APPLICATION OF GEOMETRIC PROGRAMMING
TO INDUSTRIAL SYSTEMS

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I. INTRODUCTION

The increased use of mathematical models in the analysis and optimization of industrial systems is one of the significant developments of modern engineering practice. Optimal desing of process equipment often involves finding numerical values for the design parameters to minimize a cost function, usually nonlinear, subject to design constraints. Most of the models which accurately describe the real-life systems prove to be too complex for solution by available algorithms. This is especially true of problems in which the constraints are nonlinear.

Recently, the geometric programming technique, has been developed which can handle a subclass of the above problems in which the cost function and the constraints are generalized polynomials.

In 1961 Zener [19] observed that the sum of the component costs sometimes may be minimized almost by inspection when each cost depends only on the products of the design variables, each raised to an arbitrary but known power. Duffin and Peterson [6] extended Zener's work. Zener and his associates' work had been restricted to functions they called 'Posynomials,' which are generalized polynomials with positive coefficients. Passy and Wilde [12] further generalized the method to include negative coefficients and reversed inequalities.

Geometric programming is specially suitable for engineering optimization problems based on desing relations developed either by dimensional analysis or by fitting power functions to experimental results.

An important feature of geometric programming is its computational convenience. When the number of terms exceeds the number of variables by a small number, the computations are much simpler than the highly nonlinear character of the problem would lead one to expect. To minimize an unconstrained polynomial of m variables, the conventional method of calculus involves the solution of m nonlinear equations. On the other hand, if the function to be minimized containes exactly m + 1 terms, the problem can be solved by geometric programming by solving m + 1 linear equations, a far easier task. This is advantageous when the problem involve inequality constraints.

Although Passy and Wilde [12] have extended the geometric programming algerithm to handle objective functions and constraints with negative coefficients, difficulty is often encountered in numerical analysis except in the special case where there is exactly one more term than there are independent variables.

Recently Blau and Wilde [5] developed a Lagrangian algorithm for generalized polynomial optimization with equality constraints. The method optimizes the Lagrangian function with the Newton-Raphson procedure. This algorithm can handle negative coefficients efficiently and converges rapidly. One difficulty with this method is its occasional use of too large a step, which prevents convergence. This difficulty was overcome in this work by using a forcing procedure which restricts the maximum step size to a predetermined percentage of the variables.

The perpose of this thesis is to apply geometric programming to different engineering design and industrial management systems in production planning and to analyze geometric programming's merits and faults. In the following chapter the basic algorithm of geometric programming with extensions and the algorithm of Lagrangian polynomial optimization technique are discussed. A brief review of computational procedure and approximation technique follows. In Chapter V, various possible fields of application of the above algorithms are analyzed and finally the advantages and disadvantages of geometric programming are highlighted.

II. GEOMETRIC PROGRAMMING

The theory of geometric programming is based on the arithmetic-geometric mean inequality. The set of functions comprising the mathematical model, when expressed in terms of the primal variables, is called the primal problem. A dual formulation of the primal problem can be obtained. Minimization of the primal problem is equivalent to maximization of the dual problem and the two extreme values are equal.

In this chapter only the algorithms of geometric programming and its extensions are stated and the computational procedures for them are discussed. A detailed derivation of the algorithm and the proof of the theory can be found in [6] and [18].

A set of p + 1 generalized polynomials consisting of m real positive variables x_j can be expressed as:

$$g_k = \sum_{i \in J(k)} C_i \prod_{j=1}^{m} x_j^{A_{ij}} \tag{1}$$

where $k = 0, 1, \ldots, P$

and J(k) is a set of integers ranging from m_k to n_k, thus:

$$J(k) = \{m_k, m_k + 1, \ldots, n_k\} \tag{2}$$

and $$m_o = 1,\ m_1 = n_o + 1,\ \ldots,\ m_p = n_p + 1,\ n_p = n \tag{3}$$

$$C_i > 0 \tag{4}$$

$$x_j > 0 \tag{5}$$

n is the total number of terms in the set of polynomials.

A_{ij} are any real numbers.

The primal problem is to minimize

$$g \equiv g_o \tag{6}$$

subject to the constraints

$$g_k \leq 1; \quad k = 1, 2, \ldots, P \tag{7}$$

The associated dual problem can be formed consisting of a set of n dual variables $\underline{\delta}$ satisfying a normality condition

$$\sum_{i \in J(0)} \delta_i = 1 \tag{8}$$

and m orthogonality conditions:

$$\sum_{i=1}^{n} A_{ij} \delta_i = 0 \qquad j = 1, 2, \ldots, m \tag{9}$$

as well as n non-negativity conditions

$$\delta_i \geq 0 \qquad i = 1, 2, \ldots, n \tag{10}$$

The corresponding dual problem can be written as

$$V(\delta) = \left(\prod_{i=1}^{n} \left(\frac{C_i}{\delta_i}\right)^{\delta_i} \right) \prod_{k=1}^{P} \lambda_k^{\lambda_k} \tag{11}$$

where $\lambda_k = \sum_{i \in J(k)} \delta_i \qquad k = 1, 2, \ldots, P$ (12)

The logarithm of the dual function (11) is strictly concave and hence it has only one stationary point - a global maximum. So the minimum of g_o is obtained by maximizing the dual function (11) subject to the normality and orthogonality conditions (8) to (10).

Once the dual variables $\underline{\delta}$ are known, the corresponding values of the primal variables x_j are found from the following relations:

$$C_i \prod_{j=1}^{m} x_j^{A_{ij}} = \delta_i \, g_o^* \qquad (13)$$

for $i \epsilon J(0)$

and

$$C_i \prod_{j=1}^{m} x_j^{A_{ij}} = \delta_i / \lambda_k \qquad (14)$$

for $i \epsilon J\ (k)$
$k \neq 0$

where g_o^* = minimum value of the objective function.

EXTENSION OF GEOMETRIC PROGRAMMING

The following algorithm is obtained by Passy and Wilde (12); it extends the theory of geometric programming to take into account negative coefficients and reversed inequalities.

P + 1 generalized polynomial functions $g_k(x)$ can be expressed as

$$g_k = \sum_{t=1}^{T_k} \sigma_{kt} \, C_{kt} \prod_{j=1}^{m} x_j^{A_{ktj}} \qquad k = 0, 1, \ldots, P \qquad (15)$$

where

$$\sigma_{kt} = \pm 1 \qquad (16)$$

$$C_{kt} > 0 \qquad (17)$$

$$x_j > 0 \tag{18}$$

and A_{ktj} are real numbers. The signum functions σ_{kt}, the coefficients C_{kt}, T_k (the number of terms in g_k), and the A_{ktj} are all given. Then the typical optimization problem can be written as

$$\min g_o (x) = g_o (x^*) \equiv g_o^* \neq 0, \pm \infty \tag{19}$$

(* corresponds to the optimal solution).

subject to P inequality constraints.

$$0 < \sigma_k g_k^{\sigma_k} \leq 1 \qquad k = 1, \ldots, P \tag{20}$$

where σ_k are known signum functions.

This problem can be solved by working with a set of real finite dual variables δ_{kt}, one for each term of the g_k, which satisfies the following
nonnegativity condition

$$\delta_{kt} \geq 0 \qquad \text{for all k and t} \tag{21}$$

and the normality condition:

$$\delta_{oo} = \sigma_o \sum_{t=1}^{T_o} \sigma_{ot} \delta_{ot} = 1 \tag{22}$$

the m orthogonality conditions:

$$\sum_{k=0}^{P} \sum_{t=1}^{T_k} \sigma_{kt} A_{ktj} \delta_{kt} = 0 \qquad j = 1, \ldots, m \tag{23}$$

and P inequality constraints:

$$\delta_{ko} = \sigma_k \sum_{t=1}^{T_k} \sigma_{kt} \delta_{kt} \geq 0 \quad k = 1, \ldots, P \tag{24}$$

with the qualification that

$$\delta_{kt} = 0 \tag{25}$$

if and only if

$$\delta_{ko} = 0 \qquad k = 1, \ldots, P \tag{26}$$

σ_o must be chosen to satisfy the constraints.

The dual function can be written as

$$V(\delta, \sigma_o) = \sigma_o \left[\prod_{k=0}^{P} \prod_{t=1}^{T_k} \left(\frac{C_{kt}\,\delta_{ko}}{\delta_{kt}}\right)^{\sigma_{kt}\,\delta_{kt}} \right]^{\sigma_o} \tag{27}$$

with the assumption that

$$\lim_{\delta_{kt} \to 0} \left(\frac{C_{kt}\,\delta_{ko}}{\delta_{kt}}\right)^{\sigma_{kt}\,\delta_{kt}} = 1 \tag{28}$$

When all signum functions are not positive $g_o(x)$ is not, in general, convex and may have several constrained local minima, maxima or saddle poi ts, and no simple duality relation holds. It is proved instead that to each critical point (called a pseudominimum) x^o of g_o, there corresponds a dual point (δ^o, σ_o) where V is a pseudomaximum and such that

$$g_o(x^o) = V(\delta^o, \sigma_o) \tag{29}$$

Roughly speaking, a pseudominimum is a point where g_o satisfies the Kuhn-Tucker constraint qualification as well as the differential form of the Kuhn-Tucker necessary conditions for a constrained local minimum.

Then $g_o(x^o) \equiv \text{Pmin}\, g_o(x) = \text{Pmax}\, V(\delta^o, \sigma_o) \equiv V(\delta_o, \sigma_o)$
where Pmin is an abbreviation for pseudominimum. (30)

Then at a global minimum:

$$\text{Min}\, g_o(x^*) = \text{Min}\left[\text{Pmax}\, V(\delta^*, \sigma_o)\right] \tag{31}$$

Once the dual variables $\underline{\delta}^*$ are known the primal variables are found from the following relations:

$$C_{ot} \prod_{j=1}^{m} x_j^{A_{otj}} = \delta_{ot}\, \sigma_o\, g_o^* \qquad t = 1, \ldots, T_o \tag{32}$$

and

$$C_{kt} \prod_{j=1}^{m} x_j^{A_{ktj}} = \frac{\delta_{kt}}{\delta_{ko}} \qquad \begin{array}{l} t = 1, \ldots, T_m \\ k = 1, \ldots, P \end{array} \tag{33}$$

From equation (32), it can be seen that σ_o will have the same sign as g^o. Since there will always be more terms than variable, x_j, m equations can be found which are solvable for m primals. The solution of these equations is not difficult since they are linear in log x_j.

(* corresponds to optimal solution).

COMPUTATIONAL PROCEDURE

The detailed computer flow charts and programs are provided in the appendix. The present discussion on the method is to aid the understanding of the subsequent discussions.

The computer algorithm finds the minimum of the primal function (6) subject to primal constraints (7) by maximizing the dual function (11) subject to dual constraints (8) through (10). Having found this maximum a transformation is made to obtain the primal variables x_j.

As can be seen the dual problem has n variables and m + 1 linear equality constraints. This gives the problem n - (m + 1) degrees of freedom. Zener and his associates call this the degree of difficulty.

The dual problem with nonlinear objective function and linear equality constraints can be maximized by any conventional method, such as, by Lagrange multipliers or the gradient projection method. As suggested by Duffin [6] the dual function can be transformed to eliminate the linear equalities to result in a 'd' dimensional optimization problem, where 'd' is the degree of freedom. The transformation is done as follows:

The dual variables $\underline{\delta}$ satisfying the equations (8) and (9) can be written as sum of a normality and a set of nullity vectors by the method of linear algebra.

$$\text{or} \quad \underline{\delta} = \underline{b}_o + \sum_{j=1}^{d} r_j \, \underline{b}_j \tag{34}$$

where $\underline{b}_o$ = normality vector

$\underline{b}_j$ = nullity vectors $j = 1, \ldots, d.$

r_j = are arbitrary real numbers

satisfying the positivity constraints

$$b_i^o + \sum_{j=1}^{d} r_j \, b_i^j \geq 0 \qquad i = 1, \ldots, n \tag{35}$$

writing the dual function (11) in transformed form

$$V(r) = k_o \left(\prod_{j=1}^{d} k_j^{r_j} \right) \left(\prod_{i=1}^{n} \delta_i(r)^{-\delta_i(r)} \right) \prod_{k=1}^{P} \lambda_k^{\lambda_k} \tag{36}$$

where

$$k_j = \prod_{i=1}^{n} C_i^{b_i^j} \qquad j = 0, 1, \ldots, d \tag{37}$$

$$\delta_i(r) = b_i^o + \sum_{j=1}^{d} r_j \, b_i^j \qquad i = 1, \ldots, n \tag{38}$$

This function can be maximized with respect to r_j by any direct search technique. It has been found that Hooke and Jeeves (10) direct search is quite efficient. The first four problems in chapter V have been solved by this method. Another approach is to obtain a set of 'd' equations by differentiating this function with respect to r_j and setting the result equal to zero:

$$k_j - \left(\prod_{i=1}^{d} \delta_i^{b_i^j} \right) \prod_{k=1}^{P} \lambda_k^{\lambda_k^j} = 0 \qquad j = 1, \ldots, d \tag{39}$$

where

$$\lambda_k^j = \sum_{i\ J(k)} r_j\ b_i^j \qquad j = 1, \ldots, d \qquad (40)$$

These sets of equations can be solved by the Newton-Raphson method to give optimum values of r_j and hence the dual variables. Convergence by this method is not guaranteed, but when the method works the function converges very rapidly. Problems 2 and 5 are solved by this method.

The computational procedure can be briefly summarized as follows:

a) Mathematical formulation of the dual problem.

To obtain the dual problem in the form of equation (11), the primal problem and the constraints have to be in the form of equations (6) and (7). Many problems which are not in this form can be transformed into the required form by various techniques discussed in a later chapter.

b) Calculation of normality and nullity vectors.

The normality and nullity vectors of the form of equation (34) can be obtained by the usual method of linear algebra (9).

c) Obtaining the initial feasible solution of r_j.

The nonnegarivity constraints (35) have to be satisfied for the initial feasible solution. This is achieved by adjusting all variables simultaneously.

d) Optimization of the dual function:

Any suitable method for optimization is applicable. Hooke and Jeeves pattern search [10] is mostly used in the problems that are solved in this work. Differentiation and the Newton-Raphson method [14] are also used where the function converges.

Since $V(\delta)$ is only defined for $\delta_i \geq 0$ any intermediate step in the search procedure not meating this condition is avoided.

The accuracy obtained in the solutions depends on the compromise between improvement in objective function and required computer time. Different accuracies ϵ, (ranging from .01 to .0001) are assumed for different problems. The function is assumed to converge when the function value changes by ϵ or less in two successive iterations of Newton-Raphson or step size is equal to or smaller than ϵ in Hooke and Jeeve search.

e) Transformation from dual to primal problem:

The primal variables are calculated from the optimum dual variables by equation (13) and (14).

The above procedure is effective for functions with positive coefficients. For functions with negative coefficients and reversed inequalities along positivity conditions (21), P inequality constraints (24) are to be satisfied and the minimum of all the pseudomaxima of the dual function (27) is to be found. It was found that the above procedure fails in this case because of limitations of search procedures. An efficient algorithm has been presented by Blau and Wilde [3] to handle polynomials with negative coefficients and with equality constraints. This algorithm is presented in the next chapter.

III. A LAGRANGIAN ALGORITHM

In this chapter an algorithm for optimization of generalized polynomials with equality constraints is presented. This algorithm is of high practical importance since it is the only algorithm which handles negative coefficients in the polynomials effectively; most physical restrictions occurring in practice are often strict equalities.

The basic idea is to use a Newton-Raphson procedure [18] to drive to zero the components of the gradient of a Lagrangian function formed from the logarithms of the original objective function and the constraints. A nonlinear transformation, which amounts to substituting a weighting variable for each term, makes the Lagrangian gradient linear in the weights as well as in the Lagrange multipliers.

No proof for local convergence is yet available. For justification of the procedure the reader is referred to [3].

A set of M + 1 generalized polynomials of N variables x_n g_m can be defined as:

$$g_m = \sum_{t=1}^{T_m} \sigma_{mt} C_{mt} \prod_{n=1}^{N} x_n^{A_{mtn}} \qquad m = 0, 1, \ldots, M \tag{1}$$

where $0 < x_n < \infty$ (2)

and $C_{mt} > 0 \qquad t = 1, \ldots, T_m$ (3)

$\sigma_{mt} = \pm 1$ (4)

A_{mtn} is any real number

The mininization problem can be stated as

$$\operatorname*{Min}_{x}\ g_0 \tag{5}$$

$$\text{subject to } g_m = 1 \quad (m \neq 0) \tag{6}$$

To initiate the algorithm, finite positive values x_n^o of x_n have to be chosen not necessarily satisfying the constraints (6). The initial value of the objective function is calculated as g_o^o and the initial weights as

$$W_{mt}^{o} = \begin{cases} (g_o^o)^{-1}\, C_{ot} \prod_{n=1}^{N} (x_n^o)^{A_{otn}} & \text{for } m = 0 \\ C_{mt} \prod_{n=1}^{N} (x_n^o)^{A_{mtn}} & \text{for } m = 1, \ldots, M \end{cases} \tag{7}$$

At the ith iteration the following sums are calculated as

$$S_{mn}^{i} = \sum_{t=1}^{T_m} \sigma_{mt}\, A_{mtn}\, W_{mt}^{i} \tag{8}$$

From them the N x 1 dimensional vector is formed as

$$\underset{\sim}{S}_o^i = (S_{io}^i \;.\;.\; S_{No}^i)^T \tag{9}$$

and the N x M matrix

$$S^i = \begin{bmatrix} S_{11}^i & \cdots & S_{1M}^i \\ & & \\ S_{N1}^i & \cdots & S_{NM}^i \end{bmatrix} \tag{10}$$

This matrix is assumed to have rank M, so that $\left[S^{iT} S^{i}\right]$ is non-singular. Then the initial M x 1 vector of multipliers can be computed as

$$\underline{\lambda}^{0} = (S^{0T} S^{0})^{-1} S^{0T} S^{0}_{o} \qquad (11)$$

A N X N symmetric matrix T^i can be computed as

$$T^{i}_{nj} = - \sum_{t=1}^{T_o} \sigma_{ot} A_{otn} A_{otj} W^{i}_{ot}$$
$$+ \sum_{m=1}^{M} \lambda^{i}_{m} \sum_{t=1}^{T_m} \sigma_{mt} A_{mtm} A_{mtj} W^{i}_{mi} \qquad (12)$$

At each iteration there is a value of one additional variable V^i which is also adjusted by the algorithm. To begin with, it is taken as the value of the objective function at x^o.

Let $\sigma^{i} = g^{i}_{o} / |g^{i}_{o}|$

Then the $(M + N + 1)^2$ symmetric Newton-Raphson matrix R^i is assembled as

$$R^{i} = \begin{bmatrix} T^{i} & S^{i}_{o} & S^{i} \\ S^{iT}_{o} & -1 & \underline{0} \\ S^{iT} & \underline{0} & \underline{0} \end{bmatrix} \qquad (13)$$

where $\underline{0}$ represents the null matrices with appropriate dimensiona. Next a (M + N + 1) x 1 dimensional error vector e^i is formed as

$$e^i = \begin{bmatrix} S_o^i - S^i \lambda^i \\ 1 - g_o^i (V^i)^{-1} \\ 1 - g_1^i \\ 1 - g_m^i \end{bmatrix} \tag{14}$$

Then the (N + M + 1) X 1 dimensional correction vector is given by

$$\begin{bmatrix} \Delta \ln \underline{X}^i \\ \Delta \ln \sigma^i V^i \\ \Delta \underline{\lambda}^i \end{bmatrix} = (R^i)^{-1} e^i \tag{15}$$

so the next estimate of x^{i+1} is

$$x_n^{i+1} = X_n^i \exp (\ln X_n^i) \tag{16}$$

whereas

$$V^{i+1} = V^i \exp (\ln \sigma^i V^i) \tag{17}$$

$$\underline{\lambda}^{i+1} = \underline{\lambda}^i + \Delta \underline{\lambda}^i \tag{18}$$

These quantities are used to compute the new values of weights defined in equation (7) for m = 1, ..., M.

For m = 0, the following equation is used.

$$W_{ot}^{i+1} = (V^{i+1})^{-1} C_{ot} \prod_{n=1}^{N} X_n^{A_{otn}} \tag{19}$$

Thus the algorithm completes the ith iteration. The procedure continues until all components of the error vector are acceptably close to zero.

IV. APPROXIMATION TECHNIQUES

Many optimization problems can be transformed into standard geometric programming problems, even though they are not explicitly expressed in posynomial form. This fact is illustrated in the following examples.

Example 1.

Minimize the function

$$G(\underline{x}) = f\ (\underline{x}) + [q\ (\underline{x})]^a\ h\ (\underline{x}) \tag{1}$$

where f $(\underline{x})$, q $(\underline{x})$ and h $(\underline{x})$ are posynomials in the vector variable $\underline{x} = (x_1, \ldots, x_m)$ and $a > 0$.

The above problem can be expressed as:

$$\text{minimize } g\ (\Upsilon) = f\ (x) + x_o^a\ h\ (x) \tag{2}$$

$$\text{subject to } \quad x_o^{-1}\ q\ (x) \leq 1 \tag{3}$$

where x_o is an additional independent variable and $\Upsilon = (x_o, x_1, \ldots, x_m)$. It can be seen from the construction of g (Υ) and the constraint that $(x_1^1, x_2^1, \ldots, x_m^1)$ minimizes G (x) if and only if, $(q\ (x^1), x_1^1, \ldots, x_m^1)$ minimizes g (Υ) subject to the given constraint. The constrained minimum value of g (Υ) is equal to the minimum value of G $(\underline{x})$. Thus the problem of minimizing G $(\underline{x})$ which is not necessarily a posynomial can be transformed to the form which permits the use of geometric programming.

Example 2.

Minimize the function

$$G(\underline{x}) = f(\underline{x}) + \frac{q(\underline{x})}{[V(\underline{x}) - h(\underline{x})]^{a}} \tag{4}$$

where f, q, h are posynomials, u is a posynomial with one term and $a > 0$.

The equivalent problem can be formed as

$$g(\Upsilon) = f(\underline{x}) + \frac{g(x)}{x_o^a} \tag{5}$$

subject to the constraint

$$\frac{x_o}{U(\underline{x})} + \frac{h(\underline{x})}{u(\underline{x})} \leq 1 \tag{6}$$

where x_o is an additional independent variable and $\Upsilon = (x_o, x_1, \ldots, x_m)$. Since u $(\underline{x})$ has only one term the form of the constraint permits use of geometric programming.

Example 3.

Minimize the function

$$G(\underline{x}) = f(\underline{x}) - u(\underline{x}) \tag{7}$$

where f and u are posynomials and u has one term. If it can be assumed that the minimum value of G is negative then the constraint.

$$x_o + f(\underline{x}) - u(\underline{x}) \leq 0 \quad \text{is consistent} \tag{8}$$

It can be seen that x^1 minimizes G $(\underline{x})$ if and only if $\Upsilon^1 = [\, u(x^1) - f(x^1),\ x_1^1,\ \ldots,\ x_m^1 \,]$ maximizes the function

$$h(\Upsilon) = x_o \tag{9}$$

subject to the constraint

$$x_o + f\ (\underline{x}) - u\ (\underline{x}) \leq 0$$

This maximization problem is equivalent to the problem:

$$\text{Minimize } g\ (\gamma) = \frac{1}{h\ (\gamma)} = \frac{1}{x_o} \tag{10}$$

subject to the constraint

$$\frac{x_o}{U\ (\underline{x})} + \frac{f\ (\underline{x})}{u\ (\underline{x})} \leq 1 \tag{11}$$

Thus this reduces the problem to standard geometric programming form.

So far the examples showed the transformation which gives the exact solution of the problems. Following are some examples of approximate transformation which permits use of functions other than posynomials.

Example 4.

Any function h $(\underline{x})$ which is not a posynomial can be approximated to a single term posynomial. To do this it is necessary to make a rough estimate of the range ofvariability of each variable x_j. Let x_j^* be the geometric mean of this range. Then $(x_1^*, x_2^*, \ldots, x_m^*)$ may be termed the operating point. Then h (x) can be approximated as

$$h\ (\underline{x}) \simeq h\ (x^*)\ \left(\frac{x_m}{x_1^*}\right)^{A_1} \left(\frac{x_m}{x_2^*}\right)^{A_2}, \ldots, \left(\frac{x_m}{x_m^*}\right)^{A_m} \tag{12}$$

where

$$A_j = \left(\frac{x_j}{h} \frac{\partial h}{\partial x_j}\right)\ x = x^* \qquad j = 1, \ldots, m \tag{13}$$

This approximation is equivalent to expanding log h in a power series in terms of variables $Z_j = \log (x_j/x_j^*)$ and neglecting all but linear terms.

Example 5.

Approximation of log u

The function log u is defined as:

$$\log u = \int_1^u \frac{1}{x}\, dx \tag{14}$$

On the other hand, if E is a positive number, then

$$\frac{U^E}{E} - \frac{1}{E} = \int_1^u \frac{X^E}{X}\, dx \tag{15}$$

On any interval between positive numbers, the function x^E is a uniform approximation to unity, provided E is sufficiently small. Hence, log u can be approximated by $E^{-1}(U^E - 1)$ for u in the same interval.

Example 6.

Approximation of an exponential function

Let the primal problem involve a function of the form

$$f(\underline{x}) = g(\underline{x}) + Ce^{u}(\underline{x}) \tag{16}$$

where g is a posynomial, C is a positive constant and $u(\underline{x})$ is a single term posynomial.

Using the well known relationship

$$e^u = \lim_{E\to\infty} \left(1 + \frac{u}{E}\right)^E \tag{17}$$

the function f ($\underline{x}$) can be written as

$$f_E(\underline{x}) = g(\underline{x}) + C\left(1 + \frac{u(\underline{x})}{E}\right)^E \tag{18}$$

where E is sufficiently large. This function is in the form of example 1. This can be reduced to standard geometric programming form by introducing a new variable $x_o = 1 + u/E$. (19)

f ($\underline{x}$) can be replaced by the function $g(\underline{x}) + C\,x_o^E$ (20)

and the additional constraint

$$x_o^{-1} + g^{-1}x_o^{-1}\,u(\underline{x}) \leq 1 \tag{21}$$

V. APPLICATIONS

In this section six different problems are considered to illustrate the procedure. The first is a simple hypothatical problem. The next four models are engineering design problems with different degrees of freedom and the last is a production scheduling model. The models are in posynomial form except for the last model, which is in the generalized posynomial form. Different optimization techniques are used to maximize the dual function as required by the nature of the problem. The results are compared and various computational difficulties are discussed. The last problem is solved by the Lagrangian algorithm.

A Simple Problem [17]

This simple problem can be stated as:

Minimize

$$y_o = 1000\ x_1 + 4 \times 10^9\ x_1^{-1}\ x_2^{-1} \tag{1}$$

subject to

$$2.5 \times 10^5\ x_2 + 9000\ x_1^{-1} x_2^{-1} \leq 1 \tag{2}$$

$$x_1 > 0$$

$$x_2 > 0$$

The problem has 4 terms and 2 variables. Hence it has one degree of freedom. The dual function which is of the form of Eqn. (11) of Chapter II, can be written as:

$$V(\delta) = \left(\frac{C_1}{\delta_1}\right)^{\delta_1} \left(\frac{C_2}{\delta_2}\right)^{\delta_2} \left(\frac{C_3}{\delta_3}\right)^{\delta_3} \left(\frac{C_4}{\delta_4}\right)^{\delta_4} \times (\delta_3 + \delta_4)^{(\delta_3 + \delta_4)} \qquad (3)$$

where $C_1 = 1000$

$C_2 = 4 \times 10^5$

$C_3 = 2.5 \times 10^5$

$C_4 = 9000$

The objective function has 2 terms. Hence the orthogonality condition is given by:

$$\delta_1 + \delta_2 = 1 \qquad (4)$$

and the normality conditions are given by:

$$\delta_1 - \delta_2 - \delta_4 = 0 \qquad (5)$$

$$\delta_2 + \delta_3 - \delta_4 = 0 \qquad (6)$$

The minimum value of the function given by equation (1) is obtained by maximizing the dual function given by equation (3) subject to equality constraints (4), (5) and (6). These equality constraints are eliminated by expressing vector $\underline{\delta}$ as sum of a normality and a nullity vector as explained in Chapter II. Thus,

$$\underline{\delta} = \underline{b}^o + r\underline{b}^1 \qquad (7)$$

By the above substitution the problem is transformed to maximization with respect to single variable r and the positivity constraint

Table 1. Convergence rate of the sample problem by the Hooke and Jeeves method.

r_1	$V(\delta) \times 10^{12}$	No. of functional evaluation
.80	2.15	1
.90	4.57	5
.95	6.53	10
.97	7.73	16
.98	8.38	20
.99	8.70	26
1.00	8.99	30

$$\underline{\delta} \geq \underline{0} \tag{8}$$

The problem was solved by using Hooke and Jeeves search procedure. The initial value chosen for r was 0.8. The accuracy, ϵ, as defined in Chapter II was chosen as .01. The convergence rate, i.e., the change of value of equation (3) with the number of functional evaluations is shown in Table 1.

The optimum value obtained was:

$$y_o^* = 8.99 \times 10^{12}$$

and the optimum primal variables were:

$$x_1 = 8.996 \times 10^9$$
$$x_2 = 2.0 \times 10^{-6}$$

The optimum primal variables were obtained using the following equations.

$$x_1 = \frac{y_o^* \, \delta_1}{C_1} \tag{9}$$

$$x_2 = \frac{\delta_3}{(\delta_3 + \delta_4)} \times C_3 \tag{10}$$

SEA POWER - HEAT EXCHANGER PROBLEM [6]

The conversion of the sun's radiant energy into useful power is a challenging field to many engineers. A stumbling block has been the extremely high capital cost of the equipment required to collect and concentrate this radiant energy.

Most of the solar energy is received by the upper layers of the ocean. Energy from the sea could passibly be collected by a heat engine cycle which consists of direct evaporation of water from the upper layers, and, later, after passing through turbines, condensation on the cooler underlying water. The steam vapor pressure in equilibrium with the cool deep water is so low that extremely large turbines are required. An approach that circumvents the need for extremely large turbines is the use of an intermediate fluid, such as ammonia, which has a high vapor pressure at room temperature.

The avoidance of costly, large turbine is achieved only by introduction costly item, namely, the heat exchangers that allow heat to flow from the warm surface of the water to boiling ammonia and allow the same heat to flow form the condensing ammonia to cool deep water. Since the economic feasibility of this cycle depends primarily on the size of the required heat exchanger, our objective is to minimize the required surface area of heat exchangers for a sea power plant of a specific power capacity.

For derivation of the model the following nomenclature has been used:

A = area of heat exchangers

C = specific heat of water

f = friction coefficient

h = film coefficient of water

h' = film coefficient of ammonia

m = mass flow of water

P = power output heat engines

P_N = net power output

P_{Hx} = friction loss in heat exchangers

P_{kE} = power loss by mass flow

P_r = Prandtl number

Q = heat extraction rate

T = temperature of hot reservoir

U = water flow rate

$\alpha \Delta T$ = temperature gradient to heat engine

$\beta \Delta T$ = total temperature drop across water boundary layers

$\beta' \Delta T$ = total temperature drop across liquid ammonia

$\gamma \Delta T$ = change in water temperature in heat exchangers

ΔT = temperature difference in hot and cold reservoir

ρ = density of water

η = diffuser degradation

ϵ = engine efficiency

ϵ^1 = primemover's efficiency.

From thermodynamics the available power can be expressed as

$$P = \epsilon \alpha \frac{\Delta T}{T} Q \qquad (11)$$

The heat flux Q is restricted by the impedence of the water layer of essentially laminar flow that clings to the surface across which water is flowing. The heat characteristic of this film is specified as

$$Q = hA\tfrac{1}{2}\, \beta \Delta T \qquad (12)$$

The film coefficient h can be expressed as

$$h = \frac{f}{2\,P_r^{2/3}}\,\rho\,CU \qquad (13)$$

Thus an improvement of h can be obtained by increasing the velocity U but this is obtained only at the cost of an increase in power required to drive water through the heat exchangers. The power is expressed as

$$P_{Hx} = \tfrac{1}{2}\,f\,\rho\,U^3\,(2A) \qquad (14)$$

The heat flux Q is restricted also by the impedence of boiling ammonia; to boil ammonia at a finite rate, ammonia adjacent to the boiling heat exchanger must be slightly superheated. The relation between rate of boiling and the degree of superheat is given by

$$Q\ h^1\ A\ \tfrac{1}{2}\ \beta^1 \Delta T \qquad (15)$$

The overall drop of water temperature in the heat exchangers is inversely proportional to the mass flow. We would like γ to approach zero so that α, β and β^1 could be larger. However, the smaller the value of γ is, the larger is the mass flow. Thus we must have

$$m\,C\,\gamma\Delta T = Q \qquad (16)$$

and the loss of kinetic energy is

$$P_{kE} = 2\eta\left(\tfrac{1}{2}\,m\,U^2\right) \qquad (17)$$

Summarizing, the objective function to be minimized is the

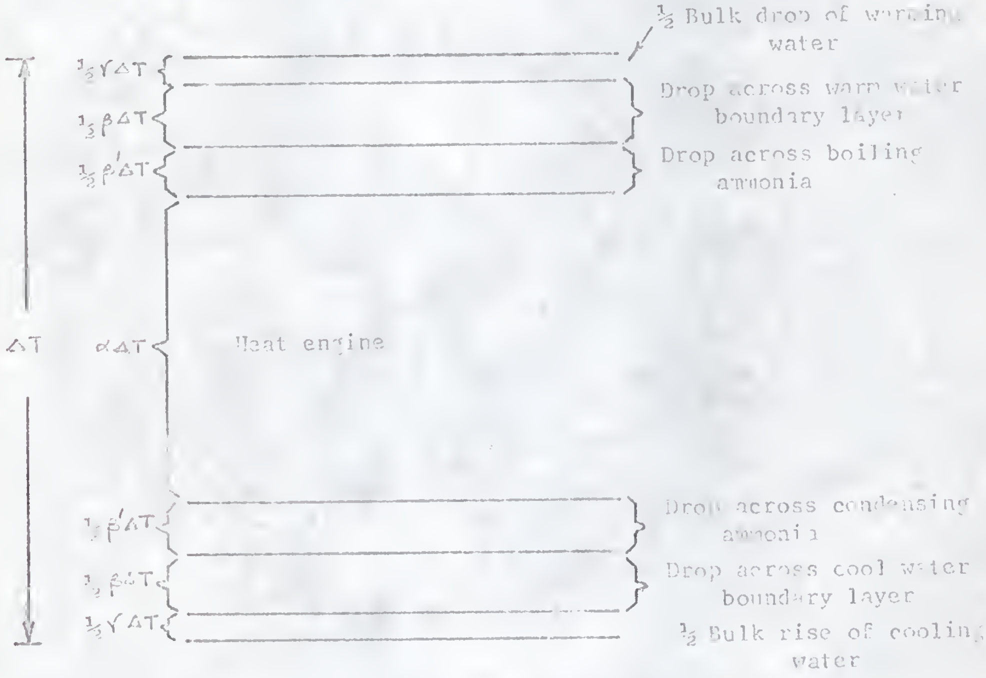


Fig. 1. Temperature distribution in the sea power problem [6].

area A, subject to constraints that A must be large enough to provide an adequate heat flux Q across the water boundary layers and across the liquid ammonia, and the heat flux Q must be large enough to provide not only P_N but also P_{Hx} and P_{kE}. Thus,

$$\frac{\epsilon \alpha \Delta T}{T} Q = P_N + \frac{1}{\epsilon^1} (P_{Hx} + P_{kE}) \tag{18}$$

The minimization of is to be made with respect to the variables Q, U, α, β, β^1, γ subject to the preceding relationship as well as to the temperature distribution relationship.

$$\alpha + \beta + \beta^1 + \gamma = 1 \tag{19}$$

The temperature distribution is depicted in Fig. 1. The formulation of this problem is due to Zener and his associates and for detailed derivation the interested reader is referred to [6].

Solution by Geometric Programming

In geometric programming all constraints must be represented by inequalities rather than equalities. Although the constraints in the preceding section have been formulated as equalities, it is obvious they can be formulated as inequalities. Thus the heat flux across the thermal barriers must be equal to or greater than the heat flux Q through the heat engine, because some of the heat flux across the heat exchanger can bypass the heat engine if this permits a reduction in A.

Hence the constraint (12) can be formulated as

$$\frac{Q}{h\ A\ \frac{1}{2}\ \beta\ \Delta T} \leq 1$$

and the constraint (15) as

$$\frac{Q}{h^{1}\ A\ \frac{1}{2}\ \beta^{1}\ \Delta T} \leq 1$$

and the constraint (18) can be expressed as

$$\frac{P_N + (1/\epsilon^{1})(P_{Hx} + P_{kE})}{(\epsilon\alpha\ \Delta T/T)\ Q} \leq 1$$

the = sign of constraint (19) can be changed into the ≤ sign. Hence the primal problem can be defined as

Minimize

$$C_1\ A$$

subject to

$$C_2\ \frac{Q}{UA\ \beta} \leq 1 \tag{20}$$

$$C_3\ \frac{Q}{A\ \beta^{1}} \leq 1 \tag{21}$$

$$C_4\ \frac{1}{Q\alpha} + C_5\ \frac{AU^3}{Q\alpha} + C_6\ \frac{U^2}{\alpha\gamma} \leq 1 \tag{22}$$

$$C_7\ \alpha + C_8\beta + C_9\beta^{1} + C_{10}\gamma \leq 1 \tag{23}$$

where h has been eliminated from (20) and m from (22) by using (13) and (16) respectively. The constants C_1, C_7, C_8, C_9, C_{10} are all unity and the constants C_2 through C_6 are

$$C_2 = \frac{4\ P_r^{2/3}}{f\rho C\Delta T}$$

$$C_3 = \frac{2}{h^1 \Delta T}$$

$$C_4 = \frac{P_N}{\epsilon (\Delta T/T)}$$

$$C_5 = \frac{f \rho \times 10^7}{\epsilon \epsilon^1 (\Delta T/T)}$$

$$C_6 = \frac{\eta T \times 10^{-7}}{\epsilon \epsilon^1 C \Delta T^2}$$

The following numerical values are taken for the constants appropriate to water at room temperature:

$P_r = 7$

$f = 1/125$

$\rho = 1$ gm/cm^3

$C = 4.18$ Joules/gm

$T = 300$ ^{o}k

$\epsilon = 0.6$

$\epsilon^1 = 0.6$

$\eta = 0.2$

$h^1 = 1.0$ watt/cm^2 ^{o}C

$\Delta T = 11$ ^{o}C

This gives the following values of the constants:

$C_2 = 40$

$C_3 = 0.18$

$C_4 = 44.5\ P_N$

$C_5 = 6.0 \times 10^{-8}$

$C_6 = 2.15 \times 10^{-8}$

The above problem has 7 variables and a total of 10 terms. Hence it has 2 degrees of freedom. The dual problem to be maximized, V (δ), has the same form as Eqn. (11) which is subject to normality and orthogonality constraints having the same form as Eqn. (8) to Eqn. (10) of Chapter II. As the problem has 2 degrees of freedom the objective function of the dual problem can be expressed as a function of two independent variables r_1 and r_2.

The problem has been solved by maximizing the dual function V(δ) by using Hooke and Jeeves search procedure. The convergence rate, i.e., the change in value of V (δ) with functional evaluation is shown in Table 3.

The problem has also been solved by differentiation. Two equations are obtained by differentiating V (δ) with respect to the independent variables and putting them equal to zero. Thus:

$$F_1 = \frac{dV(\delta)}{dr_1} = 0 \tag{24}$$

$$F_2 = \frac{dV(\delta)}{dr_2} = 0 \tag{25}$$

Equations (24) and (25) are solved by Newton-Raphson procedure. The convergence rate i.e., the change in the function values and change of the variables, are shown in Table 2 and plotted in Fig. 2 and 3.

The accuracy ϵ is chosen 0.001 for both Newton-Raphson and Hooke and Jeeves search procedure. The computer program is

Table 2. Convergence rate of the sea power problem by the Newton-Raphson method.

Iteration number	Variables		Function values	
	r_1	r_2	F_1	F_2
1	.64	.05	1.1	6.8
2	.46	.05	1.4	1.6
3	.42	.04	0.2	0.3
4	.42	.04	0.0	0.0

Table 3. Convergence rate of the sea power problem by the Hooke and Jeeves method.

Variables		$V(\delta)$	No. of functional evaluation
r_1	r_2		
.45	.02	124.69	0
.42	.07	124.79	7
.40	.04	126.03	33
.42	.04	126.72	41
.42	.04	126.75	59

Table 4. Optimum values of design parameters of the sea-power problem.

A	Q	U	α	β	β^1	γ
126.75	114.8	2.8	0.5	0.32	0.16	0.02

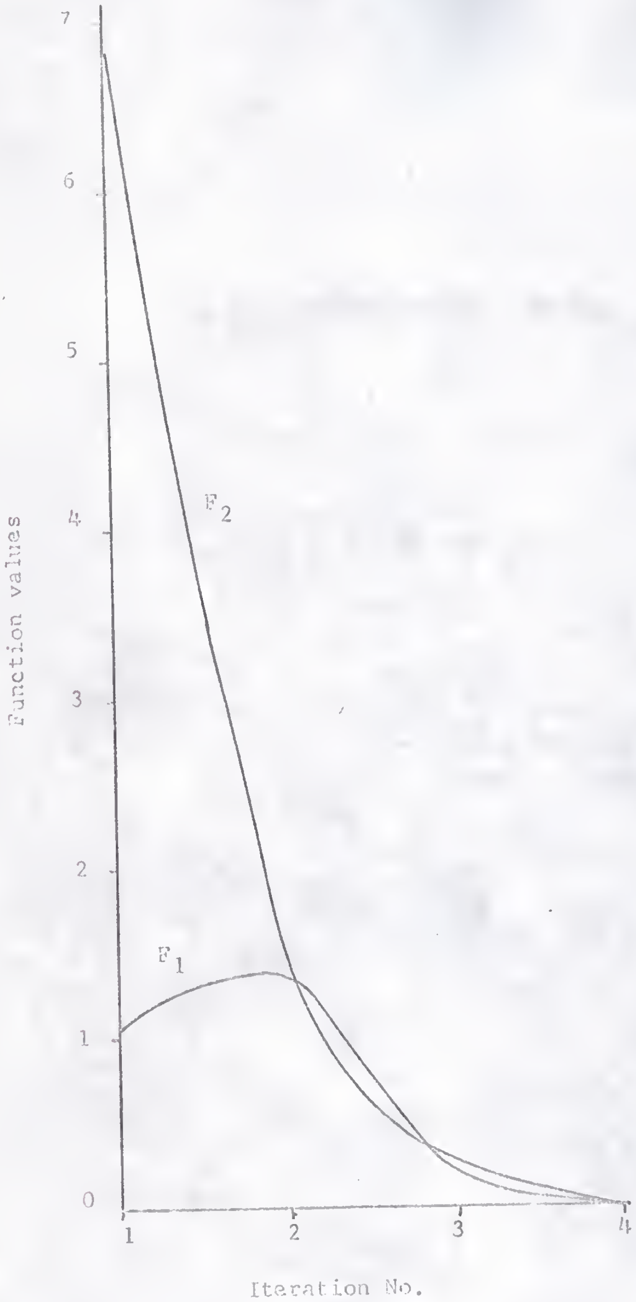


Fig. 2. The convergence rate of the sea power problem by the Newton-Raphson method.

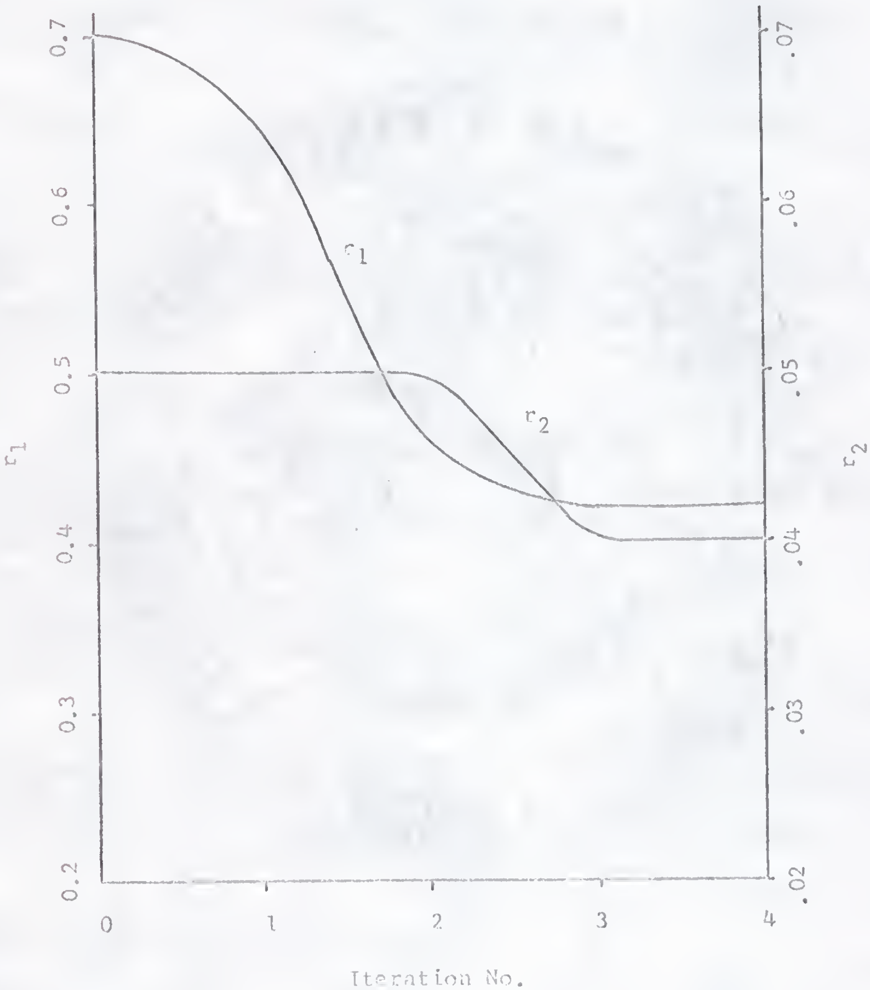


Fig. 3. The rate of change of variables in the sea power problem by the Newton-Raphson method.

written in FORTRAN language and is given in the Appendix. The results are given in Table 4.

There is no significant difference in the results between the two methods. But the number of iterations required is much less in the Newton-Raphson procedure than in the search procedure. The computer time needed was 22 secs for the Hooke and Jeeves procedure and about 19 secs for the Newton-Raphson method. This is reasonable because the Newton-Raphson method requires more computation time. The Newton-Raphson method seems to be more efficient but, as will be seen later, it may not always converge.

CONDENSER DESIGN PROBLEM

This model is taken from a pater of Wilde and Avriel [1]. Detailed derivation of the formulation can be obtained from the original pater. In this model the design of a vapor condenser with fixed heat load is considered.

Consider a horizontal condenser in which a fluid having a given flow rate W is heated without phase change from temperature T_{b1} to T_{b2} by condensing saturated steam. Optimal design involves minimizing the annual cost of the condenser, consisting of three terms:

1) Cost of steam
2) Fixed charges on the condenser
3) Cost of pumping fluid through the condenser tube.

In the derivation of the model, the following nomenclature has been used:

A_i = inside heat transfer area

A_o = outside heat transfer area

B = pressure drop factor

C = annual cost

C_E = cost of electricity

C_F = fixed charges

C_H = unit cost of condenser surface

C_p = specific heat

C_{Pu} = pumping cost

C_S = cost of steam

D_i = inside tube diameter

D_o = outside tube diameter

f = fanning friction factor

g = specific gravity

k = thermal conductivity

l = tube wall thickness

L = tube length

N = No. of tubes in condenser

P_c = depreciation rate

P_F = plant factor

Q = condenser heat load

V = rate of heat transfer

R_F = fouling resistance

T_{bm} = mean bulk temperature

T_s = steam temperature

W = flow rate inside tubes

α_o = coefficient in steam cost equation

α_1 = coefficient in steam cost equation

ΔP = pressure drop

ΔT_b = temperature rise in fluid in condenser

ΔT_{mi} = mean drop through inside tube film

ΔT_{mf} = mean drop through inside tube fouling

ΔT_{mo} = mean drop through condensing film.

Assuming that the cost of steam can be expressed as a linear function of its saturation temperature, we can write:

$$C_S = \alpha_o Q + \alpha_1 T_s Q \quad (\$/year) \tag{26}$$

The fixed charges on the condenser are expressed as

$$C_F = C_H P_c A_o \quad (\$/year) \tag{27}$$

and the pumping cost is given by

$$C_{Pu} = \frac{C_E P W P_F}{P_N} \quad (\$/year) \tag{28}$$

The objective is to select the values of T_s, A_o and ΔP which minimize the total annual cost given by

$$C = C_s + C_F + C_{Pu} \tag{29}$$

The steam temperature can be written as

$$T_s = T_{bm} + T_{mo} + \Delta T_{mi} + \Delta T_{mf} \tag{30}$$

Without going into mathematical detail, from the theory of heat transfer, the first component, the cost of steam, can be expressed as

$$C_s = \alpha_o Q + \alpha_1 T_{bm} Q + \frac{\beta_1}{N^{7/6} D_o L^{4/3}} + \frac{\beta_2 D_i^{0.8}}{N^{0.2} L} + \frac{\beta_5}{N D_i L} \tag{31}$$

where

$$\beta_1 = \left(\frac{\alpha_1 P_F}{K_f}\right)(W C_p \Delta T_b)^{7/3}\left(\frac{\mu_f}{2 \lambda \rho_f^2 g}\right)^{1/3}\left(\frac{1}{0.725}\right)^{4/3} \tag{32}$$

and

$$\beta_2 = \frac{(\alpha_1 P_F \Delta T_b)^2 (\mu)^{0.4} (C_p)^{1.5} W^{1.2}}{4^{0.8} (0.023)\, k^{0.6} \pi^{0.2}} \tag{33}$$

and

$$\beta_5 = \frac{\alpha_1 Q q R_F}{\pi} \tag{34}$$

The fixed changes can be expressed as

$$C_F = \beta_3 N D_o L \tag{35}$$

where

$$\beta_3 = \pi C_H P_c \tag{36}$$

and the pumping cost can be expressed as

$$C_{Pu} = \frac{\beta_4 L}{D_i^{4.8} N^{1.8}} \tag{37}$$

where

$$\beta_4 = \frac{32 \times 0.046\, C_E\, B\, P_F\, W^{2.8} (\mu/4)^{0.2}}{g \eta \rho^2 (\pi)^{1.8}} \tag{38}$$

Total annual cost is given by

$$C = \alpha_o Q + \alpha_1 T_{bm} Q + \frac{\beta_1}{N^{7/6} D_o L^{4/3}} + \frac{\beta_2 D_i^{0.8}}{N^{0.2} L} + \beta_3 N D_o L + \frac{\beta_4 L}{D_i^{4.8} N^{1.8}} + \frac{\beta_5}{N D_i L} \tag{39}$$

Since the first two terms are constants only the last five terms may vary and are subject to optimization. Thus the variable cost function,

$$C'' = \frac{\beta_1}{N^{7/6} D_o L^{4/3}} + \frac{\beta_2 D_i^{0.8}}{N^{0.2} L} + \beta_3 N D_o L + \frac{\beta_4 L}{D_i^{4.8} N^{1.8}} + \frac{\beta_5}{N D_i L} \tag{40}$$

From practical consideration the constraint on inside and outside diameter of the tubes is:

$$D_o - D_i \geq 21 \tag{41}$$

This can be written in the form of a geometric programming constraint as

$$\frac{\beta_6}{D_o} + \frac{\beta_7 D_i}{D_o} \leq 1 \tag{42}$$

where $\beta_6 = 21$ and $\beta_7 = 1$

Also from a practical standpoint it was found D_o can not exceed 1 inch. Hence this constraint can be included as:

$$D_o \leq D_o \max$$

$$\text{or } \beta_8 D_o \leq 1 \tag{43}$$

where $\beta_8 = 1/D_o \max = 12$

The following numerical values have been taken

$W = 500{,}000$ lbs/hr.

$T_{b1} = 195\ ^oF$

T_{b2} = 205 °F

T_{bm} = 200 °F

α_1 = 10^{-9} \$/BTU - °F

C_H = 5 \$/sq. ft.

C_E = 10^{-2} \$/kw-hr.

Pc = 0.1

P_f = 7884 hr/yr

η = 0.8

ρ = 60.13 lb/cu.ft.

μ = 0.20 cp.

k = 0.393 BTU/hr-sq.ft.-°F/ft.

C_p = 1.01 BTU/lb-°F

T_f = 210 °F

ρ_f = 59.88 lb/cu.ft.

μ_f = 0.26 cp.

k_f = .393 BTU/hr-sq.ft.°F/ft.

λ = 960 BTU/lb.

l = .049 inch

B = 1.2

R_f = 5.68×10^{-4}

The following values of the constants are obtained:

$\beta_1 = 172,400$

$\beta_2 = 97,790$

$\beta_3 = 1.57$

$\beta_4 = 0.0382$

$\beta_5 = 38380$

$\beta_6 = 0.00817$

$\beta_7 = 1.0$

$\beta_8 = 12.0$

Solution by geometric programming

The problem consists of a total of 8 terms and 4 variables. Hence it has 3 degrees of freedom. The dual problem to be maximized has the same form as Eqn. (11) which is subject to normality and orthogonality constraints having the form of Eqn. (8) to Eqn. (10) of Chapter II. As the problem has 3 degrees of freedom the dual objective function can be expressed as a function of 3 independent variables r_1, r_2, r_3. Other variables are eliminated by the use of linear equality constraints.

The problem has been solved by maximizing the dual function $V(\delta)$ by using Hooke and Jeeves search procedure. The convergence rate, i.e., the change of value of $V(\delta)$ with the number of functional evaluations during the search is shown in Table 5 and is plotted in Fig. 4.

The problem did not converge to an optimal with the Newton-Raphson method. The initial value $r_i = 1 \quad i = 1, \ldots, 3$ was tried. The difficulty with the Newton-Raphson method was in the first step where the values of r_i violated the positivity constraints

$$\underline{b}^o + \sum_{i=1}^{3} r_i \, \underline{b}^i \geq 0$$

so widely that they could not be corrected.

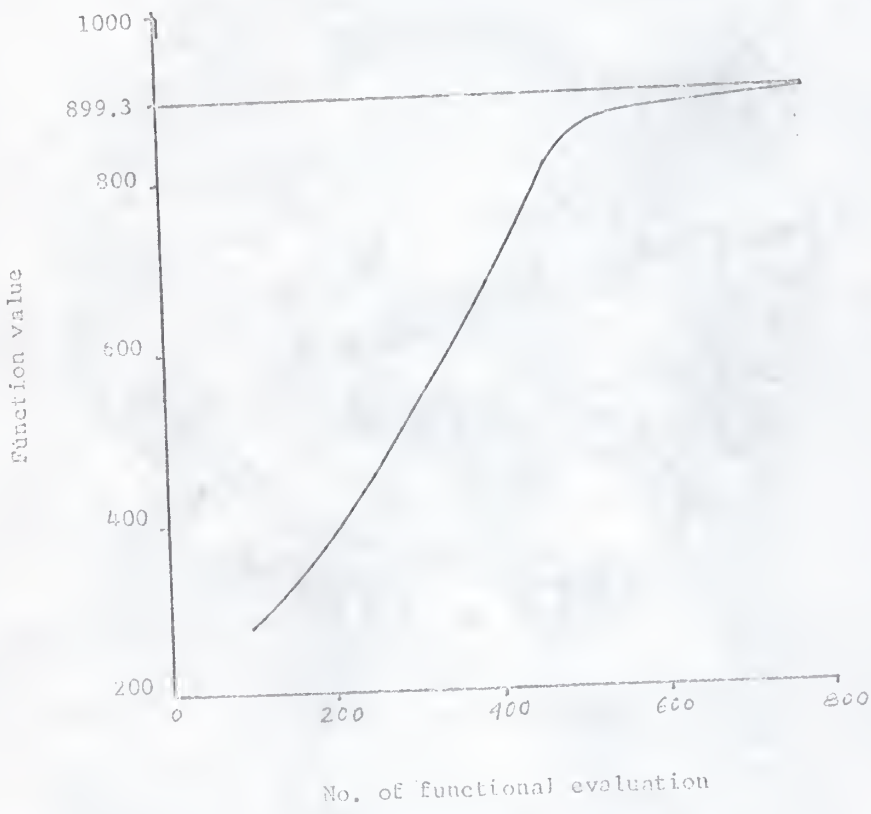


Fig. 4. The convergence rate of the condenser problem by the Hooke and Jeeves method.

Table 5. Convergence rate of the condenser design problem by the Hooke and Jeeves method.

Variables			Function Value	No. of Functional Evaluation
r_1	r_2	r_3		
.810	.210	.790	196.64	5
.800	.370	.775	272.83	99
.620	.365	.595	384.53	200
.440	.355	.415	532.16	304
.360	.355	.335	608.99	352
.270	.345	.245	702.24	403
.190	.340	.165	786.06	449
.100	.330	.075	867.08	501
.085	.325	.060	879.67	555
.083	.326	.057	880.69	602
.067	.325	.041	890.07	650
.045	.322	0.18	898.26	701
.035	.320	.008	899.33	783

The following results were obtained:

Minimum variable annual cost = 899.33 \$/year. The optimal design parameters were:

D_o = 1 inch

D_i = .90 inch

N = 114.16

L = 27.48 feet

The optimal dual variables were:

δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
.1095	.1934	.4564	.0617	.1790	.0351	.3203	.0085

The accuracy, ϵ, for Hooke and Jeeves search was chosen 0.001. The computation time taken was 115 secs.

CHEMICAL EQUILIBRIUM PROBLEM

This model is taken from a paper of Passy and Wilde [13]. According to the minimum free energy principle, a chemical system is in equilibrium at constant pressure and temperature if and only if its free energy is a minimum. To formulate this problem mathematically, let a chemical system have P phases, G_1, G_2, ..., G_p and let the chemical species occuring in phase G_k be A_{k1}, A_{k2}, ..., $A_{k,I(k)}$ where I(k) is the number of species in G_k. Let n_{ki} be the number of moles of species A_{ki} in phase G_k. Then define a row vector N_k representing the composition of phase G_k whose components are n_{k1}, n_{k2}, ..., $n_{I(k)}$. From these construct n:

$$n = [N_1, N_2, \ldots, N_p]$$

In this notation the mass balance equation for each chemical element B_j is given by

$$\sum_{m=1}^{p} \sum_{k=1}^{I(m)} a_{mkj} n_{mk} = b_j \qquad j = 1, 2, \ldots, r \tag{44}$$

where a_{mkj} is the number of atoms of element B_j in chemical species A_{mk}, b_j is the mass in gram atoms of chemical element B_j, and r is the number of different chemical elements in the system.

The Gibbs free energy $G(n)$ of the system is given by

$$G(n) = RT \sum_{k=1}^{p} \sum_{i=1}^{I(k)} n_{ki} \left(\ln \frac{n_{ki}}{N_k} - \ln C_{ki}\right) \tag{45}$$

where $N_k = \sum_{i=1}^{I(k)} n_{ki}$ for $k = 1, \ldots, P$

and the second term is the standard free energy. The equilibrium concentration n^* is found by minimizing $G(n)$ subject to (32) and the natural constraints,

$$n_{ki} \geq 0 \quad \text{for all } i \text{ and } k$$

This minimizing problem, which has a unique solution in the trivial case, is equivalent to the dual geometric programming problem since

$$V(n_{o1}, n) = \exp\left(\frac{-G(n_{o1}, n)}{RT}\right) = \exp\left(\frac{-G(n)}{RT}\right) \tag{46}$$

where (n_{o1}, n) is the vector generated by augmenting one component n_{o1} to the vector n; the corresponding coefficient C_{o1} is set equal to unity.

The mass balance equation can be written as

$$n_{o1} = 1 \tag{47}$$

$$\sum_{m=0}^{p} \sum_{k=1}^{I(m)} A_{mkj} \, n_{mk} = 0 \qquad j = 1, \ldots, r \tag{48}$$

where $a_{oij} = -b_j$ and $I(o) = 1$

These are identified as the normality and orgonality conditions as equations (9) and (10) of Chapter II. Hence the primal objective function of a chemical equilibrium problem can be identified as

$$\prod_{j=1}^{r} t_j^{-b_j} \tag{49}$$

subject to constraints

$$h_m(t) \leq 1 \qquad m = 1, \ldots, P \tag{50}$$

$$\text{where} \quad h_m(t) = \sum_{k=1}^{I(m)} C_{mk} \prod_{j=1}^{r} t_j^{a_{mkj}} \tag{51}$$

and r is the number of different chemical elements B_j, and b_j is the mass of chemical element B_j. t_j is the corresponding primal variable.

The problem discussed here is due to White, Johnson and Danzing [16]. The stoichiometric mixture of hydrazine and oxygen at 3500 ^oK and a pressure of 750 psi is considered. The initial amounts of hydrogen, oxygen and nitrogen are 2, 1 and 1 respectively.

So the objective function is

$$g_o(t) = t_1^{-2} \times t_2^{-1} \times t_3^{-1} \tag{52}$$

$$g_1(t) = C_1\, t_1 + C_2\, t_1^2 + C_3\, t_1^2\, t_2 + C_4\, t_3 + C_5\, t_3^2 + C_6\, t_3\, t_1$$

$$+ C_7\, t_3\, t_2 + C_8\, t_2 + C_9\, t_2^2 + C_{10}\, t_1\, t_2 \le 1 \tag{53}$$

The various possible constituents at equilibrium and the corresponding C_s are given as

$H = 4.411 \times 10^2$

$H_2 = 2.846 \times 10^7$

$H_2O = 6.160 \times 10^{14}$

$N = 3.703 \circ 10^2$

$N_2 = 7.107 \times 10^{10}$

$NH = 3.225 \times 10^6$

$NO = 2.930 \times 10^6$

$O = 4.471 \times 10^4$

$O_2 = 3.796 \times 10^{11}$

$OH = 4.289 \times 10^9$

putting $y_1 = 10^3\, t_1 \quad y_2 = 10^6\, t_2 \quad y_3 = 10^5\, t_3$

The objective is to minimize

$$g_o(y) = 10^{16}\, y_1^{-2}\, y_2^{-1}\, y_3^{-1} \tag{54}$$

subject to

$$g_1(y) = .4411\ y_1 + 28.46\ y_1^2 + 616\ y_1^2 y_2 + 0.03703\ y_3$$
$$+ 710.7\ y_3^2 + .3225\ y_1 y_3 + 2.93\ y_2 y_3 + .04471 y_2$$
$$+ 0.3796\ y_2^2 + 4.289\ y_1 y_2 \leq 1 \qquad (55)$$

Solution by geometric programming

The problem has 11 terms and 3 variables, and hence a degree of freedom 7. The problem is solved by geometric programming using Hooke and Jeeves search procedure. As the degree of freedom is 7, the dual problem can be expressed as a function of 7 independent variables, r_i, $i = 1, \ldots, 7$. The convergence rate of Hooke and Jeeves is shown in Table 6 and the same data is plotted in Fig. 5.

The Newton-Raphson method did not converge for this problem. An initial value of $r_i =$, $i = 1, \ldots, 7$ was tried. The difficulty with the Newton-Raphson method occurred after the first step when the value of the variables r_i violated the positivity constraints $\underline{b}^o + \sum_{i=1}^{7} r_i\, \underline{b}^i \leq 0$ so widely that they could not be corrected.

The computation time for Hooke and Jeeves procedure was 239 secs. The accuracy ϵ was chosen as 0.001.

The following values of primal variables were obtained at the optimum.

y_1	y_2	y_3
0.056229	0.245371	0.025817

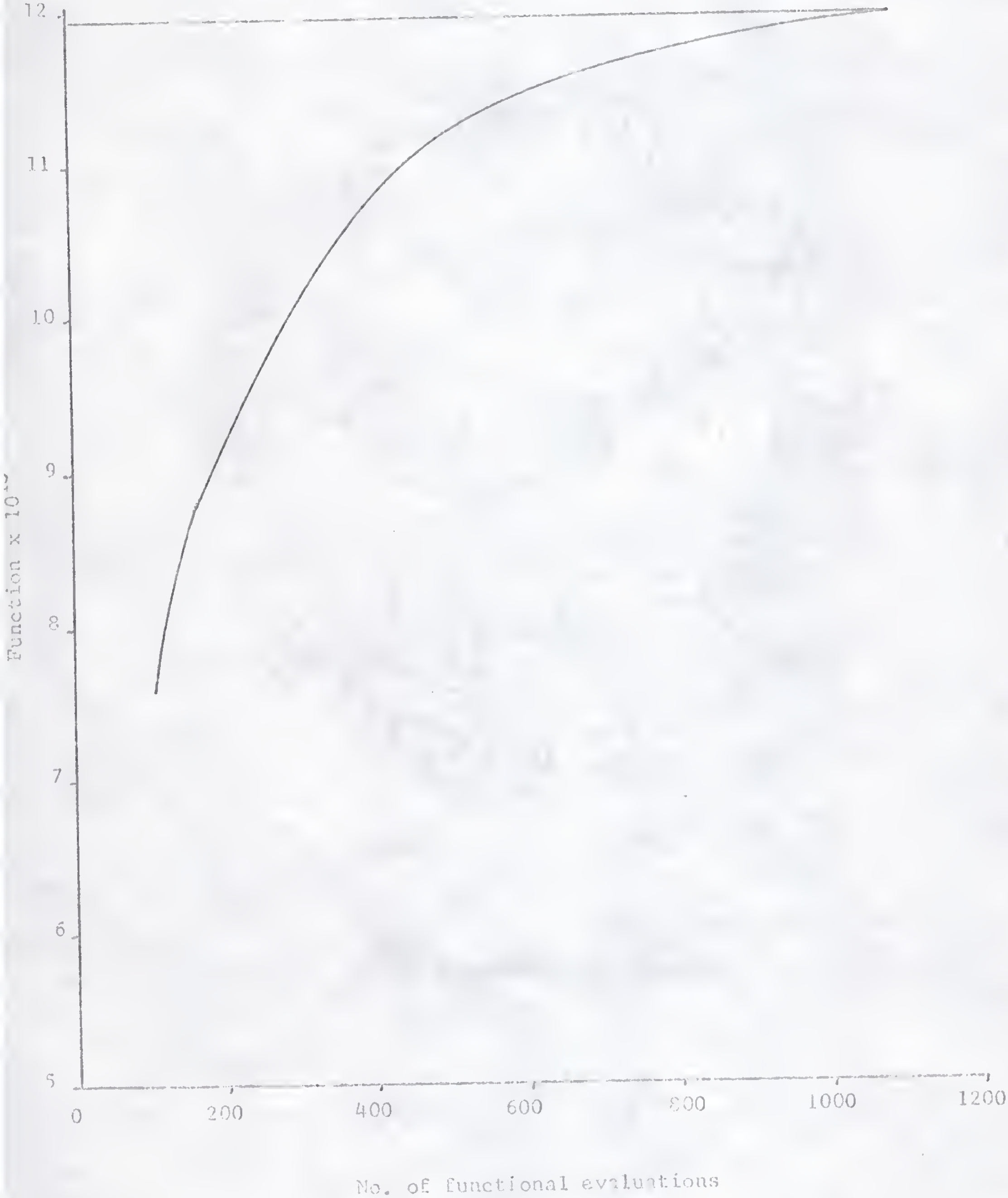


Fig. 5. The convergence rate of the chemical equilibrium problem by the Hooke and Jeeves method.

Table 6. Convergence rate of the chemical equilibrium problem by the Hooke and Jeeves method.

Variables							Function Value x 10^{13}	No. of functional evaluation
r_1	r_2	r_3	r_4	r_5	r_6	r_7		
.150	.250	.150	.050	.100	.100	.150	1.08	14
.050	.400	.050	.100	.050	.050	.050	7.61	112
.050	.412	.012	.150	.012	.012	.075	9.49	224
.044	.431	.006	.125	.012	.019	.081	10.28	318
.050	.447	.003	.097	.016	.022	.084	10.89	404
.044	.459	.003	.075	.016	.028	.087	11.30	522
.044	.461	.002	.073	.016	.028	.089	11.35	606
.056	.473	.002	.048	.017	.034	.094	11.62	701
.050	.478	.002	.041	.017	.036	.095	11.71	804
.044	.483	.001	.032	.018	.037	.096	11.77	1001
.041	.486	.001	.026	.018	.037	.098	11.79	1091

The optimum value of objective function was 11.79×10^{13}. The optimal dual variables were

$\delta_1 = 1.00$

$\delta_2 = .002$

$\delta_3 = .146$

$\delta_4 = .784$

$\delta_5 = .041$

$\delta_6 = .486$

$\delta_7 = .001$

$\delta_8 = .026$

$\delta_9 = .018$

$\delta_{10} = .037$

$\delta_{11} = .097$

The dual variables δ_2 to δ_{11} represent the optimum equilibrium concentrations in moles of the corresponding species.

TRANSFORMER PROBLEM

This model is taken from a Westinghouse Research Report [7]. The explanation of the model and identification of variables are not disclosed. The problem is stated as follows:

Minimize

$$U(t) = .2007\, t_3\, t_4\, t_5 + .2597\, t_1\, t_2\, t_6 + 3.69 \times 10^9 t_6 / t_1\, t_2\, t_3^2\, t_4^2 \qquad (56)$$

subject to

$$g_1(t) = 4\, t_1/t_5 + 6\, t_2/t_5 + 4\, t_3/t_5 \leq 1 \qquad (57)$$

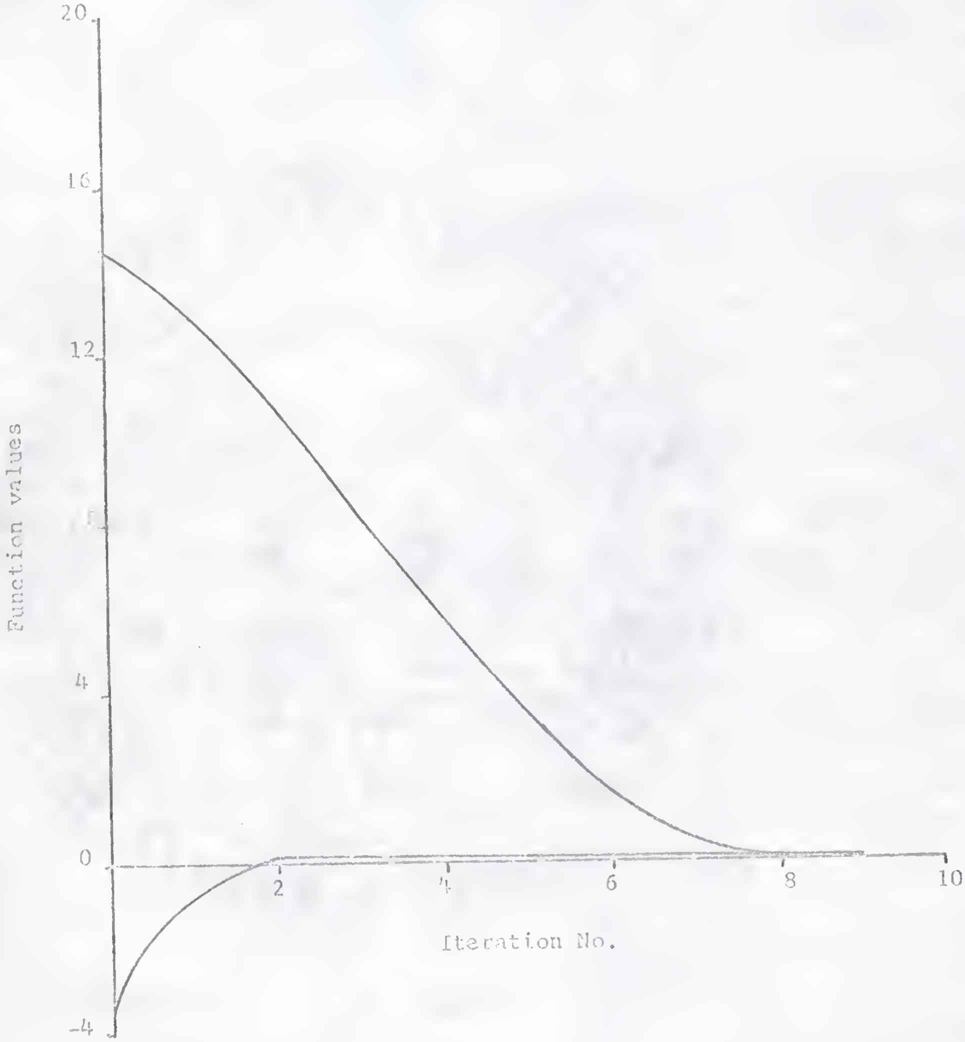


Fig. 6. The convergence rate of the transformer problem by the Newton-Raphson method.

Table 7. Convergence rate of the Transformer problem by the Hooke and Jeeves method.

Variables		$V(\delta)$	No. of Functional Evaluation
r_1	r_2		
.200	.200	55955.00	1
.400	.100	58391.38	10
.325	.075	65932.50	20
.337	.100	66420.06	30
.312	.106	66671.75	40
.322	.100	66694.62	50
.317	.103	66698.93	60
.320	.103	66703.75	70

Table 8. Convergence rate of the Transformer problem by the Newton-Raphson method.

Iteration	Variables		Function Values	
	r_1	r_2	F_1	F_2
0	.1500	.2500	-3.5099	14.5100
1	.2051	.2500	- .8765	12.8205
2	.2243	.[illegible]00	.0999	10.8018
3	.2228	.2497	.0021	8.2646
4	.2248	.2469	.0002	5.9995
5	.2372	.2294	.0065	3.8335
6	.2793	.1691	.0547	1.6322
7	.3173	.1091	.0492	.1677
8	.3193	.1033	.0010	.0004
9	.3193	.1033	0.000	0.000

Table 8a. Computational aspects of problems 1 to 5.

Problem No.	Total No. of terms	No. of variables	Degree of difficulty	Procedure for maximizing dual function			
				Newton - Raphson		Hooke and Jeeves	
				Computer time (sec.)	No. of iterations	Computer time (sec.)	No. of functional evaluation
1	4	2	1	-	-	2.5	30
2	10	7	2	19	4	22	59
3	8	4	3	-	-	115	783
4	11	3	7	-	-	239	1091
5	9	6	2	16	9	13	70

$$g_2(t) = 6\ t_3/t_6 + 6\ t_4/t_6 = 9.424\ t_1/t_6 \leq 1 \tag{58}$$

Solution by geometric programming

The above problem has 9 terms and 6 variables, and hence there are 2 degrees of freedom. The dual function can be expressed with two independent variables r_1 and r_2. The problem has been solved by maximizing the dual function by Hooke and Jeeves procedure. The convergence rate for the search is shown in Table 7. The problem has also been solved by the Newton-Raphson procedure and the convergence rate is shown in Table 8 in which the functions F_1 and F_2 are derivatives of the dual function with respect to r_1 and r_2 respectively. The same data are plotted in Fig. 6. The accuracy is chosen as 0.001 in both the Newton-Raphson and the Hooke and Jeeves procedure.

The following results are obtained:

Minimum value of the objective function is 66703.93

Optimum primal variables are:

t_1	t_2	t_3	t_4	t_5	t_6
19.074	29.690	42.524	6.601	426.532	70.625

And the optimum dual variables are:

δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	δ_9
.43	.19	.38	.08	.18	.17	.15	.32	.10

PRODUCTION - INVENTORY PROBLEM

This is a hypothetical model in which the optimum policy regarding production and inventory levels are to be determined

Let

$$x(t) = I(t) \tag{62}$$

$$z(t) = P(t) \tag{63}$$

Then Equation (59) becomes

$$dx(t)/dt = z(t) - Q(t)$$

From this the difference equation can be written as

$$x(t + \Delta t) = x(t) + (z(t) - Q(t))\Delta t \tag{64}$$

Dividing the entire time period into five stages, the integral equation for cost (50) can be written as

$$C_T = \sum_{i=1}^{5} [C_I(I_m - x_i)^2 + C_p(P_m - z_i)^2]\Delta t \tag{65}$$

and difference Eqn. (64) can be written as

$$x_i = x_{i-1} + (z_i - Q_i)\Delta t \quad i = 1, \ldots, 5 \tag{66}$$

The numerical values assumed are

$a = 2$	$b = 1$	$c = 5$
$C_I = 0.1$	$I_m = 10$	$P_m = 5$
$C_p = .01$	$t_o = 0$	$t_f = 1$
$t = .2$		

To apply geometric programming the objective function has to be expressed in polynomial form. Thus rewriting Eqn. (65)

$$C_T = \sum_{i=1}^{5} C_I \Delta t\, x_i^2 + \sum_{i=1}^{5} C_p \Delta t\, z_i^2 - \sum_{i=1}^{5} 2\, C_I\, I_m \Delta t\, x_i - \sum_{i=1}^{5} 2\, C_p\, P_m \Delta t\, z_i + \text{Constant} \tag{67}$$

where Constant $= C_I I_m^2 + C_p P_m^2$

The problem is to minimize the variable portion of Eqn. (67) subject to the constraints (66). An attempt was made to solve the problem by geometric programming. The problem has 34 terms and 10 variables, and hence the degree of freedom is 23. The problem is not in posynomial form; hence the extension of geometric programming as discussed in Chapter II had to be used. The attempt was unsuccessful because of the difficulty in obtaining an initial feasible solution r_j. As the problem has 23 degrees of freedom. A feasible solution of r_j, j = 1, 23 has to be found which satisfies the inequality constraints given by Equations (21) and (24) of Chapter II, which are

$$\infty > \delta_{kt} \geq 0 \tag{68}$$

and $$\delta_{ko} = \sigma_k \sum_{t=1}^{T_k} \sigma_{kt} \delta_{kt} \geq 0 \quad k = 1, \ldots, p. \tag{69}$$

where

$$\underline{\delta} = \underline{b}^o + \sum_{j=1}^{d} r_j \underline{b}^j$$

$$\sigma_k = \pm 1$$

$$\sigma_{kt} = \pm 1$$

and d is degree of freedom.

The problem has 34 terms and 5 constraints. So an initial solution of $\underline{r}$ has to satisfy 39 inequality constraints. The Hooke and Jeeves search was used to maximize each δ_{ko} given by Equation (69) subject to constraints (68) until $\delta_{ko} \geq 0$. Hooke and Jeeves search is not efficient in handling a large number of inequality constraints. Due to this difficulty an initial feasible solution could not be obtained by this method.

The above problem was solved successfully by Lagrangian polynomial optimization technique [3]. To use this technique, the problem has to be expressed in the following form

$$\text{Minimize } g_o \tag{70}$$

$$\text{subject to} \quad g_m = 1 \qquad m = 1, \ldots, M \tag{71}$$

$$\text{where} \quad g_m = \sum_{t=1}^{T_m} \sigma_{mt} C_{mt} \prod_{n=1}^{N} X_n^{A_{mtn}} \qquad m = 0, 1, \ldots, M \tag{72}$$

where $C_{mt} > 0$

A_{mtn} is any real number

$\sigma_{mt} = \pm 1$

The objective function of the above problem given by Equation (67) is in the form of (70). The constraints as given by Eqn. (66) can be transformed according to the requirement for this technique as follows:

rewriting Equation (66) as

$$\frac{x_1}{x_o - Q_1 t} - \frac{z_1 t}{x_o - Q_1 t} = 1 \tag{73}$$

and

$$\frac{x_{i-1}}{Q_i t} - \frac{x_i}{Q_i t} + \frac{z_i}{Q_i t} = 1 \qquad i = 2, \ldots, 5 \tag{74}$$

constraints (73) and (74) are in the required form and the algorithm as described in Chapter III can directly be applied.

A difficulty in this problem, was that the function overshot the minimum for some starting values because of too large a step size. This difficulty was overcome by using a forcing procedure which restricts the step size to a certain maximum of the variables. Various limits were tried on different starting values. The number of iterations required to converge to the optimum for each of these cases is given in Table 11. The convergence rates for two typical starting values are given in Tables 9 and 10. The optimum values of productions and inventories are given in Table 12, and are plotted in Figs. 7 and 8. The function is assumed to have converged when all components of the error vector as defined in Chapter III are less than or equal to 0.0001.

The optimum total cost obtained is

$$C_T = .374$$

Table 9. Typical convergence rate of the inventory problem with starting values $x_i = 10$, $i=1,\ldots 10$.

Iteration No.	Cost Percentage forcing			
	No forcing	20	50	100
0	-9.9999	-9.9999	-9.9999	-9.9999
1	-9.8864	-9.9290	-9.8864	-9.8864
2	-9.8738	-9. 616	-9.8738	-9.8738
3	-9.9759	-9.8750	-9.8759	-9.8759
4	-9.8759	-9.8759	-9.8759	-9.8759
5	----	-9.8759	-9.8759	-9.8759

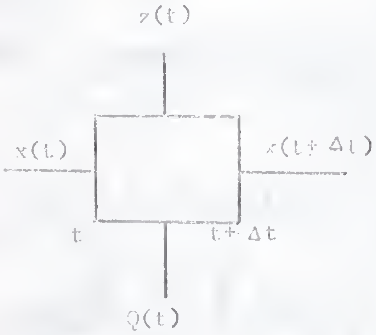
Table 10. Typical convergence rate for the inventory problem with starting values $x_i = 5.0 \quad i = 1, \ldots, 10.$

Iteration No.	Cost			
	Percentage forcing			
	No forcing	20	50	100
0	-7.7499	-7.7499	-7.7499	-7.7499
1	703.2946	-8.6153	-9.2914	-9.5404
2	1022.4440	-9.3822	-7.4436	-9.6252
3	378.3039	-9.0118	-8.2156	-9.8360
4	93.0727	-9.5885	-6.3799	-9.8680
5	9.3208	-9.6459	-7.7937	-9.8754
6	-7.2251	-9.0581	-7.4099	-9.8758
7	-9.0035	-8.9299	-7.4086	-9.8758
8	-8.2692	-9.3047	-7.3310	
9	-9.5499	-9.3213	-7.3515	
10	-9.7332	-9.2770	-7.3502	
11	-9.7480	-9.1200	-7.3435	
12	-9.7644	-9.1454	-7.[illegible]397	
13	-9.7679	-9.1501	-7.[illegible]374	
14	-9.7678	-9.1504	-7.3360	
15	-9.7677	-9.1506	-7.3353	
16	-9.7677	-9.1503	-7.3348	

Table 11. Effect of forcing on convergence.

Starting Value	No. of Iterations				
	Percentage forcing				
	No forcing	10	20	50	100
1	No. Conv.	--	No. Conv.	No. Conv.	No. Conv.
4.0	10	--	No. Conv.	--	--
4.4	9	--	--	--	--
5.0	No. Conv.	20	No. Conv.	No. Conv.	8
6.0	6	--	--	--	--
6.5	5	--	--	--	--
8	No. Conv.	--	7	7	No. Conv.
9	No. Conv.	--	7	11	No. Conv.
10	4	--	5	5	5

Table 12. Optimum inventory and production levels.



t - t + t	x (t)	z (t)	Q (t)
0.0 - 0.2	5.0	14.01	2 [illegible].0
.2 - .4	7.38	8.77	2.30
.4 - .6	8.68	[illegible].1[illegible]	2.50
.6 - .8	9.40	[illegible].93	2.70
.8 - 1.	9.84	4.62	2.90
1. -	10.20	--	

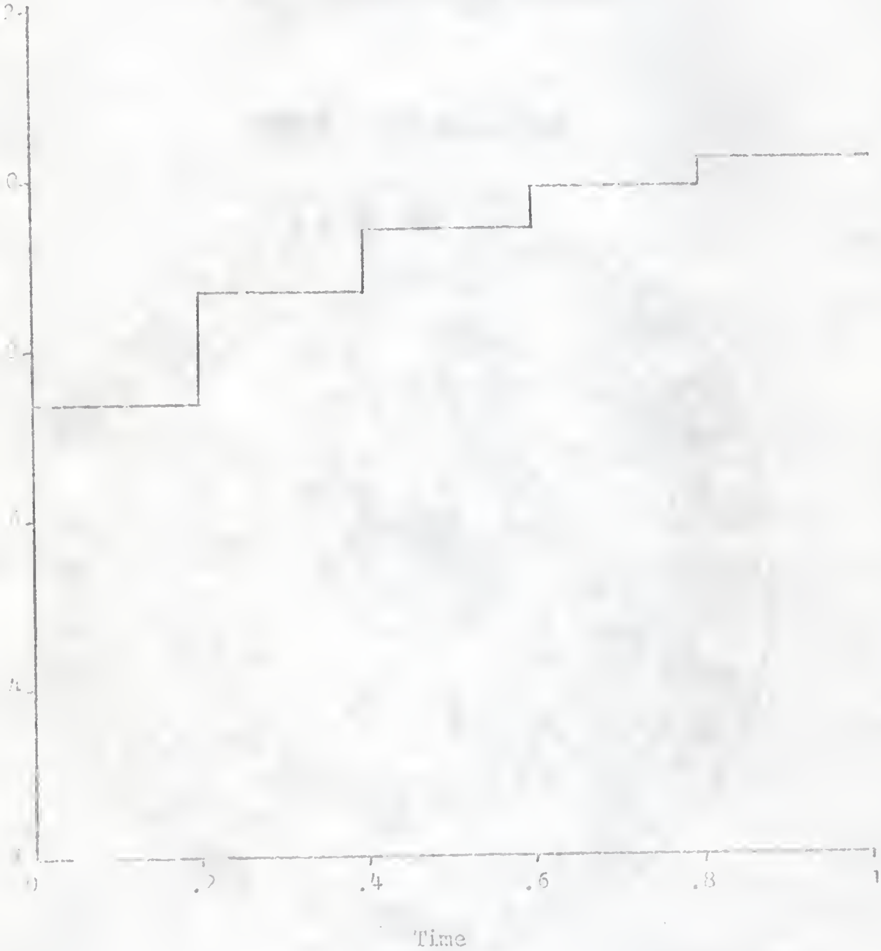


Fig. 7. Optimum inventory levels.

Fig. 8. Optimum production levels.

VI. DISCUSSION

The geometric programming algorithm in its present form can handle a large class of problems often found in practice.

In the posynomial case the method always produces a global minimum, not just a relative minimum. The minimum is equal to the maximum of the dual function whose constraints are linear. If the primal problem has zero degree of difficulty, the solution of the dual problem, hence the solution of primal problem, is obtained by solving a system of linear equations. In the case of zero degree of difficulty, each term in the optimal objective function has an invariant weight represented by the unique solution of the linear constraints. The mathematical importance of this property is that the weight of each term in the objective function is independent of the coefficients.

The extension of geometric programming, as developed by Wilde and Passy by using Kuhn-Tucker conditions, is applicable to any problem involving generalized polynomials. But any deviation from the full posynomial situation invalidates the arithmetic-geometric mean inequality and its useful applications. The optimal weights occur at stationary points of unspecified character in the general case, and this precludes direct search. Another difficulty in the general case is that one no longer has a guarantee that the solution obtained by working with the dual function corresponds to a minimum of the objective.

The existing theory of generalized geometric programming for polynomial optimization gives no way to compute optima except in the special case where there is exactly one more term there are independent variables. Also the theory is formulated in terms of inequality constraints, although the physical restrictions occuring in practice are more often strict equalities. The Lagrangian algorithm for generalized polynomial optimization [3] is suitable for equality constrained problems, which was shown by rapid convergence for the production scheduling model with 23 degrees of freedom.

The convergence is not guaranteed for the above method. Moreover, even when the algorithm does converge, the point found may be a saddle point or even a maximum. Finally, a local minimum may not be the global minimum.

Sometimes during the initial iterations, the method takes too big a step and overshoots the minimum. This results in no convergence. This difficulty was overcome by restricting the step size to a predetermined percent of the variable.

Despite these difficulties, both geometric programming and the Lagrangian algorithm can be regarded as poineering fields in nonlinear optimization with nonlinear constraints and they have great potentials in engineering design and system analysis.

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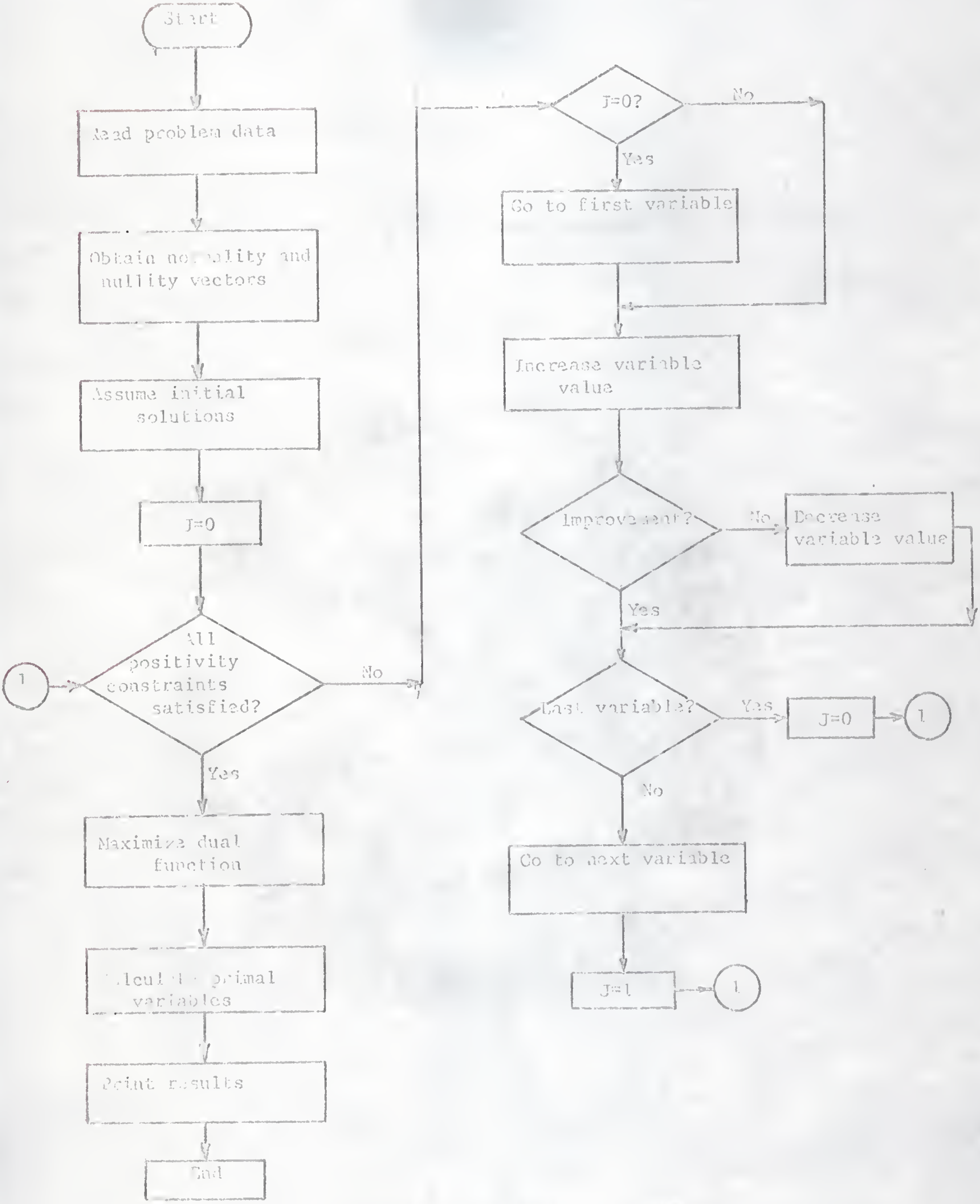

Fig. 9. Flow diagram for Geometric programming

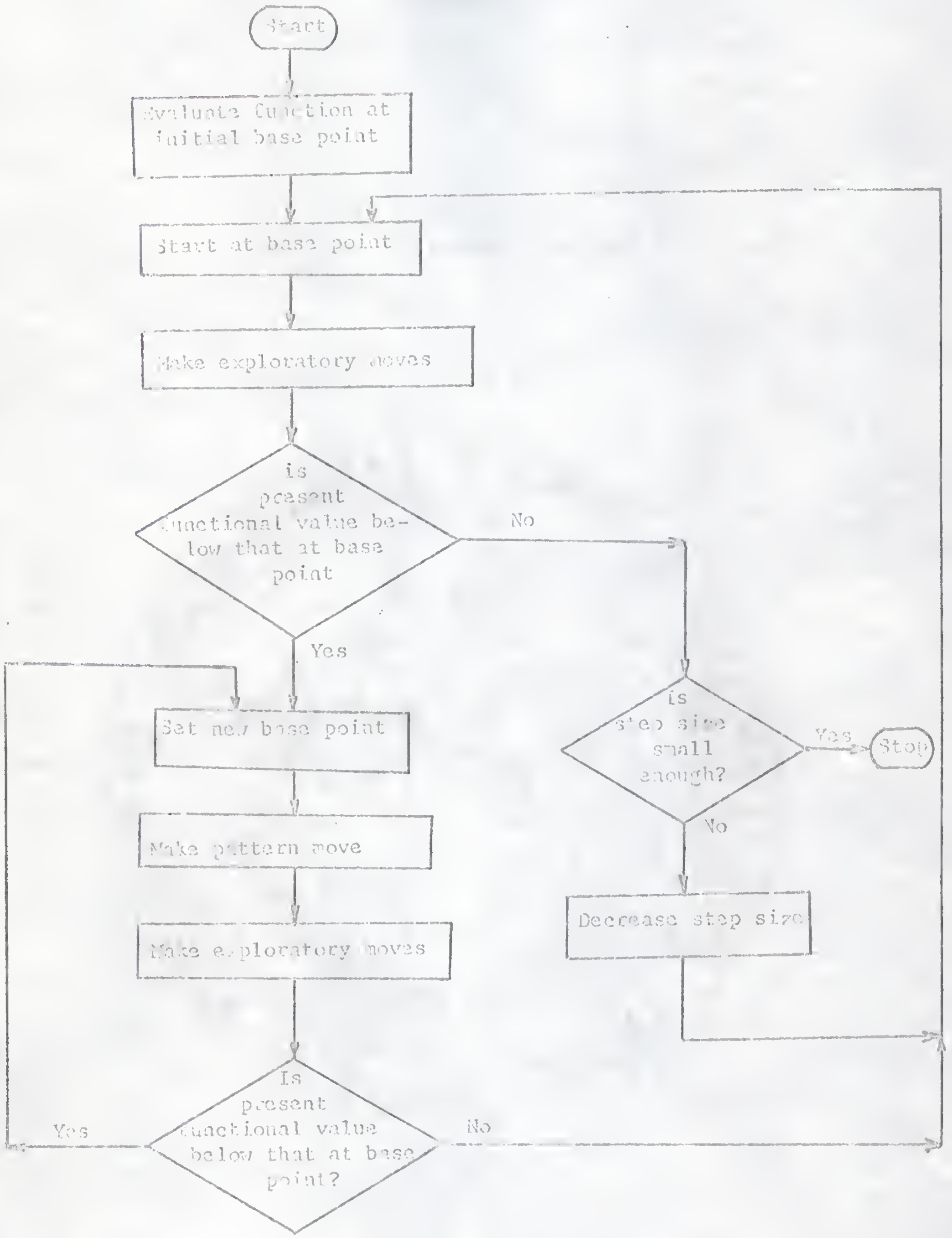

Fig. 10. Flow diagram for Hooke and Jeeves pattern search. [10]

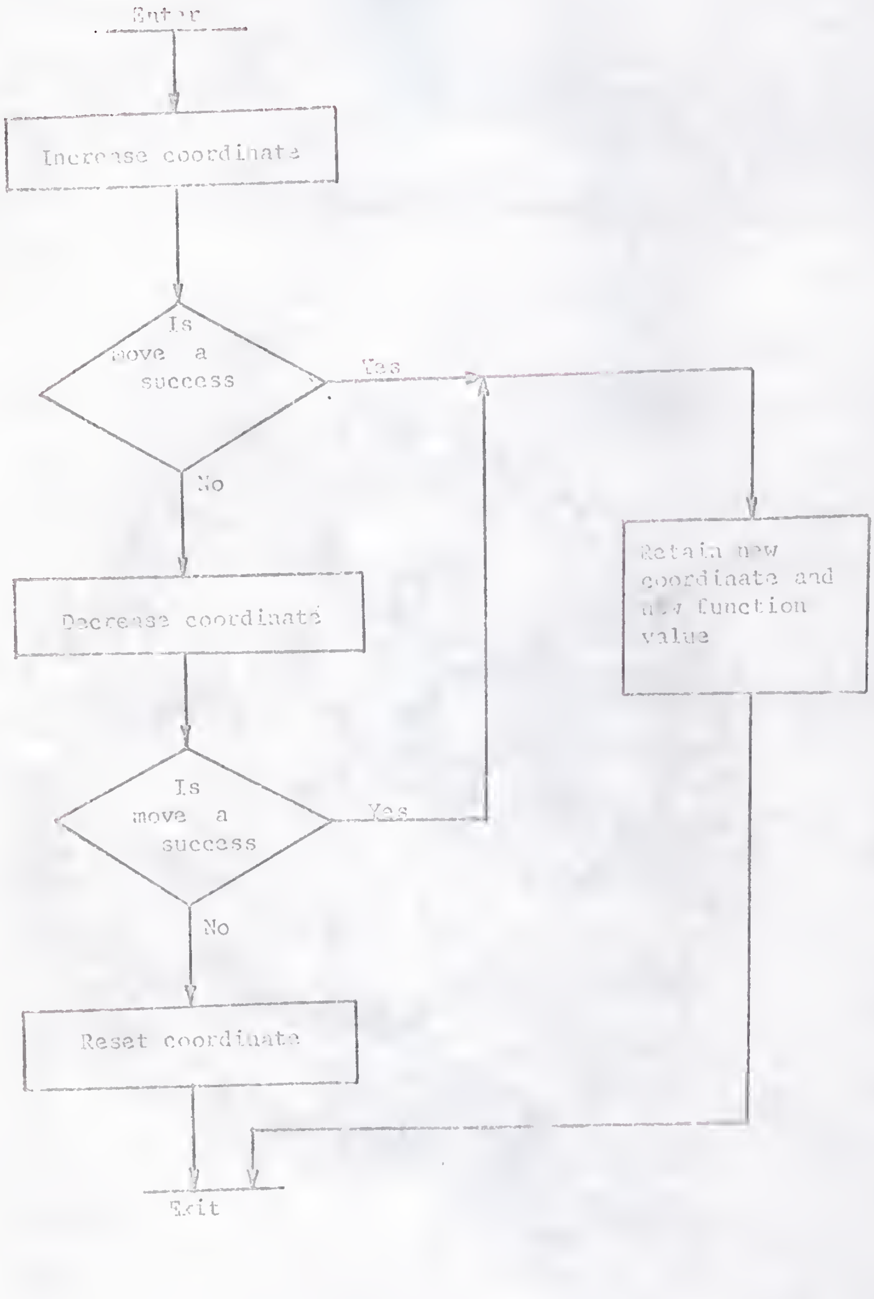

Fig.11. Flow diagram for exploratory moves. [10]

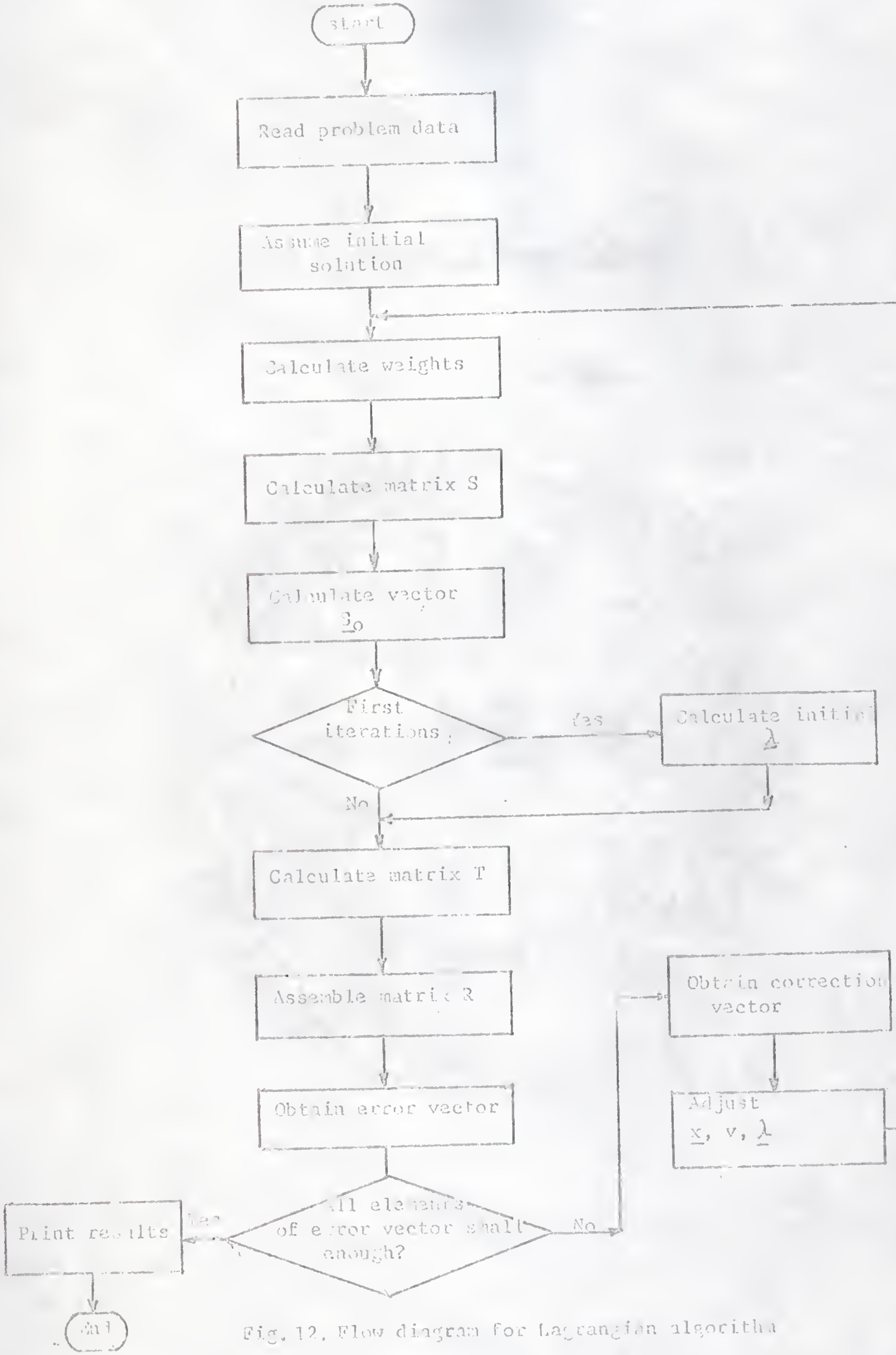

Fig. 12. Flow diagram for Lagrangian algorithm

APPENDIX B

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$JOB          GEOMETRIC,RUN=CHECK,TIME=15,PAGES=60
C
C
C      THE COMPUTER PROGRAM FOR
C      THE SAMPLE PROBLEM BY GEOMTRIC PROGRAMMING
C      USING HOOKE AND JEEVES PATTERN SEARCH
C
      DIMENSION A(50,25),AA(35,45),A1(30),A2(30),H(700),L2(30),L3(30),
     1CM(50),ROLD(35),      E(5),BASE1(35),BASE2(35),B1(50),B2(50),FA(5)
      COMMON KA(20),KB(20),SUM(20),DEL(50),SIG(50),BET(20),ALAM(50),
     1C(50),R(35),S(50),B(50,35),NX,N3,K1,NN,KOUNT,KX
    1 FORMAT (3I3)
   11 FORMAT (I4)
   12 FORMAT (1H-,'      OPTIMUM ANNUAL COST'F25.2)
   13 FORMAT (2F3.0)
   18 FORMAT ('      EXPONENT MATRIX SINGULAR')
   22 FORMAT (F22.6)
      READ 1,M,NX,K1
      READ 11,(KA(I),I=1,K1)
      READ 11,(KB(I),I=1,K1)
      READ 13,((A(I,J),J=1,M),I=1,NX)
      KOUNT=0
      N1=NX-M-1
      N2=M+1
      N3=N1+1
      N4=N2+1
      NM=KA(1)-1
      ND=KA(1)
      DO 600 I=1,15
  600 SIG(I)=1.
      BET(1)=1.
      BET(2)=1
      C(1)=1000.
      C(2)=4.*(10.)**9
      C(3)=2.5*(10.)**5
      C(4)=9000.
      DO 100 I=1,NX
      DO 100 J=1,M
  100 AA(J+1,I)=A(I,J)*SIG(I)
      DO 36 J=1,NM
   36 AA(1,J)=SIG(J)
      DO 101 J=ND,NX
  101 AA(1,J)=0.
      DO 98 I=1,N2
      DO 98 J=1,N2
      NA=N2*(J-1)
   98 H(I+NA)=AA(I,J)
      A1(1)=1.
      DO 102 I=2,N2
  102 A1(I)=0.
C      OBTAINING NORMALLITY AND NULLITY VECTORS
C      THE SUBROUTINES USED HERE ARE PROVIDED BY IBM
      CALL MINV (H,N2,D,L2,L3)
      IF(D.EQ.0.) GO TO 125
      CALL GMPRD (H,A1,B1,N2,N2,1)
      J1=1
      DO 104 J=N4,NX
      J1=J1+1
      DO 103 I=1,N2
  103 A2(I)=-AA(I,J)
```

```
      CALL GMPRD (H,A2,B2,N2,N2,1)
      DO 104 I=1,N2
  104 B(I,J1)=B2(I)
      DO 50 I=1,N2
   50 B(I,1)=B1(I)
      I1=0
      DO 51 I=N4,NX
      I1=I1+1
      DO 51 J=1,N3
      IF(J-I1-1) 52,53,52
   53 B(I,J)=1.
      GO TO 51
   52 B(I,J)=0.
   51 CONTINUE
      DO 5 I=1,N1
    5 R(I+1)=.8
C     OBTAINING INITIAL FEASIBLE SOLUTION
      I1=1
      ITER=0
      NTER=0
  126 DO 128 J=2,N3
  128 ROLD(J)=R(J)
  127 L1=1
      J1=2
      NC=1
  200 DO 111 I=1,NX
      S(I)=0.
      DO 110 J=2,N3
  110 S(I)=S(I)+R(J)*B(I,J)
      DEL(I)=B(I,1)+S(I)
  111 CONTINUE
      IF(ITER-1000) 210,129,129
  210 IF(DEL(I1)) 206,207,207
  206 CH(I1)=DEL(I1)
      IF(NC) 217,217,218
  217 IF(CH(I1)-OLD) 218,209,209
  218 GO TO (310,211),L1
  310 R(J1)=ROLD(J1)-.1
      ITER=ITER+1
      NC=0
      OLD=CH(I1)
      L1=2
      GO TO 200
  211 R(J1)=ROLD(J1)+.1
      ITER=ITER+1
      GO TO 200
  209 OLD=CH(I1)
      ROLD(J1)=R(J1)
      L1=1
      NC=1
      IF(J1-N3) 214,220,209
  214 J1=J1+1
      GO TO 200
  220 J1=2
      GO TO 200
  207 I1=1
      ROLD(J1)=R(J1)
  212 IF(DEL(I1)) 127,213,213
  213 IF(I1-NX) 215,216,216
  215 I1=I1+1
```

```
      GO TO 212
  216 CONTINUE
C     HOOKE AND JEEVES PATTERN SEARCH
      DE=.1
      DO 189 L=2,N3
  189 BASE1(L)=R(L)
  190 CALL FUNC(V)
      T=V
      L=1
      N=1
  191 DO 307 J=2,N3
      R(J)=BASE1(J)+DE
      CALL FUNC(V)
      GO TO (301,302),N
  301 IF(V-T) 303,303,304
  302 IF(V-ALD) 305,305,304
  303 ALD=T
  305 R(J)=BASE1(J)-DE
      CALL FUNC(V)
      IF(V-ALD) 306,306,304
  306 ROLD(J)=BASE1(J)
      R(J)=BASE1(J)
      GO TO 307
  304 ROLD(J)=R(J)
      ALD=V
  307 N=2
      GO TO (312,313),L
  313 IF(ALD-VMAX) 315,315,309
  312 IF(ALD-T) 308,308,309
  308 IF(DE-.001) 410,410,311
  311 DE=DE/2.
      DO 528 J=2,N3
  528 R(J)=ROLD(J)
      GO TO 191
  309 VMAX=ALD
      DO 314 J=2,N3
      BASE2(J)=ROLD(J)
  314 BASE1(J)=2.*BASE2(J)-BASE1(J)
      L=2
      DO 529 J=2,N3
  529 R(J)=BASE1(J)
      NTER=NTER+1
      IF(NTER-1000) 191,191,129
  315 DO 316 J=2,N3
  316 BASE1(J)=BASE2(J)
      DO 130 J=2,N3
  130 R(J)=BASE1(J)
      NTER=NTER+1
      IF(NTER-1000) 190,190,129
  410 DO 317 J=2,N3
  317 R(J)=BASE1(J)
      VD=T
      PRINT 12,VD
C     OBTAINING OPTIMUM PRIMAL VARIABLES
      X1=DEL(1)*VD/C(1)
      X2=DEL(3)/(ALAM(1)*C(3))
      PRINT 22,X1
      PRINT 22,X2
      GO TO 119
  129 CONTINUE
```

```
      GO TO 119
  125 PRINT 18
  119 STOP
      END
```

```
      SUBROUTINE FUNC(V)
      COMMON KA(20),KB(20),SUM(20),DEL(50),SIG(50),BET(20),ALAM(50),
     1C(50),R(35),S(50),B(50,35),NX,N3,K1,NN,KOUNT,KX
   22 FORMAT (F22.6,I5,F8.3)
      SIGMA=1.
      KOUNT=KOUNT+1
      DO 111 I=1,NX
      S(I)=0.
      DO 110 J=2,N3
  110 S(I)=S(I)+R(J)*B(I,J)
  111 DEL(I)=B(I,1)+S(I)
      I1=1
  121 IF(DEL(I1)) 117,117,118
  118 IF(I1-NX)119,120,120
  119 I1=I1+1
      GO TO 121
  117 V=-999
      RETURN
  120 DO 108 K=1,K1
      L7=KA(K)
      L8=KB(K)
      SUM(K)=0.
      DO 20 I=L7,L8
   20 SUM(K)=SUM(K)+SIG(I)*DEL(I)
      ALAM(K)=BET(K)*SUM(K)
      IF(ALAM(K).LT.0.) GO TO 117
  108 CONTINUE
      P1=1.
      DO 112 I=1,NN
      IF(DEL(I).EQ.0.) GO TO 112
      P1=P1*(C(I)/DEL(I))**(SIG(I)*DEL(I))
  112 CONTINUE
      P2=1.
      DO 114 K=1,K1
      L7=KA(K)
      L8=KB(K)
      DO 114 I=L7,L8
      IF(DEL(I).EQ.0.) GO TO 114
      P2=P2*(C(I)*ALAM(K)/DEL(I))**(SIG(I)*DEL(I))
  114 CONTINUE
      V=SIGMA*(P1*P2)**SIGMA
      PRINT 22,V,KOUNT,R(2)
      RETURN
      END
```

```
$JOB           GEOMETRIC,RUN CHECK,TIME=5,PAGES=50
C      COMPUTER PROGRAM FOR SEA POWER PROBLEM BY GEOMETRIC PROGRAMMING
C      USING THE NEWTON RAPHSON PROCEDURE
       DIMENSION C(50),A(50,25),AA(25,25),A1(25),A2(25),H(700),B(50,30),
      1B1(50),B2(50),AK(30),DLAM(30,30),R(30),S(50),SS(30,30),SA(30),
      1SB(30,30),AE(30,30),SC(30),SD(30),F(30),AF(30,30),F1(30),EPS(30),
      1DEL(50),ALAM(30),L2(30),L3(30),CH(50),ROLD(30),T(5),E(5),SIG(50),
      1BET(20),KA(20),KB(20),SUM(20)
   10 FORMAT (3I3)
   11 FORMAT (4I4)
   12 FORMAT (F12.3)
   13 FORMAT (7F3.0)
   14 FORMAT (I5)
   15 FORMAT (7F8.3)
   18 FORMAT ('      EXPONENT MATRIX SINGULAR')
      READ 10,M,N,K1
      READ 11,(KA(I),I=1,K1)
      READ 11,(KB(I),I=1,K1)
      READ 13,((A(I,J),J=1,M),I=1,N)
      N1=N-M-1
      N2=M+1
      N3=N1+1
      N4=N2+1
      C(1)=1.
      C(2)=40.
      C(3)=0.18
      C(4)=44.5
      C(5)=0.00000006
      C(6)=0.0000000215
      C(7)=1.
      C(8)=1.
      C(9)=1.
      C(10)=1.
      DO 44 I=1,10
   44 SIG(I)=1.
      DO 30 I=1,4
   30 BET(I)=1.
      DO 99 I=1,N1
   99 F1(I)=0.
      DO 100 I=1,N
      DO 100 J=1,M
  100 AA(J+1,I)=A(I,J)
      AA(1,1)=1.
      DO 101 J=2,N
  101 AA(1,J)=0.
      DO 98 I=1,N2
      DO 98 J=1,N2
      NA=N2*(J-1)
   98 H(I+NA)=AA(I,J)
      A1(1)=1.
      DO 102 I=2,N2
  102 A1(I)=0.
C     OBTAINING THE NORMALITY AND NULLITY VECTORS
C     THE SUBROUTINES USED HERE ARE PROVIDED BY IBM
      CALL MINV (H,N2,D,L2,L3)
      IF(D.EQ.0.) GO TO 125
      CALL GMPRD (H,A1,B1,N2,N2,1)
      J1=1
      DO 104 J=N4,N
      J1=J1+1
```

```
      DO 103 I=1,N2
  103 A2(I)=-AA(I,J)
      CALL GMPRD (H,A2,B2,N2,N2,1)
      DO 104 I=1,N2
  104 B(I,J1)=B2(I)
      DO 50 I=1,N2
   50 B(I,1)=B1(I)
      I1=0
      DO 51 I=N4,N
      I1=I1+1
      DO 51 J=1,N3
      IF(J-I1-1) 52,53,52
   53 B(I,J)=1.
      GO TO 51
   52 B(I,J)=0.
   51 CONTINUE
      DO 105 J=1,N3
      AK(J)=1.
      DO 105 I=1,N
      AK1=C(I)**(B(I,J)*SIG(I))
  105 AK(J)=AK(J)*AK1
      DO 108 I=1,K1
      L7=KA(I)
      L8=KB(I)
      DO 108 J=1,N3
      SUM(I)=0.
      DO 20 K=L7,L8
   20 SUM(I)=SUM(I)+B(K,J)
      DLAM(I,J)=SUM(I)
  108 CONTINUE
      DO 5 I=1,M1
    5 R(I+1)=.4
C     OBTAINING INITIAL FEASIBLE SOLUTION
      I1=1
      ITER=0
      NTER=0
  126 DO 128 J=2,N3
  128 ROLD(J)=R(J)
  127 L1=1
      J1=2
      NC=1
  200 DO 111 I=1,N
      S(I)=0.
      DO 110 J=2,N3
  110 S(I)=S(I)+R(J)*B(I,J)
      DEL(I)=B(I,1)+S(I)
  111 CONTINUE
      IF(ITER-100) 210,210,129
  210 IF(DEL(I1)) 206,206,207
  206 CH(I1)=DEL(I1)
      IF(NC) 217,217,218
  217 IF(CH(I1)-OLD) 218,209,209
  218 GO TO (310,211),L1
  310 R(J1)=ROLD(J1)-.05
      ITER=ITER+1
      NC=0
      OLD=CH(I1)
      L1=2
      GO TO 200
  211 R(J1)=ROLD(J1)+.05
```

```
      ITER=ITER+1
      GO TO 200
  209 OLD=CH(I1)
      ROLD(J1)=R(J1)
      L1=1
      NC=1
      IF(J1-N3) 214,220,200
  214 J1=J1+1
      GO TO 200
  220 J1=2
      GO TO 200
  207 I1=1
      ROLD(J1)=R(J1)
  212 IF(DEL(I1)) 127,127,213
  213 IF(I1-N) 215,216,216
  215 I1=I1+1
      GO TO 212
  216 DO 112 K=1,K1
      SA(K)=0.
      DO 113 J=2,N3
  113 SA(K)=SA(K)+R(J)*DLAM(K,J)
  112 ALAM(K)=DLAM(K,1)+SA(K)
      DO 121 L=2,N3
      DO 121 K=2,N3
      SS(L,K)=0.
      DO 121 I=1,N
  121 SS(L,K)=SS(L,K)+SIG(I)*B(I,L)*B(I,K)/DEL(I)
      DO 122 I=2,N3
      DO 122 L=2,N3
      SB(L,I)=0.
      DO 122 K=1,K1
  122 SB(L,I)=SB(L,I)+BET(K)*DLAM(K,L)*DLAM(K,I)/ALAM(K)
      DO 114 I=2,N3
      DO 114 J=2,N3
  114 AE(I-1,J-1)=SS(I,J)-SB(I,J)
      DO 118 J=2,N3
      SC(J)=0.
      SD(J)=0.
      DO 115 I=1,N
  115 SC(J)=SC(J)+SIG(I)*B(I,J)*ALOG(DEL(I))
      DO 117 K=1,K1
  117 SD(J)=SD(J)+BET(K)*DLAM(K,J)*ALOG(ALAM(K))
  118 F(J-1)=SC(J)-SD(J)-ALOG(AK(J))
      GO TO 201
  204 DO 116 I=1,N1
      DO 116 J=1,N1
  116 AF(I,J)=AE(I,J)
C     NEWTON RAPHSON PROCEDURE FOR SOLUTION OF SIMULTANEOUS EQUATIONS
      DO 500 J=1,N1
  500 F1(J)=F(J)
      CALL DETR(AE,N1,D1)
      DO 55 I=1,N1
      DO 56 J=1,N1
   56 AF(J,I)=-F(J)
      CALL DETR(AF,N1,D2)
      E1=D2/D1
      R(I+1)=R(I+1)+E1
      DO 57 J=1,N1
   57 AF(J,I)=AE(J,I)
   55 CONTINUE
```

```
      NTER=NTER+1
      IF(NTER-50) 126,126,129
  201 DO 130 I=1,N1
  130 EPS(I)=ABS(F1(I)-F(I))
      I=1
  202 IF(EPS(I)-.01) 203,203,204
  203 I=I+1
      IF(I-N1) 202,202,205
  205 P=1.
      DO 123 I=1,N
  123 P=P*(C(I)/DEL(I))**(STG(I)*DEL(I))
      P1=1.
      DO 124 K=1,K1
  124 P1=P1*ALAM(K)**(ALAM(K)*BET(K))
      VD=P*P1
      PRINT 12,VD
C     OBTAINING OPTIMUM PRIMAL VARIABLES
      AY=DEL(1)*VD/C(1)
      ALF=DEL(7)/(ALAM(4)*C(7))
      BE =DEL(8)/(ALAM(4)*C(8))
      BETD=DEL(9)/(ALAM(4)*C(9))
      GAM=DEL(10)/(ALAM(4)*C(10))
      Q=C(4)*ALAM(3)/(ALF*DEL(4))
      U=Q*ALAM(1)/(AY*BE *DEL(2))
      PRINT 15,AY,ALF,BE,BETD,GAM,Q,U
      GO TO 119
  129 PRINT 14, NTER
      PRINT 14, ITER
      GO TO 119
  125 PRINT 18
  119 STOP
      END
```

```
      SUBROUTINE DETP(A,K,DET)
      DIMENSION A(6,6)
    1 Z=1.0
      DO 9 M=2,K
      IF (A(M-1,M-1))3,4,3
    4 DO 5 I1=M,K
      IF(A(M-1,I1))6,7,6
    7 DET=0.
    5 CONTINUE
      RETURN
    6 I3=M-1
      DO 8 I2=I3,K
      TEMP=A(I2,I3)
      A(I2,I3)=A(I2,I1)
    8 A(I2,I1)=TEMP
      Z=Z*(-1.)
    3 DO 9 I=M,K
      R=A(I,M-1)/A(M-1,M-1)
      DO 9 J=M,K
    9 A(I,J)=A(I,J)-A(M-1,J)*R
      DET=1.0
      DO 18 I=1,K
   18 DET=DET*A(I,I)
      DET=DET*Z
      RETURN
      END
```

```
     $JOB            =CHECK,TIME=30,PAGES=50
     C         COMPUTER PROGRAM FOR THE PRODUCTION SCHEDULING PROBLEM
     C         BY THE LAGRANGIAN ALGORITHM USING 50 PERCENT FORCING
 1             DIMENSION KA(6),X(10),SIG(10,20),A(10,20,10),W(10,20),G(10),S(15,
              115),SB(15),SC(150),L6(25),L7(25),R5(150),R6(150),R7(15),ALAM(15),
              1WB(40),T(15,15),TB(250),R(25,25),TEM(15),E(20),RW(700),COR(50),Q(5
              1),C(8,20),OUT(30)
 2          10 FORMAT (2I3)
 3          11 FORMAT (6I3)
 4          12 FORMAT (10F3.0)
 5          13 FORMAT (5F5.2)
 6          15 FORMAT (F15.6)
 7          16 FORMAT (10F10.4)
 8          17 FORMAT (10F12.6)
 9          18 FORMAT (5F8.2)
10          19 FORMAT (16F8.2)
11          20 FORMAT (1H-,'          ITERATION NUMBER'I5)
12          21 FORMAT (6F10.3)
13          22 FORMAT (10F10.2)
14          23 FORMAT (F15.6)
15             READ 10,MC,NC
16             READ 11,(KA(I),I=1,MC)
17             DO  1 I=1,MC
18             LL=KA(I)
19             DO 61 J=1,LL
20             READ 12,(A(I,J,K),K=1,NC)
21          61 PRINT 12,(A(I,J,K),K=1,NC)
22             READ 13,C1,C2,C3,C4,DT
23             Q(1)=2.10*DT
24             Q(2)=2.30*DT
25             Q(3)=2.50*DT
26             Q(4)=2.70*DT
27             Q(5)=2.90*DT
28             DO 50 J=1,5
29          50 C(1,J)=C1*DT
30             DO 51 J=11,15
31          51 C(1,J)=2.*C1*C2*DT
32             DO 53 J=6,10
33          53 C(1,J)=C3*DT
34             DO 52 J=16,20
35          52 C(1,J)=2.*C3*C4*DT
36             C(2,1)=1/(5-Q(1))
37             C(2,2)=DT/(5-Q(1))
38             C(3,1)=1/Q(2)
39             C(3,2)=1/Q(2)
40             C(3,3)=DT/Q(2)
41             C(4,1)=1/Q(3)
42             C(4,2)= 1/Q(3)
43             C(4,3)=DT/Q(3)
44             C(5,1)=1/Q(4)
45             C(5,2)=1/Q(4)
46             C(5,3)=DT/Q(4)
47             C(6,1)=1/Q(5)
48             C(6,2)=1/Q(5)
49             C(6,3)=DT/Q(5)
50             CONS=C1*C2**2+C3*C4**2
51             DO 56 K=1,10
52          56 SIG(1,K)=1.
53             DO 57 K=11,20
54          57 SIG(1,K)=-1.
```

```
      DO 60 M=2,MC
      L1=KA(M)
      DO 60 K=1,L1
   60 SIG(M,K)=1.
      SIG(2,2)=-1.
      SIG(3,1)=-1.
      SIG(4,1)=-1.
      SIG(5,1)=-1.
      SIG(6,1)=-1.
      DO 62 I=1,10
   62 X(I)=5.0
 1001 I=0
  500 DO 140 M=1,MC
      SUM=0.
      L1=KA(M)
      DO 141 K=1,L1
      P=1.
      DO 142 N=1,NC
  142 P=P*(X(N)**A(M,K,N))
  141 SUM=SUM+SIG(M,K)*C(M,K)*P
  140 G(M)=SUM
      IF(I.GT.1) GO TO 138
      V=G(1)
      L1=KA(1)
      DO 100 K=1,L1
      P=1.
      DO 101 N=1,NC
  101 P=P*(X(N)**A(1,K,N))
  100 W(1,K)=C(1,K)*P/G(1)
  138 DO 102 M=2,MC
      L1=KA(M)
      DO 102 K=1,L1
      P=1.
      DO 103 N=1,NC
  103 P=P*(X(N)**A(M,K,N))
  102 W(M,K)=C(M,K)*P
      DO 105 N=1,NC
      DO 105 M=1,MC
      P2=0.
      L2=KA(M)
      DO 104 K=1,L2
  104 P2=P2+SIG(M,K)*A(M,K,N)*W(M,K)
  105 S(N,M)=P2
      DO 106 N=1,NC
  106 SB(N)=S(N,1)
      KX=0
      MD=MC-1
      DO 107 M=2,MC
      DO 107 N=1,NC
      KX=KX+1
  107 SC(KX)=S(N,M)
      CALL GMTRA (SC,R5,NC,MD)
      CALL GMPRD (R5,SC,OUT,MD,NC,MD)
      CALL MINV (OUT,MD,D,L6,L7)
      CALL GMPRD (OUT,R5,R6,MD,MD,NC)
      CALL GMPRD (R6,SB ,R7,MD,NC,1)
      DO 410 J=1,MD
  410 ALAM(J)=R7(J)
  600 DO 110 N=1,NC
      DO 110 J=1,NC
```

```
      S1=0.
      S2=0.
      S3=0
      L1=KA(1)
      DO 111 K=1,L1
  111 S1=S1+SIG(1,K)*A(1,K,N)*A(1,K,J)*W(1,K)
      DO 112 M=2,MC
      L2=KA(M)
      DO 113 K=1,L2
  113 S2=S2+SIG(M,K)*A(M,K,N)*A(M,K,J)*W(M,K)
  112 S3=S3+ALAM(M-1)*S2
  11[illegible] T(N,J)=S3-S1
      K3=NC+MC
      K2=0
      N=NC
      DO 115 J=1,K3
      K1=1
      DO 115 K=1,K3
      IF(J-N) 116,116,118
  116 IF(K-N) 117,117,124
  117 R(J,K)=T(J,K)
      GO TO 115
  118 IF (J-N-1) 119,119,127
  119 IF(K-N) 120,120,121
  120 R(J,K)=SB(K)
      GO TO 115
  121 IF(K-N-1) 122,122,123
  122 R(J,K)=-1
      GO TO 115
  123 R(J,K)=0.
      GO TO 115
  124 IF(K-N-1) 125,125,126
  125 R(J,K)=SB(J)
      GO TO 115
  126 K1=K1+1
      R(J,K)=S(J,K1)
      GO TO 115
  127 IF(K-N) 128,128,129
  128 K2=K2+1
      R(J,K)=SC(K2)
      GO TO 115
  129 R(J,K)=0.
  115 CONTINUE
      SIGM=G(1)/ABS(G(1))
      CALL GMPRD (SC,ALAM,TEM,NC,MD,1)
      DO 130 J=1,NC
  130 E(J)=SB(J)-TEM(J)
      E(NC+1)=1-G(1)/V
      J2=1
      KE=NC+2
      DO 131 J=KE,K3
      J2=J2+1
  131 E(J)=1-G(J2)
      J=1
  503 IF(ABS(E(J))-.0001) 501,501,502
  501 J=J+1
      IF(J-K3) 503,503,504
  502 J2=0
      DO 132 K=1,K3
      DO 132 J=1,K3
```

```
      J2=J2+1
  132 RW(J2)=R(J,K)
      CALL MINV (RW,K3,D,LA,L7)
      CALL GMPRD (RW,E,COR,K3,K3,1)
      PRINT 20,I
      PRINT 17,(X(N),N=1,NC)
      PRINT 19,(E(N),N=1,K3)
      PRINT 15,G(1)
      IA=0
      DO 133 N=1,NC
      WB(N)=X(N)
  133 X(N)=X(N)*EXP(COR(N))
      PRINT 17,(X(N),N=1,NC)
      DO 150 N=1,NC
      CH=ABS(X(N)-WB(N))
      CH1=X(N)-WB(N)
      IF(CH.GT.(.5*WB(N))) GO TO 1002
      GO TO 150
 1002 AMU=CH1/CH
      X(N)=WB(N)+.5*AMU*WB(N)
      IA=1
  150 CONTINUE
      IF(IA.GT.0) GO TO 1001
      V=V*EXP(COR(NC+1))
      J2=0
      ND=NC+2
      DO 134 M=ND,K3
      J2=J2+1
  134 ALAM(J2)=ALAM(J2)+COR(M)
      L1=KA(1)
      DO 135 K=1,L1
      P=1.
      DO 136 N=1,NC
  136 P=P*X(N)**A(1,K,N)
  135 W(1,K)=C(1,K)*P/V
  145 I=I+1
      GO TO 500
  504 G(1)=G(1)+CONS
      PRINT 15,G(1)
      PRINT 17,(X(N),N=1,NC)
      STOP
      END
```

APPLICATION OF GEOMETRIC PROGRAMMING
TO INDUSTRIAL SYSTEMS

by

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AN ABSTRACT OF A MASTER'S THESIS

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1969

Most of the optimization problems which occur in real-life systems are nonlinear in nature with nonlinear constraints. Conventional optimization techniques can solve these problems only after linearizing the constraints and hence at the sacrifice of the accuracy. Geometric programming is a recently developed technique which can handle very efficiently a subclass of the above problems characterized by functions as polynomials with positive coefficients. Wilde's extension of geometric programming makes geometric programming applicable to even a broader class in which the functions are expressed as generalized polynomials. But difficulty in numerical analysis often restricts the use of extended geometric programming. Wilde's Lagrangian algorithm is useful in cases of generalized polynomials with equality constraints.

The purpose of this thesis is to apply these recently developed techniques to different engineering and industrial management systems and to make a critical analysis of the results and computational procedure used.

First a brief review of geometric programming and its extension is made, and computational procedure is discussed. A review of the Lagrangian algorithm follows. Then various approximation techniques which were applied to transform a general problem to the required form of geometric programming are discussed. Finally six problems are solved. The first problem is for illustration purposes, while the next four are engineering design problems with different degrees of complexity. The last is a production scheduling problem which is solved by the

Lagrangian algorithm.

The advantages as well as disadvantages of geometric programming and the Lagrangian algorithm are highlighted. A modification of the Lagrangian algorithm has been suggested.