Colliding Bose-Einstein Condensates to Observe Efimov Physics

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We explore the manifestation of Efimov physics through the collision energy dependence of the three-body scattering observables and propose that it can be measured by observing atom loss in collisions of Bose-Einstein condensates. Our study shows that log-periodic Efimov features in the scattering observables extend beyond the usual threshold regime to nonzero collision energies and result from two interfering pathways. Further, these oscillations have a one-to-one connection with the scattering length oscillations at zero energy and thus to Efimov states themselves.

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Since its prediction in the early 1970s, the Efimov effect has had a profound impact on both nuclear and atomic physics [1–4]. Based on very general grounds, Efimov predicted that a three-body system with no bound two-body subsystems can, in some cases, form an infinite sequence of three-body bound states with energies

$$E_{n+1}/E_n = e^{-2\pi/s_0} \qquad (n = 0, 1, 2, \ldots). \tag{1}$$

The ground state energy, E_0, typically depends on the details of the interparticle interactions. Once determined, though, it fixes the positions of all other states via Eq. (1). The universal constant s_0 in the factor $e^{2\pi/s_0}$ is determined by the number and kind of indistinguishable particles as well as by the mass ratio of the distinguishable particles [5,6]. In ultracold gases, however, evidence of the Efimov effect does not stem from observation of the bound spectrum. Rather, it traces the signature of such states through their impact on ultracold three-body scattering processes, usually through the loss of atoms or molecules. Therefore, the "smoking gun" for Efimov physics has become the observation of log-periodic features in any physical observable—not just the bound spectrum. Moreover, in most experimental observations to date [7–11], it is the comparison with theory that finally serves to establish the connection with Efimov physics.

Nevertheless, ultracold gases have a significant advantage over other weakly bound systems since the interatomic interactions—and thus the Efimov physics—can be controlled. Most current experiments use Feshbach resonances to achieve this control by applying an external magnetic field to tune the two-body interactions which are characterized at low energies by the s-wave scattering length a. Near a Feshbach resonance, a can vary from $-\infty$ to $+\infty$, thus providing access to universal Efimov physics which appears when $|a|$ greatly exceeds the characteristic range r_0 of the interatomic interaction. Thus, the signature of Efimov physics in experiments so far has been the appearance of features as a function of a that follow the predicted behavior—that is, a feature appears each time $|a|$ is increased by a factor of e^{π/s_0}.

One of the most dramatic consequences of the Efimov effect is the infinity of three-body features. Unfortunately, no more than two of these features in any single sequence have yet been observed, so the fundamental log-periodic behavior has not been experimentally verified. The main reasons are the technical challenges of fine tuning a near a Feshbach resonance to achieve extremely large $|a|$ *and* keeping the temperature T in the threshold regime $T \lesssim \hbar^2/2\mu_3 a^2$ to prevent thermal and unitarity effects [12], where $\mu_3^2 = m_1 m_2 m_3/(m_1 + m_2 + m_3)$ for atomic masses m_i.

In this Letter, we propose an entirely new way to observe the features associated with Efimov states: tuning the relative energy of two colliding Bose-Einstein condensates (BECs). The core of our proposal relies on the fact that Efimov physics extends to finite energy three-body scattering observables. Current thinking is that Efimov physics appears only in the a dependence of *zero*-energy scattering observables. We show, however, that at fixed a, the energy-dependent rate for three-body recombination of bosons B, $B + B + B \to B_2 + B$, shows the log-periodic oscillations characteristic of Efimov physics. With a simple theoretical treatment, we connect the energy-dependent oscillations one-to-one with the previously known a-dependent oscillations and the Efimov states themselves. We also show that the recently realized ability to collide BECs [13–15] is ideal for observing Efimov oscillations since their collision energy can be precisely tuned through a large range.

The Efimov effect can be readily understood in the hyperspherical framework [16]. With the overall size of the three-particle system represented by the hyperradius R, the Efimov effect is a consequence of the attractive $1/R^2$ potential that emerges when $|a| \gg r_0$. For $a > 0$, when there exists a weakly bound two-body state, the zero energy three-body recombination rate assumes the universal form [2–4,17]

$$K_3 \propto a^4 \sin^2[s_0 \ln(a/r_0) + \Phi], \quad (2)$$

which displays a series of minima separated in a by factors of e^{π/s_0}. Along with r_0, the short-range three-body phase Φ parametrizes the details of the interparticle interactions, which vary drastically from one system to another. The log-periodic oscillation in K_3 reflects the Efimov effect and can be interpreted as the interference of two different recombination pathways [3,4,18].

In fact, these same two recombination pathways provide a natural way to understand how Efimov physics extends to finite energies. Using an analysis very similar to that presented in Ref. [18], we trace the pathways through the idealized adiabatic hyperspherical potentials for total orbital angular momentum $J = 0$ and sketch the result in Fig. 1. As described in [18], these potentials are universal for $R \geq r_0$, so we expect the rates to be universal for collision energies E less than about $\hbar^2/2\mu_3 r_0^2$. For clarity, we present here a somewhat heuristic derivation that nevertheless captures the essential details and agrees with the fully numerical results.

When $a \gg r_0$, recombination occurs predominantly at $R \approx a$ [18] which leads to the two pathways indicated in Fig. 1. We can write the amplitudes for these two pathways as $\mathbf{A}_j = A_j e^{i\phi_j}$; A_j^2 includes the probability both for the three free particles to get from $R = \infty$ to $R \approx a$ and for the transition to the atom-dimer channel at $R \approx a$. Near zero energy A_j is determined mainly by tunneling and scales like k^2a^2 [18], where $k = \sqrt{2\mu_3 E}/\hbar$ is the three-body incident wave number. For higher energies such that $a^{-1} \ll k \ll r_0{}^{-1}$, the A_j are essentially independent of both a and k.

The phases ϕ_j from Fig. 1 can be obtained via the WKB approximation [18]. Since the recombination probability P is proportional to $|\mathbf{A}_1 + \mathbf{A}_2|^2$, only the phase difference matters, as expected, giving

$$\begin{aligned}\phi_1 - \phi_2 = {}& \sqrt{k^2a^2 + s_0^2} - \sqrt{k^2a^2 - s_1^2} - \sqrt{k^2r_0^2 + s_0^2} \\ & - s_0 \ln\left(\frac{s_0 + \sqrt{k^2a^2 + s_0^2}}{s_0 + \sqrt{k^2r_0^2 + s_0^2}}\frac{r_0}{a}\right) \\ & + \Phi' - \frac{\pi}{2}s_1 + s_1 \tan^{-1}\left(\frac{s_1}{\sqrt{k^2a^2 - s_1^2}}\right). \end{aligned} \quad (3)$$

For identical boson systems, $s_0 = 1.006$ and $s_1 = 4.46$; for Cs + Cs + Li, $s_0 = 1.85$ and $s_1 = 2.76$ [5].

For incident energies such that $a^{-1} \ll k \ll r_0{}^{-1}$, the phase difference can be simplified as

$$\phi_1 - \phi_2 = -s_0 \ln(kr_0) + \Phi \quad (4)$$

to a good approximation (Φ combines Φ' with other universal constants for compactness). Using $K_3 \propto P/k^4$, the

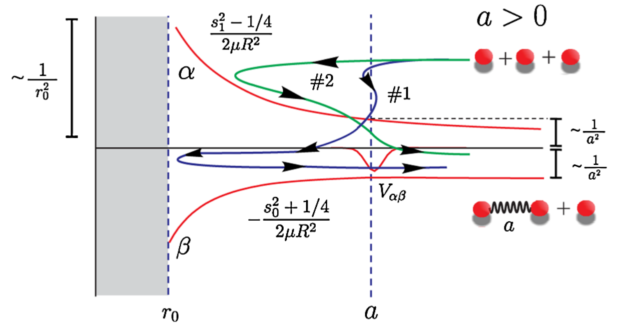

FIG. 1 (color online). Schematic potentials and pathways—#1 and #2—for recombination with $J = 0$. Channel α represents three free atoms; channel β, the atom plus dimer; and $V_{\alpha\beta}$, the coupling between the two channels.

recombination rate in this energy range is thus

$$K_3 \propto k^{-4}\{c_1 + c_2 \sin^2[-s_0 \ln(kr_0) + \Phi]\}, \quad (5)$$

where c_1 and c_2 can be written in terms of A_1 and A_2. They can also be obtained—along with Φ—by fitting to the numerically obtained K_3.

Equation (5) thus summarizes one of our main results: energy-dependent log-periodic oscillations that are intimately connected with Efimov physics. An important feature of Eq. (5) is that the oscillatory structure in the energy range $a^{-1} \ll k \ll r_0{}^{-1}$ is independent of a. So, the energy dependence measured at different, convenient a can, in principle, be combined to produce the full energy range.

The one-to-one connection between the well-known a-dependent oscillations at zero energy and the E-dependent oscillations from Eq. (5) is contained in Eq. (3) but is more easily seen in Fig. 2. Figure 2 shows Eq. (3) through the whole energy range $0 < k \lesssim r_0{}^{-1}$

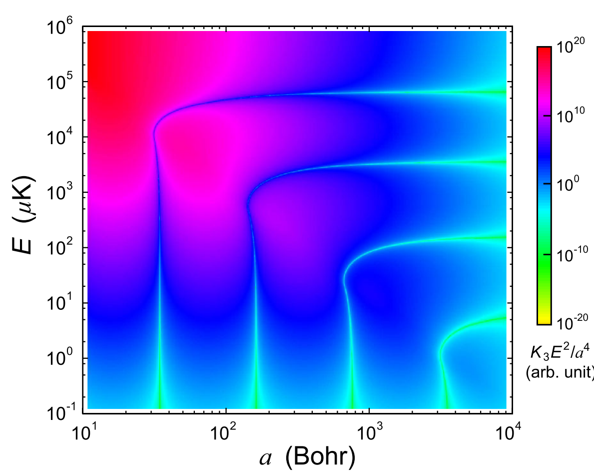

FIG. 2 (color online). The $J = 0$ recombination rate K_3 scaled as K_3E^2/a^4 [28] to emphasize the Efimov oscillations, especially the connection between the a dependence and the E dependence from Eqs. (2) and (5), respectively. Equation (3) was used with Cs + Cs + Li masses, $\Phi' = 0$, and $r_0 = 15$ bohr. The amplitudes of the two interfering pathways were assumed equal to emphasize the oscillations.

where recombination is expected to be universal. Each oscillation in E corresponds with one oscillation in a at zero energy, and by extension to a particular Efimov state. Note that Eq. (2) applies in the lower left half of Fig. (2) while Eq. (5) applies in the upper right half.

The discussion so far has been limited to $J = 0$. At finite collision energies, however, the total K_3 has contributions from $J > 0$ that must be included. Should these higher partial waves also support Efimov states—which does occur in some systems—then the corresponding oscillations in their partial recombination rates would generally add out of phase, washing out all evidence of Efimov physics. In fact, this is why no such oscillations in the total rate have been observed in electron-polar molecule scattering, where the scattering potentials also have an attractive $1/R^2$ behavior for several angular momenta [19,20]. In the cases we consider here, however, there are no attractive $1/R^2$ Efimov potentials for $J > 0$. Thus, the oscillatory structure predicted in Eq. (5) and shown in Fig. (2) should be visible in the total rate.

Besides $J > 0$, higher collision energies also require taking into account the thermal distribution of velocities [12,21]. In this case, the energy-dependent rate $K_3(E)$ must be converted to a temperature-dependent rate $\langle K_3(T)\rangle$ via thermal averaging [21].

To verify the predictions above, we calculate K_3 by solving the Schrödinger equation numerically [22]. We first consider three bosonic ^{133}Cs atoms, assuming a pairwise sum of short-range, single-channel, two-body model potentials. To illustrate the effect of higher partial waves, we have calculated recombination for the lowest two angular momenta relevant for recombination. The results are shown in Fig. 3(a). To more clearly show the oscillations, we plot K_3E^2, which is proportional to the recombination probability. The oscillatory modulation in the total rate, though small, is still discernible. Upon thermal averaging, however, the oscillations become invisible. Also, three identical bosons require temperature changes of $e^{2\pi/s_0} \approx 515$ to observe a single period.

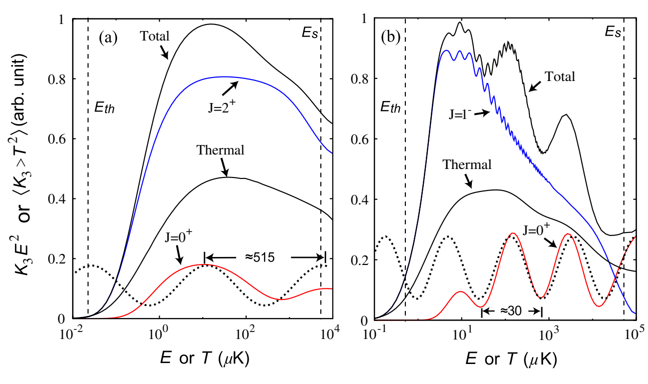

FIG. 3 (color online). Scaled three-body recombination rates $K_3(E)E^2$ for (a) Cs + Cs + Cs with $a_{\mathrm{Cs+Cs}} = 8000$ bohr and (b) Cs + Cs + Li with $a_{\mathrm{Cs+Li}} = 5000$ bohr. The lowest two contributing angular momenta are shown along with their sum and its thermal average, $\langle K_3(T)\rangle T^2$. The dotted lines are from Eq. (5). The expected lower and upper limits of universal behavior are indicated by the dashed lines at $E_{\mathrm{th}} \approx \hbar^2/2\mu_3 a^2$ and $E_s \approx \hbar^2/2\mu_3 r_0^2$, respectively. The rapid oscillations in the $J = 1^-$ contribution in (b) are not related to the Efimov effect.

This factor can be reduced, however, by seeking three-body systems with larger s_0. Fortunately, this is readily achieved for two heavy, identical bosons and a third lighter partner with resonant interactions between the heavy-light pairs. The larger the mass ratio, the better. One of the best available candidates is the Cs + Cs + Li system with a period of $e^{2\pi/s_0} \approx 30$. The numerically calculated Cs + Cs + Li recombination rates are shown in Fig. 3(b). The oscillatory Efimov structure appears very clearly in the total rate, but is still virtually eliminated by thermal averaging. The period of the oscillations has indeed shrunk to about 30, making the observation of multiple cycles much more accessible experimentally.

While Fig. 3 verifies our basic prediction of energy-dependent oscillations, it does not address the interplay between a and E. Figure 4, however, shows the numerical results that confirm this component of our prediction. It shows that as a increases, the Efimov features appear at lower energies—in agreement with Eq. (3) and Fig. 2.

To avoid the thermal averaging that blurs the Efimov features, we propose to measure the losses during collisions of two BECs instead of simply raising the temperature. Using this approach, the collision energy can be set accurately and can be tuned easily through orders of magnitude with a so-called BEC accelerator [23]. Because of the complicated many-body dynamics during such a collision, though, it is not completely obvious that the final atomic loss will correctly reflect the Efimov physics we seek. We thus solved the coupled time-dependent mean-field equations for colliding condensates with loss terms [24] corresponding to all possible recombination processes. The three-body collision energies are assumed to be time independent and are calculated as the total initial kinetic energy in the center-of-mass frame of the three relevant atoms. We take $a_{\mathrm{Cs+Li}} = 5000$ bohr, which produces three full oscillations, yet is still likely achievable experimentally. We use typical values for $a_{\mathrm{Cs+Cs}}$ and

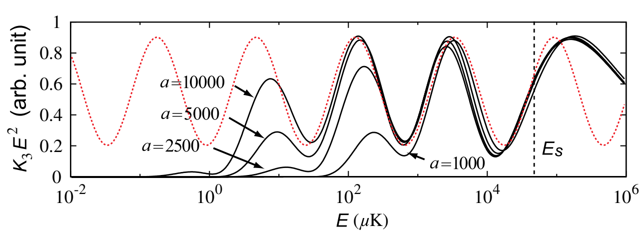

FIG. 4 (color online). The $J = 0^+$ contribution to $K_3(E)E^2$ for Cs + Cs + Li with different $a_{\mathrm{Cs+Li}}$ (in bohr). At $a_{\mathrm{Cs+Li}} = 1000$ bohr, the two oscillations as E decreases from E_s correspond to the ground and first excited Efimov states, respectively. At $a_{\mathrm{Cs+Li}} = 5000$ bohr, the second excited Efimov state appears since the number of oscillations increases to three. The dotted line is from Eq. (5).

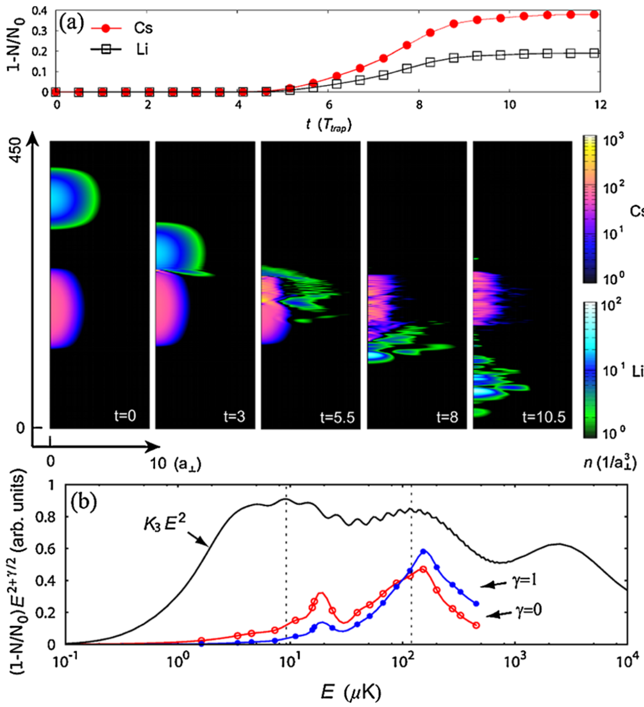

FIG. 5 (color online). (a) The time-dependent fractional atom loss, $1 - N(t)/N_0$, during the collision of a Li BEC with a Cs BEC at $E = 4\ \mu$K along with snapshots of the atomic densities in the overall center-of-mass frame. Though the condensate dynamics is quite complicated, the loss is always smooth during the collision. Initially, there are 10^4 atoms in each condensate and $a_\perp = \sqrt{\hbar/m_{\mathrm{Cs}}\omega_{\mathrm{Cs}}}$ with $\omega_{\mathrm{Cs}} = 2\pi 800$ Hz. (b) The final loss fraction of Cs atoms multiplied by $E^{2+\gamma/2}$ (filled and open circles) to show the Efimov oscillations. Oscillations are seen in both cases, and their positions correspond to those seen in K_3E^2 (no symbols).

$a_{\mathrm{Li+Li}}$, namely, 500 and 4 bohr, respectively [25,26]. The condensate collision takes place within a quasi-one-dimensional geometry with transverse trapping frequencies of 800 Hz for ^{133}Cs and 2240 Hz for ^{7}Li [27] and no longitudinal confinement.

In Fig. 5, we show the results of the mean-field calculations. Figure 5(a) shows the Cs and Li densities as a function of time for a typical collision and the time-dependent loss associated with it. We find that Cs and Li are lost at approximately a 2:1 ratio at all energies we have calculated, indicating that the Cs + Cs + Li process dominates. The number of atoms lost, though, depends on the interaction time as well as the recombination rates. If the relative velocity of the condensates is initially v, then the interaction time scales as $v^{-\gamma}$ with γ between 0 (corresponding to the condensates sticking to each other) and 1 (the condensates pass without interacting). Since K_3 itself scales like E^{-2}, the final loss will scale like $E^{-2-\gamma/2}$. Figure 5(b) thus shows the loss multiplied by $E^{2+\gamma/2}$ for $\gamma = 0$, 1. The Efimov oscillations are seen in both cases with only the relative magnitudes of the peaks changing slightly.

Before concluding, we note that the energy-dependent oscillations also occur for $a < 0$, and we have verified this numerically and analytically. Moreover, we expect Efimov physics to appear in the energy dependence of essentially any observable that shows features at zero temperature.

To summarize, we have identified a novel new manifestation of the Efimov effect in the energy dependence of the three-body recombination rate. These features can be used to confirm the geometric scaling of Efimov physics—a property not experimentally seen so far due to the difficulty of changing the scattering length over many orders of magnitude. To take advantage of this new prediction, we proposed to utilize a novel new tool: the ability to controllably collide two Bose-Einstein condensates. This technique has promise for the study of few-body collisions at low energies beyond the ultracold regime.

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