

OPTIMAL REGIONAL WATER QUALITY MANAGEMENT BY
AT-SOURCE TREATMENT AND EFFLUENT CHARGES

by

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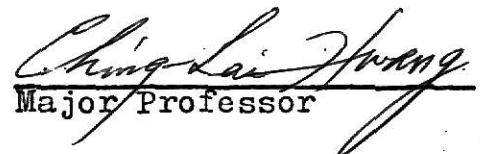
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1. INTRODUCTION

Water pollution has received increased attention in recent years and the broad problem facing water resources planners is to supply ample quantities of water of adequate quality for a wide range of users. The water quality problem is becoming more severe because of the degradation of the water quality level. Continuous and rapid industrial growth over the past decades has resulted in the excessive degradation of water quality in streams and even the largest rivers can not be treated as infinite sinks for municipal and industrial wastes.

The problem considered in this report is to manage efficiently, the satisfaction of a specified set of water quality standards. Our objective is to allocate a scarce resource, the assimilative capacity of water, most efficiently.

One approach to the problem of pollution is the use of effluent charges (taxes) to regulate the discharge of pollutants. At present, polluters are not paying any tax proportional to the damage they cause by discharging the untreated waste into the river. The mathematical formulation of this approach results in the decentralization model.

Kneese (1964) and Kneese and Bower (1968) defined the basin wide firm which owns and operates all waste and water treatment plants and carries on all water based activities within a particular region. All costs associated with pollution

and treatment are covered by the firm and it tries to maximize net profits and consequently the marginal treatment cost, at the optimum, equals the marginal damage imposed by discharge. This is the centralization model.

Literature survey

In the recent years, the role of some sort of central water authority in determining the minimum cost method of meeting the stream standard has been stressed upon, and the use of effluent charges to abate pollution has been advocated extensively. Merits of such proposition has been presented by Kneese and Bower (1968) and Freeman and Haveman (1972). Baumol and Oates (1971) concluded that the use of a price system to combat pollution is feasible and for any set of desired pollution central standards, there exists a tax scheme which will achieve the specified standards efficiently. Upton (1968) has developed an optimal tax structure and the implementation of a taxing scheme consistent with this structure has been outlined by Hass (1970), Haines et al. (1971, 1972 a,b,c), Upton (1971) and Foley and Haines (1973). Similar to Hass (1970), Taylor (1973) has suggested a scheme which is a combination of an effluent charge and the waste allotment. However, Byrne and Spiro (1973) have demonstrated that taxation is a necessary but not sufficient condition for an efficient solution under certain

circumstances when the firm has at its disposal a number of production processes where low levels of pollution require high capital input and vice versa. Bundgaard-Nielson and Hwang (1975) extended the work of Byrne and Spiro and concluded that the interaction between available technology and the enforcement strategy must be taken into account in order to prevent the use of taxation levels which do not produce any environmental quality improvement through reduced discharges, but tend to increase the production costs and thus consumer prices.

The first attempt to model a river basin was by Deininger (1965), using the basic Streeter-Phelps (1925) oxygen deficit equation. Loucks et al. (1967) extended Deininger's work by developing a modular component technique for modeling river basins. Day et al. (1965) discuss how a central water authority might achieve the imposed dissolved oxygen standards at minimum cost by using taxes.

The thorough review and classification on literature of optimal decision of regional water quality management can be seen in Bundgaard-Nielson and Hwang (1976).

Objectives

The objective of this report is to apply the non-linear programming techniques, quadratic programming and separable programming, to the basin wide river problem which has non-

linear objective function and linear constraints. The centralization model problem (basin wide firm) has been solved by both separable programming and quadratic programming. While for the decentralization model (effluent charges) the non-linear objective function has been approximated by piecewise linear functions using separable programming and the optimal at-source treatment configuration has been obtained by linear programming.

The problem considered in this report is the optimal regional waste water management of a basin wide river by at-source treatment. Both centralization model (basin wide firm) and decentralization model (uses effluent charges) have been reviewed. The Miami River basin in Ohio has been considered as an example and the problem has been formulated both according to centralization and decentralization models. To demonstrate the approach, quadratic programming, separable programming and linear programming have been used.

There are many ways for reducing the effects of pollution other than treatment at-source. The most notable are flow augmentation and mechanical reaeration. Flow augmentation can dilute both degradable and non-degradable wastage, add oxygen to the water to reduce the effects of decomposition and alter the hydrological characteristics of the body of water, whereas mechanical reaeration can only increase the oxygen content of the water. In order to meet the required dissolved oxygen

standards, we have considered both the treatment-augmentation policy as well as at-source treatment policy.

2. STATEMENT OF THE PROBLEM

The problem is to find the optimal treatment configuration for meeting water quality standards along a river basin when the central authority has complete knowledge of the treatment cost functions (centralization model) and to determine optimal pollution taxes along with the optimal treatment configuration when the central authority does not know the treatment cost functions (decentralization model). Our objective being the allocation of the assimilative capacity of water most efficiently. In summary, the decision is to determine the amount of at-source waste water treatment required to achieve a minimum cost for any particular set of stream dissolved oxygen standards within a river basin.

2.1 THE BASIN WIDE FIRM (CENTRALIZATION MODEL)

The objective can be achieved by considering the operation of a basin wide firm. This firm would own all waste and water treatment plants and would carry on all water based activities in the region. All the costs associated with pollution and with treatment would then be borne by the firm. Such a firm would operate so as to maximize net profits and, at the optimum, the marginal costs of treating an additional unit of wastage from any source by any means would equal the damage imposed by the unit being discharged into the water, where the damage would be either the value of the marginal product lost and/or

purification costs imposed elsewhere.

The algorithm presented here is a systematic method of determining the amount of at-source waste water treatment required to achieve a particular set of stream dissolved oxygen standards such that the cost of treatment and flow augmentation is minimum. In this case the central authority knows the various treatment cost functions, the quality standards desired and the low flow augmentation cost function.

A river basin may be modeled by dividing it into n reaches, one at each point of discontinuity (see Fig. 1). Examples of discontinuities are: (1) organic and/or thermal inputs, (2) a state line, (3) a change in the flow regime such as changes in stream width or depth, (4) increase in the flow such as tributary inputs and (5) change in water quality standards. Let us introduce the following notation:

- n = number of reaches;
- w_j = gross biological oxygen demand (BOD) load introduced at the beginning of the j th reach (lbs/day);
- x_j = percentage of w_j removed through treatment at-source j , if nontreatable, $x_j = 0$;
- a_{ij} = pounds of oxygen demanded in reach i by the decomposition of a pound of BOD discharged at the beginning of reach j ; $a_{ij} = 0$, $j > i$;
- y = flow rate; if no augmentation, $y = 1$;
- b_i = amount of dissolved oxygen available for decomposition process (total available less standard requirement)

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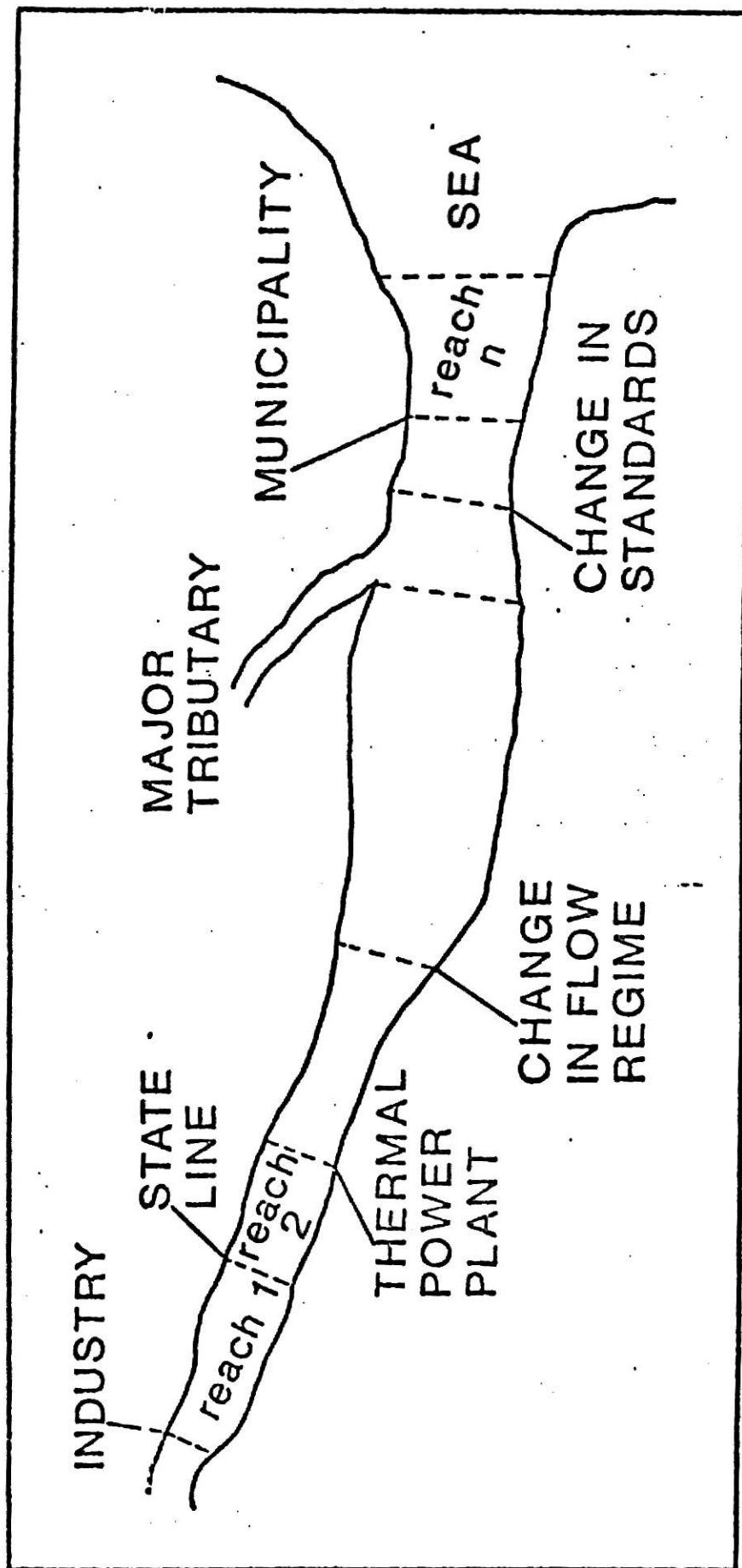


Figure 1 Generalized river basin.

in reach i per unit flow.

To maintain the water quality standard in the river the supply of available oxygen for the decomposition process in each reach must be equal to or exceed the demand imposed by BOD loads discharged into the reach and all reaches preceeding it, that is , for reach i it is required

$$a_{i1}w_1(1-x_1) + a_{i2}w_2(1-x_2) + \dots + a_{ii}w_i(1-x_i) \leq yb_i \quad (1)$$

or

$$a_{i1}w_1x_1 + a_{i2}w_2x_2 + \dots + a_{ii}w_ix_i + b_iy \geq S_i \quad (2)$$

where

$$S_i = a_{i1}w_1 + a_{i2}w_2 + \dots + a_{ii}w_i = \sum_{j=1}^i a_{ij}w_j$$

Equation (2) states that the supply of dissolved oxygen (DO) created by treatment and flow augmentation must exceed that demanded by the gross waste loads.

Let us denote the treatment cost function at treatable source i by $C_i(x_i)$ and the low flow augmentation cost function by $C_o(y)$, then the river basin problem can be formulated as

$$\min (C_1(x_1) + C_2(x_2) + \dots + C_n(x_n) + C_o(y))$$

subject to

$$\begin{array}{ll} a_{11}w_1x_1 & + b_1y \geq s_1 \\ a_{21}w_1x_1 + a_{22}w_2x_2 & + b_2y \geq s_2 \\ \vdots & \\ a_{n1}w_1x_1 + a_{n2}w_2x_2 + \dots + a_{nn}w_nx_n & + b_ny \geq s_n \end{array} \quad (3)$$

$$x_i \geq 0, i=1, \dots, n$$

$$y \geq 1$$

where

$$s_i = \sum_{k=1}^i a_{ik} w_k$$

Note that the model for the oxygen demanded by the decomposition of BOD discharge is linear. If the cost functions $C_i(x_i)$ are approximated by linear or piecewise linear functions, then (3) is a linear programming problem. The amount of at-source waste water treatment for a particular set of stream dissolved oxygen standards within a river basin is determined and its cost is minimized.

2.2 EFFLUENT CHARGES (TAXATION) FOR THE ABATEMENT OF WATER POLLUTION (DECENTRALIZATION MODEL)

In a free economy system, the central authority does not have a complete knowledge of the cost function for each individual polluter. And a freedom of choice should be offered to the polluter whether to treat his effluent by his own waste water treatment plant or to pay the effluent charge imposed by the central authority. This is a decentralization system.

The implementation of an efficient taxation or effluent charge scheme has been developed by Hass (1970). The basic assumption is that the central authority knows the quality standards desired and the low flow augmentation cost function ($C_0(y)$) while the polluters know the individual treatment cost functions ($C_i(x_i)$'s). The problem confronting the central authority is then to find an optimal allocation of treatment

costs which secure the quality requirements.

Let us examine again the river basin considered in section 2.1. Suppose that the central authority has imposed various hypothetical tax rates on the polluters that are to be applied to the percentage of their gross discharge they leave untreated and consequently discharge into the river. The authority has requested that each polluter respond to these possible imposed taxes by transmitting to the authority the percentage of waste removed by treatment and the total cost of such treatment. With his gross waste production constant, each polluter will attempt to minimize the total cost of waste disposal where total cost is the sum of treatment cost plus taxes imposed, subject to the fact that he must treat at least a minimum treatment requirement (e.g. 45% which is equivalent to at least primary treatment) and can treat no more than 99%. Then the polluter's optimization problem is:

$$\min (C_j(x_j) + (1-x_j)T_j)$$

subject to

(4)

$$0.45 \leq x_j \leq 0.99$$

where x_j is the percentage of BOD removed through treatment at source j , and T_j is the tax that would be imposed if the polluter were to discharge all the waste his plant generates. Let us denote the k^{th} response of the j^{th} polluter as x_j^k and the treatment cost associated with x_j^k as C_j^k .

The central authority must then, according to the algorithmic

approximation, find the convex combination of each polluter's treatment plans and the level of flow augmentation which minimizes the cost of achieving the standards imposed upon the river at the end of each reach. Suppose each polluter has responded with k_j distinct treatment plans. The linear programming problem the CA must solve is:

$$\min \sum_{j=1}^n \sum_{i=1}^{k_j} U_j^i C_j^i + C_0(y)$$

subject to

$$\sum_{i=1}^{k_1} a_{11} w_1 x_1^i U_1^i + b_1 y \geq S_1 \quad (5)$$

$$\sum_{i=1}^{k_1} a_{21} w_1 x_1^i U_1^i + \sum_{i=1}^{k_2} a_{22} w_2 x_2^i U_2^i + b_2 y \geq S_2$$

⋮

$$\sum_{i=1}^{k_1} a_{n1} w_1 x_1^i U_1^i + \sum_{i=1}^{k_2} a_{n2} w_2 x_2^i U_2^i + \dots + \sum_{i=1}^{k_n} a_{nn} w_n x_n^i U_n^i + b_n y \geq S_n$$

$$\sum_{i=1}^{k_j} U_j^i = 1 \quad \forall j$$

$$y \geq 1.0$$

$$y \leq 4.47$$

$$U_j^i \geq 0 \quad \forall i, j$$

All the notations used in (4) and (5) are described in section 2.1. The upper limit of the augmentation rate, y , is determined by the reservoir size. Upon solving (5) the central

authority has an approximate measure of the economic value (marginal cost) associated with supplying the j^{th} reach with an additional pound of oxygen per day; this is the shadow or dual price associated with the j^{th} constraint. For those constraints that are binding, these dual evaluators are non-zero.

Let the shadow prices on the j^{th} reach constraint be denoted by λ_{ij} . Note that the authority wishes to minimize the cost to society while achieving the water quality standards, and that $\sum_{i=1}^n \lambda_{ij} a_{ij} w_j$ is the cost to society that would accrue at the margin if polluter j were not to treat his waste before discharging it into the river. The authority then wished to discover what percentage of waste polluter j can treat at a cost below that of the rest of the society. That is, he wished to have polluter j treat his waste to the point at which the marginal cost to society of disposing that waste is equal to the marginal cost to the polluter of waste disposal. Thus he will set the tax to polluter j at

$$T_j = \sum_{i=1}^n \lambda_{ij} a_{ij} w_j \quad (6)$$

to elicit the desired information. This taxing scheme is consistent with the optimal tax structure developed by Upton (1968). If λ_i , $i = 1, 2, \dots, n$ is the true cost to society of a pound of oxygen in each reach per day, when each polluter solves (4) an optimal treatment plan for the basin will be

achieved. Since (5) is only an approximation λ is not the true cost but rather an approximation. This approximation will be improved by incorporating the new plans from (4) into (5) and recalculating the marginal costs to society of supplying oxygen to each reach.

The applications of decomposition procedures appear not only feasible but may also be more efficient than central planning under imperfect information. These decomposition procedures furthermore require less a priori information in the hands of the central authority.

In the above model it has been assumed that the amount of waste generated by a polluter will not change as the efficient taxes vary or if a change in process is the response to changing taxes, that the cost of this change is incorporated into the treatment cost function.

2.3 AN EXAMPLE (THE MIAMI RIVER PROBLEM)

The Miami River basin in Ohio is modeled according to (3) (centralization model) as well as according to (4) and (5) (decentralization model), as presented by Hass (1970). Although it is desirable to use projected BOD loads or existing loads than the loads that existed in the past, existing BOD loads are not available.

Here we wish to determine what kind of treatment plant to build at each treatable source of pollution, for example primary, secondary or tertiary treatment plants, and what size

reservoir the central authority should build and operate for flow augmentation, our objective being to achieve the given set of water quality standards at the minimum cost to society.

Taking the data from Anderson (1967) and Paraska (1967) and substituting in the constraints developed by Hass (1970), we obtain the constraint set matrix shown in Table 1.

The 58-mile length of the river under consideration was divided into 27 reaches and has 15 polluters at discrete points along the river bank. Table 2 gives the polluter index j and the beginning of the reach where it is located. The decision variables x_1 through x_{15} refer to the percent of BOD load to be removed at each of the 15 municipal and industrial waste discharge sites, and y is the proportionality constant (flow augmentation rate) to be determined and then applied to the initial flow, which is considered to be 1.0 (datum or reference level), in order to increase the flow by reservoir release.

In order to consider the constraint on flow augmentation that reservoirs must be recharged before they can be discharged again, we have to take the appropriate time span. The yearly cycle of a basin would appear to be the proper time span in this case. The months of July, August and September are considered for flow augmentation since these months are the months of lowest flow for the Miami River basin. To limit the size of the problem, the month of August is chosen as the representative month and it is assumed that $1/90^{\text{th}}$ of the reservoir

TABLE 2

<u>Pollutor index j, j=1,...,15</u>	<u>Location (Beginning of the reach i, i=1,...,27)</u>
1	Reach 4
2	Reach 6
3	Reach 7
4	Reach 8
5	Reach 9
6	Reach 11
7	Reach 14
8	Reach 15
9	Reach 16
10	Reach 18
11	Reach 21
12	Reach 22
13	Reach 23
14	Reach 25
15	Reach 26

capacity would be used during any given day of this month. With this assumption we make certain that the capacity constraint will not be violated, by leaving the other nine months for recharging. It must be noted here that with so many assumptions to simplify the problem, we might be lead to suboptimization. This treatment scheme would be different from the one in which the problem is formulated over the twelve month cycle.

To solve problems (3), and (4) and (5) for the Miami River basin, we need at source treatment and flow augmentation cost functions. These cost functions were developed by Hass (1970) for the Miami River problem from the data provided by Frankel (1965). The treatment cost function employed is as follows:

$$C_j(x_j) = 160.8 + 26.7Q_j + 640.7(x_j-0.45)^2 + 225.7Q_j(x_j-0.45)^2 \quad (7)$$

where Q_j is the plant size in mgd, x_j is the percentage of gross waste removed by at-source treatment, and $C_j(x_j)$ is the estimated cost of such treatment. The Q_j 's for the 15 pollution sources are given in Table 3.

Based on the data provided by the Miami Conservancy District and the engineering firm employed by the conservancy in designing and costing a low flow augmentation reservoir, Hass developed the augmentation cost function as:

$$C_o(y) = 726.6 (y) \quad (8)$$

where y is the flow augmentation rate and lies between 1.0 and 4.47.

If there is no flow augmentation, $y = 1.0$ the normal flow

TABLE 3: THE PLANT SIZE IN MGD FOR THE 15 SOURCES

<u>Source</u>	<u>Plant Size in Mgd.</u>
1	45.2
2	4.7
3	4.2
4	3.6
5	0.5
6	1.2
7	0.8
8	0.6
9	0.5
10	3.2
11	8.4
12	2.7
13	0.6
14	12.1
15	8.4

in the river, and if the flow is augmented, $y > 1.0$ but less than or equal to 4.47. $C_0(y)$ is the estimated cost of such augmentation. It must be noted here that the reservoir used for flow augmentation is limited in size and at the most provides flow 4.47 times the normal or average flow in the river in the month of August, the representative month for this study.

The cost data used is not very accurate and the solution of the problem is of limited practical value since it has been assumed that the month of August is representative of the yearly cycle and the optimal solution for August is optimal over the whole year, and the waste loads are those existing in 1970 rather than existing under the current conditions or future projected loads.

The imposition of a standard implies that the damages occurring to the society when the stream has a dissolved oxygen content less than the standard are so costly as to make their attainment necessary regardless of treatment cost. In this problem a standard of 4 mg/l has been chosen because this is the level of DO necessary to assure survival of aquatic life. These constraints require that the supply of available oxygen for the decomposition process in each reach (that available over and above the quality standard requirement) must be equal to or exceed the demand imposed by BOD loads discharged into that reach and all reaches preceeding it. These constraints are given by (3).

In addition we require that at least 45% of the gross waste load will be treated by each polluter and the percentage of waste treated will not exceed 99% under any circumstances. Mathematically it can be written as:

$$\begin{aligned} x_i &\geq 0.45 \\ x_i &\leq 0.99 \quad , \quad i = 1, \dots, 15 \end{aligned} \tag{9}$$

The first 15 constraints of this set insure that all discharges be subjected to at least primary treatment, which involves chlorination, filtering, and settling to reduce bacteria and remove bulky solids. Such sewage treatment usually removes about 45% of the gross BOD waste load. The second set of 15 constraints indicate the tertiary treatment, but limit the percentage treated to 99%. Beyond 99% removal the cost of treatment rises very steeply.

We now have all the information to formulate the Miami River basin problem, both according to centralization and decentralization model.

Centralization Model

Problem (3) can be written as:

$$\min \sum_{j=1}^{15} (160.8 + 26.7Q_j + 640.7(x_j - .45)^2 + 255.7Q_j(x_j - .45)^2) + 726.6y$$

subject to the constraints

$$\sum_{j=1}^{15} a_{ij}x_j + a_{i16}y \geq b_i \quad , i = 1, \dots, 27 \tag{10}$$

$$\begin{aligned}
y &\geq 1.0 \\
y &\leq 4.47 \\
x_j &\geq 0.45 \\
x_j &\leq 0.99 \quad , j = 1, \dots, 15
\end{aligned} \tag{10}$$

In the above problem, the left hand side gives the supply of available oxygen for the decomposition process in each reach.

For no flow augmentation case, we do not have any flow augmentation cost and $y = 1.0$, so problem (10) reduces to:

$$\min \sum_{j=1}^{15} (160.8 + 26.7Q_j + 640.7(x_j - .45)^2 + 255.7Q_j(x_j - .45)^2)$$

subject to the constraints

$$\begin{aligned}
\sum_{j=1}^{15} a_{ij}x_j &\geq b_i - a_{i16} \quad , i = 1, \dots, 27 \\
x_j &\geq 0.45 \\
x_j &\leq 0.99 \quad , j = 1, \dots, 15
\end{aligned} \tag{11}$$

Decentralization Model

The general problem for the decentralization model is given by (4) and (5). So for j^{th} polluter of the Miami River basin, (4) can be written as:

$$\min 160.8 + 26.7Q_j + 640.7(x_j - .45)^2 + 255.7Q_j(x_j - .45)^2 + (1 - x_j)T_j$$

subject to the constraint

$$0.45 \leq x_j \leq 0.99 \tag{12}$$

In the above problem, T_j is the tax that would be imposed if the polluter j were to dispose all the waste his plant

generates and is given by (6).

The treatment cost and the percentage treated, obtained by solving (12) for each polluter, would be input to the CA's problem given by (5) which can be written as:

$$\min \sum_{j=1}^{15} \sum_{i=1}^{k_j} U_j^i (160.8 + 26.7Q_j + 640.7(x_j^i - .45)^2 + 255.7Q_j(x_j^i - .45)^2) + 726.6y$$

subject to the constraints

$$\sum_{j=1}^{15} \sum_{i=1}^{k_j} a_j x_j^i U_j^i + a_{16} y \geq b$$

where a_j , a_{16} and b are the column vectors of size 27×1 . a_j is the j^{th} column of Table 1, b is the right hand side column of Table 1 and a_{16} is the sixteenth column of Table 1.

and

$$\begin{aligned} \sum_{i=1}^{k_j} U_j^i &= 1 \quad \forall j, j = 1, \dots, 15 \\ y &\geq 1.0 \\ y &\leq 4.47 \\ U_j^i &\geq 0 \quad \forall i, j \end{aligned} \tag{13}$$

In the above problem x_j^i is the percentage of waste water treated by polluter j under plan i . U_j^i and y are the unknown variables which the CA tries to solve for. U_j^i 's are the weights attached to each of the k_j plans reported by the polluter j and are fractions (or percentages). The sum of these fractions can not exceed 1, i.e., 100%. This means that in the convex combination, which the CA determines, a certain percentage of

the waste U_j^1 might be treated according to plan 1, a certain percentage U_j^2 according to plan 2, and in a certain percentage of the waste U_j^k according to plan k_j such that the sum of these percentages or fractions is 100 or 1.

For example if, according to CA's convex combination, the polluter number 1 has to treat 80 percent of his gross waste, i.e., $x_1 = 80\%$ and the polluter has originally responded with say two plans, i.e., $k_j = 2$, then plan 1 might have a weight of 0.9 and plan 2 might have a weight of 0.1. In this case $U_1^1 = .9$ and $U_1^2 = .1$. Under plan 1, the polluter removes say 84% of the waste and removes 45% waste under plan 2.

So

$$U_1^1 + U_1^2 = 0.9 + 0.1 = 1.0$$

$$\text{and } 0.9(x_1^1 = 0.84) + 0.1(x_1^2 = 0.45) = 0.8 \text{ or } 80\% \text{ (same as found by the CA)}$$

For the no flow augmentation case, (13) reduces to:

$$\min \sum_{j=1}^k \sum_{i=1}^{k_j} (U_j^i (160.8 + 26.7Q_j + 640.7(x_j^i - .45)^2 + 255.7Q_j(x_j^i - .45)^2)$$

subject to the constraints

$$\sum_{j=1}^{15} \sum_{i=1}^{k_j} a_j x_j^i U_j^i \geq b - a_{16} \quad (14)$$

$$\sum_{i=1}^{k_j} U_j^i = 1 \quad \forall j, j = 1, \dots, 15$$

$$U_j^i \geq 0 \quad \forall i, j$$

3. SEPARABLE PROGRAMMING

A separable function is one that has the form

$$\phi(\bar{x}) = \sum_{i=1}^m h_i(x_i) \quad (15)$$

The separable programming can be defined as finding a set of x_i , $i = 1, 2, \dots, m$, which maximizes (or minimizes)

$$C(\bar{x}) = \sum_{i=1}^m f_i(x_i) = f_1(x_1) + \dots + f_m(x_m)$$

subject to the constraints

$$\sum_{i=1}^m g_{ki}(x_i) \leq b_k, \quad k = 1, \dots, P$$

and

$$x_i \geq 0, \quad i = 1, \dots, m$$

The objective function and the constraints are constructed of separable functions of the form given by equation (15).

These non-linear functions of one variable can be approximated by piecewise linear function over a specified interval domain and can be solved by slightly revised simplex method, using MPS/360. The approximation is done by partitioning this interval domain into n_i disjoint, but contiguous intervals. The $(n_i + 1)$ points of the partitions are represented by the set:

$$S = (x_i^0, x_i^1, \dots, x_i^{n_i})$$

'Delta Method' is used to represent the piecewise linear approximation of a continuous non-linear function of one variable and uses the differences of adjacent points of the set, S ,

and the differences of the functional values at the adjacent points in developing an approximation equation of a function, $f_i(x_i)$. The differences are given by:

$$\Delta x_i^j = x_i^j - x_i^{j-1}$$

$$\Delta f_i^j = f_i(x_i^j) - f_i(x_i^{j-1}), \quad i = 1, \dots, m; \quad j = 1, \dots, n_i$$

where the subscript refers to a function and variable and the superscript refers to partitioning of a variable. For example, $f_i(x_i^j)$ is the value of $f_i(x_i)$ at $x_i = x_i^j$.

To represent the variable x_i and the approximation of $f_i(x_i)$, a set of variables, D_i^j , $j = 1, 2, \dots, n_i$, is created, that follows what is known as the 'restricted basis-entry rule'. The rule is satisfied for any of the following conditions:

$$(i) \quad 0 \leq D_i^1 \leq 1 \quad \text{iff} \quad D_i^j = 0, \quad j = 2, \dots, n_i$$

$$(ii) \quad 0 \leq D_i^j \leq 1 \quad \text{iff} \quad D_i^1 = 1, \quad i = 1, 2, \dots, j-1 \text{ and} \\ D_i^k = 0, \quad k = j+1, \dots, n_i$$

$$(iii) \quad 0 \leq D_i^{n_i} \quad \text{iff} \quad D_i^j = 1, \quad j = 1, 2, \dots, n_i-1$$

There is no upper bound here, because many times we do not know the upper bound.

D_i^j represents a variable created for the i^{th} partition of variable x_i . For any $D_i^j = 1$, all previous D_i variables ($i = 1, \dots, j-1$) must be a value of one and all following values ($i = j+1, \dots, n_i$) must be zero.

The variable x_i can be represented as:

$$x_i = \Delta x_i^1 D_i^1 + \Delta x_i^2 D_i^2 + \dots + \Delta x_i^j D_i^j$$

and the function $f_i(x_i)$ as

$$f_i(x_i) = \Delta f_i^1 D_i^1 + \Delta f_i^2 D_i^2 + \dots + \Delta f_i^j D_i^j$$

In the above estimations, x_i is represented exactly but $f_i(x_i)$ is approximated by linear interpolation.

'Delta Method' is developed in G. Hadley's Non-linear and Dynamic Programming. Details of the separable programming are given by Williams (1972). Using the methods of data input described, the non-linear function can be linearized and the output is obtained in the form of punched cards which are input for MPS/360 linear programming package.

3.1 APPLICATION OF SEPARABLE PROGRAMMING TO CENTRALIZATION MODEL.

The centralization model has been discussed in detail in section 2.1 and the Miami River basin problem has been formulated in section 2.3. For flow augmentation case, the Miami River basin problem is given as:

$$\min \sum_{j=1}^{15} (160.8 + 26.7Q_j + 640.7(x_j - 45)^2 + 255.7Q_j(x_j - 45)^2) + 726.6y$$

subject to the constraints

$$\sum_{j=1}^{15} a_{ij}x_j + a_{i16}y \geq b_i, \quad i = 1, \dots, 27$$

$$y \geq 1.0$$

$$y \leq 4.47$$

$$x_j \geq 0.45$$

$$x_j \leq 0.99, \quad j = 1, \dots, 15$$

where the values of Q_j are given in Table 3 and the values of a_{ij} 's, a_{i16} and b_i in Table 1.

In the above problem, the objective function is comprised of fifteen non-linear terms and all of them are separable functions of the form given by equation (15). These can be linearized by using separable programming described in section 3. The interval domain is from 0.45 to 0.99, as fixed by the constraints. Starting point was given as $x_1 = 0.45$, the upper limit as $x_1 = 0.99$ and the number of partitions is 30.

Miami River basin problem without augmentation

If the flow is not to be augmented, the augmentation cost becomes zero and $y = 1.0$. In this case we remove the flow augmentation cost function, $726.6y$, from the objective function and substitute $y = 1.0$ in all the constraints. The inequality constraints on y become redundant. We can write the Miami River basin problem as:

$$\min \sum_{j=1}^{15} (160.8 + 26.7Q_j + 640.7(x_j - .45)^2 + 255.7Q_j(x_j - .45)^2)$$

subject to the constraints

$$\begin{aligned} \sum_{j=1}^{15} a_{ij}x_j &\geq b_i - a_{i16}, \quad i = 1, \dots, 27 \\ x_j &\geq 0.45 \\ x_j &\leq 0.99, \quad j = 1, \dots, 15 \end{aligned}$$

All the notations used above have been described earlier.

Results with flow augmentation

The Miami River basin problem was solved using MPS/360 linear programming package. The amount of waste that different polluters will treat is shown in Table 4. Note that the polluters 7 to 15 treat only 45% of their waste and are located beyond the twelfth reach, for which the constraint is binding. These polluters do not pay any revenue to the CA for discharging 55% of their waste into the river. We found that the flow rate, y , is 3.0085 times the normal flow rate and this augmented flow will cost the CA approximately \$2186 per day. The total cost of treatment would be about \$6024 per day. The shadow price for the twelfth reach constraints, which is binding, has been obtained as, $\lambda_{12} = 0.21606$. The other twenty-six reach constraints are not binding and so the shadow price for them is zero. This means that the CA will charge revenue from the individual polluters, using the shadow price on the twelfth reach constraint, for disposing $(1-x_j)$ percent of their waste into the river.

TABLE 4: SOLUTION OF THE MIAMI RIVER BASIN PROBLEM (CENTRALIZATION MODEL) WITH FLOW AUGMENTATION, USING SEPARABLE PROGRAMMING.

<u>POLLUTION SOURCE</u>	<u>TREATMENT PERCENTAGE</u>
1	64.8
2	57.6
3	93.6
4	72.0
5	48.6
6	50.4
7	45.0
8	45.0
9	45.0
10	45.0
11	45.0
12	45.0
13	45.0
14	45.0
15	45.0
Augmentation Rate	$y = 3.00885$
Total Cost Per Day	\$8210
Shadow price of the binding constraint	$\lambda_{12} = 0.21606$

Results without flow augmentation

For this case the total cost of treatment for achieving the required standards of 4 mg/l DO is \$8343 per day. The percentage of waste treated by most of the polluters is higher in this case than that with flow augmentation. The twelfth and twenty-sixth reach constraints are binding in this case and the shadow price for them is given as, $\lambda_{12} = 0.40544$ and $\lambda_{26} = 0.28561$. These shadow prices are used by the CA to charge revenue from each polluter according to (6). The various treatment percentages are tabulated in Table 5.

Computer outputs for flow augmentation and no augmentation cases are attached in Appendix 1.

For the physical meaning of shadow price, refer to section 3.2.

3.2 APPLICATION OF SEPARABLE PROGRAMMING TO DECENTRALIZATION MODEL

The decentralization model, which uses effluent charges (taxation) for the abatement of pollution, has been discussed in section 2.2 and the Miami River basin problem has been formulated in section 2.3. The polluters and the CA's optimization problems, respectively are:

For flow augmentation case

$$\min 160.8 + 26.7Q_j + 640.7(x_j - .45)^2 + 255.7Q_j(x_j - .45)^2 + (1 - x_j)T_j$$

TABLE 5: SOLUTION OF THE MIAMI RIVER BASIN PROBLEM (CENTRALIZATION MODEL) WITHOUT FLOW AUGMENTATION, USING SEPARABLE PROGRAMMING.

<u>POLLUTION SOURCE</u>	<u>TREATMENT PERCENTAGE</u>
1	86.8
2	70.2
3	99.0
4	99.0
5	55.8
6	57.6
7	48.6
8	54.0
9	48.6
10	64.8
11	54.0
12	53.54
13	48.6
14	45.0
15	54.0
Total Cost Per Day	\$8343

Shadow prices on the
binding constraints

$$\lambda_{12} = 0.40544$$

$$\lambda_{26} = 0.28561$$

subject to the constraint

$$0.45 \leq x_j \leq 0.99$$

where T_j is the tax imposed in the polluter j . Values of Q_j are given in Table 3.

and

$$\min \sum_{j=1}^{15} \sum_{i=1}^{k_j} U_j^i (160.8 + 26.7Q_j + 640.7(x_j^i - .45)^2 + 255.7Q_j(x_j^i - .45)^2 + 726.6y)$$

subject to the constraints

$$\sum_{j=1}^{15} \sum_{i=1}^{k_j} a_j x_j^i U_j^i + a_{16} y \geq b$$

$$\sum_{i=1}^{k_j} U_j^i = 1 \quad \forall j, j = 1, \dots, 15$$

$$y \geq 1.0$$

$$y \leq 4.47$$

$$U_j^i \geq 0 \quad \forall j, i$$

All the notations used have been described in section 2.3.

For no augmentation case

$$\min 160.8 + 26.7Q_j + 640.7(x_j - .45)^2 + 255.7Q_j(x_j - .45)^2 + (1 - x_j)T_j$$

subject to the constraint

$$0.45 \leq x_j \leq 0.99$$

and

$$\min \sum_{j=1}^{15} \sum_{i=1}^{k_j} (U_j^i (160.8 + 26.7Q_j + 640.7(x_j^i - .45)^2 + 255.7Q_j(x_j^i - .45)^2))$$

subject to the constraints

$$\sum_{j=1}^{15} \sum_{i=1}^{k_j} a_j x_j^i U_j^i \geq b - a_{16}$$

$$\sum_{i=1}^{k_j} U_j^i = 1 \quad \forall j, j = 1, \dots, 15$$

$$U_j^i \geq 0 \quad \forall i, j$$

In the above problem, the non-linear term of the polluter's optimization problem can be linearized using separable programming, described in section 3. The interval domain is from 0.45 to 0.99 as given by the constraint. Starting point was given as $x_j = 0.45$, the upper limit as $x_j = 0.99$, and the number of partitions as 50.

The physical meaning of shadow price

When solving a linear programming problem, we end up with many constraints that are binding. For these binding constraints, the dual evaluators are non-zero. It was noted by Deininger (1965) that the dual evaluators associated with the optimal solution were of use in determining standards since they reflect the marginal cost of changing the required level of dissolved oxygen at various points along the river. The dual evaluator associated with the t^{th} constraint gives an approximate measure of the marginal cost associated with supplying the t^{th} reach with an additional point of oxygen per day. This is the shadow price associated with t^{th} reach constraint. It was suggested by Upton (1968) that this shadow or dual price

can be used in building an optimal tax structure. The relationship between the shadow price and the tax rate is given by equation (6). Thus the CA can use the shadow price of the binding constraint to charge revenue from the polluters (in case of centralization model) or impose new tax rates (in case of decentralization model) to elicit the desired information.

Procedure

As discussed in section 2.2, the iterative process to solve the decentralization model problem consists of make-believe rounds between the central authority and polluters in which discharge taxes are imposed by the central authority and polluters respond with treatment levels and costs. As the iterative scheme makes progress we learn what we need to know about the local treatment costs, and an optimal tax structure evolves. It is optimal in the sense that if such a set of taxes is imposed on the individual polluters, they would build and operate the optimal size treatment facilities.

With flow augmentation

Step 1

To apply the procedure to the Miami River basin, we need to obtain the starting sequences, that is, at least two plans of at-source treatment and the cost of such treatment must be obtained from each of the 15 polluters for two arbitrary tax

rates. To obtain the starting sequences, we have to solve the polluter's optimization problem twice for each of the 15 polluters along the river. The first solution was obtained by setting $\lambda_i = 0$, $i = 1, \dots, 27$. In this case the tax imposed on the polluter j is:

$$T_j = (0 \ 0 \ \dots \ 0)(a_j), \quad j = 1, \dots, 15$$

where a_j is the j^{th} column of the constraint matrix given in Table 1 and gives the total oxygen requirement.

$$a_j = \sum_{i=1}^n a_{ij} w_j$$

then $T_j = 0$, $j = 1, \dots, 15$

So the polluter's optimization problem is reduced to:

$$\min 160.8 + 26.7Q_j + 640.7(x_j - .45)^2 + 255.7Q_j(x_j - .45)^2$$

subject to the constraint

$$0.45 \leq x_j \leq 0.99$$

It is clear that the above objective function would be minimum when x_j is minimum, i.e., x_j is at its lower limit.

So by setting $\lambda_i = 0$, $i = 1, \dots, 27$, we get the costs associated with 45% treatment.

The second solution was obtained by arbitrarily setting $\lambda_{27} = 10$ and all other $\lambda_i = 0$, $i = 1, \dots, 26$. In this case the tax imposed on polluter j is:

$$T_j = (0 \ 0 \ \dots \ 0 \ 10)(a_j), \quad j = 1, \dots, 15$$

So for the j^{th} polluter the tax is equal to the product of 10 and the element in the 27^{th} row and j^{th} column of the constraint matrix shown in Table 1. Clearly, T_j is a very big number in this case. When we solved the individual polluter's optimization problem, given by equation (12), we found that because of the large value of T_j all the polluters are forced to treat 99% of their gross waste. Table 6 lists each polluter's response to the various shadow prices.

Step 2

With these two entries, the CA can simulate the basin problem and can have an approximate measure of the marginal cost of supplying the t^{th} reach with an additional pound of oxygen by trying to find the convex combination of the two plans for all the 15 polluters in the basin. The CA now tries to find the weights U_j^i (percentage or fraction), associated with each plan of all the polluters.

Although the objective function described in equation (5), which when applied to the Miami River basin problem takes form of equation (13), is non-linear in the sense that C_j^i is non-linear, this value, the treatment cost is a constant reported by the polluter j for his plan i . Other constant is the value of $\sum_{i=1}^k a_j x_j^i$, reported by all the polluters for each plan. Thus the CA tries to find U_j^i 's and y , which are the only unknowns.

For the CA's iteration 1, i varies from 1 to 2, since there

TABLE 6: POLLUTER'S ITERATIVE SOLUTIONS AND COSTS

POLLUTION SOURCE	ITERATION NUMBERS*		
	1	2	3
1	45.0	99.0	65.52
	\$1368	\$4924.65	\$1881.28
2	45.0	99.0	56.88
	\$286	\$823.55	\$312.30
3	45.0	99.0	92.52
	\$273	\$772.92	\$660.13
4	45.0	99.0	72.0
	\$257	\$712.17	\$370.73
5	45.0	99.0	49.32
	\$174	\$355.67	\$175.58
6	45.0	99.0	50.4
	\$193	\$469.12	\$195.60
7	45.0	99.0	45.0
	\$182	\$428.63	\$182.
8	45.0	99.0	45.0
	\$177	\$408.38	\$177.
9	45.0	99.0	45.0
	\$174	\$398.25	\$174.
10	45.0	99.0	45.0
	\$246	\$671.66	\$246
11	45.0	99.0	45.0
	\$385	\$1198.2	\$385.
12	45.0	99.0	45.0
	\$233	\$621.63	\$233.

TABLE 6: POLLUTER'S ITERATIVE SOLUTIONS AND COSTS (cont.)

POLLUTION SOURCE	ITERATION NUMBERS*		
	1	2	3
13	45.0 \$177	99.0 \$408.38	45.0 \$177.
14	45.0 \$484	99.0 \$1572.89	45.0 \$484.
15	45.0 \$385	99.0 \$1198.2	45.0 \$385.

*The upper number is the percent of waste removed through treatment. The lower number is the daily cost associated with the treatment level.

are only two plans to begin with. When we solved the CA's problem for the first time, the twelfth constraint was obtained as binding with a shadow price, $\lambda_{12} = 0.21606$. According to the different weights obtained (U_j^i 's), we calculated the percentage of waste that should be treated by each polluter, to obtain an overall minimum cost for the CA's optimization problem. These treatment percentages are listed under iteration 1, in Table 7.

Step 3

The new shadow price obtained in the CA's iteration 1, is now used to impose a new tax rate on the polluters. According to equation (6), the new tax rate T_j is given as:

$$T_j = (0 \ 0 \ \dots 0.21606 \ 0 \ \dots 0)(a_j), \ j = 1, \dots, 15$$

From above, it is clear that this tax rate will affect only those polluters which are located in and before reach twelve. Polluters beyond the twelfth reach will have no tax and consequently will treat only 45% of their gross waste produced.

When this new tax rate is imposed on the polluters, they respond with a new treatment plan, i.e., the treatment percentage and its cost. The treatment percentages of all the 15 polluters are listed under iteration 3, in Table 6.

TABLE 7: CENTRAL AUTHORITY'S ITERATIVE SOLUTION

POLLUTION SOURCE	ITERATION NUMBER	
	1	2
1	45.0	65.52
2	45.0	56.88
3	99.0	92.52
4	99.0	72.0
5	45.0	49.32
6	45.0	50.4
7	45.0	45.0
8	45.0	45.0
9	45.0	45.0
10	45.0	45.0
11	45.0	45.0
12	45.0	45.0
13	45.0	45.0
14	45.0	45.0
15	45.0	45.0
AUGMENTATION RATE	$y = 3.986$	$y = 2.988$
TOTAL COST PER DAY	\$8845	\$8210
SHADOW PRICE OF BINDING		
CONSTRAINTS*	$\lambda_{12} = 0.21606$	$\lambda_{12} = 0.21606$

*Subscript on shadow prices refer to reach constraint to which the shadow price applies.

Step 4

It is the repetition of Step 2. The only difference is that the CA has now three plans at its disposal. So k_j is equal to 3 and i varies from 1 to 3. When the CA's problem was solved for the second time, the shadow price on the twelfth reach constraint did not change. Since the shadow price remained constant, the taxes did not change and so the policies obtained by the CA in his iteration 2 are optimal and are identical to polluters plan 3. The percentage treatment obtained, for all the 15 polluters, by using the weights U_j^i found by the CA in his iteration 2 are listed under iteration 2 in Table 7.

Results with flow augmentation

The flow rate y has been found to be equal to 2.988 times the normal flow and this augmented flow will cost the CA approximately \$2171 per day. The CA has the authority to charge taxes, leading to a revenue of \$2321, according to equation (6) with twelfth shadow price equaling 0.21606. The total cost of waste treatment, for all the 15 polluters, is \$6039 per day. The total cost of maintaining the DO level of at least 4 mg/l in the 58-mile section of the river is \$8210 per day. Detailed discussion and comparison of the results is given in section 5.

Without flow augmentation

The iterative procedure for the Miami River basin problem

without flow augmentation is exactly identical to the one described above. We initialized the procedure with the same two taxing schemes, as discussed in step 1. Once the procedure was initialized, steps 2, 3 and 4 were carried out. The only difference in this case was that steps 3 and 4 were carried out many times before the near optimal solution was obtained. Table 8 lists the response of all the 15 polluters to the tax rates generated from the various shadow prices. Table 9 lists the treatment percentages for the 15 polluters, obtained from the weights determined by the CA in each iteration.

Results without flow augmentation

The total cost of treating the required standards is approximately \$8317 per day. The cost is approximate because only a near optimal solution was reached even when the CA's problem was solved five times and seven individual treatment plans were generated.

It is to be noted here that the treatment augmentation policy is less costly than the treatment policy alone (no augmentation).

Polluter's response to various shadow prices (results of steps 1 and 3) and the CA's convex combination plans (steps 2 and 4 results) are tabulated in Table 8 and 9 respectively. Details of the results are discussed in section 5.

A sample output of first polluter's optimization problem

TABLE 8: POLLUTER'S ITERATIVE SOLUTIONS AND COSTS

POLLUTION SOURCE	ITERATION NUMBER*						
	1	2	3	4	5	6	7
1	45.0 \$1368	99.0 \$4925	72.0 \$2557	84.96 \$3315	92.52 \$4122	88.2 \$3644	86.04 \$3422
2	45.0 \$286	99.0 \$824	67.78 \$341	69.84 \$400	74.16 \$443	70.92 \$410	70.92 \$410
3	45.0 \$273	99.0 \$773	99.0 \$773	99.0 \$773	99.0 \$773	99.0 \$773	99.0 \$773
4	45.0 \$257	99.0 \$712	86.04 \$520	99.0 \$712	99.0 \$712	99.0 \$712	99.0 \$712
5	45.0 \$174	99.0 \$356	51.48 \$177	54.72 \$181	55.8 \$183	54.72 \$181	54.72 \$181
6	45.0 \$193	99.0 \$469	54.72 \$202	56.88 \$216	60.12 \$215	57.96 \$209	57.96 \$209
7	45.0 \$182	99.0 \$429	49.32 \$184	47.16 \$182.5	48.24 \$183	47.16 \$182.5	48.24 \$183
8	45.0 \$177	99.0 \$408	56.88 \$188	50.4 \$179	53.64 \$183	51.48 \$180	52.56 \$181
9	45.0 \$174	99.0 \$398	50.4 \$176	48.24 \$175	49.32 \$176	48.24 \$175	48.24 \$175
10	45.0 \$246	99.0 \$672	72.0 \$352	59.04 \$275	65.52 \$308	60.12 \$280	61.2 \$285
11	45.0 \$385	99.0 \$1198	56.88 \$424	51.48 \$397	53.64 \$406	52.56 \$401	52.56 \$401
12	45.0 \$233	99.0 \$621	55.8 \$248	50.4 \$237	52.56 \$240	51.48 \$238	51.48 \$238

TABLE 8: POLLUTER'S ITERATIVE SOLUTIONS AND COSTS (cont.)

POLLUTION SOURCE	ITERATION NUMBER*						
	1	2	3	4	5	6	7
13	45.0 \$177	99.0 \$408	50.4 \$179	48.24 \$178	49.32 \$178	48.24 \$177	48.24 \$177
14	45.0 \$484	99.0 \$1573	59.04 \$557	51.48 \$500	55.8 \$527	55.8 \$527	53.64 \$512
15	45.0 \$385	99.0 \$1198	56.88 \$424	50.4 \$393	53.64 \$406	54.72 \$411	51.48 \$397
SHADOW PRICES OF BINDING CON- STRAINTS	NONE	$\lambda_{27}=10$	$\lambda_{12}=22329$	$\lambda_{12}=40751$	$\lambda_{12}=45712$	$\lambda_{12}=43701$	$\lambda_{12}=40766$
			$\lambda_{26}=37734$	$\lambda_{26}=18906$	$\lambda_{26}=27969$	$\lambda_{26}=23057$	$\lambda_{26}=23057$
					$\lambda_{23}=0.0009$		

*The upper number is the percent of waste removed through treatment.
The lower number is the daily cost associated with the treatment level.

TABLE 9: CENTRAL AUTHORITIES' ITERATIVE SOLUTIONS

POLLUTION SOURCE	ITERATION NUMBER				
	1	2	3	4	5
1	89.52	87.64	86.86	86.79	86.73
2	45.0	62.28	69.84	69.84	70.92
3	99.0	99.0	99.0	99.0	99.0
4	99.0	99.0	99.0	99.0	99.0
5	45.0	54.48	54.72	54.72	54.72
6	45.0	54.72	56.88	60.12	57.96
7	45.0	49.32	49.32	48.24	48.24
8	45.0	56.88	56.88	53.64	51.48
9	45.0	45.0	48.24	48.24	48.24
10	87.79	72.0	59.04	59.04	60.12
11	45.0	56.88	51.48	52.18	52.56
12	45.0	46.41	50.4	52.56	51.48
13	45.0	50.4	50.4	48.24	48.24
14	45.0	45.0	53.5	51.48	51.48
15	45.0	45.0	50.4	53.12	52.61
TOTAL COST PER DAY	\$9218	\$8614	\$8349	\$8329	\$8317
SHADOW PRICES OF BINDING CON- STRAINTS	$\lambda_{12}=.22329, \lambda_{12}=.40351, \lambda_{12}=.45712, \lambda_{12}=.43071, \lambda_{12}=.40766$ $\lambda_{26}=.37734, \lambda_{26}=.18906, \lambda_{26}=.27969, \lambda_{26}=.23057, \lambda_{26}=.23057$ $\lambda_{23}=.0009$				

for shadow price $\lambda_{27} = 10$ and CA's iteration one are attached in Appendix 2.

General Procedure for problems based on decentralization model

After solving the Miami River basin problem, we can now write a general procedure when the central authority does not know the treatment cost functions. In this case the CA has complete information about the quality standards desired and the low flow augmentation cost function while the polluters know the individual treatment cost functions.

Step 1

Initialize the procedure by obtaining at least two starting sequences, for all the polluters in the river basin, by giving arbitrary values to the shadow price vector λ_i , $i = 1, \dots, n$, and solving the polluter's optimization problem.

Step 2

Knowing all the starting sequences with at least two entries each, find the convex combination of the available plans by solving the CA's optimization problem. In this case $k_j \geq 2$ and i varies from 1 to 2 or more, depending upon the number of entries in the starting sequence.

Step 3

Use the shadow price, obtained in step 2, to generate

new tax rates according to equation (6), which when imposed on the individual polluter results in the polluter responding with a new treatment plan.

Step 4

Incorporate the new plan, developed in step 3, in the CA's optimization problem to again find the convex combination of all the available plans. In this case $k_j \geq 3$ and i varies from 1 to 3 or more.

Step 5

Repeat steps 3 and 4 till the convergence occurs, i.e., the shadow price of the binding constraints stops changing.

4. QUADRATIC PROGRAMMING

A quadratic programming problem is a non-linear programming problem having linear constraints and an objective function which is a quadratic form. The general quadratic problem can be written as:

min (or max)

$$f(x) = \sum_{i=1}^n c_i x_i + \frac{1}{2} \sum_{i=1}^n \sum_{k=1}^n q_{ik} x_i x_k$$

subject to the linear constraints

$$g_j(x) = \sum_{i=1}^n a_{ji} x_i \geq b_j, \quad j = 1, 2, \dots, m \quad (18)$$

$$x_i \geq 0, \quad i = 1, \dots, n$$

where $q_{ik} = q_{ki}$ are the given constants and elements of Q matrix.

In the vector form, (18) becomes

$$\min Z = c^T x + x^T Q x$$

subject to

$$Ax \geq b \quad (19)$$

$$x \geq 0$$

Various computational algorithms have been devised to solve (19). Here we present Lemke's complementary pivot algorithm (1965), which solves the problem of the form:

$$\text{Find } w, Z \geq 0$$

such that

$$w = MZ + q \quad (20)$$

$$w^T Z = 0$$

An optimal solution to the problem given by (19) may be obtained by solving a complementary problem of the form:

$$\begin{bmatrix} v \\ u \end{bmatrix} = \begin{bmatrix} Q+Q^T & -A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ -b \end{bmatrix} \quad (21)$$

$$u, v, x, y \geq 0$$

$$v^T x + u^T y = 0$$

where u denotes the slack variables of the given quadratic problem and (y, v) denotes the variables of the dual problem.

Comparing (20) with (21), we note that

$$w = \begin{bmatrix} v \\ u \end{bmatrix}, \quad z = \begin{bmatrix} x \\ y \end{bmatrix}, \quad M = \begin{bmatrix} Q+Q^T & -A^T \\ A & 0 \end{bmatrix} \text{ and } q = \begin{bmatrix} c \\ -b \end{bmatrix}$$

Complementary Pivot Algorithm

Consider problem (19).

$$A = [a_{ij}] \quad , \quad i = 1, \dots, m ; \quad j = 1, \dots, n$$

Let

$u_1, \dots, u_m$ denote primal slacks

$y_1, \dots, y_m$ denote dual variables

$v_1, \dots, v_m$ denote dual slacks

the vectors u and y (≥ 0) are complementary pair of vectors, i.e., in the solution $u_i y_i = 0 \quad \forall i=1, \dots, m$ (u_i and y_i are called complementary variables). Also $x_j v_j = 0 \quad \forall j=1, \dots, n$.

Complementary Rule. When a variable leaves the basis, the complement of that variable is brought into the basis. Define:

$$a'_{ij} = a_{ij} \quad \forall i,j ; \quad b'_i = -b_i \quad \forall i$$

The initial basis contains $u_1, \dots, u_m, v_1, \dots, v_n$ (primal and dual slacks) as basic variables.

Basic Variables		u_1	$\dots$	u_m	x_1	$\dots$	x_n	b'	d
*	u_1	1			a'_{11}		a'_{1n}	b'_1	d_1
$\vdots$	$\vdots$				$\vdots$		$\vdots$	$\vdots$	$\vdots$
*	u_m			1	a'_{m1}		a'_{mn}	b'_m	d_m
		C_{n+1}	$\dots$	C_{n+m}	C_1	$\dots$	C_n	C	
		f_{n+1}	$\dots$	f_{n+m}	f_1	$\dots$	f_n	f	
		y_1	$\dots$	y_m	v_{*1}		v_{*n}	Basic Variables	

'*' denotes basic variables.

In the tableau (initial)

$$d_i = 1 \quad \text{if } b'_i < 0 \quad (\Rightarrow b_i > 0) \\ = 0 \quad \text{otherwise}$$

$$C_{n+1} = \dots = C_{n+m} = 0 \quad (\text{slack coefficients})$$

$$f_j = 1 \quad \text{if } C_j < 0, \quad j = 1, \dots, n, n+1, \dots, n+m \\ = 0 \quad \text{otherwise}$$

Step 1

$$\text{Call } M_1 = \max_{d_i > 0} \frac{-b'_i}{d_i} = \max_{b'_i < 0} (-b'_i), \quad i = 1, \dots, m$$

$$M_2 = \max_{f_j > 0} \frac{-C_j}{f_j} = \max_{C_j < 0} (-C_j), \quad j = 1, \dots, n+m$$

If $b_i \geq 0 \forall i$ and $C_j \geq 0 \forall j$, then $x = y = 0$ is optimal.

Let

$$Z_0(1) = \max(M_1, M_2)$$

the max value of $Z_0(1)$ may occur

(i) corresponding to some $j = 1, \dots, n+m$

(ii) corresponding to some $i = 1, \dots, m$

Case (i). $Z_0(1)$ occurs corresponding to some j , say $j = 1$

(i.e., $Z_0(1) = -C_1/f_1$). This implies that v_1 leaves the basis.

Since a dual variable leaves the basis, we apply the following dual complementary pivot.

By the complementary rule, complementary variable x_1 enters the basis. The basic variable being replaced by x_1 is found by the minimum ratio rule. For this find ratios

$$\frac{b_i + Z_0(1)d_i}{a_{i1}}, \quad i = 1, \dots, n \text{ and } a_{i1} > 0 \quad (22)$$

Let the minimum of (22) occurs when $i=m$ (if no such minimum exists, solution is unbounded). From the min-ratio rule, we find x_1 replaces u_m , and y_m replaces v_1 in the dual basis.

Pivoting on a_{m1} yields the tableau:

	u_1	$\dots$	u_{m-1}	u_m	x_1	x_2	$\dots$	x_n	$\dots$
* u_1	1			$-\frac{a_{11}}{a_{m1}}$	0	$\bar{a}_{12}$	$\dots$	$\bar{a}_{1n}$	$\bar{b}_1$
$\vdots$				$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\bar{d}_1$

$$\begin{array}{ccccccc}
 \vdots & & \vdots & \vdots & \vdots & \vdots & \\
 *u_{m-1} & 1 & -\frac{a'_{m-1,1}}{a'_{m1}} & 0 & \bar{a}_{m-1,2} \dots \bar{a}_{m-1,n} & \bar{b}_{m-1} & \bar{d}_{m-1} \\
 *u_m & 0 & 1/a'_{m1} & 1 & \bar{a}_{m,2} \dots \bar{a}_{m,n} & \bar{b}_m & \bar{d}_m \\
 \hline
 & \bar{c}_{n+1} & \dots & \bar{c}_{n+m-1} & \bar{c}_{n+m} & 0 & \bar{c}_2 \dots \bar{c}_n \\
 \hline
 & \bar{f}_{n+1} & \dots & \bar{f}_{n+m-1} & \bar{f}_{n+m} & 0 & \bar{f}_2 \dots \bar{f}_n \\
 \hline
 & y_1 & & y_m & v_1 & v_2 & \dots v_n \\
 & & & * & * & * & *
 \end{array}$$

where

$$\begin{aligned}
 \bar{a}_{mj} &= \frac{a'_{mj}}{a'_{m1}} \quad \forall j = 2, \dots, n; \quad \bar{a}_{ij} = a'_{ij} - a'_{i1} \frac{a'_{mj}}{a'_{m1}} \\
 &\quad \forall j=2, \dots, n \\
 \bar{b}_m &= \frac{b'_m}{a'_{m1}}; \quad \bar{d}_m = \frac{d'_m}{a'_{m1}}; \quad \bar{b}_i = b'_i - a'_{i1} \frac{b'_m}{a'_{m1}} \\
 &\quad \forall i=1, \dots, m-1 \\
 \bar{d}_i &= d'_i - a'_{i1} \frac{d'_m}{a'_{m1}} \quad \forall i=1, \dots, m-1; \quad \bar{c}_j = c'_j - c'_1 \frac{a'_{mj}}{a'_{m1}}, \\
 &\quad \forall j=2, \dots, m+n \\
 f_j &= f'_j - f'_1 \frac{a'_{mj}}{a'_{m1}}, \quad j=2, \dots, n+m
 \end{aligned}$$

Case (ii). Let value of $Z_0(1)$ occurs corresponding to say $i = 1$, i.e., u_1 leaves the basis. Since a primal variable leaves the basis, we apply a primal complementary pivot.

By the complementary rule, since U_1 leaves the basis we bring y_1 into the basis which replaces one of the v_j 's, $j=1, \dots, n$. To find the basis variable being replaced by y_1 , apply the max-ratio rule. That is

$$\max \frac{C_j + Z_0(1)f_j}{a'_{ij}} \quad \forall j=1, \dots, n \text{ and } a'_{ij} < 0 \quad (23)$$

Let the max of (23) occurs corresponding to v_n (if no max exists, solution is unbounded). So y_1 replaces v_n , and X_n (the complementary variable of v_n) replaces u_1 . Using a'_{1n} as the pivot element, perform the pivot operation as in Case (i).

Step 2

With new tableau, form the ratios as in step 1 to find

$$\bar{M}_1 = \max_{\substack{\bar{b}_i \\ \bar{a}_i > 0}} \frac{\bar{b}_i}{\bar{a}_i}, \quad i = 1, \dots, m$$

$$\bar{M}_2 = \max_{\substack{\bar{C}_j \\ \bar{f}_j > 0}} \frac{\bar{C}_j}{\bar{f}_j}, \quad j = 1, \dots, n+m$$

Define

$$Z_0(2) = \max(\bar{M}_1, \bar{M}_2)$$

Here $Z_0(2) \leq Z_0(1)$. If $Z_0(2) > 0$, then according to the basic variable which leaves the basis either a primal or dual complementary pivot is done as in step 1. This yields $Z_0(3)$ and so on.

An optimum solution to (19) may be obtained by reading off the values of $(Z(1), \dots, Z(n), W_{n+1}, \dots, W_{n+m})$ from the complementary solution, as per equation (20) and (21).

Stopping Criteria

There are two possible ways of terminating:

- (i) At step n , $Z_0(n) \leq 0$. This implies that we have reached the optimal solution.
- (ii) At step n , $Z_0(n) > 0$, but we are not able to perform the complementary pivot (primal or dual) to find $Z_0(n+1)$ since the ratio rule fails. This indicates that there is no solution to the problem.

4.1 SOLUTION OF THE BASIN WIDE FIRM (CENTRALIZATION MODEL) USING QUADRATIC PROGRAMMING

The basin wide firm problem for the Miami River basin has been described in section 2.1 and, for the flow augmentation case, is given as:

$$\min \sum_{j=1}^{15} (160.8 + 26.7Q_j + 640.7(x_j - .45)^2 + 255.7Q_j(x_j - .45)^2) + 726.6y$$

subject to the constraints

$$\sum_{j=1}^{15} a_{ij}x_j + a_{i16}y \geq b_i, \quad i = 1, \dots, 27$$

$$y \geq 1.0$$

$$y \leq 4.47$$

$$x_j \geq 0.45$$

$$x_j \leq 0.99, \quad j = 1, \dots, 15$$

For no augmentation case, the problem becomes:

$$\min \sum_{j=1}^{15} (160.8 + 26.7Q_j + 640.7(x_j - .45)^2 + 255.7Q_j(x_j - .45)^2)$$

subject to the constraints

$$\sum_{j=1}^{15} a_{ij}x_j \geq b_i - a_{i16} \quad , \quad i = 1, \dots, 27$$

$$x_j \geq 0.45$$

$$x_j \leq 0.99 \quad , \quad j = 1, \dots, 15$$

The above problem is a quadratic programming problem of the form given by (18). A computer program, developed by Ravindran (1972), was used to solve the Miami River basin problem with and without flow augmentation. Computer output for the problem is attached in Appendix 3.

Results with flow augmentation

A flow rate, y , equal to 3.0098 times the normal flow was obtained in the optimal solution. This augmented flow will cost the CA approximately \$2186 per day. Various treatment percentages, for the 15 pollution sources are listed in Table 10, to meet the required standard of 4 mg/l of DO. The total at-source treatment cost is about \$6027 per day. In all, it would cost the central authority a daily cost of \$8213 to meet the required standards. The CA can charge revenue, on $(1-x_j)$ percent of the untreated waste disposed into the river, from the individual polluters according to the shadow price of the twelfth reach constraint ($\lambda_{12}=0.21605$), which is binding. Since no other reach constraint is binding, shadow price for other twenty-six reach constraints is zero. This means that

TABLE 10: SOLUTION OF THE BASIN WIDE FIRM, USING QUADRATIC

POLLUTION SOURCE	WITH AUGMENTATION	WITHOUT AUGMENTATION
1	64.9	86.7
2	57.0	70.4
3	92.9	99.0
4	72.2	99.0
5	49.4	54.7
6	50.5	57.8
7	45.2	47.7
8	45.2	51.6
9	45.3	48.1
10	45.7	60.3
11	46.6	51.9
12	45.2	51.0
13	45.3	48.3
14	46.8	52.6
15	47.1	51.3
SHADOW PRICE OF THE BINDING CONSTRAINT	$\lambda_{12}=0.21605$	$\lambda_{12}=0.4114$
TOTAL COST PER DAY	\$8213	\$8310
AUGMENTATION RATE	$y=3.0098$	

the CA will charge revenue from only those polluters which are located in and before twelfth reach. Details of the results are discussed in section 5.

Results without flow augmentation

For no augmentation case, the total treatment cost to meet the required standards was obtained as approximately \$8310 per day. The amount of waste to be treated at different pollution sources is listed in Table 10. The CA has the authority to charge revenue according to the shadow price on the twelfth and twenty-sixth reach constraints ($\lambda_{12}=0.4114, \lambda_{26}=0.20785$), which are binding. No other reach constraints are binding and consequently have zero shadow price. Since all the pollution sources are located in and before the twenty-sixth reach, revenue will be charged from all of them on the amount of untreated waste disposed into the river. Detailed discussion of the results is given in section 5. Computer output for both cases are attached in Appendix 3.

4.2 A FORTRAN PROGRAM FOR SOLVING THE QUADRATIC PROGRAMMING PROBLEMS

This program is based on the computer code developed by Ravindran (1972) and uses the complementary pivot method of Lemke (1965), discussed in section 4. Any convex quadratic programming problem expressed in the following form:

$$\begin{aligned} \min \quad & Z = c^T x + x^T Q x \\ \text{subject to} \quad & \\ & Ax \geq b \\ & x \geq 0 \end{aligned}$$

can be solved by this program. In the above problem A is an $(m \times n)$ constraint matrix, b is an $(m \times 1)$ requirement vector, c is an $(1 \times n)$ cost vector (comprising of linear terms in the objective function), x is an $(n \times 1)$ vector of decision variables, and D is the matrix of quadratic form which should be positive definite or positive semi-definite for the algorithm to guarantee an optimal solution.

Equality constraints in the problem can be easily changed to inequality form as described below:

$$\sum_j a_{ij} x_j = b_i, \quad i = 1, \dots, k$$

This can be converted to equivalent set of $2k$ inequalities as below:

$$\begin{aligned} \sum_j a_{ij} x_j &\geq b_i, \quad i = 1, \dots, k \\ - \sum_j a_{ij} x_j &\geq -b_i, \quad i = 1, \dots, k \end{aligned}$$

This program can solve problems of size $(m+n) \leq 75$, i.e., the sum of variables and constraints should be less than or equal to 75. For larger problems, a change in the DIMENSION statement is required. A listing of the program is attached in Appendix 3.

4.3 METHOD OF DATA INPUT

The program will solve any number of problems in succession.

1. The first card must contain the title for the problem in (20A4) format.
2. The second card contains the following information in (I2) format:

NVAR = number of decision variables.

NCON = number of constraints.

IPACK = 0 or blank, if matrices D and A are to be input in their entirety.

= 1 if only the nonzero elements of D and A are to be input in packed form.

NCHCK = -1 if D is already known to be positive definite or semi-definite.

= 0 if D is not known to be positive definite or semi-definite.

LP = 0 or blank, if the problem is a quadratic programming problem.

= 1, if it is a linear programming problem.

NPRNT = 1, if printout of input is desired.

= 0, if no such print is desired.

3. If IPACK = 0

(i) The next set of data cards contains the quadratic form matrix D, columnwise in (7F10.4) format.

(ii) The next set of cards contains the constraint matrix A,

columnwise in (7F10.4) format.

4. If IPACK = 1

use of packed form input: For each column, in (7(I2,F8.4)) format, enter the row in which the nonzero element occurs, followed by the element. Note that each column must end with a blank or zero, e.g., if there are seven nonzero elements in a column, a blank should follow the card containing the elements. Each column (including zero elements) must have at least one card.

- (i) The next set of data cards contains the nonzero elements of the quadratic form matrix D, columnwise in packed form (remove this card if LP = 1).
- (ii) The next set of data cards contains the nonzero elements of the constraint matrix A, columnwise in packed form.

5. The next set of data cards contains the cost vector C in (7F10.4) format.

6. The next set of data cards should contain the right hand side vector b in (7F10.4) format.

7. If there is another problem, repeat steps 1 to 6; otherwise, terminate with a blank card as the last card in the data deck

4.4 OUTPUT OF THE PROBLEM

The output of the problem is obtained in the following format:

1. The output of each problem starts on a new page.

2. Title of the problem is printed first.
3. If NPRNT = 1, the input is printed first in the following order:
 - (a) The cost vector C.
 - (b) The right hand side vector b.
 - (c) The quadratic form matrix D.
4. The optimal solution and the number of iterations.
 (Note: if the problem does not have an optimal solution, the message 'PROBLEM HAS NO SOLUTION' and the 'ITERATION NUMBER' will be printed).
5. The Lagrange multipliers which may be interpreted as shadow prices for the constraints.
6. The value of the objective function.

4.5 NUMERICAL EXAMPLE

Consider the quadratic programming problem:

$$\min -6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$$

subject to

$$-x_1 - x_2 \geq -2$$

$$x_1, x_2 \geq 0$$

comparing the above problem with the notations described before,

$$C = (-6 \ 0); \quad D = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}; \quad A = (-1 \ -1); \quad b = (-2)$$

Here D is positive definite matrix and hence Lemke's algorithm is guaranteed to find an optimal solution if one exists.

The input cards would be punched as follows:

<u>CARD NO.</u>	<u>PUNCHED DATA (STARTING COLUMN 1)</u>
1	AN EXAMPLE PROBLEM
2	020100-10001
3	2.0 -1.0
4	-1.0 2.0
5	-1.0
6	-1.0
7	-6.0 0.0
8	-2.0
9	(BLANK CARD)

The output of the above problem is attached in Appendix 3.

The optimal solution is,

$$x_1 = 1.5$$

$$x_2 = 0.5$$

The value of the objective function at the optimal point is,

$$f = -5.5$$

5. DISCUSSION OF THE RESULTS AND CONCLUSION

Comparison of centralization model results

The basin wide firm problem has been solved both by separable programming and quadratic programming. Giving the allowance for linear approximation in separable programming, results of the two methods are almost same. The amount of waste to be treated at different pollution sources, the total daily cost of achieving the standards, flow rate in case of flow augmentation, and the shadow price on the binding constraints are shown in Table 11. The total cost of achieving the standards, with flow augmentation, by separable programming is \$8210/day, and by quadratic programming is \$8213/day. For no augmentation case, the daily cost is \$8310 for quadratic programming and \$8343 for separable programming.

Comparison of decentralization model results with centralization model results (Quadratic programming)

with flow augmentation

The total cost of achieving the standards for the decentralization model is \$8210/day and for the centralization model is \$8213. The flow rate, y , is 2.998 times the normal flow for the decentralization model and 3.0098 times the normal flow for the centralization model. The polluter's response to various shadow prices and the convex combination generated by the CA,

TABLE 11: CENTRALIZATION MODEL RESULTS FOR THE MIAMI RIVER
BASIN PROBLEM

POLLUTION SOURCE	WITH FLOW AUGMENTATION Quadratic Programming	Seperable Programming	WITHOUT Quadratic Programming	AUGMENTATION Seperable Programming
1	64.9	64.8	86.7	86.8
2	57.0	57.6	70.4	70.2
3	92.9	93.6	99.0	99.0
4	72.2	72.0	99.0	99.0
5	49.4	48.6	54.7	55.8
6	50.5	50.4	57.8	57.6
7	45.2	45.0	47.7	48.6
8	45.2	45.0	51.6	54.0
9	45.3	45.0	48.1	48.6
10	45.7	45.0	60.3	64.8
11	46.6	45.0	51.9	54.0
12	45.2	45.0	51.0	53.54
13	45.3	45.0	48.3	48.6
14	46.8	45.0	52.6	45.0
15	47.1	45.0	51.3	54.0
AUGMENTATION RATE	$y=3.0098$	$y=3.00885$		
TOTAL COST PER DAY	\$8213	\$8210	\$8310	\$8343
SHADOW PRICE OF THE BINDING CONSTRAINTS	$\lambda_{12}=0.21605$	$\lambda_{12}=0.21606$	$\lambda_{12}=0.4114$	$\lambda_{12}=0.40544$
			$\lambda_{26}=0.20785$	$\lambda_{26}=0.2854$

for decentralization model, are shown in Table 6 and 7 respectively. Treatment percentages for the various pollution sources, for centralization model, are shown in Table 11. The difference in the numbers is so small that it can be neglected and we can say that the results obtained for both, centralization and decentralization model, are same.

Without flow augmentation

The total treatment cost for the decentralization model is \$8317 per day and for the centralization model is \$8310. The polluter's response shadow prices and the CA's convex combination, for the decentralization model, are shown in Table 8 and 9, respectively. Table 11 shows the treatment percentages for the centralization model. Since the difference in daily costs and treatment percentages is negligible, we can say that the decentralization model and centralization model lead to the same optimal treatment configuration.

Comparison of the decentralization model results with Hass (1970) results

Hass has also solved the Miami River basin problem (decentralization model) to find an optimal taxation scheme for meeting the required standards of 4 mg/l of DO along the river.

With flow augmentation

The total cost of achieving the standards, as obtained by

Hass, is \$7828 per day as compared to our daily cost of \$8210. He has obtained an augmentation rate, y , equal to 2.4 at a daily cost of \$1282, which is not true. When we substitute $y = 2.4$ in the flow augmentation cost function developed by him and given by equation (8), we get the augmentation cost as \$1744 per day. This when added to his treatment cost (\$6548) gives a total cost of \$8292 per day, not \$7828 per day as given by him. Even this total cost (\$8292) does not tally with our cost (\$8210). However our cost for decentralization model, is the same as for centralization model solution with quadratic and separable programming. The polluter's response and CA's convex combination as obtained by Hass in the optimal solution are shown in Table 12.

Without flow augmentation

The total treatment cost obtained by Hass is \$8312 per day as compared to our cost of \$8317 per day. The polluter's response and CA's convex combination are almost identical. For our case, these are shown in Table 8 and 9, respectively.

Treatment-augmentation policy vs. treatment policy

It should be noted at this point that treatment-augmentation policy is cheaper to practice than treatment policy alone. A higher percentage of waste water needs treatment, for no augmentation case, to achieve the required standards.

TABLE 12: OPTIMUM POLICIES OBTAINED BY HASS FOR THE MIAMI
RIVER PROBLEM WITH FLOW AUGMENTATION (DECENTRALI-
ZATION MODEL)

POLLUTION SOURCE	POLLUTER'S ITERATIVE SOLUTION NUMBER 3	CA'S ITERATIVE SOLUTION NUMBER 2
1	70.7 (\$2176)	70.7
2	60.6 (\$331)	60.6
3	99.0 (\$773)	99.0
4	80.1 (\$460)	80.1
5	50.8 (\$177)	50.8
6	53.0 (\$198)	53.0
7	45.0 (\$182)	45.0
8	45.0 (\$177)	45.0
9	45.0 (\$174)	45.0
10	45.0 (\$246)	45.0
11	45.0 (\$385)	45.0
12	45.0 (\$233)	45.0
13	45.0 (\$177)	45.0
14	45.0 (\$484)	45.0
15	45.0 (\$385)	45.0
AUGMENTATION RATE		2.4
TOTAL DAILY COST		\$7828
SHADOW PRICE OF THE BINDING CONSTRAINT*		$\lambda_{12}=0.27832$

*Subscript refers to reach constraint to which the shadow price applies.

The total daily cost increases by about \$100 if the standards are to be achieved by treatment alone. This is because the cost of treatment rises exponentially as the waste water treatment percentage increases, while the flow augmentation cost rises linearly.

The procedure converges rather rapidly for flow augmentation case than for no augmentation, although there is one less variable. If we look at the treatment percentages for various pollution sources, given in Table 6 through 9, we find that for flow augmentation case, the variable x_j settles at the boundary or very near it. While for no augmentation case it is somewhere in the middle. Consequently, convergence takes more iterations.

Conclusion

The Miami River basin problem has been solved considering both the operation of a basin wide firm (centralization model) and imposition of effluent charges (decentralization model) to find the optimal treatment configuration for meeting water quality standard of 4 mg/l of DO. It is concluded that a central water authority would achieve the required stream standards irrespective of whether he knows the treatment cost functions or not. In case the CA does not know the treatment cost functions, it is possible to build an optimal tax structure, which will meet the required standards in a systematic and

efficient way.

5.1 PROPOSED FUTURE INVESTIGATIONS

In this report the waste loads used are those used in the previous studies and so the results have only limited normative value. Furthermore, we have assumed that the amount of waste generated by a polluter will not change as efficient taxes vary or, if a change in process is the response to changing taxes, that the cost of this change is incorporated into the cost of treatment function. Another assumption is that complete mixing occurs at all points where a tributary or waste water effluent enters a stream. These assumptions, though make the model simple, push it away from reality. The model should consider the non-homogeneity in the reaches. Another problem that we face is about the percentage of waste treatment. In practice we are going to build either primary, secondary or tertiary treatment plants. This means that either we subject the waste to primary treatment and then discharge it into the river or treat the whole of it in primary and secondary treatment plants and then discharge it or subject it to primary, secondary and tertiary treatment before discharging it into the river. In other words we can treat either about 45% of the waste or about 90% of the waste or about 99% of the waste. We can not treat 60.1% or 65.52% of the waste. Thus each polluter will have three alternatives of treatment, depending upon the optimal policy, i.e., primary, secondary or tertiary treatment. When

this type of alterantive is incorporated into the model, the problem changes and become more like 0-1 integer programming. This approach is more practical as all the plants will operate to the maximum efficiency rather than when 15% of the waste is removed in the secondary treatment plant after 45% of it has been removed in the primary treatment.

Another point to be considered is that in this study we have chosen August as the representative month for the flow during July, August and September. This assumption leaves nine months for recharging, assuming that reservoir flow is used only during these months. This greatly simplifies the problem, but leads to suboptimization. The solution of the problem will be different when we consider the formulation over the twelve month cycle.

For a more practical model, we must consider non-homogeneity in reaches, formulate the problem over 12-month cycle and include the three alternatives of either primary, secondary or tertiary treatment plants.

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APPENDIX 1. COMPUTER OUTPUT OF THE MIAMI RIVER BASIN
PROBLEM (CENTRALIZATION MODEL), USING SEPARABLE
PROGRAMMING.

This appendix contains the results of the Miami River basin problem. The contents are:

1. Computer output of the Miami River basin problem, with flow augmentation.
2. Computer output of the Miami River basin problem, without flow augmentation.

APPENDIX 1.1. COMPUTER OUTPUT OF THE MIAMI RIVER BASIN PROBLEM,
WITH FLOW AUGMENTATION

PRIMAL-LOCAL OPTIMUM

SOLUTION (OPTIMAL)

TIME = 0.24 MINS. ITERATION NUMBER = 129

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	3216.26157	ROW101
RESTRAINTS		LIMITS
BOUNDS....		SEFBOUND

NOTE: The total cost = const. cost term + functional
 $= \$(4993.9 + 3216)$
 $= \$8210$

PRIMAL-LOCAL OPTIMUM

SECTION 2 - COLUMNS

NUMBER	COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.
76	X1	BS	.64800	.	.	NCNE
77	X2	BS	.57600	.	.	NONE
78	X3	BS	.53600	.	.	NONE
79	X4	BS	.72000	.	.	NCNE
80	X5	BS	.48600	.	.	NCNE
81	X6	BS	.50400	.	.	NONE
82	X7	BS	.45000	.	.	NCNE
83	X8	BS	.45000	.	.	NONE
84	X9	BS	.45000	.	.	NCNE
85	X10	BS	.45000	.	.	NONE
86	X11	BS	.45000	.	.	NONE
87	X12	BS	.45000	.	.	NCNE
88	X13	BS	.45000	.	.	NONE
89	X14	BS	.45000	.	.	NCNE
90	X15	BS	.45000	.	.	NCNE
91	X16	BS	3.00000	726.60000	.	NONE
92	P1001	UL	1.00000	3.95224	.	1.00000
93	P1002	UL	1.00000	11.85671	.	1.00000
94	P1003	UL	1.00000	19.76120	.	1.00000
95	P1004	UL	1.00000	27.66570	.	1.00000
96	P1005	UL	1.00000	35.57022	.	1.00000
97	P1006	UL	1.00000	43.47475	.	1.00000
98	P1007	UL	1.00000	51.37523	.	1.00000
99	P1008	UL	1.00000	59.28369	.	1.00000
100	P1009	UL	1.00000	67.18805	.	1.00000
101	P1010	UL	1.00000	75.09277	.	1.00000
102	P1011	UL	1.00000	82.99731	.	1.00000
103	P1012	LL	.	90.90161	.	1.00000
104	P1013	LL	.	98.80615	.	1.00000
105	P1014	LL	.	106.71069	.	1.00000
106	P1015	LL	.	114.61499	.	1.00000
107	P1016	LL	.	122.52002	.	1.00000
108	P1017	LL	.	130.42456	.	1.00000
109	P1018	LL	.	138.32861	.	1.00000
110	P1019	LL	.	146.23364	.	1.00000
111	P1020	LL	.	154.13745	.	1.00000
112	P1021	LL	.	162.04248	.	1.00000
113	P1022	LL	.	169.94629	.	1.00000
114	P1023	LL	.	177.85132	.	1.00000
115	P1024	LL	.	185.75586	.	1.00000
116	P1025	LL	.	193.66016	.	1.00000
117	P1026	LL	.	201.56470	.	1.00000
118	P1027	LL	.	209.46899	.	1.00000
119	P1028	LL	.	217.37378	.	1.00000
120	P1029	LL	.	225.27832	.	1.00000
121	P1030	LL	.	233.18262	.	1.00000
122	P1031	UL	1.00000	.55656	.	1.00000
123	P1032	UL	1.00000	1.75089	.	1.00000
124	P1033	UL	1.00000	2.58482	.	1.00000

Note: x_i , $i=1,2,\dots,15$ = the treatment percentage at each pollution source.

x_{16} = the flow augmentation rate

APPENDIX 1.2. COMPUTER OUTPUT OF THE MIAMI RIVER BASIN PROBLEM,
WITHOUT FLOW AUGMENTATION.

PRIMAL-LOCAL OPTIMUM

SOLUTION (OPTIMAL)

TIME = 0.36 MINS. ITERATION NUMBER = 274

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	3349.53303	ROW1C1
RESTRAINTS		LIMITS
BOUNDS....		SEFBCUND

NOTE: The total cash = const. cost term + functional
 $= \$(4993.9 + 3349.5)$
 $= \$8343$

PRIMAL-LOCAL OPTIMUM

SECTION 2 - COLUMNS

NUMBER	COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.
71	X1	BS	.66756	.	.	NONE
72	X2	BS	.70200	.	.	NONE
73	X3	BS	.59000	.	.	NONE
74	X4	BS	.59000	.	.	NONE
75	X5	BS	.55800	.	.	NONE
76	X6	BS	.57600	.	.	NONE
77	X7	BS	.46600	.	.	NCNE
78	X8	BS	.54000	.	.	NONE
79	X9	BS	.46600	.	.	NONE
80	X10	BS	.64800	.	.	NONE
81	X11	BS	.54000	.	.	NONE
82	X12	BS	.53540	.	.	NCNE
83	X13	BS	.48600	.	.	NONE
84	X14	BS	.45000	.	.	NCNE
85	X15	BS	.54000	.	.	NCNE
86	P1001	UL	1.00000	3.95224	.	1.00000
87	P1002	UL	1.00000	11.85671	.	1.00000
88	P1003	UL	1.00000	19.76120	.	1.00000
89	P1004	UL	1.00000	27.66570	.	1.00000
90	P1005	UL	1.00000	35.57022	.	1.00000
91	P1006	UL	1.00000	43.47475	.	1.00000
92	P1007	UL	1.00000	51.37923	.	1.00000
93	P1008	UL	1.00000	59.28369	.	1.00000
94	P1009	UL	1.00000	67.18805	.	1.00000
95	P1010	UL	1.00000	75.09277	.	1.00000
96	P1011	UL	1.00000	82.99731	.	1.00000
97	P1012	UL	1.00000	90.90161	.	1.00000
98	P1013	UL	1.00000	98.80615	.	1.00000
99	P1014	UL	1.00000	106.71069	.	1.00000
100	P1015	UL	1.00000	114.61459	.	1.00000
101	P1016	UL	1.00000	122.52002	.	1.00000
102	P1017	UL	1.00000	130.42456	.	1.00000
103	P1018	UL	1.00000	138.32861	.	1.00000
104	P1019	UL	1.00000	146.23364	.	1.00000
105	P1020	UL	1.00000	154.13745	.	1.00000
106	P1021	UL	1.00000	162.04248	.	1.00000
107	P1022	UL	1.00000	169.94629	.	1.00000
108	P1023	UL	1.00000	177.85132	.	1.00000
109	P1024	BS	.21980	185.75586	.	1.00000
110	P1025	LL	.	193.66016	.	1.00000
111	P1026	LL	.	201.56470	.	1.00000
112	P1027	LL	.	209.46859	.	1.00000
113	P1028	LL	.	217.37378	.	1.00000
114	P1029	LL	.	225.27832	.	1.00000
115	P1030	LL	.	233.18262	.	1.00000
116	P1031	UL	1.00000	.55656	.	1.00000
117	P1032	UL	1.00000	1.75089	.	1.00000
118	P1033	UL	1.00000	2.98482	.	1.00000
119	P1034	UL	1.00000	4.17875	.	1.00000

Note:

x_i , $i=1,2,\dots,15$ = the treatment percentage at each
pollution source.

APPENDIX 2. SAMPLE OUTPUT OF THE MIAMI RIVER BASIN PROBLEM
(DECENTRALIZATION MODEL)

This appendix contains a part of the results of the Miami River basin problem. The contents are:

1. Computer output of the first polluter's response to the shadow price $\lambda_i=0$, $i=1,\dots,26$, $\lambda_{27}=10$.
2. Computer output of the CA's iteration 1, with flow augmentation.

APPENDIX 2.1. COMPUTER OUTPUT OF THE FIRST POLLUTER'S RESPONSE
TO THE SHADOW PRICE $\lambda_i=0, i=1,2,\dots,26, \lambda_{27}=10$.

PRIMAL-LOCAL OPTIMUM

SOLUTION (OPTIMAL)

TIME = 0.31 MINS. ITERATION NUMBER = 51

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	29429.78433-	ROW101
RESTRAINTS		LIMITS
BOUNDS....		SEFBCUND

Note: The total cost = const. cost term + functional
 $= \$(34687,64 - 29429.8)$
 $= \$4925$

PRIMAL-LOCAL OPTIMUM

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT CCST..	..LOWER LIMIT.	..UPPER LIMIT.
5	X	BS	.99000	33320.00000-	.	NONE
6	P1001	UL	1.00000	1.42280	.	1.00000
7	P1002	UL	1.00000	4.26840	.	1.00000
8	P1003	UL	1.00000	7.11401	.	1.00000
9	P1004	UL	1.00000	9.95962	.	1.00000
10	P1005	UL	1.00000	12.80525	.	1.00000
11	P1006	UL	1.00000	15.65082	.	1.00000
12	P1007	UL	1.00000	18.45649	.	1.00000
13	P1008	UL	1.00000	21.34210	.	1.00000
14	P1009	UL	1.00000	24.18768	.	1.00000
15	P1010	UL	1.00000	27.03334	.	1.00000
16	P1011	UL	1.00000	29.87855	.	1.00000
17	P1012	UL	1.00000	32.72453	.	1.00000
18	P1013	UL	1.00000	35.57019	.	1.00000
19	P1014	UL	1.00000	38.41573	.	1.00000
20	P1015	UL	1.00000	41.26123	.	1.00000
21	P1016	UL	1.00000	44.10653	.	1.00000
22	P1017	UL	1.00000	46.95264	.	1.00000
23	P1018	UL	1.00000	49.79834	.	1.00000
24	P1019	UL	1.00000	52.64380	.	1.00000
25	P1020	UL	1.00000	55.48950	.	1.00000
26	P1021	UL	1.00000	58.33521	.	1.00000
27	P1022	UL	1.00000	61.18066	.	1.00000
28	P1023	UL	1.00000	64.02612	.	1.00000
29	P1024	UL	1.00000	66.87109	.	1.00000
30	P1025	UL	1.00000	69.71729	.	1.00000
31	P1026	UL	1.00000	72.56323	.	1.00000
32	P1027	UL	1.00000	75.40869	.	1.00000
33	P1028	UL	1.00000	78.25464	.	1.00000
34	P1029	UL	1.00000	81.10059	.	1.00000
35	P1030	UL	1.00000	83.94604	.	1.00000
36	P1031	UL	1.00000	86.79126	.	1.00000
37	P1032	UL	1.00000	89.63647	.	1.00000
38	P1033	UL	1.00000	92.48267	.	1.00000
39	P1034	UL	1.00000	95.32788	.	1.00000
40	P1035	UL	1.00000	98.17310	.	1.00000
41	P1036	UL	1.00000	101.01929	.	1.00000
42	P1037	UL	1.00000	103.86450	.	1.00000
43	P1038	UL	1.00000	106.71045	.	1.00000
44	P1039	UL	1.00000	109.55591	.	1.00000
45	P1040	UL	1.00000	112.40186	.	1.00000
46	P1041	UL	1.00000	115.24707	.	1.00000
47	P1042	UL	1.00000	118.09326	.	1.00000
48	P1043	UL	1.00000	120.93921	.	1.00000
49	P1044	UL	1.00000	123.78418	.	1.00000
50	P1045	UL	1.00000	126.63013	.	1.00000
51	P1046	UL	1.00000	129.47510	.	1.00000
52	P1047	UL	1.00000	132.32080	.	1.00000
53	P1048	UL	1.00000	135.16602	.	1.00000

Note: This is a sample output for each polluter. The first polluter treats $x=0.99$ in response to the shadow price (taxation) of $\lambda_i=0$, $i=1, \dots, 26$, and $\lambda_{27}=10$.

APPENDIX 2.2. COMPUTER OUTPUT OF THE CA'S ITERATION 1, WITH
FLOW AUGMENTATION.

EXECUTOR. MPS/360 V2-M11

SOLUTION (OPTIMAL)

TIME = 0.47 MINS. ITERATION NUMBER = 19

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL RESTRAINTS	\$845.84817	CBJFA RHVAL

Note: The total cost is \$8845.8.

EXECUTOR. MPS/360 V2-M11

SECTION 2 - COLUMNS

NUMBER	COLUMN.	AT	...ACTIVITY...	..INPUT CCST..	..LOWER LIMIT.	..UPPER LIMIT.
46	VA1	BS	$U_1^1 = 1.00000$	$c(x_1^1) = 1368.00000$	.	NONE
47	VA2	LL	$U_1^2 = .$	$c(x_1^2) = 4924.65000$	.	NCNE
48	VA3	BS	$U_2^1 = 1.00000$	$c(x_2^1) = 286.00000$	.	NONE
49	VA4	LL	$U_2^2 = .$	$c(x_2^2) = 823.55000$	.	NONE
50	VA5	LL	$U_2^3 = .$	$c(x_2^3) = 273.00000$	.	NONE
51	VA6	BS	$U_3^1 = 1.00000$	$c(x_3^1) = 772.92000$	.	NONE
52	VA7	LL	$U_3^2 = .$	$c(x_3^2) = 257.00000$	.	NCNE
53	VA8	BS	$U_3^3 = 1.00000$	$c(x_3^3) = 712.16000$	.	NONE
54	VA9	BS	$U_4^1 = 1.00000$	$c(x_4^1) = 174.00000$	.	NONE
55	VA10	LL	$U_4^2 = .$	$c(x_4^2) = 355.67000$	.	NONE
56	VA11	BS	$U_4^3 = 1.00000$	$c(x_4^3) = 193.00000$	.	NONE
57	VA12	LL	$U_4^4 = .$	$c(x_4^4) = 469.12000$	.	NONE
58	VA13	BS	$U_5^1 = 1.00000$	$c(x_5^1) = 182.00000$	.	NONE
59	VA14	LL	$U_5^2 = .$	$c(x_5^2) = 428.63000$	.	NONE
60	VA15	BS	$U_5^3 = 1.00000$	$c(x_5^3) = 177.00000$	.	NONE
61	VA16	LL	$U_5^4 = .$	$c(x_5^4) = 408.38000$	.	NONE
62	VA17	BS	$U_6^1 = 1.00000$	$c(x_6^1) = 174.00000$	.	NONE
63	VA18	LL	$U_6^2 = .$	$c(x_6^2) = 358.25000$	.	NCNE
64	VA19	BS	$U_6^3 = 1.00000$	$c(x_6^3) = 246.00000$	.	NONE
65	VA20	LL	$U_6^4 = .$	$c(x_6^4) = 671.66000$	.	NONE
66	VA21	BS	$U_7^1 = 1.00000$	$c(x_7^1) = 385.00000$	.	NCNE
67	VA22	LL	$U_7^2 = .$	$c(x_7^2) = 1198.20000$	.	NONE
68	VA23	BS	$U_7^3 = 1.00000$	$c(x_7^3) = 233.00000$	.	NCNE
69	VA24	LL	$U_7^4 = .$	$c(x_7^4) = 621.03000$	.	NONE
70	VA25	BS	$U_8^1 = 1.00000$	$c(x_8^1) = 177.00000$	.	NONE
71	VA26	LL	$U_8^2 = .$	$c(x_8^2) = 408.38000$	.	NONE
72	VA27	BS	$U_8^3 = 1.00000$	$c(x_8^3) = 484.00000$	.	NONE
73	VA28	LL	$U_8^4 = .$	$c(x_8^4) = 1572.85000$	.	NONE
74	VA29	BS	$U_9^1 = 1.00000$	$c(x_9^1) = 385.00000$	.	NONE
75	VA30	LL	$U_9^2 = .$	$c(x_9^2) = 1198.20000$	.	NONE
76	VA31	BS	$y = 3.58674$	$c_0(y) = 726.60000$	.	NONE

APPENDIX 3. COMPUTER PROGRAM FOR QUADRATIC PROGRAMMING AND OUTPUT OF THE EXAMPLE PROBLEMS.

This appendix contains a listing of the FORTRAN program for quadratic programming and results of the example problems.

The contents are:

1. The main FORTRAN program and its supporting subroutines.
2. Computer output of the numerical example considered in section 5.5.
3. Computer output of the Miami River basin problem (decentralization model), with flow augmentation.
4. Computer output of the Miami River basin problem (centralization model), without flow augmentation.

FORTRAN IV G LEVEL 21

MAIN

DATE = 76127

01/02

```

C      IPACK      AN INTEGER VARIABLE INDICATING IF THE INPUT IS
C                  IN THE PACKED OR REGULAR FORM.
C      NCHCK      AN INTEGER VARIABLE INDICATING IF A DETERMINATION
C                  OF POSITIVE(SEMI)DEFINITENESS IS DESIRED.
C      LP         AN INTEGER VARIABLE INDICATING IF THE PROBLEM
C                  IS AN L.P. OR C.F. PROBLEM.
C      NPRNT      AN INTEGER VARIABLE INDICATING IF A PRINTOUT OF THE
C                  INPUT IS DESIRED.
C
C      READ IN THE UNIT NOS. IN THE COMPUTING CENTER FOR I/O UNITS
C
C
C
C
C      READ IN THE TITLE FOR THE PROBLEM
C
0001      DIMENSION AM(75,75),Q(75),B(75,75),A(75),MBASIS(150),W(75),
0002      1Z(75),D(75,75),C1(75),EV(75),X(75),Y(75),TITLE(20)
      COMMON IPACK,IR,IZR,L1,LP,NCHCK,NE1,NE2,NL1,NL2,NPRNT,NREAD,
      XNPRINT
0003      DATA BLNK/4H      /
0004      NREAD=5
0005      NPRNT=6
0006      100 READ (NREAD,150) (TITLE(I),I=1,20)
0007      DO 110 I=1,20
0008      IF (TITLE(I).NE.BLNK) GO TO 120
0009      110 CONTINUE
C
C      IF THE TITLE CARD IS A BLANK CARD , THEN STOP.
C
0010      GO TO 140
0011      120 WRITE(NPRNT,170) (TITLE(I),I=1,20)
0012      READ (NREAD,160) NVAR,NCCN,IPACK,NCHCK,LP,NPRNT
0013      N=NVAR+NCCN
0014      N2=N*2
0015      CALL MATRIX (AM,D,B,C1,N,NVAR,Q,EV)
0016      CALL INITIA (B,MBASIS,Q,W,Z,N,N2)
C
C      SINCE FOR ANY PROBLEM TERMINATION CAN OCCUR IN INITIA,
C      NEWBAS OR SORT SUBROUTINE,THE VALUE OF IR IS MATCHED WITH
C      1000 TO CHECK WHETHER TO CONTINUE OR GO TO NEXT PROBLEM.
C
0017      IF (IR.EQ.1000) GO TO 100
0018      130 CALL NEWBAS (A,AM,B,N,NCCN,NVAR,N2,C,C1,MBASIS,Q,X,Y)
0019      IF (IR.EQ.1000) GO TO 100
0020      CALL SORT (A,N,NCCN,NVAR,N2,Q,D,C1,MBASIS,X,Y)
0021      IF (IR.EQ.1000) GO TO 100
0022      CALL PIVOT (A,B,MBASIS,N,N2,Q)
0023      GO TO 130
0024      140 PRINT 180
0025      STOP
C
0026      150 FORMAT (20A4)

```

FORTRAN IV G LEVEL 21

MAIN

DATE = 76127

01/02

```
0027      160 FORMAT (6I2)
0028      170 FCRMAT (1F1,10X,20A4)
0029      180 FCRMAT('1')
      C
0030      END
```

FORTRAN IV G LEVEL 21

MATRIX

DATE = 76127

01/02

```

0001      SUBROUTINE PATRIX (AM,D,B,C1,N,NVAR,Q,EV)
      C
      C      PURPOSE - TO INITIALIZE AND READ IN THE VARIOUS INPUT DATA
      C
0002      DIMENSION AP(N,N),D(NVAR,NVAR),B(N,N),C1(NVAR),Q(N),EV(NVAR)
0003      DIMENSION K(7),TEMP(7)
0004      COMMON IPACK,IR,IZR,L1,LP,NCHCK,NE1,NE2,NL1,NL2,NPRNT,NREAD,
      XNPRINT
0005      IF (LP.LE.0) GO TO 100
0006      PRINT 550
0007      NCHCK=-10
0008      100 NCPI=NVAR+1
0009      DO 110 I=1,N
0010      DO 110 J=1,N
0011      110 AM(I,J)=0.0
0012      IF (IPACK.GE.1) GO TO 280
0013      IF (LP.GE.1) GO TO 150
      C
      C      READ THE QUADRATIC FORM MATRIX COLUMNWISE
      C
0014      DO 120 J=1,NVAR
0015      120 READ (NREAD,460) (AM(I,J),I=1,NVAR)
0016      DO 140 I=1,NVAR
0017      DO 130 J=1,NVAR
0018      AM(I,J)=AM(I,J)+AM(J,I)
0019      130 AM(J,I)=AM(I,J)
0020      140 CONTINUE
      C
      C      READ THE CONSTRAINT MATRIX COLUMNWISE
      C
0021      150 DO 170 J=1,NVAR
0022      READ (NREAD,460) (AM(I,J),I=NCPI,N)
0023      DO 160 I=NCPI,N
0024      160 AM(J,I)=-AM(I,J)
0025      170 CCNTINUE
0026      DO 190 I=1,NVAR
0027      DO 190 J=1,NVAR
0028      D(I,J)=AM(I,J)*C.5
0029      190 C(J,I)=0(I,J)
0030      IF (NCHCK) 250,200,210
0031      200 PRINT 530
0032      GO TO 250
0033      210 IND=C
0034      ACC=3E-04
0035      NCHCK=-10
      C
      C      SUBROUTINE RSEIG RETURNS THE EIGENVALUES (EV) OF D ORDERED
      C      FROM MIN TO MAX.SETTING IND=1 WOULD ALSO RETURN THE
      C      EIGENVECTORS (E).SINCE EIGENVECTORS ARE NOT NEEDED, IND
      C      IS SET TO ZERO,AND E IS A DUMMY VARIABLE.
      C
      C      SINCE RSEIG IS A SYSTEM DEPENDENT SUBROUTINE AND IS NOT
      C      AVAILABLE AT THE K-STATE COMPUTING CENTER,IT IS REPLACED
      C      BY A DUMMY SUBROUTINE.EIGENVALUES OF MATRIX D HAVE BEEN
      C      COMPUTED BY JACOBI'S METHOD,SEPERATELY.
      C
0036      CALL RSEIG (NVAR,NVAR,D,IND,EV,E)

```

FORTRAN IV G LEVEL 21

MATRIX

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```

0037      IF (IND.EQ.C) GO TO 220
0038      PRINT 570
0039      GO TO 180

C
C      IF MIN(EV) IS NEGATIVE, D IS NEITHER P.D. NOR P.S.D. IF
C      IT IS ZERC, D IS P.S.D. IF IT IS POSITIVE, D IS P.D. TO
C      ACCOUNT FOR ROUND OFF ERROR, IF MIN(EV) IS BETWEEN (-3E-04)
C      AND (+3E-04) IT IS TAKEN TO BE ZERC.
C
0040      220 IF (EV(1).LT.-ACC) GO TO 240
0041      IF (EV(1).GT.ACC) GO TO 230
0042      PRINT 550
0043      GO TO 180
0044      230 PRINT 540
0045      GO TO 180
0046      240 PRINT 560
0047      GO TO 180

C
C      READ THE CCST VECTOR
C
0048      250 READ(NREAD,460) (Q(I),I=1,NVAR)
0049      IF (NPRNT.GT.0) PRINT 490
0050      IF (NPRNT.GT.C) WRITE (6,48C) (Q(I),I=1,NVAR)
0051      DO 260 I=1,NVAR
0052      260 C1(I)=Q(I)

C
C      READ THE RIGHT HAND SIDE VECTOR
C
0053      READ (NREAD,460) (Q(I),I=NCP1,N)
0054      IF (NPRNT.GT.0) PRINT 500
0055      IF (NPRNT.GT.C) WRITE (NPRINT,480) (Q(I),I=NCP1,N)
0056      DO 270 I=NCP1,N
0057      270 Q(I)=-Q(I)
0058      GO TO 400
0059      280 IF (LP.GT.1) GO TO 350

C
C      READ THE QUADRATIC FORM MATRIX IN PACKED FORM
C
0060      DO 320 I=1,NVAR
0061      290 DO 300 J=1,7
0062      K(J)=0
0063      300 TEMP(J)=0.0
0064      READ (NREAD,470) (K(J),TEMP(J),J=1,7)
0065      DO 310 J=1,7
0066      IF (K(J).LT.1) GO TO 320
0067      KT=K(J)
0068      310 AM(KT,I)=TEMP(J)
0069      GO TO 290
0070      320 CONTINUE
0071      DO 340 I=1,NVAR
0072      DO 330 J=1,NVAR
0073      AM(I,J)=AM(I,J)+AM(J,I)
0074      330 AM(J,I)=AM(I,J)
0075      340 CONTINUE

C
C      READ THE CCNSTRANT MATRIX IN PACKED FORM
C
0076      350 DO 390 I=1,NVAR

```

FORTRAN IV G LEVEL 21

MATRIX

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```

0077      360 DO 370 J=1,7
0078          K(J)=0
0079      370 TEMP(J)=0.0
0080          READ (NREAD,470) (K(J),TEMP(J),J=1,7)
0081          DO 380 J=1,7
0082              IF (K(J).LT.1) GO TO 390
0083              KT=NVAR+K(J)
0084              AM(KT, I)=TEMP(J)
0085      380  AM(I,KT)=-AM(KT,I)
0086              GO TO 360
0087      390  CCNTINUE
0088              GO TO 180

C
C      IN ITERATION 1,BASIS INVERSE IS AN ICENTITY MATRIX.
C

0089      400 DO 430 J=1,N
0090          DO 420 I=1,N
0091              IF (I.EQ.J) GO TO 410
0092              B(I,J)=0.0
0093              GO TO 420
0094      410  B(I,J)=1.0
0095      420  CCNTINUE
0096      430  CCNTINUE
0097          IF (NPRNT.LE.0) RETURN
0098          PRINT 510
0099          DO 440 J=NGP1,N
0100      440  WRITE (APRINT,480) (AM(J,I),I=1,NVAR)
0101          IF (LP.GT.0) RETURN
0102          PRINT 520
0103          DO 450 I=1,NVAR
0104      450  WRITE (NPRINT,480) (D(J,I),J=1,NVAR)
0105          RETURN

C
0106      460  FORMAT (7F10.4)
0107      470  FORMAT (7(I2,F8.4))
0108      480  FORMAT (/6(1X,F11.4))
0109      490  FORMAT (16H0THE COST VECTOR)
0110      500  FORMAT (27H0THE RIGHT HAND SIDE VECTOR)
0111      510  FORMAT (32H0THE CONSTRAINT MATRIX (ROWWISE))
0112      520  FORMAT (26H0THE QUADRATIC FORM MATRIX)
0113      530  FORMAT (71H0***** WITH NO CHECK ON THE POSITIVE (SEMI) DEFINIT
          1NESS OF D *****/77H ***** THERE IS NO GAURENTEE THAT ANY
          2SOLUTION FOUND IS OPTIMAL *****)
0114      540  FORMAT (70HC***** D IS A POSITIVE DEFINITE MATRIX.LEMKE S ALGO
          1RITHM IS *****/71H ***** GUARANTEED TO FIND AN OPTIMAL SOLU
          2TION IF ONE EXISTS *****)
0115      550  FORMAT (70HC***** D IS A POSITIVE SEMIDEFINITE MATRIX.LEMKE S
          1ALGORITHM *****/72H ***** IS GUARANTEED TO FIND AN OPTIMAL SOL
          2UTION IF ONE EXISTS *****)
0116      560  FORMAT (74H0***** D IS NEITHER POSITIVE DEFINITE NOR POSITIVE
          1SEMIDEFINITE *****/71H ***** LEMKE S ALGORITHM IS NOT GUARANT
          2EED TO FIND A SOLUTION *****)
0117      570  FORMAT (55H0SUBROUTINE RSEIG WAS UNABLE TO FIND THE EIGENVALUES
          1/38H OF D IN FEWER THAN THIRTY ITERATIONS.)

C
0118      END

```

FORTRAN IV G LEVEL 21

INITIA

DATE = 76127

01/02

```

0001      SUBROUTINE INITIA (B,MBASIS,C,W,Z,N,N2)
      C
      C      PURPOSE-TO FIND THE INITIAL ALMOST COMPLEMENTARY SOLUTION
      C      BY ADDING AN ARTIFICIAL VARIABLE ZC.
      C
0002      DIMENSION B(N,N),MBASIS(N2),Q(N),W(N),Z(N)
0003      COMMON IPACK,IR,IZR,L1,LP,NCHK,NE1,NE2,NL1,NL2,NPRNT,NREAD,
      XNPRINT
      C
      C      SET ZC EQUAL TO THE MOST NEGATIVE Q(I)
      C
0004      I=1
0005      J=2
0006      100 IF (Q(I).LE.Q(J)) GO TO 110
0007      I=J
0008      110 J=J+1
0009      IF (J.LE.N) GO TO 100
      C
      C      UPDATE Q VECTOR
      C
0010      IR=I
0011      T1=-Q(IR)
0012      IF (T1.LE.0.0) GO TO 140
0013      DO 120 I=1,N
0014      Q(I)=Q(I)+T1
0015      120 CCNTINUE
0016      Q(IR)=T1
      C
      C      UPDATE BASIS INVERSE AND INDICATOR VECTOR
      C      OF BASIC VARIABLES.
      C
0017      DO 130 J=1,N
0018      B(J,IR)=-1.0
0019      W(J)=C(J)
0020      Z(J)=0.0
0021      MBASIS(J)=1
0022      L=N+J
0023      MBASIS(L)=J
0024      130 CCNTINUE
0025      IZR=IR
0026      NL1=1
0027      L=N+IR
0028      NL2=IR
0029      MBASIS(IR)=2
0030      MBASIS(L)=0
0031      W(IR)=0.0
0032      ZC=Q(IR)
0033      L1=1
0034      RETURN
0035      140 WRITE(NPRINT,160)
0036      IR=1000
0037      RETURN
      C
0038      160 FORMAT (5X,40F)PROBLEM HAS A TRIVIAL SOLUTION WITH X=0.)
      C
0039      END

```

FORTRAN IV G LEVEL 21

NEWBAS

DATE = 76127

01/02

```

0001      SUBROUTINE NEWBAS (A,AM,B,N,NCON,NVAR,N2,D,C1,MBASIS,Q,X,Y)
      C
      C      PURPOSE - TO FIND THE NEW BASIS COLUMN TO ENTER IN
      C      TERMS OF THE CURRENT BASIS.
      C
0002      DIMENSION A(N),AM(N,N),B(N,N),D(NVAR,NVAR),C1(NVAR),MBASIS(N2),
      1Q(N),X(NVAR),Y(NCON)
0003      COMMON IPACK,IR,IZR,L1,LP,NCPCK,NE1,NE2,NL1,NL2,NPRNT,NREAD,
      XNPRINT
      C
      C      IF NL1 IS NEITHER 1 NOR 2 THEN THE VARIABLE ZO LEAVES THE
      C      BASIS INDICATING TERMINATION WITH A COMPLEMENTARY SOLUTION.
      C
0004      IF (NL1.EQ.1) GO TO 100
0005      IF (NL1.EQ.2) GO TO 130
0006      PRINT 150
0007      CALL PPRINT (D,C1,MBASIS,N,ACCN,NVAR,N2,Q,X,Y)
0008      IR=1000
0009      RETURN
0010      100 NE1=2
0011      NE2=NL2
      C
      C      UPDATE NEW BASIC COLUMN BY MULTIPLYING BY BASIS INVERSE.
      C
0012      DO 120 I=1,N
0013      T1=0.0
0014      DO 110 J=1,N
0015      110 T1=T1-B(I,J)*AM(J,NE2)
0016      A(I)=T1
0017      120 CONTINUE
0018      RETURN
0019      130 NE1=1
0020      NE2=NL2
0021      DO 140 I=1,N
0022      A(I)=B(I,NE2)
0023      140 CONTINUE
0024      RETURN
      C
0025      150 FORMAT (13H0      SOLUTION)
      C
0026      END

```


FORTRAN IV G LEVEL 21

SORT

DATE = 76127

01/02

```

0001      SUBROUTINE SORT (A,N,NCCN,NVAR,N2,Q,C,C1,MBASIS,X,Y)
      C
      C      PURPOSE - TO FIND THE PIVOT ROW FOR NEXT ITERATION BY THE
      C      USE OF (SIMPLEX-TYPE) MINIMUM RATIO RULE.
      C
0002      DIMENSION A(N),Q(N),D(NVAR,NVAR),C1(NVAR),MBASIS(N2),X(NVAR),
      C      1Y(NCON)
0003      COMMON IPACK,IR,IZR,L1,LP,NCHCK,NE1,NE2,NL1,NL2,NPRNT,NREAD,
      C      XNPRINT
0004      DOUBLE PRECISION TOL
0005      AMAX=ABS(A(1))
0006      DO 100 I=2,N
0007      IF (AMAX.GE.ABS(A(I))) GO TO 100
0008      AMAX=ABS(A(I))
0009      100 CONTINUE
      C
      C      SET TOL=AMAX*2.0**(-(B-1)) WHERE B IS THE NUMBER OF
      C      BITS IN THE FLOATING POINT MANTISSA AS CLASEN SUGGESTS.
      C
0010      TOL=AMAX*2.0**(-37)
0011      I=1
0012      110 IF (A(I).GT.TOL) GO TO 120
0013      I=I+1
0014      IF (I.GT.N) GO TO 160
0015      GO TO 110
0016      120 T1=Q(I)/A(I)
0017      IR=I
0018      130 I=I+1
0019      IF (I.GT.N) GO TO 150
0020      IF (A(I).GT.TOL) GO TO 140
0021      GO TO 130
0022      140 T2=Q(I)/A(I)
0023      IF (T2.GE.T1) GO TO 130
0024      IR=I
0025      T1=T2
0026      GO TO 130
0027      150 RETURN
0028      160 IF (Q(IZR).GT.TOL) GO TO 170
0029      WRITE (NPRINT,190)
0030      CALL PPRINT (D,C1,MBASIS,N,NCCN,NVAR,N2,C,X,Y)
0031      IR=1000
0032      RETURN
      C
      C      FAILURE OF THE RATIO RULE INDICATES TERMINATION WITH
      C      NO COMPLEMENTARY SOLUTION.
      C
0033      170 PRINT 200
0034      PRINT 210, L1
0035      IR=1000
0036      RETURN
      C
0037      190 FORMAT (13H      SOLUTION)
0038      200 FORMAT (5X,23HPROBLEM HAS NO SOLUTION)
0039      210 FORMAT (10X,13HITERATION NO.,I4)
      C
0040      END

```

FORTRAN IV G LEVEL 21

PIVOT

DATE = 76127

01/02

```

0001      SUBROUTINE PIVCT (A,B,MBASIS,N,N2,Q)
      C
      C      PURPOSE - TO PERFORM THE PIVCT OPERATION BY UPDATING THE
      C      INVERSE OF THE BASIS AND Q VECTOR.
      C
0002      DIMENSION A(N),B(N,N),MBASIS(N2),Q(N)
0003      COMMON IPACK,IR,IZR,L1,LP,NCHCK,NE1,NE2,NL1,NL2,NPRNT,NREAD,
      XNPRINT
0004      DO 100 I=1,N
0005      100 B(IR,I)=B(IR,I)/A(IR)
0006      C(IR)=Q(IR)/A(IR)
0007      DO 120 I=1,N
0008      IF (I.EQ. IR) GO TO 120
0009      Q(I)=Q(I)-C(IR)*A(I)
0010      DO 110 J=1,N
0011      B(I,J)=B(I,J)-B(IR,J)*A(I)
0012      110 CONTINUE
0013      120 CCNTINUE
      C
      C      UPDATE THE INDICATOR VECTOR OF BASIC VARIABLES.
      C
0014      NL1=MBASIS(IR)
0015      L=N+IR
0016      NL2=MBASIS(L)
0017      MBASIS(IR)=NE1
0018      MBASIS(L)=NE2
0019      L1=L1+1
0020      RETURN
      C
0021      END

```

FORTRAN IV C LEVEL 21

PPRINT

DATE = 76127

01/02

```

0001      SUBROUTINE PPRINT (D,C1,MEASIS,N,NCCN,NVAR,N2,C,X,Y)
      C
      C
      C
      C
      C
0002      DIMENSION D(NVAR,NVAR),C1(NVAR),MBASIS(N2),C(N),X(NVAR),Y(NCO
0003      IN)
      COMMON IPACK,IR,IZR,L1,LP,NCTCK,NE1,NE2,NL1,NL2,NPRNT,NREAD,
      XNPRINT
0004      DO 100 I=1,NVAR
0005      100 X(I)=0.0
0006      NCP1=NVAR+1
0007      DO 110 I=NCP1,N
0008      J=I-NVAR
0009      110 Y(J)=0.0
0010      PRINT 230, L1
0011      I=N+1
0012      J=1
0013      120 K1=MBASIS(I)
0014      K2=MBASIS(J)
0015      IF (Q(J).GE.0.0) GO TO 130
0016      Q(J)=0.0
0017      130 IF (K2.EQ.1) GO TO 150
0018      IF (K1.GT.NVAR) GO TO 140
0019      X(K1)=Q(J)
0020      GO TO 150
0021      140 Y(K1-NVAR)=Q(J)
0022      150 I=I+1
0023      J=J+1
0024      IF (J.LE.N) GO TO 120
0025      DO 160 I=1,NVAR
0026      160 PRINT 200,I,X(I)
0027      PRINT 240
0028      DO 170 I=1,NCCN
0029      170 PRINT 210, I,Y(I)
0030      OBJ=0.0
0031      DO 190 I=1,NVAR
0032      OBJ=OBJ+C1(I)*X(I)
0033      IF (LP.GT.C) GO TO 190
0034      DO 180 J=1,NVAR
0035      TEMP=D(I,J)*X(I)*X(J)
0036      IF (I.NE.J) TEMP=TEMP*2.0
0037      OBJ=OBJ+TEMP
0038      180 CONTINUE
0039      190 CONTINUE
0040      PRINT 250,OBJ
0041      RETURN
      C
0042      200 FORMAT (/10X,2F(X,(I3,2H)=,F15.5)
0043      210 FORMAT (/10X,2F(Y,(I3,2H)=,F15.5)
0044      230 FORMAT (1H+,13X,16H-- ITERATION NO.,I4)
0045      240 FORMAT (/5X,28HTHE LAGRANGE MULTIPLIERS ARE)
0046      250 FORMAT (/6X,64HTHE VALUE OF THE OBJECTIVE FUNCTION AT THE OPT
      11FAL POINT IS ,F13.5)
      C
0047      END

```

FORTRAN IV G LEVEL 21

RSEIG

DATE = 76127

01/02

```
0001      SUBROUTINE RSEIG (NVAR,IVAR,D,IND,EV,E)
0002      DIMENSION NVAR(20),IVAR(20),D(20,20),IND(75),EV(75),E(75)
0003      RETURN
0004      END
```

APPENDIX 3.2. COMPUTER OUTPUT OF THE NUMERICAL EXAMPLE
CONSIDERED IN SECTION 5.5.

AN EXAMPLE PROBLEM

THE COST VECTOR

-6.0000 0.0

THE RIGHT HAND SIDE VECTOR

-2.0000

THE CONSTRAINT MATRIX (ROWWISE)

-1.0000 -1.0000

THE QUADRATIC FORM MATRIX

2.0000 -1.0000

-1.0000 2.0000

SOLUTION -- ITERATION NO. 4

X(1)= 1.50000

X(2)= 0.50000

THE LAGRANGE MULTIPLIERS ARE

Y(1)= 1.00000

THE VALUE OF THE OBJECTIVE FUNCTION AT THE OPTIMAL POINT IS

-5.5

Note: The problem for this input and output is

$$\text{Minimize } f(x) = -6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$$

subject to

$$-x_1 - x_2 \geq -2$$

$$x_1, x_2 \geq 0$$

APPENDIX 3.3. COMPUTATION

OUTPUT OF THE MIAMI RIVER BASIN PROBLEM

(CENTRAL

ION MODEL) WITH FLOW AUGMENTATION.

MIAMI RIVER PROBLEM

FLOW AUGMENTATION (Y IS GE 1.0 OR LE 4.47)

THE COST VECTOR

-10978.5078	-1658.2400	-1543	-1405.0995	-691.7000	-852.7898
-760.7300	-714.7100	-651	-1313.0498	-2509.7200	-1157.9800
-714.7100	-3361.2100	-2509	726.5999		

THE RIGHT HAND SIDE VECTOR

1.2000	268.7000	24	6345.6992	9180.0000	17903.6592
18353.5577	15422.5000	2389	27109.5000	31796.6592	36289.1992
38439.5000	38460.2969	3367	33598.6592	24285.0577	25210.1592
25527.3584	22380.7969	2224	22023.0577	17739.6992	11300.8584
11430.0000	14844.6592	1106	0.4500	0.4500	0.4500
0.4500	0.4500	0	0.4500	0.4500	0.4500
0.4500	0.4500	0	0.4500	0.4500	0.4500
-0.9900	-0.9900	-0	-0.9900	-0.9900	-0.9900
-0.9900	-0.9900	-0	-0.9900	-0.9900	-0.9900
-0.9900	-0.9900	-0	1.0000	-4.4700	

THE CONSTRAINT MATRIX (ROWWISE)

0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3880.0000	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3602.0000	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3680.0000	0.0	0.0
5989.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3775.0000	0.0	0.0
8840.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3781.0000	0.0	0.0
16553.0000	982.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3483.0000	0.0	0.0
15585.0000	1102.0000	1578	0.0	0.0	0.0
0.0	0.0	0	0.0	0.0	0.0
0.0	0.0	0	3772.0000	0.0	0.0
16394.0000	1161.0000	1947	242.0000	0.0	0.0
0.0	0.0	0	0.0	0.0	0.0
0.0	0.0	0	3793.0000	0.0	0.0
18029.0000	1406.0000	351	1278.0000	89.0000	0.0
0.0	0.0	0	0.0	0.0	0.0
0.0	0.0	0	3678.0000		
19147.0000	1590.0000	47	2108.0000	161.0000	0.0

Note: Input data is the coefficients of objective function and constraint matrix.

0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3693.0000		
21004.0000	1832.0000	6181.0000	3006.0000	238.0000	230.0000
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3610.0000		
22563.0000	2059.0000	7614.0000	3943.0000	318.0000	483.0000
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3363.0000		
22959.0000	2176.0000	8610.0000	4643.0000	380.0000	701.0000
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3471.0000		
22753.0000	2172.0000	8699.0000	4723.0000	387.0000	735.0000
108.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3499.0000		
19846.0000	1904.0000	7651.0000	4155.0000	344.0000	666.0000
162.0000	169.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	3449.0000		
15255.0000	1871.0000	7681.0000	4227.0000	348.0000	657.0000
290.0000	495.0000	164.0000	0.0	0.0	0.0
0.0	0.0	0.0	3319.0000		
14268.0000	1397.0000	5820.0000	3227.0000	266.0000	548.0000
301.0000	581.0000	227.0000	0.0	0.0	0.0
0.0	0.0	0.0	3632.0000		
14166.0000	1394.0000	5857.0000	3263.0000	269.0000	564.0000
355.0000	718.0000	295.0000	909.0000	0.0	0.0
0.0	0.0	0.0	3856.0000		
14028.0000	1385.0000	5856.0000	3272.0000	270.0000	573.0000
389.0000	803.0000	338.0000	1455.0000	0.0	0.0
0.0	0.0	0.0	3834.0000		
12447.0000	1233.0000	5234.0000	2931.0000	242.0000	517.0000
365.0000	774.0000	330.0000	1720.0000	0.0	0.0
0.0	0.0	0.0	3920.0000		
12249.0000	1214.0000	5156.0000	2888.0000	239.0000	510.0000
367.0000	770.0000	329.0000	1747.0000	65.0000	0.0
0.0	0.0	0.0	3946.0000		
11967.0000	1186.0000	5045.0000	2827.0000	234.0000	500.0000
363.0000	764.0000	327.0000	1786.0000	158.0000	40.0000
0.0	0.0	0.0	3818.0000		
7551.0000	758.0000	3289.0000	1862.0000	154.0000	341.0000
255.0000	656.0000	292.0000	2241.0000	1435.0000	585.0000
168.0000	0.0	0.0	2161.0000		
5534.0000	561.0000	2468.0000	1406.0000	117.0000	264.0000
256.0000	574.0000	260.0000	2245.0000	1774.0000	732.0000
239.0000	0.0	0.0	3859.0000		
5509.0000	559.0000	2458.0000	1401.0000	116.0000	263.0000

256.0000	573.0000	260.0000	2244.0000	1770.0000	733.0000
239.0000	57.0000	0.0	3856.0000		
4103.0000	420.0000	1871.0000	1072.0000	89.0000	206.0000
216.0000	492.0000	226.0000	2089.0000	1823.0000	755.0000
249.0000	2740.0000	1698.0000	2413.0000		
3332.0000	343.0000	1540.0000	886.0000	74.0000	172.0000
185.0000	433.0000	200.0000	1914.0000	1742.0000	723.0000
239.0000	3634.0000	2267.0000	3832.0000		
1.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	1.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	1.0000	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	1.0000	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	1.0000	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	1.0000
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
1.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0000	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0000	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0000	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0000
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0

0.0	0.0	0.0	0.0	0.0	0.0
1.0000	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0000	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0000	0.0		
-1.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	-1.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	-1.0000	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	-1.0000	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	-1.0000	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	-1.0000
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
-1.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	-1.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	-1.0000	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	-1.0000
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0

0.0	0.0	0.0	0.0	0.0	0.0
-1.0000	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	-1.0000	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0000		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	-1.0000		

THE QUADRATIC FORM MATRIX

12198.3358	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	1842.4900	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	1714.6355	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	1561.2200	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	768.5458	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	947.5398
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		
845.2598	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	794.1199	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	768.5498	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1458.9395	0.0	0.0
0.0	0.0	0.0	0.0		

0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	2788.5781	0.0
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1331.0898
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
754.1199	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	3734.6699	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	2788.5781	0.0		
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0		

SOLUTION -- ITERATION NO. 35

X(1)=	0.64981
X(2)=	0.57072
X(3)=	0.92970
X(4)=	0.72284
X(5)=	0.49471
X(6)=	0.50508
X(7)=	0.45269
X(8)=	0.45218
X(9)=	0.45318
X(10)=	0.45740
X(11)=	0.46601
X(12)=	0.45270
X(13)=	0.45318
X(14)=	0.46833
X(15)=	0.47100
X(16)=	3.00984

THE LAGRANGE MULTIPLIERS ARE

Note: The optimal solution gives the treatment percentage x_i , $i=1, \dots, 15$, and x_{16} , the flow augmentation rate.

Y(1)=	0.0
Y(2)=	0.0
Y(3)=	0.0
Y(4)=	0.0
Y(5)=	0.0
Y(6)=	0.0
Y(7)=	0.0
Y(8)=	0.0
Y(9)=	0.0
Y(10)=	0.0
Y(11)=	0.0
Y(12)=	0.21605
Y(13)=	0.0
Y(14)=	0.0
Y(15)=	0.0
Y(16)=	0.0
Y(17)=	0.0
Y(18)=	0.0
Y(19)=	0.0
Y(20)=	0.0
Y(21)=	0.0
Y(22)=	0.0
Y(23)=	0.0
Y(24)=	0.0
Y(25)=	0.0
Y(26)=	0.0
Y(27)=	0.0
Y(28)=	0.0
Y(29)=	0.0
Y(30)=	0.0

Note: The shadow price of the 27 reaches. The 12th reach is the only one in which the constraint is active.

Y(31)=	0.0
Y(32)=	0.0
Y(33)=	0.0
Y(34)=	4.52501
Y(35)=	3.44739
Y(36)=	4.85540
Y(37)=	21.58177
Y(38)=	89.25864
Y(39)=	7.17597
Y(40)=	5.01583
Y(41)=	136.91217
Y(42)=	117.08432
Y(43)=	0.0
Y(44)=	0.0
Y(45)=	0.0
Y(46)=	0.0
Y(47)=	0.0
Y(48)=	0.0
Y(49)=	0.0
Y(50)=	0.0
Y(51)=	0.0
Y(52)=	0.0
Y(53)=	0.0
Y(54)=	0.0
Y(55)=	0.0
Y(56)=	0.0
Y(57)=	0.0
Y(58)=	0.0
Y(59)=	0.0

THE VALUE OF THE OBJECTIVE FUNCTION AT THE OPTIMAL POINT IS -3733.83984

Note: Method Cost = const. cost term + objective function
value given above.
= \$(11947.08 - 3733.84)
= \$8213

APPENDIX 3.4. COMPUTER OUTPUT OF THE MIAMI RIVER BASIN PROBLEM
(CENTRALIZATION MODEL) WITHOUT FLOW AUGMENTATION.

MIAMI RIVER PROBLEM WITHOUT FLOW AUGMENTATION (Y=1.0)

THE COST VECTOR

-10978.5078	-1658.2400	-1543.1799	-1405.0595	-691.7000	-852.7898
-760.7300	-714.7100	-691.7000	-1313.0498	-2509.7200	-1197.9800
-714.7100	-3361.2100	-2509.7200			

THE RIGHT HAND SIDE VECTOR

2570.7000	5399.0000	14420.6552	14621.5977	15629.5000	20219.7969
23416.5000	28186.6552	32926.1992	34968.5000	34961.2569	30221.8584
30279.6952	20653.0977	21354.1952	21693.3984	18460.7569	18296.3584
18205.0577	15578.6952	7441.8584	7574.0000	12431.6552	7233.1592
0.4500	0.4500	0.4500	0.4500	0.4500	0.4500
0.4500	0.4500	0.4500	0.4500	0.4500	0.4500
0.4500	0.4500	0.4500	-0.9500	-0.9900	-0.9500
-0.9900	-0.9900	-0.9900	-0.9900	-0.9900	-0.9900
-0.9900	-0.9900	-0.9900	-0.9900	-0.9900	-0.9900

THE CONSTRAINT MATRIX (ROWWISE)

5989.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
8840.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
16553.0000	982.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
15985.0000	1102.0000	1576.0000	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
16354.0000	1161.0000	1947.0000	242.0000	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
18029.0000	1406.0000	3524.0000	1278.0000	89.0000	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
15147.0000	1590.0000	4770.0000	2108.0000	161.0000	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
21004.0000	1832.0000	6181.0000	3006.0000	238.0000	230.0000
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
22563.0000	2059.0000	7614.0000	3943.0000	318.0000	483.0000
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
22955.0000	2176.0000	8610.0000	4643.0000	380.0000	701.0000
0.0	0.0	0.0	0.0	0.0	0.0

0.0	0.0	0.0			
22753.0000	2172.0000	8699.0000	4723.0000	387.0000	735.0000
108.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
19846.0000	1904.0000	7691.0000	4195.0000	344.0000	666.0000
162.0000	169.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
19295.0000	1871.0000	7681.0000	4227.0000	348.0000	697.0000
290.0000	495.0000	164.0000	0.0	0.0	0.0
0.0	0.0	0.0			
14288.0000	1397.0000	5820.0000	3227.0000	266.0000	548.0000
301.0000	581.0000	227.0000	0.0	0.0	0.0
0.0	0.0	0.0			
14166.0000	1394.0000	5857.0000	3263.0000	269.0000	564.0000
355.0000	718.0000	295.0000	909.0000	0.0	0.0
0.0	0.0	0.0			
14028.0000	1385.0000	5856.0000	3272.0000	270.0000	573.0000
389.0000	803.0000	338.0000	1495.0000	0.0	0.0
0.0	0.0	0.0			
12447.0000	1233.0000	5234.0000	2931.0000	242.0000	517.0000
369.0000	774.0000	330.0000	1720.0000	0.0	0.0
0.0	0.0	0.0			
12249.0000	1214.0000	5156.0000	2888.0000	239.0000	510.0000
367.0000	770.0000	329.0000	1747.0000	65.0000	0.0
0.0	0.0	0.0			
11967.0000	1186.0000	5045.0000	2827.0000	234.0000	500.0000
363.0000	764.0000	327.0000	1786.0000	158.0000	40.0000
0.0	0.0	0.0			
7551.0000	758.0000	3289.0000	1862.0000	154.0000	341.0000
259.0000	656.0000	292.0000	2241.0000	1435.0000	585.0000
188.0000	0.0	0.0			
5534.0000	561.0000	2468.0000	1406.0000	117.0000	264.0000
256.0000	574.0000	260.0000	2245.0000	1774.0000	732.0000
235.0000	0.0	0.0			
5509.0000	559.0000	2458.0000	1401.0000	116.0000	263.0000
256.0000	573.0000	260.0000	2244.0000	1777.0000	733.0000
239.0000	57.0000	0.0			
4103.0000	420.0000	1871.0000	1072.0000	89.0000	206.0000
216.0000	492.0000	226.0000	2089.0000	1823.0000	755.0000
249.0000	2740.0000	1698.0000			
3332.0000	343.0000	1540.0000	886.0000	74.0000	172.0000
189.0000	433.0000	200.0000	1914.0000	1742.0000	723.0000
239.0000	3634.0000	2267.0000			
1.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0

0.0	0.0	0.0			
0.0	-1.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	-1.0000	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	-1.0000	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	-1.0000	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	-1.0000
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
-1.0000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
0.0	-1.0000	0.0	0.0	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	-1.0000	0.0	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	-1.0000	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	-1.0000	0.0	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	-1.0000	0.0
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	-1.0000
0.0	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
-1.0000	0.0	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	-1.0000	0.0			
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	-1.0000			

THE QUADRATIC FORM MATRIX

12198.3398					
0.0					
0.0					
0.0					
0.0	1842.4900				
0.0	0.0				
0.0	0.0				
0.0	0.0				
0.0	0.0	1714.6399			
0.0	0.0	0.0			
0.0	0.0	0.0			
0.0	0.0	0.0	1561.2200		
0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	768.5498	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	947.5398
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
845.2558	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	794.1199	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	768.5498	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1458.5395	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	2788.5781	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1331.0898
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
754.1199	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	3734.6699	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	2788.5781	0.0	0.0	0.0

SOLUTION -- ITERATION NO. 29

X(1)=	0.86735
X(2)=	0.70485
X(3)=	0.99160
X(4)=	0.98967
X(5)=	0.54776
X(6)=	0.57853
X(7)=	0.47743
X(8)=	0.51688
X(9)=	0.48171
X(10)=	0.60344
X(11)=	0.51549
X(12)=	0.51027
X(13)=	0.48330
X(14)=	0.52625
X(15)=	0.51328

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Y(1)=	0.0
Y(2)=	0.0
Y(3)=	0.0
Y(4)=	0.0
Y(5)=	0.0
Y(6)=	0.0
Y(7)=	0.0
Y(8)=	0.0
Y(9)=	0.41140
Y(10)=	0.0
Y(11)=	0.0
Y(12)=	0.0
Y(13)=	0.0

Y(14)=	0.0
Y(15)=	0.0
Y(16)=	0.0
Y(17)=	0.0
Y(18)=	0.0
Y(19)=	0.0
Y(20)=	0.00604
Y(21)=	0.0
Y(22)=	0.0
Y(23)=	0.20785
Y(24)=	0.0
Y(25)=	0.0
Y(26)=	0.0
Y(27)=	0.0
Y(28)=	0.0
Y(29)=	0.0
Y(30)=	0.0
Y(31)=	0.0
Y(32)=	0.0
Y(33)=	0.0
Y(34)=	0.0
Y(35)=	0.0
Y(36)=	0.0
Y(37)=	0.0
Y(38)=	0.0
Y(39)=	0.0
Y(40)=	0.0
Y(41)=	0.0
Y(42)=	1683.69609
Y(43)=	170.99631

Y(44)= 0.0
Y(45)= 0.0
Y(46)= 0.0
Y(47)= 0.0
Y(48)= 0.0
Y(49)= 0.0
Y(50)= 0.0
Y(51)= 0.0
Y(52)= 0.0
Y(53)= 0.0
Y(54)= 0.0

THE VALUE OF THE OBJECTIVE FUNCTION AT THE OPTIMAL POINT IS -3636.80933

Note: The total cost = const. cost term + objective function
value given above.

= \$(11947.08 - 3636.81)

= \$8310

OPTIMAL REGIONAL WATER QUALITY MANAGEMENT BY
AT-SOURCE TREATMENT AND EFFLUENT CHARGES

by

ANIL MITAL

B.E. (Mech.)(I.O.)(Hons.), Allahabad University,
Allahabad, India, 1974

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1976

The problem considered in this report is to find the optimal at-source treatment policy of the individual polluters for regional water quality management such as a basin wide river. Two decision models are considered. First one is the centralization model (basin wide firm), related to the operation of a basin wide firm which owns and operates all waste and water treatment plants and carries on all water based activities within a particular region. The objective of the basin wide firm being the maximization of net profits. Second one is the decentralization model which assumes that the central authority knows the quality standards derived and the low flow augmentation cost function while the polluters know the individual treatment cost functions. In the decentralization model the central authority gives a choice to the polluter to either treat his waste or pay tax proportional to the damage caused by his untreated waste. In this case the objective of the central authority is to minimize the total cost, treatment cost and augmentation cost, of meeting the required standards to the society.

The mathematical formulation of the problem belongs to the quadratic programming, separable programming and the combination of separable programming and linear programming, which are used to find the optimum decision.

In order to demonstrate the approach, 58-mile section of the Miami River basin in Ohio is considered, which has 15

pollution sources and 27 reaches.

It was demonstrated that it is possible to find the optimal at-source treatment configuration, for the basin wide firm, to maintain the required standards. It also demonstrates that an optimal taxation scheme can be built for the decentralization model, in a systematic and efficient manner.