AN INVESTIGATION OF NUCLEAR EXCURSIONS TO DETERMINE THE SELF-SHUTDOWN EFFECTS IN THERMAL, HETEROGENEOUS, HIGHLY ENRICHED, LIQUID-MODERATED REACTORS

by

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B.S., University of Nebraska, 1957

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1962

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Nomenclature of Terms Not Defined in Text

k_{eff}	Effective multiplication factor.
ε	Fast fission factor.
p	Resonance escape probability.
η	Neutrons born per thermal absorption in fuel.
f	Thermal utilization.
B^2	Buckling, cm^{-2}.
L	Thermal diffusion length, cm.
κ	$1/L$, cm^{-1}.
τ	Fermi age, cm^2.
v	Neutron velocity, cm/sec.
q_{oo}	Heat generation rate in center of fuel at start of pulse, $btu/hrft^3$.
τ	Reciprocal period of power rise, sec^{-1}.
A_j, λ_j	Parameters for empirical fit of heat generation rate, $btu/hrft^3$ and sec^{-1} respectively.
B_i, β_i	Parameters for empirical fit of fuel plate surface boundary condition, ^oF and sec^{-1} respectively.
L, L_1	Thickness of fuel and moderator, respectively in slab geometry, cm.
R, R_1	Thickness of fuel and moderator, respectively in cylindrical geometry, cm.
x	Distance in fuel from the center of unit cell, slab geometry, cm.
x_1	Distance in moderator from outside of unit cell, slab geometry, cm.
r	Distance in fuel and moderator from center of unit cell, cylindrical geometry, cm.
i, j, n	Summation indices.

f,m	Subscripts denoting fuel and moderator, respectively.
p,s	No. of terms in heat generation rate and interface boundary condition approximations.

1.0 INTRODUCTION

The concept of reactor safety is extremely important in the engineering application of nuclear power systems. The United States Atomic Energy Commission has therefore authorized an extensive study in this area. This investigation uses experimental data resulting from that study to attempt to define the mechanisms of reactor shutdown.

The safety of a nuclear reactor system is usually considered in terms of its void and temperature coefficients of reactivity. If one designs a reactor in such a manner as to make both of these coefficients negative, the system will tend to stabilize itself if some external perturbation is placed on the system. This is due to the fact that when the excess reactivity, $k_{eff}-1$, of a system is increased, the power tends to rise, thereby increasing the temperature of the system, and this in turn causes a decrease in reactivity. In the case of a liquid moderated system, voids may be introduced which will further decrease the reactivity.

If one considers the bare reactor age-diffusion criticality equation, $k_{eff} = \dfrac{\varepsilon p \eta f \, e^{-B^2\tau}}{1 + L^2B^2}$, it is possible to see how these effects manifest themselves in the nuclear constants (22). In the case of the void coefficient, the significant effect is on the Fermi age, τ. Since part of the moderator is removed, the age increases, thus decreasing the fast non-leakage factor, $e^{-B^2\tau}$. In the case of the temperature coefficient there are several effects that must be considered. First, the moderator expands because of the positive coefficient of expansion of the liquid.

This results in a decrease in the density of the moderator, and if the reactor is under-moderated the fast non-leakage factor and p will decrease. Thus it is evident that one safety criteria is that the core should always be slightly under-moderated even though this will increase the critical mass. Second, the fuel elements expand, expelling more of the moderator from the core causing k_{eff} to decrease; however, simultaneously, the effective size of the core increases, causing a decrease in the buckling, B^2, of the system. This decrease in buckling has both a positive and a negative effect upon reactivity or k_{eff}. The increase in size increases the non-leakage factor for both fast and thermal leakage, $e^{-B^2\tau}$ and $1/(1 + L^2B^2)$, respectively. The removal of moderator increases τ reducing reactivity as described in the void coefficient discussion. All of the effects described above with the exception of the buckling, must be considered to be of a delayed nature.

Another group of effects exists which affect the reactivity immediately and these effects are therefore classified as prompt (41). The first of these prompt effects is Doppler broadening (38). Because of the increased kinetic energy of U^{238} target nuclei with increased temperature, the width of resonance absorption is increased, but the height of the peak is decreased (22), because the total area beneath the resonance curve remains constant If the resonance absorption cross sections are large, so that essentially all neutrons with energies in the resonance region are captured, the widening of the region will result in a decrease in the resonance escape probability, p, and thus the reactivity decreases as the temperature rises. A second prompt effect is caused by hardening of the thermal neutron spectrum as the moderator temperature rises. The result of this hardened spectrum is that the average thermal-neutron velocity increases, $\underline{L^2}$ increases and the thermal non-leakage factor, $\frac{1}{1 + L^2B^2}$, decreases. One would expect that the neutrons per absorption would also be affected by this spectral change but assuming the normal 1/v dependence for all cross sections, this effect cancels since the terms comprising this effect are composed of the ratios of cross sections. One final effect which is neither part of the temperature coefficient nor the void coefficient must be considered. This is the formation of radiolytic gases.

The reasons that one is inclined to speak of prompt and delayed coefficients of reactivity is that for power bursts of low reactivity and correspondingly long periods, the delayed effects may play a great part in the shut-down mechanism. However, if one supposes a burst with a very short period then it is obvious that these delayed effects will not have had time to act. What constitutes a "short time" can be answered by determining the heat transfer time constant of the fuel elements. This, in part, is the subject of the proposed investigation. Iriarte (27) reported heat transfer time constants for infinitely long cylindrical UO_2 fuel elements surrounded by a helium film which served as a thermal bond and which were clad with zirconium, stainless steel or aluminum. While the data presented by Iriarte are not applicable to the SPERT-I or TRIGA Reactors, the techniques may be useful in determining the relative importance of the delayed effects.

Before considering further the scope of this project it is informative to investigate the prior work in the field. During the early summer of 1954 a series of experiments were made on the BORAX-I Reactor to investigate the ability of the reactor, when operated in the subcooled (non-boiling) condition, to protect itself against the results of sudden, artificially induced increases in reactivity. Inasmuch as this set of experiments completed the program for the BORAX-I Reactor, the final runaway experiment was intentionally made under conditions which led to destruction of the reactor. In the final experiment, a control rod worth four per cent $\underline{k_{eff}}$ was ejected from the reactor core, inducing an exponential power increase which had a period of 2.6 milliseconds. This final experiment resulted in a melting of most of the fuel plates and failure of the reactor tank. Fuel plate fragments were scattered for a distance of 200 to 300 feet (8). This set of experiments, along with the earlier operation of the BORAX-I, (7), established two important safety axioms for water moderated reactors. First, for any given system there is a reactivity insertion beyond which the reactor cannot react fast enough to shut itself down before damage is done, and second, water moderated reactors can be designed to have a high degree of inherent self-protection against the effects of sudden large reactivity increases. A less critical fact that resulted from these tests was that if the transients are started at boiling conditions (such as in a boiling water reactor), the maximum power and fuel-plate temperature reached are less than if the transient is started with the reactor in a subcooled condition. This is as would be expected if void formation due to boiling were the shut-down mechanism, since for the subcooled system the reactor could actually achieve a stable positive period before any negative reactivity would result. This, along with the observation of large quantities of steam and water being expelled from the system, led to the conclusion that it was the void formation which was shutting the reactor down. With this background in mind, the Atomic Energy Commission set out on an intensified program to determine empirically the safe upper operating limits on each class of reactors including the pressurized and unpressurized, boiling and non-boiling thermal reactors, both heterogeneous and homogeneous, and fast-reactor systems (23). This program initiated the SPERT (Special Power Excursion Reactor Tests) and KEWB (Kinetic Experiments on Water Boilers) programs. SPERT is the heterogeneous reactor test facility and KEWB is the design for the same type of tests on a homogeneous reactor test facility. In 1956, W. B. Nyer, et. al. (39), reported the results of the initial transient test on the SPERT-I facility and concluded that the SPERT-I (43) reactor demonstrated qualitatively the results of the BORAX-I experiments although SPERT-I was more stable after the initial power burst. Factors which could contribute to this difference in behavior were the known differences in the fuel assembly construction, the possible differences in the effective void coefficients, and/or the differences in the reflector-tank environment. Another important point discovered at this time was that there was a unique relationship between the peak power and the transient period. The data were fitted rather well by two straight lines on a plot of log reciprocal period versus peak power. The slopes of these lines were approximately 0.8 and 1.7 for the lower and upper regions respectively. The point of intersection occurred at $\underline{\Delta k} = 0.74\%$, or about prompt critical. In June of 1957, R. W. Miller reported on some interesting work in analyzing the reactivity behavior during SPERT-I transients (34). Miller pointed out that it was not necessary to remove all of the initially inserted reactivity to limit a power excursion. This is a result of the delayed neutrons, which for a very short period excursion, do not contribute to the flux during the rise in power. Thus, for short excursions the reactivity compensation at maximum power need only be $\Delta k(1-\beta)-\beta$. Δk is the initial change in k_{eff} and β is the delayed neutron fraction. The total Δk must be accounted for in very long period transients. In the case of intermediate period transients the compensated reactivity was calculated as a function of the reciprocal period by numerical solution of the reactor kinetics equations using the experimentally determined power traces. Later in 1957, G. O. Bright, et. al., suggested a model for reactor burst behavior (2), based on the earlier work of Klaus Fuchs at Los Alamos (21). This model postulated a shutdown effect proportional to the energy release. However, the model provided no method for the shutdown energy to be removed and allowed for no time-delays between the energy release and the appearance of the shutdown effect. Much later this model was further modified by S. G. Forbes (13) who let the shutdown effect be proportional to the energy release raised to some power, n, and allowed for some arbitrary delay time. In this analysis the model was tested against some experimental transients from SPERT-I and values of n from 1.5 to 2 were successively used. The results also showed that the delay time was significant in matching the data; however no exact value of the delay time was determined. The overall effect of these works is to convince one that the primary shutdown mechanisms are intimately tied up with the energy release although no real information on the exact phenomena can be found. In 1958, Griffing and Deverall (10) coupled the energy shut-down model with the reactor kinetics equation including six delay groups and again showed that the energy model could describe qualitatively the power traces obtained in the SPERT-I transients even long after the initial burst. This work used a mathematical structure of the shut-down equation considered much earlier (1951) by Chernick (4) for no delay groups and in 1956 by Margulies (31) with one delay group.

In December of 1958, Deverall and Griffing (9) reported the first attempts at trying to relate the shutdown reactivity with the thermodynamic characteristics of the SPERT-I system. In this report the change in reactivity for transients with long periods was related to the temperature rise in the moderator alone although the fact was recognized that the fuel element temperature rise should also be considered. Considering only the moderator temperature rise, they found reactivity compensations at peak power that were roughly one-half of those reported by Miller (34). Since they felt that the data with which they were working were only accurate to within a factor of two, they did not pursue the investigation further. In January of 1958, Horning of Ramo-Woolridge reported on a model for transients in SPERT-I (18). This report develops a general model, taking into account the void formation and the thermal expansion. However, no real attempt was made to interpret these constants in terms of the distribution of energy in the fuel and moderator or the nuclear and thermodynamic constants of the system. Another report, bound under the same cover, by H. C. Corben (26) treats the problem of oscillations found after the burst as the power approaches some steady state level. Although the mechanism responsible for the oscillations need not be the same as the shut-down mechanism, there is certainly the possibility that they are one and the same. A third report by G. Birkhoff (26) treats the problem of void formation from the point of view of the growth of bubbles. The "Bubble Void" is probably the dominant shutdown effect in a certain type of excursion such as in the initial BORAX-I experiments. The analytic representation of this effect is the least known and thus is being widely sought. An extensive study of bubble formation including a critical review of the literature, an evaluation of the merits of purely theoretical approaches to the development of a void model, and an investigation of the possible formulation of the nucleate boiling void was reported by the Vitro Engineering Company in May of 1959 (28).

Although an exact definition of bubble formation may not be within the scope of this investigation, the determination of the heat flow into the moderator and the transient temperature distribution in the moderator should shed some light on even this difficult problem.

In July of 1958, J. C. Haire (25) reported the results of a great number of the SPERT-I transients. This report presented data on the reactor power, fuel plate surface temperatures and pressures as a function of time during the transients. These are the data that will be used extensively in the initial phases of the investigation proposed herein.

During late 1958 and 1959, several models were proposed to investigate and explain the inherent shutdown characteristics of the SPERT-I reactor. The "Conduction Boiling" model suggested by S. G. Forbes (14) is certainly credible in that it takes into account the flow of heat into the moderator in a much more exact manner than any of the earlier investigations. This model was quite successful in predicting the power, energy release and temperature at the time of peak power as a function of the reciprocal period. However the model still represents the shutdown mechanism in terms of empirical parameters. Also the non-boiling shutdown effects are not taken into account. The "Clipped Exponential" model suggested by R. W. Miller (35) made some very useful assumptions on the shape of the reactor power burst to ease the analytical solution of kinetics equations. While this model produced some useful criteria in the understanding of self-shutdown it again made use of lumped parameters which were not easily interpreted in terms of the thermodynamic and nuclear characteristics of the system. E. T. Clark (5,6) as early as 1956 had postulated a prompt fission product having a large absorption cross section for thermal neutrons as an explanation of the self-shutdown of power excursions observed in the SPERT-I Reactor. Later evidence (42) seems to indicate that this model is less likely to be valid than the more conventional models.

Also in 1958, General Atomic introduced their TRIGA Reactor with zirconium hydride moderator which demonstrated a larger prompt shutdown mechanism than either the BORAX-I or the SPERT-I (41). In this case the reactor was designed to have a shutdown mechanism which would act by hardening the thermal neutron energy spectrum thus increasing the thermal leakage to cause shutdown. The reason for choosing this effect was that they felt that it would act more quickly and thus provide a safer reactor than the accepted moderator expansion and expulsion mechanisms. The spectrum effect, so important in the TRIGA Reactors must also act to some extent in SPERT-I. The amount of this effect has apparently never been determined.

P. French, in 1959, (18) reported an attempted solution of the transient heat conduction equation in the fuel and moderator to determine the temperature distribution. However, this work appears to be in error in that he forced a separation of variables solution on the equations whereas the spatial and time dependence cannot be expressed as a simple product except after sufficiently long times so that the transient term has disappeared. H. L. McMurry (32) reported the temperature distribution in a fuel plate, cladding and moderator with exponentially rising power for pure conduction. This is an excellent piece of work but the mathematical model turns out to be more difficult than is either warranted or necessary for the analysis of the SPERT-I data. The McMurry report makes mention of earlier work by H. Greenspan (24) on the same problem with similar results. Several investigators, Kattwinkel (29), Kirchenmayer (30), Stein (40), Epel (11), Arpaci and Clark (1), Ermakov and Ivanov (12), report analytical solutions to the transient heat conduction equation. However, none of them were working on the problem with reference to the SPERT-I investigations and their results are not directly applicable to the use of the SPERT-I experimental measurements. As a result the temperature distribution in the fuel and moderator during a transient is not well known to date.

In July of 1959, Forbes, et. al. (16), summarized the work done up to that time. They showed that by fitting empirically the "Conduction Boiling" model to the SPERT-I data and including the effect of moderator and fuel element expansion they could fit the experimental compensated reactivity at peak power versus reciprocal period curves for values of the reciprocal period greater than 5 seconds^{-1}. They also showed that for values of the reciprocal period greater than 20 seconds^{-1}, the steam void contribution to reactivity was considerable. For the longer period region they postulated an additional shutdown effect from radiolytic gases.

This, of course, lends considerable credence to the "Boiling Conduction" model, however it does have one glaring shortcoming. To extend it to another reactor requires at least one transient burst experiment to determine the parameters. However, if the fundamental mechanism were understood exclusively in terms of the basic nuclear, thermodynamic and hydrodynamic characteristics of the system, one could confidently predict the limits of safe operations for different reactor systems.

The satisfactory application of the "Boiling Conduction" model led to two sets of experiments designed to test that model and the postulated radiolytic gas effect. The first set of experiments (20) showed with reasonable certainty that radiolytic gas formation was not a primary contributor to self-shutdown in the SPERT-I Reactor. The second set of experiments (19) consisted of coating all of the fuel plates with approximately five mils of insulation, Lithocote LC-34, and of performing transient tests on the reactor. Some of the tests were run with transients of such magnitude that boiling would occur in the bare core and not in the insulated core. The remaining tests involved transients in which there would be boiling in both cores, and the differences in heat transfer rates were expected to be reflected as changes in the reactor behavior. Power burst shapes for transients of the same period in both the bare and insulated cores were essentially the same. In view of the identical reactor behavior for the bare and insulated tests, it would appear that the core insulation produced no appreciable effects on the shutdown mechanism. Since it seems reasonable to assume that any shutdown effect due to boiling would be effected by the core insulation, boiling would not seem to play any part in the self-shutdown mechanism. However, if the heat transfer rate was small enough there would be negligible temperature drop across the insulation and thus the effect on boiling might be unchanged by the insulation. This experiment shows clearly the need for detailed calculation of the temperature distributions in the fuel and moderator during the transients. One final report should be cited in this summary. In April of 1960, Miller (36) reported on some photographic investigations of boiling during transients in SPERT-I. These results clearly indicated that boiling was an important agent in the initial reactor self-shutdown whenever the fuel plate temperature was sufficiently high.

2.0 THEORY

2.1 Derivation of Equations

The direct approach to determining the self-shutdown effects must include the determination of the temperature distributions in the fuel elements and moderator throughout the core during a transient. This problem can be accomplished by investigating the exact temperature distribution in a center element only and relating all effects to this center element. The exact solution of the multi-region, transient heat conduction equations even for a single fuel element, making use of only the power versus time data from the SPERT-I transients, is exceedingly difficult as pointed out in the work by McMurry (32). However, if use is made of the available fuel plate surface temperature data as well as the power data during a transient, the problem is reduced to two single region problems. Although the solutions are much simplified over the two region problem, they are still complicated and therefore have been programmed for the Kansas State University IBM 650 computer.

The methods for determining the steady state temperature distribution throughout a unit cell of thermal, heterogeneous liquid-moderated reactor are discussed in Nuclear Engineering (41) by C. F. Bonilla and in Nuclear Reactor Physics (37) by R. L. Murray. It will be considered sufficient for this work to outline the differences that must be accounted for in transient operation.

The partial differential equation for conductive heat transfer applicable in the fuel and moderator of a nuclear power reactor during a transient but before boiling is established is

$$\nabla^2\theta(x,t) + \frac{q(x,t)}{k} = 1/\alpha \frac{\partial\theta(x,t)}{\partial t} \tag{1}$$

where ∇^2 is the Laplacian operator (44), $\underline{\theta}$ is the temperature rise above the initial temperature $[T(x,t)-T_o]$, $\underline{x}$ is the position variable, $\underline{k}$ is the mean thermal conductivity, $\underline{\alpha}$ is the mean thermal diffusivity, $\underline{t}$ is the time variable and $\underline{q}$ is the volumetric heat generation rate. This equation in slab and cylindrical geometries is directly applicable to the analysis of transient behavior in a reactor following a change in reactivity. The derivation of solutions to this equation in the fuel and moderator during power transients will be shown in detail for slab geometry (Appendix A) and the important elements of the solution in cylindrical geometry will be tabulated in Section 2.2.1.

In any nuclear reactor there are heating effects due directly to fission fragments and to the attenuation of other nuclear particles. Within the fuel element, fission heating far overrides the other attenuation effects. Therefore, the heat generation rate, $\underline{q_f(x,t)}$, in the fuel elements is proportional to the thermal neutron flux. The neutron flux during a transient can be expressed as a simple product of the spatial and the time dependencies. Thus the heat generation rate is also separable in space and time as shown in equation (2).

$$q_f(x,t) = f_f(x) \quad g_f(t) \tag{2}$$

The spatial dependence, $\underline{f_f(x)}$, can be obtained easily from the steady state analysis (40) and is given in equation (3).

$$f_f(x) = q_{oo} \cosh \kappa x \tag{3}$$

Here $\underline{q_{oo}}$ is the heat generation rate at the center of the fuel, $\underline{\kappa}$ is the inverse thermal neutron diffusion length and $\underline{x}$ is the distance from the center of the fuel.

The time dependence, $\underline{g_f(t)}$, of the neutron flux and thus the heat generation rate is expressed as the sum of exponentials, thus

$$g_f(t) = \sum_{j=1}^{s} a_j e^{\lambda_j t} \tag{4}$$

Substituting equations (3) and (4) into equation (2), an expression for $\underline{q_f(x,t)}$ is obtained; that is

$$q_f(x,t) = \sum_{j=1}^{s} q_{oo} \cosh(\kappa x)\, a_j e^{\lambda_j t} \tag{5}$$

Substituting equation (5) into equation (1) yields the differential equations which must be solved to obtain the temperature distribution in the fuel, that is

$$\nabla^2 \theta_f(x,t) + \sum_{j=1}^{s} \frac{q_{oo} \cosh(\kappa x)\, a_j e^{\lambda_j t}}{k} = 1/\alpha \, \frac{\partial \theta_f(x,t)}{\partial t} \tag{6}$$

Now from investigation of the heat generation rate, $\underline{q_m(k,t)}$ in the moderator, it is noted that there is no heating due to fission, but there is heating due to nuclear particles which stream out of the fuel and are attenuated in the moderator. The moderator heating is approximately 5 to 7% (3) of the recoverable energy from fission and is sufficiently uniform in space to be so considered. Therefore, $\underline{q_m(x,t)}$ in the moderator is independent of the spatial variable but still is time dependent as shown in quation (7).

$$q_m(x,t) = \sum_{j=1}^{s} F\, a_j e^{\lambda_j t} \tag{7}$$

The heating effects in the moderator are proportional to the neutron flux in the fuel; thus F is the fraction of the recoverable energy released in the fuel which is dissipated in the moderator.

Substitution of equation (7) into equation (1) yields the differential equation to be solved for the temperature distribution in the moderator, that is

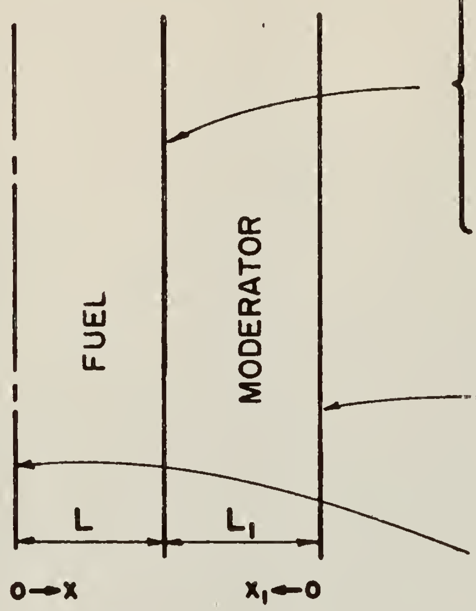
$$\nabla^2\theta_m(x,t) + \sum_{j=1}^{s} \frac{F\, a_j e^{\lambda_j t}}{k} = 1/\alpha \, \frac{\partial\, \theta_m(x,t)}{\partial t} \tag{8}$$

Equations (6) and (8) form the general set which must be solved to obtain the temperature distribution in a unit cell of the reactor during a transient. The geometry of the unit cell in which these equations must be applied is shown in Figure 1. The simultaneous solution of these equations is extremely difficult. The availability of the fuel element surface temperature as a function of time during the transients greatly simplifies this situation. The problem is reduced to solving the equations independently in the fuel and moderator, using the experimentally measured temperatures at the interface as a boundary condition for both equations. The other boundary condition necessary in each case is a zero heat flow condition at the center of the unit cell for the fuel regions and at the boundary of the unit cell for the moderator.

2.2 Analytical Solutions

The time dependent thermal diffusion equations in the fuel and moderator can be solved for the temperature distribution assuming that conduction is the primary mode of heat transfer. The equation will hold for all time in the fuel plate. In an attempt to represent as well as

SLAB GEOMETRY

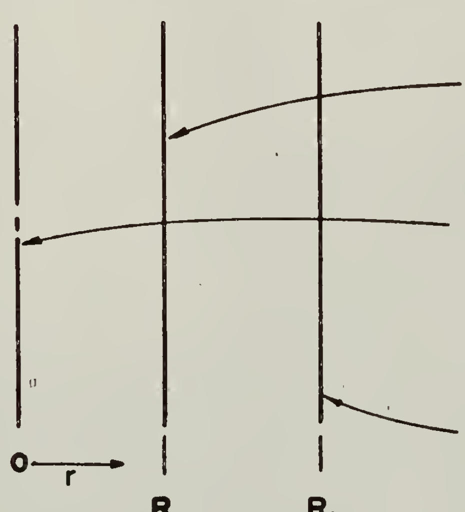


$$\left\{\begin{array}{l} \left(\frac{q}{A}\right)_f (L,t) = \left(\frac{q}{A}\right)_m (L_1,t) = \sum_{i=1}^{p} B_i \, e^{\beta_i t} \\ \text{or} \\ \theta_f \, (L,t) = \theta_m (L_1,t) = \left\{\begin{array}{l} \sum_{i=1}^{p} B_i \, e^{\beta_i t} \\ \text{or} \\ \sum_{i=1}^{p} B_i \, \mathrm{Cos}\,\beta_i t \end{array}\right. \end{array}\right.$$

$$\left.\frac{\partial \theta_m (x_1,t)}{\partial x_1}\right|_{x_1=0} = 0$$

$$\left.\frac{\partial \theta_f (x,t)}{\partial x}\right|_{x=0} = 0$$

CYLINDRICAL GEOMETRY



$$\theta_f \, (R,t) = \theta_m (R,t) = \sum_{i=1}^{p} B_i \, e^{\beta_i t}$$

$$\left.\frac{\partial \theta_f \, (r,t)}{\partial r}\right|_{r=0} = 0$$

$$\left.\frac{\partial \theta_m (r,t)}{\partial r}\right|_{r=R_1} = 0$$

Figure 1. Geometry and boundary conditions in the unit cell used to determine the temperature distributions.

possible the experimental data which is used as input to this problem and to allow some flexibility in the application of these equations, the differential equations were solved subject to several forms of representation of the boundary conditions and the forcing functions. In one case the problem was solved in cylindrical geometry.

In slab geometry the one-dimensional solutions to the transient heat transfer equation, in which the time dependence of heat generation rate and the fuel element surface temperature are represented empirically as $\sum_{j=1}^{s} A_j e^{\lambda_j t}$ and $\sum_{i=1}^{p} B_i e^{\beta_i t}$ respectively, are derived in Appendix A and are given here as

$$\theta_f(x,t) = \sum_{i=1}^{p} \frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\, x\right) e^{\beta_i t}}{\cosh \sqrt{\frac{\beta_i}{\alpha}}\, L} - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\left(\frac{n\pi x}{2L}\right) e^{\frac{n^2\pi^2\alpha}{4L^2} t}}{(L^2/n\pi\alpha) \sin \frac{n\pi}{2}}$$

$$x \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} + \sum_{j=1}^{s} \frac{q_{oo}\, \alpha\, A_j \cosh(\kappa L)}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)} \right\} \quad (9)$$

$$- \sum_{j=1}^{s} \frac{A_j\, q_{oo}\, \alpha e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{ \cosh \kappa x - \frac{\cosh(\kappa L) \cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, x\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)} \right\}$$

and

$$\theta_m(x,t) = \sum_{i=1}^{p} \frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\, x'\right) e^{\beta_i t}}{\cosh \sqrt{\frac{\beta_i}{\alpha}}\, L'} - \sum_{n=1,3,5,\ldots}^{\infty} \frac{\cos \frac{n\pi x'}{L}\, e^{\frac{-n^2\pi^2\alpha}{4L^2} t}}{(L'^2/n\pi\alpha) \sin\left(\frac{n\pi}{2}\right)}$$

$$x \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L'^2} + \beta_i} + \sum_{j=1}^{s} \frac{\alpha F A_j}{k\left(\frac{n^2\pi^2\alpha}{4L'^2}\right)\left(\frac{n^2\pi^2\alpha}{4L'^2} + \lambda_j\right)} \right\} \quad (10)$$

$$+\sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k \lambda_j} \left\{ 1 - \frac{\cosh \sqrt{\frac{\lambda_j}{\alpha}}\, x_1}{\cosh \sqrt{\frac{\lambda_j}{\alpha}}\, L_1} \right\}$$

in the fuel and moderator, respectively.

The equivalent solutions in cylindrical geometry are derived in Appendix A and are given here as

$$\theta_f(x,t) = \sum_{i=1}^{p} \frac{B_i I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right) e^{\beta_i t}}{I_o \sqrt{\frac{\beta_i}{\alpha}}\, R} + \sum_{n=1}^{\infty} \frac{J_o\left(\frac{\omega_n r}{R}\right) e^{-\frac{\omega_n^2 \alpha}{R^2} t}}{\frac{R^2}{2\omega_n \alpha} J_1(\omega_n)}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{\omega_n^2 \alpha}{R^2} + \beta_i} + \sum_{j=1}^{s} \frac{q_{oo}\, \alpha A_j I_o(\kappa R)}{k\left(\frac{\omega_n^2 \alpha}{R^2} + \lambda_j\right)\left(\frac{\omega_n^2 \alpha}{R^2} + \alpha\kappa^2\right)} \right\} \tag{11}$$

$$- \sum_{j=1}^{s} \frac{A_j q_{oo}\, \alpha e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{ I_o(\kappa r) - \frac{I_o(\kappa R)\, I_o\left(\sqrt{\frac{\lambda_j}{\alpha}}\, r\right)}{I_o\left(\sqrt{\frac{\lambda_j}{\alpha}}\, R\right)} \right\}$$

and

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i \left\{ \frac{K_1\left(\sqrt{\frac{\beta_i}{\alpha}} R_1\right) I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right) + I_1\left(\sqrt{\frac{\beta_i}{\alpha}} R_1\right) K_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right)}{K_1\left(\sqrt{\frac{\beta_i}{\alpha}} R_1\right) I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right) + I_1\left(\sqrt{\frac{\beta_i}{\alpha}} R_1\right) K_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right)} \right\} e^{\beta_i t}$$

$$+ \sum_{n=1}^{\infty} \frac{2\sqrt{\rho_n \alpha}\left[K_1\left(\sqrt{\frac{\rho_n}{\alpha}} R\right) I_o\left(\sqrt{\frac{\rho_n}{\alpha}}\, r\right) + I_1\left(\sqrt{\frac{\rho_n}{\alpha}} R\right) K_o\left(\sqrt{\frac{\rho_n}{\alpha}}\, r\right)\right] e^{\rho_n t}}{R\left[K_1\left(\sqrt{\frac{\rho_n}{\alpha}} R_1\right) I_1\left(\sqrt{\frac{\rho_n}{\alpha}} R\right) - K_1\left(\sqrt{\frac{\rho_n}{\alpha}} R\right) I_1\left(\sqrt{\frac{\rho_n}{\alpha}} R_1\right) - I_o\left(\sqrt{\frac{\rho_n}{\alpha}} R\right) + I_o\left(\sqrt{\frac{\rho_n}{\alpha}} R\right) K_o\left(\sqrt{\frac{\rho_n}{\alpha}} R\right)\right]}$$

$$\times \sum_{i=1}^{p} \left\{ \frac{B_i}{\rho_n - \beta_i} - \sum_{j=1}^{s} \frac{\alpha F A_j}{k \rho_n(\rho_n - \lambda_j)} \right\} \tag{12}$$

$$+ \sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k} \left\{ \frac{K_1\sqrt{\frac{\lambda_j}{\alpha}} R_1 I_o \sqrt{\frac{\lambda_j}{\alpha}} r + I_1\sqrt{\frac{\lambda_j}{\alpha}} R_1 K_o \sqrt{\frac{\lambda_j}{\alpha}} r}{K_1\sqrt{\frac{\lambda_j}{\alpha}} R_1) I_o \sqrt{\frac{\lambda_j}{\alpha}} R) + I_1(\sqrt{\frac{\lambda_j}{\alpha}} R_1)K_o (\sqrt{\frac{\lambda_j}{\alpha}} R)} \right\}$$

where ω_n's are the roots of the equation, $J_o(x) = 0$

and ρ_n's are the roots of the equation,

$$[K_1(\sqrt{\tfrac{S}{\alpha}} R_1) I_o (\sqrt{\tfrac{S}{\alpha}} R) + I_1(\sqrt{\tfrac{S}{\alpha}} R_1) K_o(\sqrt{\tfrac{S}{\alpha}} R)] = 0$$

The solutions to the transient heat transfer equations in slab geometry in which the time dependence of the heat generation rate and the first derivative of the surface temperature with respect to x are $\sum_{j=1}^{s'} A_j e^{\lambda_j t}$ and $\sum_{i=1}^{p'} B_i e^{\beta_i t}$ respectively are derived in Appendix A and are given here as

$$\theta_f(x,t) = \sum_{j=1}^{p'} \frac{B_i \cosh (\sqrt{\frac{\beta_i}{\alpha}} x) e^{\beta_i t}}{\sqrt{\frac{\beta_i}{\alpha}} \sinh(\sqrt{\frac{\beta_i}{\alpha}} L)} - \frac{B_i \alpha}{B_i L} - \sum_{n=1}^{\infty} \frac{\cos \frac{n\pi x}{2L} e^{\frac{n^2\pi^2\alpha}{4L^2} t}}{(L/\alpha) \cos n\pi}$$

$$\times \left\{ \sum_{i=1}^{p'} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} + \sum_{j=1}^{s'} \frac{q_{oo} \alpha A_j \kappa \sinh \kappa L}{k(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j)(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2)} \right\} \tag{13}$$

$$-\sum_{j=1}^{s'} \frac{q_{oo} \alpha A_j e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{ \cosh \kappa x - \frac{\kappa \sinh (\kappa L) \cosh (\sqrt{\frac{\lambda_j}{\alpha}} x)}{\sqrt{\frac{\lambda_j}{\alpha}} \sinh (\sqrt{\frac{\lambda_j}{\alpha}} L)} \right\}$$

$$- \frac{q_{oo} \alpha A_j \sinh \kappa L}{k \lambda_j L\kappa}$$

and

$$\theta_m(s,t) = \sum_{i=1}^{p'} \frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\, x\right) e^{\beta_i t}}{\sqrt{\frac{\beta_i}{\alpha}} \sinh \sqrt{\frac{\beta_i}{\alpha}}\, L} - \frac{B_i \alpha}{\beta_i L}$$

$$- \sum_{n=1}^{\infty} \frac{\cos \frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{(L/\alpha) \cos n\pi} \left\{ \sum_{i=1}^{p'} \frac{B_i \alpha}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} \right\} \quad (14)$$

$$+ \sum_{j=1}^{s'} \frac{\alpha F A_i}{k(\lambda_j)} \left(e^{\lambda_j t} - 1\right)$$

in the fuel and moderator, respectively.

The solution to the transient heat transfer equation in slab geometry in which the time dependence of the heat generation rate and the surface temperature are represented by $\sum_{j=1}^{s} A_j e^{\lambda_j t}$ and $\sum_{i=1}^{p} B_i \cos \beta_i t$ respectively are derived in Appendix A and are given here as

$$\theta_f(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos \frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{(L^2/n\pi\alpha) \sin \frac{n\pi}{2}}$$

$$X \left\{ \sum_{i=1}^{p} \frac{B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\left(\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2\right)} + \sum_{j=1}^{s} \frac{q_{oo}\, \alpha A_j \cosh \kappa L}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)} \right\} \quad (15)$$

$$- \sum_{j=1}^{s} \frac{q_{\infty} \alpha A_j e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{ \cosh \kappa x - \frac{\cosh(\kappa L)\cosh(\sqrt{\frac{\lambda_j}{\alpha}}\, x)}{\cosh(\sqrt{\frac{\lambda_j}{\alpha}}\, L)} \right\}$$

and

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos \frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{(L^2/n\pi\alpha)\sin \frac{n\pi}{2}}$$

$$X \left\{ \sum_{i=1}^{p} \frac{B_i \frac{n^2\pi^2\alpha}{4L^2}}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} - \sum_{j=1}^{s} \frac{\alpha F A_j}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)} \right\} \tag{16}$$

$$+ \sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k\lambda_j} \left\{ 1 - \frac{\cosh(\sqrt{\frac{\lambda_j}{\alpha}}\, x)}{\cosh(\sqrt{\frac{\lambda_j}{\alpha}}\, L)} \right\}$$

where

$$Z_i = \frac{\cos^2(\sqrt{\frac{\beta_i}{2\alpha}}\, x)\cosh^2(\sqrt{\frac{\beta_i}{2\alpha}}\, x) + \sin^2(\sqrt{\frac{\beta_i}{2\alpha}}\, x)\sinh^2(\sqrt{\frac{\beta_i}{2\alpha}}\, x)}{\cos^2(\sqrt{\frac{\beta_i}{2\alpha}}\, L)\cosh^2(\sqrt{\frac{\beta_i}{2\alpha}}\, L) + \sin^2(\sqrt{\frac{\beta_i}{2\alpha}}\, L)\sinh^2(\sqrt{\frac{\beta_i}{2\alpha}}\, L)}$$

and

$$\varphi_i = \tan^{-1} \frac{\sin(\sqrt{\frac{\beta_i}{2\alpha}}\, x)\sinh(\sqrt{\frac{\beta_i}{2\alpha}}\, x)}{\cos(\sqrt{\frac{\beta_i}{2\alpha}}\, L)\cosh(\sqrt{\frac{\beta_i}{2\alpha}}\, L)} - \tan^{-1} \frac{\sin(\sqrt{\frac{\beta_i}{2\alpha}}\, L)\sinh(\sqrt{\frac{\beta_i}{2\alpha}}\, L)}{\cos(\sqrt{\frac{\beta_i}{2\alpha}}\, L)\cosh(\sqrt{\frac{\beta_i}{2\alpha}}\, L)}$$

2.2.1 Temperature Distributions. Equations (15) and (16) of the previous section can be evaluated to obtain the temperature as a function of position and time in any unit cell of a reactor if the heat generation rate, $q(x,t)$, and the fuel surface temperature, $\theta_f(L,t)$, are known and can be expressed in the appropriate analytical form. The experimental values of these variables during applicable transient tests on the SPERT-I reactor system were obtained in graphical form from "Sub-cooled Transient Tests in the SPERT-I-A Reactor - Experimental Data" by J. C. Haire (25). Numberical power and temperature data were obtained from the graphs. These numerical data were normalized to a zero initial temperature then fit empirically by an even trigonometric series, $\sum_{i=1}^{p} A_i \cos \beta_i t$, for the temperature traces. The power traces were reduced to give the heat generation rate in the center of a central fuel element and moderator region and then fit empirically by an exponential series, $\sum_{j=1}^{s} A_j e^{\lambda_j t}$. The reduction of the power data to give the appropriate heat generation rate in the fuel region was accomplished in the following manner. The SPERT-I core contained 28 assemblies, 51 plates per assembly and an active volume of 7.523 cm^3 per plate. Therefore,

$$\bar{H}_{plate}(t) = \left(\frac{P(t) \times 10^6}{28 \text{ assemblies}}\right)\left(\frac{1 \text{ assembly}}{51 \text{ plates}}\right)\left(\frac{1 \text{ plate}}{7.523 \text{ cm}^3}\right)$$
$$= 97.78\, P(t) \text{ watts/cm}^3 \qquad (16)$$

was the average heat generation rate in an average fuel plate, where $P(t)$ is the total power in megawatts. Converting this to the required dimensions of cal/sec-cm^3 yielded $\bar{H}_{plate}(t) = 23.37\, P(t)$ cal/sec cm^3. The heat generation rate in the center of the fuel plate is found in terms of the average heat generation rate since the heat generation rate is proportional to the neutron flux distribution.

$$\bar{H}_{plate}(t) \doteq \frac{\int_o^L C\,\Phi\,(x)\ Adx}{\int_o^L Adx} = \frac{\int_o^L H_o \cosh \kappa x\ dx}{L} = \frac{H_o \sinh \kappa L}{\kappa\ L} \qquad (17)$$

where L is half-thickness of plate, 0.0254 cm,

and $\bar{H}_o$ is heat generation rate at center of the average fuel plate.

κ was determined from the neutron transport theory relationship for heavy absorbers (44),

$$\frac{\kappa}{\Sigma_{tot}} = \tanh \frac{\kappa}{\Sigma_s} \qquad (18)$$

to be 0.7973 cm^{-1} .

$$\kappa L/\sinh \kappa L \doteq \frac{0.02025}{0.02025} = 1.0 \qquad (19)$$

Therefore,

$$\bar{H}_{of}\ (t) = \bar{H}_{plate}(t) = 23.37\ P(t)\ cal/sec\ cm^3. \qquad (20)$$

The correction from the average fuel element to the one of interest, a central fuel element, required a maximum to average correction. Therefore,

$$H'_{of}(t) = \bar{H}_{of}(t)\ (\Phi_{max}/\bar{\Phi}) = 23.37\ P(t)\ (1.9) = 44.38\ P(t)\ cal/cm^3 sec. \qquad (21)$$

The final correction was to assume that approximately 5% of the power was generated in the moderator. Therefore,

$$H_{of}(t) = 0.95\ H'_{of}(t) = 42.16\ P(t)\ cal/cm^3 sec. \qquad (22)$$

The relation used to calculate the heat generation rate in the moderator was

$$q_m(t) = 0.05\ \bar{q}_f(t)\ \frac{V_f}{V_m} = 0.05\ \bar{q}_f(t)\ \frac{L_f}{L_m} = 0.05\ H_{of}(t)\ (\frac{0.0254}{0.071755}) = 0.017695\ H_{of}(t) \qquad (23)$$

where $\bar{q}_f$ is average heating rate in the fuel,

V_f is volume of the fuel,

V_m is volume of the moderator,

L_f is half-thickness of the fuel,

L_m is half-thickness of the moderator,

and q_m is the heat generation rate in the moderator.

The above equation assumes that there is a flat spatial distribution of the heating rate, that approximately 5% of the total heat generation takes place in the moderator and that the average heat generation rate in the fuel is well approximated by the heat generation rate in the center of the fuel.

As previously mentioned that data from the temperature traces were fit with a finite number of terms of a Fourier series of the form

$$\sum_{n=0}^{p} b_n \cos \frac{2\pi nt}{a} \tag{24}$$

where

$$b_o = \frac{2}{a} \int_o^a y\,(t)\,dt,$$

$$b_n = 1\ a \int_o^a y(t) \cos \frac{2\pi nt}{a}\,dt,$$

$y(t)$ = experimental temperature trace data,

and a = interval of periodicity.

The data reduced from the power traces, actually $H_{of}(t)$, were fitted with a finite number of terms of an exponential function of the form,

$$\sum_{j=1}^{s} A_j\ e^{\lambda_j t}, \tag{25}$$

where $\underline{A_i}$ and $\underline{\lambda_i}$ were parameters which were determined by trial and error to give the best fit. The best fit parameters were determined by means of an IBM-650 computer program described in Appendix B. This program resulted from a very minor modification of one written by L. R. Foulke (17). The data for $H_{of}(t)$ and the approximating equations are shown in Figures 8 through 11.

The thermal, nuclear and geometric constants used in determining the temperature distributions are given in Table 1.

Table 1. Constants Used to Evaluate the Temperature Distributions

Constant	Fuel	Moderator
α, Thermal Diffusivity, cm^2/sec	0.82	0.001512
κ, Inverse Diffusion Length, cm^{-1}	0.7973	0.0
L, Half-thickness of Region, cm	0.0254	0.071755
k, Thermal Conductivity, cal/cm sec ^oC	0.5002	0.001488

<u>2.2.2 Surface Heat Flow</u>. The heat flow rate out of the fuel and into the moderator as a function of time was evaluated by forming the partial derivative with respect to position evaluating it at the outside of the respective region and multiplying by the respective thermal conductivity. The heat flow out of the fuel and into the moderator are, respectively,

$$(q/A)_f(t) = -k_f \left. \frac{\partial \theta_f(x,t)}{\partial x} \right|_{x=L} \tag{26}$$

and

$$(q/A)_m(t) = -k_m \left. \frac{\partial \theta_m(x,t)}{\partial x} \right|_{x=L}, \tag{27}$$

Evaluating the above equations yields in the fuel

$$(q/A)_f(t) = -k_f\left\{\sum_{i=1}^{p}\left(\sqrt{\frac{\beta_i}{2\alpha}}\right)B_i\ (D_i \cos\beta_i t + E_i \sin\beta_i t)\right.$$

$$+\sum_{n=1,3,5,\cdots(L^2/n\pi\alpha)}^{\infty}\frac{\left(\frac{n\pi}{2L}\right)e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{}\left[\sum_{i=1}^{p}\frac{B_i\left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4}+\beta_i^2}+\sum_{j=1}^{s}\frac{q_{\infty}\ \alpha\ A_j\ \cosh\kappa L}{k_f\left(\frac{n^2\pi^2\alpha}{4L^2}+\lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2}+\alpha\kappa^2\right)}\right]$$

$$\left.+\sum_{j=1}^{s}\frac{q_{\infty}\alpha\ A_j\ e^{\lambda_j t}}{k_f\ (\alpha\kappa^2-\lambda_j)}\left[\frac{\sqrt{\frac{\lambda_j}{\alpha}}\cosh(\kappa L)\sinh\left(\sqrt{\frac{\lambda_j}{\alpha}}L\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}L\right)}-\kappa\sinh\kappa L\right]\right\}$$

and in the moderator

$$(q/A)_m(t) = -k_m\left\{\sum_{i=1}^{p}\left(\sqrt{\frac{\beta_i}{2\alpha}}\right)B_i\ (D_i \cos\beta_i t + E_i \sin\beta_i t)\right.$$

$$+\sum_{n=1,3,5,\cdots L^2/n\pi\alpha}^{\infty}\frac{\left(\frac{n\pi}{2L}\right)e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{}\left[\sum_{j=1}^{s}\frac{\alpha\ F\ A_j}{k_m\left(\frac{n^2\pi^2\alpha}{4L^2}+\lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2}\right)}+\sum_{i=1}^{p}\frac{B_i\left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4}+\beta_i^2}\right]$$

$$\left.-\sum_{j=1}^{s}\frac{\alpha\ F\ A_j}{k_m\ \lambda_j}\left(\frac{\sqrt{\frac{\lambda_j}{\alpha}}\sinh\left(\sqrt{\frac{\lambda_j}{\alpha}}L\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}L\right)}\right)\right\},$$

where
$$D_i = \frac{\cosh\left(\sqrt{\frac{\beta_i}{2\alpha}}L\right)\sinh\left(\sqrt{\frac{\beta_i}{2\alpha}}L\right)-\cos\left(\sqrt{\frac{\beta_i}{2\alpha}}L\right)\sin\left(\sqrt{\frac{\beta_i}{2\alpha}}L\right)}{\cos^2\left(\sqrt{\frac{\beta_i}{2\alpha}}L\right)\cosh^2\left(\sqrt{\frac{\beta_i}{2\alpha}}L\right)+\sin^2\left(\sqrt{\frac{\beta_i}{2\alpha}}L\right)\sinh^2\left(\sqrt{\frac{\beta_i}{2\alpha}}L\right)}$$

$$\text{and } E_1 = \frac{\cosh(\sqrt{\frac{\beta_1}{2\alpha}}L)\sinh(\sqrt{\frac{\beta_1}{2\alpha}}L) + \cos(\sqrt{\frac{\beta_1}{2\alpha}}L)\sin(\sqrt{\frac{\beta_1}{2\alpha}}L)}{\cos^2(\sqrt{\frac{\beta_1}{2\alpha}}L)\cosh^2(\sqrt{\frac{\beta_1}{2\alpha}}L) + \sin^2(\sqrt{\frac{\beta_1}{2\alpha}}L)\sinh^2(\sqrt{\frac{\beta_1}{2\alpha}}L)} .$$

2.3 Reactivity Effects Due to Temperature Coefficient and Fuel Expansion

It is pointed out by Deverall and Griffing (9) that the temperature rise in the moderator in the central unit cell cannot be used directly to determine reactivity changes. "Since the temperature coefficient of reactivity, α, was determined under conditions of a uniform temperature throughout the core - a condition that does not exist in a transient - it is necessary to define a properly weighted average temperature. This average temperature would then produce the same change in reactivity as if an actual uniform temperature change of this amount had been made. This average is defined by

$$\overline{\Delta T} = \frac{\int I(\vec{x})\,\Delta T(\vec{x})\,d\vec{x}}{\int I(\vec{x})\,d\vec{x}} \tag{30}$$

where

$\Delta T(\vec{x})$ is the change of temperature at position $\vec{x}$,

$I(\vec{x})$ is the statistical importance at position $\vec{x}$,

and the integration is carried out over the whole volume of the reactor."

The authors also show that for SPERT I-A (17/28) core, the system under consideration,

$$\frac{\overline{\Delta T}}{\Delta T_{max}} = 0.65 \tag{31}$$

Therefore,

$$\Delta k = 0.65\,\alpha(T)\,\Delta T_{max}. \tag{32}$$

A value for α (t) of 0.9 x 10^{-4} $(\Delta k/C^{o})$ was used yielding

$$\Delta k = (-5.85 \times 10^{-5} / C^{o}) \Delta T_{max}. \qquad (33)$$

A similar problem was faced by Forbes (13). In determining the reactivity effect due to fuel plate expansion he stated,

"In order to obtain the reactivity change, the temperature distribution and void importance function in the core must be combined to obtain the dynamic reactivity coefficient as opposed to the static coefficient which applies only to uniform void distributions. Applying the observed distribution functions for temperature and void worth, it is found that the effective average temperature rise under dynamic conditions can be obtained from the temperature rise at the center of the core by the relation

$$\Delta\theta = 0.7\ \Delta\theta_{max}. \qquad (34)$$

The reactivity change due to plate expansion, Δk_1, will be

$$\Delta k_1 = \overline{\left(\frac{\partial k}{\partial v}\right)}\ 3\ a\ v\ (0.7\ \Delta\theta_{max}), \qquad (35)$$

where $\overline{\left(\frac{\partial k}{\partial v}\right)}$ is the average void coefficient for the core,

a is the linear expansion coefficient of aluminum,

v is the volume of the aluminum which is heated (i.e., the volume of fuel plates proper),

$\overline{\Delta\theta}$" is the average temperature rise of the aluminum

and $\Delta\theta_{max}$ is the temperature rise at center of the core.

For the SPERT I-A (17/28) core the appropriate constants are the following:

$$a = 2.5 \times 10^{-5}/^{o}C$$

$$v = 2.8 \times 10^4 \text{ cm}^3$$

$$\overline{\left(\frac{\partial k}{\partial v}\right)} = -3.5 \times 10^{-6} \ \Delta k/\text{cm}^3$$

Therefore, the expression for the reactivity change becomes

$$\Delta k_1 = (-5 \times 10^{-6} \frac{\Delta k}{^{o}C}) \ (\Delta\theta_{max})." \tag{36}$$

In order to avoid erroneously taking into account the void formation due to fuel element expansion in both the temperature coefficient and in the fuel element expansion calculation, the fuel element expansion was calculated only for the temperature rise in the fuel over the temperature rise in the moderator. Therefore, the reactivity effects due to the temperature coefficient, Δk_T, and due to the fuel plate expansion, Δk_E, are

$$\Delta k_T(t) = -5.85 \times 10^{-5} \ (\Delta k/^{o}C) \ \bar{\theta}_{mod} \ (t) \tag{37}$$

and

$$\Delta k_E(t) = -5 \times 10^{-6} \ (\Delta k/^{o}C) \ (\bar{\theta}_{fuel}(t) - \bar{\theta}_{mod} \ (t)) \tag{38}$$

where $\bar{\theta}_{mod}$ (t) is the average temperature rise in the moderator at time t

$\bar{\theta}_{fuel}(t)$ is the average temperature rise in the fuel at time t as obtained from Tables 2 through 5.

2.4 Reactivity Effects Due to Steam Formation

The calculation of the steam production was based on the same assumptions as those used in the "Conduction Boiling Model for Reactor Self-Shutdown" suggested by S. G. Forbes (15). In Forbes' work the steam volume, V_s, was assumed to be proportional to a fraction, f_{as}, of the energy, E_s, transferred to the moderator after the time boiling first occurred in the core. The steam volume is given by

$$V_s = \frac{f_{as} E_s}{h_s} , \tag{39}$$

where h_s is the energy required to form a unit volume of steam from boiling water at standard pressure (1.35 watt-sec/cm^3 steam). The term f_{as} was regarded as a combination of factors involving the fraction of the energy actually forming steam during nucleate boiling (about 1%) and the fraction of the core heat transfer area, A, which is involved in boiling heat transfer (about 10%) (15). The reactivity effect of the steam is

$$\Delta k_s = V_s C_v = \frac{f_{as} E_s}{h_s} C_v , \tag{40}$$

where C_v is the void coefficient in the center of the core, (C_v = -7.2 x 10^{-6} $\Delta k/cm^3$ for Spert I-A). In this investigation the factor f_{as} was divided into its two components, the fraction of the core involved in boiling, f_a, and the fraction of the energy actually forming steam during nucleate boiling, f_s. This was done since it was possible to approximate f_a directly from the fuel surface temperature and the assumption that the gross temperature distribution over the core was proportional to the bare core power distribution. It is expected that the final factor, f_s, will be independent of the pulse parameters for a particular system and that it will prove essentially independent of the reactor parameters in any heterogeneous water moderated system. In effect, the final parameter, f_s, was left to be calibrated by any particular pulse. The test of this model was of course a constant f_s. For the two boiling runs considered, the values of f_s calculated were 3.7 x 10^{-3} and 3.3 x 10^{-3} which differ by less than 2%. The 2% difference is less than would be expected for the accuracy of the input data. The reactivity effect of the steam is then

$$\Delta k_s(t) = \left(\frac{-7.28 \times 10^{-6}}{1.35 \times 10^{-6}} \Delta k/\text{MW-sec}\right) (f_a f_s \; E_s(t)) \tag{41}$$

$$= -1.87 \times 10^{-2} \; f_a E_s(t)$$

f_a is fraction of core heat transfer area involved in boiling,

f_s is fraction of energy actually forming steam,

and $E_s(t)$ is total energy into core after initial boiling in MW-sec.

The fraction f_a was calculated assuming the gross core temperature distribution had reached a dynamic equilibrium with the power and the power distribution could be approximated by that of an equivalent bare core with an effective height, $2Z_e$, and radius, R_e. The gross temperature distribution in the core is then

$$\theta\ (r,Z) = \theta_o \; J_o\left(\frac{2.4048\ r}{R_e}\right) \cos \frac{\pi Z}{2Z_e} \tag{42}$$

where θ_o is the surface temperature at the time of interest on the axial center line of a central fuel element. The maximum value of Z, Z_{max}, for which boiling will occur on any plate can then be obtained knowing the boiling temperature, θ_b, and the axial centerline surface temperature,

$$\theta_o \; J_o\left(\frac{2.4048r}{R_e}\right).$$

That is

$$\cos \frac{\pi Z_{max}}{2\ Z_e} = \frac{\theta_b}{\theta_o \; J_o\left(\frac{2.4048r}{R_e}\right)} \tag{43}$$

so that

$$\frac{Z_{max}}{Z_e} = \frac{2}{\pi} \quad \cos^{-1} \left(\frac{\theta_b}{\theta_o \; J_o(\frac{2.4048r}{R_e})} \right) \quad .$$

The maximum value of r for which boiling will occur, r_{max}, on the core axial centerline can be obtained from

$$\theta_b = \theta_o \quad J_o \left(\frac{2.4048 r_{max}}{R_e} \right)$$

so that (44)

$$\frac{r_{max}}{R_e} = \frac{1}{2.4048} \quad J_o \left(\frac{\theta_b}{\theta_o} \right)$$

where $y = J_o^{-1}(x)$ is the inverse of $x = J_o(y)$.

The volume fraction of the core having a fuel surface temperature above the boiling temperature is

$$f_v = \frac{1}{\pi R_e^2 \, (2Z_e)} \int_o^{r_{max}} 2 \, Z_{max} \; 2\pi r dr$$

$$= \frac{2}{R_e^{\;2}} \int_o^{r_{max}} \frac{Z_{max}}{Z_e} \quad rdr$$

Making the change in variable $\eta = r/R_e$, the integral becomes

$$f_v = 2 \int_o^{\eta_{max}} \frac{Z_{max}(\eta)}{Z_e} \quad \eta d\eta \; . \tag{46}$$

Substituting the value of Z_{max} / Z_e from equation (43) yields

$$f_v = 4/\pi \int_o^{\eta_{max}} \cos^{-1} \frac{\theta_b}{\theta_\infty J_o(2.4048 \eta)} \quad \eta d\eta \; . \tag{47}$$

This integration was then carried out numerically using the value of $\eta_{max} = r_{max} / R_e$ determined from equation (44). Since there is a constant heat transfer area per unit volume in the core then $\underline{f_v} = \underline{f_a}$. For the two boiling runs $\underline{\tau}$ = 15.8 msec and 23 msec, $\underline{f_a}$ was equal to 0.116 and 0.084, respectively.

The total energy transferred to the moderator after the time of initial boiling was obtained by considering the moderator volume associated with unit surface area in the central fuel element. The heat content of the moderator at the time boiling temperatures were reached at the surface and at the time of peak power were calculated based on the conduction model. While this model gave a somewhat erroneous temperature distribution above boiling temperatures it accurately represented the heat flow into the moderator. The difference between the moderator heat content at the time of interest and at the time boiling temperatures were first reached was the energy available for boiling per unit fuel surface area. Plots of the central fuel surface temperature and the average moderator temperature in a central element used to calculate the moderator heat contents are shown in Figures 2 and 3. The total energy available for steam formation is obtained by multiplying by the total heat transfer surface area of the core. This somewhat overestimates the total energy but is probably the best estimate of the energy of interest since the boiling region is confined to a rather small central portion of the core. Therefore

$$E_s(t) = (\overline{\theta}_m(t) - \overline{\theta}_m(t_b))\, C_p M$$

where C_p is the heat capacity of the moderator
M is the mass of the moderator in the system
$\overline{\theta}_m(t)$ is the average moderator

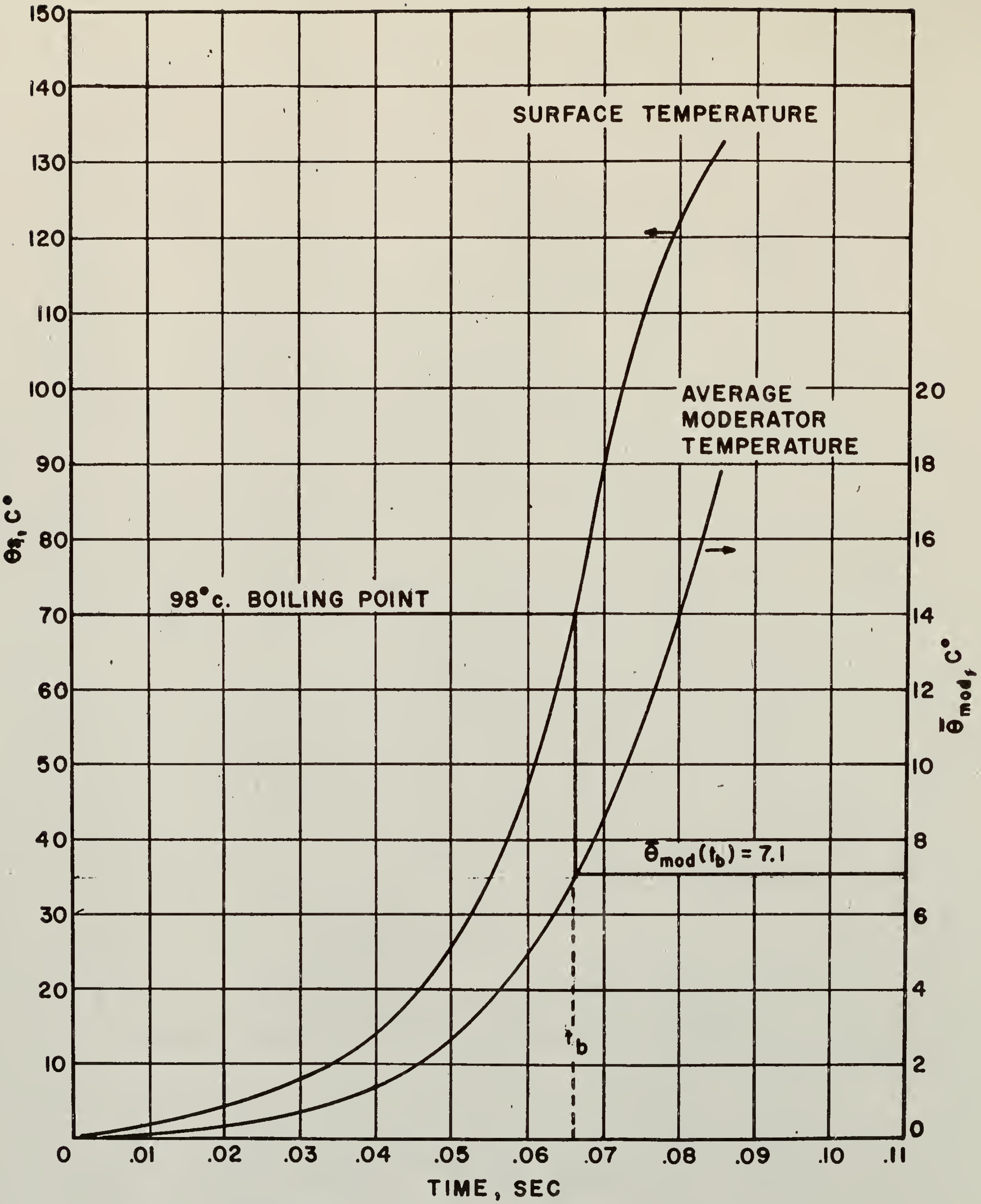


Figure 2. Graph used to determine the moderator energy for boiling calculations during a transient with an initial period of 15.8 msec.

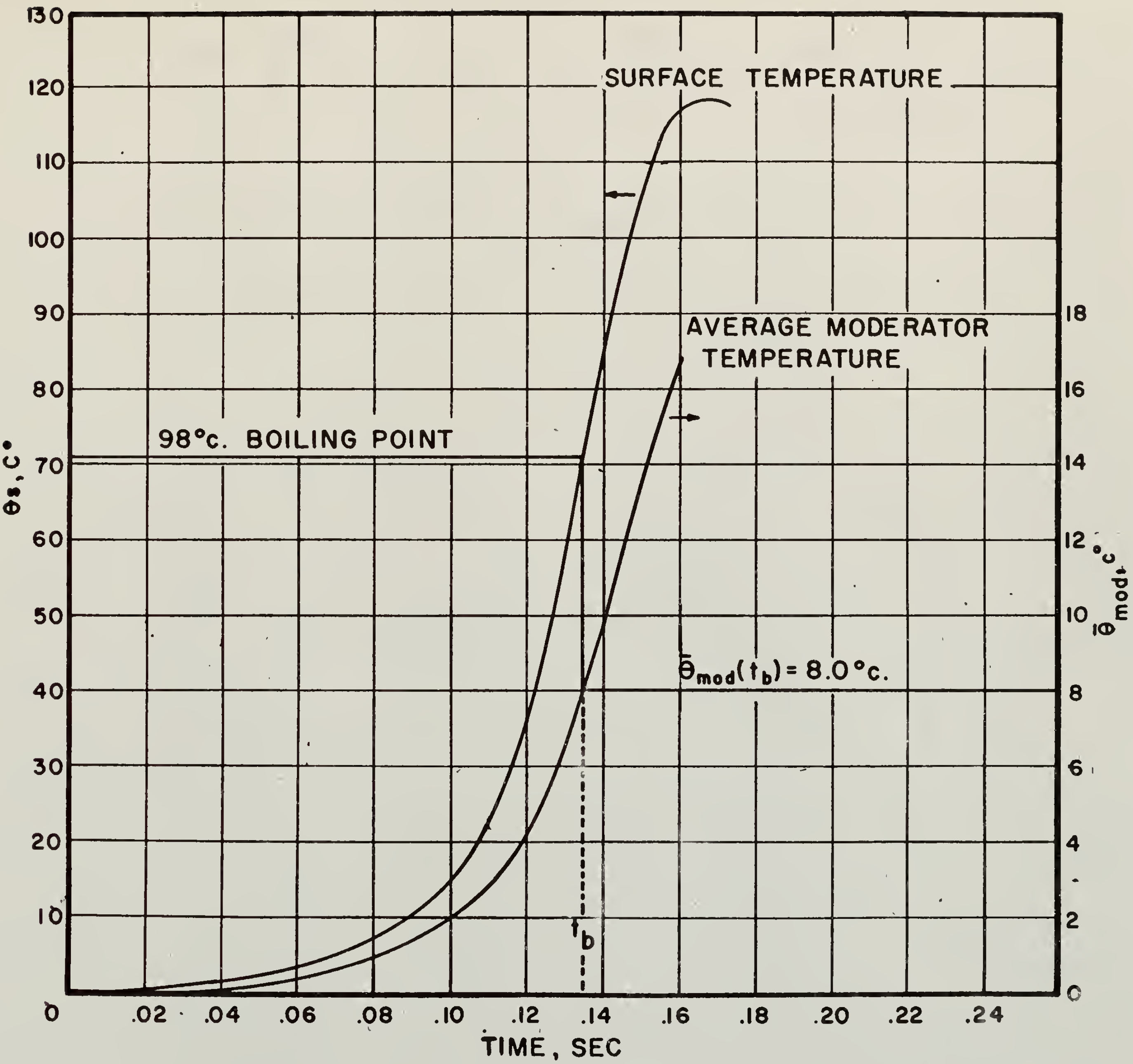


Figure 3. Graph used to determine the moderator energy content for boiling calculations during a transient with an initial period of 23 msec.

3.0 RESULTS AND DISCUSSION

3.1 Temperature Distribution and Surface Heat Flow

The temperature distributions as obtained from equations 15 and 16 are shown in Figures 4 through 7. These plots were obtained using experimental data from four transient tests on the SPERT I-A (17/28) reactor having initial periods, τ, of 15.8, 23, 120, and 150 msec, respectively. The transient burst for τ equal 15.8 and 23 msec show regions in the moderator which have temperatures above the boiling point at a pressure of one atmosphere. It is not believed that this superheating takes place. These temperature distributions are shown since such a small portion of the moderator is above the saturation point that it is not likely that it will materially affect the temperature in the remainder of the moderator or the average moderator temperature. One possibility which must be considered in calculating the reactivity effects if that pressure transients are developed which raise the boiling point above the temperatures observed in the core. This appears not to be the case for two reasons. First, experimental measurements of the pressures do not indicate sufficient rises in pressure and second, the total reactivity compensations at peak power indicate that boiling must have taken place.

The experimental surface temperature traces and the approximate analytical fits for the four transient tests are shown in Figures 8 through 11. The parameters for the analytical fits, $\theta(L,t) = \sum_{i=1}^{p} B_i \cos \beta_i t$,

are shown in Table 2. The experimental power traces, actually $H_{of}(t)$, and the approximate analytical fits for the same four transient tests

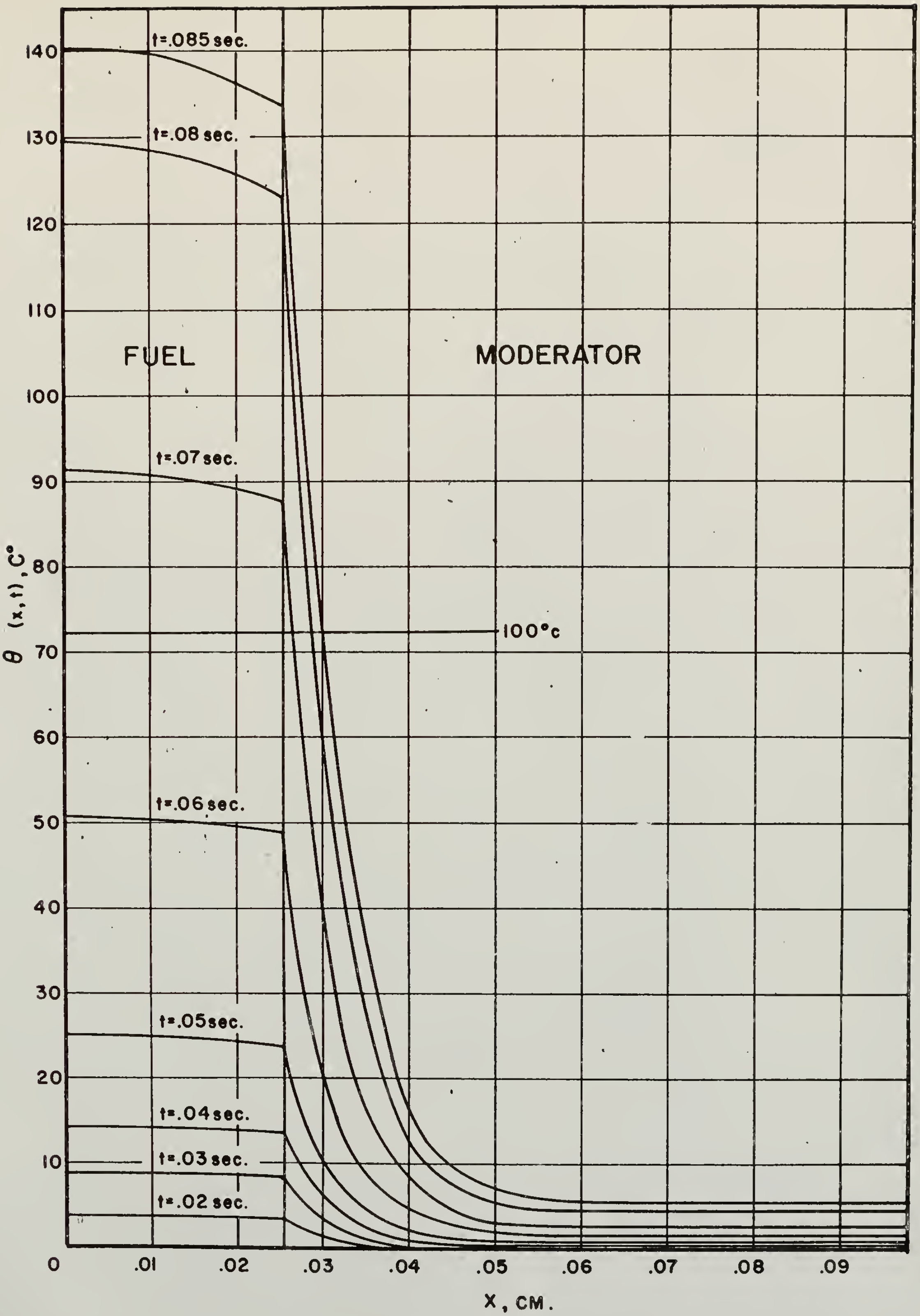


Figure 4. Temperature distributions, $\theta(x,t)$, vs position in fuel and moderator based on pure conduction during a transient with an initial period of 15.8 msec.

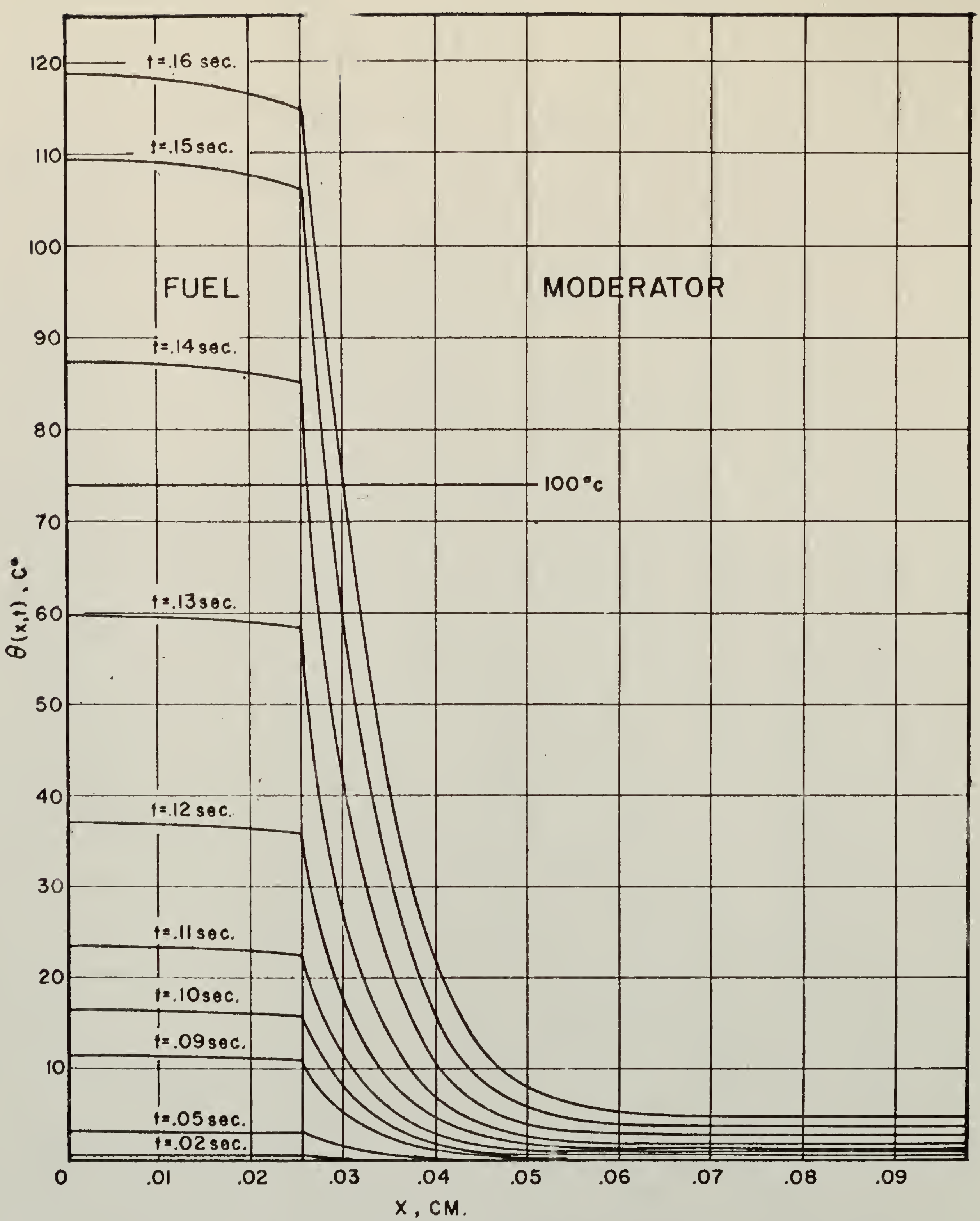


Figure 5. Temperature distributions, $\theta(x,t)$, vs position in fuel and moderator based on pure conduction during a transient with an initial period of 23 msec.

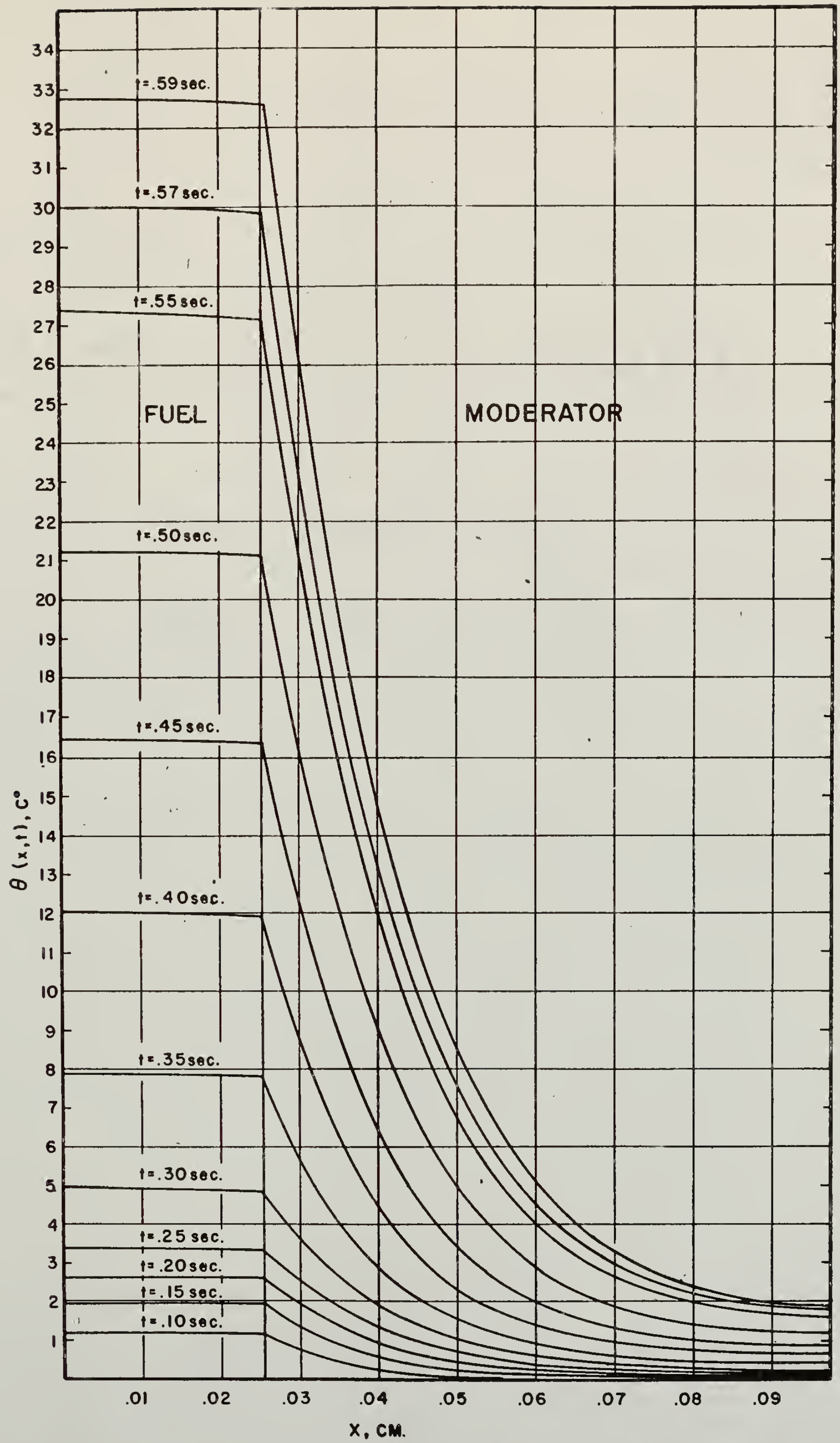


Figure 6. Temperature distributions, $\theta(x,t)$, vs position in fuel and moderator based on pure conduction during a transient with an initial period of 120 msec.

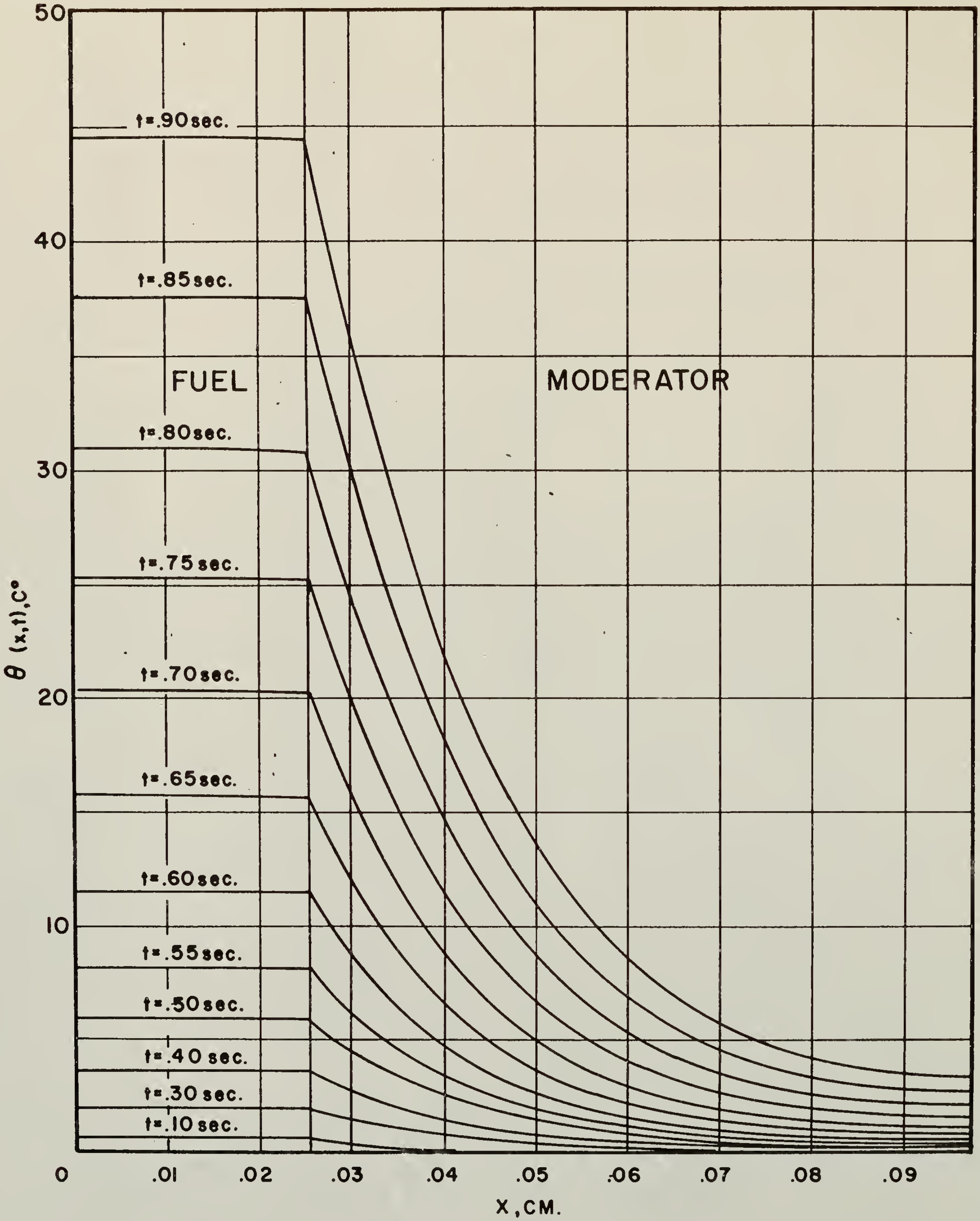


Figure 7. Temperature distributions, θ (x,t), vs position in fuel and moderator based on pure conduction during a transient with an initial period of 150 msec.

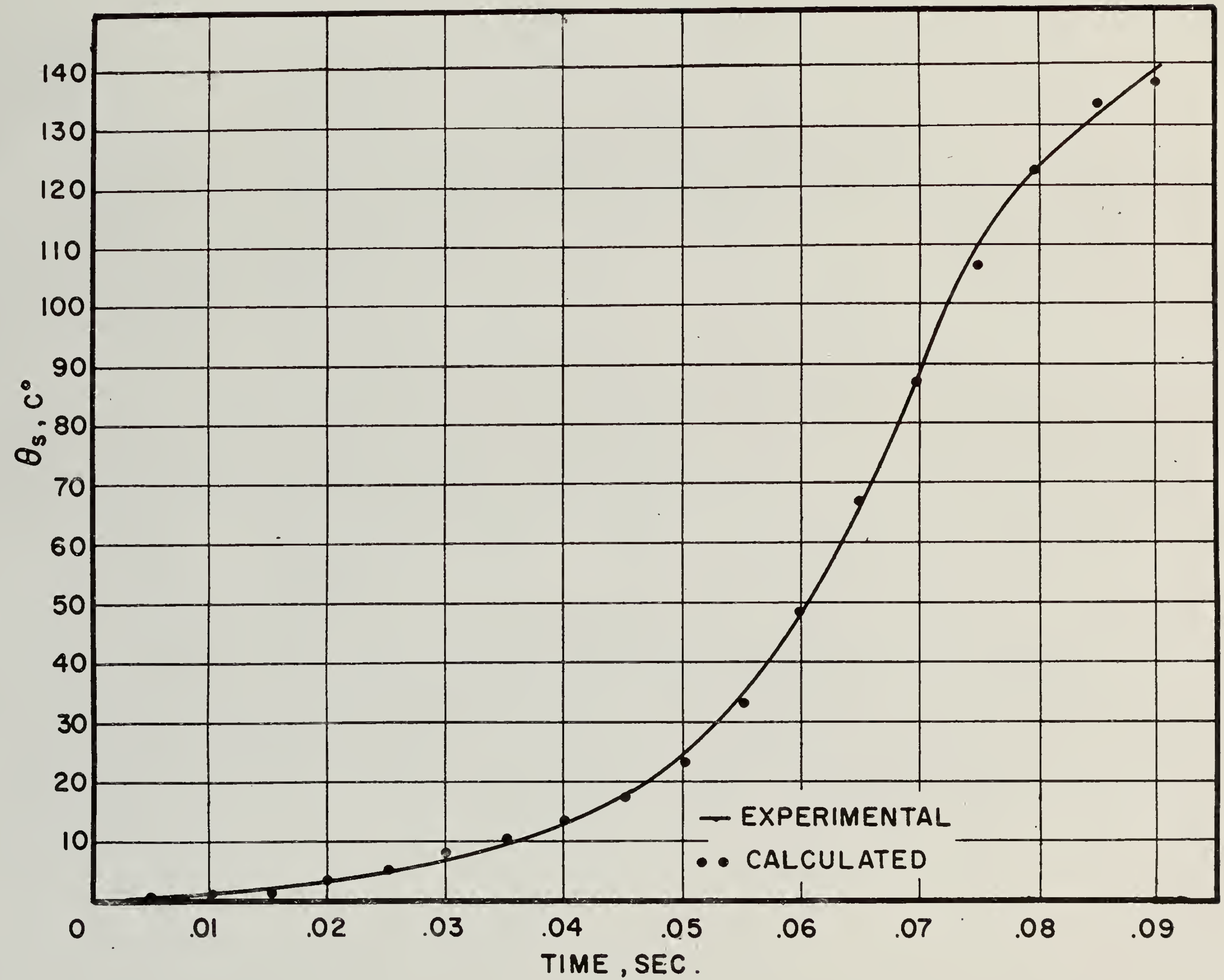


Figure 8. Interface temperatures, θ_s, vs arbitrary time during a transient with an initial period of 15.8 msec.

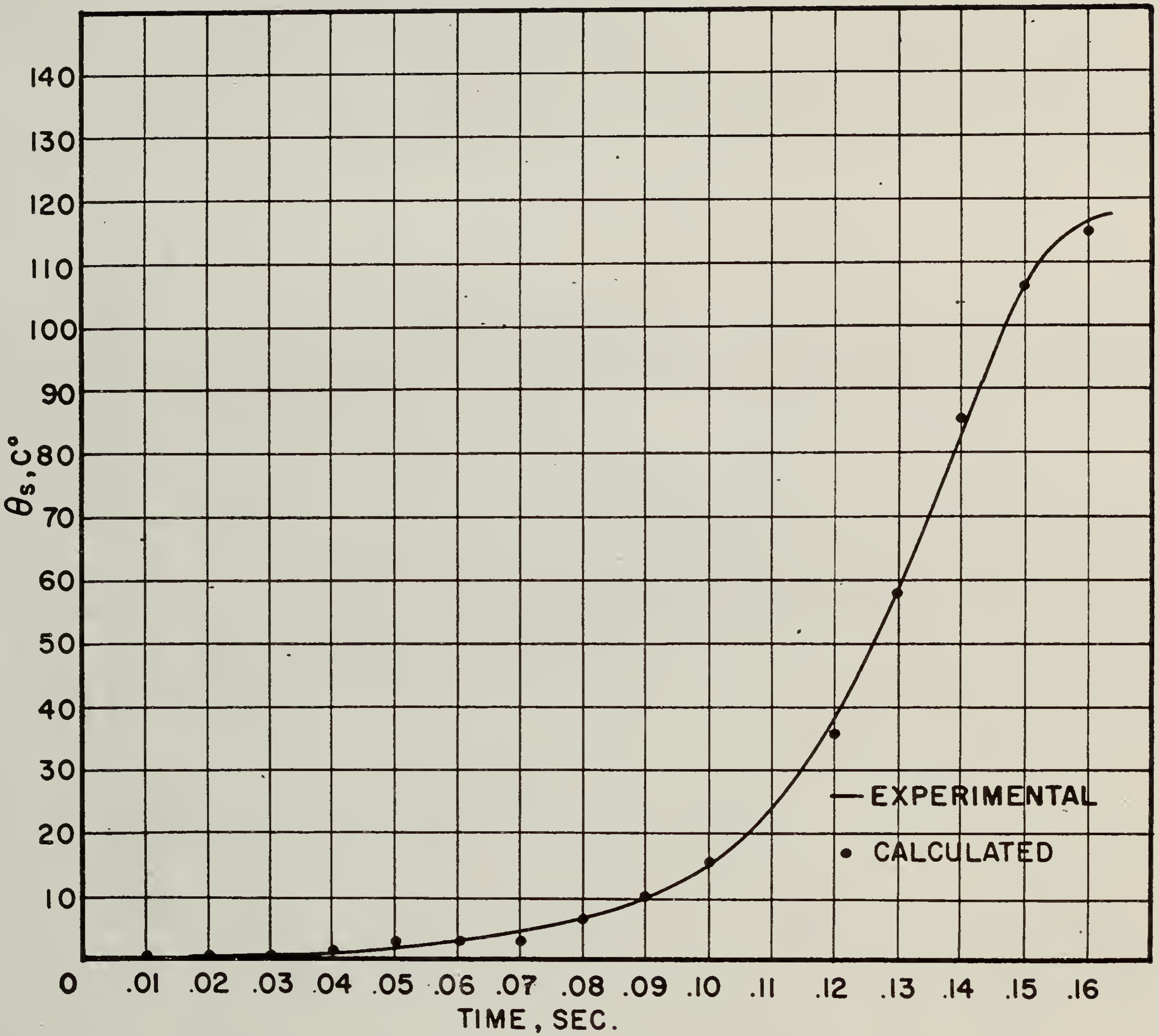


Figure 9. Interface temperatures, θ_s, vs arbitrary time during a transient with an initial period of 23 msec.

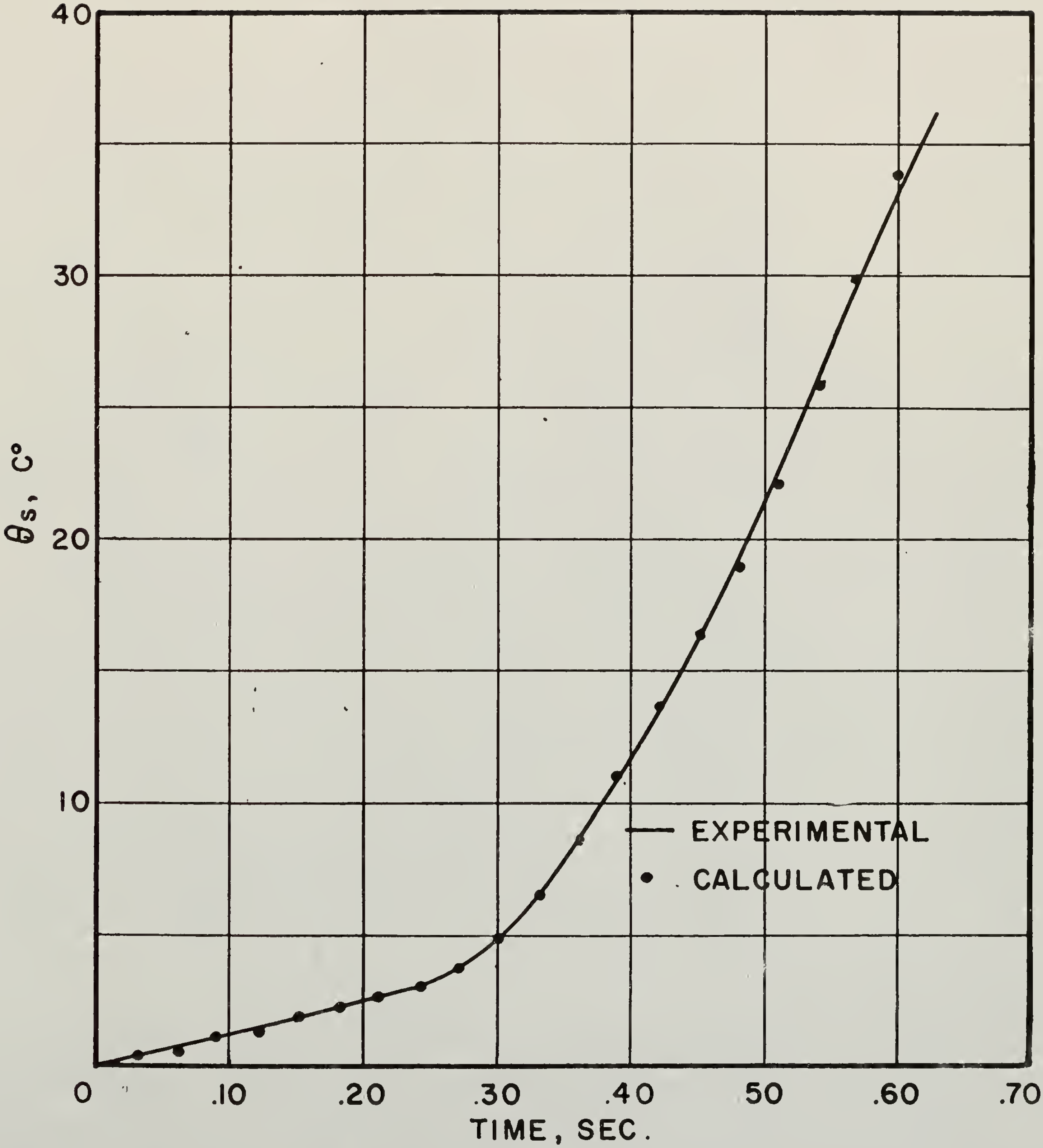


Figure 10. Interface temperatures, θ_s, vs arbitrary time during a transient with an initial period of 120 msec.

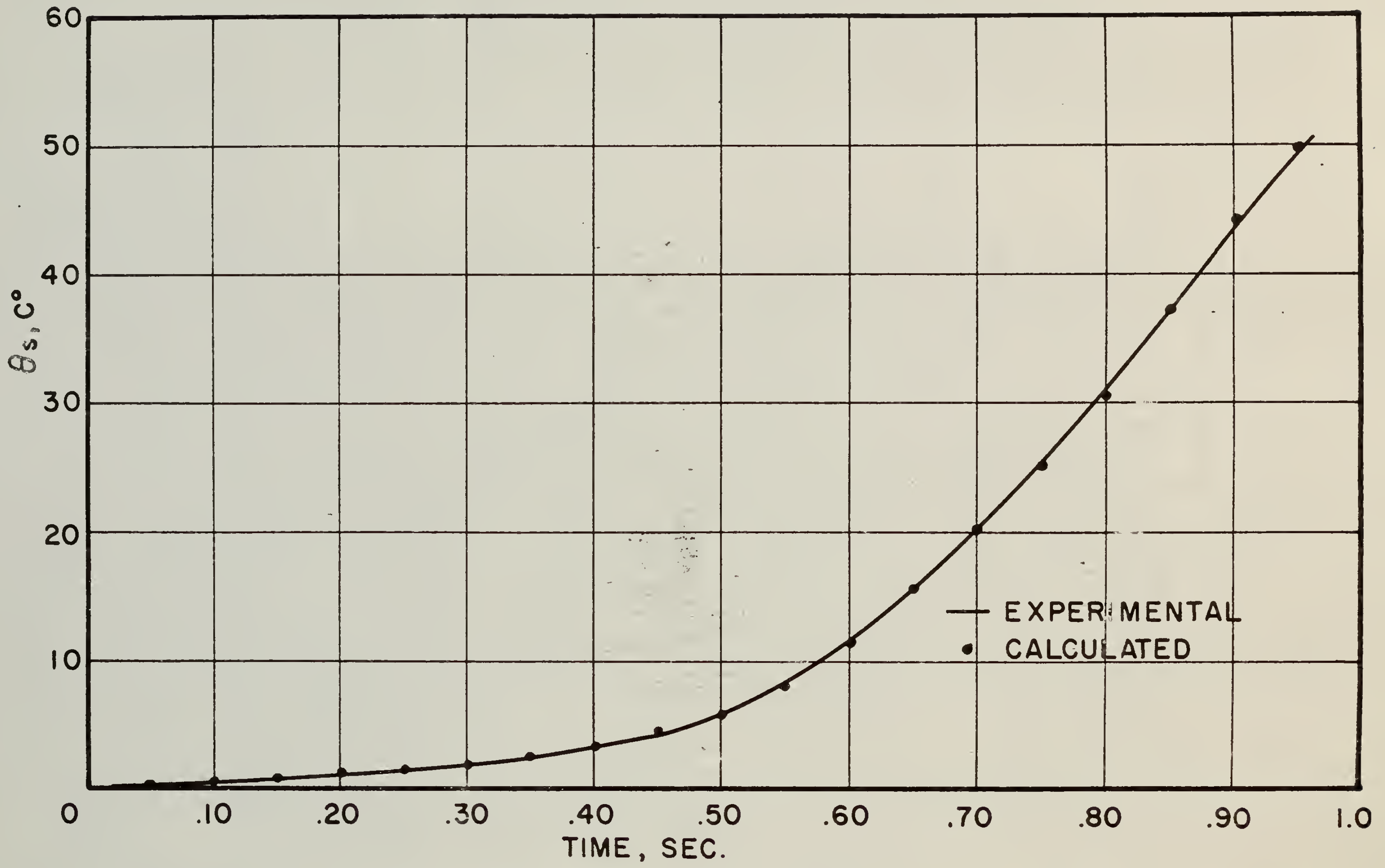


Figure 11. Interface temperatures, θ_S, vs arbitrary time during a transient with an initial period of 150 msec.

Table 2. Numerical Values of Parameters for Empirical Fits of $\theta(L,t)$ Used in Equations (15) and (16)

τ, msec	B_1 / β_1	B_2 / β_2	B_3 / β_3	B_4 / β_4	B_5 / β_5	B_6 / β_6	B_7 / β_7	B_8 / β_8
150	+14.593 / 0.00	-20.958 / 3.1416	+9.474 / 6.2832	-3.763 / 9.4279	+1.313 / 12.566	-0.871 / 15.708	+0.582 / 18.850	-0.447 / 21.991
120	+12.033 / 0.00	-15.831 / 4.760	+5.875 / 9.520	-2.067 / 14.280	+6.817 / 19.040	-0.829 / 23.800	+0.399 / 28.560	-0.100 / 33.320
23	+35.752 / 0.00	-54.937 / 17.405	+26.246 / 34.810	-7.248 / 52.215	-2.118 / 69.620	+4.476 / 87.024	-3.802 / 104.429	+2.413 / 121.834
15.8	42.074 / 0.00	-59.957 / 34.906	+25.821 / 69.812	-9.279 / 104.718	+1.094 / 139.624	0.838 / 174.530	-- / --	-- / --

are shown in Figures 12 through 15. The parameters for the analytical fits, $H_{of}(t) = \sum_{j=1}^{s} A_j e^{\lambda_j t}$ are shown in Table 3.

One physical check on the solutions not required by the mathematical formulation of the problem is that the heat flow out of the fuel must equal the heat flow into the moderator. The heat flow data are shown in Figures 16 through 19. It is obvious from inspection of these data that the equivalent heat flow condition is not well satisfied. There are several possible explanations for this discrepancy. First, the heat flow data is somewhat more sensitive to the analytical fits of the surface temperature and the heat generation rates than the average temperatures. Second, the cladding between the meat and the moderator was neglected in calculating the temperature distributions. Again, the average temperatures are far less sensitive to this approximation than the heat flow calculations. Finally, it can be seen by investigating the heat flow equations that the discrepancies could be decreased by introducing a positive phase angle to the surface temperature fits. This could be attributed to a delay time in the surface temperature measurements.

The average temperatures in the fuel and moderator as a function of time were calculated by means of a numerical integration of the calculated temperature distributions. These data are given in Tables 4 through 7.

3.2 Reactivity Effects

The reactivity compensations

$$\Delta k_c(t) = \Delta k(o) - \Delta k(t)$$

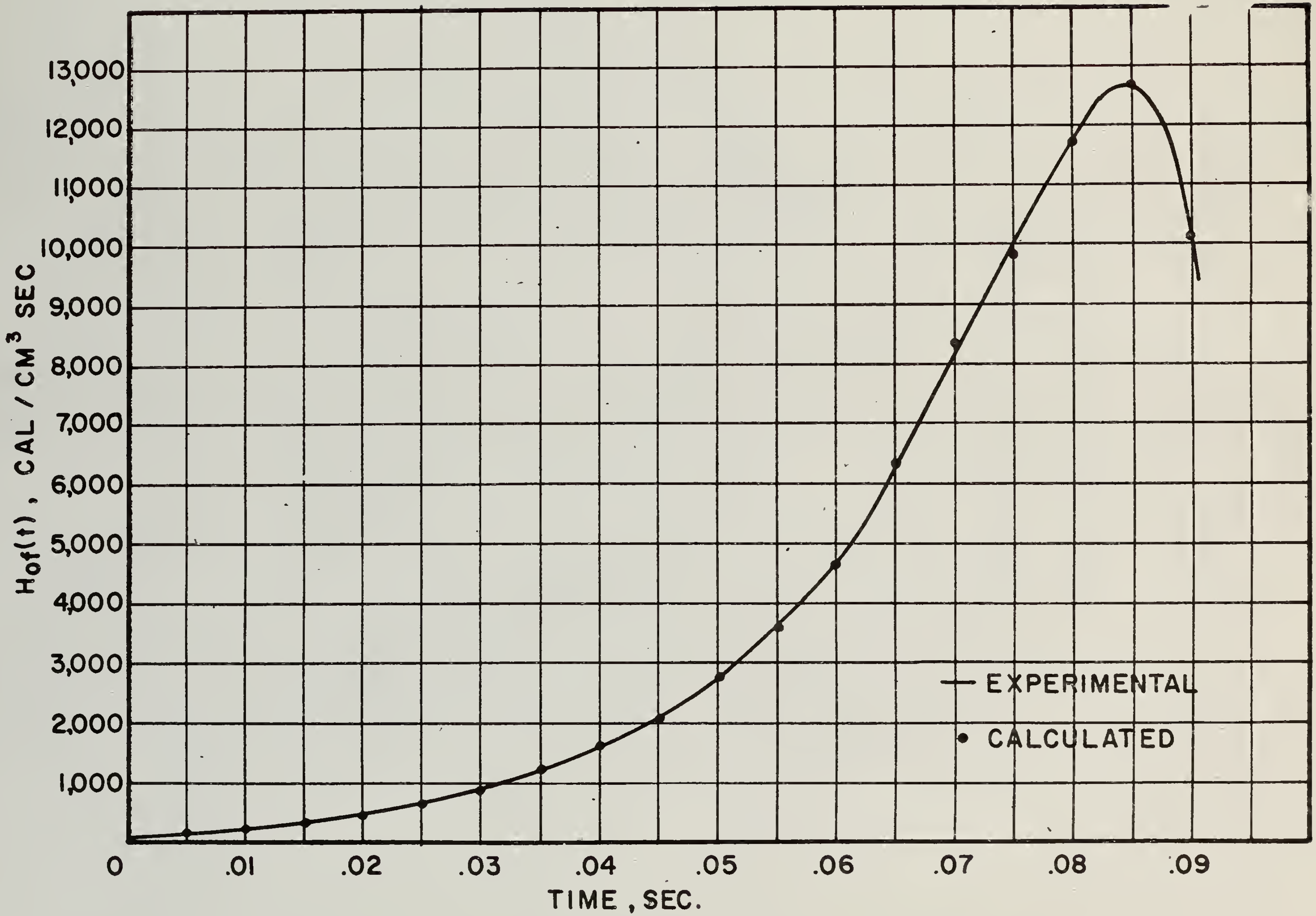


Figure 12. Internal plate heat generation rate, $H_{of}(t)$, vs arbitrary time during a transient with an initial period of 15.8 msec.

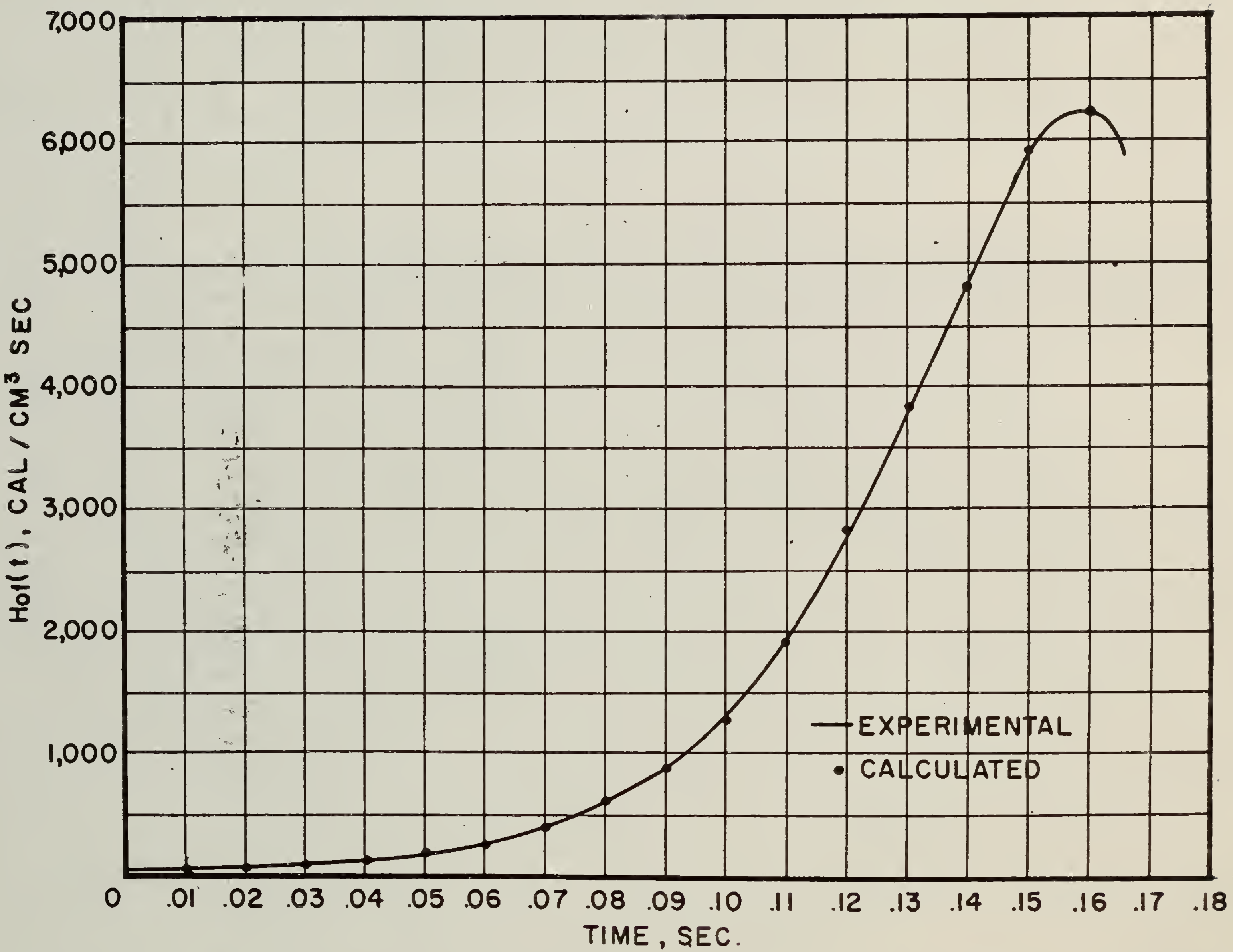


Figure 13. Internal plate heat generation rate, $H_{of}(t)$, vs arbitrary time during a transient with an initial period of 23 msec.

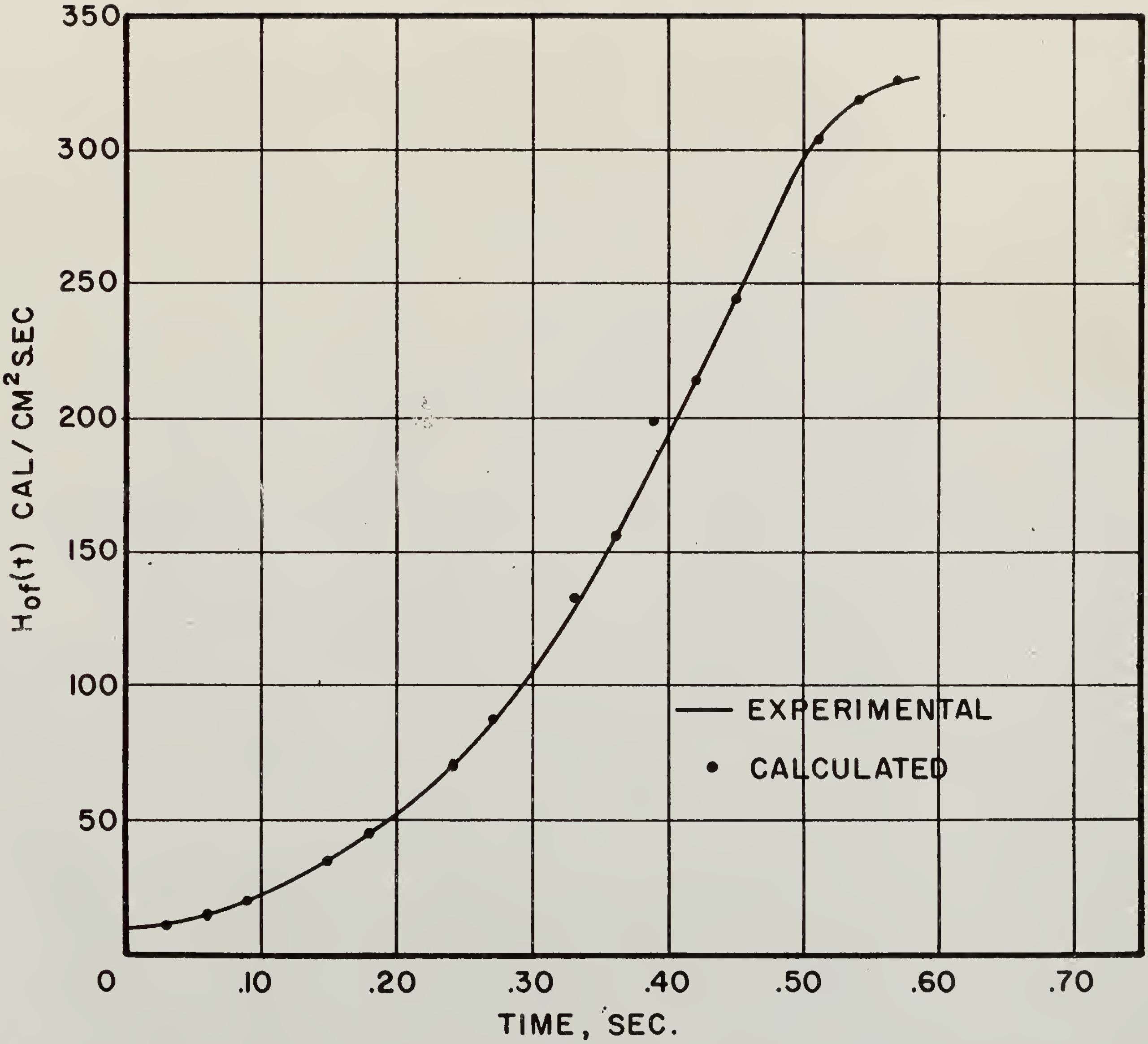


Figure 14. Internal plate heat generation rate, $H_{of}(t)$, vs arbitrary time during a transient with an initial period of 120 msec.

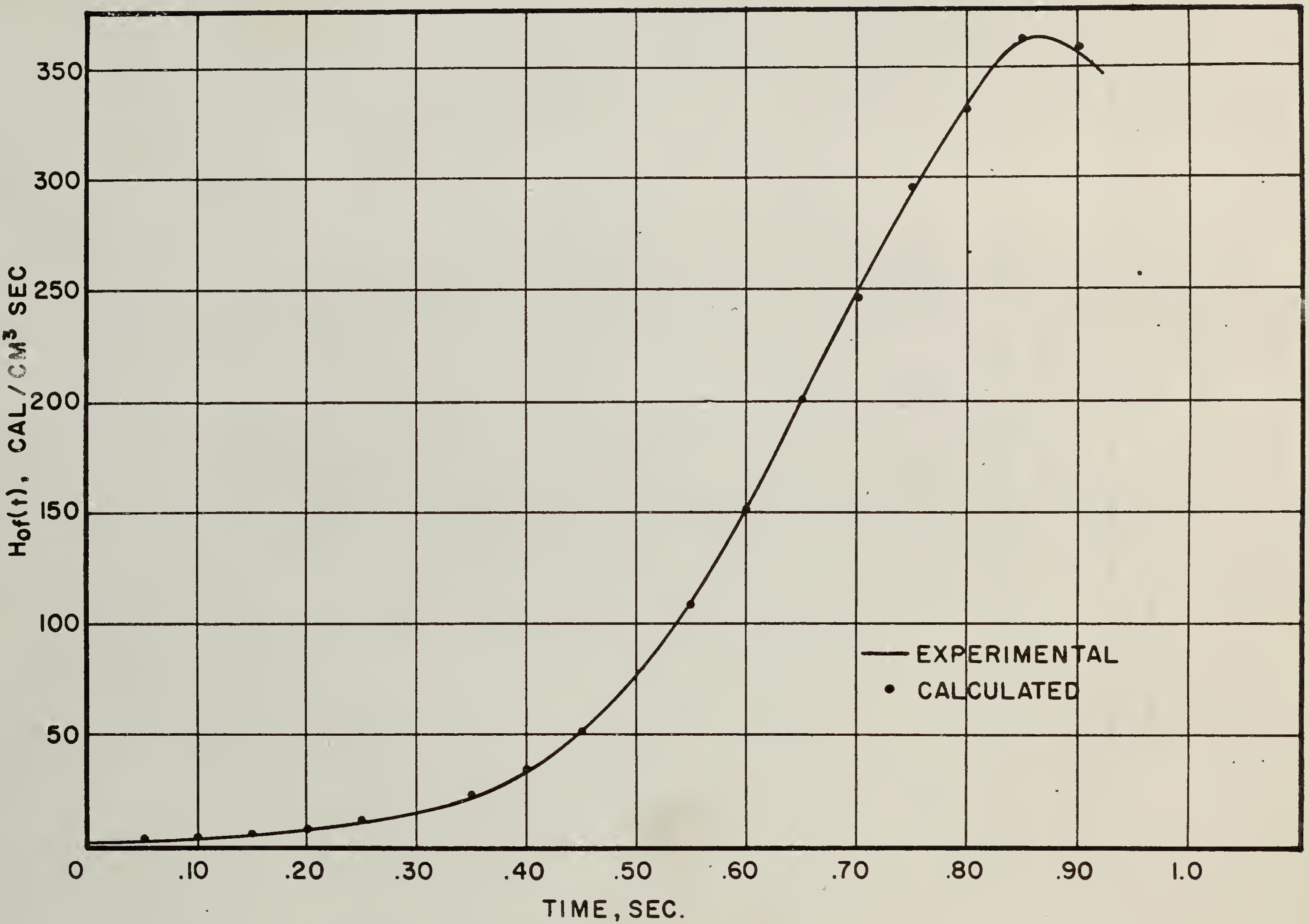


Figure 15. Internal plate heat generation rate, $H_{of}(t)$, vs arbitrary time during a transient with an initial period of 150 msec.

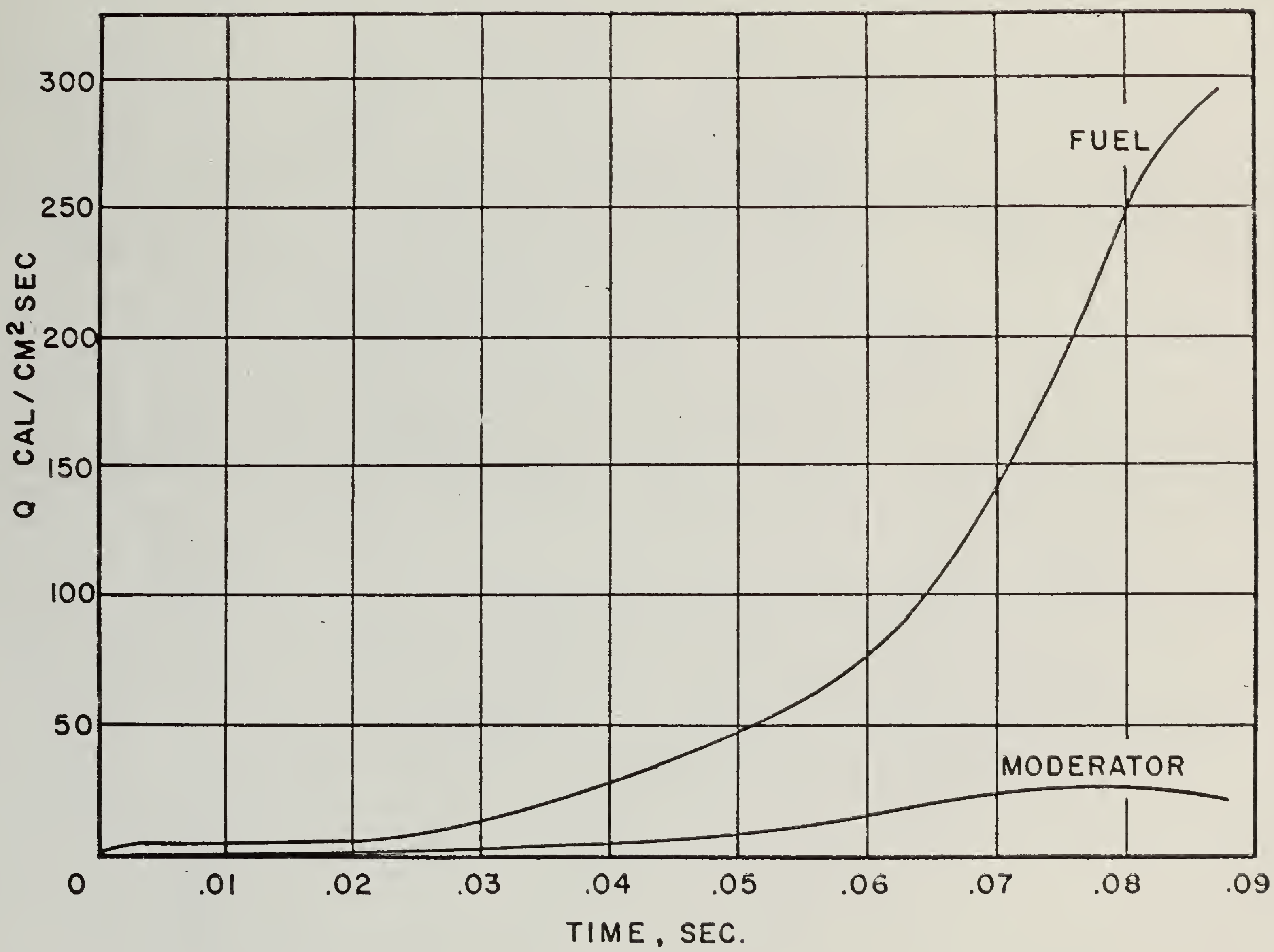


Figure 16. Comparison of, Q, heat flow rates per unit area out of the fuel and into the moderator vs arbitrary time during a transient with an initial period of 15.8 msec.

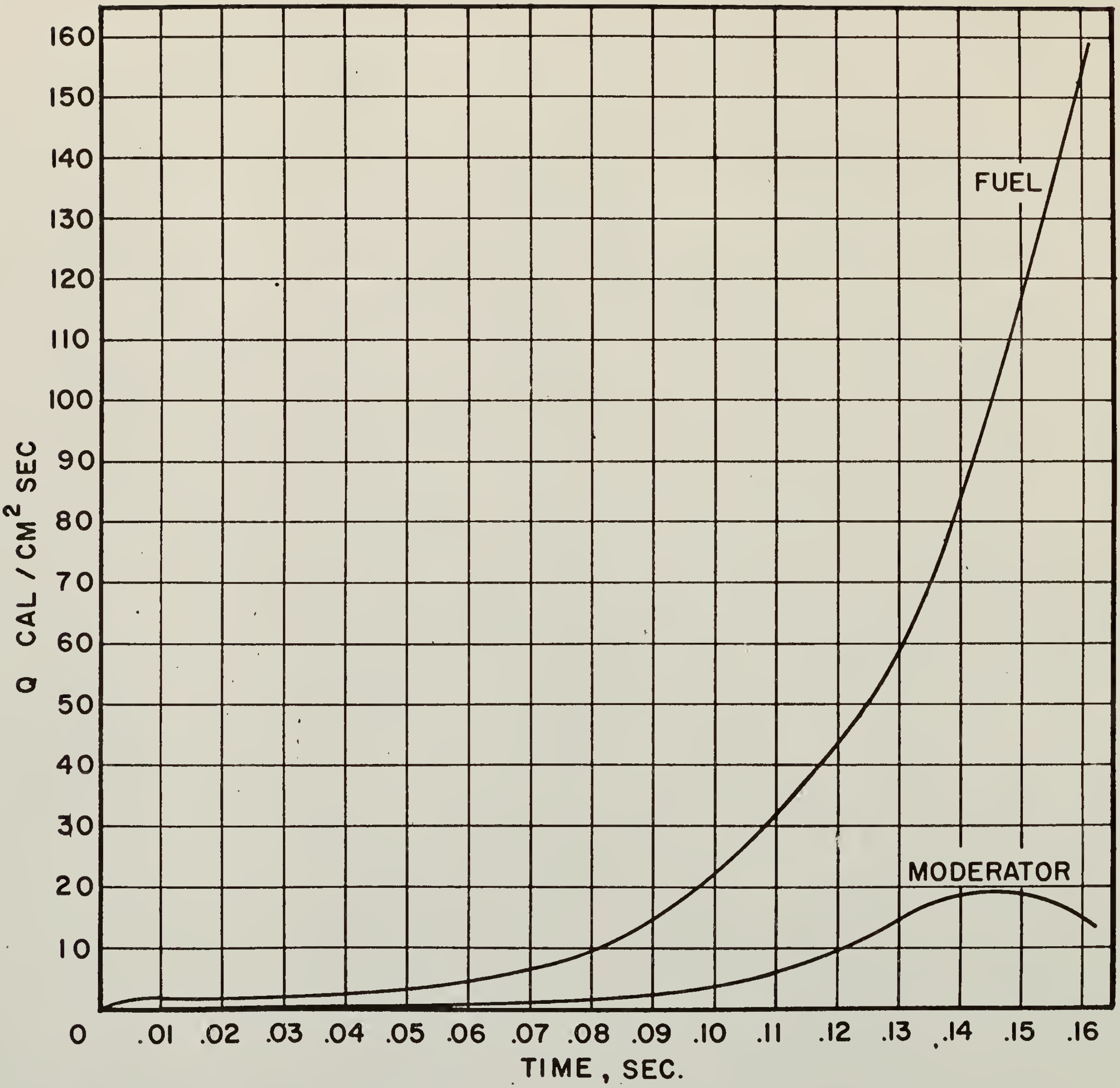


Figure 17. Comparision of, Q, heat flow rates per unit area out of the fuel and into the moderator vs arbitrary time during a transient with an initial period of 23 msec.

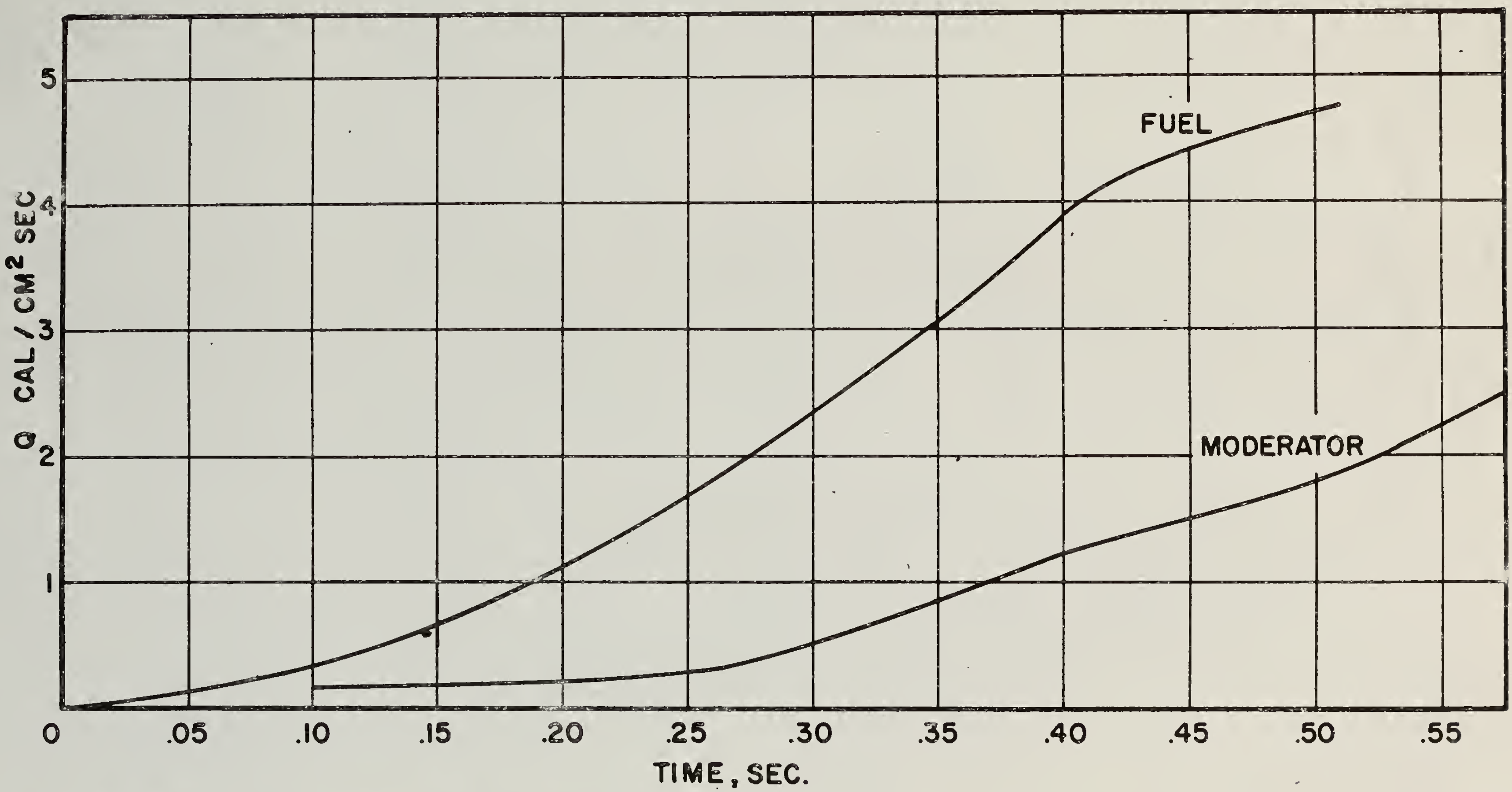


Figure 18. Comparison of, Q, heat flow rates per unit area out of the fuel and into the moderator vs arbitrary time during a transient with an initial period of 120 msec.

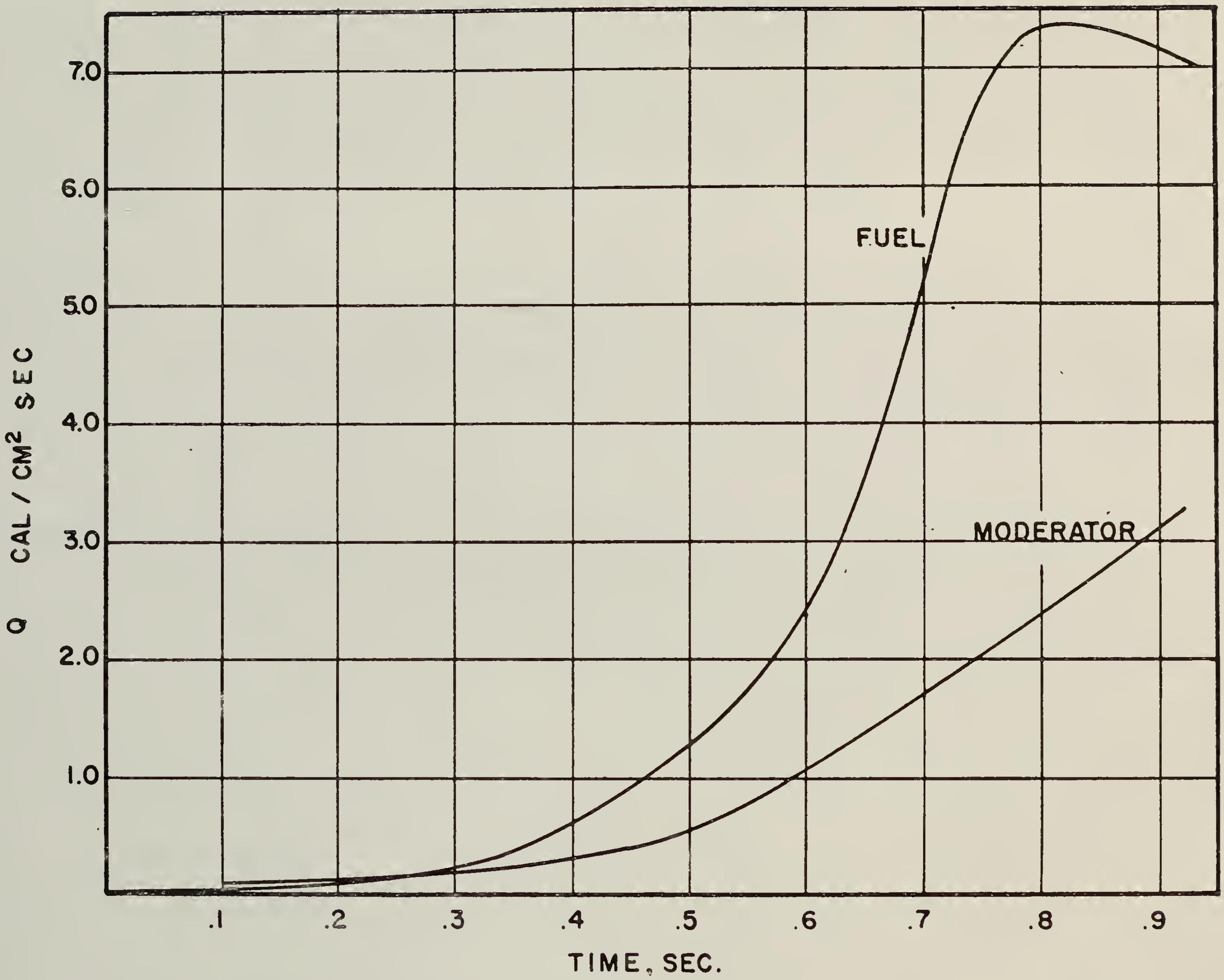


Figure 19. Comparision of, Q, heat flow rates per unit area out of the fuel and into the moderator vs arbitrary time during a transient with an initial period of 150 msec.

Table 3. Numerical Values of Parameters for Empirical Fits of $H_{of}(t)$ Used in Equations (15) and (16)

τ, msec	A_1 / λ_1	A_2 / λ_2	A_3 / λ_3	A_4 / λ_4	A_5 / λ_5
150	+1.912 / 7.426	-2.097×10^{-5} / 21.090	$+6.062\times10^{-8}$ / 27.260	--- / ---	--- / ---
120	9.218 / 9.204	-6.486×10^{-2} / 20.233	$+3.564\times10^{-4}$ / 30.093	-1.856×10^{-7} / 42.208	$+3.019\times10^{-19}$ / 85.147
23	2.414×10^{1} / 39.761	-4.364×10^{-5} / 124.03	9.563×10^{-10} / 187.61	--- / ---	--- / ---
15.8	1.364×10^{2} / 64.573	-2.993 / 104.11	--- / ---	--- / ---	--- / ---

Table 4. Average Temperature Rise in the Fuel and Moderator for τ = 15.8 msec.

t, sec.	$\overline{\theta}_{fuel}(t)$ ^{o}C	$\overline{\theta}_{mod}(t)$, ^{o}C	$(\overline{\theta}_{fuel}-\overline{\theta}_{mod})$, ^{o}C
.085	138.53	16.732	121.80
.080	127.34	13.99	113.35
0.075	110.34	10.81	99.53
.070	89.99	8.664	81.33
.060	50.12	4.87	45.25
.050	24.58	2.446	22.13
.040	13.91	1.333	12.58
.030	8.51	0.692	7.82
.020	3.52	.271	3.25
.010	1.039	.0767	.962

Table 5. Average Temperature Rise in the Fuel and Moderator for τ = 23 msec.

t, sec.	$\overline{\theta}_{fuel}(t)$, ^{o}C	$\overline{\theta}_{mod}(t)$, ^{o}C	$(\overline{\theta}_{fuel}-\overline{\theta}_{mod})$, ^{o}C
.16	117.76	16.832	100.93
.15	108.71	13.371	95.34
.14	86.92	9.764	77.16
.13	59.48	6.459	53.02
.12	36.58	4.218	32.35
.11	23.17	2.759	20.41
.10	16.28	1.869	15.21
.09	11.25	1.231	10.02
.05	3.13	.260	2.87

Table 6. Average Temperature Rise in the Fuel and Moderator for τ = 120 msec.

t, sec.	$\bar{\theta}_{fuel}(t)$, °C	$\bar{\theta}_{mod}(t)$, °C	$(\bar{\theta}_{fuel}-\bar{\theta}_{mod})$, °C
.59	32.66	8.572	24.09
.57	29.95	7.738	22.21
.55	27.33	6.935	20.39
.50	21.15	5.212	15.94
.45	16.40	3.834	12.57
.40	11.98	2.660	9.32
.35	7.84	1.744	6.10
.30	4.89	1.143	3.75
.20	2.59	.534	2.06
.10	1.17	.164	1.01

Table 7. Average Temperature Rise in the Fuel and Moderator for τ = 150 msec.

t, sec.	$\bar{\theta}_{fuel}(t)$, °C	$\bar{\theta}_{mod}(t)$, °C	$(\bar{\theta}_{fuel}-\bar{\theta}_{mod})$, °C
.90	44.46	12.994	31.47
.85	37.49	10.630	26.86
.80	30.87	8.510	22.36
.75	25.23	6.678	18.55
.70	20.31	5.111	15.20
.65	15.70	4.644	11.06
.60	11.48	2.750	8.73
.55	8.11	1.989	6.12
.50	4.86	1.472	4.39
.40	3.58	.834	2.75
.30	1.91	.431	1.48
.10	.602	.0652	.537

where $\underline{\Delta k(o)}$ is the initial reactivity insertion to start the transient and $\underline{\Delta k(t)}$ is the excess reactivity of the system at any time, $\underline{t}$, are shown as a function of time in Tables 8 through 11. Tables 8 through 11 also show the components of the compensated reactivity due to the temperature coefficient $\underline{\Delta k_T(t)}$, fuel plate expansion $\underline{\Delta k_E(t)}$ and steam formation, $\underline{\Delta k_s(t)}$. The excess reactivity $\underline{\Delta k(t)}$ for two of the transients, $\underline{\tau}$ equals 120 and 150 msec respectively, is compared with equivalent data obtained from a kinetic analysis of the power burst shapes by Miller (34) in Figures 20 and 21. The reactivity compensation at peak power, is shown along with comparable data from the kinetic analysis in Figure 22. Figure 22 also includes the components of the reactivity compensations for a model suggested by S. G. Forbes (15) as well as for the model suggested in this report.

3.3 Conclusions

The forms of all of the solutions shown in equations (9) through (16) are such that three terms are developed. The first term represents the steady state solution resulting from the surface temperature boundary condition. The second term includes the transient portion of both the surface temperature boundary condition and the forcing function, the heat generation rate. The final term represents the steady state or equilibrium solution resulting from the forcing function. It has been common in several previous works (27,36) to assume that the temperature is separable in space and time. It can be seen from the derived solutions that this will be a good approximation of the temperature distribution only when the second term, the transient solution, has died out. The

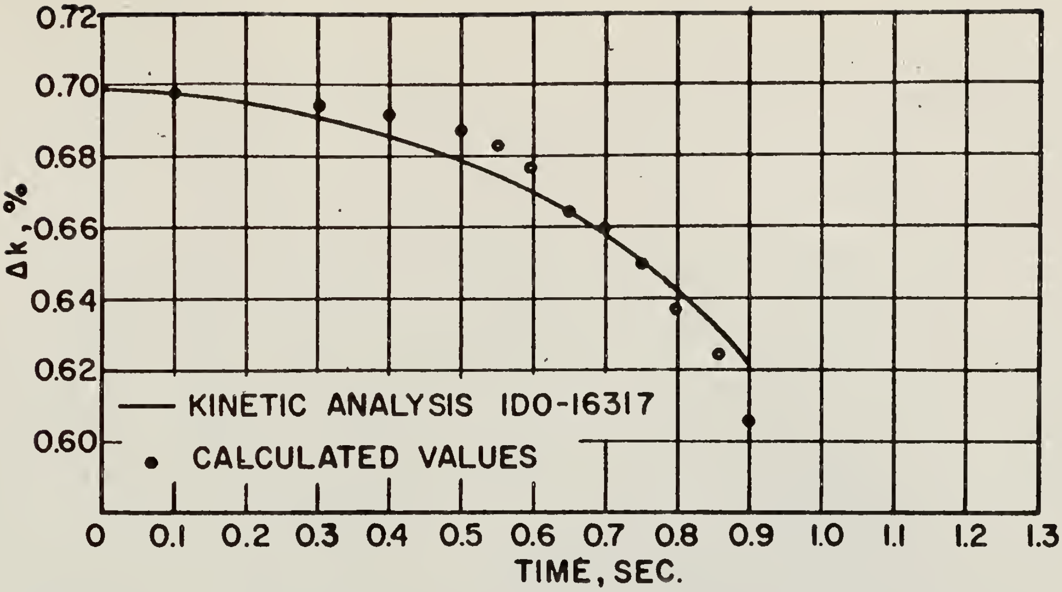


Figure 20. Comparison of calculated percent excess reactivities, Δk, and those obtained by kinetic analysis vs arbitrary time during a transient with an initial period of 120 msec.

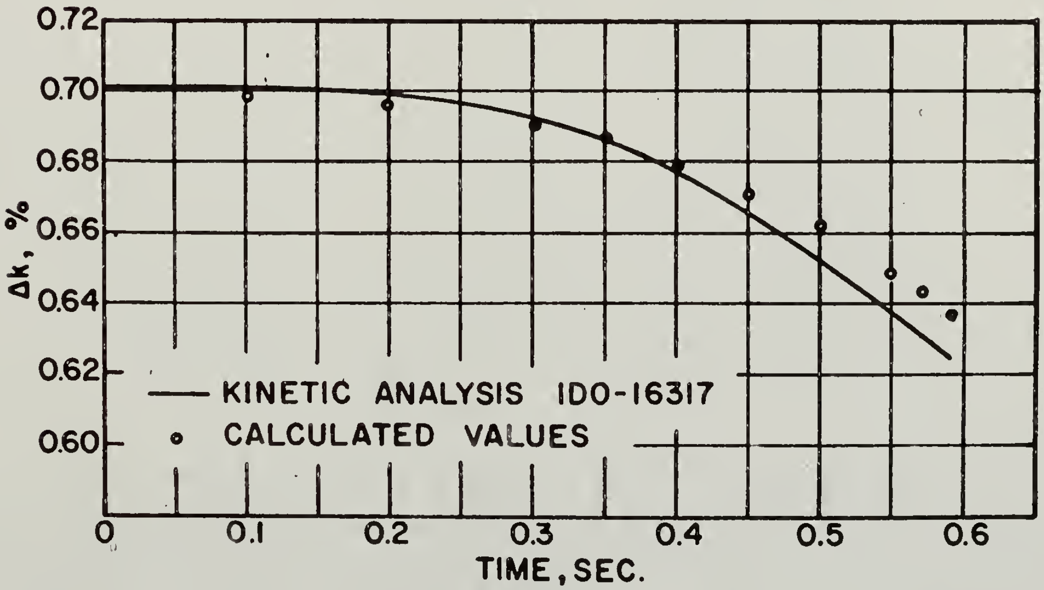


Figure 21. Comparison of calculated percent excess reactivities, Δk, and those obtained by kinetic analysis vs arbitrary time during a transient with an initial period of 150 msec.

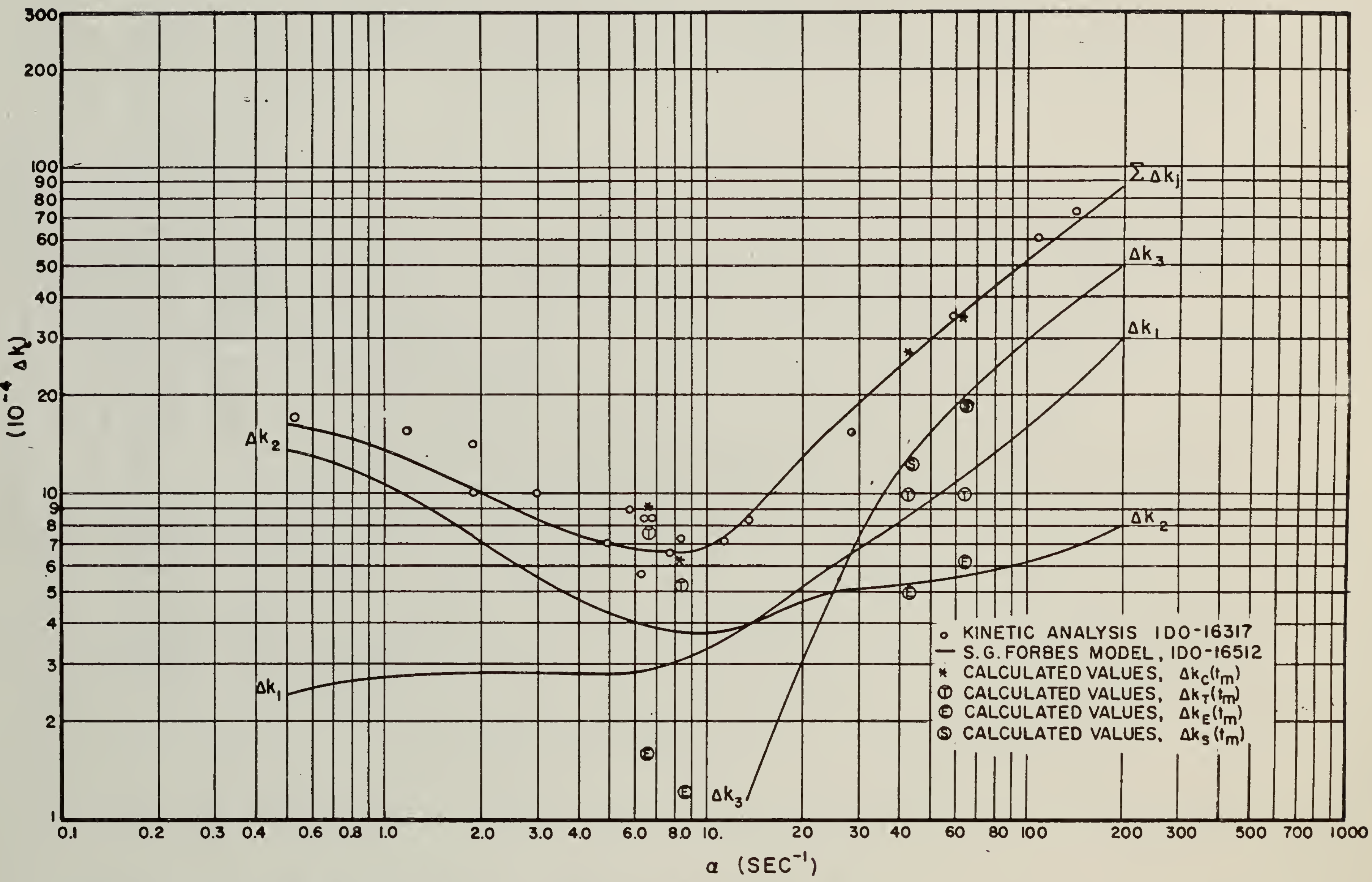


Figure 22. Peak power compensated reactivities, Δk_C, vs reciprocal period, α.

Table 8. Compensated Reactivities for τ = 15.8 msec Run

t,sec	$\Delta k_T(t)$	$\Delta k_E(t)$	$\Delta k_S(t)$	$\Delta k_C(t)$
0.084	0.979×10^{-3}	0.609×10^{-3}	1.86×10^{-3}	3.45×10^{-3}
0.080	0.818×10^{-3}	0.546×10^{-3}	1.21×10^{-3}	2.60×10^{-3}
0.075	0.632×10^{-3}	0.498×10^{-3}	0.46×10^{-3}	1.59×10^{-3}
0.070	0.684×10^{-3}	0.407×10^{-3}	0.13×10^{-3}	1.22×10^{-3}
0.060	0.285×10^{-3}	0.226×10^{-3}	0.0	0.51×10^{-3}
0.050	0.143×10^{-3}	0.111×10^{-3}	0.0	0.25×10^{-3}
0.040	0.078×10^{-3}	0.063×10^{-3}	0.0	0.14×10^{-3}
0.030	0.050×10^{-3}	0.039×10^{-3}	0.0	0.08×10^{-3}
0.020	0.016×10^{-3}	0.016×10^{-3}	0.0	0.03×10^{-3}
0.010	0.004×10^{-3}	0.005×10^{-3}	0.0	0.01×10^{-3}

Table 9. Compensated Reactivites for τ = 23 msec Run

t,sec	$\Delta k_T(t)$	$\Delta k_E(t)$	$\Delta k_S(t)$	$\Delta k_C(t)$
0.16	0.985×10^{-3}	0.505×10^{-3}	1.21×10^{-3}	2.70×10^{-3}
0.15	0.782×10^{-3}	0.468×10^{-3}	0.54×10^{-3}	1.79×10^{-3}
0.14	0.571×10^{-3}	0.386×10^{-3}	0.003×10^{-3}	0.96×10^{-3}
0.13	0.378×10^{-3}	0.265×10^{-3}	0.0	0.64×10^{-3}
0.12	0.247×10^{-3}	0.162×10^{-3}	0.0	0.41×10^{-3}
0.11	0.161×10^{-3}	0.102×10^{-3}	0.0	0.26×10^{-3}
0.10	0.109×10^{-3}	0.076×10^{-3}	0.0	0.18×10^{-3}
0.09	0.072×10^{-3}	0.050×10^{-3}	0.0	0.12×10^{-3}
0.05	0.015×10^{-3}	0.014×10^{-3}	0.0	0.03×10^{-3}

Table 10. Compensated Reactivities for τ = 120 msec Run

t, sec	$\Delta k_T(t)$	$\Delta k_E(t)$	$\Delta k_S(t)$	$\Delta k_C(t)$
0.59	0.501×10^{-3}	0.120×10^{-3}	0.0	0.621×10^{-3}
0.57	0.453×10^{-3}	0.111×10^{-3}	0.0	0.564×10^{-3}
0.55	0.406×10^{-3}	0.100×10^{-3}	0.0	0.506×10^{-3}
0.50	0.305×10^{-3}	0.080×10^{-3}	0.0	0.385×10^{-3}
0.45	0.224×10^{-3}	0.063×10^{-3}	0.0	0.287×10^{-3}
0.40	0.156×10^{-3}	0.047×10^{-3}	0.0	0.203×10^{-3}
0.35	0.102×10^{-3}	0.030×10^{-3}	0.0	0.132×10^{-3}
0.30	0.067×10^{-3}	0.019×10^{-3}	0.0	0.086×10^{-3}
0.20	0.031×10^{-3}	0.010×10^{-3}	0.0	0.041×10^{-3}
0.10	0.010×10^{-3}	0.005×10^{-3}	0.0	0.015×10^{-3}

Table 11. Compensated Reactivities for τ = 150 msec Run

t, sec	$\Delta k_T(t)$	$\Delta k_E(t)$	$\Delta k_S(t)$	$\Delta k_C(t)$
0.90	0.760×10^{-3}	0.157×10^{-3}	0.0	0.915×10^{-3}
0.85	0.622×10^{-3}	0.134×10^{-3}	0.0	0.756×10^{-3}
0.80	0.498×10^{-3}	0.112×10^{-3}	0.0	0.610×10^{-3}
0.75	0.391×10^{-3}	0.093×10^{-3}	0.0	0.484×10^{-3}
0.70	0.299×10^{-3}	0.076×10^{-3}	0.0	0.375×10^{-3}
0.65	0.272×10^{-3}	0.055×10^{-3}	0.0	0.327×10^{-3}
0.60	0.161×10^{-3}	0.044×10^{-3}	0.0	0.205×10^{-3}
0.55	0.116×10^{-3}	0.031×10^{-3}	0.0	0.147×10^{-3}
0.50	0.086×10^{-3}	0.022×10^{-3}	0.0	0.108×10^{-3}
0.40	0.049×10^{-3}	0.014×10^{-3}	0.0	0.063×10^{-3}
0.30	0.025×10^{-3}	0.007×10^{-3}	0.0	0.032×10^{-3}
0.10	0.004×10^{-3}	0.003×10^{-3}	0.0	0.007×10^{-3}

numerical results obtained from evaluating the set of equations (9) and (10) and the set of equations (15) and (16) show that the transient term is negligible for all times of interest in the fuel region but it makes a significant contribution for all times of interest in the moderator.

In one sense it would be more informative to investigate the reactor burst behavior using the minimum of input data (i.e. the physical dimensions and the initial reactivity insertion) and test for corroboration of all of the experimentally measured variables. However, it seemed better in analyzing for the shutdown mechanisms to use as much of the data as possible leaving only the compensated reactivities as a check on the validity of the model. The compensated reactivity was established as a criterion because of its extreme sensitivity to the state of the system and because of its direct influence on the safety of nuclear reactors.

The results of this work are two-fold. First, a more accurate view of distribution of the energy during a transient burst is presented and second, a model based on the energy distribution was shown to predict the reactivity effects as well as any of the existing models. The advantage of this model is that assuming the fractional energy associated with void formation can be determined as the mechanism of transient boiling becomes better understood, the final empiricism can be removed from the model.

3.4 Further Investigation

There are several avenues of attack for further work in determining the inherent shutdown mechanisms. First, additional data on Spert I-A should be tested on the model proposed in this report to make certain that it is as capable of determining reactivity effects as these preliminary runs indicate. Second, application of this model to any new system will mean that the surface temperatures and power traces would not be available. This problem can be circumvented by studying the two region conduction problem subject only to the heat generation rate forcing function. The heat generation rate can be calculated from the initial reactivity insertion, allowing a feedback from the induced negative reactivity to the heat generation rate through the reactor kinetics equations. This suggests an analog solution or possibly a digital analog combination. Third, application of the heat transfer equations developed in this report should be used to determine the mode of heat transfer during transient operation by investigating a single plate in as much detail as possible. The application of this study can probably be done more simply using electrical heating. Fourth, a detailed study of nucleate boiling at low heat fluxes is necessary before complete understanding of the mechanisms of shutdown can be obtained. This study should provide a direct measurement of the fraction of the energy used to produce steam, f_s. Fifth, work on this model should be extended to investigate further available evidence on other Spert reactors to see if it will account for changes in other parameters such as neutron lifetime, pressure and coolant flow. Finally, experimental and analytical work should be done on the zirconium hydride moderated Triga systems since qualitatively they show the greatest inherent safety that has been demonstrated to date.

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Dr. W. R. Kimel and Dr. J. O. Mingle under whose direction this work has been done. In addition, I wish to extend sincerest thanks to Dr. C. A. Halijak and Professor R. C. Bailie for their efforts in the most difficult phases of this work. Sincere appreciation is given to the Kansas State University Engineering Experiment Station, the Department of Nuclear Engineering and the National Science Foundation for their financial support of this research.

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APPENDICES

APPENDIX A

Derivations of Solutions for the Temperature Distribution in the Fuel and Moderator of a Unit Cell

The differential equations governing the time dependent temperature distribution in the fuel and moderator are

$$\nabla^2\theta_f(x,t) + \sum_{j=1}^{s} \frac{q_{oo}\cosh(\kappa x)a_j e^{\lambda_j t}}{k} = \frac{1}{\alpha}\,\frac{\partial\theta(x,t)}{\partial t} \qquad \text{(A-1)}$$

and

$$\nabla^2\theta_m(x,t) + \sum_{j=1}^{s} \frac{F\,A_j\,e^{\lambda_j t}}{k} = \frac{1}{\alpha}\,\frac{\partial\theta_m(x,t)}{\partial t} \qquad \text{(A-2)}$$

The differential equations, (A-1) and (A-2), are most easily solved by use of Laplace transforms. Considering the fuel region first and transforming the time variable in equation (1) yields

$$\nabla^2\Phi_f(x,S) + \sum_{j=1}^{s} \frac{q_{oo}\cosh(\kappa x)A_j}{k\,(S-\lambda_j)} = \frac{1}{\alpha}\left[S\,\Phi_f(x,S) - \theta\,(x,o)\right]. \qquad \text{(A-3)}$$

$\Phi(x,S)$ is the transform of $\theta(x,t)$, and $\theta(x,o)$ is the initial temperature distribution. Assuming the initial temperature distribution to be flat, the equation can be normalized by letting $\theta(x,o)$ equal zero. Thus $\theta(x,t)$ is the temperature excess in the fuel over the initial temperature. The initially flat temperature distribution in the fuel is unreal but it is a good approximation if the transients are started from low power levels.

Letting $\theta(x,o) = 0$ and rearranging the terms in equation (A-3) yields

$$\nabla^2\Phi_f(x,S) - \frac{S}{\alpha}\Phi_f\,(x,S) = -\sum_{j=1}^{s} \frac{q_{oo}\cosh(\kappa x)A_j}{k\,(S-\lambda_j)} \qquad \text{(A-4)}$$

The usual one-dimensional slab assumption is made and ∇^2 becomes $\frac{\partial^2}{\partial x^2}$. This assumption neglects the axial flow of heat in the fuel compared to the radial. While this might be the limiting assumption in the analysis, it is probably not seriously in error.

The homogeneous solution to the one dimensional form of equation (A-4) is well known and is derived in many standard texts. This solution is

$$\Phi_{fh}(x,S) = A \cosh \sqrt{\frac{S}{\alpha}}\, x + B \sinh \sqrt{\frac{S}{\alpha}}\, x \tag{A-5}$$

The particular solution is found by the method of undetermined coefficients and is

$$\Phi_{fp}(x,S) = \sum_{j=1}^{s} \frac{q_{oo}\, \alpha\, A_j \cosh \kappa x}{k(S-\lambda_j)(S-\alpha\kappa^2)} \tag{A-6}$$

Therefore,

$$\Phi_{fp}(x,S) = A \cosh \sqrt{\frac{S}{\alpha}}\, x + B \sinh \sqrt{\frac{S}{\alpha}}\, x + \sum_{j=1}^{s} \frac{q_{oo}\, \alpha\, A_j \cosh(\kappa x)}{k(S-\lambda_j)\,(S-\alpha\kappa_2)} . \tag{A-7}$$

The boundary conditions used to determine the constants A and B are as follows First, the temperature gradient in the center of the region (x = 0) is zero for all time. Second, the surface temperature is matched with the experimental data. The surface temperature is expressed as a sum of exponentials, $\sum_{i=1}^{p} B_i e^{\beta_i t}$ or a Fourier series, $\sum_{i=1}^{p} B_i \cos \beta_i t$. First consider the solution in which the exponential boundary condition is used.

The transformed boundary conditions are

$$\mathcal{L}\left\{\frac{\partial\theta_f(x,t)}{\partial x}\Bigg|_{x=0}\right\} = \mathcal{L}\{0\} \quad \text{or} \quad \frac{d\Phi_f(x,S)}{dx}\Bigg|_{x=0} = 0 \tag{A-8}$$

and

$$\mathcal{L}\left\{\theta_f(L,t)\right\} = \mathcal{L}\left\{\sum_{i=1}^{p} B_i\, e^{\beta_i t}\right\} \quad \text{or} \quad \Phi_f(L,S) = \sum_{i=1}^{p}\frac{B_i}{(S-\beta_i)} \,. \tag{A-9}$$

Applying equation (A-8) to equation (A-7) shows that B must equal zero. A is determined by evaluating equation (A-9). The solution in the transform domain is shown in equation (A-10).

$$\Phi_f(x,S) = \sum_{i=1}^{p} \frac{B_i \cosh\sqrt{\frac{S}{\alpha}}\,x}{(S-\beta_i)\cosh\sqrt{\frac{S}{\alpha}}\,L} - \sum_{j=1}^{s} \frac{q_{oo}\,\alpha A_j}{k(S-\lambda_j)(S-\alpha\kappa^2)}\left\{\cosh(\kappa x) - \frac{\cosh(\kappa L)\cosh\sqrt{\frac{S}{\alpha}}\,x}{\cosh\sqrt{\frac{S}{\alpha}}\,L}\right\} \tag{A-10}$$

Transforming equation (A-10) back into the time domain is greatly simplified since no poles of order greater than one occur in the inversion integral. It is interesting to note that it is the occurence of higher order poles which complicates the solution to the multiple region problem. Equation (A-11), shown below, (33) can be used to invert transformed functions of the form $\bar{f}(s) = j(s) / \ell(s)$ if the degree of $\ell(s)$ is at least one greater than j(s) and only poles of order one occur,

$$\mathcal{L}^{-1}\left\{\bar{f}(s)\right\} = \sum_{n=1}^{m} \frac{j(\rho_n)}{\ell'(\rho_n)}\, e^{\rho_n t} \tag{A-11}$$

ρ_n denotes the n simple poles of $\bar{f}(S)$ and $\ell'(\rho_n)$ denotes the value of $\frac{d\ell(S)}{dS}$ evaluated at $S = \rho_n$.

The inversion of equation (A-10) is shown in detail below. First consider the third term of equation (A-10).

$$\mathcal{L}^{-1}\left\{\sum_{j=1}^{s} \frac{q_{\infty}\,\alpha \cosh \kappa L\, A_j \cosh \sqrt{\frac{S}{\alpha}}\,x}{k(S-\lambda_j)(S-\alpha\kappa^2)\cosh\sqrt{\frac{S}{\alpha}}\,L}\right\} = \sum_{j=1}^{s} \frac{q_{\infty}\,\alpha \cosh(\kappa L)\,A_j}{k}\,\mathcal{L}^{-1}\{I\} \tag{A-12}$$

The obvious poles of I are at $S = \lambda_j$ and $S = \alpha\kappa^2$. Additional roots of the denominator exist for $\cosh\sqrt{\frac{S}{\alpha}}\,L = 0$. The roots of $\cosh\sqrt{\frac{S}{\alpha}}\,L$ are obtained by making the transformation $S = -s$. This is done since $\cosh\sqrt{\frac{S}{\alpha}}\,L$ cannot equal zero for real values of the argument and α and L are always positive, real constants.

$$\cosh\sqrt{\frac{S}{\alpha}}\,L = \cosh i\sqrt{\frac{s}{\alpha}}\,L = \cos\sqrt{\frac{-s}{\alpha}}\,L = 0 \tag{A-13}$$

therefore

$$\sqrt{\frac{-s}{\alpha}}\,R_o = \frac{n\pi}{2}, \quad n \text{ odd}$$

$$s_n = \frac{n^2\pi^2\alpha}{4L^2} \tag{A-14}$$

$$S_n = -\frac{n^2\pi^2\alpha}{4L^2}$$

Evaluating $\mathcal{L}^{-1}\{I\}$ by means of equation (A-11) at the poles $\rho = \alpha\kappa^2$, $\rho = \lambda_j$ and $\rho = -\frac{n^2\pi^2\alpha}{4L^2}$ yields equation (A-15).

$$\mathcal{L}^{-1}\left\{I\right\} = \frac{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\,x\right)e^{\lambda_j t}}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\,L\right)(\lambda_j - \alpha\kappa^2)} + \frac{\cosh(\kappa x)\,e^{\alpha\kappa^2 t}}{\cosh(\kappa L)\,(\alpha\kappa^2 - \lambda_j)}$$

$$+\sum_{n=1,3,5\ldots}^{\infty} \frac{\cos\frac{n\pi x}{2L}\,e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{\left(\frac{n^2\pi^2\alpha}{4L^2}+\lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2}+\alpha\kappa^2\right)\left(\frac{2L^2}{n\pi\alpha}\right)\sin\left(\frac{n\pi}{2}\right)} \tag{A-15}$$

Considering the second term in equation (A-10), the inversion as obtained through use of equation (A-11) is given in equation (A-16).

$$\mathcal{L}^{-1}\sum_{j=1}^{s}\frac{q_{\infty}\alpha A_j}{k}\;\frac{1}{(S-\lambda_j)(S-\alpha\kappa^2)} = \sum_{j=1}^{s}\frac{q_{\infty}\alpha A_j}{k}\left(\frac{e^{\lambda_j t}}{(\lambda_j-\alpha\kappa^2)}+\frac{e^{\alpha\kappa^2 t}}{(\alpha\kappa^2-\lambda_j)}\right) \tag{A-16}$$

In a similar manner the first term of equation (A-10), the inversion, again using equation (A-11), is given in equation (A-17).

$$\mathcal{L}^{-1}\left\{\sum_{i=1}^{p}\frac{C_i\cosh\sqrt{\frac{s}{\alpha}}\,x}{(S-\beta_i)\cosh\sqrt{\frac{s}{\alpha}}\,L}\right\} = \sum_{i=1}^{p}\frac{B_i\cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,x\right)e^{\beta_i t}}{\cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,L\right)}$$

$$+\sum_{i=1}^{p}\sum_{n=1,3,5,\cdots}^{\infty}\frac{B_i\cos\frac{n\pi x}{2L}\,e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{\left(-\frac{n^2\pi^2\alpha}{4L^2}-\beta_i\right)\frac{L^2}{n\pi\alpha}\left(\sin\frac{n\pi}{2}\right)} \tag{A-17}$$

The temperature distribution in the fuel plate, $\underline{\theta_f(x,t)}$ is obtained by substituting equation (A-15) in equation (A-12) and adding equations (A-12), (A-16) and (A-17). The result is

$$\theta_f(x,t) = \sum_{i=1}^{p} \frac{B_i \cosh\sqrt{\frac{\beta_i}{\alpha}}\, x}{\cosh\sqrt{\frac{\beta_i}{\alpha}}\, L}\, e^{\beta_i t} - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{\left(\frac{L^2}{\alpha n\pi}\right)\sin\left(\frac{n\pi}{2}\right)}$$

$$X \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} + \sum_{j=1}^{s} \frac{q_{oo}\,\alpha\, A_j \cosh \kappa L}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)} \right\} \qquad \text{(A-18)}$$

$$- \sum_{j=1}^{s} \frac{A_j\, q_{oo}\, \alpha\, e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{ \cosh(\kappa x) - \frac{\cosh(\kappa L)\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, x\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)} \right\}$$

As seen by comparing (A-1) and (A-2) the differential equations to be solved in the fuel and moderator are almost the same, the only difference being in the heat generation term. Thus, the total differential equation for the moderator in the Laplace transform domain after having applied the zero initial temperature condition is given in equation (A-19)

$$\nabla^2 \Phi_m(x,s) - \frac{S}{\alpha}\Phi_m(x,S) = - \sum_{j=1}^{s} \frac{F\, A_j}{k\,(S-\lambda_j)} \qquad \text{(A-19)}$$

The homogeneous solutions are the same as before and particular solutions are easily obtained, as before, from the method of undetermined coefficients. Therefore, the solutions to equation (A-19) in the transform domain are

$$\Phi_m(x,S) = C \cosh\sqrt{\frac{S}{\alpha}}\, X + D \sinh\sqrt{\frac{S}{\alpha}}\, X + \sum_{j=1}^{s} \frac{F\,\alpha\, A_j}{k\, S(S-\lambda_j)} \qquad \text{(A-20)}$$

The transform boundary conditions for the moderator are same as those for the fuel if the origin in the moderator is taken at the outside of the unit cell, i.e.,

$$\Phi_m(L,S) = \sum_{i=1}^{p} \frac{B_i}{S-\beta_i} \qquad \text{and} \qquad \left.\frac{d\Phi\,(x_1\,,\,S)}{d\,x_1}\right|_{x_1=0} = 0\,. \qquad \text{(A-21)}$$

Thus $\underline{D}$ equals zero and $\underline{A}$ takes the same form as for the solution in the fuel. The complete solution in the transform domain is given in equation (A-22).

$$\Phi_m(x,S) = \sum_{i=1}^{p} \frac{B_i \cosh\sqrt{\frac{S}{\alpha}}\,x_1}{(S-\beta_i)\cosh\sqrt{\frac{S}{\alpha}}\,L_1} + \sum_{j=1}^{s} \frac{\alpha\,F\,A_j}{k\,S\,(S-\lambda_j)}$$

$$\left[\,1 - \frac{\cosh\sqrt{\frac{S}{\alpha}}\,x_1}{\cosh\sqrt{\frac{S}{\alpha}}\,L_1}\,\right]. \qquad \text{(A-22)}$$

The inversion of this solution is easily accomplished by the same method used for the fuel region. The solution for the temperature is

$$\theta_m(x,t) = \sum_{i=1}^{p} \frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,x_1\right) e^{\beta_i t}}{\cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,L_1\right)} - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\frac{n\pi x_1}{2L_1} \cdot e^{-\frac{n^2\pi^2\alpha}{4L_1^2}t}}{\left(\frac{L_1^2}{n\pi\alpha}\right)\sin\left(\frac{n\pi}{2}\right)}$$

$$\times\left\{\sum_{i=1}^{p} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L_1^2} + \beta_i} + \sum_{j=1}^{s} \frac{\alpha\,F\,A_j}{\frac{n^2\pi^2\alpha}{4L_1^2}\left(\frac{n^2\pi^2\alpha}{4L_1^2} + \lambda_j\right)}\right\} \qquad \text{(A-23)}$$

$$+\sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k\lambda_j}\left\{1 - \frac{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,x_1}{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,L_1}\right\}.$$

This solution could have been obtained from the solution in the fuel by setting $\underline{\kappa} = 0$ and $\underline{q_{oo}} = \underline{F}$.

The equivalent solution in cylindrical geometry (r dependence only) for the exponential boundary condition is obtained in the same general manner, however, several important differences do occur. The equations governing the temperature in the fuel and moderator are

$$\nabla^2\theta_f(r,t) + \sum_{j=1}^{s} \frac{q_{oo}\, I_o(\kappa r)\, A_j\, e^{\lambda_j t}}{k} = \frac{1}{\alpha} \frac{\partial\theta_f(r,t)}{\partial t} \tag{A-24}$$

and

$$\nabla^2\theta_m(r,t) + \sum_{j=1}^{s} \frac{F\, A_j\, e^{\lambda_j t}}{k} = \frac{1}{\alpha} \frac{\partial\theta_m(x,t)}{\partial t} \tag{A-25}$$

Considering the fuel region first, transforming with respect to time and applying the zero initial condition yields

$$\nabla^2\Phi(r,S) - \frac{S}{\alpha}\Phi(r,S) = -\sum_{j=1}^{s} \frac{q_{oo}\, A_j\, I_o(\kappa r)}{k\,(S - \lambda_j)} \tag{A-26}$$

The spatial operator for this case is

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{A-27}$$

The particular solution, again by the method of undetermined coefficients, is

$$\Phi_p(r,S) = \frac{q_{oo}\,\alpha\, A_j\, I_o(\kappa r)}{k(S-\lambda_j)\,(S-\alpha\kappa^2)} \tag{A-28}$$

The homogeneous solution, noting the fact that $\Phi(o,S)$ if finite is

$$\Phi_h(r,s) = A\, I_o\left(\sqrt{\frac{S}{\alpha}}\, r\right) \tag{A-29}$$

The final constant $\underline{A}$ is evaluated by use of the surface temperature boundary condition and the solution in the transform domain is

$$\bar{\phi}_f(r,S) = \sum_{i=1}^{p} \frac{B_i}{S-\beta_i} \; \frac{I_o\left(\sqrt{\frac{S}{\alpha}}\; r\right)}{I_o\left(\sqrt{\frac{S}{\alpha}}\; R_o\right)} + \sum_{j=1}^{s} \frac{q_\infty\, \alpha\, A_j}{k(S-\lambda_j)(S-\alpha\kappa^2)} \left(I_o(\kappa r) - \frac{I_o(\kappa R_o)\, I_o\left(\sqrt{\frac{S}{\alpha}}\; r\right)}{I_o\left(\sqrt{\frac{S}{\alpha}}\; R_o\right)} \right) . \tag{A-30}$$

Inverting this expression is entirely analogous to the inversion of the equivalent expression, equation (A-9) of this appendix. The analogy carries even to the point that in determining the zeros of $I_o\sqrt{\frac{S}{\alpha}}\,R_o$ as in determining the zeros of $\text{Cosh}\sqrt{\frac{S}{\alpha}}\,L$ imaginary values of S yield an infinite set of zeros. In this case the zeros are

$$I_o\sqrt{\frac{S}{\alpha}}\,R_o \doteq I_o\sqrt{\frac{-s}{\alpha}}\,R_o = I_o\, i\sqrt{\frac{s}{\alpha}}\,R_o = J_o\sqrt{\frac{s}{\alpha}}\,R_o = 0 \tag{A-31}$$

therefore,

$\sqrt{\frac{s}{\alpha}}\,R_o = \omega_n$, where $\omega_n = 2.4048,\ 5.5201$, etc. (the zeros of $J(\omega_n)=0$)

$$\frac{s}{\alpha} R_o^2 = \omega_n^2 ,$$

$$s_n = \frac{\omega_n^2 \alpha}{R_o^2} \tag{A-32}$$

and

$$S_n = \frac{\omega_n^2\, \alpha}{R_o^2}$$

The details of the remainder of the inversion are not included. The result is

$$\theta_f(x,t) = \sum_{i=1}^{p} \frac{B_i\, I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right) e^{\beta_i t}}{I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right)} + \sum_{n=1,2,3,\cdots}^{\infty} \frac{J_o\left(\frac{\omega_n r}{R}\right) e^{-\frac{\omega_n^2 \alpha}{R^2} t}}{\frac{R^2}{2\omega_n \alpha}\, J\,(\omega_n)}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{\omega_n^2 \alpha}{R^2} + \beta_i} + \sum_{j=1}^{s} \frac{q_{\infty}\, \alpha\, A_j\, I_o\,(\kappa R_o)}{k\left(\frac{\omega_n^2 \alpha}{R^2} + \lambda_j\right)\left(\frac{\omega_n^2 \alpha}{R^2} + \alpha\kappa^2\right)} \right\} \qquad \text{(A-33)}$$

$$- \sum_{j=1}^{s} \frac{A_j\, q_{\infty}\, \alpha\, e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j t)} \left\{ [I_o\,(\kappa r)] - \frac{[I_o(\kappa R)\, I_o \sqrt{\frac{\lambda_j}{\alpha}}\, r]}{I_o \sqrt{\frac{\lambda_j}{\alpha}}\, R} \right\},$$

where the ω_n's are the roots of the equation, $J_o(X) = 0$.

The solution for the cylindrical geometry in the moderator is complicated only by the fact that the symmetry condition cannot be located at the coordinate $r = 0$. Thus both terms $I_o \sqrt{\frac{S}{\alpha}}\, r$ and $K_o \sqrt{\frac{S}{\alpha}}\, r$ of the homogeneous solution must be retained. The details of this solution are not included. The result is

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i \left\{ \frac{K_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right) + I_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) K_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right)}{\left(K_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right) + I_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) K_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right)\right.} \right\} e^{\beta_i t}$$

$$+\sum_{n=1}^{\infty} \frac{2\sqrt{\rho_n\alpha}\;\left[K_1\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right) I_o\left(\sqrt{\frac{\rho_n}{\alpha}}r\right) + I_1\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right)K_o\left(\sqrt{\frac{\rho_n}{\alpha}}r\right)\right] e^{\rho_n t}}{R\left[K_1\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right)I_1\left(\sqrt{\frac{\rho_n}{\alpha}}R\right) - K_1\left(\sqrt{\frac{\rho_n}{\alpha}}R\right)I_1\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right) - I_o\left(\sqrt{\frac{\rho_n}{\alpha}}R\right)K_o\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right) + I_o\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right)K_o\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right)\right]}$$

$$\times\left\{\sum_{i=1}^{p} \frac{B_i}{\rho_n-\beta_i} - \sum_{j=1}^{s} \frac{\alpha F A_j}{k\,\rho_n(\rho_n-\lambda_j)}\right\} + \sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k} \qquad \text{(A-34)}$$

$$\times\left\{\frac{K_1\left(\sqrt{\frac{\lambda_j}{\alpha}}R_1\right) I_o\left(\sqrt{\frac{\lambda_j}{\alpha}}r\right) + I_1\left(\sqrt{\frac{\lambda_j}{\alpha}}R_1\right)K_o\left(\sqrt{\frac{\lambda_j}{\alpha}}r\right)}{K_1\left(\sqrt{\frac{\lambda_j}{\alpha}}R_1\right) I_o\left(\sqrt{\frac{\lambda_j}{\alpha}}R\right) I_1\left(\sqrt{\frac{\lambda_j}{\alpha}}R_1\right)K_o\left(\sqrt{\frac{\lambda_j}{\alpha}}R\right)}\right\},$$

where the ρ_n's are the roots of the equation,

$$K_1\left(\sqrt{\frac{S}{\alpha}}R_1\right)I_o\left(\sqrt{\frac{S}{\alpha}}R\right) + I_1\left(\sqrt{\frac{S}{\alpha}}R_1\right)K_o\left(\sqrt{\frac{S}{\alpha}}R\right) = 0$$

The next solution considered again uses an exponential fit for the heat generation rate, however, the boundary condition was that the first derivative with respect to x evaluated at the outside of the plate or effectively the heat flow out of the plate could be expressed as a sum of exponentials. The general solution with the exception of the evaluation of the final constant $\underline{A}$ is exactly the same as the first derivation in this appendix. Including the one undetermined coefficient the solution is

$$\Phi_f(x,S) = A\cosh\sqrt{\frac{S}{\alpha}}\,x + \sum_{j=1}^{s} \frac{q_{oo}\,\alpha\,A_j\,\cosh\kappa x}{k(S-\lambda_j)\,(S-\alpha\kappa^2)}\;. \qquad \text{(A-35)}$$

Evaluation of A through use of the boundary condition leads to

$$\Phi_f(x,S) = \sum_{i=1}^{p'} \frac{B_i \cosh\left(\sqrt{\frac{S}{\alpha}}\, x\right)}{(S-\beta_i)\sqrt{\frac{S}{\alpha}}\,\sinh\left(\sqrt{\frac{S}{\alpha}}\, L\right)} + \sum_{j=1}^{s'} \frac{q_{oo}\alpha A_j}{k(S-\lambda_j)(S-\alpha\kappa^2)}$$

$$\left(\cosh \kappa x - \frac{\cosh(\kappa L)\cosh\left(\sqrt{\frac{S}{\alpha}}\, x\right)}{\cosh\left(\sqrt{\frac{S}{\alpha}}\, L\right)} \right) . \tag{A-36}$$

The details of this inversion are not included. The result is

$$\theta_f(x,t) = \sum_{i=1}^{p'} \left(\frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\, x\right) e^{\beta_i t}}{\sqrt{\frac{\beta_i}{\alpha}}\,\sinh\left(\sqrt{\frac{\beta_i}{\alpha}}\, L\right)} - \frac{B_i \alpha}{\beta_i L} \right) - \sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi x}{2L}\right) e^{-\frac{n^2\pi^2 x}{4L^2}}}{\frac{L}{\alpha}\cos n\pi}$$

$$X \left\{ \sum_{i=1}^{p'} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} + \sum_{j=1}^{s'} \frac{q_{\infty}\, \alpha A_j\, \kappa \sinh \kappa L}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)} \right\} \tag{A-37}$$

$$-\sum_{j=1}^{s'} \frac{q_{\infty}\, \alpha A_j\, e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left[\cosh \kappa x - \frac{\kappa \sinh(\lambda L)\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, x\right)}{\sqrt{\frac{\lambda_j}{\alpha}}\,\sinh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)}\right] - \frac{q_{\infty}\, \alpha A_j \sinh \kappa L}{k\,\lambda_j\, \kappa L} .$$

The time dependent temperature distribution in the moderator for the comparable boundary conditions is obtained by setting κ equal to zero and q_{∞} equal to F in equation (A-37). The result is

$$\theta_m(x,t) = \sum_{i=1}^{p'} \left(\frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\, x\right) e^{\beta_i t}}{\sqrt{\frac{\beta_i}{\alpha}}\,\sinh\left(\sqrt{\frac{\beta_i}{\alpha}}\, L\right)} - \frac{B_i \alpha}{\beta_i L} \right)$$

$$- \sum_{n=1}^{\infty} \frac{\cos \frac{n\pi x}{2L} \; e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{\left(\frac{L}{\alpha}\right) \cos n\pi} \left\{ \sum_{i=1}^{p'} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} \right\}$$

$$+ \sum_{j=1}^{s'} \frac{\alpha F A_j}{k(\lambda_j)} \left(e^{\lambda_j t} - 1 \right). \tag{A-38}$$

The final solution considered again used a sum of exponentials to represent the time dependence of the heat generation rate, however, the surface temperature boundary condition was approximated by an even Fourier series, $\sum_{i=1}^{p} B_i \cos \beta_i t$. With the exception of the steady state term resulting from the surface temperature boundary condition the derivation follows exactly the first derivation of this appendix. The steady state term is handled most easily in a slightly different manner as shown below. The general solution in the transform domain is

$$\Phi_f(x,S) = \sum_{i=1}^{p} \frac{B_i S \cosh\left(\sqrt{\frac{S}{\alpha}}\, x\right)}{(S^2+\beta_i^2) \cosh\left(\sqrt{\frac{S}{\alpha}}\, L\right)} + \sum_{j=1}^{s} \frac{q_{\infty} \alpha A_j}{k(S-\lambda_j)(S-\alpha\kappa^2)}$$

$$\left[\cosh \kappa x - \frac{\cosh(\kappa L) \cosh\left(\sqrt{\frac{S}{\alpha}}\, x\right)}{\cosh\left(\sqrt{\frac{S}{\alpha}}\, L\right)}\right] \tag{A-39}$$

The first term presents the only change and there only for the poles at $S = \pm j\beta_i$. The terms generated from the inversion integral by these two poles are

$$\rho_1 \Big|_{S=j\beta_i} = \frac{B_i \, j \, \beta_i \cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\, x\right)}{2 \, j \, \beta_i \cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\, L\right)} e^{j\beta_i t} \tag{A-40}$$

and

$$\rho_2 \Bigg|_{S=-j\beta_i} = \frac{B_i\ (-j\beta_i)\ \cosh\left(\sqrt{\frac{-j\beta_i}{\alpha}}\ x\right)}{2\ (-j\beta_i)\ \cosh\left(\sqrt{\frac{-j\beta_i}{\alpha}}\ L\right)}\ e^{-j\beta_i t} \tag{A-41}$$

These terms are most easily handled by recognizing the fact that the sum $\rho_1 + \rho_2$ is the sum of a function and its conjugate. That is $\rho_1 + \rho_2 = f(z) + f(\bar{z}) = f(z) + \overline{f(z)}$ since z is a pure imaginary number. Therefore

$$\rho_1 + \rho_2 = 2\ \mathrm{Re}\left\{ f(z) \right\} = 2\ \mathrm{Re}\left\{ \rho_1 \right\}$$

$$= 2\ \mathrm{Re}\left\{ \frac{B_i\ \cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ x\right)}{2\ \cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ L\right)}\ e^{j\beta_i t} \right\} \tag{A-42}$$

$$= B_i\ R_e \left\{ \frac{\cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ x\right)}{\cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ L\right)}\ e^{j\beta_i t} \right\}$$

$$= B_i\ R_e \left| \frac{\cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ x\right)}{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ L} \right| e^{j\left(\beta_i t + \arg\left\{ \frac{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ x}{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ L} \right\}\right)}$$

$$= B_i\ Z_i^{\frac{1}{2}}(x)\ \cos\left[\beta_i t + \varphi_i(x)\right],$$

where

$$Z_i(x) = \left| \frac{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ x}{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ L} \right|^2 = \left| \frac{\cosh\left(\sqrt{\frac{\beta_i}{2\alpha}}\ x\right)\cos\left(\sqrt{\frac{\beta_i}{2\alpha}}\ x\right) + j\ \sin\left(\sqrt{\frac{\beta_i}{2\alpha}}\ x\right)\sinh\left(\sqrt{\frac{\beta_i}{2\alpha}}\ x\right)}{\cosh\left(\sqrt{\frac{\beta_i}{2\alpha}}\ L\right)\cos\left(\sqrt{\frac{\beta_i}{2\alpha}}\ L\right) + j\ \sin\left(\sqrt{\frac{\beta_i}{2\alpha}}\ L\right)\sinh\left(\sqrt{\frac{\beta_i}{2\alpha}}\ L\right)} \right|^2$$

$$= \left\{ \frac{\cos^2(\sqrt{\frac{\beta_i}{2\alpha}}\, x)\cosh^2(\sqrt{\frac{\beta_i}{2\alpha}}\, x) + \sin^2(\sqrt{\frac{\beta_i}{2\alpha}}\, x)\sinh^2(\sqrt{\frac{\beta_i}{2\alpha}}\, x)}{\cos^2(\sqrt{\frac{\beta_i}{2\alpha}}\, L)\cosh^2(\sqrt{\frac{\beta_i}{2\alpha}}\, L) + \sin^2(\sqrt{\frac{\beta_i}{2\alpha}}\, L)\sinh^2(\sqrt{\frac{\beta_i}{2\alpha}}\, L)} \right\} \tag{A-43}$$

and

$$\varphi_i(x) = \arg\left\{\frac{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\, x}{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\, L}\right\} = \arg\left\{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\, x\right\} - \left\{\arg\ \cosh\sqrt{\frac{j\beta_i}{\alpha}}\, L\right\}$$

$$= \tan^{-1}\left(\frac{\sin\sqrt{\frac{\beta_i}{2\alpha}}\, x\ \sinh(\sqrt{\frac{\beta_i}{2\alpha}}\, x)}{\cos\sqrt{\frac{\beta_i}{2\alpha}}\, x\ \cosh(\sqrt{\frac{\beta_i}{2\alpha}}\, x)}\right) - \tan^{-1}\left(\frac{\sin(\sqrt{\frac{\beta_i}{2\alpha}}\, L)\sinh(\sqrt{\frac{\beta_i}{2\alpha}}\, L)}{\cos(\sqrt{\frac{\beta_i}{2\alpha}}\, L)\cosh(\sqrt{\frac{\beta_i}{2\alpha}}\, L)}\right). \tag{A-44}$$

The resultant time dependent temperature distributions are

$$\theta_f(x,t) = \sum_{i=1}^{p} B_i\, Z_i^{\frac{1}{2}}\ \mathrm{Cos}\,(\beta_i t + \varphi_i) - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\frac{n\pi x}{2L}\ e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{(L^2/n\pi\alpha)\ \sin\frac{n\pi}{2}}$$

$$\times\left\{\sum_{i=1}^{p} \frac{B_i\,(\frac{n^2\pi^2\alpha}{4L^2})}{(\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2)} + \sum_{j=1}^{s} \frac{q_{oo}\ \alpha\ A_j\ \cosh\kappa L}{k(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2)(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j)}\right\}. \tag{A-45}$$

$$-\sum_{j=1}^{s} \frac{q_{oo}\alpha\ A_j\ e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{\cosh\kappa x - \frac{\cosh(\kappa L)\cosh(\sqrt{\frac{\lambda_j}{\alpha}}\, x)}{\cosh(\sqrt{\frac{\lambda_j}{\alpha}}\, L)}\right\}$$

and

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos \frac{n\pi x}{2L} \, e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{(L^2/n\pi\alpha) \sin \frac{n\pi}{2}}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} - \sum_{j=1}^{p} \frac{\alpha F A_j}{k \left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)} \right\} \qquad \text{(A-46)}$$

$$+ \sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k(\lambda_j)} \left\{ 1 - \frac{\cosh \sqrt{\frac{\lambda_j}{\alpha}}\, x}{\cosh \sqrt{\frac{\lambda_j}{\alpha}}\, L} \right\}$$

in the fuel and moderator, respectively.

APPENDIX B

Description and Explanation of the IBM-650 Computer Program Used for Fitting Empirically Experimental Data with the Sum of Several Terms of Exponential Form

The computer code was written to fit an analytical function of the form of the sum of exponentials to the experimentally determined power traces during a transient burst. The program was written in SOAP II and floating point form. The object program is listed and the logic diagram is shown in this appendix.

The criteria that the machine inspected was that the sum of the squares of the residuals between the experimental data and the calculated values should be made as small as possible. Each of the fitting parameters was varied in turn by a specified increment, holding all other parameters constant, until such a time that a specified increment could make no further reduction in the sum of the squares of the residuals. This parameter was then stored as the best available estimate of the particular empirical parameter. When none of the parameters could be varied by the specified increment to give a smaller sum of the squares of the residuals, the increments were refined and the trial and error process was repeated with the refined increments. This procedure continued until the increments were less than a specified precision.

The data were fit empirically with a function of the form

$$P_i = \sum_{j=1}^{s} A_j \, e^{\lambda_j t_i} \qquad s \leqq 10 \tag{B-1}$$

As stated above, the best fit criterion was that

$$\text{ERROR} = \sum_{i=1}^{c} \frac{1}{\omega_i^2} \left(\sum_{j=1}^{s} A_j \, e^{\lambda_j t_i} - \Phi_i \right)^2 \tag{B-2}$$

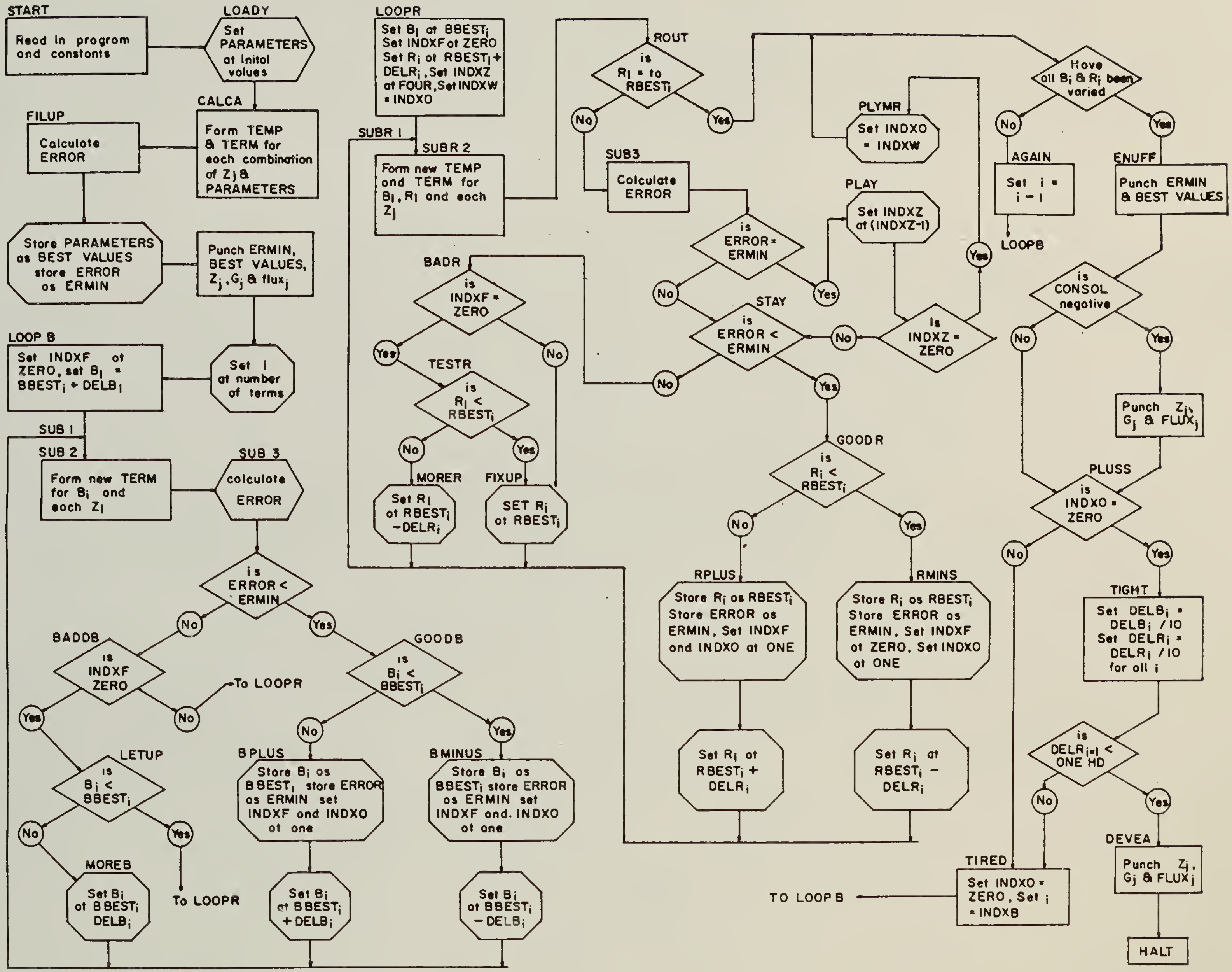
be a minimum.

The program required, in addition to the experimental data and their respective times, initial estimates for a specific number of parameters. The program had a capacity for up to 20 data points and 10 terms in the summation of equation (B-1). These input data were read into the machine along with the program deck on one-word load cards. Each one-word load card contained a particular constant or an initial value and its specified storage location. Table B-1 lists the various input data needed for this program.

Table B-1. Input data required for use of the IBM-650 program which fit empirically experimental data with several terms of Exponential form.

Symbol	Explanation	Storage Location
ZERD	0.00	0073
FPONE	1.00	0129
FOUR	4.00	0185
TEN	10.00	0158
HNDRD	100.00	0090
ONE	Index Number 1 (0000000001)	0392
EIGHT	Index Number 8 (0000000008)	0024
INDXB	Number of Exponential Terms (00000000xx)	0412
INDXA	Number of Data Points (00000000xx)	0062
ONEHD	Precision	0168
$DELB_j$	Initial increments of the Amplitude, B_j	(1800 + i)
$DELR_j$	Initial increments of R_j	(1810 + j)
$BINIT_j$	Initial estimate of B_j	(1300 + j)
$RINIT_j$	Initial estimate of R_j	(1310 + j)
Z_i	Time of ith data point	(1200 + i)
$FLUX_i$	Experimental data at ith point	(1200 + i)
W_i	Weighting function at ith point	(1750 + i)

The machine yielded an answer card having a capacity of 8 words, a word being ten digit numbers and a sign. For the first answer the machine punched out the initial estimates of the fitting parameters on as many cards as was necessary to accommodate them. $\underline{A}$ and $\underline{\lambda}$ for the first term were stored in word locations 1 and 2, respectively. After a card was filled to its 8 word capacity, it was punched and a new card began to fill. This procedure continued until all of the fitting parameters had been punched out. Then a separate card was punched giving in the word 8 location, the value of the sum of the weighted of the residuals between the experimental data and the calculated values. The machine then punched out values of the time, residual and correct values at the last data point in word locations 1, 2 and 3, respectively and the next to last data point in word locations 5, 6 and 7, respectively. The same information for two previous data points was punched out on a second card in the same format and this procedure was continued until the position residual and correct value was punched out for each data point. Subsequent improvements in the parameters and the weighted sum of squares of the residuals were printed out after each cycle of trying to vary each parameter. The positions, residuals and correct values at each data point were obtained at this time if the console instruction was negative. When the fitting parameters could not be further improved with the most refined increment specified, the punching of the best fit parameters, the sum of the weighted square of the residuals, the position, residual and correct values took place according to the procedure described above.



LOGIC DIAGRAM-APPENDIX B

OBJECT PROGRAM-APPENDIX 8

```
            8LR  1200      1830                 1   0000   00 0000 0000
            8LR  1951      1960                 2   0000   00 0000 0000
            8LR  1977      1985                 3   0000   00 0000 U000
            8YN  8TART     1999                 4   0000   00 0000 0000
            8YN  Z         1200                 5   0000   00 0000 0000
            8YN  FLUX      1220                 6   0000   00 0000 0000
            8YN  G         1240                 7   0000   00 0000 0000
            8YN  8         1260                 8   0000   00 0000 0000
            8YN  8BEST     1270                 9   0000   00 0000 0000
            8YN  R         1280                10   0000   00 0000 0000
            8YN  R8EST     1290                11   0000   00 0000 0000
            8YN  8INIT     1300                12   0000   00 0000 0000
            8YN  RINIT     1310                13   0000   00 0000 0000
            8YN  DEL8      1800                14   0000   00 0000 0000
            8YN  DELR      1810                15   0000   00 0000 0000
            8YN  TEMP      1320                16   0000   00 0000 0000
            8YN  TERM      1520                17   0000   00 0000 0000
            8YN  C         1720                18   0000   00 0000 0000
            8YN  FRAC      1730                19   0000   00 0000 0000
            8YN  QUOT      1740                20   0000   00 0000 0000
            8YN  W         1750                21   0000   00 0000 0000
PRNTM       8TO  EXXXT                         22   0000   24 0003 0006
            LDO            EDDCL               23   0006   69 0009 0012
            LOD  INOXA                         24   0009   69 0062 0015
            RAA  8001      RE88T               25   0015   80 8001 0021
RE88T       LDO  EIGHT                         26   0021   69 0024 U027
            R88  8001      8TUUF               27   0027   83 8001 0033
8TUUF       LDO  Z       A                     28   0033   69 3200 0053
            8TO  1985    8                     29   0053   24 5985 0038
            LDO  G       A                     30   0038   69 3240 0043
            8TD  1986    8                     31   0043   24 5986 0039
            LDD  FLUX    A                     32   0039   69 3220 0023
            8TO  1987    8                     33   0023   24 5987 0040
            8XA  0001                          34   0040   51 0001 0046
            AX8  0004                          35   0046   52 0004 0002
            NZA  MDRRG     PONNM               36   0002   40 0005 0056
PONNM       PCH  1977      EXXXT               37   0056   71 1977 0003
MDRRG       NZB  8TUUF     CARRO               38   0005   42 0033 0059
CARRU       PCH  1977                          39   0059   71 1977 0077
            LDO  RE88T     EDDCL               40   0077   69 0021 0012
EOOEA       8TD  AAA1                E         41   0050   24 0103 0106
            8TL  AAA14                         42   0106   20 0011 0014
            RAU  AAA16               Z         43   0014   60 0017 0071
            FAM  AAA14                         44   0071   37 0011 0037
            8TU  AAA2                          45   0037   21 0042 0045
            LOD  AAA3                O         46   0045   69 0040 0001
            8TD  AAA4      AAA8      C         47   0001   24 0004 0007
AAA8        RAU  AAA2                          48   0007   60 0042 0047
            F88  AAA5                F         49   0047   33 0100 0127
            8MI  AAA6                          50   0127   46 0030 0031
            8TU  AAA2                          51   0031   21 0042 0095
            RAU  AAA4                          52   0095   60 0004 0109
            FMP  AAA7                E         53   0109   39 0112 0162
            8TU  AAA4      AAA8                54   0162   21 0004 0007
AAA6        RAU  AAA2                          55   0030   60 0042 0097
            F88  AAA3                O         56   0097   33 0048 0025
            8MI  AAA28                         57   0025   46 0028 0029
            8TU  AAA2                          58   0029   21 0042 0145
            RAU  AAA4                          59   0145   60 0004 0159
            FMP  AAA9                E         60   0159   39 0212 0262
            8TU  AAA4      AAA6                61   0262   21 0004 0030
AAA28       RAU  AAA2                          62   0028   60 0042 0147
            F88  AAA10               P         63   0147   33 0150 0177
            8MI  AAA11                         64   0177   46 0080 0081
            8TU  AAA2                          65   0081   21 0042 0195
            RAU  AAA4                          66   0195   60 0004 0209
            FMP  AAA12               E         67   0209   39 0312 0362
            8TU  AAA4      AAA28               68   0362   21 0004 0028
AAA11       RAU  AAA2                          69   0080   60 0042 0197
            LOD            AAA17               70   0197   69 0200 0153
            FMP  AAA4                          71   0200   39 0004 0054
            8TU  AAA13                         72   0054   21 0008 0061
            RAU  AAA14                         73   0061   60 0011 0065
            8MI  AAA15                         74   0065   46 0018 0019
            RAL  AAA13     AAA1                75   0019   65 0008 0103
AAA15       RAU  AAA3                          76   0018   60 0048 0203
            FOV  AAA13                         77   0203   34 0008 0058
            RAL  8003      AAA1                78   0058   65 8003 0103
AAA17       8TO  AAA18               E         79   0153   24 0156 0259
            RAU  AAA3                D         80   0259   60 0048 0253
            FAD  AAA2                          81   0253   32 0042 0069
            8TU  AAA19                         82   0069   21 0074 0227
            LOO  AAA27               T         83   0227   69 0130 0083
            8TO  AAA20               N         84   0083   24 0036 0089
            8TO  AAA21               O         85   0089   24 0092 0245
            RAU  AAA2                          86   0245   60 0042 0247
            FMP  AAA2                          87   0247   39 0042 0142
            8TU  AAA23     AAA22     N         88   0142   21 0096 0049
AAA22       FOV  AAA21                         89   0049   34 0092 0192
            8TU  AAA24               N         90   0192   21 0146 0099
            FAO  AAA19                         91   0099   32 0074 0051
            8TU  AAA19               T         92   0051   21 0074 0277
            RAU  AAA24                         93   0277   60 0146 0101
            FDV  AAA19                         94   0101   34 0074 0124
            F88  AAA25               C         95   0124   33 0327 0303
            8MI            AAA26               96   0303   46 0206 0057
            RAU  AAA19     AAA18               97   0206   60 0074 0156
AAA26       RAU  AAA20                         98   0057   60 0036 0041
            FAD  AAA3                          99   0041   32 0048 0075
            8TU  AAA20               N        100   0075   21 0036 0139
            FMP  AAA21                        101   0139   39 0092 0242
            8TU  AAA21               N        102   0242   21 0092 0295
            RAU  AAA23                        103   0295   60 0096 0151
            FMP  AAA2                         104   0151   39 0042 0292
            8TU  AAA23     AAA22              105   0292   21 0096 0049
AAA3         10  0000      0051      O        106   0048   10 0000 0051
AAA5         50  0000      0051      F        107   0100   50 0000 0051
AAA7         14  8410      0053      E        108   0112   14 8410 0053
AAA9         27  1830      0051      E        109   0212   27 1830 0051
AAA10        20  0000      0050      P        110   0150   20 0000 0050
AAA12        12  2140      0051      E        111   0312   12 2140 0051
AAA16        00  0000      0000      Z        112   0017   00 0000 0000
AAA25        70  0000      0047      C        113   0327   70 0000 0047
AAA27        20  0000      0051      T        114   0130   20 0000 0051
EOOCL       8TD  ZZZ1                         115   0012   24 0115 0068
```

```
        LDD  ZZZ10             116  0068  69 0121 0174
        STO   1977             117  0174  24 1977 0180
        STO   1978             118  0180  24 1978 0131
        STO   1979             119  0131  24 1979 0032
        STO   1980             120  0032  24 1980 0133
        STO   1981             121  0133  24 1981 0034
        STO   1982             122  0034  24 1982 0035
        STO   1983             123  0035  24 1983 0086
        STO   1984     ZZZ1    124  0086  24 1984 0115
ZZZ10    00   0000     0000    125  0121  00 0000 0000
TEMPP   STO  EGSIT             126  0250  24 0353 0256
        RAU  Z       A         127  0256  60 3200 0055
        FMP  R       B         128  0055  39 5280 0230
        STU  RZEE              129  0230  21 0084 0087
        FAD  HNDRD             130  0087  32 0090 0067
        BMI  STO       GO      131  0067  46 0020 0171
STO     LDD  ZERO              132  0020  69 0073 0026
        STO  TEMP    C EGSIT   133  0026  24 7320 0353
GO      RAL  RZEE              134  0171  65 0084 0189
        LDD            EOOEA   135  0189  69 0342 0050
        STL  TEMP    C EGSIT   136  0342  20 7320 0353
TURMM   STO  EXXIT             137  0300  24 0403 0306
        RAU  TEMP    C         138  0306  60 7320 0125
        FMP  B       B         139  0125  39 5260 0010
        STU  TERM    C EXXIT   140  0010  21 7520 0403
ERR     STO  EXIT              141  0350  24 0453 0356
        LDD  ZERO              142  0356  69 0073 0076
        RAC   8001             143  0076  88 8001 0082
        STO  ERROR             144  0082  24 0085 0088
        LDD  INDXA             145  0088  69 0062 0165
        RAA   8001     READY   146  0165  80 8001 0221
READY   LDD  INDXC             147  0221  69 0224 0377
        AXC   8001             148  0377  58 8001 0183
        LDD  ZERO              149  0183  69 0073 0126
        STO  ACCUM     DOONE   150  0126  24 0079 0132
DOONE   RAU  TERM    C         151  0132  60 7520 0175
        FAD  ACCUM             152  0175  32 0079 0105
        STU  ACCUM             153  0105  21 0079 0182
        LDD  INDXA             154  0182  69 0062 0215
        SXC   8001             155  0215  59 8001 0271
        NZC  REPET     DIDIT   156  0271  40 0274 0225
REPET   BMC  DIDIT     DOONE   157  0274  49 0225 0132
DIDIT   RAU  ACCUM             158  0225  60 0079 0233
        FSB  FLOX    A         159  0233  33 3220 0297
        STU  G       A         160  0297  21 3240 0093
        FMP  G       A         161  0093  39 3240 0140
        FDV  W       A         162  0140  34 3750 0400
        FAD  ERROR             163  0400  32 0085 0111
        STU  ERROR             164  0111  21 0085 0138
        SXC   0001             165  0138  59 0001 0044
        SXA   0001             166  0044  51 0001 0450
        NZA  READY     EXIT    167  0450  40 0221 0453
PRINT   STO  EXET              168  0500  24 0503 0406
        LDD            EODCL   169  0406  69 0309 0012
        LDD  INDXB             170  0309  69 0412 0265
        RAB   8001     RESIT   171  0265  82 8001 0321
RESIT   LDD  EIGHT             172  0321  69 0024 0427
        RSA   8001     FILL    173  0427  81 8001 0283
FILL    LDD  BBEST   B         174  0283  69 5270 0123
        STO   1985   A         175  0123  24 3985 0188
        LDD  RBEST   B         176  0188  69 5290 0143
        STO   1986   A         177  0143  24 3986 0239
        SXB   0001             178  0239  53 0001 0345
        AXA   0002             179  0345  50 0002 0201
        NZB  NEXTB     WHOPE   180  0201  42 0104 0155
NEXTB   NZA  FILL      CARDZ   181  0104  40 0283 0108
CARDZ   PCH   1977             182  0108  71 1977 0477
        LDD  RESIT     EODCL   183  0477  69 0321 0012
WHOPE   PCH   1977             184  0155  71 1977 0527
        LDD            EODCL   185  0527  69 0280 0012
        LDD  ERMIN             186  0280  69 0333 0136
        STO   1984             187  0136  24 1984 0137
        PCH   1977     EXET    188  0137  71 1977 0503
PRIME   STO  TIXE              189  0550  24 0553 0456
        LDD  B       B         190  0456  69 5260 0013
        STO  BBEST   B         191  0013  24 5270 0173
        LDD  R       B         192  0173  69 5280 0383
        STO  RBEST   B         193  0383  24 5290 0193
        LDD  ERROR             194  0193  69 0085 0238
        STO  ERMIN     TIXE    195  0238  24 0333 0553
START   LDD  INDXB             196  1999  69 0412 0315
        RAB   8001     LOADY   197  0315  82 8001 0371
LOADY   LDD  BINIT   B         198  0371  69 5300 0603
        STO  B       B         199  0603  24 5260 0063
        STO  BBEST   B         200  0063  24 5270 0223
        LDD  RINIT   B         201  0223  69 5310 0113
        STO  R       B         202  0113  24 5280 0433
        STU  RBEST   B         203  0433  24 5290 0243
        SXB   0001             204  0243  53 0001 0149
        NZB  LOADY     SETUP   205  0149  42 0371 0653
SETUP   RAU  INDXA             206  0653  60 0062 0117
        RAA   8001             207  0117  80 8001 0273
        LDD  INDXB             208  0273  69 0412 0365
        RAB   8001             209  0365  82 8001 0421
        MPY   8001             210  0421  19 8001 0094
        STL  INDXC             211  0094  20 0224 0577
        RAC   8001     CALCA   212  0577  88 8001 0483
CALCA   LDD            TEMPP   213  0483  69 0186 0250
        LDD            TURMM   214  0186  69 0289 0300
        SXC   0001             215  0289  59 0001 0395
        SXA   0001             216  0395  51 0001 0251
        NZA  CALCA     CALCB   217  0251  40 0483 0205
CALCB   LDD  INDXA             218  0205  69 0062 0415
        RAA   8001             219  0415  80 8001 0471
        SXB   0001             220  0471  53 0001 0627
        NZB  CALCA     FILUP   221  0627  42 0483 0101
FILUP   LDD            ERR     222  0181  69 0134 0350
        LDD            PRIME   223  0134  69 0187 0550
        LDD            PRINT   224  0187  69 0190 0500
        LDD            PRNTM   225  0190  69 0293 0000
        LDD  INDXB             226  0293  69 0412 0465
        RAB   8001             227  0465  82 8001 0521
        LDD  INDXC             228  0521  69 0224 0677
        RAC   8001     LOOPB   229  0677  88 8001 0533
LOOPB   LDD  ZERO              230  0533  69 0073 0176
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          BTD  INDXF                  231   0176   24 01.    .232
          RAU  BBEST  B               232   0232   60 527     275
          FAD  DELB   B  SUBB1        233   0275   32 5800   .727
SUBB1     BTU  B      B               234   0727   21 5260   0163
          LDD   8007                  235   0163   69 8007   0119
          BTD  CTEMP                  236   0119   24 0022   0325
          LDO   8006                  237   0325   69 8006   0231
          STD  BTEMP                  238   D231   24 01R4   0237
          LDD  INDXA                  239   0237   69 0062   0515
          RAA   8001     BUBB2        240   0515   80 8001   0571
BUBB2     LDD            TURMM        241   0571   69 0324   0300
          BXC   0001                  242   0324   59 0001   0330
          SXA   0001                  243   0330   51 0001   0236
          NZA  SUBB2     SUBB3        244   0236   40 0571   0240
SUBR3     LDD            ERR          245   0240   69 0343   0350
          RAU  ERMIN                  246   0343   60 0333   0287
          FBR  ERRDR                  247   0287   33 00R5   0161
          RMI  BADDR     GOODR        248   0161   46 0064   0565
GODDR     LDD  BTEMP                  249   0565   69 01R4   0337
          RAB   8001                  250   0337   82 8001   0393
          LDD  CTEMP                  251   0393   69 0022   0375
          RAC   8001                  252   0375   8R 8001   0281
          RAU  BBEST  B               253   02R1   60 5270   0425
          FSR  B      B               254   0425   33 5260   0387
          RMI  RPLUS     BMINS        255   0387   46 0290   0091
BPLUB     LDD            PRIME        256   0290   69 0443   0550
          LDD  FPONE                  257   0443   69 0196   0199
          BTD  INDXF                  258   0199   24 0129   02R2
          STD  INDXD                  259   0282   24 0135   0288
          RAU  BBEBT  B               260   0288   60 5270   0475
          FAD  DELB   B  SUBB1        261   0475   32 5800   0727
BMINB     LDD            PRIME        262   0091   69 0144   0550
          LDD  ZERO                   263   0144   69 0073   0226
          BTD  INDXF                  264   0226   24 0129   0332
          LDD  FPDNE                  265   0332   69 0196   0249
          BTD  INDXO                  266   0249   24 0135   033R
          RAU  BBEST  B               267   0338   60 5270   0525
          FSB  DELB   B  BURB1        268   0525   33 5800   0727
BADDB     LDD  BTEMP                  269   0064   69 01R4   0437
          RAB   8001                  270   0437   82 8001   0493
          RAO  INDXF                  271   0493   60 0129   0583
          NZU  LDDPR     LETUP        272   0583   44 0487   038R
LETUP     LDD  BTEMP                  273   0388   69 0184   0537
          RAB   8001                  274   0537   R2 8001   0543
          RAU  BBEST  B               275   0543   60 5270   0575
          FBR  B      B               276   0575   33 5260   0587
          BMI  MDREB     LODPR        277   0587   46 0340   04R7
MDREB     LDD  BTEMP                  278   0340   69 01R4   0637
          RAB   B001                  279   0637   82 8001   0593
          LDD  CTEMP                  280   0593   69 0022   0625
          RAC   8001                  281   0625   88 8001   0331
          RAU  BBEST  B               282   0331   60 5270   0675
          FSR  DELB   B  BUBB1        283   0675   33 5800   0727
LDDPR     LDD  ZERD                   284   0487   69 0073   0276
          BTD  INDXF                  285   0276   24 0129   0382
          LDD  FDUR                   286   0382   69 0185   0438
          BTD  INDXZ                  287   0438   24 0141   0194
          LDD  INDXD                  288   0194   69 0135   0488
          BTD  INOXW                  289   0488   24 0191   0244
          LDD  BBEST  R               290   0244   69 5270   0323
          STD  B      B               291   0323   24 5260   0213
          STD  BINIT  B               292   0213   24 5300   0703
          RAU  RBEST  B               293   0703   60 5290   0445
          FAD  DELR   B  BUBR1        294   0445   32 5810   0687
BUBR1     BTU  R      B               295   0687   21 52R0   0633
          LDD  CTEMP                  296   0633   69 0022   0725
          RAC   8001                  297   0725   8R 8001   0381
          LDD  BTEMP                  298   0381   69 0184   0737
          RAB   8001                  299   0737   82 8001   0643
          LDD  INDXA                  300   0643   69 0062   0615
          RAA   8001     SUBR2        301   0615   80 8001   0621
BUBR2     LDD            TEMPP        302   0621   69 0374   0250
          LDD            TURMM        303   0374   69 0777   0300
          BXC   0001                  304   0777   59 0001   0683
          BXA   0001                  305   0683   51 0001   0339
          NZA  BUBR2     RDUT         306   0339   40 0621   0693
RDUT      RAU  RREBT  B               307   0693   60 5290   0495
          FSB  R      B               308   0495   33 52R0   0107
          NZU  SUBR3     NEXT         309   0107   44 0211   0462
BUBR3     LDD            ERR          310   0211   69 0114   0350
          RAU  ERMIN                  311   0114   60 0333   0787
          FSB  ERRDR                  312   0787   33 00R5   0261
          NZU  BTAY      PLAY         313   0261   44 0665   0016
PLAY      RAU  INDXZ                  314   0016   60 0141   0545
          FBB  FPONE                  315   0545   33 0196   0373
          BTU  INDXZ                  316   0373   21 0141   0294
          NZU  STAY      PLYMR        317   0294   44 0665   009R
BTAY      BMI  BADDR     GODDR        318   0665   46 0118   0169
PLYMR     LDD  INDXW                  319   0098   69 0191   0344
          STD  INDXO     NEXT         320   0344   24 0135   0462
GOODR     LDD  BTEMP                  321   0169   69 01R4   0837
          RAB   8001                  322   0837   82 8001   0743
          LDD  CTEMP                  323   0743   69 0022   0775
          RAC   8001                  324   0775   88 8001   0431
          RAU  RBEBT  B               325   0431   60 5290   0595
          FBB  R      B               326   0595   33 52R0   0157
          BMI  RPLUB     RMINS        327   0157   46 0060   0311
RPLUB     LDD            PRIME        328   0060   69 0263   0550
          LDD  FPONE                  329   0263   69 0196   0299
          BTD  INDXF                  330   0299   24 0129   0432
          BTD  INDXD                  331   0432   24 0135   053R
          RAO  RBEST  B               332   0538   60 5290   0645
          FAD  DELR   B  SOBR1        333   0645   32 5810   0687
RMINB     LDD            PRIME        334   0311   69 0164   0550
          LDD  ZERO                   335   0164   69 0073   0326
          STD  INDXF                  336   0326   24 0129   0482
          LDD  FPDNE                  337   0482   69 0196   0349
          BTD  INDXD                  338   0349   24 0135   0588
          RAU  RBEBT  B               339   0588   60 5290   0695
          FBB  DELR   B  SOBR1        340   0695   33 5810   0687
BADDR     LDD  FDUR                   341   0118   69 01R5   063R
          BTD  INDXZ                  342   063R   24 0141   0394
          RAU  INDXF                  343   0394   60 0129   0733
          NZU  FIXUP     TESTR        344   0733   44 08R7   0688
TESTR     LDD  BTEMP                  345   0688   69 01R4   0937
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          RAB    8001                 346   0937   82 8001 0793
          RAU  RBEST  B               347   0793   60 5290 0745
          FSB  R      B               348   0745   33 5280 0207
          BMI  MORER       FIXUP      349   0207   46 0110 0887
MORER     LDD  STEMP                  350   0110   69 0184 0987
          RAB    8001                 351   0987   82 8001 0843
          LDD  CTEMP                  352   0843   69 0022 0825
          RAC    8001                 353   0825   88 8001 0481
          RAU  RBEST  B               354   0481   60 5290 0795
          FSB  DELR   B    SUBR1      355   0795   33 5810 0687
FIXUP     RAU  RBEST  B    SUBR1      356   0887   60 5290 0687
NEXT      LDD  RBEST  B               357   0462   69 5290 0893
          STD  RINIT  B               358   0893   24 5310 0313
          SXB    0001                 359   0313   53 0001 0219
          NZB  AGAIN       ENUFF      360   0219   42 0072 0423
AGAIN     RAU  CTEMP                  361   0072   60 0022 0827
          SUP  INDXA                  362   0827   11 0062 0167
          STU  CTEMP                  363   0167   21 0022 0875
          RAU  BTEMP                  364   0875   60 0184 0389
          SUP  ONE                    365   0389   11 0392 0347
          STU  BTEMP                  366   0347   21 0184 1037
          LDD  CTEMP                  367   1037   69 0022 0925
          RAC    8001                 368   0925   88 8001 0531
          LDD  BTEMP                  369   0531   69 0184 1087
          RAB    8001      LOOPB      370   1087   82 8001 0533
ENUFF     LDD              PRINT      371   0423   69 0376 0500
          LDD    8000                 372   0376   69 8000 0532
          RAU    8001                 373   0532   60 8001 0439
          BMI  OEVEA       PLUSS      374   0439   46 0442 0943
PLUSB     RAU  INDXO                  375   0943   60 0135 0489
          NZU  TIREO       TIGHT      376   0489   44 0993 0444
TIRED     LDD  INDXB                  377   0993   69 0412 0715
          RAB    8001                 378   0715   82 8001 0671
          LDD  INDXC                  379   0671   69 0224 0877
          RAC    8001                 380   0877   88 8001 0783
          LDD  ZERO                   381   0783   69 0073 0426
          STD  INDXO       LOOPS      382   0426   24 0135 0533
TIGHT     LDD  INOXB                  383   0444   69 0412 0765
          RAB    8001      RETTN      384   0765   82 8001 0721
RETTN     RAU  OELB   B               385   0721   60 5800 0255
          FDV  TEN                    386   0255   34 0158 0208
          STU  DELB   B               387   0208   21 5800 0753
          RAU  DELR   B               388   0753   60 5810 0815
          FDV  TEN                    389   0815   34 0158 0258
          STU  DELR   B               390   0258   21 5810 0363
          SXB    0001                 391   0363   53 0001 0269
          NZB  RETTN       QUAAD      392   0269   42 0721 0473
QUAAD     RAB    0001                 393   0473   82 0001 0179
          RAU  DELR   B               394   0179   60 5810 0865
          FSB  ONEHD                  395   0865   33 0168 0845
          BMI  OEVEA       TIREO      396   0845   46 0442 0993
DEVEA     LDD              EDDCL      397   0442   69 0895 0012
          LDD  INDXA                  398   0895   69 0062 0915
          RAA    8001      RESET      399   0915   80 8001 0771
RESET     LDD  EIGHT                  400   0771   69 0024 0927
          RSB    8001      STUFF      401   0927   83 8001 0833
STUFF     LDD  Z      A               402   0833   69 3200 0803
          STD    1985 B               403   0803   24 5985 0738
          LDD  G      A               404   0738   69 3240 1043
          STD    1986 B               405   1043   24 5986 0539
          LDD  FLUX   A               406   0539   69 3220 0523
          STD    1987 B               407   0523   24 5987 0390
          SXA    0001                 408   0390   51 0001 0246
          AXB    0004                 409   0246   52 0004 0052
          NZA  MORES       FINIS      410   0052   40 0305 0506
MORES     NZB  STUFF       CARDS      411   0305   42 0833 0359
CARDS     PCH    1977                 412   0359   71 1977 0977
          LDD  RESET       EDDCL      413   0977   69 0771 0012
FINIS     RAB    0001                 414   0506   82 0001 0518
          RAU  OELR   B               415   0512   60 5810 0965
          FSB  ONEHO                  416   0965   33 0168 0945
          BMI  END         SOMOR      417   0945   46 0148 0399
SOMOR     PCH    1977      PLUSB      418   0399   71 1977 0943
END       PCH    1977       8000      419   0148   71 1977 8000
                                            0073   00 0000 0000
                                            0196   10 0000 0051
                                            0185   40 0000 0051
                                            0158   10 0000 0052
                                            0090   10 0000 0053
                                            0392   00 0000 0001
                                            0024   00 0000 0008
```

APPENDIX C

Description and Explanation of the IBM-650 Computer Program Used for Fitting Empirically Experimental Data with an Even Fourier Series

The computer code was written to fit an even Fourier series to the experimentally determined surface temperature traces during a transient burst. The program was written in SOAP II and floating point form. The object program is listed and the logic diagram is shown in this appendix.

The data were fit empirically by a finite number of terms of the even trigonometric series.

$$\theta(t) = \sum_{i=1}^{p} B_i \cos \frac{2\pi i t}{a} , \qquad \text{(C-1)}$$

where

$$B_o = 2/a \int_o^a y(t)\, dt$$

and

$$B_i = 1/a \int_o^a y(t) \cos \frac{2\pi i t}{a}\, dt .$$

The integrations were carried out numerically by means of Simpson's rule thus requiring an odd number of data points.

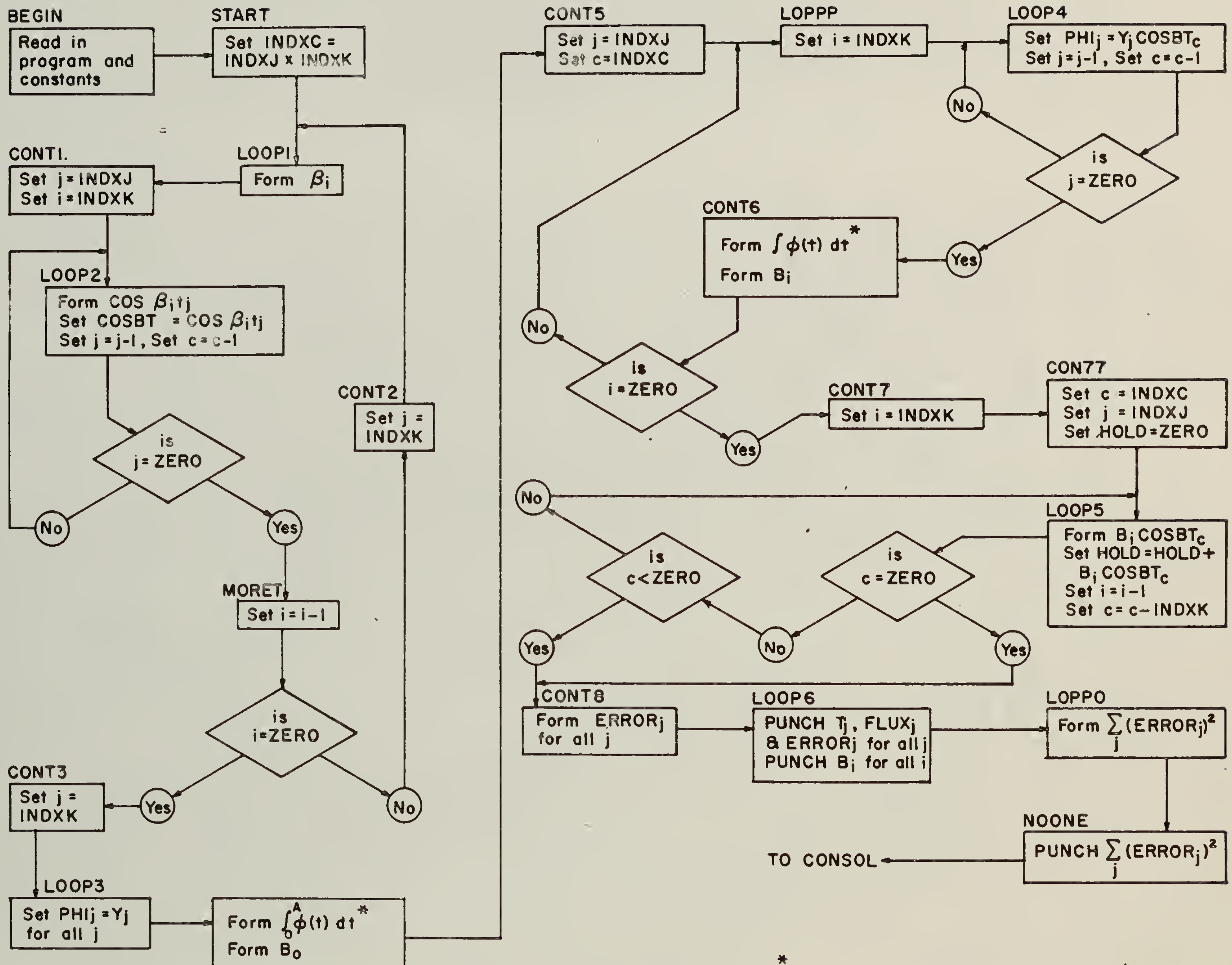
The program input consisted of the experimental data and their respective times, the period, the time increment between data points and a specification of the number of terms. These data were read into the machine on one-word load cards. Each one-word load card contained a particular constant or piece of data and its specific storage location. Table C-1 lists the input data needed for this program. Storage locations limit the product of the number of terms and the number of data points to less than 550. The number of terms and the number of data points are each limited to less than 50.

Table C-1. Input data required for use of the IBM-650 program which fits empirically experimental data with a finite number of terms of an even Fourier series.

Symbol	Expanation	Storage Location
ZERO	0.00	0083
FPONE	1.00	0034
FPTWO	2.00	0108
FPTRE	3.00	0207
FPFOR	4.00	0008
PI	3.14159	0261
INDX2	Index Number 2 (0000000002)	0160
INDXJ	Number of Terms (00000000xx)	1001
INDXK	Number of Data Points (00000000xx)	1002
A	Period of Cosine Terms	1003
H	Time Increment Between Data Points	1004
T_i	Time of ith data point	(1100 + i)
Y_i	Experimental data at ith point	(1150 + i)

The machine punched out a card having an eight word capacity, each work consisting of 10 digits and a sign. The first output consisted of the time, the calculated value and the residual between the calculated values and the experimental data for the last data point in word locations 1, 2 and 3, respectively. The same information for the next to last data point was punched out in columns 5, 6 and 7 of the same card. The same information for the two previous data points was punched out on the next card. The above procedure was continued until the time, residuals and calculated values were punched for each data point. $\underline{B_1}$ was then stored in word location 1, $\underline{B_2}$ in word location two, etc. until all of the <u>B's</u>

had been stored and punched. If more than 8 <u>B's</u> were calculated the first 8 were stored and a card punched. Additional <u>B's</u> were punched in succeeding cards with the lower number <u>B's</u> starting on the left of each card. Finally the summation of the square of the residual at each data point was punched out in word location 8 of a final card.



* Numerical integration using Simpson's Rule.

LOGIC DIAGRAM-APPENDIX C

OBJECT PROGRAM-APPENDIX C

```
         BLR   1000      1999                      1   0000   00 0000 0000
         SYN  ERROR      1050                      2   0000   00 0000 0000
         BLR   0977      0985                      3   0000   00 0000 0000
         SYN  T          1100   T1 SMALL           4   0000   00 0000 0000
         SYN  Y          1150   Y1 WITH T1         5   0000   00 0000 0000
         SYN  S          1200                      6   0000   00 0000 0000
         SYN  STA        1250                      7   0000   00 0000 0000
         SYN  PHI        1300                      8   0000   00 0000 0000
         SYN  FLUX       1350                      9   0000   00 0000 0000
         SYN  COSBT      1400                     10   0000   00 0000 0000
         SYN  START      1000                     11   0000   00 0000 0000
         SYN  INDXJ      1001   NO TERMS          12   0000   00 0000 0000
         SYN 'INOXK      1002   NO DATA PT        13   0000   00 0000 0000
         SYN  A          1003                     14   0000   00 0000 0000
         SYN  H          1004                     15   0000   00 0000 0000
EOOCR    STO  EXIT                                16   0000   24 0003 0006
         BMI  NEGAT      REDUC                    17   0006   46 0009 0010
NEGAT    FAO  TWOPI                               18   0009   32 0012 0039
         BMI  NEGAT                               19   0039   46 0009 0043
         FSB  ONEPI      COSIO                    20   0043   33 0046 0023
REDUC    FSB  TWOPI                               21   0010   33 0012 0089
         BMI             REOUC                    22   0089   46 0042 0010
         FAD  ONEPI      COSIU                    23   0042   32 0046 0023
COSIO    STU  THETA                               24   0023   21 0028 0031
         RSU  FPDNE                               25   0031   61 0034 0139
         STU  TERMM                               26   0139   21 0044 0047
         STU  FUNKT                               27   0047   21 0002 0005
         STL  ENN        NEGST                    28   0005   20 0059 0062
EOOSR    STO  EXIT                                29   0050   24 0003 0056
         BMI  NEGAV      REOUD                    30   0056   46 0109 0060
NEGAV    FAD  TWOPI                               31   0109   32 0012 0189
         BMI  NEGAV                               32  ·0189   46 0109 0093
         FSB  ONEPI      SINET                    33   0093   33 0046 0073
REDUU    FSB  TWOPI                               34   0060   33 0012 0239
         BMI             REDUD                    35   0239   46 0092 0060
         FAO  ONEPI      SINET                    36   0092   32 0046 0073
SINET    STU  THETA                               37   0073   21 0028 0001
         RSU   8003                               38   0081   61 8003 0289
         STU  TERMM                               39   0289   21 0044 0097
         STU  FUNKT                               40   0097   21 0002 0055
         LOD  FPONE                               41   0055   69 0034 0037
         STO  ENN        NEGST                    42   0037   24 0059 0062
NEGST    RAU  ENN                                 43   0062   60 0059 0013
         FAO  FPONE                               44   0013   32 0034 0011
         STU  NPDNE                               45   0011   21 0016 0019
         FAO  FPONE                               46   0019   32 0034 0061
         STU  ENN                                 47   0061   21 0059 0112
         RSU  TERMM                               48   0112   61 0044 0049
         FMP  THETA                               49   0049   39 0028 0078
         FMP  THETA                               50   0078   39 0028 0128
         FOV  NPONE                               51   0128   34 0016 0066
         FOY  ENN                                 52   0066   34 0059 0159
         STU  TERMM                               53   0159   21 0044 0147
         RAM  FUNKT                               54   0147   67 0002 0007
         STL  FMAG                                55   0007   20 0111 0014
         RAM  TERMM                               56   0014   67 0044 0099
         RAU   8002                               57   0099   60 8002 0057
         FOV  FMAG                                58   0057   34 0111 0161
         FSB  SIZES                               59   0161   33 0064 0041
         BMI  ENUFF                               60   0041   46 0094 0045
         RAU  FUNKT                               61   0045   60 0002 0107
         FAD  TERMM                               62   0107   32 0044 0021
         STU  FUNKT      NEGST                    63   0021   21 0002 0062
ENUFF    RAU  FUNKT      EXIT                     64   0094   60 0002 0003
SIZES     10   0000      0043                     65   0064   10 0000 0043
TWOPI     62   8318      5351                     66   0012   62 8318 5351
ONEPI     31   4159      2751                     67   0046   31 4159 2751
FPONE     10   0000      0051                     68   0034   10 0000 0051
COS      STO  EXITC                               69   0100   24 '0053 0106
         RAU  STA     A                           70   0106   60 3250 0105
         FMP  T       S                           71   0105   39 5100 0150
         LOD             EOOCH  SUSLDSUP          72   0150   69 0103 0000
         STU  COSBT   ○  EXITC                    73   0103   21 7400 0053
INTGT    STD  EGSIT                               74   0200   24 0153 0156
         RAB   0001                               75   0156   82 0001 0162
         RAU  PHI     S                           76   0162   60 5300 0155
         STU  ACCUM      LOP33                    77   0155   21 0110 0063
LOP33    AXB   0001                               78   0063   52 0001 0069
         RAU  PHI     S                           79   0069   60 5300 0205
         FMP  FPFDR             INPUT             80   0205   39 0008 0058
         FAO  ACCUM                               81   0058   32 0110 0087
         STU  ACCUM                               82   0087   21 0110 0113
         AXB   0001                               83   0113   52 0001 0119
         RAU  PHI     S                           84   0119   60 5300 0255
         FMP  FPTWO             INPUT             85   0255   39 0108 0158
         FAD  ACCUM                               86   0158   32 0110 0137
         STU  ACCUM                               87   0137   21 0110 0163
         RAU   8006                               88   0163   60 8006 0071
         SUP  INDXK                               89   0071   11 1002 0157
         AUP  INOX2               INPUT           90   0157   10 0160 0015
         NZU  LOP33      G0111                    91   0015   44 0063 0020
G0111    AXB   0001                               92   0020   52 0001 0026
         RAU  PHI     S                           93   0026   60 5300 0305
         FMP  FPFOR                               94   0305   39 0008 0208
         FAO  ACCUM                               95   0208   32 0110 0187
         STU  ACCUM                               96   0187   21 0110 0213
         AXB   0001                               97   0213   52 0001 0169
         RAU  PHI     S                           98   0169   60 5300 0355
         FAO  ACCUM                               99   0355   32 0110 0237
         FMP  H                 INPUT            100   0237   39 1004 0004
         FOV  FPTRE      EGSIT  INPUT            101   0004   34 0207 0153
EOOCL    STO  ZZZ1              EXIT INSTR       102   0250   24 0203 0206
         LDO  ZZZ10                              103   0206   69 0209 0212
         STO   0977                              104   0212   24 0977 0030
         STO   0978                              105   0030   24 0978 0131
         STO   0979                              106   0131   24 0979 0032
         STO   0980                              107   0032   24 0980 0033
         STD   0981                              108   0033   24 0981 0084
         STO   0982                              109   0084   24 0982 0035
         STD   0983                              110   0035   24 0983 0036
         STO   0984      ZZZ1                    111   0036   24 0984 0203
ZZZ10     00   0000       0000                   112   0209   00 0000 0000
START    RAU  INDXK                              113   1000   60 1002 0257
         MPY  INDXJ                              114   0257   19 1001 0022
         STL  INDXC                              115   0022   20 0027 0080
```

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         LDD  ZERO                  INPUT       116  0080  69 0083 0086
         RAA   8001                             117  0086  80 8001 0142
         STD  N          LOOP1                  118  0142  24 0095 0048
LOOP1    AXA   0001                             119  0048  50 0001 0054
         RAU  N                                 120  0054  60 0095 0149
         FAD  FPONE                 INPUT       121  0149  32 0034 0211
         STU  N                                 122  0211  21 0095 0098
         FMP  FPTWO                 INPUT       123  0098  39 0108 0258
         FMP  PI                    INPUT       124  0258  39 0261 0311
         FDV  A                     INPUT       125  0311  34 1003 0253
         STU  BTA       A                       126  0253  21 3250 0303
         RAU   8005                             127  0303  60 8005 0361
         SUP  INDXJ                             128  0361  11 1001 0405
         NZU  LOOP1      CONT1                  129  0405  44 0048 0210
CONT1    LDD  INDXJ                             130  0210  69 1001 0104
         RAA   8001                             131  0104  80 8001 0260
         LDD  INDXK                             132  0260  69 1002 0455
         RAB   8001                             133  0455  82 8001 0411
         LDD  INDXC                             134  0411  69 0027 0130
         RAC   8001      LOOP2                  135  0130  88 8001 0136
LOOP2    LDD             COS        SUBROUTINE  136  0136  69 0339 0100
         SXB   0001                             137  0339  53 0001 0145
         SXC   0001                             138  0145  59 0001 0001
         NZB  LOOP2      MORET                  139  0001  42 0136 0505
MORET    SXA   0001                             140  0505  51 0001 0461
         NZA  CONT2      CONT3                  141  0461  40 0114 0065
CONT2    LDD  INDXK                             142  0114  69 1002 0555
         AXB   8001      LOOP2                  143  0555  52 8001 0136
CONT3    RAA   0000                             144  0065  80 0000 0121
         LDD  INDXK                             145  0121  69 1002 0605
         AXB   8001      LOOP3                  146  0605  52 8001 0511
LOOP3   *LDD  Y         B                       147  0511  69 5150 0353
         STD  PHI       B                       148  0353  24 5300 0403
         SXB   0001                             149  0403  53 0001 0259
         NZB  LOOP3      CONT4                  150  0259  42 0511 0263
CONT4    LDD             INTGT      SUBROUTINE  151  0263  69 0116 0200
         FMP  FPTWO                             152  0116  39 0108 0308
         FDV  A                                 153  0308  34 1003 0453
         STU  BZERO      CONT5                  154  0453  21 0358 0561
CONT5    RAA   0000                             155  0561  80 0000 0017
         RAB   0000                             156  0017  82 0000 0123
         RAC   0000      CONS5                  157  0123  88 0000 0029
CONS5    LDD  INDXC                             158  0029  69 0027 0180
         RAC   8001                             159  0180  88 8001 0186
         LDD  INDXJ                             160  0186  69 1001 0154
         RAA   8001      LOPPP                  161  0154  80 8001 0310
LOPPP    LDD  INDXK                             162  0310  69 1002 0655
         RAB   8001      LOOP4                  163  0655  82 8001 0611
LOOP4    RAU  Y         B                       164  0611  60 5150 0705
         FMP  COSBT     C                       165  0705  39 7400 0300
         STU  PHI       R                       166  0300  21 5300 0503
         SXB   0001                             167  0503  53 0001 0309
         SXC   0001                             168  0309  59 0001 0115
         NZR  LOOP4      CONT6                  169  0115  42 0611 0219
CONT6    LDD             INTDT                  170  0219  69 0072 0200
         FMP  FPFOR                             171  0072  39 0008 0408
         FDV  A                                 172  0408  34 1003 0553
         STU  B         A                       173  0553  21 3200 0603
         SXA   0001                             174  0603  51 0001 0359
         NZA  LOPPP      CONT7                  175  0359  40 0310 0313
CONT7    RAA   0000                             176  0313  80 0000 0269
         RAB   0000                             177  0269  82 0000 0025
         LDD  INDXK                             178  0025  69 1002 0755
         AXB   8001                             179  0755  52 8001 0661
         LDD  INDXJ                             180  0661  69 1001 0204
         AXA   8001                             181  0204  50 8001 0360
         RAC   0000      CON77                  182  0360  88 0000 0166
CON77    LDD  INDXC                             183  0166  69 0027 0230
         AXC   8001                             184  0230  58 8001 0236
         LDD  INDXJ                             185  0236  69 1001 0254
         RAA   8001                             186  0254  80 0001 0410
         LDD  ZERO                              187  0410  69 0083 0286
         STD  HOLD       LOOP5                  188  0286  24 0389 0192
LOOP5    RAU  B         A                       189  0192  60 3200 0805
         FMP  COSBT     C                       190  0805  39 7400 0350
         FAD  HOLD                              191  0350  32 0389 0165
         STU  HOLD                              192  0165  21 0389 0242
         SXA   0001                             193  0242  51 0001 0148
         LDD  INDXK                             194  0148  69 1002 0855
         SXC   8001                             195  0855  59 8001 0711
         NZC             CONT8                  196  0711  48 0164 0215
         BMC  CONT8      LOOP5                  197  0164  49 0215 0192
CONT8    RAU  HOLD                              198  0215  60 0389 0143
         FAD  BZERO                             199  0143  32 0358 0085
         STU  FLUX      B                       200  0085  21 5350 0653
         FSB  Y         B                       201  0653  33 5150 0077
         STU  ERROR     B                       202  0077  21 5050 0703
         SXB   0001                             203  0703  53 0001 0409
         SXC   0001                             204  0409  59 0001 0265
         NZB  CON77      PRINT                  205  0265  42 0166 0319
PRINT    LDD  INDXK                             206  0319  69 1002 0905
         AXB   8001      LOOP7                  207  0905  52 8001 0761
LOOP7    LDD             EOOCL      SUBROUTINE  208  0761  69 0214 0250
         RSA   0008      LOOP6                  209  0214  81 0008 0070
LOOP6    LDD  T         B                       210  0070  69 5100 0753
         STD   0985     A                       211  0753  24 2985 0038
         LDD  FLUX      B                       212  0038  69 5350 0803
         STD   0986     A                       213  0803  24 2986 0439
         LDD  ERROR     B                       214  0439  69 5050 0853
         STD   0987     A                       215  0853  24 2987 0040
         AXA   0004                             216  0040  50 0004 0096
         SXB   0001                             217  0096  53 0001 0052
         NZB             FINIS                  218  0052  42 0955 0256
         NZA  LOOP6      CONT9                  219  0955  40 0070 0459
CONT9    PCH   0977      LOOP7                  220  0459  71 0977 0761
FINIS    PCH   0977                             221  0256  71 0977 0127
         RSB   0007                             222  0127  83 0007 0133
         LDD             EOOCL                  223  0133  69 0336 0250
         LDD  BZERO                             224  0336  69 0358 0811
         STD   0977                             225  0811  24 0977 0280
         RAA   0001      LOOP8                  226  0280  80 0001 0386
LOOP8    LDD  B         A                       227  0386  69 3200 0903
         STD   0985     B                       228  0903  24 4985 0088
         AXA   0001                             229  0088  50 0001 0144
         AXB   0001                             230  0144  52 0001 0400
```

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         RAU  8006                231   0400   60 8005 0307
         SUP  INDXJ               232   0307   11 1001 0306
         NZU            FINAL     233   0306   44 0509 0460
         NZB  LOOPB     MOST      234   0509   42 0386 0363
MOST     PCH   0977               235   0363   71 0977 0177
         LDD            EOOCL     236   0177   69 0330 0250
         RSB   0007     LDOPB     237   0330   83 0007 0386
FINAL    LDD  8       A           238   0460   69 3200 0953
         STD   0985   8           239   0953   24 4985 0138
         PCH   0977               240   0138   71 0977 0227
         LDD  INDXK               241   0227   69 1002 0356
         XAB   8001               242   0356   82 8001 0262
         LDD  ZERO                243   0262   69 0083 0436
         STD  SOMME     LOPPU     244   0436   24 0489 0292
LOPPO    RAU  ERROR   8           245   0292   60 5050 0406
         FMP  ERROR   8           246   0406   39 5050 0450
         FAD  SOMME               247   0450   32 0489 0315
         STU  SOMME               248   0315   21 0489 0342
         SXB   0001               249   0342   53 0001 0198
         NZB  LOPPO     NDONE     250   0198   42 0292 0102
NOONE    LDD            EOOCL     251   0102   69 0456 0250
         LDD  SOMME               252   0456   69 0489 0392
         STD   0984               253   0392   24 0984 0287
         PCH   0977      8000     254   0287   71 0977 8000
                                        0083   00 0000 0000
                                        0034   10 0000 0051
                                        0108   20 0000 0051
                                        0207   30 0000 0051
                                        0008   40 0000 0051
                                        0160   00 0000 0002
                                        0261   31 4159 2751
```

APPENDIX D

Description and Explanation of the IBM-650 Computer Program Used to Calculate Temperature Distributions.

The computer program was written to calculate the temperature distribution in a unit cell of a nuclear reactor system given the heat generation rate and fuel element surface temperature as a function of time. The temperature rise over the initial temperature is given by

$$\theta_f(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\left(\frac{n\pi x}{2L}\right) e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{(L^2/n\pi\alpha)\left(\sin\frac{n\pi}{2}\right)}$$

$$\times\left\{\sum_{i=1}^{p} \frac{B_i\left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^2} + \beta_i^2} + \sum_{j=1}^{s} \frac{q_{oo}\alpha A_j \cosh \kappa L}{k\left(\frac{n^2\pi^2\alpha}{4L^2}+\lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)}\right\} \qquad \text{(D-1)}$$

$$+\sum_{j=1}^{s} \frac{q_{oo}\alpha A_j e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{\frac{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}x\right)\cosh \kappa x}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}L\right)} - \cosh \kappa x\right\} .$$

in the fuel and by

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\left(\frac{n\pi x}{2L}\right) e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{(L^2/n\pi\alpha)\left(\sin\frac{n\pi}{2}\right)}$$

$$\times\left\{\sum_{i=1}^{p} \frac{B_i\left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i} - \sum_{j=1}^{s} \frac{F\alpha A_j}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)}\right\} \quad \text{(D-2)}$$

$$+\sum_{j=1}^{s} \frac{F\alpha A_j e^{\rho_j t}}{k(-\lambda_j)} \left\{ \frac{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,x}{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,L} - 1 \right\}$$

in the moderator. The moderator equation is obtained from the fuel temperature distribution by setting q_{oo} equal to F and κ equal to zero.

The equivalence between elements of the algebraic equations and the symbolic logic of the computer program is shown in Table D-1.

Table D-1. Definition of symbolic terms of the IBM-650 computer program for calculating temperature distributions.

$A1_i = B_i Z_i(x) \cos \beta_i t$

$A3P_j = q_{\infty} \alpha A_j e^{\lambda_j t} / k(\alpha\kappa^2 - \lambda_j)$

$$A3SUM_j = \frac{\cosh \kappa L \cosh\sqrt{\frac{\lambda_j}{\alpha}}\,x}{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,L} - \cosh \kappa x$$

$A3_j = (A3P_j)(A3SUM_j)$

$CSHLL_j = \cosh\sqrt{\frac{\lambda_j}{\alpha}}\,L$

$CSHLX_j = \cosh\sqrt{\frac{\lambda_j}{\alpha}}\,x$

$COSLL_j = \cos\sqrt{\frac{\lambda_j}{\alpha}}\,L$

$COSLX_j = \cos\sqrt{\frac{\lambda_j}{\alpha}}\,x$

Table D-1 cont.

$$ARG1_n = n\pi \quad 2L$$

$$ARG2_n = n^2\pi^2\alpha \quad 4L^2$$

$$A2ST1_i = B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right) \Big/ \left(\frac{n^4\pi^4\alpha^2}{16L^2} + \beta_i^2\right)$$

$$A2ST1_j = q_\infty \alpha A_j' \cosh \kappa L / k \left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right) \left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)$$

$$A2DDT_n = \frac{\cos \frac{n\pi x}{2L} \quad e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{\left(\frac{L^2}{n\pi\alpha}\right) \sin \frac{n\pi}{2}}$$

$$A2SUM_n = \left(\sum_{i=1}^{p} A2ST1_i + \sum_{j=1}^{s} A2ST1_j\right)_n$$

$$TERM_n = (A2DDT_n)(A2SUM_n)$$

Table D-2. Input Data Required for Use of the IBM-650 Computer Program Used to Calculate Temperature Distributions.

Symbol	Explanation	Storage Location
ZERO	0.00	0264
ONE	1.00	0662
TWO	2.00	0520
PI	3.14159	0018
FIFTY	50.00	0361
CRIT	0.0001	0788
ALPHA	Thermal Diffusivity	0278

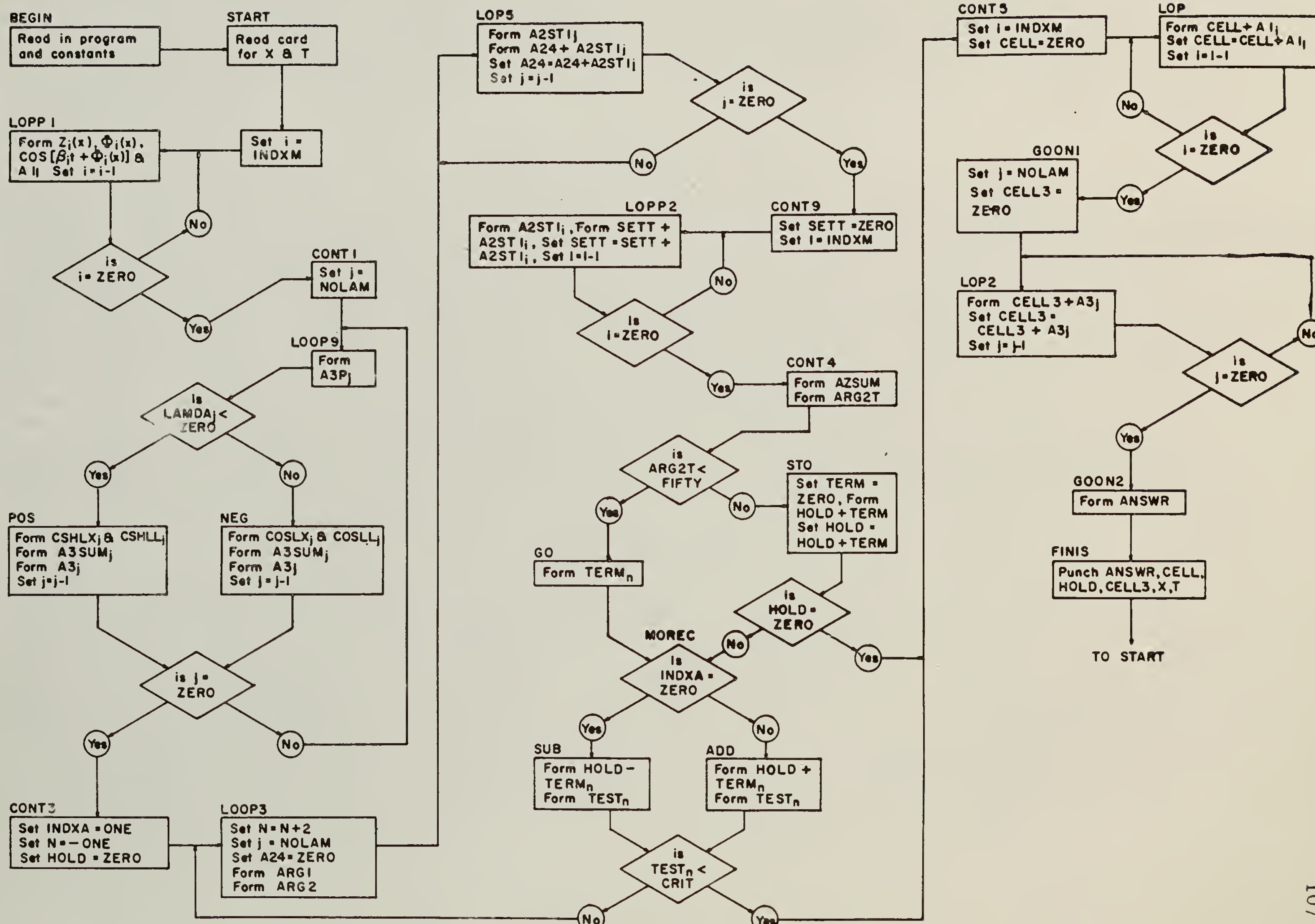
Table D-2 cont.

Symbol	Explanation	Storage Location
KAPPA	Reciprocal of Thermal Neutron Diffusion Length in Fuel	0436
Qoo	Normalization Factor for Heat Generation	0324
KAY	Thermal Conductivity	0581
L	Half-Thickness of Region	0186
INDXM	No. of Terms, Surface Temperature Fit (00000000xx)	0076
NOLAM	No. of Terms, Heat Generation Fit (00000000xx)	0456
$AYEJ_j$	Amplitude Parameter, Heat Generation Fit, A_j	(0200 + j)
$LAMDA_j$	Exponential Parameter, Heat Generation Fit, $_j$	(0220 + j)
$AMMM_i$	Amplitude Parameter, Surface Temperature Fit, B_i	(0100 + i)
$BTAA_i$	Period Parameter Surface Temperature Fit, B_i	(0120 + i)

The output from this program is punched out on one card having an eight word capacity, one word consisting of 10 digits and a sign. The form of the output is shown in Table D-3.

Table D-3. Output Form for IBM-650 Computer Program Used to Calculate Temperature Distribution

WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
$\theta(x,t)$	$\sum_{i=1}^{p} A1_i$	$\sum_{n=1,3,5}^{\infty} (\text{Term})_n$	$\sum_{j=1}^{s} A3_j$	x	t	--	--



LOGIC DIAGRAM-APPENDIX D

OBJECT PROGRAM-APPENDIX D

```
         BLR  0200     0860           1   0000   00 0000 0000   -
         BLR  1800     1899           2   0000   00 0000 0000   -
         SYN  ARTAN    1800           3   0000   00 0000 0000   -
         SYN  START    1999           4   0000   00 0000 0000   -
         SYN  AYEJ     0200           5   0000   00 0000 0000   -
         SYN  LAMDA    0280           6   0000   00 0000 0000   -
         SYN  A3       0240           7   0000   00 0000 0000   -
         BLR  1951     1960           8   0000   00 0000 0000   -
         BLR  1977     1984           9   0000   00 0000 0000   -
         BLR  0100     0160          10   0000   00 0000 0000   -
         SYN  AMMM     0100          11   0000   00 0000 0000   -
         SYN  BTAA     0120          12   0000   00 0000 0000   -
         SYN  ALPHA    0278          13   0000   00 0000 0000   -
         SYN  KAPPA    0436          14   0000   00 0000 0000   -
         SYN  L        0186          15   0000   00 0000 0000   -
         SYN  QOO      0324          16   0000   00 0000 0000   -
         SYN  KAY      0581          17   0000   00 0000 0000   -
         SYN  INOXM    0076          18   0000   00 0000 0000   -
         SYN  NOLAM    0456          19   0000   00 0000 0000   -
         SYN  A1       0140          20   0000   00 0000 0000   -
PHIS8    STD  ABCD2                  21   0000   24 0003 0006
         RAU  FCL                    22   0006   60 0009 0013
         FMP  FCX                    23   0013   39 0016 0066
         STU  DENN1                  24   0066   21 0020 0023
         RAU  FSL                    25   0023   60 0026 0031
         FMP  FSX                    26   0031   39 0034 0084
         FAD  DENN1                  27   0084   32 0020 0047
         STU  DN                     28   0047   21 0002 0005
         RAU  FSL                    29   0005   60 0026 0081
         FMP  FCX                    30   0081   39 0016 0166
         STU  NUMM2                  31   0166   21 0070 0073
         RAU  FCL                    32   0073   60 0009 0063
         FMP  FSX                    33   0063   39 0034 0184
         FSB  NUMM2                  34   0184   33 0070 0097
         FDV  DN                     35   0097   34 0002 0052
         LDD           ARTAN         36   0052   69 0055 1800
         STU  THETO                  37   0055   21 0010 0163
         RAU  DN                     38   0163   60 0002 0007
         BMI           ABCD2         39   0007   46 0060 0003
         RAU  THETO                  40   0060   60 0010 0015
         FAD  PI                     41   0015   32 0018 0045
         STU  THETO    ABCD2         42   0045   21 0010 0003
EODEA    STD  AAA1                   43   0050   24 0053 0056
         STU  AAA14                  44   0056   21 0310 0263
         RAU  AAA16                  45   0263   60 0266 0021
         FAM  AAA14                  46   0021   37 0310 0037
         STU  AAA2                   47   0037   21 0042 0095
         LDD  AAA3                   48   0095   69 0040 0001
         STU  AAA4     AAA8          49   0001   24 0004 0057
AAA8     RAU  AAA2                   50   0057   60 0042 0197
         FSB  AAA5                   51   0197   33 0300 0027
         BMI  AAA6                   52   0027   46 0030 0181
         STU  AAA2                   53   0181   21 0042 0195
         RAU  AAA4                   54   0195   60 0004 0059
         FMP  AAA7                   55   0059   39 0012 0062
         STU  AAA4     AAA8          56   0062   21 0004 0057
AAA6     RAU  AAA2                   57   0030   60 0042 0297
         FSB  AAA3                   58   0297   33 0048 0025
         BMI  AAA28                  59   0025   46 0028 0029
         STU  AAA2                   60   0029   21 0043 0295
         RAU  AAA4                   61   0295   60 0004 0309
         FMP  AAA9                   62   0309   39 0162 0262
         STU  AAA4     AAA6          63   0262   21 0004 0030
AAA28    RAU  AAA2                   64   0028   60 0042 0347
         FSB  AAA10                  65   0347   33 0350 0077
         BMI  AAA11                  66   0077   46 0080 0281
         STU  AAA2                   67   0281   21 0042 0345
         RAU  AAA4                   68   0345   60 0004 0359
         FMP  AAA12                  69   0359   39 0312 0362
         STU  AAA4     AAA28         70   0362   21 0004 0028
AAA11    RAU  AAA2                   71   0080   60 0042 0397
         LDD           AAA17         72   0397   69 0400 0303
         FMP  AAA4                   73   0400   39 0004 0054
         STU  AAA13                  74   0054   21 0008 0011
         RAU  AAA14                  75   0011   60 0310 0065
         BMI  AAA15                  76   0065   46 0068 0019
         RAU  AAA13    AAA1          77   0019   60 0008 0053
AAA15    RAU  AAA3                   78   0068   60 0048 0353
         FDV  AAA13    AAA1          79   0353   34 0008 0053
AAA17    STD  AAA18                  80   0303   24 0306 0409
         RAU  AAA3                   81   0409   60 0048 0403
         FAD  AAA2                   82   0403   32 0042 0069
         STU  AAA19                  83   0069   21 0024 0177
         LDD  AAA27                  84   0177   69 0180 0033
         STD  AAA20                  85   0033   24 0036 0039
         STD  AAA21                  86   0039   24 0092 0395
         RAU  AAA2                   87   0395   60 0042 0447
         FMP  AAA2                   88   0447   39 0042 0192
         STU  AAA23    AAA22         89   0192   21 0046 0049
AAA22    FDV  AAA21                  90   0049   34 0092 0292
         STU  AAA24                  91   0292   21 0096 0099
         FAD  AAA19                  92   0099   32 0024 0051
         STU  AAA19                  93   0051   21 0024 0277
         RAU  AAA24                  94   0277   60 0096 0301
         FDV  AAA19                  95   0301   34 0024 0074
         FSB  AAA25                  96   0074   33 0327 0453
         BMI           AAA26         97   0453   46 0356 0307
         RAU  AAA19    AAA18         98   0356   60 0024 0306
AAA26    RAU  AAA20                  99   0307   60 0036 0041
         FAD  AAA3                  100   0041   32 0048 0075
         STU  AAA20                 101   0075   21 0036 0089
         FMP  AAA21                 102   0089   39 0092 0342
         STU  AAA21                 103   0342   21 0092 0445
         RAU  AAA23                 104   0445   60 0046 0351
         FMP  AAA2                  105   0351   39 0042 0392
         STU  AAA23    AAA22        106   0392   21 0046 0049
AAA3      10  0000     0051         107   0048   10 0000 0051
AAA5      50  0000     0051         108   0300   50 0000 0051
AAA7      14  8410     0053         109   0012   14 8410 0053
AAA9      27  1830     0051         110   0162   27 1830 0051
AAA10     20  0000     0050         111   0350   20 0000 0050
AAA12     12  2140     0051         112   0312   12 2140 0051
AAA16     00  0000     0000         113   0266   00 0000 0000
AAA25     70  0000     0047         114   0327   70 0000 0047
AAA27     20  0000     0051         115   0180   20 0000 0051
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ZMXSH   STD  ABCD1              116   0450   24  0503  0406
        RAU  ARGGX              117   0406   60  0459  0313
        LDD         COSHX       118   0313   69  0316  0169
        STU  CHAAX              119   0316   21  0170  0173
        RAU  ARGGX              120   0173   60  0459  0363
        LDD         EOOCR       121   0363   69  0366  0269
        STU  CSAAX              122   0366   21  0270  0273
        RAU  ARGGX              123   0273   60  0459  0413
        LDD         SINHX       124   0413   69  0416  0319
        STU  SHAAX              125   0416   21  0320  0323
        RAU  ARGGX              126   0323   60  0459  0463
        LDD         EOOSH       127   0463   69  0466  0369
        STU  SNAAX              128   0466   21  0370  0373
        RAU  ARGGL              129   0373   60  0176  0331
        LDD         COSHX       130   0331   69  0284  0169
        STU  CHAAL              131   0284   21  0038  0091
        RAU  ARGGL              132   0091   60  0176  0381
        LDD         EOOCH       133   0381   69  0334  0269
        STU  CSAAL              134   0334   21  0088  0191
        RAU  ARGGL              135   0191   60  0176  0431
        LDD         SINHX       136   0431   69  0384  0319
        STU  SHAAL              137   0384   21  0188  0291
        RAU  ARGGL              138   0291   60  0176  0481
        LDD         EOOSH       139   0481   69  0434  0369
        STU  SNAAL              140   0434   21  0288  0341
        RAU  CHAAL              141   0341   60  0038  0043
        FMP  CSAAL              142   0043   39  0088  0338
        STU  FCL                143   0338   21  0009  0412
        FMP  CHAAL              144   0412   39  0038  0388
        FMP  CSAAL              145   0388   39  0088  0438
        STU  DEIIN              146   0438   21  0442  0495
        RAU  SNAAL              147   0495   60  0288  0093
        FMP  SHAAL              148   0093   39  0188  0488
        STU  FSL                149   0488   21  0026  0079
        FMP  SNAAL              150   0079   39  0288  0538
        FMP  SHAAL              151   0538   39  0188  0588
        FAD  DEIIN              152   0588   32  0442  0419
        STU  DENNM              153   0419   21  0174  0377
        RAU  CHAAX              154   0377   60  0170  0175
        FMP  CSAAX              155   0175   39  0270  0420
        STU  FCX                156   0420   21  0016  0469
        FMP   8003              157   0469   39  8003  0423
        STU  NUIIM              158   0423   21  0078  0531
        RAU  SNAAX              159   0531   60  0370  0275
        FMP  SHAAX              160   0275   39  0320  0470
        STU  FSX                161   0470   21  0034  0087
        FMP   8003              162   0087   39  8003  0391
        FAD  NUIIM              163   0391   32  0078  0305
        FDV  DENNM              164   0305   34  0174  0274
        LDD  ABCD1  EOOAU       165   0274   69  0503  0506
COSHX   STD  NEXTC              166   0169   24  0022  0325
        STU  ARG                167   0325   21  0280  0083
        LDD         EOOEA       168   0083   69  0086  0050
        STU  EARGP              169   0086   21  0040  0193
        RSU  ARG                170   0193   61  0280  0035
        LDD         EOOEA       171   0035   69  0638  0050
        FAD  EARGP              172   0638   32  0040  0017
        FDV  TWO    NEXTC       173   0017   34  0520  0022
SINHX   STD  NEXTS              174   0319   24  0072  0375
        STU  ARG                175   0375   21  0280  0183
        RSU  ARG                176   0183   61  0280  0085
        LDD         EOOEA       177   0085   69  0688  0050
        STU  EARGM              178   0688   21  0492  0545
        RAU  ARG                179   0545   60  0280  0185
        LDD         EOOEA       180   0185   69  0738  0050
        FSB  EARGM              181   0738   33  0492  0519
        FDV  TWO    NEXTS       182   0519   34  0520  0072
EODCR   STD  EXIT               183   0269   24  0172  0425
        BMI  NEGAT  REDUC       184   0425   46  0178  0179
NEGAT   FAD  TWOPI              185   0178   32  0631  0357
        BMI  NEGAT              186   0357   46  0178  0061
        FSB  ONEPI  COSIO       187   0061   33  0014  0441
REDUC   FSB  TWOPI              188   0179   33  0631  0407
        BMI         REDUC       189   0407   46  0360  0179
        FAD  ONEPI  COSIO       190   0360   32  0014  0441
COSIO   STU  THETA              191   0441   21  0196  0199
        RSU  FPONE              192   0199   61  0302  0457
        STU  TERMM              193   0457   21  0462  0165
        STU  FUNKT              194   0165   21  0570  0473
        STL  ENN    NEGST       195   0473   20  0427  0330
EODSR   STD  EXIT               196   0369   24  0172  0475
        BMI  NEGAV  REDUU       197   0475   46  0328  0279
NEGAV   FAD  TWOPI              198   0328   32  0631  0507
        BMI  NEGAV              199   0507   46  0328  0161
        FSB  ONEPI  SINET       200   0161   33  0014  0491
REDUO   FSB  TWOPI              201   0279   33  0631  0557
        BMI         REOUU       202   0557   46  0410  0279
        FAO  ONEPI  SINET       203   0410   32  0014  0491
SINET   STU  THETA              204   0491   21  0196  0299
        RSU   8003              205   0299   61  8003  0607
        STU  TERMM              206   0607   21  0462  0265
        STU  FUNKT              207   0265   21  0570  0523
        LDD  FPONE              208   0523   69  0302  0355
        STD  ENN    NEGST       209   0355   24  0427  0330
NEGST   RAU  ENN                210   0330   60  0427  0681
        FAD  FPONE              211   0681   32  0302  0329
        STU  NPONE              212   0329   21  0484  0187
        FAD  FPONE              213   0187   32  0302  0379
        STU  ENN                214   0379   21  0427  0380
        RSU  TERMM              215   0380   61  0462  0067
        FMP  THETA              216   0067   39  0196  0296
        FMP  THETA              217   0296   39  0196  0346
        FDV  NPONE              218   0346   34  0484  0534
        FDV  ENN                219   0534   34  0427  0477
        STU  TERMM              220   0477   21  0462  0315
        RAM  FUNKT              221   0315   67  0570  0525
        STL  FMAG               222   0525   20  0429  0032
        RAM  TERMM              223   0032   67  0462  0167
        RAU   8002              224   0167   60  8002  0575
        FDV  FMAG               225   0575   34  0429  0479
        FSB  SIZES              226   0479   33  0082  0509
        BMI  ENUFF              227   0509   46  0512  0513
        RAU  FUNKT              228   0513   60  0570  0625
        FAD  TERMM              229   0625   32  0462  0189
        STU  FUNKT  NEGST       230   0189   21  0570  0330
ENUFF   RAU  FUNKT  EXIT        231   0512   60  0570  0172
SIZES    10   0000   0043       232   0082   10  0000  0043
TWOPI    62   8318   5351       233   0631   62  8318  5351
ONEPI    31   4159   2751       234   0014   31  4159  2751
FPONE    10   0000   0051       235   0302   10  0000  0051
EOOAU   STD  SEXT               236   0506   24  0559  0562
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         BMI  SERR              237   0562   46 0365 0516
         NZE            SEXT    238   0516   45 0620 0559
         STU  SA                239   0620   21 0374 0527
         FAD  S10               240   0527   32 0430 0657
         FMP  SHAF      SB      241   0657   39 0460 0510
SB       STU  SSAV      SAB     242   0510   21 0064 0267
SAB      RAU  SA                243   0267   60 0374 0529
         FDV  SSAV              244   0529   34 0064 0164
         FAD  SSAV              245   0164   32 0064 0541
         FMP  SHAF              246   0541   39 0460 0560
         FSB  SSAV              247   0560   33 0064 0591
         NZU            SR      248   0591   44 0595 0396
         BMI            SR      249   0595   46 0098 0396
         FAD  SSAV              250   0098   32 0064 0641
         STU  SSAV      SAB     251   0641   21 0064 0267
SR       RAU  SSAV      SEXT    252   0396   60 0064 0559
SERR     HLT   0000     SEXT    253   0365   01 0000 0559
SHAF      50   0000      0050   254   0460   50 0000 0050
S10       10   0000      0051   255   0430   10 0000 0051
START    RCD   1951             256   1999   70 1951 0401
         LDD   1951             257   0401   69 1951 0304
         STD  X                 258   0304   24 0707 0610
         LDD   1952             259   0610   69 1952 0405
         STD  T                 260   0405   24 0058 0261
         LDD  ZERO              261   0261   69 0264 0317
         STD  HOLD              262   0317   24 0670 0573
         LDD  INDXM             263   0573   69 0076 0579
         RAB   8001             264   0579   82 8001 0285
         LDD  ZERO              265   0285   69 0264 0367
         STD  HOLD      LOPP1   266   0367   24 0670 0623
LOPP1    RAU  STAA  B           267   0623   60 4120 0675
         FDV  ALPHA             268   0675   34 0278 0378
         FDV  TWO               269   0378   34 0520 0720
         LDD            EOOAU   270   0720   69 0673 0506
         STU  SQRRT             271   0673   21 0428 0731
         FMP  X                 272   0731   39 0707 0757
         STU  ARGGX             273   0757   21 0459 0612
         RAU  SQRRT             274   0612   60 0428 0283
         FMP  L                 275   0283   39 0186 0286
         STU  ARGGL             276   0286   21 0176 0629
         LDD            ZMXSB   277   0629   69 0182 0450
         STU  ZMX               278   0182   21 0336 0289
         LDD            PHISB   279   0289   69 0542 0000
         RAU  STAA  B           280   0542   60 4120 0725
         FMP  T                 281   0725   39 0058 0308
         FAD  THETO             282   0308   32 0010 0287
         LDD            EOOCR   283   0287   69 0090 0269
         FMP  ZMX               284   0090   39 0336 0386
         FMP  AMMM  B           285   0386   39 4100 0500
         STU  A1    B           286   0500   21 4140 0293
         SXB   0001             287   0293   53 0001 0349
         NZB  LOPP1     CONT1   288   0349   42 0623 0553
CONT1    LDD  NOLAM             289   0553   69 0456 0609
         RAB   8001     LOOP9   290   0609   82 8001 0415
LOOP9    RAU  ALPHA             291   0415   60 0278 0333
         FMP  KAPPA             292   0333   39 0436 0486
         FMP  KAPPA             293   0486   39 0436 0536
         STU  ALKAP             294   0536   21 0190 0343
         FSB  LAMDA B           295   0343   33 4220 0497
         STU  DIFF              296   0497   21 0352 0455
         RAU  LAMDA B           297   0455   60 4220 0775
         FMP  T                 298   0775   39 0058 0358
         LDD            EOOEA   299   0358   69 0311 0050
         STU  ELAMT             300   0311   21 0566 0569
         RAU  ELAMT             301   0569   60 0566 0071
         FMP  QOO               302   0071   39 0324 0424
         FMP  AYEJ  B           303   0424   39 4200 0550
         FMP  ALPHA             304   0550   39 0278 0478
         FDV  KAY               305   0478   34 0581 0781
         FDV  DIFF              306   0781   34 0352 0402
         STU  A3P               307   0402   21 0556 0659
         RAU  KAPPA             308   0659   60 0436 0691
         FMP  X                 309   0691   39 0707 0807
         LDD            COSHX   310   0807   69 0660 0169
         STU  CSHKX             311   0660   21 0314 0417
         RAU  KAPPA             312   0417   60 0436 0741
         FMP  L                 313   0741   39 0186 0586
         LDD            COSHX   314   0586   69 0339 0169
         STU  CSHKL             315   0339   21 0044 0547
         RAU  LAMDA B           316   0547   60 4220 0825
         BMI  NEG       POS     317   0825   46 0528 0679
NEG      RSU  LAMDA B           318   0528   61 4220 0875
         FDV  ALPHA             319   0875   34 0278 0578
         LDD            EOOAU   320   0578   69 0831 0506
         STU  SQRLA             321   0831   21 0636 0389
         FMP  X                 322   0389   39 0707 0857
         LDD            EOOCH   323   0857   69 0710 0269
         STU  COSLX             324   0710   21 0364 0467
         RAU  SQRLA             325   0467   60 0636 0791
         FMP  L                 326   0791   39 0186 0686
         LDD            EOOCH   327   0686   69 0439 0269
         STU  COSLL             328   0439   21 0094 0597
         RAU  COSLX             329   0597   60 0364 0619
         FDV  COSLL             330   0619   34 0094 0194
         FMP  CSHKL             331   0194   39 0044 0294
         FBB  CSHKX             332   0294   33 0314 0841
         FMP  A3P               333   0841   39 0556 0606
         STU  A3    B           334   0606   21 4240 0393
         BXS   0001             335   0393   53 0001 0399
         NZB  LOOP9     CONT3   336   0399   42 0415 0603
POS      FDV  ALPHA             337   0679   34 0278 0628
         LDD            EOOAU   338   0628   69 0881 0506
         STU  SQRLA             339   0881   21 0636 0489
         FMP  X                 340   0489   39 0707 0907
         LDD            COSHX   341   0907   69 0760 0169
         STU  CSHLX             342   0760   21 0414 0517
         RAU  SQRLA             343   0517   60 0636 0891
         FMP  L                 344   0891   39 0186 0736
         LDD            COSHX   345   0736   69 0539 0169
         STU  CSHLL             346   0539   21 0344 0647
         RAU  CSHLX             347   0647   60 0414 0669
         FDV  CSHLL             348   0669   34 0344 0394
         FMP  CSHKL             349   0394   39 0044 0444
         FSB  CSHKX             350   0444   33 0314 0941
         FMP  A3P               351   0941   39 0556 0656
         STU  A3    B           352   0656   21 4240 0443
         SXB   0001             353   0443   53 0001 0449
         NZB  LOOP9     CONT3   354   0449   42 0415 0603
CONT3    RAA   0001             355   0603   80 0001 0709
         RAU  ONE               356   0709   60 0662 0567
         RSL   8003             357   0567   66 8003 0925
         STL  N                 358   0925   20 0729 0282
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        RAU  ZERO
        STU  HOLO        LUOP3
LOOP3   RAU  N
        FAD  TWO
        STU  N
        FMP  PI
        FDV  TWO
        FDV  L
        STU  ARG1
        FMP  ARG1
        FMP  ALPHA
        STU  ARG2
        RAU  ZERO
        STU  A24
        LDD  NDLAM
        RAB   8001       LOP5
LOP5    RAU  ARG2
        FAD  LAMDA  B
        STU  A2S41
        RAU  ARG2
        FAD  ALKAP
        STU  A2S42
        RAU  CSHKL
        FMP  ALPHA
        FMP  QOU
        FMP  AYEJ   B
        FDV  KAY
        FDV  A2S41
        FDV  A2S42
        FAD  A24
        STU  A24
        SXB   0001
        NZB  LOP5        CONT9
CONT9   RAU  ZERO
        STD  SETT
        LDD  INOXM
        RAB   8001       LUPP2
LOPP2   RAU  ARG2
        FMP  ARG2
        STU  ARGSQ
        RAU  STAA   B
        FMP  STAA   B
        FAD  ARGSQ
        STU  DENOM
        RAU  AMMM   B
        FMP  ARG2
        FDV  DENOM
        FAD  SETT
        STU  SETT
        SXR   0001
        NZB  LOPP2       CONT4
CONT4   FAD  A24
        STU  A2SUM
        RAU  ARG2
        FMP  T
        FSB  FIFTY
        BMI  GO          STO
STO     LDD  ZERO
        STD  TERM
        RAU  TERM
        FAD  HOLO
        STU  HOLO
        NZU              CONT5
        RAU  HOLO
        FSB  TERM        MOREC
GO      RSU  ARG2,
        FMP  T
        LDD              EOOEA
        STU  EAG2T
        RAU  ARG1
        FMP  X
        LDD              EOOCR
        STU  COSIX
        RAU  L
        FMP  L
        FDV  ALPHA
        FDV  N
        FDV  PI
        STU  A2DIV
        RAU  EAG2T
        FMP  COSIX
        FDV  A2DIV
        STU  A2DOT
        FMP  A2SUM
        STU  TERM        MOREC
MOREC   BMA  SUB         ADD
SUB     RAA   0001
        RAU  HOLO
        FSB  TERM
        STU  HOLO
        RAU  TERM
        FDV  HOLO
        RAM   8003
        RAU   8002
        FSB  CRIT
        BMI  CONT5       LOOP3
ADD     RSA   0001
        RAU  HOLO
        FAD  TERM
        STU  HOLO
        RAU  TERM
        FDV  HOLO
        RAM   8003
        RAU   8002
        FSB  CRIT
        BMI  CONT5       LOOP3
CONT5   LDD  INOXM
        RAB   8001
        RAU  ZERO
        STU  CELL        LOP
LOP     RAU  A1     B
        FAD  CELL
        STU  CELL
        SXB   0001
        NZB  LOP         GOON1
GOON1   LDD  NDLAM
        RAB   8001
        RAU  ZERO
        STU  CELL3       LOP2
LOP2    RAU  A3     B
        FAD  CELL3
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360   0719   21 0670 0723
361   0723   60 0729 0383
362   0383   32 0520 0697
363   0697   21 0729 0332
364   0332   39 0018 0168
365   0168   34 0520 0770
366   0770   34 0186 0786
367   0786   21 0290 0493
368   0493   39 0290 0340
369   0340   39 0278 0678
370   0678   21 0382 0335
371   0335   60 0264 0769
372   0769   21 0474 0577
373   0577   69 0456 0759
374   0759   82 8001 0465
375   0465   60 0382 0337
376   0337   32 4220 0747
377   0747   21 0452 0505
378   0505   60 0382 0387
379   0387   32 0190 0617
380   0617   21 0272 0975
381   0975   60 0044 0499
382   0499   39 0278 0728
383   0728   39 0324 0524
384   0524   39 4200 0600
385   0600   34 0581 0931
386   0931   34 0452 0502
387   0502   34 0272 0322
388   0322   32 0474 0451
389   0451   21 0474 0627
390   0627   53 0001 0433
391   0433   42 0465 0437
392   0437   60 0264 0819
393   0819   24 0372 1025
394   1025   69 0076 0779
395   0779   82 8001 0385
396   0385   60 0382 0487
397   0487   39 0382 0432
398   0432   21 0836 0589
399   0589   60 4120 1075
400   1075   39 4120 0820
401   0820   32 0836 0563
402   0563   21 0268 0171
403   0171   60 4100 0555
404   0555   39 0382 0482
405   0482   34 0268 0318
406   0318   32 0372 0549
407   0549   21 0372 1125
408   1125   53 0001 0981
409   0981   42 0385 0435
410   0435   32 0474 0501
411   0501   21 0706 0809
412   0809   60 0382 0537
413   0537   39 0058 0408
414   0408   33 0361 0587
415   0587   46 0390 0991
416   0991   69 0264 0667
417   0667   24 0870 0773
418   0773   60 0870 1175
419   1175   32 0670 0797
420   0797   21 0670 0823
421   0823   44 0677 0778
422   0677   60 0670 1225
423   1225   33 0870 0847
424   0390   61 0382 0637
425   0637   39 0058 0458
426   0458   69 0411 0050
427   0411   21 0616 0869
428   0869   60 0290 0645
429   0645   39 0707 0957
430   0957   69 0810 0269
431   0810   21 0464 0717
432   0717   60 0186 1041
433   1041   39 0186 0886
434   0886   34 0278 0828
435   0828   34 0729 0829
436   0829   34 0018 0368
437   0368   21 0422 1275
438   1275   60 0616 0271
439   0271   39 0464 0514
440   0514   34 0422 0472
441   0472   21 0276 0879
442   0879   39 0706 0756
443   0756   21 0870 0847
444   0847   41 0650 0551
445   0650   80 0001 0806
446   0806   60 0670 1325
447   1325   33 0870 0897
448   0897   21 0670 0873
449   0873   60 0870 1375
450   1375   34 0670 0920
451   0920   67 8003 0727
452   0727   60 8002 0485
453   0485   33 0788 0515
454   0515   46 0778 0723
455   0551   81 0001 1007
456   1007   60 0670 1425
457   1425   32 0870 0947
458   0947   21 0670 0923
459   0923   60 0870 1475
460   1475   34 0670 0970
461   0970   67 8003 0777
462   0777   60 8002 0535
463   0535   33 0788 0565
464   0565   46 0778 0723
465   0778   69 0076 0929
466   0929   82 8001 0585
467   0585   60 0264 0919
468   0919   21 0574 0827
469   0827   60 4140 0695
470   0695   32 0574 0601
471   0601   21 0574 0877
472   0877   53 0001 0483
473   0483   42 0827 0687
474   0687   69 0456 0859
475   0859   82 8001 0615
476   0615   60 0264 0969
477   0969   21 0624 0927
478   0927   60 4240 0745
479   0745   32 0624 0651
```

```
        STU  CELL3                480   0651   21 0624 0977
        SXB   0001                481   0977   53 0001 0533
        NZB  LOP2    GOON2        482   0533   42 0927 0737
GOON2   RSU  HOLD                 483   0737   61 0670 1525
        FAD  CELL                 484   1525   32 0574 0701
        FAD  CELL3                485   0701   32 0624 0751
        STU  ANSWR   FINIS        486   0751   21 0856 0909
FINIS   LDD  ANSWR                487   0909   69 0856 0959
        STD   1977                488   0959   24 1977 0480
        LDD  CELL                 489   0480   69 0574 1027
        STD   1978                490   1027   24 1978 1031
        LDD  HOLD                 491   1031   69 0670 0973
        STD   1979                492   0973   24 1979 0532
        LDD  CELL3                493   0532   69 0624 1077
        STD   1980                494   1077   24 1980 0583
        LDD  X                    495   0583   69 0707 0860
        STD   1981                496   0860   24 1981 0584
        LDD  T                    497   0584   69 0058 0461
        STD   1982                498   0461   24 1982 0635
        PCH   1977   START        499   0635   71 1977 1999
ONE      10   0000    0051        500   0662   10 0000 0051
TWO      20   0000    0051        501   0520   20 0000 0051
PI       31   4159    2751        502   0018   31 4159 2751
FIFTY    50   0000    0052        503   0361   50 0000 0052
ZERO     00   0000    0000        504   0264   00 0000 0000
CRIT     10   0000    0047        505   0788   10 0000 0047
ARTAN   STD  EXIT                   4   1800   24 1803 1806
        NZE          EXIT           5   1806   45 1810 1803
        SMI  MINUS                  6   1810   46 1813 1814
        STU  ARTAO                  7   1814   21 1818 1821
        LDD  FPONE                  8   1821   69 1824 1827
        STD  ENNNN                  9   1827   24 1830 1833
        STD  AYE     SUBTR         10   1833   24 1836 1839
MINUS   RSL   8003                 11   1813   66 8003 1871
        STL  ARTAO                 12   1871   20 1818 1822
        RSU  FPONE                 13   1822   61 1824 1829
        STD  ENNNN                 14   1829   24 1830 1883
        STU  AYE     SUBTR         15   1883   21 1836 1839
SUBTR   RAU  ARTAO                 16   1839   60 1818 1823
        FSB  FPONE                 17   1823   33 1824 1801
        NZE  DIFFE                 18   1801   45 1804 1805
        LDD  PIOV4                 19   1805   69 1808 1811
        STD  FUNGT   MULTA         20   1811   24 1864 1817
DIFFE   SMI  SMALL                 21   1804   46 1807 1858
        FSB  FPONE                 22   1858   33 1824 1851
        BMI  NEGAT   POSIT         23   1851   46 1854 1855
NEGAT   LDD  PIOV4                 24   1854   69 1808 1861
        STD  FUNGT                 25   1861   24 1864 1867
        RAU  ARTAO                 26   1867   60 1818 1873
        FAD  FPONE                 27   1873   32 1824 1802
        STU  TURRN                 28   1802   21 1856 1809
        FSB  FPTWO                 29   1809   33 1812 1889
        FDV  TURRN   COMBI         30   1889   34 1856 1857
SMALL   FAD  ARTAO                 31   1807   32 1818 1845
        BMI          NEGAT         32   1845   46 1848 1854
        RAU  ARTAO                 33   1848   60 1818 1874
        FSB  LOBND                 34   1874   33 1877 1853
        BMI  TINEY                 35   1853   46 1859 1860
        RAU  ARTAO                 36   1860   60 1818 1825
        STL  FUNGT   COMBI         37   1825   20 1864 1857
POSIT   LDD  PIOV2                 38   1855   69 1862 1815
        STD  FUNGT                 39   1815   24 1864 1868
        FSB  UPBND                 40   1868   33 1872 1849
        BMI          MULTA         41   1849   46 1852 1817
        RSU  FPONE                 42   1852   61 1824 1879
        FDV  ARTAO   COMBI         43   1879   34 1818 1857
COMBI   STU  TURRN                 44   1857   21 1856 1863
        STD  TURRM                 45   1863   24 1816 1819
        FMP  TURRN                 46   1819   39 1856 1865
        STU  ARGUE   FIGUR         47   1865   21 1820 1875
FIGUR   RAU  TURRM                 48   1875   60 1816 1826
        FAD  FUNGT                 49   1826   32 1864 1841
        STU  FUNGT                 50   1841   21 1864 1869
        RAU  ENNNN                 51   1869   60 1830 1835
        FAD  FPTWO                 52   1835   32 1812 1840
        STU  ENNNN                 53   1840   21 1830 1834
        RSU  TURRN                 54   1834   61 1856 1866
        FMP  ARGUE                 55   1866   39 1820 1870
        STU  TURRN                 56   1870   21 1856 1876
        FDV  ENNNN                 57   1876   34 1830 1880
        STU  TURRM                 58   1880   21 1816 1828
        RAM  FUNGT                 59   1828   67 1864 1878
        STL  FMAGG                 60   1878   20 1884 1837
        RAM  TURRM                 61   1837   67 1816 1831
        RAU   8002                 62   1831   60 8002 1890
        FDV  FMAGG                 63   1890   34 1884 1885
        FSB  SIZEB                 64   1885   33 1838 1881
        BMI  MULTA   FIGUR         65   1881   46 1817 1875
TINEY   RAU  ARTAO   MULTA         66   1859   60 1818 1817
MULTA   RAU  AYE                   67   1817   60 1836 1891
        FMP  FUNGT   EXIT          68   1891   39 1864 1803
FPONE    10   0000    0051         69   1824   10 0000 0051
FPTWO    20   0000    0051         70   1812   20 0000 0051
SIZEB    10   0000    0043         71   1838   10 0000 0043
PIOV2    15   7079    6351         72   1862   15 7079 6351
PIOV4    78   5398    1650         73   1808   78 5398 1650
UPBND    10   0000    0060         74   1872   10 0000 0060
LOBND    10   0000    0040         75   1877   10 0000 0040
```

APPENDIX E

Description and Explanation of the IBM-650 Computer Program Used to Calculate Surface Heat Flow.

The computer program was written to calculate the heat flow out of the fuel and into the moderator. The heat flow out of the fuel surface is given by

$$(q/A)_f(t) = -k_f\left\{\sum_{i=1}^{p} \sqrt{\frac{\beta_i}{2\alpha}}\; B_i\,(D_i \cos\beta_i t + E_i \sin\beta_i t)\right.$$

$$+\sum_{n=1,3,5,\cdots}^{\infty} \frac{(\frac{n}{2L})\, e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{\frac{L^2}{n\pi\alpha}} \left(\sum_{i=1}^{p} \frac{B_i\,(\frac{n^2\pi^2\alpha}{4L^2})}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} \right. \tag{E-1}$$

$$\left. + \sum_{j=1}^{s} \frac{q_{oo}\;\alpha\; A_j \cosh\kappa L}{k_m\,(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j)(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2)} \right)$$

$$\left. + \sum_{j=1}^{s} \frac{q_{oo} - A_j\, e^{\lambda_j t}}{k_f\,(\alpha\kappa^2 - \lambda_j)} \left[\frac{\sqrt{\frac{\lambda_j}{\alpha}}\cosh(\kappa L)\sinh(\sqrt{\frac{\lambda_j}{\alpha}}\,L)}{\cosh(\sqrt{\frac{\lambda_j}{\alpha}}\,L)} - \kappa\sinh\kappa L\right]\right\}$$

and the heat flow into the moderator is given by

$$(q/A)_m\,(t) = \rho\, k_m\left\{\sum_{i=1}^{p} \sqrt{\frac{\beta_i}{2\alpha}}\; B_i\,(D_i \cos\beta_i t + E_i \sin\beta_i t)\right.$$

$$+\sum_{n=1,3,5,\cdots}^{\infty} \frac{(\frac{n\pi}{2L})\, e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{\frac{L^2}{n\pi\alpha}} \left(\sum_{i=1}^{p} \frac{B_i\,(\frac{n^2\pi^2\alpha}{4L^2})}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} + \sum_{j=1}^{s} \frac{q_{oo}\;\alpha\; A_j \cosh\kappa L}{k_m(\frac{n^2\pi^2\alpha}{4L^2}+ \lambda_j)(\frac{n^2\pi^2\alpha}{4L^2})} \right)$$

$$- \sum_{j=1}^{s} \frac{\alpha F A_j}{k_m \lambda_j} \left(\frac{\sqrt{\frac{\lambda_j}{\alpha}} \sinh \sqrt{\frac{\lambda_j}{\alpha}} L}{\cosh \sqrt{\frac{\lambda_j}{\alpha}} L} \right) ,$$

(E-2)

$$\text{where } D_i = \frac{\cosh \gamma_i L \ \sinh \gamma_i L - \cos \gamma_i L \ \sin \gamma_i L}{\cos^2 \gamma_i L \ \mathrm{Cosh}^2 \gamma_i L + \sin^2 \gamma_i L \ \sinh^2 \gamma_i L} ,$$

$$E_i = \frac{\cosh \gamma_i L \ \sinh \gamma_i L + \cos \gamma_i L \ \sin \gamma_i L}{\cos^2 \gamma_i L \ \cosh^2 \gamma_i L + \sin^2 \gamma_i L \ \sinh^2 \gamma_i L}$$

$$\text{and } \gamma_i = \sqrt{\frac{\beta_i}{2\alpha}} .$$

The equivalence between elements of the algebraic equation and the symbolic logic of the computer program is shown in Table E-1.

Table E-1. Definition of symbolic terms of the IBM-650 computer program for calculating surface heat flow.

$$\gamma_i = \sqrt{\frac{\beta_i}{2\alpha}}$$

$$ZMX1_i = \frac{\cosh \gamma_i L \ \sinh \gamma_i L - \cos \gamma_i L \ \sin \gamma_i L}{\cos^2 \gamma_i L \ \cosh^2 \gamma_i L + \sin^2 \gamma_i L \ \sinh^2 \gamma_i L} = D_i$$

$$A1_i = \gamma_i A_i \ (ZMX1_i \cos \beta_i t - ZMX2_i \sin \beta_i t)$$

$$A3P_j = \frac{q_\infty \alpha A_j e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)}$$

$$A3SUM_j = \frac{\sqrt{\frac{\lambda_j}{\alpha}} \cosh \kappa L \ \sinh \sqrt{\frac{\lambda_j}{\alpha}} L}{\cosh \sqrt{\frac{\lambda_j}{\alpha}} L} - \kappa \sinh \kappa L$$

$$ZMX2_i = \frac{\cos \gamma_i L \ \sin \gamma_i L + \cosh \gamma_i L \ \sinh \gamma_i L}{\cos^2 \gamma_i L \ \cosh^2 \gamma_i L + \sin^2 \gamma_i L \ \sinh^2 \gamma_i L} = E_i$$

Table E-1 cont.

$$A3_j = (A3P_j)\ (A3SUM_j)$$

$$CSHLL_j = \cosh \sqrt{\frac{\lambda_j}{\alpha}}\ L$$

$$COSLL_j = \cos \sqrt{\frac{\lambda_j}{\alpha}}\ L$$

$$ARG1_n = n\pi / 2L$$

$$ARG2_n = n^2\pi^2\alpha / 4L^2$$

$$A2ST1_i = B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right) / \left(\frac{n^4\pi^4\alpha^2}{16L^2} + \beta_i^2\right)$$

$$A2ST1_j = q_\infty\ \alpha\ A_j \cosh \kappa L\ /\ k \left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right) \left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)$$

$$A2DDT_n = \frac{\frac{n\pi}{2L}\ e^{-\frac{n^2\pi^2\pi}{4L^2}t}}{L^2 / n\pi\alpha}$$

$$A2SUM_n = \sum_{i=1}^{p} A2ST1_i + \sum_{j=1}^{s} A2ST1_j$$

$$TERM_n = (A2DDT_n)\ (A2SUM_n)$$

The input data consists of the heat generation and surface temperature parameters, appropriate material constants, half-thickness of the region and numerical constants. Table E-2 lists input data required for the program.

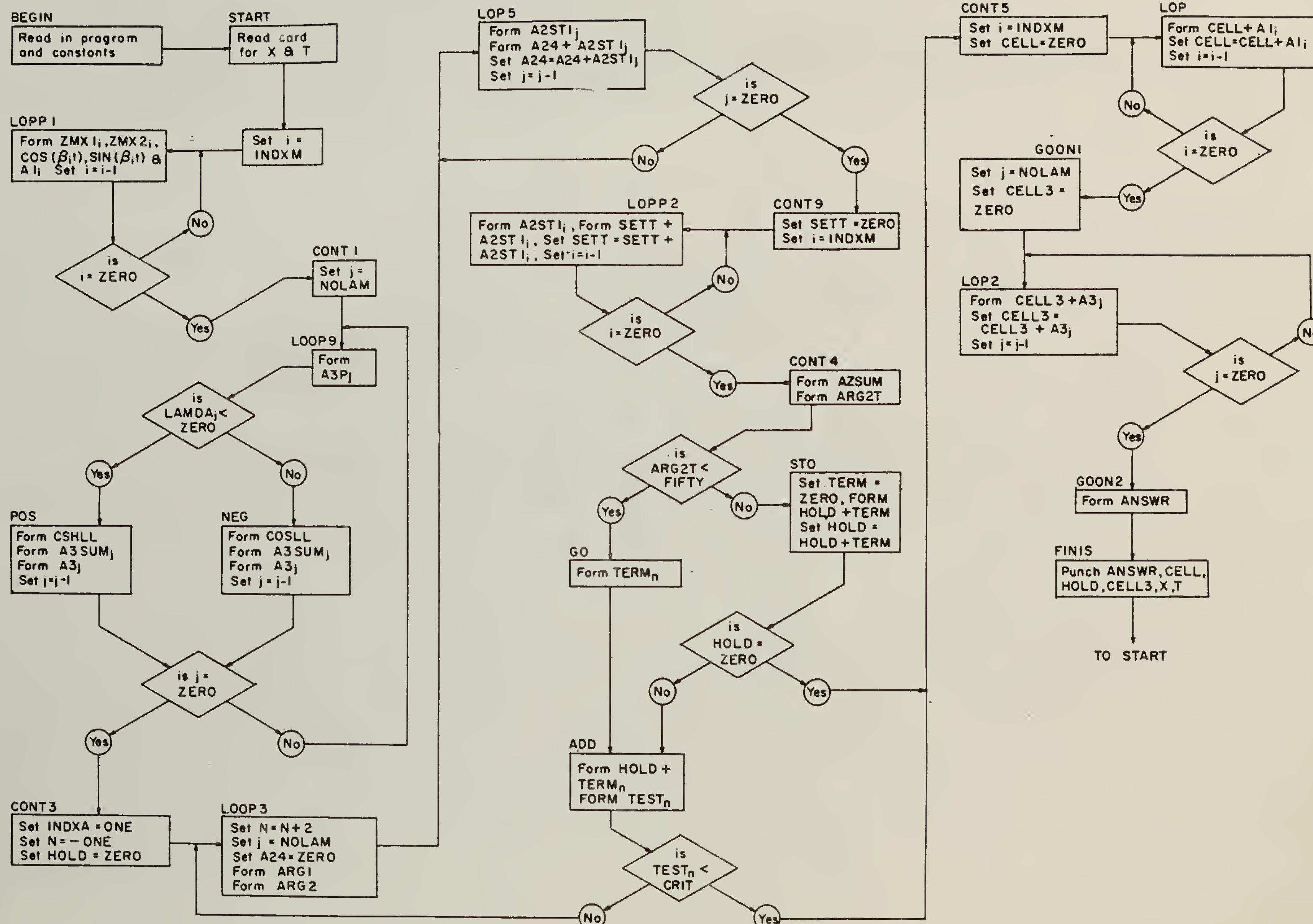
Table E-2. Input Data Required for Use of the IBM-650 Computer Program Used to Calculate Surface Heat Flow Rates.

Symbol	Explanation	Storage Location
ZERO	0.00	0164
ONE	1.00	0656
TWO	2.00	0820
Pi	3.14159	0779
FIFTY	50.00	0461
CRIT	0.0001	0388
ALPHA	Thermal Diffusivity	0278
KAPPA	Reciprocal of Thermal Neutron Diffusion Length in Fuel	0436
Q00	Normalization Factor for Heat Generation	0324
KAY	Thermal Conductivity	0581
L	Half-thickness of Region	0186
INDXM	No. of Terms, Surface Temperature Fit (00000000xx)	0076
NOLAM	No. of Terms, Heat Generation Fit (00000000xx)	0456
$AYEJ_j$	Amplitude Parameter, Heat Generation Fit	(0200 + j)
$LAMDA_j$	Exponential Parameter, Heat Generation Fit	(0020 + j)
$AMMM_j$	Amplitude Parameter, Surface Temperature Fit	(0100 + j)
$BTAA_j$	Period Parameter, Surface Temperature Fit	(0120 + j)

The output from this program is punched out on one card having an eight word capacity, one word consisting of 10 digits and a sign. The form of the output is shown in Table E-3.

Table E-3. Output form for IBM-650 Computer Program Used to Calculate Surface Heat Flow Rates.

WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
(q/A)	$\sum_{i=1}^{p} A1_i$	$\sum_{n=1,3,5}^{\infty} (\text{Term})_n$	$\sum_{j=1}^{s} A3_j$	--	t	--	--



LOGIC DIAGRAM-APPENDIX E

OBJECT PROGRAM-APPENDIX E

	BLR	0200	0260	1	0000	00	0000	0000
	BLR	1800	1899	2	0000	00	0000	0000
	SYN	ARTAN	1800	3	0000	00	0000	0000
	SYN	START	1999	4	0000	00	0000	0000
	SYN	AYEJ	0200	5	0000	00	0000	0000
	SYN	LAMDA	0220	6	0000	00	0000	0000
	SYN	A3	0240	7	0000	00	0000	0000
	BLR	1951	1960	8	0000	00	0000	0000
	BLR	1977	1984	9	0000	00	0000	0000
	BLR	0100	0160	10	0000	00	0000	0000
	SYN	AMMM	0100	11	0000	00	0000	0000
	SYN	STAA	0120	12	0000	00	0000	0000
	SYN	ALPHA	0278	13	0000	00	0000	0000
	SYN	KAPPA	0436	14	0000	00	0000	0000
	SYN	L	0186	15	0000	00	0000	0000
	SYN	QOO	0324	16	0000	00	0000	0000
	SYN	KAY	0581	17	0000	00	0000	0000
	SYN	INOXM	0076	18	0000	00	0000	0000
	SYN	NOLAM	0456	19	0000	00	0000	0000
	SYN	A1	0140	20	0000	00	0000	0000
E00EA	STO	AAA1		21	0000	24	0003	0006
	STU	AAA14		22	0006	21	0010	0013
	RAU	AAA16		23	0013	60	0016	0021
	FAM	AAA14		24	0021	37	0010	0037
	STU	AAA2		25	0037	21	0042	0045
	LOD	AAA3		26	0045	69	0048	0001
	STD	AAA4	AAA8	27	0001	24	0004	0007
AAA8	RAU	AAA2		28	0007	60	0042	0047
	FSB	AAA5		29	0047	33	0050	0027
	BMI	AAA6		30	0027	46	0030	0031
	STU	AAA2		31	0031	21	0042	0095
	RAU	AAA4		32	0095	60	0004	0009
	FMP	AAA7		33	0009	39	0012	0062
	STU	AAA4	AAA8	34	0062	21	0004	0007
AAA6	RAU	AAA2		35	0030	60	0042	0097
	FSB	AAA3		36	0097	33	0048	0025
	BMI	AAA28		37	0025	46	0028	0029
	STU	AAA2		38	0029	21	0042	0195
	RAU	AAA4		39	0195	60	0004	0059
	FMP	AAA9		40	0059	39	0162	0262
	STU	AAA4	AAA6	41	0262	21	0004	0030
AAA28	RAU	AAA2		42	0028	60	0042	0197
	FSB	AAA10		43	0197	33	0300	0077
	BMI	AAA11		44	0077	46	0080	0081
	STU	AAA2		45	0081	21	0042	0295
	RAU	AAA4		46	0295	60	0004	0309
	FMP	AAA12		47	0309	39	0312	0362
	STU	AAA4	AAA28	48	0362	21	0004	0028
AAA11	RAU	AAA2		49	0080	60	0042	0297
	LDD		AAA17	50	0297	69	0350	0053
	FMP	AAA4		51	0350	39	0004	0054
	STU	AAA13		52	0054	21	0008	0011
	RAU	AAA14		53	0011	60	0010	0015
	BMI	AAA15		54	0015	46	0018	0019
	RAU	AAA13	AAA1	55	0019	60	0008	0003
AAA15	RAU	AAA3		56	0018	60	0048	0303
	FDV	AAA13	AAA1	57	0303	34	0008	0003
AAA17	STO	AAA18		58	0053	24	0056	0359
	RAU	AAA3		59	0359	60	0048	0353
	FAO	AAA2		60	0353	32	0042	0069
	STU	AAA19		61	0069	21	0024	0177
	LOD	AAA27		62	0177	69	0180	0033
	STO	AAA20		63	0033	24	0036	0039
	STO	AAA21		64	0039	24	0092	0345
	RAU	AAA2		65	0345	60	0042	0347
	FMP	AAA2		66	0347	39	0042	0192
	STU	AAA23	AAA22	67	0192	21	0046	0049
AAA22	FDV	AAA21		68	0049	34	0092	0292
	STU	AAA24		69	0292	21	0096	0099
	FAO	AAA19		70	0099	32	0024	0051
	STU	AAA19		71	0051	21	0024	0277
	RAU	AAA24		72	0277	60	0096	0301
	FOV	AAA19		73	0301	34	0024	0074
	FSB	AAA25		74	0074	33	0327	0403
	BMI		AAA26	75	0403	46	0306	0057
	RAU	AAA19	AAA18	76	0306	60	0024	0056
AAA26	RAU	AAA20		77	0057	60	0036	0041
	FAD	AAA3		78	0041	32	0048	0075
	STU	AAA20		79	0075	21	0036	0089
	FMP	AAA21		80	0089	39	0092	0342
	STU	AAA21		81	0342	21	0092	0395
	RAU	AAA23		82	0395	60	0046	0351
	FMP	AAA2		83	0351	39	0042	0392
	STU	AAA23	AAA22	84	0392	21	0046	0049
AAA3	10	0000	0051	85	0048	10	0000	0051
AAA5	50	0000	0051	86	0050	50	0000	0051
AAA7	14	8410	0053	87	0012	14	8410	0053
AAA9	27	1830	0051	88	0162	27	1830	0051
AAA10	20	0000	0050	89	0300	20	0000	0050
AAA12	12	2140	0051	90	0312	12	2140	0051
AAA16	00	0000	0000	91	0016	00	0000	0000
AAA25	70	0000	0047	92	0327	70	0000	0047
AAA27	20	0000	0051	93	0180	20	0000	0051
ZMXSB	STO	ABC01		94	0400	24	0453	0356
	RAU	ARGGL		95	0356	60	0409	0063
	LDO		COSHX	96	0063	69	0066	0169
	STU	CHAAL		97	0066	21	0020	0023
	RAU	ARGGL		98	0023	60	0409	0163
	LDO		E00CH	99	0163	69	0166	0269
	STU	CSAAL		100	0166	21	0070	0073
	RAU	ARGGL		101	0073	60	0409	0263
	LOO		SINHX	102	0263	69	0266	0319
	STU	SHAAL		103	0266	21	0170	0173
	RAU	ARGGL		104	0173	60	0409	0313
	LDO		E00SH	105	0313	69	0316	0369
	STU	SNAAL		106	0316	21	0270	0273
	RAU	CHAAL		107	0273	60	0020	0175
	FMP	CSAAL		108	0175	39	0070	0320
	STU	FCL		109	0320	21	0174	0377
	FMP	CHAAL		110	0377	39	0020	0370
	FMP	CSAAL		111	0370	39	0070	0420
	STU	DEIIN		112	0420	21	0274	0427
	RAU	SNAAL		113	0427	60	0270	0275
	FMP	SHAAL		114	0275	39	0170	0470
	STU	FSL		115	0470	21	0374	0477

```
        RAU  CSAAL               116  0477  60 0070 0325
        FMP  SHAAL               117  0325  39 0170 0520
        STU  CSSHL               118  0520  21 0424 0527
        RAU  CSAAL               119  0527  60 0070 0375
        FMP  SNAAL               120  0375  39 0270 0570
        STU  CSSNL               121  0570  21 0474 0577
        RAU  CHAAL               122  0577  60 0020 0425
        FMP  SHAAL               123  0425  39 0170 0620
        STU  CHSHL               124  0620  21 0524 0627
        RAU  FSL                 125  0627  60 0374 0079
        FMP   8003               126  0079  39 8003 0083
        STU  D2                  127  0083  21 0038 0091
        RAU  FCL                 128  0091  60 0174 0179
        FMP   8003               129  0179  39 8003 0183
        FAD  D2                  130  0183  32 0038 0065
        STU  DENN2               131  0065  21 0670 0323
        RAU  CHSHL               132  0323  60 0524 0279
        FSS  CSSNL               133  0279  33 0474 0401
        FDV  DENN2               134  0401  34 0670 0720
        STU  ZMX1                135  0720  21 0574 0677
        RAU  CSSNL               136  0677  60 0474 0329
        FAO  CHSHL               137  0329  32 0524 0451
        FDV  DENN2               138  0451  34 0670 0770
        STU  ZMX2     ABCD1      139  0770  21 0624 0453
COSHX   STO  NEXTC               140  0169  24 0022 0475
        STU  ARG                 141  0475  21 0280 0283
        LDD           EOOEA      142  0283  69 0086 0000
        STU  EARGP               143  0086  21 0040 0043
        RSU  ARG                 144  0043  61 0280 0035
        LDO           EOOEA      145  0035  69 0088 0000
        FAD  EARGP               146  0088  32 0040 0017
        FDV  TWO      NEXTC      147  0017  34 0820 0022
SINHX   STO  NEXTS               148  0319  24 0072 0525
        STU  ARG                 149  0525  21 0280 0333
        RSU  ARG                 150  0333  61 0280 0085
        LOD           EOOEA      151  0085  69 0188 0000
        STU  EARGM               152  0188  21 0442 0445
        RAU  ARG                 153  0445  60 0280 0185
        LDD           EOOEA      154  0185  69 0288 0000
        FSB  EARGM               155  0288  33 0442 0419
        FDV  TWO      NEXTS      156  0419  34 0820 0072
EOOCR   STD  EXIT                157  0269  24 0172 0575
        SMI  NEGAT    REDUC      158  0575  46 0078 0379
NEGAT   FAO  TWOPI               159  0078  32 0181 0307
        BMI  NEGAT               160  0307  46 0078 0061
        FSB  ONEPI    COSIO      161  0061  33 0014 0191
REDUC   FSS  TWOPI               162  0379  33 0181 0357
        BMI           REDUC      163  0357  46 0060 0379
        FAO  ONEPI    COSIU      164  0060  32 0014 0191
COSIO   STU  THETA               165  0191  21 0196 0199
        RSU  FPONE               166  0199  61 0002 0407
        STU  TERMM               167  0407  21 0412 0165
        STU  FUNKT               168  0165  21 0870 0373
        STL  ENN      NEGST      169  0373  20 0727 0330
EOOSR   STO  EXIT                170  0369  24 0172 0625
        SMI  NEGAV    REDUO      171  0625  46 0178 0429
NEGAV   FAD  TWOPI               172  0178  32 0181 0457
     ,  BMI  NEGAV               173  0457  46 0178 0161
        FSS  ONEPI    SINET      174  0161  33 0014 0291
REDUD   FSB  TWOPI               175  0429  33 0181 0507
        SMI           REDUD      176  0507  46 0310 0429
        FAD  ONEPI    SINET      177  0310  32 0014 0291
SINET   STU  THETA               178  0291  21 0196 0299
        RSU   8003               179  0299  61 8003 0557
        STU  TERMM               180  0557  21 0412 0265
        STU  FUNKT               181  0265  21 0870 0423
       'LDO  FPONE               182  0423  69 0002 0005
       ,STD  ENN      NEGST      183  0005  24 0727 0330
NEGST   RAU  ENN                 184  0330  60 0727 0281
        FAU  FPONE               185  0281  32 0002 0479
        STU  NPONE               186  0479  21 0034 0087
        FAO  FPONE               187  0087  32 0002 0529
        STU  ENN                 188  0529  21 0727 0380
        RSU  TERMM               189  0380  61 0412 0067
        FMP  THETA               190  0067  39 0196 0296
        FMP  THETA               191  0296  39 0196 0346
        FOV  NPONE               192  0346  34 0034 0084
        FDV  ENN                 193  0084  34 0727 0777
        STU  TERMM               194  0777  21 0412 0315
        RAM  FUNKT               195  0315  67 0870 0675
        STL  FMAG                196  0675  20 0579 0032
        RAM  TERMM               197  0032  67 0412 0167
        RAU   8002               198  0167  60 8002 0725
        FDV  FMAG                199  0725  34 0579 0629
        FSS  SIZES               200  0629  33 0082 0459
        RMI  ENUFF               201  0459  46 0462 0363
        RAU  FUNKT               202  0363  60 0870 0775
        FAD  TERMM               203  0775  32 0412 0189
        STU  FUNKT    NEGST      204  0189  21 0870 0330
ENUFF   RAU  FUNKT    EXIT       205  0462  60 0870 0172
SIZES    10   0000     0043      206  0082  10 0000 0043
TWOPI    62   8318     5351      207  0181  62 8318 5351
ONEPI    31   4159     2751      208  0014  31 4159 2751
FPONE    10   0000     0051      209  0002  10 0000 0051
EOOAU   STO  SEXT                210  0450  24 0503 0406
        BMI  SERR                211  0406  46 0509 0360
        NZE           SEXT       212  0360  45 0064 0503
        STU  SA                  213  0064  21 0068 0071
        FAD  S10                 214  0071  32 0674 0501
        FMP  SHAF     SS         215  0501  39 0304 0354
SS      STU  SSAV     SAB        216  0354  21 0058 0261
SAB     RAU  SA                  217  0261  60 0068 0473
        FOV  SSAV                218  0473  34 0058 0308
        FAD  SSAV                219  0308  32 0058 0285
        FMP  SHAF                220  0285  39 0304 0404
        FSS  SSAV                221  0404  33 0058 0335
        NZU           SR         222  0335  44 0289 0090
        SMI           SR         223  0289  46 0492 0090
        FAD  SSAV                224  0492  32 0058 0385
        STU  SSAV     SAS        225  0385  21 0058 0261
SR      RAU  SSAV     SEXT       226  0090  60 0058 0503
SERR    HLT   0000    SEXT       227  0509  01 0000 0503
SHAF     50   0000     0050      228  0304  50 0000 0050
S10      10   0000     0051      229  0674  10 0000 0051
START   RCU   1951               230  1999  70 1951 0551
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         LDD  1952                    231  0551  69 1952 0055
         STU  T                       232  0055  24 0358 0311
         LDD  ZERO                    233  0311  69 0164 0267
         STD  HOLD                    234  0267  24 0920 0523
         LDD  INDXM                   235  0523  69 0076 0679
         RAB   8001                   236  0679  82 8001 0435
         LDU  ZERO                    237  0435  69 0164 0317
         STD  HOLD       LOPP1        238  0317  24 0920 0573
LOPP1    RAU  BTAA   B                239  0573  60 4120 0925
         FDV  ALPHA                   240  0825  34 0278 0328
         FDV  TWO                     241  0328  34 0820 0970
         LDD             EOOAU        242  0970  69 0623 0450
         STU  SQRRT                   243  0623  21 0378 0331
         FMP  L                       244  0331  39 0186 0286
         STU  ARGGL                   245  0286  21 0409 0512
         LDD             ZMXSB        246  0512  69 0365 0400
         RAU  BTAA   B                247  0365  60 4120 0875
         FMP  T                       248  0875  39 0358 0408
         STU  PHEEE                   249  0408  21 0562 0415
         LDD             EODCH        250  0415  69 0168 0269
         FMP  ZMX1                    251  0168  39 0574 0724
         STU  A11                     252  0724  21 0428 0381
         RAU  PHEEE                   253  0381  60 0562 0367
         LDD             EOOSH        254  0367  69 1020 0369
         FMP  ZMX2                    255  1020  39 0624 0774
         STU  A12                     256  0774  21 0478 0431
         RAU  A11                     257  0431  60 0428 0383
         FSB  A12                     258  0383  33 0478 0305
         FMP  AMMM   B                259  0305  39 4100 0500
         FMP  SQRRT                   260  0500  39 0378 0528
         STU  A1     B                261  0528  21 4140 0093
         SXB   0001                   262  0093  53 0001 0349
         NZB  LOPP1      CONT1        263  0349  42 0573 0553
CONT1    LDD  NOLAM                   264  0553  69 0456 0559
         RAS   8001      LOOP9        265  0559  82 8001 0465
LDOP9    RAU  ALPHA                   266  0465  60 0278 0433
         FMP  KAPPA                   267  0433  39 0436 0336
         FMP  KAPPA                   268  0336  39 0436 0386
         STU  ALKAP                   269  0386  21 0190 0193
         FSB  LAMDA  B                270  0193  33 4220 0397
         STU  DIFF                    271  0397  21 0052 0355
         RAU  LAMDA  B                272  0355  60 4220 0925
         FMP  T                       273  0925  39 0358 0458
         LDD             EOOEA        274  0458  69 0361 0000
         STU  ELAMT                   275  0361  21 0366 0469
         RAU  ELAMT                   276  0469  60 0366 0171
         FMP  QOO                     277  0171  39 0324 0824
         FMP  AYEJ   B                278  0824  39 4200 0550
         FMP  ALPHA                   279  0550  39 0278 0578
         FDV  KAY                     280  0578  34 0581 0481
         FDV  DIFF                    281  0481  34 0052 0302
         STU  A3P                     282  0302  21 0506 0609
         RAU  KAPPA                   283  0609  60 0436 0341
         FMP  L                       284  0341  39 0186 0486
         LDD             SINHX        285  0486  69 0339 0319
         STU  SNHKL                   286  0339  21 0044 0447
         FMP  KAPPA                   287  0447  39 0436 0536
         STU  KSHKL                   288  0536  21 0290 0293
         RAU  KAPPA                   289  0293  60 0436 0391
         FMP  L                       290  0391  39 0186 0586
         LDD             COSHX        291  0586  69 0389 0169
         STU  CSHKL                   292  0389  21 0094 0497
         RAU  LAMD,A B                293  0497  60 4220 0975
         BMI  NEG        POS          294  0975  46 0628 0729
NEG      RSU  LAMDA  B                295  0628  61 4220 1025
         FDV  ALPHA                   296  1025  34 0278 0678
         LDD             EODAU        297  0678  69 0531 0450
         STU  SQRLA                   298  0531  21 0636 0439
         FMP  L                       299  0439  39 0186 0686
         LDD             EDOSR        300  0686  69 0489 0369
         STU  SINLL                   301  0489  21 0194 0547
         RAU  SQRLA                   302  0547  60 0636 0441
         FMP  L                       303  0441  39 0186 0736
         LDD             EOOCR        304  0736  69 0539 0269
         STU  COSLL                   305  0539  21 0294 0597
         RSU  SINLL                   306  0597  61 0194 0399
         FMP  SQRLA                   307  0399  39 0636 0786
         FDV  COSLL                   308  0786  34 0294 0344
         FMP  CSHKL                   309  0344  39 0094 0394
         FSB  KSHKL                   310  0394  33 0290 0417
         FMP  A3P                     311  0417  39 0506 0556
         STU  A3     B                312  0556  21 4240 0343
         SXB   0001                   313  0343  53 0001 0449
         NZB  LOOP9      CONT3        314  0449  42 0465 0603
POS      FDV  ALPHA                   315  0729  34 0278 0728
         LDD             EOOAU        316  0728  69 0631 0450
         STU  SQRLA                   317  0631  21 0636 0589
         FMP  L                       318  0589  39 0186 0836
         LDD             SINHX        319  0836  69 0639 0319
         STU  SNHLL                   320  0639  21 0444 0647
         RAU  SQRLA                   321  0647  60 0636 0491
         FMP  L                       322  0491  39 0186 0886
         LDD             COSHX        323  0886  69 0689 0169
         STU  CSHLL                   324  0689  21 0494 0697
         RAU  SNHLL                   325  0697  60 0444 0499
         FDV  CSHLL                   326  0499  34 0494 0544
         FMP  CSHKL                   327  0544  39 0094 0594
         FMP  SQRLA                   328  0594  39 0636 0936
         FSB  KSHKL                   329  0936  33 0290 0467
         FMP  A3P                     330  0467  39 0506 0606
         STU  A3     B                331  0606  21 4240 0393
         SXB   0001                   332  0393  53 0001 0549
         NZB  LODP9      CONT3        333  0549  42 0465 0603
CONT3    RAU  ONE                     334  0603  60 0656 0411
         RSL   8003                   335  0411  66 8003 0519
         STL  N                       336  0519  20 0673 0026
         RAU  ZERO                    337  0026  60 0164 0569
         STU  HOLD       LOOP3        338  0569  21 0920 0723
LOOP3    RAU  N                       339  0723  60 0673 0827
         FAD  TWO                     340  0827  32 0820 0747
         STU  N                       341  0747  21 0673 0176
         FMP  PI                      342  0176  39 0779 0829
         FDV  TWO                     343  0829  34 0820 1070
         FOV  L                       344  1070  34 0186 0986
         STU  ARG1                    345  0986  21 0340 0443
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        FMP  ARG1                 346   0443   39  0340  0390
        FMP  ALPHA                347   0390   39  0278  0778
        8TU  ARG2                 348   0778   21  0182  0485
        RAU  ZERO                 349   0485   60  0164  0619
        8TU  A24                  350   0619   21  0874  0877
        LDD  NOLAM                351   0877   69  0456  0659
        RAB   8001       LOP5     352   0659   82  8001  0515
LOP5    RAU  ARG2                 353   0515   60  0182  0187
        FAD  LAMDA  B             354   0187   32  4220  0797
        8TU  A2841                355   0797   21  0352  0405
        RAU  ARG2                 356   0405   60  0182  0287
        FAD  ALKAP                357   0287   32  0190  0517
        8TU  A2842                358   0517   21  0272  1075
        RAU  C8HKL                359   1075   60  0094  0599
        FMP  ALPHA                360   0599   39  0278  0828
        FMP  000                  361   0828   39  0324  0924
        FMP  AYEJ   B             362   0924   39  4200  0600
        FDV  KAY                  363   0600   34  0581  0681
        FDV  A2841                364   0681   34  0352  0402
        FDV  A2842                365   0402   34  0272  0322
        FAD  A24                  366   0322   32  0874  0601
        8TU  A24                  367   0601   21  0874  0927
        8XB   0001                368   0927   53  0001  0483
        NZB  LOP5        CONT9    369   0483   42  0515  0337
CONT9   RAU  ZERO                 370   0337   60  0164  0659
        8TD  BETT                 371   0669   24  0372  1125
        LDD  INDXM                372   1125   69  0076  0879
        RAB   8001       LOPP2    373   0879   82  8001  0535
LOPP2   RAU  ARG2                 374   0535   60  0182  0387
        FMP  ARG2                 375   0387   39  0182  0282
        8TU  ARG8Q                376   0282   21  1036  0739
        RAU  8TAA   8             377   0739   60  4120  1175
        FMP  BTAA   8             378   1175   39  4120  1120
        FAD  ARG8Q                379   1120   32  1036  0413
        8TU  DENOM                380   0413   21  0268  0271
        RAU  AMMM   8             381   0271   60  4100  0455
        FMP  ARG2                 382   0455   39  0182  0332
        FDV  DENOM                383   0332   34  0268  0318
        FAD  BETT                 384   0318   32  0372  0649
        8TU  BETT                 385   0649   21  0372  1225
        8XB   0001                386   1225   53  0001  0731
        NZB  LOPP2       CONT4    387   0731   42  0535  0585
CONT4   FAD  A24                  388   0585   32  0874  0651
        8TU  A28UM                389   0651   21  0706  0709
        RAU  ARG2                 390   0709   60  0182  0437
        FMP  T                    391   0437   39  0358  0508
        F8B  FIFTY                392   0508   33  0461  0487
        8MI  GO          STO      393   0487   46  0440  0541
8TO     LDD  ZERO                 394   0541   69  0164  0567
        8TD  TERM                 395   0567   24  1170  0773
        RAU  TERM                 396   0773   60  1170  1275
        FAD  HOLD                 397   1275   32  0920  0847
        8TU  HOLD                 398   0847   21  0920  0823
        NZU              CONT5    399   0823   44  0977  0878
        RAU  HOLD                 400   0977   60  0920  1325
        F8B  TERM        ADD      401   1325   33  1170  0897
GO      R8U  ARG2                 402   0440   61  0182  0537
        FMP  T                    403   0537   39  0358  0558
        LDD              EOOEA    404   0558   69  0511  0000
        8TU  EAG2T                405   0511   21  0416  0719
        RAU  L                    406   0719   60  0186  0591
        FMP  L                    407   0591   39  0186  1086
        FDV  ALPHA                408   1086   34  0278  0928
        FDV  N                    409   0928   34  0673  0873
        FDV  PI                   410   0873   34  0779  0929
        8TU  A2DIV                411   0929   21  0184  0587
        RAU  EAG2T                412   0587   60  0416  0321
        FMP  ARG1                 413   0321   39  0340  0490
        FDV  A2DIV                414   0490   34  0184  0284
        8TU  A2DOT                415   0284   21  0338  0641
        FMP  A28UM                416   0641   39  0706  0756
        8TU  TERM        ADD      417   0756   21  1170  0897
ADD     RAU  HOLD                 418   0897   60  0920  1375
        FAD  TERM                 419   1375   32  1170  0947
        8TU  HOLD                 420   0947   21  0920  0923
        RAU  TERM                 421   0923   60  1170  1425
        FDV  HOLD                 422   1425   34  0920  1220
        RAM   8003                423   1220   67  8003  1027
        RAU   8002                424   1027   60  8002  0635
        F8B  CRIT                 425   0635   33  0388  0565
        BMI  CONT5       LOOP3    426   0565   46  0878  0723
CONT5   LDD  INDXM                427   0878   69  0076  0979
        RAB   8001                428   0979   82  8001  0685
        RAU  ZERO                 429   0685   60  0164  0769
        8TU  CELL        LOP      430   0769   21  0974  1077
LOP     RAU  A1     B             431   1077   60  4140  0495
        FAD  CELL                 432   0495   32  0974  0701
        8TU  CELL                 433   0701   21  0974  1127
        8XB   0001                434   1127   53  0001  0533
        NZB  LOP         GOON1    435   0533   42  1077  0637
GOON1   LDD  NOLAM                436   0637   69  0456  0759
        RAB   8001                437   0759   82  8001  0615
        RAU  ZERO                 438   0615   60  0164  0819
        8TU  CELL3       LOP2     439   0819   21  1024  1177
LOP2    RAU  A3     B             440   1177   60  4240  0545
        FAD  CELL3                441   0545   32  1024  0751
        8TU  CELL3                442   0751   21  1024  1227
        8XB   0001                443   1227   53  0001  0583
        NZB  LOP2        GOON2    444   0583   42  1177  0617
GOON2   RAU  HOLD                 445   0687   60  0920  1475
        FAD  CELL                 446   1475   32  0974  0801
        FAD  CELL3                447   0801   32  1024  0851
        FMP  KAY                  448   0851   39  0581  0781
        R8L   8003                449   0781   66  8003  0789
        8TL  AN8WR       FINI8    450   0789   20  0493  0396
FINI8   LDD  AN8WR                451   0396   69  0493  0446
        8TD   1977                452   0446   24  1977  0430
        LDD  CELL                 453   0430   69  0974  1277
        8TD   1978                454   1277   24  1978  0831
        LDD  HOLD                 455   0831   69  0920  0973
        8TD   1979                456   0973   24  1979  0382
        LDD  CELL3                457   0382   69  1024  1327
        8TD   1980                458   1327   24  1980  0633
        LDD  X                    459   0633   69  1136  0839
        8TD   1981                460   0839   24  1981  0334
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             LDD                           461   0334   69 0358 0561
             STD   1982                    462   0561   24 1982 0735
             PCH   1977    START           463   0735   71 1977 1999
ONE           10   0000     0051           464   0656   10 0000 0051
TWO           20   0000     0051           465   0820   20 0000 0051
PI            31   4159     2751           466   0779   31 4159 2751
FIFTY         50   0000     0052           467   0461   50 0000 0052
ZERO          00   0000     0000           468   0164   00 0000 0000
CRIT          10   0000     0047           469   0388   10 0000 0047
ARTAN        STD   EXIT                      4   1800   24 1803 1806
             NZE           EXIT              5   1806   45 1810 1803
             BMI   MINUS                     6   1810   46 1813 1814
             STU   ARTAO                     7   1814   21 1818 1821
             LDD   FPONE                     8   1821   69 1824 1827
             STD   ENNNN                     9   1827   24 1830 1833
             STD   AYE     SUBTR            10   1833   24 1836 1839
MINUS        RSL   8003                     11   1813   66 8003 1871
             STL   ARTAO                    12   1871   20 1818 1822
             RSU   FPONE                    13   1822   61 1824 1829
             STD   ENNNN                    14   1829   24 1830 1883
             STU   AYE     SUBTR            15   1883   21 1836 1839
SUBTR        RAU   ARTAO                    16   1839   60 1818 1823
             FSB   FPONE                    17   1823   33 1824 1801
             NZE   DIFFE                    18   1801   45 1804 1805
             LDD   PIOV4                    19   1805   69 1808 1811
             STD   FUNGT   MULTA            20   1811   24 1864 1817
DIFFE        BMI   SMALL                    21   1804   46 1807 1858
             FSB   FPONE                    22   1858   33 1824 1851
             BMI   NEGAT   POSIT            23   1851   46 1854 1855
NEGAT        LDD   PIOV4                    24   1854   69 1808 1861
             STD   FUNGT                    25   1861   24 1864 1867
             RAU   ARTAO                    26   1867   60 1818 1873
             FAD   FPONE                    27   1873   32 1824 1802
             STU   TURRN                    28   1802   21 1856 1809
             FSB   FPTWO                    29   1809   33 1812 1889
             FDV   TURRN   COMBI            30   1889   34 1856 1857
SMALL        FAD   ARTAO                    31   1807   32 1818 1845
             BMI           NEGAT            32   1845   46 1848 1854
             RAU   ARTAO                    33   1848   60 1818 1874
             FSB   LOBNO                    34   1874   33 1877 1853
             BMI   TINEY                    35   1853   46 1859 1860
             RAU   ARTAO                    36   1860   60 1818 1825
             STL   FUNGT   COMBI            37   1825   20 1864 1857
POSIT        LDD   PIOV2                    38   1855   69 1862 1815
             STD   FUNGT                    39   1815   24 1864 1868
             FSB   UPBNO                    40   1868   33 1872 1849
             BMI           MULTA            41   1849   46 1852 1817
             RSU   FPONE                    42   1852   61 1824 1879
             FDV   ARTAO   COMBI            43   1879   34 1818 1857
COMBI        STU   TURRN                    44   1857   21 1856 1863
             STD   TURRM                    45   1863   24 1816 1819
             FMP   TURRN                    46   1819   39 1856 1865
             STU   ARGUE   FIGUR            47   1865   21 1820 1875
FIGUR        RAU   TURRM                    48   1875   60 1816 1826
             FAD   FUNGT                    49   1826   32 1864 1841
             STU   FUNGT                    50   1841   21 1864 1869
             RAU   ENNNN                    51   1869   60 1830 1835
             FAD   FPTWO                    52   1835   32 1812 1840
             STU   ENNNN                    53   1840   21 1830 1834
             RSU   TURRN                    54   1834   61 1856 1866
             FMP   ARGUE                    55   1866   39 1820 1870
             STU   TUHRN                    56   1870   21 1856 1876
             FDV   ENNNN                    57   1876   34 1830 1880
             STU   TURRM                    58   1880   21 1816 1828
             RAM   FUNGT                    59   1828   67 1864 1878
             STL   FMAGG                    60   1878   20 1884 1837
             RAM   TURRM                    61   1837   67 1816 1831
             RAU   8002                     62   1831   60 8002 1890
             FDV   FMAGG                    63   1890   34 1884 1885
             FSB   SIZES                    64   1885   33 1838 1881
             BMI   MULTA   FIGUR            65   1881   46 1817 1875
TINEY        RAU   ARTAO   MULTA            66   1859   60 1818 1817
MULTA        RAU   AYE                      67   1817   60 1836 1891
             FMP   FUNGT   EXIT             68   1891   39 1864 1803
FPONE         10   0000     0051            69   1824   10 0000 0051
FPTWO         20   0000     0051            70   1812   20 0000 0051
SIZES         10   0000     0043            71   1838   10 0000 0043
PIOV2         15   7079     6351            72   1862   15 7079 6351
PIOV4         78   5398     1650            73   1808   78 5398 1650
UPBNU         10   0000     0060            74   1872   10 0000 0060
LOBNU         10   0000     0040            75   1877   10 0000 0040
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AN INVESTIGATION OF NUCLEAR EXCURSIONS TO DETERMINE THE SELF-SHUTDOWN EFFECTS IN THERMAL, HETEROGENEOUS, HIGHLY ENRICHED LIQUID-MODERATED REACTORS

by

JOHN ROBERT FAGAN

B.S., University of Nebraska, 1957

AN ABSTRACT OF
A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1962

The safe operation of nuclear reactors is imperative if there is to be increased engineering application of these systems. Transient reactor experiments, such as the SPERT tests, have demonstrated that thermal, heterogeneous, liquid-moderated reactor systems will safely shut themselves down following step and ramp insertions of limited amounts of excess reactivity. It is important that a model based on the nuclear, thermodynamic and hydrodynamic properties of the reactor system be developed to explain this phenomena so that it can be used in the design of new systems.

Equations for the fine structure of the temperature distribution in a unit cell of a heterogeneous reactor during a transient burst were derived based on the known power and fuel surface temperature distributions. A model based on recognized shutdown effects was developed to calculate the excess reactivity during a transient using the temperature distributions to define the deposition of energy. The calculated excess reactivities show this model to be satisfactory. The effect on reactivity due to steam formation required one empirical parameter which can probably be removed when a greater knowledge of transient boiling is available.