

AVAILABILITY MODELS OF MAINTAINED SYSTEMS

by

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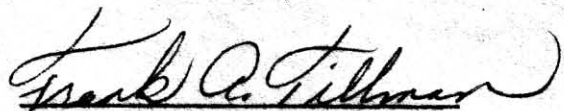
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Chapter 1

INTRODUCTION

The relatively new field of reliability engineering has been developed primarily due to the complexity, sophistication and automation which characterizes large scale systems for both military and commercial operations. The problems of field failures, repair and maintenance, became critical for military equipment used in World War II. The study of reliability in the sense we know it today began in military, industrial and space flight applications in the late 1940s and early 1950s. Reliability studies and practices were first applied as a result of the complexity of electronics and control systems in the fields of communication and transportation. The low percentage of success for the first guided missile, the NIKE, in late 1951 was an example of where the concepts of reliability were required. The Radio Electronic and Television Manufacturers Association (now known as Electronic Industries Association) define reliability as follows, "Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered".

Now it has been reported that it costs the armed services in the US about \$2 per year to maintain every dollar spent on electronic equipment. These figures may well be typical of commercial operations as well. It is only in recent years that attention has been directed to the branch of reliability which deals with the design and implementation of proper maintenance policies. The literature available on maintenance policies is neither cohesive nor standardized and hence a unified

presentation would serve a great need.

One principle area of interest in this work is to study the design, control, synthesis and improvement of corrective and preventive maintenance policies from a systems viewpoint. According to an old adage "you can't test or inspect reliability into a product; it has to be designed in". So a cradle to the grave responsibility for reliability needs to be assumed starting from the design stage through the operational stage. When maintenance is available, an appropriate measure of reliability is needed which takes into account the duration of repairs as well as the frequency of failures, this is called "availability". Other measures of reliability exist but availability seems to incorporate the most important considerations into a single measure.

A procedure is described where availability is used to determine an optimum system. A model is developed for the availability of a system consisting of stages, where each stage has identical units in parallel. The policy considered is one where corrective maintenance is performed when the system fails and preventive maintenance is performed after a fixed period of time. The exponential distribution is assumed for failure and repair times. The parameters for the model include the failure and repair rates for the units in each stage, the mission time, and the preventive maintenance period. The same approach can be used to develop availability models for systems with different configurations. The costs included in the model are:

- a. The cost for designing failure and repair rates
- b. The cost for corrective maintenance
- c. The cost for preventive maintenance

The optimum availability problem is basically a nonlinear programming one and the optimization method used for solving it is the Sequential Unconstrained Minimization Technique (SUMT). The difficulties in using the first order and second order derivatives of the complex expression for availability was bypassed by the use of the modified method of SUMT developed by Lai [42]. The program is used to determine the value of parameters which will minimize total cost of the system subject to an availability constraint. Additional constraints are included to keep the parameters within specified upper and lower bounds. The mission times are preselected.

Chapter 2

BASIC CONCEPTS

2.1 INTRODUCTION

Reliability engineering is concerned with the study of random or chance and wearout failures and the prevention, reduction, or complete elimination of them. Reliability is defined as the probability that a system will perform satisfactorily for a given period of time when used under stated conditions. The simplest and most common failure probability density function used is the exponential distribution $f(t) = \lambda e^{-\lambda t}$ where λ is the failure rate in failures per hour. The reliability function $R(t)$ is given by

$$R(t) = \int_t^{\infty} f(x) dx = e^{-\lambda t} \quad (2.1)$$

According to Rohn [56], when maintenance is always obtainable, an appropriate measure of reliability should take into account the duration of repairs as well as the frequency of failures. Consider the probability that at least one of the channels of a multichannel piece of equipment will operate for a specified period of time. If the distribution of repair times is not included, the usual reliability expression does not account for the fact that both channels could fail during the period and yet with short repair times the channel down times might not overlap. Also, the fraction of time during which at least one channel is expected to operate is dependent upon both the operating time and repair time. Hence one or more measures of system effectiveness seems appropriate. The principal measures of system reliability and effectiveness

are

1. Availability
2. Probability of Survival
3. Mean Time To Failure
4. Duration of Single Downtimes
5. Maintainability
6. Dependability
7. Mission Availability

Sometime these measures are combined with other system measures into a single measure of system effectiveness.

2.2 AVAILABILITY

This is the measure of primary concern to us and is applicable to maintained systems. By definition, "availability" is the probability that the system is operational at any time during the mission period. There are several categories of measures of availability which are used today. These are

- a. Instantaneous Availability, $A(t)$: the probability that the system will be operational at any random time t .
- b. Average Up Time or Average Availability, $A(T)$: the probability that the system is operational over a specified interval and is computed as the proportion of time in that interval that the system is operational.
- c. Steady State Availability, $A(\infty)$: the probability that the system is operational when the time interval considered is very large and is computed as the proportion of time that the system is operational.

These three availability measures are shown in Figure 2.1 and are defined as

a. Instantaneous availability, $A(t)$

b. Average Uptime, $A(T) = \frac{1}{T} \int_0^T A(t) dt$ (2.2)

c. Steady State Availability, $A(\infty) = \lim_{T \rightarrow \infty} A(T)$. (2.3)

If the time to failure and repair times are exponentially distributed with parameters λ and μ respectively for a single component, then

$$A(\infty) = \frac{\mu}{\lambda + \mu} . \quad (2.4)$$

The total mission time usually includes the operating time, the active repair time, the administrative time and the logistic time. If the administrative time and the logistic time are excluded, ARINC [70] refers to this as intrinsic availability.

For definitions of the other measures of system reliability effectiveness see Appendix I. Also covered there is, a discussion of failure rate and the choice of the distribution for failure and repair time.

2.3 CORRECTIVE AND PREVENTIVE MAINTENANCE

At one time or another all recoverable systems are subject to some form of maintenance. In general, there are two categories of maintenance actions. The first is off-schedule or corrective maintenance and is performed whenever there is an inservice failure or malfunction. The system operation is restored by replacing, repairing or adjusting the component or components which caused the interruption of service. The second category

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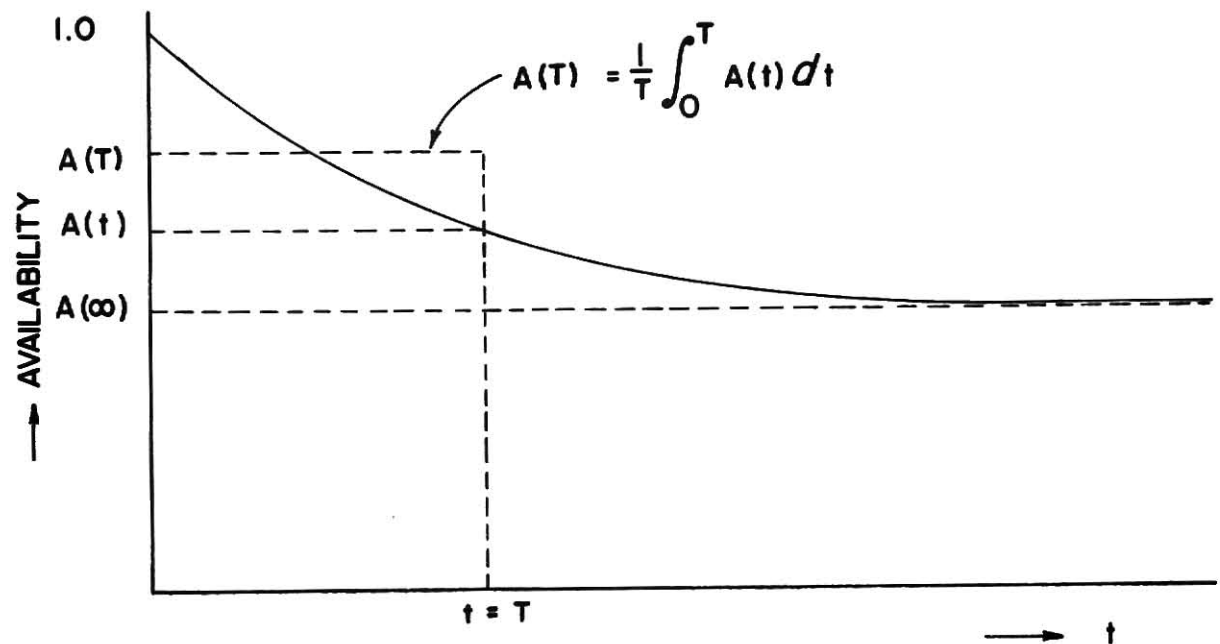


Figure 2.1. Availability versus time graph showing the three availability measures.

is the scheduled or preventive maintenance and is performed at regular intervals to keep the system in a condition consistent with its built in levels of performance reliability and safety. According to Bazovsky [11], during preventive maintenance, servicing, inspection, minor and major overhauls are done such that

- "1. regular care is provided to normally operating subsystems and components which require such attention (lubrication, refueling, cleaning, adjustment, alignment etc.)
2. failed redundant components are checked, replaced, or repaired if the system contains redundancy, and
3. components which are nearing a wearout condition are replaced or overhauled."

Though it appears that overhauls should be scheduled and accomplished to achieve the desired system reliability effectiveness, other factors affect the establishment of proper times between overhauls as pointed out by Riddick [55]. Briefly these factors are

1. the age, mission and performance requirements of the system,
2. the cost of overhauls in comparison with available budget funding,
3. the modernization program to update capabilities,
4. the availability of supplementary repair forces and spare parts.

2.4 RELIABILITY MODELS OF SYSTEMS WITH CORRECTIVE MAINTENANCE

Corrective maintenance is concerned with putting the system back into operation after it has failed either through random or wearout

failure. Let us consider the problem of developing mathematical models for the reliability of systems that can be maintained while in use. We shall employ a Markovian approach for describing stochastic behavior under a variety of failure and repair conditions. To generalize the situation, let us assume that the outcome on any trial depends upon the outcome of the directly preceding trial so that a conditional probability needs to be associated with every pair of outcomes. We also introduce space and time concepts. A space of possible states for example need to be defined and how transitions are made over a sequence of trials. We are interested in processes that are continuous in time and discrete in space. This approach was developed by Barlow and Hunter [6,7] and Epstein and Hosford [22].

Let us assume that an individual piece of equipment fails in accordance with the exponential distribution and that the times to repair are also exponentially distributed with parameters λ and μ respectively. The reason for selecting the exponential distribution for failure and repair times is that the lack of memory property is inherent and hence is a Markovian Process. A non Markovian approach would be needed if the conditional transition probabilities vary with time. Sandler [59] mentions that a full description of the reliability of a given system which can be maintained requires the following to be specified,

- "1. the equipment failure process
2. the system configuration
3. the repair policy
4. the state in which the system is defined to be failed".

A System with a Single Unit.

Suppose that we have a one unit system with a constant failure rate

λ and a constant repair rate μ . Let us define the following mutually exclusive states

State 0: The system is operating.

State 1: The system has failed and repairs have begun.

Let $P_i(t)$ for $i = 0, 1$ denote the probability that at time t the system is in state i . The expression for availability $A(t)$ is given by

$$A(t) = P_0(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} \quad (2.5)$$

and

$$P_1(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} \quad (2.6)$$

Complete details of the derivation may be seen in Rau [54] and Sandler [59].

The initial condition for this system was that at time 0 the system was in perfect condition, that is, $P_0(0) = 1$ and $P_1(0) = 0$. Somewhat different expressions for availability would be obtained if the initial conditions were different. Figure 2.1 shows a graph of $A(t)$ versus time t . With increasing time, the second term in the expression for availability becomes smaller and $A(t)$ approaches a steady state. For example for $\lambda = .01$ failures per hour, and $\mu = 1.0$ repairs per hour, the system reaches steady state in about 20 hours of continuous operation.

For the single unit system, if we are interested in the average up-time for a definite period of time T , we simply sum $A(t)$ over the time interval of interest and divide by the total time T (equation 2.2). In this instance we have

$$A(T) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2 T} - \frac{\lambda}{(\lambda + \mu)^2 T} e^{-(\lambda + \mu)T} . \quad (2.7)$$

To determine the long term or steady state availability of the system we can let $T \rightarrow \infty$ and find that

$$A(\infty) = \frac{\mu}{\lambda + \mu} . \quad (2.8)$$

In reliability literature one frequently finds availability defined as follows

$$A = \frac{MTTF}{MTTF + MRT} \quad (2.9)$$

where MTTF = Mean Time To Failure. For a single unit this is also sometimes called MTBF, Mean Time Between Failures

MRT = Mean Repair Time

Expressions (2.8) and (2.9) are the same since

$$1/\lambda = MTTF$$

$$1/\mu = MRT.$$

The availability expression for a single unit having Weibull distributed times to failure and repair can be seen in Appendix 1.

2.5 RELIABILITY MODELS OF SYSTEMS WITH PERIODIC MAINTENANCE

Preventive maintenance is usually associated with wearout failures. It is a particular category of maintenance designed to optimize the related concepts of reliability effectiveness and the costs that accrue when a system needs to be repaired. Preventive maintenance policies consist of some action depending upon either the operating age of certain components in the system, the state of system degradation or the system

configuration. In the first case a preventive maintenance policy is usually some program for the planned replacement or repair of certain critical components after they have accumulated a given number of operating hours. In the second case the preventive maintenance policies are designed to minimize the time the system will spend in the degraded state. In the third case the preventive maintenance policies consist of periodic inspection and repair to increase the mean life of the system.

Planned replacements or maintenance actions are advantageous for systems and parts whose failure rate increase with time, or are less costly to replace or repair when operating than after failure. Planned replacements or maintenance actions are also advantageous for systems whose configurations are such that the probability density function exhibits a variability of failure times less than that of the exponential distribution. Under preventive maintenance policies it may be possible either to increase a piece of equipment's availability or reliability or to minimize the total cost of replacement and repairs. To replace an item before it has aged too much may be wise on one hand, but on the other hand excessive costs will be incurred if too frequent replacements are planned. Thus one of the most important maintenance problems is that of specifying a maintenance policy which balances the cost of failures against the cost of preventive maintenance actions in order to minimize total maintenance cost. Of course, some reliability effectiveness criterion will have to be satisfied.

To realize the maximum of trouble free life, the ideal preventive maintenance policy would be to replace or repair a unit just prior to

failure but this is next to impossible in practice. It is only in recent years that a concerted effort has been made to develop a general mathematical theory of optimal preventive maintenance procedures when the components and systems are subject to failures. Several factors must be weighed simultaneously to achieve a balance between the related concepts of reliability, availability and maintenance costs for any piece of equipment. The various factors that need to be considered are mentioned in ARINC [70] and are the following

- "1. Reliability and availability index and time duration desired;
2. the cost of an inservice failure;
3. the cost of preventive maintenance before failure;
4. the most economical point in the equipments life to effect this replacement; and
5. the predictability of the failure pattern of the equipment under consideration."

Zelen [81] has outlined the different type of preventive maintenance policies. In a strictly periodic policy we may replace or take a maintenance action exactly at the time of failure and after every fixed T hours. Or we may choose to perform preventive maintenance only T hours after the last failure was repaired or preventive maintenance performed which ever comes later. Again there could be a random periodic policy in which T is a random variable. A sequentially determined replacement or repair policy is one in which the replacement or repair interval is determined at each replacement or repair in accordance with the time remaining in the total mission time. The words replacement and repair are both used in context of a failed equipment on which a maintenance action

is done to restore it to a normally operating state.

A relationship was developed by Weissshaun [14] that gives the average hourly costs in terms of two costs K_1 and K_2 and the failure probability distribution, of the item. The model is as follows

$$A(\tau) = \frac{K_1 - (K_1 - K_2) G(\tau)}{\int_0^{\tau} G(t) dt} \quad (2.10)$$

where $A(\tau)$ = the average hourly cost

K_1 = the expected cost of an inservice failure

K_2 = the expected cost for preventive maintenance

$G(\tau)$ = the probability that a repaired unit will last at least τ hours before failure

τ = the fixed period for a preventive maintenance action.

The critical factor in arriving at a decision regarding preventive maintenance is the ratio of K_1 to K_2 . Let $K = K_1/K_2$. As K increases the lowest average hourly cost is realized by performing preventive maintenance as shown in Figure 2.2. The family of curves for various ratios of K_1 to K_2 is shown for a component exhibiting normal wearout. When $K = 1$ there is no advantage of preventive maintenance and the equipment should be allowed to run to failure. Preventive maintenance is advantageous when $K > 1$.

It may be mentioned that if there is no information regarding the failure distribution then the optimal policy is not to consider any preventive maintenance.

2.6 Variability of Failure Times and Preventive Maintenance.

Equipment failure distributions were studied by Cho [18] and he found

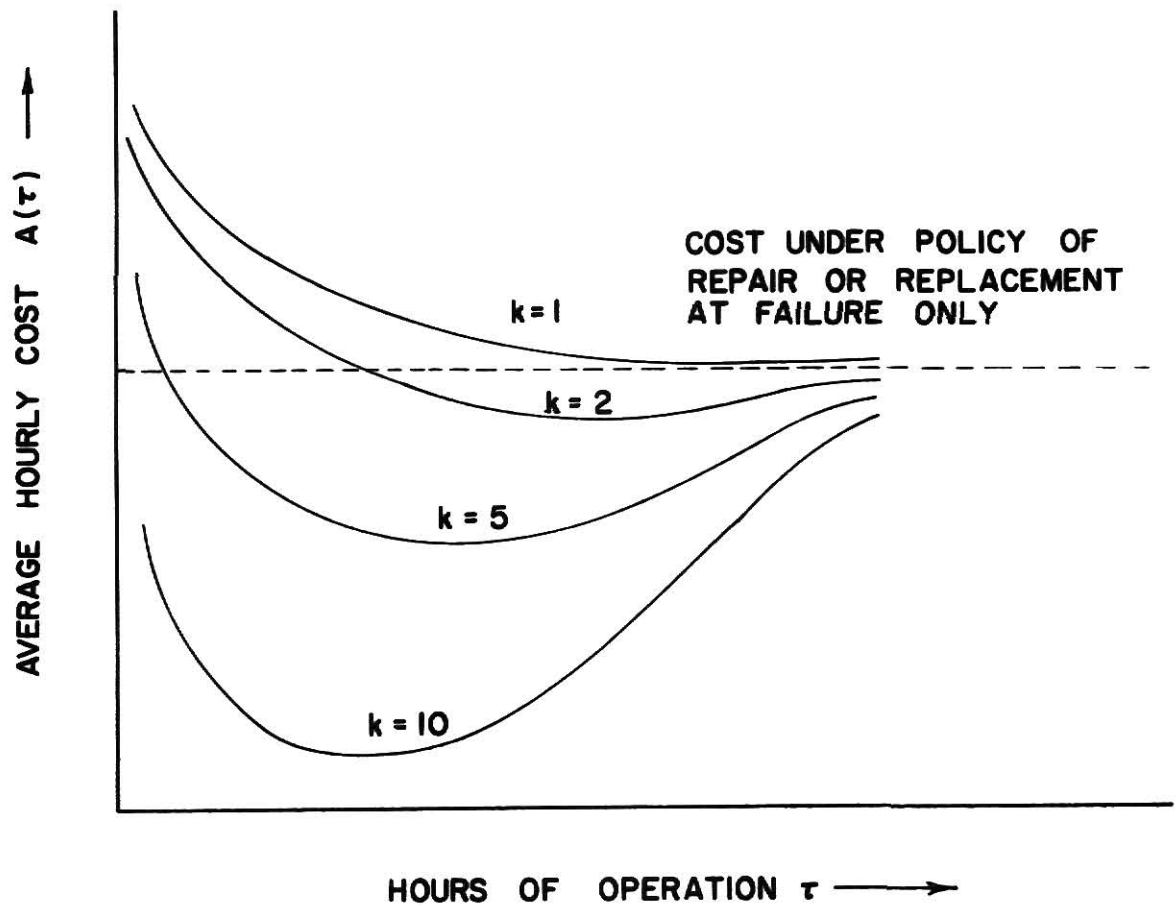


Figure 2.2. Average hourly cost of scheduled maintenance after τ hours of operation for a component exhibiting normal wear-out.

that profitability of preventive maintenance depends on one of the most important parameters which characterize any density function; namely variability, $\text{Var}(t)$. More specifically, the preventive maintenance schedule is worthwhile for that type of equipment which exhibits a probability density function of failure times with variability less than that of the exponential distribution. The variability of the exponential density function is $1/\lambda^2$ where λ is the equipment failure rate.

$$\text{Let } f(t) = \lambda e^{-\lambda t}$$

$$\begin{aligned} E(t) &= \int_0^{\infty} t f(t) dt \\ &= \frac{1}{\lambda} \Gamma(2) = \frac{1}{\lambda} \end{aligned} \quad (2.11)$$

$$\begin{aligned} E(t^2) &= \int_0^{\infty} t^2 f(t) dt \\ &= \frac{1}{\lambda^2} \Gamma(3) = \frac{2}{\lambda^2} \end{aligned} \quad (2.12)$$

$$\begin{aligned} \text{Var}(t) &= E(t^2) - [E(t)]^2 \\ &= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2} \end{aligned} \quad (2.13)$$

Next consider two identical units in parallel with each unit having an exponential failure distribution with parameter λ . If $R_s(t)$ is the reliability distribution of the system and $R_1(t)$ and $R_2(t)$ the reliability distributions for units 1 and 2, then

$$\begin{aligned}
 R_s(t) &= R_1(t) + R_2(t) - R_1(t) \cdot R_2(t) \\
 &= 2e^{-\lambda t} - e^{-2\lambda t}
 \end{aligned} \tag{2.14}$$

$$\begin{aligned}
 f_s(t) &= - \frac{dR_s(t)}{dt} \\
 &= 2\lambda e^{-\lambda t} - 2\lambda e^{-2\lambda t}
 \end{aligned} \tag{2.15}$$

$$\begin{aligned}
 E(t) &= \int_0^{\infty} t(2\lambda e^{-\lambda t} - 2\lambda e^{-2\lambda t}) dt \\
 &= \frac{2}{\lambda} - \frac{1}{2\lambda} = \frac{3}{2\lambda}
 \end{aligned} \tag{2.16}$$

$$\begin{aligned}
 E(t^2) &= \int_0^{\infty} t^2(2\lambda e^{-\lambda t} - 2\lambda e^{-2\lambda t}) dt \\
 &= \frac{4}{\lambda^2} - \frac{1}{2\lambda^2} = \frac{7}{2\lambda^2}
 \end{aligned} \tag{2.17}$$

$$\begin{aligned}
 \text{Var}(t) &= E(t^2) - [E(t)]^2 \\
 &= \frac{7}{2\lambda^2} - \frac{9}{4\lambda^2} = \frac{5}{4\lambda^2}
 \end{aligned} \tag{2.18}$$

For purpose of comparison of variability, let us assume we have a single unit with the same mean life as that of the system having two identical (Exponential)

units in parallel, then the variability of the single unit with a failure rate $\frac{2\lambda}{3}$ is

$$\text{Var}(t) = \frac{1}{\left(\frac{2\lambda}{3}\right)^2} = \frac{9}{4\lambda^2} . \quad (2.19)$$

Since $\frac{9}{4\lambda^2} > \frac{5}{4\lambda^2}$ the variability of the system is less than the variability of the one with the exponential failure distribution, hence preventive maintenance is worthwhile.

Let us next consider another system consisting of eight functional subsystems connected in series. Each subsystem has approximately the same failure rate and is individually characterized by the exponential failure distribution. Since the subsystems are in series, each of them needs to operate satisfactorily in order for the system to operate satisfactorily. Let us assume that failure can occur only at one subsystem at any random instant of time. The system can be described by an Erlang ($K = 8$) density function composed of $K = 8$ exponential subfunctions. The Erlang density function is given by

$$f(t) = \frac{(K\lambda)^K (\lambda t)^{K-1} e^{-K\lambda t}}{(K-1)!} \quad (2.20)$$

Further it can be proved that

$$R(t) = e^{-K\lambda t} \sum_{r=0}^{K-1} \left(\frac{K\lambda t}{n!}\right)^n \quad (2.21)$$

$$E(t) = \frac{1}{\lambda} \quad (2.22)$$

$$\text{Var}(t) = \frac{K}{(K\lambda)^2} = \frac{1}{K\lambda^2} \quad . \quad (2.23)$$

Since $\frac{1}{K\lambda^2} < \frac{1}{\lambda^2}$ for $K = 8$

the system variability of times to failure is less than that of the equivalent exponential case with the same mean time to failure; preventive maintenance is worth considering.

Three curves which are markedly different in their shapes are considered in Figure 2.3. The significance of their variability upon the shape of the curves is revealed. Density functions a, b, and c, represent hyperexponential, exponential and Erlang $K > 1$ respectively. Variability in times to equipment failure is the greatest for the case of curve a, then b and c in descending order. Variability is also closely related to failure rates. When the equipment failure rate is known to be decreasing over some period, preventive maintenance is not worthwhile since this corresponds to the case of greater variability than that of the exponential function. Table 2.1 outlines cases where preventive maintenance is advisable and is taken from Cho [18].

TABLE 2.1 Applicability of Preventive Maintenance

Functions and Parameters	Applicability	Preventive Maintenance		
		Advisable	Not Advisable	Borderline Case
Failure rate		increasing	decreasing	constant
Variability		less than exponential variability	greater than exponential variability	exponential variability
Applicable density function		Normal Gamma $\alpha > 1$, $\beta = 1$ Weibull $\beta > 1$, $\alpha = 1$ Erlang $K > 1$	Gamma $\alpha < 1$, $\beta = 1$ Weibull $\beta < 1$, $\alpha = 1$ Hyperexponential	Gamma $\alpha = 1$, $\beta = 1$ Weibull $\beta = 1$, $\alpha = 1$ Erlang $K = 1$ Exponential

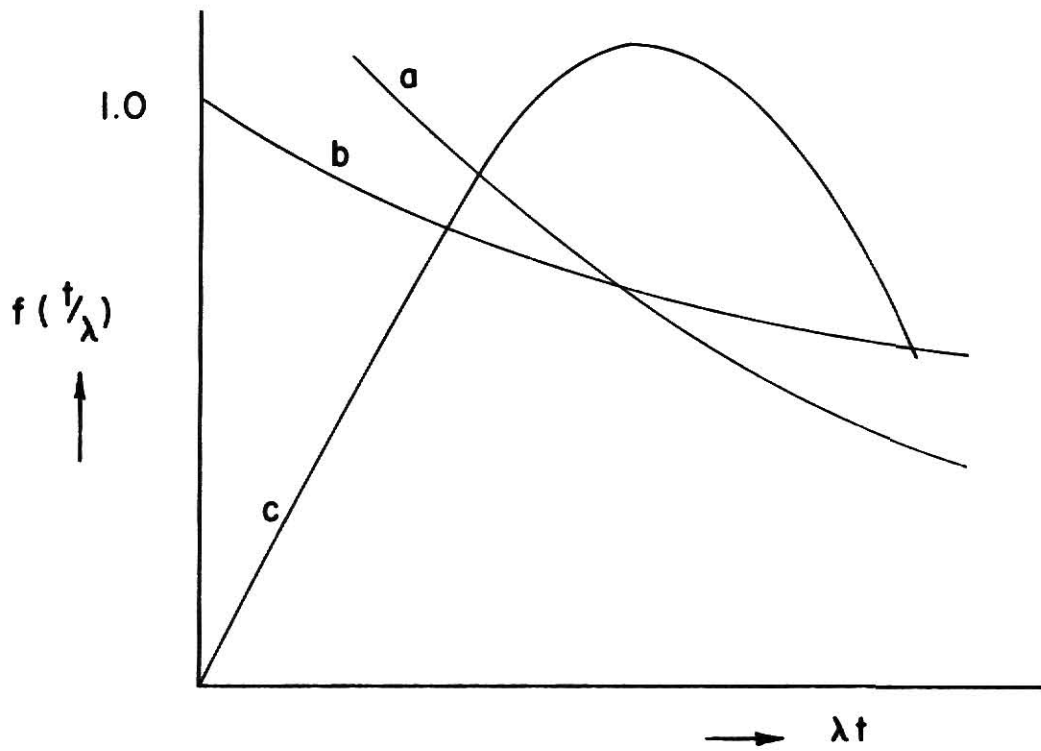


Figure 2.3. The function $f(t/\lambda)$ versus λt graph for the following distributions a) hyper-exponential b) exponential c) Erlang $k > 1$

Chapter 3

LITERATURE SURVEY

The Markovian approach in the formulation of reliability models for systems with repair was developed by Barlow and Hunter [6,7] and Epstein and Hosford [22]. Laplace transforms are used to solve the set of differential difference equations. Systems with repair indicate that corrective maintenance actions upon failed components are possible. The system are assumed to have a stochastic behavior under a variety of repair and failure conditions. This means that repair times and failure times are exponentially distributed in order to have the Markovian no memory property. Davis [19] shows that under the usual conditions of operation of equipment composed of many component parts of various types, the operating time between failures are ordinarily found to be exponentially distributed. Hall et. al. [32] have shown that after long periods of time all redundant systems behave as if their individual components had exponential failure and repair times. Rohn [56], on the assumption of exponential repair times, developed models for availability. Later on he conducted Monte Carlo simulation studies with repair times having non exponential distributions and found that the tests agreed closely with the desired expressions in which an exponential repair time was considered. Most reliability analysis permits the use of exponential distributions for repair and failure times for reasons mentioned earlier. Subsequently a variety of models utilizing a Markovian approach and having availability as the measure of system reliability effectiveness

were developed for various system configurations. Models for systems with a single unit, units in parallel and standby, and units in series under different repair policies may be seen in Sandler [59], Shooman [62] and Rau [54]. Rau has pointed out that series systems with repair offer no increase in reliability, however if the objective is to keep operating as much as possible during a specified interval then repair is valuable.

Based on the approximation $e^{-\lambda dt} \approx 1 - \lambda dt$ Rohn [56] presented a different approach in the derivation of expressions for availability of a system with channels in parallel. Since standby channels are often operated at reduced stress levels and are not completely shut down, he introduced a factor k to be associated with the transition probabilities. k ordinarily has a value between 0 and 1; when $k = 1$ it is assumed that the standby channel is operated exactly as the operating channel but the output is not used; when $k = 0$ the standby channel is shut down. Feller [25] derived an expression for an r -channel system with individual k factors employed for the standby channels. In this expression, only the average effect of overlapping repair periods were predicted but Stein and Johnsen [63] determined the distribution of these periods. It was shown that if switching time was considered, then degradation due to increased switching time overshadows the gain from the increased lifetime of the standby channel which does not operate. Even though the switching time is considerably less than the repair time, the probability that any instant will fall during a switching period is of the same magnitude as the probability that the system has failed since the occurrence of switching is more frequent than that for simultaneous repair periods. Kneale [40]

also has done some work in developing expressions for reliability of parallel systems with repair and switching.

McGregor [46] has developed good approximation formulas for availability of systems with repair and Arms and Goodfriend [3] have provided graphs and tables to obtain quick estimates of reliability measures. Gaver [30], Muth [50] and a host of others have done extensive research in the analysis of models for systems with repair. Henry [35] has studied the same problem with emphasis on weapon systems. Lewis and Gray [44] have put confidence intervals on availability. Garg [29], Finkelstein and Schafer [28], and Wohl [79] have preferred to develop models for systems with repair using dependability as a measure of system reliability effectiveness instead of the more usual availability. Weinstock [71] and Heenan [34] have elaborated on the use of matrix algebra in the development of these type of models. Since availability is composed of two factors reliability and maintainability, Westland and Hanifax [77] have shown trade off procedures between these two to determine optimum combination which (a) maximizes availability for a given dollar outlay or (b) minimizes cost for given levels of availability. The model developed in this thesis is capable of doing both and the outlook is more general in nature as will be seen later.

Due to the complexities involved, not much work has been done where the failure and repair times of the system are non-exponential. Hall et. al. [32] have investigated the development of reliability formulas for redundant configurations when failure and repair times follow combinations of the exponential, Weibull and Log normal distributions. As an

alternative to evaluating the inverse Laplace transformation needed to solve the set of difference-differential equation, the half-range Fourier cosine series was used. This method is used in computer programs.

Leibowitz [43] has developed a model for a two element redundant system with generalized repair times. Wohl [79] has developed expressions for availability of a single unit system when times to failure and repair have a Weibull distribution.

By incorporating an appropriate blend of engineering and mathematical analysis, Faragher and Watson [24] have developed highly flexible simulation techniques for availability analysis of a number of complex systems. Earlier analysis utilizing Monte Carlo simulation techniques had revealed varying degrees of lack of realism. For example the simulation was inflexible with respect to, say, configuration changes thus making it unsuitable for study of optimization of availability through equipment redundancy. Others had concentrated on mathematical aspects of the simulation and neglected the engineering aspects that are essential to obtaining a realistic evaluation of availability. An important aspect of computer simulation of models is that it yields the range of values that would occur with any desired confidence.

Up to this point, the discussion has centered on reliability models for systems with corrective maintenance. The effects of preventive maintenance along with corrective maintenance on reliability models will now be discussed. The earliest documented approach to planned replacement problem was made by Campbell [16] although previous investigations in inventory theory had posed similar questions. Campbell was concerned

with the problem of replacement of light bulbs either en masse, or as they failed. Since the treatment did not include some of the general results of renewal theory it does not have wide applicability. Campbell's problem differs from most problems of current interest in that he does not require immediate replacement to be made when failure occurs. Welker [75, 76] developed a method for determining optimum replacement intervals for certain vacuum tubes. He too was concerned with mass replacement and it was not possible to use interpolation with the plotted results. Kolner [41] developed a working method for determining optimum mandatory overhaul ages for aircraft engines; in this method, the life characteristics are introduced through the failure rate and mean life of the failed engines both as functions of mandatory overhaul age. However his assumption that repaired engines run to overhaul age is contradicted by experience. He also assumes that life characteristics are linear so that the method is useful for an extremely limited range of the overhaul age, Taylor [64] has suggested a criteria for determining whether a specific small increase in mandatory overhaul age is economically justified. Zelen [81] has given a description of how this regenerative maintenance problem was treated by Savage. Savage supposes that it is desired to replace a set of elements at a sequence of times $\{t_i\}$, that the cost of replacing these elements is A and that $F(T)$ is a loss function if the time between two successive replacements is T . The cost function is expressed in units of cost and is related to the probability of failure, or deterioration of the elements during the operating interval. The average loss per unit time defined by

$$C = \lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n [A + F(t_i - t_{i-1})]}{t_n}$$

is used to find an optimal sequence to minimize C . Under the assumption that $F(x)$ is continuous, non-negative, non-decreasing and $F(0) = 0$ he has shown that C assumes its minimum value. A relationship developed by Weissbaum [74], giving the average hourly cost in terms of the expected costs for scheduled and unscheduled maintenance actions, has been described in the previous chapter. The optimal preventive maintenance policy for a model consisting of the same factors was treated by Barlow and Proschan [8] for the case of finite and infinite time horizons. The finite time horizon is appropriate for items which become obsolete. Welker [75,76] has developed a model along similar lines for a normal failure density function with a small coefficient of variation. The coefficient of variation is the standard deviation divided by the mean and the range of interest is between 0.2 and 1.0. Due to the complexity of the equation he has employed a graphical solution. The nature of the curve for average hourly cost obtained by Welker, Barlow and Proschan, and Weissbaum are similar.

Most theoretical work to date has been done on periodic maintenance policies assuming an infinite usage horizon. The general form of the expected cost as a function of time is as follows

$$C(t, \Delta) = C_1 E \{N_1(t, \Delta)\} + C_2 E \{N_2(t, \Delta)\}$$

where C_1 is the cost of preventive maintenance or replacement, C_2 the

cost of corrective maintenance, ($C_1 \leq C_2$), $N_1(t, \Delta)$ is the number of preventive maintenance actions in time t , $N_2(t, \Delta)$ is the number of failures in time t , and Δ is the maintenance period which must be determined.

Barlow and Proschen [9] have outlined a theory of sequential replacement policies for the case of a finite time horizon. Policies that required, after each preventive maintenance action, the selection of the subsequent interval to minimize expected expenditure during the remaining time were more effective than a fixed period policy. They have further shown that for an infinite time horizon there always exists a strictly periodic maintenance policy which is superior to a random policy. Bell, Kamins and McCall [12] have conducted similar studies and have obtained specific replacement policies for parts which fail according to one of the following distributions: the normal, log normal, and Weibull. In almost all these replacement studies only deterministic models have been developed for which it is assumed that the performance of the machine can be exactly predicted before it is placed in operation. Eisen and Leibowitz [21] considered the performance as a random function of time which is more realistic since every machine has its own individual characteristic which to a large degree depends on the treatment it has received and the way it has been employed. One of the shortcomings of approaches of these types is that repair times are not taken into explicit consideration. Consequently the availability of the system has not been considered and cost is the only criteria for determining the optimum time period for preventive maintenance.

Barlow and Hunter [4] have also considered maintenance down time and derived an integral equation leading to the planned maintenance interval

which minimizes this time. The minimization of some combination of cost and down time has been analyzed by Weiss [73]. Rosenheim [57] has developed an expression for mean life under periodic maintenance which is as follows

$$\text{Mean life } \theta = \frac{\int_0^T R(x) dx}{1 - R(T)} .$$

where T is the periodic maintenance interval and R(x) is the reliability function for the system. He has shown that even when elements have constant failure rates an increase in mean life and reliability can be achieved by a preventive maintenance policy if redundancy exists. This is the basis for the development of the mathematical model for availability in this thesis.

The effects of scheduled maintenance on availability have been studied by Meyers and Dick [48] for a system with a number of pieces of equipment.

For at least "n" out of "a" pieces of equipment required for the system to be functioning

$$\text{Availability } A = t \left[\frac{\sum_{i=n}^{a-j} \binom{a-j}{i} p^i (1-p)^{a-j-i} + (T-t) \left[\sum_{i=n}^a \binom{a}{i} p^i (1-p)^{a-i} \right]}{T} \right]$$

where T = the time period between scheduled maintenance

j = the number of pieces of equipment taken down simultaneously

t = the number of hours with j pieces of equipment taken down

p = the availability of a single unit.

A similar problem, dealing with weapon systems where a major element of

effectiveness is the number of deployed systems which can be expected to perform their functions at any time has been treated by Althaus and Voegtlen [1]. The essential characteristics are a continuous alert status for a number of systems in which some equipment in each system need not be working, and an integral self test feature which monitors the status of some portion of all equipment.

Cho [18] has introduced the concept of distribution of prolongation and, based on it, formulated a preventive maintenance objective function and obtained solutions to maximize equipment availability.

If T_f = the mean time to failure of equipment

T_m = the expected time to repair. Repair may also consist of replacing the defective component by a statistically identical component.

T_p = the hours of preventive maintenance

T_a = the fixed time at which preventive maintenance is instituted after the last failure or the last preventive maintenance,

then the distribution of prolongation

$$U(x) = \frac{\int_0^{\infty} R(t') dt'}{\int_0^{\infty} R(t') dt'}$$

If

$$a = T_p/T_f, \quad b = T_m/T_f$$

and $R(t)$ = the reliability function.

then the availability, $A(T_a) = \left(1 + \frac{a R(T_a)}{1 - U(T_a)} + b \cdot \left[\frac{1 - R(T_a)}{1 - U(T_a)} \right] \right)^{-1}$.

Morse [49] has further shown that T_a^* which optimizes availability is the solution of the following equation

$$\frac{T_p}{T_m} = 1 - \frac{R(T_a)}{[f(T_a) \int_0^{\infty} R(t') dt'] [1 - U(T_a)] + [R(T_a)]^2}.$$

He has also tabulated values for $f(t)$, $R(t)$ and $U(t)$ necessary for plotting the curve of the right hand side of the above equation, with T_p/T_m along the Y axis and T_a/T_f along the X axis. This curve is used for obtaining the optimal mean time between preventive maintenance T_a^* which maximizes equipment availability. Using the same parameters Truelove [69] has developed an expression for operational readiness which is a similar measure as the average up time ratio defined earlier.

In all formulations considered so far the only information obtained from an inspection was whether or not the equipment was operative and a further condition was that a repair or replacement decision always returns the system to the same "as new" state. Klein [39] has departed from this and assumed that the deterioration of the system can be described as a discrete time, finite Markov chain and that the inspection procedure is capable of detecting which state the system is in at the time it takes place. His second assumption is that repairs, if made, can put the system in one of many possible states, the "as new" state being only one of many possible states. He has shown that for an average cost per unit time

criterion function, the problem of finding an optimal inspection - repair - replacement policy can be formulated in linear programming terms. In a similar situation Derman [20] has considered a system observed periodically and classified into one of a finite number of states. On the basis of these observations certain maintenance or replacement decisions are made. The problem is that of finding the decision rule which maximizes the expected length of time between replacements subject to the side conditions that the probabilities of replacement through certain undesirable states are bounded by prescribed numbers.

Optimal Availability for Redundant Systems

No one in the literature surveyed has developed a mathematical model to determine the maximum availability for a system consisting of two identical units in parallel, with the failure rate, repair rate, and time period for preventive maintenance, given. In addition no one has included the total cost for designing these failure and repair rates, the cost for preventive maintenance and the cost for corrective maintenance in a single model. A model which permits the inclusion of all the above has been developed in this thesis. The time between preventive maintenance actions is considered to be fixed since this is the case most often encountered in practice. The failure and repair distributions are both assumed exponential since the approach is basically Markovian. Though this limits the model to some extent, exponential failure and repair distributions are very common and also serve as good approximation for other distributions. The techniques and logic used in developing this model may easily be extended to models for systems with different configurations.

So much so for the development of models for reliability and availability of systems with corrective and preventive maintenance. To know values of system reliability effectiveness given the various parameters is not the real problem. The real problem consists of determining the parameters from a design, redesign or operating point of view such that some measure like cost, weight etc. is minimized and at the same time the system reliability effectiveness requirements are also satisfied. Tillman and Liittschwager [68] solved the problem of optimizing systems reliability subject to constraints by using integer programming. When the subsystem and the components within the same subsystem are subject to more than two modes of failure Tillman [65] has again used integer programming to obtain the optimum number and location of redundant components. Tillman et. al. [66] has used the Sequential Unconstrained Minimization Technique (SUMT) for optimizing reliability of a complex system with nonlinear constraints. In the complex system, redundant units could not be reduced to a purely parallel or series configuration and Bayes' theorem was used to obtain the overall reliability. Shershin [61] has dealt with optimizing the simultaneous apportionment of the failure rate and repair rate by means of two techniques: Lagrange multipliers and Dynamic Programming; and indicated that computer usage is possible. For more details on the Lagrange multiplier methods the reader is referred to the paper of Everett [23]. On dynamic programming, a number of books and papers are available but the books written by Bellman and Dreyfus [13] and Nemhauser [51] adequately cover the subject. Wilkinson and Walvekar [78] have also used a dynamic programming formulation for allocating availability optimally to a multi component system.

Example Problem and Solution Technique

The example used in this thesis to reflect the proposed model consists of three subsystems having two identical units in parallel. The objective is to minimize the total system cost while maintaining a given level of availability and keeping all parameters within upper and lower bounds. This is a nonlinear programming problem and the technique used to solve it is the Sequential Unconstrained Minimization Technique (SUMT). Due to difficulties in taking first and second order derivatives of the objective function and constraints when gradient search techniques are used to solve the problem, another approach suggested by Hooke and Jeeves [37] is used. This second method has been incorporated into the SUMT technique by Lai [42] and this combined approach has been used in this thesis.

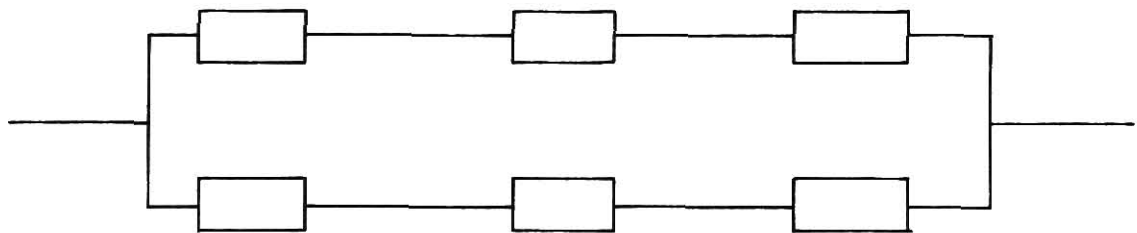
Chapter 4

DEVELOPMENT OF THE MODEL

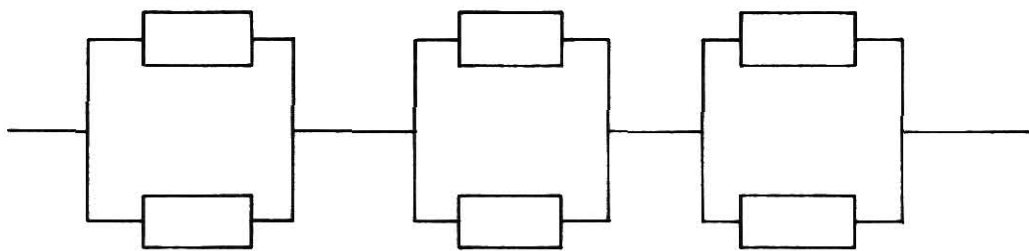
4.1 MARKOVIAN MODELS OF MAINTAINED SYSTEMS

Equipment connected in parallel redundant configurations simultaneously perform the same function, and generally the system will operate if at least one of the n units in parallel operate. Using less reliable units in redundant configurations is one of the methods of coping with the problem of designing reliable systems. For non maintained systems, redundancy is best applied at a component level rather than at a systems' level. Thus, in Figure 4.1, (b) is the best level of redundancy. However for systems whose components can be repaired as they fail, to have redundancy at the component level may not be the best policy. The reason is that if component redundancy is employed, repairs may not be possible while the system is operating whereas a failure of system redundancy, Figure 4.1 (a), could be repaired.

Let us consider a two component redundant system with two repairmen. The individual components fail according to the exponential distribution and the times to repair are also exponentially distributed with parameters λ_{ij} and μ_{ij} respectively, where i is the state it starts in and j is the state in which it ends. The four possible states for the system shown in Figure 4.2 when the status of individual components is monitored is as follows:



(a)



(b)

Figure 4.1. Redundancy levels. (a) System redundancy; (b) Component redundancy.

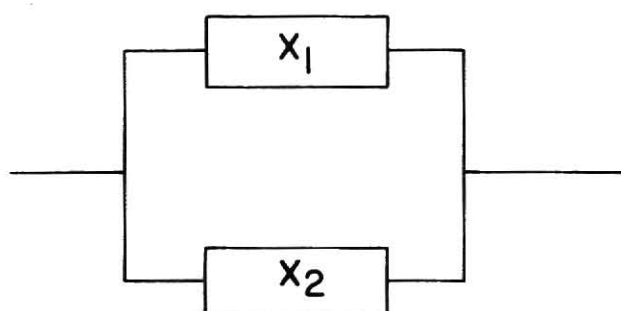


Figure 4.2. A system comprising of two units in parallel

- State 0: both units operational
- State 1: } one unit failed and under repair
 State 2: } and the other operating
- State 3: both units failed and under repair.

This is a continuous time discrete state model and for each unit the transition probabilities obey the following rules

1. The probabilities of transition in the interval $t, t + dt$ are $\lambda_{ij}dt$ in the case of failure and $\mu_{ij}dt$ in the case of repair where i and j are the two states in question. Since λ_{ij} and μ_{ij} are constant and are not functions of time, the model is called homogeneous.
2. The probabilities of more than one transition in time dt are of a higher order and hence can be neglected. The transition matrix is shown below and the Markov graph with transition probabilities is shown in Figure 4.3

The transition matrix is

		State			
		0	1	2	3
State	0	$1 - (\lambda_{01} + \lambda_{02})dt$	$\lambda_{01}dt$	$\lambda_{02}dt$	0
	1	$\mu_{10}dt$	$1 - (\mu_{10} + \lambda_{13})dt$	0	$\lambda_{13}dt$
	2	$\mu_{20}dt$	0	$1 - (\mu_{20} + \lambda_{23})dt$	$\lambda_{23}dt$
	3	0	$\mu_{31}dt$	$\mu_{32}dt$	$1 - (\mu_{32} + \mu_{31})dt$

(4.1)

For example if the system is in state 0 at a time t , it will remain there

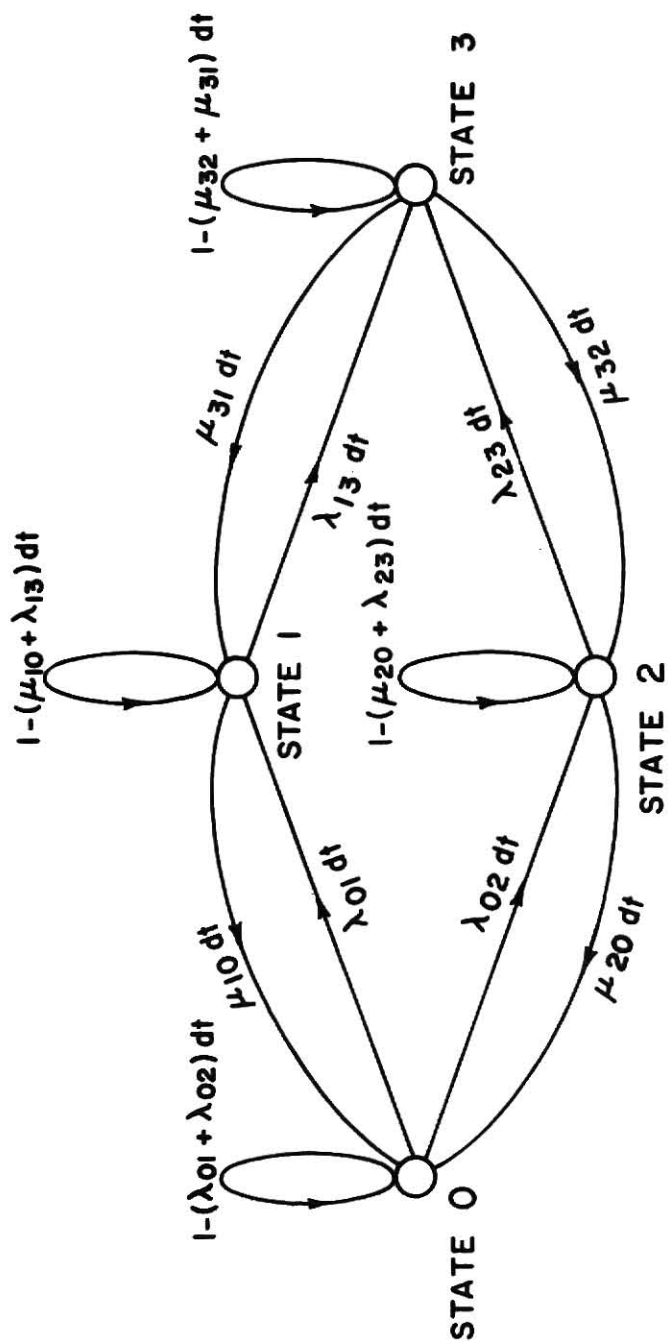


Figure 4.3. General Markov Graph for a system with two units in parallel and two repairmen

if neither units fail in the interval $t, t+dt$. The probability is

$$(1 - \lambda_{01}dt)(1 - \lambda_{02}dt) = 1 - (\lambda_{01} + \lambda_{02})dt + 0 dt.$$

Again if the system is in state 1 at time t it will remain there at the end of the interval $t, t+dt$ if x_2 does not fail or x_1 does not get repaired. The probability is

$$(1 - \mu_{10}dt)(1 - \lambda_{13}dt) = 1 - (\mu_{10} + \lambda_{13})dt + 0 dt$$

and similarly for the rest.

If we now assume that both the units are identical and that the transition probabilities are independent of the state of the system then

$$\lambda_{01} = \lambda_{02} = \lambda_{13} = \lambda_{23} = \lambda$$

$$\lambda_{01} + \lambda_{02} = 2\lambda$$

$$\mu_{10} = \mu_{20} = \mu_{31} = \mu_{32} = \mu$$

$$\mu_{31} + \mu_{32} = 2\mu$$

The model is then collapsed as shown in Figure 4.4 and the new states are

State 0: both units operating

State 1: one unit failed and under repair, the other operating

State 2: both units failed and under repair.

The transition matrix is as follows

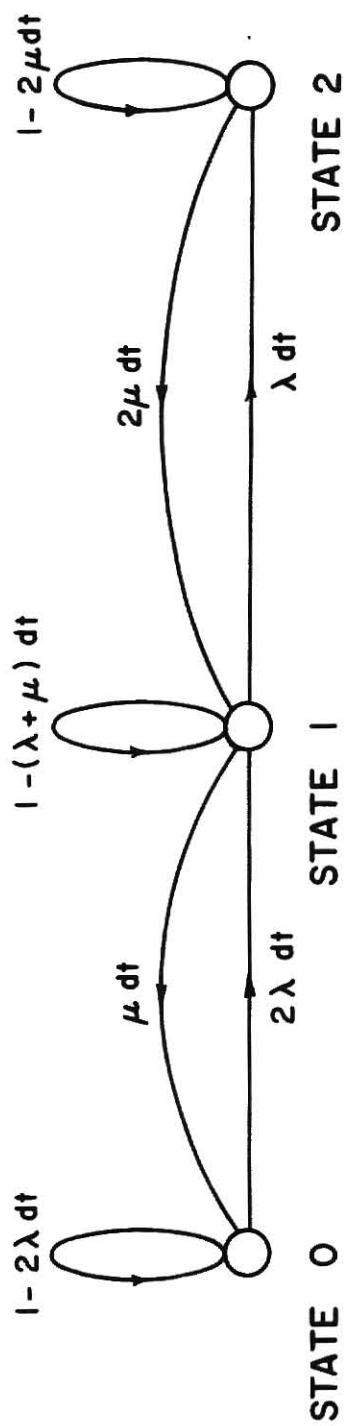


Figure 4.4. Collapsed Markov Graph for a system with two identical units in parallel and two repairmen.

		State			
		0	1	2	
State	0	$1 - 2\lambda dt$	$2\lambda dt$	0	(4.2)
1	μdt	$1 - (\mu + \lambda) dt$	λdt		
2	0	$2\mu dt$	$1 - 2\mu dt$		

The transition matrix leads directly to the system of linear homogenous differential equations which describe the stochastic behavior of this system and are as follows .

$$P'_0(t) = -2\lambda P_0(t) + \mu P_1(t)$$

$$P'_1(t) = 2\lambda P_0(t) - (\lambda + \mu) P_1(t) + 2\mu P_2(t) \quad (4.3)$$

$$P'_2(t) = \lambda P_1(t) - 2\mu P_2(t)$$

where $P_i(t)$ is the probability of being in state i at time t and $P'_i(t)$ is the first order derivative with respect to t . Shooman [62] has described a simple algorithm for writing the above equations (4.3) and it is to equate the derivative of the probability at any node to the sum of transitions coming into the node. Any unity gain factor of the self loops must first be set to zero and the dt factors are dropped from the branch gains. Let the system be in state 0 at time 0, then

$$P_0(0) = 1, \quad P_1(0) = 0, \quad P_2(0) = 0.$$

Taking Laplace transforms of equations (4.3)

$$SP_0(S) - P_0(0) = -2\lambda P_0(S) + \mu P_1(S)$$

$$SP_1(S) - P_1(0) = 2\lambda P_0(S) - (\lambda + \mu) P_1(S) + 2\mu P_2(S) \quad (4.4)$$

$$SP_2(S) - P_2(0) = \lambda P_1(S) - 2\mu P_2(S)$$

Using the initial conditions we obtain

$$\begin{aligned}
 (S+2\lambda) P_0(S) - \mu P_1(S) &= 1 \\
 -2\lambda P_0(S) + (S+\lambda+\mu) P_1(S) - 2\mu P_2(S) &= 0 \\
 -\lambda P_1(S) + (S+2\mu) P_2(S) &= 0
 \end{aligned} \tag{4.5}$$

and

$$P_2(S) = \frac{\begin{vmatrix} S+2\lambda & -\mu & 0 \\ -2\lambda & S+\lambda+\mu & 0 \\ 0 & -\lambda & 0 \end{vmatrix}}{\begin{vmatrix} S+2\lambda & -\mu & 0 \\ -2\lambda & S+\lambda+\mu & -2\mu \\ 0 & -\lambda & S+2\mu \end{vmatrix}} \tag{4.6}$$

where the numerator $= -2\lambda(-\lambda) = 2\lambda^2$ and

the denominator $= (S+2\lambda)[(S+\lambda+\mu)(S+2\mu) - 2\mu\lambda] + \mu(-2\lambda)(S+2\mu)$

$$\begin{aligned}
 &= (S+2\lambda)[S^2+2\mu S+\lambda S + \mu S + 2\mu^2] - 2\mu\lambda(S+2\mu) \\
 &= S(S^2 + 2\mu S + \lambda S + \mu S + 2\lambda S + 2\mu^2 + 4\lambda\mu + 2\lambda^2) \\
 &= S[S^2 + 3S(\lambda+\mu) + 2(\lambda+\mu)^2] \\
 &= S(S + 2\lambda + 2\mu)(S + \lambda + \mu) .
 \end{aligned}$$

Thus

$$P_2(S) = \frac{2\lambda^2}{S(S + 2\lambda + 2\mu)(S+\lambda+\mu)} . \tag{4.7}$$

Breaking this expression into partial fractions we obtain

$$\frac{2\lambda^2}{S(S+2\lambda+2\mu)(S+\lambda+\mu)} = \frac{A}{S} + \frac{B}{S+2\lambda+2\mu} + \frac{C}{S+\lambda+\mu} \quad (4.8)$$

(Let $a = \lambda + \mu$)

$$= \frac{AS^2 + 3aSA + 2a^2A + BS^2 + BSa + CS^2 + 2aSC}{S(S + 2a)(S+a)} .$$

Equating constant terms we have $A = \frac{\lambda^2}{(\lambda+\mu)^2} .$ (4.9)

Equating coefficients of S and S^2 we obtain

$$B = \frac{\lambda^2}{(\lambda+\mu)^2} , \quad (4.10)$$

$$C = - \frac{2\lambda^2}{(\lambda+\mu)^2} . \quad (4.11)$$

Hence $P_2(S) = \frac{\lambda^2}{(\lambda+\mu)^2} \cdot \frac{1}{S} + \frac{\lambda^2}{(\lambda+\mu)^2} \cdot \frac{1}{(S+2\lambda+2\mu)} - \frac{2\lambda^2}{(\lambda+\mu)^2} \cdot \frac{1}{(S+\lambda+\mu)}$ (4.12)

Taking inverse Laplace transforms,

$$P_2(t) = \frac{\lambda^2}{(\lambda+\mu)^2} + \frac{\lambda^2}{(\lambda+\mu)^2} e^{-2(\lambda+\mu)t} - \frac{2\lambda^2}{(\lambda+\mu)^2} e^{-(\lambda+\mu)t} \quad (4.13)$$

Since $P_2(t)$ is the probability of being in the failed state at time t ,
the availability at time t is given by

$$A(t) = 1 - P_2(t) = P_0(t) + P_1(t) \quad (4.14)$$

$$A(t) = \frac{\mu^2 + 2\lambda\mu}{(\lambda+\mu)^2} - \frac{\lambda^2 e^{-2(\lambda+\mu)t}}{(\lambda+\mu)^2} + \frac{2\lambda^2 e^{-(\lambda+\mu)t}}{(\lambda+\mu)^2} \quad (4.15)$$

From equation (4.15) we can obtain the steady state expression

$$A(\infty) = \lim_{T \rightarrow \infty} \int_0^T A(t) dt = \frac{\mu^2 + 2\lambda\mu}{(\lambda+\mu)^2} . \quad (4.16)$$

Instead of the parallel configuration, if we had the same two units in a series configuration and it was possible that while one failed component was being repaired, the remaining unit could fail, then we would have the same transition matrix and set of differential equations. It might be noted that in the series configuration if one unit failed then the system failed, but in the parallel configuration both units would have to be in the failed state for the system to fail.

In the two equipment parallel system with two repairmen, we might expect both of them to work together if one unit failed. However they would work independently if both units are failed. Thus we may have the case that if a single repairman services a failed unit, the repair rate is μ , but if two repairmen service the same failed equipment, the repair rate is $m\mu$. Sandler [59] assumes that two repairmen yield $m = 1.5$. If we further assume that when both repairmen are servicing a single unit and the second one fails, the second repairman immediately returns to service his own unit, then the transition matrix is as follows

		State			
		0	1	2	
State	0	$1-2\lambda dt$	$2\lambda dt$	0	} (4.17)
	1	$1.5\mu dt$	$1-(1.5\mu+\lambda)dt$	λdt	
	2	0	$2\mu dt$	$1-2\mu dt$	

If there are n elements in parallel and $k < n$ repairmen, a waiting line of failed components may build up if there are many failures in a brief period. Further details may be seen in Messinger [47]. In the previous two unit parallel system with two repairmen, failure of any unit was detected the instant it occurred. Very often this is not the case and the repair operation starts only when the entire system has failed. Let us consider the model in which only one unit is repaired if the system of two units in parallel fail due to failure of both units. It is only when preventive maintenance is undertaken that the system is restored to the state where both units are operating. There is only one repairman. The Markov graph is shown in Figure 4.5 and the transition matrix is

		State			
		0	1	2	
State	0	$1-2\lambda dt$	$2\lambda dt$	0	} (4.18)
	1	0	$1-\lambda dt$	λdt	
	2	0	μdt	$1-\mu dt$	

The differential equations are

$$\begin{aligned}
 P_0'(t) &= -2\lambda P_0(t) \\
 P_1'(t) &= 2\lambda P_0(t) - \lambda P_1(t) + \mu P_2(t) \\
 P_2'(t) &= \lambda P_1(t) - \mu P_2(t)
 \end{aligned}
 \tag{4.19}$$

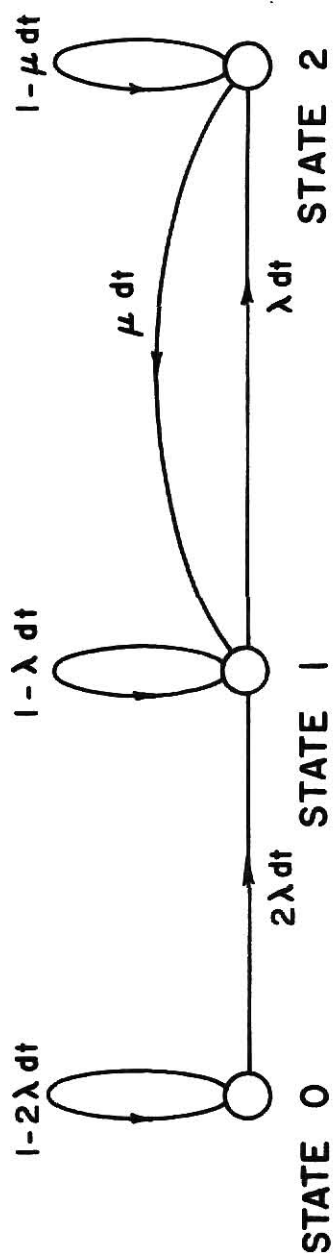


Figure 4.5. Markov Graph for a system with two identical units in parallel and one repairman, when only at system failure, one unit is repaired.

Taking Laplace transforms and using the initial conditions

$$P_0(0) = 1, \quad P_1(0) = 0, \quad P_2(0) = 0,$$

$$(S + 2\lambda) P_0(s) = 1$$

$$- 2\lambda P_0(S) + (S+\lambda) P_1(S) - \mu P_2(S) = 0 \quad (4.20)$$

$$- \lambda P_1(S) + (S+\mu) P_2(S) = 0$$

and

$$P_2(S) = \frac{\begin{vmatrix} S+2\lambda & 0 & 1 \\ -2\lambda & S+\lambda & 0 \\ 0 & -\lambda & 0 \end{vmatrix}}{\begin{vmatrix} S+2\lambda & 0 & 0 \\ -2\lambda & (S+\lambda) & -\mu \\ 0 & -\lambda & (S+\mu) \end{vmatrix}} \quad (4.21)$$

or

$$\begin{aligned} P_2(S) &= \frac{2\lambda^2}{S(S+2\lambda)(S+\lambda+\mu)} \\ &= \frac{\lambda}{\lambda+\mu} \cdot \frac{1}{S} - \frac{\lambda}{(\mu-\lambda)} \cdot \frac{1}{(S+2\lambda)} + \frac{2\lambda^2}{(\mu^2 - \lambda^2)} \cdot \frac{1}{(S + \lambda + \mu)}. \end{aligned} \quad (4.22)$$

Taking inverse Laplace transforms we obtain

$$P_2(t) = \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\mu-\lambda} e^{-2\lambda t} + \frac{2\lambda^2}{\mu^2 - \lambda^2} e^{-(\lambda+\mu)t}, \text{ and} \quad (4.23)$$

$$A(t) = 1 - P_2(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\mu - \lambda} e^{-2\lambda t} - \frac{2\lambda^2}{\mu^2 - \lambda^2} e^{-(\lambda + \mu)t}. \quad (4.24)$$

Now if in the system with two units in parallel and two repairmen, the status of the individual units is not monitored, repair will not begin until the system is in state 2 where both units have failed. We can define the four states with reference to the Markov graph shown in Figure 4.6 as follows:

State 0: both units are operating

State 1: one unit is operating, one failed and has not been detected

State 2: both units failed and under repair

State 3: one unit operating, one failed which is under repair.

The transition matrix is

		State			
		0	1	2	3
0		$1 - 2\lambda dt$	$2\lambda dt$	0	0
1		0	$1 - \lambda dt$	dt	0
State 2		0	0	$1 - 2\mu dt$	$2\mu dt$
3		μdt	0	λdt	$1 - (\mu + \lambda)dt$

(4.25)

The system of differential equations are

$$\begin{aligned}
 P_0'(t) &= -2\lambda P_0(t) && + \mu P_3(t) \\
 P_1'(t) &= 2\lambda P_0(t) && - \lambda P_1(t) \\
 P_2'(t) &= && \lambda P_1(t) - 2\mu P_2(t) + \lambda P_3(t) \\
 P_3'(t) &= && 2\mu P_2(t) - (\mu + \lambda)P_3(t).
 \end{aligned} \quad (4.26)$$

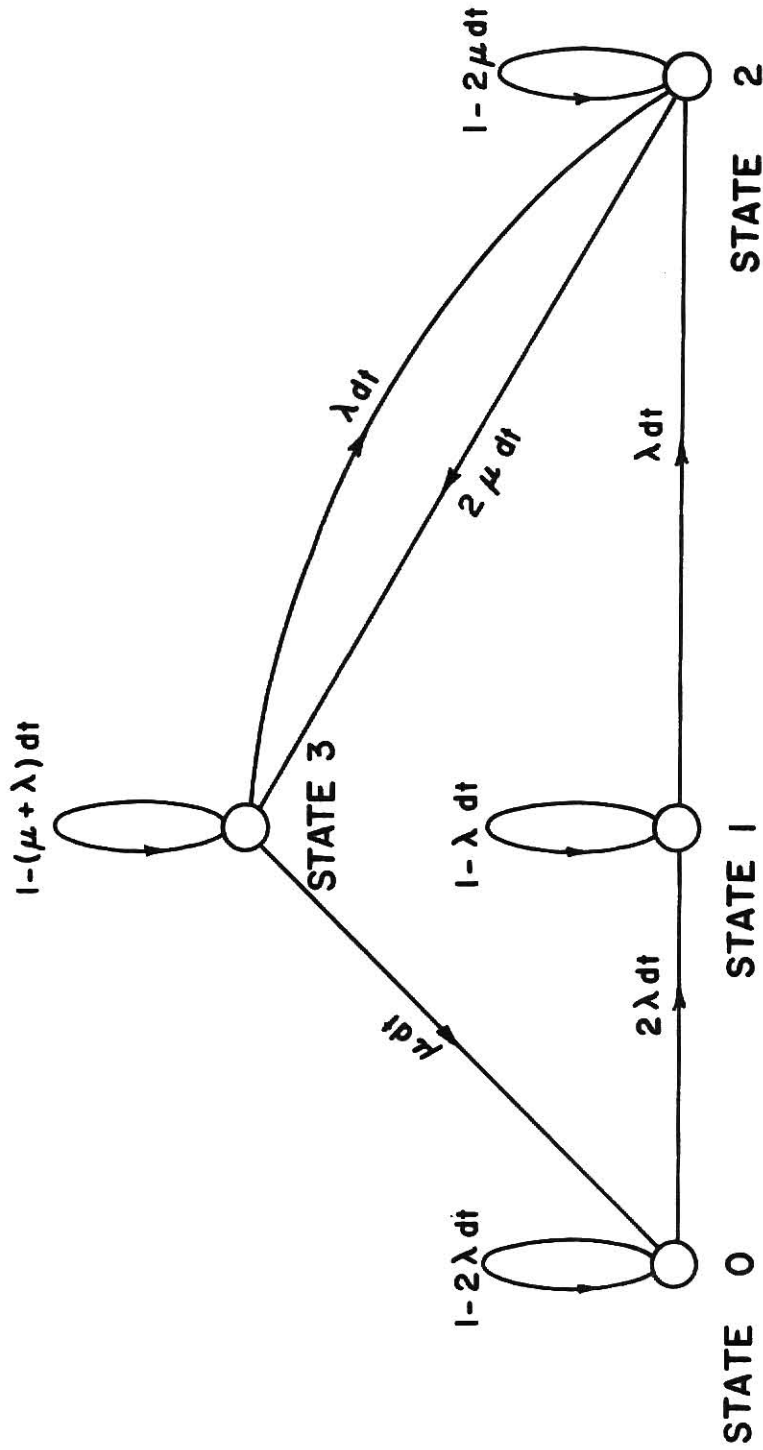


Figure 4.6. Markov Graph for a system with two identical units in parallel and two repairmen, when only at system failure, both units are repaired.

Taking the inverse Laplace transforms with the initial condition

$$P_0(0) = 1, \quad P_1(0) = 0, \quad P_2(0) = 0, \quad P_3(0) = 0, \text{ we have}$$

$$\begin{aligned}
 (S+2\lambda) P_0(S) - \mu P_3(S) &= 1 \\
 -2\lambda P_0(S) + (S+\lambda) P_1(S) &= 0 \\
 -\lambda P_1(S) - (S+2\mu) P_2(S) - \lambda P_3(S) &= 0 \\
 -2\mu P_2(S) + (S+\mu+\lambda) P_3(S) &= 0
 \end{aligned} \tag{4.27}$$

and

$$P_2(S) = \frac{
 \begin{vmatrix}
 S+2\lambda & 0 & 1 & -\mu \\
 -2\lambda & S+\lambda & 0 & 0 \\
 0 & -\lambda & 0 & -\lambda \\
 0 & 0 & 0 & S+\mu+\lambda
 \end{vmatrix}
 }{
 \begin{vmatrix}
 S+2\lambda & 0 & 0 & -\mu \\
 -2\lambda & S+\lambda & 0 & 0 \\
 0 & -\lambda & S+2\mu & -\lambda \\
 0 & 0 & -2\mu & S+\mu+\lambda
 \end{vmatrix}
 } \tag{4.28}$$

$$\text{where the numerator} = 2\lambda^2(S+\mu+\lambda) \text{ and} \tag{4.29}$$

$$\text{the denominator} = S(S+3\lambda) \{S^2 + S(3\mu+\lambda) + 2\mu^2\}. \tag{4.30}$$

The solution for the roots of $S^2 + S(3\mu+\lambda) + 2\mu^2$ yields

$$r_1, r_2 = \frac{-(3\mu+\lambda) \pm \sqrt{(3\mu+\lambda)^2 - 8\mu^2}}{2} \tag{4.31}$$

$$\text{hence } P_2(S) = \frac{2\lambda^2(S+\mu+\lambda)}{S(S+3\lambda)(S-r_1)(S-r_2)} \quad (4.32)$$

Breaking this expression into partial fractions

$$P_2(S) = \frac{A}{S} + \frac{B}{S+3\lambda} + \frac{C}{S-r_1} + \frac{D}{S-r_2} \quad (4.33)$$

The values of A, B, C, and D may be obtained from the following four equations:

$$\begin{aligned} 3\lambda r_1 r_2 A &= 2\lambda^2 (\mu+\lambda) \\ \{r_1 r_2 - 3\lambda(r_1+r_2)\}A + r_1 r_2 B - 3\lambda r_2 C - 3\lambda r_1 D &= 2\lambda^2 \quad (4.34) \\ (3\lambda - r_1 - r_2)A - (r_1+r_2)B + (3\lambda - r_2)C + (3\lambda - r_1)D &= 0 \\ A + B + C + D &= 0 \end{aligned}$$

By taking the inverse Laplace transforms, we obtain

$$P_2(t) = A + Be^{-3\lambda t} + Ce^{r_1 t} + De^{r_2 t} \quad (4.35)$$

and the availability is given by

$$A(t) = 1 - P_2(t). \quad (4.36)$$

Inspection of the quadratic equation for r_1, r_2 shows that r_1 and r_2 are always negative real numbers since λ and μ are always positive; therefore all the time functions are decaying exponentials and the instantaneous availability $A(t)$ rapidly converges to a steady state value.

Equation (4.35) is complex in nature due to r_1 and r_2 not having simple forms and consequently it is not easy to obtain the steady state availability from equation (4.36). But the steady state availability may be obtained by studying the steady state behavior.

Over a long period of time where it is possible to go from one state to another it can be shown for all cases that the limit of $P_i(t)$

$$P_i = \lim_{t \rightarrow \infty} P_i(t) \quad (4.37)$$

* always exists. This means that the steady state solutions can be found by setting the derivatives $P'_i(t)$ equal to zero. Then the system of differential equations reduces to a system of algebraic equations. The additional fact that P_i 's are a probability distribution and hence

$$\sum_{i=0}^n P_i = 1 \text{ needs to be used where } n \text{ is the number of possible states.}$$

So the same set of equations with the derivatives at each of the nodes set equal to zero need to be solved to obtain the steady state availability. The set of equations are

$$\begin{aligned} 0 &= -2\lambda P_0 && +\mu P_3 \\ 0 &= 2\lambda P_0 && -\lambda P_1 \\ 0 &= && \lambda P_1 && -2\mu P_2 && +\lambda P_3 \\ 0 &= && && 2\mu P_2 && -(\lambda+\mu) P_3 \\ 1 &= P_0 + P_1 && + P_2 && + P_3 \end{aligned} \quad (4.38)$$

Solving for P_2 using the last four equations,

$$P_2 = \frac{\begin{vmatrix} 2\lambda & -\lambda & 0 & 0 \\ 0 & \lambda & 0 & \lambda \\ 0 & 0 & 0 & -(\lambda+\mu) \\ 1 & 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 2\lambda & -\lambda & 0 & 0 \\ 0 & \lambda & -2\mu & \lambda \\ 0 & 0 & 2\mu & -(\lambda+\mu) \\ 1 & 1 & 1 & 1 \end{vmatrix}} = \frac{2\lambda(\lambda^2 + \lambda\mu)}{6\mu\lambda^2 + 2\lambda^3 + 6\lambda\mu^2}$$

$$P_2 = \frac{\lambda^2 + \lambda\mu}{\lambda^2 + 3\mu\lambda + 3\mu^2} \quad (4.39)$$

Likewise the steady state availability is

$$A(\infty) = 1 - P_2 = P_0 + P_1 + P_3 = \frac{2\mu\lambda + 3\mu^2}{\lambda^2 + 3\mu\lambda + 3\mu^2} \quad (4.40)$$

Many complex problems can similarly be solved in the steady state without much difficulty.

It is interesting to note that if the number of repairmen is equal to the number of pieces of equipment in a parallel configuration and a unit starts getting repaired the instant it fails, then each piece of equipment has a steady state availability which is independent of the others. Since the system is down when all the units are in failed states, if we let the steady state availability be $\mu/(\lambda+\mu)$, then for a two unit system

$$A(\infty) = P_0 + P_1 = 1 - (\text{Probability both are unavailable})$$

$$= 1 - \left(1 - \frac{\mu}{\mu + \lambda}\right)^2$$

$$= \frac{\mu^2 + 2\lambda\mu}{(\lambda + \mu)^2} \quad (4.41)$$

And in general when the independency condition is valid, the steady state availability that at least m out of n pieces of equipment will be available is

$$A(\infty) = P_0 + P_1 + \dots + P_m = \sum_{i=m}^n \binom{n}{i} \left(\frac{\mu}{\lambda + \mu}\right)^i \left(\frac{\lambda}{\lambda + \mu}\right)^{n-i} \quad (4.42)$$

When the number of repairmen is less than the number of components in parallel in the system, the independency condition is not true anymore. For example in the system with two units in parallel and a single repairmen the availability is given by

$$A(\infty) = \frac{\mu^2 + 2\lambda\mu}{\mu^2 + 2\lambda\mu + 2\lambda^2} \quad (4.43)$$

It has been shown by Saaty [58] for any n -equipment parallel redundant system with single repairmen

$$P_n = \frac{1}{\sum_{j=1}^n \frac{X^j}{j!}} \quad (4.44)$$

where $X = \mu/\lambda$,

and $A(\infty) = 1 - P_n$.

μ/λ is called the dependability ratio and is the inverse of λ/μ which is called service factor in queuing theory. It might be mentioned that in queuing theory this is called the one server problem with finite queue length.

When there is standby redundancy, the off-line equipment either cannot fail or have failure rates less than the on-line equipment, and hence we may assume that the system would have availability greater than that for a similar parallel redundant system. For a n-equipment standby redundant system with n-1 components off-line and assuming perfect switching reliability and that off-line equipment cannot fail until it is switched to an on-line position, we have the following expression for availability for a single repairmen

$$A(\infty) = 1 - \frac{1}{\sum_{j=0}^n X^j} . \quad (4.45)$$

where $X = \mu/\lambda$.

In the case where there are n repairmen

$$A(\infty) = 1 - \frac{1}{\sum_{j=0}^n \frac{n}{j!} X^{n-j}} . \quad (4.46)$$

4.2 EQUIVALENT FAILURE AND REPAIR RATES

Consider a nonmaintained system with two identical units in series. If the failure times of each unit is exponentially distributed with the parameter λ , then the reliability of the system is as follows

$$\begin{aligned} R_s(t) &= e^{-\lambda t} e^{-\lambda t} \\ &= e^{-2\lambda t} . \end{aligned} \quad (4.47)$$

Now if this system is replaced by a single unit with the equivalent failure rate λ_e , then

$$e^{-\lambda_e t} = e^{-2\lambda t}$$

$$\lambda_e = 2\lambda. \quad (4.48)$$

For a system having two units in parallel

$$R_s(t) = e^{-\lambda t} + e^{-\lambda t} - e^{-\lambda t} \cdot e^{-\lambda t} \quad (4.49)$$

$$= 2e^{-\lambda t} - e^{-2\lambda t}.$$

$$\begin{aligned} \text{The mean life } E(t) &= \int_0^{\infty} [2e^{-\lambda t} - e^{-2\lambda t}] dt \\ &= \frac{3}{2\lambda}. \end{aligned} \quad (4.50)$$

A component which replaces this system and has the equivalent failure rate, would have the same mean life as that of the system, hence

$$\begin{aligned} \frac{1}{\lambda_e} &= \frac{3}{2\lambda} \\ \lambda_e &= \frac{2\lambda}{3} \end{aligned} \quad (4.51)$$

In a series hook-up, exponential failure is preserved but in a parallel hook-up the failure is no longer exponential. Again if there were two

repairmen each having a repair rate μ , they would both attempt to service the equipment that failed and the equivalent repair rate would be $\mu_e = 2\mu$. For a system with two units in parallel and two repairmen, when the status of the individual units are monitored it was seen in equation (4.16) that

$$A(\infty) = 1 - P_2 = \frac{\mu^2 + 2\lambda\mu}{(\lambda+\mu)^2}, \text{ and}$$

$$P_2 = \frac{\lambda^2}{(\lambda+\mu)^2}. \quad (4.52)$$

So in time T , the system is expected to be down $\frac{\lambda^2}{(\lambda+\mu)^2} \cdot T$. (4.53)

For 2 repairmen each having a repair rate μ , the effective or equivalent repair rate $\mu_e = 2\mu$, thus

$$\text{the effective time to repair} = \frac{1}{2\mu}.$$

$$\text{The expected number of failures} = \left[\frac{\lambda^2}{(\lambda+\mu)^2} \cdot T \right] / \frac{1}{2\mu}. \quad (4.54)$$

If λ_e was the equivalent failure rate of this system, the expected number of failures in time T is $\lambda_e T$, hence

$$\lambda_e T = \frac{\lambda^2}{(\lambda+\mu)^2} \cdot 2\mu T, \text{ and}$$

$$\lambda_e = \frac{2\mu\lambda^2}{(\lambda+\mu)^2}. \quad (4.55)$$

This is approximately equal to the equivalent failure rate for the same set up calculated by Epstein and Hosford [22] using more rigorous mathematical methods. The Epstein equivalent failure rate is

$$\lambda_e = \frac{2\lambda^2}{3\lambda + \mu} . \quad (4.56)$$

Now again for a two equipment parallel redundant system where the status of individual pieces of equipment is not monitored and service, by two repairmen, begins only when the system fails was seen in equation (4.40) to be

$$A(\infty) = 1 - P_2 = \frac{2\mu\lambda + 3\mu^2}{\lambda^2 + 3\mu\lambda + 3\mu^2} ,$$

where

$$P_2 = \frac{\lambda^2 + \lambda\mu}{\lambda^2 + 3\mu\lambda + 3\mu^2} . \quad (4.57)$$

So in time T the equipment is expected to be down $\frac{\lambda^2 + \lambda\mu}{\lambda^2 + 3\mu\lambda + 3\mu^2} \cdot T$.

(4.58)

For 2 repairmen, each having a repair rate μ ,

the effective time to repair = $\frac{1}{2\mu}$.

$$\text{The expected number of failures} = \left[\frac{\lambda^2 + \lambda\mu}{\lambda^2 + 3\mu\lambda + 3\mu^2} \cdot T \right] / \frac{1}{2\mu} . \quad (4.59)$$

If λ_e was the equivalent failure rate of this system, the expected number of failures = $\lambda_e T$.

$$\text{Hence, } \lambda_e T = \frac{2\mu(\lambda^2 + \lambda\mu)}{\lambda^2 + 3\mu\lambda + 3\mu^2} \cdot T$$

and

$$\lambda_e = \frac{2\mu(\lambda^2 + \lambda\mu)}{\lambda^2 + 3\mu\lambda + 3\mu^2} \quad (4.60)$$

$$\text{and } \mu_e = 2\mu. \quad (4.61)$$

4.3 INCREASE IN MEAN LIFE DUE TO PREVENTIVE MAINTENANCE

Rosenheim [57] has shown that the mean life of a system having redundant units can be increased by performing periodic maintenance. Assume that all components obey the exponential failure law and the system is restored to "as new" condition after each periodic maintenance since no deterioration takes place. Every T hrs, starting at time 0, periodic maintenance is performed. All the elements are checked and any, which has failed, is repaired to its original condition or replaced by a new and statistically identical component.

If $f(t)$ is the failure density function then the mean life θ for any system or component is given by

$$\theta = \int_0^{\infty} t f(t) dt \quad (4.62)$$

Alternatively, if $R(t)$ is the reliability function of the system or component, then Shooman [62] has shown that

$$\theta = \int_0^{\infty} R(t) dt. \quad (4.63)$$

A period of time t hours can be written as

$$t = jT + \tau \quad j = 0, 1, 2, \dots \quad 0 \leq \tau < T$$

Let the reliability function of a redundant system, where maintenance is

performed every T hours, be denoted by $R_T(t)$. For a time period t where $j = 1$ and $\tau = 0$

$$R_T(t = T) = R(T) \quad (4.64)$$

If $j = 2$ and $\tau = 0$ the system has to operate the first T hours without failure of the system. After repair or replacement of all failed components, another T hours of failure free system operation is required, so $R_T(t = 2T) = R(T) \cdot R(T)$

If $0 < \tau < T$, then a further τ hours of failure free system operation is required and

$$R_T(t = 2T + \tau) = [R(T)]^2 R(\tau) .$$

$$\text{In general, } R_T(t = jT + \tau) = [R(T)]^j R(\tau) . \quad (4.65)$$

The mean life of the redundant system when periodic maintenance is performed every T hours is

$$\theta_s = \int_0^{\infty} R_T(t) dt .$$

The integral over the range $0 < t < \infty$ can be expressed as sum of integrals over intervals of time T .

$$\text{or } \theta_s = \sum_{j=0}^{\infty} \int_{jT}^{(j+1)T} R_T(t) dt . \quad (4.66)$$

Since $t = jT + \tau$, $dt = d\tau$ and by transformation, the limits of the integral become 0 to T .

$$\text{Hence } \theta_s = \sum_{j=0}^{\infty} \int_0^T R_T(t) d\tau = \sum_{j=0}^{\infty} \int_0^T [R(T)]^j R(\tau) d\tau$$

$$= \sum_{j=0}^{\infty} [R(T)]^j \int_0^T R(\tau) d\tau.$$

$$\text{Since } \frac{1}{1-x} = \sum_{j=0}^{\infty} x^j$$

$$\text{hence } \sum_{j=0}^{\infty} [R(T)]^j = \frac{1}{1-R(T)}$$

$$\text{and so } \theta_s = \frac{\int_0^T R(t) dt}{1 - R(T)} \quad (4.67)$$

Consider a system consisting of two units in parallel, each individual unit having a failure distribution which is exponential with parameter λ . Periodic maintenance is performed every T hrs starting from time 0. The reliability function of the system is given by

$$R(t) = 2e^{-\lambda t} - e^{-2\lambda t} \quad (4.68)$$

$$\text{The mean life of the system is } \theta_s = \frac{\int_0^T R(t) dt}{1 - R(T)}$$

$$\theta_s = \frac{\int_0^T [2e^{-\lambda t} - e^{-2\lambda t}] dt}{1 - [2e^{-\lambda T} - e^{-2\lambda T}]}$$

$$= \frac{\frac{3}{2\lambda} - \frac{2}{\lambda} e^{-\lambda T} + \frac{1}{2\lambda} e^{-2\lambda T}}{1 - e^{-\lambda T} [2 - e^{-\lambda T}]} \quad (4.69)$$

If periodic maintenance is not performed, $T = \infty$ then

$$\theta_s = \frac{3}{2\lambda} . \quad (4.70)$$

For example when $\lambda = .01$ failures/hr if

$$T = \infty: \quad \theta_s = 150 \text{ hrs}$$

$$T = 150 \text{ hrs:} \quad \theta_s = 179 \text{ hrs}$$

$$T = 100 \text{ hrs:} \quad \theta_s = 208 \text{ hrs}$$

$$T = 50 \text{ hrs:} \quad \theta_s = 304 \text{ hrs}$$

$$T = 10 \text{ hrs:} \quad \theta_s = 1097 \text{ hrs}$$

As expected, when the time period between preventive maintenance decreases, the mean life of the redundant system increases. An exponential distribution has a constant failure rate with time which is to say that the age of a unit has nothing to do with its failure rate. An old unit and a brand new one are equally likely to go on operating for some particular time period, and consequently, we gain nothing by applying planned replacement or repair to units having an exponential distribution of failures; the units we install are no better than the one we take out. This can also be seen if a single unit with constant failure rate λ undergoes periodic maintenance every T hours.

$$\begin{aligned} \text{The mean life without periodic maintenance} &= \int_0^{\infty} R(t) dt \\ &= \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda} . \quad (4.71) \\ \text{The mean life with periodic maintenance} &= \frac{\int_0^T R(t) dt}{1 - R(T)} \end{aligned}$$

$$\begin{aligned}
 &= \frac{\int_0^T e^{-\lambda t} dt}{1 - e^{-\lambda T}} \\
 &= \frac{1}{\lambda} \frac{[1 - e^{-\lambda T}]}{1 - e^{-\lambda T}} \\
 &= \frac{1}{\lambda} . \qquad (4.72)
 \end{aligned}$$

Thus for a single unit having an exponential failure distribution there is no increase in mean life by performing periodic maintenance.

Nevertheless, if a system has units in parallel, even though the individual units have an exponential failure distribution, a periodic maintenance policy can achieve an increased mean life. An increase in mean life implies that the system reliability has increased. The reason for the increase in reliability of a redundant system is that when periodic maintenance is performed the system might have been working with some redundant units in the failed state. These failed units are now repaired and restored to 'new' condition. Figure 4.7 is a plot of a reliability function, where maintenance is performed every T hours, denoted by $R_T(t)$ versus time for a redundant system. Note the increase in reliability that can be achieved by a preventive maintenance policy.

4.4 INCREASE IN AVAILABILITY DUE TO PREVENTIVE MAINTENANCE

Now with regard to the redundant system with repair, the major concern is the effect of performing preventive maintenance at fixed intervals. In a system with repair, the measure of the system reliability effectiveness is availability. By introducing periodic maintenance the

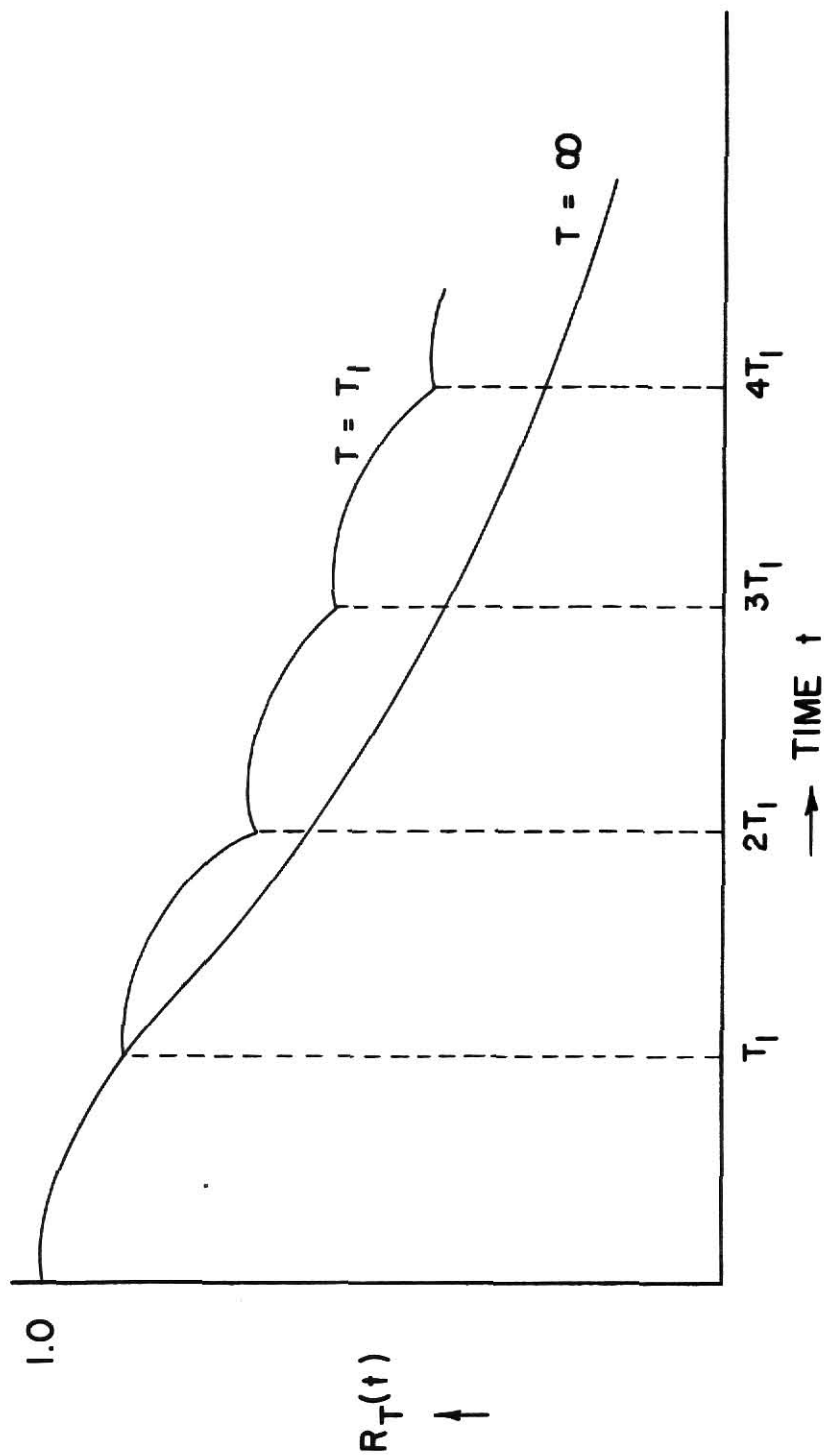


Figure 4.7. Reliability function for active parallel configuration on which maintenance is performed every T hours.

availability would increase depending upon the corrective maintenance policy that is pursued. If the status of the individual units is monitored and repair begins whenever any unit fails, then periodic maintenance will not increase availability. Periodic maintenance will, though, provide such attention as lubrication, refueling, alignment, etc., and overhaul components to prevent wearout. If the corrective maintenance policy is such that repair begins only when the system has failed due to failure of all units in parallel then availability will increase with the introduction of periodic maintenance. As before the reason for the increase in availability is that each time periodic maintenance is performed, all the units are checked and if one has failed it is repaired, or replaced by a new and statistically identical component.

For a system with corrective maintenance, expressions for instantaneous availability and steady state availability were obtained using a Markovian approach. Expressions for $P_n(t)$ and P_n were also obtained. Note that $P_n(t)$ is the probability for the system being in a failed state at any instant t , and P_n is the steady state probability of the system being in a failed state. With the introduction of periodic maintenance the availability increases and consequently the probability of the system being in a failed state decreases. On an intuitive basis it is felt that the decrease in $P_n(t)$ or P_n is directly proportional to the increase in the mean life obtained by introducing periodic maintenance. That is

$$\frac{P_n \text{ or } P_n(t) \text{ for system with corrective maintenance and periodic maintenance}}{P_n \text{ or } P_n(t) \text{ for system with only corrective maintenance}}$$

$$= \frac{\text{Mean life of system } (\theta \text{ n.p.m.})}{\text{Mean life of system with periodic maintenance } (\theta \text{ p.m.)}} \quad (4.73)$$

where

n.p.m. stands for no periodic maintenance

p.m. stands for periodic maintenance.

This is the principal assumption upon which the availability models are developed in this thesis. The justification of this assumption is made on an intuitive basis. Given that a failed state is when the system will not operate, it is felt that this is an appropriate assumption.

For example consider a system with a mean life of 150 hours. Suppose the steady state availability of this system is 0.95. The introduction of periodic maintenance every 50 hours increases the mean life to 300 hours. Then the steady state availability if periodic maintenance is performed every 50 hours is obtained as follows

$$P_n = 1 - A(\infty) = 1 - 0.95 = 0.05$$

$$P_n \text{ with periodic maintenance} = \frac{\theta_{n.p.m.}}{\theta_{p.m.}} \cdot P_n \quad (4.74)$$

$$= \frac{150}{300} \cdot 0.05 = .025.$$

$$A(\infty) \text{ with periodic maintenance} = 1 - 0.025$$

$$= 0.975.$$

So, by introducing periodic maintenance every 50 hours, there is a .025 increase in availability. In the literature covering models for systems with repair, expressions for availability for various types of redundant configurations have been derived. But the effects on availability by introducing periodic maintenance have not been stated nor has the change been shown in Mathematical terms. Some thought needs to be exercised on the existing corrective maintenance policy before introducing periodic

maintenance since it will not always increase availability. As mentioned earlier, if the status of the units is monitored and repair begins the moment any unit fails then periodic maintenance will not increase availability.

Consider a system with two units in parallel. Using equations (4.69) and (4.70) and introducing periodic maintenance every T hours, the ratio of increase in mean life is

$$\begin{aligned} \frac{\theta_{\text{n.p.m.}}}{\theta_{\text{p.m.}}} &= \frac{\frac{3}{2\lambda}}{\frac{\frac{3}{2\lambda} - \frac{2}{\lambda} e^{-\lambda T} + \frac{1}{2\lambda} e^{-2\lambda T}}{1 - e^{-\lambda T} [2 - e^{-\lambda T}]}} \\ &= \frac{\frac{3}{2\lambda} [1 - e^{-\lambda T} [2 - e^{-\lambda T}]]}{\frac{3}{2\lambda} - \frac{2}{\lambda} e^{-\lambda T} + \frac{1}{2\lambda} e^{-2\lambda T}}. \end{aligned} \quad (4.75)$$

In the same system, with a corrective maintenance policy of starting repair only when the system is in a failed state due to failure of both the units, by equation (4.39)

$$P_2 = \frac{\lambda^2 + \lambda\mu}{\lambda^2 + 3\mu\lambda + 3\mu^2}.$$

Hence P_2 with corrective maintenance and periodic maintenance every T hours is

$$P_2^{\text{p.m.}} = \frac{\frac{3}{2\lambda} [1 - e^{-\lambda T} (2 - e^{-\lambda T})]}{\frac{3}{2\lambda} - \frac{2}{\lambda} e^{-\lambda T} + \frac{1}{2\lambda} e^{-2\lambda T}} \cdot \frac{\lambda^2 + \lambda\mu}{\lambda^2 + 3\mu\lambda + 3\mu^2}. \quad (4.76)$$

The steady state availability for this system with corrective and preventive maintenance is

$$A(\infty) = 1 - P_{2\text{p.m.}} = 1 - \frac{\frac{3}{2\lambda} [1 - e^{-\lambda T} (2 - e^{-\lambda T})]}{\frac{3}{2\lambda} - \frac{2}{\lambda} e^{-\lambda T} + \frac{1}{2\lambda} e^{-2\lambda T}} \cdot \frac{\lambda^2 + \lambda\mu}{\lambda^2 + 3\mu\lambda + 3\mu^2} \quad (4.77)$$

For the system with two parallel units where only one unit is repaired when the system fails we have the following expression for availability

$$A(t) = 1 - \frac{\frac{3}{2\lambda} [1 - e^{-\lambda T} (2 - e^{-\lambda T})]}{\frac{3}{2\lambda} - \frac{2}{\lambda} e^{-\lambda T} + \frac{1}{2\lambda} e^{-2\lambda T}} \cdot \left(\frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\mu-\lambda} e^{-2\lambda t} + \frac{2\lambda^2}{\mu^2 - \lambda^2} e^{-(\lambda+\mu)t} \right). \quad (4.78)$$

The expressions for availability shown in equations (4.15) and (4.16) do not change with periodic maintenance because of the existing corrective maintenance policy. However the total cost will increase due to the additional cost for preventive maintenance.

So far in the discussions, the time required for periodic or preventive maintenance was not included. Since preventive maintenance is usually performed at times other than the normal operating hours, it was not necessary to include it. The time required for preventive maintenance is generally taken to be constant (t^*). In a period of T hours the system will be operational for $T-t^*$ hours and for t^* hours the system is down for periodic maintenance. Taking this into consideration and setting $T-t^* = T_1$ equation (4.77) becomes

$$A(\infty) = \frac{T_1}{T} \left[1 - \frac{\frac{3}{2\lambda} [1 - e^{-\lambda T_1} (2 - e^{-\lambda T_1})]}{\frac{3}{2\lambda} - \frac{2}{\lambda} e^{-\lambda T_1} + \frac{1}{2\lambda} e^{-2\lambda T_1}} \cdot \frac{\lambda^2 + \lambda\mu}{\lambda^2 + 3\mu\lambda + 3\mu^2} \right] \quad (4.79)$$

At the end of each preventive maintenance, the system is in a state with no failed components regardless of what state it was in at the beginning of preventive maintenance. This is shown in Figure 4.8. Since the availability curve repeats itself every T hours a different approach as follows may be used.

$$\text{Let } t = t' \bmod (T)$$

$$\begin{aligned} \text{Then } A(t) = & A(t') & \text{if } & 0 < t' < T_1 \\ & 0 & \text{if } & T_1 < t' < T \end{aligned} \quad (4.80)$$

If t' is known, then $A(t')$ may be obtained from equation (4.36). Equation (4.80) is for the normal case where preventive maintenance cannot be performed while the system is operating.

4.5 COST STRUCTURE

Let a single equipment have exponentially distributed failure and repair times with parameters λ and μ respectively. The total cost for design and maintenance is broken up into three components namely, the cost of designing for a failure rate λ and repair rate μ in the system, the cost of corrective maintenance and the cost of preventive maintenance. Shershin [61] has shown that the nature of the functions for each of these costs generally approximate realistic situations. General cost information for these quantities may be found in Ankenbrandt [2] and ARINC [70]. Using

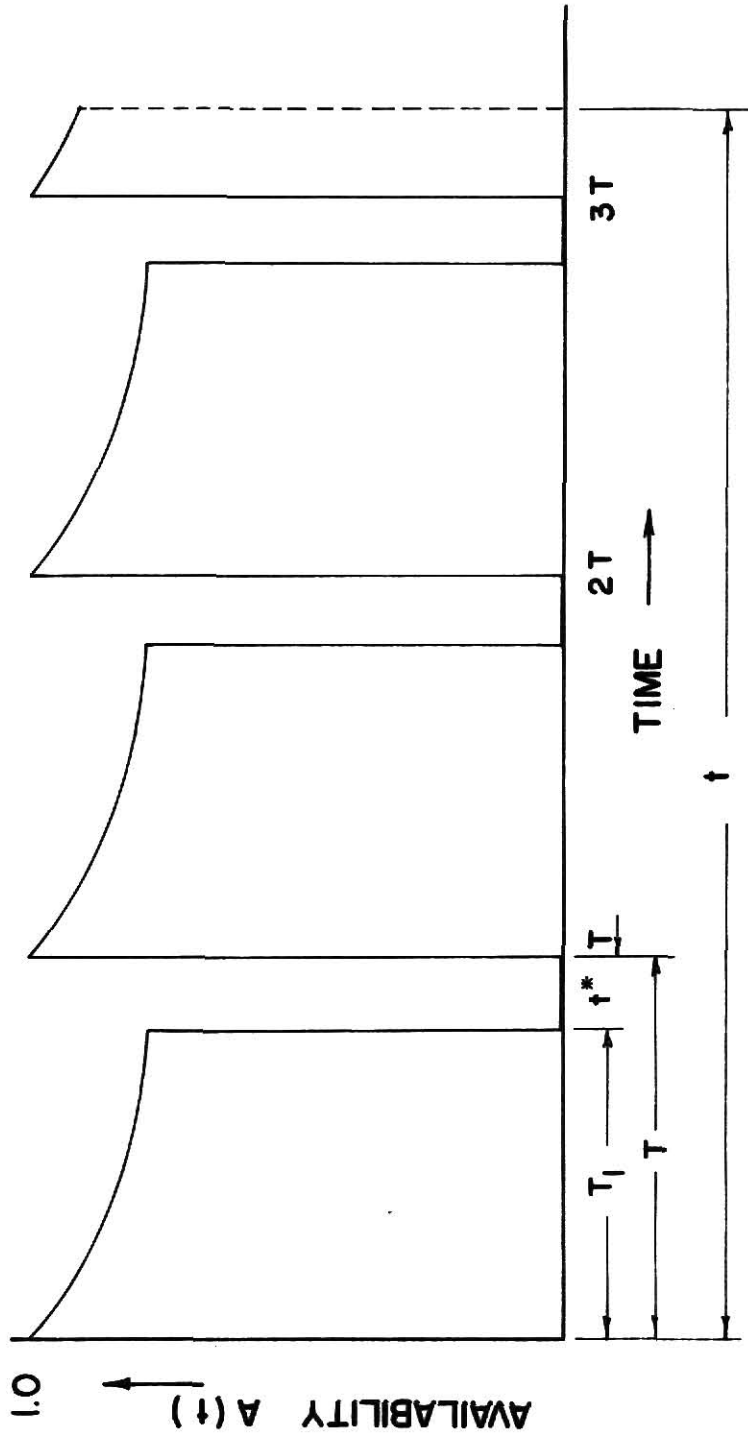


Figure 4.8 Availability versus time graph with constant time t^* for periodic maintenance

the parameters λ and μ and given constants a , b , c , d_1 , d_2 and d_3 the different cost functions can be stated as

1. the cost of corrective maintenance = $(\frac{a}{\mu})^2$

2. the cost of preventive maintenance = $\frac{b}{\mu} = c$

3. the cost of designing a failure rate

$$\lambda \text{ and repair rate } \mu \text{ in the system} = \mu d_1 + \frac{d_2}{\lambda} - d_3$$

When the value of λ decreases the design cost should increase since now the unit will take a longer time to fail and when the value of μ increases the design cost should increase while the maintenance cost should fall since now the unit is repaired in a shorter time. This is the reasoning upon which the three cost functions are based. For a system with a complex configuration the equivalent failure rate λ_e and the equivalent repair rate μ_e may be used in place of λ and μ respectively. Consider a system consisting of n subsystems in series where λ_{e_i} and μ_{e_i} are the equivalent failure and repair rates for the i^{th} subsystem. Let t be the mission time, and T the time interval for periodic maintenance. Then the total cost for the system is

$$\text{Total cost} = \sum_{i=1}^n (d_1 \mu_{e_i} + \frac{d_2}{\lambda_{e_i}} - d_3) + \sum_{i=1}^n t \lambda_{e_i} (\frac{a}{\mu_{e_i}})^2 + \frac{t}{T} \sum_{i=1}^n (\frac{b}{\mu_{e_i}} - c). \quad (4.81)$$

$\frac{t}{T}$ is a multiplier for the preventive maintenance cost function to show that if T is small then periodic maintenance is performed more often and the preventive maintenance cost component is higher.

Each of the subsystems is given the weighting λe_i for its corrective maintenance cost since the subsystem with a smaller λe_i will fail often and will need fewer corrective maintenance actions.

The design cost is the initial cost and is independent of the mission time. The total cost divided by the mission time gives the expected total cost per operating hour.

4.6 MATHEMATICAL STATEMENT OF PROBLEM

Consider a system with n subsystems in series as shown in Figure 4.9. Each subsystem consists of two identical units in parallel with the failure rate λ_i and the repair rate μ_i for the i^{th} subsystem. The corrective maintenance policy is to start servicing a subsystem when it fails due to failure of both the units. Due to the series connection, when a subsystem fails the entire system is down and two repairmen are adequate for corrective maintenance. The system will function with one unit of each subsystem in a failed state but will fail with more than one failure at any stage. Periodic maintenance is performed on the entire system every T hours. So at the time of performing periodic maintenance a maximum of $n+1$ units may be in failed states. A repair crew capable of performing periodic maintenance is assumed to be available. From equation (4.77) availability of the i^{th} subsystem is given by

$$A_i = 1 - \frac{\frac{3}{2\lambda_i} [1 - e^{-\lambda_i T} (2 - e^{-\lambda_i T})]}{\frac{3}{2\lambda_i} - \frac{2}{\lambda_i} e^{-\lambda_i T} + \frac{1}{2\lambda_i} e^{-2\lambda_i T}} \cdot \frac{\lambda_i^2 + \lambda_i \mu_i}{\lambda_i^2 + 3\mu_i \lambda_i + 3\mu_i^2}$$

From equations (4.60) and (4.61) the equivalent failure rate and repair

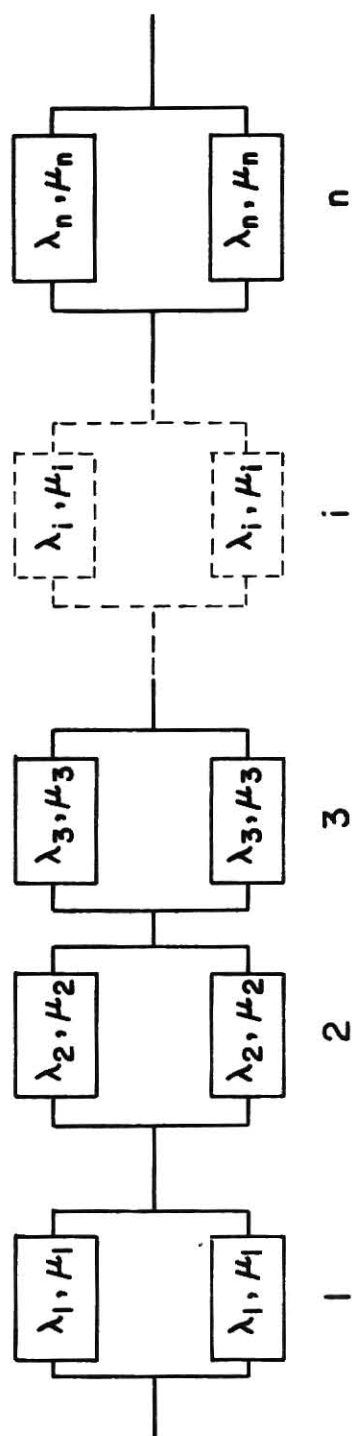


Figure 4.9. n subsystems, each with two identical units in parallel, are connected in series.

rate of the i^{th} subsystem

$$\lambda_{e_i} = \frac{2\mu_i (\lambda_i^2 + \lambda_i \mu_i)}{\lambda_i^2 + 3\mu_i \lambda_i + 3\mu_i^2}$$

$$\mu_{e_i} = 2\mu_i$$

The total cost for design and operation for a mission time t is given by equation (4.81)

$$C = \sum_{i=1}^n (d_1 \mu_{e_i} - \frac{d_2}{\lambda_{e_i}} - d_3) + \sum_{i=1}^n t \lambda_{e_i} \left(\frac{a}{\mu_{e_i}} \right)^2 + \frac{t}{T} \sum_{i=1}^n \left(\frac{b}{\mu_{e_i}} - c \right) .$$

The nonlinear programming problem may be formulated as follows:

Determine λ_i , μ_i , and T , $i = 1, \dots, n$

such that the total cost C is minimized

subject to $\ell \leq \lambda_i \leq m$ $i = 1, \dots, n$

$q \leq \mu_i \leq r$ $i = 1, \dots, n$

$v \leq T \leq w$

and

$\prod_{i=1}^n A_i \geq A_0$, the minimum availability requirement

where ℓ , m , q , r , v , w , and A_0 are known constants.

Sequential Unconstrained Minimization Technique (SUMT) is one of the optimization techniques for dealing with this problem. Due to difficulties in

taking the first order and second order derivatives with respect to each of the variables, the computer program implementing SUMT developed by Lai [42] is recommended in this thesis, instead of the standard RAC program (IBM SHARE number 3189). This optimization method may be used to appor-
tion or improve availability during the design or redesign phase, and may also be employed for improvement purposes after the initial testing has been performed.

Chapter 5

SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE (SUMT)

The Sequential Unconstrained **Minimization** Technique (SUMT) is a simple and efficient method for **solving** constrained nonlinear programming problems. The transformation of a constrained minimization problem into a sequence of unconstrained minimization problems is the principle behind SUMT. The method was first proposed by Carroll [17] in 1959, and further developed by Fiacco and McCormick [26, 27]. In 1964 Fiacco and McCormick developed their general algorithm, and in 1965 they extended their method which they called SUMT. McCormick, Mylander and Fiacco developed a general computer program called "RAC Computer Program Implementing the Sequential Unconstrained Minimization Technique for Nonlinear Programming", and the IBM SHARE number is 3189. In this computer program, the unconstrained minimization technique uses a second order gradient search method.

In large size or complex nonlinear programming problems difficulties arise when one has to find the first order and second order derivatives of the converted objective function. Since most practical problems fall into this category, a modified version was developed by Lai [41], which bypasses this difficulty. Basically it incorporates the Hooke and Jeeves pattern search method which requires taking no derivatives. The direction of search in the gradient method is the steepest descent direction, whereas in the Hooke and Jeeves pattern search technique it is determined by a direct comparison of two values of the objective function at two points separated from each other by a finite step.

For this reason when the pattern search is close to the boundary of some inequality constraint, it falls into the infeasible region. A heuristic technique developed by Paviani and Himmelblau [53] is then used to direct the search back into the feasible region.

The general nonlinear programming problem with nonlinear inequality and equality constraints is formulated as the problem of finding the n dimension vector x ,

$$\begin{aligned} x &= (x_1, x_2, \dots, x_n) \text{ which} \\ \text{minimizes} \quad & f(x) \end{aligned} \tag{5.1}$$

$$\text{subject to} \quad g_i(x) \geq 0 \quad i = 1, \dots, m \tag{5.2}$$

$$h_j(x) = 0 \quad j = 1, \dots, \ell \tag{5.3}$$

The SUMT formulation is based on the minimization of a function

$$P(x, r_k) = f(x) + r_k \sum_{i=1}^m \frac{1}{g_i(x)} + r_k^{-\frac{1}{2}} \sum_{j=1}^{\ell} h_j^2(x) \tag{5.4}$$

over a strictly monotonic decreasing sequence of the penalty coefficient r_k . Under certain conditions the sequence of values of the P function, $P(x, r_k)$, are respectively minimized by a sequence of $\{x(r_k)\}$ over a strictly monotonic decreasing sequence $\{r_k\}$, and converges to the constrained optimum values of the original objective function $f(x)$. The essential requirement is that the P function should be convex.

The second term of the P function $r_k \sum_{i=1}^m \frac{1}{g_i(x)}$ will approach infinity as the value of x approaches any one of the boundaries given by $g_i(x) \geq 0$ and hence the value of x will tend to remain within the inequality constrained feasible space.

The third term $r_k - \frac{1}{2} \sum_{j=1}^P h_j^2(x)$ will approach infinity as r_k tends to zero since the sequence $\{r_k\}$ is strictly monotonic decreasing and this consideration will force all equality constraints $h_j(x) = 0$ to be zero.

The solution process for the nonlinear programming problem as defined by the P function in equation (5.4) is started by selecting an arbitrary point inside the feasible region and selecting a value of r_k . A search is made for the minimum value of the P function. After a minimum value is obtained, the value of r_k is reduced and the search is repeated starting from the previous minimum point of the P function. By employing a strictly monotonic decreasing sequence $\{r_k\}$, a monotonic decreasing sequence $\{P_{\min}(x, r_k)\}$ inside the feasible region is obtained. As r_k tends to zero the equality constraints are satisfied and the second term of equation (5.4) $r_k \sum_{i=1}^m \frac{1}{g_i(x)}$ also approaches zero. That is as $r_k \rightarrow 0$, $P(x, r_k) \rightarrow f(x)$, where x is the optimum point which yields the minimum $P(x, r_k)$, and is also the optimum point of the original problem.

Details of the computational procedures, the flow diagrams, explanations and numerical examples may be seen in Lai [42]. The complete computer program for the numerical example solved in this thesis is listed in Appendix 2. In the computational procedures two stopping criteria are needed to obtain a meaningful optimal solution. The first stopping criteria is for terminating the minimization of the P function for the current value of r_k . For the Hooke and Jeeves pattern search

solution, it is the number of reduced stepsize operations. For the computer program one of the values (2, 3 or 4) is selected for this reduction. The second stopping criteria is to terminate the overall minimization of $f[x(r_k)]$. This stopping criteria generally ranges from 10^{-3} to 10^{-5} .

The program designed by Lai uses the WATFOR compiler and consists of the following routines

Main Program

Subroutine BACK	- used to pull back from infeasible region
Subroutine PENAT	- used to compute the penalty terms
Subroutine WEIGH	- used to compute the weight of violations
Subroutine READIN	- used to read in additional data if needed
Subroutine OUTPUT	- used to output additional information if desired
Subroutine OBRES	- used to compute the objective function and the constraints

Since the Fortran H level compiler is considerably faster, the control cards, the READ and WRITE statements and some FORMAT statements have been changed in this work in order to use it. Also since the main program and the first four subroutines are unaltered in most runs, an object deck was used for these portions of the program, thus saving some computer time. The program requires the following information.

- 1 N the number of variables in the problem.
- 2 MG the number of inequality constraints.
- 3 MH the number of equality constraints.

- 4 R r_k , the penalty coefficient for the SUMT formulation. If the value of R which is specified is less than or equal to zero, then a computed value is used. If a suitable value is not known, the value of R should be 0.
- 5 RATIO the reducing rate for R from stage to stage. If the value of RATIO which is specified is less than or equal to zero, then a computed value is used. Again the value of RATIO specified is 0, if a suitable value is not known.
- 6 INCUT the stopping criteria for the stage interation and is the number of reducing step size operations. Values of 2, 3, and 4 maybe used.
- 7 THETA the final stopping criteria and the value used is usually between 10^{-3} and 10^{-5} .
- 8 X(I) the initial starting point with values for each of the N variables. If the initial point provided is not feasible then the program computes its own initial starting point.
- 9 D(I) the initial step size for each of the N variables. 1/10th of the value of the variables of the estimated optimum point is usually good for starting.
- 10 DX(I) the estimated optimum point with values for each of the N variables.
- 11 NOPM the number of subproblems. Usually only one subproblem is run at one time.
- 12 ITMAX the maximum number of iterations within one stage. A message is printed out when the value specified is exceeded.
- 13 MAXP the maximum number of stages which if exceeded the computation will stop.

14 ISIZE the option code for the initial step size set up.

0 - use the input $D(I)$ values

1 - use the computed $D(I) = 0.02 \cdot DX(I)$

The value 0 is commonly used.

15 ICUT the option code for the step size in each of the stages

0 - use the input $D(I)$ values for all stages

1 - use the initial $D(I)/K$ for the K th stage

16 The objective function and the constraints using $X(I)$, $I = 1 \dots N$ as the variables are to be placed in the block provided in sub-routine OBRES in the following form.

$Y =$ function of $X(I)$ for the objective function

$G(J) =$ function of $X(I)$ for the constraints greater than 0

$H(K) =$ function of $X(I)$ for the constraints equal to 0

The double precision procedure is used when the objective function of the problem considered is too flat. As explained earlier the optimal x value is obtained when the P function tends to the same value as that of the f function. The program computes a final stopping criteria called $YSTOP$ at the end of each stage of the monotonically decreasing sequence of $R(r_k)$. When $YSTOP$ becomes less than $THETA$ at any stage, the computation stops, and the x vector values are printed as the final optimum point. However further improvement in the f function maybe possible when the program stops, due to the nature of the formulation of the computational procedure. Thus a modest improvement can usually be made by using the final optimum x values as the initial starting point, and running the computer program again. This process is continued until subsequent computer trials show an insignificant improvement in the f function. Three computer trials seem adequate to obtain a stabilized optimum solution for the f function and x vector.

Chapter 6

NUMERICAL EXAMPLE

6.1 PROBLEM STATEMENT

Consider a system with three subsystems connected in a series configuration. Each subsystem consists of two identical components in parallel. Let the times to failure and repair times be exponentially distributed with parameters λ_i and μ_i for the i^{th} subsystem. The status of individual units is not monitored and repair commences only when the system fails. The system will function with one unit of each subsystem in a failed state but will fail with more than one failure at any stage. A repair crew capable of repairing the system at any time is assumed to be available. Periodic maintenance is performed every T hours, starting at time 0. During periodic maintenance every element is checked and any unit which has failed is repaired. Performance requirements for the system specify that the steady state availability should be at least 0.99. The time T for periodic maintenance is suggested to fall between 75 hours and 800 hours for the mission time t equal to 1500 hours. Limits are imposed on the individual λ 's to be between 0.001 and 0.02 failures per hours. Likewise each of the μ 's are required to function between 0.01 and 0.6 repairs per hour. In designing this system the total overall costs are separated into three categories. The first cost is the cost of designing the system for particular values of the parameters λ_i 's and μ_i 's the failure rate and the repair rate for each unit of the i^{th} subsystem. The second and third costs are the costs for corrective maintenance and

preventive maintenance respectively for the system with the particular values of the parameters λ_i 's and μ_i 's. The problem is one of finding the combination of values of λ_i ($i = 1, 2, 3$), μ_i ($i = 1, 2, 3$) and T which will minimize the total overall cost for a mission time t equal to 1500 hours.

6.2 PROBLEM FORMULATION

$$\text{The equivalent failure rate of 1st subsystem } \lambda_{e_1} = \frac{2\mu_1 (\lambda_1^2 + \lambda_1\mu_1)}{3\mu_1^2 + 3\lambda_1\mu_1 + \lambda_1^2}$$

$$\text{The equivalent failure rate of 2nd subsystem } \lambda_{e_2} = \frac{2\mu_2 (\lambda_2^2 + \lambda_2\mu_2)}{3\mu_2^2 + 3\lambda_2\mu_2 + \lambda_2^2}$$

$$\text{The equivalent failure rate of 3rd subsystem } \lambda_{e_3} = \frac{2\mu_3 (\lambda_3^2 + \lambda_3\mu_3)}{3\mu_3^2 + 3\lambda_3\mu_3 + \lambda_3^2} \quad (6.1)$$

$$\text{The equivalent repair rate of 1st subsystem } \mu_{e_1} = 2\mu_1$$

$$\text{The equivalent repair rate of 2nd subsystem } \mu_{e_2} = 2\mu_2 \quad (6.2)$$

$$\text{The equivalent repair rate of 3rd subsystem } \mu_{e_3} = 2\mu_3$$

These equivalent failure and repair rates are obtained from equations (4.60) to (4.61) in Chapter 4.

The objective function is obtained from equation (4.81) in Chapter 4, in which the following values for the constants are used

$$i = 3$$

$$a = 1.5$$

$$b = 5.0$$

$$c = 5.0$$

$$d_1 = 0.15$$

$$d_2 = 150.0$$

$$d_3 = 10.0$$

and the mission time $t = 1500$ hrs.

The objective function $f(\lambda e_1, \lambda e_2, \lambda e_3, \mu e_1, \mu e_2, \mu e_3, t, T)$ is then given by

$$f = \sum_{i=1}^3 \left(\frac{0.15}{\lambda e_i} + 150 \mu e_i - 10 \right) + \sum_{i=1}^3 1500 \lambda e_i \left(\frac{1.5}{\mu e_i} \right)^2 + \frac{1500}{T} \sum_{i=1}^3 \left(\frac{5}{\mu e_i} - 5 \right) \quad (6.3)$$

The steady state availability expression for each of the subsystems is obtained from equation (4.77) in chapter 4.

$$A_1(\infty) = 1 - \frac{\frac{3}{2\lambda_1} [1 - e^{-\lambda_1 T} (2 - e^{-\lambda_1 T})]}{\frac{3}{2\lambda_1} - \frac{2}{\lambda_1} e^{-\lambda_1 T} + \frac{1}{2\lambda_1} e^{-2\lambda_1 T}} \cdot \frac{\lambda_1^2 + \lambda_1 \mu_1}{3\mu_1^2 + 3\lambda_1 \mu_1 + \lambda_1^2}$$

$$A_2(\infty) = 1 - \frac{\frac{3}{2\lambda_2} [1 - e^{-\lambda_2 T} (2 - e^{-\lambda_2 T})]}{\frac{3}{2\lambda_2} - \frac{2}{\lambda_2} e^{-\lambda_2 T} + \frac{1}{2\lambda_2} e^{-2\lambda_2 T}} \cdot \frac{\lambda_2^2 + \lambda_2 \mu_2}{3\mu_2^2 + 3\lambda_2 \mu_2 + \lambda_2^2} \quad (6.4)$$

$$A_3(\infty) = 1 - \frac{\frac{3}{2\lambda_3} [1 - e^{-\lambda_3 T} (2 - e^{-\lambda_3 T})]}{\frac{3}{2\lambda_3} - \frac{2}{\lambda_3} e^{-\lambda_3 T} + \frac{1}{2\lambda_3} e^{-2\lambda_3 T}} \cdot \frac{\lambda_3^2 + \lambda_3 \mu_3}{3\mu_3^2 + 3\lambda_3 \mu_3 + \lambda_3^2}$$

Since the three subsystems are in series, the failure of any subsystem would cause the system to fail. Therefore, the system is operational only when all three subsystems are operational and the steady state availability of the system is given by

$$A_s(\infty) = A_1(\infty) \cdot A_2(\infty) \cdot A_3(\infty) . \quad (6.5)$$

The nonlinear programming problem in the SUMT format is stated as follows

$$\begin{aligned} \text{minimize } f = & \sum_{i=1}^3 \left(\frac{0.15}{\lambda e_i} + 150 \mu e_i - 10 \right) + \sum_{i=1}^3 1500 \lambda e_i \left(\frac{1.5}{\mu e_i} \right)^2 \\ & + \frac{1500}{T} \sum_{i=1}^3 \left(\frac{5}{\mu e_i} - 5 \right) \end{aligned}$$

subject to the constraints

$$\begin{aligned} g(1) &= \lambda_1 - .001 > 0 \\ g(2) &= \lambda_2 - .001 > 0 \\ g(3) &= \lambda_3 - .001 > 0 \\ g(4) &= .02 - \lambda_1 > 0 \\ g(5) &= .02 - \lambda_2 > 0 \\ g(6) &= .02 - \lambda_3 > 0 \\ g(7) &= \mu_1 - .01 > 0 \\ g(8) &= \mu_2 - .01 > 0 \\ g(9) &= \mu_3 - .01 > 0 \\ g(10) &= .6 - \mu_1 > 0 \\ g(11) &= .6 - \mu_2 > 0 \end{aligned}$$

$$\begin{aligned}
g(12) &= .6 - \mu_3 > 0 \\
g(13) &= T - 75 > 0 \\
g(14) &= 800 - T > 0 \\
g(15) &= A_s(\infty) - .99 > 0
\end{aligned} \tag{6.6}$$

where the P function as defined in equation (5.4) is given by

$$P = f + r_k \sum_{i=1}^{15} \frac{1}{g_i} . \tag{6.7}$$

6.3 PROBLEM DEFINITION FOR THE SUMT PROGRAM

The complete SUMT computer program for solving this problem is shown in Appendix 2. For more details on how this particular program works see Lai [41]. The objective function and constraints are put in the block provided in the subroutine OBRES. Values given to the following SUMT variables and parameters are specified in the data cards.

```

NOPM      = 1
NAME      = RELPRB
N         = 7
MG        = 15
MH        = 0
R         = 0
RATIO     = 0
ITMAX     = 7000
INCUT     = 4
THETA     = 2 x 10-5
MAXP      = 30

```

ISIZE = 0

ICUT = 1

The individual variables are specified by X(I) in the program as follows:

$$\lambda_1 = X(1)$$

$$\lambda_2 = X(2)$$

$$\lambda_3 = X(3)$$

$$\mu_1 = X(4)$$

$$\mu_2 = X(5)$$

$$\mu_3 = X(6)$$

$$T = X(7)$$

Y is used to specify the objective function f,

$$f = Y.$$

In the data cards the value of R is specified to be zero so an actual value for R is calculated by the program. The initial starting point, the step size, and the estimated optimum point are also specified in the data cards.

The estimated optimum values are:

$$\lambda_1 = .003$$

$$\lambda_2 = .003$$

$$\lambda_3 = .003$$

$$\mu_1 = .20$$

$$\mu_2 = .20$$

$$\mu_3 = .20$$

$$T = 200.$$

The step size (0.1 times estimated optimum values) are:

$$D(1) = .0003$$

$$D(2) = .0003$$

$$D(3) = .0003$$

$$D(4) = .02$$

$$D(5) = .02$$

$$D(6) = .02$$

$$D(7) = 20.0.$$

6.4 RESULTS

The initial starting point, the computed value of R , the values of $RATIO$, the objective function f , and the P function are shown in Table 6.1. Also shown is the phase wise minimization of the P function, values of the parameters $\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2, \mu_3$ and T, R , the availability, the objective function f , the P function, the costs for corrective maintenance, preventive maintenance and design, and the number of iterations within each phase.

After a minimum P function value is reached at each phase the value of R is reduced and the search is repeated again starting from the previous minimum point of the P function. Through the phases a decreasing sequence of P function values are obtained corresponding to the optimum points at each phase and lying within the feasible region. As R is reduced, the P function approaches the f function and in this example at phase 9 the final stopping criteria $YSTOP$ is less than the specified value ($THETA$). In other words at phase 9 the P function and f function are sufficiently close and the optimum of the problem has been reached.

To test the solution for the optimum, the optimum parameter values just

obtained are used as the initial starting point for another trial. Table 6.2 shows the phase-wise minimization of the P function for this second trial. Very little improvement in the f function is achieved and thus it is concluded that the optimum values for the parameters and the f function have been obtained. In actuality there are other points which achieve approximately the same cost and satisfy the specified constraints. To test whether the solution is a global optimum rather than a local optimum other starting points are selected and the solutions obtained. If the other solutions converge to the first, or are inferior then the optimum has been achieved. Otherwise a local optimum has been reached and the search continues. Table 6.3 shows the phase-wise optimization when a different starting point is used where the value of the overall stopping criteria (THETA) has been reduced to 5×10^{-4} . The initial starting point provided does not lie in the feasible region and so a different starting point has been selected by the program. The optimum point obtained after nine phases is some what inferior to that obtained in Table 6.2. Table 6.4 shows an optimum point obtained from yet another starting point in which objective function has a value slightly better than that in Table 6.2. This improvement is not significant and may result from numerical round off error in the computational methods. If it were significantly better we would suspect that the other solutions were local optimums. It may be seen in the first three tables that improvements in the objective function occur mostly in the initial stages.

Table 6.1 Phases in Optimization of the Objective Function Subject to Inequality Constraints

No.	Parameters Costs etc.	Starting Point	Phase 1	Phase 2	Phase 3	Phase 4
1	λ_1	$.3500 \times 10^{-2}$	$.4400 \times 10^{-2}$	$.4156 \times 10^{-2}$	$.3994 \times 10^{-2}$	$.3675 \times 10^{-2}$
2	λ_2	$.3500 \times 10^{-2}$	$.4400 \times 10^{-2}$	$.4137 \times 10^{-2}$	$.4262 \times 10^{-2}$	$.4094 \times 10^{-2}$
3	λ_3	$.3500 \times 10^{-2}$	$.4400 \times 10^{-2}$	$.4137 \times 10^{-2}$	$.4262 \times 10^{-2}$	$.3925 \times 10^{-2}$
4	μ_1	.2500	.3300	.3525	.3408	.3246
5	μ_2	.2500	.3400	.3525	.3492	.3354
6	μ_3	.2500	.3400	.3525	.3475	.3337
7	T	100.0	180.0	267.5	292.5	368.7
8	R	.1092	.1092	$.1364 \times 10^{-1}$	$.3411 \times 10^{-2}$	$.8528 \times 10^{-3}$
9	No. of iterations within this phase		90	370	555	809
10	Cost for corrective maintenance		65.54	56.27	58.80	59.91
11	Cost for preventive maintenance		60.68	35.20	34.30	31.10
12	Cost for design		426.42	450.15	443.16	441.64
13	Value of the obj. function f	707.4	552.64	541.62	536.32	532.65
14	Value of the P function P	884.2	730.02	569.10	545.98	536.62
15	Availability		.9916	.9912	.9906	.9903

Table 6.1 (continued)

No.	Parameters Costs etc.	Phase 5	Phase 6	Phase 7	Phase 8	Phase 9 Final
1	λ_1	$.3675 \times 10^{-2}$	$.3801 \times 10^{-2}$	$.3801 \times 10^{-2}$	$.3895 \times 10^{-2}$	$.3895 \times 10^{-2}$
2	λ_2	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$
3	λ_3	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$
4	μ_1	.3246	.3246	.3246	.3246	.3246
5	μ_2	.3354	.3354	.3354	.3354	.3354
6	μ_3	.3337	.3337	.3337	.3337	.3337
7	T	368.7	368.7	368.7	368.7	368.7
8	R	$.2132 \times 10^{-3}$	$.2665 \times 10^{-4}$	$.6663 \times 10^{-5}$	$.1666 \times 10^{-5}$	$.4164 \times 10^{-6}$
9	No. of iterations within this phase	7815	8070	11049	11221	11666
10	Cost for corrective maintenance	59.91	60.58	60.58	61.08	61.08
11	Cost for preventive maintenance	31.10	31.10	31.10	31.10	31.10
12	Cost for design	441.64	439.61	439.61	438.18	438.18
13	Value of the obj. function f	532.65	531.29	531.29	530.37	530.37
14	Value of the P function P	533.65	531.50	531.35	530.40	530.37
15	Availability	.9903	.99015	.99015	.99005	.99005

Table 6.2 Phases in Optimization of the Objective Function Subject to Inequality Constraints (Starting Values Taken from Final Phase of Table 6.1)

No.	Parameters Costs etc.	Starting Point	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5	Phase 6 (Final)
1	λ_1	$.3875 \times 10^{-2}$	$.3895 \times 10^{-2}$	$.3926 \times 10^{-2}$	$.3926 \times 10^{-2}$	$.3926 \times 10^{-2}$	$.3926 \times 10^{-2}$	$.3926 \times 10^{-2}$
2	λ_2	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$	$.4094 \times 10^{-2}$
3	λ_3	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$	$.3925 \times 10^{-2}$
4	μ_1	.3246	.3246	.3246	.3246	.3246	.3246	.3246
5	μ_2	.3354	.3354	.3354	.3354	.3354	.3354	.3354
6	μ_3	.3337	.3337	.3337	.3337	.3337	.3337	.3337
7	T	368.7	.368.7	368.7	368.7	368.7	.368.7	368.7
8	R	$.5976 \times 10^{-2}$	$.5976 \times 10^{-2}$	$.7470 \times 10^{-2}$	$.1867 \times 10^{-3}$	$.4669 \times 10^{-4}$	$.1167 \times 10^{-4}$	$.1459 \times 10^{-5}$
9	No. of iterations with- in this phase	86	260	7266	7398	7575	7919	
10	Cost for corrective main- tenance	61.09	61.26	61.26	61.26	61.26	61.26	61.26
11	Cost for preventive main- tenance	31.11	31.11	31.11	31.11	31.11	31.11	31.11
12	Cost for design	438.17	437.70	437.70	437.70	437.70	437.70	437.70
13	Value of the obj. function f	530.37	530.07	530.07	530.07	530.07	530.07	530.07
14	Value of the P function P	662.96	591.00	545.30	533.88	531.02	530.19	530.19
15	Availability	.99005	.99001	.99001	.99001	.99001	.99001	.99001

Table 6.3 Phases in Optimization of the Objective Function Subject to Inequality Constraints (second set of starting values)

No.	Parameters Costs etc.	Starting Point Given	Starting Point Selected	Phase 1	Phase 2	Phase 3	Phase 4
1	λ_1	$.6000 \times 10^{-2}$	$.3600 \times 10^{-2}$	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$
2	λ_2	$.6000 \times 10^{-2}$	$.3600 \times 10^{-2}$	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$
3	λ_3	$.6000 \times 10^{-2}$	$.3600 \times 10^{-2}$	$.3600 \times 10^{-2}$	$.3600 \times 10^{-2}$	$.3600 \times 10^{-2}$	$.3600 \times 10^{-2}$
4	μ_1	.1500	.2900	.2900	.2900	.2900	.2900
5	μ_2	.1500	.2900	.2900	.2900	.2900	.2900
6	μ_3	.1500	.2900	.2900	.2900	.2900	.2900
7	T	500.0	340.0	340.0	340.0	340.0	340.0
8	R	.2199	.2199	.2199	$.2749 \times 10^{-1}$	$.6873 \times 10^{-2}$	$.1718 \times 10^{-2}$
9	No. of iterations within this phase			90	471	589	1103
10	Cost of corrective maintenance			73.57	73.57	73.57	73.57
11	Cost of preventive maintenance			47.92	47.92	47.92	47.92
12	Cost of Design			415.13	415.13	415.13	415.13
13	Value of the obj. function f	772.3	538.7	536.62	536.62	536.62	536.62
14	Value of the P function P	965.4	1533.0	3992.68	968.62	644.62	563.62
15	Availability			.99007	.99007	.99007	.99007

Table 6.3 (continued)

No.	Parameters Costs etc.	Phase 5	Phase 6	Phase 7	Phase 8	Phase 9 Final
1	λ_1	$.3750 \times 10^{-2}$	$.3750 \times 10^{-2}$	$.3750 \times 10^{-2}$	$.3750 \times 10^{-2}$	$.3750 \times 10^{-2}$
2	λ_2	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$	$.3700 \times 10^{-2}$
3	λ_3	$.3600 \times 10^{-2}$	$.3600 \times 10^{-2}$	$.3600 \times 10^{-2}$	$.3600 \times 10^{-2}$	$.3600 \times 10^{-2}$
4	μ_1	.2900	.2900	.2900	.2900	.2900
5	μ_2	.2900	.2900	.2900	.2900	.2900
6	μ_3	.2900	.2900	.2900	.2900	.2900
7	T	340.0	340.0	340.0	340.0	340.0
8	R	$.4295 \times 10^{-3}$	$.1074 \times 10^{-3}$	$.1342 \times 10^{-4}$	$.3356 \times 10^{-5}$	$.8390 \times 10^{-6}$
9	No. of iterations within this phase	1247	2147	2757	4157	4387
10	Cost of corrective maintenance	73.91	73.91	73.91	73.91	73.91
11	Cost of preventive maintenance	47.92	47.92	47.92	47.92	47.92
12	Cost of design	414.31	414.31	414.31	414.31	414.31
13	Value of the obj. function f	536.14	536.14	536.14	536.14	536.14
14	Value of the P function P	596.23	551.16	538.02	536.61	536.26
15	Availability	.990007	.99007	.99007	.99007	.99007

**Table 6.4 Final Phase Values in Optimization of the Objective Function
Subject to Inequality Constraints (third set of starting values)**

$$\lambda_1 = .3774 \times 10^{-2}$$

$$\lambda_2 = .3788 \times 10^{-2}$$

$$\lambda_3 = .3803 \times 10^{-2}$$

$$\mu_1 = .3249$$

$$\mu_2 = .3281$$

$$\mu_3 = .3253$$

$$T = 431.9$$

Value of R	$.2082 \times 10^{-6}$
Number of iterations within this phase	16468
Cost for corrective maintenance	60.13
Cost for preventive maintenance	27.79
Cost for design	441.66
Value of the objective function f	529.57
Value of the P function P	529.58
Availability	.99003

Chapter 7

DISCUSSION AND CONCLUSIONS

The introduction of maintenance is one of the major approaches for increasing system reliability effectiveness. In this thesis, the design, control and improvement of corrective and preventive maintenance policies and their associated cost have been emphasized. A complete procedure has been outlined for quantitatively employing availability as the principal parameter in the determination of an optimum system. A model has been developed for the availability of a system comprising of stages where each stage has two, identical units in parallel. A policy is established for preventive maintenance particularly when it is to be performed. It has been demonstrated that for a redundant non-maintained system, an increase in the mean life is obtained by performing periodic maintenance, and that the amount of the increase depends upon the frequency of periodic maintenance. On an intuitive basis it was felt that if corrective maintenance is performed on the same system then the decrease in the probability of the system being down is proportional to the increase in mean life achieved by periodic maintenance. This is the principle assumption in the development of the availability model. It is suggested that a topic for further work would be a complete simulation study of the system to test the validity of this assumption. The logic may be extended in developing availability models for systems with different redundant configurations. With complex maintained systems, steady state solutions were obtained without much difficulty by setting the derivatives $P'_i(t)$ equal to zero as shown in equation (4.38).

The availability model is structured on a strictly periodic maintenance policy. In practical cases having the same fixed time between preventive maintenance actions is more common and is preferred from an administrative point of view. The proposed model is inadequate if a random periodic, or a sequentially determined preventive maintenance policy is in effect. The increase in mean life of a redundant non-maintained system with the introduction of a random periodic, or a sequentially determined preventive maintenance policy has not been investigated. If such studies are conducted at a later date the same conceptual approach as used here may be used to develop these availability models.

The functions for the total systems costs categorized under a) the cost for corrective maintenance, b) the cost for preventive maintenance, c) the cost for design, have general forms that approximate realistic situations. For example the design costs for a unit which can be repaired in a short time is high, while the maintenance costs for such a unit would be low. A typical set of values for the constants in the cost functions is shown in the numerical example. These constants may be changed to obtain different cost functions. In the 'Mathematical Statement of the Problem' in Chapter 4 the objective is to minimize the total system cost while maintaining a given level of availability and keeping all the parameters within given upper and lower bounds. This type of analysis is valuable during final design or redesign when the availability requirements have been established. During preliminary design analysis it would be more realistic to maximize availability while a specified total system cost is not to be exceeded. Since the cost of purchase and

installation is directly related to the design cost of the equipment, the total system cost could very well be categorized under a) the cost for purchasing and installing the equipment b) the cost for corrective maintenance and c) the cost for preventive maintenance. Such a cost model may be used in the selection of equipment from different manufacturers.

The SUMT formulation is based on the minimization of the P function (equation (5.4)) over a strictly monotonic decreasing sequence of the penalty coefficient r_k . The number of phases required to obtain the optimal solution increases with an increase in r_k . The reason for this is that the optimal solution is obtained when the decreasing P function is sufficiently close to the objective function. The essential requirement for the decreasing sequence of values of the P function to converge to the constrained optimum value of the objective function is that the P function be convex. The availability constraint shown in equation (6.6) is exceedingly complex in form and is part of the second term of the P function. So there is a chance that the P function is not convex. Thus the optimal solution obtained in the numerical example may not be a global optimum. It is possible to obtain separate local optimum points having approximately the same value of the objective function and satisfying all the specified constraints. Reduction of the step sizes for the Hooke and Jeeves search restrict the area in which the search is conducted with a good possibility that the P function is convex in that area. The amount of reduction depends upon the problem. With large step sizes the objective function does not monotonically decrease over

the phases. Though the objective function has a decreasing trend, some increases in value along the successive phases may be experienced. An extensive search of the whole domain has to be made to obtain the global optimum. The time, effort and computational work needed for such a search was not warranted for the numerical example since the primary purpose of this work was to establish a procedure for analyzing problems of this type.

In the numerical example the mission time t was given, but under different conditions it might be treated as a variable. For example, it might be necessary to determine the expected mission time, for the system considered in this thesis, before the total system cost exceeds a set value. Again in the numerical example the availability constraint $A(\infty) = .99$ (equation 6.7) decreases over the phases and has the final value .00001 as shown in Table 6.2. This indicates that the availability constraint is active while the other constraints are inactive. It might be mentioned that in another model studied but not shown in this thesis the constraint on the upper limit of the time between periodic maintenance actions was active. In the availability expression shown in equation (4.40), if $\lambda = .01$ failures per hour and $\mu = 1.0$ repairs per hour, then $A(\infty) = .9977$. If the availability constraint was say .998 the minimization of total cost would push the time period for preventive maintenance to its upper limit because of the factor $\frac{t}{T}$ in the third term of equation (4.81) for $i = 1$.

The model for availability and total system cost are based on the exponential distribution for failure and repair times. This choice is not too restrictive as shown in the choice of the distribution for failure

and repair times (Appendix 1). The exponential distribution is convenient, simple and usually describes the physical nature of many problems. Considering exponential distributed failure and repair times, a Markovian approach may be employed in obtaining instantaneous or steady state solutions for maintained systems. On the assumption that the decrease in the probability of the system being down is proportional to the increase in mean life achieved by periodic maintenance, availability models for systems with corrective and preventive maintenance may be developed. SUMT can then be used to determine values for the failure rates, the repair rates, the time interval for preventive maintenance and the mission time such that associated costs are minimized subject to specified constraints. This is the general nature of the approach and the effort of this work is to show a method of availability analysis so that other related models can be developed and optimized.

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APPENDIX 1

A1.1 MEASURES OF SYSTEM RELIABILITY EFFECTIVENESS

Probability of Survival

The probability of system survival is a measure of the probability that a system will not reach a completely failed state during a given time interval given that the system was fully operable at the beginning of the interval. In systems where maintenance is either not possible during operation or can only be performed at different times for example missiles and satellites, the probability of survival is an appropriate measure of system reliability effectiveness.

Mean Time to System Failure

In reliability engineering, one frequently encounters Mean-Time-Between Failures (MTBF), Mean-Time-To-Failure (MTTF), and Mean-Time-To-First-Failure (MTTFF). MTBF is specifically applicable to a large number of pieces of equipment where we are interested in the average time between the individual equipment failures. MTTF is applicable to non maintained systems and is the expected time the system is in an operable state given that at the start of time all equipments comprising the system were in perfect working condition. MTTFF is applicable to maintained systems and the only difference with MTTF is that individual pieces of equipment are repaired when they fail.

Duration of Single Down Times

For systems like the early warning radar network, the duration of single down times may be the most meaningful measure of system reliability effectiveness. The reason for this is that if the enemy knew the

system was to be down on the average of an hour each time a system failure occurred, he could arrange a sneak attack.

Maintainability

Calabro [15] defines maintainability as the probability that a device will be restored to operational effectiveness within a given period of time when the maintenance action is performed in accordance with prescribed procedures. Since a device does not always fail to accomplish its mission because of failure requiring a repair action, the definition refers to maintenance action and not to repair. However the words repair and maintenance action is often used interchangeably. If we have an exponential repair distribution with μ as the maintenance action rate or repair rate measured in number of maintenance actions per hour and t is the maintenance time constraint in hours, the maintenance equation is expressed as

$$M(t) = 1 - e^{-\mu t} . \quad (A1)$$

The mission time T , also expressed in hours, is usually very large in comparison with t . The relationship between t and T is brought out in the Maintainability Increment.

Maintainability Increment

Maintainability Increment is defined as the proportion of failures in time T which will be restored to operational effectiveness in an interval of time t as a result of the maintenance activity. Maintainability Increment is a function of μ , λ , t and T where λ is the failure rate for an exponential distribution, whereas maintainability is a function

of only μ and t . Maintainability Increment M_{Δ} is expressed as

$$\begin{aligned}
 M_{\Delta} &= (\text{Probability of one or more failures in time } T)(\text{Maintainability}) \\
 &= (1 - e^{-\lambda T})(1 - e^{-\mu t}) \\
 &= 1 - e^{-\lambda T} - e^{-\mu t} (1 - e^{-\lambda T}). \quad (A2)
 \end{aligned}$$

Availability actually consists of two components, maintainability and reliability, since poor reliability can be offset by correspondingly improved maintainability. Also if the maintenance action rate is faster, then higher availability is obtained.

Dependability

Dependability is a measure applied to missions with specified duration and allowable down times. As such, it expresses the probability of successfully completing a mission of duration T when t is the allowable down time during the mission.

$$\text{Dependability } D = e^{-\lambda T} + (1 - e^{-\lambda T})(1 - e^{-\mu t}) \quad (A3)$$

$e^{-\lambda T}$ = the probability that failure will not occur during
given time interval T for the exponential failure parameter λ .

$1 - e^{-\mu t}$ = the probability that the system will be restored to operation
within allowable down time t for the exponential repair
parameter μ .

Figure A1 illustrates the relationship between μ and λ for a given t and T and this suggests that the dependability model may be used for trade off procedures.

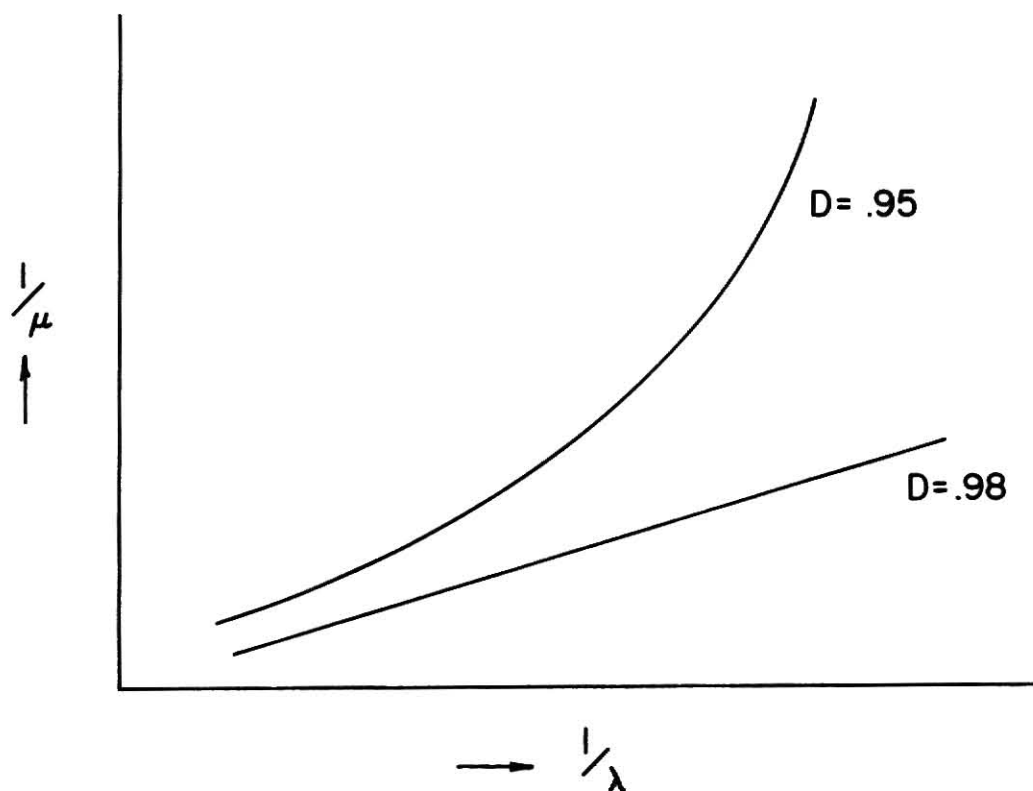


Figure A1. Dependability trade-off graph for given t and T

Mission Availability

Mission Availability has been defined by Howard [36] for a specified period of time t , as the product of the expected availability and the probability of survival for the period t . It seems intuitive that frequent failures, even though they are minor, can be more damaging than they appear to be. A car that runs a month and takes a day to fix is preferable to a car that runs a half hour and takes a minute to fix, even though these have the same availability in the steady state. For illustration let us assume that a system must operate continuously for 1/2 hour if it is to fulfill its purpose.

TABLE A1 Mission Availability for changes in mean failure and repair times

Mean Time to Failure $1/\lambda$ T hrs	Mean Time to Repair $1/\mu$ t hrs	Availability $A = \frac{T}{t+T}$	Mission Availability $A \cdot e^{-0.5/T}$
100	10	.91	.91
10	1	.91	.885
1	.1	.91	.55

As is seen in Table A1, even with equal steady state availabilities, the last system has more of a chance to fail.

A1.2 FAILURE RATE

Let the time to failure of a component be the random variable T . The failure density function is defined by

$$P(t < T \leq t + dt) = f(t)dt \quad (A4)$$

The probability of failure between time t and $t+dt$ given that there were no failures up to time t is given by

$$P(t < T \leq t + dt \mid T > t) = \frac{P(t < T \leq t + dt)}{P(T > t)} \quad (A5)$$

where $P(T > t) = 1 - P(T < t) = 1 - F(t) = R(t)$.

The conditional probability on the left side gives rise to the conditional density function $\lambda(t)$ as shown by Shooman [62] and is defined as

$$\lambda(t) = \lim_{dt \rightarrow 0} \frac{P(t < T \leq t + dt \mid T > t)}{dt} = \frac{f(t)}{R(t)} \quad (A6)$$

The conditional density function is generally called the hazard function or (instantaneous) failure rate. Figure A2 is the plot of the curve of the failure rate against the lifetime T of a very large sample from a homogenous component population. The failure rate stabilizes to an approximately constant value at time T_B . Noticeable wearout starts occurring when components reach life T_W . M is the mean wearout life of the population.

For the Weibull failure law we have

$$\lambda(t) = (\alpha\beta)t^{\beta-1} \quad (A7)$$

where α and β are positive constants. Thus λ is an increasing, decreasing or constant function of t , depending on the value of β as shown in

Figure A3. We note that the exponential distribution is a special case of the Weibull distribution since we obtain the exponential distribution if we let $\beta = 1$. The failure rate vs time plot for the normal and log normal distributions are shown in Figure A4. Another density function of

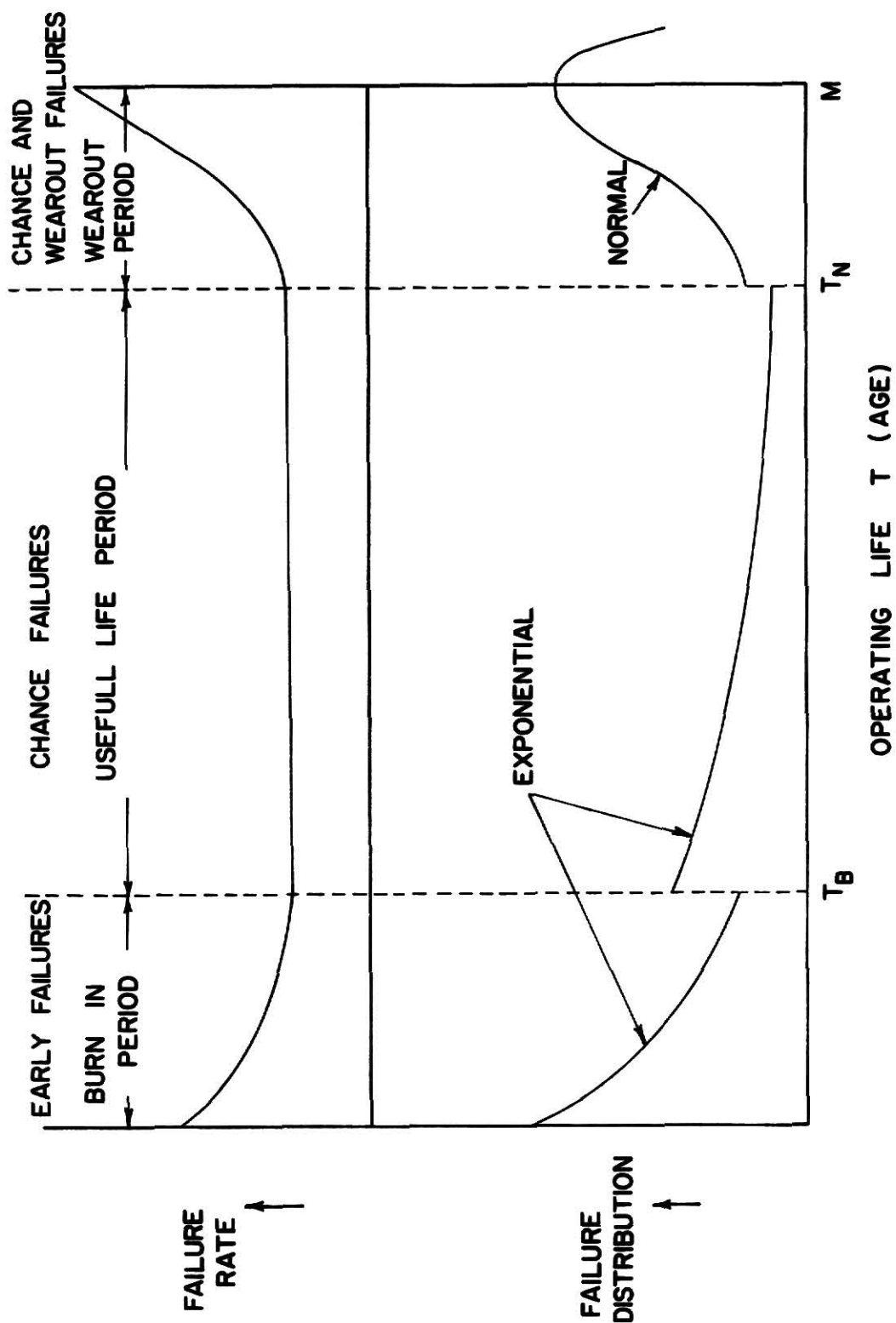


Figure A2. Component failure rate and failure distribution as a function of age.

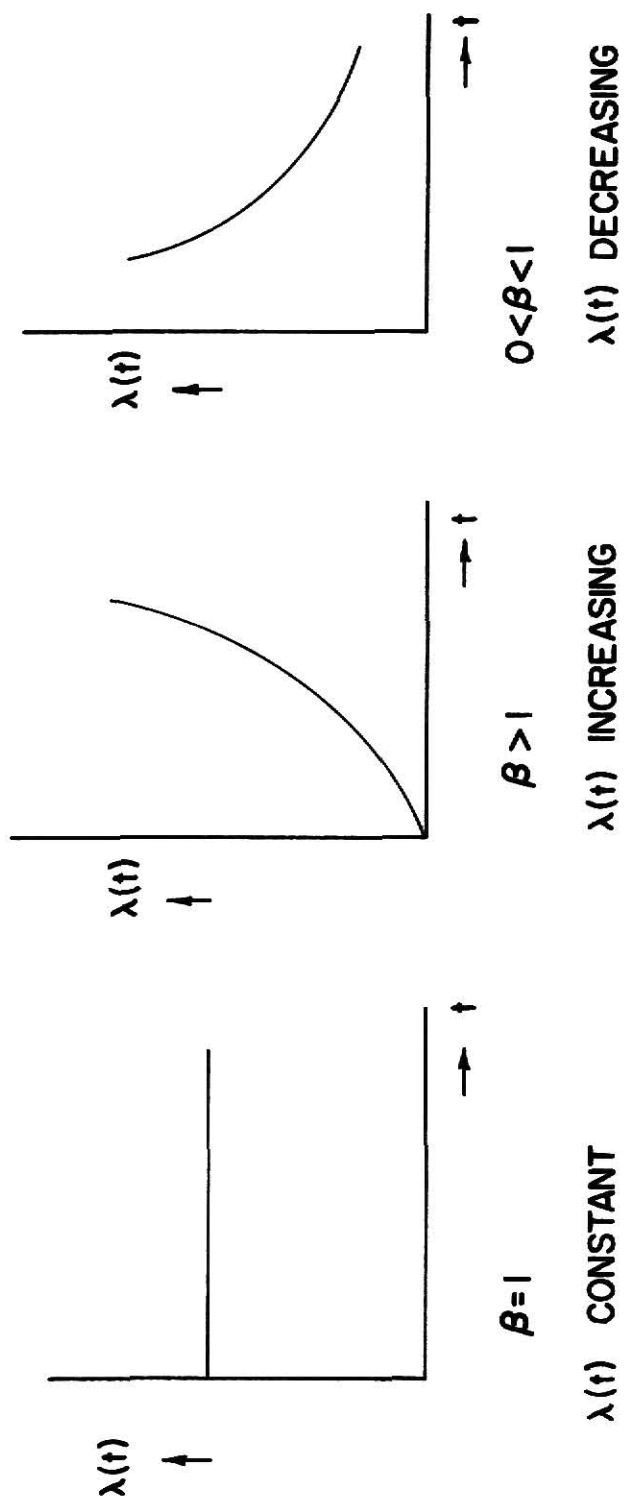


Figure A3. Changes in the nature of the failure rate $\lambda(t)$, for different values of the Weibull distribution parameter β .

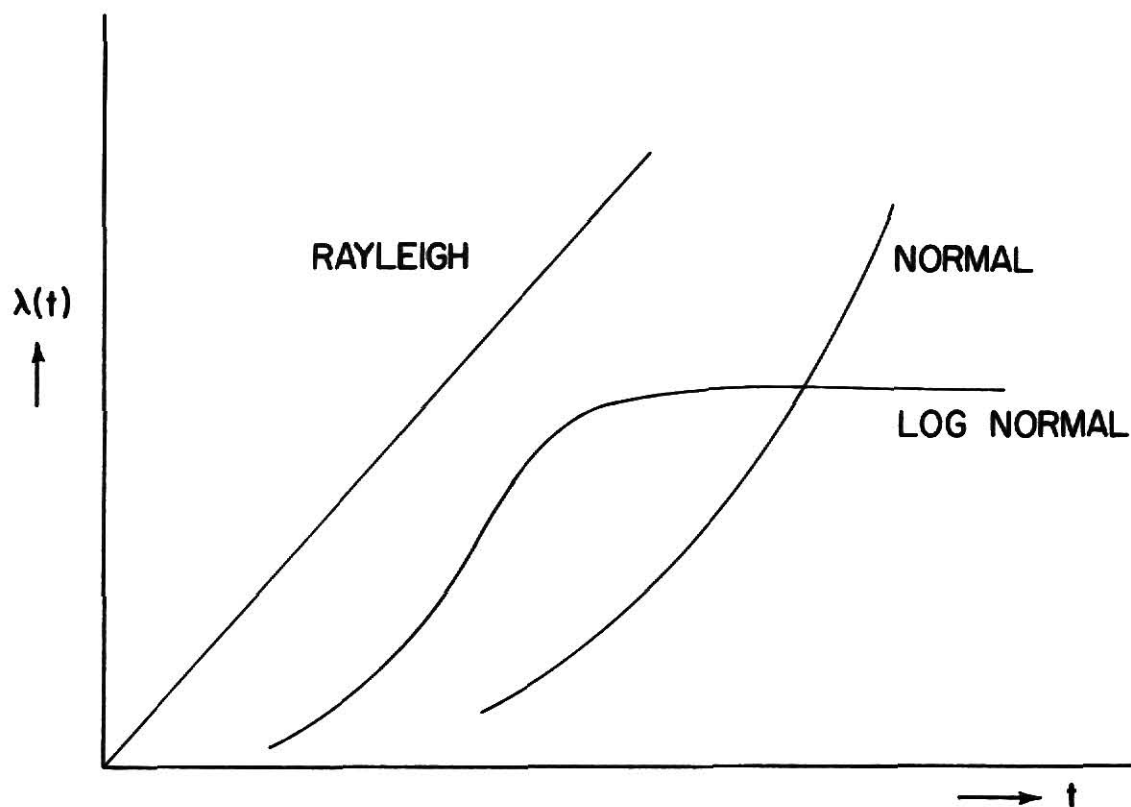


Figure A4. Graphs of failure rate $\lambda(t)$ versus time t for the Rayleigh, Normal and Log normal distributions.

considerable interest is the single parameter Rayleigh distribution which is given as

$$f(t) = Kt e^{-Kt^2/2} \quad (A8)$$

The Rayleigh distribution as shown in Figure A4 has a linearly increasing failure rate.

All that has been said with reference to failures is equally applicable in the case of repairs and the repair rate is the equivalent of the failure rate. If the failure rate is known then the reliability function $R(t)$ and the failure density function $f(t)$ may be computed by the following expressions

$$R(t) = e^{-\int_0^t \lambda(x) dx} \quad (A9)$$

$$f(t) = \lambda(t) e^{-\int_0^t \lambda(x) dx} \quad (A10)$$

The main reason for defining the $\lambda(t)$ function is that it is often more convenient to work with than $f(t)$.

A1.3 CHOICE OF THE DISTRIBUTION FOR FAILURE AND REPAIR TIMES

One chooses a model for a continuous distribution function on the basis of the following criteria.

1. The underlying assumptions associated with a particular distribution satisfy the physical nature of the problem.
2. Data is available and curve fitting is done from the plot of data.
3. A convenient and simple model is chosen based on engineering judgement.

The exponential distribution is the most important distribution for failure times since it seems to apply to most electronic equipment. All military standards are based on it and 90% of all military reliability calculations use it. The rationale is that electronic components do not fail from wearout or fatigue but from overstress (voltage, temperature, current etc) and these overstress conditions have the Poisson distribution. Under the usual conditions of operation for equipment composed of many component parts, the time between failures are exponentially distributed as shown by Davis [19].

Repair times appear to be described best by a log normal distribution but it can for computational purposes, usually be approximated by an exponential function according to Westland [77]. Shelley [60] after a study of maintenance man hour distributions for cargo air craft concluded that it adheres closely to the cumulative log normal distribution especially at the upper percentile points. The hypothesis that, "as long as the start of the repair periods occur at random, the results for availability would be independent of the type of distribution of the length of repair periods", was tested for a duplex system by a Monte Carlo method by Rohn [56]. Separate tests were made for the following distributions (1) a constant distribution (2) a normal distribution (3) a rectangular distribution (4) an exponential distribution. All the distributions were chosen to have the same average value. The fractional simultaneous repair times were very nearly the same for all the tests and agreed closely with the derived expressions in which an exponential distribution had been considered.

A1.4 SINGLE UNIT AVAILABILITY FOR WEIBULL DISTRIBUTED TIME TO FAILURE AND REPAIR.

Wohl [79] has developed expressions for availability of a single unit system when times to failure and repair have a Weibull distribution. The two parameter Weibull distribution is generally given by

$$P(t) = 1 - e^{-(t/x)^y} \quad (A11)$$

where $P(t)$ is the probability of occurrence of an event by time t , and x and y are the Weibull parameters (x is the median and y is the shape parameter). The expected time of occurrence of an event is given as

$$\bar{t} = x\Gamma\left(\frac{1}{y} + 1\right). \quad (A12)$$

Thus $MTTF = \theta\Gamma\left(\frac{1}{\beta} + 1\right)$

$$MRT = \psi\left(\frac{1}{\alpha} + 1\right)$$

where θ and ψ are the median values and β and α are the shape parameters of the failure and repair time distributions respectively.

Since the steady state availability $A(\infty) = \frac{MTTR}{MTTR + MRT}$,

$$A(\infty) = \frac{1}{1 + \frac{\psi\Gamma\left(\frac{1}{\alpha} + 1\right)}{\theta\Gamma\left(\frac{1}{\beta} + 1\right)}}.$$

Case I: $\alpha = \beta$

For any $\alpha = \beta$, $A(\infty) = \frac{1}{1 + \frac{\psi}{\theta}}$

Case II : $\beta = \frac{\alpha}{1+\alpha}$

For any $\beta = \frac{\alpha}{1+\alpha}$, $A(\infty) = \frac{1}{1 + \frac{\psi\beta}{\theta}}$

Bredeman [14] has shown that for the active tail defense system of a strategic bomber the Weibull model describes the reliability performance very well, and that Weibull parameters determined from the performance data are relatively insensitive to equipment age. The shape parameters in the Weibull failure distribution were less than unity, indicating a failure rate decreasing with time.

APPENDIX 2

The SUMT computer program listing. This version was written by
LAI [42].

[illegible]

```

C      1 -- USE COMPUTED D(I) = 0.02*OX(I).
C      ICUT .. OPTION CODE FOR STAGE STARTING STEP-SIZE
C      SET UP .. 0 -- ALL USE INPUT D(I) VALUE.
C      1 -- USE INITIAL D(I)/K FOR (K)TH STAGE.
C      P .. P FUNCTION VALUE .
C      Y .. F FUNCTION VALUE .
C      YSTOP .. COMPUTED VALUE OF FINAL-STOPPING DETERMINATOR .
C      IDPM .. SEQUENCE NO. OF SUBPROBLEMS OUTPUT .
C      NOR .. NO. OF STAGES UP TO CURRENT STAGE .
C      B .. TOLERANCE LIMIT FOR VIOLATIONS .
C      FY .. MINIMUM Y GOT SO FAR .
C      FP .. MINIMUM P GOT SO FAR .
C      G(J) .. (J)TH INEQUALITY CONSTRAINT VALUE .
C      H(K) .. (K)TH EQUALITY CONSTRAINT VALUE .
C      ITER .. WITHIN STAGE ITERATION NO.
C      NOIT .. CUMULATED ITERATION NO.
C      NOCUT .. NO. OF CUT DOWN STEP-SIZE OPERATION WITHIN STAGE.
C      NOEXP .. NO. OF SUCCESSFUL EXPLORATORY MOVES.
C      NOPAT .. NO. OF SUCCESSFUL PATTERN MOVES.
C      NOR .. NO. OF TIMES OF PULLING BACK PROCEDURE.
C      NOP .. NO. OF SUCCESSFUL MOVES INSIDE FEASIBLE REGION.
C      NOITB .. NO. OF SUCCESSFUL MOVES OUT OF FEASIBLE REGION.
C
C *****
C **SEQUENCE OF INPUT DECK ...
C (1) PROBLEM ID CARD .. ONE CARD, FORMAT 1000 .
C      PARAMETERS -- NPM,NAME( COMPOSED BY 3 PARTS ),
C      N,MG AND MH .
C (2) PROBLEM ADDITIONAL DATA CARDS .. SPECIFIED IN THE
C      SUBROUTINE READIN BY USER HIMSELF, ( OPTIONAL ).
C (3) SUBPRCBLEM 1 INITIAL DATA CARDS ..
C      FIRST -- ONE CARD, FORMAT 1002 .
C      PARAMETERS - R,RATIO,ITMAX,INCUT,THETA
C      MAXP,ISIZE, AND ICUT.
C      SECOND -- N CARDS, FORMAT 1004 .
C      PARAMETERS - J,X(I),D(I),AND OX(I).
C      *NOTE -- 1. J IS ONLY FOR USER TO
C      CHECK THE SEQUENCE OF CARDS.
C      2. CARDS SHOULD BE IN ORDER
C
C *****
C      HJS00410
C      HJS00420
C      HJS00430
C      HJS00440
C      HJS00450
C      HJS00460
C      HJS00470
C      HJS00480
C      HJS00490
C      HJS00500
C      HJS00510
C      HJS00520
C      HJS00530
C      HJS00540
C      HJS00550
C      HJS00560
C      HJS00570
C      HJS00580
C      HJS00590
C      HJS00600
C      HJS00610
C      HJS00620
C      HJS00630
C      HJS00640
C      HJS00650
C      HJS00660
C      HJS00670
C      HJS00680
C      HJS00690
C      HJS00700
C      HJS00710
C      HJS00720
C      HJS00730
C      HJS00740
C      HJS00750
C      HJS00760
C      HJS00770
C      HJS00780
C      HJS00790
C      HJS00800

```

[illegible]

```

1FG(50),H(20),FH(2C)
COMMON /CHAY/ PD1,PD2,PD3,PD4,PD5,PD6,PD7,PD8,PD9,PD10,PD11,PD12
COMMON /BLOGY/ N,MG,MH,ITER,ITMAX,ICHECK,IB,LOST
COMMON /BLOGB/ NOITP,NCITB,B,D,ISKIP
C **Q(10) ARE NOT NEEDED FOR RUNNING THIS PROGRAM, USER MAY TAKE
C THEM AWAY.
COMMON /BLOGR/ Q(10)
C **FG(20) IN BLOGO ARE USED FOR OUTPUT ADDITIONAL DATA CONCERN
C FG(20) AT SUB-OPTIMUM. USER MAY TAKE THEM AWAY.
COMMON /BLOGO/ FG
1000 FORMAT(15,5X,A2,A2,A2,3I5)
1001 FORMAT(31X,1H*,A2,A2,A2,10H* PROBLEMS/30X,20(1H*)//25X,'NO. OF
11) ...',I4/25X,'NC. OF G(J) ...',I4/25X,'NO. OF H(K) ...',I4,///,
2 NO. OF PROBLEMS ...',I4)
1002 FORMAT(20I5.4,2I5,D15.4,3I5)
1003 FORMAT(1H1,5X,7HPROBLEM,I4////////)
1004 FORMAT(15,3D15.4)
1005 FORMAT(20X,13HINITIAL POINT/5X,4HY = ,D11.4,7H, P = ,D11.4,7H, RHJS01380
1 = ,D11.4,11H, RATIO = ,D11.4,2H, /5X,4HB = ,D11.4,11H, INCUT =
2,I4, 11H, THETA = ,D11.4,2H .)
1006 FORMAT(10X,2HX(,I3,4H) = ,D14.6,7H, D(,I3,4H) = ,D14.6,2H .)
1007 FORMAT(3X,75(1H*))
1008 FORMAT(3X,15H**P (PTIMUM.. (,I4,1H)
1H,FP = ,D13.6,7H, R = ,D11.4,10H, ITER = ,I5,1H,/5X,8HNOIT = ,IHJS01440
25,7H,NOB = ,I4,8H, NOP = ,I4,10H, NOBP = ,I4/5X,8HNOEXP = ,I4,11HHJS01450
3, NOPAT = ,I4,11H, NOCUT = ,I4,2H . /5X,8HYSTOP = ,D13.6,1H.)
1011 FORMAT(5X/5X,16H**CONSTRAINTS ..)
1012 FORMAT(10X,2HG(,I3,4H) = ,D14.6,2H ,)
1013 FORMAT(10X,2HH(,I3,4H) = ,D14.6,2H ,)
1015 FORMAT(3X,46H**THE ABOVE RESULTS ARE THE FINAL OPTIMUM .)
1016 FORMAT(3X,28H**NO. OF P OPTIMUM EXCEEDED ,I5,2H .)
1020 FORMAT(5X//5X,47H**SELECTED FEASIBLE STARTING POINT ..
1021 FORMAT( ////////////////5X)
1022 FORMAT(1H 5X,44H**THE PROBLEM MIGHT BE TOO FLAT, CHECK TIMES,I4,
127H, R AND RATIO BE ADJUSTED, /7X,43HPROBABLY A DOUBLE PRECISION
2ILL BE NEEDED.)
C
C **READ IN PROBLEM NUMBER, PROBLEM NAME, AND DIMENSIONS .
C READ(5,1000) NOPM,NAME1,NAME2,NAME3,N,MG,MH

```

HJS01210

HJS01215

HJS01220

HJS01230

HJS01240

HJS01250

HJS01260

HJS01270

HJS01280

HJS01290

HJS01300

HJS01310

HJS01320

HJS01335

HJS01350

HJS01360

HJS01370

RHJS01380

HJS01390

HJS01400

HJS01410

HJS01420

HJS01430

HJS01440

HJS01450

HJS01460

HJS01470

HJS01480

HJS01490

HJS01500

HJS01510

HJS01520

HJS01530

HJS01540

HJS01550

HJS01560

HJS01570

HJS01580

HJS01590

```

WRITE(6,1021)
WRITE(6,1001) NAME1,NAME2,NAME3,N,MG,MH,NOPM
IDPM=1
C **READ IN ADDITIONAL DATA ( USED FOR ALL SUB-PROBLEMS ).
CALL READIN(N,MG,MH)
C
C **READ IN INITIAL PARAMETERS AND STOPPING CRITERIA .
1 READ(5,1002) R,RATIO,ITMAX,INCUT,THETA,MAXP,ISIZE,ICUT
WRITE(6,1003) IDPM
MP=1
MULT=1
NOEXP=0
NOPAT=0
NOCUT=0
NOR=1
FNOR=NOR
NOBP=0
NOITP=0
NOITB=0
ITER=0
NOIT=0
LOST=0
LLOST=0
IB=0
ICHECK=0
B=0.0D0
FN=N
C
C **READ IN INITIAL PCINT, INITIAL STEP-SIZES AND ESTIMATED OPTIMUM.
DO 4 I=1,N
  READ(5,1004) J,X(I),D(I),OX(I)
C **VARIABLE (J) IS USED FOR CHECKING THE SEQUENCE OF CARDS BY THE
C USER HIMSELF, AND HAS NO INFERENCE TO THE PROGRAM ( USER MAY
C USE ANY INTERGER NUMBER FOR (J) ).
  IF(ISIZE) 3,3,2
2 D(I)=OX(I)*0.02
3 BX(I)=X(I)
  FX(I)=X(I)
  PD(I)=D(I)
  OX(I)=X(I)
4 B=B+0.5D0*D(I)

```

HJS01600
HJS01610
HJS01620
HJS01630
HJS01640
HJS01650
HJS01660
HJS01670
HJS01680
HJS01690
HJS01700
HJS01710
HJS01720
HJS01730
HJS01740
HJS01745
HJS01750
HJS01760
HJS01770
HJS01780
HJS01790
HJS01800
HJS01810
HJS01820
HJS01830
HJS01840
HJS01850
HJS01860
HJS01870
HJS01880
HJS01890
HJS01900
HJS01910
HJS01920
HJS01930
HJS01940
HJS01950
HJS01960
HJS01970
HJS01980
HJS01990

```

C  **DECIDE THE STARTING VALUE OF TOLERANCE LIMIT FOR G(J) .LT. 0. .
    B=B/FN
    B=2.000*B
    PB=B
    CALL OBRES(FX,FY,FG,FH)
    CALL WEIGH(STGH,MC,FG,MH,FH)
    ITER=0
      11 CALL PENAT(FG,FH,PENAL,PENAL2)
C  **COMPUTE AN INITIAL VALUE OF R WHEN INPUT R VALUE IS .LE. 0. .
    IF(R) 12,12,13
      12 R=DABS(FY/(PENAL+PENAL2))
    R=R/4.000
C  **USE RATIO=4.0 WHEN INPUT RATIO VALUE IS .LE. 0. .
      13 IF(RATIO)14,14,15
      14 RATIO=4.0
      15 FP=FY+R*PENAL+R*(-0.5)*PENAL2
    WRITE(6,1005) FY,FP,R,RATIO,B,INCUT,THETA
    WRITE(6,1006) (I,FX(I),I,D(I),I=1,N)
    WRITE(6,1007)
    IF(LOST-2) 50,16,16
C  **SELECT AFEASIBLE STARTING POINT WHEN INPUT INITIAL POINT IS
C  NOT FEASIBLE SUBJECT TO INEQUALITY CONSTRAINTS .
C  **MAKE EXPLORATORY MOVE FOR SELECTING A FEASIBLE STARTING POINT .
      16 NOF=0
      DO 28 I=1,N
        FX(I)=X(I)+2.000*C(I)
        CALL OBRES(FX,FY,FG,FH)
        CALL WEIGH(TGH,MC,FG,MH,FH)
        IF(LOST-2) 44,18,18
      18 IF(STGH-TGH) 20,20,26
      20 FX(I)=FX(I)-4.000*D(I)
        CALL OBRES(FX,FY,FG,FH)
        CALL WEIGH(TGH,MC,FG,MH,FH)
        IF(LOST-2) 44,22,22
      22 IF(STGH-TGH) 24,24,26
      24 FX(I)=FX(I)+2.000*D(I)
        NOF=NOF+1
      GO TO 28
      26 STGH=TGH

```

HJS02000

HJS02010

HJS02020

HJS02030

HJS02040

HJS02050

HJS02060

HJS02070

HJS02080

HJS02090

HJS02100

HJS02110

HJS02120

HJS02130

HJS02140

HJS02150

HJS02160

HJS02170

HJS02180

HJS02190

HJS02200

HJS02210

HJS02220

HJS02230

HJS02240

HJS02250

HJS02260

HJS02270

HJS02280

HJS02290

HJS02300

HJS02310

HJS02320

HJS02330

HJS02340

HJS02350

HJS02360

HJS02370

HJS02380

HJS02390


```

      X(I)=FX(I)
      28 CONTINUE
C
      IF(NOF-N) 34,30,3C
C  **CUT STEP-SIZES FOR SELECTING A FEASIBLE STARTING POINT .
      30 DO 32 I=1,N
      32 D(I)=D(I)*0.500
      GO TO 16
C  **MAKE PATTERN MOVE FOR SELECTING A FEASIBLE STARTING POINT .
      34 DO 36 I=1,N
      36 PX(I)=FX(I)+(FX(I)-BX(I))
      CALL OBRES(PX,FY,FG,FH)
      CALL WEIGH(TGH,MG,FG,MH,FH)
      IF(STGH-TGH) 16,16,40
      40 DO 42 I=1,N
      X(I)=PX(I)
      42 FX(I)=PX(I)
      43 STGH=TGH
      GO TO 16
      44 DO 46 I=1,N
      D(I)=PD(I)
      OX(I)=FX(I)
      46 BX(I)=FX(I)
      LOST=0
C  **OUTPUT THE MESSAGE OF THE SELECTED FEASIBLE STARTING POINT .
      WRITE(6,1020)
      GO TO 11
      48 DO 49 I=1,N
      49 X(I)=FX(I)
      LOST=LOST
C  **START TO MINIMIZE THE CURRENT P-FUNCTION .
C
C  **MAKE EXPLORATORY MOVE FOR MINIMIZING THE P-FUNCTION .
      50 IDIFF=0
      MCUT=1
      51 NOF=0
      GO TO (52,102,52), MCUT
      52 IDIFF=IDIFF+1
      DO 101 I=1,N

```

HJS02400
 HJS02410
 HJS02420
 HJS02430
 HJS02440
 HJS02450
 HJS02460
 HJS02470
 HJS02480
 HJS02490
 HJS02500
 HJS02510
 HJS02520
 HJS02530
 HJS02540
 HJS02550
 HJS02560
 HJS02570
 HJS02580
 HJS02590
 HJS02600
 HJS02610
 HJS02620
 HJS02630
 HJS02640
 HJS02650
 HJS02660
 HJS02670
 HJS02680
 HJS02690
 HJS02700
 HJS02710
 HJS02720
 HJS02730
 HJS02740
 HJS02750
 HJS02760
 HJS02770
 HJS02780
 HJS02790


```

HJS02800      X(I)=FX(I)+D(I)
HJS02810      LOST=0
HJS02820      CALL OBRES(X,Y,G,T)
HJS02830      IF(LOST-1) 62,62,53
HJS02840      53 IF(Y-FY) 55,55,68
HJS02850      55 CALL BACK(X,X,Y,G,H)
HJS02860      NOITB=NOITB+1
HJS02870      NOBP=NOBP+1
HJS02880      C  **CHECK THE ITMAX IS EXCEEDED OR NOT IN (BACK)( LOST=1 MEANS THE
HJS02890      C  RETURNED POINT IS INFEASIBLE )
HJS02900      IF(LOST-1) 56,150,56
HJS02910      56 LOST=0
HJS02920      C  **CHECK THE ITMAX IS EXCEEDED OR NOT IN (BACK)( LOST .NE. 1 MEANS
HJS02930      C  THE ENTERED POINT IS NEAR-FEASIBLE )
HJS02940      62 IF(ICHECK-1) 64,140,140
HJS02950      64 CALL PENAT(G,H,PENAL,PENAL2)
HJS02960      P=Y+R*PENAL+R**(-C.5)*PENAL2
HJS02970      IF(P-FP) 88,68,68
HJS02980      68 X(I)=FX(I)-D(I)
HJS02990      LOST=0
HJS03000      CALL OBRES(X,Y,G,T)
HJS03010      IF(LOST-1) 80,80,70
HJS03020      70 IF(Y-FY) 73,73,86
HJS03030      73 CALL BACK(X,X,Y,G,H)
HJS03040      NOITB=NOITB+1
HJS03050      NOBP=NOBP+1
HJS03060      C  **CHECK THE ITMAX IS EXCEEDED OR NOT IN (BACK)( LOST=1 MEANS THE
HJS03070      C  RETURNED POINT IS INFEASIBLE )
HJS03080      IF(LOST-1) 74,150,74
HJS03090      74 LOST=0
HJS03100      C  **CHECK THE ITMAX IS EXCEEDED OR NOT IN (BACK)( LOST .NE. 1 MEANS
HJS03110      C  THE ENTERED POINT IS NEAR-FEASIBLE )
HJS03120      80 IF(ICHECK-1) 82,140,140
HJS03130      82 CALL PENAT(G,H,PENAL,PENAL2)
HJS03140      P=Y+R*PENAL+R**(-C.5)*PENAL2
HJS03150      IF(P-FP) 88,86,86
HJS03160      86 X(I)=FX(I)
HJS03170      NOF=NOF+1
HJS03180      GO TO 99
HJS03190      88 FY=Y

```

```

FP=P
NOITP=NOITP+1
FX(I)=X(I)
LLOST=LLOST
IF(MG) 94,94,90
90 DO 92 JJ=1,MG
92 FG(JJ)=G(JJ)
94 IF(MH) 99,99,96
96 DO 98 KK=1,MH
98 FH(KK)=H(KK)
C
C **CHECK THE STAGE STOPPING CRITERION IS SATISFIED OR NOT .
99 IF(NOCUT-INCUT) 100,150,150
100 IF(ICHECK-1) 101,150,101
101 CONTINUE
IF(NOF-N) 111,104,104
102 DO 103 I=1,N
103 X(I)=FX(I)+D(I)
CALL OBRES(X,Y,G,t)
IF(LOST-1) 1107,1107,1104
1104 IF(Y-FY) 1105,1105,1108
1105 CALL BACK(X,X,Y,G,H)
NOITB=NOITB+1
NOBP=NOBP+1
IF(LOST-1) 1106,150,1106
1106 LOST=0
IF(ICHECK-1) 1107,140,140
1107 CALL PENAT(G,H,PENAL,PENAL2)
P=Y+R*PENAL+R*(-C.5)*PENAL2
IF(P-FP) 1115,1108,1108
1108 DO 1109 I=1,N
1109 X(I)=FX(I)-D(I)
CALL OBRES(X,Y,G,t)
IF(LOST-1) 1113,1113,1110
1110 IF(Y-FY) 1111,1111,1114
1111 CALL BACK(X,X,Y,G,H)
NOITB=NOITB+1
NOBP=NOBP+1
IF(LOST-1) 1112,150,1112
1112 LOST=0
HJS03200
HJS03210
HJS03220
HJS03230
HJS03240
HJS03250
HJS03260
HJS03270
HJS03280
HJS03290
HJS03300
HJS03310
HJS03320
HJS03330
HJS03340
HJS03350
HJS03360
HJS03370
HJS03380
HJS03390
HJS03400
HJS03410
HJS03420
HJS03430
HJS03440
HJS03450
HJS03460
HJS03470
HJS03480
HJS03490
HJS03500
HJS03510
HJS03520
HJS03530
HJS03540
HJS03550
HJS03560
HJS03570
HJS03580
HJS03590

```

```

1113 IF(ICHECK-1) 1113,140,140
1113 CALL PENAT(G,H,PENAL,PENAL2)
1113 P=Y+R*PENAL+R**(-C.5)*PENAL2
1113 IF(P-FP) 1115,1114,1114
1114 MCUT=3
1114 GO TO 51
1115 FP=P
1115 FY=Y
1115 MCUT=1
1115 DO 1116 I=1,N
1116 FX(I)=X(I)
1116 IF(MG) 1119,1119,1117
1117 DO 1118 J=1,MG
1118 FG(J)=G(J)
1119 IF(MH) 50,50,1120
1120 DO 1121 K=1,MH
1121 FH(K)=H(K)
1121 GO TO 50
1121 C
1121 C **CUT STEP-SIZES FOR MINIMIZING THE P-FUNCTION .
104 DO 105 I=1,N
105 D(I)=0.5D0*D(I)
105 NOCUT=NOCUT+1
105 IF(IDIFF-INCUT) 51,106,106
106 IF(MCUT-1) 107,107,110
107 MCUT=2
108 R=R/2.0D0
108 CALL PENAT(FG,FH,PENAL,PENAL2)
108 FP=FY+R*PENAL+R**(-0.5)*PENAL2
108 INCUT=INCUT+1
108 NOCUT=0
108 DO 109 I=1,N
109 PD(I)=PD(I)*4.0D0
109 D(I)=PD(I)
109 WRITE(6,1022) MCUT
109 IF(ISIZE) 2109,2109,51
2109 DO 2110 I=1,N
2110 D(I)=D(I)/FNOR
2110 GO TO 51
110 IF(NOCUT-INCUT) 1114,150,150
HJS03600
HJS03610
HJS03620
HJS03630
HJS03640
HJS03650
HJS03660
HJS03670
HJS03680
HJS03690
HJS03700
HJS03710
HJS03720
HJS03730
HJS03740
HJS03750
HJS03760
HJS03770
HJS03780
HJS03790
HJS03800
HJS03810
HJS03820
HJS03830
HJS03840
HJS03850
HJS03860
HJS03870
HJS03880
HJS03890
HJS03900
HJS03910
HJS03920
HJS03930
HJS03940
HJS03942
HJS03944
HJS03946
HJS03950
HJS03960

```

```

HJS03970
HJS03980
HJS03990
HJS04000
HJS04010
HJS04020
HJS04030
HJS04040
HJS04050
HJS04060
HJS04070
HJS04080
HJS04090
HJS04100
HJS04110
HJS04120
HJS04130
HJS04140
HJS04150
HJS04160
HJS04170
HJS04180
HJS04185
HJS04190
HJS04200
HJS04210
HJS04220
HJS04230
HJS04240
HJS04250
HJS04260
HJS04270
HJS04280
HJS04290
HJS04300
HJS04310
HJS04320
HJS04330
HJS04340
HJS04350

```

```

C
C
111 NOEXP=NOEXP+1
    MCUT=3
    **MAKE PATTERN MOVE FOR MINIMIZING THE P-FUNCTION .
      DO 112 I=1,N
        PX(I)=FX(I)+(FX(I)-BX(I))
112   BX(I)=FX(I)
        LOST=0
        CALL OBRES(PX,Y,G,H)
          IF(LOST-1) 124,124,113
113   IF(Y-FY) 114,114,51
114   CALL BACK(PX,X,Y,G,H)
        NOITB=NOITB+1
        NOBP=NOBP+1
      **CHECK THE ITMAX IS EXCEEDED OR NOT IN (BACK)( LOST=1 MEANS THE
RETURNED POINT IS INFEASIBLE )
      IF(LOST-1) 115,15C,115
C
115 LOST=0
      **CHECK THE ITMAX IS EXCEEDED OR NOT IN (BACK)( LOST .NE. 1 MEANS
THE ENTERED POINT IS NEAR-FEASIBLE )
122 IF(ICHECK-1) 123,140,140
123 IF(ISKIP-1) 124,48,48
124 CALL PENAT(G,H,PENAL,PENA2)
P=Y+R*PENAL+R**(-C.5)*PENA2
IF(P-FP) 128,48,48
128 NOPAT=NOPAT+1
    NOITP=NOITP+1
    DO 129 II=1,N
      FX(II)=PX(II)
      LLOST=LOST
130 IF(MG) 133,133,131
131 DO 132 J=1,MG
132 FG(J)=G(J)
133 IF(MH) 136,136,134
134 DO 135 K=1,MH
135 FH(K)=H(K)
136 FY=Y
    FP=P
C

```

```

C  **CHECK THE STAGE STOPPING CRITERION IS SATISFIED OR NOT .
C  IF(NOCUT-INCUT) 128,150,150
C  138 IF(ICHECK-1) 50,150,150
C
C  **CHECK THE ITMAX EXCEEDED POINT( WHEN IT IS RETURNED FROM BACK )
C  IS BETTER OR NOT AND SET PROPER STAGE-OPTIMUM .
C  140 CALL OBRES(X,Y,G,T)
C  CALL PENAT(G,H,PENAL,PENA2)
C  P=Y+R*PENAL+R**(-C.5)*PENA2
C  IF(P-FP) 142,150,150
C
C  142 DO 144 I=1,N
C  144 FX(I)=X(I)
C  LLOST=LOST
C  GO TO 130
C
C  **SET THE SUB-OPTIMUM GOT BEFORE ENTERED TO BACK BE THE
C  STAGE-OPTIMUM .
C  150 NOPULL=0
C  PULL=0.63D0
C  IF(MG) 15,15,151
C  151 DO 152 J=1,MG
C  IF(FG(J)) 162,162,152
C  152 CONTINUE
C
C  **CHECK THE STAGE OPTIMUM IS FEASIBLE OR NOT .
C  160 IF(LLOST-1) 170,162,162
C  **PULL BACK THE INFEASIBLE STAGE-OPTIMUM INTO THE FEASIBLE REGION
C  162 DO 163 I=1,N
C  163 FX(I)=PULL*(FX(I)-OX(I))+OX(I)
C  NOPULL=NOPULL+1
C  CALL OBRES(FX,FY,FG,FH)
C  LLOST=LOST
C  NOITB=NOITB+1
C  IF(NOPULL-5) 160,164,164
C  164 NOPULL=0
C  165 DO 166 I=1,N
C  166 FX(I)=OX(I)
C  CALL OBRES(FX,FY,FG,FH)
C  170 LOST=0
C  CALL PENAT(FG,FH,PENAL,PENA2)
C  FP=FY+R*PENAL+R**(-0.5)*PENA2

```

```

HJS04760
HJS04770
HJS04780
HJS04785
HJS04790
HJS04800
HJS04810
HJS04820
HJS04830
HJS04840
HJS04850
HJS04860
HJS04870
HJS04880
HJS04890
HJS04900
HJS04910
HJS04920
HJS04930
HJS04940
HJS04950
HJS04960
HJS04970
HJS04980
HJS04990
HJS05000
HJS05010
HJS05020
HJS05030
HJS05040
HJS05050
HJS05060
HJS05070
HJS05080
HJS05090
HJS05100
HJS05110
HJS05120
HJS05130
HJS05140

208 NOIT=NOIT+ITER
   YSTOP=DABS(FY/(FY-R*PENAL+R**(-0.5)*PENAL2))
   YSTOP=DABS(YSTOP-1.0)
   CALL OBRES(FX,FY,FG,FH)
   WRITE(6,1008) NOR,FY,FP,R,ITER,NOIT,NOBP,NOITP,NOITB,NOEXP,
   1NOPAT,NOICUT,YSTOP
   WRITE(6,1006) (I,FX(I),I,D(I),I=1,N)
   WRITE(6,1011)
   IF(MG) 216,216,215
215 WRITE(6,1012) (J,FG(J),J=1,MG)
216 IF(MH) 218,218,217
217 WRITE(6,1013) (K,FH(K),K=1,MH)
   C **OUTPUT ADDITIONAL INFORMATION.
218 CALL OUTPUT(N,MG,MH)
   WRITE(6,1007)

C
C **CHECK THE FINAL STOPPING CRITERION IS SATISFIED OR NOT.
   IF(YSTOP-THETA) 230,230,220
C **CHECK THE MAXP IS EXCEEDED OR NOT .
220 IF(NOR-MAXP) 221,232,232
C **STORE LAST SUB-OPTIMUM PCINT .
221 DO 222 I=1,N
   D(I)=PD(I)
222 OX(I)=FX(I)
C **SHIFT TO THE NEXT STAGE SEARCH .
   R=R/RATIO
   FP=FY+R*PENAL+R**(-0.5)*PENAL2
   NOR=NOR+1
   IF(NOR-5*MP) 224,224,223
223 INCUT=INCUT+1
   MP=MP+1
224 IF(NOBP) 226,226,225
225 INCUT=INCUT+1
226 NOBP=0
   MULT=1
   NOITB=0
   NOITP=0
   ICHECK=0
   NOEXP=0
   NOPAT=0

```

HJS05150
HJS05160
HJS05170
HJS05180
HJS05190
HJS05200
HJS05210
HJS05220
HJS05230
HJS05240
HJS05250
HJS05260
HJS05270
HJS05280
HJS05290
HJS05300
HJS05310
HJS05320
HJS05330
HJS05340
HJS05350
HJS05360
HJS05370
HJS05380

```

NOCUT=0
ITER=0
IB=0
FNOR=NOR
B=0.000
MCUT=1
IDIFF=0

C
C  **DECIDE THE INITIAL STEP-SIZES AND TOLERANCE LIMIT.
C  IF(ICUT) 229,229,227
227 DO 228 I=1,N
D(I)=PD(I)/FNOR
228 B=B+0.500*D(I)
B=B/FN
GO TO 50
229 B=PB
GO TO 50
230 WRITE(6,1015)
GO TO 234
232 WRITE(6,1016) MAXF
234 IDPM=IDPM+1
IF(IDPM-NOPM) 1,1,236
236 STOP
END

```

HJS05390
HJS05400
HJS05410
HJS05420
HJS05430
HJS05440
HJS05450
HJS05460
HJS05470
HJS05480
HJS05490
HJS05495
HJS05500

```

SUBROUTINE BACK(XE,X,Y,G,H)

C
C  THIS SUBROUTINE PULLS INFEASIBLE POINTS BACK INTO THE
C  FEASIBLE OR NEAR-FEASIBLE REGION .
C
C  **DEFINITION **
C  FEASIBLE .. ALL G(I) .GE. 0. ,
C  NEAR-FEASIBLE.. (B-TGH) .GE. 0. .
C
C  IMPLICIT REAL*8(A-H,O-Z)
C  DIMENSION XB(20),X(20),G(50),H(20),D(20)
C  COMMON /CHAY/ PD1,PD2,PD3,PD4,PD5,PD6,PD7,PD8,PD9,PD10,PD11,PD12
C  COMMON /BLOGY/ N,PG,MH,ITER,ITMAX,ICHECK,IB,LOST

```

```

COMMON /BLOG8/ NOITP,NOITB,B,D,ISKIP
ITERB=ITER
ISKIP=0
FRAC=0.5
CALL WEIGH(TGH,MG,G,MH,H)
IF(TGH) 8,8,4
C **DECREASE THE VALUE OF B IN RETURN .
  4 IF(B-TGH) 12,12,6
  6 IF(0.70D0*B-TGH) 10,8,8
  8 B=0.75D0*B
 10 LOST=0
  RETURN
 12 FTGH=TGH
C
C **MAKE EXPLORATORY MOVE FOR MINIMIZING TGH .
22 NOF=0
  DO 38 NB=1,N
    XB(NB)=XB(NB)-FRAC*D(NB)
    CALL OBRES(XB,Y,G,H)
    CALL WEIGH(TGH,MG,G,MH,H)
    IF(LOST-2) 24,26,26
    24 NOITP=NOITP+1
    25 LOST=0
    GO TO 46
    26 NOITB=NOITB+1
    IF(ICHECK-1) 27,45,45
    27 IF(TGH-FTGH) 28,32,32
    28 FTGH=TGH
    IF(B-TGH) 38,38,25
C
    32 XB(NB)=XB(NB)+D(NE)*2.0*FRAC
    CALL OBRES(XB,Y,G,H)
    CALL WEIGH(TGH,MG,G,MH,H)
    IF(LOST-2) 24,34,34
    34 NOITB=NOITB+1
    IF(ICHECK-1) 35,45,45
    35 IF(TGH-FTGH) 28,36,36
    36 XB(NB)=XB(NB)-FRAC*D(NB)
    NOF=NOF+1
    38 CONTINUE

```

HJS05510
HJS05520
HJS05525
HJS05530
HJS05540
HJS05550
HJS05560
HJS05570
HJS05580
HJS05590
HJS05600
HJS05610
HJS05620
HJS05630
HJS05640
HJS05650
HJS05660
HJS05670
HJS05680
HJS05690
HJS05700
HJS05710
HJS05720
HJS05730
HJS05740
HJS05750
HJS05760
HJS05770
HJS05780
HJS05790
HJS05800
HJS05810
HJS05820
HJS05830
HJS05840
HJS05850
HJS05860
HJS05870
HJS05880
HJS05890


```

C      IF(NOF-N) 22,42,42
C      **ADD STEP-SIZES FOR MINIMIZING TGH .
42 IF(ITER-ITERB-4*N) 44,43,59
43 FRAC=FRAC#5.O00
    GO TO 22
44 FRAC=FRAC#1.5
    GO TO 22
45 LOST=1
C
C      **SET BASE POINT TO RETURN .
46 DO 50 NB=1,N
   D(NB)=D(NB)#0.55DC
50 X(NB)=XB(NB)
C      **DECREASE THE VALUE OF B IN RETURN .
   IF(0.7DO*B-TGH) 6C,58,58
58 B=0.75DO*B
   RETURN
59 LOST=0
   ISKIP=1
60 RETURN
   END
HJS05900
HJS05910
HJS05920
HJS05930
HJS05940
HJS05950
HJS05960
HJS05970
HJS05980
HJS05990
HJS06000
HJS06010
HJS06020
HJS06030
HJS06040
HJS06050
HJS06060
HJS06062
HJS06064
HJS06066
HJS06070
HJS06080
```

```

C
C
C
C
C
C
SUBROUTINE PENAT(G,H,PENAL,PENA2)
      THIS SUBROUTINE CCMPUTES THE PENALTY TERMS FOR SUMT FORMULATION .
      PENAL FOR INEQUALITY CONSTRAINTS .
      PENA2 FOR EQUALITY CONSTRAINTS .

      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION G(50),H(20)
      COMMON /CHAY/ PD1,PD2,PD3,PD4,PD5,PD6,PD7,PD8,PD9,PD10,PD11,PD12
      COMMON /BLOGY/ N,MG,MH,ITER,ITMAX,ICHECK,IB,LOST
      PENAL=0.00
      PENA2=0.00
      IF(MG) 5,5,1
1    DO 4 I=1,MG
      IF(G(I)) 4,2,4
      HJS06090
      HJS06100
      HJS06110
      HJS06120
      HJS06130
      HJS06140
      HJS06150
      HJS06160
      HJS06165
      HJS06170
      HJS06180
      HJS06190
      HJS06200
      HJS06210
      HJS06220

```

HJS06230
HJS06240
HJS06250
HJS06260
HJS06270
HJS06280
HJS06290
HJS06300
HJS06310

```

C  **SET G(I)=0.1E-48  WHEN G(I)=0. ( ON THE BOUNDARY )
2  G(I)=0.1D-48
4  PENAL=PENAL+DABS(1.0D0/G(I))
5  IF(MH) 10,10,6
6  DO 9 K=1,MH
8  PENAL2=PENAL2+H(K)**2
9  CONTINUE
10 RETURN
    END

```

HJS06320
HJS06330
HJS06340
HJS06350
HJS06360
HJS06370
HJS06380
HJS06385
HJS06390
HJS06400
HJS06410
HJS06420
HJS06430
HJS06440
HJS06450
HJS06460
HJS06470
HJS06480
HJS06490
HJS06500
HJS06510
HJS06520

```

C  SUBROUTINE WEIGH(TGH,MG,G,MH,H)
C
C  THIS SUBROUTINE CCMPUTES THE TOTAL WEIGHT OF VIOLATION
C  TO THE INEQUALITY CCNSTRANTS .
C
C  IMPLICIT REAL*8(A-H,O-Z)
C  DIMENSION G(50),H(20)
C  COMMON /CHAY/ PD1,PD2,PD3,PD4,PD5,PD6,PD7,PD8,PD9,PD10,PD11,PD12
C  TGH=0.
C  IF(MG) 4,4,1
1  DO 3 IR=1,MG
C  IF(G(IR)) 2,3,3
2  TGH=TGH+G(IR)**2
3  CONTINUE
4  IF(MH) 8,8,5
5  DO 7 IR=1,MH
C  IF(H(IR)) 6,7,6
6  TGH=TGH+H(IR)**2
7  CONTINUE
8  TGH=TGH**0.5D0
    RETURN
    END

```

HJS06530
HJS06540
HJS06550

```

C  SUBROUTINE READIN(N,MG,MH)
C  THIS SUBROUTINE IS FOR READ IN ADDITIONAL DATA .
C  USER SUPPLIES HIS OWN READ STATEMENT AND FORMAT .

```

```

C ARGUMENTS N,MG,MH ARE NUMBERS OF VARIABLES,OF INEQUALITY CONSTRAIN
C AND OF EQUALITY CONSTRAINTS .
C COMMON /BLOGR/ ..... STATEMENT IS FOR TRANSFER DATA USE .
C
C IMPLICIT REAL*8(A-H,O-Z)
C COMMON /BLOGR/ Q(10)
C RETURN
C END

```

```

SUBROUTINE OUTPUT(N,MG,MH)
C THIS SUBROUTINE IS FOR USER TO PRINT OUT ADDITIONAL INFORMATION
C WANTED. ARGUMENTS N,MG,MH ARE NUMBERS OF VARIABLES,OF INEQUALITY
C CONSTRAINTS,AND OF EQUALITY CONSTRAINTS .
C THE NEEDED DATA INFORMATION
C COMMON /BLOGO/..... IS FOR TRANSFER NEEDED DATA IN MAIN TO
C THE SUBROUTINE OUTPUT .
C COMMON /CHAY/ HAS BEEN USED INSTEAD OF COMMON /BLOGO/
C USER SUPPLIES ALL NECESSARY FORMATS .
C
C IMPLICIT REAL*8(A-H,O-Z)
C COMMON /CHAY/ PD1,PD2,PD3,PD4,PD5,PD6,PD7,PD8,PD9,PD10,PD11,PD12
C COMMON /BLOGO/ G(50)
C
9020 FORMAT(//,10X,' COST = ',D15.6)
C WRITE(6,9020) PD1
C WRITE(6,9020) PD2
C WRITE(6,9020) PD3
C RETURN
C END

```

```

HJS06750
HJS06760

```

```

SUBROUTINE OBRES(X,Y,G,H)
C THIS SUBROUTINE CCMPUTES OBJ. AND CONSTRAINT VALUES .
C USER SHOULD SUPPLY ALL NECESSARY STATEMENTS IN THE FORM ..
C Y=....., FUNCTION OF X(I) , FOR OBJECTIVE FUNCTION .
C G(J)=....., J FROM 1 TO MG , FOR CONSTRAINTS G(J) .GT. 0.0 .
C H(K)=....., K FROM 1 TO MH , FOR CONSTRAINTS H(K) .EQ. 0.0 .
C
HJS06770
HJS06780
HJS06790
HJS06800
HJS06810
HJS06820
HJS06830

```

```

C      INSERT THESE STATEMENTS IN THE BLOCK BELOW LINED BY ***** . HJS06840
C      IMPLICIT REAL*8(A-H,O-Z) HJS06850
C      DIMENSION X(20),G(50),H(20),Q(10) HJS06860
C      COMMON /BLOGY/ N,PG,MH,ITER,ITMAX,ICHECK,IB,LOST HJS06870
C      COMMON /CHAY/ PD1,PD2,PD3,PD4,PD5,PD6,PD7,PD8,PD9,PD10,PD11,PD12 HJS06880
C      COMMON /BLOGR/ Q HJS06885
C      100 FORMAT(3X,25H**THE ITERATION EXCEEDED ,I5,IH.) HJS06890
C      HJS06900
C      HJS06910
C      ***** STATEMENT NUMBERS 1,2,3,4,5,6,7,8,100 HAVE BEEN USED. HJS06920
C      ST=1500.0 HJS06930
C      A1=2.0*X(4)*((X(1)**2+X(1)*X(4))/(3.0*X(4)**2+3.0*X(1)*X(4)+X(1)**
12))
C      A2=2.0*X(5)*((X(2)**2+X(2)*X(5))/(3.0*X(5)**2+3.0*X(2)*X(5)+X(2)**
12))
C      A3=2.0*X(6)*((X(3)**2+X(3)*X(6))/(3.0*X(6)**2+3.0*X(3)*X(6)+X(3)**
12))
C      B1=2.0*X(4)
C      B2=2.0*X(5)
C      B3=2.0*X(6)
C      C1=ST*A1
C      C2=ST*A2
C      C3=ST*A3
C      C4=ST/X(7)
C      E11=1.0/DEXP(X(1)*X(7))
C      E12=1.0/DEXP(X(2)*X(7))
C      E13=1.0/DEXP(X(3)*X(7))
C      E21=1.0/DEXP(2.0*X(1)*X(7))
C      E22=1.0/DEXP(2.0*X(2)*X(7))
C      E23=1.0/DEXP(2.0*X(3)*X(7))
C      D11=(1.5/X(1))*(1.0-E11*(2.0-E11))
C      D12=(1.5/X(2))*(1.0-E12*(2.0-E12))
C      D13=(1.5/X(3))*(1.0-E13*(2.0-E13))
C      D21=(1.5/X(1))-(2.0/X(1))*E11+(0.5/X(1))*E21
C      D22=(1.5/X(2))-(2.0/X(2))*E12+(0.5/X(2))*E22
C      D23=(1.5/X(3))-(2.0/X(3))*E13+(0.5/X(3))*E23
C      D31=X(1)**2+X(1)*X(4)
C      D32=X(2)**2+X(2)*X(5)
C      D33=X(3)**2+X(3)*X(6)

```

```

D41=3.0*X(4)**2+3.0*X(1)*X(4)+X(1)**2
D42=3.0*X(5)**2+3.0*X(2)*X(5)+X(2)**2
D43=3.0*X(6)**2+3.0*X(3)*X(6)+X(3)**2
AV1=1.0-(D11*D31)/(D21*D41)
AV2=1.0-(D12*D32)/(D22*D42)
AV3=1.0-(D13*D33)/(D23*D43)
Y1=C1*(1.5/B1)**2+C2*(1.5/B2)**2+C3*(1.5/B3)**2
Y2=C4*(5.0/B1+5.0/B2+5.0/B3-15.0)
Y3=.15/A1+.15/A2+.15/A3+150.0*(B1+B2+B3)-30.0
PD1=Y1
PD2=Y2
PD3=Y3
Y=Y1+Y2+Y3
G(1)=X(1)-.001
G(2)=X(2)-.001
G(3)=X(3)-.001
G(4)=.020-X(1)
G(5)=.020-X(2)
G(6)=.020-X(3)
G(7)=X(4)-.01
G(8)=X(5)-.01
G(9)=X(6)-.01
G(10)=0.6-X(4)
G(11)=0.6-X(5)
G(12)=0.6-X(6)
G(13)=AV1*AV2*AV3-0.990
G(14)=X(7)-75.0
G(15)=800.0-X(7)

```

```
IF(G(I)) 5,6,7
5  LOST=2
   GO TO 7
6  IB=1
7  CONTINUE
8  RETURN
   END
```

```
HJS07060
HJS07070
HJS07080
HJS07090
HJS07100
HJS07110
HJS07120
```

AVAILABILITY MODELS OF MAINTAINED SYSTEMS

by

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AN ABSTRACT OF A MASTER'S THESIS

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In this thesis, a procedure is presented which uses availability as the principal consideration in the design of an optimum system. A system is considered which has n parallel subsystems connected in series and corrective maintenance is practiced. Each subsystem consists of two identical units in parallel, where the failure and repair times are exponentially distributed, with parameters λ and μ respectively. A Markovian approach is employed in obtaining the instantaneous and steady state availability expressions for each of the subsystems. It is shown that periodic maintenance increases the mean life of nonmaintained redundant systems. Based on the assumption that the probability of the system being down is proportional to the increase in the mean life achieved by periodic maintenance, availability models for systems with corrective and preventive maintenance are developed. The total system costs that are included in the model are a) the cost for designing failure and repair rates; b) the cost for corrective maintenance; and c) the cost for preventive maintenance. The total cost expression is a function of the failure and repair rates of the individual units, the time interval for preventive maintenance and the mission time.

The problem is one in which the total system cost is minimized while maintaining a given level of availability. This is a nonlinear programming problem and is solved with the Sequential Unconstrained Minimization Technique (SUMT). A version of this technique suggested by Lai (42) is used which does not require finding the first order and second order derivatives of the objective and constraint functions as when the gradient search versions of the technique are used. The version of Lai used in