

THE EVOLUTION AND COMPARISON
OF DESIGN METHODS FOR CONCRETE SLABS

by

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MASTER OF SCIENCE

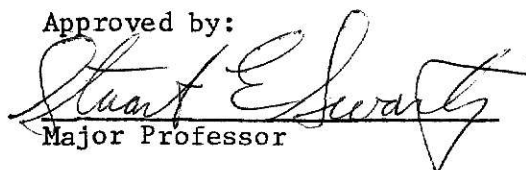
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I INTRODUCTION

Reinforced concrete floor slabs are common structural elements. Among the types often used, those spanning in more than one direction are classified into two different design categories - the two-way slab and the flat slab. Two-way slabs are supported on all four sides by reinforced concrete or steel beams or walls. Flat slabs are supported directly on columns which may have capitals and the slabs may have drop panels.

While these slabs have been analyzed and successfully designed using many different methods for a long time, the design methods for beam-supported slabs have all been based on the results of analyses of slabs on rigid beams. The possibility of designing, on a reasonable basis, slabs supported on beams of intermediate stiffness was not even admitted.

In view of this, ACI 318-71(1)^{*} combined the two categories into one, with beam stiffness ranging from zero, that is the "flat slab", to some non-zero value representing a "stiff" support such as a heavy beam or wall. It included provisions for the effect of the stiffness of the supporting beams on the distribution of moments within the slab.

It is inevitable that the design methods would be inconsistent with each other. The purpose of this report is (1) to describe the developments that led to the different methods of design for the two-way slab and flat slab and then to the unique methods in ACI 318-71(1)

^{*} Numerals in parentheses refer to items listed in References.

for both categories; (2) to compare the different methods of design by using numerical examples; (3) to compare these methods with some test results listed in the references.

II EVOLUTION OF SLAB DESIGN METHODS

Part A Two-Way Slabs

The 1912 Progress Report of the Joint Committee (10) suggested that square spans may well be reinforced in both directions. A footnote gave the proportion of load to be considered in each direction, provided the aspect ratio of the panel was less than 1.5:

"The exact distribution of load on square and rectangular slabs, supported on four sides and reinforced in both directions cannot readily be determined. The following method of calculation is recognized as faulty but it is offered as a tentative method which will give results on the safe side. The distribution of load is first determined by the formula:

$$k = \frac{L^4}{L^4 + S^4} \quad \text{II-1}$$

where

k = proportion of load carried by the transverse reinforcement

(short span);

L = length of slab (long span);

S = breadth of slab (short span)."

In the 1916 final report of the First Joint Committee(11), Eq. II-1 based on the crossing beam analogy was replaced by

$$k = \frac{L}{S} - 0.5 \quad \text{II-2}$$

a linear equation approximating Eq. II-1.

The publication in 1921 of the paper "Moments and Stresses in Slabs" (12), by Westergaard and Slater, showed quite clearly the fallacy of the crossing beam analogy. Nevertheless, the 1928 ACI Building Code (8) adopted equation II-2 since no further investigation had been made in the time being. The 1925 edition of the book "Concrete, Plain and Reinforced" (13), by Taylor, Thompson and Smulski, warned that the load distribution could be affected also by the end restraints. This approach culminated in Method 1 of the ACI Building Code. This method for the design of two-way slabs was developed for the New York City Building Code and proposed to the ACI in 1935. It was incorporated in the 1936 ACI Building Code (6) and its derivation was explained in a paper by Distasio and van Buren (14). The 1936 version of Method 1 provided formulas for the determination of points of contraflexure under various conditions of end restraint. Later studies (15) showed that this was unnecessary and the formulas were replaced by a set of constants in the 1947 ACI Building Code (4).

In 1926, Westergaard published a paper proposing a method of design for two-way slabs (16). This paper contained moment coefficients for slabs and supporting beams. The coefficients were based on analyses of continuous plates on rigid supports providing no torsional restraint. However, they were modified in view of the fact that exceeding the permissible stress locally was not critical and that flat slabs were designed satisfactorily for only 72 percent of the static moment.

Westergaard's moment coefficients were not adopted directly in a design method since they were given for a series of equal adjacent panels, a condition rarely met in practice. Hence, the Joint Committee (17) modified and extended Westergaard's procedure to give moment coefficients for isolated panels with various edge conditions. The modified procedure was adopted as Method 2 in the 1947 ACI Building Code (4).

The 1963 ACI Building Code (2) adopted a new method defined as "Method 3". It followed the same idea of Method 2. However, it took into account the inelastic moment redistribution and modified the moment coefficients. It also subdivided the 5 cases of continuity into 9 cases.*

The 1971 ACI Building Code (1) used the basic idea of the two methods for flat slabs - the empirical method and the elastic analysis method, and modified them by introducing the effects of supporting beams. Two unique methods for both the two-way slab and the flat slab were adopted.*

*The writer, in his time-limited searching for reference materials, is not able to find sufficient references which followed the detailed trail of the evolution of ACI Method 3 and the two methods in ACI 318-71.

Part B Flat Slabs

In 1906, C. A. P. Turner built the first flat slab, but he found entrenched resistance against his invention. A few years later, the situation changed. By 1913, over 1000 flat slab buildings had been built around the world.

In 1910, A. R. Lord made testing history when he reported the strain readings he had made during the load test of a flat slab floor of the Deere and Webber Building in Minneapolis, Minnesota (32). Unfortunately, interpretations of the data from this test and a series of others did little toward resolving the mystery of the flat slab. For the structural engineer, plate action was still an entirely new concept.

In 1912, the progress report of the Joint Committee on Concrete and Reinforced Concrete had only one paragraph on the flat slab (10).

In 1914, while the majority of the engineering profession classified the flat slab as being "beyond the range of pure analysis", J. R. Nichols, a young engineer from Boston, Massachusetts, solved the problem with a simple, straightforward analysis (18).

Consider one rectangular panel of a plate having an infinite number of identical, uniformly-loaded panels. The plate rests on pin supports. The equilibrium conditions for one-half of the panel are shown in Fig. II-1. Because of symmetry, shears and twisting moments do not exist along the edges of the portion considered. Thus,

$$M_A + M_B = M_O = \frac{Wl^2}{8}$$

in which,

W = total load on whole panel;

ℓ_1 = span length in the direction considered.

Eq. II-3 does not indicate the relative magnitudes of the positive and negative moments but it does stipulate that the total moment resistance provided at the two sections should add up to $W\ell_1/8$. For the interior panel of a flat slab with a round capital and with the assumption that the reaction is distributed uniformly around the capital, Eq. II-3 becomes

$$M_o = \frac{W\ell_1}{8} \left[1 - \frac{4c}{\pi\ell_1} + \frac{1}{3} \left(\frac{c}{\ell_1} \right)^3 \right] \quad \text{II-4}$$

in which

c = effective support size.

In the closure to his paper, Nichols suggested that Eq. II-4 could be approximated by

$$M_o = \frac{W\ell_1}{8} \left(1 - \frac{2c}{3\ell_1} \right)^2 \quad \text{II-5}$$

However, it was observed, when the measured steel strains were converted to bending moments through the use of the straight-line formula

$$M = A_s f_s j d \quad \text{II-6}$$

the sum of the moments across the negative and positive moment sections was only a fraction of M_o as defined by Eq. II-5.

In 1916, in the final report of the First Joint Committee (11), the design static moment was given as 85 percent of that based on Nichol's analysis, or

$$M_o = 0.107 W l_1 \left(1 - \frac{2c}{3l_1}\right)^2. \quad \text{II-7}$$

The 1917 proposed revision (30) and the 1920 approved version (31) of the ACI Building Code contained an even more flagrant violation of statics:

$$M_o = 0.09 q_t l_2 (l_1 - qc)^2 \quad \text{II-8}$$

where qc was twice the distance from center line of the capital to the center of gravity of the periphery of the half capital.

In 1921, Westergaard and Slater published their monumental work on the analysis and design of slabs (12). One of the accomplishments of the paper was to show conclusively that the moments calculated from measured strains in the flat slab tests were not as small as they were interpreted to be. Previously, the moment had been calculated from the measured steel strain using the straight line formula

$$M = A_s E_s \epsilon_s j d \quad \text{II-6a}$$

which gives a linear relationship between M and ϵ_s for a given section as shown by line A in Fig. II-2. Due to the tensile strength of the concrete, the actual moment-strain curve in a load test is as shown by line B in the same figure.

The Westergaard-Slater treatise on slabs consummated the compromise between analysis and design. The Second Joint Committee (19) and 1928 ACI Building Code (7) adopted a slightly modified version of Eq. II-8 by which the rectangular capital was not recognized,

$$M_o = 0.09 W \ell_1 \left(1 - \frac{2c}{3\ell_1}\right)^2. \quad \text{II-9}$$

Before 1941, all design methods for flat slabs were "empirical". Due to the limitations of the empirical method and the anticipated effects of pattern loading on slab and column bending moments, there was a need for a "rational" method. The ACI frame analysis finds its beginnings in a paper by H. D. Dewell and H. B. Hammill published in 1938 (20). The final ACI elastic analysis method for flat slabs was developed under the direction of R. L. Bertin and was adopted in 1941 ACI Building Code (5).

In the 1956 ACI Building Code (3), modifications were made on both the elastic analysis method and empirical method. For the empirical method, a new factor F , was introduced based on the fear that the old tests on flat slabs which had a c/ℓ_1 ratio on the order of 0.2 might not apply to the modern flat plate which had rather low c/ℓ_1 ratios. Thus, Eq. II-9 was changed to

$$M_o = 0.09 W \ell_1 F \left(1 - \frac{2c}{3\ell_1}\right)^2 \quad \text{II-10}$$

where

$$F = 1.15 - c/\ell_1 \text{ but not less than } 1.$$

The 1963 ACI Building Code (2) remained unchanged on flat slab design except for a slight modification in the pattern loading when determining the maximum positive and negative moments in the frame analysis method.

The 1971 ACI Building Code (1) used the basic idea of the two methods for flat slab - the empirical method and the elastic analysis method, and modified them by introducing the effects of supporting beams. Two unique methods for both the two-way slab and the flat slab were adopted.

**THIS BOOK
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WITH DIAGRAMS
THAT ARE CROOKED
COMPARED TO THE
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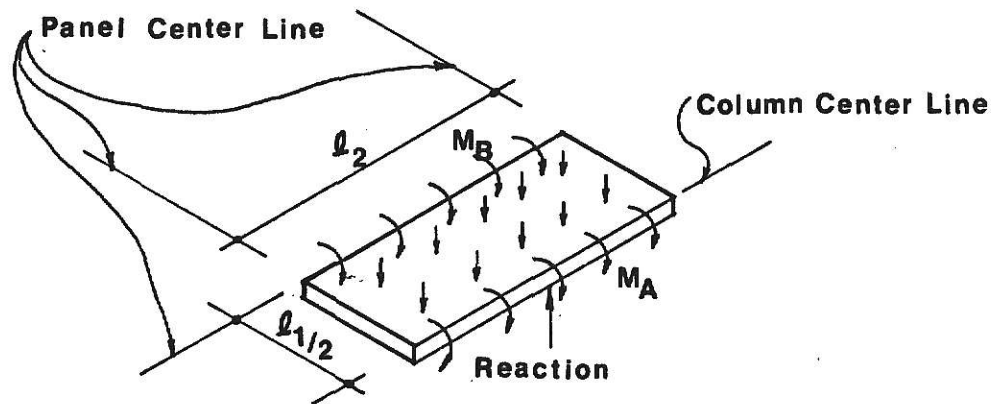


Fig. II - 1 Free Body Diagram of One-Half of a Typical Interior Panel

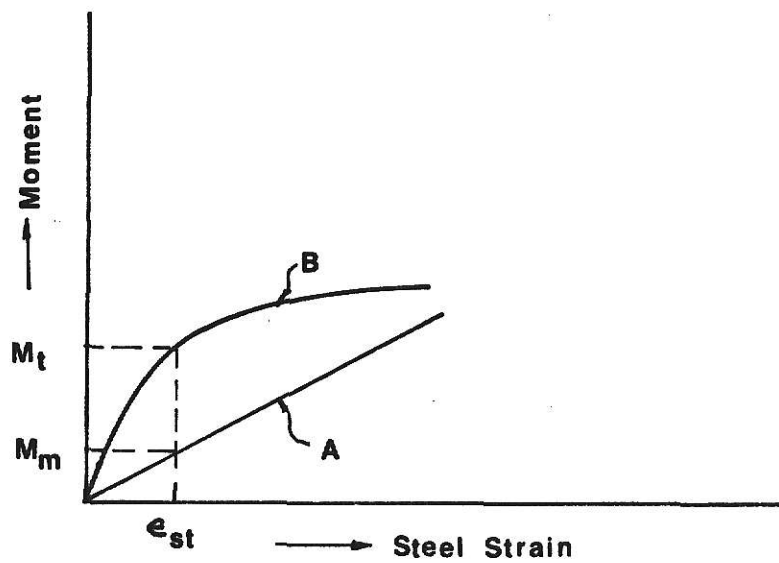


Fig. II - 2 Relationship Between Bending Moment & Steel Strain

III DESIGN METHODS FOR SLABS

Part A Two-Way Slabs

(1) ACI Method 1:

Method 1 of the ACI Building Code for the design of two-way slabs was developed for the New York City Building Code and proposed to the ACI in 1935. It was incorporated in the 1936 ACI Building Code (6). It remained in the ACI Code until 1971. In this Code, a unique method for both two-way slabs and flat slabs has been proposed instead of all the methods previously proposed by ACI.

Method 1 of the previous Code is kindred to the "elastic analysis" for flat slabs in that it is a two-dimensional approximation to the three-dimensional problem of a plate on a rigid support. It was based on two simplifications: an assumption about the division of load to the two slab spans and an assumption about the distribution of load along a given span. The division is determined by the following formula :

$$k = \frac{L^3}{L^3 + S^3} \quad \text{III-1}$$

in which

k = proportion of load carried by the transverse reinforcement
(short span);

L = length of slab (long span);

S = breadth of slab (short span).

To account for the end restraints, the span between the points of contraflexure are used as effective span. The lines of inflection are

determined by elastic analysis of the continuous structure in each direction, when only the span under consideration is loaded.

When either span is at least $2/3$ and at most $3/2$ of the adjacent continuous span or spans, the values of g or g_1 may be taken as 0.87 for exterior spans, 0.76 for interior spans, and 1.00 for spans discontinuous at both ends; in which

g = ratio of span between lines of inflection to L_1 in the direction of span L_1 , when span L_1 only is loaded;

g_1 = ratio of span between lines of inflection to L_1' in the direction of span L_1' , when span L_1' only is loaded;

L_1 = length of clear span;

L_1' = length of clear span in the direction normal to L_1 .

Thus, the proportion between two effective spans, that is $r = gL_1/g_1L_1'$, is determined as shown in Fig. III-1.

The bending moments are determined in each direction with the coefficients prescribed for one-way construction, B , and modified by factor C_1 or C_1' from Table III-1. The bending moment formulas for slab strips are as follows:

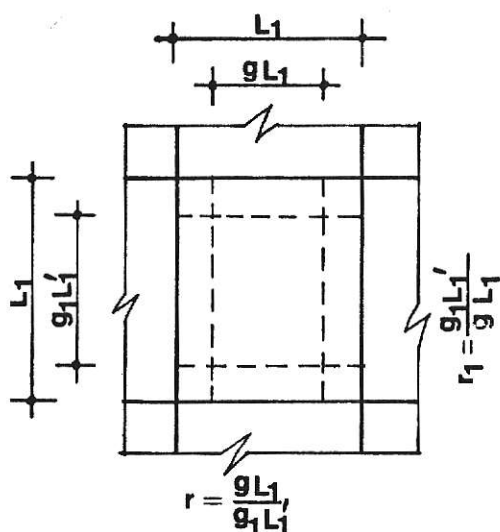
in L_1 direction	in L_1' direction
$M = C_1 B W_1 L_1$	$M = C_1' B W_1' L_1'$

III-2

in which

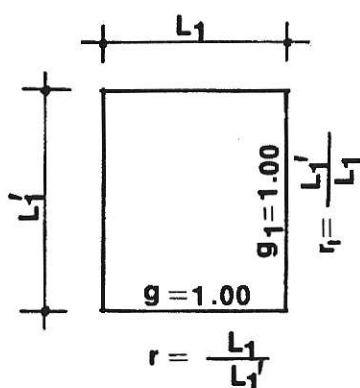
W_1 = total load between opposite supports on slab strip in L_1 direction of any width;

W_1' = corresponding value of W_1 in direction L_1' .

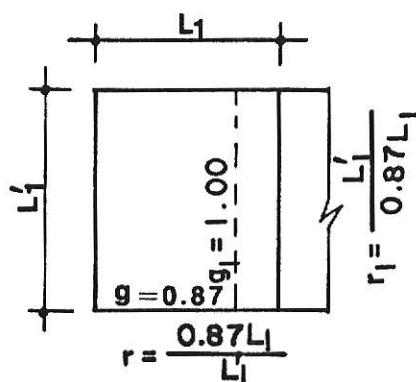


General Case

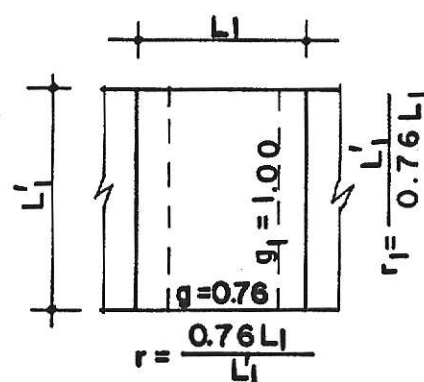
When L_1 or L_1' is at least $2/3$ or at most $3/2$ of the adjacent continuous span or spans, obtain r or r_1 from Case 1 to 6. Beyond these limits, Compute g and g_1 by elastic analysis when only the span under consideration is loaded.



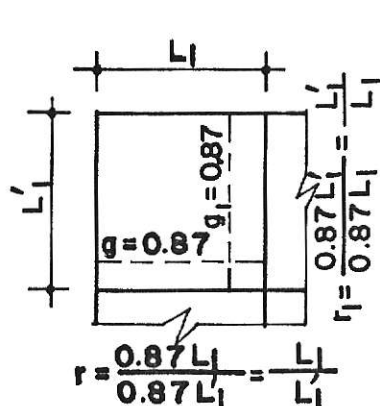
Case 1



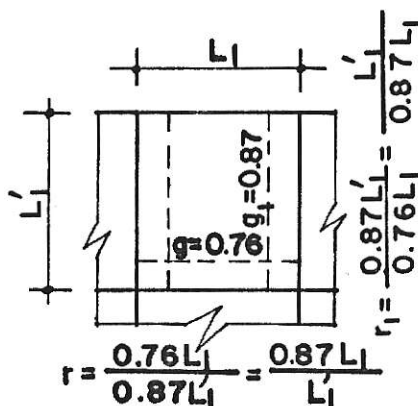
Case 2



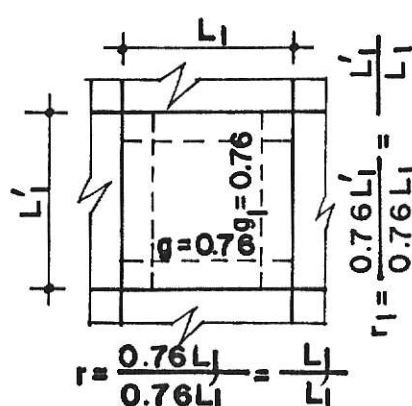
Case 3



Case 4



Case 5



Case 6

Fig. III-1 Proportion Between Two Effective Spans

In determining B, if the larger of two adjacent spans does not exceed the shorter by more than 20 percent with loads uniformly distributed, where the unit live load does not exceed three times the unit dead load, the following coefficients may be used in lieu of more accurate analysis. Beyond these limits, elastic analysis must be employed to determine B.

Positive moment:

End spans:

If discontinuous end is unrestrained	$\frac{1}{11}$
--------------------------------------	----------------

If discontinuous end is integral with the support	$\frac{1}{14}$
---	----------------

Interior spans	$\frac{1}{16}$
----------------	----------------

Negative moment at exterior face of first interior support:

Two spans	$\frac{1}{9}$
-----------	---------------

More than two spans	$\frac{1}{10}$
---------------------	----------------

Negative moment at other faces of interior supports	$\frac{1}{11}$
---	----------------

Negative moment at face of all supports for slab with spans not exceeding 10 ft	$\frac{1}{12}$
--	----------------

Negative moment at interior faces of exterior supports for members built integrally with their supports, where the support is a spandrel beam or girder	$\frac{1}{24}$
---	----------------

Since this report is concerned only with slab bending moments, all details about moments in beams and shearing force both in slab and beams are neglected here.

Table III-1 Moment Coefficients C_1 and C_1' for ACI Method 1

r	$r_1 = \frac{1}{r}$	C_1	C_1'	r	$r_1 = \frac{1}{r}$	C_1	C_1'
0.00	∞	1.00	0.00	1.10	0.91	0.28	0.39
0.50	2.00	0.89	0.06	1.20	0.83	0.23	0.45
0.55	1.82	0.79	0.08	1.30	0.77	0.19	0.51
0.60	1.67	0.70	0.10	1.40	0.71	0.16	0.57
0.65	1.54	0.64	0.13	1.50	0.67	0.14	0.61
0.70	1.43	0.58	0.15	1.60	0.63	0.12	0.66
0.80	1.25	0.48	0.21	1.80	0.55	0.08	0.79
0.90	1.11	0.40	0.27	2.00	0.50	0.06	0.89
1.00	1.00	0.33	0.33	∞	0.00	0.00	1.00

(2) ACI Method 2:

The 1940 Joint Committee (17) modified and extended Westergaard's design method (12) to give moment coefficients for isolated panels with various edge conditions. This method for two-way slabs was adopted as "Method 2" in the 1947 ACI Building Code (4).

"Method 2" was based on one basic assumption, namely that the slab was built monolithically with the supporting beams. Thus, moments were prescribed at the discontinuous edges and if an unbalanced moment occurred at adjacent negative moment sections, one-third of the difference was assumed to be resisted by the beam in torsion. However, the moment coefficients had been modified, taking into account the effect of pattern loading and that the effect of local overstress was not critical.

In this method, a two-way slab is considered as consisting of strips in each direction as shown in Fig. III-2. The critical sections for bending moments are along the edges of the panel at the faces of the supporting beams for negative moment, and along the center lines of the panels for positive moment. The bending moments for the middle strips are computed from the formula:

$$M = C_2 q_t S^2 \quad \text{III-3}$$

in which

C_2 = moment coefficient for two-way slabs as given in Table III-2;

q_t = total uniform load on slab;

S = length of short span for two-way slabs. The span is considered as the center-to-center distance between supports or the clear span plus twice the thickness of slab, whichever value is the smaller.

The average moments per foot of width in the column strip are two-thirds of the corresponding moments in the middle strip, varying from a maximum at the edge of the middle strip to a minimum at the edge of the panel. Where the negative moment on one side of a support is less than 80 percent of that on the other side, two-thirds of the difference is distributed in proportion to the relative stiffness of the slabs.

(3) ACI Method 3:

ACI Method 3 is similar to Method 2. However, it also takes into account the inelastic moment redistribution and subdivides the 5 cases

Table III-2 Moment Coefficients C_2 for ACI Method 2

Moments	Short Span						Long span, all values of m
	Value of m = S/L						
	1.0	0.9	0.8	0.7	0.6	0.5 and less	
Case 1-Interior panels							
Negative moment at -							
Continuous edge	0.033	0.040	0.048	0.055	0.063	0.083	0.033
Discontinuous edge	-	-	-	-	-	-	-
Positive moment at midspan	0.025	0.030	0.036	0.041	0.047	0.062	0.025
Case 2-One edge discontinuous							
Negative moment at -							
Continuous edge	0.041	0.048	0.055	0.062	0.069	0.085	0.041
Discontinuous edge	0.021	0.024	0.027	0.031	0.035	0.042	0.021
Positive moment at midspan	0.031	0.036	0.041	0.047	0.052	0.064	0.031
Case 3-Two edges discontinuous							
Negative moment at -							
Continuous edge	0.049	0.057	0.064	0.071	0.078	0.090	0.049
Discontinuous edge	0.025	0.028	0.032	0.036	0.039	0.045	0.025
Positive moment at midspan	0.037	0.043	0.048	0.054	0.059	0.068	0.037
Case 4-Three edges discontinuous							
Negative moment at -							
Continuous edge	0.058	0.066	0.074	0.082	0.090	0.098	0.058
Discontinuous edge	0.029	0.033	0.037	0.041	0.045	0.049	0.029
Positive moment at midspan	0.044	0.050	0.056	0.062	0.068	0.074	0.044
Case 5-Four edges discontinuous							
Negative moment at -							
Continuous edge	-	-	-	-	-	-	-
Discontinuous edge	0.033	0.038	0.043	0.047	0.053	0.055	0.033
Positive moment at midspan	0.050	0.057	0.064	0.072	0.080	0.083	0.050

of continuity in Method 2 into 9 cases.

In this method, the way of dividing strips and the critical sections for bending moment are the same as in Method 2. The bending moments for the middle strips are computed by the use of Tables III-3, III-4 and III-5 from:

$$M_s^- = C_3 q_t S^2 \quad \text{III-4}$$

$$M_\ell^- = C_3 q_t L^2 \quad \text{III-5}$$

$$M_s^+ = C_3' q_d S^2 + C_3'' q_\ell S^2 \quad \text{III-6}$$

$$M_\ell^+ = C_3' q_d L^2 + C_3'' q_\ell L^2 \quad \text{III-7}$$

in which

M_s^- = critical negative moment in short direction;

M_ℓ^- = critical negative moment in long direction;

M_s^+ = critical positive moment in short direction;

M_ℓ^+ = critical positive moment in long direction;

q_t = uniform total load on slab;

q_d = uniform dead load on slab;

q_ℓ = uniform live load on slab;

C_3, C_3', C_3'' = moment coefficients, from Tables III-3, III-4 and

III-5 respectively.

Table III-3 Coefficients for Negative Moments in Slabs, C_3 , ACI Method 3

m span	s	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
										
1.00	S		0.045		0.050	0.075	0.071		0.033	0.061
	L		0.045	0.076	0.050			0.071	0.061	0.033
0.95	S		0.050		0.055	0.079	0.075		0.038	0.065
	L		0.041	0.072	0.045			0.067	0.056	0.029
0.90	S		0.055		0.060	0.080	0.079		0.043	0.068
	L		0.037	0.070	0.040			0.062	0.052	0.025
0.85	S		0.060		0.066	0.082	0.083		0.049	0.072
	L		0.031	0.065	0.034			0.057	0.046	0.021
0.80	S		0.065		0.071	0.083	0.086		0.055	0.075
	L		0.027	0.061	0.029			0.051	0.041	0.017
0.75	S		0.069		0.076	0.085	0.088		0.061	0.078
	L		0.022	0.056	0.024			0.044	0.036	0.014
0.70	S		0.074		0.081	0.086	0.091		0.068	0.081
	L		0.017	0.050	0.019			0.038	0.029	0.011
0.65	S		0.077		0.085	0.087	0.093		0.074	0.083
	L		0.014	0.043	0.015			0.031	0.024	0.008
0.60	S		0.081		0.089	0.088	0.095		0.080	0.085
	L		0.010	0.035	0.011			0.024	0.018	0.006
0.55	S		0.084		0.092	0.089	0.096		0.085	0.086
	L		0.007	0.028	0.008			0.019	0.014	0.005
0.50	S		0.086		0.094	0.090	0.097		0.089	0.088
	L		0.066	0.022	0.006			0.014	0.010	0.003

Table III-4 Coefficients for Dead Load Positive Moments in Slabs, C_3' ,
ACI Method 3

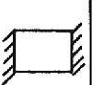
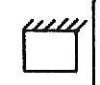
m	span	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
										
1.00	S	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	L	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	S	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.023	0.024
	L	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	S	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	L	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	S	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	L	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	S	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	L	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	S	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	L	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	S	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	L	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	S	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	L	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	S	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	L	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	S	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	L	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	S	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	L	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

Table III-5 Coefficients for Live Load Positive Moments in Slabs, C_3'' ,
ACI Method 3

m	span	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
		S 	L 							
1.00	S	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	L	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	S	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	L	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	S	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	L	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	S	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	L	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	S	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	L	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	S	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	L	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	S	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	L	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	S	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	L	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	S	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	L	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	S	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	L	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	S	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	L	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

The bending moments in the column strips are gradually reduced from the full value from the edges of the middle strip to one third of these values at the edges of the panel, resulting in an average value of two-thirds. When the negative moment on one side of a support is less than 80 percent of that on the other side, the difference is distributed in proportion to the relative stiffness of the slabs.

(4) ACI 318-63 One-Way Method:

If a two-way slab has the same restraint condition on all four sides, from the crossing beam analogy, it is evident that

$$\frac{q_l}{q_s} = \frac{S^4}{L^4}$$

III-8

in which

q_s = share of load q_t carried in short direction;

q_l = share of load q_t carried in long direction.

In equation III-8, if $m = S/L = 1/2$, then $q_l/q_s = 1/16$. That is, when the short span is one-half of the long span, the load shared in the long span is only one-sixteenth of that in the short span. Hence, such a slab acts almost as if it were spanning in the short direction only. In agreement with this, the ACI Code provides that any slab more oblong than 2:1 shall be treated as a one-way slab provided that negative reinforcement, as required for a ratio of $m = 0.5$, shall be provided along the short edge.

The bending moment for the short span per linear foot of width is computed from

$$M = Bq_t S^2 \quad \text{III-9}$$

in which B is the moment coefficient for one-way slab. It is determined in the same way as that described in ACI Method 1.

(5) Elasticity Analysis Mathematical Solution:

The deflection of a plate subjected to transverse load is given by Timoshenko (21) as satisfying the following differential equation:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^2 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad \text{III-10}$$

and the bending moment is calculated from

$$M_x = -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad \text{III-11}$$

$$M_y = -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \quad \text{III-12}$$

in which

w = deflection in the transverse direction;

x,y = orthogonal coordinates for middle plane of plate;

q = transverse load;

D = flexural rigidity of the plate;

$$= \frac{Eh^3}{12(1-\nu^2)}, \text{ for uniform thickness;}$$

ν = poisson's ratio;

M_x = bending moment in the x direction;

M_y = bending moment in the y direction.

The problem now turns to solving the differential eq. III-10.

Different ways for solving this problem have been discussed by Timoshenko (21). For a rectangular plate simply supported on all four sides, Fig III-3, two solutions are given. The first solution is by Navier, who assumes for any kind of loading given by the equation

$$q = f(x,y) \quad \text{III-3}$$

one can represent the function $f(x,y)$ in the form of a double trigonometric series:

$$f(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad \text{III-14}$$

in which, a_{mn} is determined by

$$a_{mn} = \frac{4}{ab} \int_0^a \int_0^b f(x,y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dx dy. \quad \text{III-15}$$

From the above assumption, the solution for the deflection w is obtained

$$w = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}. \quad \text{III-16}$$

Another solution is given by M. Levy who assumed

$$w = \sum_{m=1}^{\infty} Y_m \sin \frac{m\pi x}{a} \quad \text{III-17}$$

in which Y_m is a function of y only. An expression for w is derived from this assumption,

$$w = \frac{4qa^4}{\pi^5 D} \sum_{m=1,3,5,\dots}^{\infty} \frac{1}{m^5} \left(1 - \frac{\alpha_m \tanh \alpha_m + 2}{2 \cosh \alpha_m} \cosh \frac{2\alpha_m y}{b} + \frac{\alpha_m}{2 \cosh \alpha_m} \frac{2y}{b} \sinh \frac{2\alpha_m y}{b} \right) \sin \frac{m\pi x}{a} \quad \text{III-18}$$

in which

$$\alpha_m = \frac{m\pi b}{2a}.$$

Further discussion on rectangular plates with various edge conditions is also presented by Timoshenko (21). For comparative purposes in the next chapter of this report, rectangular plates with all edges built in are discussed here. A different coordinate axes is shown in Fig. III-4. First, we divide this support condition into three cases:

case 1: simply supported plate subjected to load q

case 2: simply supported plate subjected to M_y along the edges

$$y = \pm \frac{b}{2}$$

case 3: simply supported plate subjected to M_x along the edges

$$x = \pm \frac{a}{2}$$

Superimpose these three cases together and make them satisfy the boundary conditions

$$\left(\frac{\partial w}{\partial y}\right)_y = \pm \frac{b}{2} + \left(\frac{\partial w_1}{\partial y} + \frac{\partial w_2}{\partial y}\right)_y = \pm \frac{b}{2} = 0 \quad \text{III-19}$$

$$\left(\frac{\partial w}{\partial x}\right)_x = \pm \frac{a}{2} + \left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial x}\right)_x = \pm \frac{a}{2} = 0 \quad \text{III-20}$$

one will find the solution to this problem.

Case 1: Simply supported plate subjected to transverse load q

Use M. Levy's expression for the new coordinate system,

$$w = \frac{4qa^4}{\pi^5 D} \sum_{m=1,3,5,\dots}^{\infty} \frac{(-1)^{(m-1)/2}}{m^5} \cos \frac{m\pi x}{a} \left(1 - \frac{\alpha_m \tanh \alpha_m + 2}{2 \cosh \alpha_m} \cosh \frac{m\pi y}{a} + \frac{1}{2 \cosh \alpha_m} \frac{m\pi y}{a} \sinh \frac{m\pi y}{a}\right) \quad \text{III-21}$$

and

$$\left(\frac{\partial w}{\partial y}\right)_y = \pm \frac{b}{2} = \frac{24a^3}{\pi^4 D} \sum_{m=1,3,5,\dots}^{\infty} \frac{(-1)^{(m-1)/2}}{m^4} \cos \frac{m\pi x}{a} \left(\frac{\alpha_m}{\cosh^2 \alpha_m} - \tanh \alpha_m\right). \quad \text{III-22}$$

Case 2: Simply supported plate subjected to M_y along the edges $y = \pm \frac{b}{2}$

From considerations of symmetry, M_y can be represented by the following series:

$$(M_y)_y = \pm \frac{b}{2} = \sum_{m=1,3,5,\dots}^{\infty} (-1)^{(m-1)/2} E_m \cos \frac{m\pi x}{a} . \quad \text{III-23}$$

The corresponding deflection w_1 is obtained as

$$w_1 = - \frac{a^2}{2\pi^2 D} \sum_{m=1,3,5,\dots}^{\infty} E_m \frac{(-1)^{(m-1)/2}}{m^2 \cosh \alpha_m} \cos \frac{m\pi x}{a} \left(\frac{m\pi y}{a} \sinh \frac{m\pi y}{a} - \alpha_m \tanh \alpha_m \cosh \frac{m\pi y}{a} \right) . \quad \text{III-24}$$

The rotations are

$$\begin{aligned} \left(\frac{\partial w_1}{\partial y} \right)_y = \frac{b}{2} = - \frac{a}{2\pi b} \sum_{m=1,3,5,\dots}^{\infty} E_m \frac{(-1)^{(m-1)/2}}{m} \cos \frac{m\pi x}{a} \left(\tanh \alpha_m + \frac{\alpha_m}{\cosh^2 \alpha_m} \right) \end{aligned} \quad \text{III-25}$$

$$\left(\frac{\partial w_1}{\partial x} \right)_x = \frac{a}{2} = - \frac{4b^2}{\pi^2 D a} \sum_{m=1,3,5,\dots}^{\infty} \frac{E_m}{m^3} \sum_{i=1,3,5,\dots}^{\infty} \frac{i(-1)^{(i-1)/2}}{\left(\frac{b^2}{a^2} + \frac{i^2}{m^2} \right)^2} \cos \frac{i\pi y}{b} . \quad \text{III-26}$$

Case 3: Simply supported plate subjected to M_x along edges $x = \pm \frac{a}{2}$

Assuming a symmetrical distribution and taking

$$(M_x)_x = \pm \frac{a}{2} = \sum_{m=1,3,5,\dots}^{\infty} (-1)^{(m-1)/2} F_m \cos \frac{m\pi y}{b} \quad \text{III-27}$$

then, the corresponding deflection w_2 is obtained

$$w_2 = - \frac{b^2}{2\pi^2 D} \sum_{m=1,3,5,\dots}^{\infty} F_m \frac{(-1)^{(m-1)/2}}{m^2 \cosh \beta_m} \cos \frac{m\pi y}{b} \left(\frac{m\pi x}{b} \sinh \frac{m\pi x}{b} - \beta_m \tanh \beta_m \cosh \frac{m\pi x}{b} \right) \quad \text{III-28}$$

in which

$$\beta_m = \frac{m\pi a}{2b}.$$

The rotations are

$$\begin{aligned} \left(\frac{\partial w_2}{\partial x} \right)_x = \frac{a}{2} = - \frac{b}{2\pi D} \sum_{m=1,3,5,\dots}^{\infty} F_m \frac{(-1)^{(m-1)/2}}{m} \cos \frac{m\pi y}{b} \left(\tanh \beta_m + \frac{\beta_m}{\cosh^2 \beta_m} \right) \end{aligned} \quad \text{III-29}$$

$$\left(\frac{\partial w_2}{\partial y} \right)_y = \frac{b}{2} = - \frac{4a^2}{\pi^2 D b} \sum_{m=1,3,5,\dots}^{\infty} \frac{F_m}{m^3} \sum_{i=1,3,5,\dots}^{\infty} \frac{i(-1)^{(i-1)/2}}{\left(\frac{a^2}{b^2} + \frac{i^2}{m^2} \right)} \cos \frac{i\pi x}{a}. \quad \text{III-30}$$

Superimpose these three cases and apply eq. III-19 to obtain

$$\frac{4qa^3}{\pi^3} \frac{1}{i^4} \left(\frac{\alpha_i}{\cosh^2 \alpha_i} - \tanh \alpha_i \right) - \frac{E_i}{i} \left(\tanh \alpha_i + \frac{\alpha_i}{\cosh^2 \alpha_i} \right)$$

$$- \frac{8ia}{\pi b} \sum_{m=1,3,5,\dots}^{\infty} \frac{F_m}{m^3} \frac{1}{\left(\frac{a^2}{b^2} + \frac{i^2}{m^2} \right)} = 0. \quad \text{III-31}$$

A similar equation is obtained also by applying eq. III-20. The constants $E_1, E_3, \dots, F_1, F_3, \dots$ can be determined in each particular case from these two systems of equations by the method of successive approximations. Substituting the numerical values of the coefficients in these equations and considering only the first four coefficients, we obtain eight linear equations with eight unknowns, $E_1, E_3, E_5, E_7, F_1, F_3, F_5$, and F_7 . Solving for these coefficients and substituting them into eq. III-23 and eq. III-27, one can find the moment distribution along the built-in edges. From this, the deflection of the built-in plate can be found and the moments computed. The comparison of this result with Boobnov's table, calculated with a much larger number of equations, shows that the error in the maximum bending moment, by taking only four terms, is less than 1 percent. Table III-6 shows the calculated results based on four terms given by Timoshenko (21).

(6) Direct Design Method - ACI 318-71(1)

All methods previously described in this chapter are based on the results of analyses of slabs on rigid beams. This is not actually the case since ordinary supporting beams are not heavy enough to satisfy

Table III-6 Deflections and Bending Moments in a Uniformly Loaded Rectangular Plate with Built-in Edges (Coordinates refer to Fig. III-4)

b/a	$(w)_{x=0,y=0}$ (qa^4/D)	$(M_x)_{x=a/2,y=0}$ (qa^2)	$(M_y)_{x=0,y=b/2}$ (qa^2)	$(M_x)_{x=0,y=0}$ (qa^2)	$(M_y)_{x=0,y=0}$ (qa^2)
1.0	0.00126	-0.0513	-0.0513	0.0231	0.0231
1.1	0.00150	-0.0581	-0.0538	0.0264	0.0231
1.2	0.00172	-0.0639	-0.0554	0.0299	0.0228
1.3	0.00191	-0.0687	-0.0563	0.0327	0.0222
1.4	0.00207	-0.0726	-0.0568	0.0349	0.0212
1.5	0.00220	-0.0757	-0.0570	0.0368	0.0203
1.6	0.00230	-0.0780	-0.0571	0.0381	0.0193
1.7	0.00238	-0.0799	-0.0571	0.0392	0.0182
1.8	0.00245	-0.0812	-0.0571	0.0401	0.0174
1.9	0.00249	-0.0822	-0.0571	0.0407	0.0165
2.0	0.00254	-0.0829	-0.0571	0.0412	0.0158
∞	0.00260	-0.0833	-0.0571	0.0417	0.0125

this assumption and will always undergo some amount of deflection. The idea of 318-71(1) is to take into account the influence of beam deflection when designing slabs. The beam stiffness could range from zero, ie, the flat slab, to some non-zero value representing a "stiff" support such as a heavy beam or wall. The "Direct Design Method" is one of the two design procedures given in this code. It is related to the "empirical method" of flat slabs and could be used for both flat slabs and two-way slabs.

The division into column strips and middle strips is shown in Fig. III-5. In using the direct design method, there are some limitations: there shall be a minimum of three continuous spans in each direction; the panels shall be rectangular with the ratio of the longer to shorter spans within a panel not greater than 2.0; the successive span length in each direction shall not differ by more than one-third of the longer span; columns may be offset a maximum of 10 percent of the span, in direction of the offset, from either axis between center lines of successive columns; the live load shall not exceed three times the dead load; if a panel is supported by beams on all sides, the relative stiffness of the beams in the two perpendicular directions, $\alpha_1 \ell_2^2 / \alpha_2 \ell_1^2$, shall not be less than 0.2 nor greater than 5.0, in which

α_1 = ratio of flexural stiffness of beam section to the flexural stiffness of a width of slab bounded laterally by the center line of the adjacent panel on each side of the beam in the direction of ℓ_1 ;

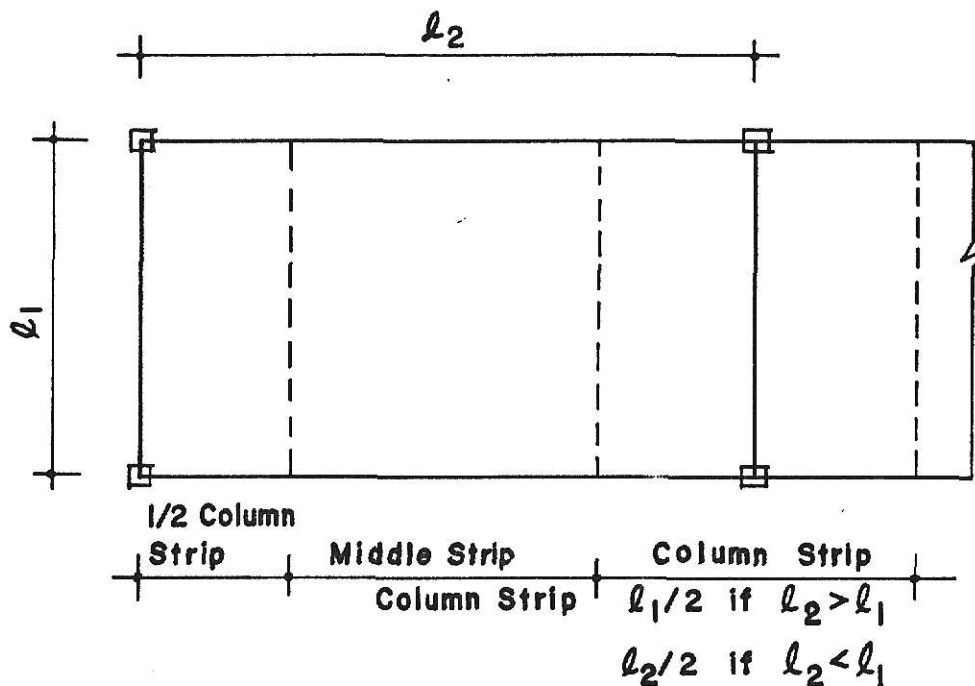
α_2 = corresponding value of α_1 in the direction of ℓ_2 .

All bending moments in slabs and beams are in proportion to the "total static design moment". This is determined in a strip bounded laterally by the center lines of the panels on each side of the center line of the supports as follows:

$$M_o = \frac{q_t \ell_2 L_1^2}{8}$$

III-32

in which



l_1 = length of span in the direction moments are being determined, measured center-to-center of supports.

l_2 = length of span transverse to l_1 , measured center-to-center of supports.

Fig. III-5 Middle and Column Strips in A C I 318-71
Direct Design Method & Equivalent Frame Method

M_o = total static design moment;

L_1 = clear span in the direction of l_1 .

The total static design moment is distributed into "negative design moment" and "positive design moment" as follows:

Interior span:

negative design moment 0.65

positive design moment 0.35

End span:

interior negative design moment $0.75 - \frac{0.1}{1 + \frac{1}{\alpha_{ec}}}$

positive design moment $0.63 - \frac{0.28}{1 + \frac{1}{\alpha_{ec}}}$

exterior negative design moment $\frac{0.65}{1 + \frac{1}{\alpha_{ec}}}$

where

α_{ec} = ratio of flexural stiffness of the columns above and below the slab to the combined flexural stiffness of the slabs and beams at exterior joint taken in the direction moments are being determined.

The negative design moment and positive design moment are then subdivided into beam, slab of column strip and slab of middle strip moments as shown in Tables III-7, III-8, and III-9. All moment distribution

Table III-7 Moment Distribution Factors for Interior Negative Design Moment

l_2/l_1		0.5	1.0	2.0
$\alpha_1 l_2/l_1 = 0$	Beam	0.75x0.00	0.75x0.00	0.75x0.00
	Slab, column strip	0.75x1.00	0.75x1.00	0.75x1.00
	Slab, middle strip	0.25	0.25	0.25
$\alpha_1 l_2/l_1 \geq 1.0$	Beam	0.90x0.85	0.75x0.85	0.45x0.85
	Slab, column strip	0.90x0.15	0.75x0.15	0.45x0.15
	Slab, middle strip	0.10	0.25	0.55

Table III-8 Moment Distribution Factors for Exterior Negative Design Moment

l_2/l_1			0.5	1.0	2.0
$\alpha_1 l_2/l_1 = 0$	$\beta_t = 0$	Beam	1.00x0.00	1.00x0.00	1.00x0.00
		Slab, column strip	1.00x1.00	1.00x1.00	1.00x1.00
		Slab, middle strip	0.00	0.00	0.00
	$\beta_t \geq 2.5$	Beam	0.75x0.00	0.75x0.00	0.75x0.00
		Slab, column strip	0.75x1.00	0.75x1.00	0.75x1.00
		Slab, middle strip	0.25	0.25	0.25
$\alpha_1 l_2/l_1 \geq 1.0$	$\beta_t = 0$	Beam	1.00x0.85	1.00x0.85	1.00x0.85
		Slab, column strip	1.00x0.15	1.00x0.15	1.00x0.15
		Slab, middle strip	0.00	0.00	0.00
	$\beta_t \geq 2.5$	Beam	0.90x0.85	0.75x0.85	0.45x0.85
		Slab, column strip	0.90x0.15	0.75x0.15	0.45x0.15
		Slab, middle strip	0.10	0.25	0.55

Table III-9 Moment Distribution Factors for Positive Design Moment

ℓ_2/ℓ_1		0.5	1.0	2.0
$\alpha_1 \ell_2 / \ell_1 = 0$	Beam	0.60x0.00	0.60x0.00	0.60x0.00
	Slab, column strip	0.60x1.00	0.60x1.00	0.60x1.00
	Slab, middle strip	0.40	0.40	0.40
$\alpha_1 \ell_2 / \ell_1 \geq 1.0$	Beam	0.90x0.85	0.75x0.85	0.45x0.85
	Slab, column strip	0.90x0.15	0.75x0.15	0.45x0.15
	Slab, middle strip	0.10	0.25	0.55

factors between the values shown are to be linearly interpolated. Those factors being products of two values, both values should be linearly interpolated. To account for inelastic moment redistribution, the Code allows 10 percent variation for design moment. However, the total static design moment should remain the same.

(7) ACI 318-71 Equivalent Frame Method (1)

The "Equivalent Frame Method" is another design procedure given in ACI 318-71 (1). It is a frame analysis method which gives results comparable to those obtained by the elasticity analysis. In this method, the structure is considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building. Each frame consists of a row of equivalent columns or supports and slab-beam strips, bounded laterally by the center line of the panel on each side of the center line of the columns or supports. Frames adjacent and parallel to an edge are considered to be bounded by the edge and the center line of the adjacent panel.

Each such frame may be analyzed in its entirety, or, for vertical loading, each floor thereof and the roof may be analyzed separately with its columns being assumed fixed at their remote ends. Where slab-beams are thus analyzed separately, it may be assumed in determining the bending moment at a given support that the slab-beam is fixed at any support two panels away provided the slab is continuous beyond that point. The moment of inertia of the slab-beam or column may be based on the cross-sectional area of the concrete. The equivalent column is

assumed to consist of the actual columns above and below the slab-beam plus an attached torsional member transverse to the direction in which moments are being determined and extending to the bounding, lateral, panel center lines on each side of the column. The flexibility (inverse of the stiffness) of the equivalent column is determined by equation III-33:

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t} \quad \text{III-33}$$

in which

K_{ec} = flexural stiffness of equivalent column;

K_c = flexural stiffness of column;

K_t = torsional stiffness of torsional member.

The stiffness K_t of the torsional member is calculated from equation III-34:

$$K_t = \sum \frac{9 E_{cs} C}{l_2 \left(1 - \frac{c_2}{l_2}\right)^3} \quad \text{III-34}$$

where

E_{cs} = modulus of elasticity for slab concrete;

C = cross-sectional constant;

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction in which moments are being determined.

Constant C is further defined as:

$$c = \sum \left(1 - 0.63 \frac{x_1}{y_1} \right) \frac{x_1^3 y_1^3}{3} \quad \text{III-35}$$

in which

x_1 = shorter overall dimension of a rectangular part of a cross section;

y_1 = longer overall dimension of a rectangular part of a cross section.

On account of the pattern loading, when live load does not exceed three-quarters of the dead load, the maximum moment may be assumed to occur at all sections when the full design live load is on the entire slab system. For other conditions, maximum positive moment near mid-span of a panel may be assumed to occur when three-quarters of the full design live load is on the panel and on alternate panels; and maximum negative moment in the slab at a support may be assumed to occur when three-quarters of the full design live load is on the adjacent panels only. However, the design moments should not be less than those occurring with full design live load on all panels.

The critical section for interior negative moment is taken at the face of rectilinear supports, but in no case at a distance greater than $0.175 \ell_1$ from the center of the column. The critical section for exterior negative moment is taken at a distance from the face of the supporting element not greater than one-half the projection of the bracket or capital beyond the face of the supporting element.

The negative design moment and positive design moment thus obtained by "frame analysis" are then distributed into beam, slab or column strip

and slab of middle strip by the same way as in the "Direct Design Method", that is as shown in Tables III-7, III-8 and III-9. The division into column strips and middle strips is also as shown in Fig. III-5.

(8) Gamble's Method

William L. Gamble presented a paper "Moments in Beam Supported Slabs" in the ACI Journal, March, 1972 (22). In that paper, he showed the theoretical distribution of elastic moments in interior panels as a function of stiffness of the beams, relative to the slab stiffness.

In his study, he assumed that the beam stiffnesses in the two directions are arbitrarily related to their spans in the following ratio:

$$\frac{\alpha_1}{\alpha_2} = \left(\frac{l_1}{l_2}\right)^2 . \quad \text{III-36}$$

This can be reduced to

$$\frac{E_{cb} I_{b1}}{E_{cb} I_{b2}} = \frac{l_1}{l_2} . \quad \text{III-37}$$

He also assumed that the Poisson's ratios were equal to zero. The moments presented were obtained from solutions of the governing plate equations previously reported by Sutherland et al. (23), and from unpublished results obtained by Vanderbilt et al. (24), in connection with a study of slab deflections. The former analyses used the Ritz energy method of analysis in conjunction with an infinite series of polynomial functions derived by Duncan (25). Vanderbilt used the finite difference

method of analysis, working with a 20x20 grid for each panel. Both methods are described by Timoshenko and Woinowsky-Krieger (21). The results of the analyses are shown in Fig. III-6 through III-11. In Fig. III-6 through III-10, it is assumed the columns and beams have no width. Fig. III-11 shows the effect of column size, yet the beams are still of zero-width.

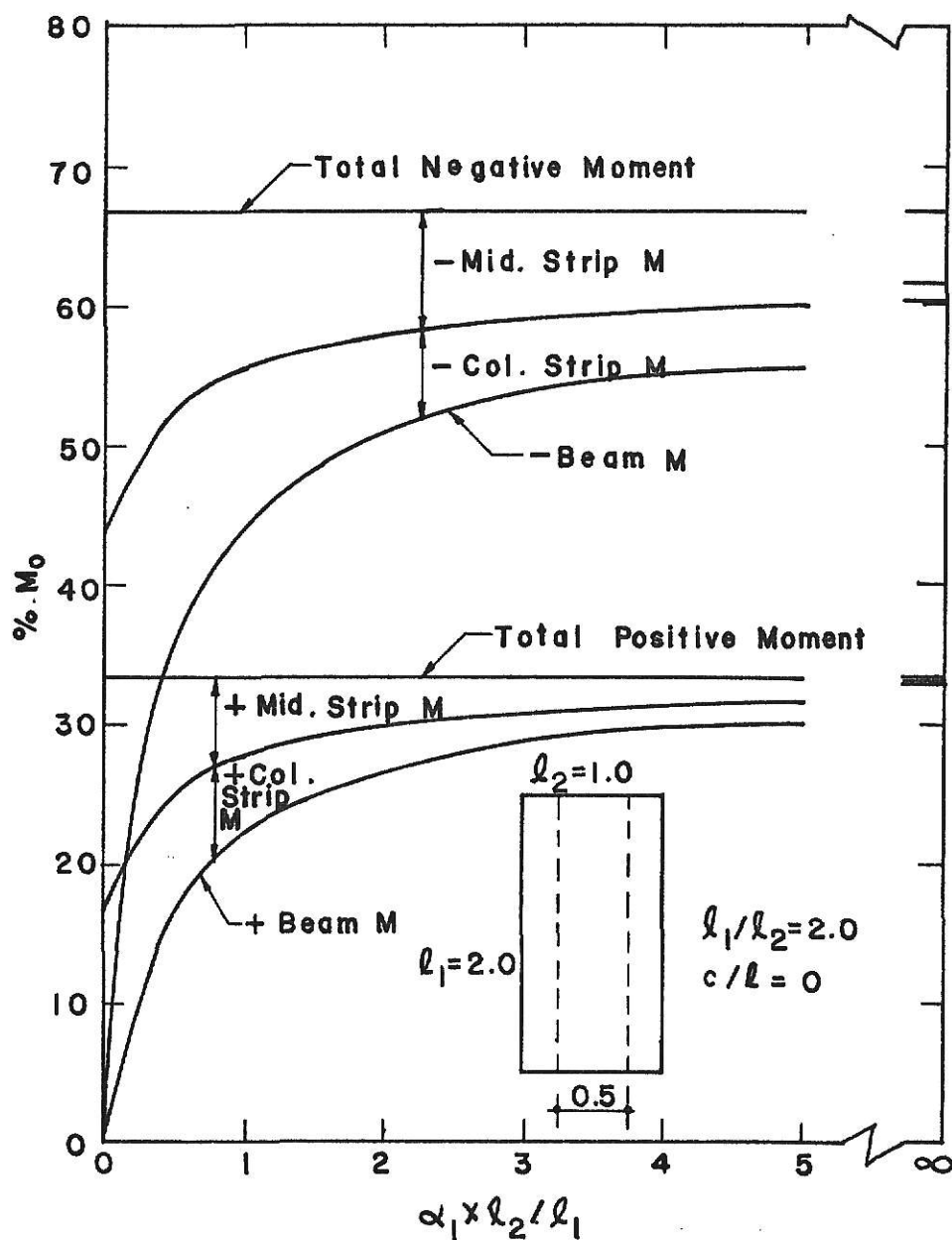


Fig. III-6 Moment Versus Beam Stiffness for $l_1/l_2 = 2.0$

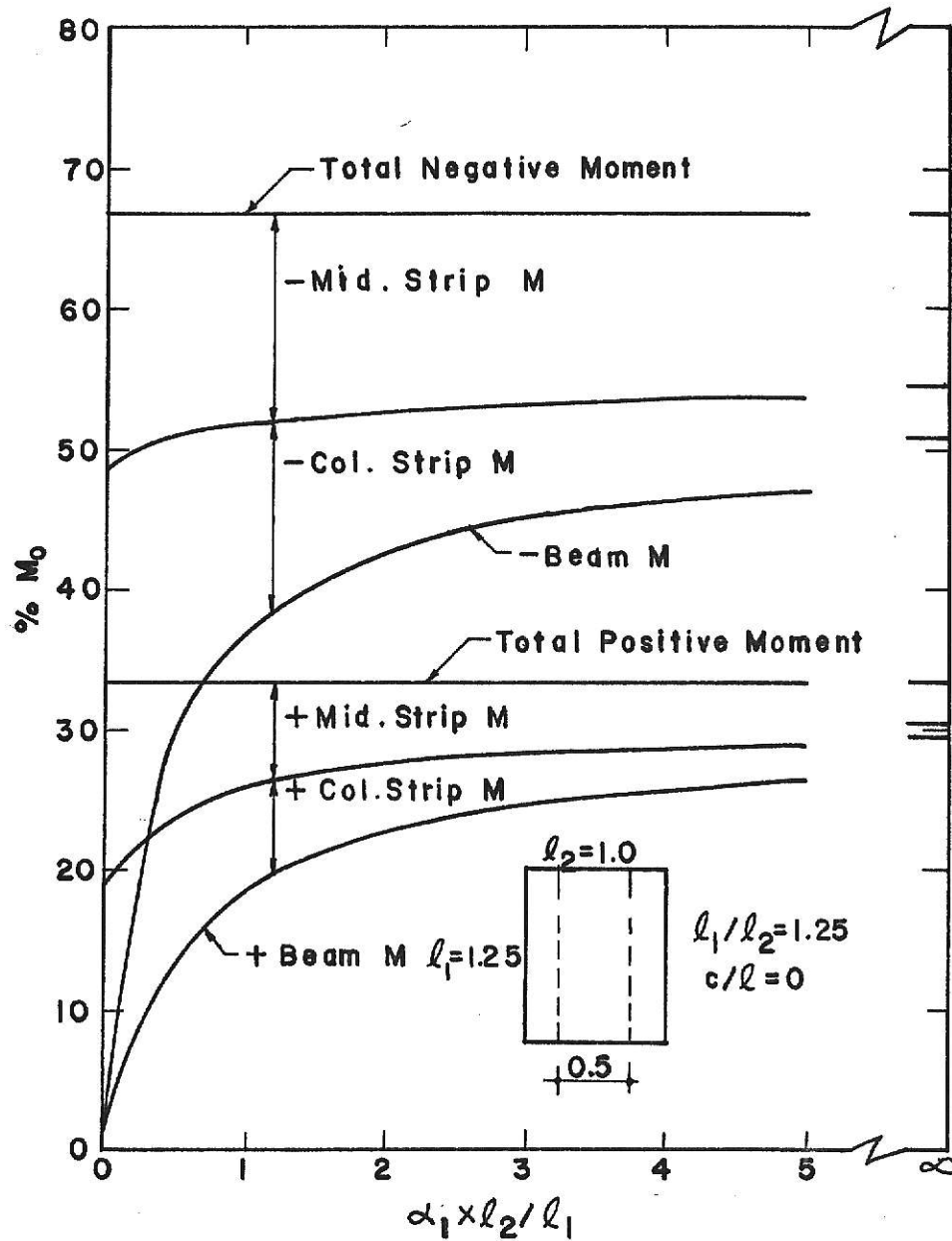


Fig. III-7 Moment Versus Beam Stiffness for $l_1/l_2 = 1.25$

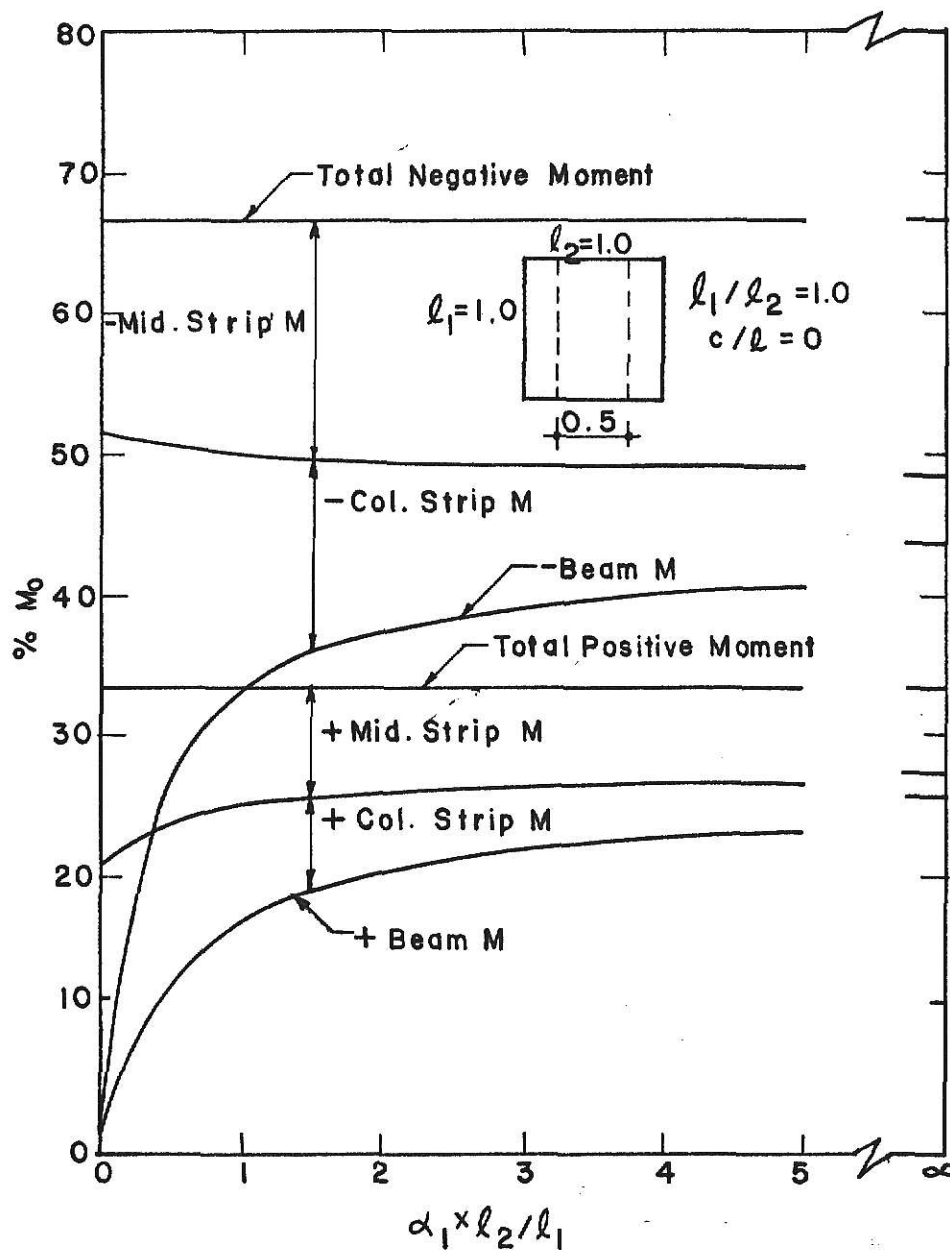


Fig. III-8 Moment Versus Beam Stiffness for $l_1/l_2 = 1.0$

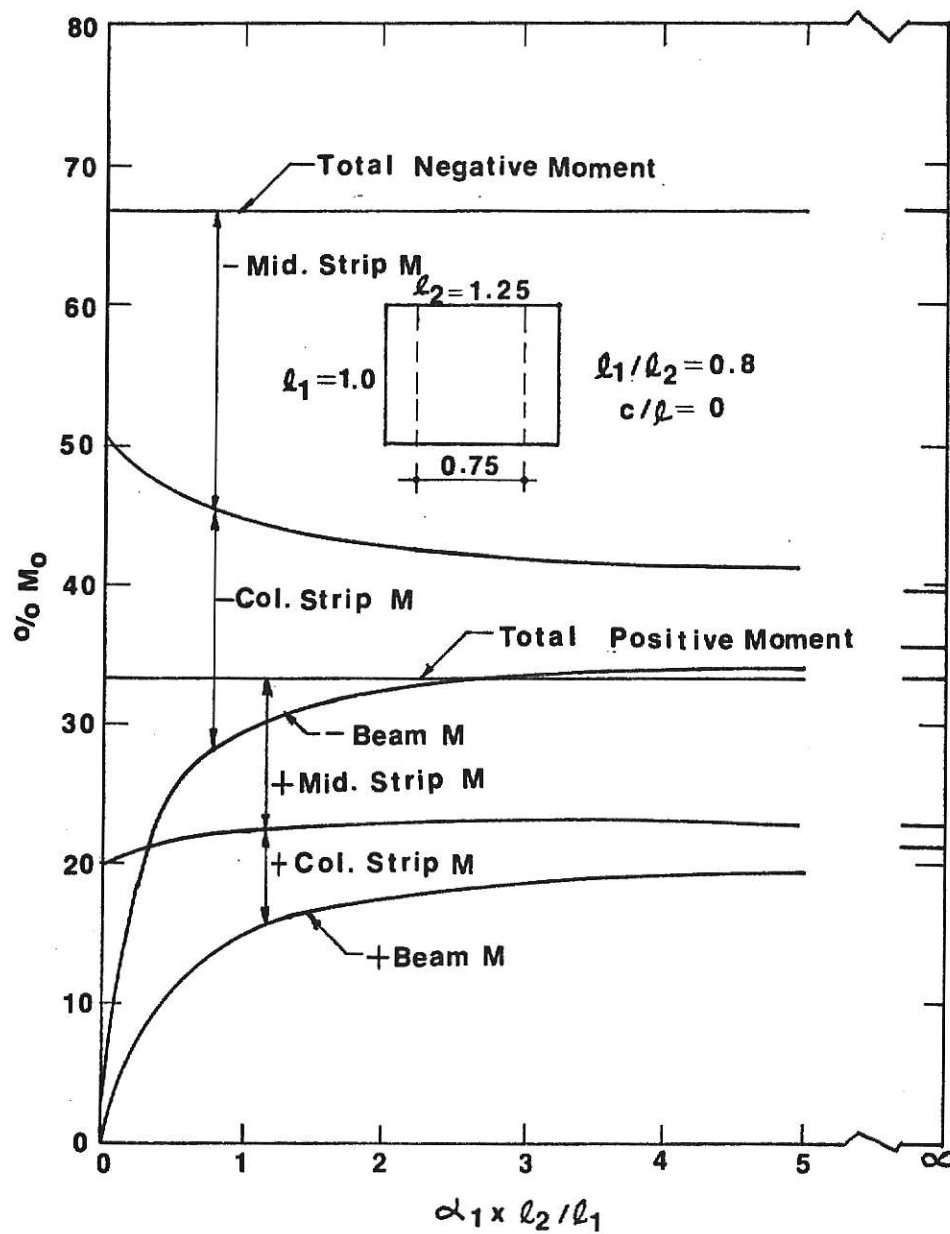


Fig.III- 9 Moment Versus Beam Stiffness for $l_1/l_2 = 0.8$

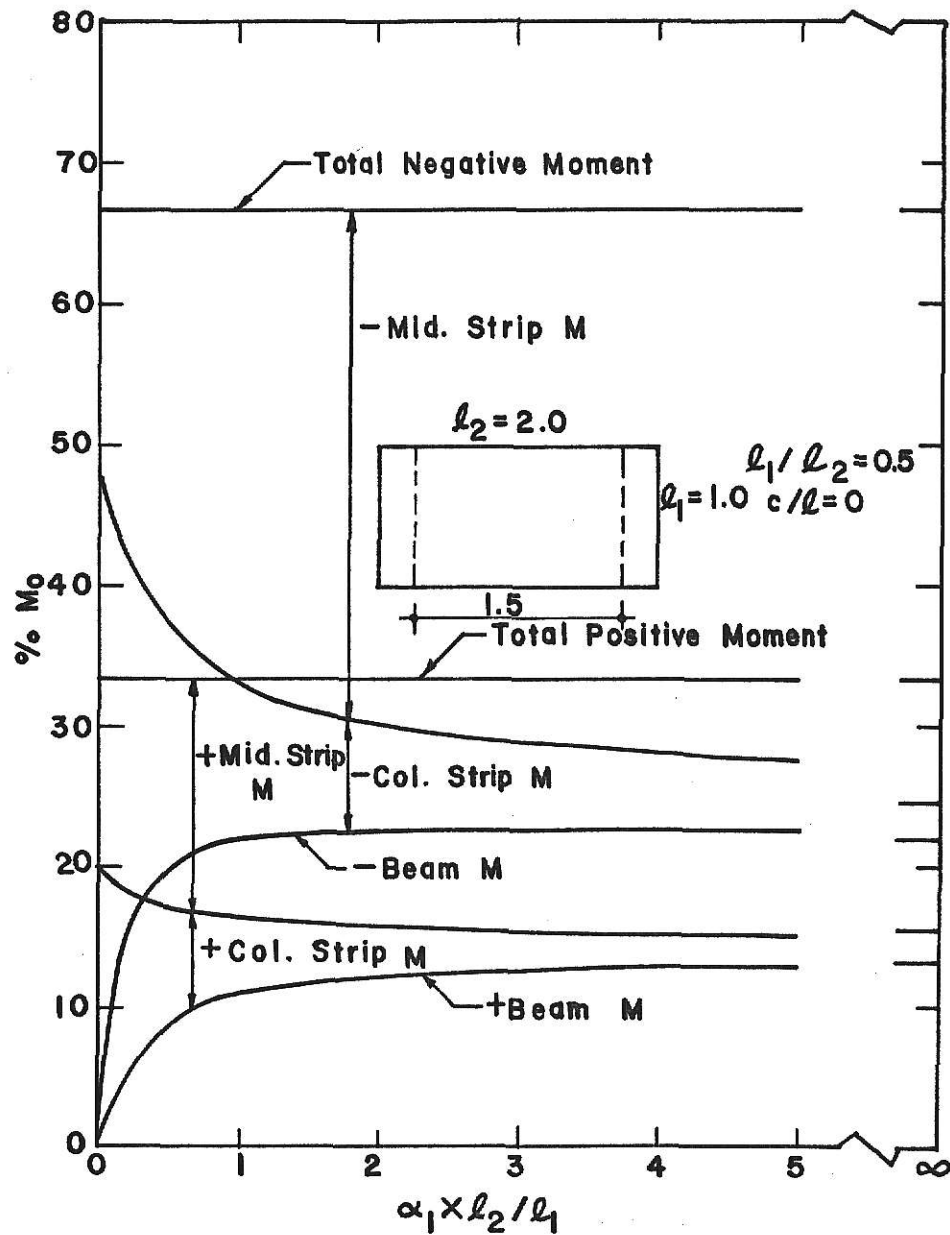


Fig. III-10 Moment Versus Beam Stiffness for $l_1/l_2=0.5$

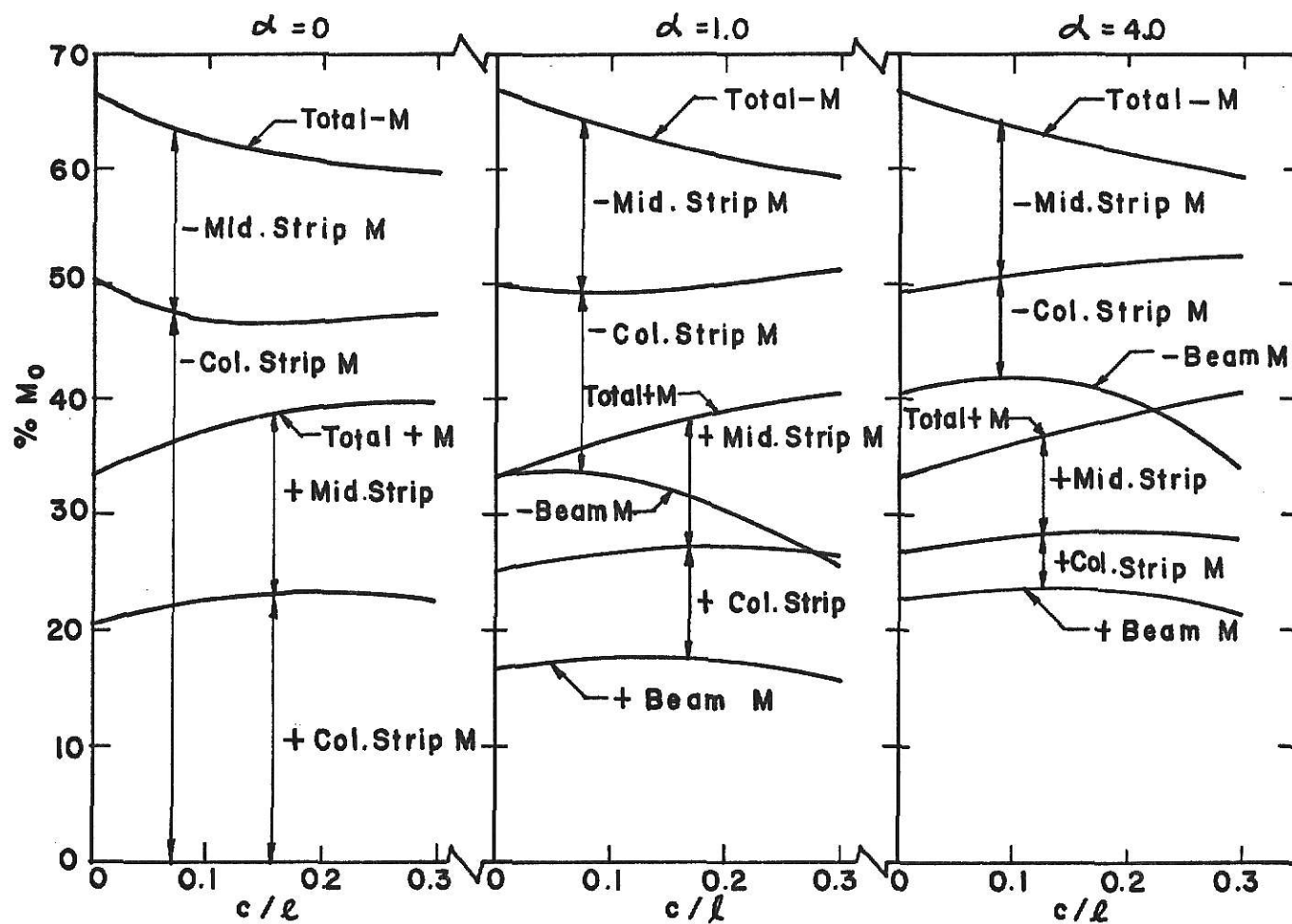


Fig. III-11 Effects of Column or Capital Size on Slab and Beam Moments in Square Panels

(9) Yield Line Theory

All methods previously described in this chapter are based upon elastic theory, although some have been modified for inelastic redistribution. While these methods of moment analysis are based essentially on elastic behavior, the actual proportioning of slabs is increasingly being performed by ultimate-strength method. This means we only take into account the inelastic nature within a section but do not take into account the rotation ability which leads to the moment redistribution within the whole structure.

It is known that reinforced concrete structures possess some rotation ability to develop moment redistribution; but it is not yet known if the rotation ability is enough to fully develop moment redistribution as in steel structures. However, a method of analysis called "yield line theory", which is based on the assumption that the structure has enough rotation ability to develop fully the moment redistribution, has been developed. Two approaches, the equilibrium method and the virtual work method, have been used. The virtual work method is more powerful and is presented here.

First, we assume the yield line pattern of the slab, that is the failure mechanism. As the structure undergoes a small virtual displacement, assuming rotation occurs only at yield lines, one can express the external work in terms of q and the internal work in terms of M_u , in which

M_u = ultimate moment resistance of a slab section of unit width.

By equating the external work to internal work, one will find the required M_u in terms of q for the assumed yield line pattern. By assuming different yield line patterns, one will obtain many values of M_u . Since yield line theory is essentially an upper bound theorem, the highest value of M_u should be the answer. In other words, given a value of M_u , the lowest possible load that can stand is the one we need.

Part B Flat Slabs

(1) ACI 318-63 Empirical Method (2)

This empirical method for the flat slab has been a basic design method from the very beginning of the development of design of flat slabs. Although the details and total static moment have changed from time to time, the basic idea remains the same. Before applying this method, there are some limitations: the construction shall consist of at least three continuous panels in each direction; the ratio of length to width of panels shall not exceed 1.33; the grid pattern shall consist of approximately rectangular panels; the successive span lengths in each direction shall differ by not more than 20 percent of the longer span; columns may be offset a maximum of 10 percent of the span, in the direction of offset, from either axis between center lines of successive columns.

The panel is divided into middle and column strips as shown in Fig. III-12. All moments in a panel are in proportion to the total static moment M_o , which is defined as:

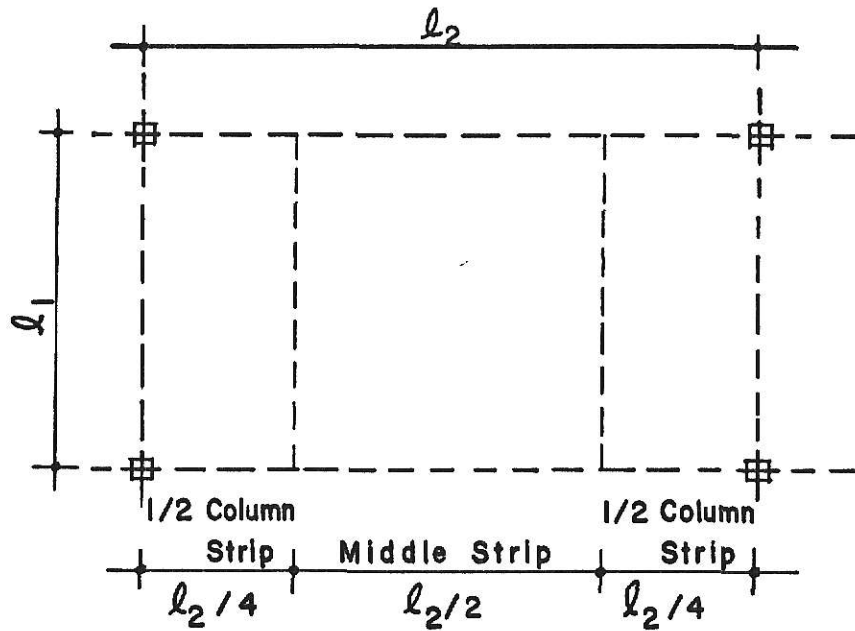
$$M_o = 0.09 F_q l_2 l_1^2 \left(1 - \frac{2c}{3l_1}\right)^2 \quad \text{III-38}$$

in which

$F = 1.15 - c/l_1$ but not less than 1;

c = effective support size.

The proportions of slab moments to M_o are shown in Table III-10. For intermediate proportions of total beam depth to slab thickness, values for loads and moments may be obtained by interpolation.



l_1 = length of span in the direction moments are being determined, measured center-to-center of supports.

l_2 = length of span transverse to l_1 , measured center-to-center of supports.

Fig. III-12 Middle and Column Strips in A C I 318-63 Empirical Method & Elastic Analysis Method for Flat Slabs

Table III-10 Moments in Flat Slab Panels in Percentages of M_o

Strip	Column head	Side support type	End support type	Exterior panel		Interior panel		
				Exterior negative moment	Positive moment	Interior negative moment	Positive moment	Negative moment
Column strip	With drop		A	44	24	56	20	50
			B	36				
			C	6				
	With-out drop		A	40	28	50	22	46
			B	32				
			C	6				
Middle strip	With drop		A	10	20	17*	15	15*
			B	20				
			C	6				
	With-out drop		A	10	20	18*	16	16*
			B	20				
			C	6				
Half column strip adjacent to marginal beam or wall	With drop	1	A	22	12	28	10	25
			B	18				
			C	3				
		2	A	17	9	21	8	19
			B	14				
			C	3				
		3	A	11	6	14	5	13
			B	9				
			C	3				
	With-out drop	1	A	20	14	25	11	23
			B	16				
			C	3				
		2	A	15	11	19	9	18
			B	12				
			C	3				
		3	A	10	7	13	6	12
			B	8				
			C	3				

* Increase negative moments 30 percent of tabulated values when middle strip is continuous across support of type B or C.

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Table III-11 Type of Support Listed in Table III-10

Percentage of panel load to be carried by marginal beam or wall in addition to loads directly superimposed thereon	Side support parallel to strip	Side or end edge condition of slabs of depth t	End support at right angles to strip
0	1	Columns with no beams	
20	2	Columns with beams of total depth $1\frac{1}{4}t$	A
40	3	Columns with beams of total depth $3t$ or more	B
		Reinforced concrete bearing walls integral with slab	
		Masonry or other walls providing negligible restraint	C

(2) ACI 318-63 Elastic Analysis Method (2)

The elastic analysis method given by ACI 318-63 for flat slabs is actually the basis of the equivalent frame method given by ACI 318-71. The procedure for taking a frame and analyzing by this method is the same as that of the equivalent frame method, which is described in part A of this chapter, except there are no supporting beams. The division into column strips and middle strips is shown in Fig. III-12. The positive moments and negative moments obtained from the frame analysis are then distributed to strips as shown in Table III-12. Linear interpolation should be used for intermediate ratios of beam depth to slab thickness.

Table III-12 Distribution Between Column Strips and Middle Strips in Percent of Total Moments at Critical Sections of a Panel

Strip		Moment section			
		Negative moment at interior support	Positive moment	Negative moment at exterior support	
				Slab supported on columns and on beams of total depth equal to the slab thickness	Slab supported on reinforced concrete bearing wall or columns with beams of total depth equal or greater than 3 times the slab thickness
Column strip		76	60	80	60
Middle strip		24	40	20	40
Half column strip adjacent and parallel to marginal beam or wall	Total depth of beam equal to slab thickness	38	30	40	30
	Total depth of beam or wall equal to or greater than 3 times slab thickness	19	15	20	15

Note: The total dead and live reaction of a panel adjacent to a marginal beam or wall may be divided between the beam or wall and the parallel half column strip in proportion to their stiffness, but the moment provided in the slab shall not be less than that given in this Table.

IV COMPARISON OF METHODS

Part A. Two-Way Slabs

Comparisons are made for moment coefficients for a unit width of slab for interior panels using all the methods described in the previous chapter. All moments are in terms of coefficients of $q_t S^2$, where

q_t = total uniform transverse load on slab;

S = length of short span of the slab.

For the sake of simplicity, all supporting beams and columns are of zero-width. In case pattern loading effects are included in computations, the live load is assumed equal to the dead load. For the direct design method and equivalent frame method of ACI 318-71 (1) and the method using Gamble's analysis (22), $\alpha_1 \ell_2 / \ell_1$ is assumed equal to 4 for all slabs. Comparisons are also made with test results (28). The computed moment coefficients are shown in Tables IV-1 through IV-6.

A comparison of ratios of maximum moment coefficients to moment coefficient of yield line theory is also made as shown in Fig. IV-1. In that figure, f is the ratio between coefficients and is defined as

$$f = \frac{C_m}{C_u}$$

where

C_m = maximum moment coefficient;

C_u = moment coefficient using yield line theory.

Comparisons of total static moments are shown in Fig. IV-2.

Table IV-1 Comparison of Moment Coefficients C per Unit Width of Two-Way Slabs for Interior Panel, $m = 0.5$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	0.081	0.056	0.081	0.056	0.022	0.015	0.022	0.015
2	0.083	0.062	0.055	0.041	0.033	0.025	0.022	0.017
3	0.086	0.053	0.057	0.035	0.024	0.012	0.016	0.008
4	0.091	0.063	0.091	0.063	0.033	-	0.022	-
5	0.083	0.041	-	-	0.057	0.016	-	-
6	0.060	0.032	0.022	0.012	0.065	0.035	0.088	0.047
7	0.062	0.034	0.023	0.012	0.067	0.036	0.090	0.049
8	0.065	0.031	0.025	0.011	0.072	0.020	0.046	0.017
9	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035

$$M = Cq_t S^2$$

- | | |
|------------------------|------------------------------------|
| * 1. ACI Method 1 | 6. ACI 318-71 Direct Design Method |
| 2. ACI Method 2 | 7. ACI 318-71 Equivalent Frame |
| 3. ACI Method 3 | Method |
| 4. ACI One-Way Method | 8. Method Using Gamble's Analysis |
| 5. Elasticity Analysis | 9. Yield Line Theory |

Table IV-2 Comparison of Moment Coefficients C per Unit Width of Two-Way Slabs for Interior Panel, $m = 0.6$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	0.064	0.044	0.064	0.044	0.025	0.017	0.025	0.017
2	0.063	0.047	0.042	0.031	0.033	0.025	0.022	0.017
3	0.081	0.047	0.054	0.031	0.028	0.020	0.019	0.013
4	-	-	-	-	-	-	-	-
5	0.079	0.039	-	-	0.057	0.019	-	-
6	0.052	0.028	0.022	0.012	0.059	0.032	0.059	0.032
7	0.054	0.029	0.023	0.012	0.060	0.033	0.060	0.033
8	0.062	0.027	0.025	0.012	0.062	0.017	0.032	0.012
9	0.032	0.032	0.032	0.032	0.032	0.032	0.032	0.032

$$M = Cq_t S^2$$

* See Table IV-1 for explanation of identifying numbers.

Table IV-3 Comparison of Moment Coefficients C per Unit Width of Two-Way Slabs for Interior Panel, $m = 0.7$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	0.053	0.036	0.053	0.036	0.028	0.019	0.028	0.019
2	0.055	0.041	0.037	0.027	0.033	0.025	0.022	0.017
3	0.074	0.040	0.049	0.027	0.035	0.020	0.023	0.013
4	-	-	-	-	-	-	-	-
5	0.074	0.036	-	-	0.057	0.021	-	-
6	0.047	0.026	0.022	0.012	0.053	0.029	0.042	0.023
7	0.048	0.027	0.023	0.012	0.055	0.030	0.043	0.024
8	0.057	0.025	0.024	0.012	0.057	0.018	0.030	0.011
9	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028

$$M = Cq_t S^2$$

* See Table IV-1 for explanation of identifying numbers.

Table IV-4 Comparison of Moment Coefficients C per Unit Width of Two-Way Slabs for Interior Panel, $m = 0.8$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	0.044	0.030	0.044	0.030	0.030	0.021	0.030	0.021
2	0.048	0.036	0.032	0.024	0.033	0.025	0.022	0.017
3	0.065	0.034	0.043	0.023	0.042	0.022	0.028	0.015
4	-	-	-	-	-	-	-	-
5	0.066	0.031	-	-	0.056	0.023	-	-
6	0.044	0.024	0.021	0.011	0.048	0.026	0.031	0.017
7	0.045	0.025	0.021	0.012	0.049	0.027	0.032	0.017
8	0.053	0.022	0.024	0.011	0.052	0.019	0.029	0.011
9	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026

$$M = Cq_t S^2$$

* See Table IV-1 for explanation of identifying numbers.

Table IV-5 Comparison of Moment Coefficients C per Unit Width of Two-Way Slabs for Interior Panel, $m = 0.9$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	0.036	0.025	0.036	0.025	0.030	0.021	0.030	0.021
2	0.040	0.030	0.027	0.020	0.033	0.025	0.022	0.017
3	0.055	0.029	0.037	0.019	0.046	0.023	0.031	0.015
4	-	-	-	-	-	-	-	-
5	0.059	0.027	-	-	0.054	0.023	-	-
6	0.042	0.023	0.019	0.011	0.044	0.024	0.023	0.013
7	0.043	0.024	0.020	0.011	0.045	0.025	0.024	0.013
8	0.048	0.019	0.023	0.011	0.047	0.018	0.027	0.010
9	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023

$$M = Cq_t S^2$$

* See Table IV-1 for explanation of identifying numbers.

Table IV-6 Comparison of Moment Coefficients C per Unit Width of Two-Way Slabs for Interior Panel, $m = 1.0$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	0.030	0.021	0.030	0.021	0.030	0.021	0.030	0.021
2	0.033	0.025	0.022	0.017	0.033	0.025	0.022	0.017
3	0.045	0.023	0.030	0.015	0.045	0.023	0.030	0.015
4	-	-	-	-	-	-	-	-
5	0.051	0.023	-	-	0.051	0.023	-	-
6	0.041	0.022	0.018	0.010	0.041	0.022	0.018	0.010
7	0.042	0.023	0.019	0.010	0.042	0.023	0.019	0.010
8	0.044	0.018	0.023	0.010	0.044	0.018	0.023	0.010
9	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021
10	0.034	0.021	-	-	-	-	-	-
11	0.046	0.025	-	-	-	-	-	-

$$M = Cq_t S^2$$

* See Table IV-1 for explanation of identifying numbers.

10. Measured bending moment coefficients when testing, uniform load on all panels (28)
11. measured maximum bending moment coefficients when testing, considering the effects of pattern loading (28)

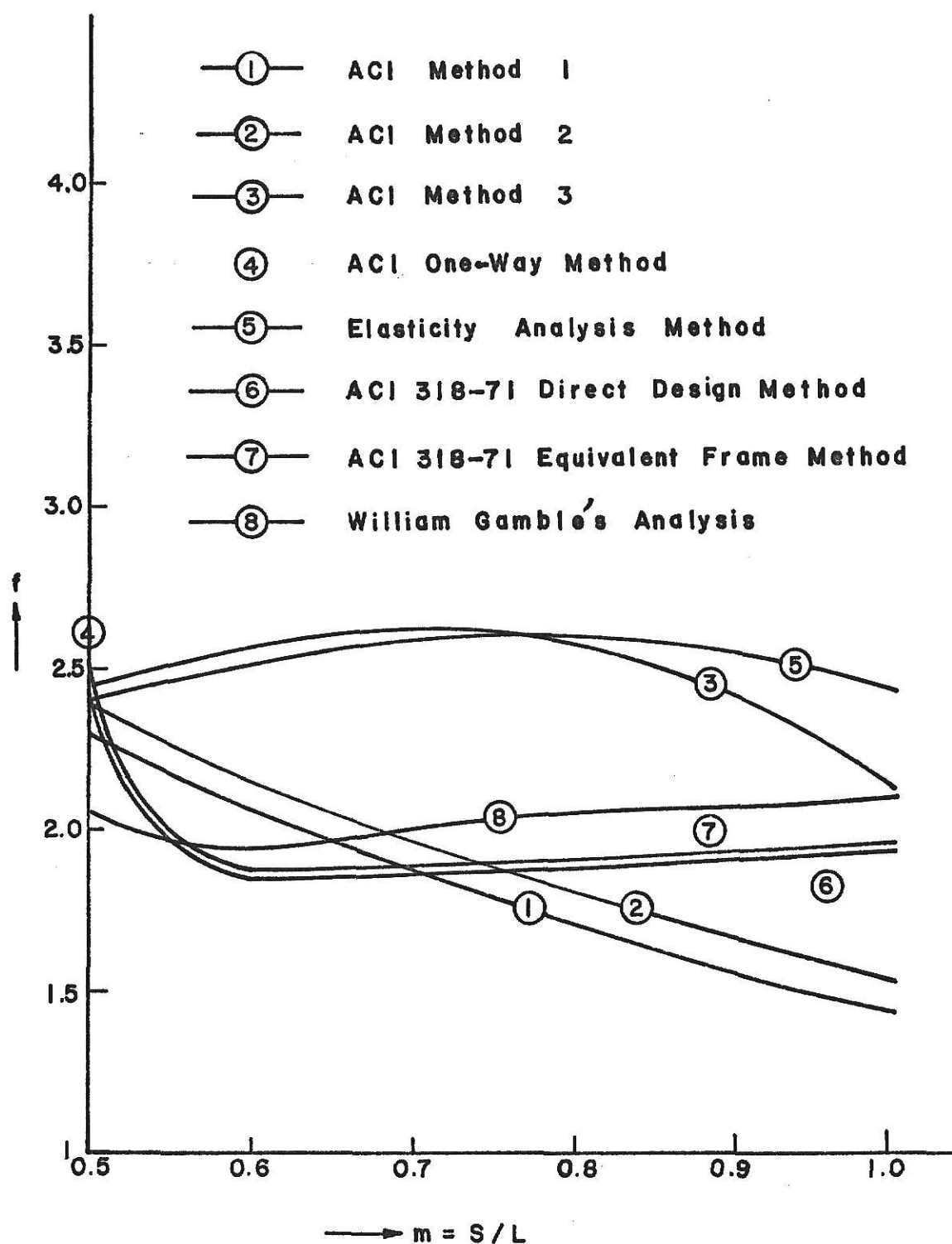


Fig. IV-1 Comparison of Ratios of Maximum Moment Coefficients to the Moment Coefficient of Yield Line Theory for Two-Way Slabs

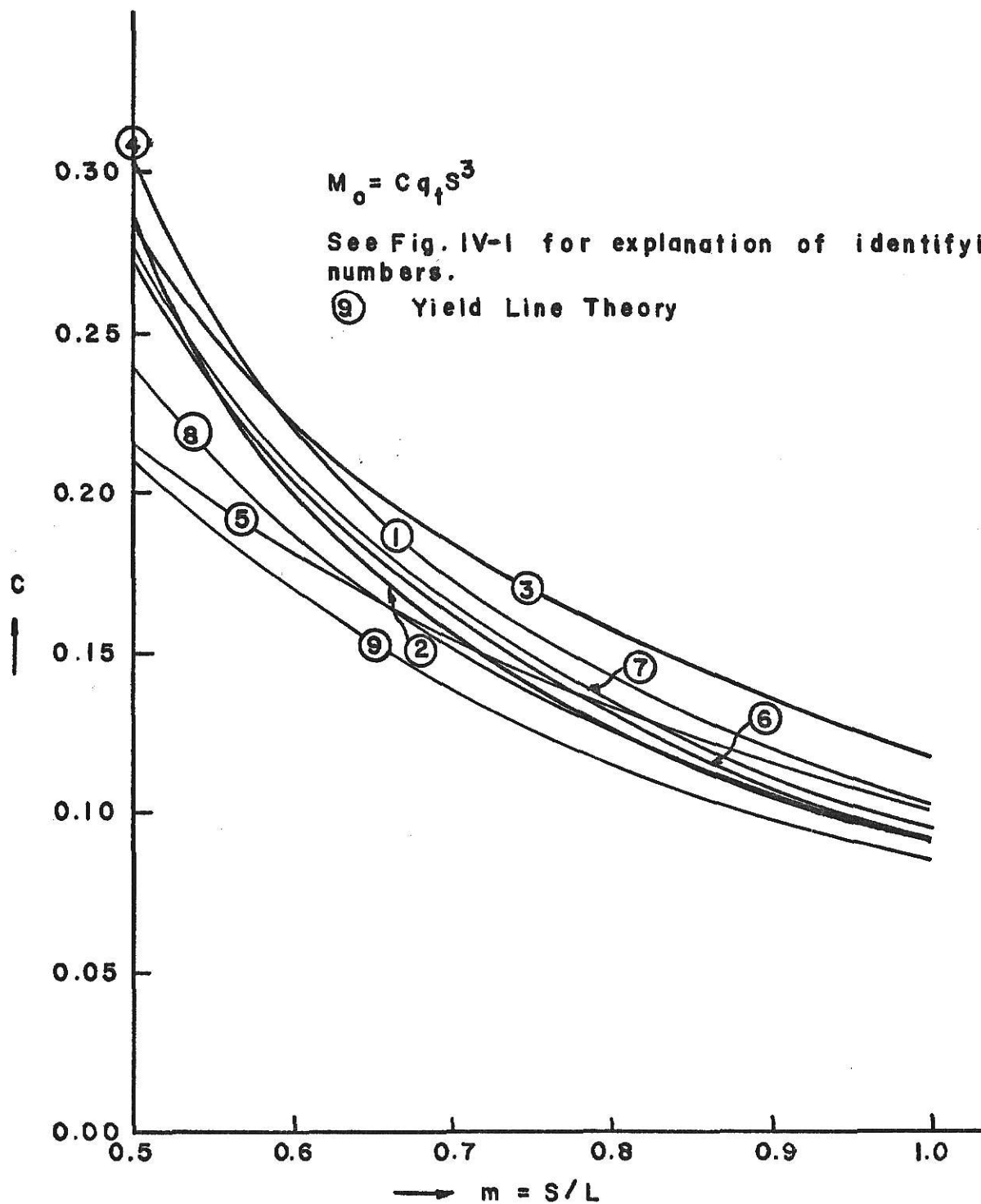


Fig. IV-2 Comparison of Total Static Moment,
Short Span + Long Span, Two-Way Slabs

Part B Flat Slabs

Comparison are made for moment coefficients for unit widths of flat slab for interior panels using the 4 methods of ACI 318-63 and ACI 318-71. All moments are in terms of the coefficients of $q_t S^2$, where

q_t = total uniform transverse load on slab;

S = length of short span of the slab.

For the sake of simplicity, all the supporting columns are of zero-width. In case pattern loading effects are included in computations, the live load is assumed equal to the dead load. Comparisons are also made with a test result (29). The computed moment coefficients are shown in Tables IV-7 through IV-12. Comparison of total static moments is shown in Fig. IV-3.

Table IV-7 Comparison of Moment Coefficients C per Unit Width of Flat Slabs for Interior Panel, $m = 0.5$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	-	-	-	-	-	-	-	-
2	0.040	0.037	0.127	0.055	0.160	0.146	0.506	0.219
3	0.026	0.024	0.248	0.105	0.156	0.139	0.486	0.209
4	0.027	0.025	0.254	0.110	0.160	0.146	0.506	0.219

$$M = C q_t S^2$$

- | | |
|--|--|
| <p>* 1. ACI 318-63 Empirical Method</p> <p>2. ACI 318-63 Elastic Analysis Method</p> | <p>3. ACI 318-71 Direct Design Method</p> <p>4. ACI 318-71 Equivalent Frame Method</p> |
|--|--|

Table IV-8 Comparison of Moment Coefficients C per Unit
Width of Flat Slabs for Interior Panel, $m = 0.6$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	-	-	-	-	-	-	-	-
2	0.040	0.037	0.127	0.055	0.111	0.101	0.352	0.152
3	0.028	0.025	0.206	0.089	0.108	0.097	0.342	0.146
4	0.029	0.026	0.211	0.092	0.111	0.101	0.352	0.152

$$M = Cq_t S^2$$

* See Table IV-7 for explanation of identifying numbers.

Table IV-9 Comparison of Moment Coefficients C per Unit
Width of Flat Slabs for Interior Panel, $m = 0.7$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	-	-	-	-	-	-	-	-
2	0.040	0.037	0.127	0.055	0.082	0.075	0.260	0.112
3	0.030	0.027	0.177	0.076	0.080	0.072	0.253	0.108
4	0.031	0.028	0.182	0.079	0.082	0.075	0.260	0.112

$$M = Cq_t S^2$$

* See Table IV-7 for explanation of identifying numbers.

Table IV-10 Comparison of Moment Coefficients C per Unit Width of Flat Slabs for Interior Panel, $m = 0.8$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	0.033	0.033	0.096	0.046	0.052	0.052	0.149	0.071
2	0.040	0.037	0.127	0.055	0.062	0.057	0.198	0.086
3	0.032	0.030	0.155	0.066	0.060	0.055	0.193	0.083
4	0.033	0.031	0.159	0.069	0.062	0.057	0.198	0.086

$$M = Cq_t S^2$$

* See Table IV-7 for explanation of identifying numbers.

Table IV-11 Comparison of Moment Coefficients C per Unit Width of Flat Slabs for Interior Panel, $m = 0.9$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	0.033	0.033	0.096	0.046	0.041	0.041	0.118	0.056
2	0.040	0.037	0.127	0.055	0.049	0.045	0.155	0.068
3	0.035	0.033	0.137	0.059	0.048	0.043	0.151	0.066
4	0.036	0.034	0.141	0.061	0.049	0.045	0.155	0.068

$$M = Cq_t S^2$$

* See Table IV-7 for explanation of identifying numbers.

Table IV-12 Comparison of Moment Coefficients C per Unit
Width of Flat Slabs for Interior Panel, $m = 1.0$

*	Short Span				Long Span			
	Middle Strip		Column Strip		Middle Strip		Column Strip	
	Ends	Center	Ends	Center	Ends	Center	Ends	Center
1	0.033	0.033	0.096	0.046	0.033	0.033	0.096	0.046
2	0.040	0.037	0.127	0.055	0.040	0.037	0.127	0.055
3	0.039	0.036	0.124	0.053	0.039	0.036	0.124	0.053
4	0.040	0.037	0.127	0.055	0.040	0.037	0.127	0.055
5	0.036	0.018	0.077	0.027	0.036	0.018	0.077	0.027

$$M = Cq_t S^2$$

* See Table IV-7 for explanation of identifying numbers.

5. measured bending moment coefficients when testing, uniform load
on all panels (29)

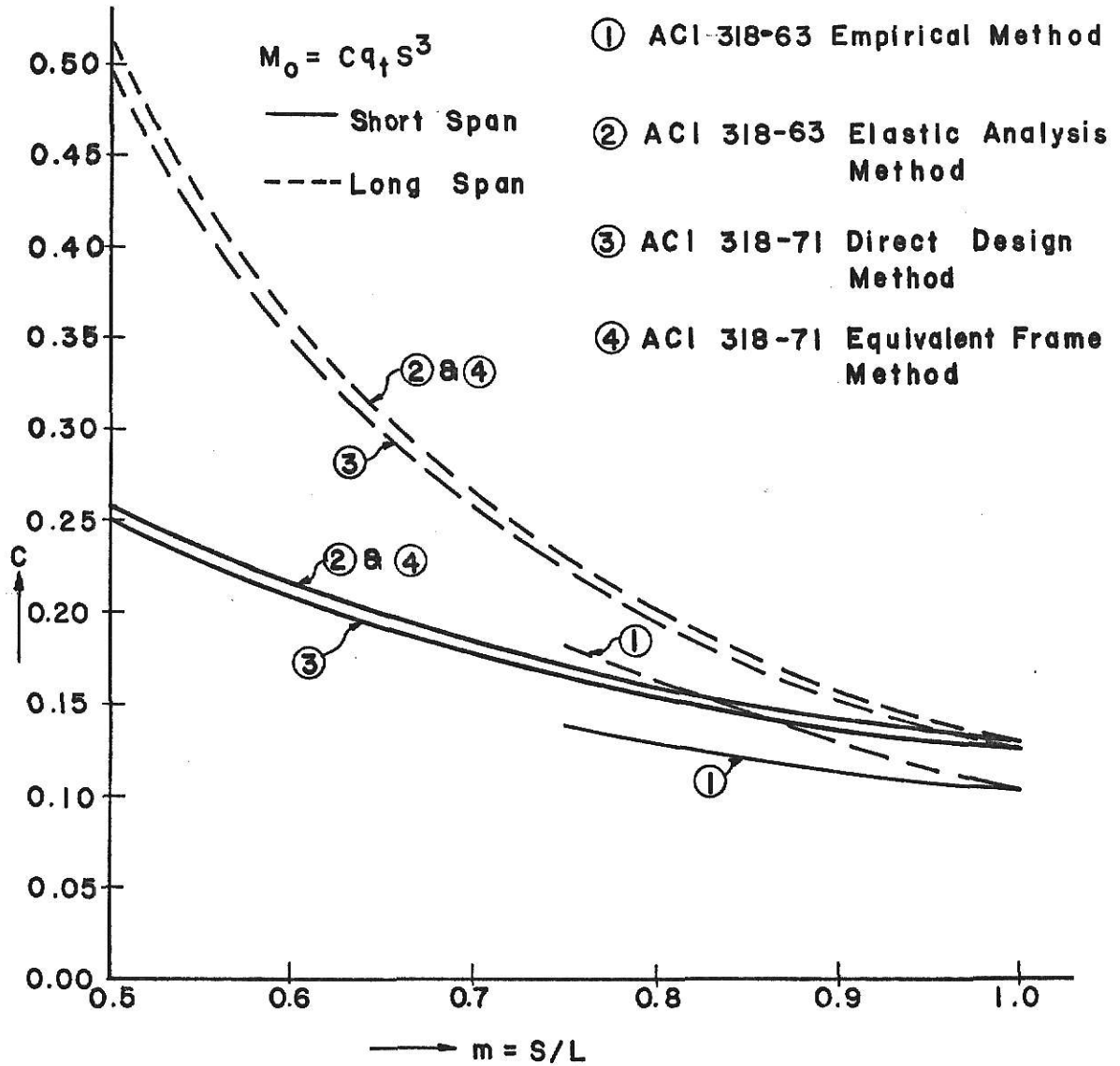


Fig. IV-3 Comparison of Total Static Moment for Flat Slabs

V CONCLUSIONS

As a rule of nature, things generally become better as they evolve. This is actually the case for the slab design methods. We know that no matter how we design the slab, it will behave in its own way. What we have done is to try to approach the true behavior of the slab by simple, practical design procedures. We began from the crossing beam analogy and ended up with a method which takes almost everything - effects of pattern loading, deflections of supporting beams, plastic moment redistribution, etc. - into consideration.

From the comparison Tables, it is interesting to note that the moment coefficients from different methods differ from each other greatly. As a rule of design, we always use the critical required thickness throughout the whole slab. A comparison of all methods shows that the ratios of maximum moment coefficients to the moment coefficient of yield line theory, f , ranges from 2.7 to 1.4. This means that the required thickness ranges from 1.64 to 1.18 times of that required by yield line theory. This is not too bad since reinforced concrete is not able to develop fully moment redistribution.

Although thickness is the same throughout the whole slab, reinforcement is usually proportioned for the moment computed along the critical sections. From this viewpoint, a comparison for total static moment has been made. It is shown from that comparison that all methods possess higher total static moment than that of yield line theory. Again, this is reasonable for reinforced concrete. The testing results shown

in the comparison Tables also agree fairly well with some methods. It is concluded then that all methods are satisfactory from the viewpoint either of economy or of safety despite the great difference between the moment coefficients.

However, in comparing the two methods in ACI 318-71 (1) and Gamble's (22) analysis which is actually based on the same assumptions, we are still able to find the great differences between moment coefficients. This is due to the fact that the two methods of ACI 318-71 are too simplified. They assume that the results are the same for $\alpha_1 l_2 / l_1$ equal or greater than 1. Actually, there is a great difference in results between $\alpha_1 l_2 / l_1 = 1$ and $\alpha_1 l_2 / l_1 \rightarrow \infty$. There is still room for argument on this point since the code cut the tail too soon. If we want to approach the true behavior of the slab better, it is likely the code needs some modifications although it is satisfactory at the present time.

New approaches are still necessary since all the theories and design methods are based on the assumption that the reinforced concrete is either perfectly elastic or perfectly plastic. Reinforced concrete is actually neither.

APPENDIX 1 NOTATION

- A_s = area of reinforcing bars
 a = span length of rectangular plate in x direction
 B = moment coefficient for one-way slab
 b = span length of rectangular plate in y direction
 C = moment coefficient in general
 or
 = cross-sectional constant
 C_1, C_1' = moment coefficients for two-way slab, ACI method 1
 C_2 = moment coefficient for two-way slab, ACI method 2
 C_3, C_3', C_3'' = moment coefficients for two-way slab, ACI method 3
 C_m = maximum moment coefficient
 C_u = moment coefficient using yield line theory
 c = effective support size for flat slab
 c_2 = size of rectangular or equivalent rectangular column,
 capital, or bracket measured transverse to the direction
 in which moments are being determined
 D = flexural rigidity of plate

$$= \frac{Eh^3}{12(1-\nu^2)}, \text{ for uniform thickness}$$

 d = effective depth of a reinforced concrete section
 E = Young's modulus in general
 E_{cb} = Young's modulus of beam concrete
 E_{cs} = Young's modulus of slab concrete
 E_s = Young modulus of steel

- F = a factor in calculation of static moment for flat slab
 = $1.15 - c/\ell_1$ but not less than 1
- f = ratio of maximum moment coefficient to moment coefficient
 of yield line theory
- f_s = tensile stress of reinforcing bars
- g = ratio of span between lines of inflection to L_1 in the
 direction of span L_1 , when span L_1 only is loaded
- g_1 = ratio of span between lines of inflection to L_1' in the
 direction of span L_1' , when span L_1' only is loaded
- h = thickness of plate
- I_b = moment inertia of beam section
- I_s = moment inertia of slab section
- j = ratio of lever arm of resisting moment of reinforced
 concrete section to effective depth
- K_{ec} = flexural stiffness of equivalent column
- K_c = flexural stiffness of column
- K_t = torsional stiffness of torsional member
- k = proportion of load carried by the transverse direction
 (short span)
- L = length of slab (long span), see ACI Code for details for
 different methods
- L_1 = length of clear span in the direction moments are being
 determined
- L_1' = length of clear span in the direction normal to L_1
- ℓ_1 = length of span in the direction moments are being determined,
 measured center-to-center of supports

- l_2 = length of span transverse to l_1 , measured center-to-center of supports
 M = bending moment in general
 M_ℓ^- = critical negative moment in the long direction
 M_ℓ^+ = critical positive moment in the long direction
 M_o = total static design moment
 M_s^- = critical negative moment in the short direction
 M_s^+ = critical positive moment in the short direction
 M_u = ultimate moment resistance of a slab section of unit width
 M_x = bending moment in the x direction
 M_y = bending moment in the y direction
 m = ratio of short span to long span, S/L
 q = transverse load in general
 q_d = uniform transverse dead load on slab
 q_ℓ = uniform transverse live load on slab
 or
 = share of transverse load carried in the long direction
 q_s = share of transverse load carried in the short direction
 q_t = uniform transverse total load on slab
 r = the proportion between two effective spans, $g_1 L_1 / g_1 L_1'$
 r_1 = $\frac{1}{r} = g_1 L_1' / g_1 L_1$
 S = breadth of slab (short span), see ACI Code for details for different methods
 ν = Poisson's ratio

- W = total load on whole panel
 W_1 = total load between opposite supports on slab strip in L_1 direction of any width
 W_1' = corresponding value of W_1 in L_1' direction
 w = deflection in the transverse direction of a plate
 x, y = orthogonal coordinates for middle plan of plate
 x_1 = shorter overall dimension of a rectangular part of a cross section
 y_1 = longer overall dimension of a rectangular part of a cross section
 ϵ_s = strain of steel
 α_1 = ratio of flexural stiffness of beam section to the flexural stiffness of a width of slab bounded laterally by the center line of the adjacent panel on each side of the beam in the direction of ℓ_1
 α_2 = corresponding value of α_1 in the direction of ℓ_2
 α_{ec} = ratio of flexural stiffness of the column above and below the slab to the combined flexural stiffness of the slabs and beams at exterior joint taken in the direction moments are being determined
 β_t = ratio of torsional stiffness of edge beam section to the flexural stiffness of a width of a width of slab equal to the span length of the beam, center-to-center of supports

$$= \frac{E_{cb} C}{2 E_{cs} I_s}$$

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THE EVOLUTION AND COMPARISON
OF DESIGN METHODS FOR CONCRETE SLABS

by

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AN ABSTRACT OF MASTER'S REPORT

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ABSTRACT

The evolution and description of 11 design methods for floor slabs spanning in two directions is presented. The purpose is to show the background of why these design methods changed and to illustrate how to apply each method. Comparisons were made for moment coefficients of interior panels using all design methods described and also with test results. Comparisons were also made for total static moments and ratios of maximum moment coefficients to moment coefficient using yield line theory. It is concluded from the comparisons that, despite the great differences between the moment coefficients, all design methods are considered satisfactory on the whole panel basis. It was also found that the two methods in ACI 318-71 still need some modification in order to approach the true behavior of the slab better.