A STUDY OF PRESTRESS LOSSES IN THE POST-TENSIONING METHOD

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by

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SYNOPSIS

This report presents a library review of the literature on the important aspects of prestress losses in the post-tensioning method, which can be broadly classified under seven types: (1) the frictional loss in the jack, (2) loss due to anchorage take up, (3) frictional losses due to the change of direction and wobble of the tendons, (4) elastic shortening of concrete, (5) shrinkage loss, (6) creep in concrete, and (7) relaxation of the prestressing steel.

First, a brief review of the history and the development of prestressed concrete is introduced, then the definitions and the sources of prestressed losses are described. The major portion of this study is devoted to the theoretical analysis, and some numerical examples based on the theoretical analysis have been presented. Some methods for reducing the prestressing losses are given.

The total amount of losses indicates that ignoring the effects of prestress losses in design problems may result in structural failure. The reduction in prestress due to elastic shortening, shrinkage, creep, relaxation and friction reveals a significant influence of prestress losses in prestressed concrete structural designs.

INTRODUCTION

Simplicity and economy in engineering technique are two very important principles of the modern manufacturing world. Prestressed concrete is probably the latest discovery in man's continuing search for new construction materials and methods in accordance with these two aspects. During the past decade, prestressed concrete has taken great strides forward in many fields of civil engineering, e.g., bridges, buildings, piles, pavements, water tanks, and pressure pipes.

One of the best definitions of prestressed concrete is given by the American Concrete Institute Committee on Prestressed Concrete. "Prestressed concrete is defined as concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from given external loading are counteracted to a desired degree."[1] Prestressed concrete structures can be classified in a number of ways, depending upon their features of design and construction. The classification based on time of stressing is pre-tensioning and post-tensioning. Post-tensioning is defined as a method of stressing in which the tendons are tensioned after the concrete has hardened, and pre-tensioning is defined as a method of stressing in which the tendons are tensioned before the concrete is placed.[1]

The art of prestressing has been practised from ancient times; however, it was not practical in general application as late as 1933. The main obstruction was that men were unable to develop an effective method of reducing the large amount of prestress loss during prestressing.

It is the purpose of this report to investigate the various aspects of

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1. Design of Prestressed Concrete Structures, T. Y. Lin, John Wiley & Sons, Inc., New York, 1965, p. 11-24.

prestress loss in the post-tensioned method in order to obtain a better understanding of this specific problem. The sources of prestress losses in the post-tensioned method can be broadly divided into seven types: (1) frictional loss in the jack; (2) loss due to anchorage take up; (3) elastic shortening of concrete; (4) shrinkage; (5) creep in concrete; (6) relaxation of prestressing steel, and, (7) frictional loss due to the change of direction and wobble of the tendons. The scope of this report is limited to a library search of the important literature, and the problem has been treated broadly rather than in depth. The conclusions, therefore, are those of the investigators whose work has been abstracted and are the most significant in the opinion of the writer.

HISTORY AND DEVELOPMENT[2]

The principle of prestressing has been employed to improve the stability of structures for centuries. The first step in the evolution of this principle was made by Doehring of Germany in 1888, who suggested prestretching steel reinforcement in a concrete slab in order to promote simultaneous failure of these two materials of distinctly different extensibilities. In the same year Jackson of the United States described a method of increasing the load-bearing capacity of concrete structures by imposing pre-service compressive stresses on concrete by tensioning the reinforcing rods set in sleeves after the concrete had hardened.

Theoretical treatment of the design of prestressed concrete was made by Mandl in Germany in 1896, and was further developed by Koenen of Germany in 1907, who also realized the existence of loss in prestressing caused by elastic shortening of concrete. However, they failed to take account of losses due to shrinkage and other factors, so the initial compression produced in concrete by Mandl and Koenen lasted for only a relatively short time.

The important fact of losses in prestressing due to shrinkage was first recognized by Steiner in the United States in 1908. He proposed re-tensioning after losses had taken place, a method known today as prestressing by stages.

The first utilization of highly tensioned piano wires to make concrete planks was first produced by Wettstein in Vienna in 1919. However, some authorities attributed this invention to Hoyer in 1939. No patents appear to have been granted to Wettstein. The utilization of high tensile steel of large size was suggested in 1928, by Emperger and Dill in Nebraska, U.S.A.

2. The historical notes in this chapter are based on "The Patents and Code Relating to Prestressed Concrete," C. Dobell, Journal of ACI, Vol. 48, 1950, p. 713-724.

It is justifiable to grant Freyssinet a place as the originator of the modern concept and technique of prestressed concrete. In 1928, he showed an appreciation of the necessity of taking creep into consideration. In 1933, Freyssinet also reported results of tests on shrinkage and creep of concrete of various strengths, and quantitatively demonstrated the advantage of employing high-strength steel to minimize the importance of these losses.

The principle of unbonded tendons was first used by Dischinger in 1928, while the principle of bond in prestressed concrete was developed by Hoyer in Germany in 1938. Modern development of expansive cement started in France in 1940 by Lossier. Intensive study on self-stressing cement has been investigated in Russia and at the University of California, Berkeley, since 1953.

Except for work on circular prestressing, the last name to consider in the development of prestressed concrete is Gustave Magnel. His work provided a basis for the understanding of successful prestressed concrete.

As can be seen, the whole picture of the history of prestressed concrete is man's continuing search to overcome the prestress loss problem, and for a rational and systematic method of design in this field.

FRICTION LOSS IN THE JACK

In both pre-tensioning and post-tensioning, the most common method for stressing the tendons is jacking. Hydraulic jacks are used very extensively because hydraulic pressure provides the simplest means of producing large prestressing forces. The tensioning force is determined as the product of the piston area and the gauge reading of the pressure of the hydraulic fluid, between 1 and 5%[3] being deducted for friction between the piston and the cylinder. Let the constant C express the friction loss in the jack which is proportional to the jack pressure, and is determined from tests on a jack and short length of prestressing tendon.

Fig. 1 shows the arrangement used for calibrating the friction loss of a Freyssinet 12-wire jack, the jack and tendon being set up with the standard anchorage units for the system under investigation in such a way that the anchorage units are made to bear against opposite ends of a reaction framework. This framework contains load-measuring pressure capsules, so that the tension

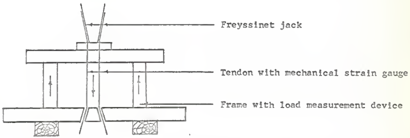

Fig. 1 Arrangement of jack, pressure capsules and reaction frame for calibration test on prestressing cable.

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3. Prestressed Concrete Design and Construction, F. Leonhardt, Wilhelm Ernst and Sohn, Berlin, Munich, 1964, p. 148.

in the tendon can be determined accurately.

The test is conducted by taking readings of jack pressure, tendon force and tendon strain. The constant C is then determined as the ratio of the tendon force to the product of the piston area and the gauge pressure of the jack. The values of C for some commonly used jacks are as follows: Freyssinet jack C = 0.95; Magnel jack C = 0.98; and, Lee-McCall jack C = 0.97.[4]

It is not possible and perhaps also not necessary to compile all the data concerning each jack system, since friction in the jack is generally small though not insignificant and normally more or less constant. It can in any event be measured reasonably accurately.

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4. "Friction in Post-tensioned Prestressing Systems", E. H. Cooley, Cement and Concrete Association, Research Report No. 1, London, 1958, p.11.

LOSS DUE TO ANCHORAGE TAKE UP

For most systems of post-tensioning the stress is created by tensioning the steel, using a jack which bears against the member to be stressed. When the tensioning force is transferred from the jacks to the anchorage devices these devices are subject to stresses which will cause them to deform, thus causing the cable to slacken slightly. If frictional wedges are used in the anchorage they will allow a small amount of slipping before the wires can be firmly gripped. Thus, a portion of the extension of the cable which was obtained during stressing is lost due to this transfer.

Theoretical Considerations

In investigating this type of loss, the following conditions should be considered:

Condition I - The stress in the cable at center span is not influenced by the deformation of the anchorage, which means the length of cable that is affected is less than one half of the length of the tendon. This condition generally occurs in the relatively long spans. The variation in prestress due to friction in a post-tensioned cable which is stressed from one end is shown in Fig. 2. It has been shown that the prestress gradient along the tendon is a flat curve slightly concave upward. However, for simplicity of computation the curves ABCD and BE are assumed to be straight lines. Tests have shown that the inclinations of lines AB and BE are approximately equal but opposite in sign.[5]

The unit stress at any point in a cable can be determined by using the

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5. Prestressed Concrete Design and Construction, James R. Libby, The Ronald Press Company, New York, 1961, p. 365.

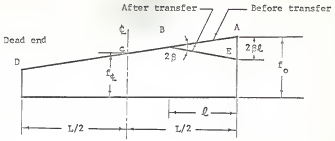

Fig. 2. Stress distribution in a post-tensioned cable before and after transfer. Condition I.

relationship

$$f_o = f_x \cdot e^{(\mu\theta + KX)} \qquad (2)$$

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where

f_o = unit stress in steel at jacking end,

f_x = unit stress at point X feet from the jacking end,

e = base of the Naperian logarithm,

K = coefficient for wobble effect per feet,

μ = coefficient of friction,

θ = angular change between the tangents to the tendon at the jacking end and at point X from the jacking end.

From this

$$f_o = f_{℄} \cdot e^{(\mu\theta + KL/2)}$$

and

6. To be discussed in the subsequent chapter.

$$\text{Slope} = \beta = \frac{f_o - f_{\mathfrak{t}}}{L/2} = \frac{2f_{\mathfrak{t}}\cdot(e^{(\mu\theta + KL/2)} - 1)}{L}$$

or

$$\beta = \frac{2f_{\mathfrak{t}}(e^{\phi} - 1)}{L} \quad \dots\dots\dots\dots\dots \quad (3)$$

where

$$\phi = \mu\theta + KL/2 .$$

For a small angle β the stress loss at the jacking end is equal to $f_o - (f_o - 2\beta\ell) = 2\beta\ell$, the average being $\beta\ell$.

Hence from Hooke's Law

$$\frac{\beta\ell}{\Delta/\ell} = E_s \quad \text{or} \quad \ell = \sqrt{\frac{E_s \cdot \Delta}{\beta}} \quad \dots\dots\dots \quad (4)$$

in which

ℓ = length of tendon which is affected by the anchorage deformation in inch,

β = slope of stress gradient in psi./in.,

Δ = total deformation of the anchorage device in inches,

E_s = modulus of elasticity of the steel in psi.

If the value of ℓ computed from Eq. (4) is less than L/2, condition I exists. The stress at the center line is not affected by the elastic deformation of the anchorage.

Condition II - The stress at the center line is affected by the deformation of the anchorage. The assumed variation in stress which is referred to as condition II is shown in Fig. 3.

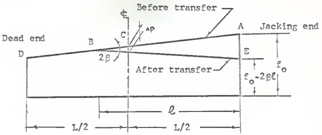

Fig. 3. Stress distribution in a post-tensioned cable before and after transfer. Condition II.

The slope of the stress line will be identical with Eq. (3). Thus,

$$\text{slope} = \beta = \frac{2f_{\mathbb{C}}\cdot(e^{\phi} - 1)}{L}$$

and

$$\ell = \sqrt{\frac{E_s\cdot\Delta}{\beta}} .$$

The stress loss at the center line resulting from this anchorage deformation is equal to

$$\Delta P = 2\beta(\ell - L/2) \qquad \ldots\ldots\ldots\ldots\ldots \qquad (5)$$

where

ΔP = stress loss at center line of the cable in psi.

Condition III - Since this loss of prestress is caused by a fixed total amount of the deformation of the anchorage devices, the most severe condition is found when the affected length ℓ is greater than the length of the member.

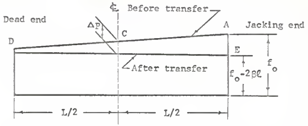

Fig. 4. Stress distribution in a post-tensioned cable before and after transfer. Condition III.

Fig. 4 shows the assumed distribution of stress for this condition; the loss at the center line is equal to

$$\Delta P = \frac{\Delta \cdot E_s}{L} \qquad \text{. . . . .} \qquad (6)$$

The amount of slippage and deformation of anchorages depend on the different types of anchorage systems used and the stress in the wires. For some anchorage systems the deformation is very small and can be reasonably neglected. However, anchorage deformation as high as one-half inch has been observed with wedge type anchors under very critical conditions.[7]

An average value for anchorage deformation is found to be about 0.06 in. for a Magnel sandwich plate wedge.[8] If condition III exists, this corresponds to a strain of 0.06 x 1/12L, where L is the length of the cable in feet. The loss of stress in percentage is consequently $0.06 \times E_s/12LF_o$,

7. Prestressed Concrete, Gustave Magnel, Concrete Publications Limited, London, 1954, 3rd. edition, p. 232.

8. Prestressed Concrete Design and Construction, James R. Libby, The Ronald Press Company, New York, 1961, p. 367.

with $E_s = 29 \times 10^6$ psi. and $F_o = 125{,}000$ psi. The reduction of stress in percentage is 1.16/L. In Table 1. are given the percentages of stress loss for cables of different lengths.

Table 1. Loss of stress for cables of different lengths.

Cable Length	10	15	30	45	60	120	150	250	350
Loss Percent	11.6	7.75	3.87	2.58	1.94	0.97	0.78	0.46	0.33

It appears that for short cables this loss is significant, and it is very difficult to prestress short wires accurately.

Numerical Example

Problem 1: A post-tensioned simple beam has a span of 80 ft, if the designed stress at the center line is 135,000 psi, the deformation of the anchorage is 0.4 in.; assume the percentage loss of prestress due to friction is given by $\phi = \mu\theta + KL/2 = 0.11$, and the modulus of elasticity of the steel is 29×10^6 psi. It is required to investigate the effect of stress at the center line due to the anchorage deformation.

Solution:

The slope of stress gradient β can be obtained by applying Eq. (3).

$$\beta = \frac{2f_{\mathcal{L}}(e^{\phi} - 1)}{L} = \frac{2 \times 135{,}000 \times (e^{0.11} - 1)}{80 \times 12} = 32.7 \text{ psi./in.}$$

Compute the length of the tendon which is affected by using Eq. (4).

$$\ell = \sqrt{\frac{E_s \cdot \Delta}{\beta}} = \sqrt{\frac{29 \times 10^6 \times 0.4}{32.7}} = 595 \text{ in.} > 480 \text{ in.} = L/2 .$$

Therefore, condition II exists and the stress loss at the center line from this anchorage deformation is given by Eq. (5).

$$\Delta P = 2\beta(\ell - L/2) = 2 \times 32.7 \times (595 - 960/2) = 7510 \text{ psi.}$$

FRICTION LOSS DUE TO CHANGE OF DIRECTION AND WOBBLE

Sources of Frictional Loss

There are two major sources of frictional loss. One is the loss produced by the initial bonding of the tendon in the duct. This could be extremely serious and every effort should be used to avoid its occurrence. Careless workmanship and unsatisfactory construction procedures may result in the sticking of cables to the surrounding materials. For example, if there are holes in the sheathing which separates the cables from the concrete, fresh mortar may force its way into the sheathing and bind the cables so that it would be extremely difficult to tension them. For cables that are to be grouted for bond, it may happen that during the grouting of some cables grout finds its way into untensioned cables through holes in the sheathing or through intersection of the conduits. Such leakage may also result in serious binding.

The second and more common source of frictional loss is the bending of cables which can be further classified into three types:[9]

1. Intentional bending of the cables to obtain a proper profile and location. This is often necessary in order to obtain a suitable position for the cables to counteract the external loadings to a desired degree.

2. The frictional loss as the cables pass through the anchorage unit at the foot of the jack. For most systems using parallel wires, the wires are twisted together along the length of the tendon but are separated near the anchorages. This loss should be very small with proper adjustment of

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9. "Cable Friction in Post-tensioning", T. Y. Lin, Proceedings of ACI, Vol. 82, Nov. 1956, p. 1107.

most types of equipment, and can be easily taken care of by overtensioning at the jack.

3. Unintentional wobbling of the cables along their length. It is caused by the misalignment of the ducts due to inadequate supports for the cables during concreting operations such as placing and vibrating. The amount of wobble usually depends upon the weight of the cable and sheathing, the spacing and rigidity of supports, and the amount of vibration. In some serious occasions it may produce more friction than the intentional curvature of the cables.

The frictional losses mentioned above can be conveniently considered in two parts; the length effect and the curvature effect. The amount of friction depends on the following factors.

1. The coefficient of friction between the prestressing cables and the surrounding materials depends on: (a) the type of cable, whether wires, strand, or bars; (b) the surrounding material, whether paper, metal, plastic, or concrete; (c) the surface conditions of the tendons, whether corrugated, or rusted; and, (d) the use of lubricants including grease, oil, and wax.

2. Pressure created by the normal reaction due to change of angle of prestressing units.

3. The sequence of tensioning. In multiple prestressing units which are not prestressed simultaneously, the tensioned wires may press against the untensioned ones and increase the frictional loss.

4. Deviation of the ducts from the required position.

5. The length and stress of the cables.

Theoretical Analysis

Consider a differential length dx of a prestressing tendon whose centroid follows the arc of a circle of radius R, Fig. 5, then

$$d\theta = dx/R$$

$$N = F d\theta = F dx/R$$

the amount of frictional loss dF around the length dx is given by the pressure

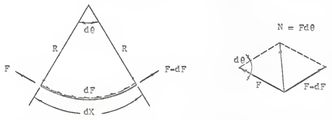


Fig. 5. Frictional loss along a differential length dx.

times a coefficient of friction μ,

$$dF = -\mu N = -\mu F dx/R = -\mu F d\theta$$

or

$$dF/F = -\mu d\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Integrating Eq. (7) on both sides gives

$$\int_{F_1}^{F_2} \frac{dF}{F} = -\mu \int_0^{\theta} d\theta$$

or

$$F_2 = F_1 \cdot e^{-\mu\theta} = F_1 \cdot e^{-\mu x/R} \quad \text{. . . . .} \quad (8)$$

where

F_1 = prestress at jacking end,

F_2 = prestress at distance L,

μ = coefficient of friction,

R = radius of the curved cable,

L = cable length.

If it is intended to combined the length and curvature effect, the remaining prestress in a cable at distance X from the jack is

$$F_2 = F_1 \cdot e^{-(\mu X/R + KX)} \quad \text{. . . . .} \quad (9)$$

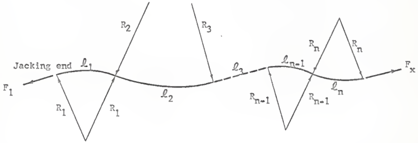


Fig. 6. Profile of a tendon consisting of several curves of different radii.

If the profile of a tendon consists of several curves of different radii as shown in Fig. 6, Eq. (9) can be expressed as

$$F_x = F_1 \cdot e^{-(KX + \mu(\ell_1/R_1 + \ell_2/R_2 + \ell_3/R_3 + \cdot \cdot \cdot + \ell_n/R_n))} \quad \text{. . .} (10)$$

where

F_x = the prestress force in a cable at distance X from the jack.

Let

$$X_1 = \ell_1$$

$$X_2 = \ell_1 + \ell_2$$

$$X_3 = \ell_1 + \ell_2 + \ell_3$$

. . . .

$$X_n = \ell_1 + \ell_2 + \ell_3 + \cdots\cdots + \ell_n$$

Hence when

$$0 < X < X_1$$

$$F_x = F_1 \cdot e^{-(KX + \mu X/R_1)}$$

$$X_1 < X < X_2$$

$$F_x = F_1 \cdot e^{-(KX + \mu(\ell_1/R_1 + X/R_2 - X_1/R_2))}$$

. . . .

$$X_{n-1} < X < X_n$$

$$F_x = F_1 \cdot e^{-(KX + u(\sum_{i=1}^{i=n-1} \frac{\ell_i}{R_i} + \frac{X - X_{n-1}}{R_n}))} \quad \cdots\cdots\cdots \quad (10a)$$

Assume Hooke's Law applies, the elongation of cable considering the effect of friction loss is given by the following expression

$$\Delta L = \int_{X}^{L} \frac{F_x}{A_s E_s} dx = \frac{F_1}{A_s E_s} \int_{X_o}^{X_1} e^{-(KX + \mu X/R_1)} dx$$

$$+ \frac{F_1}{A_s E_s} \int_{X_1}^{X_2} e^{-(KX + \mu(\ell_1/R_1 + X/R_2 - X_1/R_2))} dx + \cdots\cdots$$

$$+ \frac{F_1}{A_s E_s} \int_{X_{n-1}}^{X_n} e^{-(KX + \mu(\sum_{i=1}^{i=n-1} \frac{\ell_i}{R_i} + \frac{X - X_{n-1}}{R_n}))} dx \quad \cdots (11)$$

where

ΔL = elongation of cable due to the effect of friction,

A_s = area of the prestressing steel,

E_s = modulus of elasticity of steel.

Friction Coefficients

In order that the prestress loss and the elongation of cable due to the effect of friction can be calculated from the derived equations, values of μ and K must be known. Although data from several series of tests are available concerning the coefficient of friction in post-tensioning cables, our knowledge on this important problem is relatively limited. The difficulty lies in correlating the test conditions with actual field conditions. In an effort to determine the friction coefficient, the Cement and Concrete Association of England conducted an extensive series of experiments.[10] Average values for the Freyssinet, Magnel, and Lee-McCall systems, are listed in Table 2. Experiments were also conducted by Leonhardt[11] of Germany and

10. Friction in Post-tensioned Prestressing Systems, E. H. Cooley, Cement and Concrete Association, London, 1953, p. 19.

11. "Continuous Prestressed Concrete Beam," F. Leonhardt, ACI Proceedings, Vol. 49, 1953, p. 617.

Guyon[12] of Great Britain; their test results are listed in Tables 3 and 4 respectively.

Table 2. Frictional coefficients and wobble effect.

(From "Friction in Post-tensioned Prestressing Systems," E. H. Cooley, Cement & Concrete Association, London, 1953, p. 19.)

	Freyssinet system		Magnel system		Lee-McCall system	
	μ	K	μ	K	μ	K
Duct formed by withdrawing steel tubing	0.55	0.0000	0.30	0.0010	0.55	0.0005
Rubber core unstiffened	0.55	0.0020	0.30	0.0005	0.55	0.0010
Rubber core stiffened internally	0.55	0.0005	0.30	0.0005	0.55	0.0005
Metal sheathing	0.55	0.0010	0.30	0.0005	0.30	0.0005

Table 3. Frictional coefficients.

(From "Continuous Prestressed Concrete Beam," F. Leonhardt, ACI Proceedings, Vol. 49, 1953, p. 617.)

Type of tendon	Underlay	μ
Draw wires, 0.916 in. diameter	smoothly finished concrete	0.29-0.31
	roughly finished concrete	0.35-0.44
	new black sheet metal	0.16-0.22
Two 0.079 in. wire strands	smoothly finished concrete	0.38-0.40
	roughly finished concrete	0.40-0.46
	new black sheet metal	0.19-0.22
Seven 0.099 in. wire strand	black sheet metal	0.20-0.25
	paraffin, under pressure of 30 psi	0.10
	paraffin, under pressure of 700 psi	0.02-0.025

12. Prestressed Concrete, Y. Guyon, John Wiley and Sons, New York, 1953, p. 86.

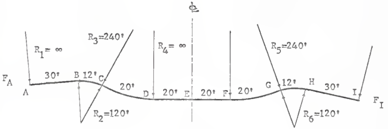
Table 4. Frictional coefficients.

(From Prestressed Concrete, Y. Guyon, John Wiley and Sons, New York, 1953, p. 86.)

	Max.	Min.	Ave.
Uncoated wires, paper wrapped	0.32	0.43	0.37
Uncoated wires, metal sheathed	0.36	0.25	0.32
Bitumen-painted wires, paper wrapped	0.08	0.10	0.09
Bitumen-painted wires, metal sheathed	0.17	0.22	0.19

Numerical Example

Problem 2: A Freyssinet cable (12 @ 0.196 in. wires), Fig. 7, is jacked at both ends. Use $\mu = 0.55$, $K = 10 \times 10^{-4}$ per ft, the modulus of elasticity $E_s = 30 \times 10^6$ psi., and the effective piston area of the Freyssinet jack of



$_1 = 30'$	$R_1 = \infty$	$X_1 = 30'$
$_2 = 12'$	$R_2 = 120'$	$X_2 = 42'$
$_3 = 20'$	$R_3 = 240'$	$X_3 = 62'$
$_4 = 20'$	$R_4 = \infty$	$X_4 = 82'$

Fig. 7. Cable profile for problem 2.

$12\ in.^2$ It is required to obtain a prestress of 120 ksi. at the center line on the cable before creep, shrinkage and elastic shortening occur. Compute the initial gauge pressure required and the corresponding elongation at each end.

(a) <u>Theoretical Solution:</u>

The cable is symmetrical about the center line and is jacked at both ends, hence it need only consider the effect from A to E.

The area for 12 @ 0.196 in. diameter cable is $0.362\ in.^2$; hence the designed prestress at center line is

$$F_E = 120 \times 0.362 = 43.7 \text{ kips.}$$

Applying Eq. (10),

$$F_E = F_A \cdot e^{-(KX_4 + \mu(\ell_1/R_1 + \ell_2/R_2 + \ell_3/R_3 + \ell_4/R_4))}$$

$$43.7 = F_A \cdot e^{-(0.001 \times 82 + 0.55(0 + 12/120 + 20/240 + 0))}$$

$$F_A = 43.7/0.83 = 52.6 \text{ kips.}$$

The corresponding gauge pressure is

$$P = F_A/A = 52.6 \times 1000/12 = 4390 \text{ psi.}$$

From Eq. (11) the total elongation at each end is

$$\Delta L = \frac{F_A}{A_s E_s}\left[\int_{X_o}^{X_1} e^{-(KX + \mu X/R_1)}\,dx + \int_{X_1}^{X_2} e^{-(KX + \mu(\ell_1/R_1 + X/R_2 - X_1/R_2))}\,dx\right.$$

$$\left. + \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \right]$$

$$= \frac{52.6}{0.362 \times 30 \times 10^6}\left[\int_0^{30} e^{-(10^{-3}X)}\,dx + \int_{30}^{42} e^{-(10^{-3}X + 0.55(0 + \frac{X-30}{120}))}\,dx\right.$$

$$+ \int_{42}^{62} e^{-(10^{-3}X + 0.55(0 + 12/120 + \frac{X - 20}{240}))} dx$$

$$+ \int_{62}^{82} e^{-(10^{-3}X + 0.55(0 + 12/120 + 20/240 + 0))} dx]$$

$$= \frac{52.6 \times 1000 \times 898.4}{0.362 \times 29 \times 10^6}$$

$$= 4.45 \text{ in.}$$

(b) Practical Method of Solution:

Let ℓ be the length of segment, from Eq. (9) we have

$$F_2 = F_1 \cdot e^{-(K + \mu/R)\ell} .$$

The prestress ratio at end of segment is

$$r = F_2/F_1 = e^{-(K + \mu/R)\ell}$$

$$= 1 - (K + \frac{\mu}{R})\ell + \frac{(K + \mu/R)^2\ell^2}{2!} - \frac{(K + \mu/R)^2\ell^3}{3!} + \ldots \quad (12)$$

In most practical solutions it is justifiable to use the first two terms of Eq. (12); thus

$$r = 1 - (K + \mu/R)\ell \quad \ldots\ldots\ldots\ldots\ldots \text{(12A)}$$

From this the remaining prestress which takes into account not only the variation from segment to segment but also that from point to point along the cable can be computed. The solution is tabulated in Table 5.

The results are very nearly the same for both solutions. The practical method is more convenient and is usually used.

When the coefficient of friction and the wobble effects are known, the consideration for frictional losses is a relatively simple problem. However, the computations for most practical design problems are quite lengthy. In a paper by M. J. Montagnon[13] a very convenient graphical solution was presented which gives the relation between friction loss, cable elongation, and the coefficient of friction. Also a series of charts for the solution of Eq. (9) have been presented by Freyssinet Prestress Company, New York. These charts are presented in the Appendix.

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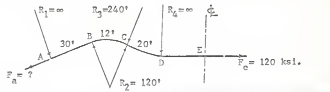
13. "Aspects Practiques de la Precontrainte Par Cables-Le Probleme des Frottements," M. J. Montagnon, Supplément aux Annales de L'Institut Technique du Batiment et des Travaux Publics, Paris, 1954, p. 504-520.

Table 5. Computation Sheet for Problem 2.



DATA

Segment	ℓ (ft)	R (ft)	K (10^{-4}/ft)	μ	$K\ell$	$\mu\ell/R$	$K\ell+\mu\ell/R$
AB	30	∞	10	0.55	0.030	0.0000	0.030
BC	12	120	10	0.55	0.012	0.0550	0.067
CD	20	240	10	0.55	0.020	0.0462	0.0662
DE	20	∞	10	0.55	0.020	0.0000	0.020

Tension Ratio

Segment	$K\ell+\mu\ell/R$	Section Ratio	Tendon Tension		
			Point	Tension Ratio	Prestress ksi.
AB	0.030	0.970	A	1.000	143
BC	0.067	0.932	B	0.970	139
CD	0.0662	0.926	C	0.910	130
DE	0.020	0.976	D	0.857	123
			E	0.840	120

Tendon Elongation

Segment	Extreme Tension Ratio		Average Tension Ratio	Section Length (ft)	Effective Length (ft)
	Jack End	Far End			
AB	1.000	0.970	0.985	30	29.6
BC	0.970	0.910	0.940	12	11.3
CD	0.910	0.857	0.8835	20	17.8
DE	0.857	0.840	0.8485	20	17.0
			Total	82	75.7

Jack Pressure	Tendon Elongation
Effective area of piston = 12 in^2.	Total effective length = 75.7 ft.
Prestress at jack end = 143 ksi.	Tendon area = 0.362 in^2.
Jack pressure = 143x1000x0.362/12	E_s = 30 x 10^6 psi.
P = 4320 psi.	Tendon elongation
	$\Delta L = \dfrac{4320 \times 12 \times 75.7 \times 12}{0.362 \times 30 \times 10^6} = 4.35$ in.

ELASTIC SHORTENING OF CONCRETE

In order to develop a compressive prestress, concrete must undergo a strain which results in a shortening and the prestressed steel shortens with it. Hence there is a loss of prestress in steel.

Modulus of Elasticity of Concrete

(a) Theoretical Considerations

It has been suggested by T. C. Hansen[14] that the internal structure of a heterogeneous material like concrete can be considered in two different ways:

1. The ideal combined hard material has a continuous lattice of an elastic component with a high modulus of elasticity and the voids are filled with another component with a lower modulus of elasticity.

2. The ideal combined soft material has grains of an elastic component with a high modulus of elasticity embedded in a continuous component with a lower modulus of elasticity.

The modulus of elasticity for a combined hard material can be obtained from the assumption that the strain is the same over the whole section.[14] Fig. 8 represents this type of material of a unit volume. From the above assumption,

$$\epsilon = \sigma_w/E_w = \sigma_h/E_h \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

where

ϵ = elastic strain of the combined material,

14. "Creep and Stress Relaxation of Concrete," Torben C. Hansen, Handlingar Proc. No. 31, Stockholm, 1960, p. 37.

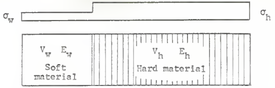


Fig. 8. Stress distribution over a unit volume of a combined hard material.

E_w = the modulus of elasticity for the soft component,

E_h = the modulus of elasticity for the hard component.

σ_w = stress on the soft component,

σ_h = stress on the hard component.

The equilibrium condition exists so we get

$$\sigma_w V_w + \sigma_h V_h = \sigma \cdot 1 \qquad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

From Eq. (14) and (13),

$$\frac{e\,E_w\,V_w}{e} + \frac{e\,E_h\,V_h}{e} = \frac{\sigma}{e} = E$$

or

$$E = E_w\,V_w + E_h\,V_h \qquad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

where

E = the modulus of elasticity for the combined material,

V_h, V_w = area of cross-section consisting of hard and soft materials respectively.

For a combined soft material the modulus of elasticity can be calculated from the assumption that the stress is identical over a whole section.

Fig. 9 represents this type of material of a unit volume.

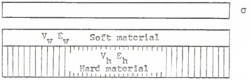


Fig. 9. Stress distribution over a unit volume of a combined soft material.

From the assumption that the stress is identical over the whole section,

$$e_w V_w + e_h V_h = e \cdot 1 \qquad \text{. . . . . (16)}$$

Since

$$e_w = \sigma/E_w, \quad e_h = \sigma/E_h$$

or

$$e = \sigma V_w/E_w + \sigma V_h/E_h = \sigma/E$$

or

$$1/E = V_w/E_w + V_h/E_h \qquad \text{. . . . . (17)}$$

Generally, cement mortar is the continuous soft component in concrete while the aggregates are the hard component which are embedded in this mass. In most cases the moduli of elasticity of the aggregates are higher than that of the mortar. Hence it may be considered as a combined soft material, so the modulus of elasticity is calculated from Eq. (17). If, however, the moduli of elasticity of the aggregates are lower than that of mortar, then the modulus of elasticity should be calculated on the assumption that concrete is a combined hard material and use Eq. (15).

(b) Practical Considerations

The term elastic strain is perhaps a little ambiguous, since the stress-strain curve of concrete is seldom straight. However, since the lower portion of the instantaneous stress-strain curve is relatively straight, it may be justifiably considered elastic, and the evaluation of the modulus of elasticity of concrete is possible. The modulus varies with several factors, notably the strength of concrete, the age of concrete, the properties of aggregates and cement, the speed of load application, and the definition of modulus of elasticity itself. Hence, it is difficult to predict with any accuracy the value of the modulus of elasticity of a given concrete.

The modulus of elasticity of concrete has been approximated by the following empirical formulas,[15]

(1) $E_c = 1000 f'_c$	1953 ACI Code
(2) $E_c = w^{1.5} \cdot 33\sqrt{f'_c}$	1963 ACI Code
(3) $E_c = 6 \times 10^6/(1 + 2000/f'_c)$	Proposed by Jensen, T. C.
(4) $E_c = 1,800,000 + 460 f'_c$	Proposed by Hognestad, E. H.
(5) $E_c = 8.15 \times 10^6 \times \dfrac{f'_c}{2300 + f'_c}$	Proposed by Komendant, A. E.

The above five proposals are plotted in Fig. 10. The results from the formulas of Jensen, Hognestad, and the ACI 1963 Code are in close agreement; Komendant's values are substantially higher; and, the ACI Code of 1953 is a different relationship and does not correlate well with any of the other four.

15. Design of Prestressed Concrete Structures, T. Y. Lin, John Wiley and Sons Inc., New York, 1965, p. 36.

Effect of Stretching Cables in Pairs

In a pre-tensioned member the elastic shortening takes place upon release of the wires from the external anchorages, as the prestressing force is transferred to the concrete. For post-tensioning, the problem is different. When prestress is applied to the first pair of cables, no loss in prestress due to that shortening need be accounted for, since the force in the cable is measured after the elastic shortening of the concrete has taken

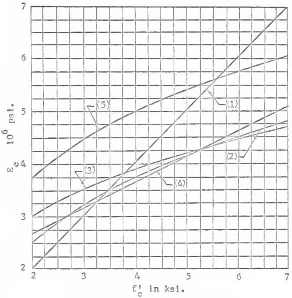


Fig. 10. Empirical formulas for E_c.

place. When prestress is applied to the second pair of cables, the loss of prestress in the first pair is assumed to be Δ_2F_1.

Equating the strains in the concrete and steel,

$$\frac{F_1 - \Delta_2F_1}{A_cE_c}\left(1 + \frac{e^2}{r^2}\right) = \frac{m\,\Delta_2F_1}{A_sE_s}$$

where

A_c = area of the concrete cross-section,

A_s = area of prestressing steel,

m = number of pairs of cables,

e = eccentricity for all cables,

r = radius of gyration.

Denoting

$$w = \frac{A_sE_s}{A_cE_c}\left(1 + \frac{e^2}{r^2}\right) \qquad \ldots\ldots\ldots\ldots\ldots \qquad (18)$$

then

$$\Delta_2F_1 = \frac{F_1\,w}{m + w} = \frac{F_1\,w/m}{1 + w/m} \; .$$

When the same force is applied to the third pair of wires, the decrease of prestress in each of the two pairs already stretched will be Δ_3F_1. Hence,

$$\Delta_3F_1 = \frac{w\,F_1/m}{1 + 2w/m}$$

and so on for the subsequent pairs of wires. It can therefore be concluded that after all wires have been stretched the prestress that remains in each pair will be:

First pair: $F_1 - (\frac{w/m}{1+w/m} + \frac{w/m}{1+2w/m} + \cdots + \frac{w/m}{1+(m-1)w/m})F_1$

Second pair: $F_1 - (\frac{w/m}{1+2w/m} + \frac{w/m}{1+3w/m} + \cdots + \frac{w/m}{1+(m-1)w/m})F_1$ . . . (19)

Third pair: $F_1 - (\frac{w/m}{1+3w/m} + \frac{w/m}{1+4w/m} + \cdots + \frac{w/m}{1+(m-1)w/m})F_1$

Pair m-1 : $F_1 - (\frac{w/m}{1+(m-1)w/m})F_1$

Pair m : F_1.

The total force in the cables will then be

$$\Sigma F = mF_1 - (\frac{w/m}{1+w/m} + \frac{2w/m}{1+2w/m} + \frac{3w/m}{1+3w/m} + \cdots + \frac{(m-1)w/m}{1+(m-1)w/m})F_1$$

or

$$\Sigma F = mF_1(1 - \frac{w}{m^2}(\frac{1}{1+w/m} + \frac{2}{1+2w/m} + \cdots + \frac{m-1}{1+(m-1)w/m})) .$$

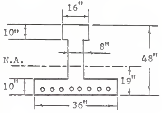
Since in most practical cases the section properties w, and the number of pairs of cables are known, the total loss in prestress can be obtained and is equal to

$$\Sigma \Delta_F = \frac{wF_1}{m}(\frac{1}{1+w/m} + \frac{2}{1+2w/m} + \frac{3}{1+3w/m} + \cdots + \frac{m-1}{1+(m-1)w/m}) \quad \ldots \quad (20)$$

In deriving these equations, it is assumed that all the cables are straight and have same eccentricity whereas in practice this is not always the case. The computation of elastic shortening is made quite complicated if a beam has curved cables and the eccentricity of the cables varies. But for most practical cases, it is accurate enough to determine the loss of the first cable and use half of that value for the average loss of all cables.

Numerical Example

PROBLEM 3: A beam of the cross-section shown in Fig. 11 is post-tensioned by four pairs of cables each of 0.296 $in.^2$, tensioned to an initial stress of 150,000 psi consecutively. If the modular ratio is 7.5, calculate the loss of prestress in the first pair and the total loss due to elastic deformation after the tensioning.



A_c = 744 $in.^2$
I_c = 123,800 $in.^4$
r = 12.75 in.
e = 14 in.
m = 4 pairs

Fig. 11. Problem 3

SOLUTION:

From Eq. (18),

$$w = \frac{A_s E_s}{A_c E_c}(1 + \frac{e^2}{r^2}) = \frac{0.296 \times 8 \times 7.5}{744} \times (1 + \frac{14^2}{12.75^2}) = 0.0665$$

$$w/m = 0.0166$$

Applying Eq. (19) the loss in first pair is

$$\Delta F_1 = \frac{F_1 w}{m}(\frac{1}{1+w/m} + \frac{1}{1+2w/m} + \frac{1}{1+3w/m})$$

$$= 150{,}000 \times 0.296 \times 0.0166 \times (\frac{1}{1+0.0166} + \frac{1}{1+0.0332} + \frac{1}{1+0.0498})$$

$$= 2150 \text{ lb.}$$

The total loss can be obtained by using Eq. (20) thus,

$$\Sigma\Delta_F = \frac{F_1 w}{m}\left(\frac{1}{1+w/m} + \frac{2}{1+2w/m} + \frac{3}{1+3w/m}\right)$$

$$= 150{,}000 \times 0.296 \times 0.0166 \times \left(\frac{1}{1+0.0166} + \frac{2}{1+0.0332} + \frac{3}{1+0.0498}\right)$$

$$= 4260 \text{ lb.}$$

SHRINKAGE LOSS

As distinguished from creep, shrinkage in concrete is its contraction due to drying and chemical changes. Concrete is a physiochemical mixture of cement, water and aggregate and contains a great number of voids and capillary tubes filled with water and air. As such, it is a porous solid to which the the capillary laws can be applied. The total surface areas which are exposed to the action of capillary forces in concrete is large; consequently, the magnitude of the resulting stresses may be very high during the period of evaporation or condensation of the water in concrete. Hence, it may cause considerable shrinkage strain.

Shrinkage Strain

An infinitesimal plane cut out of a concrete cube parallel to XY plane is shown in Fig. 12.

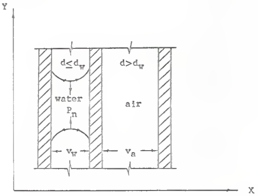


Fig. 12. An infinitesimal plane cut out of a concrete cube.

Here

v_w = space unit in concrete filled with water,

v_a = space unit in concrete filled with air,

d_w = diameter of capillary tube filled with water,

P_n = normal pressure in capillary tube caused by surface tension,

γ = capillary constant = 8 mg per mm.

According to P. S. Laplace, the normal pressure caused by the surface tension in the capillary tube is given by following equation[16]

$$P_n = P/dA = \gamma/r = 2\gamma/dw \text{ mg per mm}^2 \quad \ldots\ldots \quad (21)$$

It is also presented by Carnot and Loar Kelvin. The normal pressure in capillary tubes in terms of the relative humidity of the air, e_A, is as follows:[16]

$$P_n = 1300 \ln_e(1/e_A) \text{ kg per cm}^2 \quad \ldots\ldots\ldots\ldots \quad (22)$$

Equating Eq. (21) and (22), gives the limiting value for the diameter of the capillary tube de_A under a given air humidity. Thus

$$de_A = \frac{2\gamma \times 10^{-4}}{1300 \times \ln_e(1/e_A)} \text{ mm.} \quad \ldots\ldots\ldots\ldots\ldots \quad (23)$$

and the corresponding value of the normal pressure in these tubes will be

$$P_{e_A} = 2\gamma/de_A \text{ mg per mm}^2 \quad (24)$$

Applying Hooke's Law, the strain in any direction is

16. Prestressed Concrete Structures, August E. Komendant, McGraw-Hill Book Company, 1952, New York, p. 39.

$$e_{sh} = \frac{P_n v_w A}{A \psi_{sr} E_t} = \frac{V_w P_n}{\psi_{sh} E_t} \quad \ldots\ldots\ldots\ldots\ldots \quad (25)$$

where

E_t = modulus of elasticity at certain time t,

ψ_{sh} = coefficient of shrinkage, a function of time and relative humidity to be determined by tests.

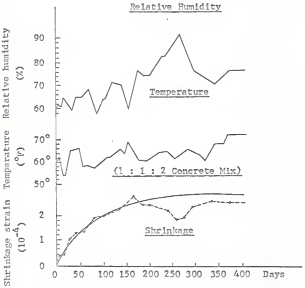


Fig. 13. Typical shrinkage curve of concrete.

Fig. 13 presents a typical shrinkage curve of concrete, the erratic points indicating the effect of temperature and relative humidity changes.[17]

17. Prestressed Concrete Theory and Design, Evans Bennett, John Wiley and Sons, 1958, New York, p. 47.

Loss of Stress Due to Shrinkage Strain

If the loss of stress in steel is denoted as ΔP_{sh}, then the reduction of stress in the concrete at the level of the wires will be

$$\frac{\Delta P_{sh} A_s}{A_c}(1 + \frac{e^2}{r^2}) = \Delta P_{sh} p(1 + \frac{e^2}{r^2})$$

The compressive strain in concrete considering only the effect of shrinkage is

$$\epsilon_{sh} - \frac{\Delta P_{sh} p}{E_c}(1 + \frac{e^2}{r^2})$$

where

ϵ_{sh} is shrinkage strain.

Equating the strains in the concrete and the steel, we get

$$\epsilon_{sh} - \frac{\Delta P_{sh} p}{E_c}(1 + \frac{e^2}{r^2}) = \frac{\Delta P_{sh}}{E_s} .$$

From Eq. (16), the section property is given by

$$w = \frac{E_s A_s}{E_c A_c}(1 + \frac{e^2}{r^2}) .$$

It indicates that the loss of prestress due to the effect of shrinkage is

$$\Delta P_{sh} = \frac{E_s \epsilon_{sh}}{1 + w} \quad \dots\dots\dots\dots\dots\dots \quad (26)$$

Numerical Example

PROBLEM 4: As in problem 3, compute the loss of stress due to a shrinkage of 2.0×10^{-4} if $E_s = 30 \times 10^6$ psi.

SOLUTION:

From problem 3, we have

$$w = \frac{E_s A_s}{E_c A_c}\left(1 + \frac{e^2}{r^2}\right) = 0.0665$$

and

$$\epsilon_{sh} = 2.0 \times 10^{-4} \text{ in./in.}$$

$$E_s = 30 \times 10^6 \text{ psi.}$$

applying Eq. (26)

$$\Delta P_{sh} = \frac{E_s \epsilon_{sh}}{1 + w} = \frac{30 \times 10^6 \times 2.0 \times 10^{-4}}{1.0665} = 5625 \text{ psi.}$$

Shrinkage is affected by the concrete mix, the type of cement, the method of curing, the type of aggregate used, and time, but the greatest uncertainty in the evaluation of shrinkage strain is perhaps the environment to which the structure will be exposed. For design purposes, it has been suggested by T. Y. Lin[18] that shrinkage strain for the pre-tensioned construction is 3.0×10^{-4} in./in. Post-tensioned construction has the advantage that part of the shrinkage has occurred before tensioning is carried out, and the suggested shrinkage strain is 2.0×10^{-4} in./in.

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18. Design of Prestressed Concrete Structures, T. Y. Lin, John Wiley and Sons, New York, 1963, p. 95.

PRESTRESS LOSS DUE TO CREEP IN CONCRETE

The Phenomenon of Creep

Concrete creep is the inelastic deformation which continues to increase over the entire period under the application of sustained load and is not to be considered as including any deformation due to other causes such as growth, shrinkage, swelling, etc. When concrete is stressed by the application of sustained load, an immediate deformation occurs which is followed by further gradual deformation at a continuously diminishing rate. The terms immediate and gradual have been used only in a descriptive sense; the initial rapid deformation is not really immediate, nor can it be precisely separated from the later gradual deformation. However, the basic characters involved in these two types of strain are sufficiently distinct that they may be separated for most practical purposes. The initial immediate deformation is generally known as "elastic shortening", its effect on prestress has been previously discussed in this report; the gradual strain which follows may be considered as "creep strain". This is perhaps one of the most significant factors which influence the design of prestressed concrete structures.

Expression for Creep Strain

According to T. C. Hansen,[19] total sustained strain in concrete may be represented by

$$\epsilon_{tot} = \epsilon_{es} + \epsilon_{creep}$$

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19. "Creep and Stress Relaxation of Concrete", Torben C. Hansen, Handlingar Proceeding No. 31 Swedish Cement and Concrete Research Institute, Stockholm, 1960, p. 15.

where

e_{tot} = total strain under sustained load at some time t at which total strain is measured

e_{es} = elastic shortening measured when transferred at time t_o

e_{creep} = creep strain taking place during the time $(t - t_o)$

and according to Z. Erzen,[20] the creep strain is considered to be a function of e_{es} and t_o/t. The above expression may be written as

$$e_{tot} = \frac{\sigma t_o}{E_{t_o}} + f\left(\frac{\sigma t_o}{E_{t_o}}, \frac{t_o}{t}\right)$$

where

σt_o = initial unit stress,

E_{t_o} = modulus of elasticity of concrete at time of transfer.

Assuming that the creep strain varies linearly with the initial unit stress σt_o; the above expression may be rewritten as

$$e_{tot} = e_{es}(1 + g(t_o/t))$$

or writing the above expression in the dimensionless form, it gives

20

$$\frac{e_{tot}}{e_{es}} = e^{\alpha\left(1 - \left(\frac{t_o}{t}\right)^{\beta}\right)} \quad \ldots\ldots\ldots\ldots\ldots \quad (27)$$

where α and β are constants to be determined by tests. In order that total strain be calculated from Eq. (27), values of modulus of elasticity E_{t_o} at

---

20. "An Expression for Creep and It's Application to Prestressed Concrete", Cevdet Z. Erzen, Proceedings of ACI Vol. 57, 1955, p. 206.

the time of loading must be known. The values of modulus of elasticity can be found approximately from the dimensionless formula[20]

$$\frac{E_{t_o}}{E_{28}} = \frac{4}{3 + 28/t} \quad \cdots\cdots\cdots\cdots \quad (28)$$

Loss of Stress in Prestressed Beam Due to Creep

Assume all the cables are stressed simultaneously and have the same eccentricity, also the beam is considered lying on the ground during prestress transfer so that the dead load moment of the beam M_G can be neglected. Equating the unit strain in concrete at the position of the center of gravity of wires and the unit strain in the wires, gives

$$\frac{\Delta F_{es}}{A_s E_s} = \frac{F_1 - \Delta F_{es}}{A_c E_{t_o}}\left(1 + \frac{e^2}{r^2}\right) \quad \cdots\cdots\cdots\cdots \quad (29)$$

From this and Eq. (27) the total strain at time t may be expressed as

$$\epsilon_I = \frac{1 + e^2/r^2}{A_c E_{t_o}}(F_1 - \Delta F_{es})\, e^{\alpha\left(1 - \left(\frac{t_o}{t}\right)^{\beta}\right)}$$

and due to the variation of stress there results a gradual loss of stress denoted by $\Delta\overline{F}(t)$, as shown in Fig. 14a, such that the total loss at any time is given by

$$\Delta F = \Delta F_{es} + \Delta\overline{F}(t).$$

Thus, as the strain due to initial stress takes place, there results a different strain which corresponds to $\Delta\overline{F}(t)$ and is given by

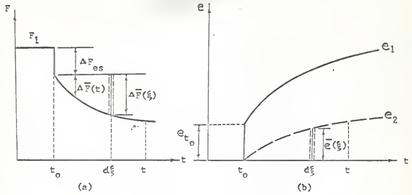

Fig. 14. Variation of stress and strain with time.

$$e_2 = \int_{t_o}^{t} e^{\alpha(1 - (\xi/t)^{\beta})} \frac{d\overline{e}(\xi)}{d\xi} d\xi$$

in which $\frac{d\overline{e}(\xi)}{d\xi} d\xi$ is the increment of strain due to $\Delta\overline{F}(\xi)$ during a certain interval $d\xi$ as shown in Fig. 14b. The net unit strain in the concrete, which is also the unit strain in the steel, at any time t is the difference in the strain e_1 and e_2. Hence

$$\frac{\Delta F}{A_s E_s} = \frac{1 + e^2/r^2}{A_c E_{t_o}}(F_1 - \Delta F_{es})\, e^{\alpha(1 - (\frac{t_o}{t})^{\beta})} - \int_{t_o}^{t} e^{\alpha(1 - (\frac{\xi}{t})^{\beta})} \frac{d\overline{e}(\xi)}{d\xi} d\xi \quad .$$

Integrating the second term on the right hand side by parts and also noting that $\overline{e}(t_o) = 0$ and $\overline{e}(\xi) = \overline{\sigma}(\xi)/E = \frac{\Delta\overline{F}(\xi)}{E_\xi A_c}(1 + e^2/r^2)$, there results

$$\frac{\Delta F}{A_s E_s} = \frac{1 + e^2/r^2}{A_c E_{t_o}}(F_1 - \Delta F_{es})\, e^{\alpha(1 - (\frac{t_o}{t})^\beta)} - \frac{\Delta \overline{F}(t)}{A_c E_t}(1 + e^2/r^2)$$

$$- \frac{\alpha\beta}{t^\beta}\frac{1 + e^2/r^2}{A_c}\int_{t_o}^{t} e^{\alpha(1 - (\frac{\xi}{t})^\beta)}\, \xi^{\beta-1}\frac{\Delta \overline{F}(\xi)}{E_\xi}\, d\xi \quad \ldots\ldots \quad (30)$$

which is an integral equation containing the unknown function $\Delta\overline{F}(\xi)$. The integration is much too difficult to perform. In practice, the solution is obtained by numerical procedures in which the last term of Eq. (30) is transformed from an integral to a series by changing the infinitesimal difference $d\xi$ to the finite difference $\Delta\xi$ and the integration is replaced by a summation with respect to t, thus

$$\frac{-\alpha\beta}{t^\beta}\frac{1 + e^2/r^2}{A_c}\sum_{t_o}^{t} e^{\alpha(1 - (\frac{\xi}{t})^\beta)}\, \xi^{\beta-1}\frac{\Delta \overline{F}(\xi)}{E_\xi}\Delta\xi \ .$$

Numerical Example

Problem 5: For the beam in problem 3, E_c is based on 28-day tests and is 3.64×10^6 psi, $E_s = 29 \times 10^6$ psi. and $\beta = 1$, $\alpha = 0.74$. The prestress is transferred on the tenth day after the casting of concrete, compute creep loss on the twelveth day after casting.

Solution: From problem 3, the beam possesses the following properties:

$A_c = 744$ sq. in.	$E_s = 29 \times 10^6$ psi.
$I_c = 123{,}800$ in.^{4}	$E_{28} = 3.64 \times 10^6$ psi.
$r = 12.75$ in.	$F_1 = 88{,}000$ lb.
$e = 14.0$ in.	$P = 0.00318$

According to Eq. (28) the modulus of elasticity of concrete on the tenth day is

$$E_{10} = \frac{4}{3 + 28/t} E_{28} = 2.56 \times 10^6 \text{ psi.}$$

$$n_{10} = 11.5$$

From Eq. (18)

$$w = n\,p\,(1 + e^2/r^2) = 11.5 \times 0.0038(1 + 14^2/12.75^2) = 0.084$$

The total prestress loss due to elastic shortening can be obtained by using Eq. (20), so

$$\Delta F_{es} = \frac{88{,}000 \times 0.084}{4}\left(\frac{1}{1 + 0.021} + \frac{2}{1 + 0.042} + \frac{3}{1 + 0.063}\right)$$

$$= 10{,}560 \text{ lb.}$$

To find the loss on the succeeding days Eq. (30) is used so that

$$\frac{\Delta F_{es} + \Delta\overline{F}(t)}{P(1 + e^2/r^2)E_s} = \frac{(F_1 - \Delta F_{es})}{E_{t_o}} e^{\alpha(1 - (t_o/t)^{\beta})} - \frac{\Delta\overline{F}(t)}{E_t}$$

$$- \frac{\alpha\beta}{t^{\beta}} e^{\alpha(1 - (\xi/t)^{\beta})} \xi^{\beta - 1} \frac{\Delta\overline{F}(\xi)}{E_{\xi}} \Delta\xi$$

in which $\Delta\overline{F}(\xi)\Delta\xi$ is the differential area as shown in Fig. 14a. To find the additional creep loss on the twelveth day the above equation t and ξ are taken as 12 and 11.5 respectively, and

$$\Delta\overline{F}(11.5)\Delta\xi = \frac{1}{2}\Delta\overline{F}(12) \times 1$$

since

$$1/E_{10} = 0.391 \times 10^{-6},$$

$$1/E_{11.5} = 0.388 \times 10^{-6},$$

$$1/E_{12} = 0.366 \times 10^{-6}.$$

Substituting all the numerical values into the above expression gives

$$4.9(10560 + \Delta\overline{F}(12)) \times 10^{-6} = 34200 \times 10^{-6} - 0.366 \times 10^{-6} \times \Delta\overline{F}(12)$$

$$- 0.0616 \times 1.056 \times 1/2 \times \Delta\overline{F}(12) \times 1 \times 0.388 \times 10^{6}.$$

Solving for $\Delta\overline{F}(12)$ the loss due to creep on the second day after transfer, there is found

$$\Delta\overline{F}(12) = 3{,}340 \text{ lb.}$$

Physical Factors Affecting Creep Strain

The values of creep are controlled by the following physical factors all of which are not of equal importance.

1. Relative humidity of the atmosphere; high percentage relative humidity during the loaded period results in a low creep strain.

2. Creep continues over the entire period, but decrease in creep is affected by increase in age of loading.

3. Creep increases with a higher water - cement ratio and with a lower aggregate - cement ratio.

4. The magnitude of the creep depends upon the mineral composition, partical shape, and surface texture of the aggregate.

5. Creep of concrete with low-heat cement is appreciably greater than that of normal cement concrete.

6. Increasing the diameter of the specimen results in a decrease in the creep.

7. Creep is influenced by both the temperature of the concrete and the temperature of the surrounding atmosphere.

8. From the viewpoint of the gel structure it is conceivable that creep should be influenced by the degree of compaction of the concrete mass. A high degree of compaction should result in less creep strain.

RELAXATION OF PRESTRESSING STEEL

Relaxation is defined as the loss of prestress in a stressed material maintained at a constant length for a period of time. Another manifestation of the same basic phenomenon, creep, is defined as the change in the length of a material under stress. The two definitions give about the same results when the relaxation is not excessive, but the tendon does not deform freely and the stress in it can change. Thus, the constant strain method is more often adopted as a basis for measurement because of its similarity to the actual conditions in prestressed concrete.

Factors Affecting Relaxation

Relaxation varies with many factors. The available experimental information reveals that the major factors affecting stress relaxation are: (a) the initial stress ratio, (b) the type of steel, (c) the program of stressing, and (d) the temperature.

When a steel is under low stress the relaxation is insignificant and is negligible. However, when it is subjected to any stress above 0.6 f_y, the influence of the initial stress ratio becomes important. Typical curves, giving the relation between time, stress relaxation, and initial stress ratio, are shown in Fig. 15. It is seen that higher stress loss is found when the initial stress ratio is high.

The relaxation losses measured in tests on steels of different compositions and treatments have been observed to be different, hence the expressions for estimating the amout of stress relaxation for particular types of steels will be different. However, this is undesirable not only because it eliminates the general objective of obtaining a useful method for estimating the effects

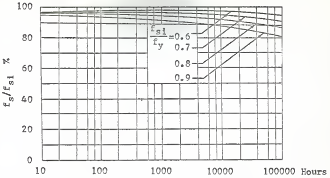

Fig. 15. Variation of stress with time according to equation 31. (From "A Study of Stress Relaxation in Prestressing Reinforcement," D. D. Magura and M. A. Sozen, PCI Journal, Vol. 9, No. 2, 1964, p.24)

of stress relaxation but also because to derive a particular stress relaxation formula for a certain type of prestressing steel will not easy to fulfill the end.

Temperature variations can have a critical effect on relaxation if the range is abnormally high. However, under ordinary temperature conditions this variable may be ignored.

Expressions for Estimating the Amount of Stress Relaxation

According to D. D. Magura and M. A. Sozen,[21] the expression for the remaining stress at any time t after prestress is

$$f_s = \frac{f_{si}}{1 + 10^n} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

21. "A Study of Stress Relaxation in Prestressing Reinforcement", D. D. Magura and M. A. Sozen, Journal of the PCI, Vol. 9, No. 2, 1964, p. 24.

where

f_{si} = the initial stress in prestressing steel,

f_s = the remaining stress at any time t after prestressing,

n = a function of time and the initial stress ratio.

The function n is described satisfactorily by the expression[21]

$$n = -1.3 + \frac{\log t}{3}\left(\frac{f_{si}}{f_y} - 0.55\right) \qquad \text{(32)}$$

where

f_y = yield stress based on 0.1% offset,

t = time in hours after prestressing.

The variation of stress with time computed by using Eq. (31) and (32) are shown in Fig. 15. The non-linear relationship of f_s and f_{si} in Eq. (31) can be approximated by the following linear equation

$$\frac{f_s}{f_{si}} = 1 - \frac{\log t}{10}\left(\frac{f_{si}}{f_y} - 0.55\right) \qquad \text{(33)}$$

In the case of pretensioned cable the loss occurring before transfer should be subtracted from the total loss predicted for the effective stress at transfer. Thus Eq. (33) may be modified as follows

$$\frac{f_s}{f_{si}} = 1 - \left(\frac{f_{si}}{f_y} - 0.55\right)\left(\frac{\log t_1 - \log t_0}{10}\right) \qquad \text{(33a)}$$

where

t_o = time at transfer measured from the wire is tensioned,

t_1 = time when the relaxation loss is estimated.

The rate at which relaxation occurs is very rapid at first. In practice, it is possible to apply a preliminary overstress of 5 - 10% to the wires, maintaining this overstress for two or three minutes and then reducing to the required initial prestress. The effects of this preliminary overstressing technique have been studied at the University of Leeds. The result has shown that prestretching is an effective technique to reduce relaxation.

REDUCTION OF THE PRESTRESS LOSSES

Concrete

Higher strength concrete is required for prestressed concrete than for conventional reinforced concrete. Present practice calls for a 28-day strength of 4000 psi. for prestressed concrete, while the corresponding value for reinforced concrete is around 2500 psi. Concrete for prestressed work should possess the properties of high strength, high modulus of elasticity, and minimum possible shrinkage and creep. All these factors result in less loss of prestressing force, which results in an important saving in the jacking process and may be more economical with respect to the amount of steel required.

Steel

High strength steel is essential for successful prestressing work. It is preferable to prestress with wires having a proportional limit of at least 100,000 psi. so that total losses in prestressing will only be a small percentage of the applied prestress. In general, prestressing steel should have the following properties:

1. High tensile strength. The ultimate strength of steel wire should be around 200,000 to 300,000 psi

2. High ratio of tensile strength to yield strength.

3. Higher proportional limit.

4. High strength wire has lost ductility; from a safety standpoint it is desirable to have sufficient deformation before failure occurs. This requires that the stress-strain curve of the steel must have a flat yielding range.

Reduction of Frictional Losses

The first step in reducing frictional losses lies in the design stage. Two simple ways to keep down frictional losses are to limit the length of tendons and to reduce their curvature. In practice, cables with length over 100 ft., or where total angular change exceeds 20 degrees, the effect of friction becomes significant, in these cases it is always preferable to jack from both ends.

The second step in reducing frictional losses can be accomplished by choosing tendons with proper surface conditions. For example, corrugeted tendons and rusted tendons will have higher frictional coefficients. In addition, the types and surface conditions of the surrounding materials, whether metal sheathing, paper, plastic, or concrete, will also have considerable effect on the frection.

Lubrication has been shown to be an effective method to reduce frictional losses.

The fourth approach in reducing frictional losses lies in the care exercised during construction. The tendons must be carefully wrapped so as to avoid any leakage of mortar through the sheathing. The wobble effect can be reduced by providing adequate supports for the tendons. The spacing of the supports depends upon the size and stiffness of the tendons and in general, the supports should not be more than 4 ft. apart.

Friction can be totally or partically balanced by over-tensioning. However, the amount of over-tensioning is limited by the strength of the tendons and should not be carried too far; a common rule is not to tension the tendons to over 75% or 80% of their ultimate strength.

Shrinkage and Creep

It has been shown that the rate of occurrence of shrinkage is comparable to that of creep. Both have a rate which is greatest during the early days after casting and which approaches a maximum value after a period of time. Since both shrinkage and creep are affected by similar factors, the following methods of reduction in prestress losses are applicable to both cases:

1. High strength concrete is desirable, since it is less liable to shrinkage and it also has a high modulus of elasticity and smaller creep strain. It has been shown that creep and shrinkage increase with a higher water-cement ratio; hence, it is practical to keep the water-cement ratio as low as possible.

2. A survey of available creep data discloses that the creep strain decreases with the elastic moduli of the aggregates. In general, the elastic moduli depend on both the size on the aggregates and the quality of the garding. Thus aggregates of larger size, which are harder, more dense, and have low absorption, are preferable. Concrete containing hard limestone is believed to have smaller shrinkage and creep strain than that containing granite, basalt, and sandstone, approximately in that order.

3. From the viewpoint of the gel theory it is believed that creep and shrinkage can be reduced by increasing the degree of compaction of the concrete mass, this can be achieved by using either internal or external vibration.

4. Since the major part of shrinkage and creep strain occurs in the first few days of the entire period, it is important to reduce the shrinkage and creep strain in this relatively short period of time, thus early hardening of concrete is often desirable. This can be obtained by using special curing

methods. For example, autoclave curing, which accelerates the formation of stable chemicals like lime-silicate hydrate, can reduce the shrinkage by a considerable amount.

Reduction of Relaxation Losses

1. The first step in reducing relaxation losses lies in choosing prestressing steel itself. Strength and ductility are the two important characteristics of the steel; an increase in carbon content increases the strength and hardness of the steel. In other words, it reduces the initial stress ratio, which is considered an important factor in relaxation loss.

2. It has been shown in most series of tests that relaxation losses can be reduced by overstressing. In practice, steel is stressed a few per cent above its initial prestress, this overstress is maintained for a few minutes and then the stress is reduced to the required initial prestress. It is believed that this prestretch technique is an effective method for reducing relaxation loss, since a large portion of the loss occurs after tensioning and before release. Since the overstressing is limited by the strength of the tendons and concrete, it should not be carried too far.

TOTAL AMOUNT OF LOSSES

The magnitude of losses can be expressed in the following ways:

1. In unit strains. This is the most convenient for losses such as creep, shrinkage, and elastic shortenings of concrete.

2. In unit stresses. If the modulus of elasticity of steel is known, all losses expressed in unit strains can be transformed into unit stresses in steel.

3. In total strains. This is more convenient for the anchorage losses.

4. In percentage of the initial prestress. Losses due to friction and the relaxation of steel can be most easily expressed in this way. This often gives a better picture of the significance of the losses.

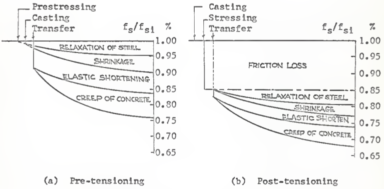

Fig. 16. Pictorial analysis of typical losses of prestress.

(From Prestressed Concrete Theory and Design, R. H. Evans, John Wiley and Sons, New York, 1958, p. 51.)

It is difficult to generalize the amount of loss of prestress, because it is dependent on so many factors. Fig. 16 shows a pictorial analysis of the various factors contributing to the total loss of stress for average steel and concrete properties, cured under average conditions.

Figure 16 is based on the assumption that proper over tensioning has been applied to reduce relaxation in steel and the loss due to anchorage take up. It is seen that post-tensioning construction possesses the advantage that the concrete is generally older and therefore of higher strength at transfer of the prestress so that both shrinkage and creep are considerably reduced.

CONCLUSIONS

The purpose of this report has been to illustrate the various aspects of prestress losses in the post-tensioned method, and to serve as an aid in establishing a sense of values concerning this specific problem.

The value of friction loss in the jack varies in different systems and the intention is not to consider this as a type of loss in prestress. Anchorage deformation for some post-tensioning systems is very small and can be reasonably neglected, while in some cases its effect is vitally significant. Frictional loss is not a serious problem for relatively short and straight cables if reasonable care is taken during prestressing. In continuous beams, frictional losses can be very high and every effort should be made to reduce the coefficient of friction. In most cases the cables are stressed in succession, hence the process in computing the loss due to elastic shortening becomes quite complicated. For practical purposes, it is accurate enough to determine the loss for the first cable and use half of that value for the average loss of all the cables.

Although it is understood that shrinkage and creep of concrete are fundamentally different, they are affected by similar factors. Despite the large amount of work accomplished by various investigators, the knowledge concerning the effects of concrete shrinkage and creep on prestress losses are relatively limited. This limitation is due to the large number of variables involved, and to the lack of correlated experimental results of various investigators.

The rate of steel relaxation for a given steel is dependent upon the magnitude of stress, temperature, duration of loading and initial stress ratio. It has been shown that by overstressing the relaxation losses can be reduced effectively.

Prestressed concrete construction does offer numerous advantages and its use in various fields is increasing. In spite of its impressive usefulness, there are no general theories of prestress losses that have been satisfactorily formulated. However, the material presented in this report should be considered applicable to most practical design of prestressed concrete structures.

ACKNOWLEDGEMENT

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APPENDICES

APPENDIX A - TABLES AND GRAPHS FOR THE SOLUTION OF FRICTIONAL LOSSES

Table 6. KX, $\mu X/R$, e^{-KX}, $e^{-\mu X/R}$ for computing friction losses

KX or $\mu X/R$	e^{-KX} or $e^{-\mu X/R}$	KX or $\mu X/R$	e^{-KX} or $e^{-\mu X/R}$
0.01	0.990	0.41	0.663
0.02	0.980	0.42	0.657
0.03	0.970	0.43	0.650
0.04	0.960	0.44	0.644
0.05	0.951	0.45	0.637
0.06	0.941	0.46	0.631
0.07	0.932	0.47	0.625
0.08	0.923	0.48	0.618
0.09	0.913	0.49	0.616
0.10	0.904	0.50	0.606
0.11	0.895	0.51	0.600
0.12	0.886	0.52	0.594
0.13	0.878	0.53	0.588
0.14	0.869	0.54	0.582
0.15	0.860	0.55	0.576
0.16	0.852	0.56	0.571
0.17	0.843	0.57	0.565
0.18	0.835	0.58	0.559
0.19	0.826	0.59	0.554
0.20	0.818	0.60	0.548
0.21	0.810	0.61	0.543
0.22	0.802	0.62	0.537
0.23	0.794	0.63	0.532
0.24	0.786	0.64	0.527
0.25	0.778	0.65	0.522
0.26	0.771	0.66	0.516
0.27	0.763	0.67	0.511
0.28	0.755	0.68	0.506
0.29	0.748	0.69	0.501
0.30	0.740	0.70	0.496
0.31	0.733	0.71	0.491
0.32	0.726	0.72	0.486
0.33	0.718	0.73	0.481
0.34	0.711	0.74	0.477
0.35	0.704	0.75	0.472
0.36	0.697	0.76	0.467
0.37	0.690	0.77	0.463
0.38	0.683	0.78	0.458
0.39	0.677	0.79	0.453
0.40	0.670	0.80	0.449

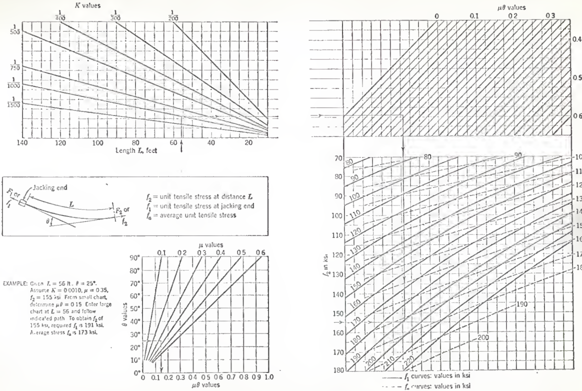


Fig. 17. Chart for Solution of Equation 9 on p. 10.

(From "Design of Prestressed Concrete Structure," T. Y. Lin, John Wiley & Sons, New York, p. 108.)

Solution for: $f_2 = f_1 e^{-(u\theta+KL)}$, $f_{av} = f_2 \dfrac{e^{(u\theta+KL)} - 1}{u\theta + KL}$.

EXAMPLE:

Cable as shown, f_2 = 155 ksi. Assume K = 0.0010, u = 0.35. It is required to find f_1 at jacking end and cable elongation for average stress.

SOLUTION:

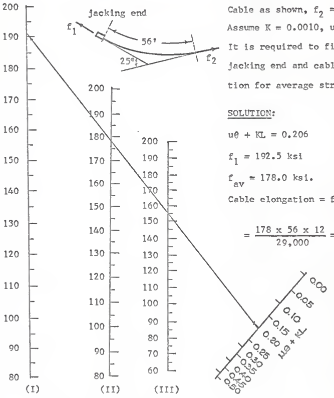
$u\theta + KL = 0.206$

f_1 = 192.5 ksi

f_{av} = 178.0 ksi.

Cable elongation = $f_{av}L/E_s$

$= \dfrac{178 \times 56 \times 12}{29{,}000} = 4.12$ in.



(I) Tension at jacking end f_1, ksi.

(II) Average tension in cable f_{av}, ksi.

(III) Tension at non-jacking end f_2, ksi.

Fig. 18 Nomograph for Frictional Loss and Cable Elongation. (From "Cable Friction in Post-tensioning," T. Y. Lin, Proceedings of ASCE, Vol. 82, 1956, p. 1107.)

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APPENDIX C - NOTATIONS

A_p	actual area of the piston of the jack.
A_c	gross area of the concrete section.
A_s	area of prestressing steel.
β	slope of the stress diagram in computing friction losses.
C	jack efficiency.
d_{eA}	diameter of the capillary tube under a given air humidity.
d_w	diameter of capillary tube filled with water.
E	the modulus of elasticity for the combined material.
E_c	modulus of elasticity of concrete.
E_s	modulus of elasticity of prestressing steel.
E_w	modulus of elasticity of the soft material.
E_h	modulus of elasticity of the hard material.
E_{t_o}	modulus of elasticity of concrete at time t_o.
e	eccentricity of the prestressing force; base of Naperian logarithms in computing effects of friction.
F_1	prestress at jacking end.
F_2	prestress at dead end.
F_x	prestress at distance x from jacking end.
f_s	the remaining stress at any time t after relaxation.
f_{si}	initial stress in prestressing steel after seating of the anchor.
f_y	nominal yield-point stress of prestressing steel.
f'_c	compressive strength of the concrete at 28 days.
f_o	unit stress in steel at jacking end during stressing.
$f_{℄}$	unit stress in the prestressing steel at the center line of the member.

f_x unit stress in the steel at a distance of x feet from the jacking end.

K friction wobble coefficient per foot of prestressing steel.

L length of prestressing steel.

l length of tendon which is affected by the anchorage deformation; segment length in computing the effects of friction.

N normal force created by prestress force due to curvature.

n ratio of E_s/E_c .

m number of pairs of wires.

P_n normal pressure in capillary tube caused by surface tension.

P ratio of prestressing steel area to concrete area.

R radius of the curved cable.

r radius of gyration.

t_o time at transfer measured from the wire is tensioned.

t_1 time when the relaxation is estimated.

$V_{h,w}$ volume of cross section consisting of hard and soft materials, respectively.

α, β constants used in computing the creep effects.

γ capillary constant equals 8 mg per mm.

e_{tot} total strain.

e_{es} strain due to elastic shortening of concrete.

e_{sh} strain due to shrinkage of concrete.

e_{creep} strain due to creep of concrete.

σ_w stress on the soft material.

σ_h stress on the hard material.

σ_{t_o} initial unit stress.

μ	coefficient of friction.
θ	angular change between the tangents to the tendon at the jacking end and at point x from the jacking end.
	total deformation of the anchorage device.
ΔL	elongation of cable due to prestress.
ΔF	total prestress loss due to creep in concrete.
ΔF_{es}	loss of prestress due to elastic shortening of concrete.
$\Delta \bar{F}(t)$	loss of prestress due to creep at any time t.
ΔP	loss of prestress at the center line of cable due to elastic deformation of anchorage device.
ΔP_{sh}	loss of prestress due to shrinkage of concrete.
w	a section constant $w = \frac{A_s E_s}{A_c E_c}(1 + e^2/r^2)$.

A STUDY OF PRESTRESS LOSSES IN THE POST-TENSIONING METHOD

by

YA-TSUEN LIU

B. S., Taiwan Provincial Taipei Institute of Technology, 1963

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

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1967

The main purpose of prestressing concrete is to achieve benefits in strength and economy. Thanks to engineers' continuing research for this new construction material and technique, prestressed concrete has become practical in general application during the past decade. In spite of its impressive success, no general theories and accurate methods in predicting the prestress losses have been formulated.

It is the purpose of this report to present a library review of the literature on the various aspects of prestressed losses in a concise and logical method to give a better understanding of this specific problem.

One of the best definitions of prestressed concrete is given by the American Concrete Institute Committee on Prestressed Concrete, "Prestressed concrete is defined as concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from given external loading are counteracted to a desired degree."

A brief review of the history and the development of prestressed concrete is presented in the first chapter. The second chapter deals with the frictional loss in the jack. This problem is somewhat simple, since it can be estimated from the discrepancy between the measured and the expected elongations, and depends only on the different types of jacking systems used. Prestress loss due to the distortion of the fixing devices and the slipping of the wires is discussed in chapter three. The amount of slippage depends on the type of wedge and the stress in wires.

Elastic shortening of concrete as the result of the applied prestress is equal to the sum of the deformations in the concrete over the whole length of the beam. The theoretical solution can be obtained by the elastic theory, though the problem is made quite complicated when considering the effect of stretching wires in pairs. This problem is treated in chapter four.

Shrinkage in concrete is the contraction due to drying and chemical changes dependent on time and moisture conditions, but not on stresses. It has been shown that the capillary forces in concrete in a low humidity are very high and may cause a correspondingly large strain in concrete. The theoretical expressions of shrinkage are presented in the report.

The phenomenon of creep is defined and briefly described. An equation is developed which defines creep as a function of stress, age at loading and modulus of elasticity of concrete. This equation is then used in determining the loss of stress in the prestressed beams due to creep of concrete. Some physical factors which affect the magnitude of creep are also discussed.

Relaxation is defined as the loss of stress in a stressed wire held at a constant length. According to Y. Guyon, the main factors determining the relaxation for a given type of steel are dependent on its chemical composition, method of manufacture, the modulus of elasticity, the initial stress and time. A general equation relating relaxation stress and time is presented.

One of the major losses of prestress in post-tensioning is that due to frictional resistance along the length of the cable. Other sources of frictional loss of prestress are discussed. Data for the coefficient of friction under various conditions are presented. The analytical expressions that are necessary in estimating the frictional losses are derived, and methods for reducing frictional losses are considered.

In reviewing the important literature on prestress losses, the writer arrives at the conclusion that no general theories of prestress losses have been satisfactorily formulated. In fact, the problem is quite complicated. In connection with this, some graphs and diagrams for practical purposes have been abstracted, and are of the most significance in the opinion of the writer.