

AN ANALYSIS OF A HORIZONTALLY PROJECTED CHILLED JET
SUBJECTED TO VERTICAL CROSSFLOW

by

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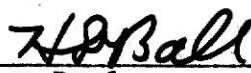
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NOMENCLATURE

Symbol	Significance	Units
A_i	An experimental constant	
B	The Guassian profile shape factor	$1/\text{ft}^2$
C_i	Cosine θ_i	
C_ϕ	Cosine ϕ	
C_v	The specific heat of air	$\text{BTU}/\text{s}^\circ\text{F}$
D	A constant of integration	
E	The entrainment function	ft^2/s
P	Pressure	lbf/ft^2
S_i	Sine θ_i	
S_ϕ	Sine ϕ	
ΔT_M	The difference between the centerline (minimum) temperature and the ambient temperature	$^\circ\text{F}$
U_∞	The free stream crossflow velocity	f/s
ΔU	The maximum (centerline) difference between free stream and jet velocity	f/s
V	The velocity vector	f/s
X	The characteristic acceleration due to the body force field	f/s^2
g	Acceleration due to gravity	f/s^2
i, j, k	The Cartesian coordinate unit vectors	
i_r, i_s, i_ϕ	The natural curvilinear coordinate unit vectors	
k	Thermal conductivity	$\text{BTU}/\text{sec-ft-}^\circ\text{F}$
r, s, ϕ	The natural curvilinear coordinates	
t	Temperature	

NOMENCLATURE (Continued)

Symbol	Significance	Units
t_{∞}	Ambient temperature	$^{\circ}\text{F}$
u, v, w	Velocity components in the Cartesian directions	ft/s
x, y, z	The Cartesian coordinates	
β	Coefficient of expansion for air	$1/^{\circ}\text{F}$
λ	Proportionality constant	
ν	Kinematic viscosity	ft^2/s
ρ	Density	sl/ft^3
ρ_0	Reference density	sl/ft^3
ρ_{∞}	Ambient density	sl/ft^3
ρ_i	Centerline radius of curvature ($1/(d\theta_i/ds)$)	
τ	Time	sec
ϕ	Dissipative effects	

CHAPTER I. INTRODUCTION

In the past several decades a great deal of work has been devoted to man's comfort in his home and work environments. In the last few years spiraling energy costs have caused an in depth scrutiny to be undertaken of the manner in which this comfort is achieved. It is obvious from this scrutiny that in certain applications total volume air conditioning is impractical because of the enormous energy costs involved.

An example of an impractical application of total volume air conditioning might be a factory with large equipment filled spaces with high ceilings. The energy cost to overcome the heating load of many large industrial machines as well as the heat gains due to human occupancy, infiltration and conduction through exterior walls could well be staggering. As a possible alternative the spot cooling of worker stations deserves considerable investigation.

Spot cooling of an individual worker station will probably best be achieved by orienting a diffuser (air conditioning outlet) in the vicinity of the worker station and directing a chilled jet from this diffuser to achieve the desired cooling effect. The process of locating a diffuser is complicated first by consideration of the conditions necessary to achieve worker comfort and second by incomplete knowledge of the behavior of the resulting jet in the presence of possible disturbances.

In considering horizontally projected jets a possible disturbance to the behavior of the jet might be vertical crossflow. This crossflow might be either approximately uniform along the entire trajectory of the jet or it might be restricted to one or more small locations along the trajectory. Figure 1 illustrates the possible origin of uniform vertical crossflow. In the figure strong sunlight heats an open expanse of floor beneath the jet resulting in a vertical crossflow that is approximately uniform along the

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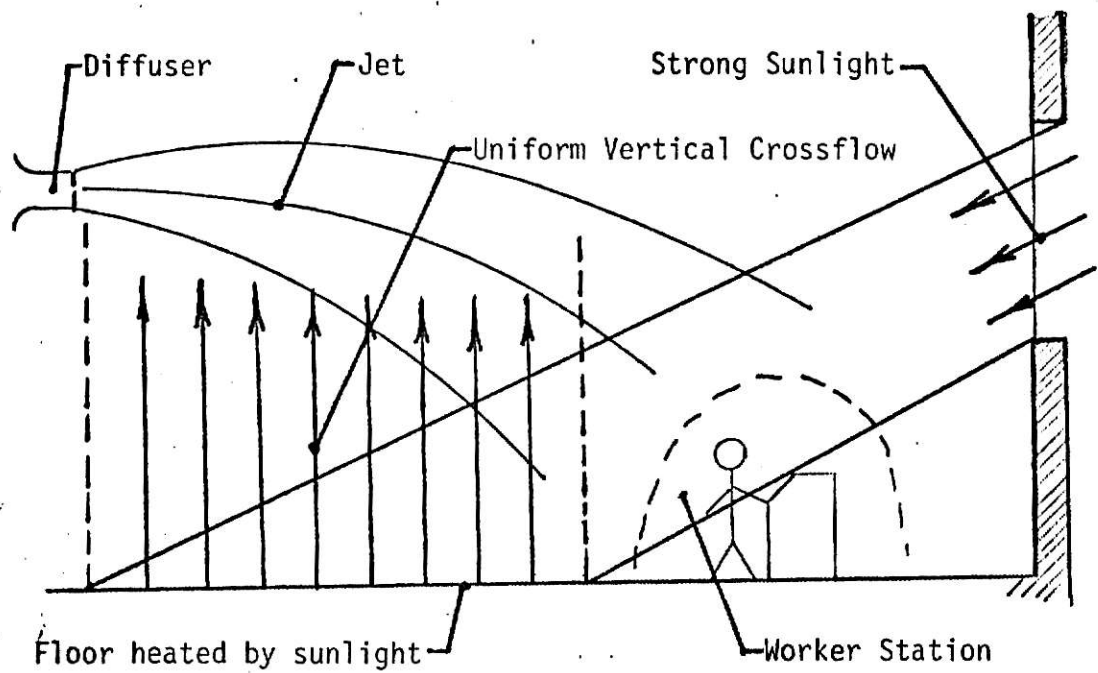


Figure 1. Possible Origin of Uniform Vertical Crossflow

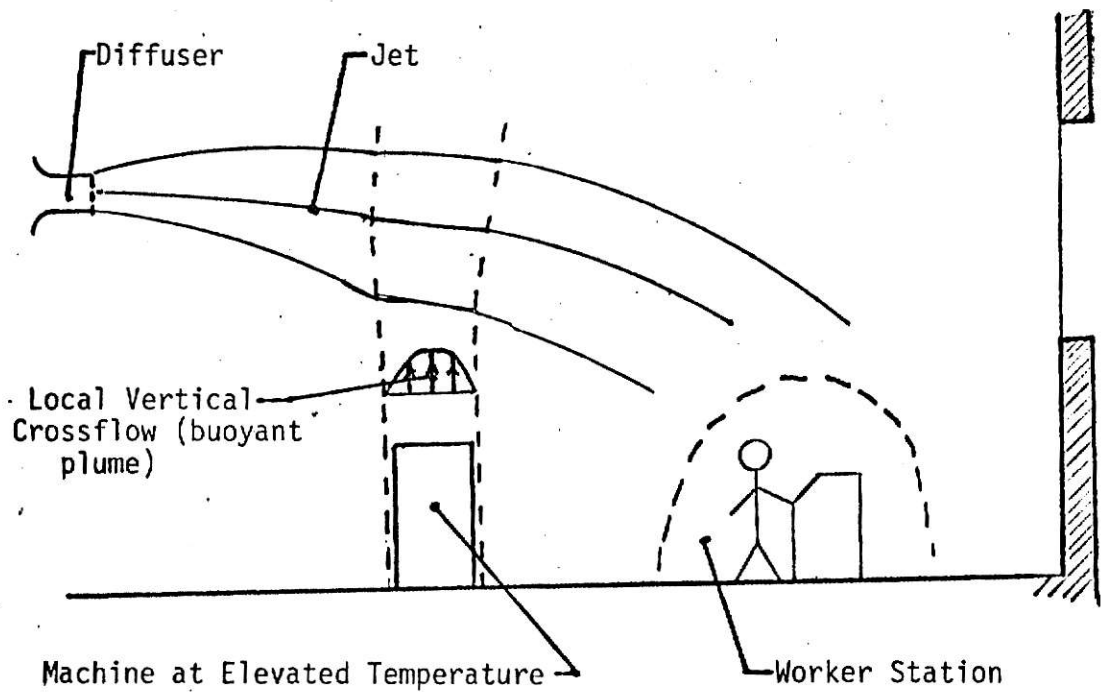


Figure 2. Possible Origin of Local Vertical Crossflow

entire trajectory of this jet. Another possible origin of uniform crossflow might be the same flow heated by radiation from a hot ceiling. Figure 2 illustrates the possible origin of localized vertical crossflow. In the figure a piece of machinery or equipment beneath the jet and at some elevated temperature heats its surrounding air resulting in a buoyant plume which rises into the path of the jet.

This work is basically concerned with the behavior of horizontally projected chilled jets in the presence of vertical crossflow.

OBJECTIVE

The objective of this work is to devise a model that will predict the behavior of a chilled free turbulent jet subjected to either uniform or localized vertical buoyant crossflow.

CHAPTER II. REVIEW OF THE LITERATURE

Most of the basic work dealing with the behavior free turbulent jets has been done for the case of isothermal jets. In more recent years attempts have been made to predict the behavior of heated and chilled jets. Since these cases are simplifications of the case of a chilled jet with vertical cross-flow much of the basic analysis applies to the work. For this reason some of that previous work is reviewed here. Additionally, a large body of work exists dealing with thermal effluents from industrial plants. For example, plumes resulting from smoke stacks have received a great deal of attention. Marked similarities between the conditions surrounding and affecting these plumes and the conditions expected to exist in the jet under consideration exist. Consequently, a portion of the literature in this area is also reviewed.

Free turbulent jets are generally considered to be divided into four distinct regions of flow. Zone I is a short initial region extending about four diameters down stream of the outlet.

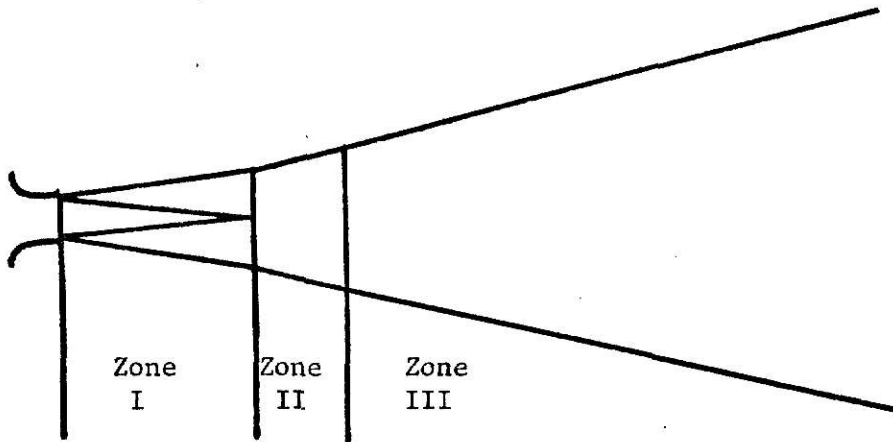


Figure 3. Diagram of a free turbulent jet (terminal zone not shown).

In Zone I the centerline velocity of the jet remains essentially constant. Zone II is a transition region extending to about eight diameters where the centerline velocity begins to decay and the velocity profiles characteristic of the fully developed region begin to emerge. Zone III is the region of fully developed flow. It is a very long zone and may vary from 25 to 100 nozzle diameters in length. It is zone III that is of major engineering significance. Zone IV is the terminal zone where velocities decrease rapidly into the range that can be considered still air. The jet described above is shown in Figure 3.

G. N. Abramovich's work [3], "The Theory of Turbulent Jets" is an in depth review of most important analytical work done by early researchers in the area of free turbulent jets. Abramovich reviews work done by such notable individuals as Prandtl, Taylor, Tolmien, Riechardt and Gortler. These individuals each applied some theory of free turbulence to the Reynold's equations and generally derived an elegant mathematical solution to describe the flow. When the added complications of the jet being at a temperature different from its surrounding and the existence of crossflow are introduced the Reynold's equations retain a degree of complexity that defies solution by the methods employed by these individuals.

In the early 1950's, H. B. Nottage [11] conducted extensive experimental studies of horizontal projected isothermal jets and a limited study of a chilled jet. Nottage's experimental work was considered extremely interesting to this author because it was the only body of experimental work located that studied both isothermal and chilled jets using the same experimental apparatus and conditions. This sameness of conditions allows the differences in performance of chilled jets and isothermal jets due to this chilled condition to be isolated. Any comparison of theory to isothermal jet and

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chilled jet data obtained by different researchers runs the risk of distorting the effects of chilling because of differences in experimental methods as well as differences in the apparatus and ambient conditions.

A. Koestel [12] developed an equation for the centerline trajectory of a chilled jet that he compared with a body of experimental work. His technique of solution was based on an integral energy and integral momentum approach to analysis. In reviewing Koestel's work this author considered that excessive simplification of the problem was made. The effects of entrainment on the spread of the jet are totally ignored; rather, the jet was assumed to spread at a constant rate. The questions arises: Do chilled and isothermal jets spread at the same rate? If they do then buoyancy has no effect on entrainment and/or spread of the jet. If they do not then Koestel's theory is inflexible and incapable of reducing to the simpler case of an isothermal jet without selection of a new constant relating to rate of spread. This author did benefit from Koestel's treatment of the transverse velocity profiles as gaussian and employed a similar technique.

In 1970 D. G. Fox [6] analyzed a forced plume in a stratified fluid starting with the general vector forms of the continuity, Navier-Stokes and energy equations. This line of investigation results in the requirement of a function to describe the rate of mass entrainment into the jet. Fox took a novel approach of mathematically forcing closure on the equations describing the flow to derive an entrainment function.

Following Fox [6], E. Hirst [5] studied liquid effluents discharged into a river, using the same technique. Hirst studied a heated or chilled jet subjected to crossflow. Hirst's problem is very similar to that being studied here, however the orientation of his jet relative to crossflow and buoyancy forces is different from the jet under consideration here. Hirst

additionally considered conservation of salinity within the jet. Consequently study of Hirst's treatment could give direction to further studies of chilled jets that might consider entrainment of dust, chemicals, or other particulate matter into the jet. The approach used by Hirst in his analysis is considered to have sufficient flexibility to handle the problem studied in this thesis and was studied very carefully by this author.

In surveying those works mentioned above and a great deal of other related works no work was found on either heated or chilled jets with vertical crossflow. This work was therefore undertaken.

CHAPTER III. ANALYSIS OF THE FLOW

The problem under consideration here involves the behavior of a free turbulent jet discharged horizontally from a round diffuser. The resultant jet behavior may be influenced by buoyancy forces and local velocities within the jet, by the diameter and orientation of the jet, by vertical crossflow, ambient turbulence levels and ambient density variations around the jet and possibly by other unidentified variables. In view of the lack of solid evidence that a complete list of factors important to jet behavior exists it was decided to begin the analysis of the flow with the general equations of fluid motion and simplify by elimination of factors on a rational basis.

The basic equations [1,2] describing the motion of the jet in general vector form are:

The Continuity Equation:

$$\frac{\partial \rho}{\partial \tau} + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (1)$$

The Energy Equation:

$$\frac{\partial t}{\partial \tau} + \mathbf{V} \cdot (\nabla t) = \frac{1}{\rho C_V} \nabla \cdot (k \nabla t) + \frac{v}{C_V} \phi - P(\nabla \cdot \mathbf{V}) \quad (2)$$

The Momentum Equation:

$$\frac{D\mathbf{V}}{D\tau} = - \frac{\nabla P + \rho \mathbf{X}}{\rho} + \nu \nabla^2 \mathbf{V} \quad (3)$$

where
$$\frac{D\mathbf{V}}{D\tau} = \frac{\partial \mathbf{V}}{\partial \tau} + \mathbf{V} \cdot \nabla \mathbf{V} = \frac{1}{2} \nabla^2 V^2 - \mathbf{V} \times (\nabla \times \mathbf{V})$$

In this analysis several simplifying assumptions will be made. The first group of assumptions are:

1. Steady flow is considered to exist in the mean.
2. Flow is considered to be fully turbulent.

$$-\frac{\nabla P + \rho X}{\rho_0} = \frac{g(\rho - \rho_\infty)}{\rho_0} - \frac{\nabla P^*}{\rho_0} \quad (9)$$

Now for an equation of state a truncated Taylor series expansion is taken about a reference density, ρ_0 :

$$\rho = \rho_0 (1 + \beta(t - t_0)) \quad (10)$$

where: $\beta = -\frac{1}{\rho_0} \left(\frac{\partial \rho}{\partial t} \right)_P$

Using equation (10) it can be shown that:

$$\frac{\rho - \rho_\infty}{\rho_0} = \beta(t - t_\infty) \quad (11)$$

Substituting equations (9) and (11) into equation (6) yields

$$\frac{1}{2} \nabla V^2 - V \times (\nabla \times V) = -\frac{\nabla P + \rho F}{\rho_0} \quad (12)$$

It is readily apparent that the equations of motion, energy and continuity in Cartesian coordinates are cumbersome and inconvenient at best when applied to the free turbulent jet under consideration. Because the path of the jet is not a straight-line, conversion to a natural curvilinear system of coordinates is most desirable. The chosen system is shown below in Figure 4.

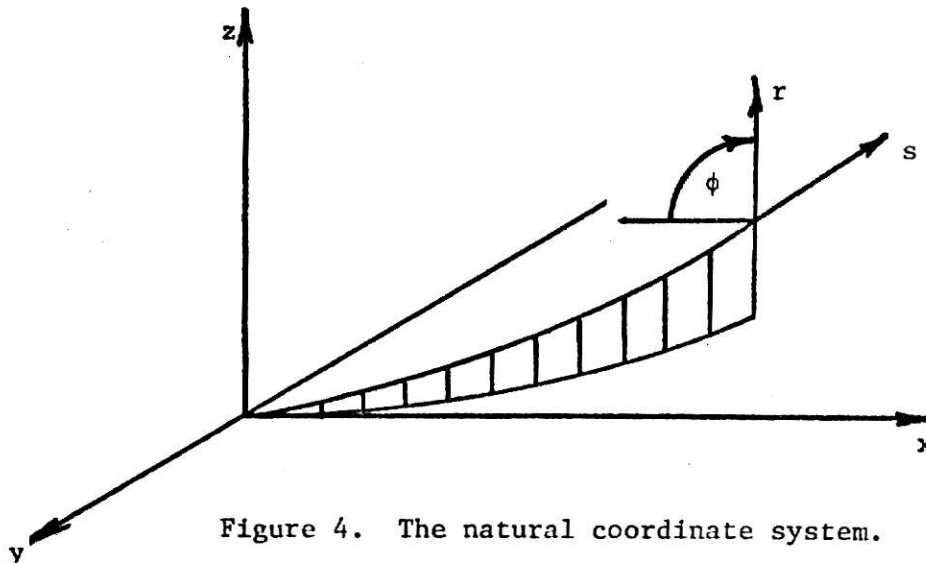


Figure 4. The natural coordinate system.

In this system s measures the distance along the centerline, r measures the distance from the axis of the jet in a plane perpendicular to the centerline and ϕ determines the orientation of r . s is zero at the origin of the jet, r is zero along the axis of the jet and ϕ is zero when the r vector is horizontal to the left when looking along the jet axis in the direction of increasing s . The system is discussed in detail in Appendix A.

The details of conversion of the Cartesian equations to the natural curvilinear system are shown in Appendix B. The resulting equations are:

The Continuity Equation:

$$\frac{1}{1-e} \frac{\partial u}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r}(rv) + \frac{1}{r} \frac{\partial w}{\partial \phi} - \frac{v}{1-e} \left(\frac{e}{r}\right) - \frac{w}{r(1-e)} \frac{\partial e}{\partial \phi} = 0 \quad (13)$$

where: $e = \frac{C_\phi C_2}{\rho_1} + \frac{S_\phi}{\rho_2}$

and $V = u\hat{i}_s + v\hat{i}_r + w\hat{i}_\phi$

The Energy Equation:

$$\frac{u}{1-e} \frac{\partial t}{\partial s} + v \frac{\partial t}{\partial r} + \frac{w}{r} \frac{\partial t}{\partial \phi} = 0 \quad (14)$$

The s-Momentum Equation:

$$\frac{u}{1-e} \frac{\partial u}{\partial s} + \frac{w}{r} \frac{\partial u}{\partial \phi} - \frac{uw}{1-e} \left(\frac{1}{r} \frac{\partial e}{\partial \phi}\right) + v \frac{\partial u}{\partial r} - \frac{uv}{1-e} \frac{e}{r} = g\beta(t - t_\infty)S_2 - P_S^* \quad (15)$$

The r-Momentum Equation:

$$v \frac{\partial v}{\partial r} + \frac{u}{1-e} \frac{\partial v}{\partial s} + \frac{u^2}{1-e} \frac{e}{r} - \frac{w^2}{r} + \frac{w}{r} \frac{\partial v}{\partial \phi} = g\beta(t - t_\infty)S_\phi C_2 - P_r^* \quad (16)$$

3. Flow is considered to be incompressible with density variation appearing only in the buoyancy term (Boussinesq' Approximation).
4. Fluid properties are considered to be constant.
5. Low velocities are considered to exist so that frictional heating and dissipative effects are neglected.

With these assumptions equations (1) through (3) reduce to the following:

The Continuity Equation:

$$\nabla \cdot \mathbf{V} = 0 \quad (4)$$

The Energy Equation:

$$\nabla \cdot (\nabla t) = 0 \quad (5)$$

The Momentum Equation:

$$\frac{1}{2} \nabla V^2 - \mathbf{V} \times (\nabla \times \mathbf{V}) = - \frac{\nabla P + \rho \mathbf{X}}{\rho_o} \quad (6)$$

where: $\mathbf{X}$ = an acceleration term characteristic of this force field in question

Solution of these equations requires one additional relation, an equation of state. Before this equation is incorporated into the analysis it is desirable to rewrite the right side of equation (6) in a more convenient form.

The pressure gradient, ∇P , can be written as:

$$\nabla P = \rho_{\infty} \mathbf{g} + \nabla P^* \quad (7)$$

where: $\rho_{\infty} \mathbf{g}$ = the hydrostatic pressure gradient

and ∇P^* = the hydrodynamic pressure gradient.

The body force term, $\rho \mathbf{X}$, is due to gravitational force and thus is simply the weight of the fluid

$$\rho \mathbf{X} = \rho \mathbf{g} \quad (8)$$

Combining (7) and (8) and dividing by ρ_o yields:

The ϕ -Momentum Equation:

$$\frac{w}{r} \frac{\partial w}{\partial \phi} + v \frac{\partial w}{\partial r} + \frac{vw}{r} + \frac{u^2}{1-e} \left(\frac{1}{r} \frac{\partial e}{\partial \phi} \right) + \frac{u}{1-e} \frac{\partial w}{\partial s} = g\beta(t - t_{\infty})C_{\phi}C_2 - P_{\phi}^* \quad (17)$$

It is recalled at this point that the jet originates from a round diffuser. It is therefore reasonable to assume that this jet is axisymmetric. Furthermore, it is recalled that the jet is assumed to have low velocity; hence the hydrodynamic pressure gradient is neglected. In the fact of the assumption of axisymmetry the ϕ momentum equation becomes meaningless and the r -equation becomes dubious in value. The momentum equations are now rewritten so that instead of s , r and ϕ equations we have s , k , and i equations. That is, an s momentum equation parallel to the centerline of the jet, a k momentum equation parallel to both crossflow and buoyancy force and an i -momentum equation that is written for the purpose of completeness.

Before the equations are rewritten as the axisymmetric equations it is convenient to include explicitly the effects of turbulence. This is done by considering the instantaneous velocity to consist of a time average component and a fluctuating component and then averaging these components over time.

These assumptions and operations are applied to equations (13) through (17) in Appendix C. The result is the axisymmetric equations:

The Continuity Equation:

$$\frac{\partial \bar{u}}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (r\bar{v}) = 0 \quad (18)$$

The Energy Equation:

$$\bar{u} \frac{\partial t}{\partial s} + \bar{v} \frac{\partial t}{\partial r} = - \frac{1}{r} \frac{\partial}{\partial r} (r \overline{v' t'}) \quad (19)$$

The s-Momentum Equation:

$$\bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{v}}{\partial r} = g\beta(\bar{t} - t_\infty)S_2 - \frac{1}{r} \frac{\partial}{\partial r} (r \overline{u'v'}) \quad (20)$$

The k-Momentum Equation is:

$$\begin{aligned} (\bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{u}}{\partial r})S_2 &= g\beta(\bar{t} - t_\infty) - \frac{1}{r} \frac{\partial}{\partial r} (r \overline{u'v'})S_2 \\ &- (\bar{u}^2 - \frac{r}{4} (\frac{\partial \bar{v}^2}{\partial r} + \frac{\partial \overline{v'^2}}{\partial r})) \frac{C_2}{\rho_2} \end{aligned} \quad (21)$$

The i-Momentum Equation is:

$$(\bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{u}}{\partial r})C_2 = (\bar{u}^2 - \frac{r}{4} (\frac{\partial \bar{v}^2}{\partial r} + \frac{\partial \overline{v'^2}}{\partial r})) \frac{S_2}{\rho_2} - \frac{1}{r} \frac{\partial}{\partial r} (r \overline{u'v'})C_2 \quad (22)$$

The axisymmetric equations above and a set of appropriate boundary conditions are sufficient to solve for u , v , t and θ_2 as a function of the single independent variables r and s . By making an integral approximation radius, r , can be eliminated from the problem. This simplifies the analysis in that the variables become functions of only the distance along the centerline, s . In Appendix D equations (18), (19), (20), (21) and (22) are integrated in a general sense radially from zero to infinity.

In performing the integration of the axisymmetric equations use must be made of certain boundary conditions on the jet. Before proceeding to write the integral equations the boundary conditions used in evaluating the integrals will be noted.

- 1) As $r \rightarrow \infty$ $\bar{v} \rightarrow -E/r$

This boundary condition serves as a definition of the entrainment function. The entrainment rate, E , is the volume rate of entrainment into the jet. The units on the entrainment function are $\text{ft}^3/\text{sec}/\text{ft}$.

As r goes to infinity the total velocity, $\bar{v}$, goes to the free stream velocity. When evaluating an integral containing $\bar{u}$ in the s -momentum equation, as r approaches infinity we seek the component of U_∞ in the s -direction.

2) In the s -direction; as $r \rightarrow \infty$ $\bar{u} \rightarrow U_\infty S_2$

When evaluating an integral in the k -direction, as r approaches infinity we seek the component of $\bar{u}$ in the k -direction.

3) In the k -direction; as $r \rightarrow \infty$ $\bar{u} \rightarrow U_\infty / S_2$

Applying the above boundary conditions in the integration of equations (18), (19), (20), (21) and (22) and neglecting ambient turbulence reduces the analysis to the following set of equations (Note that for convenience the "bars" above time average terms have now been deleted. The primes indicating fluctuating terms are retained):

The Continuity Equation is:

$$\frac{d}{ds} \left(\int_0^\infty u r dr \right) = E \quad (23)$$

The Energy Equation is:

$$\frac{d}{ds} \left(\int_0^\infty u(t - t_\infty) r dr \right) = - \frac{dt_\infty}{ds} \int_0^\infty u r dr \quad (24)$$

The s-Momentum Equation is:

$$\frac{d}{ds} \left(\int_0^\infty u^2 r dr \right) = S_2 \int_0^\infty g\beta(t - t_\infty) r dr + EU_\infty S_2 \quad (25)$$

The i-Momentum Equation is:

$$\frac{d}{ds} \left(C_2 \int_0^\infty u^2 r dr \right) = EU_\infty S_2 C_2 + \left(\int_0^\infty u^2 r dr - \frac{E}{4} \right) \frac{S_2}{\rho_2} \quad (26)$$

The Centerline Slope Equation is:

$$\frac{d\theta_2}{ds} = \frac{C_2}{q} \left(\int_0^\infty g\beta(t - t_\infty) r dr + EU_\infty \right) \quad (27)$$

where: $q = \int_0^\infty u^2 r dr - \frac{E^2}{4}$

Note that the k-momentum equation has been eliminated from the analysis and that a centerline slope equation appears in its place. Details of the integrations described above and the derivation of the centerline slope equation are shown in Appendix D.

A considerable amount of experimental data [3,4] indicates that the velocity profiles in the fully developed region are closely approximated by Gaussian (error function) type curves. Selection of a profile of this type involves the additional assumption that velocity profiles are similar in the fully developed region. Additionally the temperature profiles will be assumed Gaussian and similar.

The Velocity Profile is selected to be:

$$u = \Delta U e^{-r^2/B} + U_\infty S_2$$

where: ΔU = the difference between the centerline (maximum) velocity and the free stream velocity, ΔU varies with s .

B = a "shape" factor which determines the location of the edge of the jet. B varies with s .

The temperature profile is selected to be:

$$t = \Delta T_M e^{-r^2/\lambda B} + t_\infty \quad \text{or} \quad t - t_\infty = \Delta T_M e^{-r^2/\lambda B}$$

where ΔT_M = the difference between the centerline (minimum) temperature and the ambient temperature, t_∞ . ΔT_M varies with s .

λ = a constant that determines relationship between the thermal and hydrodynamic jet boundaries.

These velocity and temperature profiles are substituted into equations (23) through (27) in Appendix E and the appropriate integration is performed.

The results are as follows:

The Continuity Equation is:

$$\frac{d}{ds} \left(\frac{BV_M}{2} \right) = E \quad (28)$$

where: $V_M = U_M + U_2$

$$U_2 = U_\infty S_2$$

$$U_M = \Delta U + U_2$$

The Energy Equation is:

$$\frac{d}{ds} \left(\frac{\Delta T_M \lambda B}{2} \left(\frac{\Delta U}{1+\lambda} + U_2 \right) \right) = 0 \quad (29)$$

The s-Momentum Equation is:

$$\frac{d}{ds} \left(\frac{BV_M^2}{4} \right) = \frac{S_2 g \beta \Delta T_M \lambda B}{2} + EU_2 \quad (30)$$

The Centerline Slope Equation is:

$$\frac{d\theta_2}{ds} = \frac{C_2}{q} \left(\frac{gB \Delta T_M \lambda B}{2} + EU_\infty \right) \quad (31)$$

where: $q = \frac{BV_M^2}{4} - \frac{E^2}{4}$

The problem has now been reduced to four first order total differential equations in four unknowns. The unknown dependent variables are ΔU , B , ΔT_M , and θ_2 . The single independent variable is s .

These equations can be solved for the four dependent variables (numerically) provided only that an entrainment function and appropriate initial conditions are specified. The derivation of an entrainment function is the topic of the next chapter and the specification of initial conditions will be discussed in Appendix I.

CHAPTER IV. THE ENTRAINMENT FUNCTION

Entrainment is the ingestion of ambient fluid into the jet. Since density variation is considered only in the buoyancy terms, entrainment in this analysis is the space rate of volumetric ingestion, that is, the volume (in ft^3/s) ingested per foot in the axial direction.

The rate of entrainment is dependent-- and thus the entrainment function should be dependent-- on the following.

- (1) Mean local flow conditions (ΔU and B)
- (2) Local buoyancy effects (ΔT_M , ΔU and B)
- (3) Magnitude of crossflow (U_∞)
- (4) Orientation of jet (θ_2)
- (5) Ambient turbulence levels.

Since data concerning ambient turbulence is non-existent, the effects of ambient turbulence will not be considered.

The entrainment function in this analysis is defined by the boundary condition used in the radial integration of the continuity equation. The resulting continuity equation is:

$$\frac{d}{ds} \int_0^\infty \bar{u} r dr = E \quad (23)$$

The flow rate through any section of the jet is:

$$Q = 2\pi \int_0^\infty \bar{u} r dr \quad (32)$$

Therefore:

$$\frac{Q}{2\pi} = \int_0^\infty \bar{u} r dr$$

and
$$\frac{dQ}{dS} = 2\pi E .$$

The entrainment function is not equal to the rate of change of flow but is rather merely proportional to the rate of change of flow within the jet.

The entrainment function used in this analysis is that suggested by Hirst [5]. This function is obtained by combination of entrainment functions obtained by Fox [6] and by Hoult, Fay and Forney [7].

Fox's method as used by Hirst consists of forcing closure on the equations describing the flow. Hirst attempted to derive an entrainment function that would take into consideration crossflow, buoyancy effects, orientation of the jet and mean local flow conditions using Fox's closure method. This author did the same, unfortunately the presence of the crossflow velocity prevents solution for an entrainment function that is dependent on mean variables only. Rather the resulting entrainment function contains derivative terms that prevented numerical solution. It was therefore necessary to abandon crossflow effects in this initial derivation. The resulting entrainment function is

$$E_F = \frac{A_3 S_2 \Delta T_M^B}{\Delta U} + \frac{6SI}{\Delta U^2} \quad (33)$$

where: SI = the shear integral

This function is derived in Appendix F. It is also shown in Appendix F that this function is identical to that obtained by both Fox and Hirst. This function considers mean local flow conditions, local buoyancy effects and orientation of the jet.

In their paper Hoult, Fay and Forney [7] concluded that crossflow affects entrainment through two mechanisms: one is due to the difference between

the jet velocity and the crossflow velocity component parallel to the jet and the other is due to the crossflow component normal to the jet. In developing their entrainment function they simply postulated that these two mechanisms were additive. Written in terms of the notation of this thesis the entrainment function used by Hoult, Fay and Forney is:

$$E_H = A_4 \sqrt{B} \Delta U + A_5 \sqrt{B} U_\infty S_2 \quad (34)$$

where A_4 and A_5 are "entrainment parameters" independent of position, that is, A_4 and A_5 are constants.

Hirst suggests that Fox's function be combined with that of Hoult, Fay and Forney. As a first step in this direction let equation (34) be rearranged.

$$E_H = A_4 (\sqrt{B} \Delta U + A_6 \sqrt{B} U_\infty S_2) \quad (35)$$

where: $A_6 = A_5/A_4$

The final step in the combination is to factor $\Delta U \sqrt{B}$ from E_F and let $E_F/b\Delta U = A_4$. The result is:

$$E = \left(\frac{6SI}{\sqrt{B} \Delta U^3} + \frac{A_3 S_2 \Delta T_M \sqrt{B}}{\Delta U^2} \right) (\sqrt{B} \Delta U + A_6 \sqrt{B} U_\infty S_2) \quad (36)$$

Utilizing equation (41) from the next chapter ΔT_M can be eliminated from equation (36). This results in

$$E = \left(\frac{6SI}{\sqrt{B} \Delta U^3} + \frac{A_7 S_2^D}{\Delta U^2 \lambda \sqrt{B} \left(\frac{\Delta U}{1+\lambda} + U_2 \right)} \right) (\sqrt{B} \Delta U + A_6 \sqrt{B} U_\infty S_2) \quad (37)$$

It now only remains to deal with the shear stress integral SI. Several approaches were considered and tried with poor results. These are detailed

in Appendix G. Finally it was decided to resort to the following:

$$SI = A_8 \sqrt{B} \Delta U^3 \quad (38)$$

where A_8 is the result of evaluation of some similarity integral. The entrainment function becomes:

$$E = \left(A_9 + \frac{A_7 S_2 D}{\Delta U^3 \lambda \sqrt{B} \left[\frac{\Delta U}{1+\lambda} + U_2 \right]} \right) (\sqrt{B} \Delta U + A_6 \sqrt{B} U_\infty S_2) \quad (39)$$

where: $A_9 = 6A_8$.

There is considerable precedent for equation (38). Hirst cites entrainment functions used by Morton, et al. [8] and Fan [9] as resulting in identical terms. Additionally Anwar [10] uses an entrainment function with this form. The final form of the entrainment function, equation (39), is thus nothing more than that suggested by Hirst. It is this function, as shown in equation (39) that is referred to henceforth as simply E.

CHAPTER V. NUMERICAL SOLUTION

The analysis of the flow conducted in Chapter III resulted in a system of first order total differential equations. Equations of this type lend themselves readily to solution by numerical integration procedures such as Runge-Kutta-Gill provided that the derivatives of the individual variables can be isolated. For the analysis at hand the isolation of the individual derivatives can be accomplished routinely.

In equation (31) $d\theta_2/ds$ is already isolated so that no additional manipulation is required. For convenience equation (31) is repeated:

$$\frac{d\theta_2}{ds} = \frac{C_2}{q} \left(\frac{g\beta \Delta T_M \lambda_B}{2} + EU_\infty \right) \quad (31)$$

where: $q = \frac{BV_M^2}{4} - \frac{E^2}{4}.$

In the absence of ambient thermal stratification the energy equation can be solved exactly.

$$\int \frac{d}{ds} \left(\frac{\Delta T_M \lambda_B}{2} \left(\frac{\Delta U}{1+\lambda} + U_2 \right) \right) = \frac{\Delta T_M \lambda_B}{2} \left(\frac{\Delta U}{1+\lambda} + U_2 \right) - D = 0 \quad (40)$$

where: D is a constant of integration.

Rearranging equation (40) the following result is obtained:

$$\Delta T_M = \frac{2D}{\lambda_B \left(\frac{\Delta U}{1+\lambda} + U_2 \right)} \quad (41)$$

Equations (28) and (30) can be solved for dB/ds and $d\Delta U/ds$ by simply expanding the derivative terms on their left hand sides, substituting equation (41) for ΔT_M , and manipulating the two algebraically. The results are:

$$\frac{d\Delta U}{ds} = \frac{2S_2 g \beta D}{B \left(\left(\frac{\Delta U}{1+\lambda} \right) + U_\infty S_2 \right) \left(\Delta U + 2U_\infty S_2 \right)} - \frac{2E\Delta U}{B(\Delta U + 2U_\infty S_2)} - 2 \left(\frac{dU_\infty}{ds} S_2 + U_\infty C_2 \frac{d\theta}{ds} \right) \quad (42)$$

and

$$\frac{dB}{ds} = \frac{2E - B \left(\frac{d\Delta U}{ds} + 2 \left(\frac{dU_\infty}{ds} S_2 + U_\infty C_2 \frac{d\theta}{ds} \right) \right)}{\Delta U + 2U_\infty S_2} \quad (43)$$

The x and y coordinates corresponding to any point s are related to θ and s by the following:

$$x = \int_0^s \cos \theta_2 ds \quad (44)$$

$$\text{and} \quad z = \int_0^s \sin \theta_2 ds \quad (45)$$

$$\text{Thus} \quad \frac{dx}{ds} = \cos \theta_2$$

$$\text{and} \quad \frac{dz}{ds} = \sin \theta_2$$

In order to integrate equations (31), (42), (43), (44), and (45) numerically the entrainment function, E, the values of U_∞ and dU_∞/ds at all points s and the initial values of each of the dependent variables (ΔU , B, θ , x, y) must be specified. Because the energy equation was integrated exactly the magnitude of the centerline temperature difference is tied directly to the values of the centerline velocity difference, ΔU , and the shape factor, B. Thus, numerical integration of the centerline temperature difference is not necessary.

It has been noted that the equations of this analysis apply only in the region of fully developed flow. The initial conditions required for solution are therefore the conditions that exist at the beginning of the fully developed region. The initial conditions used were obtained directly from the data to which this theory is compared. The details of these initial conditions may be found in Appendix I.

The complete program for numerical integration of equations (31), (42), (43), (44), and (45) may be found in Appendix H.

CHAPTER VI. COMPARISON OF THEORETICAL PREDICTIONS FOR THE SIMPLE CASES OF ISOTHERMAL JETS AND CHILLED JETS IN QUIESCENT ENVIRONMENTS

The mathematical model of a free turbulent jet developed in this thesis has sufficient flexibility to be reduced to simpler cases. For example the model can be reduced to chilled and isothermal jets with no crossflow present. Since the ultimate goal of this analysis is prediction of behavior of a chilled jet subjected to vertical crossflow only a limited comparison of predictions made by theory to simpler cases will be made.

The comparison of predictions to experimental data will be limited to data by H. B. Nottage [11] in his dissertation "Ventilation Jets in Room Air Distribution." For isothermal jets the three parameters compared are centerline velocity decay, spread (radius growth), and flow rate. For the chilled jet centerline temperature decay and trajectory as well as the parameters above are compared.

Figures 5 through 9 are a comparison of theoretical centerline velocity decay to experimental data for six isothermal jets. With the value of $A_3 = 0.055$ the centerline velocity decay of each of the higher speed jets (6000, 2500 and 1000 fpm) are found to agree very well with experimental values. Unfortunately the comparison to the lower speed jets is not quite so good. In fact the lower the velocity of the jet the poorer the comparison.

It should be recalled that the existence of constant A_2 is not something that results directly from theory, rather its existence is essentially postulated by several researchers. Examination of the velocity decay curves led this author to suspect that the value of the constant A_2 might be tied in some way to the initial conditions in the jet. For example, the value of A_5 might be 0.055 for high speed jets and 0.075 for lower speed jets. (A value of

**THIS BOOK
CONTAINS
NUMEROUS PAGES
WITH DIAGRAMS
THAT ARE CROOKED
COMPARED TO THE
REST OF THE
INFORMATION ON
THE PAGE.**

**THIS IS AS
RECEIVED FROM
CUSTOMER.**

Figure 5. Axial Velocity Variations for Isothermal Jets (2500 & 6000 fpm comparison)

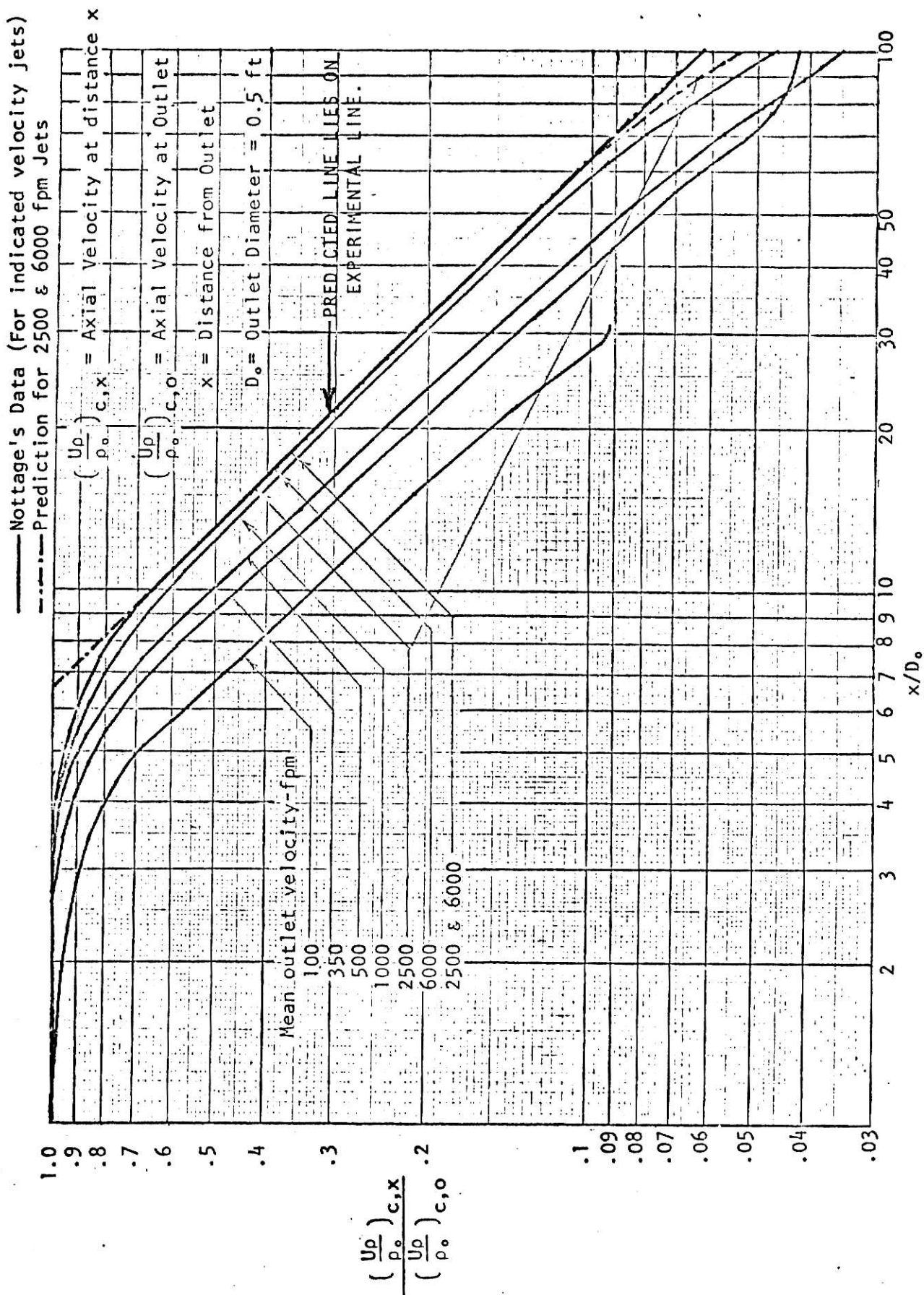
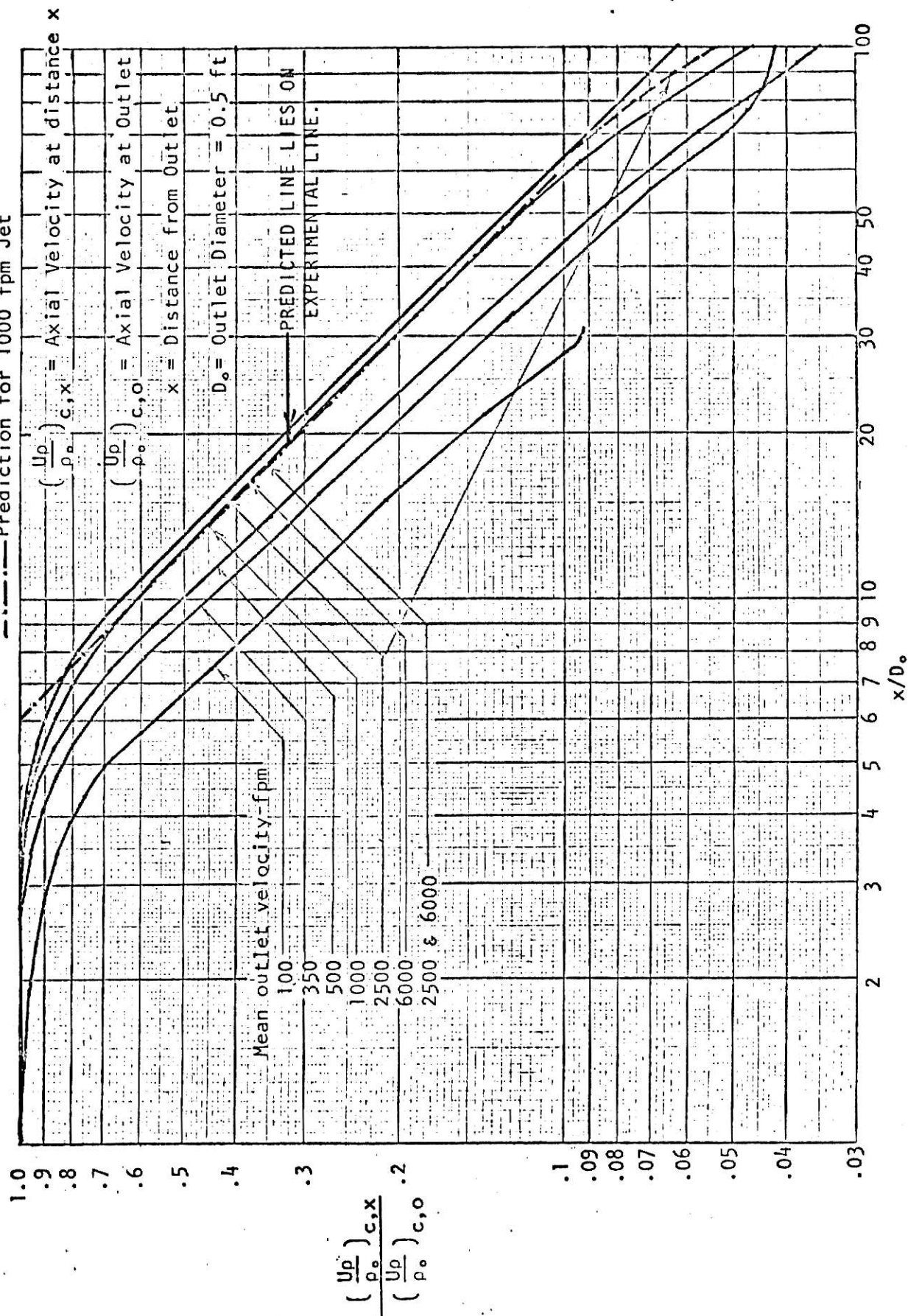


Figure 6. Axial Velocity Variations for Isothermal Jets (1000 fpm comparison)

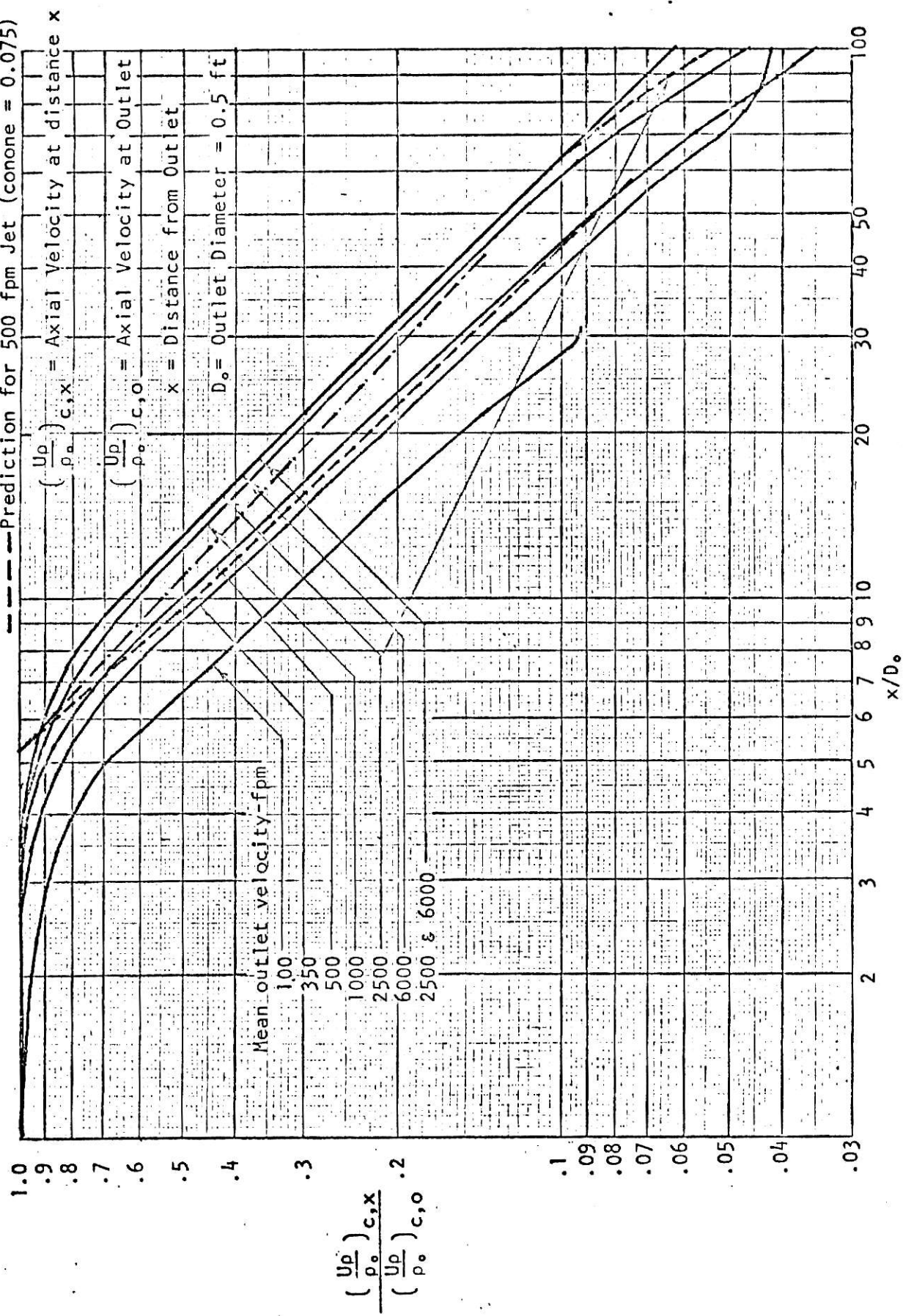
— Nottage's Data (For indicated velocity jets)
 - - - Prediction for 1000 fpm Jet



— Nottage's Data (For indicated velocity jets)

- - - Prediction for 500 fpm Jet (conone = 0.055)

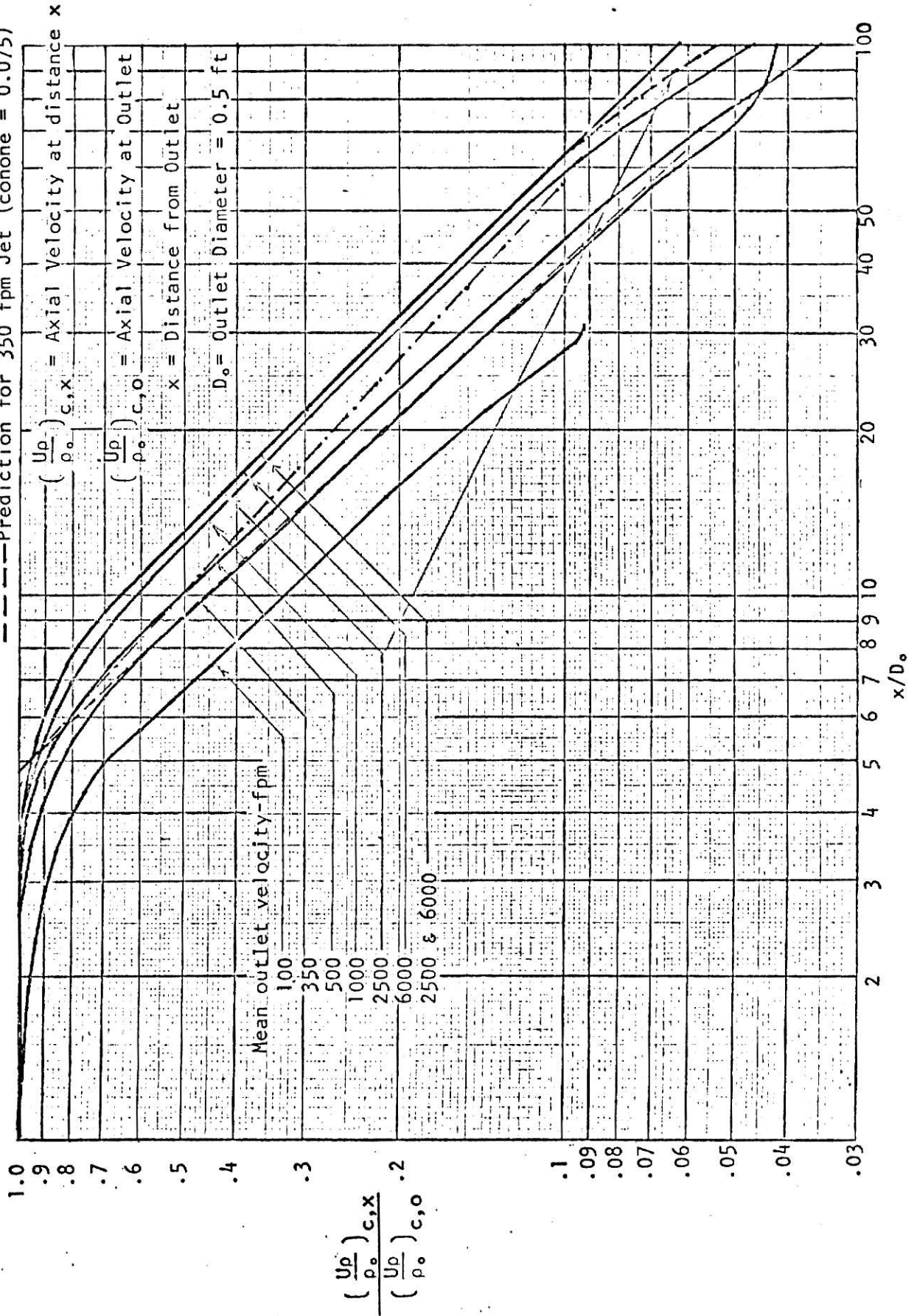
- - - Prediction for 500 fpm Jet (conone = 0.075)



— Nottage's Data (For indicated velocity jets)

--- Prediction for 350 fpm Jet (conone = 0.055)

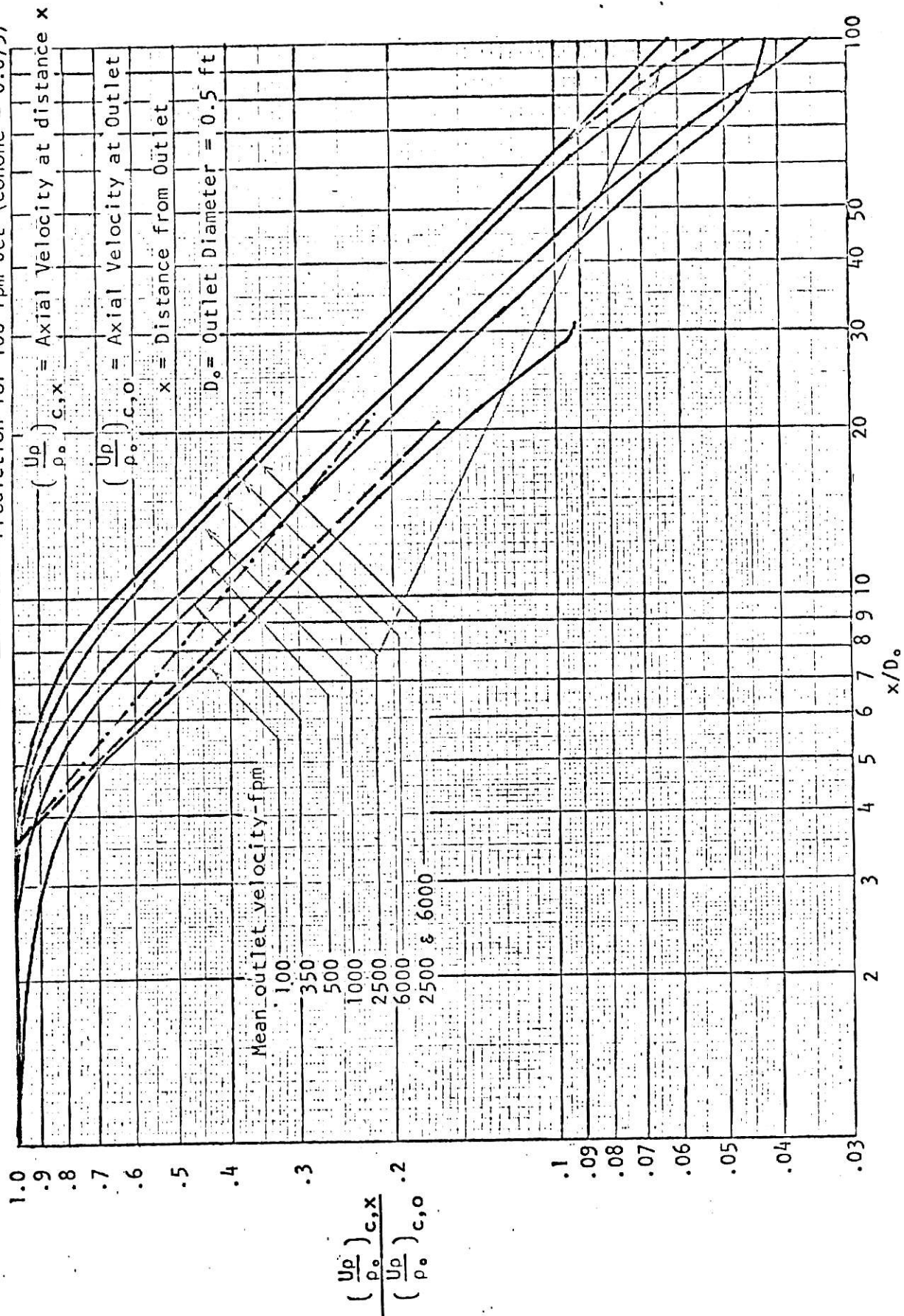
--- Prediction for 350 fpm Jet (conone = 0.075)



—— Nottage's Data (For indicated velocity jets)

--- Prediction for 100 fpm Jet (conone = 0.055)

--- Prediction for 100 fpm Jet (conone = 0.075)



$A_2 = 0.075$ causes the 350 fpm theoretical and experimental curves to agree very closely.) With $A = 0.075$ all three low speed jets (100, 350, and 500 fpm) show greatly improved comparison to experimental velocity decay data.

Figures 10 through 13 are a comparison of theory to experimental data on radius growth. When the constant A_2 is 0.055 agreement with experimental data is uniformly good, though theory predicts a greater spread than the data indicates. This is not considered a particularly important shortcoming on the part of the theory for several reasons. First the location of the jet boundary is really a matter of definition. In his data Nottage placed the boundary at that point where velocity was no longer measurable. This author was unable to determine from Nottage's writing what this lowest measureable velocity for Nottage's experimental set up was. It is very possible that the actual boundary of the jet was out farther than what was measured by Nottage. Secondly the Gaussian velocity profiles that were assumed to exist in development of this theory may vary sufficiently from the profiles that actually exist to cause some variation in the jet boundary.

Figures 14 through 19 show the normalized volumetric flow as predicted by theory and the flow rates Nottage calculated from his data. Agreement of theory to experimental data is quite good. As in the case of the velocity decay the comparison deteriorates somewhat for the low velocity jets.

Referring now to the hypothesis that a second value of the constant A_2 should be used for low velocity jets it can be seen from Figures 10, 11, 18, and 19 that predictions of theory for radius growth and flow rate do not agree as well when the "low velocity constant" ($A_2 = 0.075$) is used. It is therefore completely possible that the value of the constant does not change for low velocity jets but rather that some other unidentified mechanism

Figure 10. Spread of a
100 fpm Isothermal Jet

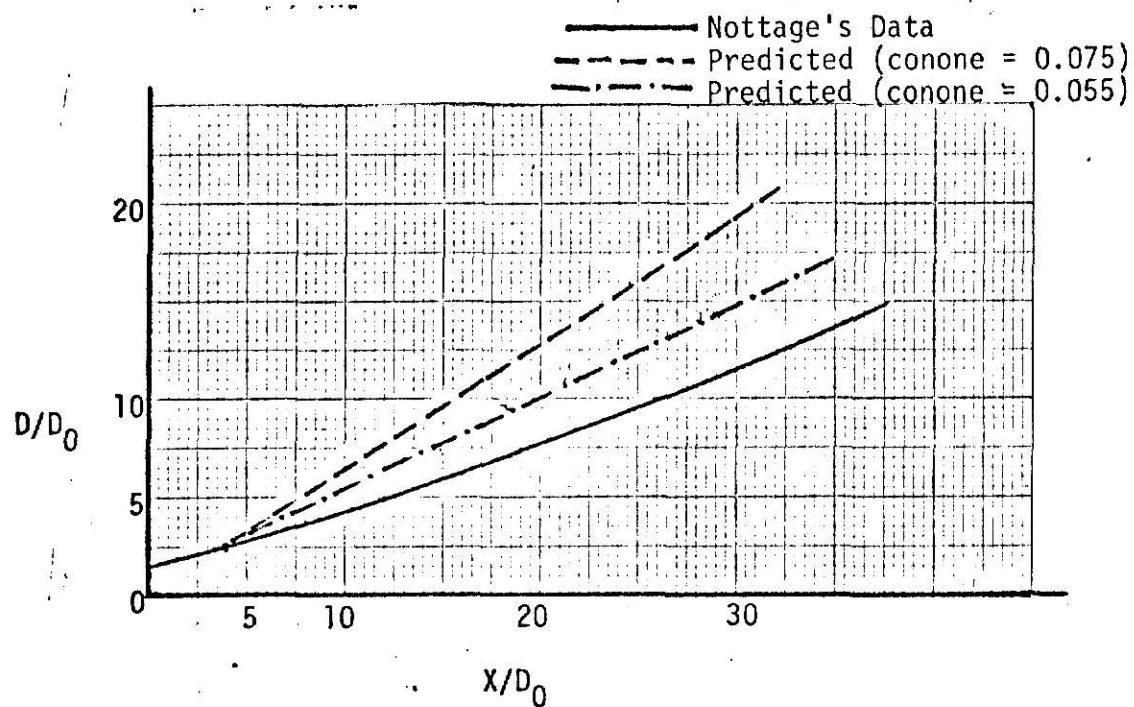


Figure 11. Spread of
350 & 500 fpm Isothermal Jets

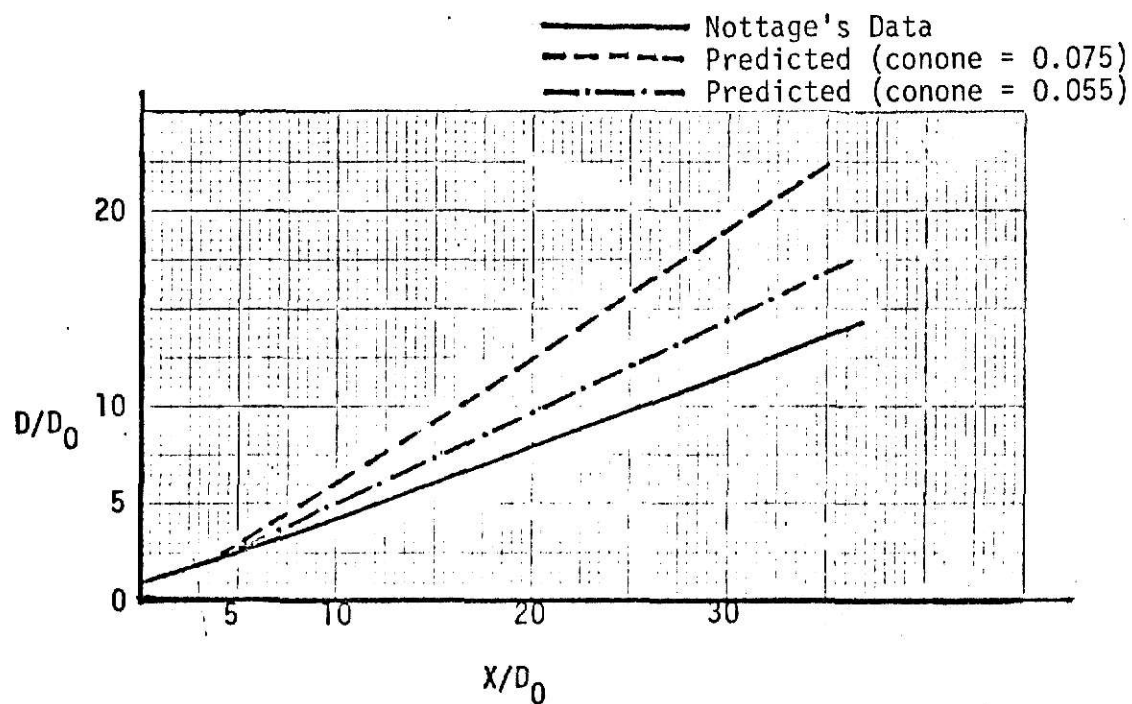


Figure 12. Spread of a
1000 fpm Isothermal Jet

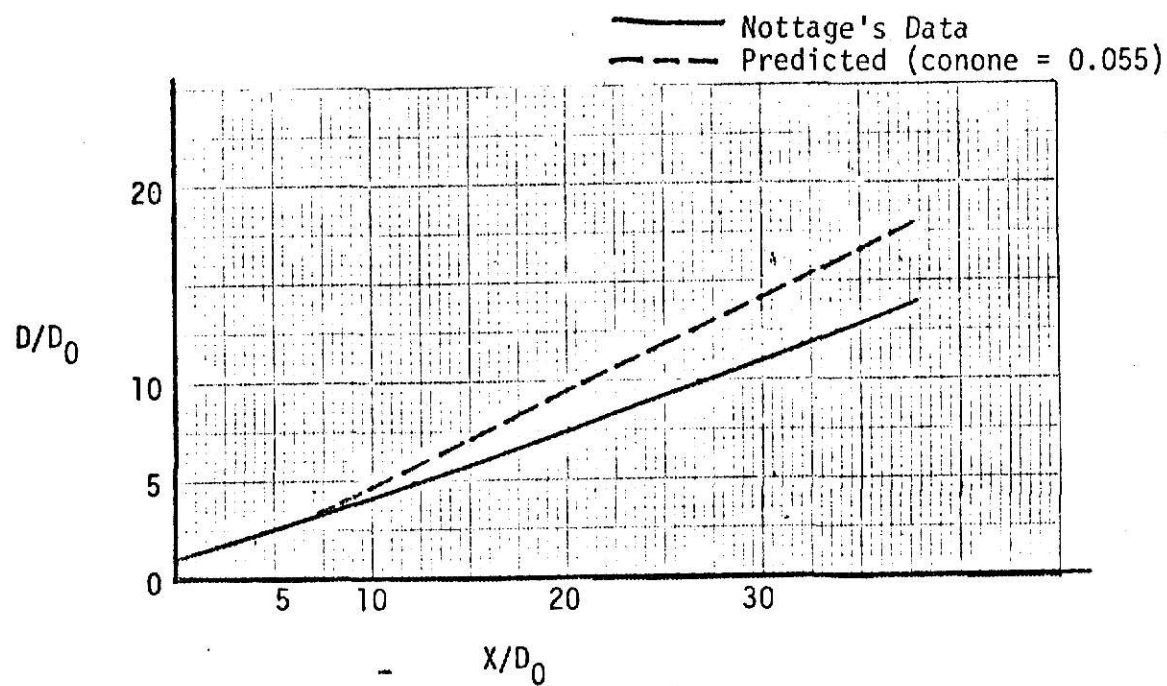


Figure 13. Spread of
2500 & 6000 fpm Isothermal Jets

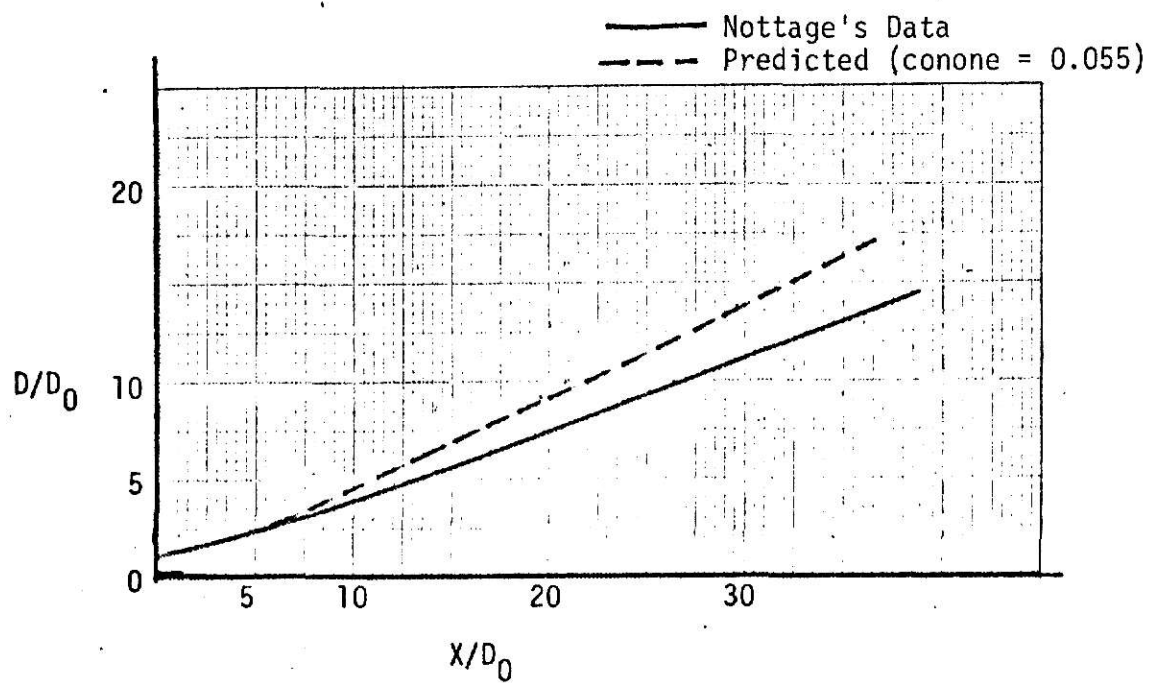


Figure 14. Flow Rate within a 6000 fpm
Isothermal Jet

--○-- Nottage's Data
—— Predicted by theory

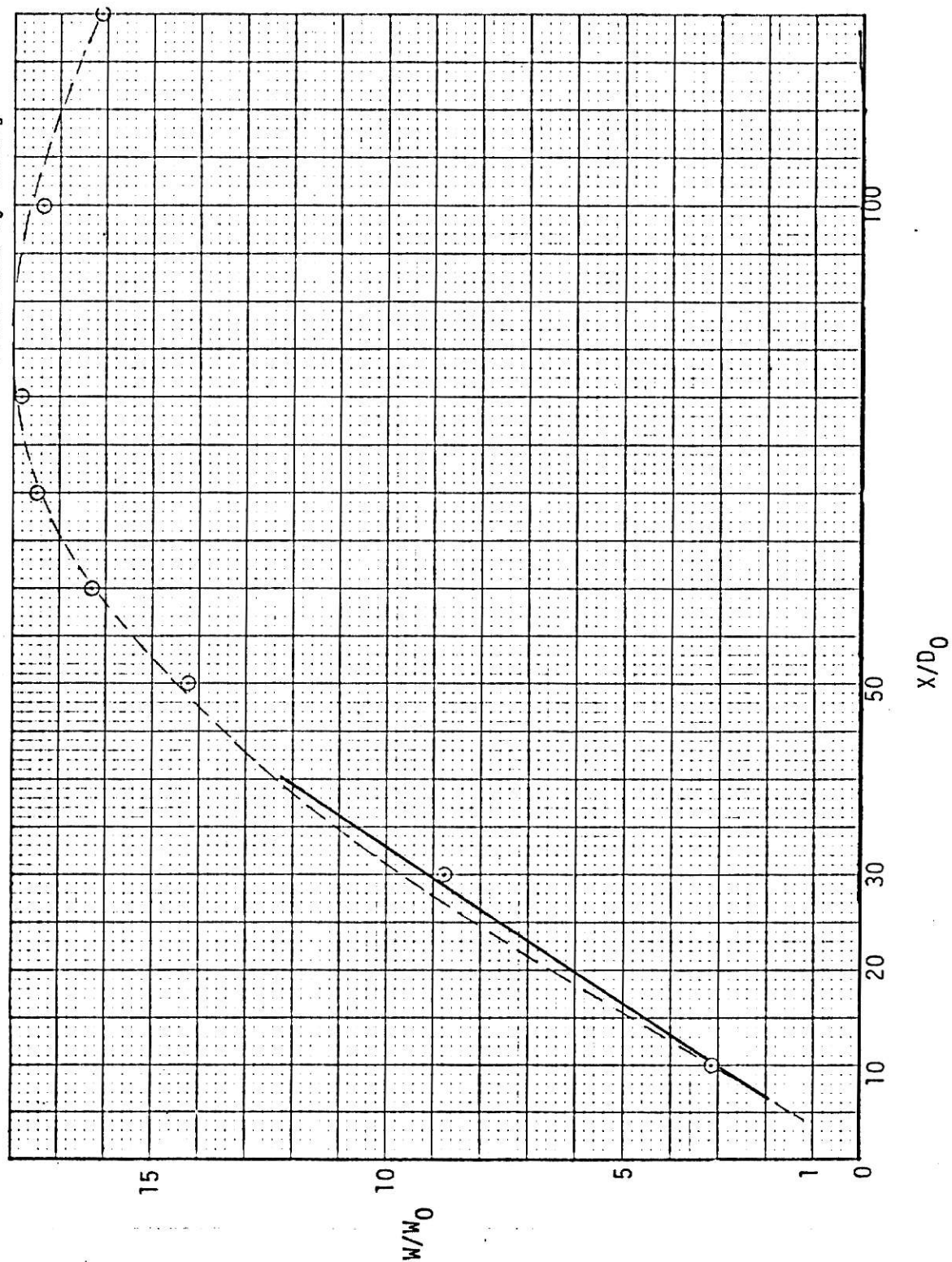


Figure 15. Flow Rate within a 2500 fpm
Isothermal Jet

--○-- Nottage's Data
— Predicted by theory

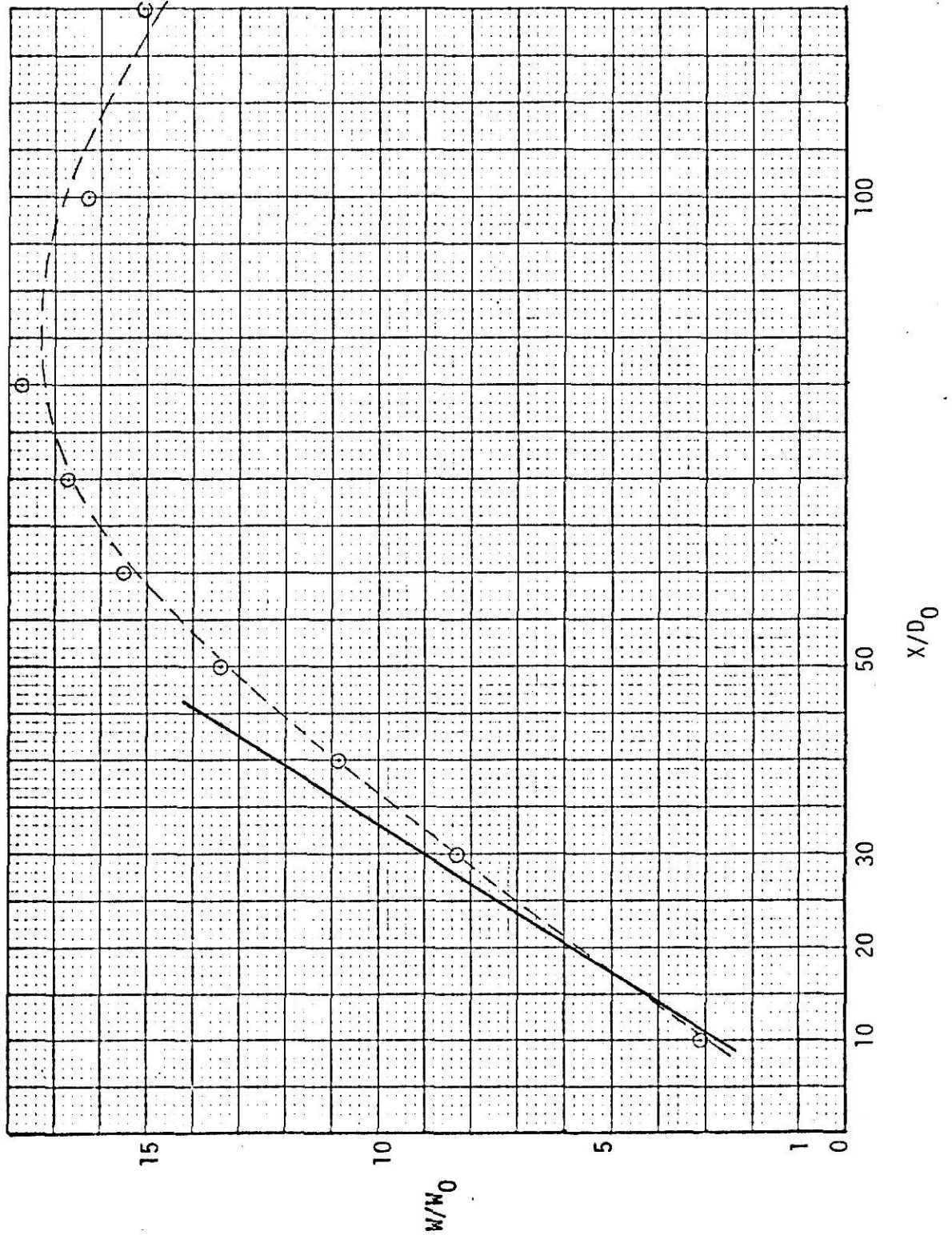


Figure 16. Flow Rate within a 1000 fpm
Isothermal Jet

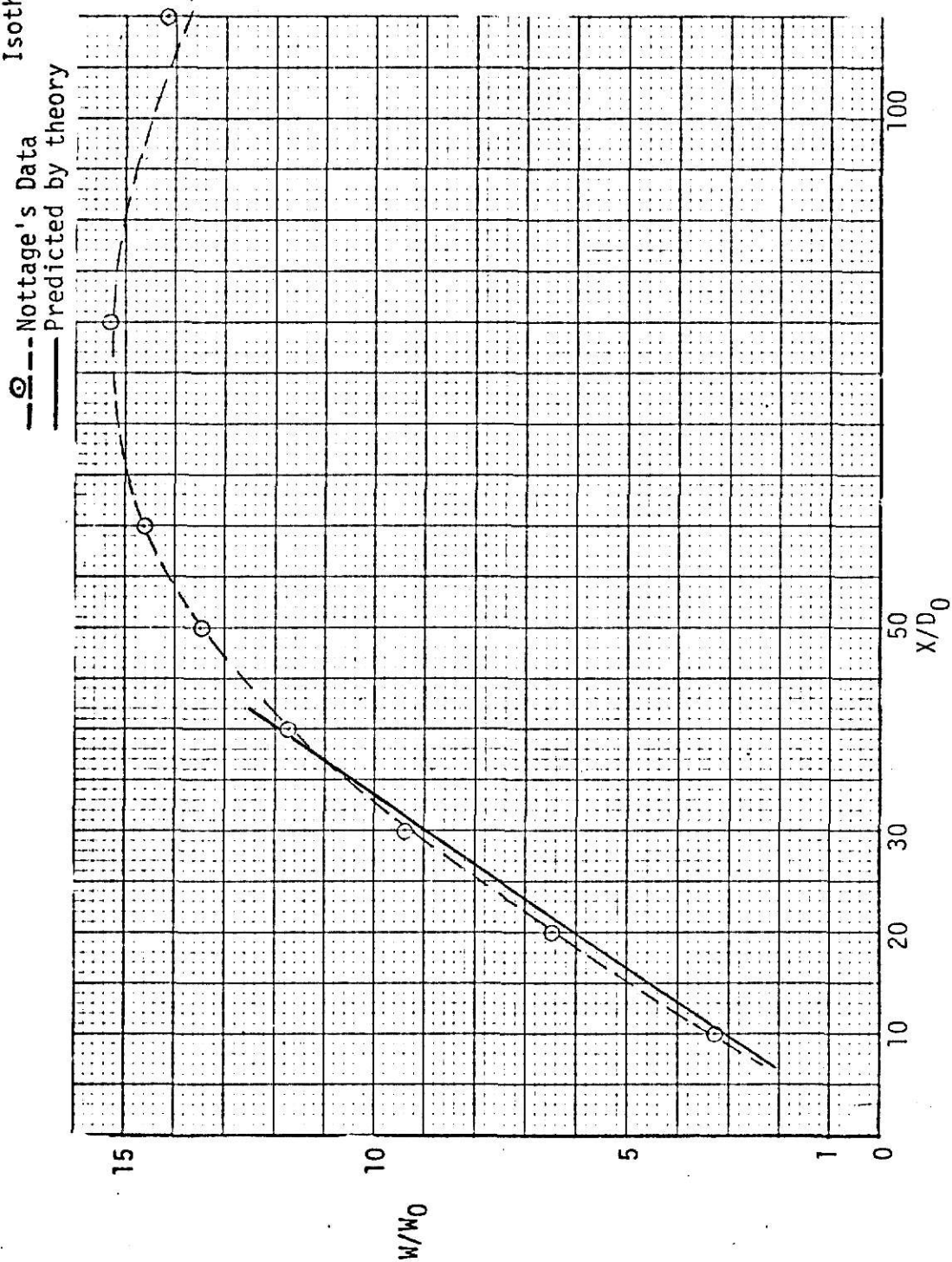


Figure 17. Flow Rate within a 500 fpm
Isothermal Jet

$\bigcirc$ --- Nottage's Data
 --- Predicted by theory (conone = 0.075)
 --- Predicted by theory (conone = 0.055)

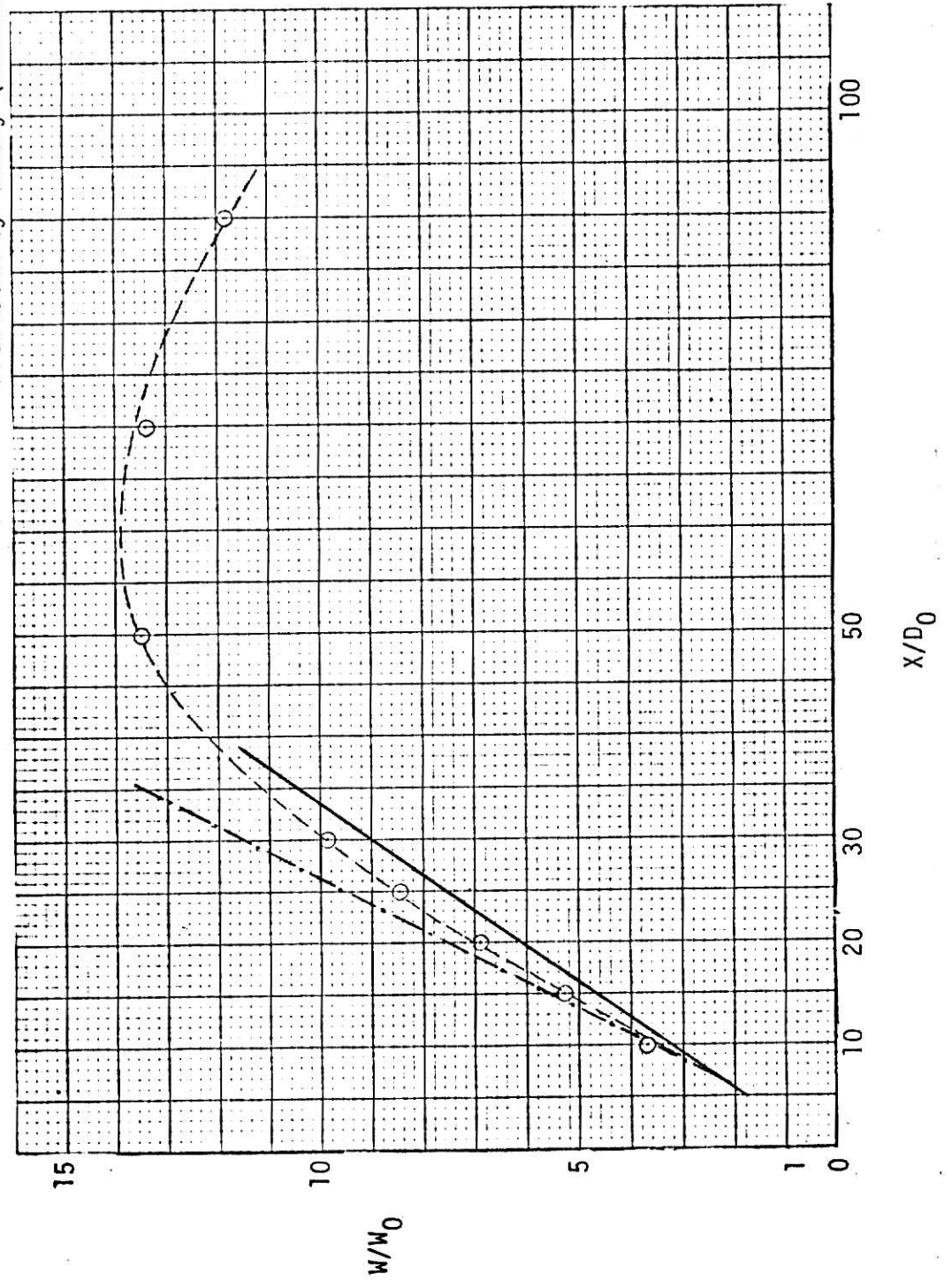


Figure 18. Flow Rate within a 350 fpm
Isothermal Jet

- Nottage's Data
- Predicted by theory (conone = 0.075)
- .-.- Predicted by theory (conone = 0.055)
- Predicted by theory (conone = 0.055)

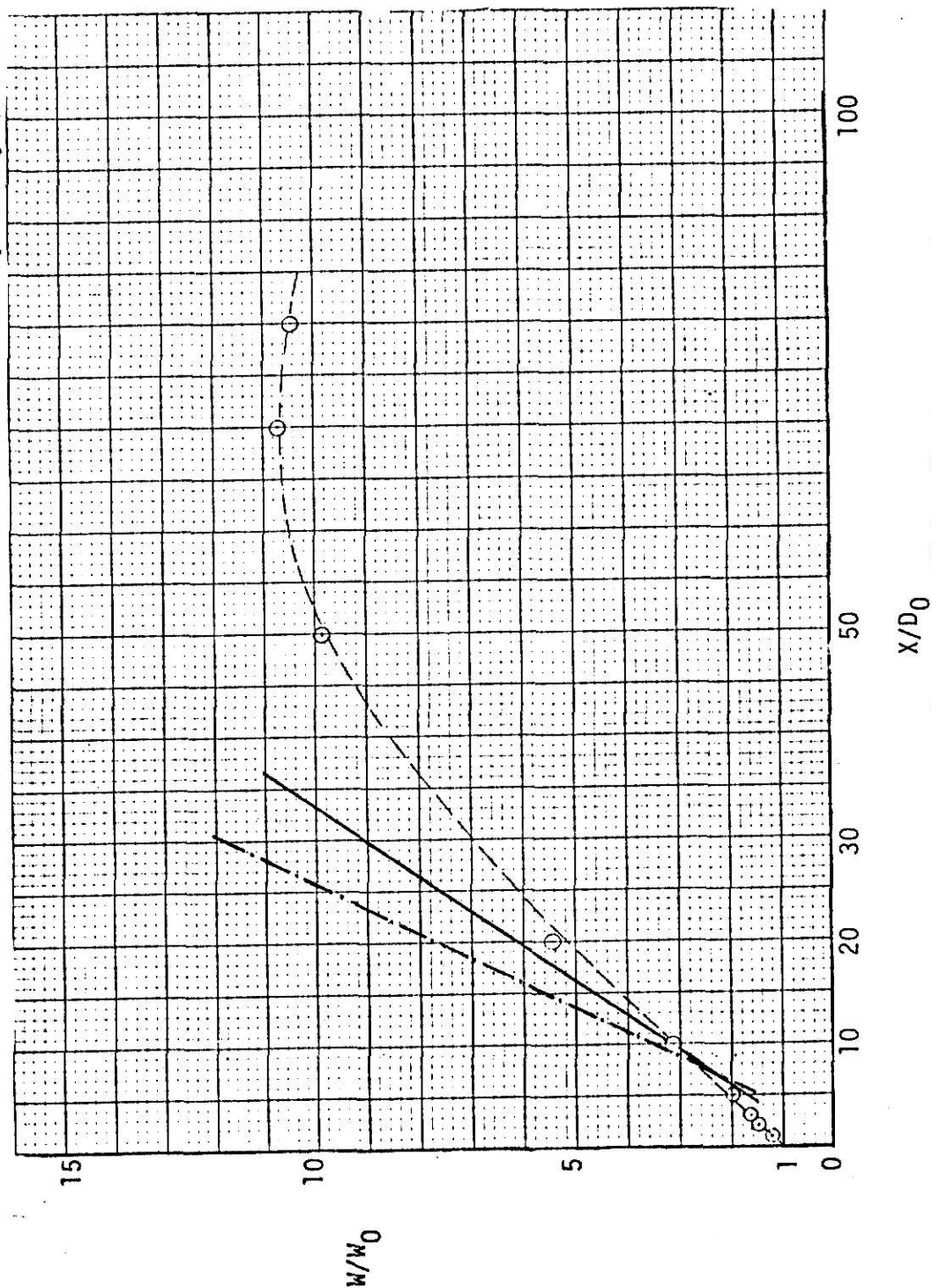
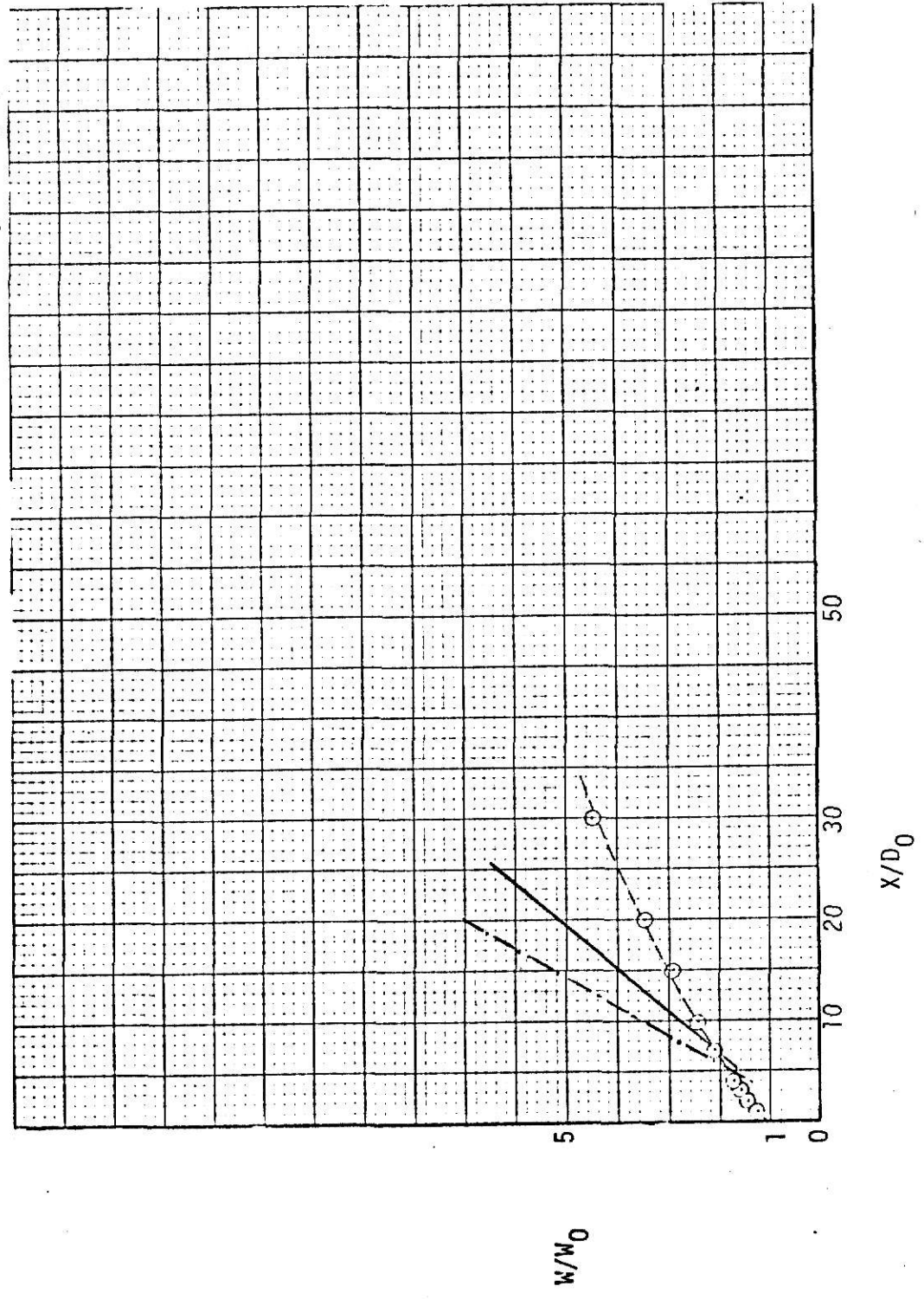


Figure 19. Flow Rate within a 100 fpm Isothermal Jet

- Nottag's Data
- Predicted by theory (conone = 0.075)
- Predicted by theory (conone = 0.055)



causes the jets to decelerate more rapidly than the present theory predicts. Considering the low velocities involved in these jets it may well be that they pass into region IV, the terminal region, much earlier than anticipated invalidating some of the assumptions made in the theory. This author elects to specify only one value for the constant, $A_1 = 0.055$. Overall it is considered that agreement between theory and experimental data for isothermal jets is good.

The predicted decay of both centerline temperature and velocity is compared to experimental data in Figure 20. Agreement again is good. Figure 21 shows predicted radius growth compared to experiment. As was the case with isothermal jets the theoretical radius growth is greater than that of the experimental data. The reasons for this greater rate of growth are the same as those discussed for the isothermal jets. Even so radius growth agrees quite well with experiment.

Theoretically predicted flow rate increases more rapidly than experimental data in Figure 22 indicates it should. This is not surprising and probably due to the use of Boussinesq approximation in the development of the theory. As a result of Boussinesq's approximation density variations are considered only in the buoyancy terms. This means that air entrained into the jet which is at a temperature greater than that of the air in the jet is considered to be as dense as the chilled air in the jet. While the density variation is small enough to justify the approximation it does lead to a somewhat inflated flow rate. This is not considered to be particularly important and comparison of flow rate to experimental data is considered moderately good.

The vertical plane trajectory predicted by the theory is compared to the experimental trajectory in Figure 23. Theory indicates that the jet will fall

Figure 20. Centerline Temperature and Velocity Variations for a 500 fpm Chilled Jet ($\Delta T_{\text{initial}} = 42^\circ\text{F}$)

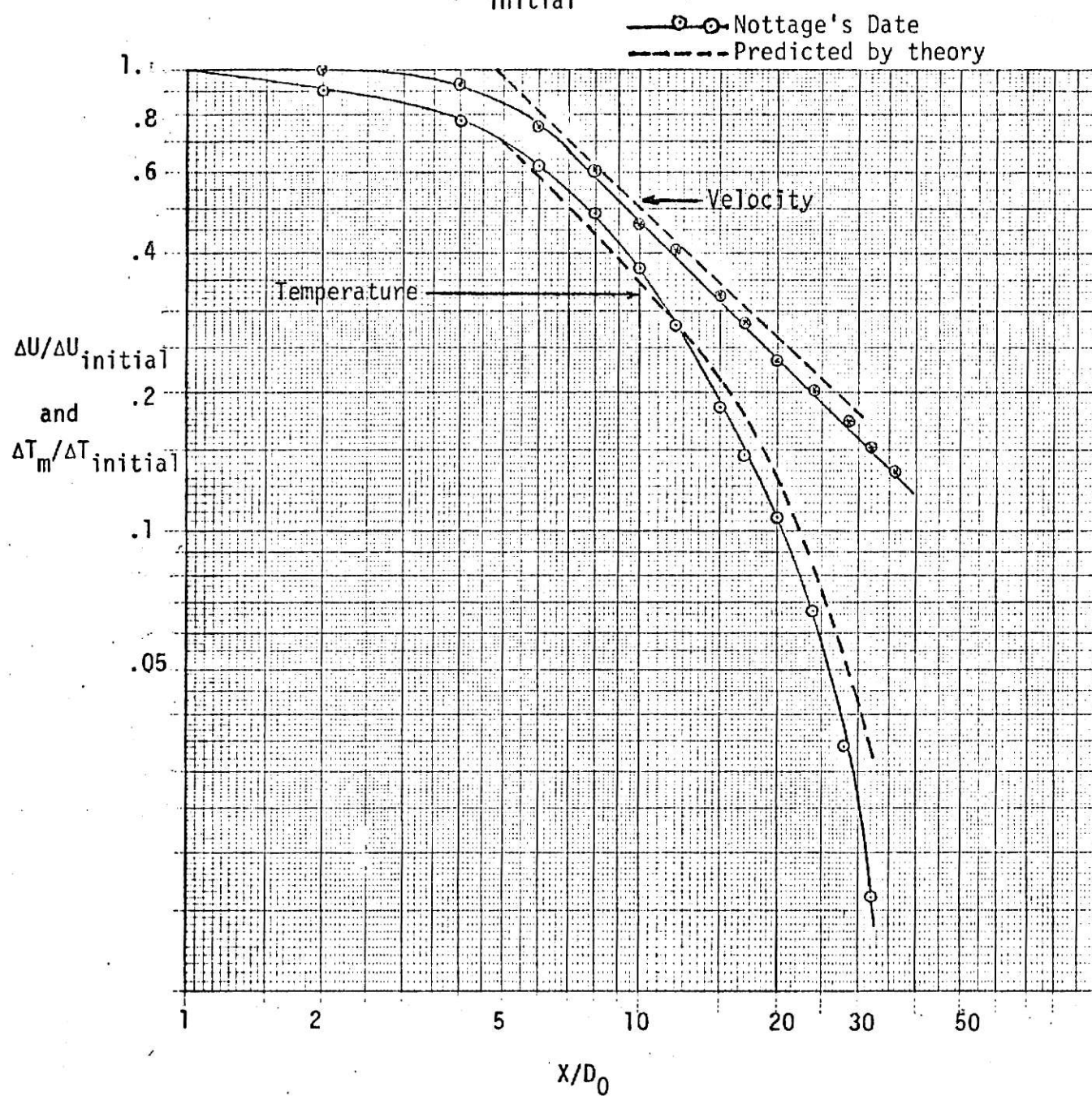


Figure 21. Thermal and Hydrodynamic Jet Radius Growth
 (500 fpm chilled jet, $\Delta T_m / \Delta T_{initial} = 42^\circ\text{F}$)

— Hydrodynamic (Nottage's Data)
 - - - Thermal (Nottage's Data)
 - · - · - Hydrodynamic (Predicted)
 - · - · - Thermal (Predicted)

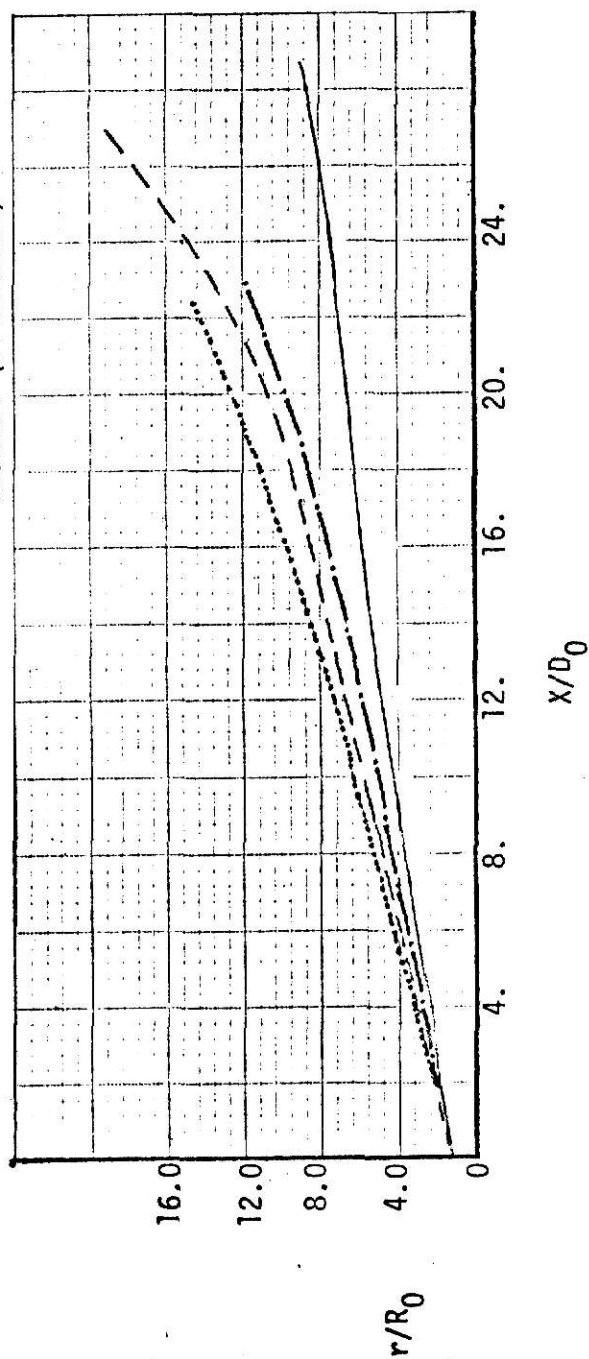


Figure 22. Flow Rate Within a 500 fpm Chilled Jet

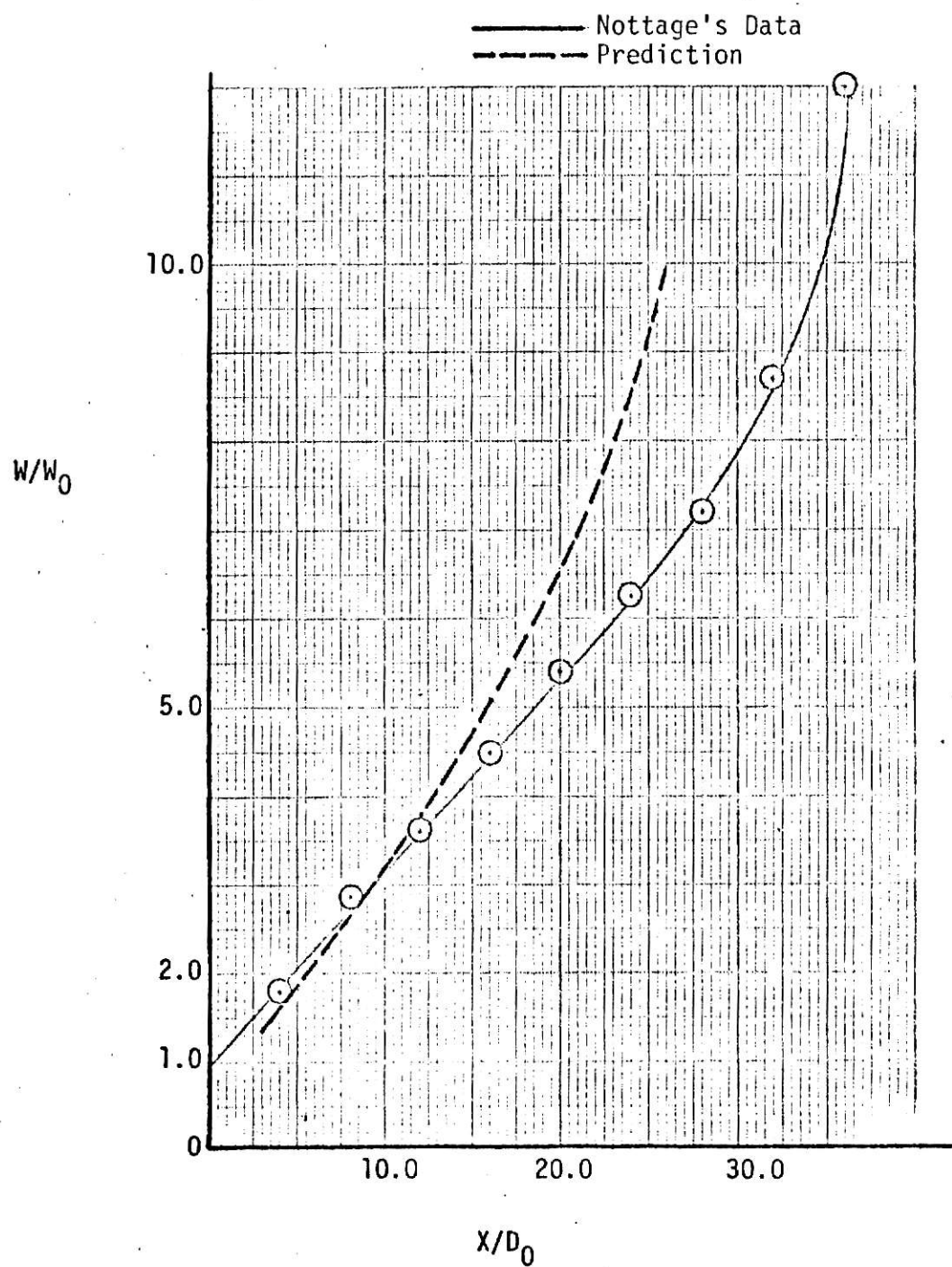
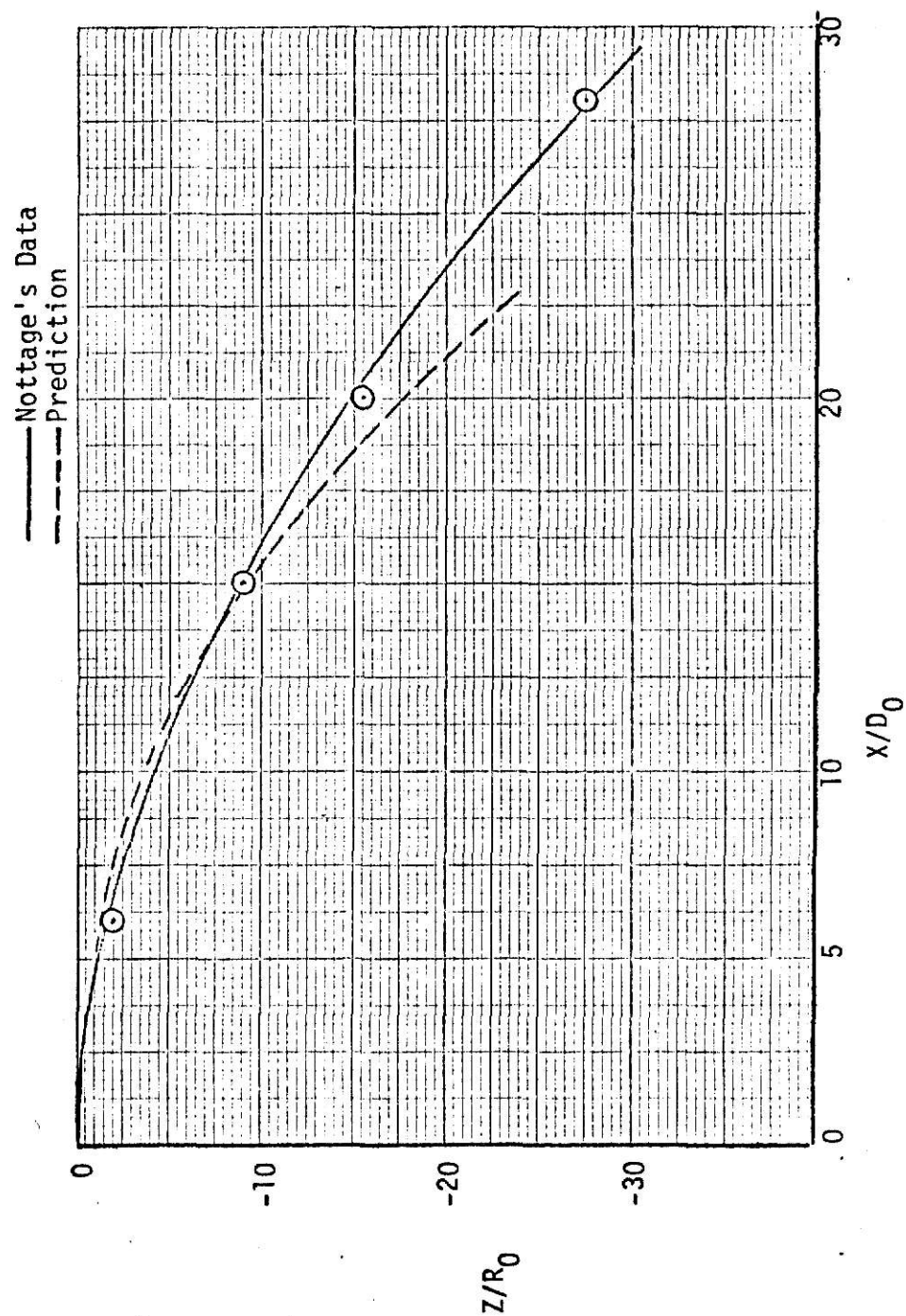


Figure 23. Vertical Plane Trajectory of a 500 fpm Chilled Jet



slightly more rapidly than it should. This rapid fall may be due to the fact that theoretically predicted temperature decays at a rate slightly slower than experiment indicates actually exists. This causes slightly higher buoyancy forces in the jet causing it to fall too rapidly. Refinement of the constants in the entrainment function might cure this difference in trajectory but unless a larger body of chilled, jet data were available this wouldn't seem to be worthwhile to invest much time in such a pursuit.

Overall agreement of theory to experiment is considered to be sufficiently good to make predictions for the flow for which the theory was developed. The predicted behavior of a chilled jet subjected to crossflow is discussed in Chapter VII.

CHAPTER VII. PREDICTED CHILLED JET BEHAVIOR UNDER THE INFLUENCE OF VERTICAL CROSSFLOW

The ultimate goal of this thesis has been prediction of the behavior of a horizontally projected chilled jet subjected to vertical buoyant crossflow. This crossflow might be either uniform along the axis of the jet or confined to one or more localities along the axis of the jet. The model predictions will now be examined.

First the case of uniform crossflow will be investigated. Figures 24 through 27 show comparisons of centerline velocity decay, centerline temperature decay, jet radius growth, and centerline trajectory for a 1000 fpm (bulk) velocity jet under the influence of a vertical crossflow of four varying velocities.

The existence of vertical crossflow on a horizontally projected crossflow should increase the shearing action along the boundaries of the jet which in turn should increase the rate of entrainment. Furthermore the greater the crossflow the greater the increase in the entrainment rate. With greater entrainment one should expect a more rapid rate of both centerline velocity and temperature decay. Figures 24 and 25 indicate that this is the case in the early portions of the curves. That is, as the crossflow velocity increases the curves shift down and left. However it is noted that the higher crossflow velocity curves began closing on the lower velocity curves in their latter regions. This may at first seem puzzling but the same shearing mechanism probably explains the shift. Figure 26 shows that in the same region where the rate of velocity decay slows the jet has been deflected so that the crossflow tends to lessen the effect of the shear mechanism.

Figure 24. Predicted Centerline Velocity Variation of a Chilled Jet in Uniform Crossflow ($\Delta U_{\text{initial}} = 17.0$; $\Delta T_{\text{initial}} = -31.2^\circ\text{F}$)

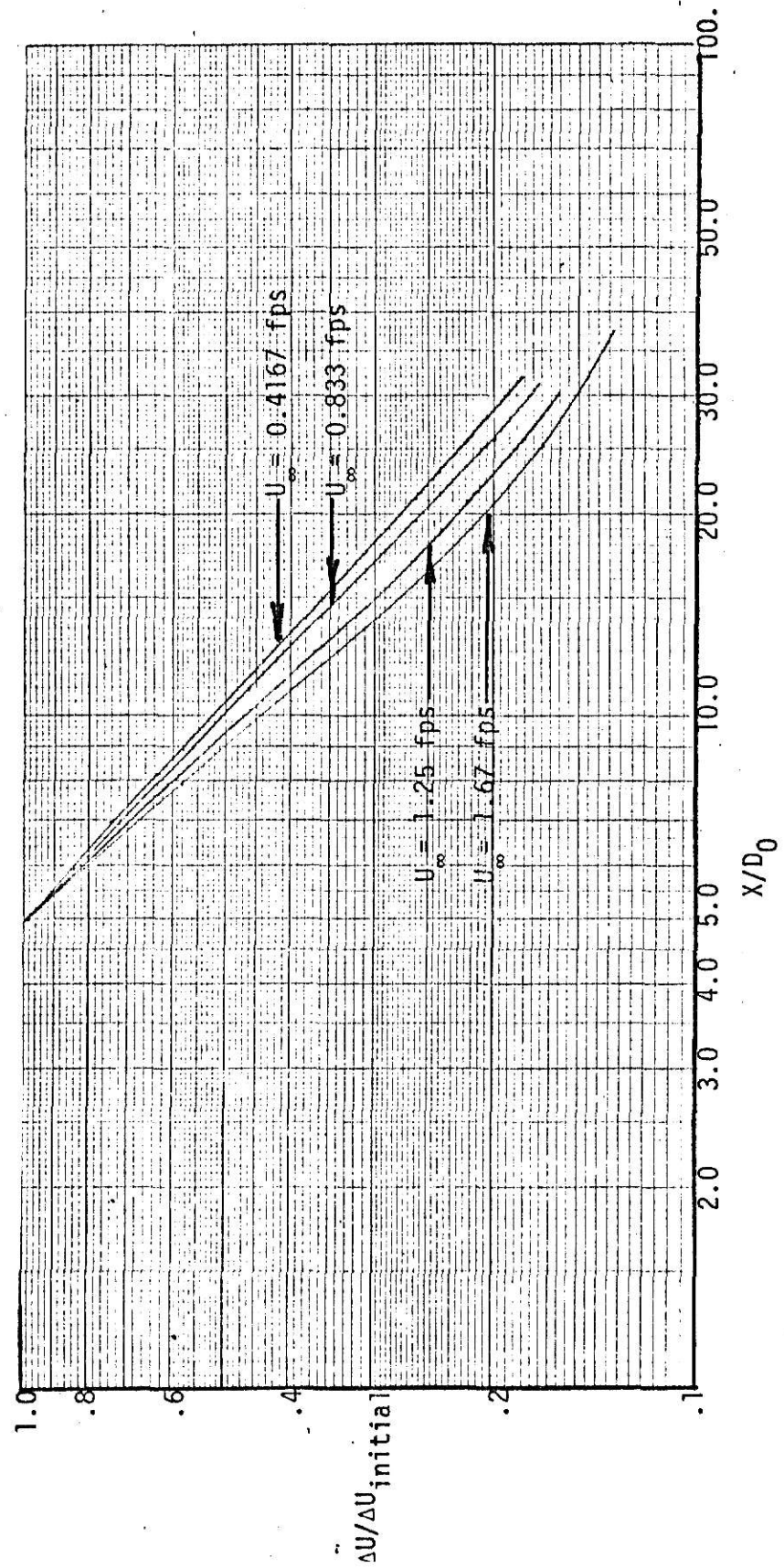


Figure 25. Predicted Centerline Temperature Variation of a Chilled Jet
in Uniform Crossflow ($\Delta U_{\text{initial}} = 17.0$ fps; $\Delta T_{\text{initial}} = -31.2^\circ\text{F}$)

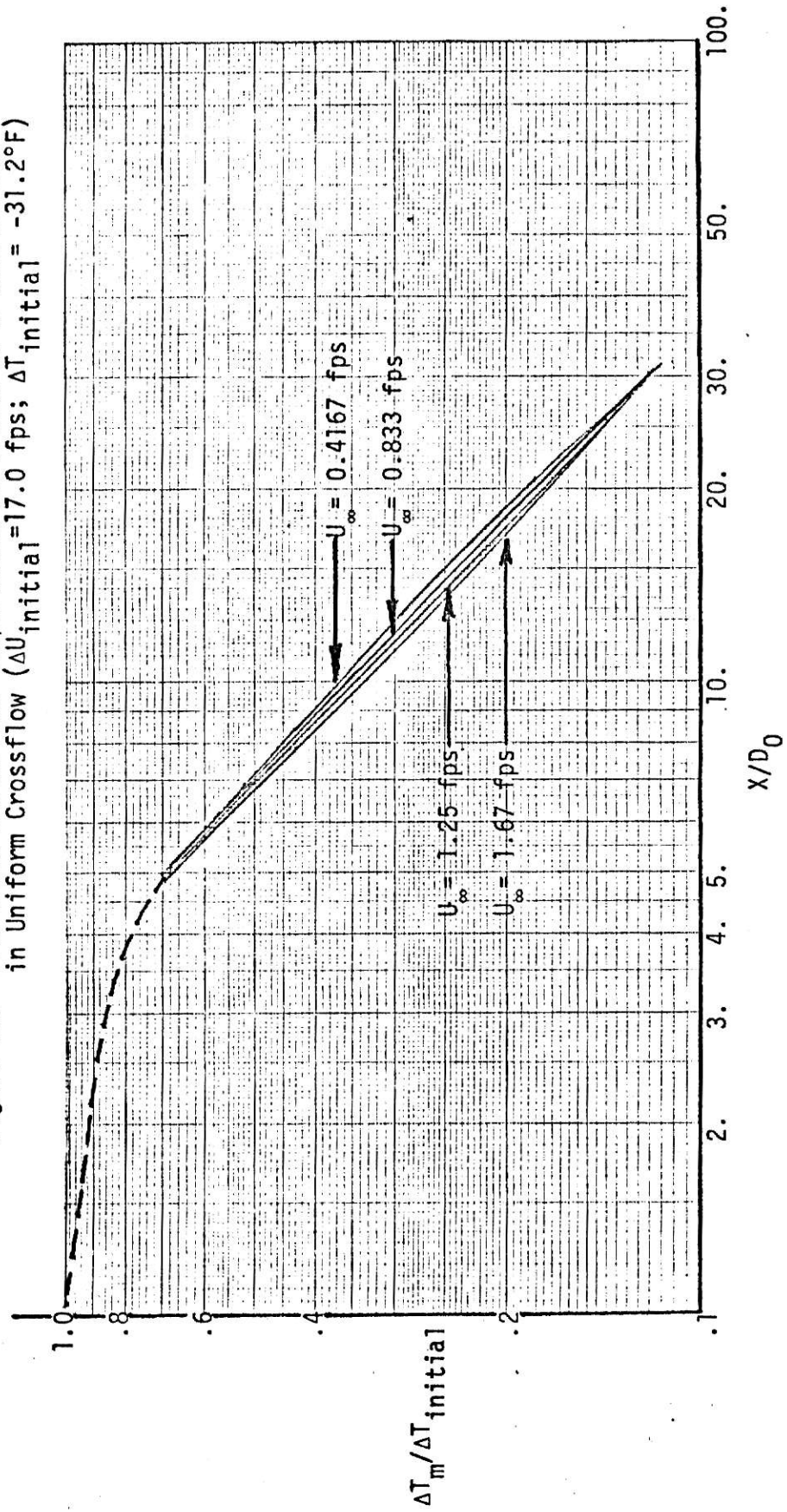


Figure 26. Predicted Vertical Plane Centerline Trajectory for a Chilled Jet in Uniform Crossflow ($\Delta U = 17.0$ fps; $\Delta T_{\text{initial}} = -31.2^\circ\text{F}$)

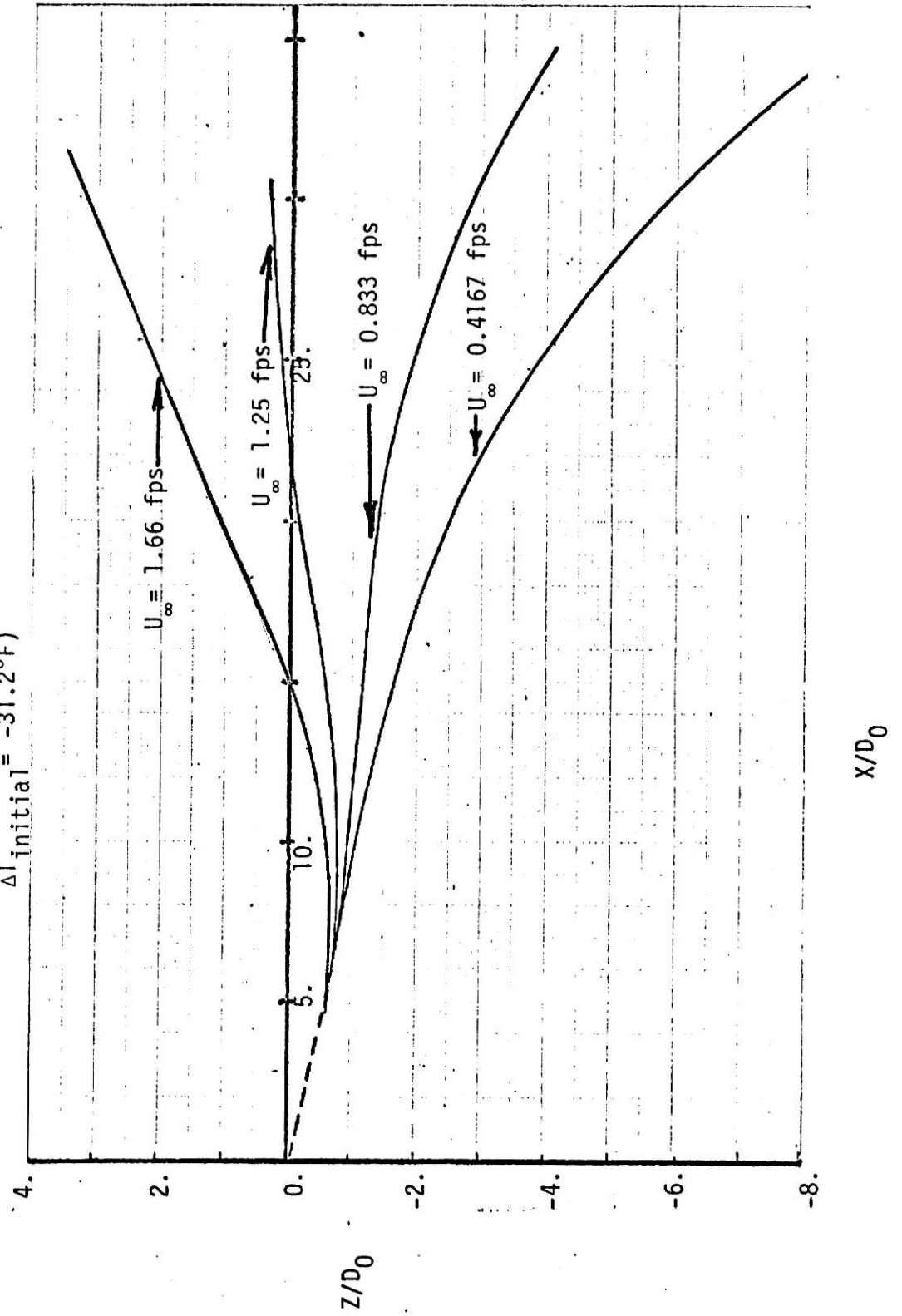


Figure 27. Predicted Radius Growth for a Chilled Jet
in Uniform Crossflow ($\Delta U_{\text{initial}} = 17.0$ fps;
 $\Delta T_{\text{initial}} = -31.2^\circ\text{F}$)

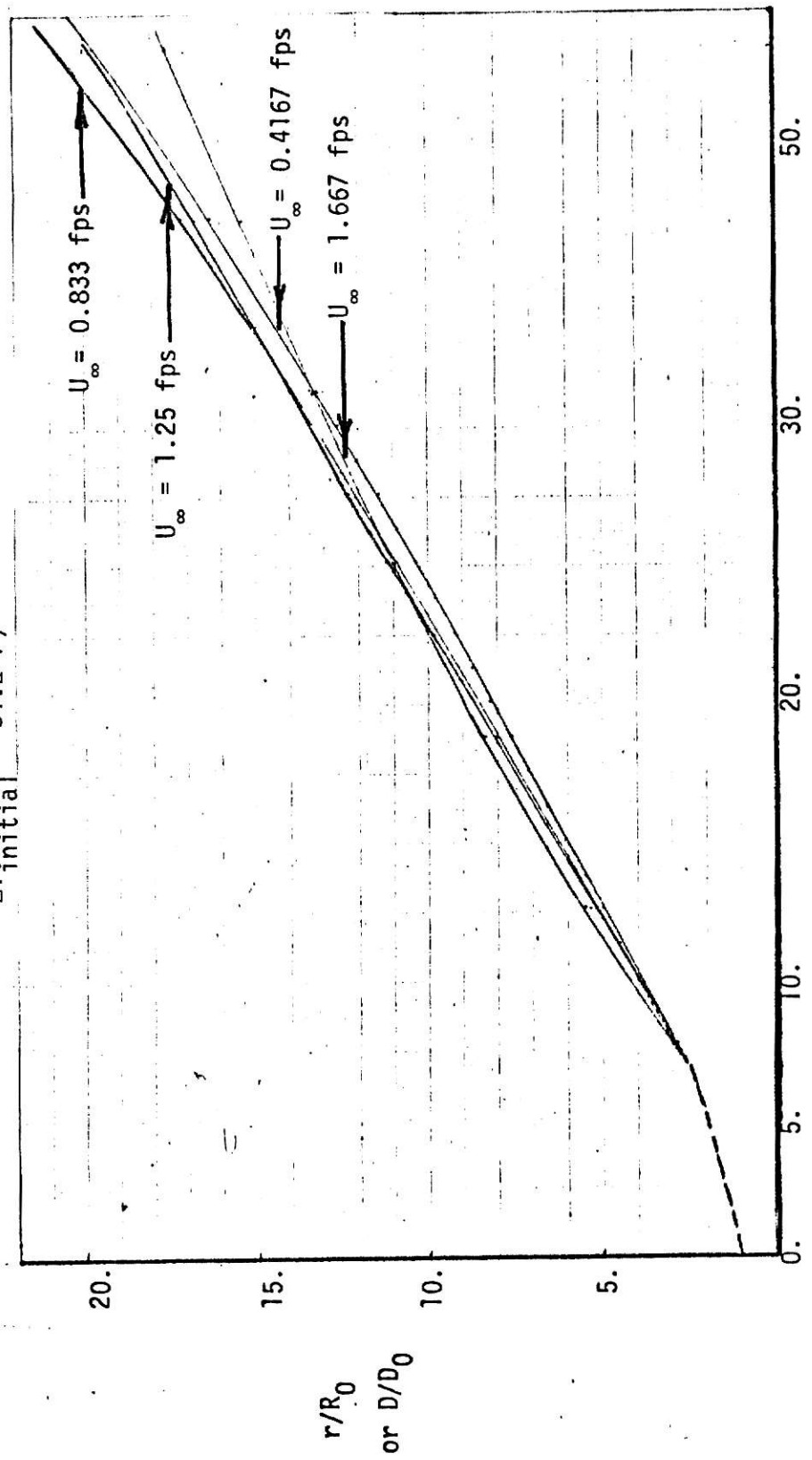


Figure 27 shows a comparison of radius growth for the four crossflows. Comparison of Figure 26 and 27 shows that in regions where the jet travels into the crossflow the radius of the jet grows more rapidly with increasing crossflow velocity. Likewise where the jet is deflected in the direction of the crossflow the rate of radius growth decreases. This phenomena is directly attributable to variations in the effect of the shear mechanism discussed above.

Finally, for uniform flow, Figure 26 compares the vertical plane centerline trajectories for the four crossflows under discussion. As the crossflow velocity increases the jet is deflected progressively farther in the direction of the crossflow. This is precisely as expected.

Figures 28 through 31 compare jet behavior for three localized crossflows of varying duration. In each case the crossflow is a buoyant plume whose velocity profile is approximated by a sine wave with a peak velocity of 100 fpm. By varying only the duration of the crossflow the effect of both duration and the rate of change of crossflow velocity can be partially isolated from changes in magnitude and orientation relative to the jet.

Figure 28 is a comparison of centerline velocity decay for crossflow durations 2.5 feet, 5.0 feet, and 7.5 feet. In all cases the existence of crossflow causes an increase in the rate of centerline velocity decay. A check of Figure 31 shows that in no case is the crossflow of sufficient magnitude or velocity to deflect the jet upward. This should cause an increase in the shear mechanism the entire time the jet is affected by the crossflow. This increase is reflected in the increased rates of velocity decay.

Examining the three crossflow curves in Figure 28 the relative magnitudes of the changes in decay rate are found to be reasonable. In the short duration crossflow the crossflow velocity builds up rapidly and the decay rate increases accordingly. When the jet passes out of the crossflow it resumes its undisturbed decay rate with its decay curve running approximately parallel to the decay curve of the jet with no crossflow. In the medium duration crossflow the crossflow velocity builds more slowly and the decay rate increases. However the rate at which the decay rate increases is not as great as in the short duration crossflow. The decay rate remains at a higher level for a longer period of time reflecting the longer duration of the crossflow. When the jet finally leaves the crossflow its centerline velocity is lower than after the shorter duration crossflow. The same effects appear in the longest duration crossflow. Furthermore identical effects are expected in the temperature decay curve and indeed examination of Figure 29 shows that they are there.

Figure 30 compares radius growth for the same jet and local crossflows. While under the influence of the crossflow the rate of radius growth shows an increase. As the duration of the crossflow increases the duration of the accelerated radius growth increases. The result predicted by the model is that the jet will have a greater diameter after leaving the longer duration crossflow.

Figure 31 shows predicted centerline trajectory for the jet under the influence of the three crossflows. The medium duration crossflow deflects the jet more than the short duration and the long duration crossflow deflects the jet more than the medium. This is as it should be. Furthermore the rate at which the jet is deflected is proportional to the rate at which the crossflow velocity increases. Again this is exactly as expected.

Figure 28. Predicted Centerline Velocity Variation in a Chilled Jet in Localized Crossflow ($\Delta T_{\text{initial}} = -31.2^\circ\text{F}$; $\Delta U_{\text{initial}} = 17.0$ fps; $U_\infty = \text{crossflow}$)

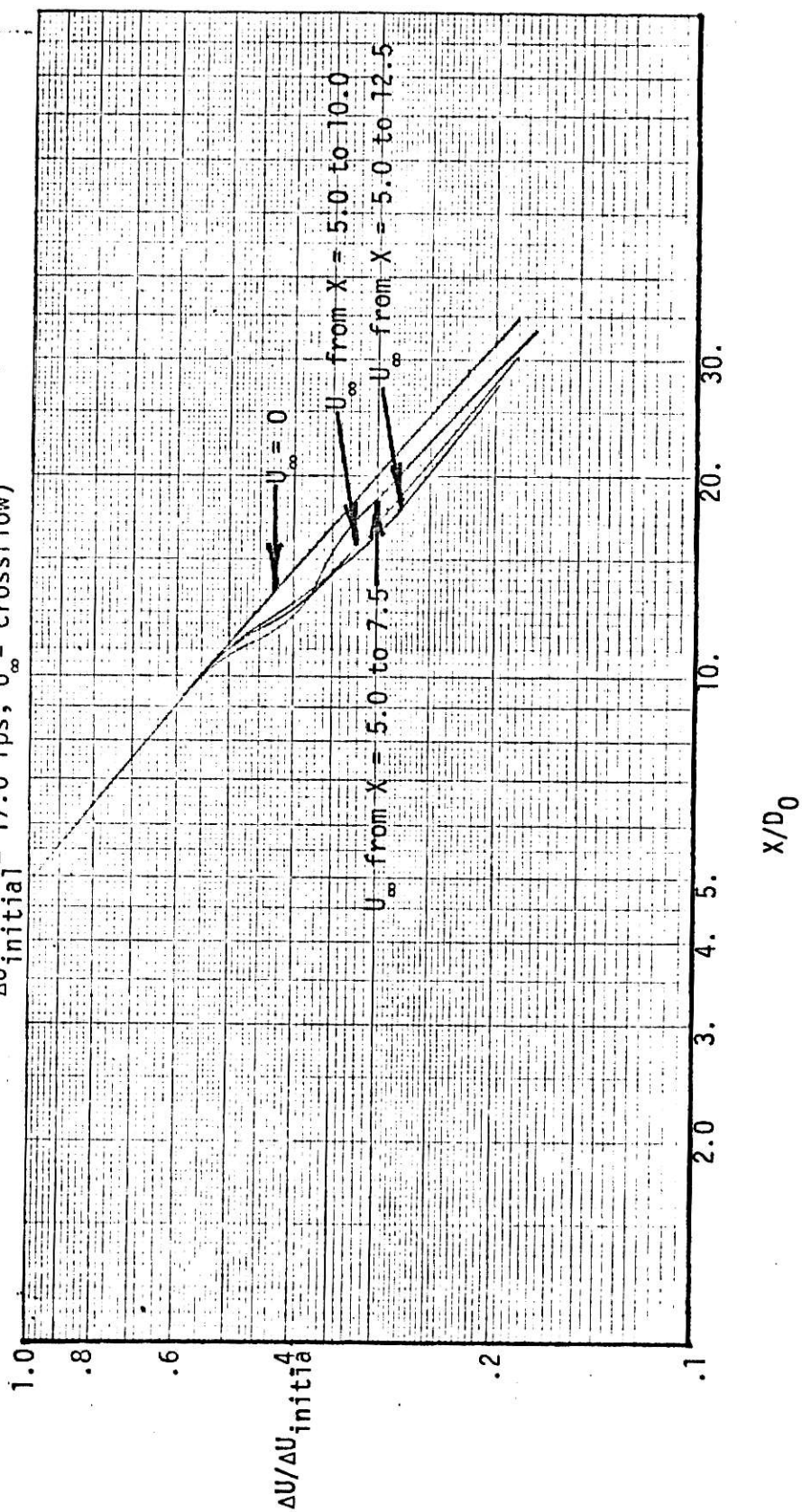


Figure 29. Predicted Centerline Temperature Variation of a Chilled Jet in Localized Crossflow ($\Delta T_{initial} = -31.2^\circ\text{F}$; $\Delta U_{initial} = 17.0$ fps; $U_\infty = \text{crossflow}$)

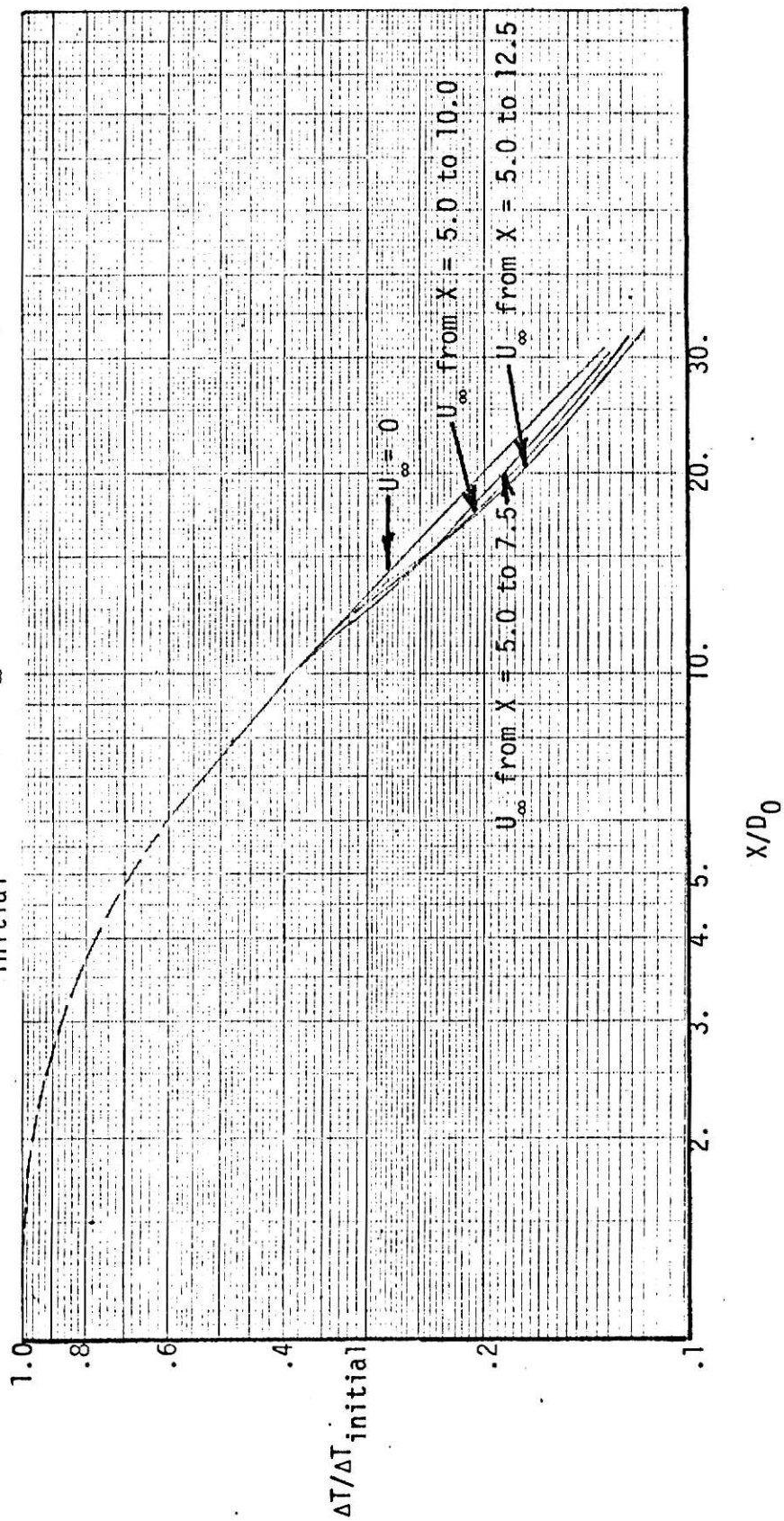


Figure 30. Predicted Radius Growth of a Chilled Jet in Localized Crossflow ($U = \text{crossflow}$; $\Delta U_{\text{initial}} = 17.0 \text{ fps}$; $\Delta T_{\text{initial}} = -31.2^\circ\text{F}$)

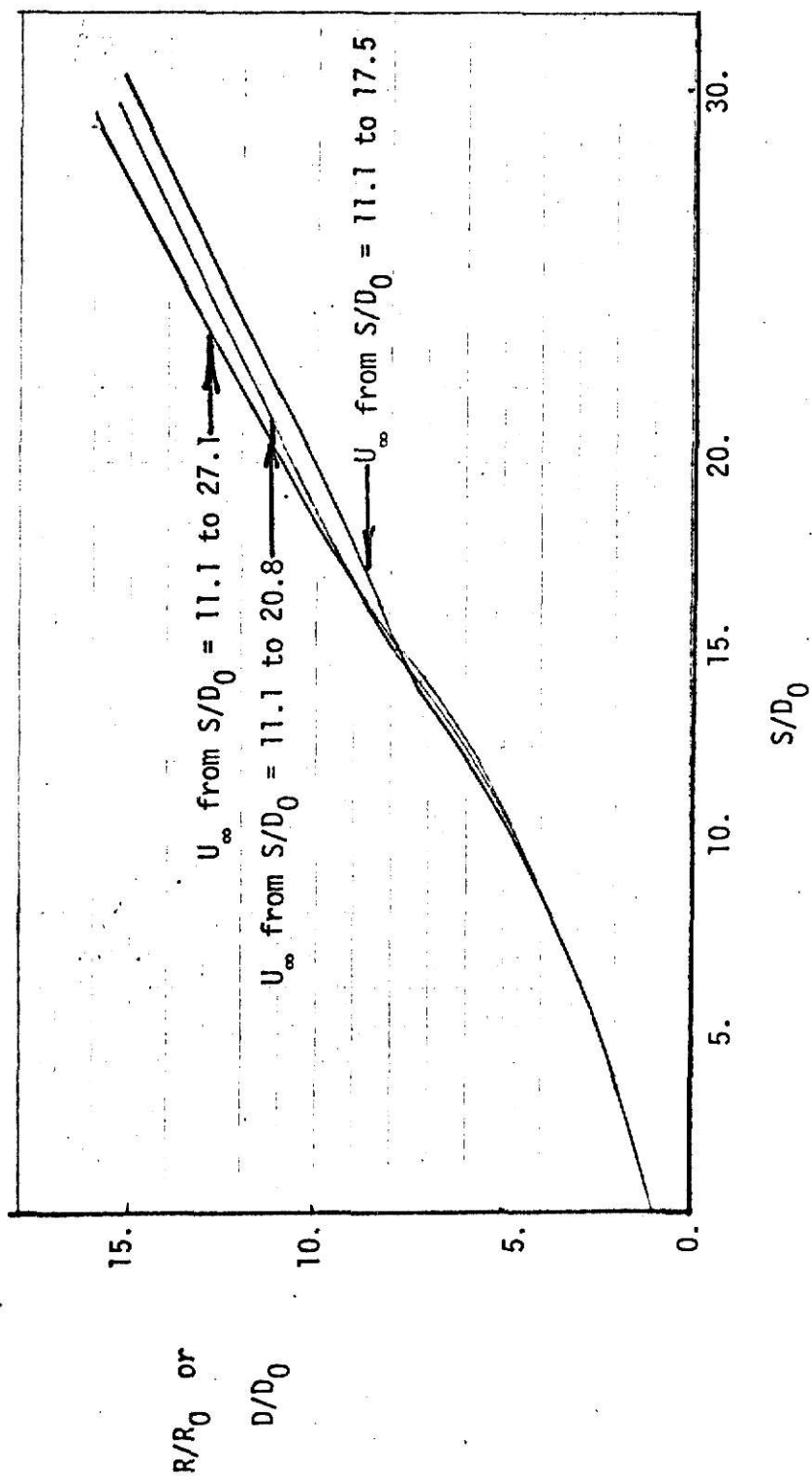
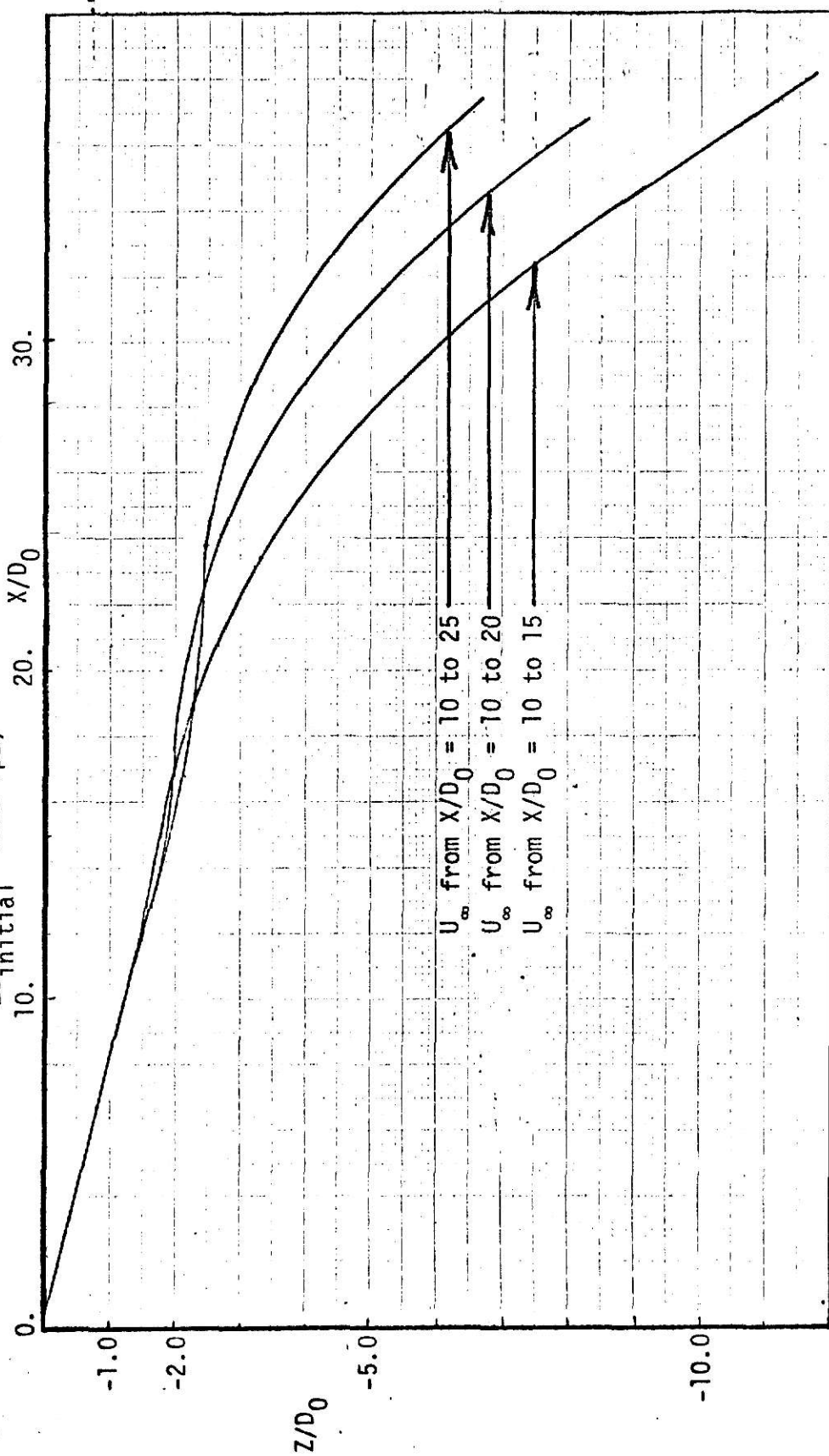


Figure 31. Predicted Vertical Plane Trajectory of a Chilled Jet in Localized Crossflow (U_∞ = crossflow; $\Delta T_{\text{initial}} = -31.2^\circ\text{F}$; $\Delta U_{\text{initial}} = 17.0$ fps)



The predictions made here should be interpreted as qualitative rather than quantitative. The value of the constant in the entrainment function that weights the shear and buoyancy effects on one hand the crossflow effects on the other was selected essentially arbitrarily since the purpose here was only to demonstrate the reasonableness of the model. The model predictions of jet behavior have been examined in detail in this and the previous chapter. The author contends that this is an extremely reasonable and believable model.

CONCLUSIONS

As a result of this analysis it was found that:

1. The model predicts the behavior of isothermal free turbulent jets with moderately good accuracy. This implies that the postulated result of integration of the shear stress integral is correct.
2. The model predicts the behavior of Nottage's chilled jet with reasonable accuracy.
3. The model responds to the influences of shear, buoyancy and crossflow in a reasonable manner.

RECOMMENDATIONS

It is considered that the jet model developed in this thesis responds to the influence of shear, buoyancy and crossflow in a sufficiently reasonable manner to warrant the investment of time and effort to secure experimental verification of its predictions. It is recommended that an experimental apparatus be designed and built and that the appropriate experimentation be performed.

It is recommended that further research be done on the nature of the relationship between turbulent fluctuations and the mean values of the flow variables. It is recommended that this work be applied directly to the evaluation of the shear stress integral encountered in the derivation of the entrainment function in this thesis.

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APPENDIX A: THE COORDINATE SYSTEM

The coordinate system used in this analysis is shown in the figure below.

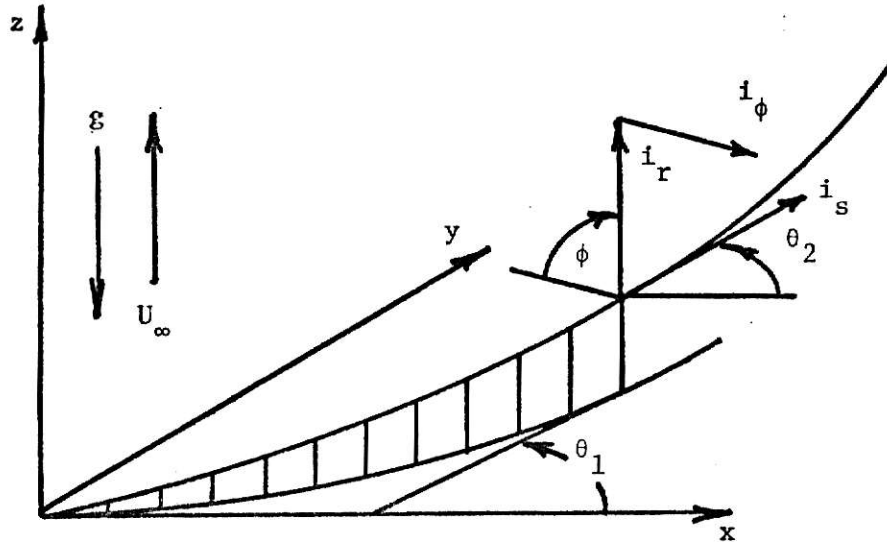


Figure 32. The Curvilinear Coordinate System.

In the figure x , y , and z are the Cartesian coordinates with i , j , and k the corresponding unit vectors. z is the vertical coordinate and is set parallel to the buoyancy force and the crossflow.

s , r , and ϕ are the natural curvilinear coordinates with i_s , i_r and i_ϕ as the corresponding unit vectors. s is a measure of distance along the jet centerline from the outlet (origin) and the corresponding unit vector i_s is always tangent to the centerline. r is a measure of the distance from the jet axis in a plane perpendicular to i_s . r is equal to zero at the centerline of the jet. The angle ϕ determines the orientation of r . When looking along the jet centerline in the direction of increasing s , ϕ equals zero when i_r is horizontal to the left. ϕ increases in the clockwise direction.

Under the definitions just specified, s , r , and ϕ form a right-handed system of orthogonal curvilinear coordinates. For any particular s , r and ϕ are simply the usual polar coordinates.

The curvilinear and Cartesian systems are related by angles θ_1 and θ_2 . θ_1 is the angle between the x-axis and the projection of i_s onto the x-y plane. θ_2 is the angle between i_s and the x-y plane. Since nothing acts in the y-direction θ_1 remains constant at zero and the jet remains in the x-z plane. The behavior of θ_2 responds to both buoyancy force and crossflow.

The relationships among the unit vectors are shown in Table A-1.

Table A-1. Relationships among the unit vectors in Cartesian natural coordinate systems.

	i	j	k
i_s	$C_1 C_2$	$S_1 S_2$	S_2
i_r	$-(C_\phi S_1 + S_\phi C_1 S_2)$	$(C_\phi C_1 - S_\phi S_1 S_2)$	$S_\phi C_2$
i_ϕ	$S_\phi S_1 - C_\phi C_1 S_2$	$-(S_\phi C_1 + C_\phi S_1 S_2)$	$C_\phi C_2$
	$C_1 = \cos \theta_1$	$C_\phi = \cos \phi$	
	$S_1 = \sin \theta_1$	$S_\phi = \sin \phi$	

Use of Table A-1 is illustrated by the following examples:

$$i_s = C_1 C_2 i + S_1 C_2 j + S_2 k$$

$$k = S_2 i_s + S_\phi C_2 i_r + C_\phi C_2 i_\phi$$

The basic differential equations in Cartesian coordinates are written in terms of the gradient divergence and curl. These operations can be defined (written) in the natural coordinate system only after a set of scale factors, h , have been obtained.

The scale factors are:

$$h_s = \frac{\partial R}{\partial r} \cdot i_s$$

$$h_r = \frac{\partial R}{\partial r} \cdot i_r$$

$$h_\phi = \frac{\partial R}{\partial \phi} \cdot i_\phi$$

where: $R = si_s + ri_r$, the position vector of any point in the jet.

The scale factors relate the differential distances to the differentials of the coordinates. In general, the scale factors vary from point to point. Only for the Cartesian system are the scale factors constant.

To evaluate h_s :

$$h_s = \frac{\partial R}{\partial s} \cdot i_s \quad \text{and} \quad R = si_s + ri_r$$

$$\frac{\partial R}{\partial s} = i_s + s \frac{\partial i_s}{\partial s} + \frac{\partial r}{\partial s} i_r + r \frac{\partial i_r}{\partial s}$$

$$\frac{\partial R}{\partial s} \cdot i_s = 1 + 0 + 0 + r \frac{\partial i_r}{\partial s} \cdot i_s$$

$$\frac{\partial i_r}{\partial s} = \frac{\partial i_r}{\partial \theta_1} \frac{\partial \theta_1}{\partial s} + \frac{\partial i_r}{\partial \theta_2} \frac{\partial \theta_2}{\partial s}$$

$$\rho_1 = \frac{\partial s}{\partial \theta_1} \quad \text{and} \quad \rho_2 = \frac{\partial s}{\partial \theta_2}$$

$$\frac{\partial i_r}{\partial s} = \frac{\partial i_r}{\partial \theta_1} \frac{1}{\rho_1} + \frac{\partial i_r}{\partial \theta_2} \frac{1}{\rho_2}$$

Using Table A-1

$$\begin{aligned} \frac{\partial i_r}{\partial s} &= \frac{1}{\rho_1} \left(-(C_\phi C_1 - S_\phi S_1 S_2) i - (C_\phi S_1 + S_\phi C_1 S_2) j \right) \\ &\quad - \frac{1}{\rho_2} (S_\phi C_1 C_2 i + S_\phi S_1 C_2 j + S_\phi S_2 k) \end{aligned}$$

$$i_s = C_1 C_2 i + S_1 C_2 j + S_2 k$$

Taking the scalar product of the above two equations and multiplying through by r the following result can be obtained.

$$r \frac{\partial i_r}{\partial s} \cdot i_s = - \frac{r}{\rho_1} C_\phi C_2 - \frac{r}{\rho_2} S_\phi$$

Finally, from the definition:

$$h_s = 1 - r \left(\frac{C_\phi C_2}{\rho_1} + \frac{S_\phi}{\rho_2} \right) \equiv 1 - e$$

The remaining scale factors can be obtained in a similar fashion. They are:

$$h_r = 1 \quad \text{and} \quad h_\theta = r$$

The gradient [1] of any scalar is:

$$\nabla A = \frac{1}{h_s} \frac{\partial A}{\partial s} i_s + \frac{1}{h_r} \frac{\partial A}{\partial r} i_r + \frac{1}{h_\phi} \frac{\partial A}{\partial \phi} i_\phi \quad (\text{A-1})$$

Substituting the scale factors into equation (A-1) produces the following result:

$$\nabla A = \frac{1}{1-e} \frac{\partial A}{\partial s} i_s + \frac{\partial A}{\partial r} i_r + \frac{1}{r} \frac{\partial A}{\partial \phi} i_\phi \quad (\text{A-2})$$

The divergence [1] of a vector is:

$$\nabla \cdot A = \frac{1}{h_s h_r h_\phi} \left(\frac{\partial (h_r h_\phi a_s)}{\partial s} + \frac{\partial (h_\phi h_s a_r)}{\partial r} + \frac{\partial (h_s h_r a_\phi)}{\partial \phi} \right) \quad (\text{A-3})$$

where $A = a_s i_s + a_r i_r + a_\phi i_\phi$.

Substituting the scale factors into equation (A-3) produces the following result:

$$\nabla \cdot A = \frac{1}{1-e} \frac{\partial a_s}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (r a_r) + \frac{1}{r} \frac{\partial a_\phi}{\partial \phi} - \frac{a_r}{1-e} \left(\frac{e}{r} \right) - \frac{a_\phi}{r(1-e)} \frac{\partial e}{\partial \phi} \quad (\text{A-4})$$

The curl [1] of a vector is:

$$\nabla \times \mathbf{A} = \frac{1}{h_s h_r h_\phi} \begin{vmatrix} h_s \mathbf{i}_s & h_r \mathbf{i}_r & h_\phi \mathbf{i}_\phi \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} \\ h_s a_s & h_r a_r & h_\phi a_\phi \end{vmatrix} \quad (\text{A-5})$$

Substituting the scale factors into equation (A-5) produces the following result:

$$\begin{aligned} \nabla \times \mathbf{A} = & \frac{1}{r} \frac{\partial}{\partial r} (r a_\phi) - \frac{\partial a_r}{\partial \phi} \mathbf{i}_s + \frac{1}{r} \frac{\partial a_s}{\partial \phi} - \frac{a_s}{1-e} \left(\frac{1}{r} \frac{\partial e}{\partial \phi} \right) - \frac{1}{1-e} \frac{\partial a_\phi}{\partial s} \mathbf{i}_r \\ & + \frac{1}{1-e} \frac{\partial a_r}{\partial s} - \frac{\partial a_s}{\partial r} + \frac{a_s}{1-e} \frac{e}{r} \mathbf{i}_\phi \end{aligned} \quad (\text{A-6})$$

APPENDIX B: CONVERSION OF THE BASIC EQUATIONS TO NATURAL CURVILINEAR COORDINATES

Under the assumptions noted in the text the basic equations of this analysis in Cartesian coordinates are as follows:

The Continuity Equation:

$$\nabla \cdot \mathbf{V} = 0 \quad (\text{B-1})$$

The Energy Equation:

$$\mathbf{V} \cdot (\nabla t) = 0 \quad (\text{B-2})$$

The Momentum Equation:

$$\frac{1}{2} \nabla V^2 - \mathbf{V} \times (\nabla \times \mathbf{V}) = g\beta(t - t_\infty) - \frac{\nabla p^*}{\rho_o} \quad (\text{B-3})$$

Applying equation (A-4) to equation (B-1) yields:

$$\frac{1}{1-e} \frac{\partial u}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (rv) + \frac{1}{r} \frac{\partial w}{\partial \phi} - \frac{v}{1-e} \left(\frac{e}{r}\right) - \frac{w}{r(1-e)} \frac{\partial e}{\partial \phi} = 0 \quad (\text{B-4})$$

where: $\mathbf{V} = ui_s + vi_r + wi_\phi$

Applying equations (A-2) and (A-4) to equation (B-2) yields:

$$\frac{u}{1-e} \frac{\partial t}{\partial s} + v \frac{\partial t}{\partial r} + \frac{w}{r} \frac{\partial t}{\partial \phi} = 0 \quad (\text{B-5})$$

It is noted that while both the continuity equation and the energy equations are scalar the momentum equation is a vector equation. Consequently the momentum equation can be separated into three scalar equations. In separating the vector momentum equation the buoyancy form term and the pressure term must be separated

$$\frac{\nabla p^*}{\rho_o} = P_s^* i_s + P_r^* i_r + P_\phi^* i_\phi$$

From this manner in which the coordinate system was defined it is noted that the buoyancy force has only one component and it lies in the k direction. Therefore utilizing Table A-1 the following result is obtained:

$$g\beta(t - t_{\infty})k = g\beta(t - t_{\infty}) [S_2 i_s + S_{\phi} C_2 i_r + C_{\phi} C_2 i_{\phi}]$$

Applying equations (A-2) and (A-5) to the left side of equation (B-3) and separating the buoyancy force and pressure terms as shown above the following three scalar momentum equations can be written:

The s-momentum equation:

$$\frac{u}{1-e} \frac{\partial u}{\partial s} + \frac{w}{r} \frac{\partial u}{\partial \phi} - \frac{uw}{1-e} \left(\frac{1}{r} \frac{\partial e}{\partial \phi} \right) + v \frac{\partial u}{\partial r} - \frac{uv}{1-e} \frac{e}{r} = g\beta(t - t_{\infty}) S_2 - P_s^* \quad (B-6)$$

The r-momentum equation:

$$v \frac{\partial v}{\partial r} + \frac{u}{1-e} \frac{\partial v}{\partial s} + \frac{u^2}{1-e} \frac{e}{r} - \frac{w^2}{r} + \frac{w}{r} \frac{\partial v}{\partial \phi} = g\beta(t - t_{\infty}) S_{\phi} C_2 - P_r^* \quad (B-7)$$

The ϕ -momentum equation:

$$\frac{w}{r} \frac{\partial w}{\partial \phi} + v \frac{\partial w}{\partial r} + \frac{vw}{r} + \frac{u^2}{1-e} \left(\frac{1}{r} \frac{\partial e}{\partial \phi} \right) + \frac{u}{1-e} \frac{\partial w}{\partial s} = g\beta(t - t_{\infty}) C_{\phi} C_2 - P_{\phi}^* \quad (B-8)$$

APPENDIX C: DERIVATION OF THE AXISYMMETRIC EQUATIONS

The assumption of axisymmetry will now be placed on the basic equations as set forth in Appendix B. This assumption dictates that $w = 0$ and $\partial/\partial\phi = 0$. Additionally the nonhydrostatic pressure gradient will now be assumed negligible.

Under the assumptions noted above the ϕ momentum equation becomes meaningless. Furthermore the nature of the flow makes it inconvenient to obtain information from the r -momentum equation. Equations will therefore be written in the s , x , and z directions rather than s , r , and ϕ directions. This is accomplished by the utilizing Table A-1: For example,

$$\text{The } k\text{-momentum equation} = (B-6)S_2 + (B-7)S_\phi C_2 + (B-8)C_\phi C_2$$

Writing equations in the s , x , and z directions gives a momentum equation parallel to the jet axis and a momentum equation parallel both to the buoyancy force and the crossflow.

The s -momentum equation is:

$$u \frac{\partial u}{\partial s} + (1-e) v \frac{\partial u}{\partial r} - \frac{uv}{r} e = g\beta(t - t_\infty)S_2(1-e) \quad (C-1)$$

The k -momentum equation is

$$\begin{aligned} u \frac{\partial u}{\partial s} S_2 + u \frac{\partial v}{\partial s} S_\phi C_2 + (1-e) v \frac{\partial u}{\partial r} S_2 + (1-e)v \frac{\partial v}{\partial r} S_\phi C_2 - \frac{uv}{r} e S_2 \\ + \frac{u^2 e}{r} S_\phi C_2 + \frac{u^2}{r} \frac{\partial e}{\partial \phi} C_\phi C_2 = g\beta(t - t_\infty) \end{aligned} \quad (C-2)$$

The i -momentum equation is

$$\begin{aligned} u \frac{\partial u}{\partial s} C_2 - u \frac{\partial v}{\partial s} C_\phi S_2 - (1-e) v \frac{\partial u}{\partial r} C_2 - (1-e)v \frac{\partial v}{\partial r} (C_\phi S_2 + S_\phi S_2) - \frac{uv}{r} e C_2 \\ - \frac{u^2 e}{r} C_\phi C_2 - \frac{u^2 e}{r} S_\phi S_2 - \frac{u^2}{r} \frac{\partial e}{\partial \phi} C_\phi S_2 = 0 \end{aligned} \quad (C-3)$$

The continuity equation is:

$$\frac{\partial u}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (rv) (1-e) - \frac{ve}{r} = 0 \quad (C-4)$$

The energy equation is:

$$u \frac{\partial t}{\partial s} + (1-e)v \frac{\partial t}{\partial r} = 0 \quad (C-5)$$

It is desired that ϕ be completely eliminated from all equations. Presently ϕ is present in each equation through the presence of e . This elimination is accomplished by multiplying through equations (C-1) through (C-5) by $d\phi$ and integrating on ϕ from 0 to 2π . Performing this integration and dividing through by 2π yields the following:

The s-momentum equation:

$$u \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial r} = g\beta(t - t_\infty)S_2 \quad (C-6)$$

The k-momentum equation:

$$\left(u \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial r} \right) S_2 + \left(u^2 - \frac{rv}{2} \frac{\partial v}{\partial r} \right) \frac{C_2}{\rho_2} = g\beta(t - t_\infty) \quad (C-7)$$

The i-momentum equation:

$$\left(u \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial r} \right) C_2 - \left(u^2 - \frac{rv}{2} \frac{\partial v}{\partial r} \right) \frac{S_2}{\rho_2} = 0 \quad (C-8)$$

The continuity equation:

$$\frac{\partial u}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (rv) = 0 \quad (C-9)$$

The energy equation:

$$u \frac{\partial t}{\partial s} + v \frac{\partial t}{\partial r} = 0 \quad (C-10)$$

To this point the effects of turbulence have not been explicitly included in the equations. The effects of turbulence will now be included by the usual method of writing the individual variables in terms of a time average and a fluctuating component. For example

$$u = \bar{u} + u' \quad \text{where} \quad \bar{u} = \lim_{\tau \rightarrow \infty} \left(\frac{1}{2\tau} \int_{-\tau}^{\tau} u d\tau \right)$$

and u' is the fluctuating component.

Substituting the mean and fluctuating components into the equations and averaging over time yields the following:

The s-momentum equation:

$$\bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{u}}{\partial r} = g\beta(\bar{t} - t_{\infty})S_2 - \left[\frac{\partial}{\partial s} (\overline{u'^2}) + \frac{1}{r} \frac{\partial}{\partial r} (\overline{ru'v'}) \right] \quad (C-11)$$

The k-momentum equation is:

$$\begin{aligned} \left(\bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{u}}{\partial r} \right) S_2 + \left(\bar{u} - \frac{r\bar{v}}{2} \frac{\partial \bar{v}}{\partial r} \right) \frac{C_2}{\rho_2} &= g\beta(\bar{t} - t_{\infty}) - \left[\frac{\partial \overline{u'^2}}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (\overline{ru'v'}) \right] S_2 \\ &- \left[\overline{u'^2} - \frac{r}{4} \frac{\partial \overline{v'^2}}{\partial r} \right] \frac{C_2}{\rho_2} \end{aligned} \quad (C-12)$$

The i-momentum equation is:

$$\begin{aligned} \left(\bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{u}}{\partial r} \right) C_2 - \left(\bar{u}^2 - \frac{r\bar{v}}{2} \frac{\partial \bar{v}}{\partial r} \right) \frac{S_2}{\rho_2} &= - \left[\frac{\partial \overline{u'^2}}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (\overline{ru'v'}) \right] C_2 \\ &+ \left[\overline{u'^2} - \frac{r}{4} \frac{\partial \overline{v'^2}}{\partial r} \right] \frac{S_2}{\rho_2} \end{aligned} \quad (C-13)$$

The continuity equation is:

$$\frac{\partial \bar{u}}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (r\bar{v}) = 0 \quad (C-14)$$

The energy equation is:

$$\bar{u} \frac{\partial \bar{t}}{\partial s} + \bar{v} \frac{\partial \bar{t}}{\partial r} = - \left[\frac{\partial}{\partial s} (\overline{u't'}) + \frac{1}{r} \frac{\partial}{\partial r} (\overline{rv't'}) \right] \quad (C-15)$$

Flow in a buoyant free turbulent jet is of the boundary layer type in that axial gradients are much smaller than radial gradients. Additionally $\bar{u}$ is much greater than $\bar{v}$. These facts result in Prandtl's boundary layer approximations which in turn allow many terms to be neglected. Application of these approximations simplifies the equations to the following forms.

The s-momentum equation is:

$$\bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{u}}{\partial r} = g\beta(\bar{t} - t_{\infty})S_2 - \frac{1}{r} \frac{\partial}{\partial r} (\overline{ru'v'}) \quad (C-16)$$

The k-momentum equation is:

$$\begin{aligned} \left(\bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{u}}{\partial r} \right) S_2 &= g\beta(\bar{t} - t_{\infty}) - \frac{1}{r} \frac{\partial}{\partial r} (\overline{ru'v'}) S_2 \\ &- \left(\bar{u}^2 - \frac{r}{4} \left(\frac{\partial \bar{v}^2}{\partial r} + \frac{\partial \bar{v}^2}{\partial r} \right) \right) \frac{C_2}{\rho_2} \end{aligned} \quad (C-17)$$

The i-momentum equation is:

$$\left(\bar{u} \frac{\partial \bar{u}}{\partial s} + \bar{v} \frac{\partial \bar{u}}{\partial r} \right) C_2 = \left(\bar{u}^2 - \frac{r}{4} \left(\frac{\partial \bar{v}^2}{\partial r} - \frac{\partial \bar{v}^2}{\partial r} \right) \right) \frac{S_2}{\rho_2} - \frac{1}{r} \frac{\partial}{\partial r} (\overline{ru'v'}) C_2 \quad (C-18)$$

The continuity equation is:

$$\frac{\partial \bar{u}}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (r\bar{v}) = 0 \quad (C-19)$$

The energy equation is:

$$\bar{u} \frac{\partial \bar{t}}{\partial s} + \bar{v} \frac{\partial \bar{t}}{\partial r} = - \frac{1}{r} \frac{\partial}{\partial r} (\overline{rv't'}) \quad (C-20)$$

These are the axisymmetric equations.

APPENDIX D: THE INTEGRAL EQUATIONS

The axisymmetric equations derived in Appendix C will now be integrated radially from r equals zero to infinity. This integration will be performed in general to allow specification of alternate velocity profiles in future work.

In writing the equations the "bar" above time average terms will be deleted at this point solely for convenience. Fluctuating terms will retain their primes.

The continuum equation is:

$$\frac{\partial u}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} (rv) = 0 \quad (D-1)$$

Integrating radially

$$2\pi \int_0^{\infty} \frac{\partial u}{\partial s} r dr + 2\pi \int_0^{\infty} \frac{1}{r} \frac{\partial}{\partial r} (rv) r dr = 0$$

Dividing through by 2π and interchanging the order of integration and differentiation in the first integral the continuity equation becomes:

$$\frac{d}{ds} \int_0^{\infty} u r dr + \int_0^{\infty} d(rv) = 0$$

In anticipation of evaluating the second integral the following are noted to be true

$$\lim_{r \rightarrow 0} rv = 0 \quad \text{and} \quad \lim_{r \rightarrow \infty} v = -\frac{E}{r}$$

The second integral is

$$\int_0^{\infty} d(r\bar{v}) = r\bar{v} \Big|_0^{\infty} = -\frac{rE}{r} - 0 = -E$$

The continuity equation is

$$\frac{d}{ds} \int_0^\infty u r dr = E \quad (D-2)$$

Similar integration processes yield the following equations:

The energy equation has the addition modification that the temperature scale is shifted by t_∞ so that the temperature variable is now $t - t_\infty$. The energy equation is

$$\frac{d}{ds} \int_0^\infty u(t - t_\infty) r dr = - \int_0^\infty \frac{dt_\infty}{ds} u r dr - \lim_{r \rightarrow \infty} r u' v' \quad (D-3)$$

The s-momentum equation is:

$$\frac{d}{ds} \int_0^\infty u^2 r dr = \int_0^\infty g\beta(t - t_\infty) r S_2 dr + EU_\infty S_2 - \lim_{r \rightarrow \infty} r u' v' \quad (D-4)$$

The k-momentum equation is:

$$\begin{aligned} \frac{d}{ds} \int_0^\infty u^2 r dr S_2 = & \int_0^\infty g\beta(t - t_\infty) r dr + EU_\infty - S_2 \lim_{r \rightarrow \infty} r u' v' \\ & - \left[\int_0^\infty u^2 r dr - \frac{E^2}{4} - \frac{1}{4} \lim_{r \rightarrow \infty} (r \overline{v'^2}) \right] \frac{C_2}{\rho_2} \end{aligned} \quad (D-5)$$

The i-momentum equation is:

$$\begin{aligned} \frac{d}{ds} \int_0^\infty u^2 r dr C_2 = & EU_\infty S_2 C_2 - C_2 \lim_{r \rightarrow \infty} r u' v' \\ & + \left[\int_0^\infty u^2 r dr - \frac{E}{4} - \frac{1}{4} \lim_{r \rightarrow \infty} (r^2 \overline{v'^2}) \right] \frac{S_2}{\rho_2} \end{aligned} \quad (D-6)$$

Examination of equations (D-2) through (D-6) shows that these equations take into consideration ambient turbulence. The ambient turbulence terms are:

$$\lim_{r \rightarrow \infty} r v' t', \quad \lim_{r \rightarrow \infty} r u' v' \quad \text{and} \quad \lim_{r \rightarrow \infty} r^2 \overline{v'^2}$$

Since no information exists regarding ambient turbulence in the data to which the theory will ultimately be compared the ambient turbulence terms will now be neglected.

Before writing the integral equations in final form the s and k momentum equations can be manipulated to obtain an equation for the rate of change of the centerline slope with s, that is $d\theta_2/ds$. This equation is obtained by multiplying equation (D-5) by S_2 and subtracting equation (D-5) from the result. Finally solution for $1/\rho_1 = d\theta_1/ds$ yields the equation in question. It is noted that the centerline slope equation, the k-momentum equation and the s-momentum equation are now dependent. The k-momentum equation is now deleted from the analysis. The final form of the integral equations is as follows:

The continuity equation is:

$$\frac{d}{ds} \int_0^\infty u r dr = E$$

The energy equation is:

$$\frac{d}{ds} \int_0^\infty u(t - t_\infty) r dr = - \int_0^\infty \frac{dt_\infty}{ds} u r dr \quad (D-7)$$

The s-momentum equation is:

$$\frac{d}{ds} \int_0^\infty u^2 r dr = S_2 \int_0^\infty g\beta(t - t_\infty) r dr + EU_\infty S_2 \quad (D-8)$$

The i-momentum equation is:

$$\frac{d}{ds} \int_0^\infty u^2 r C_2 dr = EU_\infty S_2 C_2 + \left(\int_0^\infty u^2 r dr - \frac{E}{4} \right) \frac{S_2}{\rho_2} \quad (D-9)$$

The centerline slope equation is:

$$\frac{d\theta_1}{ds} = \frac{C_2}{q} \int_0^\infty g\beta(t - t_\infty) r dr + \frac{C_2}{q} EU_\infty \quad (D-10)$$

where: $q = \int_0^\infty u^2 r dr - \frac{E^2}{4}$

These are the integral equations in general form.

APPENDIX E. INTEGRATION OF THE GENERAL INTEGRAL EQUATIONS USING GAUSSIAN TEMPERATURE AND VELOCITY PROFILES

As discussed in the text the velocity and temperature profiles selected for use in this analysis are Gaussian. They are

$$t - t_{\infty} = \Delta T_M e^{-r^2/\lambda B}$$

$$u = \Delta U_M e^{-r^2/B} + U_{\infty} S_2$$

Substituting these profiles into the general integral equations the integrations are now performed.

The continuity equation is:

$$\frac{d}{ds} \int_0^{\infty} \bar{u} r dr = E \quad (D-2)$$

Substituting in the velocity profile and integrating:

$$\frac{d}{ds} \int_0^{\infty} (\Delta U_M e^{-r^2/B} + U_{\infty} S_2) r dr = \frac{d}{ds} \left(\frac{\Delta U B}{2} + U_2 B \right) = E$$

$$U_2 = U_{\infty} S_2$$

$$\text{Let } U_M = \Delta U_M + U_2 \quad \text{and} \quad V_M = U_M + U_2$$

The continuity equation is:

$$\frac{d}{ds} \left(\frac{B V_M}{2} \right) = E \quad (E-1)$$

The energy equation is:

$$\frac{d}{ds} \left(\int_0^{\infty} u (t - t_{\infty}) r dr \right) = \frac{dt_{\infty}}{ds} \int_0^{\infty} u r dr$$

Neglecting ambient stratification, substituting the Gaussian profiles and integrating, the energy equation becomes:

$$\frac{d}{ds} \left(\int_0^\infty (\Delta t_M \Delta U e^{-r^2/B} e^{-r^2/\lambda B} + U_2 \Delta t_M e^{-r^2/\lambda B}) r dr \right) = 0$$

$$\frac{d}{ds} \left(\frac{\Delta t_M \lambda B}{2} \left(\frac{\Delta U}{1+\lambda} + U_2 \right) \right) = 0 \quad (E-2)$$

The s-momentum equation is:

$$\frac{d}{ds} \left(\int_0^\infty u^2 r dr \right) = S_2 \int_0^\infty g\beta(t - t_\infty) r dr + EU_2 \quad (D-7)$$

Substituting the Gaussian profiles and integrating the s-momentum equation becomes:

$$\begin{aligned} \frac{d}{ds} \left(\int_0^\infty (\Delta U^2 e^{-2r^2/B} + 2\Delta U U_2 e^{-r^2/B} + U_2^2) r dr \right) &= S_2 g\beta \int_0^\infty \Delta t_M e^{-r^2/\lambda B} r dr + EU_2 \\ \frac{d}{ds} \left(\frac{BV_M^2}{4} \right) &= \frac{S_2 g\beta \Delta t_M \lambda B}{2} + EU_2 \end{aligned} \quad (E-3)$$

The i-momentum equation is now deleted as unnecessary to the analysis. The centerline slope equation is:

$$\frac{d\theta_2}{ds} = \frac{C_2}{q} \int_0^\infty g\beta(t - t_\infty) r dr + \frac{C_2 EU_\infty}{q}$$

$$\text{where: } q = \int_0^\infty u^2 r dr - \frac{E^2}{4}$$

Substituting the Gaussian profiles and integrating the centerline slope equation becomes:

$$\begin{aligned} \frac{d\theta_2}{ds} &= \frac{C_2}{q} g\beta \int_0^\infty \Delta t_M e^{-r^2/\lambda B} r dr + \frac{C_2}{q} EU_\infty \\ \text{where: } q &= \int_0^\infty (\Delta U e^{-2r^2/B} + 2\Delta U U_2 e^{-r^2/B} + U_2^2) r dr - \frac{E^2}{4} \end{aligned}$$

$$\frac{d\theta_2}{ds} = \frac{C_2}{q} \frac{g\beta \Delta t_M \lambda B}{2} + \frac{C_2 EU_\infty}{q} \quad (E-4)$$

where: $q = \frac{BV_M^2}{4} - \frac{E^2}{4}$

The analysis now consists of four first order coupled total differential equations. They are equations (E-1), (E-2), (E-3), and (E-4). The four dependent variables are B , ΔU , ΔT and θ_2 . The single independent variable is s .

APPENDIX F. DERIVATION OF THE ENTRAINMENT FUNCTION

As a first step a mechanical energy equation is derived by multiplying equation (20) by U and integrating across the jet. The result is:

$$\frac{d}{ds} \left[\int_0^\infty \bar{u}^3 r dr \right] = 2S_2 g \beta \left[\int_0^\infty (\bar{t} - t_\infty) \bar{u} r dr \right] + EU_2^2 - 2SI \quad (F-1)$$

where: $U_2 = U_\infty S_2$

$$SI = \int_0^\infty \bar{u} \frac{\partial}{\partial r} (\overline{ru'v'}) dr$$

SI is henceforth referred to as the shear stress integral.

Substitution of the Gaussian profiles into equation (F-1) plus a small amount of algebraic gymnastics yields the mechanical energy equation:

$$\frac{d}{ds} \left(\frac{V_M^3}{6} \right) = \frac{d}{ds} \left(\frac{BU_2}{4} (\Delta U + U_2)^2 \right) + EU_2^2 + S_2 g \beta \Delta T_M \lambda_B \left(\frac{\Delta U}{1+\lambda} + U_2 \right) - SI \quad (F-2)$$

where: $V_M = U_M + U_2$

$U_M = \Delta U + U_2$

$U_2 = U_\infty S_2$

The units on this equation are $\text{ft}^5/\text{sec}^3\text{-ft}$. If this equation were multiplied by density in sl/ft^3 the resulting units would be $(\text{lbf-ft/s})/\text{ft}$. Hence the title the mechanical energy equation.

To facilitate solution of equations (28), (30) and (F-1) for E some way of relating the three equations is necessary. It can be shown that

$$V_M^2 \frac{d}{ds} (V_M B) = 2V_M \frac{d}{ds} (V_M^2 B) - \frac{d}{ds} (V_M^3 B) \quad (F-3)$$

For convenience equations (28) and (30) are repeated.

$$\frac{d}{ds} \left(\frac{BV_M}{2} \right) = E \quad (28)$$

$$\frac{d}{ds} \left(\frac{BV_M^2}{4} \right) = \frac{S_2 g \beta \Delta T_M^{\lambda B}}{2} + EU_2 \quad (30)$$

Using equation (F-3) equations (28), (30) and (F-1) can be related. The result is:

$$\begin{aligned} 2V_M^2 E = 2V_M \left[2S_2 g \beta \Delta T_M^{\lambda B} + 4EU_2 \right] - 6 \frac{d}{ds} \left[\frac{BU_2}{4} (\Delta U + U_2)^2 \right] \\ - 6EU_2 - 6S_2 g \beta \Delta T_M^{\lambda B} \left(\frac{\Delta U}{1+\lambda} + U_2 \right) + 12SI \end{aligned} \quad (F-4)$$

Solution of this equation would yield an entrainment function, E , that would take into consideration buoyant effects, shear effects, and crossflow as well as local conditions within the jet. Unfortunately the presence of the derivative term makes numerical solution of the basic equations impossible. Computer work using an entrainment function obtained from this equation by simply neglecting the derivative term proved totally worthless. It is therefore necessary to sacrifice one of the features of the entrainment function in order to obtain something useful. Assume that no crossflow exists, then $U_\infty = 0$. Equation (F-4) becomes:

$$\Delta U^2 E = \Delta U (2S_2 g \beta \Delta T_M^{\lambda B}) - 3S_2 g \beta \Delta T_M^{\lambda B} \left(\frac{\Delta U}{1+\lambda} \right) + 6SI \quad (F-5)$$

Solving for E yields:

$$E = \frac{S_2 g \beta \Delta T_M^{\lambda B}}{\Delta U} \left(2 - \frac{3}{1+\lambda} \right) + \frac{6SI}{\Delta U^2} \quad (F-6)$$

The local Froude number is:

$$Fr_L = \frac{\Delta U^2}{(\rho_\infty - \rho_c / \rho_o) \sqrt{B} g}$$

It can be shown that

$$\beta \Delta T_M = \frac{t_c - t_\infty}{t_o} = \frac{\rho_c - \rho_\infty}{\rho_o}$$

The local Froude number can be written:

$$Fr_L = \frac{\Delta U^2}{g \beta \Delta T_M \sqrt{B}}$$

Substituting this into equation (F-6) the entrainment function becomes:

$$E = \frac{S_2 \lambda \sqrt{B}}{Fr_L} \left(2\Delta U - \frac{3\Delta U}{1+\lambda} \right) + \frac{6SI}{\Delta U^2} \quad (F-7)$$

Let $SI = I \Delta U^3 b$ where I = a similarity integral, A constant,

$$A_1 = 6I$$

$$A_2 = 2\lambda - \frac{3\lambda}{\lambda+1}$$

Then:

$$E = \left(A_1 + \frac{A_2}{Fr_L} S_2 \right) \Delta U b \quad (F-8)$$

This latter expression is the entrainment function derived by Fox and Hirst. Hirst assumed the shear stress integral to be similar in obtaining this function.

$$\text{Let:} \quad A_3 = g\beta\lambda \left(2 - \frac{3}{1+\lambda} \right)$$

$$\text{Then:} \quad E = A_3 \frac{S_2 \Delta T_M^B}{\Delta U} + \frac{6SI}{\Delta U^2}$$

This is the entrainment function that is obtained from Fox's "New Method" as used by Hirst.

APPENDIX G. TREATMENT OF THE SHEAR STRESS INTEGRAL

The shear stress integral was encountered in Appendix F in the course of deriving an entrainment function. This shear stress integral, SI, is:

$$SI = \int_0^{\infty} u \frac{\partial}{\partial r} (ru'v') dr$$

The result of this integral is dependent upon the manner in which the turbulent fluctuations are related to the mean variables.

According to Prandtl's mixing length theory of free turbulence:

$$u'v' = L^2 \left| \frac{du}{dy} \right| \frac{du}{dy}$$

where: L = the mixing length.

Prandtl further assumes that $L(r) = a$ constant, but that $L(s) = Cs$. Substituting the above along with the Gaussian velocity profile into the shear stress integral the following result is obtained:

$$\begin{aligned} SI &= \frac{4C^2 S^2 \Delta U^3}{B^2} \left\{ 3 \int_0^{\infty} r^2 d^{-3r^2/B} - \frac{4}{B} \int_0^{\infty} r^4 e^{-3r^2/B} dr \right. \\ &= \frac{k S^2 \Delta U^3}{B^{3/2}} \end{aligned}$$

$$\text{where: } k = \frac{2C^2}{3\sqrt{3}}$$

Upon evaluation of this integral using Taylor's theory one obtains an identical result. Attempts at evaluation of the shear stress integral using Von Karman's similarity hypothesis proved fruitless. Computer work showed that the result of evaluation of the shear stress integral using Prandtl's theory gives very poor results.

As a result of the failure of available theories of turbulence to give reasonable results a simple intuitive approach to the shear stress integral was finally taken.

It can reasonably be expected that the rate of entrainment due to shear effects will be in some way proportional to the local velocity in the jet. The relationship indicated by Prandtl's theory was assumed:

$$SI \propto \Delta U^3 .$$

It can additionally be expected that the effect of shear in entraining matter should be proportional to the area over which shear is occurring; that is, the perimeter of the jet.

$$SI \propto \text{jet diameter}$$

For the purpose of this thesis the jet boundary is considered to occur at that radius where the difference between the velocity component in the direction parallel to the s-axis and the free stream velocity component in the s-axis is equal to one percent of the centerline difference.

Since:

$$U = \Delta U e^{-r^2/B}$$

$$\text{Jet Diameter} \propto B^{1/2} \Delta U^3$$

Thus:

$$SI \propto B^{1/2} \Delta U^3$$

or

$$SI = A_1 B^{1/2} \Delta U^3 .$$

This last is the form employed in all analysis in this thesis.

APPENDIX H: THE COMPUTER PROGRAM

```

DIMENSION Y(6),Q(6),DY(6),A(6)
COMMON QFLI,RI
COMMON CCNCNE,CCNTWC,EXP,CON3
COMMON E,D,G,BETA,UFREE,LAM2,DUFREE,EZERO
REAL JB,LAM2,JB1
REAL JB1,JD,JDN
C SET VALUES OF CCNstants
CCNCNE=0.055
CCNTWC=0.825
CON3=1.25
LAM2=1.11
G=32.174
BETA=0.001886792
C NUMBER OF EQUATIONS, N
N=6
C SET STEP SIZE IN FEET
H=0.05
C THE INDEPENDENT VARIABLE, S, IN FEET
S=3.05
C SET INITIAL CONDITIONS
C RI IS THE INITIAL JET RADIUS
RI=0.25
C THE INITIAL CENTERLINE TEMPERATURE DIFFERENCE IS TMI.
TMI=-31.24
TMC=-44.
C SET INITIAL VALUES OF Y-FUNCTIONS
C Y(1) = THETA, THE ANGLE BETWEEN THE JET CENTERLINE AND THE
C HORIZONTAL IN RADIANS
Y(1)=-.08723
WRITE(6,206)Y(1)
C Y(2) = DELTA U
Y(2)=17.
UDEL1=Y(2)
WRITE(6,207)Y(2)
C Y(3)=B SQUARED, THE SHAPE FACTOR IN FEET
Y(3)=0.1141
JB1=SQRT(4.605*Y(3))
JB1=RI*2.
WRITE(6,208)JB1
C X=Y(4) AND Y=Y(5) FOR CONVERSION FROM NATURAL TO RECTANGULAR COORDINATE
Y(4)=2.45
Y(5)=-.31
Y4=Y(4)
CALL CFLCH(Y4,UFREE,DUFREE)
C THE BULK VELOCITY IS UBULK
UBULK=16.6666666
WRITE(6,218)UBULK
C QFLI IS THE INITIAL VOLUME RATE OF FLOW

```

```

QFLI=UBULK*3.1415927*RI*RI
RHO=39.69/(530.+(.215*TM))
QFLI=QFLI*RHO
Y(6)=3.1415927*((UFREE*SIN(Y(1))*JB*JB)+(Y(2)*Y(3)))
THE ENTRAINMENT FUNCTION, E
E=0.0
WORK VECTORS
Q(1)=0.0
Q(2)=0.0
Q(3)=0.0
Q(4)=0.0
Q(5)=0.0
Q(6)=0.0
WRITE(6,217)CONCNE,CCNTWC
D=TM*LAM2*Y(3)*((Y(2)/(1.0+LAM2))+(UFREE*SIN(Y(1))))
WRITE(6,209)TM,D
WRITE(6,211)QFLI
WRITE(6,202)
WRITE(6,212)
WRITE(6,213)
WRITE(6,202)
WRITE(6,200)
WRITE(6,201)
WRITE(6,202)
NFLAG=1
NTEST=1
00 CALL RKG(N,H,S,Y,DY,C)
Y4=Y(4)
CALL CFLCW(Y4,UFREE,UFREE)
01 ANGLE=57.296*Y(1)
02 VCEN=Y(2)+(UFREE*SIN(Y(1)))
03 JB=SQRT(4.60517*Y(3))
JD=JB*2.0
TM=D/(LAM2*Y(3)*((Y(2)/(1.0+LAM2))+(UFREE*SIN(Y(1)))))
TMN=TM/TMC
RHOS=39.69/(530.+(.215*TM))
QFL=03.14159*((UFREE*SIN(Y(1))*JB*JB)+(0.99*Y(2)*Y(3)))
QFL=QFL*RHOS
UNGRM=Y(2)/UDELI
QFLN=QFL/QFLI
VNORM=VCEN/UDELI
Y6NORM=Y(6)*RHOS/QFLI
IF (NFLAG.EQ.1) GO TO 149
IF(NTEST.NE.10)GO TO 160
NTEST=0
49 WRITE(6,214)
Y6=Y(6)
50 WRITE(6,203)S,Y(4),Y(5),Y(3),JD,Y(2),VCEN,Y(1),ANGLE,TM,E,QFL,Y6,

```

```

XUFREE,CUFREE
  SN=S/(2.0*RI)
  XN=Y(4)/(2.0*RI)
  YN=Y(5)/(2.0*RI)
  JDN=JD/(2.0*RI)
  WRITE(6,215)
  WRITE(6,216)SN, XN, YN,JCN,UNORM,VNCRM,TMN,QFLN,Y6NORM.
  WRITE(6,202)
  NFLAG=C
0  NTEST=NTEST+1
  IF(S.LT.20.0)GO TO 100
  WRITE(6,205)
0  FORMAT(' ',28X,'CIA/    VELCCITY',9X,'SLOPE',7X,'TEMP.',
X2X,'ENTRNMNT    FLOW RATE')
1  FORMAT(' ',2X,'S',6X,'X',6X,'Y',6X,'B    METER DIFF.',3X,'CENT.',
X4X,'RAD    DEG    DIFF.    RATE    LCCAL    COLLECTIVE UFREE',
X1X,'CUFREE')
2  FORMAT('0')
3  FORMAT(' ',F5.2,1X,F6.2,1X,F6.2,2X,F5.3,1X,F5.2,2X,F5.2,3X,F5.2,
X3X,F5.2,2X,F5.1,3X,F5.1,3X,F6.3,2X,F6.3,6X,F6.3,4X,F5.2,1X,F6.2)
5  FORMAT('1')
6  FORMAT('1',5X,'THE INITIAL SLOPE IS:',1X,F4.1,1X,'RADIANS')
7  FORMAT(' ',5X,'THE INITIAL CENTERLINE VELOCITY IS:',1X,F6.1,1X,
X'FEET PER SECOND')
8  FORMAT(' ',5X,'THE INITIAL DIAMETER IS:',F5.2,1X,'FEET')
9  FORMAT(' ',5X,'THE INITIAL TEMPERATURE DIFFERENCE IS:',F5.1,1X,
X'DEGREES F. THE ENERGY CONSTANT, D, IS:',F7.2)
1  FORMAT(' ',5X,'THE INITIAL FLOW RATE WITHIN THE JET IS:',F12.7)
2  FORMAT(' ',5X,'X, Y, VELCCITY, CENTERLINE VELOCITY AND CENTERLINE
XTEMPERATURE DIFFERENCE AS NOTED BELCW ARE')
3  FORMAT(' ',5X,'ALL NORMALIZED WITH RESPECT TO INITIAL VALUES.')
4  FORMAT(' ',5X,'ACTUAL VALUES')
5  FORMAT(' ',5X,'NORMALIZED VALUES')
6  FORMAT(' ',F5.2,1X,F6.2,1X,F6.2,8X,F5.2,3X,F4.2,4X,F4.2,19X,F4.2,
X11X,F6.3,6X,F6.3)
7  FORMAT(' ',5X,'CCNCNE IS:',F6.3,2X,'CONTWO IS:',F6.3)
8  FORMAT(' ',5X,'THE INITIAL AVERAGE VELCCITY IS:',1X,F8.3,'.')
  STOP
  END
  SUBROUTINE RKG(NEQ,H,X,Y,DY,Q)
  * * * * *
  THE INDEPENDENT VARIABLE X IS INCREMENTED IN THIS PROGRAM
  Y(I) AND DY(I) ARE THE DEPENDENT VARIABLE AND ITS DERIVATIVE
  ALL THE Q(I) MUST BE INITIALLY SET TO ZERO IN THE MAIN PROGRAM
  NEQ = NUMBER OF FIRST ORDER EQUATIONS
  H = INTERVAL SIZE
  A SUBROUTINE DERIV(NEQ,X,Y,DY) MUST BE PROVIDED
  * * * * *

```



```
DY5 IS DY/DS  
DY(5)=SIN(Y(1))  
DY(6) IS THE FLOW RATE DERIVATIVE WHEN CROSSFLOW IS ZERO ONLY.  
DY(6)=6.283185*E  
RETURN  
END  
SUBROUTINE CFLCW(S,UFREE,DUFREE)  
UFREE=1.666666666  
DUFREE=0.0  
RETURN  
END
```

APPENDIX I. INITIAL CONDITIONS

The equations that have been developed in this thesis to describe free turbulent jets apply only in the region of fully developed flow. For this reason the model composed of these equations cannot be expected to make good predictions based on conditions at the nozzle or outlet. It is therefore necessary to attempt to determine the conditions that exist in the jet at the beginning of the region of fully developed flow.

The problem of determining conditions at the beginning of the fully developed region is compounded by the fact that this region begins at different downstream distances for jets of different velocities and perhaps even for jets of different diameters. Furthermore the fact that one region gradually changes into another along the axis of the jet makes the beginning of the fully developed region still more indistinct. Since the point at which the fully developed begins is not predicted by this model and is beyond the scope of this work, a set of initial conditions must be obtained from some other source before solution is possible. The beginning of the region of fully developed flow will be referred to as the initial point.

It was decided to attempt to locate the start of the fully developed region by simply examining Nottage's data and curves and estimating the location where characteristics of the fully developed region began to appear. When centerline velocity decay is plotted on log-log paper the region of fully developed flow is characterized by a straight-line. For the purpose of this model the initial point is considered to be that point where a straight line lying along the straight line portion of the velocity decay curve intersects the $V/V_0 = 1.0$ line. This location fixes the initial value of the independent variables.

For isothermal jets the velocity at the initial point is considered to be the same as the outlet velocity. The jet diameter at the initial point is extracted directly from Nottage's data and the initial value of the shape factor is computed from it.

For heated or chilled jets the initial values of the velocity, jet diameter and shape factor are specified in the same manner as isothermal jets. The presence of buoyancy forces in heated and chilled jets causes them to either rise or fall so that the slope at the initial point is something other than zero. The slope at the initial point was estimated from Nottage's curve.

For the final case considered a horizontally projected chilled jet subjected to vertical crossflow the absence of any data prevented determination of initial conditions. It was therefore decided that the initial conditions would be specified as the same as for the chilled jet with no crossflow.

Table 2 is a summary of the initial conditions used for numerical solution in this thesis.

TABLE 2. INITIAL CONDITIONS

Isothermal Jets

Mean Velocity (fpm)	Centerline Velocity (fpm)	s (ft)	Diameter (ft)	Shape Factor (1/ft ²)
100	108	1.8	1.15	.0718
350	364	2.35	1.35	.0989
500	515	2.65	1.40	.106
1000	1020	3.05	1.45	.114
2500	2525	3.30	1.50	.122
6000	6030	3.30	1.50	.122

Chilled Jet

Mean Velocity (fpm)	Centerline Velocity (fpm)	s (ft)	Diameter (ft)	Shape Factor (1/ft ²)	Initial Slope (Degrees)
500	609.93	5.12	1.29	.0896	-12.0°

Centerline temperature difference at outlet: -44°

Centerline temperature difference at initial point: -31.24°

All jets were projected from a 6 inch diameter nozzle.

VITA

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ABSTRACT

An analysis of the flow within a horizontally projected free turbulent jet subjected to vertical crossflow is made. A computer model is developed from the analysis and is compared to existing experimental data for the simple cases of isothermal jets and chilled jets in the absence of crossflow. The model is subjected to crossflow and predictions of actual jet behavior are made. The behavior of the model under the influences of the various parameters is discussed.