and governing laws

Superstructures and foundations should be designed with a sufficient margin of safety with respect to failure, thus implying that they are both likely to remain within the elastic state under working static stresses. Furthermore, Laloui et al. (2003) and Bourne-Webb et al. (2009) found from the full scale in situ tests, and Laloui et al. (2006), Knellwolf et al. (2011), Ozudogru et al. (2015) and Perić et al. (2020) found from comparisons of predictions of numerical models and observations collected during the corresponding field tests that the energy piles exhibited fully reversible behavior, thus remaining in the elastic state. Consequently, it is assumed herein that both, the pile and the soil pile interface remain in the elastic state. Additionally, Laloui et al. (2003) found that vertical displacements of energy piles were dominant in comparison to horizontal displacements. Thus, one dimensional model is used herein to capture the main features of a single energy pile response.

A positive x-axis and positive displacement are directed upwards while axial stress and strain are both negative in compression. Thus, a compressive axial force is negative while heating of the pile induces positive temperature change. Next, the assumptions and laws of mechanics, upon which the analytical solutions found herein, are based are summarized as follows:

- 1D formulation is used, whereby the single coordinate is oriented in vertical direction.
- Equilibrium in vertical direction is enforced.
- Higher order terms in the kinematic relationship are neglected.
- The energy pile is assumed to follow a thermo-elastic constitutive law.

- Soil pile interface is assumed to remain in the elastic range whereby both, the shear stress along the shaft and normal stresses at the pile tip are linear functions of vertical displacement.
- Temperature change is uniform throughout the entire pile at any given time.
- Material properties of soil and the pile are not affected by temperature.

The kinematic relationship of the pile is given by

$$\varepsilon = \frac{du}{dx} \quad (3.1)$$

where $u = u(x)$ is the axial displacement of the pile, and $\varepsilon = \varepsilon(x)$ is the axial strain in the pile. For a general case of thermo-elasticity, the corresponding constitutive law of the pile is given by

$$\sigma = E(\varepsilon - \alpha \Delta T) \quad (3.2)$$

where $\sigma = \sigma(x)$ is the axial stress, E is the Young's modulus of the pile, α is the coefficient of thermal expansion of the pile, and ΔT is a temperature change of the pile relative to the surrounding soil.

The soil adjacent to the pile is replaced by a continuous linear elastic shear spring that is attached along the pile shaft. Thus, the shear stress $\tau = \tau(x)$, which acts along the pile shaft, is given by

$$|\tau| = k_s |u| \quad (3.3)$$

where k_s is the stiffness of the shear spring that is constant with depth for a given soil layer. Similar to the Knellwolf et al. (2011) study, the self-weight of the pile is not considered in this study.

Analytical solutions

A layered soil profile is considered whereby the energy pile is surrounded by four different soil layers while its tip is flush with the top boundary of a bedrock or another soil layer. Every soil layer is characterized by a shear spring of a different stiffness (k_{sj}), where $j = 1$ to 4. Figure

3.1 depicts the pile subjected to thermal loading. The entire length of the pile is divided into four different segments having lengths L_j . Each pile segment has its own coordinate system x_j . Axial displacement, strain and stress for each pile segment are denoted by u_j , ε_j , σ_j .

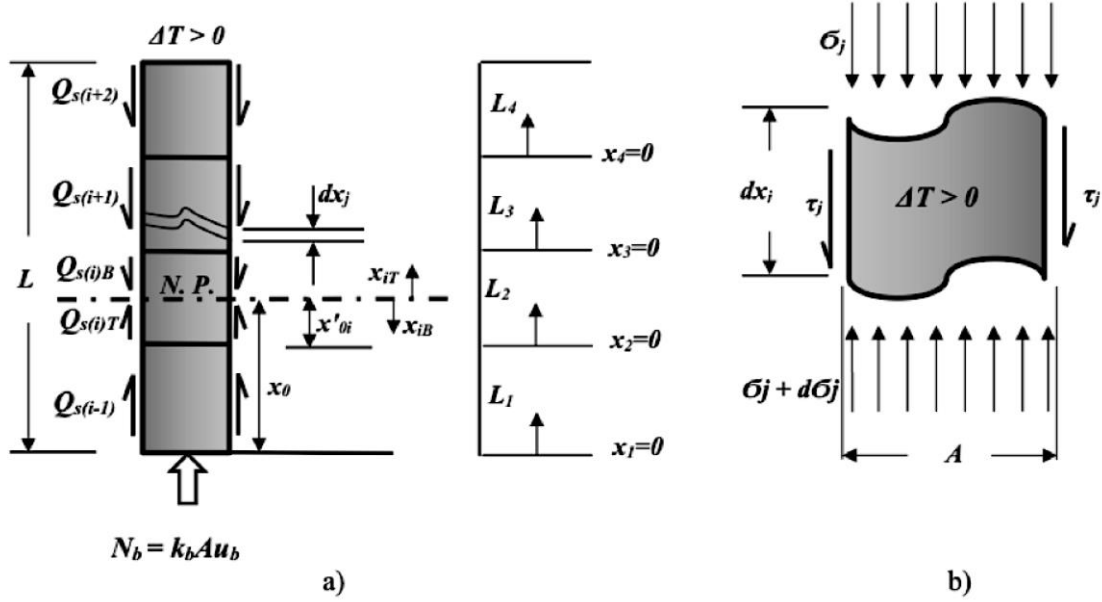


Figure 3.1. FBDs of the energy pile in layered soil profile subjected to thermal load ($\Delta T > 0$),
a) the entire energy pile, b) an infinitesimal segment of energy pile above the null point

Thermal Load

Imposing equilibrium of an infinitesimal pile segment located within any of the four soil layers surrounding the pile (Figure 3.1) results in

$$\frac{d^2 u_j}{dx_j^2} - \psi_j^2 u_j = 0 \quad j = 1, 4 \text{ (no summation)} \quad (3.4)$$

where parameter ψ_j is a constant given by

$$\psi_j^2 = \left(\frac{p}{A}\right) \left(\frac{k_{sj}}{E}\right) = \xi_g \xi_{sj} \quad (3.5)$$

and p and A are the perimeter and cross-section area of the pile respectively. Thus, the geometry of the pile is expressed by the parameter ξ_g . Furthermore, the pile radius is constant versus depth thus

leading to a constant value of ξ_g versus depth. Parameter ξ_{sj} represents the ratio of the stiffness of a shear spring replacing j -th soil layer and the stiffness of the pile.

Solving for the unknown function $u_j(x_j)$ from Eq. (3.4) gives

$$u_j(x_j) = C_{j,1}e^{\psi_j x_j} + C_{j,2}e^{-\psi_j x_j} \quad j = 1,4 \text{ (no summation)} \quad (3.6)$$

Combining Eqs. (3.1), (3.2), and (3.6) and results in

$$\varepsilon_j(x_j) = \psi_j(C_{j,1}e^{\psi_j x_j} - C_{j,2}e^{-\psi_j x_j}) \quad j = 1,4 \text{ (no summation)} \quad (3.7)$$

and

$$\sigma_j(x_j) = E[\psi_j(C_{j,1}e^{\psi_j x_j} - C_{j,2}e^{-\psi_j x_j}) - \alpha\Delta T] \quad j = 1,4 \text{ (no summation)} \quad (3.8)$$

Boundary condition at the pile tip is given by

$$\sigma_1(0) = k_b u_1(0) \quad (3.9)$$

where k_b is the stiffness of the normal spring attached to the pile tip. Additionally, continuity of displacements and stresses is enforced at the transitions between different pile segments, thus resulting in

$$u_j(L_j) = u_{j+1}(0) \quad n = 1,3 \text{ (no summation)} \quad (3.10)$$

and

$$\sigma_j(L_j) = \sigma_{j+1}(0) \quad n = 1,3 \text{ (no summation)} \quad (3.11)$$

Where L_j is the thickness of the j -th layer. Since the bedrock is not infinitely stiff the thermal null is located above the pile tip. The location of the null point is not known a priori. During the solution process it is first assumed that the null point is located within each layer and corresponding solution is obtained. The exact location of the null point for any given site is determined by substituting the relevant soil parameters determined from in situ tests, thus resulting in the null point being located in only one layer (i -th layer) out of the four layers. Coordinate x'_{oi} indicates

the distance of the null point above the bottom boundary of the i -th layer as shown in Figure 3.1. Positive x'_{0i} is directed upwards.

The continuity of the stress and displacement at the null point, which is located in the i -th layer, can be expressed in local coordinates as

$$u_{iB}(0) = u_{iT}(0) = 0 \quad \text{and} \quad \sigma_{iB}(0) = \sigma_{iT}(0) \quad (3.12)$$

Next, Eqs. (3.1)(3.2)(3.6) and (3.12) are combined whereby x_j , $C_{j,1}$, $C_{j,2}$ in Eq. (3.6) are replaced first by x_{iT} , and $C_{iT,1}$ and $C_{iT,2}$, and then by x_{iB} , and $C_{iB,1}$ and $C_{iB,2}$. This results in

$$C_{iT,2} = C_{iB,2} = -C_{iT,1} \quad (3.13)$$

$$C_{iB,1} = C_{iT,1} \quad (3.14)$$

Four possible scenarios are considered corresponding to the null point being located in the each of the four layers. If the null point is assumed to be in the i -th layer ($i = 1$ to 4), the solutions are derived for the layer containing the null point (i -th layer) and the three remaining layers. The following notation is adopted.

$$\sinh[\psi_i(L_i - x'_{0i})] = SH_{iT} \quad (3.15)$$

And

$$\cosh[\psi_i(L_i - x'_{0i})] = CH_{iT} \quad (3.16)$$

where L_i is the length of the pile within the i -th layer, and x'_{0i} is given by

$$x'_{0i} = x_0 - \sum_{n=1}^{n=i-1} L_n \quad (3.17)$$

where x_0 is a global coordinate of the null point.

Expressing the solutions for displacement, strain and stress given in Eqs. (3.6) through (3.8) for the layer containing the null point (i -th layer) results in

$$C_{i,1} = C_{iT,1} e^{-\psi_i x'_{0i}} \quad (3.18)$$

$$C_{i,2} = -C_{iT,1} e^{\psi_i x'_{oi}} \quad (3.19)$$

The constants for the first layer above the i -th layer are determined from the continuity of displacements and stresses, which were given in Eqs. (3.10) and (3.11) Thus,

$$C_{i+1,1} = C_{iT,1} \left(SH_{iT} + \frac{\psi_i}{\psi_{i+1}} CH_{iT} \right) \quad (3.20)$$

$$C_{i+2,2} = C_{iT,1} \left(SH_{iT} - \frac{\psi_i}{\psi_{i+1}} CH_{iT} \right) \quad (3.21)$$

Similarly, the constants for the second layer above the layer, in which the null point is located, are given by

$$C_{i+2,1} = C_{i,1T} \left[SH_{iT} \left(CH_{i+1} + \frac{\psi_{i+1}}{\psi_{i+2}} SH_{i+1} \right) + CH_{iT} \left(\frac{\psi_i}{\psi_{i+1}} SH_{i+1} + \frac{\psi_i}{\psi_{i+2}} CH_{i+1} \right) \right] \quad (3.22)$$

$$C_{i+2,2} = C_{i,1T} \left[SH_{iT} \left(CH_{i+1} - \frac{\psi_{i+1}}{\psi_{i+2}} SH_{i+1} \right) + CH_{iT} \left(\frac{\psi_i}{\psi_{i+1}} SH_{i+1} - \frac{\psi_i}{\psi_{i+2}} CH_{i+1} \right) \right] \quad (3.23)$$

And constants for the third layer above the layer, in which the null point is located, are

$$C_{i+3,1} = C_{iT,1} \left[(\psi_{i+1} SH_{iT} SH_{i+1} + \psi_i CH_{iT} CH_{i+1}) \left(\frac{SH_{i+2}}{\psi_{i+2}} + \frac{CH_{i+2}}{\psi_{i+3}} \right) \right. \quad (3.24)$$

$$\left. + \left(SH_{iT} CH_{i+1} + \frac{\psi_i}{\psi_{i+1}} CH_{iT} SH_{i+1} \right) \left(CH_{i+2} + \frac{\psi_{i+2}}{\psi_{i+3}} SH_{i+2} \right) \right]$$

$$C_{i+3,2} = C_{iT,1} \left[(\psi_{i+1} SH_{iT} SH_{i+1} + \psi_i CH_{iT} CH_{i+1}) \left(\frac{SH_{i+2}}{\psi_{i+2}} - \frac{CH_{i+2}}{\psi_{i+3}} \right) \right. \quad (3.25)$$

$$\left. + \left(SH_{iT} CH_{i+1} + \frac{\psi_i}{\psi_{i+1}} CH_{iT} SH_{i+1} \right) \left(CH_{i+2} - \frac{\psi_{i+2}}{\psi_{i+3}} SH_{i+2} \right) \right]$$

The boundary condition at the pile head for the four-layer system is given by

$$\sigma_4(L_4) = -k_h u_4(L_4) \quad (3.26)$$

where k_h represents the stiffness of the normal spring at the pile head that models the restraint imposed by the superstructure. By combining Eqs. (3.6) through (3.8) and Eqs. (3.18) through

(3.25), and enforcing the boundary condition from Eq. (3.26) constant $C_{iT,1}$ is determined. It is given by

$$C_{iT,1} = \frac{\alpha \Delta T}{2} [SH_{iT} * (f + g) + CH_{iT} * (h + j)]^{-1} \quad (3.27)$$

where

$$f = \psi_4(aSH_4 + bCH_4) \quad (3.28)$$

$$g = \frac{k_h}{E}(aCH_4 + bSH_4) \quad (3.29)$$

$$h = \psi_4(cSH_4 + dCH_4) \quad (3.30)$$

$$j = \frac{k_h}{E}(cCH_4 + dSH_4) \quad (3.31)$$

Constants a through d are given in Table 3.1 whereby x'_{oi} can be located in each of the four layers.

The following notation is introduced

$$\sinh(\psi_i x'_{oi}) = SH_{iB} \quad (3.32)$$

$$\cosh(\psi_i x'_{oi}) = CH_{iB} \quad (3.33)$$

$$\sinh(\psi_i L_i) = SH_i \quad (3.34)$$

$$\cosh(\psi_i L_i) = CH_i \quad (3.35)$$

where $i = 1, 2, 3$ or 4 .

Table 3.1. Coefficients for $C_{iT,1}$

Variable	Layer of x'_0 (the i -th layer)			
	1	2	3	4
a	$\frac{\psi_2}{\psi_3} SH_2 SH_3 + CH_2 CH_3$	CH_3	1	CH_4
b	$\frac{\psi_2}{\psi_4} SH_2 CH_3 + \frac{\psi_3}{\psi_4} CH_2 SH_3$	$\frac{\psi_3}{\psi_4} SH_3$	0	$-SH_4$
c	$\frac{\psi_1}{\psi_2} SH_2 CH_3 + \frac{\psi_1}{\psi_3} CH_2 SH_3$	$\frac{\psi_2}{\psi_3} SH_3$	0	$-SH_4$
d	$\frac{\psi_1}{\psi_4} \left(\frac{\psi_3}{\psi_2} SH_2 SH_3 + CH_2 CH_3 \right)$	$\frac{\psi_2}{\psi_4} CH_3$	$\frac{\psi_3}{\psi_4}$	CH_4

The constants for the first layer below the i -th layer are obtained by enforcing the continuity of stress and displacement between two subsequent layers, which was described by Eqs. (3.10)) and (3.11), and it leads to

$$C_{i-1,1} = \frac{1}{2} \left[\left(1 + \frac{\psi_i}{\psi_{i-1}} \right) C_{i,1} e^{\psi_i L_i} + \left(1 - \frac{\psi_i}{\psi_{i-1}} \right) C_{i,2} e^{-\psi_i L_i} \right] \quad (3.36)$$

$$C_{i-1,2} = \frac{1}{2} \left[\left(1 - \frac{\psi_i}{\psi_{i-1}} \right) C_{i,1} e^{\psi_i L_i} + \left(1 + \frac{\psi_i}{\psi_{i-1}} \right) C_{i,2} e^{-\psi_i L_i} \right] \quad (3.37)$$

Combining Eqs. (3.18) and (3.19) with Eqs. (3.32) through (3.37) results in

$$C_{i-1,1} = C_{iT,1} \left[SH_{iB} + \left(\frac{\psi_i}{\psi_{i-1}} \right) CH_{iB} \right] \quad (3.38)$$

$$C_{i-1,2} = C_{iT,1} \left[SH_{iB} - \left(\frac{\psi_i}{\psi_{i-1}} \right) CH_{iB} \right] \quad (3.39)$$

whereby the null point is located in the i -th layer. Similarly, the following is obtained

$$C_{i-2,1} = C_{iT,1} \left[SH_{iB} \left(CH_{i-1} + \left(\frac{\psi_{i-1}}{\psi_{i-2}} \right) SH_{i-1} \right) + CH_{iB} \left(\left(\frac{\psi_i}{\psi_{i-1}} \right) SH_{i-1} + \left(\frac{\psi_i}{\psi_{i-2}} \right) CH_{i-1} \right) \right] \quad (3.40)$$

$$C_{i-2,2} = C_{iT,1} \left[SH_{iB} \left(CH_{i-1} - \left(\frac{\psi_{i-1}}{\psi_{i-2}} \right) SH_{i-1} \right) \right. \quad (3.41)$$

$$\left. + CH_{iB} \left(\left(\frac{\psi_i}{\psi_{i-1}} \right) SH_{i-1} - \left(\frac{\psi_i}{\psi_{i-2}} \right) CH_{i-1} \right) \right]$$

$$C_{i-3,1} = C_{iT,1} \left[(\psi_{i-1} SH_{iB} SH_{i-1} + \psi_i CH_{iB} CH_{i-1}) \left(\frac{SH_{i-2}}{\psi_{i-2}} + \frac{CH_{i-2}}{\psi_{i-3}} \right) + (SH_{iB} CH_{i-1} \right. \quad (3.42)$$

$$\left. + \frac{\psi_i}{\psi_{i-1}} CH_{iB} SH_{i-1}) (CH_{i-2} + \frac{\psi_{i-2}}{\psi_{i-3}} SH_{i-2}) \right]$$

$$C_{i-3,2} = C_{iT,1} \left[(\psi_{i-1} SH_{iB} SH_{i-1} + \psi_i CH_{iB} CH_{i-1}) \left(\frac{SH_{i-2}}{\psi_{i-2}} - \frac{CH_{i-2}}{\psi_{i-3}} \right) + (SH_{iB} CH_{i-1} \right. \quad (3.43)$$

$$\left. + \frac{\psi_i}{\psi_{i-1}} CH_{iB} SH_{i-1}) (CH_{i-2} - \frac{\psi_{i-2}}{\psi_{i-3}} SH_{i-2}) \right]$$

To find the location of the null point in a multi-layered system the possibility that the null point is located within each of the four layers is first considered. The final solution for the location of the null point is the only one that gives the reasonable value of x_0 . For example, if the null point is located in the i -th layer then the following holds

$$\sum_1^{i-1} L_n \leq x_0 \leq \sum_1^i L_n \quad \text{where } i = 1, 2, 3 \text{ or } 4 \quad (3.44)$$

By using Eqs. (3.18) through (3.25), and (3.27) through (3.43), the constants for the layer containing the thermal null point, and three layers above and below it, are obtained. Substituting these values into Eqs. (3.6) through (3.8) gives solutions for u , ε , and σ within the corresponding layer. By enforcing the boundary condition at the pile tip, which was given by Eq. (3.9) the solution for the location of the null point in each of the four layers is derived. It is given by

$$x'_{0i} = \frac{1}{\psi_i} \tanh^{-1} \left[\frac{SH_i(f+g) + CH_i(h+j) - n \left(\psi_1 SH_1 + \frac{k_p}{E} CH_1 \right) - o \left(\psi_1 CH_1 + \frac{k_p}{E} SH_1 \right)}{CH_i(f+g) + SH_i(h+j) + l \left(\psi_1 SH_1 + \frac{k_b}{E} CH_1 \right) + m \left(\psi_1 CH_1 + \frac{k_p}{E} SH_1 \right)} \right] \quad (3.45)$$

where constants l , m , n and o are given in Table 3.2 for each scenario.

Table 3.2 Coefficients for x'_{0i}

Variable	Layer of x'_0 (the i -th layer)			
	1	2	3	4
<i>l</i>	CH_1	1	CH_2	$\frac{\psi_3}{\psi_2}SH_2SH_3 + CH_2CH_3$
<i>m</i>	$-SH_1$	0	$\frac{\psi_2}{\psi_1}SH_2$	$\frac{\psi_2}{\psi_1}SH_2CH_3 + \frac{\psi_3}{\psi_1}CH_2SH_3$
<i>n</i>	$-SH_1$	0	$\frac{\psi_3}{\psi_2}SH_2$	$\frac{\psi_4}{\psi_2}SH_2CH_3 + \frac{\psi_4}{\psi_3}CH_2SH_3$
<i>o</i>	CH_1	$\frac{\psi_2}{\psi_1}$	$\frac{\psi_3}{\psi_1}CH_2$	$\frac{\psi_4}{\psi_1}\left(\frac{\psi_2}{\psi_3}SH_2SH_3 + CH_2CH_3\right)$

After finding the final solution for the null point in accordance with Eq. (3.45) the corresponding x_o value is obtained by rearranging Eq. (3.17) into the following form

$$x_0 = \sum_{1}^{i-1} L_n + x'_{0i} \quad (3.46)$$

Most of layered soil profiles within the context of energy piles can be idealized by the maximum of four soil layers. Thus, herein the solutions are derived for a four-layered soil profile. Furthermore, a flowchart depicted in Figure 3.2 delineates the analytical solution procedure for a single energy pile embedded in an arbitrary number of soil layers (n), subjected to a thermal load. To this end, as described in Figure 3.2, step 3, for any given number of layers (n), solutions should be derived for $(n-1)$ layers above and $(n-1)$ layers below the layer containing the null point (i -th layer).

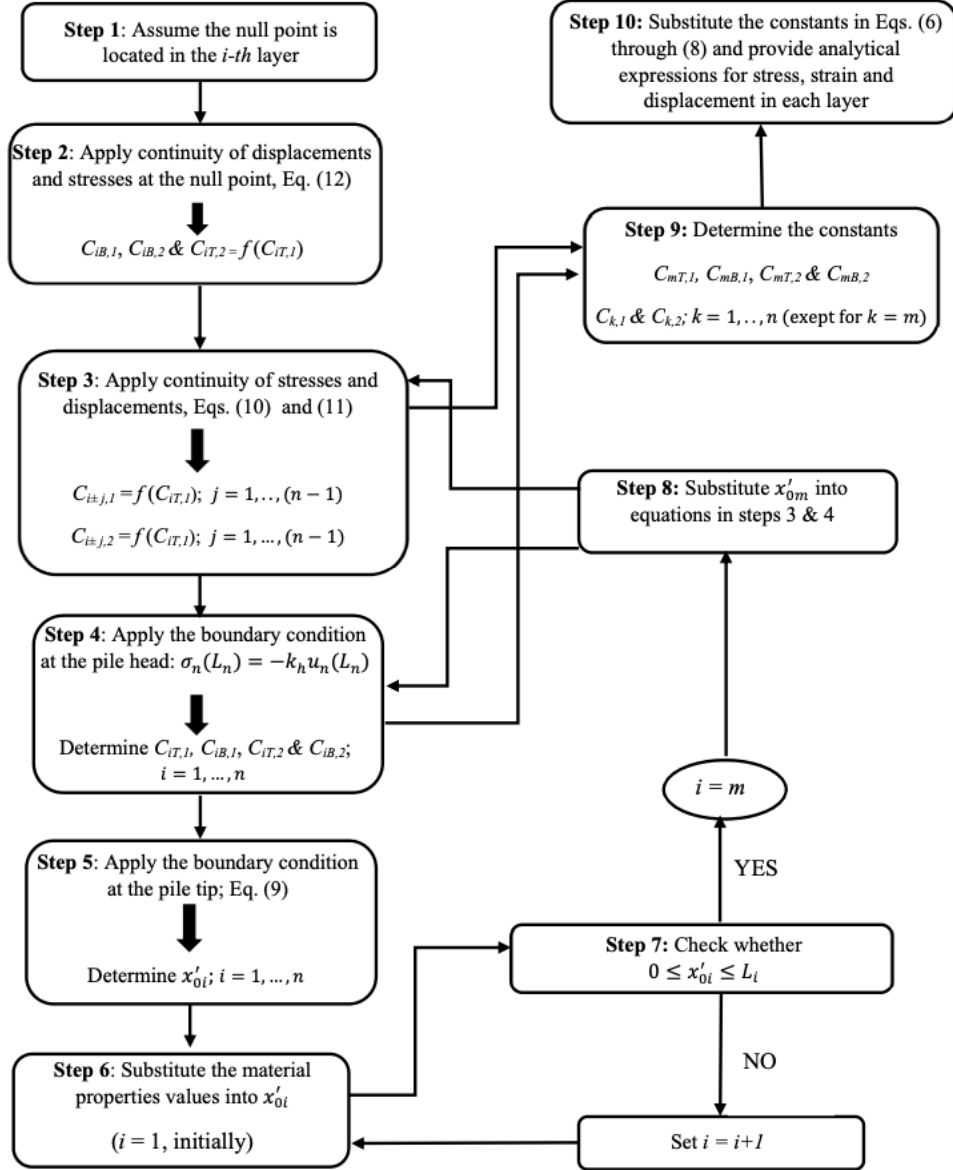


Figure 3.2 Flowchart depicting the analytical solution procedure for a single energy pile embedded in n soil layers, and subjected to thermal load

Once the boundary condition at the pile tip, which was given in Eq. (3.9), is imposed in step 5 the analytical expression for the local coordinate of the null point (x'_{0i}) for all possible scenarios is obtained. The value of i , i.e. the layer in which the null point is located, is determined in steps 6 and 7 depicted in in Figure 3.2. The constants for the remaining layers are determined subsequently.

Mechanical Load

Analytical solutions for axial displacement of a pile embedded into a homogeneous soil profile and subjected to mechanical compressive load was presented by Scott (1981). Herein more comprehensive solutions have been derived including equations for displacement, strain and stress in the pile embedded in a layered soil and subjected to mechanical loading. These solutions are based on the assumptions and governing laws presented previously. To this end, Figure 3.3 depicts the FBD of the entire energy pile and of its infinitesimal segment whereby the pile is subjected to an axial force (F).

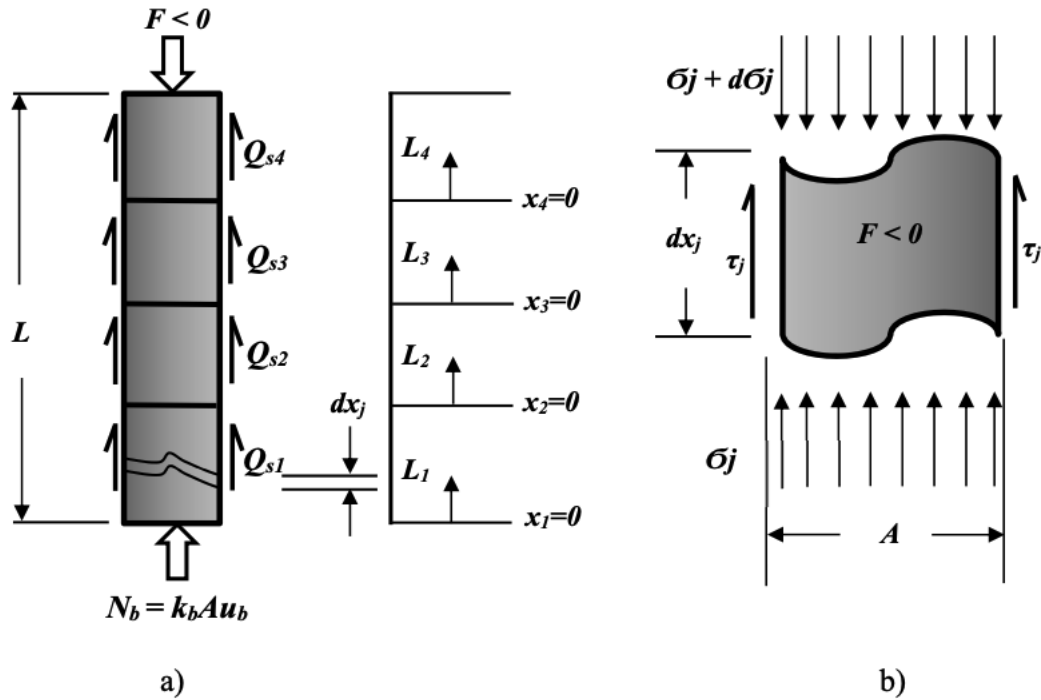


Figure 3.3 FBDs of the energy pile in layered soil subjected to mechanical load ($F < 0$), a) the entire energy pile, b) an infinitesimal segment of the energy pile

Consequently, the governing equation that results from enforcing equilibrium of an infinitesimal pile segment within each soil layer is given by Eq. (3.4) and the solution is given by Eq. (3.6).

The boundary conditions at the tip and head of a semi-floating pile, which is subjected to a mechanical load, are given by

$$\sigma_1(0) = k_b u_1(0) \quad \text{and} \quad \sigma_4(L_4) = \frac{F}{A} \quad (3.47)$$

where compressive force (F) is negative. Combining Eq. (3.47) with Eqs. (3.1), (3.2), and (3.6) leads to

$$C_{1,2} = \lambda C_{1,1} \quad (3.48)$$

where λ is given by

$$\lambda = \frac{E\psi_1 - k_b}{E\psi_1 + k_b} \quad (3.49)$$

Substituting Eq. (3.48) into Eq. (3.6) while observing the constitutive law and the kinematic relationship results in

$$u_1(x_1) = C_{1,1}(e^{\psi_1 x_1} + \lambda e^{-\psi_1 x_1}) \quad (3.50)$$

$$\varepsilon_1(x_1) = \psi_1 C_{1,1}(e^{\psi_1 x_1} - \lambda e^{-\psi_1 x_1}) \quad (3.51)$$

$$\sigma_1(x_1) = E\psi_1 C_{1,1}(e^{\psi_1 x_1} - \lambda e^{-\psi_1 x_1}) \quad (3.52)$$

Enforcing continuity of displacement and stress at the boundary between the first and the second layer, which was expressed by Eqs. (3.10) and (3.11) results in

$$C_{2,1} = \frac{C_{1,1}}{2}(ep_1 + \lambda e^{-m_1}) \quad (3.53)$$

$$C_{2,2} = \frac{C_{1,1}}{2}(em_1 + \lambda e^{-p_1}) \quad (3.54)$$

where

$$ep_j = e^{\psi_j L_j} \left(1 + \frac{\psi_j}{\psi_{j+1}} \right) \quad (3.55)$$

$$em_j = e^{\psi_j L_j} \left(1 - \frac{\psi_j}{\psi_{j+1}} \right) \quad (3.56)$$

$$e^{-p_j} = e^{-\psi_j L_j} \left(1 + \frac{\psi_j}{\psi_{j+1}} \right) \quad (3.57)$$

$$e^{-m_j} = e^{-\psi_j L_j} \left(1 - \frac{\psi_j}{\psi_{j+1}} \right) \quad (3.58)$$

where $j = 1, 2, 3$ or 4 . Substituting the constants given by Eqs. (3.53) and (3.54) into Eq. (3.6) while observing the constitutive law and kinematic relationship results in

$$u_2(x_2) = \frac{C_{1,1}}{2} [(ep_{1+}\lambda e^{-m_1})e^{\psi_2 x_2} + (em_{1+}\lambda e^{-p_1})e^{-\psi_2 x_2}] \quad (3.59)$$

$$\varepsilon_2(x_2) = \frac{\psi_2 C_{1,1}}{2} [(ep_{1+}\lambda e^{-m_1})e^{\psi_2 x_2} - (em_{1+}\lambda e^{-p_1})e^{-\psi_2 x_2}] \quad (3.60)$$

$$\sigma_2(x_2) = \frac{E\psi_2 C_{1,1}}{2} [(ep_{1+}\lambda e^{-m_1})e^{\psi_2 x_2} - (em_{1+}\lambda e^{-p_1})e^{-\psi_2 x_2}] \quad (3.61)$$

A subsequent enforcement of displacement and stress continuity at the boundary between the second and the third layer results in

$$C_{3,1} = \frac{C_{1,1}}{4} [(ep_{1+}\lambda e^{-m_1})ep_2 + (em_{1+}\lambda e^{-p_1})e^{-m_2}] \quad (3.62)$$

$$C_{3,2} = \frac{C_{1,1}}{4} [(ep_{1+}\lambda e^{-m_1})em_2 + (em_{1+}\lambda e^{-p_1})e^{-p_2}] \quad (3.63)$$

Similarly, as before substituting the constants given by Eqs. (3.62) and (3.63) into Eq. (3.6) while observing the constitutive law and kinematic relationship gives the solutions for the third layer as

$$u_3(x_3) = \frac{C_{1,1}}{4} \{ [(ep_{1+}\lambda e^{-m_1})(ep_2 e^{\psi_3 x_3} + em_2 e^{-\psi_3 x_3})] \\ + [(em_{1+}\lambda e^{-p_1})(e^{-m_2} e^{\psi_3 x_3} + e^{-p_2} e^{-\psi_3 x_3})] \} \quad (3.64)$$

$$\varepsilon_3(x_3) = \frac{\psi_3 C_{1,1}}{4} \{ [(ep_{1+}\lambda e^{-m_1})(ep_2 e^{\psi_3 x_3} - em_2 e^{-\psi_3 x_3})] \\ + [(em_{1+}\lambda e^{-p_1})(e^{-m_2} e^{\psi_3 x_3} - e^{-p_2} e^{-\psi_3 x_3})] \} \quad (3.65)$$

$$\begin{aligned}\sigma_3(x_3) = \frac{E\psi_3 C_{1,1}}{4} \{ & [(ep_{1+}\lambda e^{-m_1})(ep_2 e^{\psi_3 x_3} - em_2 e^{-\psi_3 x_3})] \\ & + [(em_{1+}\lambda e^{-p_1})(e^{-m_2} e^{\psi_3 x_3} - e^{-p_2} e^{-\psi_3 x_3})]\} \end{aligned} \quad (3.66)$$

Finally, enforcing displacement and stress continuity at the boundary of the third and the fourth layer results in

$$\begin{aligned}C_{4,1} = \frac{C_{1,1}}{8} \{ & [(ep_{1+}\lambda e^{-m_1})ep_2 + (em_{1+}\lambda e^{-p_1})e^{-m_2}]ep_3 + [(ep_{1+}\lambda e^{-m_1})em_2 \\ & + (em_{1+}\lambda e^{-p_1})e^{-p_2}]e^{-m_3}\} \end{aligned} \quad (3.67)$$

$$\begin{aligned}C_{4,2} = \frac{C_{1,1}}{8} \{ & [(ep_{1+}\lambda e^{-m_1})ep_2 + (em_{1+}\lambda e^{-p_1})e^{-m_2}]em_3 + [(ep_{1+}\lambda e^{-m_1})em_2 \\ & + (em_{1+}\lambda e^{-p_1})e^{-p_2}]e^{-p_3}\} \end{aligned} \quad (3.68)$$

Substituting the constants given in Eqs. (3.67) and (3.68) into Eq. (3.6) while observing the constitutive law and kinematic relationship results in

$$\begin{aligned}u_4(x_4) = \frac{C_{1,1}}{8} \{ & [(ep_{1+}\lambda e^{-m_1})ep_2 + (em_{1+}\lambda e^{-p_1})e^{-m_2}](ep_3 e^{\psi_4 x_4} + em_3 e^{-\psi_4 x_4}) \\ & + [(ep_{1+}\lambda e^{-m_1})em_2 + (em_{1+}\lambda e^{-p_1})e^{-p_2}](e^{-m_3} e^{\psi_4 x_4} \\ & + e^{-p_3} e^{-\psi_4 x_4})\} \end{aligned} \quad (3.69)$$

$$\begin{aligned}\varepsilon_4(x_4) = \frac{\psi_4 C_{1,1}}{8} \{ & [(ep_{1+}\lambda e^{-m_1})ep_2 + (em_{1+}\lambda e^{-p_1})e^{-m_2}](ep_3 e^{\psi_4 x_4} \\ & - em_3 e^{-\psi_4 x_4}) \\ & + [(ep_{1+}\lambda e^{-m_1})em_2 + (em_{1+}\lambda e^{-p_1})e^{-p_2}](e^{-m_3} e^{\psi_4 x_4} \\ & - e^{-p_3} e^{-\psi_4 x_4})\} \end{aligned} \quad (3.70)$$

$$\begin{aligned} \sigma_4(x_4) = \frac{E\psi_4 C_{1,1}}{8} \{ & [(ep_{1+}\lambda e^{-m_1})ep_2 + (em_{1+}\lambda e^{-p_1})e^{-m_2}](ep_3 e^{\psi_4 x_4} \\ & - em_3 e^{-\psi_4 x_4}) \\ & + [(ep_{1+}\lambda e^{-m_1})em_2 + (em_{1+}\lambda e^{-p_1})e^{-p_2}](e^{-m_3} e^{\psi_4 x_4} \\ & - e^{-p_3} e^{-\psi_4 x_4}) \} \end{aligned} \quad (3.71)$$

By enforcing the boundary condition at the pile head, which was given in Eq. (3.47) and combining it with Eq. (3.71) results in

$$\begin{aligned} C_{1,1} = \frac{8F}{AE\psi_4} \{ & (ep_{1+}\lambda e^{-m_1})[(ep_2 ep_3 + em_2 e^{-m_3})e^{\psi_4 L_4} \\ & - (ep_2 em_3 + em_2 e^{-p_3})e^{-\psi_4 L_4}] \\ & + (em_{1+}\lambda e^{-p_1})[(e^{-m_2} ep_3 + e^{-p_2} e^{-m_3})e^{\psi_4 L_4} \\ & - (e^{-m_2} em_3 + e^{-p_2} e^{-p_3})e^{-\psi_4 L_4}] \}^{-1} \end{aligned} \quad (3.72)$$

Similarly, to the solution for thermal load, the solution for mechanical load can also be extended to n soil layers. Continuity of displacements and stresses expressed in Eqs. (3.10) and (3.11) is enforced at the boundary of any two neighboring layers. The level of complexity increases tremendously as the number of layers increases and new terms emerge in the solution. As described in Figure 3.4 constants $C_{j,1}$ and $C_{j,2}$ ($j = 2, \dots, n$) can be derived for any number of layers in terms of $C_{1,1}$. Nevertheless, $C_{1,1}$ is determined only at the very end of the derivation by imposing the boundary condition at the pile head.

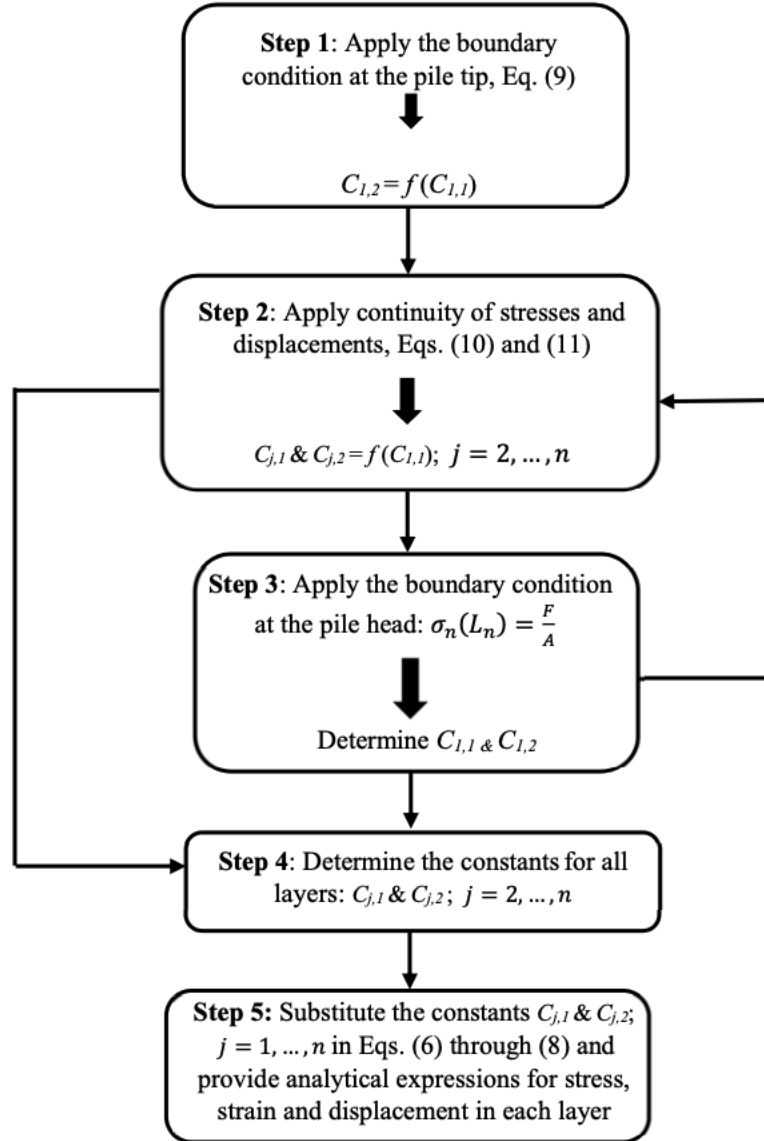


Figure 3.4. Flowchart depicting the analytical solution procedure for a single energy pile embedded in n soil layers, and subjected to mechanical load

Validation of analytical solutions

The analytical solutions derived herein are validated against the full scale pile tests that were conducted at the Swiss Federal Institute of Technology (Laloui et al. 2006; Laloui. et al. 1999). At this site a total of 97 piles support a four-story building having a plan area of 100 m x 30 m. Only one pile out of 97, which has an average diameter of 1m and approximate length of 26

m, was constructed as energy pile. Two of the pile tests that were conducted at this site were selected for the purpose of validation, test T1 and test T7. Test T1 was conducted prior to the construction of the building while test T7 was conducted upon the completion of the construction. Both tests included a heating-cooling cycle during which the pile remained in a net heated state. Nevertheless, test T7 also included a simultaneous application of the axial compressive force of 1000 kN (Knellwolf et al. 2011) imposed by the building. The temperature histories, which are shown in Table 3.3, were obtained from Laolui et al. (2006).

Table 3.3 Temperature histories for tests T1 and T7

T1	Day	0	5	12	15	28
	ΔT (°C)	0	2	21	9	3
T7	Day	0	5	22	27	62
	ΔT (°C)	0	9	14	6	2

The energy pile is embedded in the layered soil profile shown in Figure 3.5 Perić et al. (2017) classified all four soil layers as low plasticity clays (CL) based on the laboratory test results reported by Laloui et al. (1999). The bottom clay layer is underlain by sandstone/claystone. Young's modulus of the pile is equal to 29.2 GPa according to Laloui et al. (2006). Knellwolf et al. (2011) reported the stiffness of the elastic shear springs to be equal to 16.7, 10.8, 18.2 and 121.4 MPa/m for soils A1, A2, B and C respectively based on the full-scale in situ pile tests. These values are used herein for all predictions.

In situ axial pile stress was computed from the measured axial strain, based on the thermo-elastic constitutive relationship as follows

$$\sigma(x) = E [\varepsilon(x) - \alpha \Delta T(x)] \quad (3.73)$$

It is noted that Eq. (3.73) accounts for the measured temperature change versus depth. Knellwolf et al. (2011) found that the actual temperature change with depth varied ± 2 °C from the

average values at the Lausanne site that are listed in Table 3.3. It is because the analytical solutions herein were developed based on the assumption of a constant temperature versus depth while the actual temperature varied with depth, that the three different predictions are presented. They include the one based on the actual temperature provided in Table 2, and the temperatures that are two degrees higher and lower than the actual temperature.

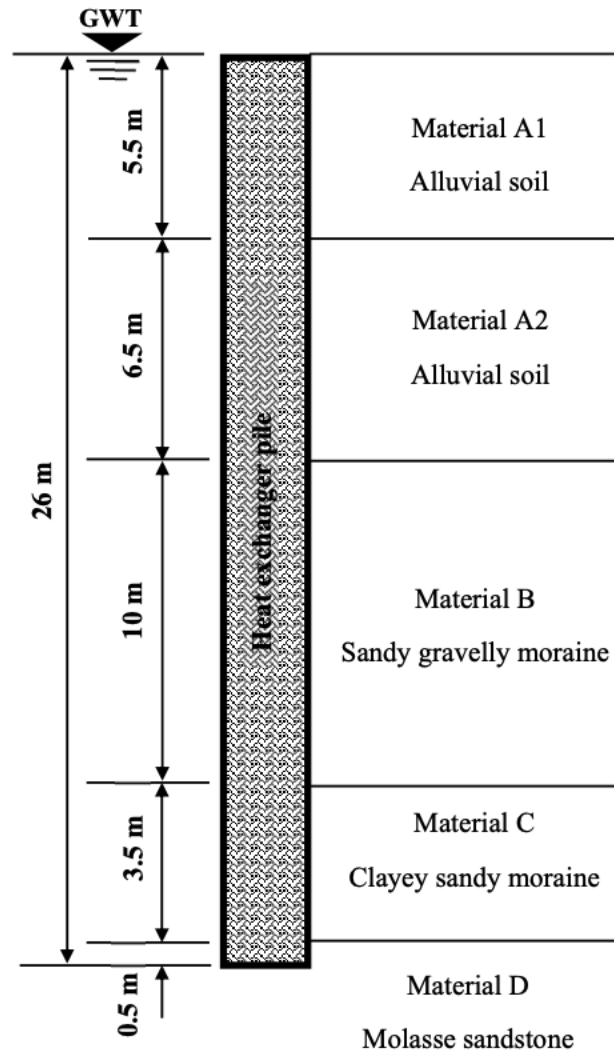


Figure 3.5. Layered soil profile at Swiss Federal Institute of Technology, Lausanne

Knellwolf et al. (2011) provided the values of stiffness of the elastic shear springs based on the in situ tests. They also pointed out that no precise information about the k_b value for the

stiffest stratum, which is layer D, was available. Furthermore, they suggested that the stiffness of the normal spring (k_b) representing the layer D should have been equal to one to two times k_b value of layer C. While Knellwolf et al. (2011) did not provide the k_b value representing layer C they provided the range of k_b values representing layer D in their Table 3.2. The provided values range from 667.7 MPa/m to 1335.4 MPa/m. Based on this information the k_b value selected in this study is 1335.4 MPa/m. As it corresponds to the upper bound of values provided by Knellwolf et al. it is denoted by k_{bUB} herein.

Prediction of thermal head displacement of the energy pile subjected to temperature history corresponding to test T1 is depicted in Figure 3.6 along with the experimental results. While analytical predictions and experimental data follow very similar patterns the predictions are closer to the optical fiber measurements than to the extensometer measurements. Figure 3.7 shows predicted and measured thermal axial strains of the energy pile versus depth during test T1 for $\Delta T = 13.4^\circ\text{C}$. For the most part the analytical predictions for the lowest and the highest temperatures bracket the experimental data. Figure 3.8 shows thermal axial stress in the energy pile versus depth during test T1 for $\Delta T = 13.4^\circ\text{C}$.

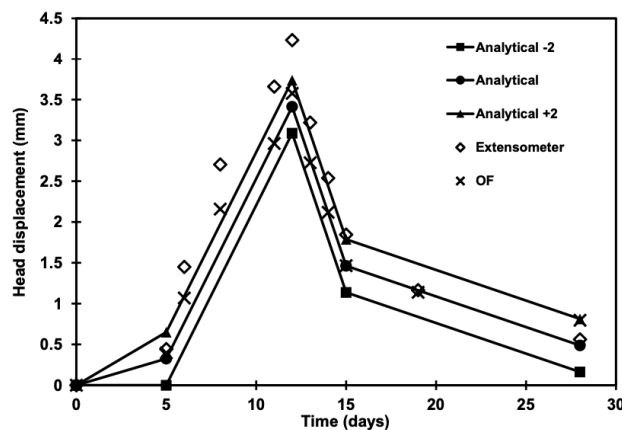


Figure 3.6. Analytical prediction of vertical pile head displacements versus time for test T1

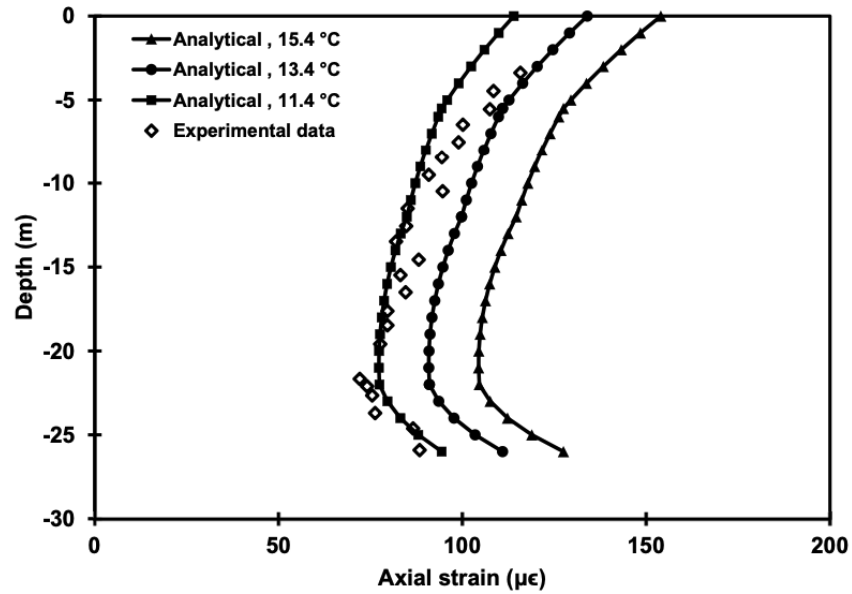


Figure 3.7. Analytical prediction of axial strain versus depth for test T1 ($\Delta T = 13.4\text{ }^{\circ}\text{C}$)

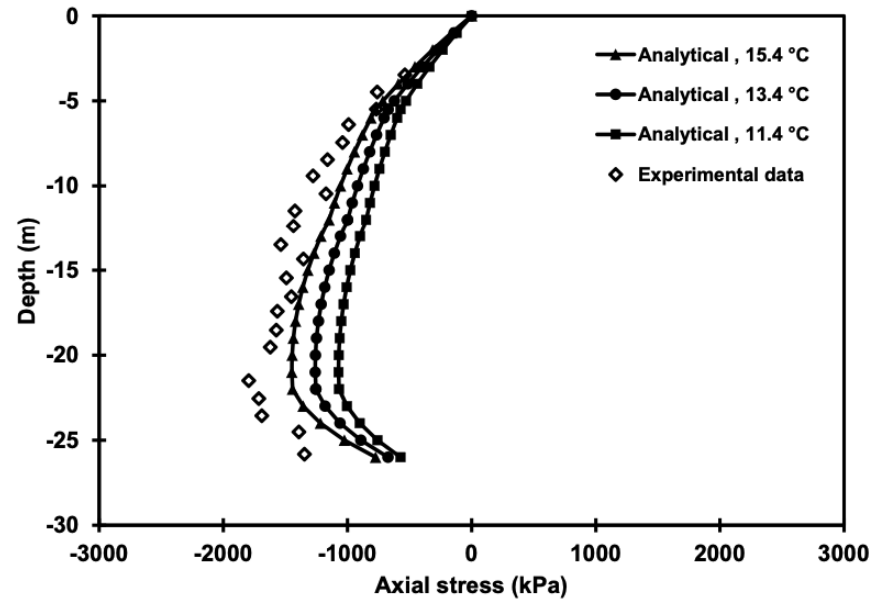


Figure 3.8. Analytical prediction of axial stress versus depth for test T1 ($\Delta T = 13.4\text{ }^{\circ}\text{C}$)

Perić et al. (2020) introduced an equivalent mechanical load (F_{eq}) in terms of a thermal load (ΔT) based on the response of end bearing energy pile in homogenous soil layer. It is given by

$$F_{eq} = \pm AE\alpha\Delta T \quad (3.74)$$

where minus sign applies for heating and plus for cooling provided that the force is compressive. Figure 3.9 depicts the analytical model predictions for mechanical and thermal axial stresses versus depth, for test T7 at $\Delta T = 14^\circ\text{C}$. The magnitude of the compressive force applied at the pile head is 1000 kN. According to Eq. (3.74) the magnitude of an equivalent force is equal to 3211 kN. Thus, the magnitude of the actual force is only 0.311 of the magnitude of the equivalent force.

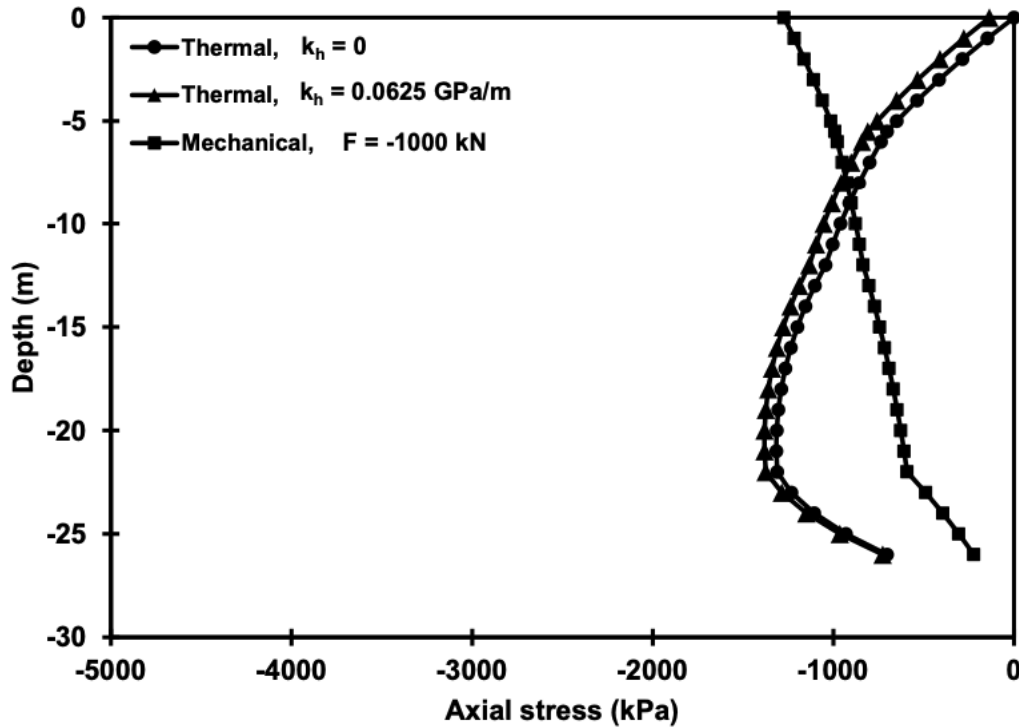


Figure 3.9. Analytical prediction of axial stress versus depth for thermal load ($\Delta T = 14^\circ\text{C}$) and mechanical load ($F = -1000\text{ kN}$), test T7

Consequently, the mechanical compressive stress is larger than thermal compressive stress only at depths smaller than 8m or 9m, depending on the value of k_h . Nevertheless, maximum thermal compressive stress is larger than the maximum mechanical compressive stress. Furthermore, presence of a head spring induces a compressive thermal stress at the pile head. The selected value of the stiffness of the spring characterizing the head restraint, which is 0.0625

GPa/m, is based on the best fit of a thermo-mechanical axial stress with experimental data that is discussed next.

Figure 3.10 and Figure 3.11 show combined axial strain and stress versus depth respectively for test T7 at $\Delta T = 14^\circ\text{C}$, $F = -1000 \text{ kN}$, and $k_b = 0.0625 \text{ GPa/m}$. Analytical predictions follow the trend of the experimental data. For comparison, Figure 3.12 and Figure 3.13 depict the combined axial strain and stress versus depth at the same conditions as those applicable to Figure 3.10 and Figure 3.11, except in this case there is no head restraint. By comparing Figure 3.11 and Figure 3.13 it is observed that slightly better prediction of axial stress is obtained in the presence of the head restraint that models the restraint imposed by the superstructure.

It is noted that the predicted stress is closer to the experimental data in case of test T7 as compared to test T1. While increasing the k_b value above the selected 1335.4 MPa would improve the prediction of the model for test T1 it would also slightly deteriorate the model prediction for test T7. Nevertheless, this is likely to equalize the errors in predictions of test T7 and test T1. It is because the tip of the actual pile at the Lausanne site is embedded 0.5 m into the bedrock while the model adopted herein assumes it to be flush with the top boundary of the bedrock that the increase of k_b beyond the maximum value provided by Knellwolf et al. (2011) would be justified. Nevertheless, the procedure required for a rational increase in k_b value is beyond the scope of this study.

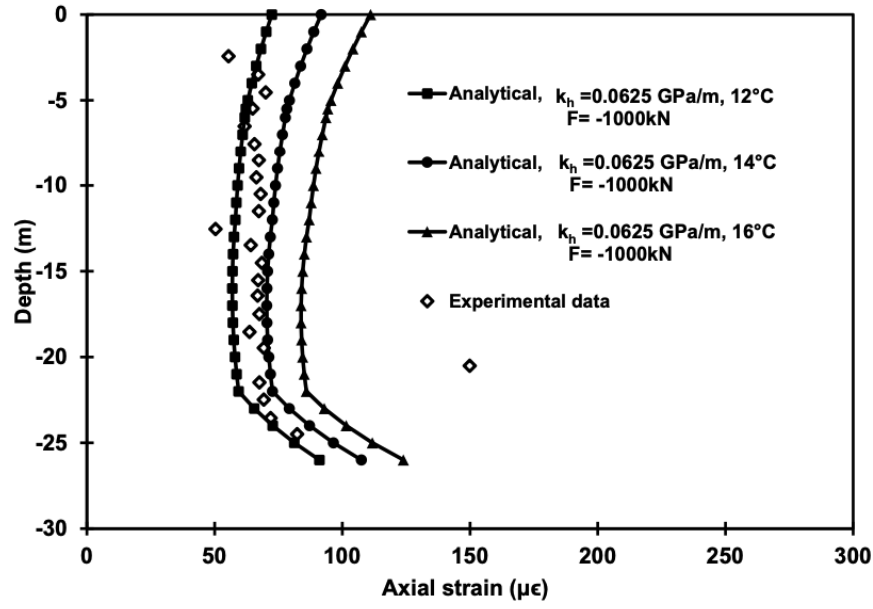


Figure 3.10. Analytical prediction of axial strain versus depth for test T7 at $\Delta T = 14^\circ\text{C}$, $k_h = 0.0625$ GPa/m

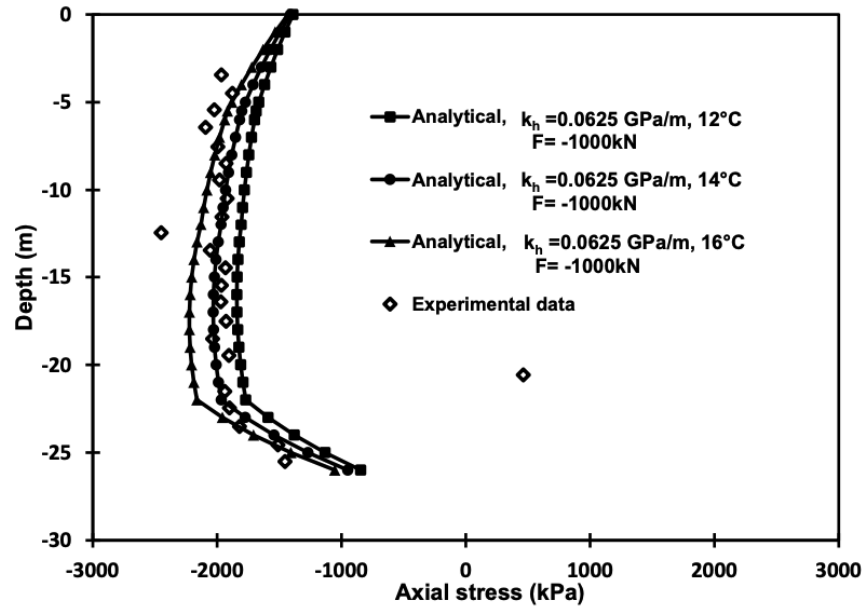


Figure 3.11. Analytical prediction of axial stress versus depth for test T7 at $\Delta T = 14^\circ\text{C}$, $k_h = 0.0625$ GPa/m

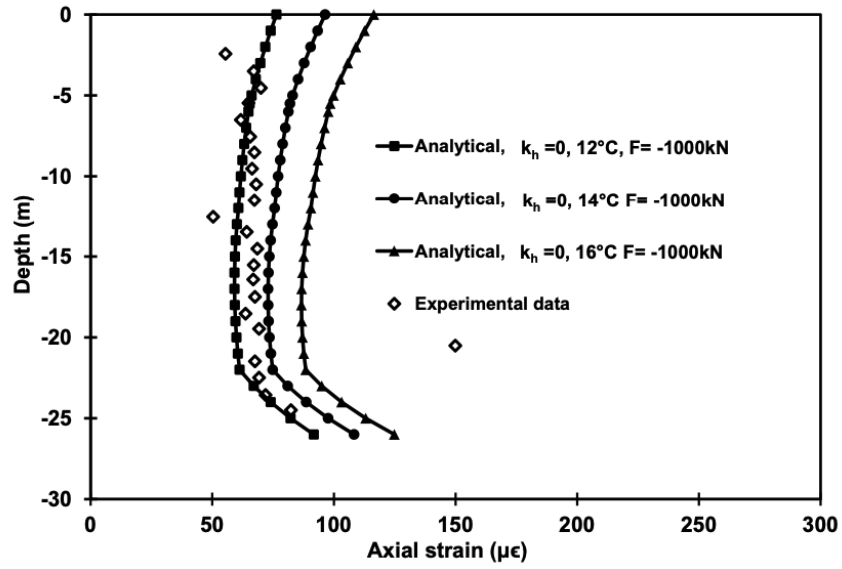


Figure 3.12. Analytical prediction of axial strain versus depth for test T7 at $\Delta T = 14^\circ\text{C}$, no head restraint

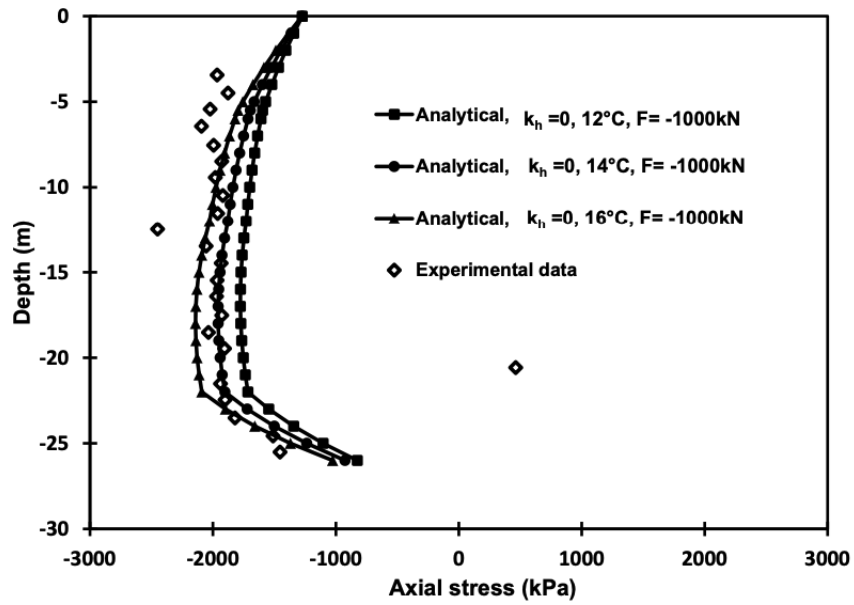


Figure 3.13. Analytical prediction of axial stress versus depth for test T7 at $\Delta T = 14^\circ\text{C}$, no head

The location of the thermal null point depends on the relationship among the values of the shear springs stiffness (k_{sj}), and the stiffness of the normal springs at the pile head and tip (k_h and k_b). The thermal null point for energy pile embedded in a homogenous soil layer is located nearly

at the mid-depth of nearly fully floating pile having a finite length. In case of a semi floating pile embedded in a homogenous soil layer with the same stiffness of head and tip springs the thermal null point is located exactly at the mid-depth (Saeidi Rashk Olia and Perić 2021a).

The effect of the tip spring stiffness on the location of the thermal null point in the absence of head restraint is shown in Figure 3.14. Based on the realistic properties of the stiffest rock (Saeidi Rashk Olia and Perić 2020a) and the relationship between the stiffness of the tip spring and shear modulus of the rock (G_b) provided by Scott (1981), k_b can be expressed as

$$k_b = \frac{2G_b}{(1 - \nu_b)d} \quad (3.75)$$

where d is the pile diameter and ν_b is the Poisson's ratio of the bedrock. According to Eq. (3.75) the corresponding maximum value of tip spring stiffness is 52.17 GPa/m. This is reflected in Figure 3.14.

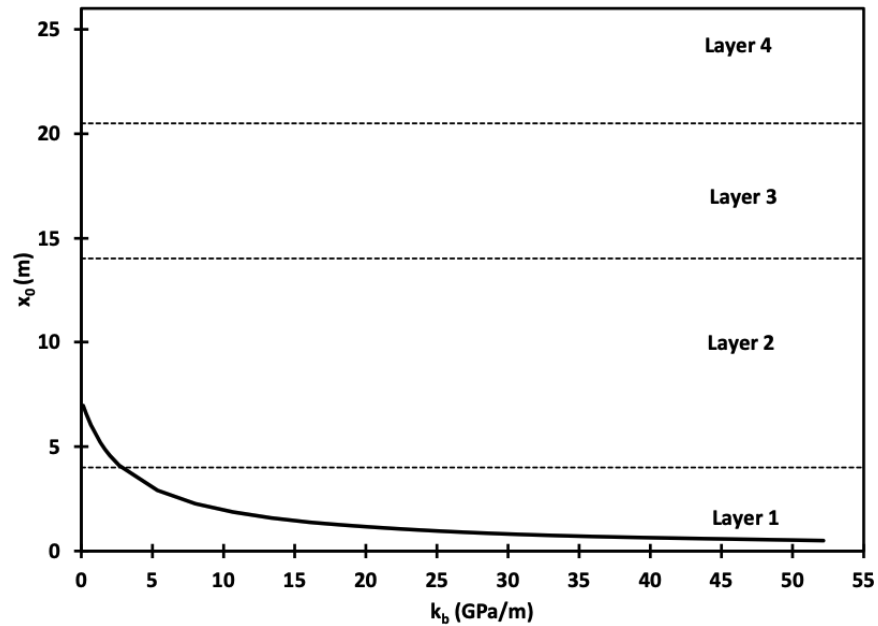


Figure 3.14. Location of the thermal null point for different values of spring tip stiffness

It is seen that the thermal null point is located at the boundary of the first and the second layer for $k_b = 2804$ MPa/m. Increase in k_b causes the null point to move downward toward the pile tip and

it is located at 0.5m once $k_b = k_{b,max} = 52.17 \text{ GPa/m}$. Saeidi Rashk Olia and Perić (2021a) have shown that increase in the head stiffness causes the null point to move upwards. The proposed model predicts that the thermal null point is located 5.2 m above the pile tip in case that $k_b = k_{bUB}$, and in the absence of head restraint. According to Fig. 9 this location coincides with the location of the maximum magnitude of thermal stress. The coincidence of the locations of the thermal null point and maximum magnitude of thermal axial stress follows directly from the mathematical model adopted herein. Namely, obtaining the maximum magnitude of thermal axial stress requires that the first derivative of Eq. (3.8) is equal to zero, which results in the same condition that results from setting the displacement given in Eq. (3.6) equal to zero. The location of zero thermal displacement is always equal to the location of zero shear stress according to Eq. (3.3).

This feature of the model presented herein does not correspond to the thermo-mechanical schemes presented by Rotta Loria and Laloui (2018). In their case the locations of zero thermal displacement and zero shear stress coincide only in the absence of head and tip restraints. Otherwise, the location of zero shear stress appears to always coincide with the location of the maximum magnitude of thermal axial stress. Rotta Loria and Laloui (2018) explain this feature of their thermo-mechanical schemes by referring to Rotta Loria et al. (2018). In the latter three different models are used to analyze the vertical displacement of energy pile groups. They are layer model, continuous model and finite element model. Among these models the layer model is the closest to the model adopted in this study due to the limited capability of both models to capture the effects of a shear stress acting at any location along the pile shaft on the surrounding soil in a continuous way. Thus, the location of the null point of thermal displacement and shear stress coincide in case of both models, the layer model and the model presented in this study.

Finally, a possibility of the slip at the pile soil interface has been investigated by using the constitutive relationship proposed by Frank and Zhao (1982). Specifically, the constitutive relationship assumes yielding of the interface at the half of the ultimate value of the interface shear stress, and yielding of the interface at the pile tip at the half of the ultimate normal stress at the tip. As seen in Table 3.4 and Table 3.5 shear stresses throughout the depth remain lower than $q_s/2$ for both loading scenarios considered herein.

Table 3.4. Yielding check at the pile soil interface along the pile depth, Test T1

Test T1: $\Delta T = 15.4$ °C							
Soil Layer	k_s (MPa/m)	u_{max} (mm)	$k_s(u_{max})$ (kPa)	$q_s/2$ (kPa)	q_s (kPa)	$k_s(u_{max}) < q_s/2$ check	$k_s(u_{max}) < q_s$ check
A1	16.7	2.50220	41.75	51	102	Yes	Yes
A2	10.8	1.73248	18.68	35	70	Yes	Yes
B	18.2	0.94942	17.28	37	74	Yes	Yes
C	121.4	0.57905	70.29	80	160	Yes	Yes

Table 3.5. Yielding check at the pile soil interface along the pile depth, Test T7

Test T7: $\Delta T = 16$ °C (with k_h)							
Soil Layer	k_s (MPa/m)	u_{max} (mm)	$k_s(u_{max})$ (kPa)	$q_s/2$ (kPa)	q_s (kPa)	$k_s(u_{max}) < q_s/2$ check	$k_s(u_{max}) < q_s$ check
A1	16.7	1.72433	28.79	51	102	Yes	Yes
A2	10.8	1.13675	12.27	35	70	Yes	Yes
B	18.2	0.52553	9.57	37	74	Yes	Yes
C	121.4	0.6496	78.86	80	160	Yes	Yes

In addition, Table 3.6 indicates that there is no yielding of the soil pile interface at the pile tip. Thus, the yielding check performed in Table 3.4 through Table 3.6 justifies the assumption of the elastic response of the pile soil interface. The values of the ultimate shear stress (q_s) and the ultimate normal stress (q_b) for the relevant soils were taken from Knellwolf et al. (2011).

Table 3.6 Yielding check at the pile soil interface at the pile tip, Tests T1 and T7

Test T1 & T7 (with k_b)							
Test	k_b (MPa/m)	u_{max} (mm)	$k_b(u_{max})$ (kPa)	$q_b/2$ (kPa)	q_b (kPa)	$k_b(u_{max}) < q_b/2$ check	$k_b(u_{max}) < q_b$ check
T1	1335.5	2.50220	3340	5500	11000	Yes	Yes
T7	1335.5	1.72433	2302	5500	11000	Yes	Yes

Conclusions

Analytical solutions for axial displacement, stress and strain in a single semi floating energy pile embedded in a four-layered soil profile have been derived based on the one-dimensional mathematical model.

- For the first time analytical solutions for both, thermal and mechanical responses of a semi floating energy pile embedded in a four-layered soil profile were derived and presented.
- Analytical solutions successfully captured the main features of axial stress and strain response in both, thermal and combined thermo-mechanical in situ tests that were conducted on the energy pile at Swiss Federal Institute of Technology, Lausanne, Switzerland, which was embedded in a four-layered soil profile.
- The soil layer containing the zero thermal displacement point known as a null point was identified. The location of the null point was obtained by enforcing the semi floating boundary condition at the pile tip.
- The analytical solutions were also capable of capturing the effect of head restraint imposed by the superstructure. Performance of the analytical model against experimental data for test T7 showed that including the head restraint results in more accurate prediction of the energy pile response subjected to thermo-mechanical load.

- Location of the thermal null point of displacement and shear stress coincides with the location of the maximum magnitude of thermal stress according to the solutions presented herein.
- Furthermore, two flowcharts depicting analytical solution procedures for a single energy pile embedded into a soil profile containing an arbitrary number of layers (n) are provided. The flowchart depicted in Fig. 2 delineates the procedure for thermal load while the flowchart shown in Fig. 4 provides details of the procedure corresponding to a mechanical load.

Chapter 4 - Analytical Solutions for Thermomechanical Soil Structure Interaction in Single Energy Piles Exhibiting Irreversible Interface Behavior

The revised version of this manuscript, after addressing the reviewers' comments has been submitted to the *ASCE-International Journal of Geomechanics*

Saeidi Rashk Olia, A. & Perić, D. (2023a). Analytical Solutions for Thermomechanical Soil Structure Interaction in Single Energy Piles Exhibiting Irreversible Interface Behavior. *Int. J. Geomech.*

Abstract:

The main purpose of this study was to derive analytical solutions for soil structure interaction in single energy piles that exhibit irreversible interface behavior. The solutions were derived for a pile embedded in a homogenous soil layer subjected to thermal, mechanical and thermo-mechanical loads. Furthermore, the solutions were subsequently validated against the experimental data obtained from the full-scale tests. Load transfer diagrams depicting evolutions of displacement, interface shear stress and axial stress versus depth for semi-floating, fully floating and end bearing piles were obtained from the analytical solutions and subsequently presented. The effects of a progression of thermally induced interface yielding on the displacement, interface shear stress and axial stress were illustrated in case of heating and cooling for different end restraints. In addition, the analytical solutions show that location of the null point changes after the initiation of the interface yielding. The analytical solutions indicate that residual displacement, strain and stress are generated in the pile upon the initiation of the interface yielding. This finding was also confirmed through a comparison of analytical predictions with experimental data. Finally, analytical solutions show that evolutions of displacement, strain and interface shear stress are hyperbolic trigonometric functions within the depth of the non-yielded interface. In contrast,

within a depth corresponding to the yielded interface axial stress and strain are linear functions, displacement is a quadratic function, while the interface shear stress is constant with depth.

Keywords: Analytical solutions, Energy pile, Interface yielding, Load transfer mechanism, Soil structure interaction, Thermo-mechanics

Introduction

Energy piles are multifunctional sustainable foundations that use a low enthalpy renewable geothermal energy for space heating and cooling, thus improving the energy efficiency of buildings while decreasing the emission of carbon dioxide into the atmosphere. Development of an in-depth understanding of effects of mechanical and thermal loads on energy piles is vitally important for the analysis and design of these foundations, thus contributing to their wider use and increased sustainability of the civil infrastructure. Thermo-mechanical soil structure interaction in energy piles refers to the interaction between the energy pile as a structural element and the surrounding soil and rock, caused by a simultaneous action of mechanical and thermal loads whereby the latter is induced by heating or cooling of the pile. Understanding of this interaction, which induces displacement, strain and stress in the energy piles, is crucial for analysis and design of energy piles.

To this end, several authors (Bourne-Webb et al., 2009; Bourne-Webb et al., 2013; Amatya et al. 2012; Rotta Loria and Laloui, 2018; and Saeidi Rashk Olia and Perić 2021a, Perić and Saeidi Rashk Olia 2022) presented load transfer diagrams for energy piles that are embedded in a single soil layer. These authors assumed that temperature was uniform throughout the pile. Bourne-Webb et al. (2009), Bourne-Webb et al. (2013), and Amatya et al. (2012) simply postulated their load transfer diagrams based on the assumption of the uniform shaft resistance (q_s) along the pile length.

These diagrams, which they referred to as “a simple set of mechanisms”, exhibit linear evolutions of axial force, strain and restrained thermal strain versus depth in the case of mechanical loading while the evolutions become bilinear in the case of combined thermomechanical loading. However, Bourne-Webb et al. (2009), Bourne-Webb et al. (2013), and Amatya et al. (2012) later neglected their own assumption of a constant magnitude of a shaft resistance versus depth in the case of a combined thermomechanical loading while Amatya et al. (2012) neglected this assumption also in the case of thermal loading. A summary showing different end restraints that were addressed by these three groups of authors along with other pertinent details is shown in Table 4.1.

The strain notation in Table 4.1 fully corresponds to the one employed by the corresponding authors. Nevertheless, it is noted that Bourne-Webb et al. (2009) used ε to denote both, a mechanical strain and restrained portion of the thermal strain. Otherwise, P denotes axial force in the pile, τ denotes interface shear stress, F denotes vertical force applied at the pile head, ΔT denotes thermal load, while k_h and k_b represent head and tip restraints respectively. Furthermore, it is noted that Bourne-Webb et al. (2013) used restrained portion of the thermal strain (ε_{T-Rstr}) to denote both, i.e. truly restrained portion of thermal strain, which does not materialize due to the presence of the soil around the pile, and mechanically induced strain that materializes. Amatya et al. (2012) presented both, ε_{T-Rstr} and ε_{T-Obs} , whereby the latter denotes the actual, i.e. observed strain. Finally, Bourne-Webb et al. (2013) also stated that the pile has a constant cross-sectional area and that it is made from a linear elastic material. Similarly, Amatya et al. (2012) also pointed out that the pile has a constant cross-sectional area, and that it is installed in a rigid plastic soil.

Table 4.1. Summary of load transfer diagrams of energy piles presented by Bourne-Webb et al. (2009), Bourne-Webb et al. (2013), and Amatya et al. (2012)

	$F < 0$	$\Delta T < 0$	$\Delta T > 0$	$F < 0$ & $\Delta T < 0$	$F < 0$ & $\Delta T > 0$	Restraints	Reference	
P	L	BL	BL	BL	BL	$k_h = 0$ $k_b = 0$ (FF)	B-W et al. (2009)	
ε	L	BL	BL	BL	BL			
$\tau(= q_s)$	const	const	const	$\neq const$	$\neq const$			
P	L	BL	BL	BL	BL	$k_h = 0$ $k_b = 0$ (FF)	B-W et al. (2013)	
ε_{T-Rstr}	L	BL	BL	BL	BL			
$\tau(= q_s)$	const	const	const	$\neq const$	$\neq const$			
P	N/A	BL	BL	BL	BL	$k_h = k_b$ $\neq 0$ (SF)		
ε_{T-Rstr}	N/A	BL	BL	BL	BL			
$\tau(= q_s)$	N/A	const	const	$\neq const$	$\neq const$			
P	N/A	N/A	N/A	BL	BL	$k_h = 0$ $k_b \neq 0$ (SF)		A et al. (2012)
ε_{T-Rstr}	N/A	N/A	N/A	BL	BL			
$\tau(= q_s)$	N/A	N/A	N/A	$\neq const$	$\neq const$			
P	L	BL	BL	BL	BL	$k_h = 0$ $k_b = 0$ (FF)		
ε	L; ε_{T-Rstr}	BL; ε_{T-Rstr}	BL; both	BL; ε_{T-Rstr}	BL; ε_{T-Rstr}			
$\tau(= q_s)$	const	const	const	$\neq const$	$\neq const$			
P	N/A	N/A	BL	N/A	BL	$k_h = k_b$ $\neq 0$ (SF)		
ε	N/A	N/A	BL; both	N/A	BL; ε_{T-Rstr}			
$\tau(= q_s)$	N/A	N/A	const	N/A	$\neq const$			
P	N/A	N/A	BL	N/A	N/A	$k_h = 0$ $k_b \neq 0$ (SF)		
ε	N/A	N/A	BL; ε_{T-Obs}	N/A	N/A			
$\tau(= q_s)$	N/A	N/A	$\neq const$	N/A	N/A			
P	N/A	N/A	BL	N/A	BL	$k_h \neq 0$ $k_b = 0$ (FF)		
ε	N/A	N/A	BL; both	N/A	BL; ε_{T-Rstr}			
$\tau(= q_s)$	N/A	N/A	$\neq const$	N/A	$\neq const$			
Note: L = linear, BL = bilinear, FF = fully floating, SF = semi floating, both = ε_{T-Rstr} and ε_{T-Obs} , B-W et al. = Bourne-Webb et al. , A et al. = Amatya et al.								

Rotta Loria and Laloui (2018) stated that the number of conditions of validity of load transfer diagrams described in Table 4.1 has never been explicated. They proposed “thermo-mechanical schemes” for response of single energy piles that, according to them, are based on the thermo-elasticity theory. They provided sixteen global pile equilibrium equations and a number of load transfer diagrams, details of which are summarized in Table 4.2. Rotta Loria and Laloui (2018) presented approximately linear evolution of axial stress in the pile and approximately constant interface shear stress in case of mechanical loading, similarly to the corresponding evolutions summarized in Table 4.1. In addition, the corresponding pile displacement (w) is also

constant with depth. Nevertheless, in case of thermal loading and combined thermo-mechanical loading the stress evolution with depth is non-linear while evolutions of shear stress and vertical displacements are linear, according to Rotta Loria and Laloui (2018). The corresponding head and tip restraints along with further details are provided in Table 4.2.

Table 4.2. Summary of load transfer diagrams of energy piles presented by Rotta Loria and Laloui (2019)

	$F < 0$	$\Delta T > 0$	$F < 0$ & $\Delta T > 0$	Restraints	Reference
σ	L	NL	NL	$k_h = 0$ $k_b = 0$ (FF)	RL&LL (2019)
w	<i>const</i>	L	L		
τ	<i>const</i>	L	L		
σ	L	NL	NL	$k_h = 0$ $k_b \neq 0$ (SF)	
w	<i>const</i>	L	L		
τ	<i>const</i>	L	L		
σ	L	NL	NL	$k_h \neq 0$ $k_b = 0$ (FF)	
w	<i>const</i>	L	L		
τ	<i>const</i>	L	L		
σ	L	NL	NL	$k_h = k_b$ $\neq 0$ (SF)	
w	<i>const</i>	L	L		
τ	<i>const</i>	L	L		
σ	L	NL	NL	$k_h \neq 0$ $k_b \neq 0$ (SF)	
w	<i>const</i>	L	L		
τ	<i>const</i>	L	L		
Note : w = vertical displacement, L = linear, NL = non-linear, FF = fully floating, SF = semi floating, RL & LL = Rotta Loria and Laloui					

Saeidi Rashk Olia and Perić (2021a) adopted a rational, consistent mechanics-based approach that lead to the analytical solutions for evolutions of axial stress, strain, displacement and shear stress. In addition to the constant cross-sectional area of the pile and uniform temperature within the pile, they assumed that the material properties of soil and pile are not affected by the temperature. Furthermore, Saeidi Rashk Olia and Perić (2021a) assumed that the pile follows thermo-elastic constitutive law while no yielding occurs at the soil pile interface, i.e. the response

of the interface is fully reversible. Their solutions are based on enforcing the equilibrium of an infinitesimal pile segment accompanied by the linear kinematic relationship and thermo-elastic constitutive behavior of the pile. The corresponding load transfer functions along with head and tip restraints are summarized in Table 4.3.

Table 4.3. Summary of load transfer diagrams of energy piles presented by Saeidi Rashk Olia and Perić(2021a)

	$F < 0$	$\Delta T < 0$	$\Delta T > 0$	$F < 0$ & $\Delta T < 0$	$F < 0$ & $\Delta T > 0$	Restraints	Reference
σ	NL; sh	NL; ch	NL; ch	NL; sh+ch	NL; sh+ch	$k_h = 0$ $k_b = 0$ (FF)	SRO & P (2021a)
ε	NL; sh	NL; ch	NL; ch	NL; sh+ch	NL; sh+ch		
τ^*	NL; ch	NL; sh	NL; sh	NL; ch+sh	NL; ch+sh		
σ	NL; ch	NL; ch	NL; ch	NL; ch	NL; ch	$k_h = 0$ $k_b \rightarrow \infty$ (EB)	
ε	NL; ch	NL; ch	NL; ch	NL; ch	NL; ch		
τ^*	NL; sh	NL; sh	NL; sh	NL; sh	NL; sh		
σ	NA	N/A	NL; ch	N/A	N/A	$k_h \neq 0$ $k_b = 0$ (FF)	
ε	N/A	N/A	N/A	N/A	N/A		
τ^*	N/A	N/A	N/A	N/A	N/A		
σ	N/A	N/A	NL; ch	N/A	N/A	$k_h \neq 0$ $k_b \rightarrow \infty$ (EB)	
ε	N/A	N/A	N/A	N/A	N/A		
τ^*	N/A	N/A	N/A	N/A	N/A		
σ	NL; sh+ch	NL; ch	NL; ch	NL; sh+ch	NL; sh+ch	$k_h = 0$ $k_b \neq 0$ (SF)	
ε	NL; sh+ch	NL; ch	NL; ch	NL; sh+ch	NL; sh+ch		
τ^*	NL; ch+sh	NL; sh	NL; sh	NL; ch+sh	NL; sh+ch		
Note: NL = non-linear, τ^* = interface shear stress inferred from axial displacement, sh = sine hyperbolic, ch =cosine hyperbolic, SRO &DP = Saeidi Rashk Olia and Perić							

The main feature of the axial stress, strain and shear stress evolutions is that they are transcendental functions of depth, i.e. trigonometric hyperbolic sine and cosine functions or their addition for all load scenarios including mechanical, thermal and thermo-mechanical loads. Clearly, in the case of fully reversible interface behavior and thermo-elastic behavior of the pile, the shear stress at the interface is not constant with depth, and neither axial strain nor axial stress are linear functions of depth.

While several authors (Iodice et al. 2020; Perić et al. 2020; Saeidi Rashk Olia and Perić 2021a, Saeidi Rashk Olia et al. 2022) presented analytical solutions for thermo-mechanical soil structure interaction in single energy piles, Saeidi Rashk Olia and Perić (2021a) were the first to present mechanics-based evolutions of axial displacement, strain and stress in the form referred to as thermo-mechanical schemes by Rotta Loria and Laloui (2018). Thus, the load transfer diagrams presented by Saeidi Rashk Olia and Perić (2021a) resulted from a strict enforcement of laws of mechanics rather than from the experimental observations or statistical analyses. Consequently, these diagrams provide mechanics based qualitative and quantitative representations of displacement, strain and stress evolutions with depth. As shown in Table 4.3, Saeidi Rashk Olia and Perić (2021a) included thermal, mechanical and combined thermo-mechanical loads in case of fully floating, semi-floating and ideally end bearing energy piles.

Previous considerations of irreversible soil-pile interface response adopted finite difference approach (Pasten and Santamarina 2014, Chen and McCartney 2017, Pei et al. 2023). Pasten and Santamarina (2014) implemented a one-dimensional load-transfer method and used elastic-perfectly plastic interface shear stress-displacement behavior to analyze the long-term displacement of an energy pile subjected to a cyclic thermal load. Chen and McCartney (2017) provided a hyperbolic interface shear stress-displacement relationship and used it to investigate a response of single energy piles subjected to monotonic thermal loads. Pei et al. (2022) further modified these models by introducing the dependence of the soil-pile interface strength on the number of thermo-mechanical cycles. They concluded that the cyclic t-z model is a useful tool for predicting the long term performance of energy piles under cyclic thermal loads that can cause the irreversible soil deformation and impact the pile's displacement.

The main goal of this study was to develop analytical solutions for displacement, strain, axial stress and interface shear stress in a single energy pile that exhibits irreversible interface behavior. To this end, an elastic-perfectly plastic shear stress-displacement relationship was adopted. The solutions developed herein were validated against the experimental data and subsequently used to clarify differences in evolutions presented in Table 4.1 and Table 4.2. Furthermore, these solutions enabled development of mechanics-based load transfer diagrams that account for irreversible interface behavior. Finally, the presented closed form solutions also enable elucidation of the salient features of energy piles that result from the irreversible interface response.

Governing laws and assumptions

Seed and Reese (1957) and Coyle and Reese (1966) proposed the first load transfer model for piles subjected to mechanical load. Since then, various load transfer functions have been proposed (Randolph and Wroth 1978; Frank and Zhao 1982; Armaleh and Desai 1987; Frank et al. 1991, Plaseied 2012; Suryatriyastuti et al. 2014, Chen and McCartney 2017).

Thermo-elastic behavior of pile was confirmed by full scale in situ tests (Laloui et al. 2003; Bourne-Webb et al. 2009), numerical simulations (Laloui et al. 2006; Knellwolf et al. 2011, Perić et al. 2017), and analytical solutions (Iodice et al. 2020 Perić et al. 2020, Saeidi Rashk Olia and Perić 2021a). Consequently, it is assumed herein that the pile obeys thermo-elastic constitutive law while the soil pile interface exhibits irreversible behavior described by an elastic perfectly plastic shear stress displacement relationship along the pile shaft (Figure 4.1a) and at the pile tip (Fig. 1b). Analytical solutions are obtained by enforcing the equilibrium on two different 1D infinitesimal pile segments exhibiting elastic (Figure 4.2b) and elastic-plastic (Figure 4.2c) interface behavior.

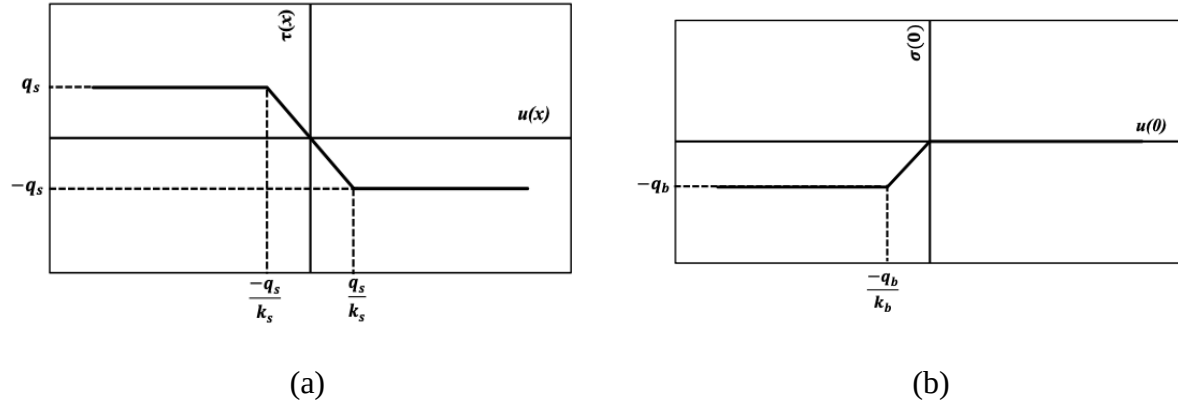


Figure 4.1 Load transfer functions: a) along the pile shaft, b) at the tip

Figure 4.2a depicts an energy pile and defines the coordinate systems used for derivation of the analytical solution. Global coordinate system (x) originates at the pile tip, and its positive direction is upward while local top (x_T) and bottom (x_B) coordinate systems originate at the null point. Conversion from global to local coordinates is described in Eq. (4.16).

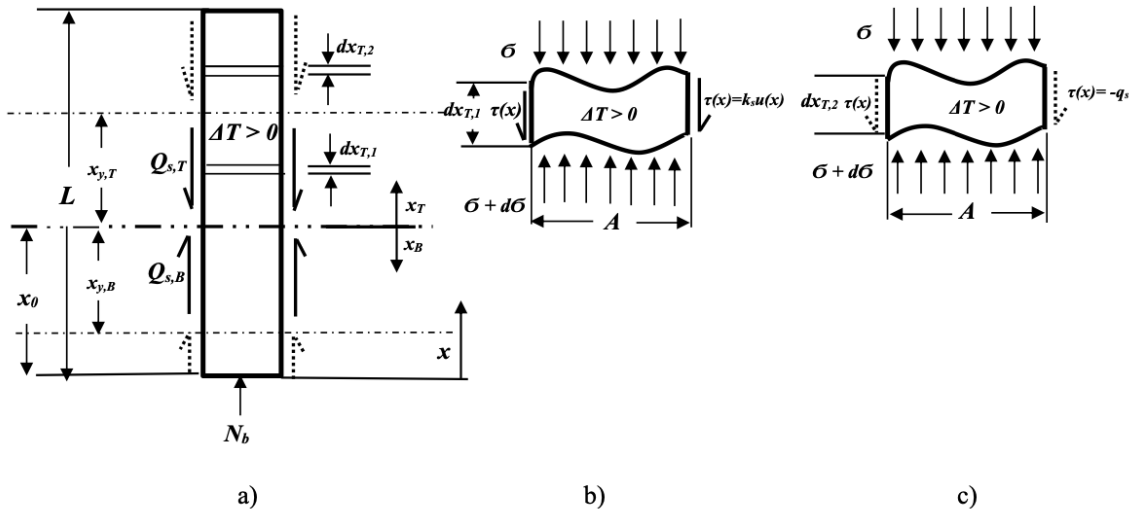


Figure 4.2. FBDs of a semi floating energy pile subjected to a thermo-mechanical load, $F < 0$ and $\Delta T > 0$: (a) the entire pile, (b) an infinitesimal segment of the pile above the null point exhibiting reversible interface behavior, and (c) an infinitesimal segment of the pile above the null point exhibiting irreversible interface behavior

Similarly, to Knellwolf et al. (2011) and Perić et al. (2020), the self-weight of the pile is not considered in this study. Temperature change (ΔT) is positive for heating and temperature is assumed to be uniform throughout the pile. Furthermore, it is assumed that material properties of soil, pile, and soil-pile interface are not affected by the temperature change. Kinematic relationship for small displacements, which is given by (4.1) is used herein

$$\varepsilon = \frac{du}{dx} \quad (4.1)$$

where $\varepsilon = \varepsilon(x)$ is vertical strain and $u = u(x)$ denotes vertical displacement. Upward displacement and tensile strain are both positive. All these assumptions, except the elastic-plastic nature of interface were validated against the experimental data (Saeidi Rashk Olia and Perić 2021a). The pile follows the thermo-elastic constitutive law given by

$$\sigma = E(\varepsilon - \alpha \Delta T) \quad (4.2)$$

where $\sigma = \sigma(x)$ represents vertical stress that is positive in tension. E and α are elastic modulus and coefficient of thermal expansion of the energy pile respectively, while ΔT is a temperature difference of the pile relative to the surrounding soil.

The interface shear stress displacement relationship depicted in Figure 4.1a is represented by the continuous linear elastic shear spring having elastic stiffness k_s and a slider that is activated upon reaching the yield shear stress of magnitude q_s . Thus, the magnitude of the shear stress $\tau = \tau(x)$ is given by

$$|\tau| = k_s |u| \quad \text{for} \quad |u| < \frac{q_s}{k_s} \quad \text{and} \quad |\tau| = q_s \quad \text{for} \quad |u| \geq \frac{q_s}{k_s} \quad (4.3)$$

Similarly, the behavior of the interface between the pile tip and soil or rock below (Figure 4.1b), in terms of global coordinates, is described by

$$\sigma(0) = k_b u(0) \quad \text{for} \quad 0 \geq u(0) > -\frac{q_b}{k_b} \quad \text{and} \quad \sigma(0) = -q_b \quad \text{for} \quad 0 \geq -\frac{q_b}{k_b} \geq u(0) \quad (4.4)$$

where q_s and q_b are magnitudes of yield shear and normal stresses. In accordance with Figure 4.1b transfer of a normal stress at the pile tip ceases once the stress transitions from compression to tension.

Eq. (4.4) represents the boundary condition for a semi-floating pile, whereby neither the stress nor the displacement at the pile tip is equal to zero. For comparison, tip displacement is equal to zero in ideally end bearing pile subjected to a compressive stress at the tip, while in the case of ideally fully floating pile the tip stress is equal to zero. It is noted that ideally fully floating pile has infinite length, but in a pile of a finite length that transfers a very small amount of normal stress at the tip the thermal null point is located near the pile's mid-length. Such a pile is referred to as fully floating pile in this study.

Analytical solutions for irreversible interface behavior

A soil profile that comprises a single soil layer underlain by a bedrock or another soil stratum whose top surface is flush with the pile tip is referred to as a homogeneous soil profile herein. The solution for a semi-floating energy pile is presented first followed by the special cases of fully floating and ideally end bearing piles.

Thermomechanical Loading

Solutions derived herein are presented for an energy pile subjected to a combined compressive mechanical load ($F < 0$) applied at its head, and thermal load (ΔT), such as heating or cooling whereby temperature is uniformly distributed within the entire pile. While the response of the pile is thermo-elastic and thus reversible, the response of the soil pile interface becomes non-

reversible once the yielding starts (Eqs. (4.3), and (4.4)). It is assumed in this section that the behavior of the soil pile interface remains elastic or reversible under mechanical loading while additional thermal loading causes yielding of the interface. This is a reasonable assumption for energy piles that encounter both, mechanical and thermal loading.

Semi Floating Energy Pile

The corresponding Free Body Diagrams (FBDs) of the entire semi-floating energy pile and its infinitesimal segments representing two regions of the pile above the thermomechanical null point are shown in Figure 4.2 for $\Delta T > 0$. The two regions are :1) the region exhibiting reversible interface behavior that surrounds the thermo-mechanical null point and 2) the region exhibiting irreversible interface behavior that is located further away, near the tip and head of the pile. The null point is located at $x = x_0$.

As mechanically induced displacement is negative throughout the entire pile length there is no mechanical null point. A thermo-mechanical null point emerges as magnitude of thermal load increases. As long as the maximum magnitudes of vertical displacements developed on either side of the null point are smaller than the yield displacement (Figure 4.1a), the vertical interface exhibits reversible behavior. Once the displacement on either side of the null point reaches the yield displacement, the vertical soil-pile interface starts to yield and it no longer exhibits reversible behavior. Upon continued thermal loading, the size of the yielded region increases. Figure 4.2a depicts the situation, in which the vertical interface has yielded within two regions, each located on a different side of the null point. The vertical interface inside the third region displays elastic behavior resulting in the corresponding interface shear stress being a linear function of the vertical displacement. Nevertheless, once the yielding has progressed from the pile tip or head the interface shear stress is no longer a linear function of displacement along the entire length of the pile.

Figure 4.2b illustrates the FBD of an infinitesimal segment of the pile above the null point and below the yield location, within which the interface responds elastically ($0 \leq x_T < x_{y,T}$). Similarly, there is another region ($0 \geq x_B > x_{y,B}$), within which the interface displays elastic behavior. In addition, $x_{y,T}$ and $x_{y,B}$ are local coordinates that describe the lengths of elastic interface regions above and below the thermo-mechanical null point respectively. By imposing the equilibrium of the infinitesimal pile segment depicted in Figure 4.2b and using Eqs. (4.1), (4.2), and (4.3) the following homogeneous second order differential equation is obtained.

$$\frac{d^2 u_i}{dx_i^2} - \psi^2 u_i = 0 \quad \text{where} \quad |x_i| < |x_{y,i}| \quad (4.5)$$

and $i=B$ and $i=T$ denote the regions of the pile located below and above the thermo-mechanical null point respectively. The following holds

$$0 \leq x_T \leq L - x_0 \quad \text{and} \quad 0 \geq x_B \geq -x_0 \quad (4.6)$$

Parameter ψ is a constant given by

$$\psi^2 = \left(\frac{p}{A}\right) \left(\frac{k_s}{E}\right) = \xi_g \xi_s \quad (4.7)$$

where p and A are the perimeter and cross-sectional area of the pile respectively. Thus, the parameter ψ contains both, the pile geometry in the form of ξ_g and the ratio of the stiffness of the soil and the pile in the term ξ_s .

Solving for the unknown function $u_i(x_i)$ from Eq. (4.5) results in

$$u_i(x_i) = C_{1i} e^{\psi x_i} + C_{2i} e^{-\psi x_i} \quad (4.8)$$

Enforcing the continuity of displacements at the null point while setting them equal to zero results in the following

$$u_i(x_i) = 2C_{1i} \sinh(\psi x_i) \quad (4.9)$$

The yield condition, expressed in the local coordinates is given by

$$u_T(x_{y,T}^-) = \frac{\pm q_s}{k_s} \quad \text{and} \quad u_B(x_{y,B}^-) = \frac{\mp q_s}{k_s} \quad (4.10)$$

where the superscript minus denotes approaching the yield location from within the pile segment displaying reversible soil pile interface behavior ($|x_i| \rightarrow |x_{y,i}|$). Top sign in Eq. (10) refers to heating while bottom sign refers to cooling. This sign convention applies throughout this derivation. Combining Eq. (4.9) and Eq. (4.10) results in

$$C_{1T} = \frac{\pm q_s}{2k_s \sinh(\psi x_{y,T})} \quad \text{and} \quad C_{1B} = \frac{\pm q_s}{2k_s \sinh(-\psi x_{y,B})} \quad (4.11)$$

After substituting Eq. (4.11) into Eq. (4.9) and rearranging the expressions for strain and stress the following is obtained

$$u_T(x_T) = \frac{\pm q_s \sinh(\psi x_T)}{k_s \sinh(\psi x_{y,T})} \quad \text{and} \quad u_B(x_B) = \frac{\pm q_s \sinh(\psi x_B)}{k_s \sinh(-\psi x_{y,B})} \quad (4.12)$$

$$\varepsilon_T(x_T) = \frac{\pm q_s \psi \cosh(\psi x_T)}{k_s \sinh(\psi x_{y,T})} \quad \text{and} \quad \varepsilon_B(x_B) = \frac{\pm q_s \psi \cosh(\psi x_B)}{k_s \sinh(-\psi x_{y,B})} \quad (4.13)$$

$$\sigma_T(x_T) = E \left\{ \frac{\pm q_s \psi \cosh(\psi x_T)}{k_s \sinh(\psi x_{y,T})} - \alpha \Delta T \right\} \quad \text{and} \quad \sigma_B(x_B) = E \left\{ \frac{\pm q_s \psi \cosh(\psi x_B)}{k_s \sinh(-\psi x_{y,B})} - \alpha \Delta T \right\} \quad (4.14)$$

After enforcing the continuity of stress at the null point Eq. (4.14) gives

$$x_{y,T} = -x_{y,B} = x_y \quad (4.15)$$

Eq. (4.15) indicates that the distance of the location above the null point that is at onset of yielding is equal to the distance of the location below the null point that is at onset of yielding.

The solution given by Eqs. (4.12) through (4.14) is converted to global coordinate by using the following coordinate transformation.

$$x_i = x - x_0 \quad \text{where} \quad i = B \quad \text{and/or} \quad T \quad (4.16)$$

thus, resulting in

$$u(x) = \frac{\pm q_s \sinh[\psi(x-x_0)]}{k_s \sinh(\psi x_y)} \quad (4.17)$$

$$\varepsilon(x) = \frac{\pm q_s \psi \cosh[\psi(x-x_0)]}{k_s \sinh(\psi x_y)} \quad (4.18)$$

$$\sigma(x) = E \left\{ \frac{\pm q_s \psi \cosh[\psi(x-x_0)]}{k_s \sinh(\psi x_y)} - \alpha \Delta T \right\} \quad (4.19)$$

The FBD of the infinitesimal segment from within the pile region where the interface has yielded, i.e. $x_T \geq x_y$ is depicted in Figure 4.2c. Imposing the equilibrium in vertical direction results in

$$\frac{d\sigma}{dx} = \frac{\pm q_s p}{A} \quad (4.20)$$

Integration of Eq. (4.20) gives

$$\sigma(x) = \pm \frac{q_s p}{A} x + C_{3T} \quad (4.21)$$

The boundary condition at the pile head is given by

$$\sigma(L) = \frac{F}{A} \quad (4.22)$$

and the yield condition at the location that corresponds to the onset of yielding and that is approached from within the irreversible interface region is given by

$$u_T(x_y^+) = \frac{\pm q_s}{k_s} \quad (4.23)$$

Combining Eq. (4.21) through (4.23) with Eqs. (4.1) and (4.2) results in the solutions for displacement, strain and stress in the region where $x \geq x_0 + x_y$ that are given by

$$u(x) = \pm \frac{q_s p}{AE} \left\{ \frac{1}{2} [x^2 - (x_0 + x_y)^2] - L(x - x_0 - x_y) \right\} + \left(\frac{F}{AE} + \alpha \Delta T \right) (x - x_0 - x_y) \pm \frac{q_s}{k_s} \quad (4.24)$$

$$\varepsilon(x) = \frac{\mp q_s p(L-x) + F}{AE} + \alpha \Delta T \quad (4.25)$$

$$\sigma(x) = \frac{\mp q_s p(L-x) + F}{A} \quad (4.26)$$

Enforcing continuity of stress at $x_T = x_y$ gives

$$\sigma_T(x_y^+) = \sigma_T(x_y^-) \quad (4.27)$$

thus, resulting in

$$\Delta T \mp \frac{q_s \psi}{k_s \alpha \tanh(\psi x_y)} \mp \frac{q_s p(L-x_0-x_y)}{AE\alpha} + \frac{F}{AE\alpha} = 0 \quad (4.28)$$

Eq. (4.28) defines the relationship between x_0 and x_y at the onset of the interface yielding and beyond.

Next, equilibrium of infinitesimal segment located below the null point, in which the vertical soil pile interface exhibits irreversible behavior, is enforced. This is combined with the boundary condition at the pile tip given (4.4) whereby it is assumed that there is no yielding at the tip, thus giving the solution for stress provided by Eq. (4.31). Strain is obtained by using the thermo-elastic constitutive law. Displacement is obtained from kinematic relationship by using the continuity of displacement given in Eq. (4.10). Consequently, the solution for displacement, strain and stress in the region where $x \leq x_0 - x_y$ is given by

$$u(x) = \mp \frac{q_s p}{2AE} [x^2 - (x_0 - x_y)^2] + \left(\frac{k_b u(0)}{E} + \alpha \Delta T \right) [x - (x_0 - x_y)] \mp \frac{q_s}{k_s} \quad (4.29)$$

$$\varepsilon(x) = \frac{\mp q_s p}{AE} x + \frac{k_b}{E} u(0) + \alpha \Delta T \quad (4.30)$$

$$\sigma(x) = \frac{\mp q_s p}{A} x + k_b u(0) \quad (4.31)$$

Displacement at the pile tip is obtained from Eq. (4.29) as

$$u(0) = \frac{\frac{\pm q_s p}{2AE}(x_0 - x_y)^2 - \alpha \Delta T(x_0 - x_y) \mp \frac{q_s}{k_s}}{1 + \frac{k_b}{E}(x_0 - x_y)} \quad (4.32)$$

Enforcing the continuity of stress at $x = x_0 - x_y$ results in

$$E \left\{ \frac{\pm q_s p}{k_s \tanh(\psi x_y)} - \alpha \Delta T \right\} = \frac{\mp q_s P}{A}(x_0 - x_y) - k_b u(0) \quad (4.33)$$

and Eq. (4.33) defines another relationship between x_0 and x_y at and beyond the onset of yielding.

Values of x_0 and x_y are obtained from Eqs. (4.28) and (4.33) for a given thermal load (ΔT).

In the case that the interface at the pile tip yields along with yielding along the pile shaft the yield condition from Eq. (4.4) is used and corresponding solution for displacement, stress and strain is given by

$$u(x) = \mp \frac{q_s p}{2AE} [x^2 - (x_0 - x_y)^2] + \left(\alpha \Delta T \mp \frac{q_b}{E} \right) [x - (x_0 - x_y)] \pm \frac{q_s}{k_s} \quad (4.34)$$

$$\varepsilon(x) = \frac{\mp q_s P}{AE} x \mp \frac{q_b}{E} + \alpha \Delta T \quad (4.35)$$

$$\sigma(x) = \frac{\mp q_s P}{A} x \mp q_b \quad (4.36)$$

Eqs. (4.24) through (4.36) are valid for $x \leq x_0 - x_y$. Enforcing continuity of stress at $x = x_0 - x_y$, in this case, results in

$$E \left\{ \frac{\pm q_s p}{k_s \tanh(\psi x_y)} - \alpha \Delta T \right\} = \frac{\mp q_s P}{A}(x_0 - x_y) - q_b \quad (4.37)$$

In accordance with the soil pile interface properties depicted in Figure 4.1b both k_b and q_b approach zero as the normal stress at the pile tip transitions from compression to tension. This is valid throughout the derivations presented herein.

Finally, for a given thermal load (ΔT) Eqs. (4.28) and (4.37) establish a relationship between x_0 and x_y . The solutions can be obtained from the MATLAB as it is not possible to obtain explicit analytical solutions. Yielding occurs for $x_y \leq x_0$ and $x_y \leq L - x_0$. Furthermore, as

shown above Eqs. (4.29) through (4.32) present the solution for a reversible interface response at the pile tip while equations (4.34) through (4.36) present a solution for an irreversible interface response at the pile tip.

Special Cases

Special cases of the solutions presented above, which represent an ideally end bearing and fully floating energy piles, are presented next. For an ideally end bearing pile solutions for displacement, strain and stress due to axial force applied at the pile head are obtained by letting $k_b \rightarrow \infty$ into the corresponding solutions for a semi floating pile (Saeidi Rashk Olia and Perić, 2021c). This is applicable only in cases where compressive stress is generated at the pile tip. It also implies that mechanical, thermal and thermo-mechanical null points are all located at the pile tip and that $x_0 = 0$. The interface starts to yield from the pile head and progresses downwards with increase in thermal load. The solution for thermo-mechanical displacement in the zone where the interface has yielded ($x \geq x_y$) is obtained by substituting $x_0 = 0$ into Eq. (4.24), thus resulting in

$$u(x) = \pm \frac{q_s p}{2AE} (x^2 - x_y^2) + \left[\frac{\mp q_s p L + F}{AE} + \alpha \Delta T \right] (x - x_y) \pm \frac{q_s}{k_s} \quad (4.38)$$

where x_y represents the distance between the pile tip and the bottom of the yielded portion of the interface. Solutions for strain and stress in this zone is given by Eqs. (4.25) and (4.26) respectively. For the pile segment with elastic soil pile interface ($x < x_y$) displacement, strain and stress are obtained from Eqs (4.17) through (4.19) as

$$u(x) = \frac{\pm q_s \sinh(\psi x)}{k_s \sinh(\psi x_y)} \quad (4.39)$$

$$\varepsilon(x) = \frac{\pm q_s \psi \cosh(\psi x)}{k_s \sinh(\psi x_y)} \quad (4.40)$$

$$\sigma(x) = E \left\{ \frac{\pm q_s \psi \cosh(\psi x)}{k_s \sinh(\psi x_y)} - \alpha \Delta T \right\} \quad (4.41)$$

The yield location (x_y) is obtained by enforcing the continuity of stress at $x = x_y$, thus resulting in

$$\Delta T \mp \frac{q_s p(L - x_y)}{AE\alpha} \mp \frac{q_s \psi}{k_s \alpha \tanh(\psi x_y)} + \frac{F}{AE\alpha} = 0 \quad (4.42)$$

Eq. (42) enables determination of x_y for given ΔT . Yielding occurs for $x_y \leq L$.

For an ideally fully floating energy pile solutions for displacement, strain and stress due to axial force applied at the pile head are obtained by letting $k_b \rightarrow 0$. In this case the solutions for the pile segment above the null point, within which the interface has yielded ($x \geq x_0 + x_y$), and for the pile segment exhibiting elastic interface behavior ($x_0 - x_y < x < x_0 + x_y$), are given by Eqs. (4.17) through (4.19) and Eqs. (4.24) through (4.26) respectively. For the pile segment below the null point, within which the interface has yielded, the solution is obtained by letting $k_b \rightarrow 0$ in Eqs. (4.29) through (4.31). Analytical solutions for an energy pile that exhibits interface yielding under thermal load can be obtained from the above solutions by setting the force F equal to zero. The procedure for deriving the analytical solutions for thermal load is presented in Figure 4.3.

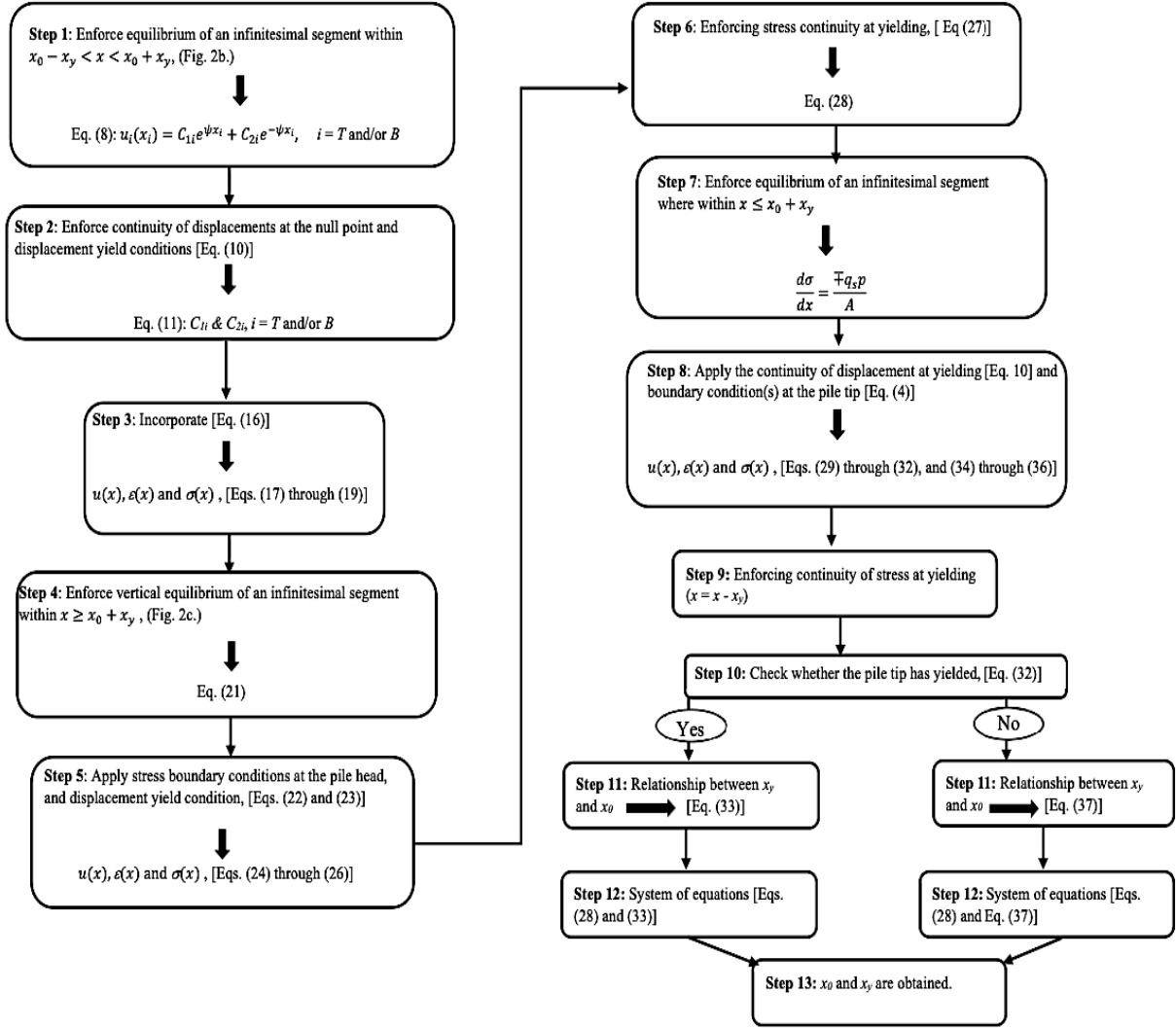


Figure 4.3. Flowchart depicting the procedure of derivation of analytical solutions for a single semi-floating energy pile exhibiting soil-pile interface yielding under thermal load.

Mechanical Loading

In case that pile is subjected to mechanical loading, yielding of a significant portion of the soil pile interface typically occurs in pile load tests that can in turn be used for validation of analytical and numerical models. To this end, analytical solutions for only mechanical load are presented next.

Semi Floating Pile

A mechanical load is applied at the pile head of a semi-floating pile in the form of a vertical force. In response the entire pile moves downward or upward for compressive or tensile load respectively. Consequently, there is no null point. The interface yielding starts at the pile head and progresses downward with increased magnitude of mechanical load. Thus, there is a region ($x < x_y$) of the vertical soil pile interface that exhibits elastic response and another region ($x \geq x_y$) that displays elastic-plastic response ($x \geq x_y$).

The solution for elastic response is obtained by enforcing the equilibrium of an infinitesimal length of the pile within the corresponding region. Similarly, to Eq. (4.8) the solution is given by:

$$u(x) = C_1 e^{\psi x} + C_2 e^{-\psi x} \quad (4.43)$$

After applying the non-yielding boundary condition at the pile tip, which was given in Eq. (4.4), and specifying the displacement at the onset of yielding at $x = x_y$, similarly to Eq. (4.23) the solution for $x < x_y$ is obtained as

$$u(x) = \mp \frac{q_s [E\psi \cosh(\psi x) + k_b \sinh(\psi x)]}{k_s [E\psi \cosh(\psi x_y) + k_b \sinh(\psi x_y)]} \quad (4.44)$$

$$\varepsilon(x) = \mp \frac{q_s \psi [E\psi \sinh(\psi x) + k_b \cosh(\psi x)]}{k_s [E\psi \cosh(\psi x_y) + k_b \sinh(\psi x_y)]} \quad (4.45)$$

$$\sigma(x) = \mp \frac{q_s \psi E [E\psi \sinh(\psi x) + k_b \cosh(\psi x)]}{k_s [E\psi \cosh(\psi x_y) + k_b \sinh(\psi x_y)]} \quad (4.46)$$

where the top sign, in this case minus, corresponds to compressive load and the bottom sign, in this case plus, to a tensile load. This sign convention applies throughout this derivation. Furthermore, as indicated previously k_b is zero in the case of tensile mechanical loading.

The solution for displacement, strain and stress that develops within the region of the pile where interface exhibits yielding ($x \geq x_y$) is obtained by enforcing the equilibrium of the corresponding infinitesimal segment of the pile. After applying the top boundary condition given

in Eq. (4.22) and specifying displacement at onset of yielding similarly to Eq. (4.23) the following solution is obtained for $x \geq x_y$

$$u(x) = \mp \frac{q_s p}{2AE} (x^2 - x_y^2) + \frac{F \pm q_s p L}{AE} (x - x_y) \mp \frac{q_s}{k_s} \quad (4.47)$$

$$\varepsilon(x) = \frac{\pm q_s p (L - x) + F}{AE} \quad (4.48)$$

$$\sigma(x) = \frac{\pm q_s p (L - x) + F}{A} \quad (4.49)$$

The yield location (x_y) is obtained by enforcing the continuity of stress at $x = x_y$, thus resulting in

$$\frac{F}{A} \pm \frac{q_s p (L - x_y)}{A} \pm \frac{q_s \psi E [E \psi \sinh(\psi x_y) + k_b \cosh(\psi x_y)]}{k_s [E \psi \cosh(\psi x_y) + k_b \sinh(\psi x_y)]} = 0 \quad (4.50)$$

where yielding occurs for $x_y \leq L$.

Special Cases

In case of ideally end bearing energy pile ($k_b \rightarrow \infty$) Eqs. (4.44) through (4.46) are transformed into

$$u(x) = \mp \frac{q_s \sinh(\psi x)}{k_s \sinh(\psi x_y)} \quad (4.51)$$

$$\varepsilon(x) = \mp \frac{q_s \psi \cosh(\psi x)}{k_s \sinh(\psi x_y)} \quad (4.52)$$

$$\sigma(x) = \mp \frac{q_s \psi E \cosh(\psi x)}{k_s \sinh(\psi x_y)} \quad (4.53)$$

As stated previously end bearing pile is applicable only to compressive mechanical load. Yield location (x_y) is obtained by enforcing the continuity of stress at $x = x_y$ that requires combining Eqs. (4.49) and (4.43), thus resulting in

$$\frac{\pm q_s p (L - x_y) + F}{A} \pm \frac{q_s \psi E}{k_s \tanh(\psi x_y)} = 0 \quad (4.54)$$

Furthermore, in case of a fully floating pile solutions for displacement, strain and stress within the portion of the pile that exhibits elastic interface behavior are obtained by modifying Eqs. (4.44) through (4.46) accordingly ($k_b \rightarrow 0$), thus resulting in

$$u(x) = \mp \frac{q_s \cosh(\psi x)}{k_s \cosh(\psi x_y)} \quad (4.55)$$

$$\varepsilon(x) = \mp \frac{q_s \psi \sinh(\psi x)}{k_s \cosh(\psi x_y)} \quad (4.56)$$

$$\sigma(x) = \mp \frac{q_s \psi E \sinh(\psi x)}{k_s \cosh(\psi x_y)} \quad (4.57)$$

Finally, yield location (x_y) is determined by setting $k_b = 0$ in Eq.(4.50).

Validation of the analytical solutions

The analytical solutions presented herein were validated against the full-scale tests that were conducted on energy piles constructed at Lambeth College in London (Bourne-Webb et al. 2009). At this site a loading frame was used to apply mechanical load at the pile head of the main pile. In addition, a heat sink pile was used only for a heat exchange. This implies that, for example, cooling of the main pile induced heating of the heat sink pile. Two tests are used herein for validation of the previously presented analytical solutions. One test involves heating of the heat sink pile and the other involves mechanical loading of the main pile. The main pile is 23 m long and its strains were measured with vibrating wire strain gauges (VWSG) and optical fiber sensors (OFS). Thermal strains of the heat sink pile were measured by continuous OFS. Both, the main test pile and the heat sink pile have a diameter of 0.61 m throughout the first 5 m of depth due to the casing that was required within the made ground and terrace deposits. Below the depth of 5 m piles have diameter of 0.55 m and they are surrounded by London clay. Bourne-Webb et al. (2009) reported elastic modulus of 40 GPa and coefficient of thermal expansion of $8.5 \times 10^{-6} / ^\circ\text{C}$ for both piles. Amatya et al. (2012) reported that the maximum observed thermal strains of the heat sink

pile occurred at the pile ends and thermally induced loads at these locations were nearly equal to zero, thus indicating negligible restraints.

According to Frank and Zhao (1982) a value for stiffness of the elastic shear spring (k_s) for fine grained soils and weak rocks including clays, marls and limestones can be determined from

$$k_s = \frac{2E_M}{D} \quad (4.58)$$

where D is the pile diameter, and E_M is the Menard pressuremeter modulus. Knellwolf et al. (2011) reported E_M of 45 MPa for the London clay at the Lambeth College site, thus resulting in $k_s = 163.63$ MPa/m. Amatya et al. (2012) reported the ultimate shaft resistance value (q_s) for the site to be equal to 50 kPa for the top half of the pile, and 70 kPa for the bottom half of the pile. Consequently, an average value of 60 kPa is adopted herein for validation of the solution for a homogeneous soil profile.

Analytical predictions of thermally induced strain and axial load in the heat sink pile subjected to heating of 29.4°C are shown in Figure 4.4a and Figure 4.4b respectively. These predictions are depicted next to the predictions of the numerical models developed by Ozudogru et al. (2014) and Abdelaziz and Ozudogru (2016). The nearly bilinear response indicates yielding of the vertical soil pile interface and mobilization of the ultimate shaft resistance along the significant portion of the pile length. For the most part the analytical prediction captured the experimentally observed response very well. Nevertheless, the analytical solution predicted the elastic interface within the middle 6 m of the pile length, at this thermal load. Furthermore, analytically predicted and experimentally observed locations of the maximum axial load, which also correspond to the location of the null point, agree closely.

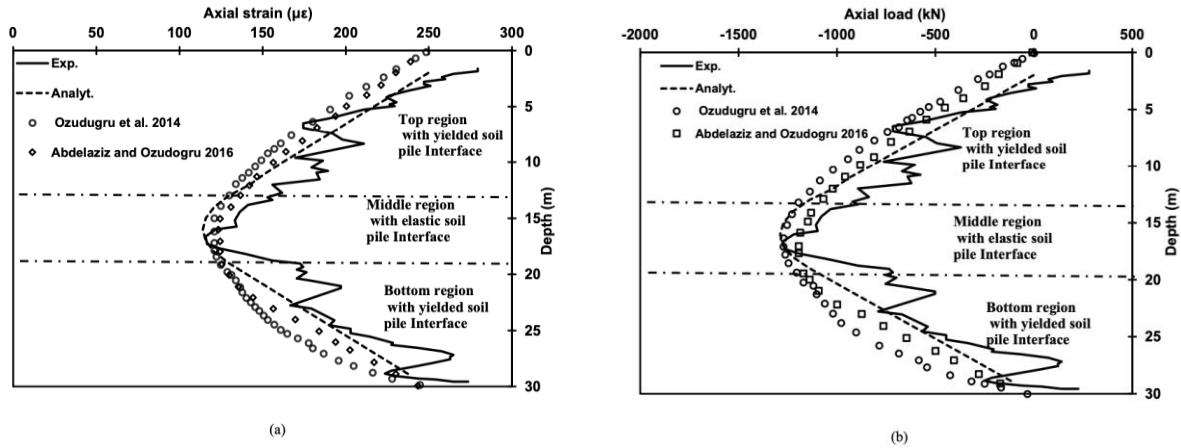


Figure 4.4. Validation of analytical solutions against experimental data for the heat sink pile at Lambeth College under heating ($\Delta T = 29.4^\circ\text{C}$): (a) thermally induced axial strain, and (b) thermally induced axial load

Prior to any heating or cooling the main test pile was loaded in compression to 1800 kN, followed by unloading to zero kN, thus resulting in a residual strain. The measured and predicted strains at 1800 kN and at unloaded state are shown in Figure 4.5. Analytical solution detected the interface yielding up to a depth of 11.81 m under 1800 kN. The interface yielding induced a residual strain that is obtained by deducting the strain due to unloading from the strain due to the original loading to 1800 kN. It is noted that the interface yield stress increases twofold during the unloading due to a direction of loading that differs from the original loading to 1800 kN (Figure 4.6c). As shown, the analytical solution successfully captured the overall trend in the experimentally observed strains. In addition, it is noted that the top four meters of the test pile are embedded in the made ground and terrace deposit (Bourne-Webb et al. 2009), the properties of which are not available. Furthermore, a steel casing was placed around the pile up to a depth of 5 m. Although these features were omitted in the analytical prediction the overall agreement between the experimental data and analytical predictions is very good.

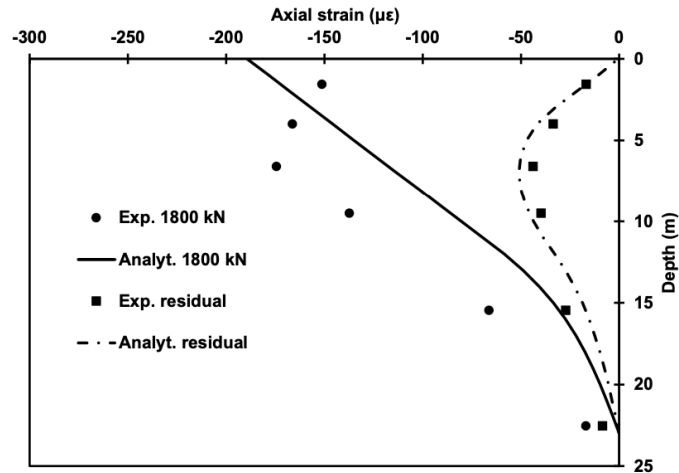
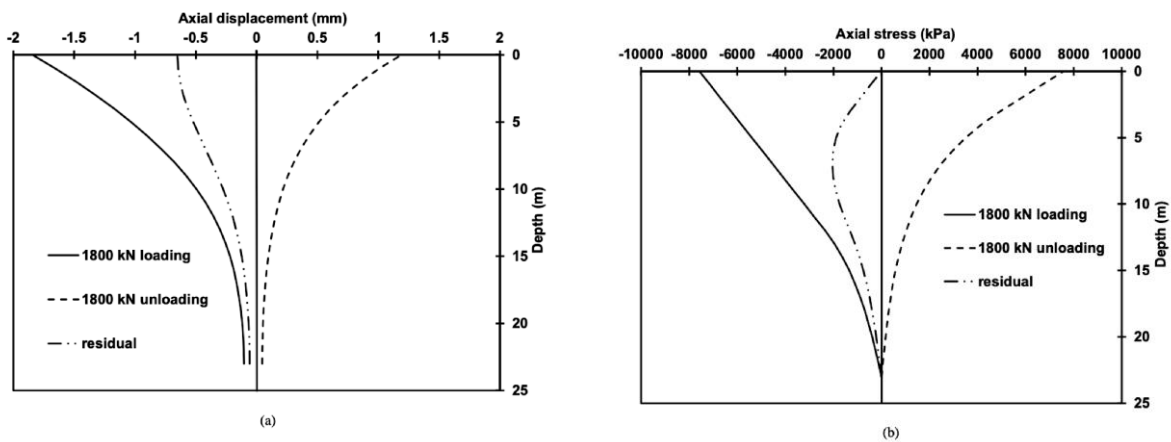


Figure 4.5 Comparison between analytical predictions and experimental data for the main test pile at Lambeth college under mechanical loading ($F < 0$)

It is important to note that the residual strain is not a plastic strain in the pile as it is assumed that the pile obeys thermo-elastic constitutive law. The residual displacement, strain and stress arise from the equilibrium with non-zero shear stress that resulted from unloading of the pile that exhibits partially yielded interface under the original loading. Figure 4.6 further elucidates the mechanism of development of residual axial displacement, strain, and shear stress, resulting from the elastic unloading of the main energy pile in Lambeth College, London.



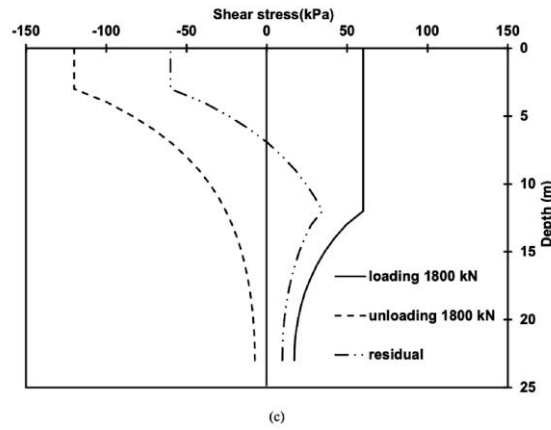


Figure 4.6 Mechanism of development of mechanical residual: (a) displacement, (b) axial stress, and (c) shear stress in Lambeth College main pile subjected to mechanical loading. ($F = -1800\text{kN}$)

Load transfer diagrams

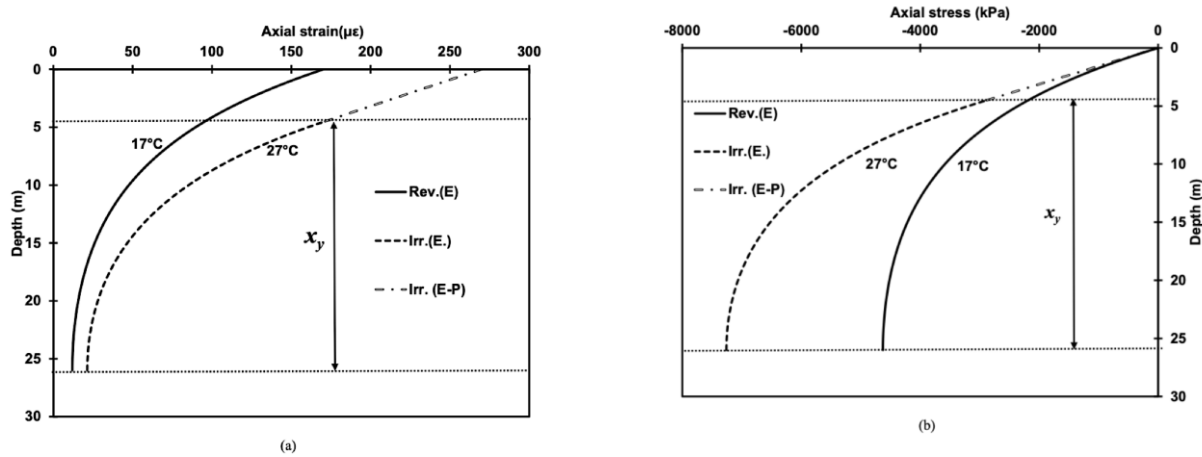
Thermal, mechanical, and thermomechanical axial displacement, strain, stress as well as shear stress along the energy pile are presented herein and referred to as load transfer diagrams. For this purpose, the diagrams are developed for a hypothetical homogeneous soil profile comprising a single soil layer underlain by a bedrock or another soil stratum, the top surface of which is flush with the pile tip.

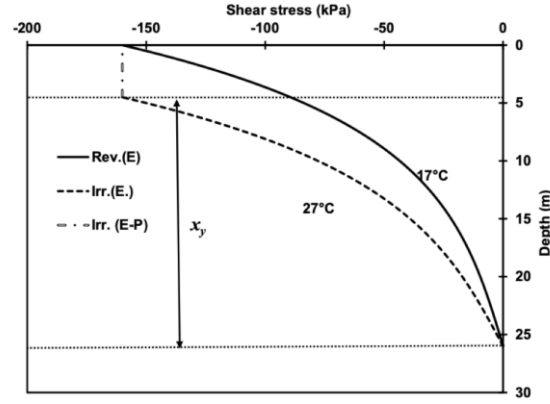
As the analytical solutions cannot be made entirely independent of the input parameters by the process of normalization the selected input parameters correspond to those encountered in the single energy pile constructed at the Swiss Federal Institute of Technology at Lausanne (Laloui et al. 2006). Thus, the length and the diameter of the pile are equal to 26 m and 1 m, respectively, while its elastic modulus and coefficient of thermal expansion are equal to 29.2 GPa and $1 \times 10^{-5}/^{\circ}\text{C}$, respectively. It is noted that the actual soil profile at the Swiss Federal Institute of Technology consists of four different soil layers located above the bedrock that were referred to as soils A1, A2, B, and C by Laloui et al. (2006). All these were classified as low plasticity clays

by Perić et al. (2017) based on the information provided by Laloui et al. (1999). Thus, the synthetic soil profile used herein is hypothetical in that it contains only a single layer, which is layer C from the Lausanne site, above the bedrock. Thus, all the figures in this section are developed for corresponding value of k_s of to 121.4 MPa/m, and ultimate shaft friction value (q_s) of 160, both reported by Knellwolf et al. (2011). The adopted soil and pile parameters resulted in the value of parameter ψ determined from Eq. (4.7) being equal to 0.1289/m. They also reported that the stiffness of the normal spring attached to the pile tip ranged between 667.7 MPa/m and 1335.54 MPa/m. In the present study, the upper bound value (1335.54 MPa/m) was used to represent the semi-floating energy pile.

Irreversible Response Mechanism

The mechanism of development of irreversible thermal strain, stress, and shear stress has been depicted in Figure 4.7 and Figure 4.8 for end bearing and fully floating piles, respectively. To this end, the response of these piles under the temperature, at which the soil pile interface is at the onset of yielding, ($\Delta T_y = 17^\circ\text{C}$) and 10°C higher than that are shown. It is noted that in Figure 4.7 and Figure 4.8 the term reversible refers to the response characterized by the entirely non-yielded interface.





(c)

Figure 4.7. Mechanism of development of thermal irreversible: (a) strain, (b) axial stress, and (c) shear stress in an end bearing energy pile subjected to heating (10°C beyond $\Delta T_{y,head}$)

Saeidi Rashk Olia and Perić (2021a) previously provided the solutions for this case. Nevertheless, the solutions presented herein cover partial or complete yielding of the interface that is characterized as an irreversible behavior. As illustrated in Figure 4.7 and Figure 4.8 not the entire interface has to yield to activate the irreversible behavior that results in the residual displacement, strain and stress that were illustrated in Figure 4.5 and Figure 4.6. Specifically, in Figure 4.7 and Figure 4.8 the term ‘Rev. (E)’ describes a completely non-yielded interface, ‘Irr.(E)’ describes the non-yielded portion of the interface in case of the overall irreversible response, and the term ‘Irr (E-P)’ describes the yielded portion of the interface in case of the overall irreversible response.

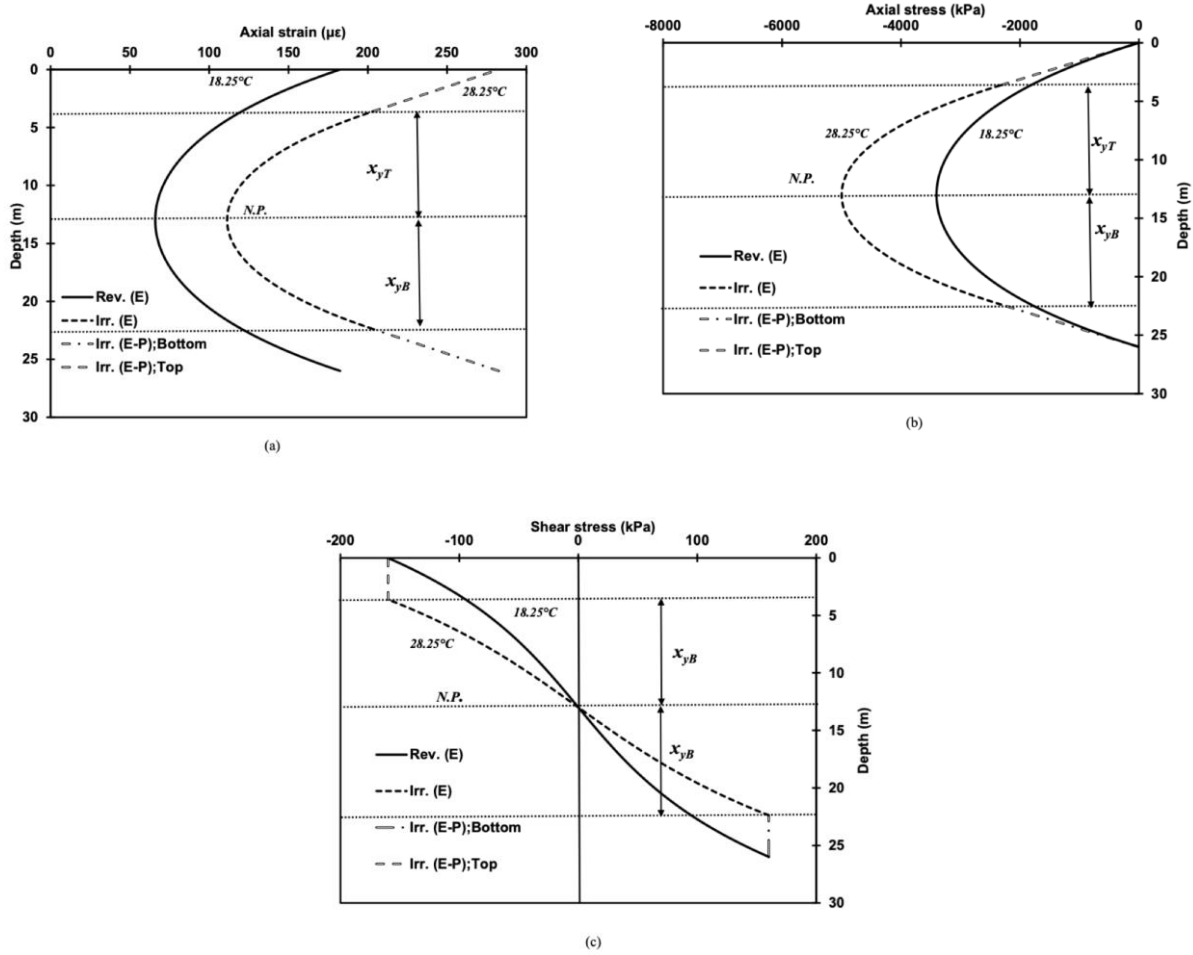


Figure 4.8. Mechanism of development of thermal irreversible: (a) strain, (b) axial stress, and (c) shear stress in a fully floating energy pile subjected to heating (10°C beyond ΔT_y)

Yielding of the soil pile interface in the end bearing pile starts at the pile head and propagates downwards. On the other hand, in a fully floating pile, the maximum thermal displacement occurs at the pile head and tip simultaneously. Therefore, the soil pile interface yielding starts from the pile head and tip simultaneously and propagates inwards towards the null point while the interior edges of the yield zones remain at the same distance from the null point (x_y). The smaller thermal load (17°C) is required to initiate the yielding of the soil pile interface in the end bearing energy pile as compared to the fully floating pile (18.25°C) (Figure 4.8). This is due to the higher magnitude of maximum thermal displacement induced by the full-fixity tip

boundary condition compared to a zero restraint in fully floating piles. The response of the energy pile interface remains fully reversible according to the corresponding constitutive model, i.e. $|\tau| = k_s|u|$ all along the pile shaft up until reaching the onset of yielding ($\Delta T < \Delta T_y$). Nevertheless, once the thermal load surpasses the yield temperature ($\Delta T \geq \Delta T_y$), the response of the pile interface becomes irreversible. Nevertheless, the portion of interface within the non-yielded region still demonstrates an elastic interface behavior while in the yielded region it exhibits elastic-plastic behavior, i.e. $|\tau| = |q_s|$.

By comparing Figure 4.7 and Figure 4.8 it is observed that the additional temperature increase of 10°C induces a longer yield zone at the pile head in the end bearing pile ($L - x_y = 4.47$ m) compared to the fully floating pile ($L_T - x_{yT} = 3.62$ m). However, the fully floating pile experiences a longer length of the yield zone (7.24 m) due to the equal yield lengths developing at both ends of the pile. Yielding causes redistribution of the induced thermal stress in the piles. It is important to note that the pile response remains thermo-elastic while yielding occurs and propagates along the soil pile interface.

Thermal and Mechanical Interface Yielding

Figure 4.9 and Figure 4.10 depict the normalized load transfer diagrams whereby the soil pile interface is yielding under thermal and mechanical load, respectively. While the mechanical responses of end bearing and fully floating energy piles demonstrate similarities, their thermal responses have fundamental differences.

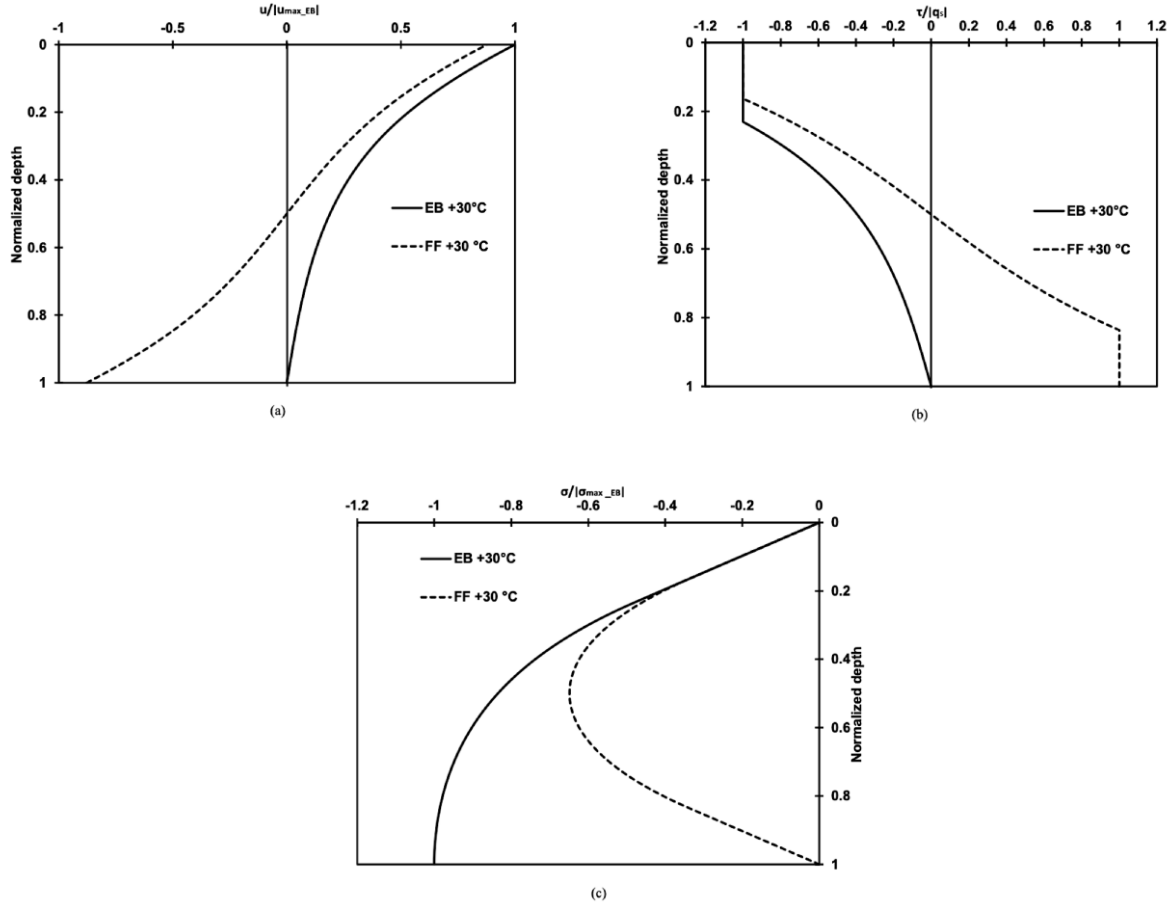


Figure 4.9. Comparison of thermal (a) displacement, (b) shear stress, and (c) axial stress under heating in ideally end bearing and fully floating energy piles ($\Delta T = 30^\circ\text{C}$)

Figure 4.9a show that the end bearing energy pile experiences larger magnitude of maximum thermal displacement along the pile shaft, while the magnitude of the maximum mechanical displacement is only slightly larger in the fully floating pile, as shown in Figure 4.10a. In thermal loading the fully floating pile experiences a longer yield zone, accounting for yielding that occurs simultaneously at both ends. The length of the yielded portion of the interface as well as the boundaries of the yielded part are better observed from the interface shear stress evolutions depicted in Figure 4.9b. Nevertheless, the propagation of mechanical yield zone exhibits a similar pattern in both piles, initiating and propagating from the pile head downwards as shown in Figure 4.10b. The fully floating pile experiences only a slightly longer mechanical yield zone that is

caused by the larger displacement at the pile head. Both thermal and mechanical axial stresses developed at the pile head, which is located within the yield zone, are the same for the end bearing and fully floating energy piles.

The constant shear stress along the pile shaft within the yield zone, as described in Eq. 26, results in equal stresses in end bearing and fully floating piles inside the yield zone. As shown in Figure 4.9c and Figure 4.10c outside the yield zone, the evolutions of axial stresses deviate from the linear and they follow the hyperbolic trigonometric functions.

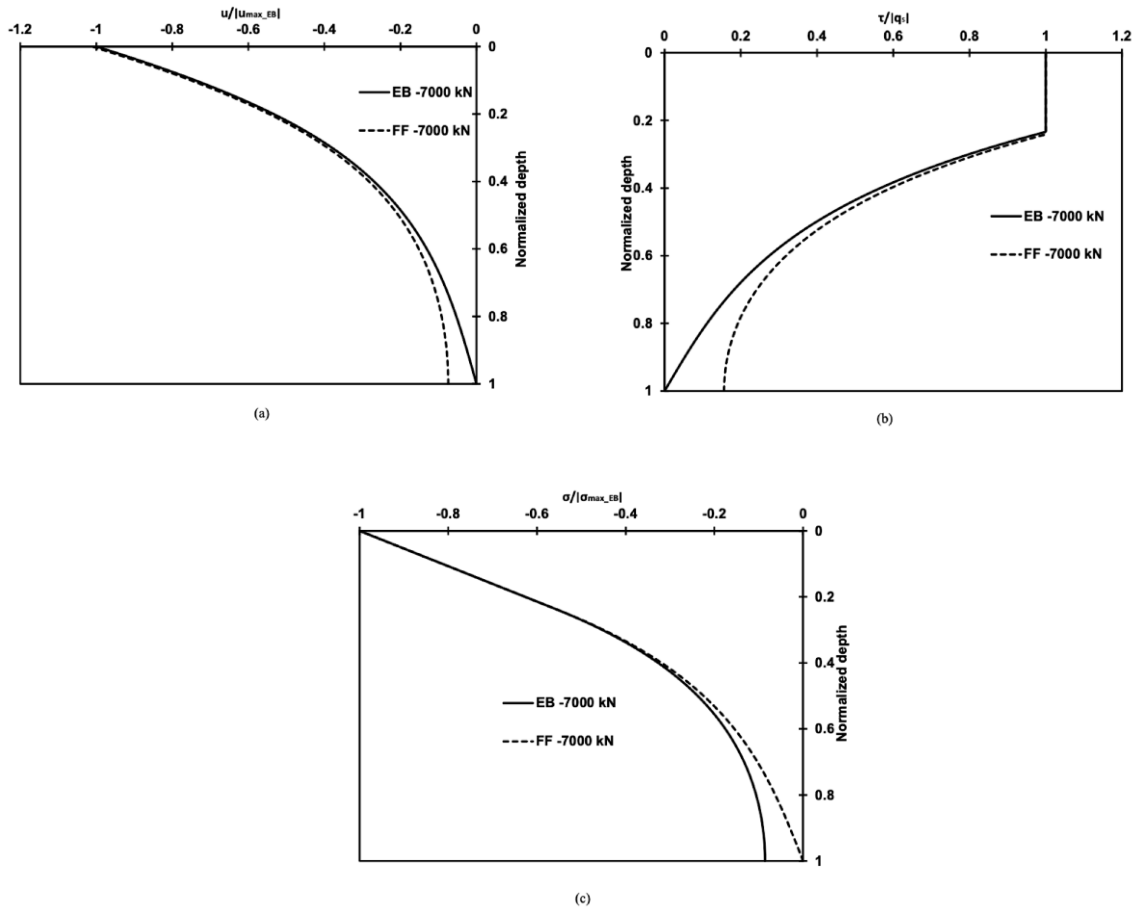


Figure 4.10. Comparison of mechanical (a) displacement; (b) shear stress, and (c) axial stress under compressive mechanical load in an ideally end bearing fully floating energy pile ($F = -7000$ kN)

Progression of Yielding under Thermomechanical Loading

Progression of the soil pile interface yielding under thermomechanical compression and heating, is shown in Figure 4.11 through Figure 4.13, and under compression and cooling in Figure 4.14. The energy pile was subjected to a 1000 kN compressive mechanical load, representing the axial force from the completely constructed building at the Lausanne site (Knellwolf et al., 2011). The soil pile interface remains in elastic state under the 1000 kN of applied mechanical load in cases of end bearing, fully floating, and semi floating energy piles.

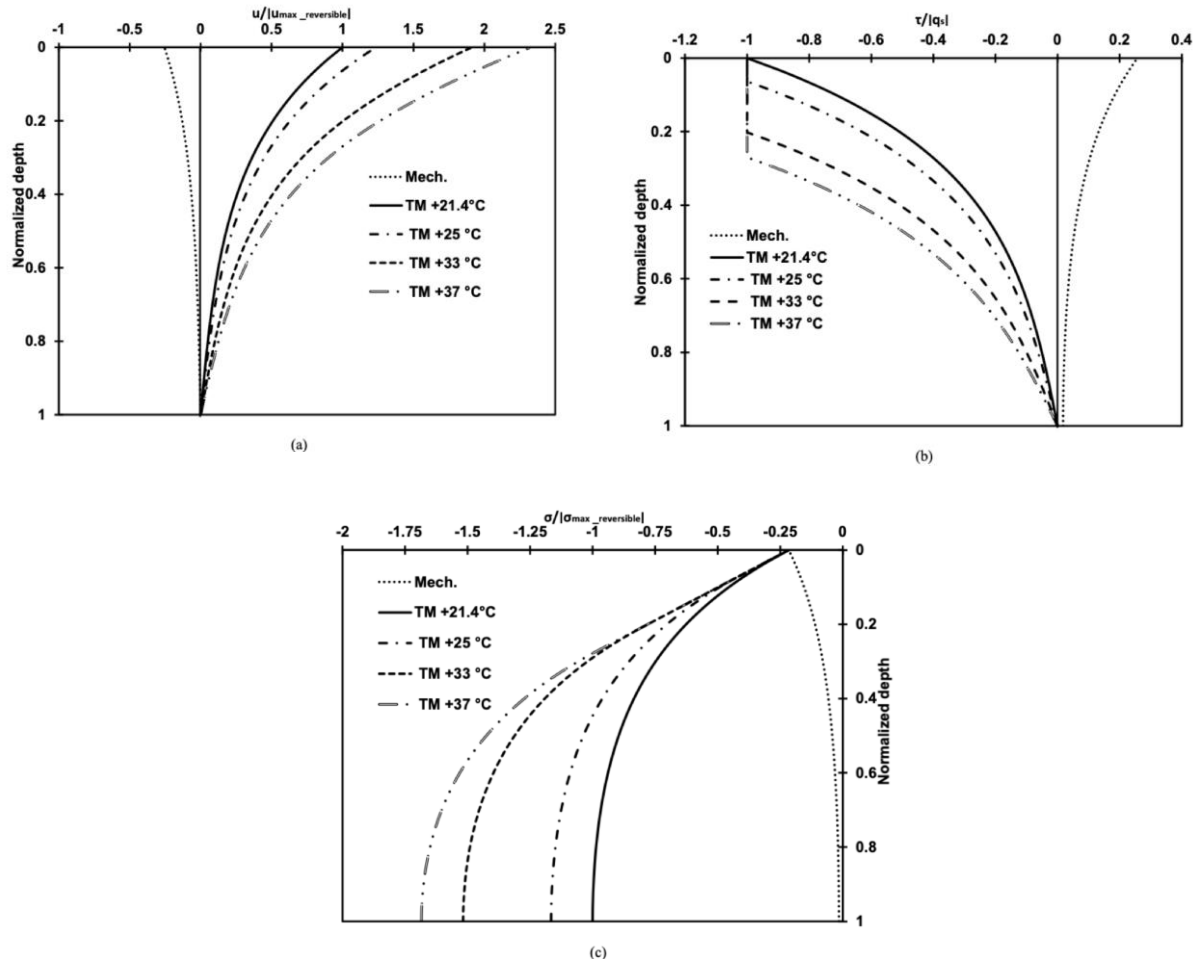
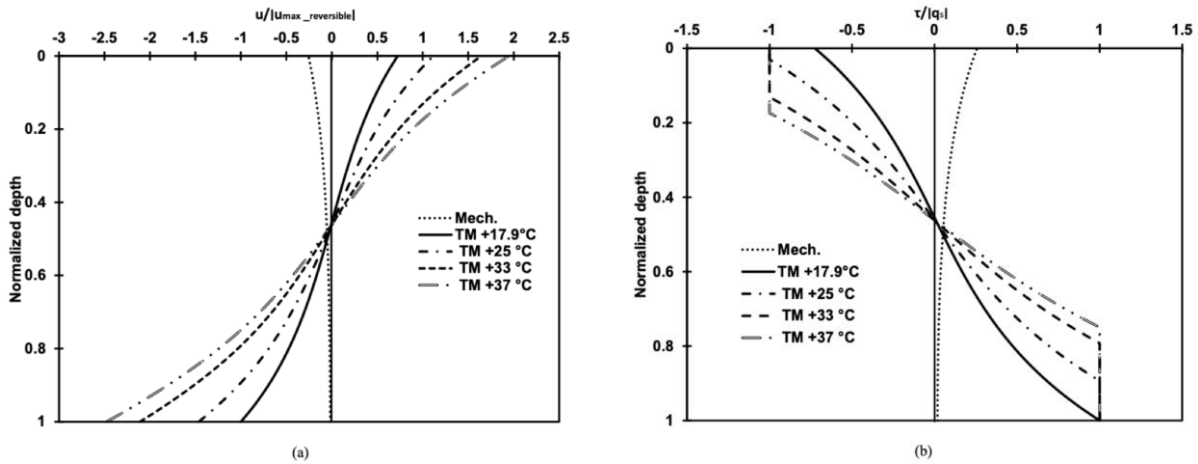
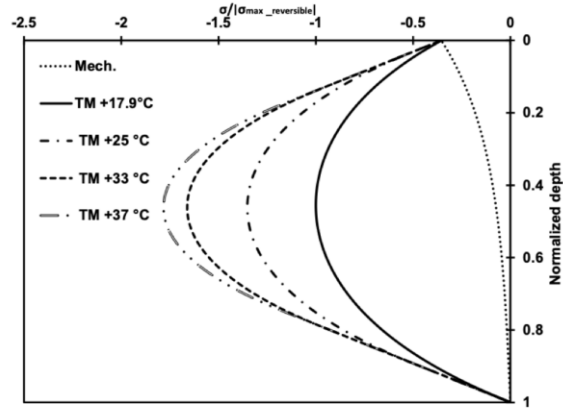


Figure 4.11. Progression of thermal soil pile interface yielding: (a) displacement, (b) shear stress, and (c) axial stress for an end bearing energy pile subjected to mechanical compression and heating

The thermal load was subsequently applied to the pile, and its response was normalized with respect to the response of the pile at the onset of soil pile interface yielding. It is noted that the maximum shear stress, which could be developed along the pile shaft is the ultimate shaft friction (q_s), is used for normalization of the shear stress. This provides a better perspective of the irreversible response of the energy pile with respect to its maximum reversible scenario. The thermal load required for each type of pile to be at the verge of soil pile interface yielding varies slightly due to different tip restraints.

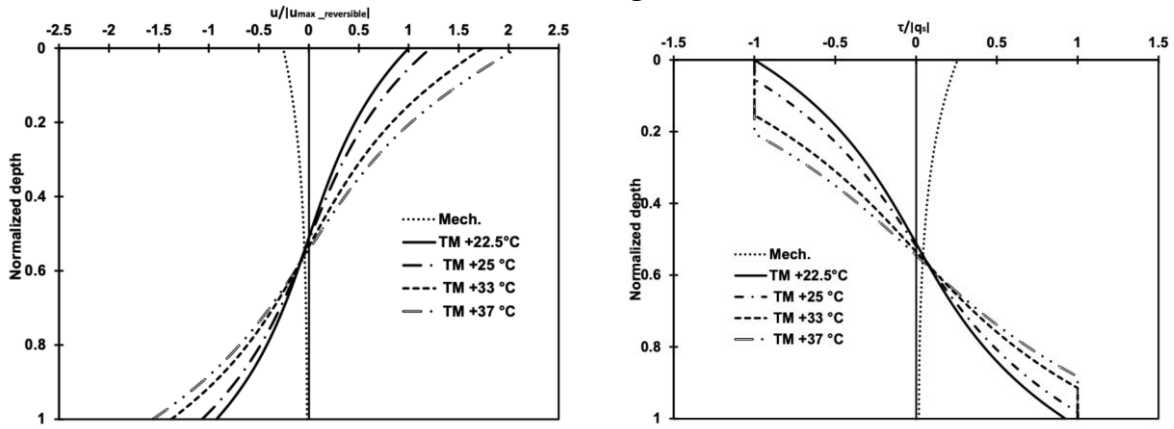
Nevertheless, comparing Figure 4.12 and Figure 4.14 indicates that the sign of thermal load has a significant effect on the magnitude of the temperature required for soil the initiation of the interface yielding. Specifically, the initiation of the yielding in compression and cooling requires smaller magnitude of temperature than in compression and heating. This is due to the mechanical and thermal displacements having the same direction within the top region of the pile, in the case of compression and cooling.





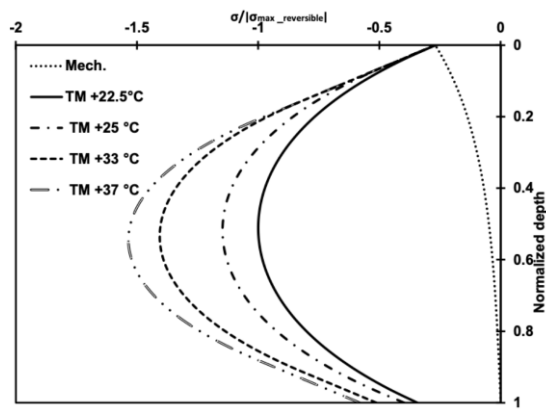
(c)

Figure 4.12. Progression of thermal soil pile interface yielding: (a) displacement, (b) shear stress, and (c) axial stress for a fully floating energy pile subjected to mechanical compression and heating



(a)

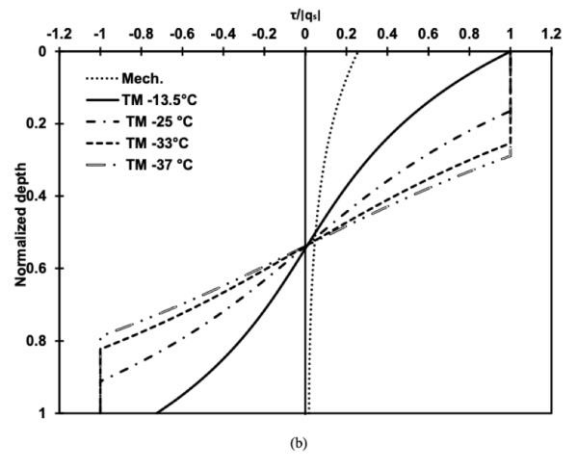
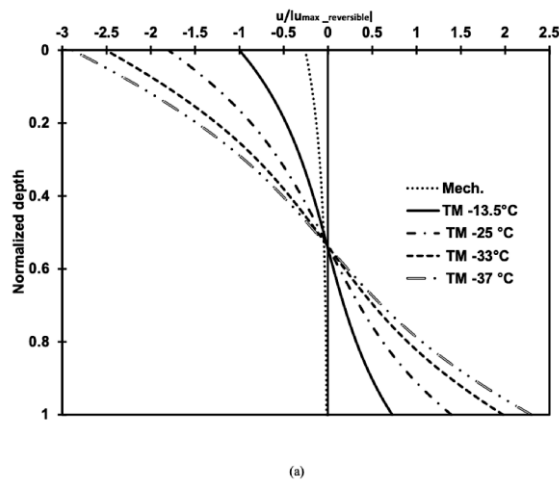
(b)



(c)

Figure 4.13. Progression of thermal soil pile interface yielding; (a) displacement, (b) shear stress, and (c) axial stress for a semi floating energy pile subjected to mechanical compression and heating

As shown in Figure 4.11 through Figure 4.13, $\Delta T = 21.4^\circ\text{C}$ corresponds to the onset of yielding for the end bearing pile under compression and heating while the fully floating pile requires only 17.9°C . Furthermore, in case of compression and heating the interface yielding is initiated at the pile head in cases of end bearing and semi floating piles and at the pile tip in the case of the fully floating pile. In this loading scenario, thermal and mechanical displacements in semi floating and fully floating piles have the same sign below the thermal null point. Nevertheless, in the case of the fully floating pile the thermal null point is located at the mid-length of the pile while in the case of the semi floating pile it is located below the mid-length, thus resulting in the larger magnitude of the displacement at the pile tip than at the pile head in the case of fully floating pile. Consequently, interface yielding starts at the tip. The opposite occurs in the case of compression and cooling, i.e. thermal and mechanical displacements have opposite signs below the thermal null point, thus resulting in the initiation of yielding at the pile head. Finally, in case of end bearing pile there is no displacement at the pile tip by definition, thus leading to the onset of interface yielding at the pile head.



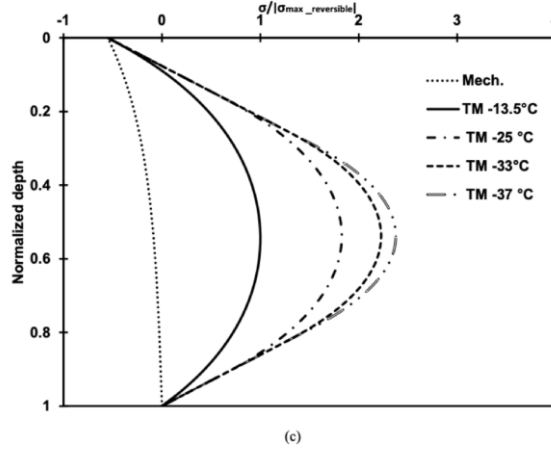


Figure 4.14. Progression of thermal soil pile interface yielding: (a) displacement, (b) shear stress, and (c) axial stress for a fully floating energy pile subjected to mechanical compression and cooling.

After the onset of interface yielding the thermomechanical null point moves slightly more downward in the semi-floating pile (Figure 4.13) than in the fully floating pile (Figure 4.12). Within the region where the interface has yielded the axial stress evolution becomes linear, thus different from the evolution corresponding to the non-yielded interface. Furthermore, the axial stress versus depth follows the same line, from which curves describing the evolutions within the non-yielded zone separate at different distances corresponding to the lengths of the yielded regions for given temperatures. The corresponding yield locations can be clearly seen in the evolutions of the interface shear stress (Figures. 4.11b, 4.12b, 4.13b, 4.14b).

Mechanically loaded fully floating energy pile is subsequently cooled to the temperature causing the onset of yielding at the soil pile interface and beyond. The corresponding load transfer diagrams are depicted in Figure 4.14. Under similar $|\Delta T|$ the length of the zone experiencing interface yielding in case of compression and cooling is larger than in case compared to compression and heating.

Saeidi Rashk Olia and Perić (2021a) proved that the location of the thermal null point constant and independent of the magnitude of thermal load in case of reversible behavior of the

soil pile interface. They also showed that in this case the location of the combined thermomechanical null point varies with the magnitude of the mechanical load (Saeidi Rashk Olia and Perić 2021b). Nevertheless, once the interface starts yielding the location of the null point changes.

Conclusions

This study presents rational, mechanics-based solutions for axial displacement, stress, strain, and interface shear stress in single thermo-elastic energy piles that exhibit an irreversible soil-pile interface behavior.

- Analytical solutions were derived for thermo-mechanical, thermal and mechanical loadings of a single energy pile embedded in a homogeneous soil layer. While the irreversible interface response of the energy pile has been investigated previously through use of numerical methods, this is the first time it has been incorporated in the analytical solutions. The solutions were subsequently validated against the full-scale pile tests conducted at the Lambeth College, London.
- The study found that the location of the null point changes after the initiation of the soil pile interface yielding. The change is the result of enforcing the equilibrium amidst the inability of the yielded portion of the interface to experience a shear stress of a magnitude larger than that of the ultimate interface shear stress.
- The analytical solutions captured residual displacement, strain and stress as features of the irreversible interface behavior accompanied with the thermo-elastic response of the pile. A comparison of analytically predicted and experimentally observed responses at Lambeth College, London further illustrates this feature.

- The presented load transfer diagrams depict evolutions of displacement, axial stress, and interface shear stress for various loading scenarios. Within the regions, where yielding of the interface occurs, displacement is a quadratic function, interface shear stress is constant, and stress and strain are linear functions of depth. In contrast, within the regions where there is no interface yielding all evolutions are hyperbolic trigonometric functions.

Chapter 5 - Computational, Analytical and Statistical Analyses for Assessing Effects of Soil Anisotropy on Response of Energy Piles

Abstract

In this study, computational, analytical, and statistical modeling were employed to evaluate the effects of soil anisotropy on the load transfer mechanism in single energy piles. Finite element analyses, which employ a cross-anisotropic elastic material model for soil, and analytical solutions predicted axial displacement, strain, and stress in a single energy pile subjected to thermal loading. Statistical analyses were conducted on the data obtained from computational and analytical models to develop correlations between the key stiffness parameters of the soil and bedrock and energy pile, and the maximum magnitude of axial stress developed along the pile shaft. correlation between the stiffness of the shear spring used in the analytical approach and elastic constants used in the finite element model was obtained by equalizing the maximum magnitudes of axial stresses obtained from the two approaches. The resulting correlation captured the effect of soil anisotropy in the analytical solution and revealed that a higher degree of anisotropy leads to a lower stiffness of the shear spring representing the soil. Finally, parameters of analytical solution obtained from these correlations were validated against the experimental results of a full-scale energy pile test, and cross validated against predictions of the computational model.

Keywords: Cross validation, Energy pile, Multiple linear regression, Soil anisotropy, Thermal stress

Introduction

Energy piles are cutting-edge geo-structures that are environmentally friendly and aim to improve the sustainability of civil infrastructure. They play a dual role comprising transferring loads from superstructures to the ground and exchanging the thermal energy between the shallow subsurface and superstructures. Ground temperature remains constant throughout the entire year beyond a certain depth that depends on the local climate and environment. The depths are in the vast majority of cases smaller than the depth, i.e., length of the pile required for a safe and effective load transfer, thus providing the piles with direct access to the thermal energy available in the shallow subsurface. A conversion of the conventional piles into energy piles, which can capture and transport renewable thermal energy, incurs only a minor additional expense associated with the insertion of polyethylene tubing that serves as a conduit for a heat exchange fluid.

Thermal stresses induced by the temperature change in the energy piles are crucially important for the structural analysis and design of these dual-functional foundations. To this end, the determination of primarily thermally, but also mechanically induced axial stress, strain and displacement, and interface shear stress has been the focus of many researchers in the past twenty years. The main emphasis has been on the experimental approach including in situ testing (Laloui et al. 2003, Laloui et al. 2006, Brandl 2006, Bourne-Webb et al. 2009, Adam, and Markiewicz 2009, Wood et al. 2009, Amatya et al. 2012, Akrouch et al. 2014, Sung et al. 2018, Fang et al. 2020, Moradshahi et al. 2021, Elzeiny and Suleiman 2021, and Yang et al. 2022), and centrifuge testing (Stewart and McCartney 2013, Murphy and McCartney 2015, Ng et al. 2020, and 2021).

More recently, multiple series of analytical solutions provided mechanics-based rational solutions that were validated against the actual experimental data. (Perić et al. 2020, Iodice et al. 2020, Saeidi Rashk Olia and Perić 2021a, Saeidi Rashk Olia et al. 2022, and Saeidi Rashk Olia

and Perić 2023b). These solutions provide qualitative and quantitative assessments of axial displacement, strain, and stress as well as of the interface shear stress in single energy piles.

The ability of the Finite Element Method (FEM), which originated in the mid-20th century, to provide detailed predictions has made it an indispensable tool for engineers and scientists.. FEM has been used to predict the behavior of geomechanical and structural systems as well as to improve the comprehension of the thermal, mechanical, and thermomechanical responses of energy piles. For example, Laloui et al. (2006) developed a fully coupled thermal hydro-mechanical FEM code that simulates a thermo-mechanical response of single energy piles. They used it to predict the response of the energy pile at ETH Lausanne (Laloui et al. 2006) and found that a 1°C temperature increase induced a 100 kN of axial force in the pile shaft. Nevertheless, as this is a numerical finding from a specific site and pile configuration it cannot be applied elsewhere. Di Donna et al. (2016) and Perić et al. (2017) subsequently corrected the values of isotropic elastic soil stiffness parameters reported by Laloui et al. (2006). Furthermore, Perić et al. (2017) discovered the anisotropic nature of the bottom two layers at the Lausanne site and calibrated the elastic material models accordingly, thus providing an improved predictions of axial strain and stress in the pile as compared to the predictions of the isotropic material model.

More recently, the FE method has been extended to advance the understanding of the behavior of energy piles embedded in isotropic soil. To this end, Arzanfudi et al. (2020) developed a novel thermo-hydro-mechanical finite element model for energy piles subjected to cyclic thermal loading. This model considered several important features, such as three-dimensionality, embedded heat exchangers, and the inclusion of the pile-soil interface model. Iodice et al. (2023) investigated the impact of different constitutive models for soils on the development of the pile's

axial force that led to the development of a practical design procedure for energy piles under thermal loading.

While there have been noteworthy advancements in the computational modeling of energy piles, there remains a lack of insight regarding the effect of soil anisotropy on the soil stiffness parameters required by analytical solutions. This study, therefore, attempts to fill this gap by focusing on the effect of soil anisotropy and developing statistical correlations between the elastic soil parameters required by computational and analytical approaches.

Methodology

A series of elastic finite element analyses was performed to study the impact of soil (E, ν_{vh}), and bedrock elastic parameters (E_b), pile length to diameter ratio (L/D), and degree of soil anisotropy (α_s) on response of single energy piles. The maximum magnitude of axial stress along the pile (σ_{max}) was recorded as it is the main input in the structural design of energy piles. A series of multiple linear regression analyses were subsequently performed to investigate the correlation between the selected variables and σ_{max} .

Additionally, a series of analytical predictions based on the analytical solutions were obtained to evaluate the effect of soil and bedrock stiffness (k_s and k_b), and the pile length to diameter (L/D) ratio on the maximum magnitude of axial stress (σ_{max}). The correlations between these variables were investigated and the impact of each parameter on the outcome was analyzed. Through a comparison of the correlations obtained from the analytical solutions and the FEM simulations, a correlation between the parameters of discrete and continuum models was obtained based on the maximum axial stress, thus providing available insight into the effects of soil anisotropy on the analytical solutions.

Computational Modeling

Coupled thermal hydro-mechanical finite element analyses were performed using the software ABAQUS/CAE 2019. Figure 5. shows the dimension of the employed model of the energy pile, where D denotes the pile diameter that was chosen to be 1 m, and L represents the length of the pile that varies to match the desired L/D ratios of 20, 30, and 40. The vertical boundaries were restricted from horizontal displacement, while the bottom boundary was fully fixed. The analysis was carried out in two stages, the first stage included a thermal analysis only while the second stage involved a coupled pore-pressure-displacement analysis. The temperature distribution obtained from the first stage was used as input into the second stage. The pile and soil are thermo-elastic materials, and it was also assumed that the temperature within the pile was uniformly distributed in analytical and computational models. The heat transfer analysis was conducted using eight-node quadratic axisymmetric heat transfer elements (DCAX8), and the fully coupled pore pressure displacement analysis was performed by using eight-node biquadratic axisymmetric elements (CAX8P) for the soil and bedrock and eight-node biquadratic axisymmetric elements (CAX8) for the pile. Figure 5.1 depicts temperature distributions in the model at several values of a temperature difference as indicated in the figure caption.

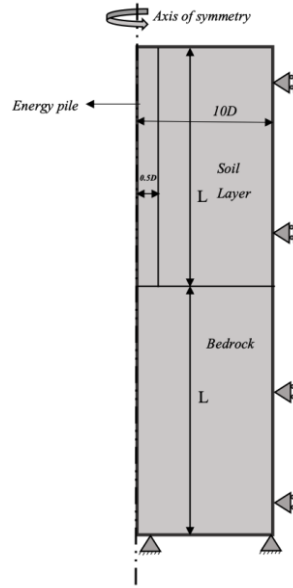


Figure 5.1. Computational model: dimensions and boundary conditions

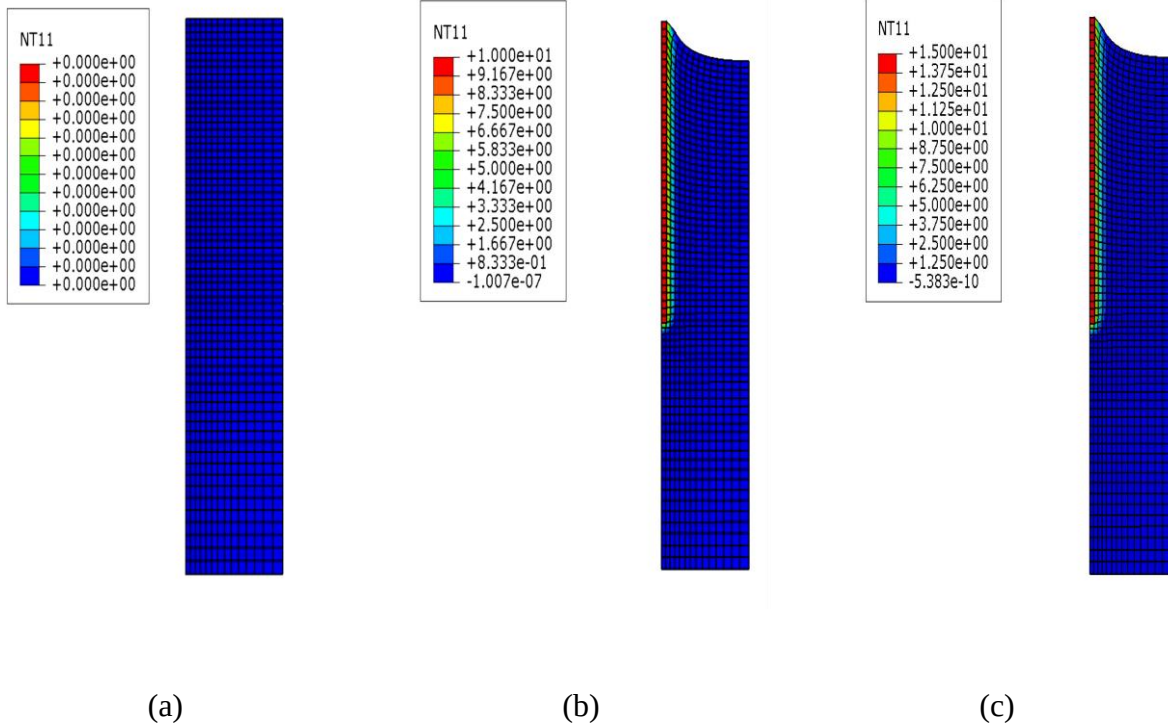


Figure 5.1. Temperature distribution in for a) $\Delta T = 0^\circ\text{C}$ b) $\Delta T = +10^\circ\text{C}$ and c) $\Delta T = +15^\circ\text{C}$ with a deformation scale factor of 1500 (NT stands for nodal temperatures in $^\circ\text{C}$)

Isotropic and anisotropic elastic soils

In soils, stiffness is often anisotropic due to the geological compression and sedimentation processes. This effect is known as horizontal bedding, and it results in soil having different stiffness in the vertical and horizontal directions. Anisotropic soils can have significant implications in geotechnical engineering, for example on the analysis and design of foundations, tunnels, and other underground structures.

By using an isotropic elastic model of soil Perić et al. (2017) found out from the laboratory test results (Laloui et al. 1999) that two layers (B and C) out of the four layers present in the soil profile at the Lausanne site (Laloui et al. 2006) have Poisson's ratio of 0.5. A subsequent consideration of cross anisotropic elastic soils B and C resulted from the value of Poisson's ratio for isotropic soils that fell out of the expected range. The final assessment of these two assumptions was made by comparing the predictions of isotropic and anisotropic elastic models with the actual measurements whereby the predictions of cross-anisotropic model provided values closer to those measured during the field test.

. Bower (2009) described the stress-strain relationship in cross-anisotropic or transversely isotropic material as

$$\begin{bmatrix} d\epsilon_{xx} \\ d\epsilon_{yy} \\ d\epsilon_{zz} \\ d\epsilon_{xz} \\ d\epsilon_{yz} \\ d\epsilon_{xy} \end{bmatrix} = \begin{bmatrix} 1/E'_h & -\nu'_{hh}/E'_h & -\nu'_{vh}/E'_v & 0 & 0 & 0 \\ -\nu'_{hh}/E'_h & 1/E'_h & -\nu'_{vh}/E'_v & 0 & 0 & 0 \\ -\nu'_{hv}/E'_h & -\nu'_{hv}/E'_h & 1/E'_v & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2G_{vh} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2G_{vh} & 0 \\ 0 & 0 & 0 & 0 & 0 & (1 + \nu'_{hh})/E'_h \end{bmatrix} \begin{bmatrix} d\sigma'_{xx} \\ d\sigma'_{yy} \\ d\sigma'_{zz} \\ d\sigma'_{xz} \\ d\sigma'_{yz} \\ d\sigma'_{xy} \end{bmatrix} \quad (5.1)$$

where

$$\frac{\nu'_{vh}}{E'_v} = \frac{\nu'_{hv}}{E'_h} \quad (5.2)$$

Thus, a transversely isotropic elastic material is characterized by five independent elastic parameters, namely the effective Young's modulus in the horizontal (E'_h) and vertical (E'_v)

directions, the effective Poisson's ratios relating normal strains in the horizontal plane (v'_{hh}) and in the vertical and horizontal planes (v'_{hv}), and the shear modulus describing the resistance to shear strains in vertical planes (G_{vh}). These elastic parameters can be determined by laboratory testing of differently oriented soil samples. Graham and Houlsby (1983) offered a simplified form of cross-anisotropy that can be employed if test results from differently oriented samples are not available. Their model requires only three independent elastic parameters that can be determined by testing vertically oriented samples exclusively. The corresponding constitutive relationship is given by

$$\begin{bmatrix} d\varepsilon_{xx} \\ d\varepsilon_{yy} \\ d\varepsilon_{zz} \\ d\varepsilon_{xz} \\ d\varepsilon_{yz} \\ d\varepsilon_{xy} \end{bmatrix} = \frac{1}{E^*} \begin{bmatrix} 1/\alpha_s^2 & -v^*/\alpha_s^2 & -v^*/\alpha_s & 0 & 0 & 0 \\ -v^*/\alpha_s^2 & 1/\alpha_s^2 & -v^*/\alpha_s & 0 & 0 & 0 \\ -v^*/\alpha_s & -v^*/\alpha_s & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1+v^*)/\alpha_s & 0 & 0 \\ 0 & 0 & 0 & 0 & (1+v^*)/\alpha_s & 0 \\ 0 & 0 & 0 & 0 & 0 & (1+v^*)/\alpha_s \end{bmatrix} \begin{bmatrix} d\sigma'_{xx} \\ d\sigma'_{yy} \\ d\sigma'_{zz} \\ d\sigma'_{xz} \\ d\sigma'_{yz} \\ d\sigma'_{xy} \end{bmatrix} \quad (5.3)$$

where α_s , represents the degree of anisotropy and is defined as :

$$\alpha_s = \frac{v_{hh}}{v_{vh}} \text{ and } v^* = v_{hh} \quad (5.4)$$

where

$$\alpha_s^2 = \frac{E'_h}{E'_v} \text{ and } E^* = E'_v \quad (5.5)$$

Thus, the parameters of the transversely isotropic model in terms of the simplified model proposed by Graham and Houlsby (1983) can be derived by comparing Eqs. (5.1) and (5.3) as

$$E'_h = \alpha_s^2 E^* = \alpha_s^2 E'_v \quad (5.6)$$

$$G_{vh} = \frac{\alpha_s E^*}{2(1+v^*)} = \frac{\alpha_s E'_v}{2(1+v'_{hh})} \quad (5.7)$$

$$G_{hh} = \frac{\alpha_s^2 E^*}{2(1+v^*)} = \frac{\alpha_s^2 E'_v}{2(1+v'_{hh})} = \frac{E'_h}{2(1+v'_{hh})} \quad (5.8)$$

$$G_{hh} = \alpha_s G_{vh} \quad (5.9)$$

$$v'_{vh} = \frac{v'_{hh}}{\alpha_s} \text{ and } v'_{hv} = \alpha_s v'_{hh} \quad (5.10)$$

Therefore, three parameters, namely E^* , ν^* , and α_s must be determined from the results of triaxial tests. When $\alpha = 1$ the isotropic model is obtained, thus leading to the elastic stress-strain relationship leading to

$$E'_h = E'_v = E \text{ and } \nu^* = \nu'_{hh} = \nu'_{hv} = \nu'_{vh} = \nu' \quad (5.11)$$

Figure 5.2 depicts the thermal axial displacement, strain, and stress of an energy pile embedded in a homogenous soil layer with varying degrees of anisotropy. The elastic modulus of the soil is 90 MPa while its effective Poisson's ratio (ν'_{vh}) is 0.3. The rest of the parameters were adopted from the energy pile constructed and tested at the Laussane site (Laloui et al. 2006 and Perić et al. 2017). Specifically, the coefficients of thermal expansion, heat capacity, and thermal conductivity are $1 \times 10^{-5}/^\circ\text{C}$, $2.4 \times 10^6 \text{ J}/(\text{m}^3^\circ\text{C})$, and $1.8 \text{ W}/(\text{m}^\circ\text{C})$ for soil and $1 \times 10^{-6}/^\circ\text{C}$, $2.0 \times 10^6 \text{ J}/(\text{m}^3^\circ\text{C})$, and $1.1 \text{ W}/(\text{m}^\circ\text{C})$, for bedrock. In addition, the effective elastic modulus of the bedrock is equal to 3864 MPa while its effective Poisson ratio is equal to 0.153. The hydraulic conductivities of the soil layer and bedrock are equal to $2 \times 10^{-6} \text{ m/s}$ and $1 \times 10^{-10} \text{ m/s}$, respectively. As shown in Figure 5.2 the energy pile embedded in an anisotropic soil experiences larger magnitude of head displacement and axial strain along with a smaller magnitude of axial stress. The trend is more pronounced with increased degree of anisotropy. The maximum difference among displacements occurs at the pile head, while the maximum differences among the strains and stresses occur near the null point. In this particular case, the maximum magnitude of axial stress decreases nearly two-fold for the maximum degree of anisotropy considered herein as compared to the pile embedded into the isotropic soil.

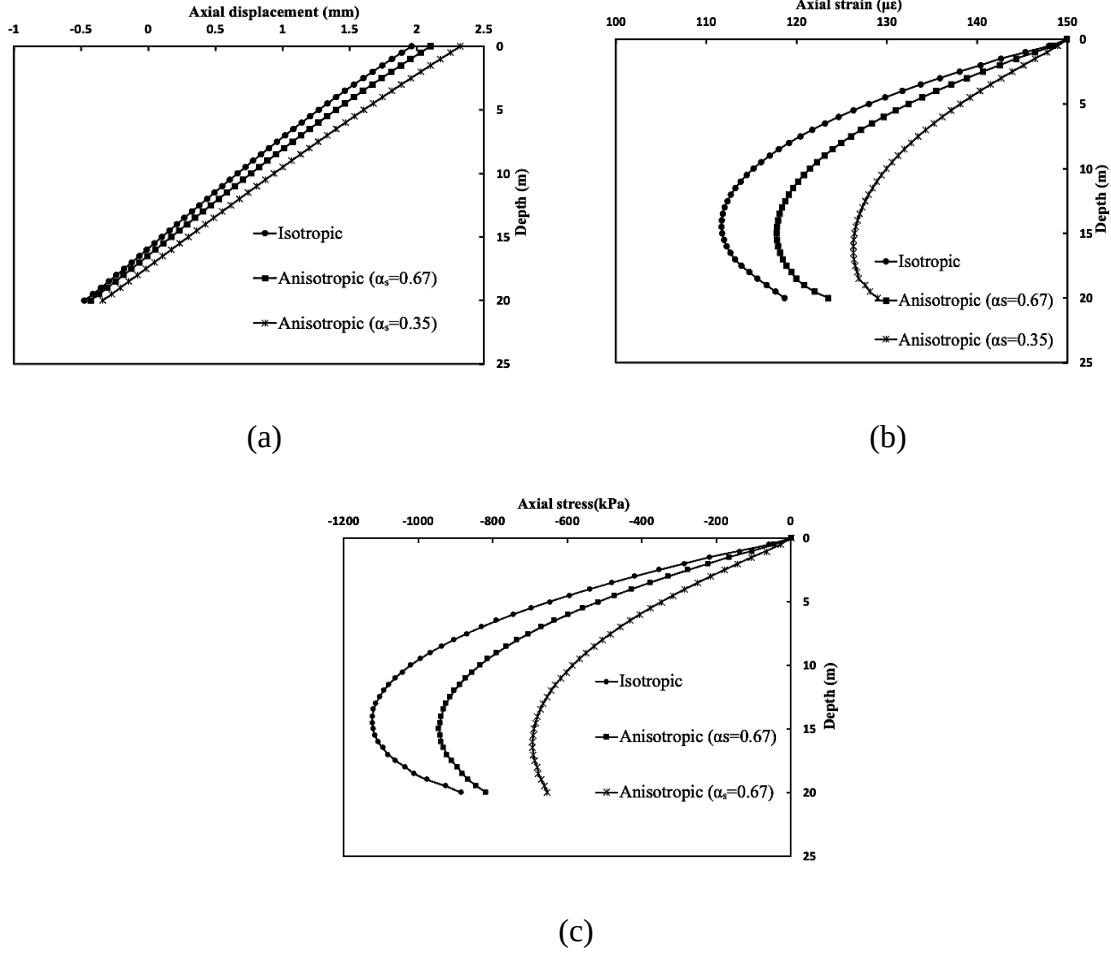


Figure 5.2. The effect of soil anisotropy, based on the computational model, on (a) displacement; (b) strain; and (c) stress for the thermal load ($\Delta T = +15^\circ\text{C}$)

Statistical Analysis

The statistical analysis examined various configurations by altering the soil stiffness (E'_v and ν'_{vh}), pile length to diameter ratio (L/D), stiffness of the bedrock or underlying layer, and a degree of anisotropy of the soil layer (α_s). A total of 432 computational analyses were conducted using different values of $L/D = 20, 30, 40$, of elastic modulus of soil $E_v = 15, 45, 90$, and 180 MPa, and of Poisson's ratio of soil $\nu_{vh} = 0.2, 0.3$, and 0.4 . The soil anisotropy is taken into consideration by setting $\alpha_s = 0.67$ and $\alpha_s = 0.35$ in addition to isotropic analysis ($\alpha_s = 1$). The rest of the parameters in the computational model were obtained from the test site at the Swiss Federal Institute of Technology in Lausanne, Switzerland (Laloui et al. 2006). To this end, the soil layer has

coefficients of thermal expansion, heat capacity, and thermal conductivity equal to $1 \times 10^{-5}/^{\circ}\text{C}$, $2.4 \times 10^6 \text{ J}/(\text{m}^3^{\circ}\text{C})$, and $1.8 \text{ W}/(\text{m}^{\circ}\text{C})$, respectively. The bedrock material has coefficients of thermal expansion, heat capacity, and thermal conductivity equal to $1 \times 10^{-6}/^{\circ}\text{C}$, $2.0 \times 10^6 \text{ J}/(\text{m}^3^{\circ}\text{C})$, and $1.1 \text{ W}/(\text{m}^{\circ}\text{C})$, respectively. In addition, the Poisson's ratio of the bedrock is equal to 0.153. The hydraulic conductivities of the soil layer and bedrock are equal to $2 \times 10^{-6} \text{ m/s}$ and $1 \times 10^{-10} \text{ m/s}$, respectively.

A similar procedure was performed by using the analytical solutions that Saeidi Rashk Olia and Perić (2021a) developed. In this case pile length to diameter ratio (L/D) values of 20, 30, and 40 were selected. In addition, the stiffness of the shear spring (k_s) varied from 5 to 180 MPa/m and the stiffness of the normal spring at the pile tip (k_b) varied from 0 to 50,000 MPa/m, thus replicating an ideally fully floating energy pile and nearly ideally end bearing energy pile. These analyses provided 2424 data points that were used as the input into statistical analyses. The elastic modulus and coefficient of thermal expansion of the pile are equal to 29.2 GPa and $1 \times 10^{-5}/^{\circ}\text{C}$, in both computational and analytical predictions whereby both values correspond to the characteristics of the energy pile at the Lausanne site (Laloui et al. 2006). The results of both analytical and computational analyses were recorded at 15°C of heating.

Multiple Linear Regression Analysis

A series of multiple linear regression analyses using Statistical Analysis System (SAS) was performed on the data obtained from analytical and computational models to find the desired correlations. To this end, the maximum magnitudes of axial stresses in both approaches were expressed as follows

$$\text{Computational approach: } \sigma_{max} = \sigma_{max} \left(E_v, E_B, \frac{L}{D}, \alpha_s, \nu_{vh} \right) \quad (5.12)$$

$$\text{Analytical approach: } \sigma_{max} = \sigma_{max} \left(k_s, k_b, \frac{L}{D} \right) \quad (5.13)$$

Hence, $E_v, E_B, \frac{L}{D}, \alpha_s$, and ν_{vh} , and k_s, k_b and $\frac{L}{D}$ are independent variables in statistical datasets respectively. Given the inherent physical independence of the variables in the two analyses, multiple forward variable selection analyses were conducted on various combinations of independent parameters, including their higher powers, and roots to investigate the best correlation between the maximum magnitude of stress and various forms of the selected variables. The analysis suggested that the best correlation of the maximum magnitude of stress has a positive correlation with $\sqrt{\frac{L}{D}}, E_v, \alpha_s$ and E_B , while it has a negative correlation with the soil Poisson's ratio (ν_{vh}). The selection method also indicated that the maximum magnitude of stress has the highest correlation with $k_s, k_b, \sqrt{\frac{L}{D}}$ in case of the analytical solutions.

To accomplish the goal of finding the correlation between the soil stiffness parameters required by finite element and analytical models,, the combination of the two variables of soil elastic modulus (E_{vh}) and Poisson's ratio (ν_{hh}) was introduced into the statistical analysis. Moreover, as the elastic modulus of the energy pile (E_p) does not vary nearly as much as the elastic modulus of soil its value was fixed in the finite element analyses. Nevertheless, it was used for the normalization of the soil elastic constants in the final selection of the variables. Thus, the final selection of the variables in the regression analysis resulted in

$$\text{Computational approach: } \sigma_{max} = \sigma_{max} \left(\frac{\left[\frac{E_v}{2(1 + \nu_{hh})} \right] \sqrt{\frac{L}{D}}}{E_p}, E_B, \alpha_s, \right) \quad (5.14)$$

$$\text{Analytical approach: } \sigma_{max} = \sigma_{max} \left(\frac{k_s \sqrt{\frac{L}{D}}}{E_p}, k_b \right) \quad (5.15)$$

A multiple linear regression analysis based on the variables given in Eqs. (5.14) and (5.15) resulted in the following the correlations.

$$\text{Computational: } \sigma_{max} (kPa) = -103,799 \frac{\left[\frac{E_v(MPa)}{2(1+\nu_{hh})} \right] \sqrt{\frac{L}{D}}}{E_p(MPa)} - 0.00464 E_B(MPa) - 569.21(\alpha_s) \quad (5.16)$$

$$R^2 = 0.9085$$

$$P_{\text{value}} < 0.0001, \alpha = 0.01$$

$$\text{Analytical: } \sigma_{max} (kPa) = -110.00849 \frac{k_s \left(\frac{kPa}{m} \right) \sqrt{\frac{L}{D}}}{E_p(MPa)} - 0.00004183 k_b \left(\frac{kPa}{m} \right) \quad (5.17)$$

$$R^2 = 0.9300$$

$$P_{\text{value}} < 0.0001, \alpha = 0.01$$

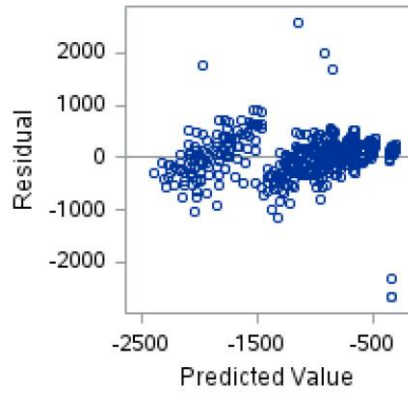
Residuals of a regression model, are defined as the observed values of the dependent variable minus the values of the dependent variable predicted by the regression model (Mendenhall et al. 2003), in this case σ_{max} , are given by

$$\hat{\varepsilon}_i = y_i - \hat{y}_i \quad (5.18)$$

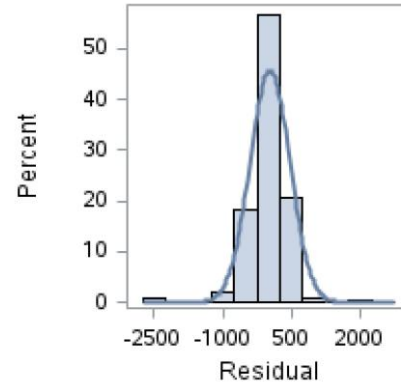
where $\hat{\varepsilon}_i$ represents the residual, y_i represents the observed dependent variable and $\hat{y}_i$ represents the value of the dependent variable predicted by the regression model that corresponds to the i^{th} observation. One of the assumptions of a regression model is that the mean probability distribution of residuals is equal to zero (Mendenhall et al. 2003).

$$\frac{1}{n} \sum_{i=1}^n \hat{\varepsilon}_i = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i) = 0 \quad (5.19)$$

As shown in Figure 5.3a and Figure 5.4a, the residuals have mean values of 0 for regression models presented in (5.16) and (5.17). Additionally, regression models are based on the assumption that the probability distribution of ε_i is normal, and that is demonstrated in Figure 5.3b and Figure 5.4b, thus proving the validity of the assumptions.

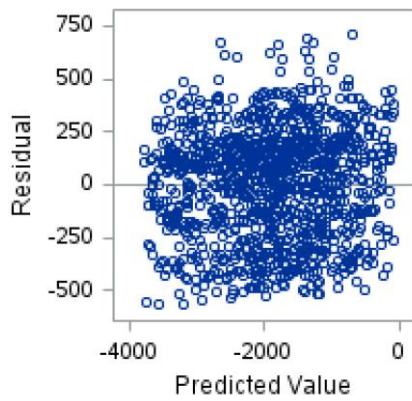


(a)

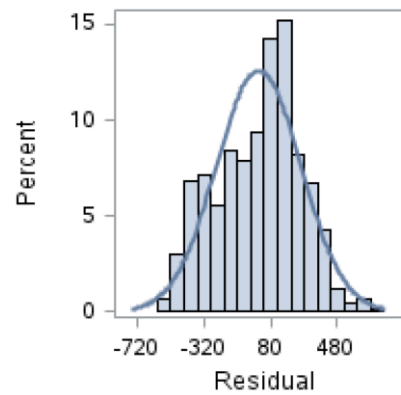


(b)

Figure 5.3. a) Residuals vs predicted values and b) distribution of residuals of the regression analysis of computational data (5.16)



(a)



(b)

Figure 5.4. a) Residuals vs predicted values and b) distribution of residuals of the regression analysis of analytical data (Eq. (5.17))

The results of the predicted versus observed maximum magnitudes of axial stress along the energy pile shaft are shown in Figure 5.5a, and b for the regression models applied to data obtained from the computational analysis and analytical solutions respectively.

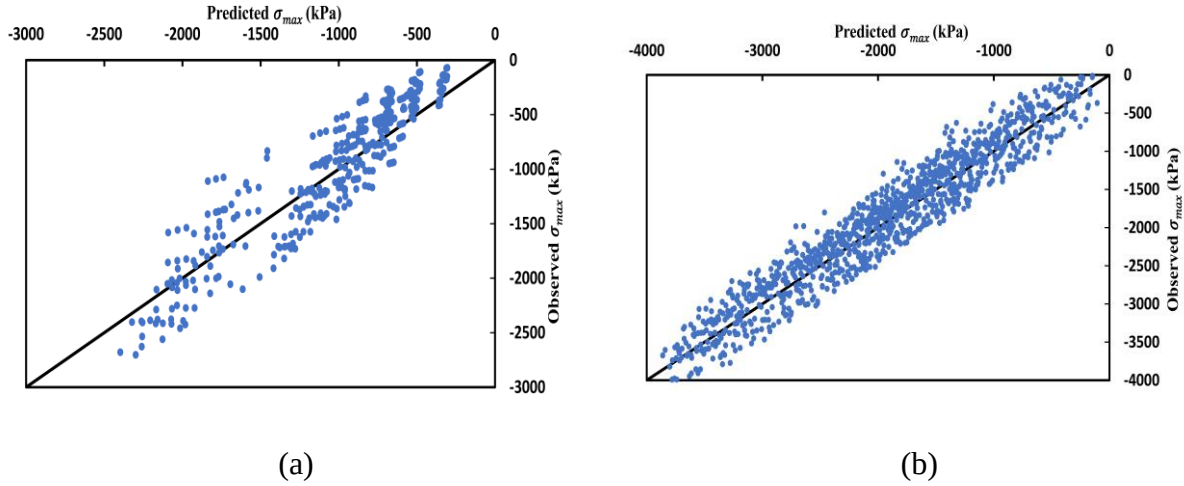


Figure 5.5. Predicted versus actual (observed) values of maximum axial stress along the pile a) computational regression model Eq. ((5.16)), b) analytical regression model Eq. ((5.17))

Figure 5.6 depicts the effect of soil anisotropy on the maximum magnitude of axial stress along the pile shaft for various values of elastic modulus, E_v based on Eq. (5.17). As shown a decrease in α_s leads to a decrease in the maximum magnitude of axial stress along the pile shaft. This agrees with the findings of Perić et al. (2017) based on the comparison of axial stresses obtained in the energy piles embedded in cross-anisotropic and isotropic soils.

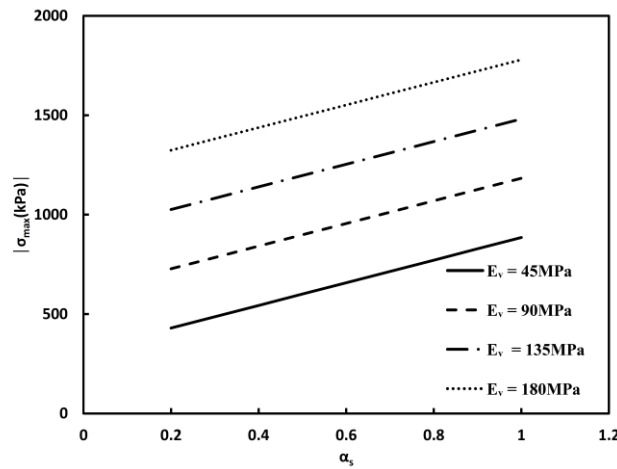


Figure 5.6. The effect of soil anisotropy on the magnitude of maximum axial stress based on Eq. (5.17)

In order to find a correlation between the elastic soil parameters required by the computational and analytical models the corresponding maximum magnitudes of axial stress along the pile are equalized, thus resulting in in Eq. (5.20). Consequently, the stiffness of the shear spring has a positive correlation with the value of soil anisotropy (α_s). Also, k_s shows a positive correlation with the soil elastic modulus and a negative correlation with Poisson's ratio that in the case of isotropic soil becomes a direct correlation between k_s and the shear modulus of the soil.

$$k_s \left(\frac{kPa}{m} \right) = 943.554 \left[\frac{E_v(MPa)}{2(1+\nu_{hh})} \right] + 5.1742 \left(\frac{\alpha_s E_P(MPa)}{\sqrt{\frac{L}{D}}} \right) + 4.217 \times 10^{-5} \frac{E_B(MPa) E_P(MPa)}{\sqrt{\frac{L}{D}}} - 3.8024 \times 10^{-7} \frac{k_b \left(\frac{kPa}{m} \right) E_P(MPa)}{\sqrt{L/D}} \quad (5.20)$$

Predictions of full-scale experimental data

The correlation provided for stiffness of the shear spring (k_s) was implemented into analytical solutions provided by Saeidi Rashk Olia and Perić (2021a) to predict the response of the full-scale tests that were conducted on energy piles constructed at Lambeth College in London (Bourne-Webb et al. 2009). The energy pile, referred to as the main pile, is 23 m long, and its strains were measured with vibrating wire strain gauges (VWSG) and optical fiber sensors (OFS (Bourne-Webb et al. 2009). A loading frame was used at the site to apply a mechanical load on the pile head of the main pile, while a heat sink pile was solely used for heat exchange purposes. This implies that cooling of the main pile causes heating of the sink pile. The pile was initially loaded and unloaded to 1800 kN mechanically, which led to recorded residual strain after unloading of the pile, thus indicating an irreversible soil-pile interface behavior. The pile was subsequently loaded to 1200kN. The reversible experimental response of the pile was obtained by deducting the residual strain along the pile shaft from the strains recorded at 1200kN during subsequent loading.

The analytical solutions obtained by Saeidi Rashk Olia and Perić (2023a) were used to verify that the reloading to 1200 kN corresponded to an entirely reversible interface response. The pile has a diameter of 0.61 m throughout the first 5 m of depth due to the casing that was required within the made ground and terrace deposits. Below the depth of 5 m it has a diameter of 0.55 m and it is surrounded by London clay. Bourne-Webb et al. (2009) reported an elastic modulus of 40 GPa and a coefficient of thermal expansion of $8.5 \times 10^{-6} / ^\circ\text{C}$ for both piles. Bourne Webb et al. (2009) also reported the strain at the pile tip to be almost zero, thus indicating that the main energy pile and the heat sink pile both acted as fully floating piles corresponding to $k_b = 0$.

Arzanfudi et al. (2020) reported Young's modulus and Poisson's ratio of the London clay to be 70MPa, and Poisson's ratio of 0.3. Approximating the cross-section of the pile as a uniform cross-section with a diameter of 0.55 m and embedding the pile into a homogeneous soil profile results in $k_s = 52.667 \text{ MPa/m}$ based on Eq.

(5.20). Frank and Zhao (1982) provided an empirical correlation between the stiffness of the elastic shear spring (k_s) for fine-grained soils and weak rocks including clays, to the Menard pressuremeter modulus (E_M) through Eq. (5.21)

$$k_s = \frac{2E_M}{D} \quad (5.21)$$

Knellwolf et al. (2011) reported E_M of 45 MPa for the London clay at the Lambeth College site, thus resulting in $k_s = 163.63 \text{ MPa/m}$. Figure 5.7 shows the analytical predictions of axial strain in the pile subjected to 1200kN obtained by using values of k_s obtained from the statistical correlation developed herein (Eq.5.20) and the empirical correlation proposed by Frank and Zhao (1982) in Eq. (5.21). While both methods provide acceptable predictions, it is shown that the statistical correlation developed herein provides a better agreement with the experimental data. It is noted that the deviations of the predictions from the experimentally observed data within the top

5 m are due to omitting the presence of the made ground and terrace layers in the employed analytical model. Furthermore, the characteristics of these layers are not available.

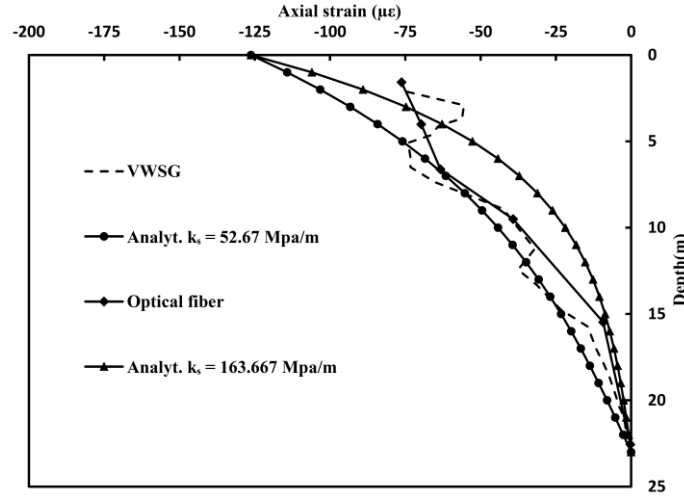


Figure 5.7. Predictions of axial strain in the full-scale Lambeth College main energy pile under 1200kN of compressive mechanical load

To better understand how the soil anisotropy affects analytical solutions, a presence of a synthetic anisotropic soil was also included. To this end, the values of $\alpha_s = 0.67$ & 0.35 were selected, thus leading to the k_s values of to be 42.104 MPa/m and 31.861 MPa/m respectively. The predictions for displacement, stress, and strain under $\Delta T = 15^\circ\text{C}$ is depicted in Figure 5.8. As expected, the decreased value of α_s lead to smaller magnitude of thermal stress while it increased the magnitude of the corresponding thermal strain and displacement. A similar effect was also observed from the results of computational models shown in Figure 5.2. Thus, the correlation provided in Eq. (5.20) can account for the effect of soil anisotropy on the analytical solutions.

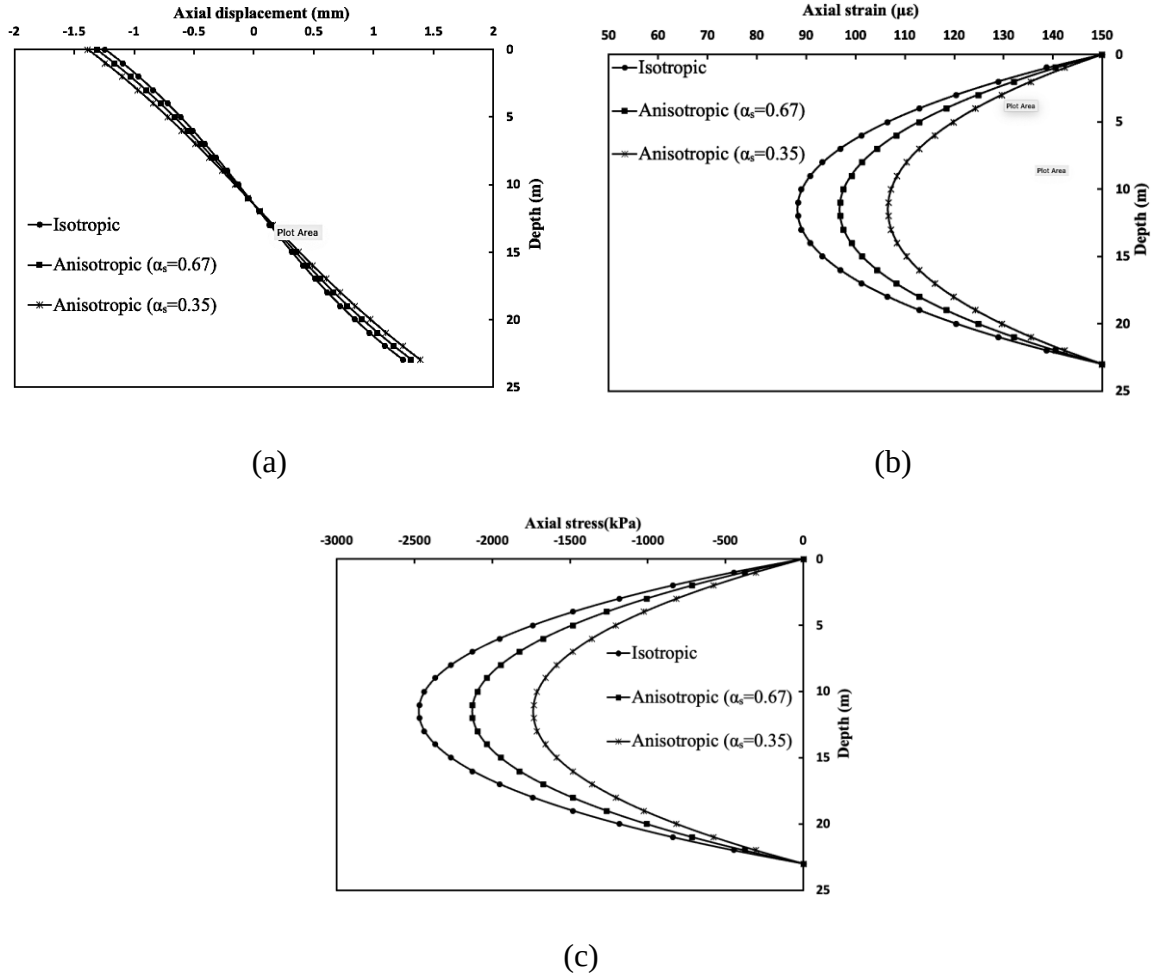


Figure 5.8. The effect of soil anisotropy reflected in analytical solutions on (a) displacement; (b) strain; and (c) stress for the thermal load ($\Delta T = +15^\circ\text{C}$)

Cross-validation of FEM and analytical predictions

Predictions of axial displacement, strain, and stress in the energy pile obtained from the analytical and computational models are presented for the purpose of cross-validation. In this section, one isotropic one anisotropic soil was chosen from the Lausanne site to construct 20-m synthetic homogeneous soil profiles resting on the bedrock. Parameters of soils A1 and B representing the isotropic and anisotropic soils respectively were taken from the results of independent model calibrations conducted by Perić et al. (2017). They are provided in Table 5.1 while the properties of the pile are provided in Table 5.2

Table 5.1. Properties of soil and bedrock (Perić et al. 2017)

Material	E_v (MPa)	ν_{hh}	α_s	Mass density, ρ (kg/m ³)	Coefficient of thermal expansion, β (°C ⁻¹)	Heat Capacity C (J/m ³ °C)	Thermal conductivity λ (W/m/°C)
Soil A1	190	0.22	1	2000	1×10^{-5}	2.4×10^6	1.8
Soil B	56.06	0.0524	0.4	2000	1×10^{-4}	2.4×10^6	1.8
Bedrock	3864	0.153	1	2550	1×10^{-6}	2×10^6	1.1

Table 5.2. Properties of reinforced concrete (Laloui et al. 2006)

Property	E_p (MPa)	Poisson's ratio (ν)	Mass density, ρ_p (kg/m ³)	Coefficient of thermal expansion, β_p (°C ⁻¹)	Heat Capacity C_p (J/m ³ °C)	Thermal conductivity λ_p (W/m/°C)	L/D
	29200	0.177	2500	1×10^{-5}	2.4×10^6	2.1	20

The maximum stiffness of the normal spring at the pile tip (k_b) equal to 1335. 4 MPa/m provided for the Lausanne site (Knellwolf et al. 2011) was used herein The values of k_s obtained from Eq. (5.20) for the isotropic soil (A1) and anisotropic soil (B) are 93.78 MPa/m and 29.61 MPa/m, respectively. FEM and analytical predictions of the response of the energy pile heated to 15°C and embedded into soil A1 are shown in Figure 5.9. The analytical solutions captured the trend of the predictions for stress and strain and displacement successfully. While the magnitude of displacement predicted by FEM is slightly higher than the one predicted by the analytical solutions this difference decreases throughout the depth and disappears at the pile tip.

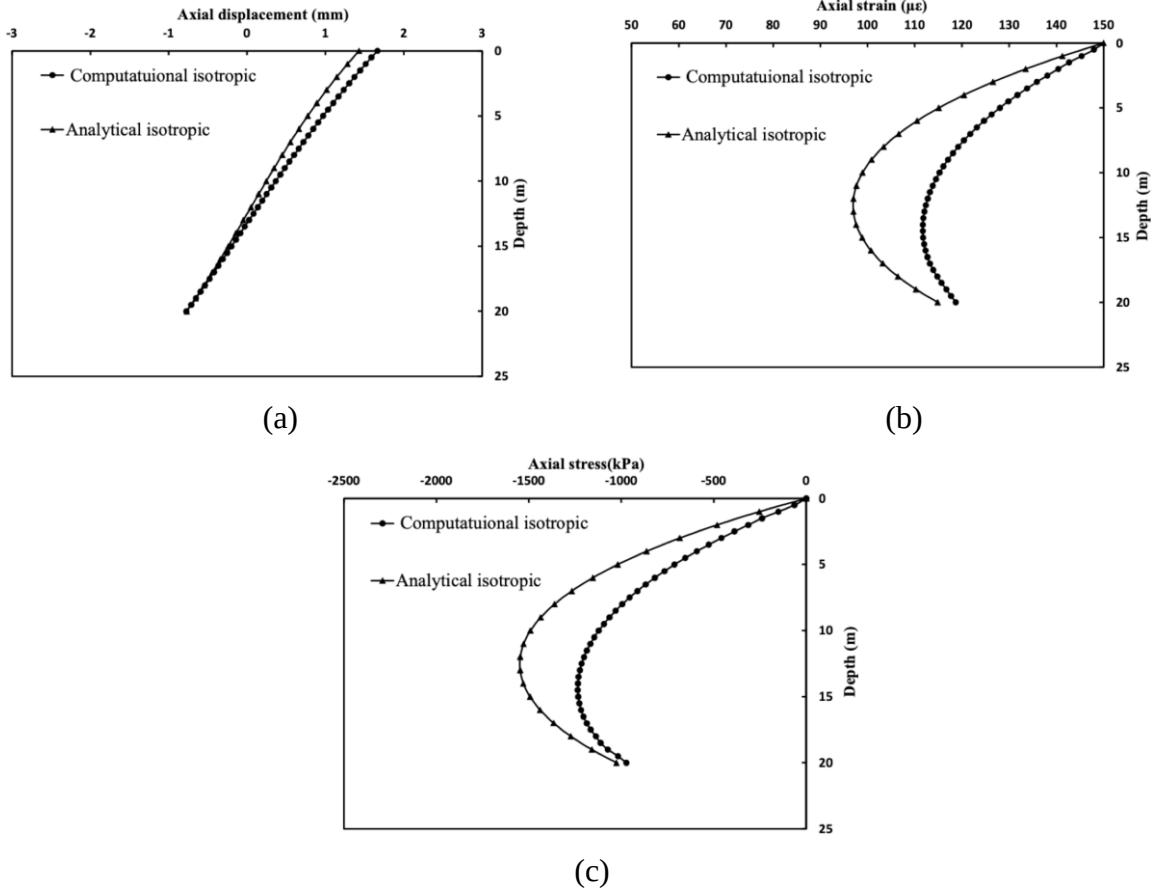


Figure 5.9. Comparison of computational and analytical predictions for energy pile subjected to $\Delta T = +15^\circ\text{C}$ embedded into isotropic soil (A1) a) displacement b) strain and c) stress

Figure 5.10 shows the FEM response of energy pile in an anisotropic soil profile (Soil B), and the analytical counterpart by using the k_s value calculated from Eq. (5.20) for a degree of anisotropy equal to $\alpha_s = 0.4$. Consequently, the two methods provide close predictions with analytical solutions again resulting in smaller magnitude of displacement and strain, but larger magnitude of stress as compared with finite element prediction. Smaller magnitude of displacement in analytical solutions result in the zero-displacement location known as a null point being located at a lower depth than the one obtained from FEM results. This causes the location of the maximum magnitudes of stress and strain predicted by analytical solutions to develop at lower depths than those predicted by the computational model.

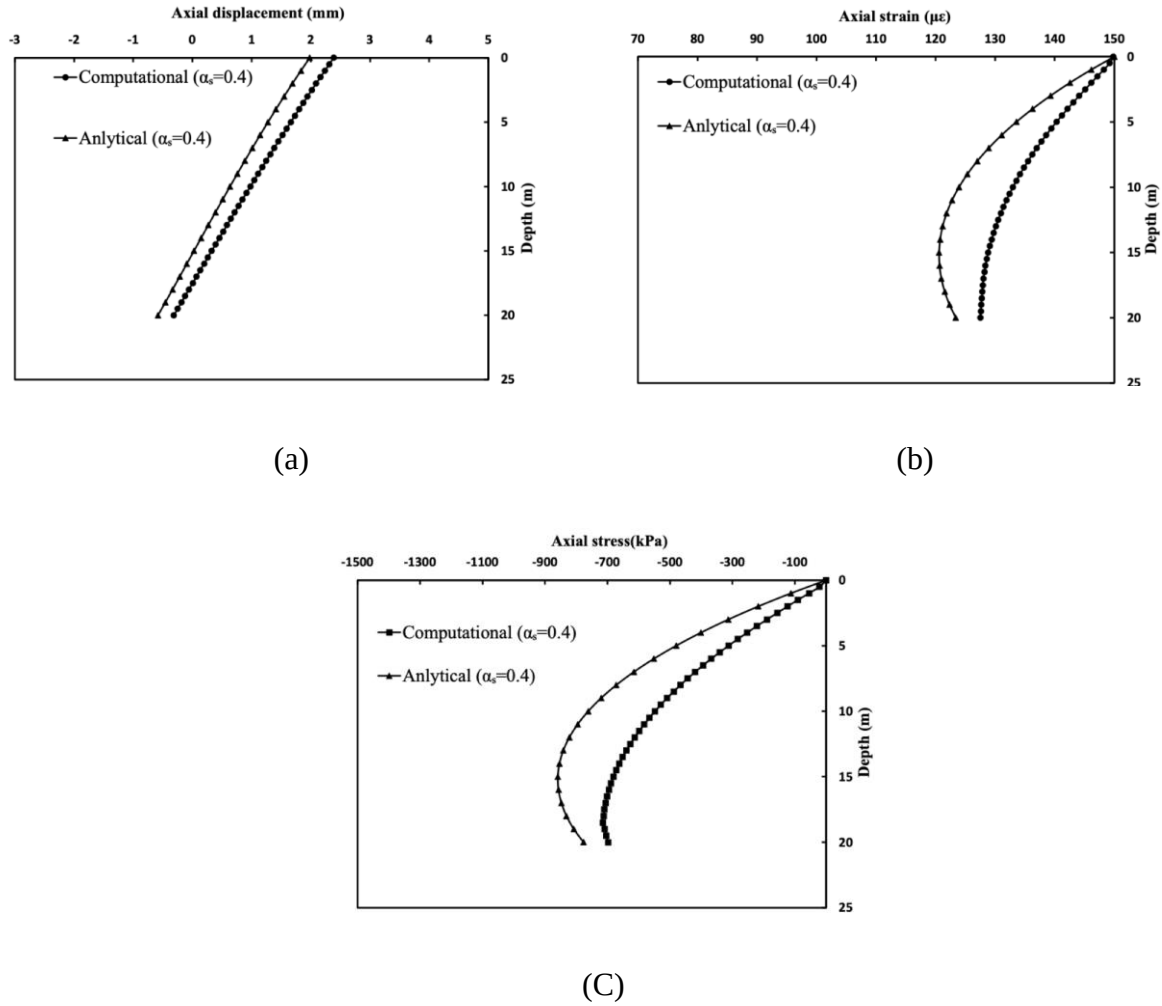


Figure 5.10. Comparison of computational and analytical predictions for energy pile subjected to $\Delta T = +15^\circ\text{C}$ embedded in anisotropic soil (B) of a) displacement b) strain and c) stress

Conclusions

In this study, the effect of soil stiffness anisotropy on the response of single energy piles was investigated through computational, analytical, and statistical modeling. Soil anisotropy was modeled by the cross anisotropic elastic soil model in FE analysis, whereby the results showed that for considered range of anisotropy a higher amount of anisotropy resulted in a smaller magnitude of axial stress developed along the pile shaft and larger magnitude of displacement.

Multiple linear regression analyses provided the correlations between the maximum magnitude of thermal axial stress developed along the pile shaft, and the correlation between the elastic parameters of computational and analytical models. A positive correlation between the maximum magnitude of stress and a degree of anisotropy was also found.

A correlation between the stiffness of the shear spring (k_s) used in analytical solutions and the continuum elastic parameters employed by FEM was obtained by equalizing the maximum magnitudes of axial stresses obtained from FE analysis and analytical predictions. Analytical solutions obtained by using the value of k_s calculated from the derived statistical correlation successfully predicted the reversible response of the full-scale energy pile test conducted at Lambeth College, London. In addition, the analytical solutions were successfully cross validated against the computational predictions for thermal loading of the energy piles embedded in the homogeneous isotropic and anisotropic profiles whereby the elastic stiffness parameters required by the analytical model were obtained from the statistical correlations developed in this study.

Chapter 6 - Conclusions and Recommendations

Within this research, mechanics-based analytical solutions for soil-structure interaction in energy piles exhibiting reversible and irreversible interface behavior have been derived. The solutions were subsequently validated against the results of full-scale and geotechnical centrifuge tests. Consequently, they provide a rational basis for load transfer diagrams that were prepared and presented, thus aiding in analysis and design of energy piles experiencing a variety of loading scenarios. Furthermore, combining fully coupled thermo-hydro-mechanical finite element analyses, analytical solutions, and advanced statistical analyses ultimately lead to the correlation between the elastic parameters employed by finite element model and analytical solutions, for isotropic and cross-anisotropic soils. Additionally,

Multiple linear regression models were developed for the maximum axial stresses obtained by use of finite element models and analytical solutions.

Consequently, the results of this study directly contribute to the advancement of sustainable, eco-friendly deep energy foundations that support structural loads while facilitating the exchange of renewable thermal energy between the subsurface and superstructure. The major findings of this study are discussed below.

The analytical expressions for the exact location of the zero-displacement point known as the null point were obtained. It was shown that the null point corresponds to the location of the maximum magnitude of the axial stress along the pile shaft. Furthermore, the analytical solutions for the location of the null point in piles exhibiting irreversible interface response were derived for the first time. The solutions show that unlike in the piles with reversible interface behavior the location of the null point changes after the onset of the interface yielding. Load transfer diagrams were created based on the analytical solutions, thus offering qualitative and quantitative

characterizations of displacement, strain, axial and shear stress evolutions with depth. The load transfer diagrams provided in this study have overcome the limitations of previous load transfer diagrams provided in that literature that were largely based on the observed trends of experimental data. The effect of head and tip restraints on developed stresses along the pile shaft, and progression of interface yielding culminating in the development of constant shear stress were further elucidated with the aid of the presented load transfer diagrams. Furthermore, the solutions accounting for interface yielding have led to the in depth understanding of development of a residual displacement, strain, and axial and interface shear stresses in a thermo-elastic energy pile after the inception of the interface yielding. The outcomes of this research filled the long-standing gap of missing analytical solutions for displacement, strain, axial and interface shear stresses in energy piles exhibiting reversible and irreversible interface behavior.

Finally, the combined use of finite element modeling, analytical solutions and advanced statistical methods provided multiple linear regression models for maximum axial stress in finite element models and analytical predictions. Equalizing the maximum stresses obtained by both approaches enabled expressing stiffness parameters of analytical solutions in terms of elastic stiffness parameters of finite element models for both, isotropic and cross-anisotropic models.

Based on the findings of this research, it is recommended that future studies focus on expanding the correlations between the parameters of finite element model and analytical solutions for irreversible interface behavior. By establishing these correlations, future researchers can gain better understanding of the behavior of irreversible interface and develop more accurate analytical and numerical models for predicting the response of energy piles under different loading conditions. This will ultimately improve the design and performance of these geo-structures and contribute to the sustainable development of infrastructure systems.

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