ULTIMATE LOAD CAPACITY OF STEEL BEAMS WITH WEB OPENINGS
BY THE FINITE ELEMENT METHOD

by

Aslam G. Porbandarwala

B. Tech., Indian Institute of Technology, Bombay, 1974

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree
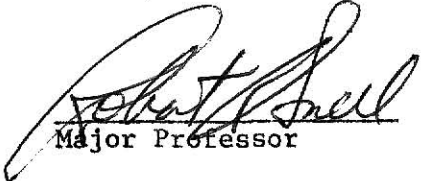
MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1975

Approved by:

Major Professor

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (Continued)

INTRODUCTION

In the past few years, openings in the webs of steel beams for access to or passing of utility components have been a much discussed subject.

In the construction of multistoryed steel buildings, openings through the webs of wide flange beams are frequently necessary to accommodate the passage of ductwork for heating, ventilation and air-conditioning. This helps in reducing the height of each story and thus significant savings are realized through the reduction of the materials used.

When a portion of the web is removed, the beam may be weakened in the vicinity of the opening and often it becomes essential to reinforce the hole. A considerable amount of analytical and experimental work has been done on this topic yielding several theoretical solutions which have been verified.

The purpose of this study was to use the finite element method of analysis to observe the behavior of steel beams with reinforced eccentric web openings under ultimate load conditions, using a computer program developed at the Illinois Institute of Technology (1). The objective was to obtain the ultimate load data, the yield patterns and the modes of failure and to compare them with the results of an experimental program carried out at Kansas State University (2).

Five W-shape steel beams with eccentric rectangular web openings were tested, all but one being reinforced at the opening. In all cases, the moment to shear ratio was 30 inches and the eccentricity was 2 inches. The variables in the analysis were the reinforcing area, the opening length and the opening height.

LITERATURE REVIEW

In 1932, Muskhelishvilli (3) introduced the application of the conformal mapping technique and complex integration to plane problems of the theory of elasticity and in particular to the problem of the stress distribution in a plane or thin plate which is weakened by any type of hole.

In 1950, Joseph and Brock (4) used this complex variable method to obtain an exact solution for the stress concentrations around small openings of several shapes subjected to pure bending. The problem of stress concentration due to such openings has been much studied and many papers concerning this problem have been published (5,6,7).

Using the same method of complex variables Heller, Brock and Bart (8), in 1958, presented a solution for the stress around a rectangular opening with rounded corners in a uniformly loaded plate. In 1962 (9), they modified the procedure and obtained stress distributions due to bending with shear.

Snell (10), in 1962, used the finite element method to study the effects of various reinforcing configurations for rectangular openings in plates subjected to uniaxial tension. His work was published in 1965.

In 1963, MuCutcheon, So and Gersovitz (11) submitted a report describing the test program carried out at McGill University. Tests were performed on beams with unreinforced circular openings and theoretical versus measured strains were indicated.

Segner (12,13) conducted tests on six A36 steel WF beams having rectangular openings of various sizes. The openings were all centered on the neutral axis of the beams and many different reinforcing schemes were investigated. His theoretical approach was based on the theory that a member having such openings centered on the neutral axis acts as a

Vierendeel truss and thus has a point of contraflexure at mid-length of each opening above and below the opening in the tee-section.

An analytical method for calculating stresses around elliptical holes in a wide-flange beam under a uniform load was presented by Bower (14,15) in 1966. The applicability of the analysis depends on the hole size and on the magnitude of the M/V ratio at the hole. Later in the same year, he conducted tests on simply supported wide-flange beams with and without cantilever action having circular or rectangular web openings loaded by concentrated loads.

A comparison of the stresses from the elasticity solution to the experimental tangential stress measurements, for cases of pure bending and bending with shear, was conducted by Redwood (16), in 1967. There seemed to be a good agreement for cases of pure bending but the stresses were underestimated when shear was present.

In the same year the ultimate moment capacity of a beam was investigated by Redwood and McCutcheon (17) using several test specimens. The parameters investigated were shape, number of openings, and the ratio of moment to shear at the opening. The results showed that large reductions in ultimate capacity occur and that the size of those reductions increases with the amount of shear present. Openings that were circular seemed to perform better than rectangular openings and the presence of a second opening nearby seemed to further reduce the strength of the beam.

In 1968, Redwood and McCutcheon (18) reported on tests to failure of wide-flanged steel beams containing one or two unreinforced openings. Different shear to moment ratios were investigated. The openings were of various shapes but all had the same height which was 57% of the beam depth. The experimental results indicated that under pure bending the moment capacity

of the beams with one or two openings could be calculated based on the plastic

modulus of the net section through the opening. The presence of shear reduced

the moment capacity of the beam at the opening below that for pure bending.

The reduction was a function of the opening shape, dimension, the spacing of

openings and the shear to moment ratio. When a single rectangular opening

was present, the moment capacity of the beam was reduced to approximately 40%

of Mp, but when an identical opening was added at a spacing equal to the

opening depth, only a small additional reduction resulted. At an M/V ratio

of 0.425, the moment capacity reduced to 64% to 72% for both simple and

double circular openings.

Also in 1968, Bower (19) suggested criteria for elastic, plastic and

buckling design. He concluded that, for elastic design, beams with web holes

should be designed using the same basic factors of safety against yielding as

in the AISC specifications, except that the maximum allowable bending and

shear stresses should be computed using the actual stresses causing yielding

at the hole rather than nominal beam stresses. For plastic design, beams

with web holes should be designed using the AISC load factor of 1.70, except

that the maximum allowable loads should be computed using the actual ultimate

strength of the beam at the hole rather than the strength of the gross beam.

For large spacings of holes, the effects of each hole should be computed

individually. For more than two adjacent holes, the Vierendeel frame analysis

should be used while for geometrically dissimilar adjacent holes a frame

analysis may be used. With regards to buckling, the same type of buckling

analysis should be used as in case of beams without web holes, so long as the

edge of the hole is at least four inches from the edge of bearing. When a

hole is located in a region of pure bending, the possibility of vertical

flange buckling could be checked by assuming that the compression T-section

at the hole acts as a column.

In the same year Redwood (20) presented a method of deriving an interaction curve relating moments and shears, as well as a simplified, slightly conservative solution to obtain an approximation to the interaction curve. Bower (21) provided the information on the plastic behavior of beams with web openings along with equations predicting their ultimate strength. As the load on the beam increases, the first yielding in the vicinity of the hole occurs at the corners because of the stress concentration at that location.

In 1969, Cheng (22) experimentally analyzed the stresses around a rectangular web opening in a W shape beam using the photostress method and electrical resistance strain gage technique.

Several papers (23,24,25,26) have been published concerning the plastic behavior and ultimate strength of beams with web openings.

In 1970, Congdon and Redwood (27) conducted an investigation concerning the plastic analysis of reinforced openings through beam webs based on the assumptions of perfectly plastic material behavior. A series of tests were carried out to determine the effects of M/V ratio, reinforced area, hole aspect ratio, ratio of hole depth to beam depth, and location of reinforcement on the beam behavior. The experimental results confirmed the assumption of simple stress distribution at failure. An equation for obtaining the area of reinforcement to take up the maximum shear capacity was also provided. The effects due to one-sided reinforcement for the web opening were not much different from those obtained from reinforcements on both sides of the web at critical points.

An analytical method of determining the moment carrying capacity of beams with eccentric rectangular web holes was given by Richard (28) in 1971.

The effects of varying the opening eccentricity, opening length and opening height were investigated. By increasing the opening eccentricity, the moment carrying capacity for high shear values increased. As the opening length and opening height increased, the moment carrying capacity decreased. When the opening height became larger than the opening length, the moment carrying capacity of the beam increased. It was also found that the shear forces were unequally distributed across unequal web areas.

In 1972, Cooper and Snell (29) performed tests on beams with reinforced web openings and confirmed the validity of the Vierendeel Analysis for the estimation of the normal stresses in the vicinity of the hole. Ultimate load tests were run on three beams and the results were consistent with the predictions obtained from the ultimate strength theory for reinforced web openings presented in Ref. (26).

Frost (30), 1973, conducted an experimental investigation on eight beams to determine their ultimate strength. The web openings were tested with an eccentricity of 1 in. and 2 in. and with the moment to shear ratio at the hole at values of 0 in. and 40 in. The Vierendeel Method was used for the theoretical analysis and several different formulas for determining the shear force distribution were presented and compared.

THE FINITE ELEMENT METHOD

The concept of the finite element method was originally introduced by Turner, et al. in 1956 (31). The method has proved to be quite convenient from an automation point of view, for the solution of problems in continuum mechanics. The first applications were in plane stress problems (32). O. C. Zienkiewicz and Y. K. Cheng (33) also presented the theory necessary for the analysis of a plane elastic continua. The finite element method has since then been extended to axi-symmetric stress analysis, flat plate bending, three-dimensional stress analysis and shell analysis.

The basic concept of the finite element method is that every structure may be considered to be an assemblage of individual structural components or elements. A plane continuum is divided into elements interconnected at a finite number of nodes. Certain approximations have been introduced into the formulation of this discretization of the original continuum and the evaluation of element properties. Judgement is required in making the proper subdivisions, such as element shape and degree of freedom, so that the substitute structure can simulate the actual structure. It is also important to choose a suitable displacement function which can satisfy the requirements of displacement continuity between adjacent elements. All of these factors will determine whether the substitute structure is stiffer or more flexible than the real structure and to what degree the approximation simulates the behavior of the actual structure.

In brief, the finite element analysis of an elastic continuum has the following characteristics:

(1) Structural discretization

(2) The necessity for choosing an appropriate displacement function.

(3) The evaluation of element properties

(4) The assemblage of finite elements and the following of standard

displacement method procedures.

In applying this method, the following requirements must be satisfied

simultaneously.

(1) Force equilibrium in each element

(2) Displacement compatibility at nodal points between adjacent elements

(3) The internal forces and deformations are related through the

geometric and material property characteristics.

Various shapes of finite elements were employed in these analyses. In general, applicable rectangular elements give a little better approximation of stresses and deflections for a given nodal pattern than triangular elements, because they employ a closer deformation approximation. However, the use of quadrilateral elements could entail arithmetical difficulty and consequently a disproportionate increase of computing time in deriving the element characteristics. Because of the greater adaptability of the triangular shape in fitting arbitrary boundary geometrics, triangular elements have been used more widely in the development of general purpose analysis programs.

THE FINITE ELEMENT PROGRAM

Introduction

The computer program used (1) in this project is for the stress analysis of plane structures in the elastic-plastic range by the finite element method. It was developed jointly by the Illinois Institute of Technology Research Institute and the Air Force Flight Dynamics Laboratory. The program can handle bar and triangular plate elements so that it is applicable to trusses and to the analysis of in-plane stresses in reinforced plates. The material behavior is assumed to be isotropic and the user has a choice of three types of stress-strain laws namely Ramberg-Osgood, Goldberg-Richard and the Bilinear laws. In this project the Bilinear law has been used. The program is developed to handle up to ten different materials.

A numerical step by step procedure for obtaining solutions which satisfy the requirements of the incremental theory of plasticity for materials which obey the Mises yield condition and the associated flow rule is used in the program. At each step in the solution, an iterative procedure is used to find the correct values of the strain increments. Changes in plastic strain are accounted for by the addition of fictitious plastic forces to the actual loading on the structure in such a way that the deflections of the structure under the modified loading with assumed elastic behavior are equal to the actual deflections.

Element Properties

The two types of elements used in the analysis (the bar and the triangular plate) are shown in Fig. (1). The coordinates of the end points of the bar and the vertices of the triangle are referred to a fixed coordinate system in a plane. The nodes of each element are numbered in the anti-clockwise

direction as shown in the figure. The geometry of the structure is determined by specifying the x and y coordinates of each node, with respect to a fixed set of coordinate axes and by specifying the thickness of the triangular element or the cross-sectional area in the case of a bar element. The cartesian components of the nodal displacements for each of these elements comprise the element displacement vector $\bar{X}$. The total element strain designated by the vector ε can be expressed in terms of the nodal displacement by an equation of the form

$$\varepsilon = B\bar{X}$$

The stresses are related to the elastic strains by Hooke's Law

$$\sigma = C\varepsilon^e$$

The nodal forces, F, corresponding to given displacement, $\bar{X}$, are found by the principle of virtual work.

Elastic-Plastic Analysis:

In the elastic range of material behavior the equilibrium equations for a structure composed of plate and bar elements of the type considered can be written as

$$F = K\bar{X} \tag{1}$$

where the force and displacement vectors now have as their components, the cartesian components of force and displacement at all the nodes and K is the assembled stiffness matrix for the whole structure. The solution of Eq. (1) for the unknown displacement is given symbolically by $\bar{X} = K^{-1} F$.

The displacements known, the element strain and stresses can be obtained. However, when the stresses reach the intensity required to cause plastic flow, it becomes necessary to determine the increments of plastic strain caused by the load increment. The material is assumed to obey the Mises yield condition and the associated flow rule. For plane stress the following equations apply:

$$\bar{\sigma} = (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2} = H(\bar{\epsilon}^P) \qquad (2)$$

$$\Delta\bar{\epsilon}^P = \frac{2}{\sqrt{3}} (\Delta\epsilon_x^{P^2} + \Delta\epsilon_x^P \Delta\epsilon_y^P + \Delta\epsilon_y^{P^2} + \frac{1}{4}\gamma_{xy}^{P^2}) \qquad (3)$$

$$\left. \begin{aligned} \Delta\epsilon_x^P &= \frac{\Delta\bar{\epsilon}^P}{2\bar{\sigma}} (2\sigma_x - \sigma_y) \\[2ex] \Delta\epsilon_y^P &= \frac{\Delta\bar{\epsilon}^P}{2\bar{\sigma}} (2\sigma_y - \sigma_x) \\[2ex] \Delta\gamma_{xy}^P &= 3\frac{\Delta\bar{\epsilon}^P}{\bar{\sigma}} \tau_{xy} \end{aligned} \right\} \qquad (4)$$

where $\bar{\sigma}$ and $\bar{\epsilon}^P$ are the effective stress and the effective plastic strain, respectively, and where $H(\bar{\epsilon}^P)$ is the stress-plastic strain relation for uniaxial stress.

If it is assumed that the response of the structure to the removal of a load increment will be completely elastic then Equation (1) can be modified to account for plastic flow as follows

$$K\bar{X} = F + F^P \qquad (5)$$

where X and F are the displacement and load after the application of the increment and F^P is the vector of plastic forces corresponding to the plastic strains. The plastic strain increments caused by the increment of load must satisfy Equations (4) and for an element undergoing plastic flow the stresses must satisfy the yield condition (Equation (2)).

In general, the following steps give the iterative method used to obtain solution. For the details refer to the program in Appendix I:

1. An increment is given to the applied loads.

2. New values of displacement are found from Equation (5) using the current values of the plastic forces (these will be zero for the first step).

3. The displacements are used to compute total strains, elastic strains, stresses, and the effective stress.

4. If the new value of the effective stress is greater than the largest previous value, the element is plastic and the effective stress is used to determine a new value of the effective strain.

5. Plastic strain increments computed from Equations (4) are added to the current values of the plastic strain and new values of the plastic forces are calculated.

6. If the increment in effective plastic strain is sufficiently small the iteration is complete and a return to step 1 is made, if not a return is made to step 2 and a new cycle begun.

This procedure is applied to each of the elements and the decision to start a new step (apply a load increment) is based on the largest plastic strain increment found among all the elements.

An important feature of the method is the way in which the effective plastic strain is computed from the new value of the effective stress at each iteration. If the inverse of Equation (2) is used to give $\bar{\epsilon}^p$ as a function of $\bar{\sigma}$ the solution may become unstable. This becomes obvious when one considers the case of the elastic, perfectly plastic material for which the inverse of the function $H(\bar{\epsilon}^p)$ does not exist. To avoid this difficulty, the total strain ϵ_t is taken equal to the sum of the value of $\bar{\epsilon}^p$ computed in the previous iteration and $\bar{\sigma}/E$.

The stress-strain law can be written in the form

$$\epsilon_t = \frac{\bar{\sigma}}{E} + \bar{\epsilon}^p$$

or

$$\epsilon_t = \frac{H(\bar{\epsilon}^p)}{E} + \bar{\epsilon}^p \tag{6}$$

The new value of $\bar{\epsilon}^p$ can be found from Equation (6) without difficulty.

The criterion used in Step 6 of the iterative procedure given above, to decide whether the plastic strains have been determined with sufficient accuracy, is the size of the ratio of the increment in effective plastic strain to $\bar{\sigma}/E$. This ratio is a measure of the difference between the ordinates to the theoretical stress-strain curve and the curve that is actually being used at that step in the calculations.

LABORATORY EXPERIMENTS

Ultimate load tests were conducted on five A36 steel beams, two of which were W16 x 45 shapes and three were W16 x 40 shapes. Though the length of the beam was not the same in all the cases, the moment-to-shear ratio, M/V, at the centerline of the opening was kept constant at 30 inches. The test set up, as depicted in Fig. 2a, consisted of simple supports at the ends and a concentrated load applied at midspan. A variable, X, describes the variation of the span length as illustrated in the figure and tabulated in Table 1 for the various test specimens. Beams 3 and 4 are not listed in Table 1 since they were subjected to elastic tests only.

The size of an opening, one of the experimental variables, is given by half its length, a, half its depth, h, and the corner radius, r. Table 1 gives these values for each of the test specimens. However the eccentricity of the opening, that is the distance between the mid-depth of the beam and the centerline of the opening was 2 inches for all the beams. The eccentricity was towards the compression flange in Beams 1, 2 and 5. In Beams 6 and 7, it was towards the tension flange.

Except for Beam 1, all Beams were reinforced at the opening with two bars, one above and one below the opening and at a distance 1/4 inch from the edge of the opening. Beam 5 was reinforced on both sides of the web and the others on one side only. Figure 2b shows the general layout of the reinforcement used. The reinforcement was comprised of bars 2 x 1/4 inches which were considered as the practical minimum size in accordance with the AISC Specifications (34). In all cases the reinforcement was extended 3 inches beyond the edges of the opening. This 3 inches was calculated to be sufficient to develop the strength of the bar using a 3/16 inch fillet weld.

Bars of 3 x 1/2 inches, welded to the web and the flanges with a 3/16 inch fillet weld served as bearing stiffeners in all the beams. All except Beam 1 were provided with bearing stiffeners at the supports and at the load point. Beam 1 was provided with bearing stiffeners only at the load point.

In Beams 2 and 5, cover plates were attached with 1/4" fillet welds to both the flanges at the centerline of the beam. The function of these cover plates was to strengthen the beam at the center and force the failure to occur at the opening. Table 1 gives the dimensions of the cover plates. In Beams 1, 6 and 7 no cover plates were used because the opening sizes and the spans used were such that failure occurred at the opening before a plastic hinge could form at mid-span.

The actual static yield stresses were determined from tensile tests on coupons which were cut from the web and flanges. Similar tests were also conducted to obtain the static yield stresses of the reinforcing bars. The average static yield stresses and the maximum deviation from the average are listed in Table 2.

Load was applied in increments with a Tinius-Olsen screw type machine until specified deflections were reached, and the load then allowed to drop off to the static level. A detailed description of the experimental work can be found in Ref. (2). The experimental ultimate loads were measured and the load-deflection curves plotted. The ultimate loads were corrected for strain hardening and the corrected values are given in Table 6.

PREPARATION FOR COMPUTER ANALYSIS

Idealization

In order to apply a plane stress finite element method to a three-dimensional structure, certain modifications must be made. The procedure followed was to substitute equivalent bar members pin-connected to the appropriate nodes of the plate elements for the flanges and reinforcing bars. These bar members do not have an associated thickness but have one dimensional material properties which can transfer only axial forces.

To find the equivalent flange element, therefore, there is a conflict between maintaining the area of the flange and maintaining the moment of inertia of the beam. Since retaining the actual strength of the beam is more important compared to maintaining the actual area, the equivalent flanges were determined in such a way that the moment of inertia was kept a constant, Fig. 3.

The moment of inertia, I, of the beam is given as:

$$I = \frac{t_w}{12} (d - 2t_f)^3 + 2\left\{ b_f \frac{t_f^3}{12} + b_f\, t_f \left[\frac{d}{2} - \frac{t_f}{2} \right]^2 \right\}$$

and the moment of inertia, I_{eq}, of the equivalent idealized beam is given as:

$$I_{eq} = \frac{t_w\, d^3}{12} + 2\left\{ A_{eff} \left[\frac{d}{2} \right]^2 \right\}$$

If $I_{eq} = I$, then the effective flange area, A_{eff}, of the idealized beam is given as

$$A_{eff} = \frac{2I}{d^2} - \frac{t_w\, d}{6}$$

where t_w is the web thickness of the actual and idealized beam sections, d is the total depth of the beam, t_f is the thickness of the flange and b_f is the width of the flange. The idealized properties of the beams are listed in Table 3.

The bar elements representing the flanges had to be increased in the corresponding length of the beam to account for the cover plates. The actual modified moment of inertia of the beam at that section is

$$I_{mod} = I + I_{cp}$$

where $I_{cp} = 2A_{cp} \left(\frac{d}{2} - \frac{d'}{2}\right)^2$; being the moment of inertia of the cover plates about the x-axis of the beam,

$$d' = \text{thickness of the cover plate.}$$

The equivalent modified moment of inertia is given as $I_{eq.mod.} = \frac{t_w \cdot d^3}{12} + 2\{A_{mod}(d/2)^2\}$. If $I_{eq.mod} = I_{mod}$ then the modified area for the flange elements to account for cover plates is given by

$$A_{mod} = \frac{2I}{d^2} - \frac{t_w d}{6} + A_{cp}\left(1 - \frac{d'}{d}\right)^2$$

The idealization using a plane stress procedure to simulate the beam behavior was similar for one and two sided reinforcement. This is based on the conclusions of other investigators (27,29), that one sided reinforcement has no significant effects different from those of two sided reinforcement. Therefore, the results of the study can represent both types of reinforcement with the same total cross-sectional area. The equivalent reinforcing elements were obtained by replacing the reinforcement by bars which have the same location and cross-sectional area as the reinforcement and shown in Fig. 3b.

Discretization

Though no general rule can be stated as how to best disect a given structure, it is obvious that the accuracy improves as the size of the mesh decreases. Good results are frequently obtained with rather coarse

subdivisions Ref. (35). Consider a case of a deep rectangular beam subjected to pure bending. Two types of rectangular discretizations are shown in Fig. 4. Now, the assumptions of the beam theory states that plane cross-sections remain plane in bending though the axial fibers of the beam become curved. Thus in order to best approximate the straight transverse sections and the curved axial sections that occur during deformation, the discretization (b) in Fig. 4 is preferred over the discretization (a).

From the results of the examples considered in Ref. (35), it is found that if triangular elements are well formed, i.e. essentially equilateral, better results are expected. The poorest overall displacement patterns are produced with the discretization which contains many weak triangles.

Keeping the above mentioned points in mind, the test beams were divided into a mesh of triangular and bar elements as given in Table 3. Smaller triangular elements were used near the opening in order to get a better picture of the yield pattern near the opening. The rounded corners of the hole were approximated by the best suited triangular elements. The element configuration for the fillets at the corners of the opening were slightly different in beams 6 and 7. This change made no significant difference in terms of the yield pattern or the ultimate load.

Solid Beams

A regular beam without any opening was called a solid beam. Every beam with an opening had a corresponding solid beam. Test runs, by the finite element method only, were made on each of the solid beams in order to aid in comprehending the effects of the openings. The details of the idealization of the solid beams are given in Table 4.

PRESENTATION OF RESULTS

The solid beams were tested first, in order to get an idea of the ultimate capacity of the beams without an opening. Table 5 gives the loading details of the beams with the opening and the solid beams.

The actual experimental ultimate loads, the ultimate loads corrected for strain hardening and the ultimate loads obtained from the finite element analysis are given in Table 6.

From the computer output, the yielded elements were determined and were then plotted in order to show the yield patterns. A triangular element having any effective strain was considered yielded. With an error tolerance of 0.03 in the effective strain the effective stresses of the yielded elements were not exactly equal to the yield stress of the material but rather close to it. For a bar element to yield, its stress value must reach the yield point, however, due to the error tolerance used, it was decided to consider a bar element yielded whenever its stress was within 0.15 kip/in^2 of the yield stress.

The usual sign convention is used in the program, namely tensile stresses are positive and compressive stresses are negative. If the stresses in both the directions, that is x and y, are positive then the element is considered to have been yielded in tension and likewise, in compression if both are negative. But if the stress in one direction is positive and in the other direction is negative, the element is considered to be yielded in a combination of tensile and compressive stresses. Compressive yielding is shown by horizontal lines while tensile yielding is shown by verticle lines. A combination type of yielding is shown by solid shading.

Since the elements near the opening are very small, the figures showing the yield pattern have been divided into two types. One showing the enlarged

view in the vicinity of the opening and the other showing the remaining portion of the beam. None of the stiffeners in any of the beams were close to yielding. Hence they have been omitted in order to simplify the figures.

The yield pattern for the ultimate load is shown for all the beams. Additional plots for intermediate loads are also shown for beams 1, 2 and 6. The details of the figures are given below:

Figure 5:   Yield pattern for Solid Beam 1 at ultimate load of 144 kips.

Figure 6:   Yield pattern in the vicinity of its opening for Beam 1 at a load of 80 kips. No other elements in the Beam have yielded at this load.

Figure 7:   Yield pattern in the vicinity of its opening for Beam 1 at a load of 96 kips. No other elements being yielded in the Beam.

Figure 8:   Yield pattern in the vicinity of its opening for Beam 1 at a load of 112 kips. No other elements being yielded in the Beam.

Figure 9:   Full view of Beam 1 showing yield pattern at ultimate load of 144 kips.

Figure 10:   Yield pattern in the vicinity of its opening for Beam 1 at ultimate load of 144 kips.

Figure 11:   Yield pattern for Solid Beam 2 at ultimate load of 168 kips. There are cover plates at midspan and stiffeners at the supports. This being the only difference between Solid Beam 2 and Solid Beam 1.

Figure 12:   Yield pattern in the vicinity of its opening for Beam 2 at a load of 96 kips. No other elements being yielded in the Beam.

Figure 13:   Full view of Beam 2 showing yield pattern at a load of 144 kips.

Figure 14:   Yield pattern in the vicinity of its opening for Beam 2 at a load of 144 kips.

Figure 15:   Full view of Beam 2 showing yield pattern at ultimate load of 152 kips.

Figure 16:   Yield pattern in the vicinity of its opening for Beam 2 at ultimate load of 152 kips.

Figure 17:   Yield pattern for Solid Beam 5 at ultimate load of 144 kips.

DISCUSSION

Ultimate Loads

The ultimate loads obtained from the finite element results are given in
Table 6. The accuracy of these ultimate loads depends on the size of the load
increments used in the analysis. The smaller the increment size the better
the accuracy, but the cost of running the computer program increases as the
increment size is decreased. The ultimate load predicted by the finite element
method is an upper bound value. Theoretically it should satisfy the inequality:

$$Pu^{true} \leq Pu^{FE} < (Pu^{true} + i)$$

where i = increment size.

The experimental ultimate loads corrected for the effects of strain
hardening (2), along with their ratios to those obtained from the finite
element method are also listed in Table 6. These ratios indicate that the
values predicted by the finite element method are within 6% of those obtained
experimentally in all cases.

A further comparison of the finite element results is made with those
obtained from an ultimate strength analysis (2). These theoretical ultimate
loads were found through the use of interaction diagrams which were obtained
from a computer program developed by Wang (36). Once again the ratios of the
theoretical and the finite element ultimate loads show a good correlation.
Thus it can be concluded that the finite element method predicts the ultimate
loads with reasonably good accuracy.

The ultimate loads of the solid beams and their ratios to the ultimate
loads of the beams with the openings are also given in Table 6. These ratios
show the reductions in the load carrying capacities of the beams due to the
web openings. The value of this ratio for Beam 1 is 1.00, thus indicating that

the opening has very little or no effect on the ultimate load of the beam under the existing conditions. This is consistent with the experimental findings (2).

Yield Patterns and Modes of Failure

To begin with, a brief discussion of the effects of secondary moments will aid in a better understanding of the yield patterns.

Consider a section of a loaded beam at the opening. See Fig. 27a. The high moment edge of the opening is at the right, and hence primary moment $M_2 > M_1$. Equilibrium requires that $2aV = M_2 - M_1$. The shear forces V_t and V_B are distributed to the top and bottom of the opening, Fig. 27b. To maintain equilibrium, secondary moments M_T^S and M_B^S are generated,

where

$$2M_T^S = 2aV_T$$

and

$$2M_B^S = 2aV_B$$

Thus the individual sections, above and below the opening, behave as fixed ended beams. Due to the secondary moments there exists compressive normal stresses in the regions near b, d, f and h and tension in the region near a, c, e and g. Due to the primary moment effects only, the entire section above the opening is in compression while the entire section below the opening is in tension. Therefore the corners of the opening at the low moment edge, namely d and e, yield first due to the additive effect of the primary and secondary moments. At the corners c and f the effect of the primary and secondary moments is in the opposite sense and hence the yielding is not so pronounced.

Figures 7 and 12 show the plots of the yielded elements of Beams 1 and 2 at a load of 96 kips. The number of elements yielded in the reinforced case,

that is beam 2, is nearly half that of the unreinforced case, that is, Beam 1.
The effect of the reinforcement is such that the yielding is limited to the
area in the web between the reinforcement and the edge of the opening. In
both cases there is greater yielding at the low moment edge because of the
secondary moment effect. Yield patterns for the same beams at a load of
144 kips are shown in Figs. 10 and 14. While this is the ultimate load for
the unreinforced case, the reinforced case is 8 kips below its ultimate load.
In general there exists a greater amount of yielding in the unreinforced case
with a substantial amount of yielding in the flanges, though, there is very
little flange yielding at this stage in the reinforced case. This is attributed
to the cover plates. The flange yielding in Beam 2 is only on the tension
side, between the opening and the end of the cover plate. A closer look at
Fig. 10 reveals the existence of a compression element in the tension region.
This is due to the secondary moment effect. There are no stiffeners at the
supports in Beam 1 and hence the elements at the supports have yielded. This
is not so in the Solid Beam 1 because of larger elements used in the discretiza-
tion of the Solid Beam. Solid Beam 2, Fig. 11, has a greater moment carrying
capacity because of the cover plates at mid-span. The ultimate load for solid
Beam 2 being higher, the number of elements yielded in Solid Beam 2 is
substantially greater than in Solid Beam 1.

Figures 17 through 19 are for Beam 5 and reveal a reduction of 16 kips in
the ultimate load due to the opening. Once again in this case the cover plates
prevent flange element yielding at mid-span. The quantity of reinforcement
used being large, there is no yielding in the reinforcement.

Looking at the yield patterns of Beams 6 and 7, Figs. 23 through 26, a
great deal of similarity is observed. There is very little yielding at midspan
and the failure in both cases is essentially at the opening.

In all the solid beams it can be observed that the yielding is more extensive towards one side of the centerline even though the loading and support conditions are symmetrical, see Figs. 5, 11, 17 and 20. This is due to the asymmetric element configuration. If the elements were symmetric about the centerline, then the yielding also would be symmetrical. The failure is a typical one hinged mechanism, with the plastic hinge developing at midspan under the load.

In all the beams the first sign of yielding is observed in the web adjacent to the corners of the opening. These yielded zones enlarge as the load increases. At the ultimate load the yielding takes up a pattern which can schematically be shown as in Fig. 28. This type of a pattern generally confirms that a four hinged mechanism develops at failure as assumed in the theoretical analysis (36).

CONCLUSIONS

From the comparison of the results obtained by the finite element method with the experimental and theoretical values, the conclusions reached were:

1) The ultimate loads obtained from the finite element analysis are in reasonably good agreement with those obtained from the experiments and also with those obtained theoretically from an ultimate strength analysis.

2) The yield patterns at various loads agree closely with those obtained in the experiments.

3) The failure at the opening is a four hinged mechanism as assumed in the theory, with a plastic hinge at each corner of the opening.

ACKNOWLEDGEMENTS

APPENDIX I

Details of the Computer Program

A brief description of the Fortran IV program for the elastic-plastic

analysis of plane structures composed of bar and triangular plate elements is

given here.

Table AI-I.   Input Data Format

Card 1   TITLE CARD (72H)
Col   1-72   Any alphanumeric information

Card 2   PROPERTIES CARD (14I5)
Col   1-5   NNODE –   number of nodes (maximum 450)
          6-10   NELEM –   number of elements (maximum 800)
          11-15   ILAW   –   1 Ramberg-Osgood Law
                           –   2 Goldberg-Richard Law
                           –   3 Bilinear Law
          16-20   IUNLD –   1 Unloading following loading
                           –   0 Loading only
          21-25   MAT   –   number of materials used (maximum 10)
          26-30   MAXBND–   maximum bandwidth, MAXBND = 60
                               for this program
          31-35   NBC   –   number of boundary conditions with prescribed
                               displacement. The maximum number is 30 in this
                               program.

Card 3   MATERIAL PROPERTIES CARDS (E15.8, 3F10.5)
Col   1-15   EE   –   modulus of elasticity
          16-25   EE1   –   secant yield stress, ultimate stress, yield stress
          26-35   PRR   –   Poisson's ratio
          36-45   EE2   –   shape parameter, plastic modulus

Card 4   CONTROL CARD (6I5, F10.0)
Col   1-5   NDIV   –   number of load increments
          6-10   NIT   –   maximum number of iterations per step
          11-15   NPRINT–   print output for each NPRINT increment.
                               (e.g., if NPRINT = 3, for increments 3, 6, 9 etc.)
          16-20   KSTART–   number of increments at which solution is to start
          21-25   KSTOP –   number of increments at which solution is to stop
          26-30   NLOAD –   number of nodes at which loads are specified
          31-40   TOL   –   error tolerance

Table AI-I (Continued)

Card 5   NODE CARDS (4I5, 5F10.0)
Col   1-5   Node number
   6-10   IBCX = 1, if displacement in x-direction is specified
   11-15   IBCY = 1, if displacement in y-direction is specified
   16-20   IBCS = 1, if slope is specified
   21-30   XCORD - x coordinate of the node
   31-40   YCORD - y coordinate of this node
   41-50   BC1 - specified displacement in x-direction
   51-60   BC2 - specified displacement in y-direction
   61-70   BC3 - specified slope at the node

Card 6   ELEMENT CARDS (5I5, F10.0)
Col   1-5   Element number
   6-10   I1 - nodes defining the element
   11-15   I2 - nodes defining the element
   16-20   I3 - nodes defining the element
   21-25   NTYPE - material type
   26-35   Z - element thickness or cross-section area

Card 7   LOAD CARDS (I5, 2F10.0)
Col   1-5   Node number
   6-15   x-component of force
   16-25   y-component of force

The program uses a subroutine for the solution of simultaneous equations in band form written by Professor E. L. Wilson of the University of California. Great economies in storage requirements and in time required for solution are achieved in this way.

Three types of stress-strain laws are available for use in the computer program. Each of them is a three parameter law and is given below:

1. Ramberg-Osgood Law

$$\epsilon_t = \frac{\sigma}{E} + \frac{3\sigma_1}{7E}\left(\frac{\sigma}{\sigma_1}\right)^n$$

in which

     E – Young's modulus

     σ_1 – secant yield stress (stress at which the secant modulus = 0.7E)

     n – shape factor

2.   Goldberg-Richard Law

$$\sigma = E\epsilon_t \left\{ 1 + \left| \frac{E\epsilon_t}{\sigma_u} \right|^{n} \right\}^{-1/n}$$

in which

     E – Young's modulus

     σ_u – maximum stress

     n – shape factor

3.   Bilinear Law

$$\sigma = E\epsilon_t \qquad\qquad\qquad \text{for } \sigma < \sigma_y$$

$$\sigma = \sigma_y + E_1\left(\epsilon_t - \frac{\sigma_y}{E}\right) \qquad \text{for } \sigma \geq \sigma_y$$

in which

     E – Young's Modulus

     σ_y – yield stress

     E_1 – slope of the plastic portion of the stress-strain curve.

The correspondence between the program variables and the stress-strain law parameters for each of three laws is given in Table AI-II.

Table AI-II

Stress Strain Law	Program Variables				
	ILAW	E	EE1	EE2	PRR
Ramberg-Osgood	1	E	σ_1	n	ν
Goldberg-Richard	2	E	σ_u	n	ν
Bilinear	3	E	σ_y	E_1	ν

Correspondence Between Program Variables and Stress
Strain Law Parameters.

The displacement component in the x and y direction can be specified at
any node or a node can be required to move along a line with a specified slope.

The x and y components of load can be specified at any node. Distributed
loads must be treated as concentrated at the nodes.

The number of equal increments into which the applied loads and specified
displacements are to be divided is specified as input. It is also necessary
to specify the number of the increment at which the solution is to start. For
example, if a number of increments NDIV = 20 is specified and a value of the
starting increment KSTART = 5 is used, one quarter of the load (displacement)
will be applied in the first step, the rest in 15 equal increments. If it
is desired to stop the solution at an intermediate step a value of KSTOP may
be specified. If the unloading solution is desired the value 1UNLD = 1 is
used.

An error tolerance must be specified as input. After each cycle of
iteration the maximum error among all the elements is compared with the
specified tolerance. If the tolerance is met the next load increment is
applied, if not, the iteration is continued. If the tolerance on error is

is not met when the allowable number of iterations is reached the solution is stopped.

The nodal forces and displacements, the maximum error and the number of the element in which it occurs are printed out at the end of each increment. The cartesian components, principal values, and direction of stress and strain are printed out at the user's option by specifying a value of NPRNT as input. For example, a value of NPRNT = 3 will cause the stresses and strains to be printed out for increment numbers divisible by three. The directions of the principal axes of stress are defined by

$$\phi = -\frac{1}{2} \tan^{-1} \frac{2\tau}{\sigma_x - \sigma_y} \quad , \qquad \frac{\pi}{2} < \phi < \frac{\pi}{2}$$

The value of ϕ in degrees is printed out. In the case of strain the principal directions are defined by

$$\phi = -\frac{1}{2} \tan^{-1} \frac{\gamma}{\epsilon_x - \epsilon_y} \quad , \qquad -\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

This value is also printed out since in general the principal axes of stress and total strain do not coincide when plastic flow has taken place.

The effective stress and the effective plastic strain are also given as output for each element.

An example problem of a cantilever beam has been worked out to illustrate the use of the finite element program.

The Fortran IV source program is listed along with the input data and the output of the example problem.

ILLEGIBLE DOCUMENT

THE FOLLOWING
DOCUMENT(S) IS OF
POOR LEGIBILITY IN
THE ORIGINAL

THIS IS THE BEST
COPY AVAILABLE

```
C         ELASTIC PLASTIC FINITE ELEMENT PROGAM                        VER70001
C         WITH THREE STRESS STRAIN LAW OPTIONS                         VER70002
      COMMON/ADD/   EE(10),EE1(10),EE2(10),PRR(10)                     VER70003
      COMMON   E,CC,G1,F2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,         VER70004
     1         N2,NELEM,KEL,ILAW,MAT,NBC,                              VER70005
     2         B(900,60),BC(30,3),TAB(101,20),IFIX(2),
     3         X(900),XCORD(450),Y(450),ICODE(450),
     4         FP(900),F(900),
     5         I1(800),I2(800),I3(800),NTYPE(800),Z(800),I4(800),
     6         SEE(800),SET(800),EEP(800),EXPL(800),EYP(800),EXYP(800),
     7    NNODE,MBAND                                                  VER70011
      DIMENSION JX(800,3),FE(900)
      EQUIVALENCE (JX,I1)
      EQUIVALENCE (IFIX(1),IBCX),(IFIX(2),IBCY)                        VER70014
C                                                                      VER70015
C   **** READ AND PRINT DATA ****                                     VER70016
C                                                                      VER70017
   10 READ (5,20,END=700)
      IONE=1                                                           VER70019
   20 FORMAT(72H   BCD INFORMATION                                     VER70020
     1                  )                                              VER70021
      WRITE (6,30)                                                     VER70022
   30 FORMAT(1H1)                                                      VER70023
      WRITE (6,20)                                                     VER70024
      READ(5,40) NNODE,NELEM,ILAW,IUNLD,MAT,MAXBND,NBC                 VER70025
   40 FORMAT(14I5)                                                     VER70026
      READ(5,50) (EE(I),EE1(I)       ,PRR(I),EE2(I),I=1,MAT)          VER70027
   50 FORMAT(E15.8,3F10.5)                                             VER70028
      READ(5,60)NDIV,NIT,NPRNT,KSTART,KSTOP,NLOAD,TOL                  VER70029
   60 FORMAT(6I5,F10.0)                                                VER70030
      IF(KSTOP.EQ.0) KSTOP=NDIV                                        VER70031
      GO TO(70,90,110),ILAW                                           VER70032
   70 WRITE(6,80) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)          VER70033
   80 FORMAT(1H014X18HRAMBERG OSGOOD LAW/                             VER70034
     115X,30HMATERIAL---------------------- I3/                        VER70035
     215X30HMODULUS OF ELASTICITY--------- E12.4/                      VER70036
     315X30HSECANT YIELD STRESS----------- E12.4/                      VER70037
     415X30HSHAPE PARAMETER--------------- E12.4/                      VER70038
     515X30HPOISSON'S RATIO--------------- F8.4/                       VER70039
     615X30HERROR TOLERANCE--------------- F8.4)                       VER70040
      GO TO 130                                                        VER70041
   90 WRITE(6,100) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)         VER70042
  100 FORMAT(1H014X20HGOLDBERG RICHARD LAW/                           VER70043
     115X,30HMATERIAL---------------------- I3/                        VER70044
     215X30HMODULUS OF ELASTICITY--------- E12.4/                      VER70045
     315X30HSECANT YIELD STRESS----------- E12.4/                      VER70046
     415X30HSHAPE PARAMETER--------------- E12.4/                      VER70047
     515X30HPOISSON'S RATIO--------------- F8.4/                       VER70048
     615X30HERROR TOLERANCE--------------- F8.4)                       VER70049
      GO TO 130                                                        VER70050
  110 WRITE(6,120) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)         VER70051
  120 FORMAT(1H014X12HBILINEAR LAW/                                   VER70052
     115X,30HMATERIAL---------------------- I3/                        VER70053
     215X30HMODULUS OF ELASTICITY--------- E12.4/                      VER70054
     315X30HSECANT YIELD STRESS----------- E12.4/                      VER70055
     415X30HSHAPE PARAMETER--------------- E12.4/                      VER70056
     515X30HPOISSON'S RATIO--------------- F8.4/                       VER70057
     615X30HERROR TOLERANCE--------------- F8.4)                       VER70058
  130 WRITE (6,140)NNODE,NELEM,NDIV,NIT                               VER70059
  140 FORMAT(1H014X30HNO. OF NODES          NNODE =I4/15X30HNO. OF ELEMVER70060
```

```
      1FNTS          NELEM =I4/15X30HNO. OF STEPS          NDIV =I4/15X30VER70061
     2HNO. OF ITERATIONS/STEP   NIT =I4)                            VER70062
       DO 150 I=1,NBC                                               VER70063
       DO 150 J=1,3                                                 VER70064
 150 BC(I,J)=0                                                      VER70065
       IC=1                                                         VER70066
       WRITE(6,160)                                                 VER70067
 160 FORMAT(25H0BOUNDARY CONDITION ARRAY/10H0 NODAL PT15X1HX23X1HY  VER70068
     120X7HSLIDING/1H 14X4HCODE7X5HVALUE9X4HCODE7X5HVALUE9X4HCODE   VER70069
     27X5HVALUE)                                                    VER70070
C                                                                   VER70071
C   **** NODE COORDINATES AND BOUNDARY CONDITIONS ****             VER70072
C                                                                   VER70073
       DO 200 J=1,NNODE                                             VER70074
       READ(5,170) K,IBCX,IBCY,IBCS,XCORD(K),Y(K),BC1,BC2,BC3      VER70075
 170 FORMAT(4I5,5F10.0)                                             VER70076
       IF(IBCX+IBCY+IBCS.NE.0) WRITE(6,180)K,IBCX,BC1,IBCY,BC2,IBCS,BC3 VER70077
 180 FORMAT(I7,3X,3(I8,1PE17.7))                                    VER70078
       ICODE(K)=IBCS+10*IBCY+100*IBCX                              VER70079
       IF(BC1+BC2+BC3.EQ.0.) GO TO 200                             VER70080
       ICODE(K)=ICODE(K)+IC*1000                                    VER70081
       BC(IC,1)=BC1                                                 VER70082
       BC(IC,2)=BC2                                                 VER70083
       BC(IC,3)=BC3                                                 VER70084
       IC=IC+1                                                      VER70085
       IF(IC.LE.NBC)GO TO 200                                       VER70086
       WRITE(6,190)                                                 VER70087
 190 FORMAT(54H0 MORE THAN 29 NODES HAVE NON ZERO BOUNDARY CONDITIONS) VER70088
       GO TO 620                                                    VER70089
 200 CONTINUE                                                       VER70090
C                                                                   VER70091
C   **** ELEMENT PROPERTIES  ****                                  VER70092
C                                                                   VER70093
       READ(5,210)(K,I1(K),I2(K),I3(K),NTYPE(K),Z(K),J=1,NELEM)    VER70094
 210 FORMAT(5I5,F10.0)                                              VER70095
C                                                                   VER70096
C   **** LOADS ****                                                VER70097
C                                                                   VER70098
       N2=2*NNODE                                                   VER70099
       DO 220 K=1,N2                                                VER70100
 220 F(K)=0                                                         VER70101
       IF(NLOAD.EQ.0) GO TO 250                                     VER70102
       DO 230 K=1,NLOAD                                             VER70103
 230 READ(5,240)J,F(2*J-1),F(2*J)                                   VER70104
 240 FORMAT(I5,2F10.0)                                              VER70105
 250 CONTINUE                                                       VER70106
       WRITE(6,260)(K,XCORD(K),F(2*K-1), Y(K),F(2*K),ICODE(K),K=1,NNODE) VER70107
 260 FORMAT(10H0 NODAL PT8X7HX-COORD8X7HX-FORCE8X7HY-COORD8X7HY-FORCE VER70108
     111X4HCODE//(4X,I3,5X4F15.4,I15))                             VER70109
       WRITE(6,270)                                                 VER70110
 270 FORMAT(1H0///10X,90HELEMENT   NODE 1   NODE 2   NODE 3    ELEMENVER70111
     1T TYPE   AREA OR THICK.   MATERIAL TYPE,//)                  VER70112
       DO 300 K=1,NELEM                                            VER70113
       IF(I3(K).EQ.0) WRITE(6,280) K,I1(K),I2(K),I3(K),Z(K),NTYPE(K) VER70114
       IF(I3(K).NE.0) WRITE(6,290) K,I1(K),I2(K),I3(K),Z(K),NTYPE(K) VER70115
 280 FORMAT( 6X,4I9,11X,3HBAR,F22.5,I14)                           VER70116
 290 FORMAT( 6X,4I9,11X,5HPLATE,F20.5,I14)                         VER70117
 300 CONTINUE                                                       VER70118
C                                                                   VER70119
C   INITIALIZATION                                                 VER70120
```

```
C                                                                        VER70121
  310 XDIV=NDIV                                                          VER70122
      GO TO(320,340,340),ILAW                                           VER70123
  320 CONTINUE                                                          VER70124
      DO 330 I=1,MAT                                                    VER70125
      F=FF(I)                                                           VER70126
      F1=FF1(I)                                                         VER70127
      F2=FF2(I)                                                         VER70128
      CC=F1/F                                                           VER70129
      G1=(7.0*E/3.0)**(1.0/E2)*F**(1.0-1.0/E2)                         VER70130
      CALL TABLE(I)                                                     VER70131
  330 CONTINUE                                                          VER70132
  340 CONTINUE                                                          VER70133
C                                                                        VER70134
C   **** DETERMINE BAND WIDTH ****                                      VER70135
C                                                                        VER70136
      DO 350 K=1,NELEM                                                  VER70137
      I4(K)=I3(K)                                                       VER70138
  350 IF(I3(K).EQ.0) JX(K,3)=JX(K,1)                                    VER70139
      J=0                                                               VER70140
      DO 380 N=1,NELEM                                                  VER70141
      DO 380 I=1,3                                                      VER70142
      DO 370 L=1,3                                                      VER70143
      KK=IABS(JX(N,I)-JX(N,L))                                          VER70144
      IF(KK-J)370,370,360                                              VER70145
  360 J=KK                                                              VER70146
  370 CONTINUE                                                          VER70147
  380 CONTINUE                                                          VER70148
      MBAND=2*J+3                                                       VER70149
      IF(MBAND.GT.MAXBND) WRITE(6,390) MBAND                           VER70150
      IF(MBAND.GT.MAXBND) GO TO 10                                     VER70151
  390 FORMAT(1H0]0X20HBAND WIDTH TOO LARGE5X6HMBAND=I4)                VER70152
      DO 400 I=1,NELEM                                                  VER70153
  400 I3(I)=I4(I)                                                       VER70154
      DO 410 I=1,N2                                                     VER70155
      DO 410 J=1,MBAND                                                  VER70156
  410 B(I,J)=0.                                                         VER70157
C                                                                        VER70158
C     CALCULATION OF STIFFNESS MATRIX                                   VER70159
C                                                                        VER70160
      CALL STIFF                                                        VER70161
C                                                                        VER70162
C   **** REDUCE MATRIX ****                                             VER70163
C                                                                        VER70164
      CALL SYMSOL(1)                                                    VER70165
C                                                                        VER70166
C   **** INCREMENT LOADS,ADD PLASTIC FORCES AND SOLVE FOR DISPLACEMENTS**VER70167
C                                                                        VER70168
      DO 420 I=1,NELEM                                                  VER70169
      SET(I)=0                                                          VER70170
      SEF(I)=0                                                          VER70171
      EEP(I)=0                                                          VER70172
      EXPL(I)=0                                                         VER70173
      EYP(I)=0                                                          VER70174
  420 EXYP(I)=0                                                         VER70175
      KO=KSTART-1                                                       VER70176
      KU=KO                                                             VER70177
      DO 430 I=1,N2                                                     VER70178
  430 FP(I)=0                                                           VER70179
      GO TO 490                                                         VER70180
```

```
440 WRITE (6,450)KU,(I,FE(2*I-1),FE(2*I),X(2*I-1),X(2*I),I=1,NNODE)    VER70181
450 FORMAT(1H120X38HFORCES AND DISPLACEMENTS FOR INCREMENT,I4//10X4HNOVER70182
   1DE5X7HX-FORCE8X7HY-FORCE9X8HX-DISPL.7X8HY-DISPL./(9X13,2F15.3,5X2FVER70183
   215.4 ))                                                           VER70184
    WRITE (6,460)XERR,KEL,IT                                          VER70185
460 FORMAT(13HOMAX. ERROR =F8.5,5X14HIN ELEMENT NO.I4,5X17HNO. OF ITERVER70186
   1ATIONSI4)                                                         VER70187
470 IF(MOD(KO,NPRNT))490,480,490                                      VER70188
480 CALL OUTPT                                                        VER70189
    IF(KO.EQ.KSTOP)GO TO 620                                          VER70190
    GO TO 500                                                         VER70191
490 IF(KO.EQ.KSTOP) CALL OUTPT                                        VER70192
    IF(KO.EQ.KSTOP)GO TO 620                                          VER70193
500 KO=KO+IONE                                                        VER70194
    KU=KU+1                                                           VER70195
    IF(KO-NDIV)510,510,620                                            VER70196
510 XKO=KO                                                            VER70197
    DO 520 I=1,N2                                                     VER70198
520 FE(I)=XKO/XDIV*F(I)                                               VER70199
    DO 530 K=1,NELEM                                                  VER70200
530 SEF(K)=SET(K)                                                     VER70201
    IT=0                                                              VER70202
540 DO 570 I=1,NNODE                                                  VER70203
    IF(ICODE(I).EQ.0) GO TO 570                                       VER70204
    CALL DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)                   VER70205
    IF(IBCS.NE.1) GO TO 550                                           VER70206
    ALF=BC(IC,3)                                                      VER70207
    FP(IX)=FP(IX)+ALF*FP(IY)                                          VER70208
    FP(IY)=0.                                                         VER70209
550 DO 560 N=1,2                                                      VER70210
    IF(JFIX(N).NE.1) GO TO 560                                        VER70211
    IR=IX+N-1                                                         VER70212
    FP(IR)=0.                                                         VER70213
560 CONTINUE                                                          VER70214
570 CONTINUE                                                          VER70215
C                                                                     VER70216
C   **** SOLVE FOR DISPLACEMENTS ****                                 VER70217
C                                                                     VER70218
    DO 580 I=1,N2                                                     VER70219
580 X(I)=FE(I)+FP(I)                                                  VER70220
    CALL SYMSOL(2)                                                    VER70221
C                                                                     VER70222
C   CALCULATE TOTAL STRAINS,STRESSES AND PLASTIC                      VER70223
C     FORCES AND STRAINS FOR EACH ELEMENT                             VER70224
C                                                                     VER70225
    DO 590 I=1,N2                                                     VER70226
590 FP(I)=0                                                           VER70227
    XERR=0.0                                                          VER70228
    KEL=0                                                             VER70229
    CALL STRAIN                                                       VER70230
C                                                                     VER70231
C   **** PICK LARGEST ERROR AND DETERMINE WHEN TO REITERATE ****      VER70232
C                                                                     VER70233
    IT=IT+1                                                           VER70234
    IF(XERR-TOL)440,440,600                                           VER70235
600 IF(IT-NIT)540,610,610                                             VER70236
610 KO=NDIV                                                           VER70237
    IUNLD=0                                                           VER70238
    GO TO 440                                                         VER70239
620 IF(IUNLD.EQ.0) GO TO 10                                           VER70240
```

```
      IUNLD=0                                                        VER70241
      IONE=-1                                                        VER70242
      KSTOP=0                                                        VER70243
      GO TO 440                                                      VER70244
  700 WRITE (6,710)
  710 FORMAT (/10X,13H JOB FINISHED)
      STOP
      END                                                            VER70245
      SUBROUTINE ELEM(M)                                             ELM 0001
      COMMON/ADD/   FE(10),FE1(10),FE2(10),PRR(10)                   ELM 0002
      COMMON   F,CC,G,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,        ELM 0003
     1         N2,NELEM,KEL,ILAW,MAT,NBC,                            ELM 0004
     2         B(900,60),BC(30,3),TAB(101,20),IFIX(2),
     3         X(900),XCORD(450),Y(450),ICODE(450),
     4         FP(900),F(900),
     5         I1(800),I2(800),I3(800),NTYPE(800),Z(800),I4(800),
     6         SEF(800),SET(800),EEP(800),EXPL(800),EYP(800),EXYP(800),
     7         NNODE,MBAND                                            ELM 0010
      DIMENSION XX(450)                                              ELM 0011
      EQUIVALENCE (XCORD,XX)                                         ELM 0012
      J1=I1(M)                                                       ELM 0013
      J2=I2(M)                                                       ELM 0014
      J3=I3(M)                                                       ELM 0015
      X21=XX(J2)-XX(J1)                                              ELM 0016
      Y21=Y(J2)-Y(J1)                                                ELM 0017
      IF(J3.EQ.0) GO TO 10                                           ELM 0018
      Y32=Y(J3)-Y(J2)                                                ELM 0019
      Y31=Y(J3)-Y(J1)                                                ELM 0020
      X32=XX(J3)-XX(J2)                                              ELM 0021
      X31=XX(J3)-XX(J1)                                              ELM 0022
      RETURN                                                         ELM 0023
   10 Y32=SQRT(X21**2+Y21**2)                                        ELM 0024
      RETURN                                                         ELM 0025
      END                                                            ELM 0026
CDCOD                                                                DCOD0001
      SUBROUTINE DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)          DCOD0002
      DIMENSION ICODE(450)                                           DCOD0003
      IBCS=MOD(ICODE(I),10)                                          DCOD0004
      IBCX=MOD(ICODE(I),1000)/100                                    DCOD0005
      IBCY=MOD(ICODE(I),100)/10                                      DCOD0006
      IC=MOD(ICODE(I),100000)/1000                                   DCOD0007
      IX=2*I-1                                                       DCOD0008
      IY=IX+1                                                        DCOD0009
      IF(IC.EQ.0) IC=NBC                                             DCOD0010
      RETURN                                                         DCOD0011
      END                                                            DCOD0012
C2222     PLASTIC STRAIN DETERMINATION                               PNEW0001
      SUBROUTINE PNEW1 (EEPK,EET,E,E1,E2)                            PNEW0002
      J=1                                                            PNEW0003
      XU=EET                                                         PNEW0004
      XL=0                                                           PNEW0005
   10 EEPK=.5*(XL+XU)                                                PNEW0006
   20 Y=EEPK+E1/E*EEPK**(1.0/E2)-EET                                 PNEW0007
   30 YP=1.0+E1/E/E2*EEPK**(1.0/E2-1.0)                              PNEW0008
      J=J+1                                                          PNEW0009
      IF(J-50)40,40,100                                             PNEW0010
   40 IF(Y)50,100,60                                                 PNEW0011
   50 XL=EEPK                                                        PNEW0012
      GO TO 70                                                       PNEW0013
   60 XU=EEPK                                                        PNEW0014
```

```
   70 XT=EEPK-Y/YP                                                    PNEW0015
      IF(XU-XT)10,10,80                                               PNEW0016
   80 IF(XT-XL)10,10,90                                               PNEW0017
   90 EEPK=XT                                                         PNEW0018
      DIFF=ABS(Y/YP/EEPK)                                             PNEW0019
      IF(DIFF-.00001)100,100,20                                       PNEW0020
  100 RETURN                                                          PNEW0021
      END                                                             PNEW0022
CTABL      STRAIN-PLASTIC STRAIN TABLE                                PLST0001
      SUBROUTINE TABLE(K)                                             PLST0002
      COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                     PLST0003
      COMMON    E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,       PLST0004
     1          N2,NELEM,KEL,ILAW,MAT,NBC,                            PLST0005
     2          B(900,60),BC(30,3),TAB(101,20),IFIX(2),
     3          X(900),XCORD(450),Y(450),ICODE(450),
     4          FP(900),F(900),
     5          I1(800),I2(800),I3(800),NTYPE(800),Z(800),I4(800),
     6          SEE(800),SET(800),EEP(800),EXPL(800),EYP(800),EXYP(800),
     7      NNODE,MBAND                                               PLST0011
      II2=2*K                                                         PLST0012
      II1=II2-1                                                       PLST0013
      TAB(1,II1)=0.                                                   PLST0014
      TAB(1,II2)=0.                                                   PLST0015
      DO 20 I=1,101                                                   PLST0016
      IF(I-1)20,20,10                                                 PLST0017
   10 TAB(I,II1)=FLOAT(I-1)*CC/5.                                     PLST0018
      EET=TAB(I,II1)                                                  PLST0019
      CALL PNEW1(EEPK,EET,E,G1,E2)                                    PLST0020
      TAB(I,II2)=EEPK                                                 PLST0021
   20 CONTINUE                                                        PLST0022
      RETURN                                                          PLST0023
      END                                                             PLST0024
      SUBROUTINE STIFF                                                STIF0001
      COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                     STIF0002
      COMMON    E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,       STIF0003
     1          N2,NELEM,KEL,ILAW,MAT,NBC,                            STIF0004
     2          B(900,60),BC(30,3),TAB(101,20),IFIX(2),
     3          X(900),XCORD(450),Y(450),ICODE(450),
     4          FP(900),F(900),
     5          I1(800),I2(800),I3(800),NTYPE(800),Z(800),I4(800),
     6          SEE(800),SET(800),EEP(800),EXPL(800),EYP(800),EXYP(800),
     7      NNODE,MBAND                                               STIF0010
      DIMENSION FFP(2),INODE(3),LNODE(6),DSK(6,6)                     STIF0011
      EQUIVALENCE (IFIX(1),IBCX),(IFIX(2),IBCY)                       STIF0012
      EQUIVALENCE(INODE(1),J1),(INODE(2),J2),(INODE(3),J3)            STIF0013
      DO 80 M=1,NELEM                                                 STIF0014
      J1=I1(M)                                                        STIF0015
      J2=I2(M)                                                        STIF0016
      J3=I3(M)                                                        STIF0017
      CALL ELEM(M)                                                    STIF0018
      NTY=NTYPE(M)                                                    STIF0019
      E=EE(NTY)                                                       STIF0020
      PR=PRR(NTY)                                                     STIF0021
      IF(J3.EQ.0) GO TO 30                                            STIF0022
      IF(NTY.LE.MAT) GO TO10                                          STIF0023
      GO TO 160                                                       STIF0024
C                                                                     STIF0025
C  **** STIFFNESS MATRIX CALCULATIONS FOR TRIANGULAR ELEMENTS ****    STIF0026
C                                                                     STIF0027
   10 JMAT=6                                                          STIF0028
```

```
      AF=E*Z(M)                                                   STIF0029
      A123=X21*Y31-X31*Y21                                        STIF0030
      A123=ABS(A123)                                              STIF0031
      ET1=AF/(2.0*A123*(1.0-PR**2))                               STIF0032
      ET2=AF/(4.0*A123*(1.0+PR))                                  STIF0033
      DSK(1,1)= ET1*Y32**2          +ET2*X32**2                   STIF0034
      DSK(2,1)=-ET1*PR*Y32*X32      -ET2*Y32*X32                  STIF0035
      DSK(2,2)= ET1*X32**2          +ET2*Y32**2                   STIF0036
      DSK(3,1)=-ET1*Y31*Y32         -ET2*X32*X31                  STIF0037
      DSK(3,2)= ET1*PR*Y31*X32      +ET2*Y32*X31                  STIF0038
      DSK(3,3)=ET1*Y31**2           +ET2*X31**2                   STIF0039
      DSK(4,1)= ET1*PR*Y32*X31      +ET2*Y31*X32                  STIF0040
      DSK(4,2)=-ET1*X31*X32         -ET2*Y31*Y32                  STIF0041
      DSK(4,3)=-ET1*PR*Y31*X31      -ET2*Y31*X31                  STIF0042
      DSK(4,4)= ET1*X31**2          +ET2*Y31**2                   STIF0043
      DSK(5,1)= ET1*Y21*Y32         +ET2*X32*X21                  STIF0044
      DSK(5,2)=-ET1*PR*Y21*X32      -ET2*Y32*X21                  STIF0045
      DSK(5,3)=-ET1*Y31*Y21         -ET2*X31*X21                  STIF0046
      DSK(5,4)= ET1*PR*Y21*X31      +ET2*Y31*X21                  STIF0047
      DSK(5,5)= ET1*Y21**2          +ET2*X21**2                   STIF0048
      DSK(6,1)=-ET1*PR*Y32*X21      -ET2*Y21*X32                  STIF0049
      DSK(6,2)= ET1*X32*X21         +ET2*Y21*Y32                  STIF0050
      DSK(6,3)= ET1*PR*Y31*X21      +ET2*Y21*X31                  STIF0051
      DSK(6,4)=-ET1*X31*X21         -ET2*Y21*Y31                  STIF0052
      DSK(6,5)=-ET1*PR*Y21*X21      -ET2*Y21*X21                  STIF0053
      DSK(6,6)= ET1*X21**2          +ET2*Y21**2                   STIF0054
      DO 20 I=1,JMAT                                              STIF0055
      DO 20 J=1,JMAT                                              STIF0056
   20 DSK(I,J)=DSK(J,I)                                           STIF0057
      GO TO 50                                                    STIF0058
C                                                                 STIF0059
C **** STIFFNESS MATRIX CALCULATIONS FOR BARS ****               STIF0060
C                                                                 STIF0061
   30 ET1=Z(M)*E/Y32**3                                           STIF0062
      FFP(1)=X21                                                  STIF0063
      FFP(2)=Y21                                                  STIF0064
      DO 40 I=1,2                                                 STIF0065
      DO 40 J=1,2                                                 STIF0066
      DSK(I,J)=ET1*FFP(I)*FFP(J)                                  STIF0067
      DSK(I+2,J)=-DSK(I,J)                                        STIF0068
      DSK(I,J+2)=-DSK(I,J)                                        STIF0069
   40 DSK(I+2,J+2)=DSK(I,J)                                       STIF0070
      JMAT=4                                                      STIF0071
C                                                                 STIF0072
C **** INCORPORATION OF ELEMENT MATRICES INTO                    STIF0073
C                 COMPLETE STIFFNESS MATRIX ****                  STIF0074
C                                                                 STIF0075
   50 JMAT2=JMAT/2                                                STIF0076
      K=0                                                         STIF0077
      DO 60 I=1,JMAT2                                             STIF0078
      DO 60 J=1,2                                                 STIF0079
      K=K+1                                                       STIF0080
   60 LNODE(K)=2*INODE(I)-2+J                                     STIF0081
      DO 80 I=1,JMAT                                              STIF0082
      KI=LNODE(I)                                                 STIF0083
      DO 80 J=1,JMAT                                              STIF0084
      KJ=LNODE(J)                                                 STIF0085
      IF(KJ-KI)80,70,70                                           STIF0086
   70 K=KJ-KI+1                                                   STIF0087
      B(KI,K)=B(KI,K)+DSK(I,J)                                    STIF0088
```

```
      80 CONTINUE                                                          STIF0089
C                                                                          STIF0090
C     **** DISPLACEMENT BOUNDARY CONDITIONS ****                           STIF0091
C                                                                          STIF0092
         DO 150 I=1,NNODE                                                  STIF0093
         IF(ICODE(I).EQ.0) GO TO 150                                       STIF0094
         CALL DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)                   STIF0095
         IF(IBCS.NE.1) GO TO 110                                           STIF0096
         ALF=BC(IC,3)                                                      STIF0097
         B(IX,1)=B(IX,1)+ALF*(ALF*(B(IY,1)+1.)+2.*B(IX,2))                 STIF0098
         B(IX,2)=-ALF                                                      STIF0099
         B(IY,1)=1.                                                        STIF0100
         F(IX)=ALF*F(IY)+F(IX)                                            STIF0101
         F(IY)=0                                                           STIF0102
         KL=IX-MBAND+2                                                     STIF0103
         KU=IX+MBAND-1                                                     STIF0104
         IF(KL.LT.1) KL=1                                                  STIF0105
         IF(KU.GT.N2)KU=N2                                                 STIF0106
         DO 100 K=KL,KU                                                    STIF0107
         IF(K.EQ.IX.OR.K.EQ.IY) GO TO 100                                  STIF0108
         IF(K.GT.IY) GO TO 90                                              STIF0109
         L=IX-K+1                                                          STIF0110
         B(K,L)=B(K,L)+ALF*B(K,L+1)                                        STIF0111
         B(K,L+1)=0.                                                       STIF0112
         GO TO 100                                                         STIF0113
      90 L=K-IX+1                                                          STIF0114
         B(IX,L)=B(IX,L)+ALF*B(IY,L-1)                                     STIF0115
         B(IY,L-1)=0.                                                      STIF0116
     100 CONTINUE                                                          STIF0117
     110 DO 140 N=1,2                                                      STIF0118
         IF(IFIX(N).NE.1) GO TO 140                                        STIF0119
         IR=IX+N-1                                                         STIF0120
         ML=IR-MBAND+1                                                     STIF0121
         MU=IR+MBAND-1                                                     STIF0122
         IF(ML.LT.1) ML=1                                                  STIF0123
         IF(MU.GT.N2)MU=N2                                                 STIF0124
         DO 130 M=ML,MU                                                    STIF0125
         L=IR-M+1                                                          STIF0126
         IF(L.LE.1) GO TO 120                                              STIF0127
         F(M)=F(M)-B(M,L)*BC(IC,N)                                         STIF0128
         B(M,L)=0.                                                         STIF0129
         GO TO 130                                                         STIF0130
     120 L=M-IR+1                                                          STIF0131
         F(M)=F(M)-B(IR,L)*BC(IC,N)                                        STIF0132
         B(IR,L)=0.                                                        STIF0133
     130 CONTINUE                                                          STIF0134
         B(IR,1)=1.                                                        STIF0135
         F(IR)=BC(IC,N)                                                    STIF0136
     140 CONTINUE                                                          STIF0137
     150 CONTINUE                                                          STIF0138
         RETURN                                                            STIF0139
     160 WRITE(6,170) M                                                    STIF0140
     170 FORMAT(1H010X32HINVALID ELEMENT CODE ELEMENT NO.I4)               STIF0141
         STOP                                                              STIF0142
         END                                                              STIF0143
C                                                                          STIF0144
         SUBROUTINE SYMSOL(KKK)                                            SYMS0001
         COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                       SYMS0002
         COMMON   F,CC,G),F2,PR,FPR,X21,Y21,X31,Y31,X32,Y32,XERR,          SYMS0003
        1        N2,NELEM,KEL,ILAW,MAT,NBC,                                SYMS0004
```

```
      2           B(900,60),BC(30,3),TAB(101,20),IFIX(2),
      3           X(900),XCORD(450),Y(450),ICODE(450),
      4           FP(900),F(900),
      5           I1(800),I2(800),I3(800),NTYPE(800),Z(800),I4(800),
      6           SEF(800),SET(800),EEP(800),EXPL(800),EYP(800),EXYP(800),
      7         NNODE,MBAND                                                  SYMS0010
C                                                                            SYMS0011
      NN=N2                                                                  SYMS0012
      MM=MBAND                                                               SYMS0013
      GO TO (10,60),KKK                                                      SYMS0014
C                                                                            SYMS0015
C     REDUCE MATRIX                                                          SYMS0016
C                                                                            SYMS0017
   10 DO 50 N=1,NN                                                           SYMS0018
      DO 40 L=2,MM                                                           SYMS0019
      C=B(N,L)/B(N,1)                                                        SYMS0020
      I=N+L-1                                                                SYMS0021
      IF(NN-I) 40,20,20                                                      SYMS0022
   20 J=0                                                                    SYMS0023
      DO 30 K=L,MM                                                           SYMS0024
      J=J+1                                                                  SYMS0025
   30 B(I,J)=B(I,J)-C*B(N,K)                                                 SYMS0026
   40 B(N,L)=C                                                               SYMS0027
   50 CONTINUE                                                               SYMS0028
      GO TO 130                                                              SYMS0029
C                                                                            SYMS0030
C     REDUCE VECTOR                                                          SYMS0031
C                                                                            SYMS0032
   60 DO 80 N=1,NN                                                           SYMS0033
      DO 70 L=2,MM                                                           SYMS0034
      I=N+L-1                                                                SYMS0035
      IF(NN-I) 80,70,70                                                      SYMS0036
   70 X (I)=X (I)-B(N,L)*X (N)                                              SYMS0037
   80 X (N)=X (N)/B(N,1)                                                     SYMS0038
C                                                                            SYMS0039
C     BACK SUBSTITUTION                                                      SYMS0040
C                                                                            SYMS0041
      N=NN                                                                   SYMS0042
   90 N=N-1                                                                  SYMS0043
      IF(N) 100,130,100                                                      SYMS0044
  100 DO 120 K=2,MM                                                          SYMS0045
      L=N+K-1                                                                SYMS0046
      IF(NN-L) 120,110,110                                                   SYMS0047
  110 X (N)=X (N)-B(N,K)*X (L)                                              SYMS0048
  120 CONTINUE                                                               SYMS0049
      GO TO 90                                                               SYMS0050
C                                                                            SYMS0051
  130 RETURN                                                                 SYMS0052
C                                                                            SYMS0053
      END                                                                    SYMS0054
      SUBROUTINE STRAIN                                                      STRN0001
      COMMON/ADD/  EE(10),FE1(10),FE2(10),PRR(10)                           STRN0002
      COMMON    F,CC,G1,F2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,             STRN0003
      1           N2,NELEM,KEL,ILAW,MAT,NBC,                                STRN0004
      2           B(900,60),BC(30,3),TAB(101,20),IFIX(2),
      3           X(900),XCORD(450),Y(450),ICODE(450),
      4           FP(900),F(900),
      5           I1(800),I2(800),I3(800),NTYPE(800),Z(800),I4(800),
      6           SEF(800),SET(800),EEP(800),EXPL(800),EYP(800),EXYP(800),
      7         NNODE,MBAND                                                  STRN0010
```

```
      DO 130 K=1,NELEM                                              STRN0011
      J1=2*I1(K)-1                                                  STRN0012
      J2=2*I1(K)                                                    STRN0013
      J3=2*I2(K)-1                                                  STRN0014
      J4=2*I2(K)                                                    STRN0015
      J5=2*I3(K)-1                                                  STRN0016
      J6=2*I3(K)                                                    STRN0017
      CALL ELEM(K)                                                  STRN0018
      NTY=NTYPE(K)                                                  STRN0019
      E=EE(NTY)                                                     STRN0020
      E1=EE1(NTY)                                                   STRN0021
      E2=EE2(   NTY)                                                STRN0022
      CC=E1/E                                                       STRN0023
      PR=PRR(   NTY)                                                STRN0024
      IF(ILAW.EQ.1)        G1=(7.*E/3.)**(1./E2)*E1**(1.-1./E2)     STRN0025
      IF(ILAW.GT.1) G1=E1                                           STRN0026
      EPR=E/(1.0-PR*PR)                                             STRN0027
      IF(I3(K).EQ.0) GO TO 60                                       STRN0028
C                                                                   STRN0029
C    **** TRIANGULAR ELEMENT CALCULATIONS ****                      STRN0030
C                                                                   STRN0031
   10 A123=X21*Y31-X31*Y21                                          STRN0032
      SN=A123/ABS(A123)                                             STRN0033
      EXT=(-Y32*X(J1)+Y31*X(J3)-Y21*X(J5))/A123                     STRN0034
      EYT=(  X32*X(J2)-X31*X(J4)+X21*X(J6))/A123                    STRN0035
      EXYT=(X32*X(J1)-Y32*X(J2)-X31*X(J3)+Y31*X(J4)                 STRN0036
     1     +X21*X(J5)-Y21*X(J6))/A123                               STRN0037
      EXE=EXT-EXPL(K)                                               STRN0038
      EYE=EYT-EYP(K)                                                STRN0039
      EXYE=EXYT-EXYP(K)                                             STRN0040
      SX=EPR*(EXE+PR*EYE)                                           STRN0041
      SY=EPR*(EYE+PR*EXE)                                           STRN0042
      SXY=E/(1.0+PR)*EXYE/2.0                                       STRN0043
      SE=SQRT(SX**2-SX*SY+SY**2+3.0*SXY**2)                         STRN0044
      CRIT=ABS(SE)-SEF(K)                                           STRN0045
      IF(CRIT)40,40,20                                              STRN0046
   20 EET=SE/E+EEP(K)                                               STRN0047
      CALL STRSTN(EET,EEPK,SETK,NTY)                                STRN0048
      DEEP=EEPK-EEP(K)                                              STRN0049
   30 EEP(K)=EEPK                                                   STRN0050
      SET(K)=SETK                                                   STRN0051
      EXPL(K)=DEEP/SE*(SX-SY/2.0)+EXPL(K)                           STRN0052
      EYP(K)=DEEP/SE*(SY-SX/2.0)+EYP(K)                            STRN0053
      EXYP(K)=3.0*DEEP/SE*SXY+EXYP(K)                               STRN0054
      ERR=E*DEEP/SE                                                 STRN0055
      ERR=ABS(ERR)                                                  STRN0056
      GO TO 50                                                      STRN0057
   40 ERR=0.0                                                       STRN0058
   50 CONTINUE                                                      STRN0059
      Q1=E*Z(K)/(1.0-PR**2)/2.0*SN                                  STRN0060
      Q2=E*Z(K)/(1.0+PR)/4.0*SN                                     STRN0061
      EXPT=EXPL(K)                                                  STRN0062
      EYPT=EYP(K)                                                   STRN0063
      EXYPT=EXYP(K)                                                 STRN0064
      FP(J1)=-Q1*Y32*EXPT-Q1*Y32*PR*EYPT+Q2*X32*EXYPT+FP(J1)        STRN0065
      FP(J2)= Q1*X32*PR*EXPT+Q1*X32*EYPT-Q2*Y32*EXYPT+FP(J2)        STRN0066
      FP(J3)= Q1*Y31*EXPT+Q1*Y31*PR*EYPT-Q2*X31*EXYPT+FP(J3)        STRN0067
      FP(J4)=-Q1*X31*PR*EXPT-Q1*X31*EYPT+Q2*Y31*EXYPT+FP(J4)        STRN0068
      FP(J5)=-Q1*Y21*EXPT-Q1*Y21*PR*EYPT+Q2*X21*EXYPT+FP(J5)        STRN0069
      FP(J6)= Q1*X21*PR*EXPT+Q1*X21*EYPT-Q2*Y21*EXYPT+FP(J6)        STRN0070
```

```
      GO TO 110                                                     STRN0071
C                                                                   STRN0072
C     **** BAR CALCULATIONS ****                                    STRN0073
C                                                                   STRN0074
   60 EET=(X21*(X(J3)-X(J1))+Y21*(X(J4)-X(J2)))/Y32**2              STRN0075
      STRN=ABS(EET-EXPL(K))                                         STRN0076
      SIGN=(EET-EXPL(K))/STRN                                       STRN0077
      SE=E*STRN                                                     STRN0078
      CRIT=SE-SEF(K)                                                STRN0079
      EET=STRN+EEP(K)                                               STRN0080
      IF(CRIT)90,90,70                                              STRN0081
   70 CONTINUE                                                      STRN0082
      CALL STRSTN(EET,EEPK,SETK,NTY)                                STRN0083
      DEEP=EEPK-EEP(K)                                              STRN0084
   80 EEP(K)=EEPK                                                   STRN0085
      SET(K)=SETK                                                   STRN0086
      EXPL(K)=EXPL(K)+SIGN*DEEP                                     STRN0087
      ERR=E*DEEP/SE                                                 STRN0088
      ERR=ABS(ERR)                                                  STRN0089
      GO TO 100                                                     STRN0090
   90 ERR=0.0                                                       STRN0091
  100 CONTINUE                                                      STRN0092
      EXPT=EXPL(K)                                                  STRN0093
      O1=E*Z(K)/Y32                                                 STRN0094
      FP(J1)=FP(J1)-O1*X21*EXPT                                     STRN0095
      FP(J2)=FP(J2)-O1*Y21*EXPT                                     STRN0096
      FP(J3)=FP(J3)+O1*X21*EXPT                                     STRN0097
      FP(J4)=FP(J4)+O1*Y21*EXPT                                     STRN0098
  110 IF(ERR-XERR)130,130,120                                      STRN0099
  120 XERR=ERR                                                      STRN0100
      KEL=K                                                         STRN0101
  130 CONTINUE                                                      STRN0102
      RETURN                                                        STRN0103
      END                                                          STRN0104
      SUBROUTINE STRSTN(EET,EEPK,SETK,NTY)                          STRS0001
      COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                   STRS0002
      COMMON   E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,      STRS0003
     1         N2,NELEM,KEL,ILAW,MAT,NBC,                           STRS0004
     2         B(900,60),BC(30,3),TAB(101,20),IFIX(2),
     3         X(900),XCORD(450),Y(450),ICODE(450),
     4         FP(900),F(900),
     5         I1(800),I2(800),I3(800),NTYPE(800),Z(800),I4(800),
     6         SEF(800),SET(800),EEP(800),EXPL(800),EYP(800),EXYP(800),
     7         NNODE,MBAND                                           STRS0010
      GO TO(10,50,60),ILAW                                          STRS0011
   10 J=5.0*EET/CC+1.0                                              STRS0012
      NT2=2*NTY                                                     STRS0013
      NT1=NT2-1                                                     STRS0014
      IF(J-101)20,30,30                                             STRS0015
   20 EEPK=TAB(J,NT2)+(TAB(J+1,NT2)-TAB(J,NT2))*(EET-TAB(J,NT1))/   STRS0016
     1(TAB(J+1,NT1)-TAB(J,NT1))                                     STRS0017
      GO TO 40                                                      STRS0018
   30 CALL PNEW1(EEPK,EET,E,G1,E2)                                  STRS0019
   40 SETK=G1*EEPK**(1.0/E2)                                        STRS0020
      RETURN                                                        STRS0021
   50 SETK=E*EET/(1.+(ABS(E*EET/G)))**E2)**(1./E2)                  STRS0022
      EEPK=EET-SETK/E                                               STRS0023
      RETURN                                                        STRS0024
   60 FC=G1/F                                                       STRS0025
      IF(EET-FC)70,70,80                                            STRS0026
```

```
   70 EEPK=0.                                                       STRS0027
      SETK=F*EET                                                    STRS0028
      RETURN                                                        STRS0029
   80 EEPK=EET-EC                                                   STRS0030
      SETK=G1+E2*EEPK                                               STRS0031
      RETURN                                                        STRS0032
      END                                                           STRS0033
C12345            OUTPUT SUBROUTINE                                 OTPU0001
      SUBROUTINE OUTPT                                              OTPU0002
      COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                  OTPU0003
      COMMON  F,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,      OTPU0004
     1         NN2,NELEM,KEL,ILAW,MAT,NBC,                         OTPU0005
     2         B(900,60),BC(30,3),TAB(10),20),IFIX(2),
     3         X(900),XCORD(450),Y(450),ICODE(450),
     4         EP(900),F(900),
     5         I1(800),I2(800),I3(800),NTYPE(800),Z(800),I4(800),
     6         SEF(800),SET(800),EFP(800),EXPL(800),EYP(800),EXYP(800),
     7         NNODE,MBAND                                          OTPU0011
      L=0                                                           OTPU0012
      DO 70 K=1,NELEM                                               OTPU0013
C                                                                   OTPU0014
C    **** TRIANGULAR ELEMENT CALCULATIONS ****                     OTPU0015
C                                                                   OTPU0016
      NTY=NTYPE(K)                                                  OTPU0017
      E=EE(NTY)                                                     OTPU0018
      PR= PRR(NTY)                                                  OTPU0019
      EPR=E/(1.-PR*PR)                                              OTPU0020
      IF(I3(K).EQ.0) GO TO 70                                       OTPU0021
   10 CALL ELEM(K)                                                  OTPU0022
      A123=X21*Y31-X31*Y21                                          OTPU0023
      J1=2*I1(K)-1                                                  OTPU0024
      J2=2*I1(K)                                                    OTPU0025
      J3=2*I2(K)-1                                                  OTPU0026
      J4=2*I2(K)                                                    OTPU0027
      J5=2*I3(K)-1                                                  OTPU0028
      J6=2*I3(K)                                                    OTPU0029
      EXT=(-Y32*X(J1)+Y31*X(J3)-Y21*X(J5))/A123                    OTPU0030
      EYT=( X32*X(J2)-X31*X(J4)+X21*X(J6))/A123                    OTPU0031
      EXYT=(X32*X(J1)-Y32*X(J2)-X31*X(J3)+Y31*X(J4)                OTPU0032
     1     +X21*X(J5)-Y21*X(J6))/A123                              OTPU0033
      EXE=EXT-EXPL(K)                                               OTPU0034
      EYE=EYT-EYP(K)                                                OTPU0035
      EXYE=EXYT-EXYP(K)                                             OTPU0036
      SX=EPR*(EXE+PR*EYE)                                           OTPU0037
      SY=EPR*(EYE+PR*EXE)                                           OTPU0038
      SXY=E/(1.0+PR)*EXYE/2.0                                       OTPU0039
      PE2=SQRT((.5*(SX-SY))**2+SXY**2)                             OTPU0040
      PHI=.5*ATAN2((-2.0*SXY),(SX-SY))*57.29578                    OTPU0041
      PH2=.5*ATAN2((-EXYT),(EXT-EYT))*57.29578                     OTPU0042
      PS1=.5*(SX+SY)                                                OTPU0043
      SIGE1=PS1+PE2                                                 OTPU0044
      SIGE2=PS1-PE2                                                 OTPU0045
      PST1=.5*(EXT+EYT)                                             OTPU0046
      PET2=SQRT((.5*(EXT-EYT))**2+EXYT**2/4.0)                     OTPU0047
      STRE1=PST1+PET2                                               OTPU0048
      STRE2=PST1-PET2                                               OTPU0049
      PET2=2.0*PET2                                                 OTPU0050
      N1=I1(K)                                                      OTPU0051
      N2=I2(K)                                                      OTPU0052
      N3=I3(K)                                                      OTPU0053
```

```
      XC=XCORD(N))+(X21+X31)/3.0                                       OTPU0054
      YC=Y(N1)+(Y21+Y31)/3.0                                           OTPU0055
      EPE=2.*SORT((EXPL(K)**2+EXPL(K)*EYP(K)+EYP(K)**2+EXYP(K)**2/4.)/3.OTPU0056
     1)                                                                OTPU0057
      SE=SORT(SX**2-SX*SY+SY**2+3.*SXY**2)                             OTPU0058
      L=L+1                                                            OTPU0059
      IF(MOD(L-1,14))30,20,30                                          OTPU0060
   20 WRITE (6,40)                                                     OTPU0061
   30 WRITE(6,50)K,XC,YC,SX,SY,SXY,SIGE1,SIGE2,PE2,PHI,PH2,EXT,EYT,  EXOTPU0062
     1YT,STRE1,STRE2,PET2                                              OTPU0063
   40 FORMAT(9H1EL. NO./5X11HCOORDINATES28X33HS T R E S S E S / S T R A OTPU0064
     11 N S /8H     PHI7X1HX8X1HY6X9H    TAU-XX6X9H    TAU-YY6X9H    TAU-XOTPU0065
     2Y8X7HMAXIMUM8X7HMINIMUM6X9HMAX SHEAR )                           OTPU0066
   50 FORMAT(1H0I7,0PF8.3,F9.3,1P6E15.4/1H 0PF7.2,F8.2,9X1P6E15.4)      OTPU0067
      WRITE(6,60) SE,EPE                                               OTPU0068
   60 FORMAT(1H 23H***** EFFECTIVE STRESS=E12.5,23H***** EFFECTIVE STRAIOTPU0069
     1N=E12.5)                                                         OTPU0070
   70 CONTINUE                                                         OTPU0071
C                                                                     OTPU0072
C **** BAR CALCULATIONS ****                                          OTPU0073
C                                                                     OTPU0074
      J=0                                                              OTPU0075
      DO 120 K=1,NELEM                                                 OTPU0076
      NTY=NTYPE(K)                                                     OTPU0077
      E=EE(  NTY)                                                      OTPU0078
      PR=PRR(  NTY)                                                    OTPU0079
      J1=2*I1(K)-1                                                     OTPU0080
      J2=2*I1(K)                                                       OTPU0081
      J3=2*I2(K)-1                                                     OTPU0082
      J4=2*I2(K)                                                       OTPU0083
      IF(I3(K).NE.0) GO TO 120                                         OTPU0084
   80 CALL ELEM(K)                                                     OTPU0085
      EET=(X21*(X(J3)-X(J1))+Y21*(X(J4)-X(J2)))/Y32**2                 OTPU0086
      SE=E*(EET-EXPL(K))                                               OTPU0087
      SEMZK=SE*Z(K)                                                    OTPU0088
      K1=I1(K)                                                         OTPU0089
      K2=I2(K)                                                         OTPU0090
      IF(J)100,90,100                                                  OTPU0091
   90 WRITE (6,130)                                                    OTPU0092
      J=1                                                              OTPU0093
  100 CONTINUE                                                         OTPU0094
  110 WRITE(6,140)K,K1,K2,SE,EET,SEMZK                                 OTPU0095
  120 CONTINUE                                                         OTPU0096
  130 FORMAT(9H0BAR NO. 6X10HNODE NOS. 8X7HSTRESS 8X7HSTRAIN ,4X,       OTPU0097
     113HMEMBER FORCES)                                                OTPU0098
  140 FORMAT(1H0 3I8,2E15.5,2X,E15.5)                                  OTPU0099
      RETURN                                                           OTPU0100
      END                                                              OTPU0101
```

EXAMPLE PROBLEM NO. 1   W16X40   'CANTILEVER BEAM'   ULTIMATE LOAD.

```
                    BILINEAR LAW
                    MATERIAL----------------------      1
                    MODULUS CF ELASTICITY----------      0.2900E 05
                    SECANT YIELD STRESS-----------      0.3600E 02
                    SHAPE PARAMETER---------------      0.0
                    POISSEN'S RATIC---------------      0.3000
                    ERROR TOLERANCE--------------      0.0300

                    NO. CF NODES        ANODE =     21
                    NO. OF ELEMENTS     AELEM =     34
                    NC. CF STEPS        NDIV =     20
                    NC. CF ITERATICNS/STEP  NIT = 100
```

BOUNDARY CONDITION ARRAY

NODAL PT	CODE	X VALUE	CODE	Y VALUE	SLIDING CODE	VALUE
1	1	0.0	1	0.0	0	0.0
2	1	0.0	1	0.0	0	0.0
3	1	0.0	1	0.0	0	0.0
4	1	0.0	1	0.0	0	0.0
5	1	0.0	1	0.0	0	0.0

NODAL PT	X-COORD	X-FORCE	Y-COORD	Y-FORCE	CODE
1	0.0	0.0	0.0	0.0	110
2	0.0	0.0	4.0000	0.0	110
3	0.0	0.0	8.0000	0.0	110
4	0.0	0.0	12.0000	0.0	110
5	0.0	0.0	16.0000	0.0	110
6	5.0000	0.0	0.0	0.0	0
7	5.0000	0.0	4.0000	0.0	0
8	5.0000	0.0	8.0000	0.0	0
9	5.0000	0.0	12.0000	0.0	0
10	5.0000	0.0	16.0000	0.0	0
11	10.0000	0.0	0.0	0.0	0
12	10.0000	0.0	4.0000	0.0	0
13	10.0000	0.0	8.0000	0.0	0
14	10.0000	0.0	12.0000	0.0	0
15	10.0000	0.0	16.0000	0.0	0
16	20.0000	0.0	0.0	0.0	0
17	20.0000	0.0	8.0000	0.0	0
18	20.0000	0.0	16.0000	0.0	0
19	36.0000	0.0	0.0	0.0	0
20	36.0000	0.0	8.0000	0.0	0
21	36.0000	0.0	16.0000	-80.0000	0

ELEMENT	NODE 1	NODE 2	NODE 3	ELEMENT TYPE	AREA OR THICK.	MATERIAL TYPE
1	1	6	0	BAR	0.32200E 01	1
2	1	6	2	PLATE	0.30700E 00	1
3	6	7	2	PLATE	0.30700E 00	1
4	2	7	3	PLATE	0.30700E 00	1

5	7	8	3	PLATE	0.30700E 00	1
6	3	8	9	PLATE	0.30700E 00	1
7	3	9	4	PLATE	0.30700E 00	1
8	4	9	10	PLATE	0.30700E 00	1
9	4	10	5	PLATE	0.30700E 00	1
10	5	10	0	BAR	0.32200E 01	1
11	6	11	0	BAR	0.32200E 01	1
12	6	11	7	PLATE	0.30700E 00	1
13	11	12	7	PLATE	0.30700E 00	1
14	7	12	13	PLATE	0.30700E 00	1
15	7	13	8	PLATE	0.30700E 00	1
16	8	13	9	PLATE	0.30700E 00	1
17	13	14	9	PLATE	0.30700E 00	1
18	9	14	15	PLATE	0.30700E 00	1
19	9	15	10	PLATE	0.30700E 00	1
20	10	15	0	BAR	0.32200E 01	1
21	11	16	0	BAR	0.32200E 01	1
22	11	16	12	PLATE	0.30700E 00	1
23	16	17	12	PLATE	0.30700E 00	1
24	12	17	13	PLATE	0.30700E 00	1
25	13	17	14	PLATE	0.30700E 00	1
26	17	18	14	PLATE	0.30700E 00	1
27	14	18	15	PLATE	0.30700E 00	1
28	15	18	0	BAR	0.32200E 01	1
29	16	19	0	BAR	0.32200E 01	1
30	16	19	17	PLATE	0.30700E 00	1
31	19	20	17	PLATE	0.30700E 00	1
32	17	20	21	PLATE	0.30700E 00	1
33	17	21	18	PLATE	0.30700E 00	1
34	18	21	0	BAR	0.32200E 01	1

FORCES AND DISPLACEMENTS FOR INCREMENT 19

NODE	X-FORCE	Y-FORCE	X-DISPL.	Y-DISPL.
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	-0.2947E-01	-0.3895E-01
7	0.0	0.0	-0.1416E-01	-0.3616E-01
8	0.0	0.0	-0.1959E-03	-0.3600E-01
9	0.0	0.0	0.1414E-01	-0.3585E-01
10	0.0	0.0	0.2998E-01	-0.3859E-01
11	0.0	0.0	-0.3529E-01	-0.6606E-01
12	0.0	0.0	-0.1741E-01	-0.6491E-01
13	0.0	0.0	-0.3339E-03	-0.6453E-01
14	0.0	0.0	0.1730E-01	-0.6469E-01
15	0.0	0.0	0.3584E-01	-0.6584E-01
16	0.0	0.0	-0.4316E-01	-0.1304E 00
17	0.0	0.0	-0.8716E-03	-0.1295E 00
18	0.0	0.0	0.4398E-01	-0.1307E 00
19	0.0	0.0	-0.4688E-01	-0.2400E 00
20	0.0	0.0	0.1002E-02	-0.2425E 00
21	0.0	-76.000	0.4809E-01	-0.2481E 00

MAX. ERROR = 0.03939   IN ELEMENT NO.   9   NO. OF ITERATIONS 100

EL. NO./ PHI	COORDINATES X	Y	TAU-XX	TAU-YY	TAU-XY	S T R E S S E S / S T R A I N S MAXIMUM	MINIMUM	MAX SHEAR
2 63.68	1.667 63.56	1.333	-3.5012E 01 -5.8934E-03	-1.7718E 01 0.0	-1.1326E 01 -7.7697E-03	-1.2116E 01 1.9372E-03	-4.0614E 01 -7.8306E-03	1.4249E 01 9.7679E-03
***** EFFECTIVE STRESS= 0.36114E 02*****					EFFECTIVE STRAIN= 0.67364E-02			
3 67.01	3.333 58.01	2.667	-3.2921E 01 -2.8322E-03	-5.6319E 00 6.9709E-04	-1.2054E 01 -3.9055E-03	-4.5179E 00 1.3846E-03	-3.8045E 01 -3.5197E-03	1.6763E 01 4.9044E-03
***** EFFECTIVE STRESS= 0.35999E 02*****					EFFECTIVE STRAIN= 0.22554E-02			
4 55.72	1.667 55.65	5.333	-2.5433E 01 -7.8322E-03	-1.2524E 01 0.0	-1.6417E 01 -7.2320E-03	-1.3452E 00 2.4673E-03	-3.6621E 01 -5.2955E-03	1.7638E 01 7.7668E-03
***** EFFECTIVE STRESS= 0.35968E 02*****					EFFECTIVE STRAIN= 0.40526E-02			
5 45.55	3.333 45.52	6.667	-3.4548E-01 -3.5181E-05	4.2195E-01 4.0752E-05	-2.0745E 01 -3.7081E-03	2.0775E-01 1.8553E-05	-2.0723E 01 -1.8537E-03	2.0749E 01 3.7095E-03
***** EFFECTIVE STRESS= 0.35938E 02*****					EFFECTIVE STRAIN= 0.10674E-02			
6 45.53	3.333 45.60	9.333	-4.1507E-01 -3.9181E-05	3.5713E-01 3.6469E-05	-2.0746E 01 -3.6149E-03	2.0721E 01 1.8065E-03	-2.0779E 01 -1.8092E-03	2.0750E 01 3.6157E-03
***** EFFECTIVE STRESS= 0.35939E 02*****					EFFECTIVE STRAIN= 0.10135E-02			
7 34.16	1.667 34.24	10.667	2.5604E 01 2.8284E-03	1.2608E 01 0.0	-1.6551E 01 -7.1702E-03	3.6791E 01 5.2682E-03	-1.5110E 00 -2.4397E-03	1.7595E 01 7.7079E-03
***** EFFECTIVE STRESS= 0.35969E 02*****					EFFECTIVE STRAIN= 0.40200E-02			
8 22.44	3.333 21.21	13.333	3.2352E 01 2.8284E-03	5.7601E 00 -6.8546E-04	-1.1747E 01 -3.2101E-03	3.8204E 01 3.4512E-03	4.9084E 00 -1.3082E-03	1.6648E 01 4.7594E-03
***** EFFECTIVE STRESS= 0.36002E 02*****					EFFECTIVE STRAIN= 0.21856E-02			
9 26.14	1.667 26.08	14.667	3.5155E 01 5.9455E-03	1.7805E 01 0.0	-1.1218E 01 -7.7186E-03	4.0661E 01 7.8554E-03	1.2299E 01 -1.8898E-03	1.4181E 01 9.7742E-03
***** EFFECTIVE STRESS= 0.36118E 02*****					EFFECTIVE STRAIN= 0.66088E-02			
12 71.65	6.667 69.70	1.333	-2.2799E 01 -1.1650E-03	9.8885E 00 6.7096E-04	-1.2145E 01 -1.5956E-03	1.3507E 01 9.9215E-04	-2.6818E 01 -1.4601E-03	2.0363E 01 2.4522E-03
***** EFFECTIVE STRESS= 0.35855E 02*****					EFFECTIVE STRAIN= 0.40310E-03			
13 63.08	8.333 63.08	2.667	-1.9003E 01 -6.5074E-04	2.8961E 00 2.8610E-04	-1.4294E 01 -1.2816E-03	1.0153E 01 6.1142E-04	-2.5260E 01 -9.7606E-04	1.7707E 01 1.5875E-03
***** EFFECTIVE STRESS= 0.31585E 02*****					EFFECTIVE STRAIN= 0.0			
14 59.40	8.333 58.40	5.333	-1.5809E 01 -6.5074E-04	-3.1233E 00 9.7215E-05	-1.6516E 01 -1.4807E-03	7.0738E 00 5.5269E-04	-2.9969E 01 -1.1062E-03	1.8503E 01 1.6599E-03
***** EFFECTIVE STRESS= 0.34038E 02*****					EFFECTIVE STRAIN= 0.0			
15 45.66	6.667 45.88	6.667	-3.5473E-01 -2.7600E-05	1.0412E 00 4.0752E-05	-2.0770E 01 -2.2145E-03	2.1125E 01 1.1143E-03	-2.0439E 01 -1.1012E-03	2.0782E 01 2.2155E-03
***** EFFECTIVE STRESS= 0.35597E 02*****					EFFECTIVE STRAIN= 0.20346E-03			
16 45.82	6.667 45.87	9.333	-4.4408E-01 -2.7600E-05	7.4152E-01 3.6469E-05	-2.0775E 01 -2.1213E-03	2.0932E 01 1.0655E-03	-2.0635E 01 -1.0567E-03	2.0783E 01 2.1222E-03
***** EFFECTIVE STRESS= 0.35598E 02*****					EFFECTIVE STRAIN= 0.14948E-03			
17 31.82	8.333 31.82	10.667	1.9766E 01 6.2234E-04	4.7664E 00 -4.0233E-05	-1.5144E 01 -1.3578E-03	2.9164E 01 1.0526E-03	-4.6376E 00 -4.6161E-04	1.6901E 01 1.5122E-03
***** EFFECTIVE STRESS= 0.21738E 02*****					EFFECTIVE STRAIN= 0.0			

EL. NO./ PHI	COORDINATES X	Y	STRESSES / STRAINS TAU-XX	TAU-YY	TAU-XY	MAXIMUM	MINIMUM	MAX SHEAR
18	8.333	13.333	1.7397E 01	-3.1367E 00	-1.2650E 01	2.3422E 01	-9.1619E 00	1.6292E 01
25.47	25.47		6.2234E-04	-2.8813E-04	-1.1341E-03	9.0243E-04	-5.5822E-04	1.4607E-02
***** EFFECTIVE STRESS= 0.29105E 02***** EFFECTIVE STRAIN= 0.0
19	6.667	14.667	2.3383E 01	-9.5521E 00	-1.1453E 01	2.6574E 01	-1.3143E 01	2.0059E 01
17.41	19.37		1.1707E-03	-6.8449E-04	-1.4892E-03	1.4324E-03	-9.4725E-04	2.3797E-03
***** EFFECTIVE STRESS= 0.35424E 02***** EFFECTIVE STRAIN= 0.37715E-03
22	13.333	1.333	-1.6594E 01	7.2168E-01	1.7571E 01	1.1114E 01	-2.8986E 01	2.0050E 01
59.40	59.33		-7.8634E-04	2.8610E-04	-1.9626E-03	8.6815E-04	-1.3684E-03	2.2365E-03
***** EFFECTIVE STRESS= 0.35860E 02***** EFFECTIVE STRAIN= 0.26261E-05
23	16.667	4.000	-1.3563E 01	-7.5540E-01	1.3555E 01	7.8319E 00	-2.2151E 01	1.4991E 01
57.64	57.64		-4.6895E-04	1.1426E-04	-1.2126E-03	4.9921E-04	-8.4443E-04	1.3449E-03
***** EFFECTIVE STRESS= 0.26935E 02***** EFFECTIVE STRAIN= 0.0
24	13.333	6.667	-5.3830E-01	2.2509E 00	-2.0731E 01	2.1634E 01	-1.9922E 01	2.0778E 01
46.92	46.94		-5.3771E-05	9.7215E-05	-2.2237E-03	1.1361E-03	-1.0927E-03	2.2289E-03
***** EFFECTIVE STRESS= 0.35999E 02***** EFFECTIVE STRAIN= 0.21129E-03
25	13.333	9.333	-1.9179E 00	-1.7205E 00	-2.0736E 01	1.8939E 01	-2.2577E 01	2.0758E 01
45.14	45.18		-5.3771E-05	-4.0323E-05	-2.0845E-03	9.9521E-04	-1.0893E-03	2.0845E-03
***** EFFECTIVE STRESS= 0.36000E 02***** EFFECTIVE STRAIN= 0.12918E-03
26	16.667	12.000	1.2105E 01	-7.6189E-01	-1.0388E 01	1.7899E 01	-6.5472E 00	1.7219E 01
29.11	29.11		4.2528E-04	-1.9149E-04	-9.3132E-04	6.8462E-04	-4.1083E-04	1.0555E-03
***** EFFECTIVE STRESS= 0.21910E 02***** EFFECTIVE STRAIN= 0.0
27	13.333	14.667	1.5625E 01	-5.8049E 00	-1.6823E 01	2.9146E 01	-1.0101E 01	1.9623E 01
29.51	29.60		8.1480E-04	-2.8813E-04	-1.9507E-03	1.3406E-03	-8.1384E-04	2.1544E-03
***** EFFECTIVE STRESS= 0.35297E 02***** EFFECTIVE STRAIN= 0.23779E-03
30	25.333	2.667	-6.3242E 00	1.4166E 00	-1.7454E 01	1.5425E 01	-2.0322E 01	1.7878E 01
51.25	51.25		-2.3273E-04	1.1427E-04	-1.5649E-03	7.4221E-04	-8.6067E-04	1.6029E-03
***** EFFECTIVE STRESS= 0.21063E 02***** EFFECTIVE STRAIN= 0.0
31	30.667	5.333	7.7779E 00	-8.7276E 00	-1.2011E 01	8.5421E 00	-1.6992E 01	1.2917E 01
34.21	34.21		1.1711E-04	-3.0908E-04	-1.0763E-03	4.8309E-04	-6.7498E-04	1.1581E-03
***** EFFECTIVE STRESS= 0.22723E 02***** EFFECTIVE STRAIN= 0.0
32	30.667	10.667	-2.9917E 00	-2.1292E 00	-1.3118E 01	3.8519E 00	-2.8136E 01	1.5994E 01
27.55	27.55		1.1711E-04	-7.0328E-04	-1.1701E-03	4.2389E-04	-1.0101E-03	1.4335E-03
***** EFFECTIVE STRESS= 0.30247E 02***** EFFECTIVE STRAIN= 0.0
33	25.333	13.333	6.7272E 00	-2.3752E 00	-1.9305E 01	2.2010E 01	-1.7659E 01	1.9835E 01
38.37	38.37		2.5454E-04	-1.5149E-04	-1.7308E-03	9.4166E-04	-8.3661E-04	1.7783E-03
***** EFFECTIVE STRESS= 0.34423E 02***** EFFECTIVE STRAIN= 0.0

BAR NO.	NODE NOS.		STRESS	STRAIN	MEMBER FORCES
1	1	6	-0.36000E 02	-0.36924E-02	-0.11592E 03
10	5	10	0.36000E 02	0.59965E-02	0.11592E 03
11	6	11	-0.23785E 02	-0.11650E-02	-0.10879E 03
20	10	15	0.33950E 02	0.11707E-02	0.10932E 03

21	11	16	-0.228C4E 02	-0.78634E-03	-0.73428E 02
28	15	18	0.23632E 02	0.81488E-03	0.76094E 02
29	16	19	-C.67492E 01	-0.22273E-03	-0.21732E 02
34	18	21	0.74396E 01	0.25654E-03	0.23955E 02

--- JCB FINISHED

Example Problem

. To illustrate the use of the program a simple example of a cantilever beam is worked out. The beam, a W16 x 40, is 36 inches long with a load at its cantilevered tip.

The properties of the cross-section are listed below: $d = 16.0"$; $tw = 0.307$; $t_f = 0.503"$; $b_f = 7.0"$; $Z = 72.8$ in^3; $S = 64.6$ in^3.

For a yield stress of 36 ksi, the theoretical ultimate load can be calculated as follows: The plastic moment of the section = $Mp = F_y Z$. Equating it to the externally applied load: $Mp = PuL$

$$Pu = F_y Z/L = 72.8k$$

Similarly, the load at first yield can be given by:

$$P_y = F_y S/L = 64.6 \text{ k}$$

Solution by the Finite Element Method

To work out the example by the use of the finite element program, the three dimensional beam is idealized into a two dimensional plane stress problem, see Fig. AI-Ia. Having established 21 nodes in the cartisian coordinates, the web is comprised of 26 triangular elements of thickness 0.307 in. and the flanges are simulated by 8 bar elements having a cross-sectional area of 3.22 sq. in.

The beam is fixed at the left end by specifying zero displacements in the x and y directions for nodes 1 through 5.

The Bilinear law was made use of with the material properties as follows:

Modulus of Elasticity = 29 x 10^3 ksi

Yield Stress   = 36 ksi

Poisson's Ratio   = 0.3

Plastic Modulus   = 0.0

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

THIS IS AS RECEIVED FROM CUSTOMER.
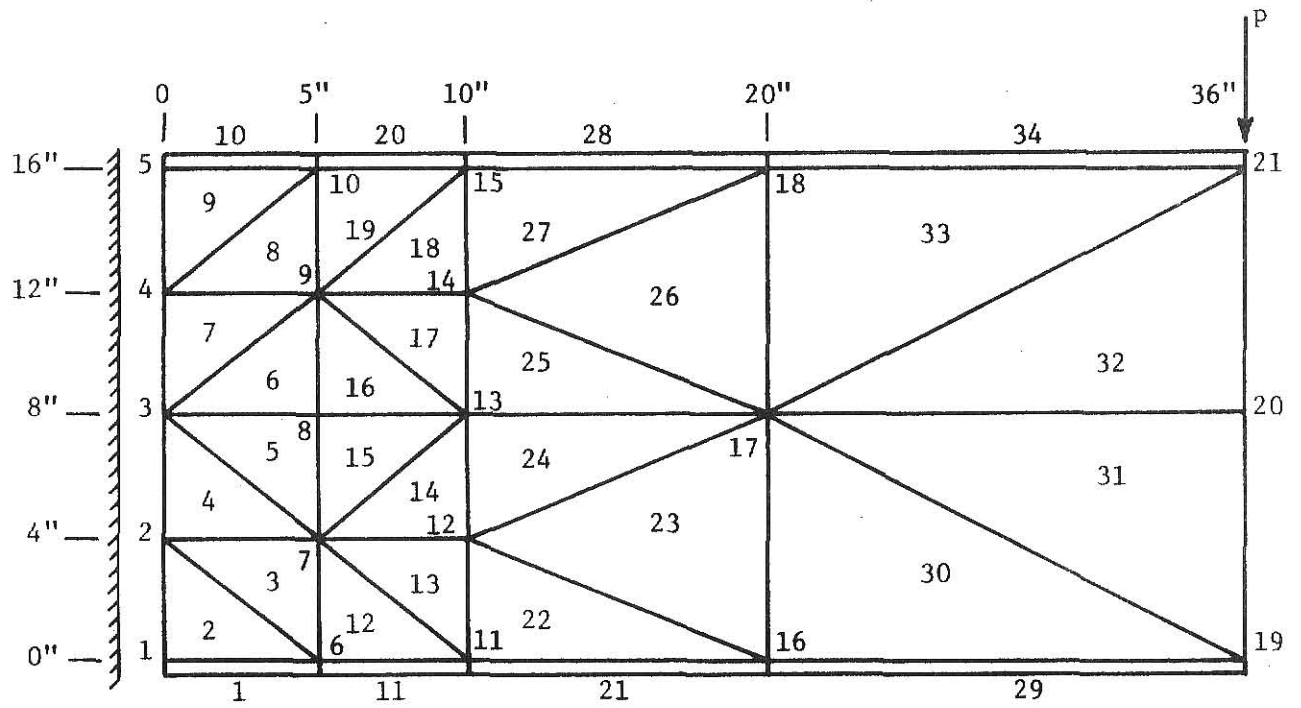
Fig. AI-Ia.   Idealized Cantilever Beam.
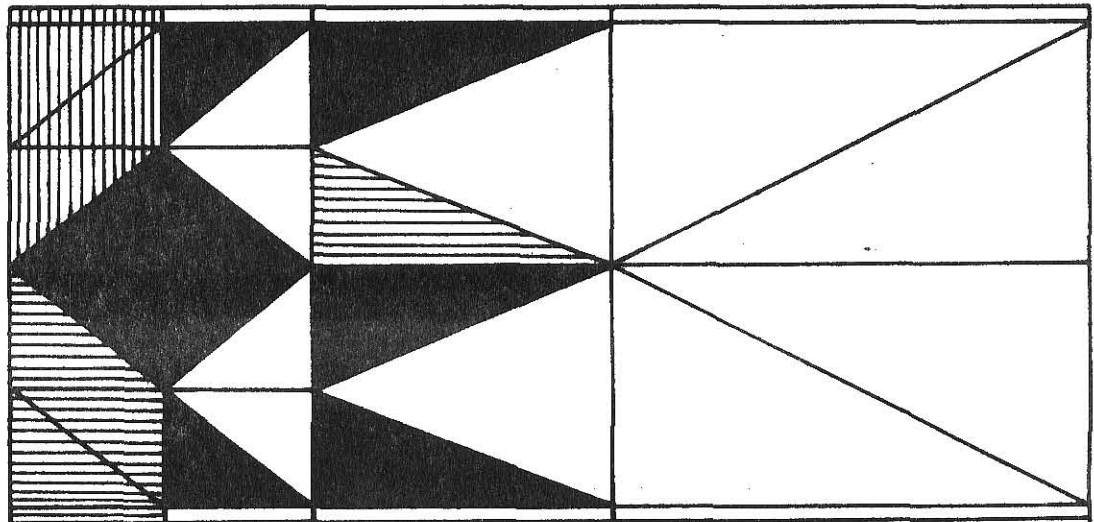
Fig. AI-Ib.   Yield pattern at ultimate load (76 kips).

A total load of 80 kips was applied in 20 increments of 4 kips each. KSTART was specified as 15 and therefore the solution started with a load of 60 kips.

A maximum of 100 iterations per step and an error tolerance of 0.03 were specified. At increment number 19, that is a load of 76 kips, the error in element number 9 could not be reduced to less than 0.03 in 100 iterations and therefore, the solution was stopped at that increment. The load of 76 kips is considered as the ultimate load with an error of 0.03 in the effective plastic strain. A plot of the yielded elements at the ultimate load is shown in Fig. AI-Ib.

The choice of the maximum number of iterations per step and the error tolerance is left to the individual and depends on the accuracy desired and the computer time available. For the purpose of this project the above values were selected based on a few trial runs.

Trial Runs

Two sets of trial runs were made. One on Beam 1 and the other on Solid Beam 1. Table AI-III gives the details of these runs. The variables were, number of nodes at which the load was applied, error tolerance and the maximum number of iterations per step.

In the actual testing of the beam the load was applied through a plate 1" x 6" x 7". To simulate this condition a three point loading was tried in a few runs, with one half of the load applied at the center and one quarter of the load at the other two points. As can be observed in the tables, the three point loading had no significant effect on the maximum number of load increments reached, the idea of 3 point loading was therefore abandoned.

With the overall computer time available for the project as a limiting constraint, a maximum of 100 iterations per step and an error tolerance of 0.03 gave a reasonably good prediction of the ultimate load and were selected for the rest of the beams.

The execution time required to test a beam with the opening on the IBM 370 model 158 ranged from 7 to 9 minutes.

Run No.	No. of Nodes at which Load is Specified	No. of Increments at which Solution Stopped	Specified Error Tolerance	Specified Maximum No. of Iterations Per Step	Maximum Residual Error
		Beam 1			
1	1	15	0.02	20	0.0244
2	1	16	0.03	25	0.0457
3	1	17	0.03	50	0.0514
4	3	17	0.03	50	0.0528
5	1	18	0.03	100	0.1057
6	1	18	0.10	25	0.1640
		Solid Beam 1			
1	1	18	0.02	100	0.0708
2	3	18	0.02	100	0.0288
3	1	18	0.03	20	0.0756
4	3	18	0.03	20	0.0323
5	1	18	0.03	100	0.0710
6*	1	21	0.03	100	0.0503
7	3	20	0.10	100	0.1354

* with cover plates and stiffeners at supports

Table AI-III.   Trial Runs on Beam 1 and Solid Beam 1.

APPENDIX II - REFERENCES

1. Salmon, M., Berke, L. and Sandhu, R., "An Application of the Finite Element Method to Elastic-Plastic Problems of Plane Stress," Illinois Institute of Technology Research Institute and the Air Force Flight Dynamics Laboratory, Project No. 1467, "Structural Analysis Methods," December 1968.

2. Kussman, R. L. and Cooper, P. B., "Ultimate Load Tests on Steel Beams with Reinforced Eccentric Web Openings," Research Project Report, Department of Civil Engineering, Kansas State University, August, 1975.

3. Muskhelishvilli, N. I., "Some Basic Problems of the Mathematical Theory of Elasticity," 2nd Edition, P. Noorhoff Ltd., Croningen, The Netherlands, 1963.

4. Joseph, J. A. and Brock, J. S., "The Stresses Around a Small Opening in a Beam Subjected to Pure Bending," Journal of Applied Mechanics, Vol. 17, No. 4, 1950.

5. Timoshenko, S., "On Stresses in a Flat Plate with a Circular Hole," Journal of the Franlin Institute, Vol. 197, 1924.

6. Heller, S. R., Jr., "The Stresses Around a Small Hole in a Beam Subjected to Bending with Shear," Proceedings 1st National Congress of Applied Mechanics, ASME, N.Y., 1951.

7. Savin, G. N., "Stress Concentration Around Holes," International Series of Monographs in Aeronautics and Astronautics, Pergamon Press, 1961.

8. Heller, S. R., Jr., Brock, J. S. and Bart, R., "The Stresses Around a Rectangular Opening with Rounded Corners in a Uniformly Loaded Plate," Proc., 3rd U.S. National Congress of Applied Mechanics, 1958.

9. Heller, S. R., Jr., Brock, J. S. and Bart, R., "The Stress Around a Rectangular Opening with Rounded Corners in a Beam Subjected to Bending with Shear," Proc., 4th U.S. National Congress of Applied Mechanics, Berkeley, California, 1962.

10. Snell, R. R., "Reinforcing for a Rectangular Opening in a Plate," Journal of Structural Division, ASCE, Vol. 91, No. ST4, August 1965.

11. McCutcheon, J. O., So, W-C. and Gersovitz, B., "A Study of the Effects of Large Circular Openings in the Webs of Wide Flange Beams," McGill University, Applied Mechanics Series No. 2, November, 1963.

12. Segner, E. P., Jr., "An Investigation of the Requirements for Reinforcement around Large Rectangular Openings in the Webs of Wide Flange Beams Subject to Bending Moment and Shear," Research Report submitted to AISC, January 1963.

13. Segner, E. P., Jr., "Reinforcement Requirements for Girder Web Openings," Journal of the Structural Division, ASCE, Vol. 90, No. ST3, June 1964.

14. Bower, J. E., "Elastic Stresses Around Holes in Wide Flange Beams," Journal of the Structural Division, ASCE, Vol. 92, No. ST2, April 1966.

15. Bower, J. E., "Experimental Stresses in Wide Flange Beams with Holes," Journal of the Structural Division, ASCE, Vol. 92, No. ST5, Proc., Paper 4945, October 1966.

16. Redwood, R. G., "Experimental Stresses in Wide Flange Beams with Holes," Journal of the Structural Division, ASCE, Vol. 93, No. ST1, Discussion, February, 1967.

17. Redwood, R. G. and McCutcheon, J. O., "Experimental Tests of Wide Flange Beams with Large Unreinforced Web Openings," McGill University, Structural Mechanics Series No. 1, April, 1967.

18. Redwood, R. G. and McCutcheon, J. O., "Beam Tests with Unreinforced Web Openings," Journal of the Structural Division, ASCE, Vol. 94, No. ST1, Proc., Paper 5706, January 1968.

19. Bower, J. E., "Design of Beams with Web Openings," Journal of the Structural Division, ASCE, Vol. 94, No. ST3, Proc., Paper 5869, March 1968.

20. Redwood, R. G., "Plastic Behavior and Design of Beams with Web Openings," Proceedings of the Canadian Structural Engineering Conference, Canadian Institute of Steel Construction, Toronto, February, 1968.

21. Bower, J. E., "Ultimate Strength of Beams with Rectangular Holes," Journal of the Structural Division, ASCE, Vol. 94, No. ST6, Proc., Paper 5982, June 1968.

22. Cheng, Kho Shu, "Experimental Study of Beam with Web Opening," M.S. Thesis, Kansas State University, 1969.

23. Redwood, R. G., "Ultimate Strength Design of Beams with Multiple Openings," ASCE, Annual Meeting and National Meeting on Structural Engineering, Pittsburg, Pennsylvania, September 30 - October 4, 1968.

24. Delesque, R., "Stability of the Web Posts of Castellated Beams," Translated from Construction Metallique, No. 3, September, 1968.

25. Redwood, R. G., "The Strength of Steel Beams with Unreinforced Web Holes," Civil Engineering and Public Works Review, Vol. 64, No. 755, London, June, 1969.

26. Congdon, J. G., "Ultimate Strength of Beams with Reinforced Rectangular Openings," A Master's Thesis, McGill University, Montreal, Quebec, Canada, July, 1969.

27. Congdon, J. G. and Redwood, R. G., "Plastic Behavior of Beams with Reinforced Holes," Journal of the Structural Division, ASCE, Vol. 96, No. ST9, September, 1970.

28. Richard, M. W., "Ultimate Strength Analysis of Beams with Eccentric Rectangular Web Openings," A Master's Thesis, Department of Civil Engineering, Kansas State University, 1971.

29. Cooper, P. B. and Snell, R. R., "Test on Beams with Reinforced Web Openings," Journal of the Structural Division, ASCE, Vol. 98, No. ST3, March 1972.

30. Frost, R. W., "Behavior of Steel Beams with Eccentric Web Holes," Technical Report 46.019-400(1), Research Laboratory, United States Steel Corporation, February, 1973.

31. Turner, M. J., Clough, R. M., Martin, H. C. and Topp, L. J., "Stiffness and Deflection Analysis of Complex Structures," Journal of Aeronautical Science, 23, No. 9, 1956.

32. Clough, R. W., "The Finite Element in Plane Stress Analysis," Proceedings of the 2nd ASCE, Conference on Electronic Computation, Pittsburgh, Pa., September, 1960.

33. Zienkiewiez, O. C. and Cheng, Y. K., "The Finite Element Method in Structural and Continum Mechanics," McGraw-Hill Publishing Company Limited, Berkshire, England, 1967.

34. "Manual of Steel Construction," 7th Edition, American Institute of Steel Construction, New York, 1970.

35. Connar, J. and Wills, G., "Computer Aided Teaching of the Finite Element Displacement Method," Research Report 69-23, Department of Civil Engineering, School of Engineering, Massachusetts Institute of Technology.

36. Wang, T. M., Snell, R. R. and Cooper, P. B., "Strength of Beams with Eccentric Reinforced Holes," Journal of the Structural Division, ASCE, Vol. 101, No. ST9, September, 1975.

APPENDIX III - NOTATIONS

A_{cp} cross sectional area of cover plate

A_{eff} effective flange area of idealized beam

A_{mod} modified area of flange element with cover plates

A_{Reff} effective reinforcement area of idealized beam

a half the opening length

b_f width of flange

d overall depth of beam

d' thickness of cover plate

F nodal force vector

F^p vector of plastic forces corresponding to plastic strains

h half the opening depth

I moment of inertia of actual beam

I_{cp} moment of inertia of cover plates about the axis of beam

I_{eq} moment of inertia of equivalent idealized beam

$I_{eq.mod.}$ moment of inertia of equivalent idealized beam at section with cover plates

I_{mod} modified moment of inertia of beam at section with cover plates

i size of load increment

K stiffness matrix

M_1, M_2 primary bending moments

M_B^S secondary bending moment in the bottom section at the opening

M_T^S secondary bending moment in the top section at the opening

Pu^{exp} experimental ultimate load corrected for strain hardening

Pu^{FE} ultimate load obtained from finite element analysis

Pu^{Theo} theoretical ultimate load

Pu^{true} true ultimate load

Pu^{solid} ultimate load of solid beam

r radius of opening corners

t_f thickness of flange

t_w thickness of web

V shear force at any section

V_B vertical shear force in the bottom section at the opening

V_T vertical shear force in the top section at the opening

X distance between center of opening and centerline of beam

$\bar{X}$ element displacement vector

γ^p_{xy} plastic angular strain

ϵ total element strain vector

$\bar{\epsilon}^p$ effective plastic strain

ϵ_t total strain

ϵ^p_x plastic strain in x-direction

ϵ^p_y plastic strain in y-direction

σ element stress vector

$\bar{\sigma}$ effective stress

σ_x normal stress in x-direction

σ_y normal stress in y-direction

τ_{xy} shear stress

Beam	1	2	5	6	7
X	24"	24"	25"	26"	26"
a	4.5"	4.5"	6"	8"	6"
h	3"	3"	3"	4"	4"
r	3/4"	3/4"	17/32"	1/2"	1/2"
Reinforcing bars	None	One Side	Both Sides	One Side	One Side
Cover Plates	None	PL $\frac{5}{16}$ x 4 x 24	PL $\frac{5}{16}$ x 4 x 31	None	None

Table 1. Test Variables.

	Beam		Reinforcing bar	
Beam	Average F_y (ksi)	Maximum Deviation from Average F_y (%)	Average F_y (ksi)	Maximum Deviation from Average F_y (%)
1	43.17	3.96	----	----
2	42.79	2.17	39.42	2.38
5	40.80	2.82	38.28	0.29
6	40.80	2.82	30.96	0.16
7	40.80	2.82	30.96	0.16

Table 2. Static Yield Stresses.

Beam	1	2	5	6	7
Beam Type	W16 x 45	W16 x 45	W16 x 40	W16 x 40	W16 x 40
Number of Nodes	302	302	296	341	341
Number of Elements	574	608	596	684	684
Web Thickness (in.)	0.346	0.346	0.307	0.307	0.307
Flange Area (in^2)	3.64	3.64	3.22	3.22	3.22
Modified Flange Area for Cover Plates (in^2)	----	4.96	4.52	----	----
Area of Reinforcement (in^2)	----	0.5	1.0	0.5	0.5
Area of each stiffener (in^2)	3.0	3.0	3.0	3.0	3.0

Table 3. Idealized Properties of Beams.

Beam	1	2	5	6 & 7
Beam Type	W16 x 45	W16 x 45	W16 x 40	W16 x 40
Number of Nodes	51	51	51	51
Number of Elements	98	102	102	102
Web Thickness (in.)	0.346	0.346	0.307	0.307
Flange Area (in^2)	3.64	3.64	3.22	3.22
Modified Flange Area for Cover Plates (in^2)	----	4.96	4.52	----
Area of each stiffener (in^2)	3.0	3.0	3.0	3.0

Table 4. Idealized Properties of Solid Beams.

Beam	Increment Size (kips)	Increment Number at Start of Solution	Increment Number at First Yield	Increment Number at which Solution Stopped
Beams with Opening				
1	8	6	6	18
2	8	6	6	19
5	8	6	6	16
6	6	3	3	14
7	6	3	4	16
Solid Beams				
1	8	6	13	18
2	8	6	13	21
5	8	12	12	18
6&7	8	12	12	14

Table 5. Loading Details for Beams with Opening and Solid Beams.

Beam	1	2	5	6	7
FE Load Increment (kips)	8	8	8	6	6
Pu^{FE} (kips)	144	152	128	84	96
*Pu^{exp} (kips)	136	155	124	84	101
Pu^{FE}/Pu^{exp}	1.06	0.98	1.03	1.00	0.95
*Pu^{Theo} (kips)	129	152	129	83	96
Pu^{FE}/Pu^{Theo}	1.12	1.00	0.99	1.01	1.00
Pu^{Solid} (kips)	144	168	144	112	112
Pu^{FE}/Pu^{Solid}	1.00	0.90	0.89	0.75	0.86

*These values are obtained from reference 2.

Table 6. Ultimate Loads.
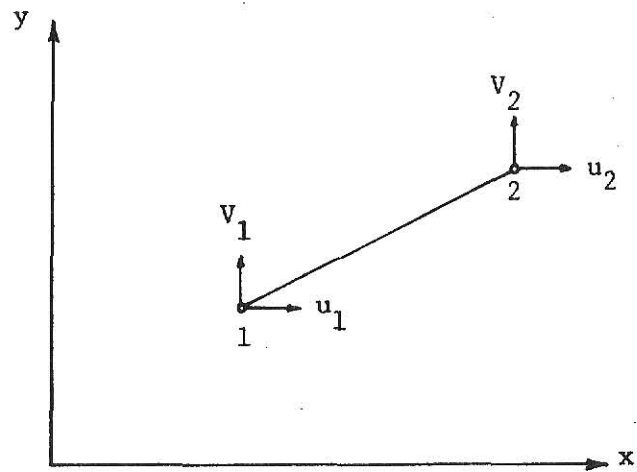
Bar Element
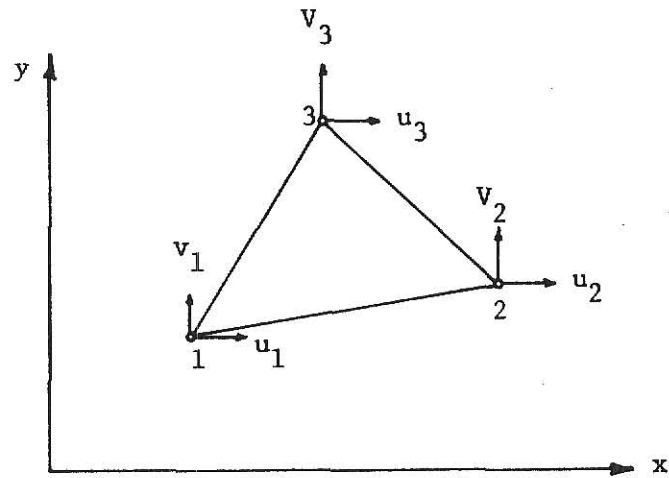
Triangular Plate Element

Figure 1. Bar and Plate Element.
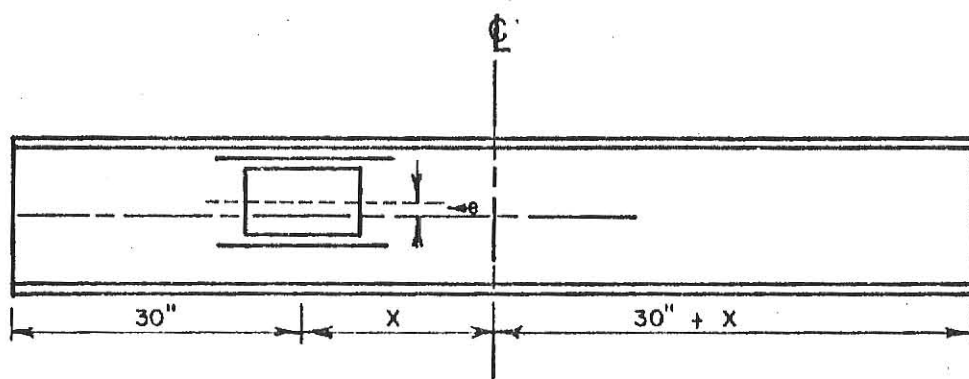
(a) Test Setup
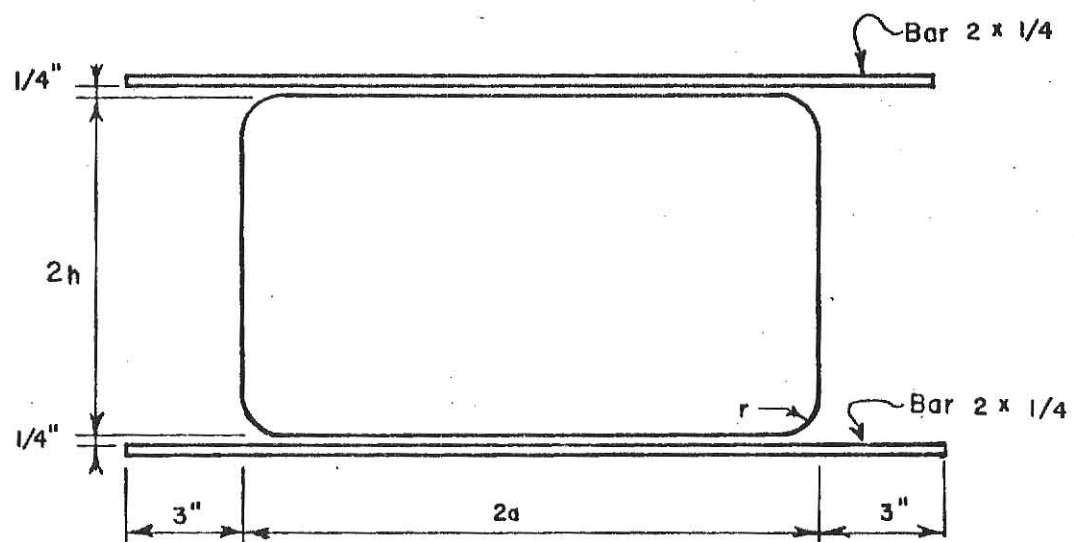
(b) Opening and Reinforcement Details

Figure 2. Test Setup and Reinforcement Details.
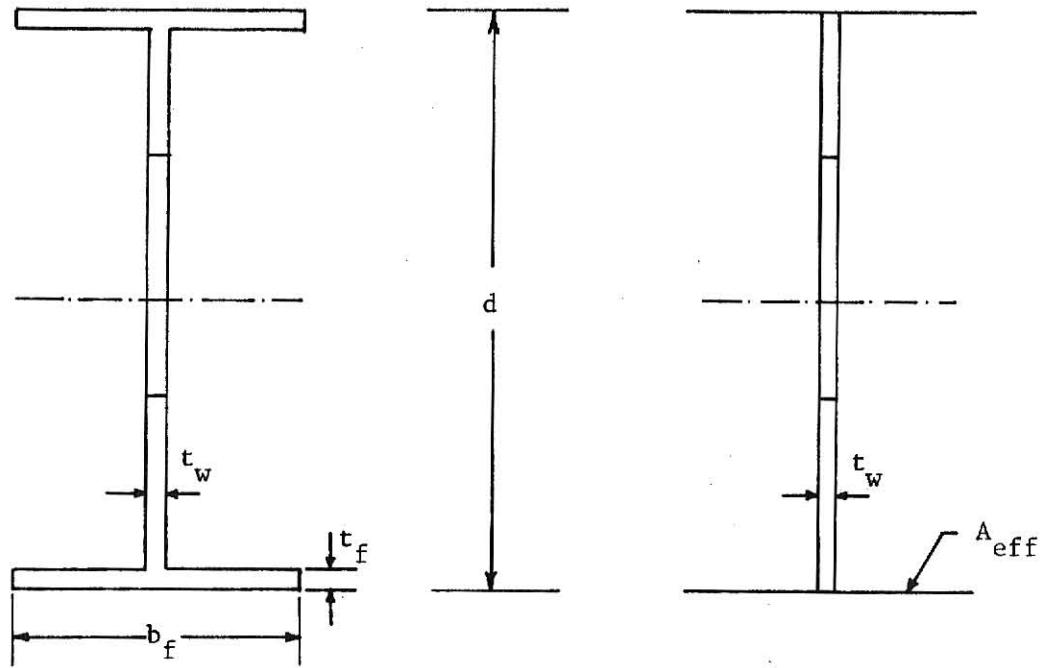
(a)   Unreinforced
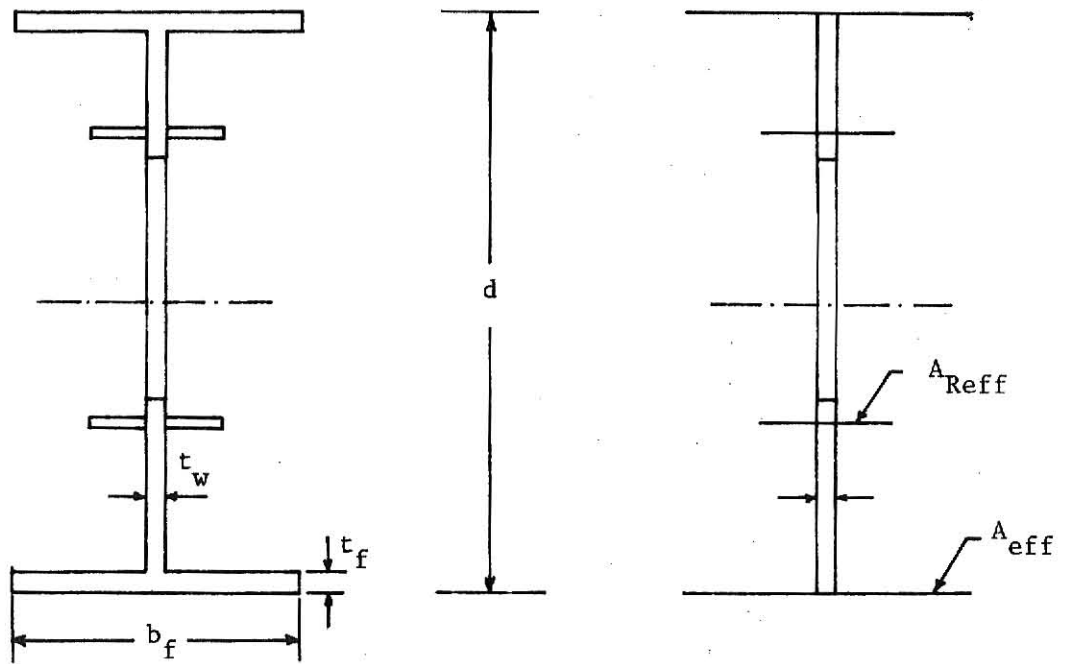
(b)   Reinforced

Figure 3.   Actual and Idealized Beam Sections.
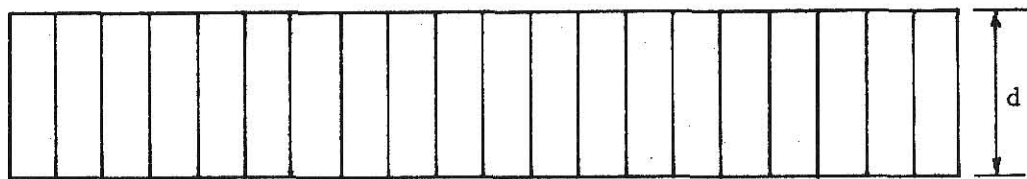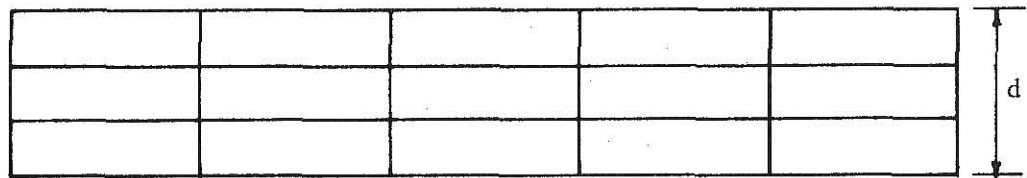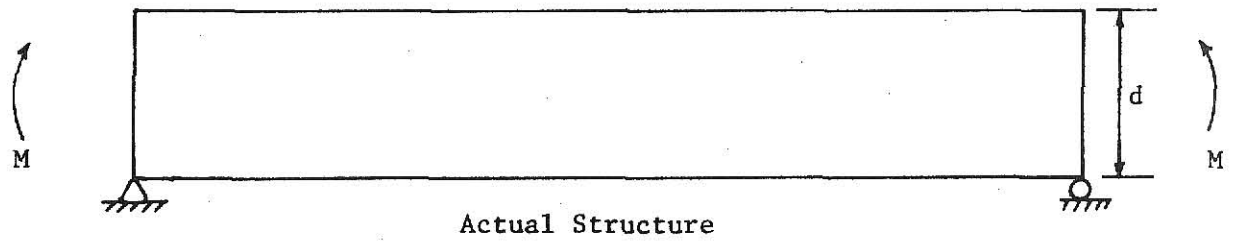
Figure 4. Discretization of a Deep Beam.

KEY TO FIGURES 5 THROUGH 26

Horizontal lines indicate compressive yielding

Verticles lines indicate tensile yielding

Solid shading indicates a combination of compressive and tensile yielding

In figures showing a full view of a beam, the portion near the opening
has been omitted and is shown enlarged in the following figure.
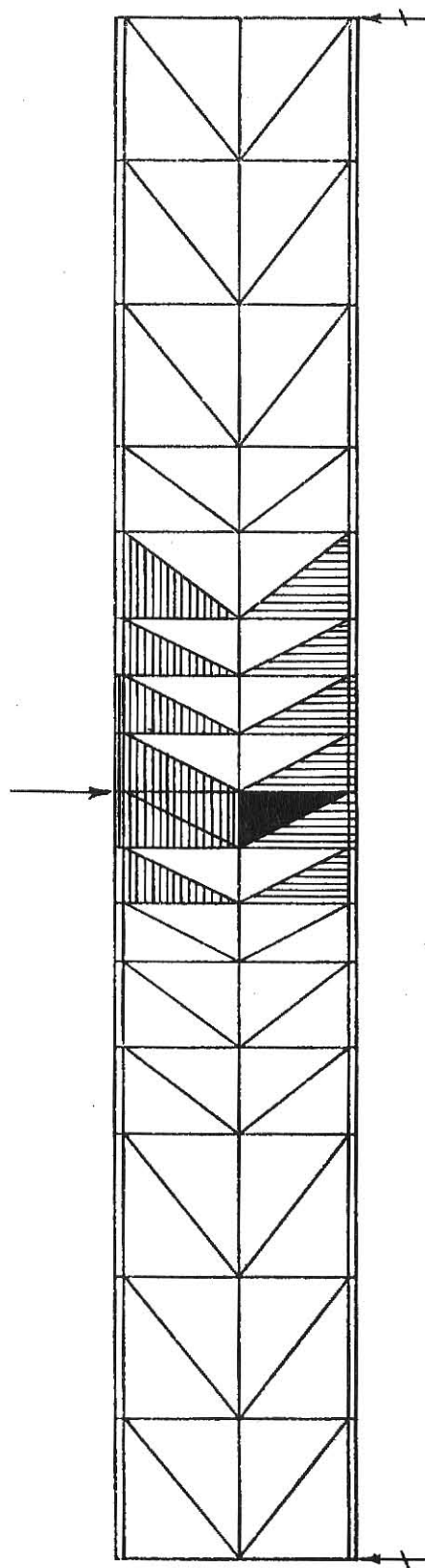
Figure 5. Yield Pattern for Solid Beam 1 at Ultimate Load (144^k).
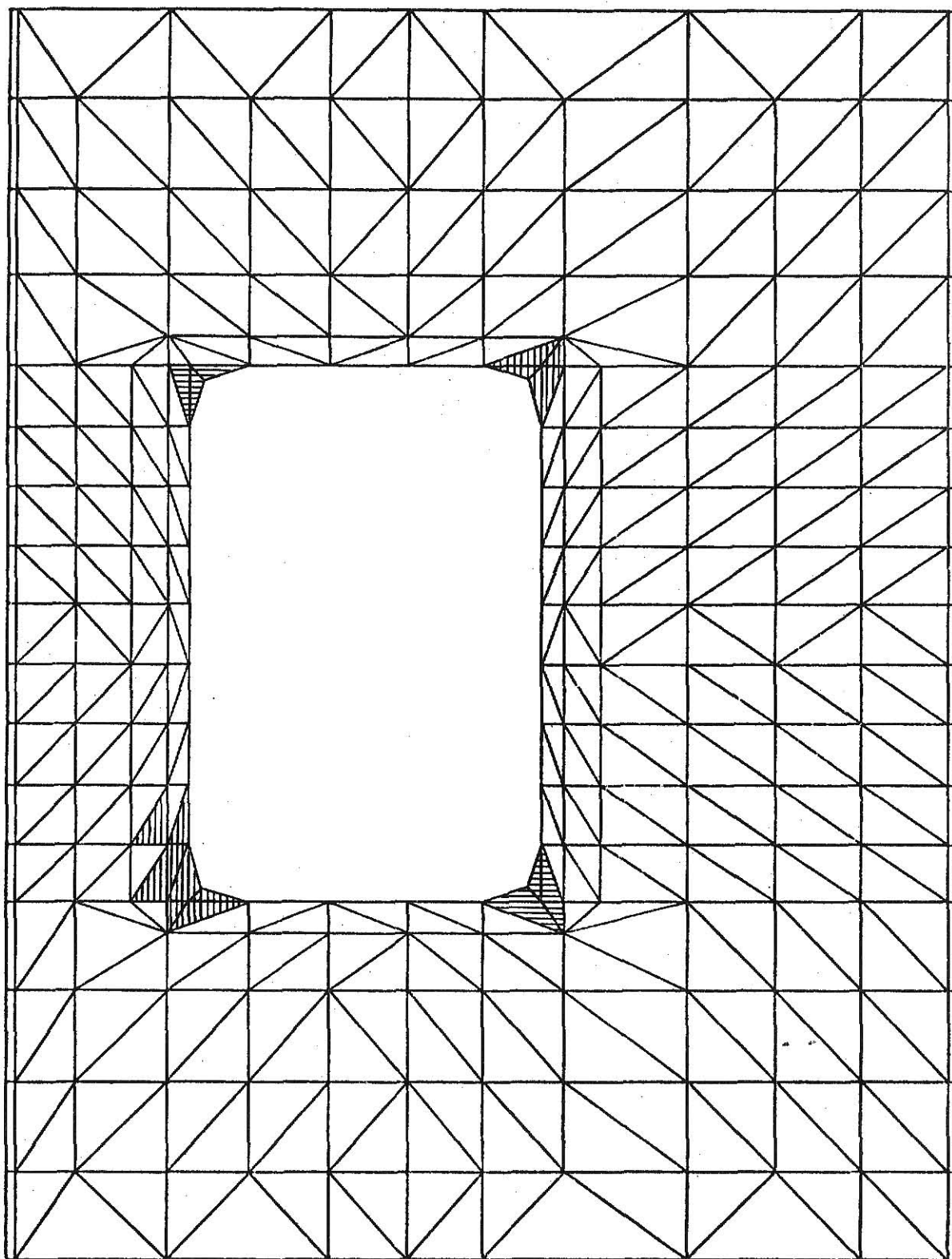
Figure 6. Yield Pattern in Vicinity of Opening for Beam 1 at 80^k.
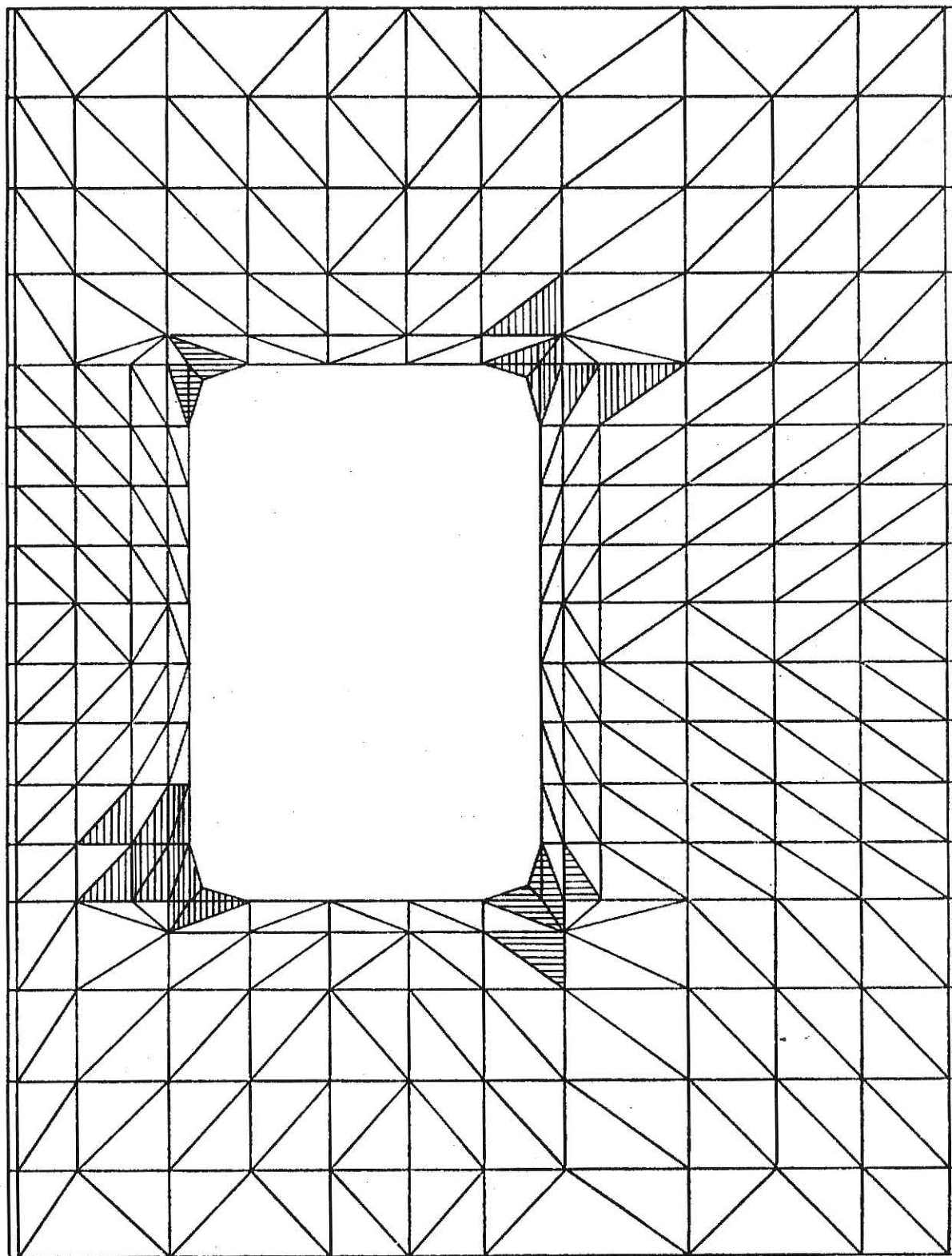
Figure 7. Yield Pattern in Vicinity of Opening for Beam 1 at 96^k.
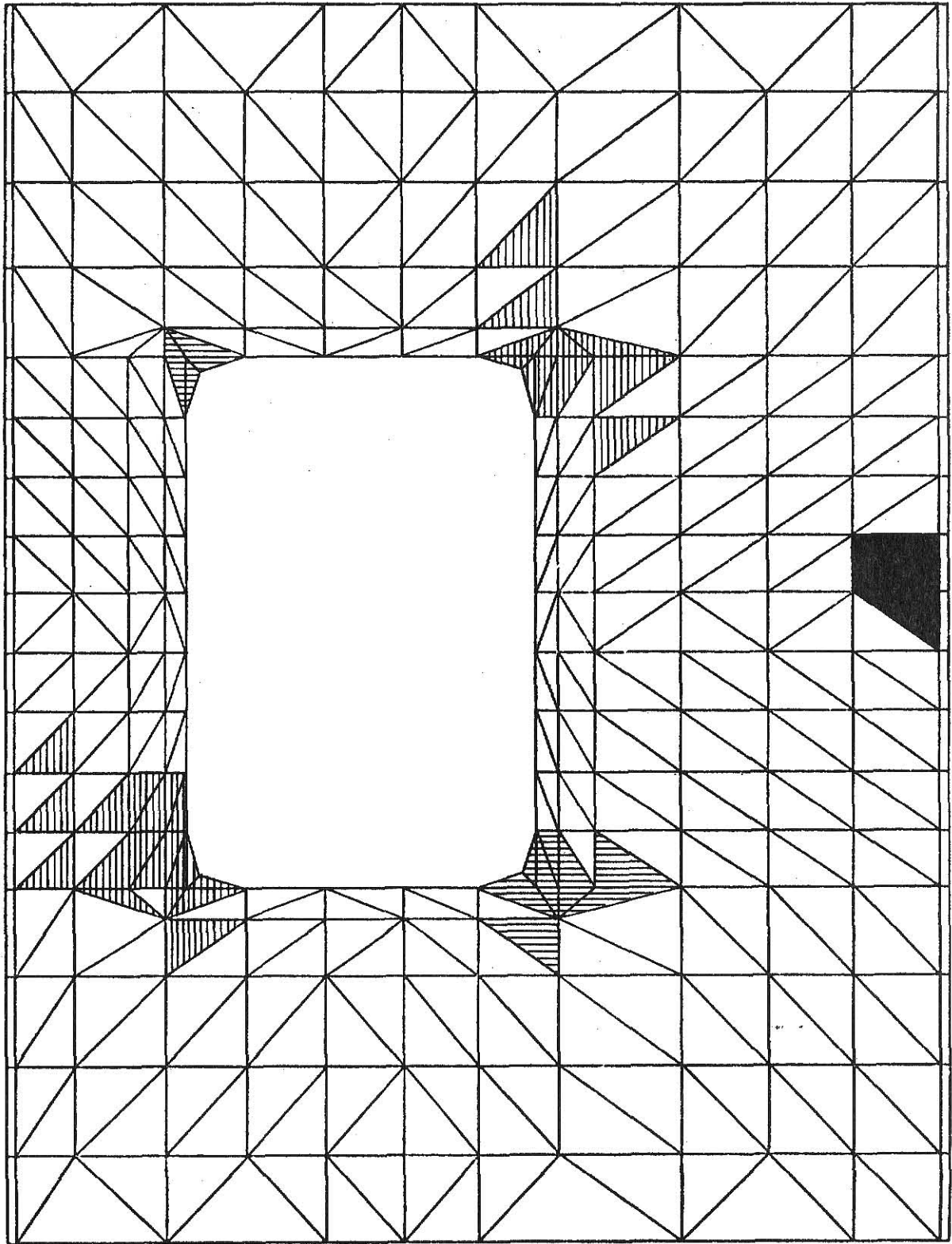
Figure 8. Yield Pattern in Vicinity of Opening for Beam 1 at 112^k.
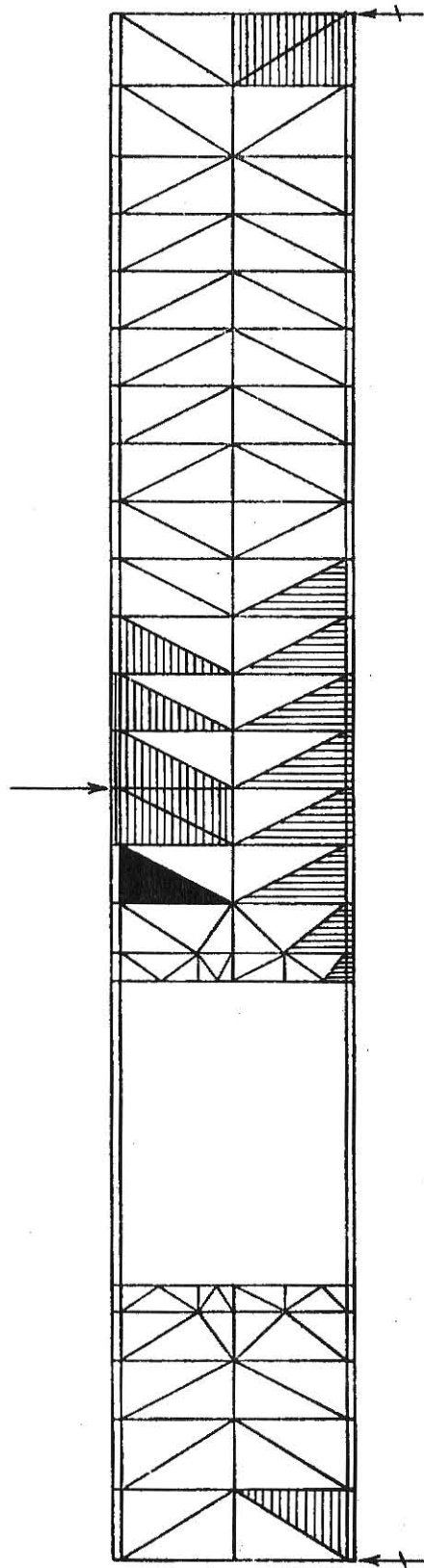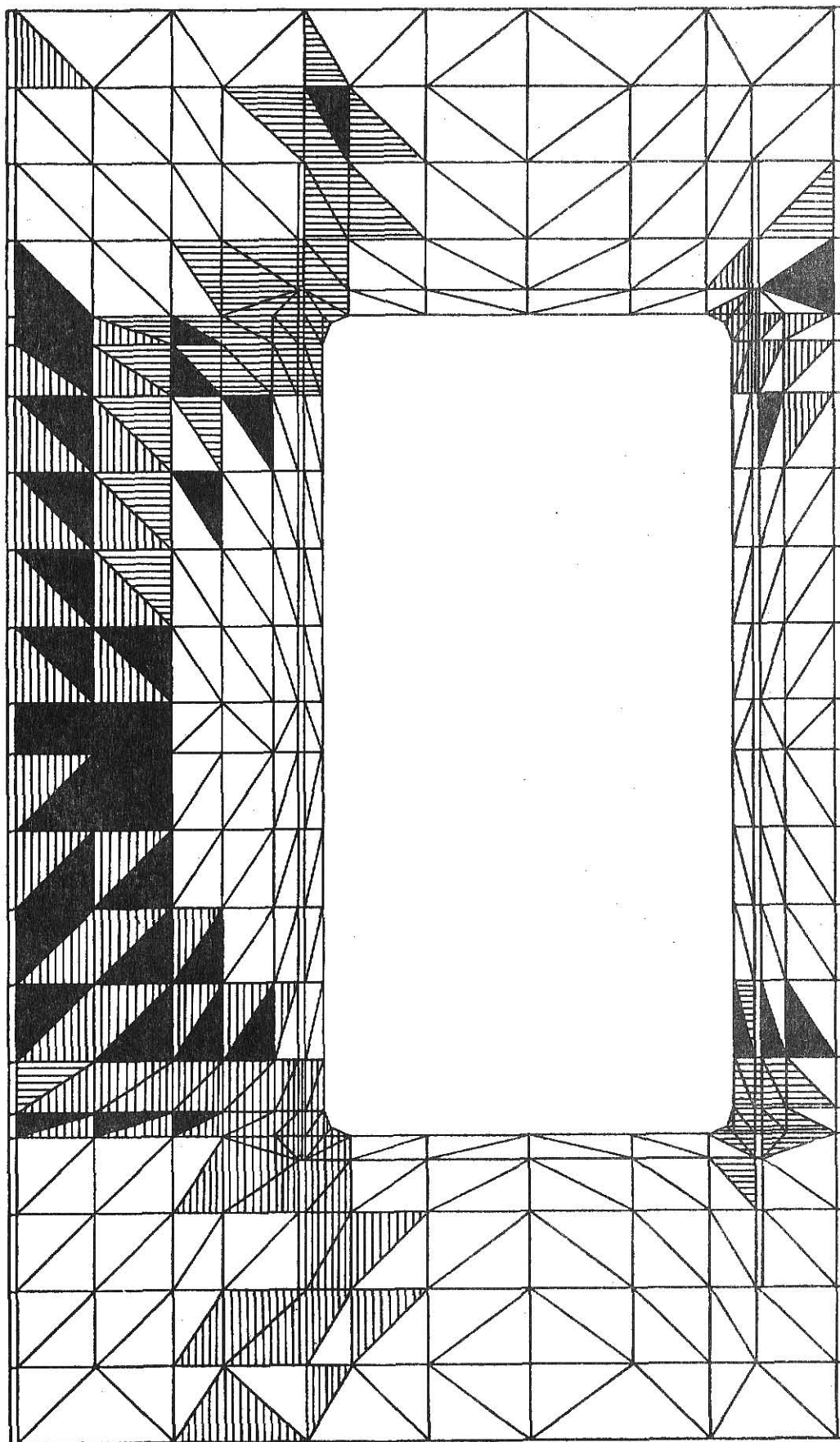
Figure 9. Yield Pattern for Beam 1 at Ultimate Load (144^k).
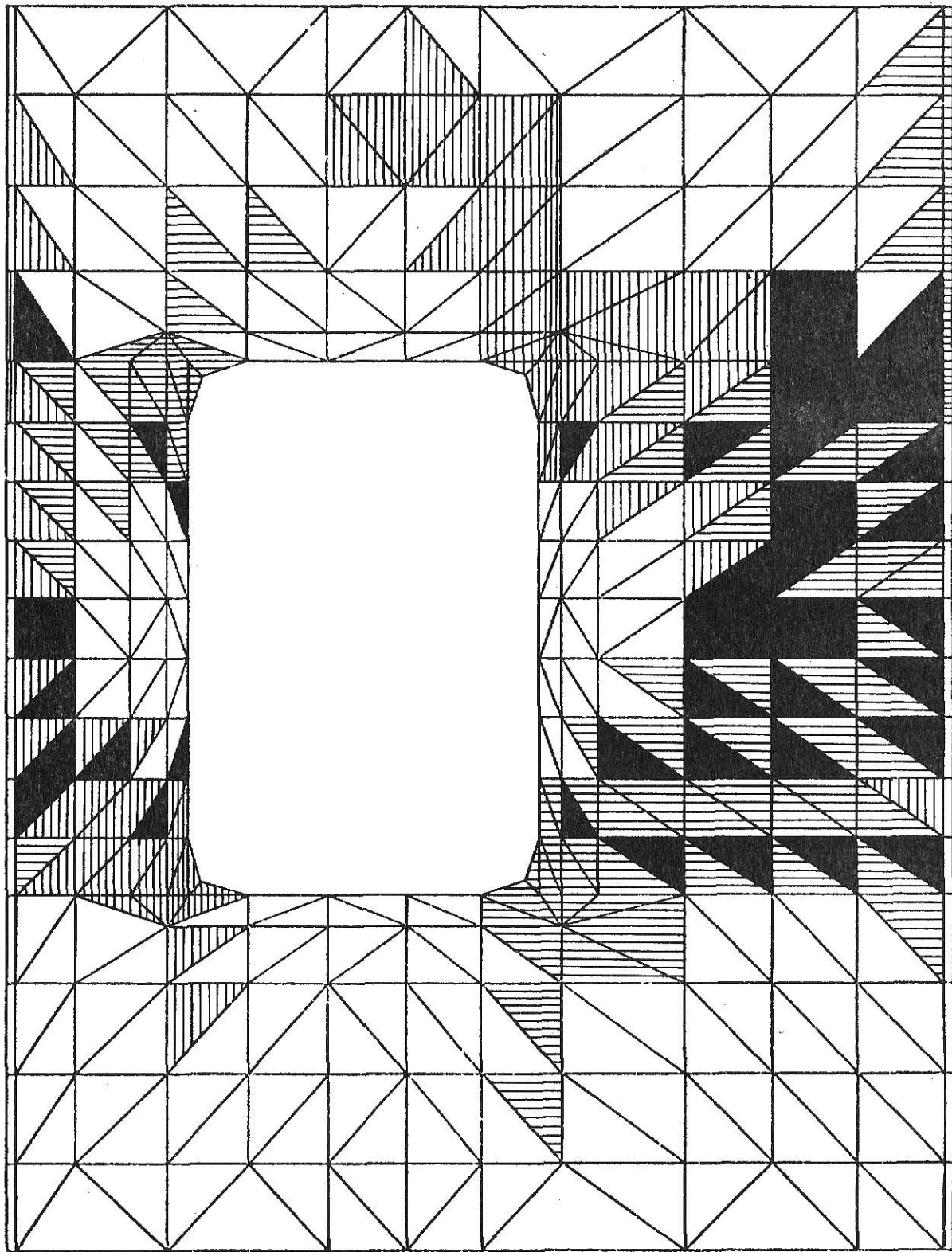
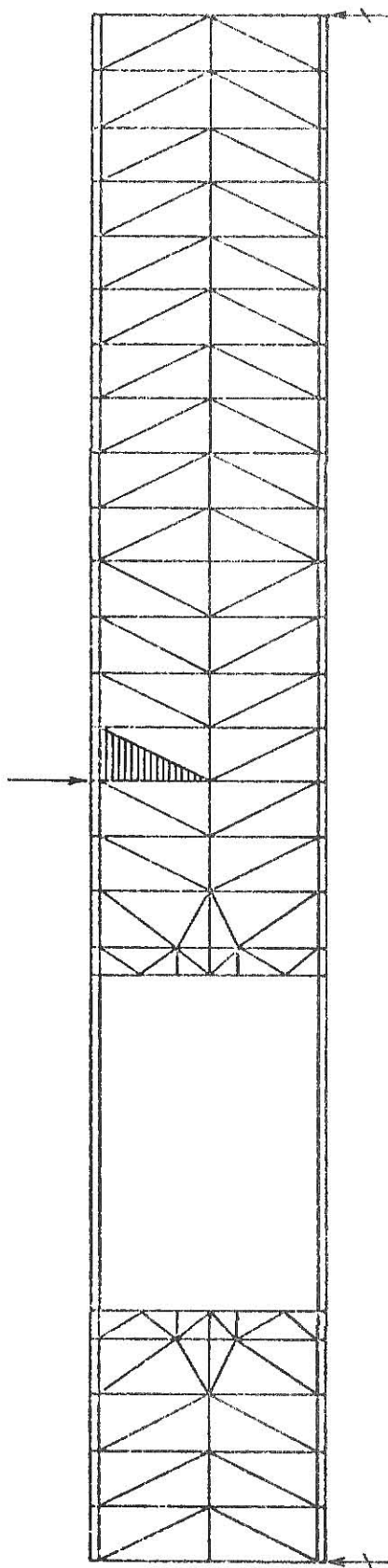Figure 10. Yield Pattern in Vicinity of Opening for Beam 1 at Ultimate Load (144^k).
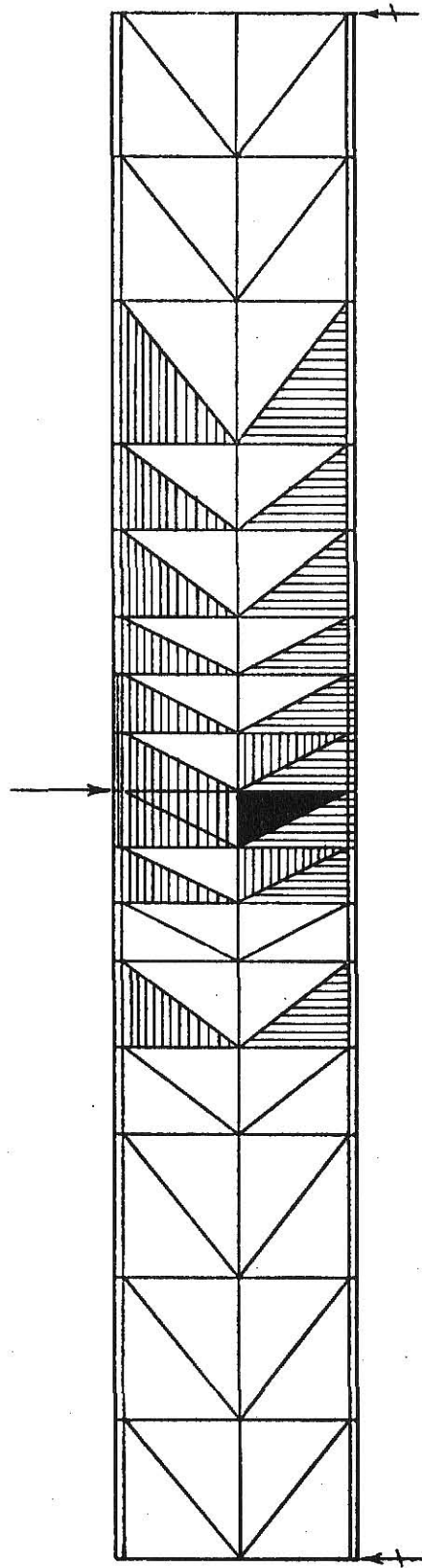
Figure 11. Yield Pattern for Solid Beam 2 at Ultimate Load (168^k).
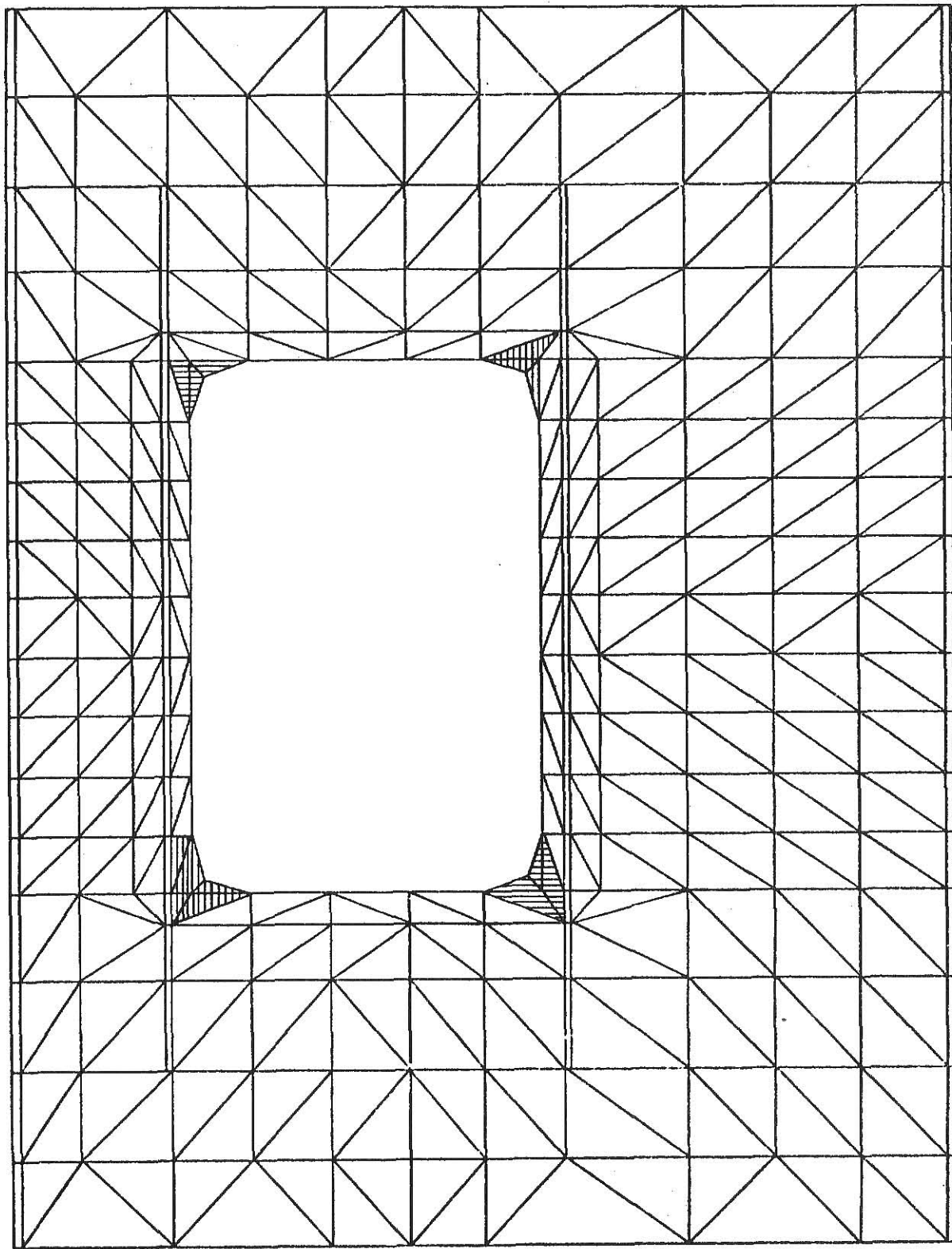
Figure 12. Yield Pattern in Vicinity of Opening for Beam 2 at 96^k.
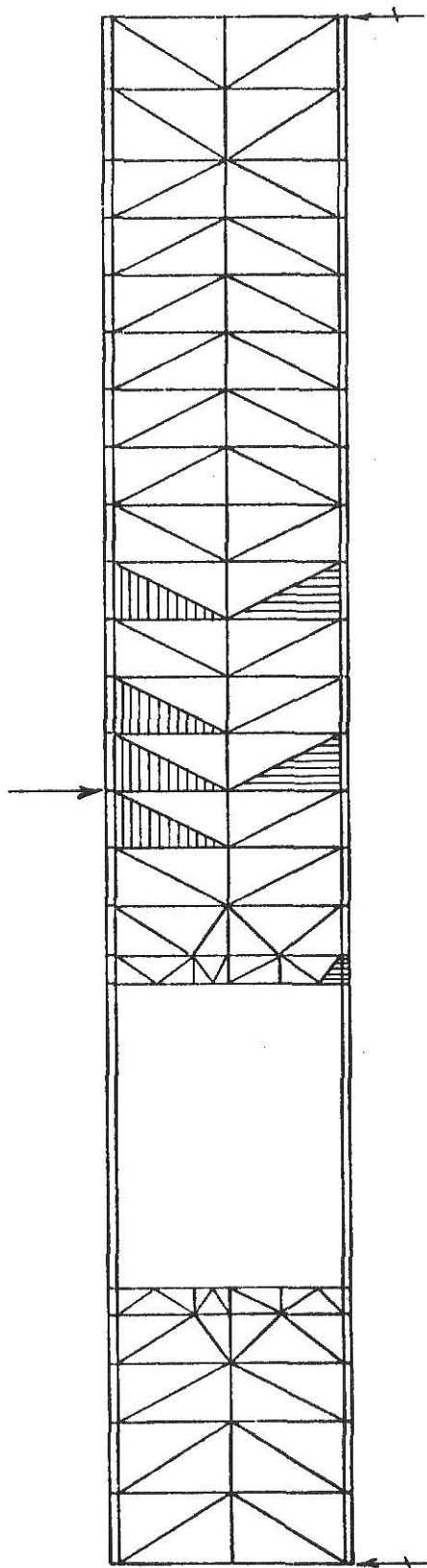
Figure 13. Yield Pattern for Beam 2 at 144^k.
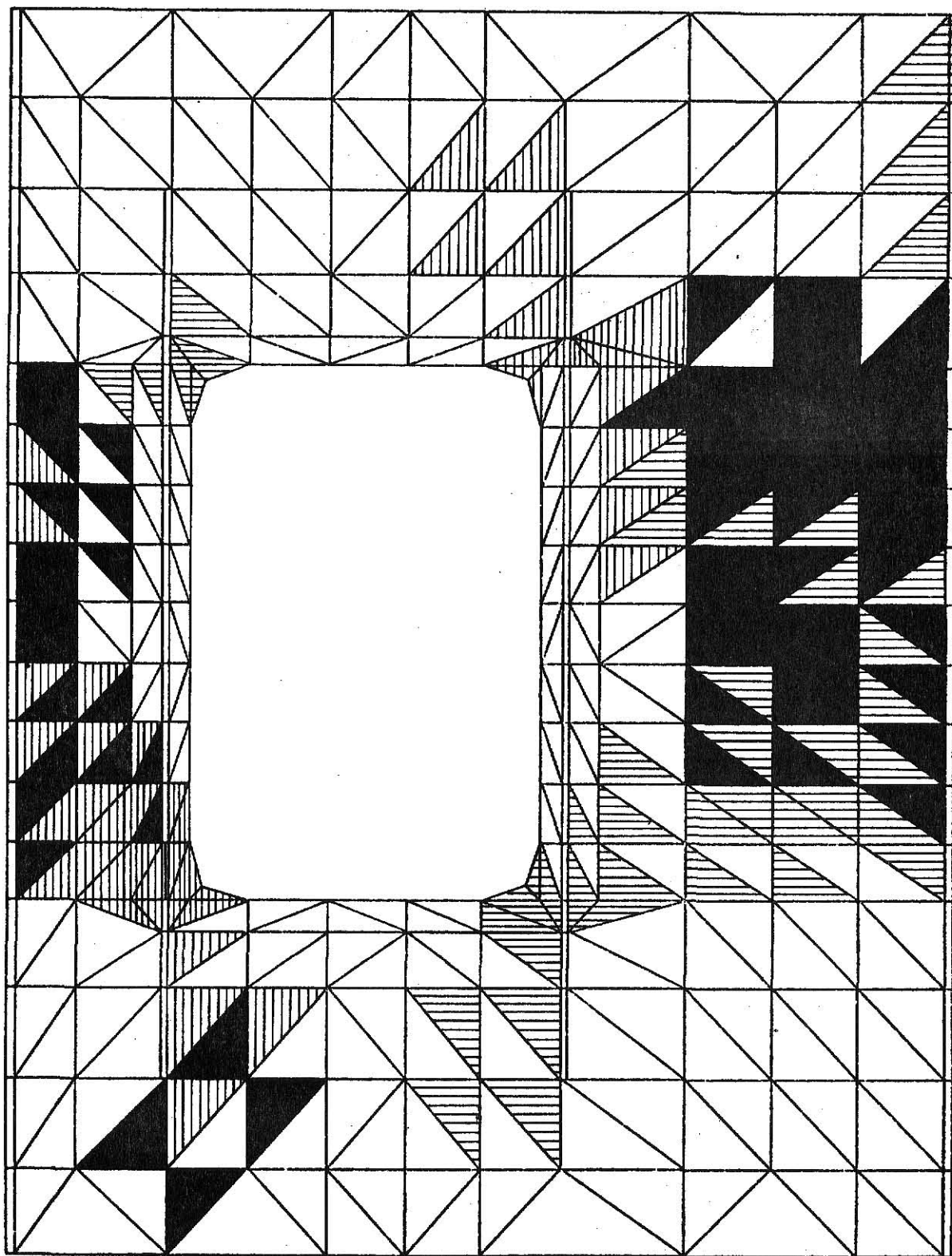
Figure 14. Yield Pattern in Vicinity of Opening for Beam 2 at 144^k.
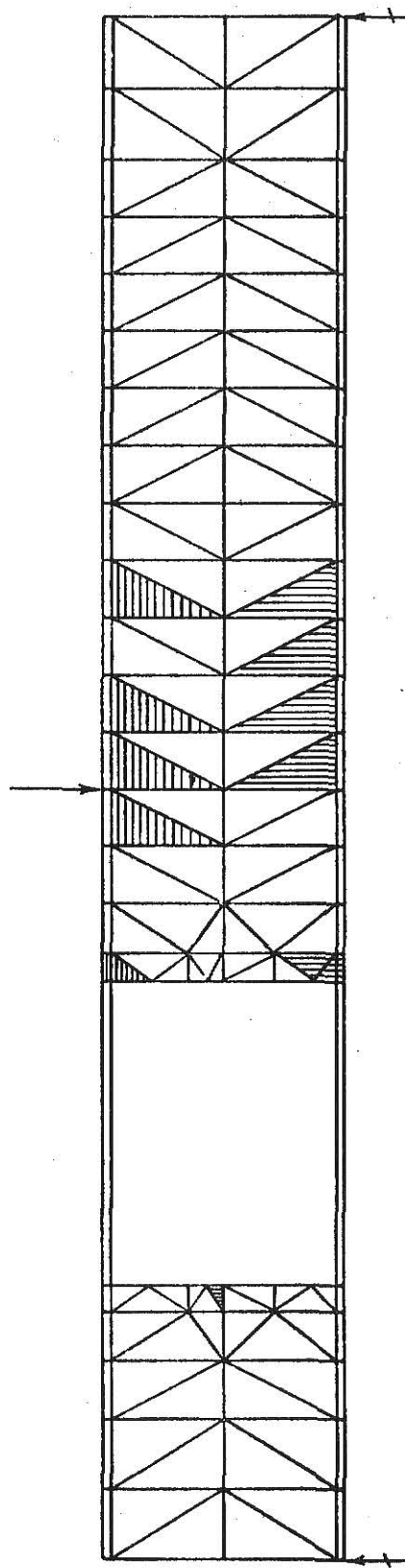
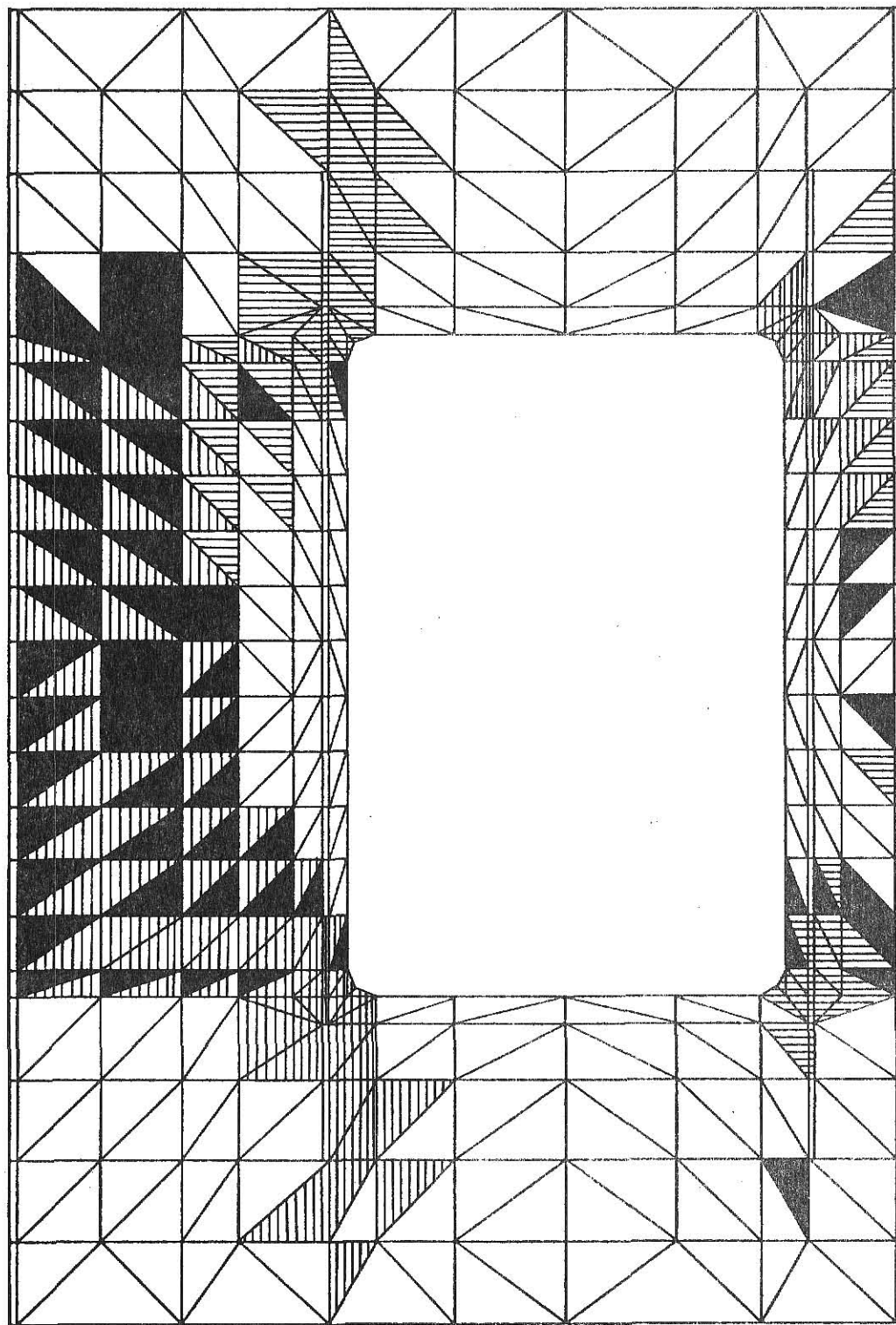Figure 15. Yield Pattern for Beam 2 at Ultimate Load (152^k).
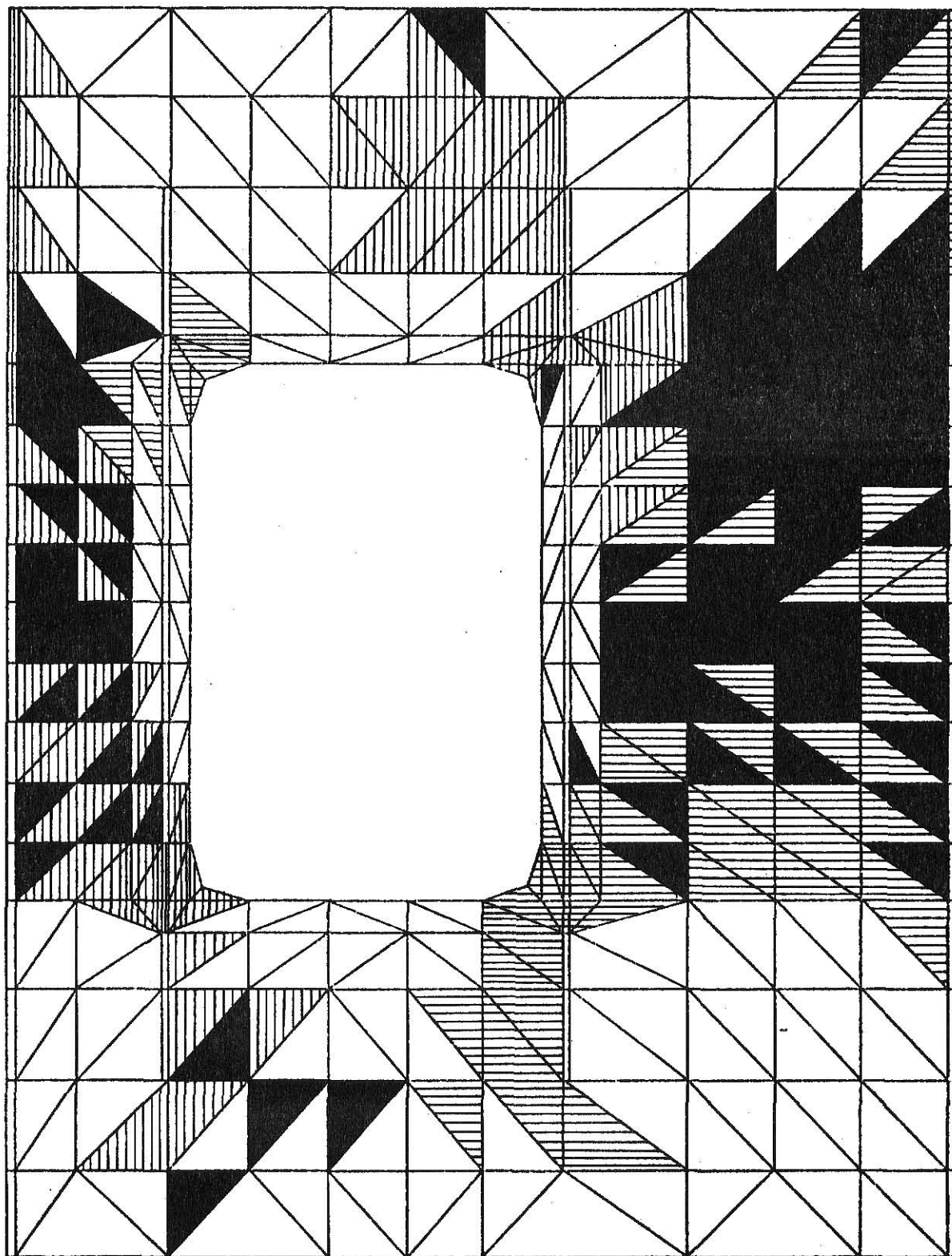
Figure 16. Yield Pattern in Vicinity of Opening for Beam 2 at Ultimate Load (152^k).
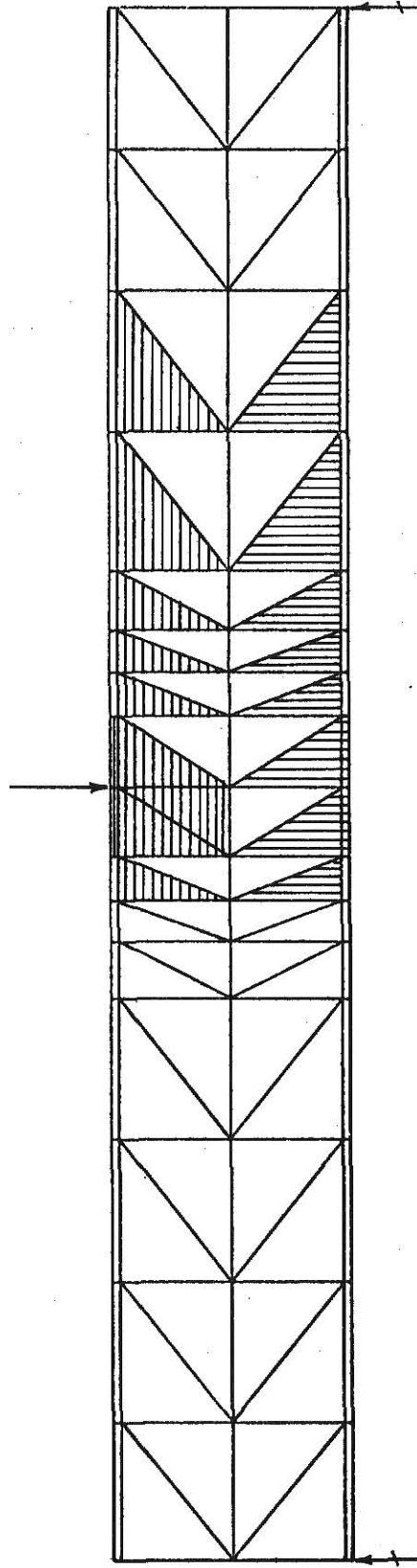
Figure 17. Yield Pattern for Solid Beam 5 at Ultimate Load (144^k).
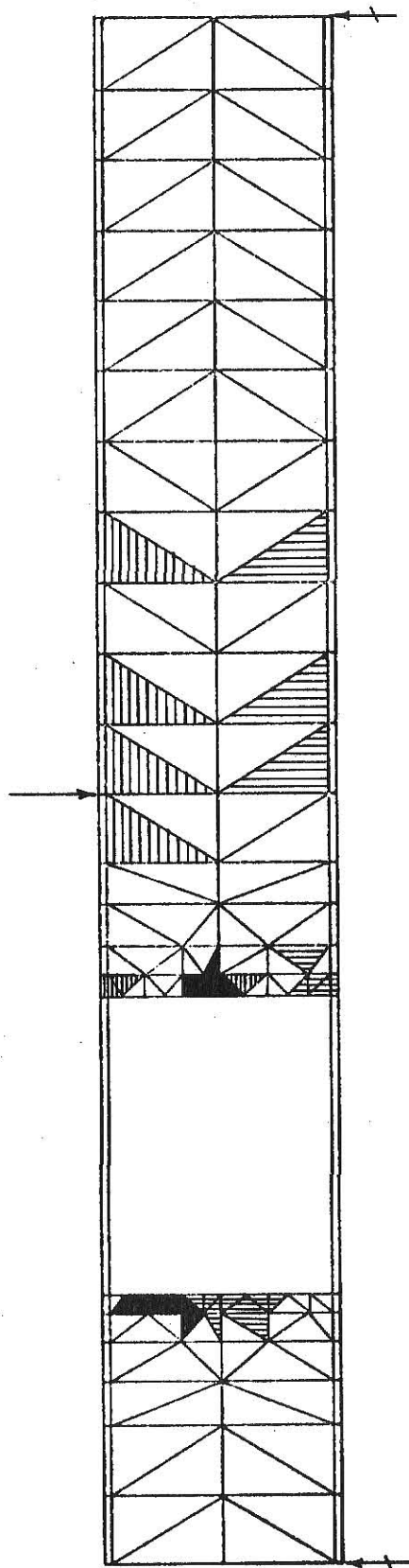
Figure 18. Yield Pattern for Beam 5 at Ultimate Load (128^k).
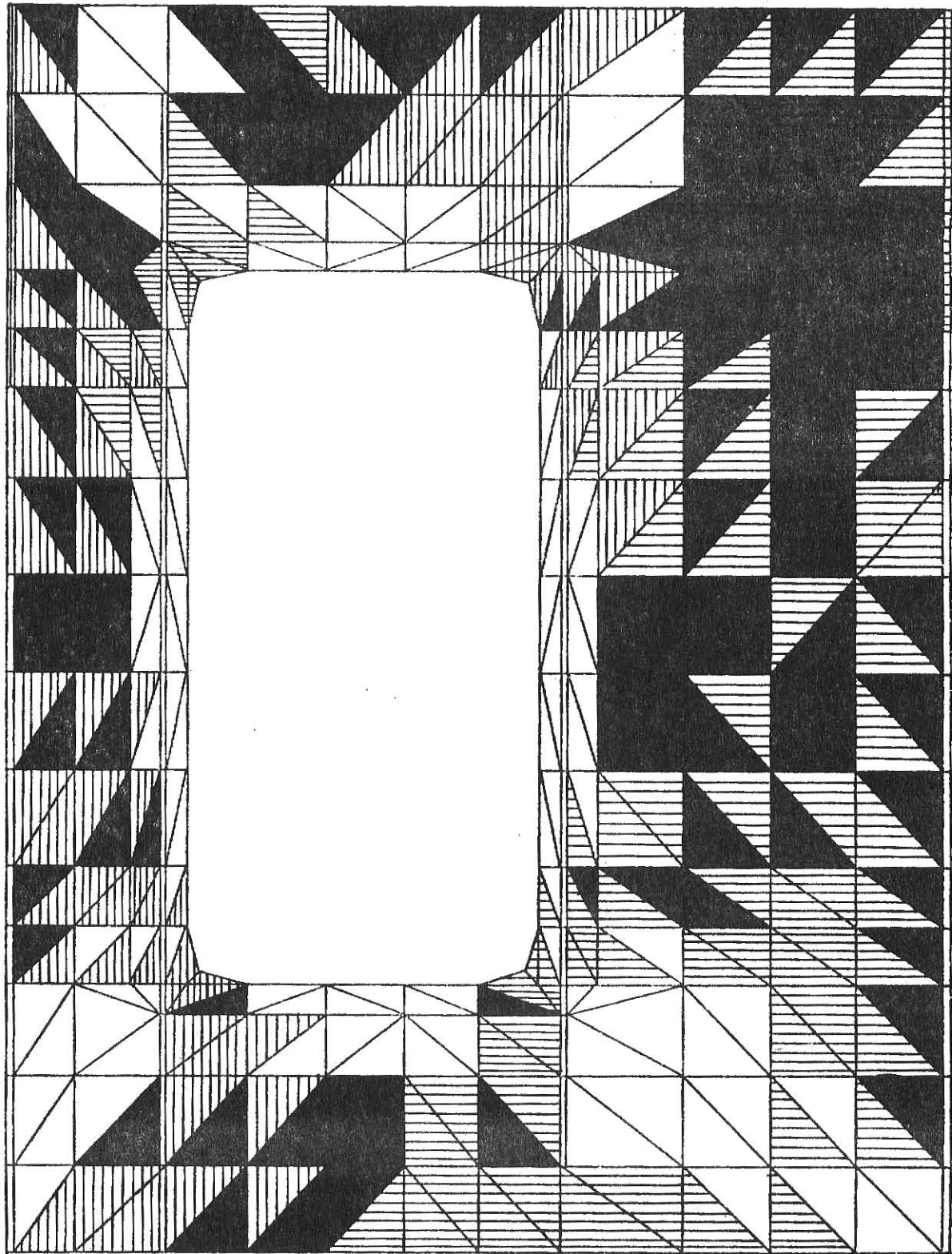
Figure 19. Yield Pattern in Vicinity of Opening for Beam 5 at Ultimate Load (128^k).
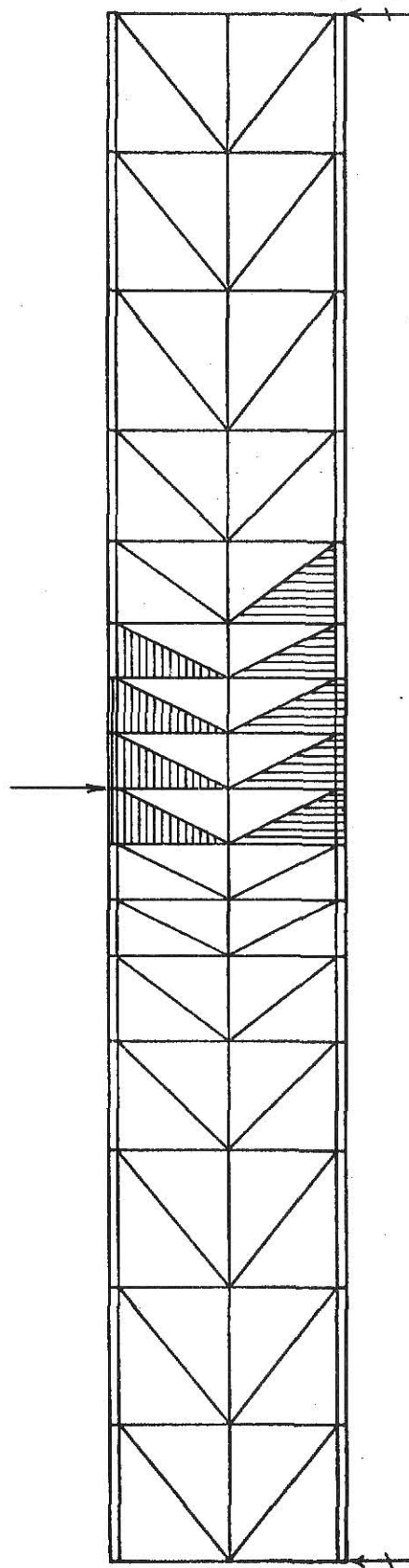
Figure 20. Yield Pattern for Solid Beam 6 and 7 at Ultimate Load (112^k).
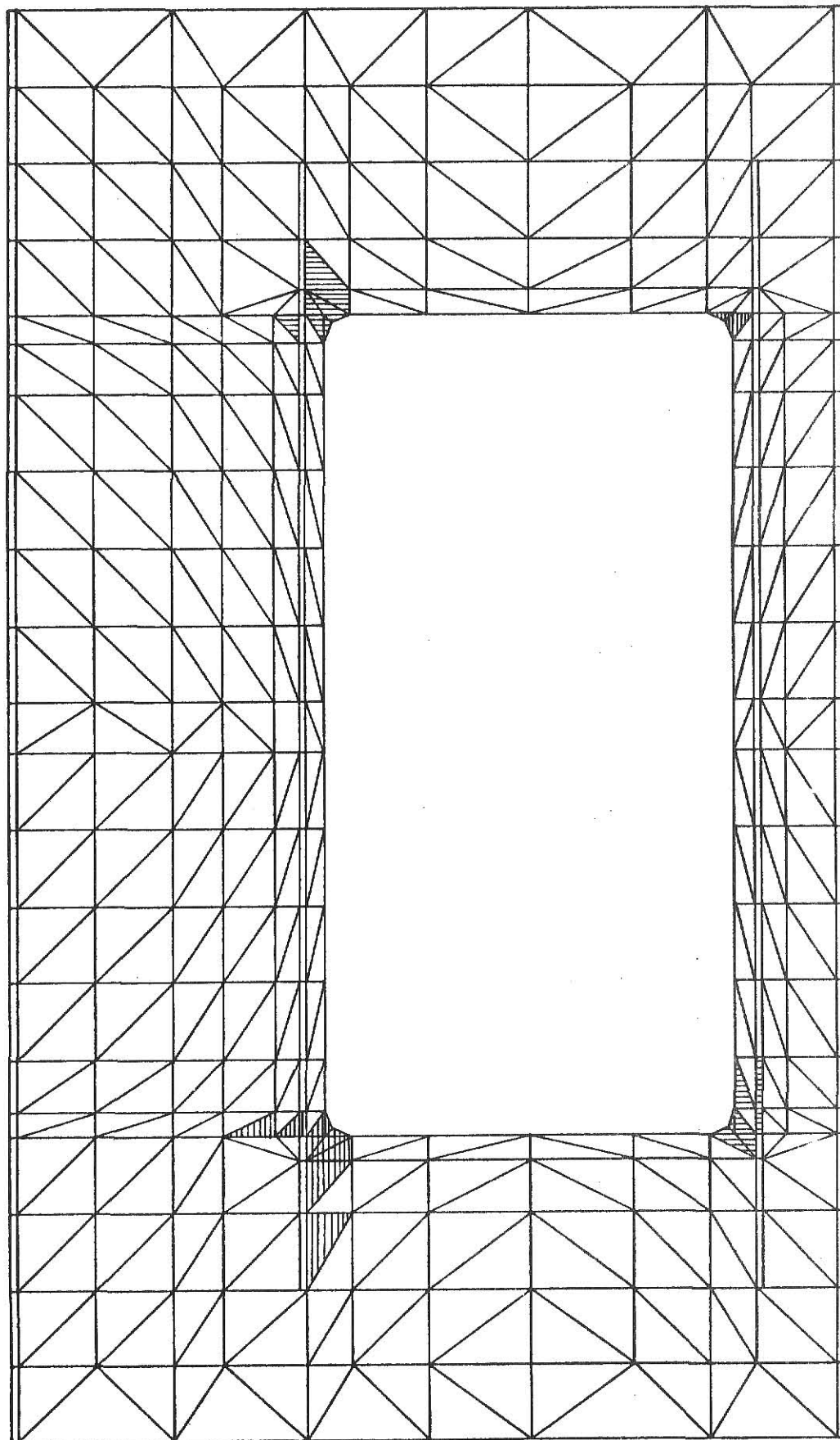
Figure 21. Yield Pattern in Vicinity of Opening for Beam 6 at 54^k.
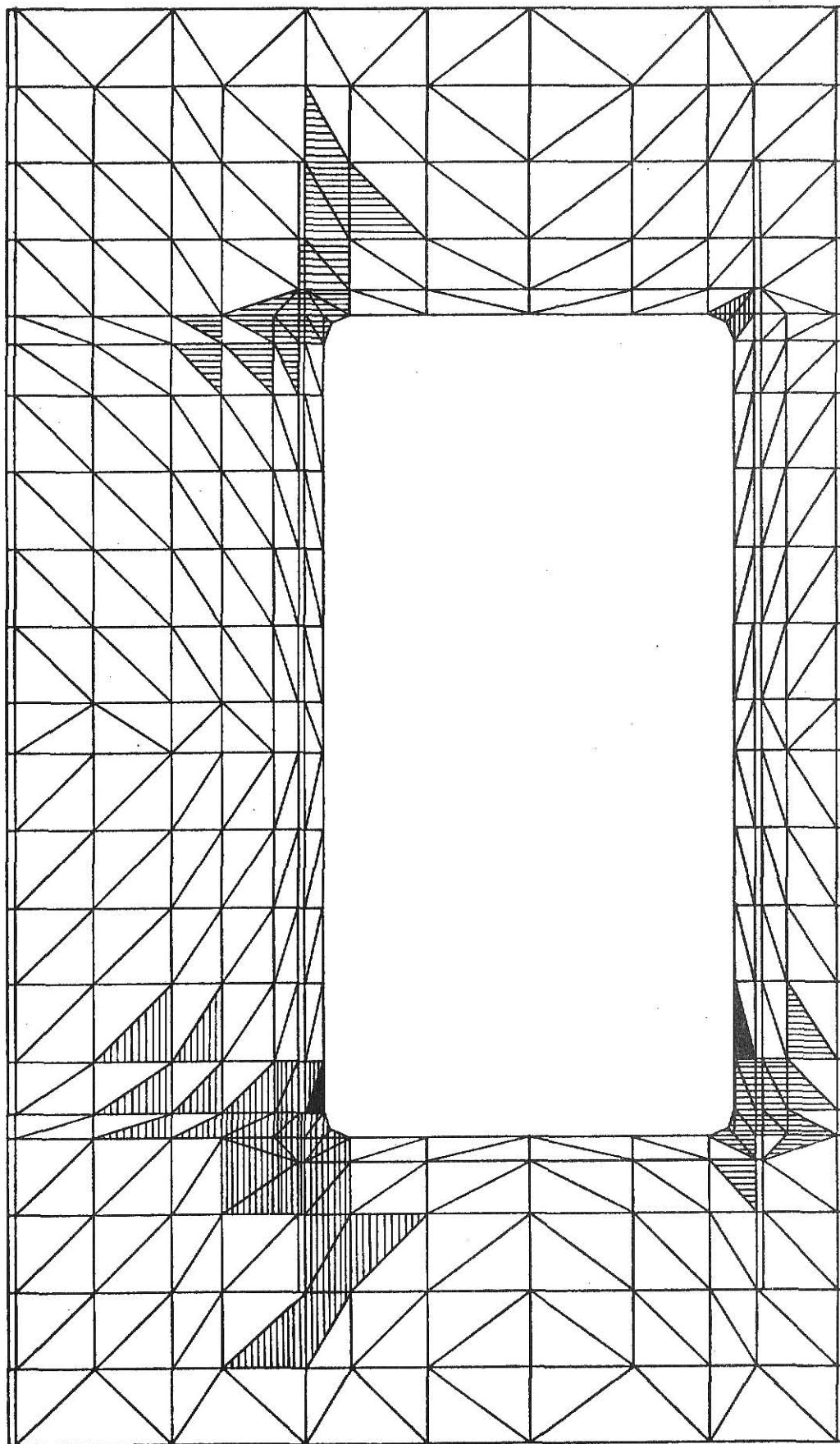
Figure 22. Yield Pattern in Vicinity of Opening for Beam 6 at 72^k.
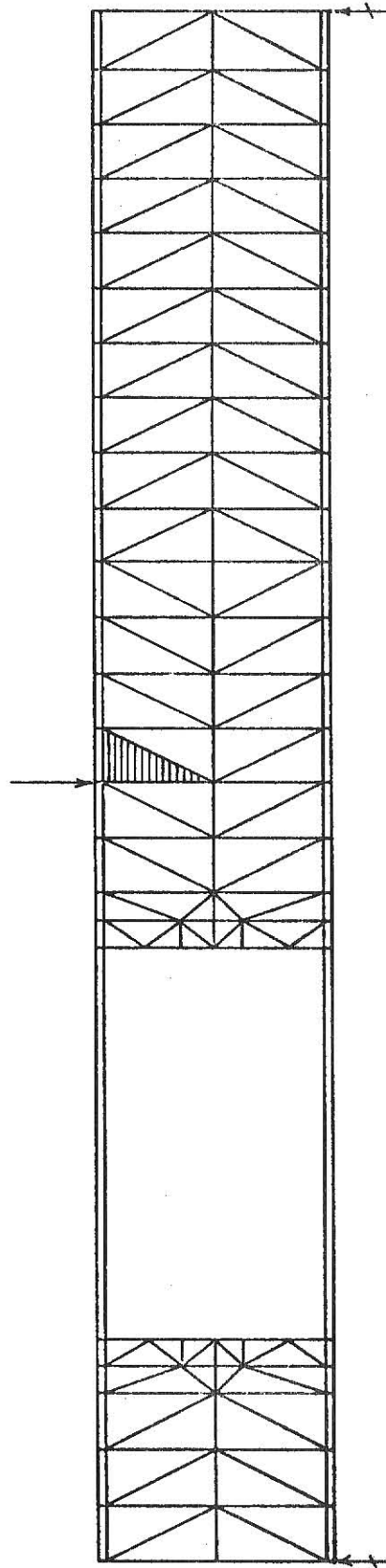
Figure 23. Yield Pattern for Beam 6 at Ultimate Load (84^k).

Figure 24. Yield Pattern in Vicinity of Opening for Beam 6 at Ultimate Load (84^k).

Figure 25. Yield Pattern for Beam 7 at Ultimate Load (96^k).

Figure 26. Yield Pattern in Vicinity of Opening for Beam 7 at Ultimate Load (96^k).
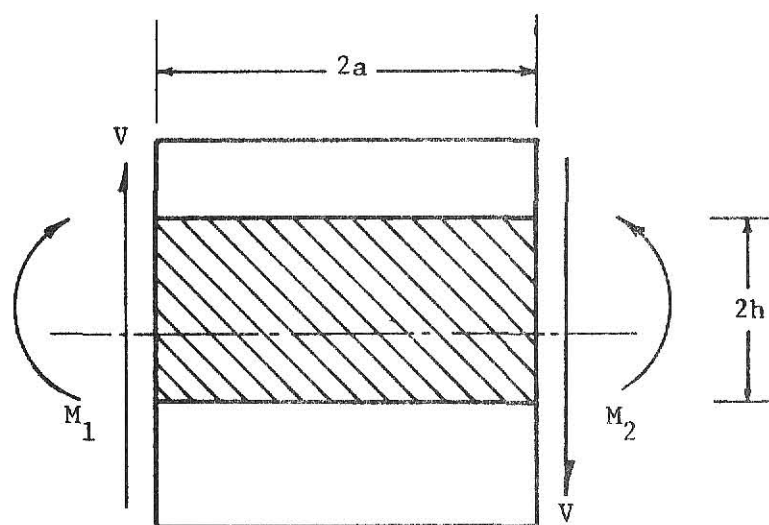
(a)

(b)

Figure 27.   Secondary Moments.

indicates location of plastic hinge
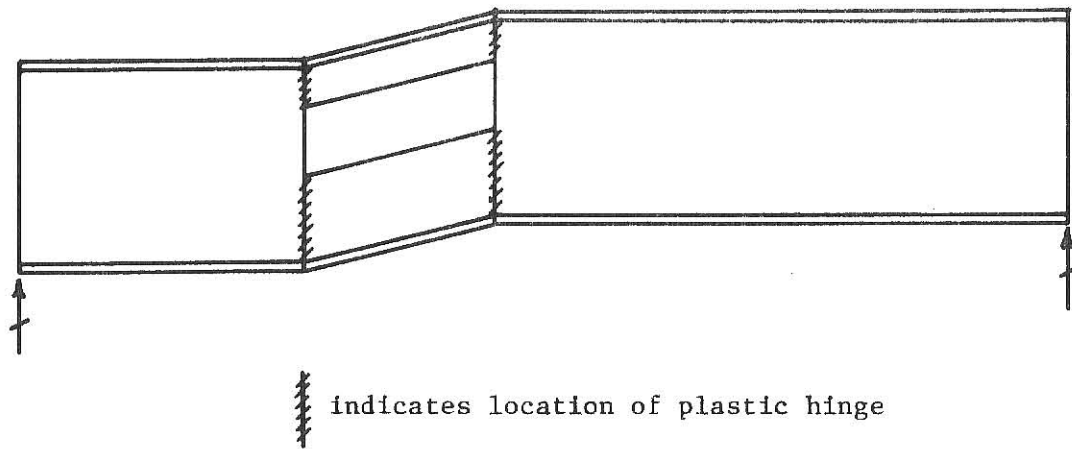
Figure 28. Four Hinged Mechanism at the Opening.

ULTIMATE LOAD CAPACITY OF STEEL BEAMS WITH WEB OPENINGS
BY THE FINITE ELEMENT METHOD

by

Aslam G. Porbandarwala

B. Tech., Indian Institute of Technology, Bombay, 1974

———————————————————————

ABSTRACT

A finite element program was used to carry out ultimate load analyses on five A36 steel beams with varying sizes of rectangular web openings. Two of the beams were W16 x 45 shapes and three were W16 x 40 shapes. In all cases the opening had an eccentricity of 2 inches and the moment to shear ratio at the centerline of the opening was 30 inches. The openings in all but one beam were reinforced.

The ultimate loads based on the finite element analysis indicated good agreement with those obtained experimentally and with those obtained from an ultimate strength analysis. The yield patterns at various loads also agree closely with those observed in the experiments. It was confirmed that the failure at the opening is a four hinged mechanism as assumed in the theory, with a plastic hinge at each corner of the opening.