ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

P-474

LABORATORY EXERCISES FOR NINTH
GRADE GENERAL MATHEMATICS

by 1264

MICHAEL DEAN GRUB

B. S., Fort Hays Kansas State College, 1965

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

Approved by:

J. Harvey Littrell

Major Professor

ACKNOWLEDGMENT

The writer wishes to express his appreciation to Dr. J. Harvey Littrell of the College of Education, Kansas State University, for the constructive criticism and advice given in the preparation of this report. The writer also wishes to thank his wife, Melinda, for her patience and endurance during the compiling of this report.

TABLE OF CONTENTS

THE PROBLEM

> Laboratory teaching is one answer to giving reality to mathematics without the loss of its abstract and theoretical aspects.[1]

The proceding was stated by Howard F. Fehr, of Columbia University, as he looked at the great potential of the laboratory in teaching mathematics.

Some teachers now use some type of laboratory exercises at different times throughout the year; however, at present no published laboratory manual for ninth grade general mathematics exists.

> There is a scarcity of mathematics laboratory lessons, however, and much more work must be done. More teachers of general mathematics must experiment with laboratory lessons and learn to construct new units that successfully motivate general mathematics students. These ideas must be exchanged, refined, and incorporated into a relevant, but varied and attractive, ninth-grade general mathematics program.[2]

In the four years that the author has been teaching junior high mathematics, it has become evident that such a manual would be invaluable as a supplement for teaching ninth-grade general mathematics students.

Statement of the problem. It is the purpose of this

---

[1]Howard F. Fehr, "The Place of Multisensory Aids in the Teacher Training Program," The Mathematics Teacher, XL (May, 1947), 212-216.

[2]Jerome A. Auclair and Thomas P. Hillman, "A Topological Problem for the Ninth-Grade Mathematics Laboratory," The Mathematics Teacher, LXI (May, 1968), 507.

report to develop a set of sample laboratory exercises which can be used as a supplement to the typical ninth-grade general mathematics books. The purpose of the laboratory exercises will be to enable students to enhance their comprehension of the various topics through concrete experiences.

Importance of the study. All too often in the traditional chalk board and eraser classroom, the teacher knows the concept behind the formula used in working the problems on the day's assignments but fails to realize that to the student such a formula is just something to make homework possible. However, with a simple laboratory exercise the student could gain insight into the how and why of the existence of the formula and hence not only know how to use it but also how to derive it, thus making the whole set of exercises take on a more meaningful and useful application for daily life. To help accomplish this, the author has assembled this report.

REVIEW OF THE LITERATURE

Much has been written in terms of laboratory exercises for the physical sciences. It has been the belief for many years that this was the way to teach these courses, but such is not true of mathematics. Prior to 1950, very few articles appeared in professional journals dealing with the mathematics laboratory; however, since that time numerous articles have been written containing favorable reports concerning the use of the laboratory and laboratory exercises in the teaching of mathematics. Only a brief summary of those which pertain to the field of mathematics will be presented here.

Literature favorable to the use of the laboratory and laboratory exercises in teaching mathematics. Other subject-matter-fields have recognized the value of laboratory teaching, e.g. science laboratories have been in existence for many years. Foreign language laboratories are becoming increasingly common. In industrial arts and homemaking the major portion of the instruction is of the laboratory type. For nearly thirty years it has been recommended to mathematics teachers that they work more with the laboratory approach. Even so, few schools today have a special period allotted to what might be described as laboratory activities in mathematics.

A few reasons for operating a mathematics laboratory are given below:

1. The laboratory method utilizes an experimental approach that requires each student's participation, and allows him to work at his own rate. It can thus be the focal point of interest for all students from the slow learner to the capable student.

2. Utilization of the laboratory method leaves the

instructor time to administer more individual help to the students.

3. Since the laboratory is more often conducted on a less formal level than regular classroom teaching, it provides the opportunity for more discussion among the students, and they learn from one another. As a result of this relaxed atmosphere the student's attitude toward mathematics and the mathematics teacher is likely to improve.

4. The similarity between the laboratory lesson and the everyday situations of life enables a greater transfer of learning from books to real life situations than the usual classroom procedure.

5. The laboratory lesson is probably most effective in getting every student to participate actively. Each student must do some thinking as he collects the data or conducts an experiment. This active involvement is the key to successful learning.[1]

Three major points of agreement concerning the learning process mentioned by Forbes[2] are: First, learning is an active rather than a passive process. Hence, an effective learning situation must provide for extensive interaction

---

[1]Donovan A. Johnson and Gerald R. Rising, Guidelines for Teaching Mathematics (Belmont: Wadsworth Publishing Company, Inc., 1967), p. 302.

[2]Jack E. Forbes, "Programmed Instructional Materials -- Past, Present and Future," The Mathematics Teacher, LVI (April, 1963), 224-226.

between the learner and the material learned. Second, the learning process is improved when the learner "knows what he is doing"; that is, when he is provided with regular reinforcement of the correct response and immediate "blocking" of incorrect responses. Third, an efficient learning situation is one in which well-defined goals are established. These goals are necessarily behavioristic in nature, for generalities can only be deduced from observed behavior. That is, if we want to know if a student understands a concept we ask him questions, record his responses, and decide whether or not understanding exists.

Kuethe says,

> One way to teach a concept is to have the learner "discover" it as the result of an inductive process. First one relation is studied and then another until the student suddenly appreciates the existance of the total structure or some general principle. For example, it is possible to have students conduct a number of experiments that illustrate some scientific principle that is involved. However the teacher should make himself available for guidance and channel the students work along the right lines.[1]

Again this points to the mathematics laboratory to reinforce the student's learning on a concept or situation.

Moore says that mathematics is a participation subject. If a student is to learn, then he must participate to the fullest extent in the discovery of mathematical

---

[1] James L. Kuethe, The Teaching-Learning Process. Keystones of Education Series (Scott Foresman and Company, 1968), 74.

concepts. Without participation he will lack the understanding of their implications.[1]

Another advantage of the laboratory approach method of teaching is that it is equally applicable to a modern or a traditional program of instruction.

Willoughby says that on occasions, students should be allowed and encouraged to go through many of the activities carried on by mathematicians in real life -- including following a blind alley to its end. After all the discovery has been in use for over two thousand years and is still very much alive today in our advanced society.[2]

In mathematics classes, according to Willoughby, one should spend more time thinking about what has been done, and less time just trying to cover a portion of a book. In short, one should spend more time thinking and preparing and less time doing. (Doing, here meaning haphazard working of problems from a book.)[3]

The laboratory approach enables the teacher to deal with students on a one-to-one basis and also lets the students communicate easily and naturally with each other. Children have an innate ability to communicate, even highly

---

[1]Richard E. Moore, "Individualized Math," <u>School and Community</u>, LIV (February, 1968), 20-21.

[2]Steven S. Willoughby, "Discovery," <u>The Mathematics Teacher</u>, LXVI (January, 1963), 22.

[3]<u>Ibid</u>.

complex ideas, to each other fluently, with highly eloquent language. One product which can be guaranteed from a mathematics laboratory is that children will be highly motivated in their mathematical studies.[1]

A very important part of a teacher's job is to help the child learn to express himself. Teachers in mathematics have a special opportunity to help the child learn to express himself clearly and concisely. A laboratory approach in mathematics gives him something to write about that he himself has thought through and developed. The pupil's statement differs in this respect from some of his writings in which he reads and then simply rearranges the words and thoughts of others.

Langford stated that, in general, young people learn best when they are taught to discover for themselves the ideas basic to the subject under study; also he stated that they learn least effectively when ideas are told to them by a teacher as bits of abstract information to be learned. Students, quite easily, can determine the teacher who simply "spoon feeds" them information. Often when asked to evaluate that teacher he will reply "He is a good teacher but he talks too much." They see that they would learn more if they were given more time to think things out for themselves.

---

[1]David M. Clarkson, "Mathematical Activity," The Arithmetic Teacher, XV (October, 1968), 497.

The method of teaching mathematics by the laboratory method gives the students a much better opportunity to think things out for themselves.[1] Furthermore, an inductive approach does not necessarily require a great deal of physical apparatus.

Kluttz cites a specific example affirming the benefit of the mathematics laboratory for general mathematics students in Valley High School, Albuquerque, New Mexico. Four mathematics laboratories were installed in an attempt to lessen the frustration of ninth-grade general mathematics students. Kluttz found that not only the outlook of the students has been favorably broadened toward mathematics but also the instructors teaching the material have found renewed interest in the subject by using the laboratory approach.[2]

In summing up the favorable literature concerned with using a laboratory approach to upgrade the meaningful learning in the mathematics classroom, the author sees the following as decisive benefits each student can receive. First, as one shifts to the realm of the student, instead of the teacher, pupils show marked improvement in their

---

[1]Francis G. Langford, Jr., "Helping Pupils to Make Discoveries in Mathematics," The Mathematics Teacher, XLVIII (January, 1955), 45.

[2]Marguerite Kluttz, "The Mathematics Laboratory - A Meaningful Approach to Mathematics Instruction," The Mathematics Teacher, LIV (March, 1963), 144.

attitude toward mathematics. Second, students are eager for the opportunity to assume responsibility in working lessons in which they may interact with each other in a meaningful manner. A great deal can be learned by this interaction which otherwise would have been missed in a formal classroom.

Literature pointing out some weaknesses of the laboratory method. The weakness of the laboratory approach, which the author found most often expressed in the literature, was that the length of time required for a student to discover a principle or process is more than can profitably be spent under the current classroom structure. Wittrock points out the lack of consistent, adequate terminology to describe the stimuli employed in discovery studies.[1]

Bittinger believes that the discovery method should be used only with those students who intend to pursue mathematics beyond the high school level for these students are the ones who will receive the most good from it. He goes on to say that we need to be sure that those who terminate their mathematic endeavors early have a good knowledge of the fundamental processes of arithmetic.[2]

---

[1]M. C. Wittrock, "Verbal Stimuli in Concept Formation: Learning by Discovery," Journal of Educational Psychology, LIV (1963), 184.

[2]Marvin L. Bittinger, "A Review of Discovery," The Mathematics Teacher, LXI (February, 1968), 145.

PROCEDURES AND LABORATORY FACILITIES

It must be remembered that the purpose of the laboratory exercise is to provide a needed meaningful experience. The exercises in this manual will, above all, attempt to help the student organize each exercise in a logical and systematic manner to reach the desired conclusion, by the time the exercise is completed.

Procedures. It is the author's primary aim to construct these sample exercises in such a way that any ninth grade student, at the appropriate time, having been presented this set of exercises, could by himself read, understand, compute and reach the desired conclusions. By doing this he can broaden his knowledge and understanding of the mathematical concepts, faced not only in mathematics textbooks but also those met in the process of everyday life in our society.

With this in mind each of the sample exercises will be constructed as follows:

1. A clear and concise statement of the principle or concepts to be studied.
2. A list of objectives pertinent to the concept or principle.
3. A list of materials and/or equipment needed to perform the operation.
4. Directions needed to arrive at the conclusion.

5. Tables or charts on which to record data obtained.
6. A set of questions relating to desired results.
7. Where applicable a statement by the student of what he has observed from his work.

Special care was taken in developing these sample exercises to allow for the time element. With each exercise the student will either have time to complete the entire exercise in class or he will have time to complete all experimental data and then be able to finish the exercise at a later time.

This plan will be seen in each of the exercises given in this report.

Laboratory facilities. The perfect mathematics laboratory is not feasible for most school systems, mainly because of the monetary outlay necessary to obtain such a laboratory.

However, an efficient mathematics laboratory is easily within the financial reach of most school systems. The main necessity is enough room to allow the students freedom of movement, both to work and to obtain and return instruments used in the calculations of the problems. Blinds or other darkening devices should be available to be used during the showing of films or overhead projection. Some type of shelving units would be desirable for storing the manipulating devices such as meter sticks, scales, compass, etc.

Many of the devices needed to establish concepts can be made or provided by the instructor with no outlay of money. However, if the money is available there are several commercial firms which manufacture and sell all the needed physical apparatus. Lists of these companies are available in almost all school offices.

If money presents a problem the interested teacher should investigate Title III of the National Defense Education Act. Since 1958 the Act has enabled local schools to purchase equipment and materials and to improve facilities for mathematics instruction.

Finally, for mathematics teachers who wish to obtain laboratories for their schools some suggestions by Kluttz are given below:[1]

1. Find out what has been done in other localities concerning mathematics laboratories.
2. Try to involve your whole staff and principal in your planning.
3. Obtain help from the mathematics consultant in the state department of instruction.
4. Draw up an NDEA project proposal showing how equipment and materials to be purchased would facilitate more effective instruction in mathematics.
5. Submit your proposal through proper channels.
6. If the proposal is approved and funds are allocated, proceed with ordering planned

---

[1]Marguerite Kluttz, "The Mathematics Laboratory — A Meaningful Approach to Mathematics Instruction," <u>The Mathematics Teacher</u>, LIV (March, 1963), 144, 145.

equipment.
7. Arrange for installation of your new equipment.
8. Evaluate the new program.
9. Alter instructional program as it becomes necessary.

SAMPLE EXERCISES

The remainder of this report will consist of a set of selected sample laboratory exercises suitable for use with ninth grade general mathematics students. These sample exercises were constructed and compiled to give teachers of ninth grade general mathematics some idea of the form of laboratory exercises and possible subject matter areas which tend to lend themselves to the laboratory approach. It is hoped that these will be an encouragement to other instructors of mathematics to work toward developing exercises of their own to use in their classes where lack of motivation is a problem.

The exercises given here by no means cover the field of potential exercises for use with general mathematics books, but are given only for your consideration. Most units in general mathematics books will easily lend themselves to at least one laboratory exercise.

LESSON ONE

A Study to Determine the Value of Pi

OBJECTIVES: The students will discover a method for determining the value of pi.

The student will find existing relationships which will yield pi.

MATERIALS NEEDED: common cord or string in three foot lengths; eighteen inch ruler graduated in tenths of an inch; three household cans, number 10, 303, $2\frac{1}{2}$

DIRECTIONS: First of all we need to be sure that we know the meanings of some important terms.
Circumference - distance around a circle
Diameter - distance across a circle passing through the center
Radius - distance from the center to edge of circle

1. Take the number 10 can and the ruler, measure the diameter at one end. Record this value in your table, correct to the nearest tenth.

2. Now take the cord and carefully measure the circumference of the can at the end and record it in your table, again correct to the nearest tenth.

3. Repeat numbers one and two for each of the two remaining cans and record the results in your table.

4. Complete the table by computing the four remaining columns each to the nearest tenth of an inch.

	diameter A	circumference B	B + A	B - A	B x A	B ÷ A
Number 10						
Number 303						
Number $2\frac{1}{2}$						

LESSON ONE (cont.)

Questions

1. Now that you have completed your table look at it. Is there any column which arouses your curiosity? ____ Which column is it? (A + B), (B x A), (B - A), (B ÷ A)
circle one

2. The value we are looking for is always a constant. Does this agree with your choice in question one? ____

3. What is your value for pi, correct to the nearest one-hundredth of an inch? ____________

4. Now in your own words explain the process necessary to arrive at the value of pi. ____________

____

____

5. Using the symbols; π, c, d, give a formula for finding pi. ____________

LESSON TWO

A Method for Finding the Formula for the Area of a Triangle

OBJECTIVES: The student will develop the formula for finding the area of a triangle.

The student will see how this formula works in finding the area of a triangle.

MATERIALS NEEDED: ruler, at least six inches in length, compass, some type of construction paper, and a pair of scissors.

DIRECTIONS:

1. Using your ruler, draw, on the construction paper, any two equal triangles such as these.

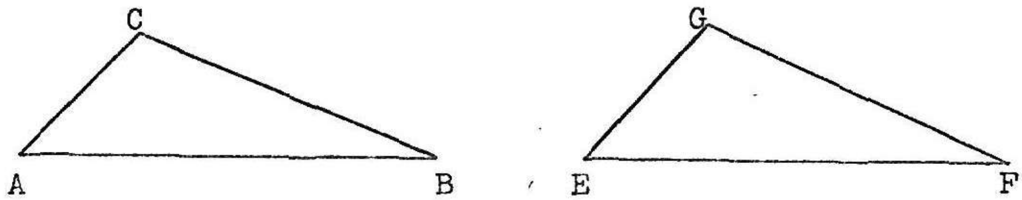


Fig 1

2. On triangle ABC construct a perpendicular from C to AB as follows: Place the point of your compass at C and choose any radius greater than or equal to AC and draw an arc as in figure 2.

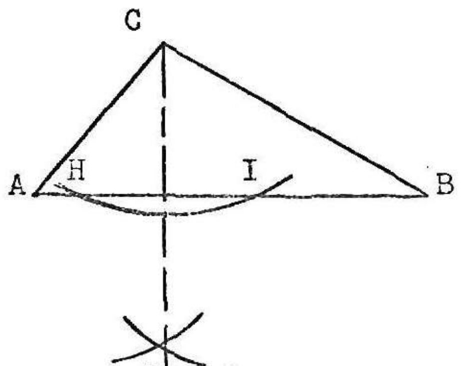


Fig 2

Now with any radius greater than or equal to HI. From point H draw an arc. From point I also draw an arc. Now using your ruler connect point C with the intersection of the last two arcs.

Now we have the perpendicular from C to AB.

LESSON TWO (cont.)

3. Cut out the two triangles and divide triangle ABC into two parts by cutting on the line we constructed CD. You should now have three triangles as shown below.

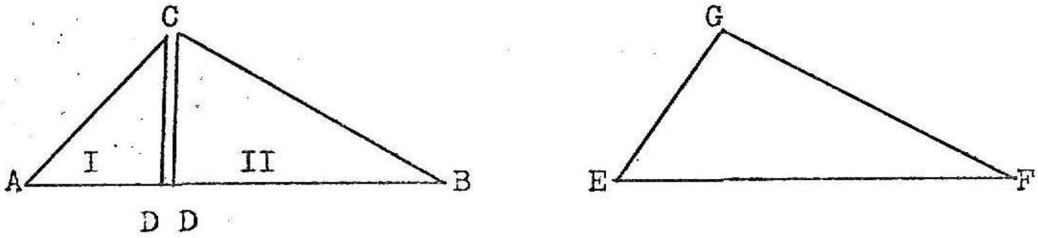


Fig 3

4. Place triangle I and triangle II above triangle EFG as shown below.

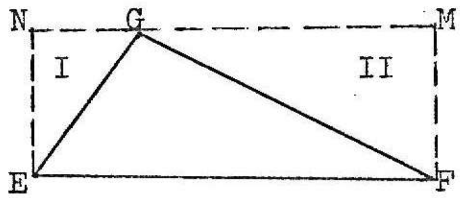


Fig 4

Questions

1. What kind of figure is EFMN? ______________________________

2. What is the formula for its area? ________________________

3. The area of EFMN is how many times the area of triangle EFG? __________________________

4. Line CD is perpendicular to AB and is called the altitude or height of the triangle ABC. (see fig 1). What dimension of the rectangle is the same length as the altitude of the triangle? _____________________________

5. Side AB of triangle ABC is the base of the triangle. Which dimension of the rectangle is equal to the base of the triangle? _____________________________________

LESSON TWO (cont.)

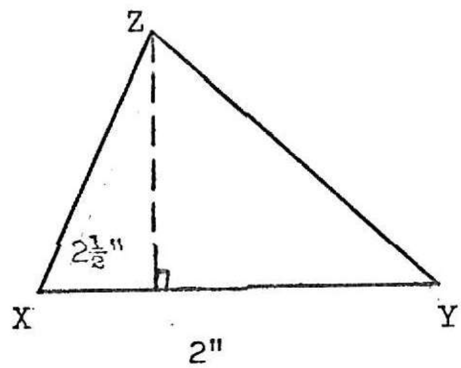
6. Questions 3, 4, and 5 show the following:
   a. The area of the rectangle EFMN is twice the area of one of the original triangles.
   b. The width of the rectangle is equal to the altitude of either of the original triangles.
   c. The length of the rectangle is equal to the length of the base of the original triangles.

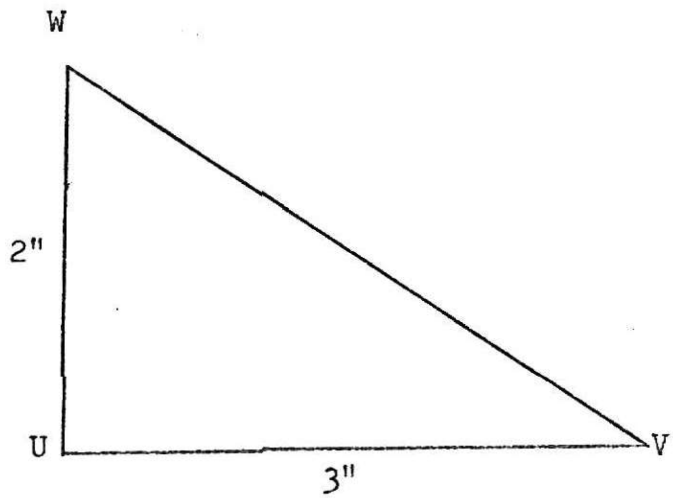
   Now from a, b, and c tell how to find the area of a triangle. ______________________

   ____

7. Using the letters A, b, and h to represent; A = the area of the triangle, b = the length of the base of the triangle, and h = the height of the triangle, now write the formula for the area of a triangle. ______________

8. Using the formula found in problem seven, now find the areas of the following triangles XYZ and UVW.





Area of triangle XYZ is ______________________

Area of triangle UVW is ______________________

LESSON THREE

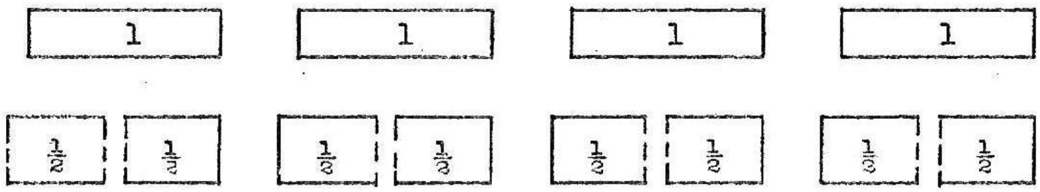
Division by a Fraction

OBJECTIVE: To give the student a better understanding of the process of division by fractions, using concrete examples.

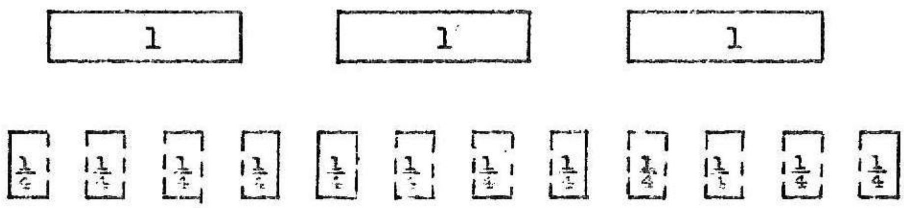
MATERIALS NEEDED: None, except this paper and a pencil.

DIRECTIONS: Carefully study the examples below and then answer the questions concerning them.

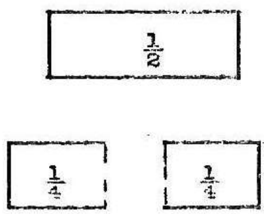
a. $4 - \frac{1}{2} =$ ________



b. $3 - \frac{1}{4} =$ ________



c. $\frac{1}{2} - \frac{1}{4} =$ ________



LESSON THREE (cont.)

Questions

1. Into how many parts has the original number of part (a) been divided? __________

2. If I take the number 4, what other arithmetic operation and what number will yield the same result or answer?

____

3. Then dividing by $\frac{1}{2}$ and multiplying by __________ yield the same result.

4. Now repeat questions 1 - 3 for parts b and c.

1(b). ________ 2(b). ________ 3(b). ________

1(c). ________ 2(c). ________ 3(c). ________

5. From your knowledge of arithmetic $\frac{1}{2}$ is called the ______________ of 2 and visa, versa.

6. Then from the above questions if the problem states to divide a number or a fraction by a fraction, I replace the fraction by its reciprocal and __________________.

Hence: $4 - \frac{1}{2} = 4 \times$ _____.

$3 - \frac{1}{4} = 3 \times$ _____.

$\frac{1}{2} - \frac{1}{4} = \frac{1}{2} \times$ _____.

7. Now state the rule for dividing by a fraction. ______

____

____

LESSON FOUR

Percentage Computer

OBJECTIVES: The student will gain a better understanding of percentage problems.

The student will work with his hands to develop a useful mathematical tool.

MATERIALS NEEDED: a piece of string, a sheet of graph paper, and a piece of stiff cardboard. The best graph paper to use would be one with ten squares to the inch.

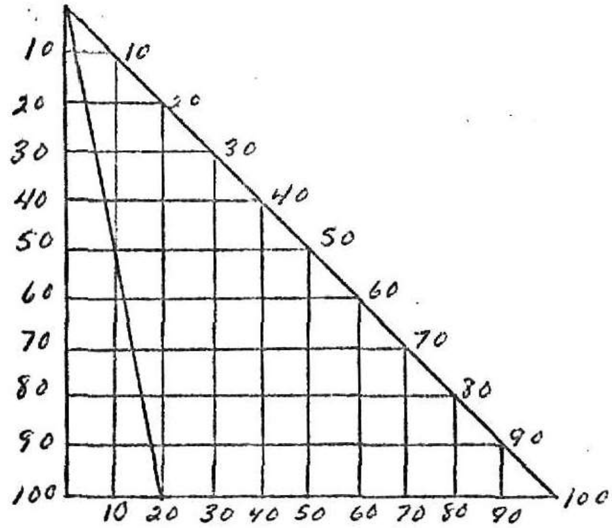
DIRECTIONS: (a). Mount the graph paper on the cardboard. Along the bottom of the graph paper draw a horizontal line $\overline{AB}$, 100 units long. This line represents the per cent scale and should be labeled in convenient units from 0 to 100, from left to right. (Let each vertical line represent 10 units. i.e. 0, 10, 20, $\cdots$, 100).

(b). At A (the point 0) darken the perpendicular line $\overline{AC}$ (vertical). $\overline{AC}$ should also be 100 units long. Starting at C again label the units from 0 to 100 at A in groups of 10 (0, 10, 20, $\cdots$, 100).

(c). At C, punch a small hole and attach a string slightly longer than the distance $\overline{CB}$. Now draw the line $\overline{BC}$. Now label the other ends on the line segments which are parallel to $\overline{AB}$ as you did when you labeled from A to C.

At this point your computer should look like figure 1, and is now ready to be used.

LESSON FOUR (cont.)



(figure 1)

EXCERCISES

Example 1. To find what per cent 8 is of 40. Find the horizontal line between $\overline{AC}$ and $\overline{BC}$ that is 40 units long. Now locate the point that is 8 units to the right of $\overline{AC}$, on this line place a pin at this point. Hold the string tight to form a straight line that passes from C across the pin point. The per cent is then found where the string crosses $\overline{AB}$ (the per cent scale). The line from C (in figure 1) represents your string and if you were correct your result should be 20%.

Example 2. To find 20% of 40 (the percentage) hold the string tight across the point 20 on the per cent scale ($\overline{AB}$). The point where it crosses the horizontal line that is 40 units long, between $\overline{AC}$ and $\overline{BC}$ is the answer, 8.

Example 3. To find the number (base) if 8 is 20% of it, hold the string tight across 20 on the per cent scale ($\overline{AB}$). Locate the line which the string crosses exactly 8 units to the right of $\overline{AC}$. The length of this line between $\overline{AC}$ and $\overline{BC}$ is the answer, 40.

LESSON FOUR (cont.)

In a typical per cent problem lets identify the three parts. Given this problem: 20% of 40 is ________. The 20% is the per cent; 40 is the base; and the answer is the percentage. Each part of a per cent problem can be identified in this manner.

Exercises:

1. Find the percentages.
   (a). 20% of 50. (b). 60% of 50.
   (c). 10% of 90. (d). 90% of 20.

2. Find the per cents.
   (a). 20 is what per cent of 50?
   (b). 40 is what per cent of 80?
   (c). 15 is what per cent of 50?
   (d). 12 is what per cent of 15?

3. Find the bases.
   (a). 40% of _____ is 20.
   (b). 30% of _____ is 27.
   (c). 15% of _____ is 6.
   (d). 25% of _____ is 20.

Now that you have completed this exercise check your answers by the conventional method.

LESSON FIVE

Discovering the Pythagorean Theorem

and some of its applications

OBJECTIVES: The student will discover the Pythagorean Theorem and its applications.

The student will discover some patterns contained in right triangles.

MATERIALS NEEDED: None, except this paper and a pencil.

DIRECTIONS:

1. Observe the right triangle in figure 1. Notice that its sides are 3, 4, and 5 respectively.

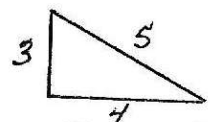


figure 1

2. Now notice that in figure 2, we have formed squares on all three of the sides of the triangle in number 1.

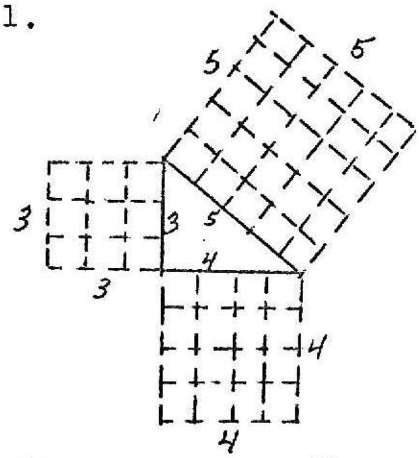


figure 2

3. How many small or unit squares are there in the area of the square drawn on the three inch side? ________

   How many small or unit squares are there in the area of the square drawn on the four inch side? ________

   How many small or unit squares are there in the area of the square drawn on the five inch side? ________

4. What is the sum of the unit squares in side 3 and side 4? ________

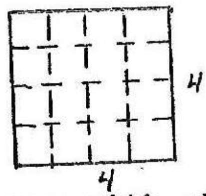
LESSON FIVE (cont.)

What is the difference in the number of unit squares in side 4 and side 3? ________

5. From number 4, do you see any relationship between either the sum or the difference in number of unit squares in side 3 and side 4 compared to the number in side 5? yes or no
If yes what is the relationship? ____________________

____

6. As a matter of review, if we have a figure 4 units on a side, how do we find its area?
Is it 4×4 or 4^2 which equals 16?



____

So in a shorter notation could we say that in figures 1 and 2 that $3^2 + 4^2 = 5^2$? yes or no

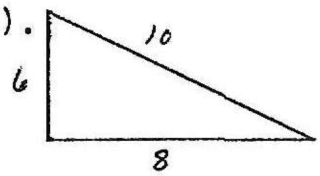
Keep in mind that the Pythagorean Theorem with which we are working, works only with right triangles. Also remember that when dealing with right triangles, the two sides which form the right angle are called the legs and the side opposite the right angle is called the hypotenuse.

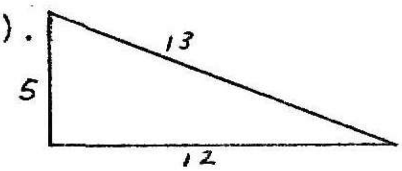
7. Now state in terms of a, b, and c the Pythagorean Theorem for this right triangle, where a and b are the legs and c is the hypotenuse.

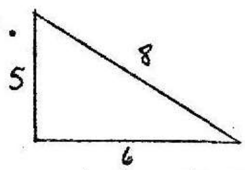
____

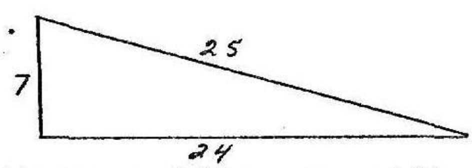
8. Was your answer $a^2 + b^2 = c^2$? yes or no

9. Now using the Pythagorean Theorem, check the following triangles to see if they are right triangles or not. (hint; if $a^2 + b^2 = c^2$, then they are right triangles if they are not equal then the triangle is not a right triangle)

(a).


(b).


(c).


(d).


If you have correctly solved these problems you will find that; a, b, d are right triangles, c is not.

LESSON SIX

Napier's Rods

OBJECTIVES: To show the students the complete set of Napier's Rods.

The student will use Napier's Rods to increase the speed and accuracy of his multiplication work.

MATERIALS NEEDED: Straight edge, one sheet of heavy construction paper, and scissors.

DIRECTIONS:

Observe the set of Napier's Rods below, with the index at the left. Each member on the left is multiplied by the index number in that column.

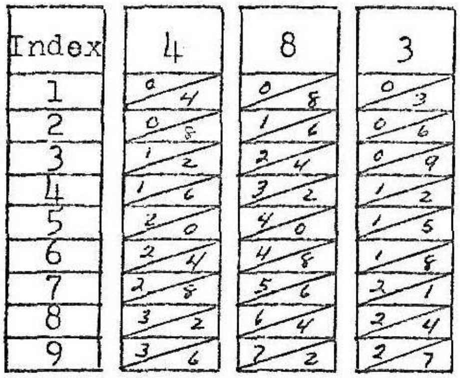
Index	1	2	3	4	5	6	7	8	9
1	0/1	0/2	0/3	0/4	0/5	0/6	0/7	0/8	0/9
2	0/2	0/4	0/6	0/8	1/0	1/2	1/4	1/6	1/8
3	0/3	0/6	0/9	1/2	1/5	1/8	2/1	2/4	2/7
4	0/4	0/8	1/2	1/6	2/0	2/4	2/8	3/2	3/6
5	0/5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5
6	0/6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4
7	0/7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3
8	0/8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2
9	0/9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1

Notice that in each multiplication the upper $\frac{1}{2}$ is the tens digit of the number and the lower $\frac{1}{2}$ is the units digit of the number. This is the reason 1 x 5 is recorded as 0/5.

Copy this set of Rods on the heavy construction paper and separate them with the scissors.

Now to multiply two numbers like 7 x 483 we arrange the rods against the index as so:

LESSON SIX (cont.)



Of this set of rods we need only this row.

 = 3381

Now add on the diagonals and move from right to left carrying where necessary.

With this information complete the multiplication of the following numbers by arranging the rods correctly.

1. 5 x 267
2. 3 x 2678
3. 6 x 5115
4. 8 x 989
5. 9 x 5432
6. 4 x 21286
7. 6 x 411
8. 2 x 212
9. 3 x 26364

Now check your answers by using conventional multiplication.

LESSON SEVEN

Experimental Probability

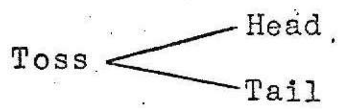
OBJECTIVES: The student will have a basic understanding of probability through the tossing of two pennies.

MATERIALS NEEDED: This paper, a pencil, and two pennies.

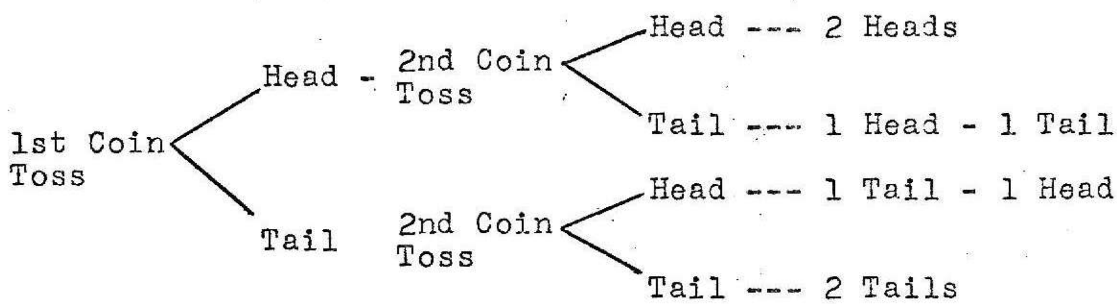
DIRECTIONS:

In order to look at the experimental probability of this event we first need to look at a sample space for the tossing of two coins.

If I toss one coin I will either get 1 head or 1 tail. In a tree diagram it would look like this:



Now for two coins I have this tree diagram:



Event	Probability
0-Heads	$\frac{1}{4}$
1-Head	$\frac{1}{2}$
2-Heads	$\frac{1}{4}$

If we apply this distribution to a set of twenty tosses of two coins, we find that the <u>expected</u> distribution of heads is:

Event
0-Heads -- 5
1-Head -- 10
2-Heads -- 5

The same would be true for the probability of tails.

LESSON SEVEN (cont.)

Now we will break into groups of four students each. Each group will have two coins and each student will toss the two coins five times and record the results in his table for each member of the group.

	1st Student					2nd Student					3rd Student					4th Student					
Trials	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	Totals
2 Heads																					
1 Head																					
0 Heads																					

Take total of 2 Heads row and put it over 20 (the number of trials). Now change this common fraction to a decimal fraction by dividing the numerator by the denominator (20). At the completion of this you have the probability of getting 2 Heads out of two tosses.
Probability of getting two heads is ____________________.

Repeat this for the 1 Head row and for the 0 Heads row.
Probability of getting one head is ____________________.

Probability of getting zero heads is ____________________.

How does this compare with the answers of the other groups?

How does it compare with the prediction we made before we started?

LESSON EIGHT

Saturnian Numbers

OBJECTIVES: The student will develop the ability to use deductive reasoning to solve problems.

MATERIALS NEEDED: None, except this paper and a pencil.

DIRECTIONS:

On the planet Saturn, the inhabitants use a system of numeration that consists of the following set of symbols.

{*, $, &, #, Z}

These symbols stand for digits which we use here on earth.

Here are some Saturnian addition facts:

$ + * = * + $ = $

& + * = * + & = &

+ * = * + # =

Z + * = * + Z = Z

What number does the numeral * represent? __________
What can be said about the sum of any Saturnian number and *? ____

Here are some more facts about these numerals:

$ + $ = &

& + $ = #

+ $ = Z

What is the smallest numeral $ could represent? ________
What numeral represents the sum of $ + $ + $? __________
What numeral represents the sum of $ + $ + $ + $? ______

In Saturnian addition we also have:

Z + $ = $*

Z + & = $$

LESSON EIGHT (cont.)

From the bottom of page before, what is the sum of Z + #? ______________

What is the sum of Z + Z? __________

Now complete the following table of facts about these Saturnian Numerals:

+	*	$	&	#	Z
*					
$					
&					
#					
Z					

How does this Saturnian system of numeration compare with our base five system of numeration? ______________

____.

LESSON NINE

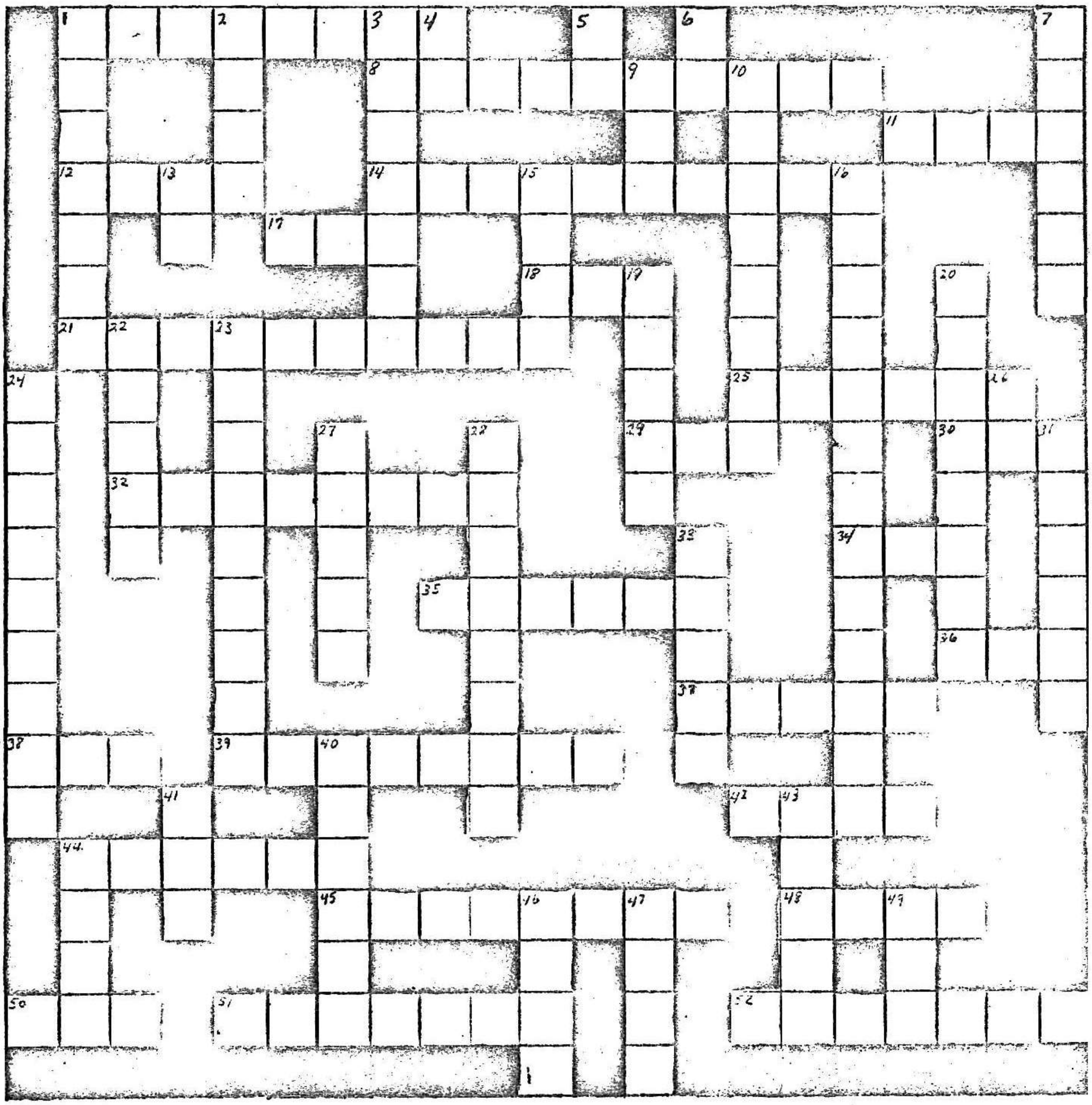
End of Year Vocabulary

Refresher

OBJECTIVES: By the use of this puzzle the student will be able to strengthen his technical vocabulary.

MATERIALS NEEDED: None, except this paper and a pencil.

DIRECTIONS: Complete the following crossword puzzle with the correct vocabulary words.



LESSON NINE (cont.)

CLUES

Across

1. One of the equal parts of a whole
8. Per cent of price an agent receives for selling merchandise
11. Another name for an ellipse
12. Number used most frequently in a group of numbers
14. The study of numbers
17. Abbreviation for logarithm
18. Drawing of a country, continent, etc.
21. Having a measure of less than 90°
25. A special rectangle
29. Abbreviation for metric
30. To find the sum
32. The volume of something is its ________
34. Little-used unit of length
35. Pertaining to lines; involving measurement in one dimension only
36. Abbreviation for least common denominator
37. A flat or level surface
38. Any rule that must be obeyed
39. A number that is divided by another number
42. A stands for ___ in A = lw
44. The middle point of a circle
45. A three sided polygon
48. An algebraic expression with its sign

Down

1. A short way of writing a rule in which letters are used instead of words
2. A solid whose base is a circle and whose curved surface comes to a point
3. An eight sided figure
4. Abbreviation for the word number
5. The name of the letter
6. Mind your p's and _____
7. Capacity
9. Add to find the sum
10. The formula for finding _____ is I = prt
13. Perform
15. t stands for ____ in I = prt
16. Formula for ____ of a circle is C = d
19. A number dividible evenly only by one and itself
20. Two lines that will never meet are _____
22. Written form directing bank to pay money to someone
23. Four sided polygon having one pair of parallel sides
24. Parallelogram having one right angle
26. Abbreviation for edition
27. _________ drawing
28. Both bases are circles and it has curved sides
31. Add, Subtract, Multiply, _________
33. A way of showing percent or ratio with pictures is a picto________

LESSON NINE (cont.)

CLUES (cont.)

Across

50. One tenth of one hundred
51. A six sided polygon
52. Same as one down

Down

40. Point where the sides of an angle meet
41. _____, two, three, four, five
43. Comparison of one number to another
44. Formula for volume of a _____ is $V = e^3$
46. Eight, _____, ten, eleven
47. Shortest distance between two points
49. Edge of a wheel

Adapted from DeJong, Linda, "Mathematics Crossword," *School Science and Mathematics* (January, 1962), 45.

SUMMARY

It was the purpose of this report to develop a set of exercises using the laboratory approach in the teaching of ninth grade general mathematics.

The laboratory approach to mathematics gives the student a chance to discover for himself much of the how and why involved in mathematics, as apposed to the traditional chalkboard and eraser classroom where he is presented with a formula or other type problem, told that it is true, and simply must put values into the formula and grind out answers. By using a simple laboratory exercise he is exposed to the development of the formula, why and how it works, and then reinforces this learning by working exercises using the formula.

In the field of the physical sciences, the laboratory approach to learning has been used for many years. Foreign languages, industrial arts and home living spend a large part of each instructional period in the laboratory setting, such has not been the case in the field of mathematics.

Some of the advantages found in the literature were: The laboratory method utilizes an experimental approach which requires each student's participation, allows each student to work at his own rate, leaves the instructor more time to give individual help, and since it is conducted on a less formal basis allows for more student interaction which gives them the chance to learn from each other. It was pointed out that most people agree that learning is an active rather than a passive process and by using the laboratory approach the student becomes actively involved. The laboratory approach also works equally well with either a modern or the traditional approach. A laboratory approach in mathematics gives the student a chance to write about what he himself has thought through and developed. As one uses the laboratory approach the material is shifted from the realm of the teacher to the realm of the student, which can greatly improve the attitude of the student toward the teacher and the subject matter. Most students are eager to assume responsibility and by using laboratory lessons they are allowed to do this.

The main disadvantages found in the literature were that of the time element and lack of consistent, adequate terminology used to describe the stimuli employed in discovery studies.

Each of the sample laboratory exercises were constructed with the following items:

1. A clear and concise statement of the principle or concept to be studied.
2. A list of objectives pertinent to the concept.
3. A list of materials and/or equipment needed to perform the operation.
4. Directions needed to arrive at the conclusion.

5. Tables or charts in which to record data obtained.
6. A set of questions relating to desired results.
7. Where applicable, a statement by the student of what he has observed from his work.

The sample exercises were:

1. A Study to Determine the Value of Pi.
2. A Method for Finding the Formula for the Area of a Triangle.
3. Division by a Fraction.
4. The Percentage Computer.
5. Discovering the Pythagorean Theorem.
6. Napier's Rods.
7. Experimental Probability.
8. Saturian Numbers.
9. End of Year Vocabulary Refresher.

Each of the sample exercises listed can be used in a classroom with no special equipment necessary. Every item is readily available almost everywhere.

BIBLIOGRAPHY

A. BOOKS

Bruner, Jerome S. Toward a Theory of Instruction. W. W. Norton and Company, Inc., 1968. 187 pp.

Johnson, Donovan A., and Gerald R. Rising. Guidelines for Teaching Mathematics. Belmont: Wadsworth Publishing Company, Inc., 1967. 446 pp.

Kuethe, James L. The Teaching-Learning Process. Keystones Education Series. Scott Foresman and Company, 1968. 168 pp.

B. PERIODICALS

Auclair, Jerome A., and Thomas P. Hillman. "A Topological Problem for the Ninth-Grade Mathematics Laboratory," The Mathematics Teacher, LXI (May, 1968), 507.

Barkdoll, O. R. "Teaching Mathematics in the Laboratory," The Clearing House, XXXII (October, 1957), 77-79.

Biggs, Edith E. "Mathematics Laboratories and Teacher Centres — The Mathematics Revolution in Britain," The Arithmetic Teacher, XV (May, 1968), 400-408.

Bittinger, Marvin L. "A Review of Discovery," The Mathematics Teacher, LXI (February, 1968), 145.

Bolding, James. "A Look at Discovery," The Mathematics Teacher, LVII (February, 1964), 105-106.

Bruner, Jerome S. "The Act of Discovery," Harvard Educational Review, XXXI (1961), 21-32.

Cambridge Conference on School Mathematics. "Goals for School Mathematics," American Mathematical Monthly, LXXXI (1964), 196-199.

Clarkson, David M. "Mathematical Activity," The Arithmetic Teacher, XV (October, 1968), 493-497.

Craig, R. C. "Directed Versus Independent Discovery of Established Relations," Journal of Educational Psychology, XLVII (1956) 223-234.

DeJong, Linda. "Mathematics Crossword," School Science and Mathematics, LI (January, 1962), 45.

Fehr, Howard F. "The Place of Multisensory Aids in the Teacher Training Program," The Mathematics Teacher, XL (May, 1947), 212-216.

Forbes, Jack E. "Programmed Instructional Materials — Past, Present, and Future," The Mathematics Teacher, LXI (April, 1963), 224-226.

Johnson, Donovan A. "Enriching Mathematics Instruction with Creative Activities," The Mathematics Teacher, LV (April, 1962), 238-242.

Johnson, Larry K. "The Mathematics Laboratory in Today's Schools," School Science and Mathematics, LXII (January, 1962), 586-592.

Kersh, Bert Y. "The Adequacy of Meaning As an Explanation for the Superiority of Learning by Directed Discovery," Journal of Educational Psychology, XLIX (1958), 282-292.

Kluttz, Marguerite. "The Mathematics Laboratory — A Meaningful Approach to Mathematics Instruction," The Mathematics Teacher, LIV (March, 1963), 144-145.

Langford, Francis G., Jr. "Helping Pupils to Make Discoveries in Mathematics," The Mathematics Teacher, XLVIII (January, 1955), 45.

Lowry, William C. "Pupil Discovery in Junior High School Mathematics," The Mathematics Teacher, XLIX (April, 1956), 201-203.

Matthews, Geoffery. "The Nuffield Mathematics Teaching Project," The Arithmetic Teacher, XV (February, 1968), 101-102.

May, Lola J. "Learning Laboratories in Elementary Schools in Winnetka," The Arithmetic Teacher, XV (October, 1968), 501-503.

Moore, Richard E. "Individualized Math," School and Community, LIV (February, 1968), 20-21.

Sparks, Jack N. "Designing Research Studies in Elementary School Mathematics Education," The Arithmetic Teacher, XV (January, 1968), 60-63.

Sweet, Raymond. "Organizing A Mathematics Laboratory," The Mathematics Teacher, LX (February, 1967), 571-575.

_______. "The Madison Project of Syracuse University," The Mathematics Teacher, LIII (November, 1960), 571-575.

Willoughby, S. S. "Discovery," The Mathematics Teacher, LVI (January, 1963), 22.

Wittrock, M. C. "Verbal Stimuli in Concept Formation: Learning by Discovery," Journal of Educational Psychology, LIV (1963), 184.

LABORATORY EXERCISES FOR NINTH
GRADE GENERAL MATHEMATICS

by

MICHAEL DEAN GRUB

B. S., Fort Hays Kansas State College, 1965

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

It was the purpose of this report to develop a set of exercises using the laboratory approach to the teaching of ninth grade general mathematics.

The laboratory approach to mathematics gives the student a chance to discover for himself much of the what and how involved in mathematics, as opposed to the traditional chalkboard and eraser classroom where he is presented with a formula or other type of problem, told that it is true, and simply must put values in the formula and grind out answers. While by using a simple laboratory exercise he is exposed to the development of the formula, why and how it works, and then reinforces this learning by working exercises using the formula.

Some advantages found in the literature were: The laboratory method utilizes an experimental approach which requires each student's participation, allows each student to work at his own rate, leaves the instructor more time to give individual help, since it is conducted on a less formal basis, allows for more student interaction which gives them the chance to learn from each other. Learning is an active rather than a passive process and by using the laboratory approach the student becomes actively involved. A laboratory approach in mathematics gives the student a chance to write about what he himself has thought through and developed. It works equally well with either a modern or a traditional approach to subject matter. Most students are eager to accept responsibility and by using this approach they are allowed to do this.

The main disadvantages found in the literature were that of the time element and lack of consistent, adequate terminology used to describe the stimulus employed in discovery studies.

Each of the sample laboratory exercises were constructed with the following items included: A clear and concise statement to be studied, a list of objectives pertinent to the concept or principle, a list of materials and/or equipment needed to perform the operation, directions needed to arrive at the conclusion, tables or charts in which to record data obtained, and where applicable, a statement by the student of what he has observed from his work.

The sample exercises were: A Study to Determine the Value of Pi, A Method for Finding the Formula for the Area of a Triangle, Division by a Fraction, the Percentage Computer, Discovering the Pythagorean Theorem, Napier's Rods, Experimental Probability, Saturian Numbers, and End of Year Vocabulary Refresher.

Each of these exercises listed can be used in a classroom with no special equipment necessary. Every item needed is readily available almost everywhere.