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CONDENSING HEAT TRANSFER IN A HORIZONTAL TUBE

by

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## INTRODUCTION

In 1916 Nusselt first formulated the mathematics involved in condensing pure vapors. Since then great progress has been made in both theoretical analysis and empirical correlation of condensing heat transfer data. However, the analysis of condensing phenomena inside a horizontal tube has not been completely formulated.

Many investigators in their analysis of this problem neglect the shear force which exists at the vapor-liquid interface inasmuch as this factor increases the complexity of the analysis. In this report, an effort is made to determine a consistent equation for correlation and determination of condensing heat transfer coefficients by considering the effect of shear force at vapor-liquid interface.

Experimental data for Freon 12 and Methanol were taken from KSU-ASHRAE cooperative research project and other workers to verify the study.

## ANALYSIS OF FLOW PATTERNS

### Definition and Classification

Two-phase flow patterns can be characterized by the gas-liquid ratio and the rate of mass flow. At a constant gas-liquid ratio with a fixed mass flow rate, the flow pattern is constant. This is defined as fixed two-phase flow. Air-water flow is a typical case of fixed two-phase flow except when large pressure drops occur. On the other hand, at a fixed mass flow rate but with variable gas-liquid ratio, the flow pattern

is variable. This is defined as variable two-phase flow. Condensing flow is a typical case of variable two-phase flow.

Two-phase flow patterns have been investigated by Martinelli (3), Hoogendoorn (1), Borgelin and Gazley (25) and many other workers. In general these patterns may be classified in the following categories: (Plate I and II)

- A. Stratified Flow  
The liquid flows in the lower part of the pipe and the vapor over it with a smooth interface between the two-phases.
- B. Wave Flow  
Similar to stratified flow, except for a wavy interface due to the velocity difference between the two phases.
- C. Wave-mist Flow  
Wave flow will turn to wave-mist flow when the total flow rate is so increased that a certain amount of atomization takes place.
- D. Plug Flow  
The vapor moves in bubbles or plugs along the upper side of the pipe.
- E. Slug Flow  
Splashes or slugs of liquid move at higher velocity than the bulk of the liquid. Pressure fluctuations are typical for this type of flow.
- F. Mis-annular Flow  
When the total flow rate is so increased that the vapor velocity is higher than that of wave-mist flow, the liquid is partly atomized in the vapor phase and partly flowing in an annular film along the pipe wall.
- G. Froth Flow  
The gas is dispersed in fine bubbles through the liquid phase.

#### Condensing Flow Patterns

From Plate I and II it is seen that without heat transfer between the tube wall and the fluid, and with constant mass flow, the flow pattern is fixed. That is, at any fixed mass flow and vapor-liquid ratio, the flow pattern is represented by a fixed point on the diagram.



## EXPLANATION OF PLATE I

Schematic diagram of two-phase flow patterns

$C_g$  : gas percentage

$G_g$  : gas flow rate by volume,  $m^3/sec$

$G_l$  : liquid flow rate by volume,  $m^3/sec$

$V_m$  : mixture velocity,  $m/sec$

$D$  : diameter of tube,  $m$

----- : constant mass flow line

\_\_\_\_\_ : transition line

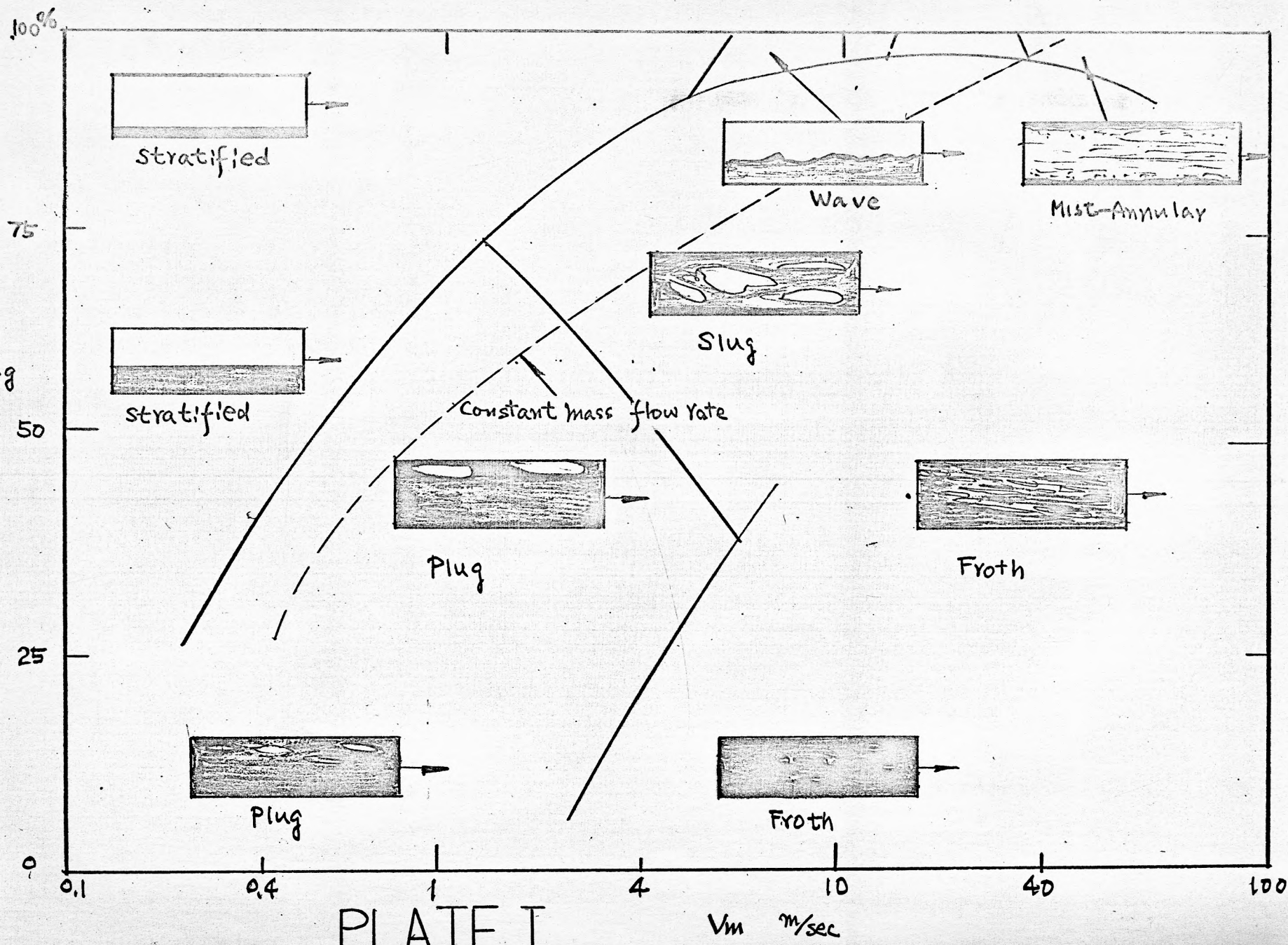


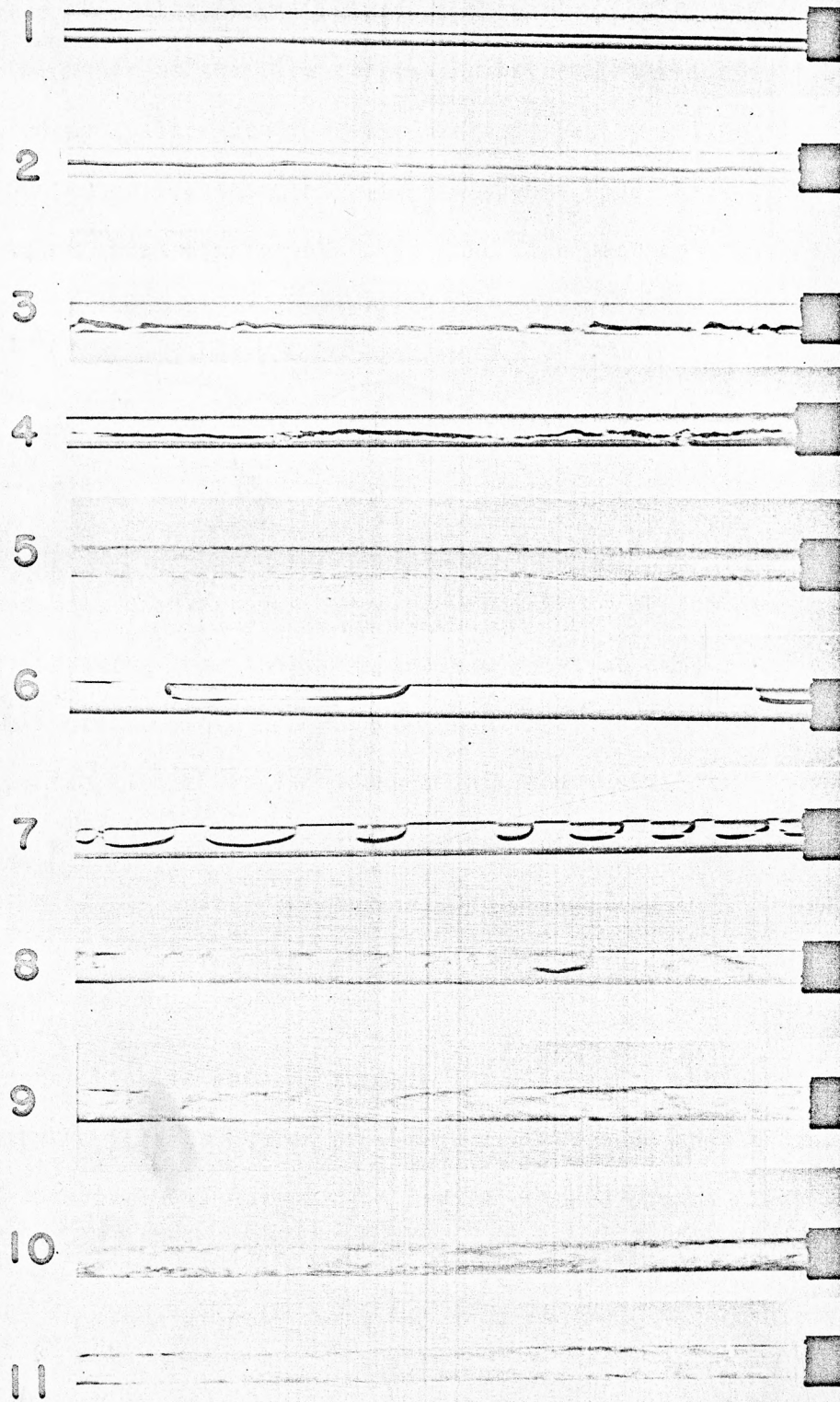
PLATE I



## EXPLANATION OF PLATE II

The picture of air-water flow patterns

- 1: stratified flow
- 2: transition of stratified to wave flow
- 3: wave flow
- 4: mist-wave flow
- 5: annular flow
- 6: slow plug flow (high liquid percentage)
- 7: plug flow
- 8: slug flow
- 9: slug flow
- 10: froth flow
- 11: froth flow





When there is heat transfer between the tube wall and the fluid so as to condense the vapor as the flow travels downstream, the flow pattern is not represented by a fixed point on the diagram, but by a line of constant mass flow rate. Consequently, the flow pattern depends upon the rate of heat transfer. At low rates of heat transfer, the flow pattern tends to converge to a fixed point, i.e., fixed two-phase flow. On the other hand, at very high rates of heat transfer, the flow pattern tends to change rapidly.

The pressure drop in condensing flow is partly due to condensation and partly due to friction. At low rates of condensation, the pressure drop may be approximated by the pressure drop in fixed two-phase flow which the Martinelli and Nelson's method<sup>(6)</sup> applies. To predict the pressure drop at high rates of heat transfer, Martinelli and Nelson's method<sup>(6)</sup> should be modified by considering the pressure drop due to condensation.

In condensing flow there is always a thin condensate film formed on the tube wall. As a result of condensate in circumferential flow, transition of flow patterns will take place earlier in condensing flow than in the fixed two-phase flow; that is from mist-annular to mist-wave, or mist wave to wave, etc.

A phenomena which is not reported in the literature was observed in these experiments. In the region of wave, mist-wave and annular flow, the vapor-liquid interface was observed to become concave upward along the tube wall as vapor velocity was increased. It is assumed that the down flow of condensate due to gravitational force may be a factor. Nevertheless, even with air-water flow, the interface still exhibited a trend to become concave upward but to a less degree.

## ANALYSIS OF HEAT TRANSFER

### Literature Survey

Many equations for heat transfer coefficients have been either semi-theoretically derived or empirically correlated. For the purpose of comparison several possible assumptions which have been used are listed below. The assumptions which apply specifically to the correlation equations which follow are listed with the equation.

1. Laminar flow
2. A film of condensate covered the cooling surface
3. Only latent heat was transferred through the film
4. Negligible surface curvature
5. Negligible vapor shear on the vapor-liquid interface
6. Physical properties of the condensate taken at the mean film temperature
7. Uniform wall temperature
8. Negligible accumulation of condensate on the bottom of tube
9. Turbulent flow
10. Vertical tube

Any other assumptions used in specific equations will be described separately.

### General Equations for Two-Phase Flow

#### 1. Nusselt's Equation

Nusselt first presented an equation for condensation on a horizontal tube.

$$h_m = 0.728 \left[ \frac{k_l^3 \rho_l (\rho_l - \rho_v) g h_{fg}}{D \Delta T \mu_l} \right]^{1/4}$$



The equation may apply to condensation inside a horizontal tube.

Assumptions: 1 through 8.

2. Carpenter and Corburn's equation (12).

$$h_m = 0.065 \left[ \frac{C_p \rho_l K_1 f}{2 \mu_l \rho_v} \right]^{1/2} G_m$$

Where  $G_m$  is the mean value of vapor mass flow rate,  $\text{lb}_m/\text{hr ft}^2$

Assumptions: 9, 10.

The equation studies the effect of vapor velocity on condensation inside a vertical tube. It is also assumed that all resistance to heat transfer takes place in a liquid laminar sublayer which is characterized by use of the universal velocity profile.

3. Crosser's Equation (13)

$$\frac{h_m D}{K_l} = B (P_{rl})^{1/3} \left[ \frac{D G_m}{\mu_l} \left( \frac{\rho_l}{\rho_v} \right)^{1/2} \right]^n$$

Where  $n$  varies from 0.2 at low Reynolds number to a constant value of 0.8 at Reynolds number of  $10^5$ .  $B$  is a constant.

Assumptions: 2 through 9.

4. Chaddock's Equation (26)

$$h_{m\psi} = (2\psi)^{1/3} \beta^{4/3} \left[ \frac{K_l \rho_l g}{\mu_l} \right]^{1/3} \left[ \frac{4\Gamma}{\mu_l} \right]^{1/3}$$

$$\beta = \frac{0.9036}{\psi} \int_0^\psi \frac{d\theta}{S^{1/4}}$$

Assumptions: 1, 2, 5, 7.

The equation assumed that a film which formed over an angle inside the tube would be identical with a film formed over an angle outside of the tube.

When  $\psi = 2\pi$  the equation becomes

$$h_{m\psi} = \beta \left[ \frac{K_1^3 \rho_1^2 h_{fg} g}{\mu_1} \right]^{1/4} \left[ \frac{1}{D \Delta T} \right]^{1/4}$$

which is Nusselt's Equation.

### 5. Chato's Equation (Stratified Flow only) (7)

For slope  $\Gamma \leq 0.002$

$$h_m = 0.468 K' \left[ \frac{g \rho_1 (\rho_1 - \rho_v) h_{fg} (1 + 0.68 \xi) K_1^3}{\mu_1 r_o \Delta T} \right]^{1/4}$$

Assumptions: 1, 2, 3, 5, 6, 7.

where  $\xi = \left( \frac{C_p \Delta T}{h_{fg}} \right)$

For slope  $\Gamma \geq 0.002$

$$h_m = 0.3 K' \left[ \frac{g \rho_1 (\rho_1 - \rho_v) h_{fg} (1 + 0.68 \xi) K_1^3}{\mu_1 r_o \Delta T} \right]^{1/4} \left[ \int_0^\phi \sin^{1/3} \theta d\theta \right]^{3/4} \cos^{1/4} (\sin^{-1} \Gamma)$$

The equation employs the same method as used by Chaddock by using an angle factor which applies only to laminar flow (stratified). However, an important result obtained by Chato is that when the tube is inclined ten to twenty degrees from the horizontal line, maximum heat transfer may be resulted. In the equation  $K'$  is a correction factor for low Prandtl number.

For heat transfer coefficients for Freon 12, the following equations have been obtained.

### 6. White's Equation (14)

$$h_m = 0.630 \left[ \frac{K_1^3 \rho_1^2 g h_{fg}}{D \mu_1 \Delta T} \right]^{1/4}$$

At elevated pressure the equation gives values 13 per cent below those predicted by Nusselt's equation.

Assumptions: 1, 2, 3, 4, 5, 6, 7.



## 7. Rosson's Equation (Stratified and Wave Flow) (8)

$$N_{ul} = 0.388 (P_{rl})^{1/3} \left[ \frac{g(\rho_l - \rho_v) \rho_l r^2}{K_1 \mu_1 \Delta T} \right]^{1/6} \left[ \frac{D G_v}{\mu_1} \left( \frac{\rho_l}{\rho_v} \right)^{1/2} \right]^{1/4}$$

Assumptions: 1, 2, 3, 4, 5.

Also the equation assumed no acceleration and no radial velocity in the liquid. The equation shows a maximum deviation of  $\pm 20$  per cent.

## 8. Patel's Equation (15)

$$N_{ul} = 10.17 \left( \frac{D G_m}{\mu_1} \right)^{0.4913}$$

## 9. Sun's Equation (11)

$$\frac{h_m D}{K_1} = 27.8 \left( \frac{D G_m}{\mu_1} \right)^{0.29366}$$

This equation was correlated by Sun (11) in 1959. Data were taken from the experiments performed at Kansas State University. The equation allows a maximum error of 30 per cent and the data plotted  $\frac{D G_m}{\mu_1}$  against  $N_{ul}$  shows considerable scattering particularly when the inlet condition was wet. That is when condensate had accumulated on the bottom of the tube. Comparison will be made later with a new equation presented in this report by using the same data.

## 10. Akers and Rosson's Equation (9)

$$N_{ul} = C(P_{rl})^{1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/6} \left[ \frac{D G_v}{\mu_1} \left( \frac{\rho_l}{\rho_v} \right)^{1/2} \right]^n$$

where	$\frac{D G_v}{\mu_1} \left( \frac{\rho_l}{\rho_v} \right)^{1/2}$	n	C
	1,000	0.2	13.8
	100,000	0.67	0.1

A chart of data correlated by this equation is also reproduced in Plate VII to compare with the equation presented in this report.

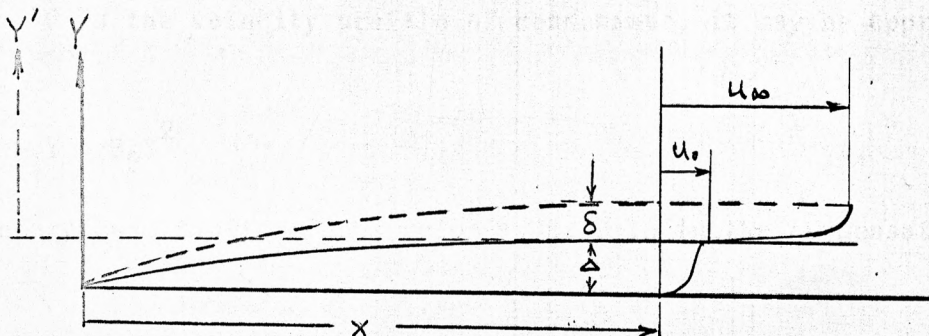
Assumptions: 2, 3, 4, 7, 8, 9.

### Derivation

For the purpose of analysis an equation for the heat transfer coefficient for a horizontal plate was first attempted. Following some modification, these equations may be used for condensing heat transfer in a horizontal tube. Equation for laminar and turbulent flow are also obtained in a form suggested for experimental correlation.

Consider the vapor boundary formed at the vapor liquid interface when the vapor is flowing along the plate and while the vapor is condensing on the plate. The following assumptions are made:

1. Constant plate temperature
2. Constant vapor velocity in X direction at Y greater than the thickness of vapor boundary layer, i.e.,  $u_{\infty} = \text{constant}$
3. Vapor boundary layer is independent of liquid film
4. Velocity at vapor-liquid interface is constant
5. No slip at vapor condensate interface
6. Saturated temperature exists at vapor-liquid interface.





Assume that the velocity profile for the vapor boundary layer takes the form

$$V = u - u_0 = C_1 Y' + C_2 Y'^3 \quad (1)$$

Where  $u_0$  is the velocity of the interface and  $u$  is absolute velocity. Then boundary conditions are

$$\begin{array}{lll} Y' = 0 & V = 0 & u = u_0 \\ Y' = \delta & V = V_\infty & u = u_\infty \\ Y' = \delta & \frac{\partial V}{\partial Y'} \Big|_{\delta} = 0 & \frac{\partial u}{\partial Y'} \Big|_{\delta} = 0 \end{array}$$

From the boundary conditions Equation (1) becomes

$$\frac{V}{V_\infty} = \frac{u - u_0}{u_\infty - u_0} = 1.5 \frac{Y'}{\delta} - \frac{1}{2} \left( \frac{Y'}{\delta} \right)^3 \quad (2)$$

Applying Von Kármán's integral method, the thickness of boundary layer is obtained

$$\delta = 4.64 \frac{X}{(Re_v)^{1/2}} \quad (3)$$

When no slip at vapor-liquid interface is assumed, the shear force in the vapor at the interface must be equal to that in the liquid, or

$$\mu_v \frac{\partial V}{\partial Y'} \Big|_{Y'=0} = \mu_l \frac{\partial U}{\partial Y} \Big|_{Y=\delta} = \mu_v \frac{1.5 V_\infty}{\delta} \quad (4)$$

Where  $U$  is the velocity profile of condensate, it may be approximated by

$$U = B_1 Y + B_2 Y^2 \quad (5)$$

Boundary conditions for the velocity profile in the condensate are

1.  $Y = 0 \quad U = 0$
2.  $Y = \Delta \quad U = u_0$
3.  $Y = \Delta \quad \left. \frac{\partial U}{\partial Y} \right|_{\Delta} = \frac{\mu_v}{\mu_l} \frac{1.5}{\delta} V_{\infty} = \mu^* \frac{1.5}{\delta} V_{\infty}$

Thus equation (5) with these boundary conditions becomes

$$U = \left( \frac{2U_0}{\Delta} - \mu^* \frac{1.5V_{\infty}}{\delta} \right) Y + \left( \mu^* \frac{1.5V_{\infty}}{\delta \Delta} - \frac{U_0}{\Delta^2} \right) Y^2 \quad (6)$$

The mass rate of flow of condensate per unit breadth of the plate,  $\Gamma$ , is obtained by integrating the local mass flow rate at the distance  $X$  between the limits  $Y = 0$  and  $Y = \Delta$ , or

$$\begin{aligned} \Gamma &= \int_0^{\Delta} \rho_l \left[ \left( 2 \frac{U_0}{\Delta} - \mu^* \frac{1.5V_{\infty}}{\delta} \right) Y + \left( \mu^* \frac{1.5V_{\infty}}{\delta \Delta} - \frac{U_0}{\Delta^2} \right) Y^2 \right] dY \\ &= \rho_l \left[ \frac{2}{3} U_0 \Delta - \frac{1}{4} \mu^* \frac{V_{\infty} \Delta^2}{\delta} \right] \end{aligned} \quad (7)$$

Since heat is transferred through the condensate film solely by conduction in laminar flow, the rate of heat flow through  $dX$  to the plate is

$$\frac{K_l (T_w - T_v) dX}{\Delta} \quad (8)$$

But between the distance  $X+dX$  and  $X$  the film thickness increased by an amount  $d\Delta$  as a result of condensation. The liquid condensed in this distance increase the liquid flow rate by the amount

$$d\Gamma = \rho \left( \frac{2}{3} U_0 - \frac{1}{2} \mu^* \frac{V_{\infty} \Delta}{\delta} \right) d\Delta \quad (9)$$

At the same time there must be maintained a rate of heat flow to the cold tube wall equal to the rate at which latent heat of condensation is released between  $X$  and  $dX$ . Therefore, the energy balance between the rate of heat liberation as a result of condensation at the vapor liquid interface and the



rate of heat conduction through the film is

$$\frac{K_1 (T_w - T_v) dX}{\Delta} = h_{fg} d\Gamma$$

or

$$-K_1 (T_w - T_v) dX = h_{fg} \rho_l \left[ -\frac{2}{3} u_o \Delta + \frac{1}{2} \mu^* \frac{v_\infty \Delta^2}{\delta} \right] d\Delta \quad (10)$$

Equation (10) turns out to be a nonlinear differential equation and is difficult to integrate. Assuming for an approximation

$$\Delta^2 = cX \quad (11)$$

where  $c$  is constant for certain conditions of condensation, then, with Equation (3), Equation (10) may be reduced to an ordinary differential equation and  $\Delta$  is obtained.

$$\Delta = \left[ \frac{4.64 \times 4 \left( \frac{\gamma_v}{v_\infty} \right)^{1/2} \left\{ K_1 (T_w - T_v) + \frac{h_{fg} \rho_l u_o c}{3} X^{3/2} \right\}}{h_{fg} \rho_l v_\infty \mu^*} \right]^{1/3} \quad (12)$$

Equation (12) has the same form as Equation (11). When the heat transfer coefficient is defined as  $h = \frac{K_1}{\Delta}$  then

$$h = \left[ \frac{h_{fg} K_1^3 u_o \left( 1 - \frac{u_o}{u_\infty} \right) \rho_l \mu^*}{4 \times K_1 (T_w - T_v) \left( \frac{\rho_l h_{fg} c u_o^2}{3 K_1 \Delta T} + 1 \right) \delta} \right]^{1/3} \quad (13)$$

Thus, local Nusselt number takes a form of

$$N_{ul} = \left( \frac{1}{18.56} \right)^{1/3} (R_{ev})^{1/2} \left( \frac{\gamma_v}{\gamma_l} \right)^{2/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/3} \left( \frac{\rho_l}{\rho_v} \right)^{1/3} (P_{rl})^{1/3} B^{1/3} \quad (14)$$

where

$$B = \frac{\left( 1 - \frac{u_o}{u} \right)^{1/2}}{1 + \frac{2 h_{fg} c u_o \rho_l}{3 K_1 \Delta T}}$$

In the term B,  $c$  is a constant from the film profile approximation given in Equation (11). That is under certain conditions of condensation the profile of the film thickness is a parabolic function of  $X$ . When  $X$  is held fixed,  $c$  is found to be proportional to  $\Delta^2$ . But  $\Delta^2$  is proportional to the temperature difference across the condensate film,  $\Delta T$ , and thermal conductivity,  $K_l$ . On the other hand,  $\Delta^2$  is also inversely proportional to the latent heat of the vapor,  $h_{fg}$ , density,  $\rho_l$ , and the velocity at the vapor liquid interface,  $u_o$ . Therefore

$$c \propto \Delta^2 \propto \frac{\Delta T K_l}{h_{fg} \rho_l u_o}$$

Let  $E$  be a constant such that

$$c = E \frac{\Delta T K_l}{h_{fg} \rho_l u_o}$$

Then,  $B$  becomes

$$B = \frac{(1 - \frac{u_o}{u_\infty})^{1/2}}{1 + 2/3 E}$$

Furthermore, when  $u_\infty$  increases,  $u_o$  also increases. If  $u_o/u_\infty$  is assumed to be a constant for a certain range of  $u_\infty$ ,  $B$  is always less than one and also can be treated as a constant. Accordingly, an equation useful for correlation is obtained and the value of undetermined constant is less than one. The average value of the heat transfer coefficient expressed in Nusselt number  $\bar{N}_{ul}$  is then

$$\bar{N}_{ul} = 3/2 N_{ul} \quad (15)$$

In the case of turbulent flow, a suggested form may be obtained by modifying the term which contains the eddy viscosity. From Equation (14)



$$N_{ul} = F (R_{ev} P_{rl} \frac{h_{fg}}{C_p \Delta T} \frac{\nu_v}{\nu_l} \frac{\rho_l}{\rho_v}) \quad (16)$$

Where  $P_{rl}$  and  $\rho_l/\rho_v$  are fluid properties,  $\frac{h_{fg}}{C_p \Delta T}$  is thermal potential, and  $R_{ev}$  and  $\nu_v/\nu_l$  are the dimensionless group related to the flow condition. To examine more carefully on  $R_{ev}$  and  $\nu_v/\nu_l$ ,  $R_{ev}$  depends upon a characteristic length  $L$  or  $D$  while  $\nu_v/\nu_l$  is independent of geometric factor. In condensing flow when the vapor flows in the continuous phase, the main difference between laminar and turbulent flow is that the shear force at the interface is no more governed by kinematic viscosity of fluid but by eddy viscosity. The eddy viscosity is greater than the kinematic viscosity which is negligible in turbulent flow. From this analysis it is concluded that term,  $\nu_v/\nu_l$ , in Equation (14) should be modified for turbulent flow using the eddy viscosity.

$$\frac{\nu_v}{\nu_l} \longrightarrow \frac{\epsilon_{mv} + \nu_v}{\epsilon_{ml} + \nu_l} \longrightarrow \frac{\epsilon_{mv}}{\epsilon_{ml}}$$

However, from Prandtl's mixing-length theory eddy viscosity may be expressed by  $L^2 \frac{du}{dy}$ . Where  $L$  is mixing length. In other words, the eddy viscosity is independent of the fluid properties. Therefore, two assumptions may be made:

1. The term  $\nu_v/\nu_l$  transforms to  $\epsilon_{mv}/\epsilon_{ml}$  and may be considered as a constant in turbulent flow.
2. In turbulent flow with  $\epsilon_{mv}/\epsilon_{ml}$  constant, the exponent of Reynolds number may be affected.

The equation for condensing flow in turbulent is then suggested in the following forms:

$$N_{ul} = \text{Constant} (R_{ev})^N (P_{rl})^{1/3} \left(\frac{h_{fg}}{C_p \Delta T}\right)^{1/3} \left(\frac{\rho_l}{\rho_v}\right)^{1/3} \quad (17)$$

Where  $N$  is  $1/3$  or an undetermined constant.

For heat transfer inside a horizontal tube, an analogue to the horizontal plate may be employed if mean film thickness  $\Delta_m$  and a characteristic velocity  $U_m$  are assumed for  $\Delta$  and  $u_w$  in Equation (10). Mean film thickness is defined as the film thickness which would form in a horizontal tube when the gravitational force is neglected. The characteristic velocity here is defined as the mean velocity of vapor in the tube as it flows in single phase. In another word, it is the vapor mass flow rate divided by the cross section area of the tube.

On the other hand, however, the vapor boundary layer in a tube is no longer the vapor boundary layer on the flat plate when the velocity profile in the tube is fully developed. The vapor boundary layer in the tube may be taken as

$$\delta = R - \Delta_m \quad (18)$$

for an approximation. Therefore, Equation (10) may be transformed to

$$\frac{K_1 \Delta T (R - \Delta_m) dX}{h_{fg} \rho_1} = \left[ \frac{1}{2} \frac{\mu_v u_m \Delta_m^2}{\mu_1} - \frac{2}{3} u_0 (R - \Delta_m) \Delta_m \right] d\Delta_m \quad (19)$$

Integrating as in Equation (12) from  $X = 0$  to  $X = X$  and  $\Delta_m = 0$  to

$\Delta_m = \Delta_m$ , film thickness  $\Delta_m$  is then

$$\Delta_m = \left[ \frac{3 \left\{ \frac{K_1 \Delta T}{h_{fg} \rho_1} \left( R - \frac{2\sqrt{cX}}{3} \right) + \frac{cR}{3} \right\} X}{\frac{1}{2} \frac{\mu_v}{\mu_1} u_m + \frac{2}{3} U_0} \right]^{1/3} \quad (20)$$

The heat transfer coefficient is then

$$h = \left[ \frac{K_1^3 \left( \frac{\mu_v u_m}{2 \mu_1} + \frac{2}{3} u_0 \right)}{3X \left\{ \frac{K_1 \Delta T}{h_{fg}} \left( R - \frac{2\sqrt{cX}}{3} \right) + \frac{cR}{3} \right\}} \right]^{1/3} \quad (21)$$



The Nusselt number is then

$$N_{ul} = \left(\frac{1}{6}\right)^{1/3} \left(\frac{4\Gamma}{\mu_l}\right)^{1/3} (P_{rl})^{1/3} \left(\frac{h_{fg}}{c_p \Delta T}\right)^{1/3} \left(\frac{\gamma_v}{\gamma_l}\right)^{1/3} B^{1/3} \quad (22)$$

Where

$$B' = \frac{1 + \frac{4\mu_l u_0}{3\mu_v u_m}}{\frac{X}{D} \left(1 - \frac{2\sqrt{cX}}{3R} + \frac{c h_{fg}}{3K_l \Delta T}\right)} \quad \text{and} \quad \Gamma = \frac{4 A U_m \rho_v}{\pi D} = \frac{W}{\pi D}$$

To treat  $B'$  as a constant is rather complicated. Nevertheless,  $c$  in the term  $B'$  may be treated in a manner similar to that in Equation (14).

Let  $E'$  be a constant and equal to  $\frac{c h_{fg}}{3 K_l \Delta T}$  and note that the relation

of shear force at the vapor-liquid interface is  $\mu_l \frac{du}{dy} \Big|_{\Delta} = \mu_v \frac{au}{ay} \Big|_0$ . At a small distance from the interface the velocity in the liquid side is approximately  $u_0$  while on the vapor side, the velocity increases approximately to  $u_m$ . Then,

$$\mu_l u_0 = \mu_v u_m \quad \text{or} \quad 1 = \frac{\mu_l u_0}{\mu_v u_m}$$

Hence the numerator of  $B'$  in Equation (22) is a constant. The remainder in the denominator of term  $B'$  is

$$\frac{X}{D} \left(1 - \frac{2\sqrt{cX}}{3R} + E'\right)$$

Although the distance  $X$  cannot be considered as a constant, in many practical cases there is no way to predict the distance at which the flow condition at the inlet to a condenser is wet. That is the point at which some condensate has accumulated on the bottom of the tube. Moreover, when  $\frac{X}{D}$  increases,  $\left(1 + E' - \frac{2\sqrt{cX}}{3R}\right)$  decreases. Some degree of stability is, therefore, provided. If  $B'$  is assumed to be constant, an equation for laminar condensation inside a horizontal tube is:

$$\checkmark N_{ul} = \text{constant} \left( \frac{4\Gamma}{\mu_1} \right)^{1/3} (P_{rl})^{1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/3} \left( \frac{\gamma_v}{\gamma_l} \right)^{1/3} \quad (23)$$

In the case of turbulent condensation, the same assumption in Equation (17) may be employed. That is to transform  $\gamma_v/\gamma_l$  to  $\epsilon_{mv}/\epsilon_{ml}$  which according to Dessieler and Von Karman's assumption for a duct flow is

$$\epsilon_m = n^2 u y \quad \text{when } y^+ \text{ is less than 26 and } n \text{ is } 0.109$$

where  $y^+ = \frac{y u^*}{\nu}$  of the universal velocity profile in a tube

$$\epsilon_m = K \frac{\left( \frac{du}{dy} \right)^3}{\left( \frac{d^2 u}{dy^2} \right)^2} \quad \text{when } y^+ \text{ is greater than 26 and } K \text{ is } 0.36.$$

Hence by analog to Equation (17) the equation for the Nusselt number in a horizontal tube is as follows:

$$N_{ul} = \text{constant} \left( \frac{4\Gamma}{\mu_1} \right)^{1/3} (P_{rl})^{1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/3} \quad (24)$$

or

$$N_{ul} = \text{constant} \left( \frac{4\Gamma}{\mu_1} \right)^n (P_{rl})^{1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/3}$$

From this analysis it may be concluded that for a single phase flow

$$N_u = F(R_e, P_r)$$

for fixed two-phase flow such as air-water flow

$$N_u = F(R_e, P_r, \frac{\gamma_v}{\gamma_l})$$

for variable two-phase flow such as condensation

$$N_{ul} = F\left(\frac{4\Gamma}{\mu_1}, P_r, \frac{\gamma_v}{\gamma_l}, \frac{h_{fg}}{C_p \Delta T}\right)$$

Furthermore, it is proposed that for two-phase flow the Stanton number be defined as follows.



For fixed two-phase flow such as air-water flow

$$St_{tf} = \frac{N_u}{R_e P_{rl} \frac{\gamma_v}{\gamma_l}}$$

For variable two-phase flow such as condensation

$$St_{tv} = \frac{N_u}{\frac{4\Gamma}{M_l} P_{rl} \frac{\gamma_v}{\gamma_l} \frac{h_{fg}}{C_p \Delta T}}$$

These relationships may prove to be helpful in the correlation of pressure drop or skin drag data.

## EXPERIMENTAL APPARATUS AND EQUIPMENT

### System

The apparatus used in this investigation was originally constructed by Patel (15), Hwang (16), and Sun (11) in 1956, 1957, and 1958 respectively. It was successively modified in order to obtain better data and results. The major components are:

1. Vapor generator
2. Pre-condenser
3. Test section
4. Pre-accumulating tank
5. Condensate tank
6. After-condenser
7. Condensed exit vapor tank

The major components are supplemented by the necessary cooling-water metering devices, temperature measuring instruments, thermocouple elements, and electrical power measuring instruments.

All sections of the equipment containing vapor were insulated. A schematic diagram of the equipment is shown in Plate III and details of components and their construction are given in (15), (16), and (11). Modifications are given in Appendix C.

The vapor from the vapor generator rose vertically approximately five feet before entering a horizontal condensing section. In this condensing section, the fluid first passed through a three foot double-pipe pre-condenser where the liquid loading was controlled by partial condensation. The amount of partial condensation was measured by the pre-accumulating tank. In order to make sure that all condensate was measured by the tank a trap and a by-pass pipe were used at the end of the pre-condenser. The flow then passed through a six-inch test section where all condensing coefficients were measured by a Copper-Bismuth heatmeter. Two sight glasses before and after the test section provide an observation of the flow pattern.

The exit flow from the test section was separated into vapor and condensate so that all the condensate was accumulated in the condensate tank. The vapor then passed through the after-condenser and accumulated in the condensed vapor tank. After measurement, the condensate in the three tanks was returned to the vapor generator. All valves on the three tanks were open except when the measurement was taken. Heat input to the system was measured by a wattmeter.






Three water tanks were divided so that the cooling water rate to the pre-condenser, the test section, and the after-condenser could be measured simultaneously. A Copper-Bismuth heatmeter was used for the test section. Details of the construction and method of measurement are given in Appendix C. The heatmeter was divided into four segments so that the circumferential temperature profile could be obtained. However, the axial temperature profile along the test section could not be determined.



# EXPLANATION OF PLATE III

Schematic diagram of experimental apparatus and system

Legend:

A: generator	 : thermocouple
B: superheater	 : valve
C: precondenser	 : pressure gauge
D: preaccumulating tank	 : pressure control gauge
E: sight glass	 : cooling water flow direction
F: test section	
G: condensate tank	
H: after-condenser	
J: condensed vapor tank	

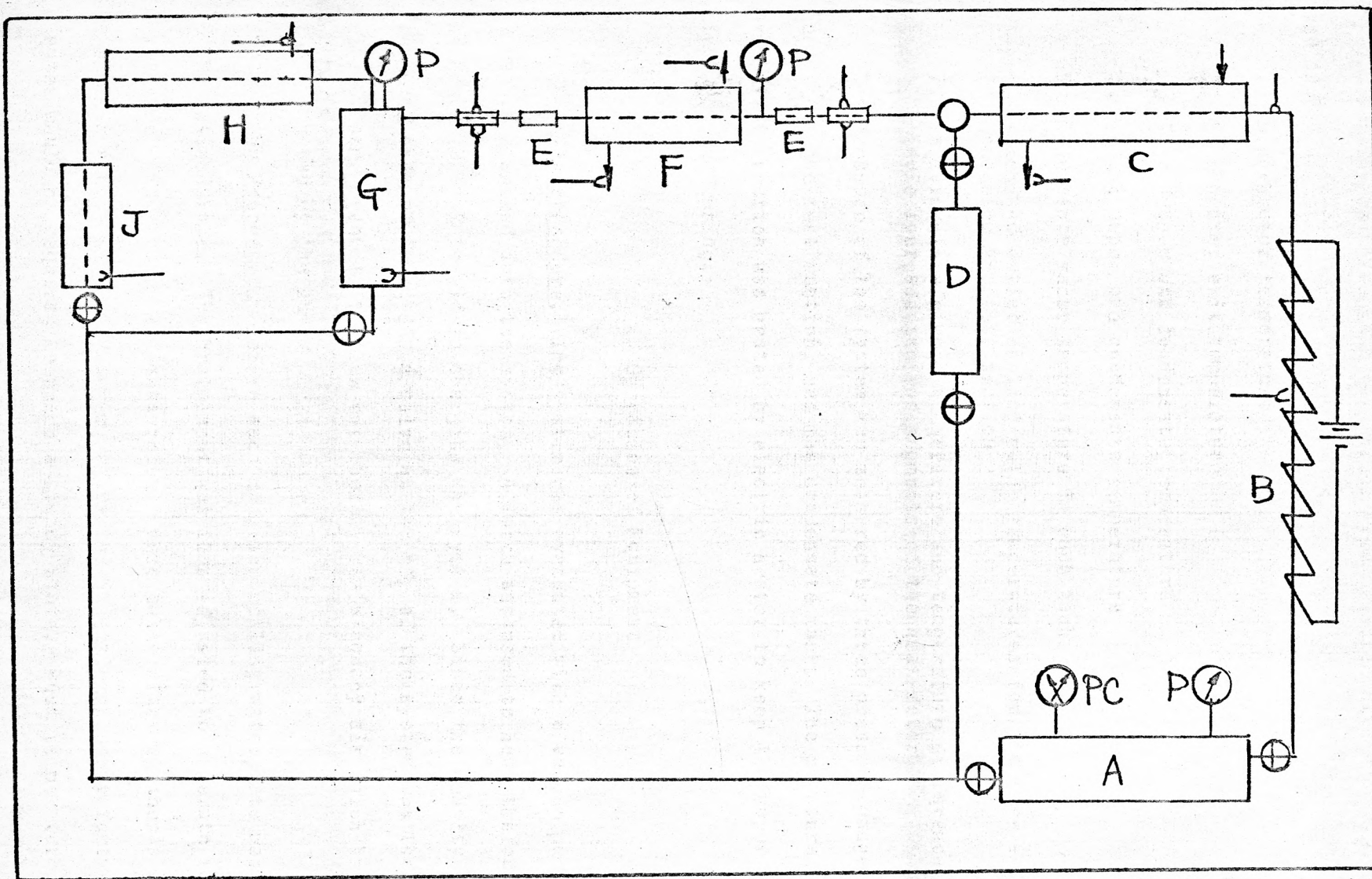


PLATE III

Temperatures throughout the system were measured with a potentiometer and included:

1. Superheater temperature
2. Superheater exit temperature
3. Inlet vapor and condensate temperature
4. Outlet vapor and condensate temperature
5. Condensate vapor temperature in each tank
6. Inlet and outlet cooling water temperatures for the pre- and after-condenser and test section
7. Individual reference temperature and temperature difference of the middle four and two end segments of the heat-meter

The pressure of the system was measured by three pressure gauges at the generator, the test section, and the condensate tank. The pressure gauge line at the test section was heated by electric heater to keep the vapor in the line from condensing.

### Operation of Equipment

Charging Procedure. The system was evacuated with a vacuum pump. Freon 12 was then sucked into the system. A torch was used to heat the outside of the Freon 12 tank to aid in charging the system. Since the density of air is less than that of Freon 12, a valve at the top of the system was then opened to allow some Freon 12 to flow out thereby removing any air which had not been evacuated by the pump.

Running Procedure. Before each run, the cooling water rate was adjusted to the desired rate. The heat input was then adjusted to generate the desired vapor flow rate. All measurements were taken after an hour's operation to reach a steady state. However, after the initial run, only forty-five minutes would be required to reach a steady state. Room temperature, heat



input, boiler pressure, and cooling water rate were first recorded. In order to obtain a steady state, the level of each Freon tank was read, one at a time, so that the pressure throughout the system would not be affected by closing the valves of the Freon tank. After the liquid level had been read, the valve was then opened to let the condensate return to the generator. A level should be maintained in the generator such that the heater is not in contact with the Freon vapor. It was found that when the heater in the generator was exposed to the Freon vapor, a white film would be formed on the sight glasses. The flow pattern observed in the sight glass was recorded after all thermocouples and heat meter readings had been taken.

A typical sample calculation is given in the Appendix D.

#### EXPERIMENTAL RESULTS

Nine runs of data were taken using the present system. These data are all in the region of high flow rate. That is, wave flow, mist-wave, incomplete annular, and mist-annular flow. The data with the data of Sun (11) and Rosson (8) provide a wide range of Reynolds number and various vapor-liquid ratios for comparison. The data and results are summarized in the Appendix B.

The resulting circumferential temperature profiles in the test section for typical flow patterns are plotted in Plate V. The temperature difference between top and bottom of the tube may be as high as six degrees Fahrenheit in wave, stratified, and incomplete annular flow patterns. However, in the mist-annular flow pattern, as may be expected, the temperature difference may be as low as one or two degrees Fahrenheit.

The pressure ranged from 65 psig to 144 psig. The temperature across the film from 3.39 to 27.25 degree in Fahrenheit. Mass flow rates of Freon 12 vapor are from 14.820 to 197.200 pound mass per hour per foot square.

#### EXPLANATION OF PLATE IV

Relation of the average film coefficient of heat transfer  $h_m$ ,  
to the Freon vapor mass flow rate, G

Legend:

- : Present data
- ① : Sun's data (11)

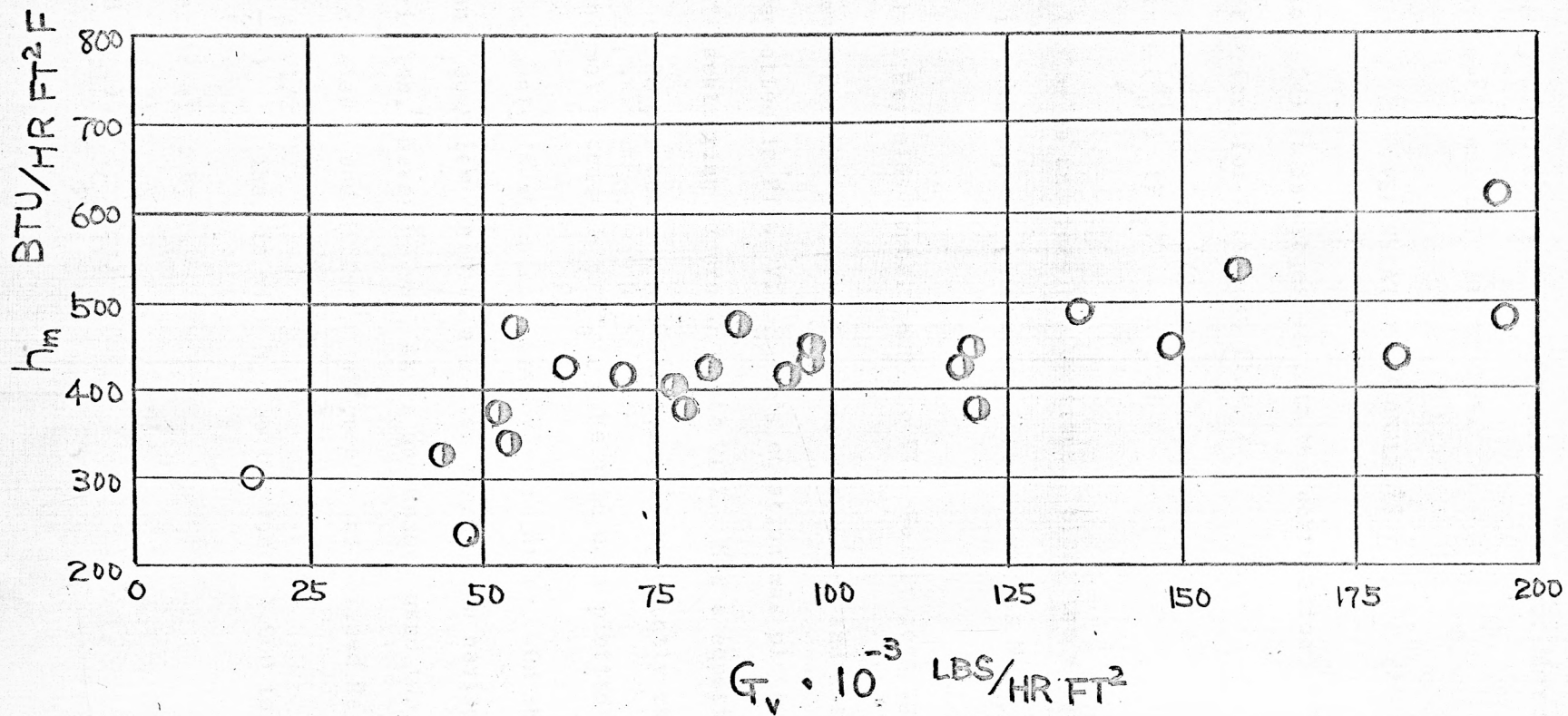


PLATE IV



The resulting heat transfer coefficients are plotted against mass flow rate in Plate IV. The pressure drop in the test section was not observed since the length of heat meter was only six inches.

#### VERIFICATION OF ANALYSIS

From Equation (24) it is suggested that the correlation of Nusselt's number may take the form for turbulent flow.

$$N_{ul} = \text{constant} \left( \frac{4\Gamma}{\mu_l} \right)^N (P_{rl})^{1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/3}$$

Where N is 1/3 or an undetermined constant. Since the exponent of Reynolds number is unknown, the data were first plotted as  $N_{ul} P_{rl}^{-1/3}$

$\left( \frac{h_{fg}}{C_p \Delta T} \right)^{-1/3}$  against Reynolds number in Plate VI. Fairly good agreement with the data is shown. For the purpose of comparison, Akers and Rosson's equation and data were replotted in Plate VII. Sun's equation was plotted in Plate VIII.

Considerable scattering occurs in Plate VIII. Sun's equation has an error deviation of 30 per cent. This deviation is apparently caused by neglecting the factor of thermal potential,  $\frac{h_{fg}}{C_p \Delta T}$ . The same data plotted in Plate VI using the present analysis shows much better correlation. On the other hand, Akers and Rosson's equation exhibits a better correlation having only 20 per cent of deviation. Nevertheless, Akers and Rosson used two equations with different constants and different exponents and defined Reynolds number

as  $N_{re} = \frac{D G_v}{\mu_l} \left( \frac{\rho_l}{\rho_v} \right)^{1/2}$ . That is with  $N_{re}$  in the range 1,000 to 20,000,

Nusselt's number is given by

$$N_{ul} = 13.8 (P_{rl})^{1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/6} (N_{re})^{0.2}$$

# EXPLANATION OF PLATE V

Circumferential temperature profile of  
the tube surface in the test section

Fig. 1. stratified flow (Run No. 8)

Fig. 2. wave flow (Run No. 2)

Fig. 3. incomplete annular flow (Run No. 5)

Fig. 4. mist-annular flow (Run No. 4)

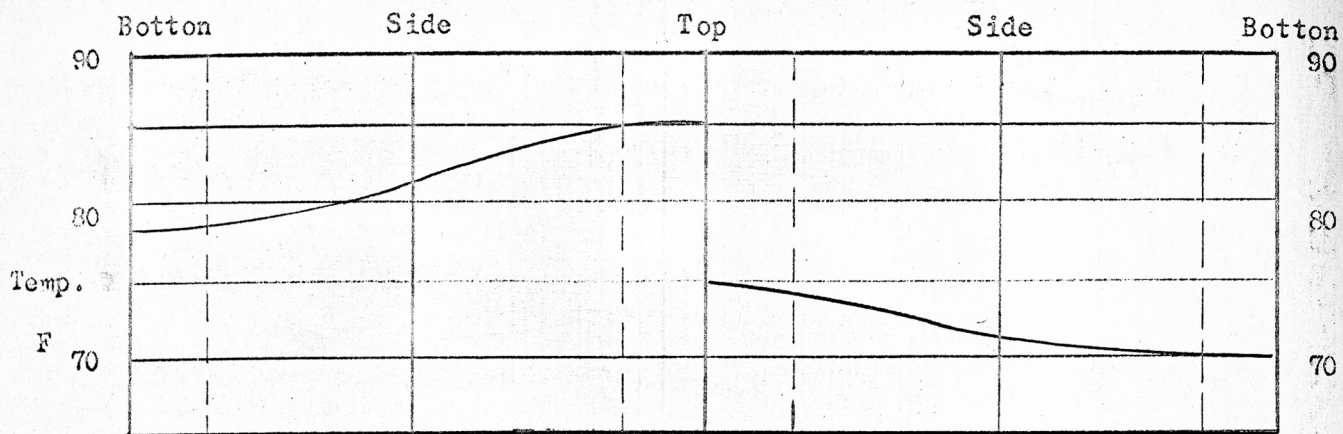


Fig. 1

Fig. 2

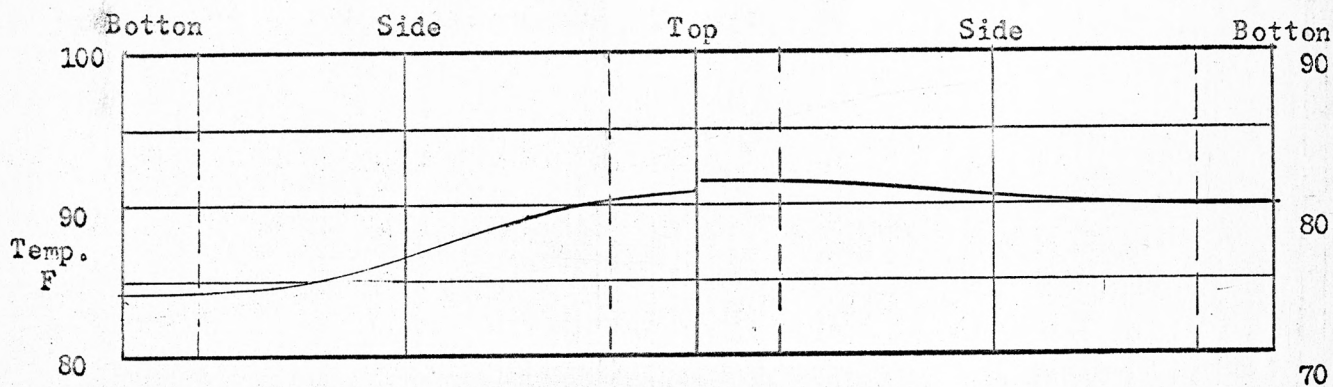


Fig. 3

Fig. 4

PLATE V

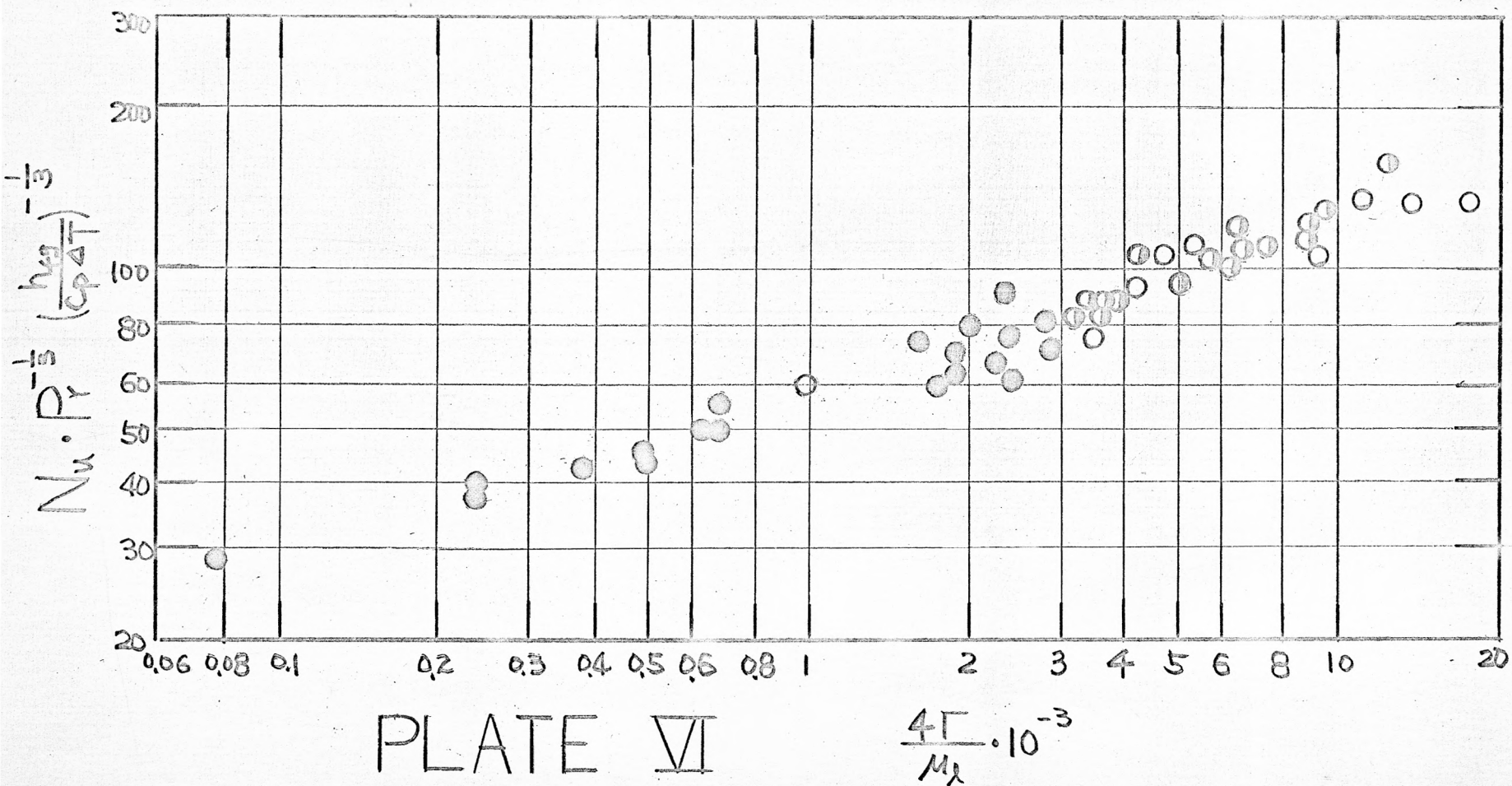


## EXPLANATION OF PLATE VI

$$N_{ul} P_{rl}^{-1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{-1/3} \text{ vs. } \frac{4\Gamma}{\mu_1} \text{ correlation of}$$

Freon 12 and Methanol by theoretical derivation in this report.

- Legend:
- : Freon 12
  - ① : Freon 12 (11)
  - : Methanol (8)



# EXPLANATION OF PLATE VII

$$N_{ul} P_{rl}^{-1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{-1/6} \text{ vs. } \frac{D G_v}{\mu_1} \left( \frac{\rho_l}{\rho_v} \right)^{1/2} \text{ correlation}$$

of Freon 12 and Methanol by Akers and Rosson (8).

Legend: ○ : Freon 12

● : Methanol



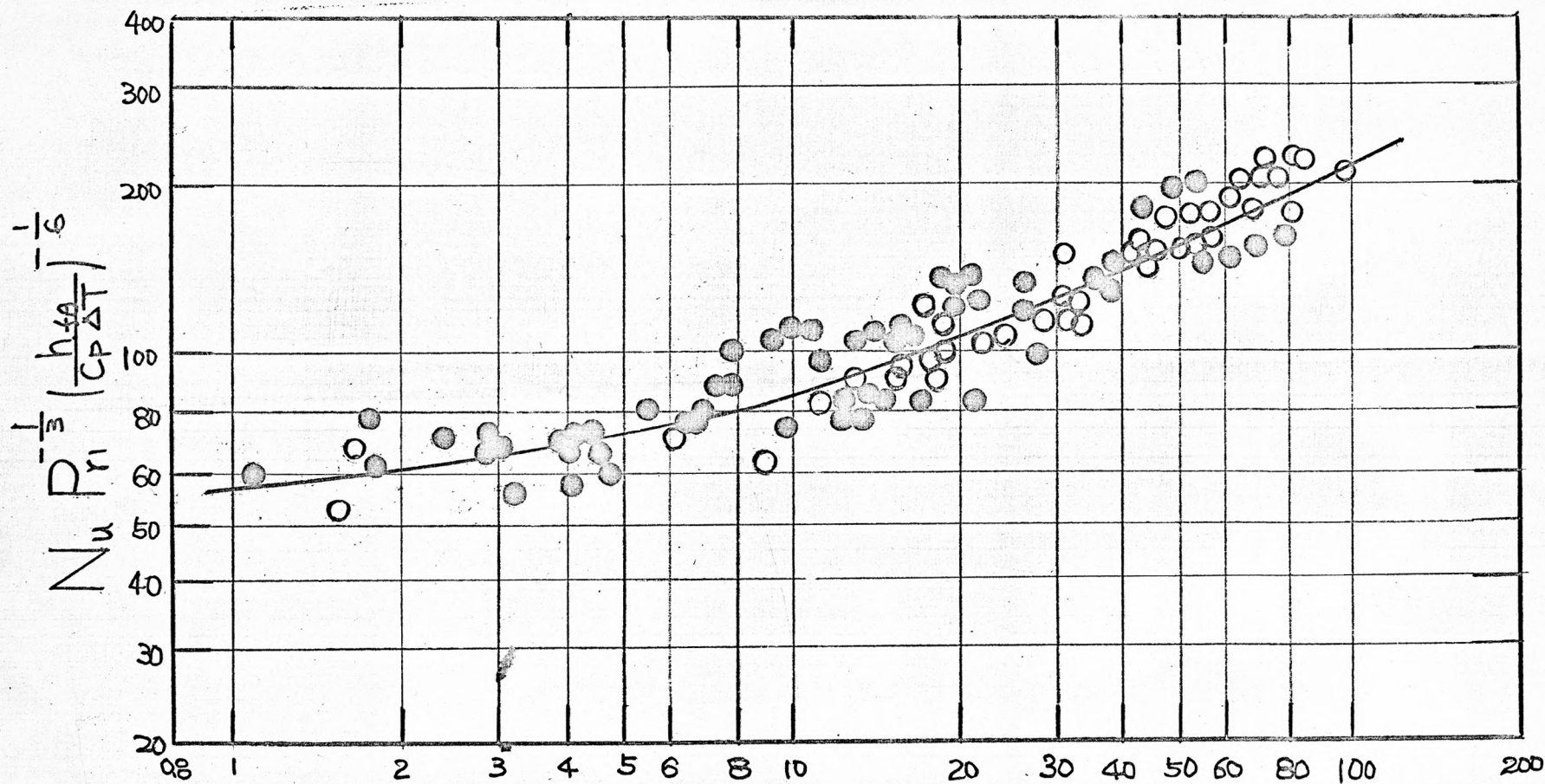


PLATE VII

$$\frac{D Gr}{\mu \lambda} \left( \frac{Pr}{Pr_s} \right)^{1/2} \cdot 10^{-3}$$

EXPLANATION OF PLATE VIII

$N_{ul}$  vs.  $-\frac{D G_m}{\mu_1}$  correlation of Freon 12 by Sun (11)

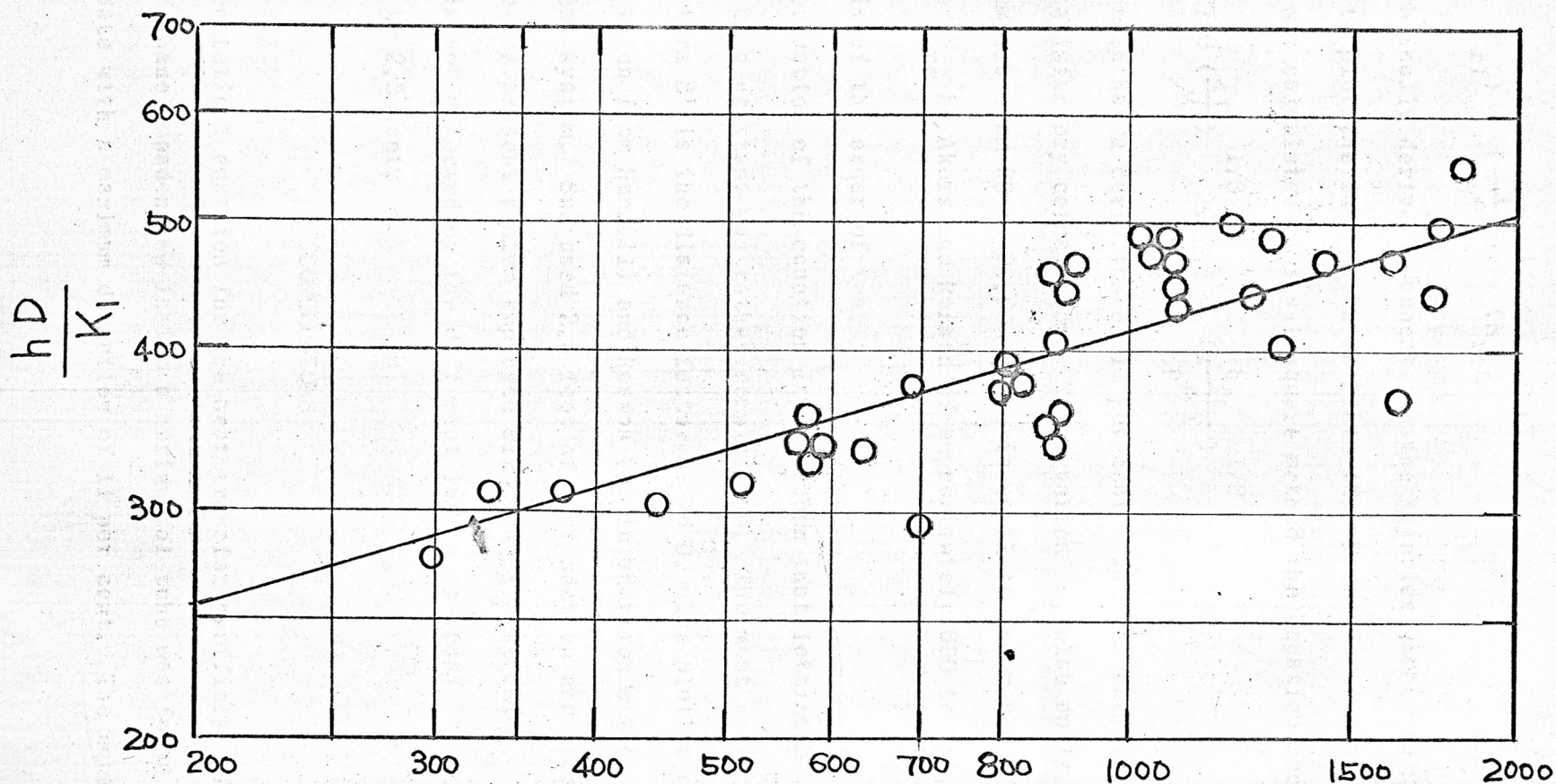


PLATE VIII  $\frac{D\epsilon_{fm}}{\mu_1}$



and from 20,000 to 100,000, Nusselt number is given by

$$N_{ul} = 0.1 (P_{rl})^{1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/6} (N_{re})^{0.67}$$

The theoretically derived exponent for the Reynolds number in Equation (24) was used to plot the data shown in Plate IX. When the constant of the Equation (24) was calculated by least square method the equation becomes

$$N_{ul} = 5.8718 \left( \frac{4T}{\mu_1} \right)^{1/3} (P_{rl})^{1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/3}$$

This equation has a deviation of 11 per cent. This is normally considered as a satisfactory correlation. The equation may also apply a range of Reynolds numbers from 80 to 20,000 which is equal to the range of the defined Reynolds number  $N_{re}$  by Akers and Rosson of approximately 800 to 200,000, since  $(\rho_1/\rho_v)^{1/2}$  is of the order of ten.

In the derivation of the equation it was shown that letting  $B'$  of Equation (22) be a constant was rather complicated. The most unstable factor in the term  $B'$  is the distance factor  $L$  and  $D$ . The plotted data in Plate IX were taken from Sun (11) and Rosson (8) in addition to the data taken from the present system. Sun used 2.5 feet of test section and Rosson used one foot while the present system employed six inch of heat meter. Therefore the equation is not affected by the different length at least in the range of six inches to 2.5 feet.

### CONCLUSIONS

A semi-theoretical equation for the heat transfer coefficient for Freon 12 and Methanol condensing inside a horizontal tube was found to correlate experimental data with a maximum deviation of 11 per cent. The equation is:

# EXPLANATION OF PLATE IX

$$N_{ul} P_{rl}^{-1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{-1/3} \text{ vs. } \left( \frac{4 \Gamma}{\mu_1} \right)^{1/3} \text{ correlation}$$

of Freon 12 and Methanol by theoretically derived exponent to the Reynolds number.

Legend:      ○ : Freon 12  
                  ⊙ : Freon 12 (11)  
                  ● : Methanol (8)

----- : maximum deviation of line of 11%

$$Nu \cdot Pr^{-\frac{1}{3}} \cdot \left( \frac{h_g}{C_p \Delta T} \right)^{-\frac{1}{3}} \cdot 10^{-1}$$

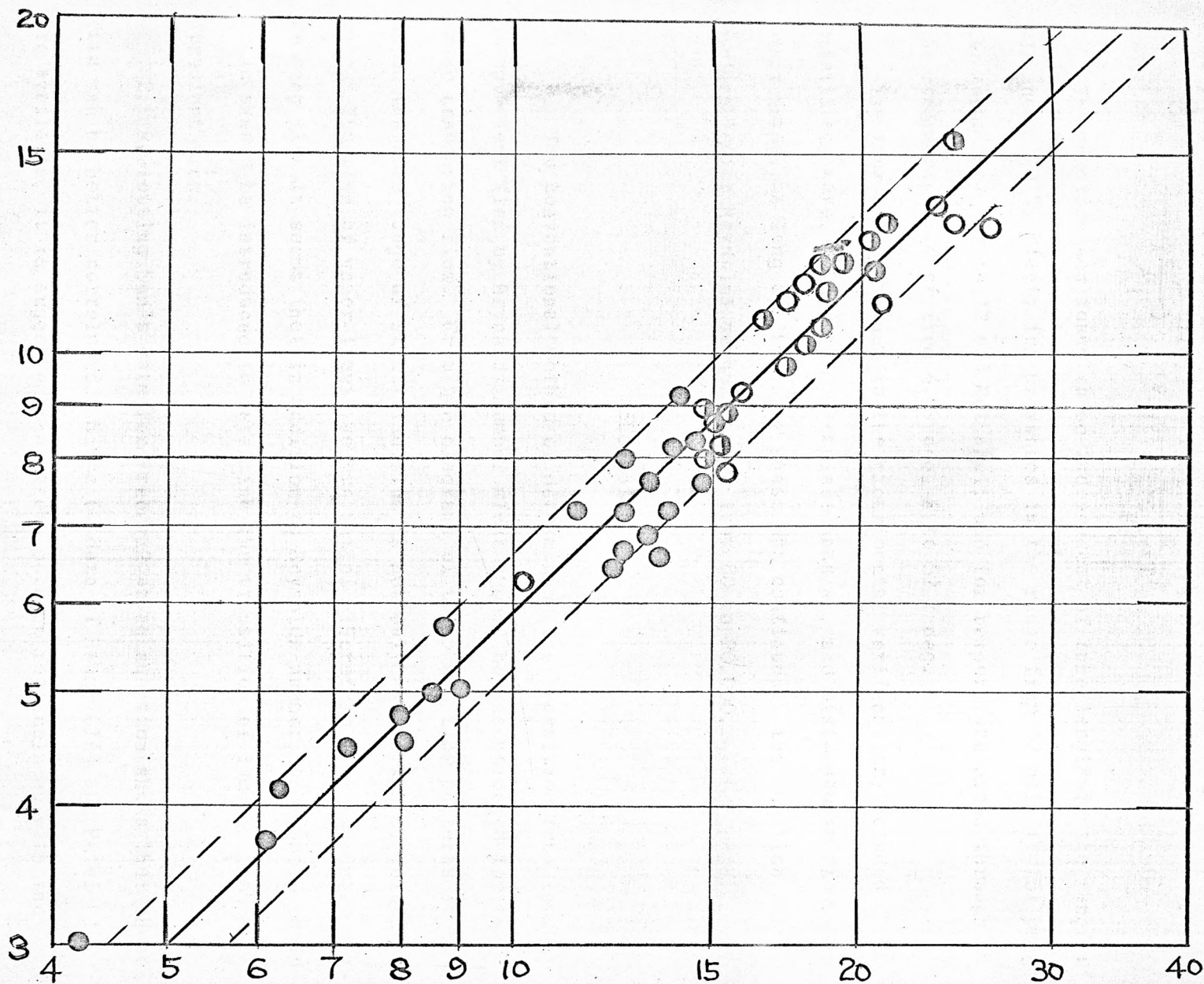


PLATE IX  $\left( \frac{4 \Gamma}{\mu_1} \right)^{\frac{1}{3}}$



$$N_{ul} = 5.8718 \left( \frac{4\Gamma}{\mu_1} \right)^{1/3} (P_{rl})^{1/3} \left( \frac{h_{fg}}{C_{pl} \Delta T} \right)^{1/3}$$

The equation was found to be independent of tube length in the range of six inches to 2.5 feet; for pressures in the range from 12 psig to 151 psig; for two fluids, Freon 12 and Methanol; and for temperature drops through the condensate film of from 4.29 deg. F to 52.7 deg.

The flow patterns used in the experimental verification included stratified, wave, mist-wave, incomplete annular, and mist-annular flow. In other words, as long as the vapor phase was continuous, the equation was applicable for Reynolds numbers  $\left( \frac{4\Gamma}{\mu_1} \right)$  from 80 to 20,000.

### SUGGESTIONS

1. The conventional method of determining the transition from laminar to turbulent flow by Reynolds number alone should be reconsidered for two-phase condensing flow. From the diagrams of two-phase flow patterns it is shown that the range of stratified flow is not only a function of velocity but also a function of vapor-liquid volume ratio. Therefore, for two-phase condensing flow it seems that in addition to Reynolds number, the vapor-liquid ratio should be introduced to determine the transition of laminar flow to turbulent flow.

2. Replotting Sun's data for Freon 12 using equation shows that these data can be better correlated. Since 60 data of Hwang (16) and Patel (15) are available, it is suggested that these data be recalculated with the present equation.

3. To study the pressure drop of two-phase condensing flow, the use of a two-phase Stanton number is proposed. The Stanton number to be defined as follows:

for fixed two-phase flow

$$St_{tf} = \frac{Nu}{Re Pr_l} \left( \frac{\gamma_l}{\gamma_v} \right)$$

for variable two-phase flow such as condensation

$$St_{tv} = \frac{Nu}{\frac{4\Gamma}{\mu_l} Pr_l} \left( \frac{\gamma_l}{\gamma_v} \right) \left( \frac{h_{fg}}{C_p \Delta T} \right)$$

4. A study of the vapor-liquid interface being concave upward when the velocity of vapor is increased is proposed. An effort to derive a function which would relate the vapor velocity with the shape of the interface was attempted. Although not successful the relation appears to be a combination of trigonometric and elliptic functions.

### ACKNOWLEDGMENT

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## APPENDIX A

## Properties of Iron 12

(Diluted 100%)

1. 1-1 Density  $\rho$  vs. Temperature  $T$ vs. Temperature  $T$ 10. 1-10 Thermal Conductivity  $k$  vs. Temperature  $T$ vs. Temperature  $T$ 11. 1-11 Thermal Expansion  $\alpha$  vs. Temperature  $T$ vs. Temperature  $T$ 12. 1-12 Specific Heat  $c_p$  vs. Temperature  $T$ vs. Temperature  $T$ 

## APPENDIX

13. 1-13 Thermal Expansion  $\alpha$  vs. Temperature  $T$ vs. Temperature  $T$



## APPENDIX A

Properties of Freon 12  
(Dichlorodifluoromethane)

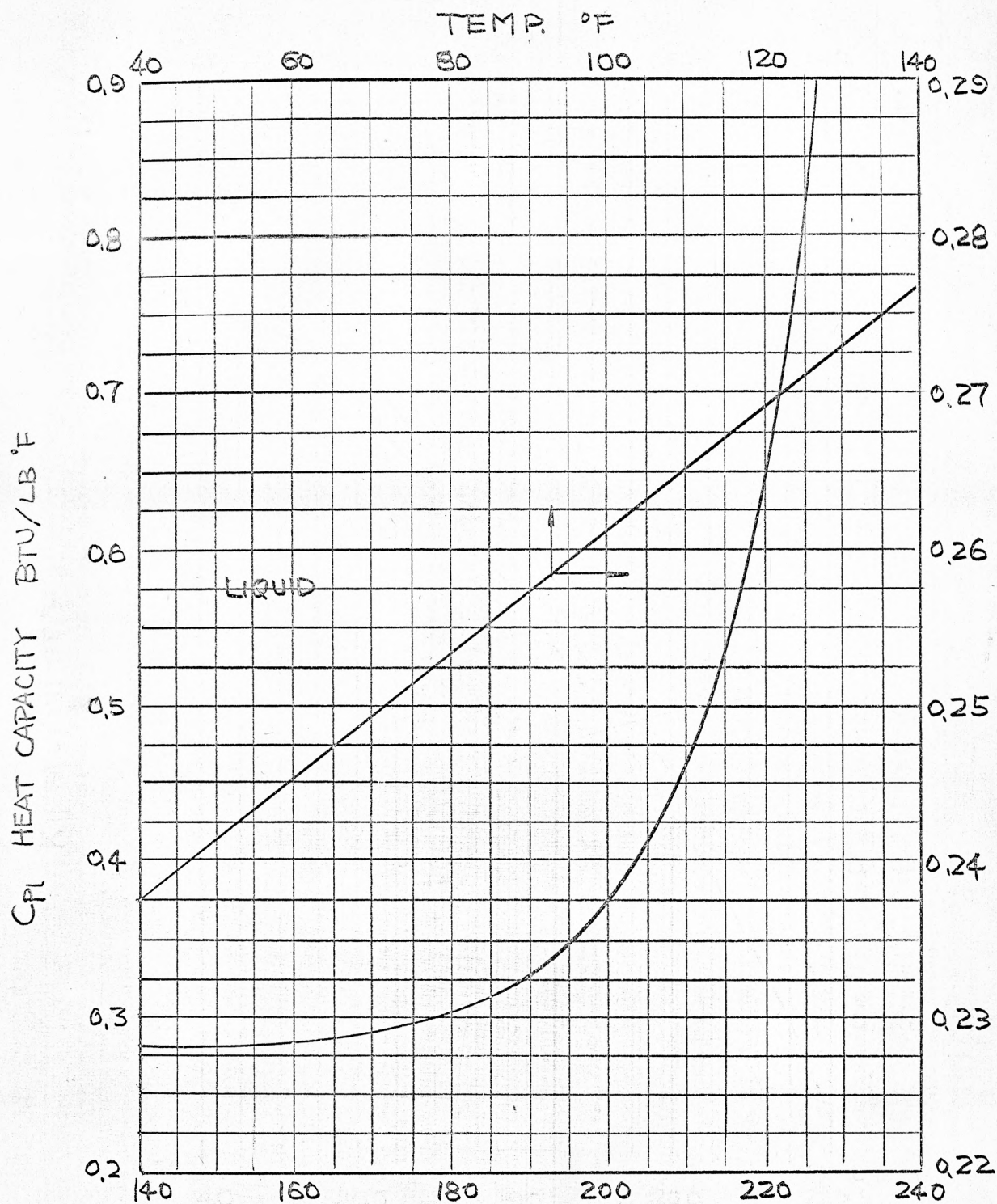
Fig. A-1 Heat Capacity  $\text{Btu/lb}_m \text{ F}$   
vs. Temperature F.

Fig. A-2 Thermal Conductivity  $\text{Btu/hr ft F}$   
vs. Temperature F.

Fig. A-3 Latent Heat of Vaporization  $\text{Btu/lb}_m$   
vs. Temperature F.

Fig. A-4 Density of Saturation  $\text{lb}_m/\text{ft}^3$   
vs. Temperature F.

Fig. A-5 Viscosity  $\text{lb}_m/\text{hr ft}$   
vs. Temperature F.



TEMP. °F

Fig. A-1

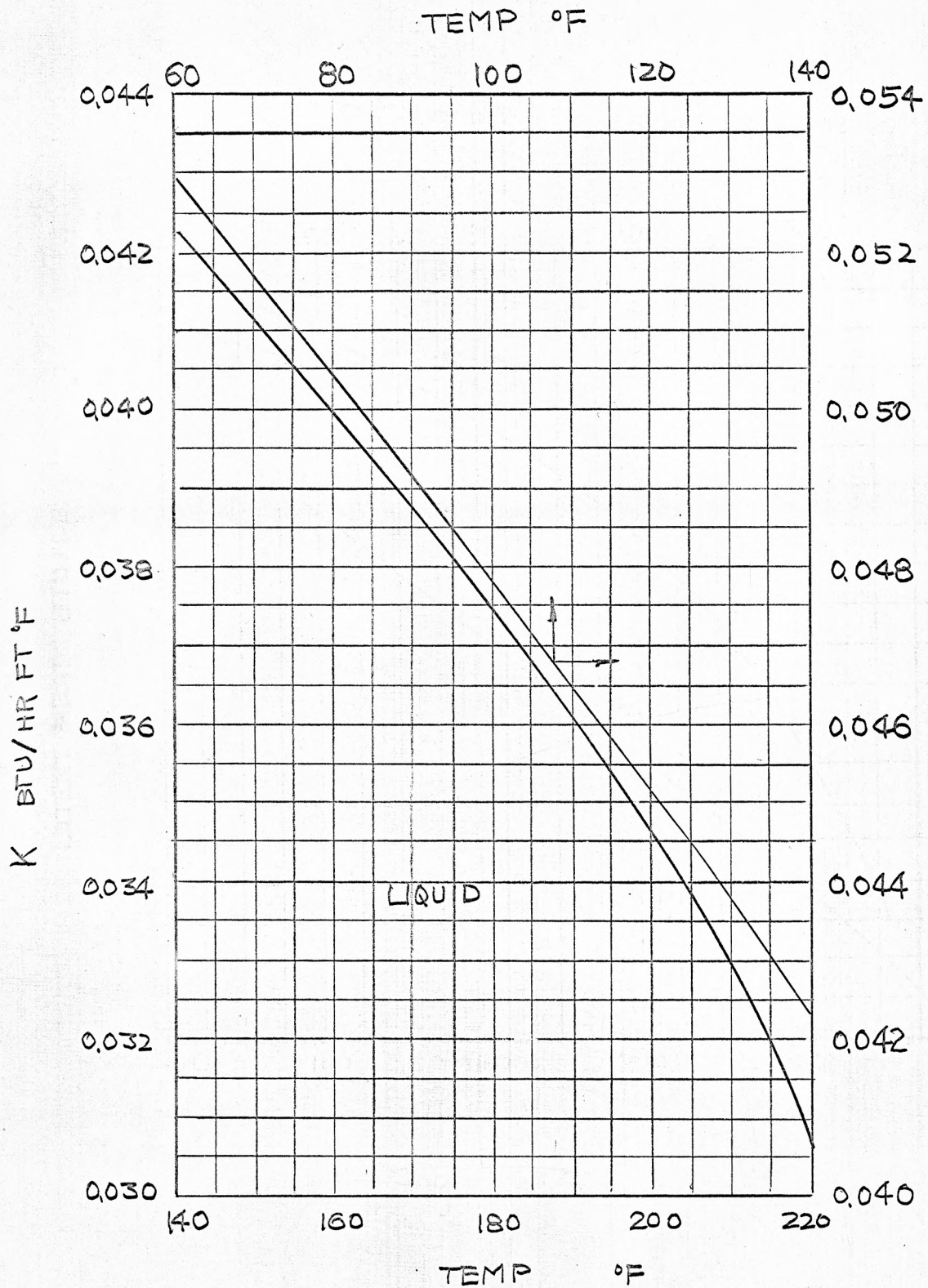


Fig. A-2



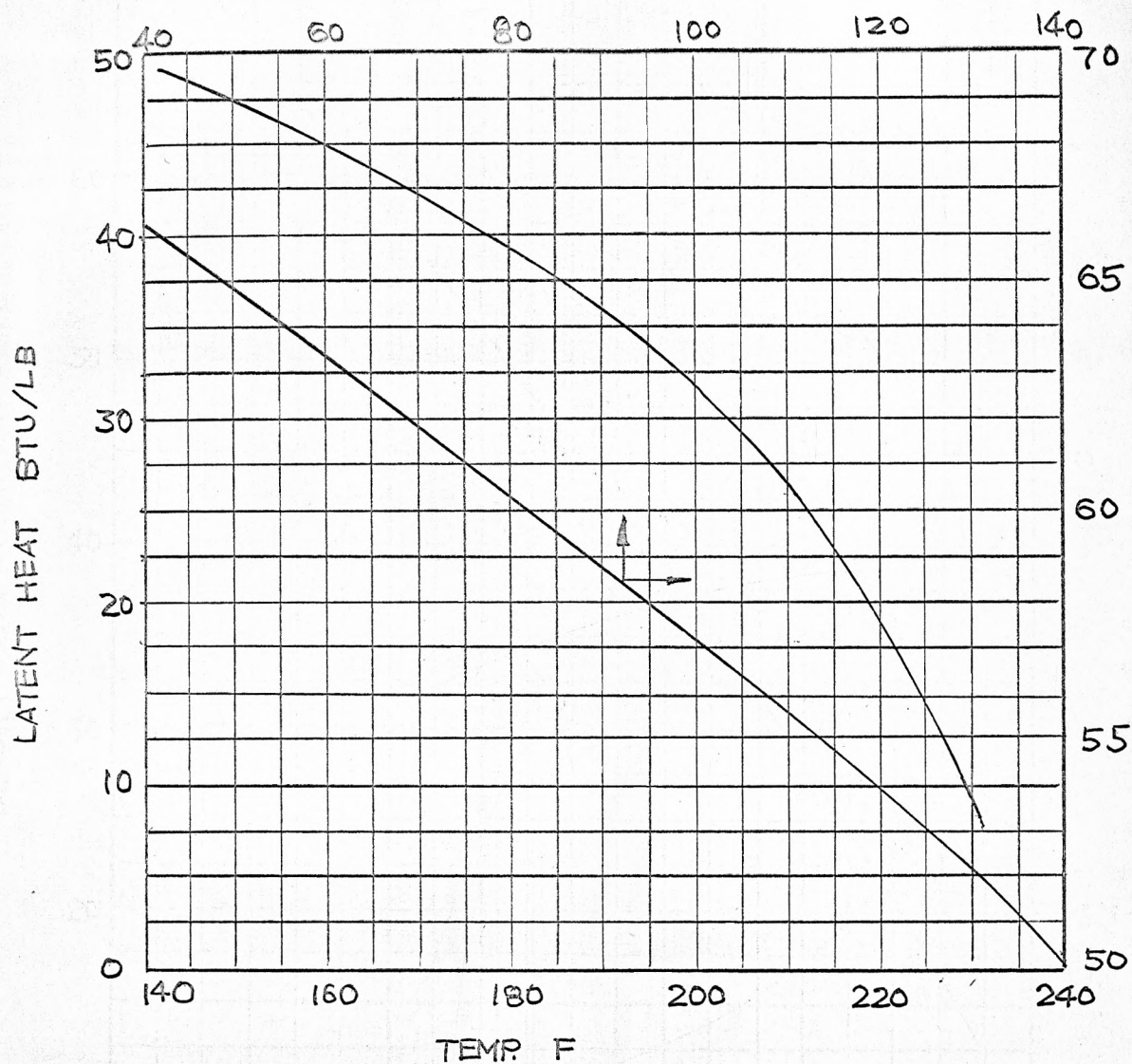


Fig. A-3

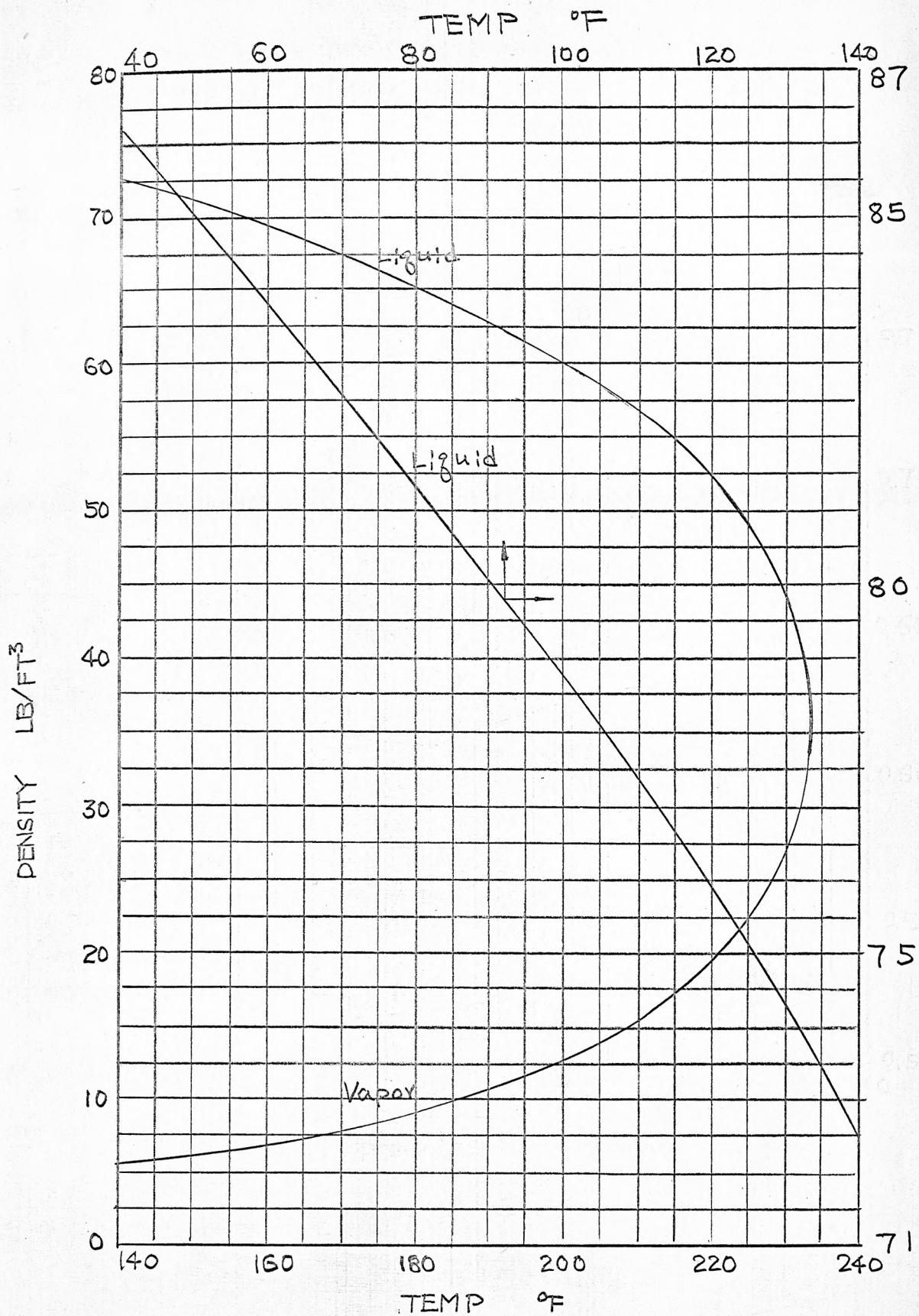


Fig. A-4

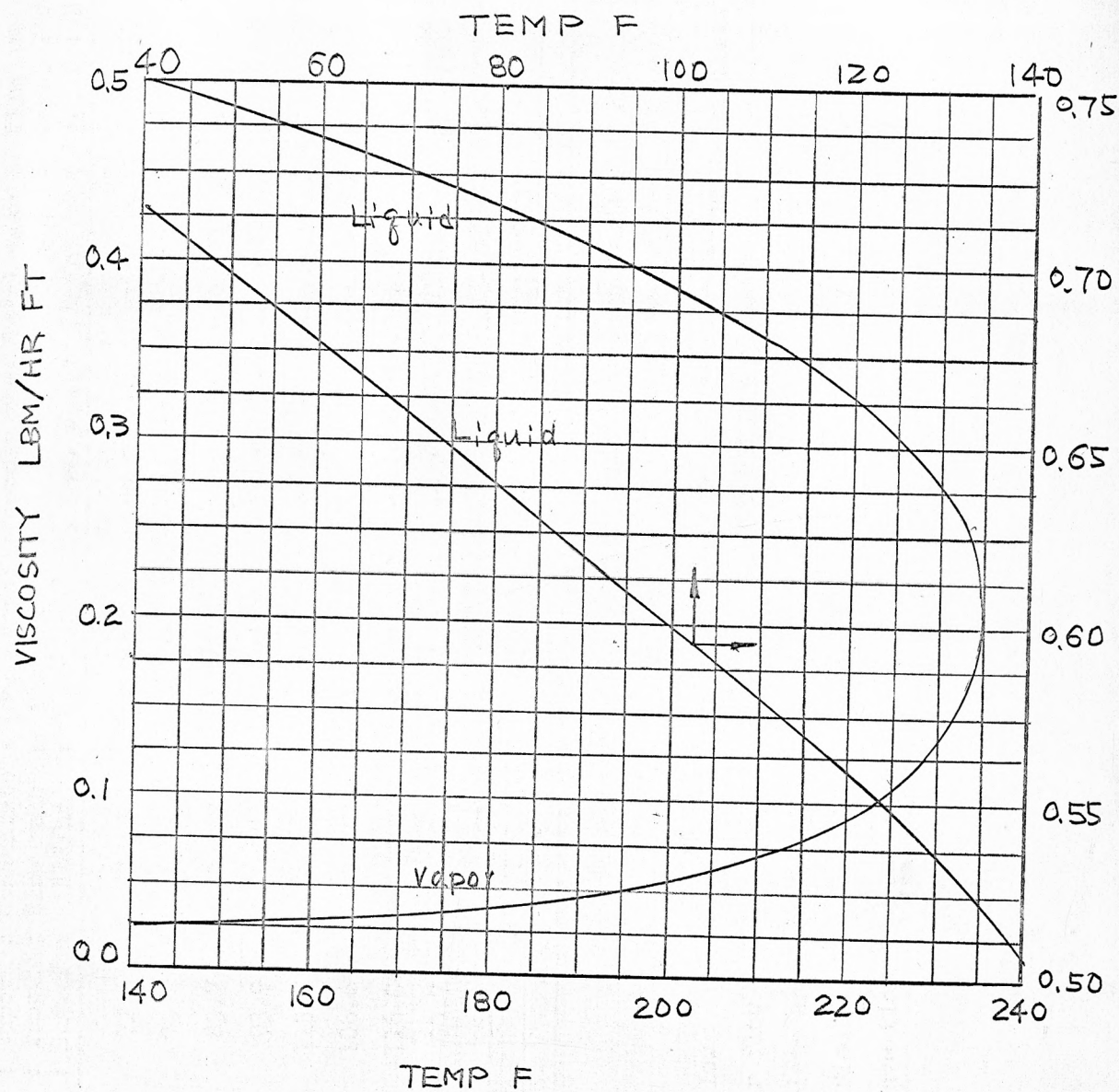


Fig. A-5



# Appendix B. Summary of Freon data

Run No.	Boiler Pressure : psig	Vapor Temp. : Deg. f	Condensate flow rate : Ft <sup>3</sup> /hr	Vapor (condensed) flow rate : Ft <sup>3</sup> /hr	Temp drop across the film : Deg. F	Total mass flow rate : lb <sub>m</sub> /hr	Film Temp. : Deg. f	Heat transfer coefficient : Btu/ft <sup>2</sup> hr F	Note
1.	65	72.66	0	0.2605	7.36	21.75	68.77	372	W
2.	69	70	1.03	0.775	4.29	233	67.85	680	W
3.	90.3	83.7	0.788	1.72	9.4	202.3	79	600	A
4.	109	93.9	1.28	2.382	13.37	289.9	87.22	568	A
5.	144	113.5	1.26	2.245	25.57	269.5	100.68	423	IA
6.	132	106.6	0.912	1.88	19.77	218	96.79	516	IA
7.	138	116.06	0.7	0.217	27.25	71	99.3	259	S
8.	104	90.7	0.88	0.463	17.6	107.2	86.71	486	S
9.	97	96	0.557	0.543	15.56	88.9	88.22	509.2	MW

- \* W Wave flow
- A Annular flow
- IA Incomplete annular flow
- S Stratified
- MW Mist wave flow

\* All observed flow patterns were taken from the outlet sight glass.

## APPENDIX C

## Copper-Bismuth Heatmeter

A copper-bismuth heatmeter was employed as a test section in this investigation. The main advantages of the heatmeter are as follows:

1. The heatmeter measures heat flux directly.
2. It is not necessary to insert thermocouples into the test section. The pattern of the condensing flow is therefore not disturbed.
3. The heatmeter is able to measure more precisely the surface temperature of the tube.
4. The heatmeter may be cut into many segments along an axial line to provide a means for determining the circumferential temperature profile.

The copper-bismuth heatmeter is based upon the same principle as the conventional thermocouple. It is a composite cylinder consisting of three metallic layers (copper-bismuth-copper) which form an infinite number of thermocouples connected in parallel. When heat flows through these layers as a result of a temperature difference existing at the interfaces, a thermoelectric potential (the Seebeck Emf) is developed. The relation between the interface temperature and the developed Emf is represented by the equation: (10)

$$\text{Emf} = B_1(T-32) + 1/2 B_2(T-32)^2 - B_1(T'-32) - 1/2 B_2(T'-32)^2 \quad (\text{C-1})$$

where  $T$  = Hotside interface temperature, in F

$T'$  = Cold-side interface temperature, in F

and  $B_1$  and  $B_2$  are constants dependent upon the metals used. For copper-bismuth  $B_1$  is 0.0258 mv/F, and  $B_2$  is 0.0001488mv/F. The Emf output has been computed for a wide range of temperatures by using an IBM 650 digital computer and the results are shown in the accompanying chart. SOAP II program are also

included in the Appendix. These theoretical values were found to be within 5 per cent by actual calibration of the heat meter.

When the temperature difference between the interfaces and conductivity of the intervening metal ( $K_m$ ) are known, the heat flux can be evaluated by the equation:

$$Q = \frac{2 L K_m (T - T')}{\ln (D_o / D_i)}$$

where  $D_o$  is outside diameter and  $D_i$  is inside diameter.

The inner tube surface temperature ( $T_w$ ) may be obtained by using the same equation above since the heat flux through the inner copper tube will be the same as that through the bismuth.

The heatmeter used in this investigation is six inches in length with an inner diameter of 5/8 inches and outer diameter of one inch. The copper tubes are of hard (L) type copper tubing manufactured by Lewin Mathe Company with a purity of 99.9 per cent. The intermediate layer, bismuth was purchased from the Fisher Scientific Company of St. Louis, Missouri, with a purity of 99.98 per cent.

The procedure followed in constructing the heatmeter was to make an aluminum base on which the two copper tubes may be held in a concentric position. The base and tubes were then set in a vertical position in a sand filled container. Before the liquid bismuth poured into the annular space between the two tubes, solder flux was applied and the sand was heated up to 350 degrees F. After the liquid bismuth was poured, the base and the tubes and the base were held for 24 hours to let free cooling to prevent bubble formation in the bismuth. Also after it had been removed from the aluminum base, an x-ray photograph of the heatmeter was made. The heatmeter was then cut into



the desired shape as shown in Plate X. The final assembly is shown in plate XI. The meter was calibrated by running hot water inside the heatmeter and cooling water outside. The heat flux measured by the meter checked the heat flux calculated from the cooling water flow rate and temperature difference within 5 per cent.

To calculate the heat transfer coefficient, the amount of heat transferred through the heatmeter is equal to the heat transfer across the condensate film or

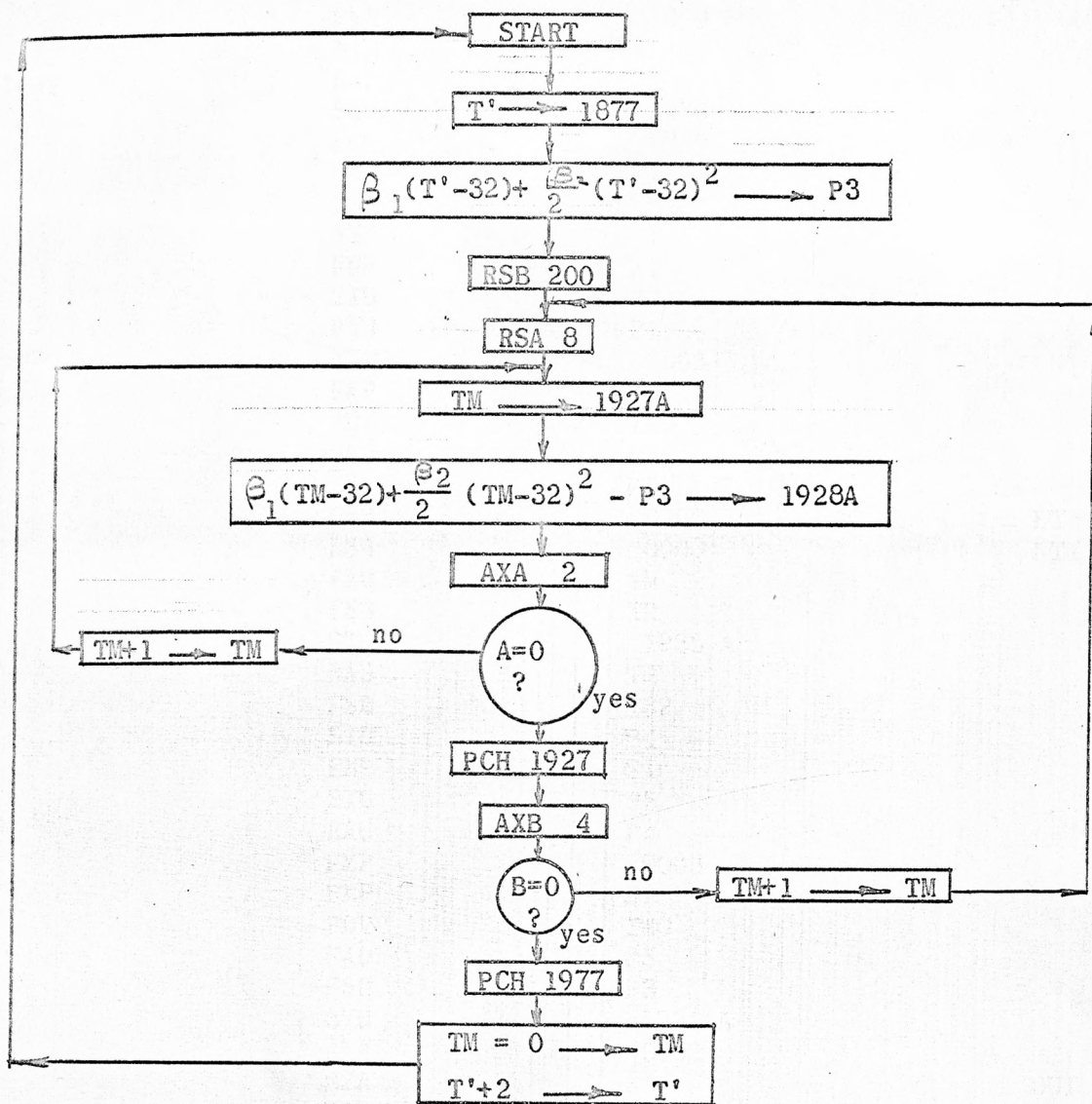
$$h A \Delta T_f = \frac{2 \pi K_b L \Delta T_b}{\ln D_o / D_i}$$

where A is the inner surface area of the test section,  $\Delta T_f$  is the temperature difference between vapor and tube wall,  $\Delta T_b$  is the temperature difference between hot side and cold side interface of the copper-bismuth heatmeter, and  $D_o$  and  $D_i$  are outside and inside diameter of the bismuth layer of the heatmeter.

Since the heatmeter employs water as the coolant, the temperature of the bismuth layer is always in the range of 60 to 150 degrees Fahrenheit. As reported by Hsu (10), the thermal conductivity of the bismuth layer may be considered as a constant equal to 4.7 BTU/hr ft F. Therefore, the final equation for the heat transfer coefficient is:

$$h = \frac{2 \times 3.1416 \times 4.7 \times L \times T_b}{\frac{1}{2} \times \frac{1}{12} \times L \times \frac{1}{\ln 5/8} \times T_f} = 480 \frac{\Delta T_b}{\Delta T_f} \text{ BTU/HR FT}^2\text{F}$$

A SOAP II program for Eq(C-1) is as follows:



43	48	51	57
Location	Op.	D. Address	I. Address
START	BLR	1800	1999
	SYN	START	1999
	RAU	TR	
	STU	1830	
	PCH	1830	
	PCH	1977	
	FSB	T32	
	STU	P1	
	FMP	S1	
	STU	P2	
	RAU	P1	
	FMP	8003	
	FMP	S2	
	FDV	TWO	
	FAD	P2	
	STU	P3	
	RSB	0200	HT4
	RSA	0008	HT1
	RAU	TM	
HT4	FSB	TR	
HT1	STU	1935 A	
	RAU	TM	
	FSB	T32	
	STU	P4	
	FMP	S1	
	STU	P5	
	RAU	P4	
	FMP	8003	
	FMP	S2	
	FDV	TWO	
	FAD	P5	
	FSB	P3	
	STU	1936 A	
	AXA	0002	
	NZA		OUT
	RAU	TM	
	FAD	ONE	
	STU	TM	HT1
OUT	PCH	1927	
	AXB	0004	
	NZB	HT2	HT3
HT3	PCH	1977	
	RAU	TR	
	FAD	TWO	
	STU	TR	
	STU	TM	START
HT2	RAU	TM	
	FAD	ONE	
	STU	TM	HT4
T32	32	0000	0052
S1	25	8000	0049
S2	14	8800	0047
TWO	20	0000	0051
ONE	10	0000	0051



# EXPLANATION OF PLATE X

Circuits of Copper-Bismuth Heatmeter and its segments

Legend:        • : Junction  
                 T' : Reference temperature  
                 P : Potentiometer

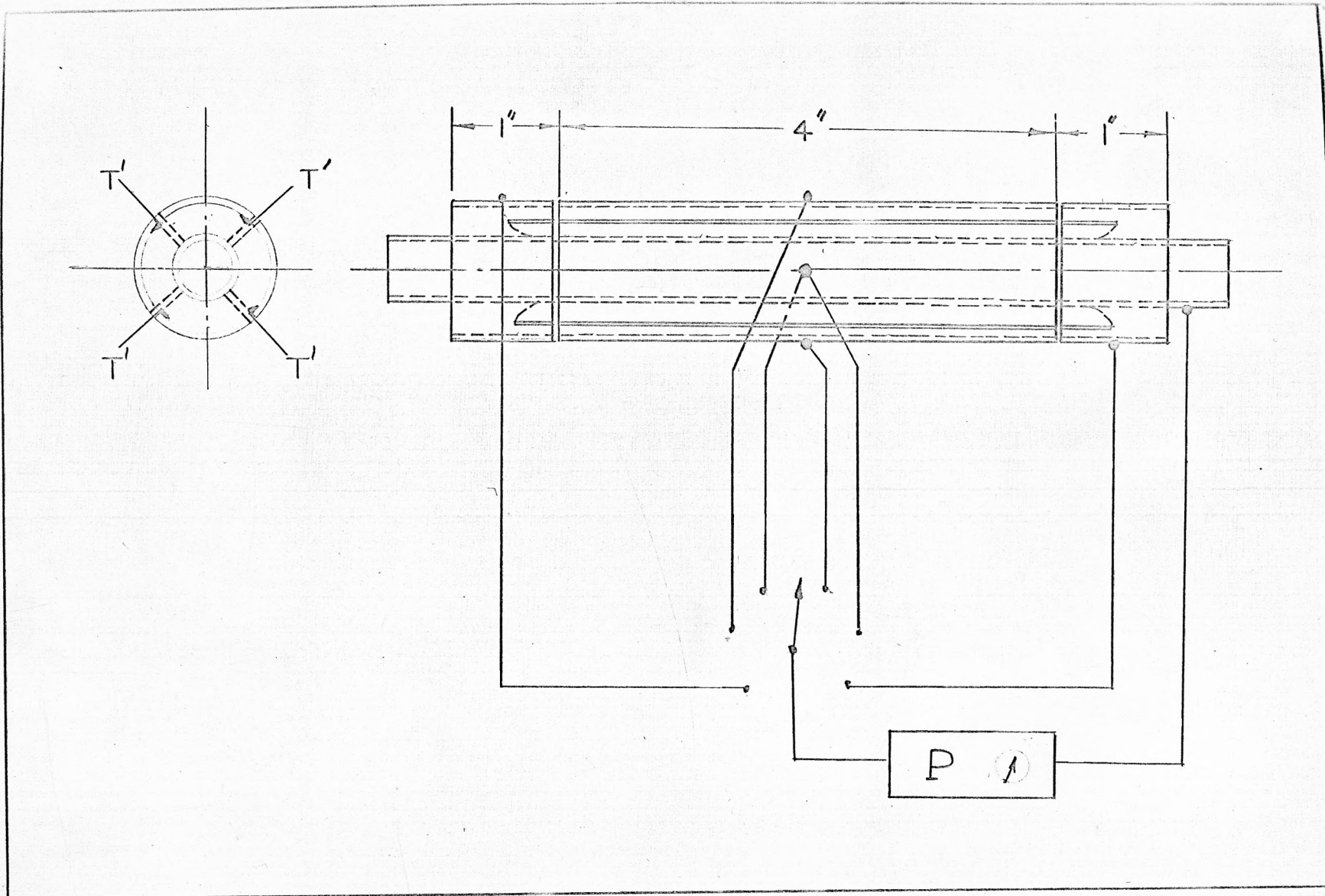


PLATE X

# EXPLANATION OF PLATE XI

Schematic diagram of Copper Bismuth heatmeter

Legend:      W: Cooling water  
              Bi: Bismuth layer  
              R: Rubber gasket  
              V: Vapor





## APPENDIX D

## SAMPLE CALCULATION

Run No. 5

The average temperature of vapor in the test section

$$T_v = \frac{T_{v \text{ in}} + T_{v \text{ out}}}{2} = \frac{114.17 + 112.784}{2} = 113.477 \text{ deg. F}$$

The average temperature of the inner surface is  $T_s = 87.95 \text{ deg. F}$ .The average temperature drop of the heatmeter is  $T_s - T' = 22.47 \text{ deg. F}$ .

The average temperature drop across the condensate film is  $T_v - T_s = 113.477 - 87.95 = 25.527 \text{ deg. F}$ .

Heat transfer coefficient is then

$$h = 480 \times 22.47 / 25.527 = 423 \text{ BTU/HR FT}^2\text{F}$$

To evaluate the properties of the condensate, the film temperature should be used. The average film temperature is

$$T_f = \frac{T_v + T_s}{2} = \frac{113.477 + 87.95}{2} = 100.763 \text{ deg. F.}$$

Then Viscosity  $\mu_1 = 0.586 \text{ lb}_m/\text{ft hr}$ Specific volume  $v_1 = 0.013 \text{ Ft}^3/\text{lb}_m$ Specific heat  $C_p = 0.2665 \text{ BTU/lb}_m \text{ F}$ Latent heat  $h_{fg} = 55.08 \text{ BTU/lb}_m$ Prandtl number  $P_{r1} = 3.5$ Thermal conductivity  $K_1 = 0.0463 \text{ BTU/hr Ft F}$ 

the pressure in the system = 144 psig, total flow rate =  $3.506 \text{ Ft}^3/\text{hr}$ . Tube diameter is one half inch.

mass flow rate  $W_t = 3.506 / 0.013 = 269.5 \text{ lb}_m/\text{hr}$ .

$$\frac{4 \Gamma}{\mu_1} = \frac{4 W_t}{\pi D \mu_1} = 1.4 \times 10^4$$

Thermal potential is  $\frac{h_{fg}}{C_p \Delta T_f} = 55.08 / 0.2665 \cdot 25.577 = 8.05$

Nusselt number  $N_{ul} = \frac{h D}{K_1} = 423 \cdot 0.0417 / 0.0463 = 380$



## APPENDIX E

## Nomenclature

A	area, $\text{ft}^2$
B	constant
C	constant or $C_g$ gas percentage
c	constant defined in Equation (11)
D	diameter ft
E	constant
f	fanning factor
$G_g$	gas flow rate by volume $\text{ft}^3/\text{hr}$
$G_l$	liquid flow rate by volume $\text{ft}^3/\text{hr}$
$h_{fg}$	latent heat $\text{BTU}/\text{lb}_m$
h	heat transfer coefficient $\text{BTU}/\text{hr ft}^2\text{F}$
$K_l$	thermal conductivity of condensate $\text{BTU}/\text{hr ft F}$
$K'$	correction factor for low Prantl number
k	constant defined in eddy viscosity $m$
L	tube or plate length, ft.
n	constant or exponential constant
Q	heat flux $\text{BTU per hr.}$
R	radius, ft. or inch.
T	temperature deg. in Fahrenheit
u	vapor velocity in the boundary layer; u free stream velocity
$u_0$	vapor liquid interface velocity; $u_m$ mean vapor velocity in a tube
U	condensate velocity
V	relative velocity of vapor to the interface velocity $u_0$

$W$	mass flow rate of vapor or condensate $\text{lb}_m/\text{hr.}$
$x$	distance from the edge of plate or tube, ft.
$y$	vertical distance from the tube wall or plate, ft.
$y'$	vertical distance from the condensate to the vapor boundary layer, ft.
$C_p$	specific heat, BTU per lb. per deg. F.
$N_u$	Nusselt number
$N_{Re}$	Reynolds number defined by Akers and Rosson
$R_e$	Reynolds number
$P_r$	Prantl number

#### Greek Letters

$\Gamma$	mass flow rate $W/D\pi$ defined in Reynolds number $\frac{4T}{\mu_1}$
$\delta$	thickness of vapor boundary layer, ft.
$\Delta$	difference, or thickness of condensate film, ft.
$E_m$	eddy viscosity, $\text{ft}^2/\text{hr.}$
$\mu$	dynamic viscosity, lb. per $\text{ft}^2$ per hr.
$\nu$	kinematic viscosity, $\text{ft}^2$ per hr.
$\rho$	density, $\text{lb}_m$ per $\text{ft}^3$ .
$\tau$	shear force $\text{lb}_f$ per $\text{ft}^2$
$\xi$	$\frac{C_p \Delta T}{h_{fg}}$
$\psi$	180 deg. - vapor subtended half angle
$\sigma$	slope of tube from horizontal line
$\mu^*$	dimensionless number $\mu_v/\mu_1$

## Subscripts

l	liquid
m	mean value
o	liquid-vapor interface
v	vapor
w	wall
s	saturated temperature
$\infty$	free stream
-	mean value
x	local



# ERRATA

Page 4

$$C_g = \frac{G_g}{G_g + G_l}$$

$$V_m = \frac{G_g + G_l}{A}$$

Page 7 line 11,12

Lockhart and Martinelli's method (6)

Page 10 line 18

White's Equation (14)

Page 11 line 1

Rosson's Equation (8)

Page 14 line 5

$$U =$$

Page 17 line 14

Prandtl's

Page 19 line 4

$$\Gamma = \frac{A U_m \rho_v}{\pi D}$$

Page 18 Eq.(19)

$$2/3 U_o$$

Page 45 reference 2 2

McAdams, William H.

Abstract line 9

5.8713

Chart for Appendix C

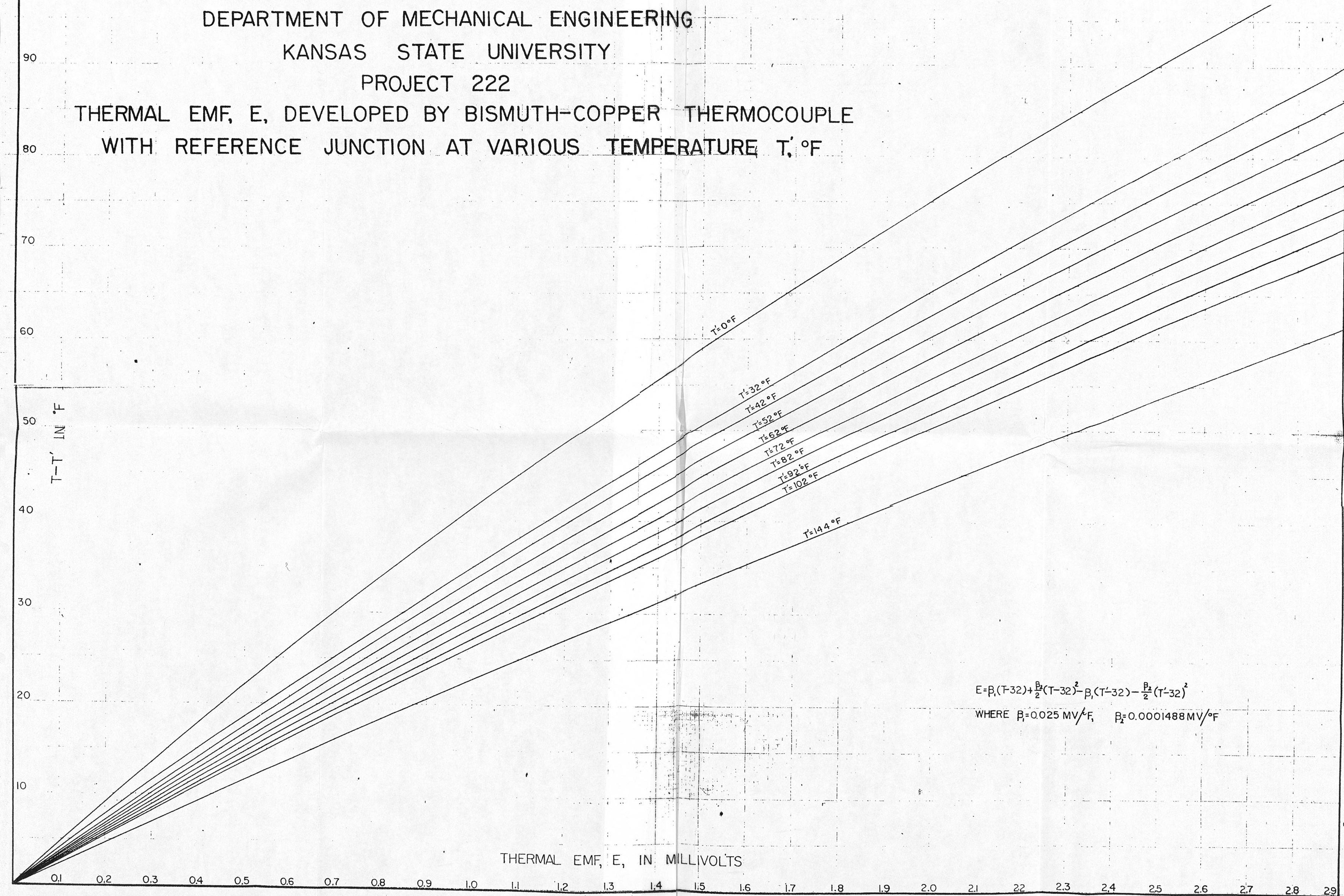


# DEPARTMENT OF MECHANICAL ENGINEERING

KANSAS STATE UNIVERSITY

PROJECT 222

THERMAL EMF, E, DEVELOPED BY BISMUTH-COPPER THERMOCOUPLE  
WITH REFERENCE JUNCTION AT VARIOUS TEMPERATURE  $T', ^\circ\text{F}$



$$E = \beta_1(T-32) + \frac{\beta_2}{2}(T-32)^2 - \beta_1(T'-32) - \frac{\beta_2}{2}(T'-32)^2$$

WHERE  $\beta_1 = 0.025 \text{ MV}/^\circ\text{F}$ ,  $\beta_2 = 0.0001488 \text{ MV}/^\circ\text{F}^2$



CONDENSING HEAT TRANSFER IN A HORIZONTAL TUBE

by

CHING-JEN CHEN

Diploma, Taipei Institute of Technology  
Taiwan, Formosa, 1957

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AN ABSTRACT OF  
A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY  
Manhattan, Kansas

1962

In this report an effort was made to determine a consistent equation for correlation and determination of condensing heat transfer coefficients in a horizontal tube by considering the effect of shear force at vapor-liquid interface. In order to understand the condensing flow in a horizontal tube, an analysis of general two-phase flow patterns was included. A semi-theoretical equation for the heat transfer coefficient was found to correlate experimental data with a maximum deviation of 11 per cent. The equation is:

$$N_{ul} = 5.8713 \left( \frac{4\Gamma}{\mu_1} \right)^{1/3} (Pr_l)^{1/3} \left( \frac{h_{fg}}{C_p \Delta T} \right)^{1/3}$$

The equation was found to be independent of tube length in the range of six inches to 2.5 feet; for pressures in the range from 12 psig to 151 psig; for two fluids, Freon 12 and Methanol, and for temperature drops through the condensate film of from 4.29 deg. F to 52.7 deg. F.

The flow patterns used in the experimental verification included stratified, wave, mist-wave, incomplete-annular, and mist-annular flow. Also the equation was applicable for Reynolds numbers,  $\left( \frac{4\Gamma}{\mu_1} \right)$ , from 80 to 20,000.