A COMPREHENSIVE STUDY ON SOIL CONSOLIDATION

by

SHAHIN NAYYERI AMIRI

B.Sc., University of Tabriz, 2000 M.Sc., University of Tabriz, 2003 M.Phil., University of Tabriz, 2006

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Abstract

In this research, soil consolidation is explored in a comprehensive analytical and experimental study.

The pore pressure development and dissipation for clay at its liquid limit under one-dimensional compression was investigated using the mid-plane pore pressure measurements.

In general, the Terzaghi's theory of consolidation predicted the pore pressure dissipation and the percent consolidation accurately as long as the sample was in a normally loaded state. For a preconsolidated state however, the results obtained by Terzaghi theory are doubtful. Coefficient of consolidation c_{ν} for smaller pressures varied during consolidation, and although the soil was in a fully saturated state for relatively high pressure increments, the pore pressure developed was less than the applied pressure. Then, the effect of different pressure increment ratios on one dimensional consolidation tests has been studied. The secondary compression effects have been founded to increase as the pressure increment ratio is reduced.

Consolidation of a clay layer delimited between sheets with small permeability was also investigated in this study. The consolidation theory of compressible soils usually assumes drainage-free boundaries. This change in boundary conditions at the drainage surface necessitates the use of an approximate technique for solution of the governing partial differential equation. In this study, the solution was obtained by using the Galerkin Method and compared with the "free drainage" case. As expected, the consolidation in the case of restricted drainage proceeds at a much lower rate.

The compression consolidation behavior of trampled clays in a semi- saturated state was also analyzed in this research program. It is generally known that the type and energy of compaction bring about deviations in the soil structure and hence, in its engineering properties. Therefore, in the experimental phase of this study, soils were prepared by different trampling efforts and also by different compaction methods.

Finally, a reasonably realistic theory of soil consolidation has been proposed and the effect of variable permeability and compressibility on the consolidation behavior was investigated followed by a mathematical treatment of the behavior. Subsequently, laboratory consolidation tests with mid-plane pore pressure measurements were conducted on different kinds of clay.

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Dedication

This dissertation is dedicated to *my mother* who has always given me the encouragement to complete all tasks that I undertake and continiously supports me whenever I face any difficulty in my life as well as for her love and sacrifice.

Preface

This study includes a comprehensive study on soil consolidation and is presented in the following episodes:

In the first chapter, the pore pressure development and dissipation under conditions of one dimensional compression in the oedometer for clay prepared at its liquid limit is investigated, making use of the mid-plane pore pressures measured during compression. In general, the following points have been observed a) Terzaghi's theory of consolidation predicts accurately the pore pressure dissipation and the percent consolidation as long as the sample is in a normally loaded state. For a preconsolidated state however, the results obtained by Terzaghi theory may be doubtful. b) Coefficient of consolidation c_v for smaller pressures is observed to vary during consolidation. c) It has been observed that although the soil is in a fully saturated state for relatively high pressure increments, the pore pressure developed is less than the applied pressure.

In the second chapter, an experimental program is carried out to investigate the effect of the pressure increment ratio on the shape of compression time curves and secondary compression characteristics. Therefore, one dimensional consolidation tests on remodeled soft clay using different pressure increment ratios of one or less, with load increment duration of one week, are performed. Five different methods are used to evaluate the coefficient of consolidation c_v which yielded reasonably close values. The secondary compression effects have been found to increase as the pressure increment ratio is reduced. The c_v values and the end of primary void ratio effective stress relationship appear to be independent of the pressure increment ratio.

In the third chapter, consolidation of a clay layer delimited between sheets with small permeability is investigated. The consolidation theory of compressible soils usually assumes that at the boundaries the drainage is free. That is, the surrounding layers are pervious. When free drainage is thus inhabited, the boundary conditions at the drainage surface changes necessitating the use of an approximate technique of solution of the governing partial differential equation. In this chapter, the solution is obtained by using the Galerkin Method. The solution obtained is compared with the "free drainage" case and consolidation is drawn therefrom. As expected, the consolidation in the case of restricted drainage proceeds at a much lower rate.

In the fourth chapter, investigation has been undertaken in order to analyze the compression consolidation behavior of trampled clays in a semi- saturated state. Furthermore, it is known that the type and energy of compaction bring about deviations in the soil structure and hence, in its engineering properties. Therefore, in the experimental phase of the study, soils are prepared by different trampling efforts and also by different compaction methods.

In the fifth chapter, the effect of variable permeability and compressibility on the consolidation behavior is investigated. For this objective, a mathematical treatment of the behavior is presented. Subsequently, laboratory consolidations tests with mid plane pore pressure measurements are conducted on soft, remolded, preconcolidated and undistributed samples of Tabriz clay. The test results, when compared with the theoretical findings, indicate that most of inherent discrepancies may be explained via the use of the theory developed in this study.

CHAPTER 1 - One Dimensional Consolidation with Special Reference to Pore Water Pressures

1.1 Introduction and Purpose of this chapter

The Soil consolidation under tall and heavy structures and development of new highway systems necessitates detailed study of soil behavior under stress. To study the rate and amount of settlement of structures founded on clay soil is a very important aspect of soil mechanics. This settlement results from the interaction of variety phenomena.

- a) Shear strain that develop simultaneously with change in load (immediate or initial settlement).
- b) Time dependent shear strain (creep).
- c) Time dependent volume changes that occur during dissipation of excess pore pressure (consolidation).
- d) Time dependent volume changes after excess pore pressures are essentially dissipated (secondary compression).

In this chapter, the one dimensional consolidation theory proposed by Terzaghi will be studied as far as its applicability to a highly plastic soil at its liquid limit, representing the normally consolidated case. Mid-plane pressure measurements will be made use of in this investigation. Mainly, this chapter is to be carried out to serve the following purposes.

- a) To investigate the inducement and subsequent dissipation of pore water pressure for one dimensional compression realized in the oedometer, by means of mid plane pore pressure measurements, and hence to check the applicability of Terzaghi's theory of consolidation to the oedometer testing conditions.
- b) To check the applicability of the common empirical methods used to evaluate the time rate of compression of oedometer samples (i.e. Casagrande Logarithmic fitting method and Taylor square root fitting method).

1.2 The Organization of this chapter work

So far various studies on consolidation relevant with pore pressure measurement have been carried out. In many of such studies striking departure from Terzaghi theory of consolidation has been observed, particularly for pressure increment ratios other than one and for pressures greater than the preconsolidated pressure. In this chapter in order to fulfill the preliminary assumptions of Terzaghi consolidation theory, that is complete saturation, and a normally loaded soil, the grey clay used has been prepared at its liquid limit. An oedometer of larger size than conventional consolidometers has been used in order to eliminate side friction and arching effect. Single drainage at the top was provided and mid plane pore pressure were measured by means of an automatic pore pressure device. In this chapter the general theory of conventional consolidation has been given and also it includes an account of the tests performed and the results obtained therefrom and then the last section of this chapter is devoted to discussion and conclusion.

1.3 The Mechanism of One Dimensional Consolidation

A soil may be considered to be a skeleton of solid grains enclosing voids which may be filled with gas, with liquid or with combination of gas and liquid. If a sample of soil is placed under sustained stress so that its volume is decreased in a drained manner, there are these possible factors to which this decrease might be attributed.

- a) A compression of solid matter.
- b) A compression of water and air within the voids.
- c) An escape of water and air from the voids.

Under the loads usually encountered in soil masses, the solid matter and the pore water, being relatively incompressible do not undergo appreciable volume change. For this reason, it is sufficiently accurate to consider the decease in volume of mass, if it is completely saturated, as due entirely to an escape of water from the voids. Sedimentary clay deposits are usually saturated. Compressibility of soil mass depends on the rigidity of the soil skeleton. The rigidity in turn, is dependent on the structural arrangement of particles, and in fine grained soils on degree to which adjacent particles are bonded together.

A honeycombed structure or in general any structure with high porosity, is more compressible than a dense structure. A soil composed predominantly of flat grains is more compressible than the one containing mostly spherical grains. A soil in remolded state may be much more compressible that the same soil in undisturbed state.

When the pressure on a soil is increased equally in all direction the volume decreases. If the pressure is later decreased to its previous value some expansion will take place, but the volume rebound will not be any means so great as the proceeding compression. In other words, soils show some elastic tendency but they are elastic to a small degree.

As the compression occurs, the pore water is drained according to Darcy's law, (Taylor 1948). The gradual process which involves, simultaneously, a slow escape of water and a gradual compression and which will be shown later to involve also a gradual pressure adjustment is called consolidation.

The consolidation of clay under a load does not take place instantaneously; clays are so impervious that the water is almost trapped into the pores. When an increment of load is applied the pore water can not escape immediately. Since clay particles tend to approach one another and pressure develops in the pore water which is called the excess pore pressure. The hydraulic gradients set up the due to this excess pressure cause the fluid to drain from the soil. As drainage continuous, the excess pressures dissipate and since the externally applied total pressure is constant pressure is gradually transferred to the soil skeleton. The part of stress carried by soil skeleton is called effective stress. Soil skeleton then deforms under the increase in effective stresses. This is called consolidation.

The rate of settlement is rapid at first and then decreases to a small fairly constant value. Due to a decrease in excess pore pressure on the one hand during consolidation and due to the decrease impermeability on the other hand. In Terzaghi's theory, the decrease in permeability during consolidation is not taken into account. However, later on Barden (1965) has shown that this phenomenon is important. The progress of consolidation can be observed by measuring decrease of excess pore water pressure which is the main object of this chapter. This decrease occurs at different rates in different parts of the sample. It occurs more rapidly at the place where drainage is facilitated.

It is well known characteristics of clay considerable time is required for the occurrence of the compression caused by a given increment of load.

This large time lag can be attributed to two phenomena. First is due to time required for the escape of pore water, this is called hydrodynamic lag. It is due basically to permeability which controls the flow of the pore water. Second factor is complex one and is called plastic lag. It is only partially understood and it is due to plastic action in adsorbed water near grain to grain contacts or points of nearest approach to contact. Terzaghi theory neglects such secondary effects as the plastic lag.

1.4 Terzaghi's Consolidation Theory

The compression of soil is classified into two stages.

- a) Primary consolidation which is due to the dissipation of excess pore pressures. In this stage it is assumed that no plastic lag exists and all the time lag is due to a low value of the coefficient of permeability.
- b) Secondary compression which occurs subsequent to the primary. The causes of this is rather complex.

Terzaghi (1948) developed a theory in which it leads to functions that may be recognized as analogous to those expressing the flow of heat.

The assumptions which he used in this theory are:

- 1. Homogeneous soil
- 2. Complete saturation
- 3. Negligible compressibility of soil grain and water
- 4. Action of infinitesimal masses no different from that of larger, representative masses.
- 5. One dimensional flow.
- 6. One dimensional compression.
- 7. Validity of Darcy's law.
- 8. Constant values for certain soil properties which actually vary somewhat with pressure.

- 9. The greatly idealized pressure void ratio relation ship $a_v = \frac{de}{dp}$ constant.
- 10. No plastic time lags during primary compression.

The fundamental expression for flow in saturated soil masses representing the time rate of change of volume $is\left(k_x\frac{\partial^2 h}{\partial x^2} + k_y\frac{\partial^2 h}{\partial y^2} + k_z\frac{\partial^2 h}{\partial z^2}\right)dx.dy.dz = 0$. This expression is dependent only on assumptions one to four inclusive and seven. For one dimensional flow, which is assumption six, the absence of gradient in the x, and y directions eliminates the first two terms of the parenthesis. The permeability k_z may, from this point on, be designated simply by k, giving

$$k\frac{\partial^2 h}{\partial z^2}dx.dy.dz \tag{1.1}$$

The volume of element is dx.dy.dz, the pore volume is $dx.dy.dz\frac{e}{1+e}$, and since all changes in volume must be changes in pore volume, a second expression for the time rate of change of volume may be written $\frac{\partial}{\partial t} \left(dx.dy.dz\frac{e}{1+e} \right)$. Since $\frac{dx.dy.dz}{1+e}$ is the constant volume of solids, the above expression may be written $\frac{dx.dy.dz}{1+e} \times \frac{\partial e}{\partial t}$ equating this expression to expression (1.1) and canceling dx.dy.dz gives:

$$k\frac{\partial^2 h}{\partial z^2} = \frac{1}{1+e} \times \frac{\partial e}{\partial t}$$

only heads due to hydrostatic excess pressure will tend to cause flow in the case under consideration. Thus, *h* in the above equation may be replaced by $\frac{u}{\gamma_w}$, giving

$$\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e} \times \frac{\partial e}{\partial t}$$
(1.2)

The hydrostatic excess pressure u is not necessarily the only pressure in the water. In addition static water pressure of unrestricted magnitudes may exist, but they play no part in

consolidation because they do not tend to cause flow. Total pressure is $p_T = \overline{p} + u$ if we differentiate this expression

$$dp_T = d\overline{p} + du$$

Since
$$p_T$$
 is constant $dp_T = 0$, then $d\overline{p} = -du$ (1.3)

and coefficient of compressibility a_v which is the slope of straight line curve of pressure versus void ratio is negative

$$a_{\nu} = -\frac{de}{d\overline{p}} \tag{1.4}$$

Substitution of equation (1.3) into equation (1.4) gives following expression of assumption nine

$$de = a_{y} du \tag{1.5}$$

Substitution of this relationship in equation (1.2) gives

$$\frac{k(1+e)}{a_v \gamma_w} \times \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$
(1.6)

The group of terms in the bracket may be written

$$\frac{k(1+e)}{a_v \gamma_w} = c_v \tag{1.7}$$

The soil property designated by c_v is called the coefficient of consolidation. Its insertion in the equation (1.6) gives

$$c_{\nu}\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$
(1.8)

which is partial differential equation of consolidation. In the consolidation theory the z coordinate distance is measured downward from the surface of the clay sample. The thickness of the sample is designated by 2H, the distance H thus being the length of the longest drainage path.

A dimensionless time factor is defined as

$$T = \frac{c_v t}{H^2}$$

Dimensionless pressure variable W is defined as $W = \frac{u}{u_i}$ where u_i is initial excess pore

pressure, and dimensionless variable Z, $Z = \frac{z}{H}$, z is measured from the surface of the compressing layer, and substituting in equation (1.8).

$$\frac{\partial W}{\partial T} = \frac{\partial^2 W}{\partial Z^2} \tag{1.9}$$

is obtained.

The solution of equation (1.9) for relatively simple oedometer conditions is given by

$$W_{Z,T} = \frac{4}{\pi} \sum_{m=0}^{\infty} \frac{1}{(2m+1)} e^{-\left[\frac{\pi^2 (2m+1)^2}{4}\right]^T} \sin \frac{\pi}{2} (2m+1)Z$$
(1.10)

Then, the average value of *W* is given by

$$W = \frac{\int_{0}^{1} W(Z,T) dZ}{\int_{0}^{1} W_{i} dZ}$$
(1.11)

So that
$$W = \frac{8}{\pi^2} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} e^{-\left[\frac{\pi^2 (2m+1)^2}{4}\right]T}$$
 (1.12)

For mid plane pore pressure Z = 1.0 in (1.10) so that

$$W_m = \frac{4}{\pi} \sum_{m=0}^{\infty} \frac{1}{(2m+1)} e^{-\left[\frac{\pi^2 (2m+1)^2}{4}\right]^T}$$
(1.13)

and defining the percent consolidation U as

$$U = 1 - W = 1 - \frac{\int_{0}^{1} W(Z,T) dZ}{\int_{0}^{1} W_{i} dZ} = 1 - \frac{8}{\pi^{2}} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^{2}} e^{-\left[\frac{\pi^{2}(2m+1)^{2}}{4}\right]^{T}}$$
(1.14)

The degree of mid plane pore pressure dissipation may then be given by

$$D = 1 - W_m = 1 - \frac{4}{\pi} \sum_{m=0}^{\infty} \frac{1}{(2m+1)} e^{-\left[\frac{\pi^2 (2m+1)^2}{4}\right]^T}$$
(1.15)



Figure 1. 1: Mid-Plane dissipation versus time factor (1) and average percent consolidation versus time factor (2)

This expression represents a mathematical series with solutions obtained by successive values of the integer m from zero to infinity the term (2m+1) does not depend on any physical characteristics of clay but it is introduced to facilitate the solution.

Since equations (1.14) and (1.15) simply show the relation between U vs T and D vs T it is possible and more convenient to represent them by a curve. Therefore, instead of solving the equation whenever a time factor value is required the value may be read directly from the curve.

These are shown in Fig 1.1. It is also convenient to know for the section from U = 0 to U = 60 percent, the curve is very closely approximated by the parabola

$$T = \frac{\pi}{4}U^2 \tag{1.16}$$

1.5 Criticism of Terzaghi Theory under the Light of Past Research

Leonards and Ramiah (1959) noted the striking changes in the shape of compression time curves as the load increment ratio was reduced, and also values of c_v calculated from conventional curve fitting procedures were as the load increment ratio was reduced.

Pore pressure measurement made by Leonnards G. A. and Altschaeffl A. G. (1964) during the consolidation test demonstrated that:

1. The ratio of initial excess pore pressure u_0 to the increment of applied pressure $\Delta \sigma$ is essentially one for all conditions of loading provided air does not come out of solution as a result of sampling or rebounding in oedometer.

2. The rate of pore pressure dissipation is reliably predicted by the Terzaghi theory when the load increment ratio is sufficiently large, and essentially the same value of c_v is obtained from the compression time curve as from the pore pressure dissipation curve.

3. For the curves which result from small load increment that straddles the preconsolidation pressure, p_c , the rate of pore pressure dissipation. (Measured pore pressures generally dissipate more rapidly than those predicted from the Terzaghi model).

Consequently, it is no longer meaningful to calculate c_v from the compression time curve using curve fitting procedures based on the Terzaghi model.

Crawford (1964) suggested that laboratory consolidation test be conducted at a steady rate of compression, sufficiently slow to prevent development of significant pore pressures.

He concluded that there is substantial field evidence that the prediction of consolidation settlement from laboratory tests is not always satisfactory. Much of the difficulty in predicting consolidation settlement from laboratory tests may be due to great differences between rates of compression in the laboratory and in the field.

It is shown that in order to create hydrodynamic effect the laboratory rate may be as much as several million time as fast as the field rate.

Pore pressure measurements show that the maximum pressure developed in the specimen is 80% of the applied load and that, this pressure dissipates as a direct function of deflection.

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Primary consolidation by direct measurement is found to be completed a little earlier than suggested by usual empirical methods.

Taylor (1948) in some tests observed time patterns of overall sample compression and pore pressure at the bottom of the sample which do not agree with those predicted by usual consolidation theory and hence attempted to develop a theory for taking into account the effects of interparticle bonding and structural viscosity.

Leonards and Girault (1961) measured pore pressure at the base of fixed ring consolidometer with the aid of the null meter in the low pressures range a mercury manometer was used to measure the pressure. They made comparison of theoretical and experimental pore pressure dissipation vs time.

Some values of coefficient of consolidation have been computed for different pressures for different methods.

| Ta | bl | le | 1 | 1 |
|----|----|----|---|---|
| | | | | |

| $\sigma^{kg}_{cm^2}$ | $\frac{\Delta\sigma}{\sigma}$ | $c_v \qquad \frac{cm^2}{\sec} \times 10^{-4}$ | | | |
|----------------------|-------------------------------|---|------|------|--|
| | | Ι | II | III | |
| 0.2 | 2.0 | 12.5 | 12.9 | 15.6 | |
| 1.1 | 3.0 | 1.46 | 0.93 | 1.41 | |
| 5.3 | 0.15 | 0.13 | 9.14 | 0.67 | |

Col I. Values calculated from dial reading time curves using Casagrande log time method.

Col II. Values calculated from the pore pressure curves at $\frac{u}{\Delta\sigma} = 0.5$

Col III. Values calculated from the pore pressure curves at a time corresponding to t_{50} on the dial reading time curve.

They stated that anomalous behavior using small load increment ratios can not be attributed to side friction as suggested by Taylor.

It is also dear that the values of c_v calculated from any particular test procedure can be greatly in error when applied in situ conditions where the load increment ratios varies with depth.

Finally the rate of pore pressure dissipation can be predicted reliably from Terzaghi theory only if comparatively large load increment ratios are applied. If the load increment ratio is smaller than the critical value Terzaghi theory can not predict even approximately the rate of excess pore pressure dissipation.

L. Barden and P.L Berry (1965) made a series of consolidation tests on normally consolidated clays and they stated that in tests with thin sample and small pressure increment ratio; $H \prec 0.3, \Delta \sigma / \sigma \prec 1$ the effect of structural viscosity is extremely marked and causes a complete departure from theoretical Terzaghi behavior. The shape of settlement time plot departs from the characteristic Terzaghi shape, because of large secondary compression, and the pore water pressure dissipation curves also departs from the characteristic Terzaghi shape. For thicker sample H = 2.5 in. and $\Delta \sigma / \sigma = 1$ behavior closely resembles the theoretical Terzaghi shape, including negligible secondary compression.

They mainly dealt with the study of effects of varying permeability and nonlinear void ratio effective stress relation on the consolidation process.

It was seen that for various type of clays dissipation of pore pressure are different. The small variation in permeability under $\Delta \sigma / \sigma = 1$ meant that the Terzaghi theory was not considerable in error.

For thin samples and small pressure increment ratios the effect of structural viscosity dominates the consolidation process and obscures the effects of other important factors such as varying permeability etc.

Test on the thicker sample with $\Delta \sigma / \sigma \ge 1$ minimize the effects of structural viscosity and suggests that these effects may be negligible at the field scale and that undue preoccupation with test on small laboratory samples may be misleading.

The proposed nonlinear theory incorporating a permeability varying as $k = k_f (1 + b\sqrt{u})$ gives also very close agreement with experimental results.

For tests in which the variation of permeability is small, curve fitting based on Terzaghi theory is found to be suitable method of extrapolating laboratory results, even permitting the prediction of pore pressure behavior from settlement observation.

At the field scale, where structural viscosity is minimized. The simple linear Terzaghi theory may therefore be generally applicable to the majority of clays exhibiting no unusual properties.

Robinson (2000) prepared an interesting technical note that concerns the use of pore water pressure measurements to estimate the coefficient of consolidation.

Singh (2008) developed diagnostic curve methods for simultaneously identifying consolidation coefficient, final settlement, and ratio of top and bottom excess pore-water pressures from observed settlements, in the case of linear excess pore-water pressure. Simple equations has been proposed for estimating consolidation coefficient and final settlement.

1.6. Experimental Procedure

1.6.1 Soil Used

The soil used in the study is Tabriz grey Clay whose properties have been reported by Khak Kavan Soli test laboratory. Samples taken at different depths from different locations show that the clay has well defined consistency limits. In general, the natural water content and plastic limit vary within 20 and 35 percent, and the liquid limit between 55 and 75 percent. The unit weight of the clay ranges between 1.75 and 1.95 t/m^3 , and the specific gravity of particles is between 2.6 and 2.7. The shrinkage limit varies between 15 and 20 percent.

The results of laboratory consolidation tests indicate that the coefficient of volume compressibility, m_v , does not change considerably with depth.

The consistencies limits of the soil are used in this study are as shown in Table 1.2.

Table 1. 2: The soil consistency limits that used in testes

| LL | PL | SL | G_{s} |
|----|----|----|---------|
| 75 | 36 | 13 | 2.70 |

1.6.2 Details of apparatus

a) Oedometer:

Oedometer used, is made of steel with inside diameter of 102.6 mm and height of 25.4 mm. Details of the oedometer is shown in Fig. 1.2. At the bottom a porous stone was housed in order to measure mid plane pore pressures. The porous stone at the top of the sample is for drainage of water as the consolidation proceedings.

Application of the vertical stress was provided by a steel sphere which was placed on the steel circular plate. One dimensional vertical drainage was provided by porous stone and a top drain valve.

Settlements were followed by a dial gage placed on the plate where oedometer was put.

The mid plane pore pressure was measured at a ceramic disc placed in the center of the base, using pore pressure apparatus which was de-aired and periodically checked by the parts of the oedometer are shown in Fig. 1.2.



Figure 1. 2: The parts of the oedometer that used for test

P_1 – Steel ring

- P_2 Steel base for soil to be placed
- P_3 Steel plate
- P_4 Copper valve for connecting nylon tube

- P_5 Porous stone
- P_6 Porous stone
- P_7 Drainage valve

To minimize effects of side friction, the side walls were smeared with Vazeline grease. To understand the magnitude of side friction L. Barden and P. L. Berry (1965) made some series of tests. Under lower pressure (0 psi 30 psi), there was no arching at the transducers. Under high pressure (60 psi- to 140 psi) immediate readings of total stress were approximately 3% lower because of the side arching effect, but as pore pressure dissipated and stress became effective this arching was destroyed. The central transducers indicated that side friction had no effect in the area pore pressure was measured.

The reading at the edge showed that the reduction in stress caused by side friction was less than 3% even at the worst location. Thus it can be accepted that the average reduction in total stress across the base of the sample will be less than 2% and anomalous pore pressure behavior can not therefore be attributed side friction effects.

b) Pore pressure device:

For measuring pore water pressure, Model No. 350 Mark II Leonardo Farnell automatic pore water pressure measurement apparatus was used. The equipment automatically measures the pore water pressure requiring almost no flow of pore fluid from the sample.

The layout of the apparatus is shown in Fig. 1.3. The pressure change is sensed by the monitoring unit which consists of a Perspex block attached by a pipe to the oedometer.

Within the perspex block is a mercury "U" tube and a pair of platinum contacts. The monitoring unit is connected via an oil filled tube to the control unit, which houses the pressure gauge together with the control mechanism.

Any increase in pore pressure causes the mercury within the "U" tube of the monitoring unit to move and establish contact with platinum wire on the high side of the "U" tube. This electrical contact is used to operate a relay in the control unit, which switches on a heater connected in an oil circuit.

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Figure 1. 3: The layout of the apparatus

The oil expansion restores the mercury in the "U" tube to its original position and at the same time the pressure change is indicated on the gauge. When the expanding oil restores the mercury to its original position the heater is switched off. Since the heater is maintained above ambient temperature it now cools and contracts. It will be seen therefore that mercury maintains a constant position in "U" tube by alternate expansion and contraction of the oil.

1.6.3 Time Lags in Pore Pressure Measurement

In these test results it is seen that there was a certain time elapsed for the pore pressure to reach its peak value. These time lags in measurement of pore pressure were also researched by Whitman, Richardson and Healy (1961). They stated the factors affecting measured pore pressures as,

a) Excessive flexibility in the measuring system. Such flexibility has two effects.

1. It alters the overall compressibility of the pore phase and changes the distribution of total stress between mineral skeleton and pore phase.

2. It leads to a time lag in the response of the measuring system.

b) Mineral skeleton of low compressibility interparticle bonding, structural viscosity and very tight packing of particles are possible reasons why stiffness of the mineral skeleton might approach that water.

The time required for any level of pressure to be reached in measuring chamber is a function of the permeability of porous stone of the bottom of oedometer and coefficient of compressibility m_y .

If ΔU_B is less than $\Delta \sigma$ the problem is to decide whether the cause is real (stiff mineral skeleton) or extraneous (flexibility in measuring system).

In order to make this decision there is need for a clear picture of the effect of measuring system flexibility.

Where $\Delta \sigma$ applied pressure

 ΔU_B pore pressure measured

 ΔU_A pore pressure assumed to be in reality

$$\frac{\Delta U_B}{\Delta U_A} = \frac{1}{1+B}$$
$$B = \frac{G}{AHm_v}$$

 $m_v = \text{Coefficient of compressibility } \frac{in^2}{h}$

G = Flexibility of measuring system $(in^3 \text{ per } \frac{lb}{in^2})$

H = Thickness of sample (*in*)

A = Area of oedometer

Suppose that for some reasons the minerals skeleton is quit stiff. For example if m_{y} is

about 5 times the compressibility of water then $\frac{\Delta U_A}{\Delta \sigma} = 0.83$ using values of A, H and

G, *B* becomes 0.5 and $\frac{\Delta U_B}{\Delta U_A} = 0.67$ which means recorded pressure is much less than the actual

pressure.

Pore pressures during drained compression (consolidation)

The theoretical determination of a curve such as that shown in Fig 1.4 involves the solution of the one dimensional consolidation.



Figure 1. 4: One dimensional consolidation

Equation $c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$ with the following boundary condition at the base of the sample $\Delta U_B = \frac{1}{G} \times \frac{AK}{\gamma} \int_0^t \frac{\partial^2 u}{\partial z^2} dt$

An approximate solution of this equation was obtained using the ten lump electric analogues. Solutions obtained in this manner are plotted in dimensionless form.

Fig 1.5 which shows $\Delta U_{B_{\Delta U_{A_0}}}$ as a function of the average consolidation ratios for the sample as a whole, where ΔU_{A_0} denotes the initial value of ΔU_A . These solutions are based on the assumption that m_{ν} is the same for consolidation and rebound.



Figure 1. 5: Diagram of $\Delta U_{B} / \Delta U_{A_0}$ respect to average degree of consolidation

The assumption is reasonable in the case of over consolidated soils, and the error resulting from this assumption tends to decrease calculated response time of the measuring system in normally consolidated soils. Therefore, if these curves are used in design of a measuring system a conservative indication of its performance will be obtained. During undrained compression, the time-log did not affect the peak value of ΔU_B only the time required to achieve this peak value. Now, however, the time-lag in the measuring system is important, for ΔU_A in time variant. The effect of the lag is revealed in Table 1.3.

| В | $Max \Delta U_{B} / \Delta U_{A}$ | | | |
|-------|------------------------------------|---------|--|--|
| | Undrained | Drained | | |
| 0.001 | 1 | 0.99 | | |
| 0.01 | 0.99 | 0.98 | | |
| 0.05 | 0.95 | 0.88 | | |
| 0.10 | 0.91 | 0.78 | | |

Table 1. 3: Effect of the lag

Thus, of *B* is much greater than 0.01, the peak measured ΔU_B in a drained compression test may be much less than the actual initial ΔU_A . In such test therefore, the presence of modest amount of air in the measuring system may be disastrous.

Fig.1.6 shows comparison of various measured value with Terzaghi theory.



Figure 1. 6: Comparison of various measured value with Terzaghi Theory

The flexibility of the system used in this thesis, is $G = G_1 + G_2$ where

 G_1 = Flexibility of nylon tube

 $G_2 =$ Flexibility of pore device

From Bishop and Henkel (1962) values of G_1 and G_2 are given as:



1.6.4 Preparation of Sample and Testing Procedure

The oedometer and plastic pipe were filled with water and checked that there was no air bubbles. Then ends of the pipes were closed and only very thin film of water remained on the deaired ceramic porous stone in order to prevent from entering the porous stone. After that the soil clay was prepared at its liquid limit in the form of slurry and placed into the oedometer by means of a spoon and at each thin layer of soil the oedometer was shocked to provide escape of air bubbles. When the oedometer was full the excess part of the soil was trimmed off. The important measurement has been the rate of settlement and rate of dissipation of mid plane pore pressures. In order to minimize the effect of structural viscosity it was desirable. To test with a pressure increment ratio of one and with a relatively thick sample.

Loads were applied in steps, each load doubling the previous value. $\Delta \sigma / \sigma = 1$, and each load was allowed to stand until essentially most of the base pore pressure measured had dissipated.Load increments were 0 - 0.25, 0.25 - 0.50, 0.5 - 1.0, 1.0 - 2.0, 2.0 - 4.0, and $4.0 - 8.0 \frac{kg}{cm^2}$.

After these six increments have been completed, the final load of $8.0 \frac{kg}{cm^2}$ on the

sample was reduced to $2.0 \frac{kg}{cm^2}$ and the sample was allowed to rebound for two days during which time water was provided from drainage valve for the sample to swell.

Then reloading from 2.0 to $4.0 \frac{kg}{cm^2}$ and from 4.0 to $8.0 \frac{kg}{cm^2}$ were applied and at each stress increment same procedure of loading and measurement of compression and excess pore pressure was made.

1.7 Experimental Results and Their Interpretation

Table 1.4 shows that the coefficient of consolidation c_v , computed for each stress increments by square root fitting, log fitting and from dissipation tests are more or less equal to each other indicating that prediction of consolidation by Terzaghi theory gives satisfactory results for the soil tested. The values of c_v showing discrepancies for certain stress increments by the log fitting method are due to the back of some points in the curves. More points could not be obtained due to the fact that consolidation takes a long time for completion and the pore pressure device had to be cleaned therefore, to be able to finish the tests in the shortest possible time each load was allowed to stand only for a limited period (approximately 24 hours).

Table 1. 4: The coefficient of consolidation c_v computed for each stress increments by different methods

| Pressure | Squ | uare root | Log fitt | Log fitting method | | From dissipation tests | | | From dissipation tests | | | Void Ratio |
|-------------|------------------------------|----------------------------------|--------------------------|------------------------|------------------------------|----------------------------|--------------------------|------------------------|------------------------|--|--|------------|
| kg/m^2 | Fitti | ng method | | | | | | | | | | |
| 7 CM | <i>t</i> ₉₀ (min) | $\frac{c_v}{cm^2/sec}$ | t ₅₀ (min) | $\frac{c_v}{cm^2/sec}$ | <i>t</i> ₅₀ (min) | $\frac{c_{v}}{cm^{2}/sec}$ | t ₉₀ (min) | $\frac{c_v}{cm^2/sec}$ | | | | |
| 0-0.25 | 12.7 ² | 5.5×10 ⁻⁴ | 36 | 5.7×10 ⁻⁴ | 8 ² | 6.2×10 ⁻⁴ | 14.0 ² | 5.5×10 ⁻⁴ | 2.5-1.41 | | | |
| 0.25 - 0.50 | 16.3 ² | 3.32×10 ⁻⁴ | 62 | 3.28×10 ⁻⁴ | 8 ² | 6.2×10 ⁻⁴ | 15.5 ² | 4.32×10 ⁻⁴ | 1.41-1.31 | | | |
| 0.50-1.0 | 16.2 ² | 3.40×10 ⁻⁴ | 69 | 2.95×10 ⁻⁴ | 7.7 ² | 6.75×10 ⁻⁴ | 16.4 ² | 4.0×10 ⁻⁴ | 1.31–1`.17 | | | |
| 1.0-2.0 | 16.7 ² | 3.16×10 ⁻⁴ | 68 | 3.00×10 ⁻⁴ | 10 ² | 4.0×10 ⁻⁴ | 18.4 ² | 1.91×10 ⁻⁴ | 1.17-0.945 | | | |
| 2.0-4.0 | $\overline{23.3}^{2}$ | 1. 63×10⁻⁴ | 98 | 2.08×10 ⁻⁴ | 15.3 ² | 1.70×10 ⁻⁴ | 25.2^{2} | 1.03×10 ⁻⁴ | 0.945-0.772 | | | |

| 4.0-8.0 | 29.0 ² | 1.05×10^{-4} | 170 | 1.20×10^{-4} | 18.3 ² | 1.19×10 ⁻⁴ | 28.5^{2} | 0.80×10^{-4} | 0.772 - 0.59 |
|-----------|-------------------|-----------------------|-----|-----------------------|-------------------|-----------------------|-----------------|-----------------------|--------------|
| | | | | | | | | | |
| Reloading | 19.4 ² | 2.35×10^{-4} | 25 | 8.20×10^{-4} | 13.3 ² | 2.70×10^{-4} | 19 ² | 1.81×10^{-4} | 0.65-0.63 |
| 2 - 4.00 | | | | | | | | | |
| Reloading | 22.4 ² | 1.75×10^{-4} | 87 | 2.35×10^{-4} | 13 ² | 2.35×10^{-4} | 21 ² | 1.48×10^{-4} | 0.63-0.587 |
| 4.0-8.0 | | | | | | | | | |

According to Terzaghi, for fully saturated clays (Sr = 1), at time t = 0, applied stress is equal to excess pore pressure. But tests show that it is not true for high stress increments. For the first three stress increments $0 - 0.25, 0.25 - 0.50, 0.5 - 1.0 \frac{kg}{cm^2}$ ratio of pore pressure to applied stress is equal to one, that is $C = \frac{\Delta U}{\Delta \sigma} = 1$, but for higher stress increments it was observed that this ratio C started to decrease. For reloading case very small valves of C was obtained, for the stress increment of $2.0 - 4.0 \frac{kg}{cm^2}, C = 0.25$ and for the increment of $4.0 - 8.0 \frac{kg}{cm^2}, C$ was equal to 0.41. This is illustrated in Fig. 1.7 and Table 1.5.

| 10010 1.5 | Tabl | le | 1. | 5 |
|-----------|------|----|----|---|
|-----------|------|----|----|---|

| Stress | Void | Coefficient of | Initial | Coefficient of | Water | ΔU | G |
|------------------------------|------------|--|-----------------------|-----------------------------|---------|-----------------|------------------------|
| | Ratio | Compressibility | Void | volume | Content | $\Delta \sigma$ | $\overline{A.H.M_{v}}$ |
| | | | Ratio | Compressibility | | | |
| $\Delta \sigma^{kg}/_{cm^2}$ | Δe | $a_v = \frac{\Delta e}{\Delta \sigma} \frac{cm^2}{kg}$ | <i>e</i> ₀ | $m_v = \frac{a_v}{1 + e_0}$ | W | С | В |
| 0-0.25 | 1.00 | 4.00 | 2.50 | 1.140 | 0.93 | 1.00 | 4.1×10^{-6} |
| 0.25-0.50 | 0.20 | 0.80 | 1.50 | 0.320 | 0.56 | 1.00 | 1.48×10^{-5} |
| 0.50-1.00 | 0.14 | 0.28 | 1.30 | 0.121 | 0.48 | 1.00 | 3.90×10^{-5} |
| 1.00-2.00 | 0.22 | 0.22 | 1.16 | 0.102 | 0.43 | 0.98 | 4.65×10^{-5} |
| 2.00-4.00 | 0.20 | 0.10 | 0.94 | 0.051 | 0.35 | 0.87 | 9.40×10 ⁻⁵ |
| 4.00-8.00 | 0.14 | 0.035 | 0.74 | 0.020 | 0.27 | 0.82 | 2.35×10^{-4} |
| Reload | 0.03 | 0.015 | 0.66 | 0.009 | 0.24 | 0.25 | 5.2×10^{-3} |

| 2.00-4.00 | | | | | | | |
|-----------|------|------|------|-------|------|------|-----------------------|
| 4.00-8.00 | 0.04 | 0.01 | 0.63 | 0.006 | 0.23 | 0.41 | 1.48×10^{-3} |



Figure 1. 7: Pore water pressure versus total stress

Fig. 1.8 shows the ratio $C = \frac{\Delta U}{\Delta \sigma}$ versus water content, it is seen that as the consolidation is progressed, around the plastic limit *C* starts to decrease and somewhere between the shrinkage limit and the plastic limit a very sudden drop of *C* occurs.


Figure 1. 8: Ratio of $C = \frac{\Delta U}{\Delta \sigma}$ versus water content

Also, as the coefficient of volume compressibility m_v decreases C again decreases as indicated in Fig.1.9.



Figure 1. 9: Ratio of $C = \frac{\Delta U}{\Delta \sigma}$ versus coefficient of volume compressibility m_{ν}

From these observations it seems possible that, as the water content decreases the soil sample starts to become stiff and the soil skeleton attains a more rigid state so that some of the applied stress may be taken by the soil skeleton. On the other hand Whitman and Healty (1961) state that the flexibility of the pore pressure measuring system may cause such an apparent reduced reading of the of the actual pore pressure.

The controversial point may be analyzed in the following way.

At each stress increment it was observed that a certain time clasped for the pore pressure to reach its peak value.

As it was discussed before the reduction in the measured peak pore pressure can not be attributed to the flexibility of the apparatus. This following from the fact that for each stress increment as computed in table 1.5 values of B is so small that reference to fig.1.5 indicates at once that the measured value of peak pore pressures showed be almost equal to the applied pressure. But the observations show that the ratio of peak pressure to applied pressure is 80-85% instead of 99-100%.

Furthermore, reference to the same figure indicates that some time lag is expected and this is observed in the tests performed. Calculations based on the c_v values obtained indicate that no dissipation of base pore pressure can occur within the interval of this time lag.

The pore pressure device initially was directly connected to the triaxial test chamber and within an increase of chamber pressure up to $16 \frac{kg}{cm^2}$ there was no time closed expect for several seconds and also the measured pore pressure was observed to be equal to the applied stress. This may be taken as evidence that the pore pressure apparatus itself is inflexible with the usual filling methods used in this study.

Therefore it may be concluded that the observation of developed pore pressure not being equal to applied pressure is due to rigid mineral skeleton which develops at the water content approximately near the plastic limit. (Fig. 1.8)

As Crawford (1964) reports the dissipation v.s compression curves are observed to be straight lines (Fig.1.10 and Fig. 1.11). This means that the coefficient of compression a_v may be taken as a constant during the consolidation process.



Figure 1. 10



Figure 1.11

Another interesting observation is that the calculated coefficient of consolidation c_v , corresponding to each percent of pore pressure dissipation as computed by means of fig.1.12 seemed to decrease for lower stress increments. For higher stress increments c_v seems to be constant as indicated in fig. 1.12 Terzaghi in his theory assumes c_v to be constant, however at lower stress increments do not hold.



Figure 1.12

It is known that, the coefficient of consolidation is given by the equation $c_v = \frac{(1+e)k}{a_v \gamma_w}$ and

here specific gravity of water γ_w , and void ratio *e* are constant and as indicated above coefficient of compressibility a_v is also constant during consolidation process. Then the only variable which remains is the permeability. So that the variation of coefficient of consolidation c_v during consolidation process can be attributed to the variation in permeability.

L. Barden and P.L Berry (1965) reports that for saturated, normally consolidated clays, permeability is a complex function of the void ratio and of soil structure and hence decrease during the consolidation process.

The amount of variation is of the order of $\frac{k_0}{k_f} = 2$ or $\frac{k_0}{k_f} = 3$ for most clays but can be as high as $\frac{k_0}{k_f} = 50$ for certain highly compressible clays.

The most commonly accepted variation is the linear void ratio logarithm of permeability relation.

Fig.1.13. shows percent compression versus percent consolidation. Excluding reloading and the first stress increment at which part of the very liquid sample was inevitably squeezed out of the oedometer it is seen that they fall on a line inclined as an angle of 45° means that at identical times, percent consolidation is approximately equal to percent compression.





Percent consolidations computed from c_v for 90% dissipation U_c and for each percent dissipation U_v were plotted on Figs 1.14-1.21. In addition, percent settlement $S = \frac{\delta_t}{\delta_{100}}$ where

 δ_{100} was computed as 100% compression from Fig 1.10 and Fig 1.11 by means of extrapolating the dissipation v.s. compression curves and the mid plane pore pressure dissipation *D* were platted on these figures.



Figure 1.14



Figure 1.15

Δ**σ** = 0.50 - 1.00 kg/cm



Figure 1. 16



Figure 1.17

∆⊂ = 2.00 - 4.00 kg/cm²



Figure 1. 18





Figure 1. 19



Figure 1.20



Figure 1. 21

An analysis of these curves show that the values of percent consolidation U_v computed from c_v for each percent dissipation are more close to actual settlement values *S* than the values of consolidation computed from c_v for 90% dissipation U_c .

For reloading cases these relations can not be seen. This indicates that percent consolidation and percent compression are entirely distinct entities for the preconsolidated soil. Thus, for such a case, the applicability of the Terzaghi theory seems doubtful. Fig 1.22 shows void ratio versus pressure and Fig 1.23 shows void ratio v.s log pressure. Also, an example of Casagrande log fitting and Taylor square root fitting methods to determine 100% and 90% consolidation respectively are illustrated on Fig 1.24 and Fig 1.25.



Figure 1. 22: void ratio versus pressure



Figure 1. 23: shows void ratio v.s log pressure



Figure 1.24





1. Terzaghi theory is observed to hold true in estimating the rate of compression in the oedometer of the clay used, as long as the soil is not preconsolidated.

2. For tests in which the variation of coefficient of consolidation, consequently the coefficient of permeability is small, the usual curve fitting methods are found to be suitable for the interpretation of laboratory test results, even permitting the prediction of pore pressure behavior from observations of compression.

3. The ratio of measured peak pore pressure to applied pressure decresses as the coefficient of compressibility decreases, most probably due to an increase in the rigidity of the soil skeleton

4. Percent of total compression seems to be equal to percent consolidation for each identical time as long as the clay is normally loaded.

5. If the clay is preconsolidated the pore pressure dissipation and percent compression of the sample seem to be distinct entities. Therefore, for such soils the applicability of Terzaghi's theory may be doubtful, although it may be used as an indication.

6. For lower stress increments, the coefficients of consolidation c_v computed for each percent mid plane pore pressure dissipation decreases as going from 10% to 90% dissipation. This shows that the value of permeability changes during consolidation, leading to a deviation from the conventional theory of consolidation.

CHAPTER 2 - One Dimensional Consolidation for Different Pressure Increment Ratios

2.1 Introduction and Purpose of this chapter

In this research, one dimensional consolidation tests on remodeled soft clay using different pressure increment ratios of one or less, with load increment duration of one week, were performed. Five different methods were used to evaluate the coefficient of consolidation C_v which yielded reasonably close values. The secondary compression effects have been found to increase as the pressure increment ratio is reduced. The C_v values and the end of primary void ratio effective stress relationship appear to be independent of the pressure increment ratio.

The influence of the pressure increment ratio $\left(\Delta\sigma/\sigma_{0}\right)$ on the results obtained from onedimensional consolidation tests have been studied by a number of researchers including Newland and Allely (1960), Leonards and Girault (1961), Wahls (1962), Madhav and Sridharan (1963), and Lun and Parkin (1985). The results indicate that the rate of excess pore pressure distribution can not be reliably predicted from the Terzaghi theory if the load increment ratio is small. The compression versus the logarithm of time curves lose their characteristics shapes with an inflection point for $\left(\Delta\sigma/\sigma_{0}\right)$ less than about $\frac{1}{3}$ with the result that the Casagrande time fitting method becomes inapplicable. The time required for the development of the linear secondary compression part of the compression log time curves and t_{100} have been stated to increase with decreasing $\left(\Delta\sigma/\sigma_{0}\right)$. The relative importance of primary and secondary effects and secondary compression characteristics has also been shown to be included by the pressure increment ratio. While there is agreement on the increased secondary effects if pressure increment ratio is reduced Newland et al (1960) the effect on the rate of secondary compression has been expressed in various ways and some results are controversial. Newland and Allely (1960), and Wahls (1962) noted that the slop of the secondary compression line (plotted in terms of void ratio change versus the logarithm of time) is independent of the pressure increment ratio. While Lun and Parkin (1985) results are essentially in agreement with the above, Madhav and Sridharan find the rate of secondary compression (defined as the void ratio change per logarithmic cycle of time per unit pressure increment) to increase with decreasing pressure increment ratio. It is also stated that the duration of previous increment affects secondary compression when the pressure increment ratio is small. Leonards and Girault (1961) plotted the rate of secondary compression as compression per cycle on the log time scale R_s , per unit pressure increment, per unit height of sample versus the pressure at the end of the increment $R_s / \Delta \sigma$. H vs σ and found large rates of secondary compression associate with small pressure increment ratios.

The void ratio change at the completion of primary consolidation is considered to be contributed by the compressibility of the soil structure with effective stress and the compressibility with time (secondary compression during the time required for primary consolidation). It is suggested that for any soft clay a unique of primary (EOP) void ratio effective stress relationship exists independent of the duration of primary consolidation which is related to the thickness of the consolidating layer and the pressure increment ratio.

Hence, in this study, an experimental program was carried out to investigate the effect of the pressure increment ratio on the shape of compression time curves and secondary compression characteristics. Five different curve fitting methods were used to determine coefficient of consolidation for comparison and to check their applicability when the pressure increment ratio is small.

2.2 Experimental Study Procedure

The results of three consolidation tests with $\left(\frac{\Delta\sigma}{\sigma_0}\right)$ of 1.0, 0.6 and 0.25 carried out on identical remolded samples of clay are presented. The samples were prepared by the slurry consolidation method and the maximum consolidation pressure applied was $0.5 \frac{kg}{cm^2}$. The liquid and plastic limits of the clay used were 98% and 23% respectively and the percentage of

clay size $(\langle 2\mu n \rangle)$ was 63%. The specific gravity of solid particles was 2.79 and the soil was classified as *CH*. The predominant clay mineral was illite. A floating ring type large diameter oedometer (diameter =112.7 mm and height= 19 mm) was used in performing tests. The sides of the oedometer were greased in an effort to minimize the effect of side friction. The applied pressures were maintained on the sample for a period of one week to ensure virtually the completion of the secondary consolidation under the previous increment of load which is considered to be important when the pressure increment ratio is small. The average room temperature during the tests was $22^{\circ}C$ and during load increment duration (1 week) the temperature variation was only a few degrees.

2.3 Experimental Results and Discussion

Coefficient of consolidation C_v values given in Table 2.1 to Table 2.3 were calculated by the inflection point method, the improved rectangular hyperbola method and the negative tangent method in addition to the commonly used square root of time and the logarithm of time fitting methods. All these methods, however, were developed for the conventional oedometer test procedures in which the pressure increment ratio is 1.0. The experimental results of Leonards and Girault (1961), Wahls (1962) and the present study show that the compression against the logarithm of time curves lose their characteristics shapes for small pressure increment ratios. Fig. 1 illustrates typical curves obtained for $\Delta \sigma / \sigma_0 = 1.0$, 0.6, and 0.25 for comparable pressure ranges. To eliminate the possibility of scale effects, the curve corresponding to $\Delta \sigma / \sigma_0 = 0.25$ was replotted to a larger scale which showed the absence of an inflection point even more clearly. The compression square roots of time plots have also been found to deteriorate and the initial experimental points, which should normally lie on a straight line, show a larger dispersion at smaller pressure increment ratios. As a result, it has not been possible to apply the logarithm of time and the inflection point methods, and the best fitting straight line was used in the square root of time method for $\Delta \sigma / \sigma_0 = 0.25$. Tables 2.1 to Table 2.3 give C values for $\Delta \sigma / = 10, 0.6$ and 0.25 respectively for pressure ranges greater than

2.3 give C_v values for $\frac{\Delta\sigma}{\sigma_0} = 1.0$, 0.6 and 0.25 respectively for pressure ranges greater than

the pressure applied in sample preparation stage. The results indicate agreement between various methods and no definite trend of dependency of C_{ν} or t_{100} values on pressure increment ratio.

| Table 2. 1: Coefficient of Consolidation values $\frac{\Delta \sigma}{\sigma_0} = 1$. |
|--|
|--|

| Coefficient of consolidation $C_v \left(\frac{m^2}{yr}\right)$ | | | | | | | | |
|--|-----------|----------|--------------|----------------|----------------------|--|--|--|
| Pressure | Root time | Log time | Inflection | Negative | Improved rectangular | | | |
| | method | method | point method | tangent method | hyperbola method | | | |
| $\frac{kg}{cm^2}$ | | | | | | | | |
| 0.5-1.0 | 0.38 | 0.21 | 0.35 | 0.22 | 0.37 | | | |
| 1.0-2.0 | 0.39 | 0.14 | 0.33 | 0.19 | 0.26 | | | |
| 2.0-4.0 | 0.11 | 0.09 | 0.21 | 0.14 | 0.15 | | | |
| 4.0-8.0 | 0.09 | 0.08 | 0.20 | 0.12 | 0.14 | | | |
| 8.0-16.0 | 0.13 | 0.07 | 0.18 | 0.10 | 0.14 | | | |

Table 2. 2: Coefficient of Consolidation values $\Delta \sigma / \sigma_0 = 0.60$

| Coefficient of consolidation $C_v \left(\frac{m^2}{yr}\right)$ | | | | | | | |
|--|-----------|----------|--------------|----------|------------------|--|--|
| Pressure | Root time | Log time | Inflection | Negative | Improved | | |
| | method | method | point method | tangent | rectangular | | |
| $\frac{kg}{cm^2}$ | | | | method | hyperbola method | | |
| 0.64-1.02 | 0.28 | 0.11 | 0.12 | 0.25 | 0.28 | | |
| 1.02-1.64 | 0.11 | 0.08 | 0.09 | 0.15 | 0.15 | | |
| 1.64-2.62 | 0.12 | 0.07 | 0.07 | 0.16 | 0.14 | | |
| 2.62-4.19 | 0.09 | 0.06 | 0.07 | 0.15 | 0.14 | | |
| 4.19-6.71 | 0.09 | 0.06 | 0.07 | 0.14 | 0.16 | | |
| 6.71-10.74 | 0.07 | 0.06 | 0.06 | 0.11 | 0.12 | | |
| 10.74-17.18 | 0.08 | 0.06 | 0.06 | 0.06 | 0.11 | | |

Pore pressure measurements were not available to evaluate C_v values or to assess the end of primary consolidation. So, a simple check was carried out to examine the conformity of the experimental compression curve with a theoretical curve calculated from the Terzaghi theory by employing the C_v values obtained using the square root method for two pressure ranges for tests with $\Delta \sigma / \sigma_0 = 1.0$ and $\Delta \sigma / \sigma_0 = 0.25$. The experimental and calculated points deviate a small amount after about 90% consolidation; the measured compressions being approximately 5% larger than the calculated ones for both the pressure increment ratios.

| Coefficient of consolidation $C_v \left(\frac{m^2}{yr}\right)$ | | | | | | |
|--|-----------|----------|----------------------|--|--|--|
| Pressure | Root time | Negative | Improved rectangular | | | |
| | method | tangent | hyperbola method | | | |
| $\frac{kg}{cm^2}$ | | method | | | | |
| 0.61-0.76 | 0.40 | 0.29 | 0.38 | | | |
| 0.76-0.95 | 0.55 | 0.52 | 0.35 | | | |
| 0.95-1.19 | 0.32 | 0.25 | 0.29 | | | |
| 1.19-1.49 | 0.39 | 0.24 | 0.28 | | | |
| 1.49-1.86 | 0.46 | 0.23 | 0.29 | | | |
| 1.86-2.33 | 0.23 | 0.32 | 0.19 | | | |
| 2.33-2.91 | 0.28 | 0.23 | 0.21 | | | |
| 2.91-3.64 | 0.16 | 0.19 | 0.22 | | | |
| 3.64-4.55 | 0.25 | 0.17 | 0.19 | | | |
| 4.55-5.68 | 0.21 | 0.11 | 0.19 | | | |
| 5.68-7.11 | 0.11 | 0.13 | 0.14 | | | |
| 7.11-8.88 | 0.10 | 0.09 | 0.15 | | | |
| 8.88-11.1 | 0.18 | 0.14 | 0.15 | | | |
| 11.1-13.88 | 0.10 | 0.11 | 0.14 | | | |
| 13.88-17.35 | 0.13 | 0.15 | 0.13 | | | |

Table 2. 3: Coefficient of Consolidation values $\Delta \sigma / \sigma_0 = 0.25$

Wahls (1962) developed a consolidation equation and proposed a procedure to obtain theoretical consolidation curves that closely approximate experimental curves and to determine EOP void ratio. Although not specifically stated or used by Wahls (1962), in the present study the author's method was used to determine t_{100} and C_v . Although the procedure due to Wahls predicts the form of experimental consolidation curves for $\Delta \sigma / \sigma_0 \leq 1.0$, the C_v values obtained become increasingly smaller than the ones determined by the other methods as $\Delta \sigma / \sigma_0$ is reduced. The influence of the pressure increment ratio on secondary compression was also examined. Although the compression dial readings versus logarithm of time plots indicate a fairly unique final independent of the pressure increment ratio (Fig 2.1) in agreement with Lun and Parkin [19], the coefficient of secondary compression C_{α} defined as the vertical strain per logarithmic cycle of time increases as the pressure increment ratio is reduced.



Figure 2. 1: Comparison of dial reading versus logt curves

This is illustrated in Fig 2.2 where the average C_{α} value is plotted against $\Delta \sigma / \sigma_0$. The plots of the secondary compression per cycle on the log time scale R_s , per unit pressure increment, per unit average height for the particular pressure increment versus the final effective stress are also given in Fig.3. As observed, $\frac{R_s}{\Delta \sigma \cdot H}$ decreases as the pressure increases, and at a given pressure, increase as the pressure increment ratio is reduced, in conformity with the findings of Leonards and Girault (1961).



Figure 2. 2: Variation of C_{α} with $\Delta \sigma / \sigma$

An opportunity was taken to check the concept of uniqueness of the EOP void effective stress relationship for any soft clay which is also independent of the pressure increment ratio. Fig.2.4 presents EOP void ratio effective stress curves for tests with $\Delta \sigma / \sigma_0 = 1.0$ and $\Delta \sigma / \sigma_0 = 0.25$

where the EOP void ratios were calculated from dial readings corresponding to t_{100} as found from the square root time fitting method. The close agreement exhibited in this figure and the results of other tests support the proposed concept.



Figure 2. 3: Variation of $\frac{R_s}{\Delta\sigma \cdot H}$ with pressure at different pressure increment ratios.



Figure 2. 4: EOP Void Ratio Effective Stress Relationshi

2.4 Conclusions

For a small pressure increment ratios $(\Delta \sigma / \sigma_0 = 0.25 \text{ in the present study})$ the form of experimental consolidation curves do not permit the logarithm of time and the inflection point methods of time fitting to be applied. A comparison of the square root of time, logarithm of time, inflection point, negative tangent and improved rectangular hyperbola time fitting methods indicated in general a close agreement between the C_v values obtained. The pressure increment ratio does not appear to influence C_v values in a consistent way. The secondary compression effects increase as $\Delta \sigma / \sigma_0$ is reduced. The end of primary (EOP) void ratio effective stress relationship seems to be independent of the pressure increment ratio.

CHAPTER 3 - Clay Layer Consolidation Delimited Between Sheets with Small Permeability

3.1 Introduction and Purpose of this chapter

The problem of consolidation of clayey soils is an attractive one for the soils engineers and scientists mainly due to the fact that the solution of this particular problem provides the answers to the question of the rate and amount of settlement of engineering structures.

The classical theory of consolidation due to Terzaghi (1934) is the most widely used one mainly because of its simplicity. This theory is very well-known to the soil scientists and its presentation is given in almost any textbook on soil mechanics (Taylor (1948), Lambe (1969), and Ccott (1963)). The consolidation theory of compressible soils usually assumes that at the boundaries the drainage is free. That is, the surrounding layers are pervious.

The fundamental assumptions of the Terzaghi theory have been criticized by many researchers and a number of modifications have been proposed during the last two decades (Barden (1965), Berry (1964), Taylor (1942)).

Another interesting point inherent in obtaining solutions by means of this theory is the boundary conditions prevailing for a certain problem in question. Terzaghi and Frochlich (1936) give a number of closed form solutions for different boundary conditions which are applicable to various field problems.

As far as the boundary conditions are concerned, almost all of them represent the consolidation clay soil with its pore water discharging into pervious boundaries. Therefore, in such cases, the boundary conditions are relatively simple, and closed form solutions are readily obtainable.

Kang-He et al. (1999) studied one dimensional consolidation of two-layered soil with partially drained boundaries. In their paper, a fully explicit analytical solution was presented for one-dimensional consolidation of two-layered soils with partially drained boundaries. Lee et al. (2005) studied one-dimensional consolidation of layered systems. They found a general analytical solution, which is more explicit than other solutions. Wang et al. (2004) investigated soil consolidation by vertical drains with double porosity model.

In this study, due to the case studied, the boundary conditions are not as simple, necessitating the use of an approximate solution. The case presented is a clay layer of relatively large thickness sandwiched between two thin layers of soil whose compressibility and permeability is much lower than the layer in between. Therefore, discharge during consolidation of the pore water of the sandwiched layer into the neighboring top and bottom layers is not "free" as in most usual cases, but restricted.

In summary, this chapter the case in which the boundary layers have a low permeability is studied. When free drainage is thus inhibited, the boundary conditions at the drainage surface changes necessitating the use of an approximate technique of solution of the governing partial differential equation. Therefore, the solution is obtained by using the Galerkin Method. The solution obtained is compared with the "free drainage" case and conclusions are drawn thereform. As expected, the consolidation in the case of restricted drainage proceeds at a much lower rate.

3.2 Assumptions

The assumptions made for the solution of this problem are as follows:

a) The consolidation layer is soft, saturated, normally loaded clay.

b) All the main and subsidiary assumptions made in the classical theory of consolidation are assumed to be applicable to the consolidating layer.

c) The thickness of the consolidating layer is very large in comparison to the top and bottom layers.

d) The compressibility and the permeability of the consolidating layer are much greater than that of the top and bottom layers.

e) In view of assumptions c and d given above, the contribution of the top and bottom layers to consolidation is considered negligible and these layers are regarded as semi-pervious membranes hindering the expulsion of pore water from the consolidating layer.

3.3 Mathematical Treatment of the Process

In this treatment reference is made to Fig. 3.1. The symmetric nature of the problem with respect to the centre line enables its treatment in the half space bounded by the center line and defined by the space variable z where z measured is positive downwards.



Figue 3. 1: Configuration of the problem

Under the imposed total stress σ , an initial excess pore pressure of magnitude $u_0 = \sigma$ develops in the saturated, soft clay. The hydraulic gradient then set up due to this excess pore water pressure initiates flow of pore water towards the top and bottom boundaries I-I and II-II respectively, and the excess pore pressure dissipates as a function of time *t* and depth *z*. That is u = u(z,t).

The flow occurs according to Darcy's law.

$$V_z = -\frac{k}{\gamma_w} \times \frac{\partial u}{\partial z}$$
(3.1)

where V_z is the flow velocity in z direction, k is the permeability of the soft clay. On the other hand, time rate of dissipation of excess pore water pressure $\left(\frac{\partial u}{\partial t}\right)$ at a point in the consolidation layer is proportional to the divergence of velocity (Terzaghi (1934), Taylor (1948), Lame (1969), and Ccott (1964)) that is

$$\frac{\partial v_z}{\partial z} = -\frac{a_v}{1+e_0} \times \frac{\partial u}{\partial t}$$
(3.2)

Combination of eq. (3.1) and eq. (3.2) yields

$$\frac{\partial^2 u}{\partial z^2} = \frac{a_v \gamma_w}{k(1+e_0)} \times \frac{\partial u}{\partial t}$$
(3.3)

Eq. (3. 3) is the famous equation of consolidation proposed by Terzaghi where a_v the compressibility coefficient is and e_0 is the initial void ratio of the soft clay respectively.

It is possible to render equation (3.3) dimensionless by employing dimensionless pore pressure, space variable and time as follows:

$$w = \frac{u}{u_o}, \quad Z = \frac{z}{H}, \ T = \frac{(1+e_0)k}{H^2 a_v \gamma_w} t, \tag{3.4}$$

where *H* is half thickness of the soft clay and *T* is called the time factor. Thereby, the dimensionless form of equation (3.3) is

$$\frac{\partial^2 w}{\partial Z^2} = \frac{\partial w}{\partial T}$$
(3.5)

The water expelled from the consolidation layer is transferred across the boundaries I-I and II-II to the adjoining layers according to

$$V(0,t) = \frac{1}{\gamma_{w}} \times h.u(0,t)$$
(3.6)

Where *h* may be termed the specific permeability of the semi pervious membranes where *h* has the dimensions of $\left(\frac{1}{time}\right)$. It should be noted that if the specific permeability is multiplied by

length, it yields the usual conception of permeability with dimensions $\left(\frac{length}{time}\right)$.

On the other hand, the same transference law may be stated for the lower neighborhood of the boundary:

$$\left[V(0,t) = \frac{k}{\gamma_w} \times \frac{\partial u}{\partial z}\right]_{z=0}$$
(3.7)

The combination of (3.6) and (3.7) then yields one boundary condition which express the "hindrance" to flow at the boundaries i.e.:

$$\left[h.u(0,t) = k \times \frac{\partial u}{\partial z}\right]_{z=0}$$
(3.8)

At this stage, h may be expressed in terms of the drainage length H and the permeability k of the consolidation layer in the following form:

$$k = m.h.H \tag{3.9}$$

where m is obviously a positive number greater than unity, and specifies the magnitude of h in relation to the permeability of the soft clay.

The boundary condition (3.6) may be expressed in the dimensionless form through the first expression of eq.(3.4) and when combined with equation (3.9) yields:

$$w - \frac{\partial w}{\partial z} = 0; \quad T \succ 0$$
 (3.10)

The other boundary conditions are

$$w(z,0) = 1.0$$
 $0 \le z \le 1$, $T \succ 0$
 $w_z(1,T) = 0$ $T \succ 0$ (3.11)

Equations (3.5) with the boundary conditions (3.10) and (3.11) describe the process of consolidation.

3.4. Solution of the Equation

Although the boundary conditions are homogeneous, a closed form solution of equation (3.5) is not readily available, because of the nature of the boundary condition (3.10). Therefore, an approximate method is employed, which will be described subsequently.

The function w(Z,T) will be approximated by selecting a trail solution of the form

$$w(Z,T) = \sum_{i=1}^{n} a_i(T).\varphi_i(Z)$$
(3.12)

for *n* discrete variables $a_i(T)$, where the $\varphi_i(Z)$ are known functions. The boundary conditions at Z = 0 and Z = 1 is homogenous, therefore if the $\varphi_i(Z)$ satisfy the following conditions:

$$\varphi_{i} - \frac{d\varphi_{i}}{dZ} = 0 \qquad Z = 0$$

$$\frac{d\varphi_{i}}{dZ} = 0 \qquad Z = 1$$
(3.13)

then it would be possible to make w(Z,T) comply with the boundary conditions with no restriction. A simple family of polynomials can be selected to satisfy condition (3.13), i.e.

$$\varphi_i = 1 + Z - \frac{Z^{i+1}}{i+1} \tag{3.14}$$

A trial equation (3.12) constructed with polynomial (3.14) would satisfy the boundary conditions, but it would not satisfy the initial condition, or the governing equation.

Therefore, some kind mathematical approximation is required to satisfy these conditions. In this chapter Galerkin's weighted residual method is used to fix the unknown $\alpha_i(T)$ so that the initial condition and the governing equation are approximately satisfied. The points of the Galerkin method is given in almost all text books on numerical methods (Hildebrand (1956), Hartree (1962), and Salvadori (1952)). In the analysis, equation (3.12) is limited two terms. By virtue of the initial condition of (3.11) w(Z,0)=1.0 forming the initial residual yields:

$$R[\alpha_1(0), \alpha_2(0), Z] = 1 - \left(1 + Z - \frac{Z^2}{2}\right) a_1(0) - \left(1 + Z - \frac{Z^3}{3}\right) a_2(0)$$
(3.15)

Through the application of Galerkin's criteria, a pair of constant $a_1(0)$ and $a_2(0)$ can be obtained for which the residual expression (3.15) would be the least.

Now for, $T \prec 0$ using the governing equation (3.5), it is possible to obtain the equation residual

$$R[a_1(T), a_2(T), Z] = \left(1 + Z - \frac{Z^3}{2}\right) \frac{da_1}{dT} + \left(1 + Z - \frac{Z^3}{3}\right) \frac{da_2}{dT} + 2Za_2$$
(3.16)

Applying the Galerkin criteria to this residual, a pair of differential equations for the $a_1(T)$ and $a_2(T)$ is obtained. They are considered as the propagation equations for the $a_i(T)$. Then the approximate solution is obtained by solving these equation subjects to the initial conditions obtained from the initial residual given by expression (3.15). The Galerkin criteria is applied to the equation residual by setting

$$\int_{0}^{1} R[a_{1}(T), a_{2}(T), Z] \varphi_{i}(Z) dZ = 0 \quad i = 1, 2, \dots$$
(3.17)

And it yields

$$\frac{9}{5}\frac{da_1}{dT} + \frac{691}{360}\frac{da_2}{dT} + \frac{4}{3}a_1 + \frac{17}{12}a_2 = 0$$

$$\frac{691}{360}\frac{da_1}{dT} + \frac{1291}{630}\frac{da_2}{dT} + \frac{17}{12}a_1 + \frac{23}{12}a_2 = 0$$
(3.18)

As governing equations for the a_1 and a_2

If the Galerkin criterion is also applied to the initial residual (3.15) by setting

$$\int_{0}^{1} R[a_1(T), a_2(0), Z] \varphi_i(Z) dZ = 0 \quad i = 1, 2, \dots$$
(3.19)

It yields

$$\frac{4}{3} - \frac{9}{5}a_1(0) - \frac{691}{360}a_2(0) = 0$$

$$\frac{17}{12} - \frac{691}{360}a_1(0) - \frac{1291}{630}a_2(0) = 0$$
(3.20)

For the initial value of a_1 and a_2 .

The solution to eq. (3.18) which satisfies the conditions given by expression (3.20) is found to be:

$$a_1 = 0.59e^{-0.74T} + 2.45e^{-11.7T}$$

$$a_2 = 0.14e^{-0.74T} - 2.30e^{-11.7T}$$
(3.21)

The corresponding approximate solution for w(Z,T) with these values is then:

$$w(Z,T) = a_1 \left(1 + Z - \frac{Z^2}{2} \right) + a_2 \left(1 + Z - \frac{Z^3}{3} \right).$$
(3.22)

The approximation to the initial consolidation w(Z,0)=1.0 of eq.(3.22) is shown in Fig. 3.2.



Figue 3. 2: Fit of Initial Condition by the solution

To obtain the percent consolidation, the usual definition is used, which is given by the expression

$$U = 1 - \frac{\int_{0}^{1} w(Z,T) dz}{\int_{0}^{1} w(Z,0) dz}$$
(3.23)

Substitution of equation (3.22) in equation (3.23) yields

$$U = 1 - \frac{4}{3}a_1 - \frac{17}{12}a_2 \tag{3.24}$$

The percent consolidation U thus obtained is plotted against time factor T in Fig. 3.3. For purpose of comparison the Terzaghi solution with two way free drainage is also included in the same figure.



Figue 3. 3: Equation (3.24) compared with Terzaghi case

3.5 Discussion and Conclusions

Consolidation of a soft, saturated clay soil delimited by sheets of soil with a low permeability is treated by means of employing an approximate procedure for the solution of the governing partial differential equation. Reference to Fig. 3.1 would illustrate the limitation brought forward by the approximate procedure as far as the compliance with the initial condition is concerned. The nature of this approximation indicates that the accuracy of the method used is within tolerable limits as far as this condition is concerned.

Fig. 3.3 represents the very large deviations brought about by the low permeability layers from the free drainage case in which expulsion of pore water is not inhibited by any means. As it is expected, the consolidation in the former case proceeds at a much slower rate. Particularly expansive and very plastic soils may "wash" into the coarser layers which are presumed to provide free drainage as in the Terzaghi theory, and this phenomenon may lead to semi-pervious membranes as assumed in this study. It is the author's opinion that the judgment should be left to the practicing engineer as far as the estimation of field rates of consolidation is concerned. However, the present study brings to light another aspect of the consolidation process which should be taken into consideration in situations where the free drainage is inhibited.

CHAPTER 4 - One Dimensional Consolidation Behavior of Trampled Clays in a Semi-Saturated State

4.1 Introduction and Purpose of this chapter

The extensive use of earth in engineering construction justifies research in order to understand the engineering properties of compacted soils. In general, earthen structures are constructed by compaction at or around the optimum water content. When compacted at optimum, the soil is in a semi saturated state; therefore, the conventional Terzaghi theory may not readily be applicable to such a case.

The present investigation has been undertaken in order to analyze the compression consolidation behavior of soils compacted at optimal conditions. Furthermore, it is known that the type and energy of compaction bring about deviations in the soil structure and hence, in its engineering properties. Therefore, in the experimental phase of the study, soils are prepared by different trampling efforts and also by different compaction methods.

4.2 Theoretical Discussion

Unsaturated soil usually consists of soil solids, water and air. In order to be able to make a mathematical analysis of deformation of such a soil under externally applied loads, it is necessary to determine first the state of distribution of air and water in soil voids. A survey of the accumulation of knowledge on the physical and engineering properties of soils shows that for soils with a relatively high degree of saturation (above 80%) the air is in the form of occluded bubbles. For a soil compacted at optimum therefore, the air is assumed to be in the form of occluded bubbles. Starting with atmospheric pressure in such bubbles, it is evident that the pore water would initially be in a state of tension (Lambe 1961).

If a load is applied to such a soil, the skeleton and the pore fluid share this load. On the other hand, under its share of the external pressure, the pore air compresses according to Boyle's law and goes into solution in the surrounding water according to Henry's law. This process takes a very short time in the laboratory and is called the "Instantaneous Compression". The amount of

compression of air bubbles should be equal to the amount of compression of the soil skeleton. Instantaneous compression is assumed to occur essentially in an undrained manner.

If the load increment applied is large enough to cause a positive excess pore pressure in the soil, the process of "consolidation" starts subsequent to the process of compression. The "consolidation" process, like Terzaghi's assertion is mainly governed by the pore pressure dissipation characteristics. In a partially saturated soil, it is to be expected that the part of air which dissolves in water according to Henry's law during the "Instantaneous Compression" will come out of the solution as the excess pore pressure dissipates. These expanding bubbles may become trapped by the soil skeleton and hinder flow. Such a process combined with a decrease in void ratio throughout consolidation will obviously cause a variation in the permeability (Barden 1965).

Thus, it becomes clear that the concept of a constant permeability as asserted in Terzaghi's theory of consolidation may not be justifiable for the present case (Taylor (1941)). Therefore, in the mathematical treatment of the process of consolidation, the average permeability has been assumed to vary as a function of time.

In the mathematical phase of the investigation, to calculate the amount of instantaneous compression, an expression has been derived making use of Boyle's law of compression of air and Henry's law of solubility, assuming undrained compression.

This expression is given in the form:

$$\Delta e_s = C_w e_0 \Delta u_w \tag{4.1}$$

where

 Δe_s = Amount of instantaneous compression

 C_w = Compressibility of the pore fluid

 $\Delta u_w =$ Excess pore pressure developed

And the pore compressibility C_w is given by the expression:

$$C_{w} = \frac{1}{e_{0}} \times \frac{u_{a_{0}} \left(e_{a_{0}} + h e_{w} \right)}{\left(u_{a_{0}} + \Delta u_{w} \right)^{2}}$$
(4.2)

where

 e_0 = Initial void ratio

 u_{a_0} = Initial pore air pressure

 e_{a_0} = Initial pore air volume

h = Henry's coefficient of solubility

 e_w = Pore water volume

On one hand, equations (4.1) and (4.2) enable one to calculate the amount of instantaneous compression if measurements of the excess pore pressure developed are made and on the other of the value of the excess pore water pressure if the amount of instantaneous compression has been measured.

The mathematical treatment for the following process of consolidation has been made assuming an average permeability k(t), which varies with time during consolidation. The effective stress principle is applied in its simplest form, and secondary effects are neglected. The continuity equation for the case can be written as:

$$\frac{\partial}{\partial z} \left(k(t) \frac{\partial u}{\partial z} \right) = \frac{\gamma_w}{1 + e_i} \times \frac{\partial}{\partial t} (S.e)$$
(4.3)

when S, e are the degree of saturation and void respectively, z and t are the space and time variables and u is the pore pressure. Rewriting (4.3) yields:

$$\frac{\partial^2 u}{\partial z^2} = \frac{\gamma_w}{(1+e_i)k(t)} \left(e_i \frac{\partial S}{\partial t} + S_i \frac{\partial e}{\partial t} \right)$$
(4.4)

In equation (4.4):

$$\frac{\partial S}{\partial t} = C_w \frac{\partial u}{\partial t} \tag{4.5}$$

$$\frac{\partial e}{\partial t} = a_v \frac{\partial u}{\partial t} \tag{4.6}$$

where C_w is the pore fluid compressibility and $a_v = \frac{de}{du}$, coefficient of compressibility.

Substituting (4.5) and (4.6) in (4.4), the equation governing the consolidation process is obtained.

$$\frac{\partial^2 u}{\partial z^2} = \frac{\gamma_w}{(1+e_i)k(t)} (e_i C_w + S_i a_v) \frac{\partial u}{\partial t}$$
(4.7)

Pore compressibility C_w may also be taken as a function of time. Now, to reduce (4.7) to

a dimensionless form, the following substitutions can be made: $W = \frac{u}{u_i}$; $Z = \frac{z}{H}$ where

 $u_i =$ Initial excess pore pressure

H = Sample thickness

Furthermore, a coefficient of consolidation $C_{v}(t)$ dependent on time may be defined such that:

$$C_{\nu}(t) = \frac{1+e_i}{\gamma_{\nu}} \times \frac{k(t)}{S_i a_{\nu} + e_i C_{\nu}}$$
(4.8)

It is obvious that for a fully saturated soil $(C_w = 0; S = 1.0)$ with a constant coefficient of permeability (4.8) becomes:

$$C_{\nu_0} = \frac{(1+e_i)k}{a_{\nu}\gamma_{\nu}}$$
(4.9)

And (4.7) becomes:

$$\frac{\partial u}{\partial t} = C_{v_0} \frac{\partial^2 u}{\partial z^2}$$
(4.10)

Which are the coefficient of consolidation and the partial differential equation respectively as given in Terzaghi (1923).

Moreover definition of a time factors T_B such that:

$$T_{B} = \frac{1}{H^{2}} \int_{0}^{t} C_{\nu}(t) dt$$
(4.11)

Will render (4.7) to read:

$$\frac{\partial W}{\partial T_B} = \frac{\partial^2 W}{\partial Z^2} \tag{4.12}$$

with the usual boundary conditions

$$W(z,0) = 1.0 \quad \text{with } T \succ 0$$
$$W(0,T) = 0 \quad 0 \prec Z \prec 1.0$$
$$W_z(1,T) = 0$$

This last equation is identical in form to the usual diffusion equation proposed by Terzaghi and its solution may easily be obtained in the form of Fourier series. The solution to this equation has been reported in many textbooks on soil mechanics.

Now equation (4.11) is written in the form:
$$T_B = \frac{R(t)}{H^2} \tag{4.13}$$

where

$$R(t) = \int_{0}^{t} C_{\nu}(t) dt$$
(4.14)

Defining a function for the variation of the coefficient of consolidation with time. For the Terzaghi theory of consolidation $C_v = C_{v_0}$ the coefficient of consolidation is constant, therefore (4.14) will read

$$R(t) = \int_{0}^{t} C_{\nu}(t) dt$$
(4.15)

This implies that if experimentally determined values of R(t) are plotted versus t the resulting curves will be in the form of straight lines going through the origin with a slop equal to C_{v_0} .

For R(t) being a function other than such straight lines, the implication would be that deviations from the Terzaghi theory occur. The behaviors of compacted soils which are to be discussed presently indicate such deviations.

Another deviation from the Terzaghi case of consolidation arises in the manner of computation of percent consolidation of a soil sample. In the Terzaghi case the percent consolidation U is given by:

$$\%U = \frac{S_t}{S_{100}} \times 100 \tag{4.16}$$

Whereas in case of a compacted soil the equation in modified to make allowance for the instantaneous compression

$$\%U = \frac{S_t - S_i}{S_{100}} \times 100 \tag{4.17}$$

where

 S_t = Compression at any time after the start of test

 S_{100} = Total compression at end of test

 S_i = Instantaneous compression

4.3 Experiments

In order to verify the foregoing ideas, tests have been performed on samples of Tabriz Red Clay [4] compacted at the optimum water content of the standard proctor energy and one half proctor have been made also on samples which are statically compacted to the optimal of conditions cited above. The tests involve the measurement of mid plane pore water pressure by means of an automatic pore pressure measuring device. For the purpose of testing, a special oedometer has been devised and the porous stone allowing for pore pressure measurement has been chosen to be of a special type. (Air entry value=240 psi). The oedometer was designed so as to reduce frictional effects to a minimum.

The time response characteristics of the pore pressure device have been checked by various means and have proven to be satisfactory for the purpose. Load increments have been applied using a Karol- Warner type of consolidometer using air to create the necessary pressure, it has been found that this is advantageous in applying the loads instantly. The values of compression were recorded very frequently at the beginning of the test so that the value of instantaneous compression could be obtained. Fig 4.1 gives a typical example for the use of test results to evaluate the magnitude of instantaneous compression. For the subsequent consolidation process to locate 100% consolidation following Crawford (1964); compression was plotted versus mid plane pore pressure dissipation and the extrapolation of the straight line portion was made use of to determine the desired value.



Figure 4.1: : Evaluation of instantaneous comparison, sample prepared by 15 blow proctor

Fig 4.2 is typical of the procedure. The values representing the end of consolidation have been compared by those which could be obtained by usual empirical procedures. Although reasonably close agreement has been obtained for the logarithmic fitting method, no relationship could be established with the Taylor square root fitting method. The values of instantaneous compression assessed by test results have been compared to those obtained via equations (4.1) and (4.2) and the deviations for various tests have been found to be within 0% to 20%. This implies that the equation proposed together with the assumption inherent in their derivation is reasonably correct. Pore pressure dissipation tests have been used in conjunction with the solution of equation (4.12) to obtain the forms of the function R(t) defined by equation (4.14) versus time. These forms have also been compared to the form given by equation (4.15) (Terzaghi case) which can be easily obtained by evaluating a C_{ν_0} depending on a specific percentage of consolidation (e.g. 90%).



Figure 4. 2: Pore pressure dissipation vs comparison standard proctor sample

Fig 4.3 is a typical curve of such a comparison. As is apparent in this typical curve, most of the tests indicated that Terzaghi case may be well applicable up to about 60% consolidation and from there on, it overestimates the rate of pore pressure dissipation largely due to the assumption of a constant coefficient of consolidation.



Figure 4. 3: Time factor vs. time (standard proctor sample)

The actual percentages of consolidation as observed from the dial readings have also been compared to the values that have been obtained from pore pressure dissipation observations used in conjunction with the solution of equation (4.12). The agreement is usually found to be close and tolerable within the limits of the experimental accuracy involved. Fig 4.4 is given as a typical curve to illustrate such a comparison.



Figure 4. 4: Percent consolidation vs time 15 blow proctor sample

A common and very interesting observation made through the tests is that initially the pore water is in a state of tension whose magnitude increase as the compactive effort is increased and/or as a static compaction procedure is applied in preference to the usual dynamic proctor compaction method for the same density and water contact. Furthermore, it has been observed that unless the applied pressure is sufficiently large to cause the development of a positive excess pore water pressure, no hydrodynamic consolidation process can start, that is, compression consists of the instantaneous one only, followed by creep effects. For statically compacted samples having a relatively high initial negative pore water pressure, the inducement of positive excess pore pressures may require several successive pressure increments to be applied. Fig 4.5 illustrates this point of view with respect to the statically compacted samples.



Figure 4.5: Pore pressure development statically compacted samples

4.4 Conclusions

The conclusions reached as a result of the investigation may be summarized as follows:

1. The total process of consolidation consists of an instantaneous compression which takes place in a very short time, and of a subsequent process of consolidation.

2. The observed amounts of instantaneous compression are found to be compatible with equations (4.1) and (4.2). This means that the basic assumptions of air being in an occluded form and initial air pressure being initially atmospheric are not far from the truth.

3. The comparison of test results obtained with samples prepared by different compaction methods reveals that the instantaneous compression amount is greater for statically compacted samples than those dynamically compacted.

4. For a given method of compaction, the magnitude of instantaneous compression is larger for the smaller compactive effort.

5. In general, the ratio of instantaneous compression to total compression decreases as at the compactive effort decrease.

6. Pore pressure dissipation tests indicate that the estimates of the rates of compression based on pore pressure measurements are quite close to reality with the exception of very high initial pressure applications, where part of the total stress thrown into the soil skeleton presumably accelerates the secondary effects.

7. The use of a constant coefficient of consolidation seems to be justifiable only where rough work other than research is involved and that is applicability is possible only within up to 60% consolidation.

To assess such a constant coefficient of consolidation, the empirical logarithmic fitting procedure may be employed. However, the difficulty of this curve fitting procedure lies in the long duration of testing required to reach the end of consolidation. Whenever possible, dissipation tests should be performed in preference to such empirical method.

CHAPTER 5 - A Realistic Theory of Soils Consolidation

5.1 Introduction and Purpose of this chapter

Consolidation is a process by which soils decrease in volume. It occurs when stress is applied to a soil that causes the soil particles to pack together more tightly, therefore reducing its bulk volume. When this occurs in a soil that is saturated with water, water will be squeezed out of the soil. In the Classical Method, the magnitude of consolidation usually is predicted by a theory developed by Karl von Terzaghi but Laboratory observations of the consolidation behavior exhibit discrepancies between the theory and the results. These discrepancies are usually attributed to the secondary effects that occur during primary consolidation. On the other hand, Terzaghi's theory presupposes the constancy of permeability and compressibility of the soil. In this study, the effect of variable permeability and compressibility on the consolidation behavior is investigated. For this objective, a mathematical treatment of the behavior is presented. Subsequently, laboratory consolidations tests with mid plane pore pressure measurements are conducted on soft, remolded, preconcolidated and undistributed samples of Tabriz clay. The test results, when compared with the theoretical findings, indicate that most of inherent discrepancies may be explained via the use of the theory developed in this study.

The theory of consolidation proposed by Terzaghi (1923) is a very useful tool for the determination of settlement rates and amounts. Since the proposal of this theory, various researchers have investigated its validity and applicability. These subsequent studies have led to the development of various procedures for estimating settlements. Seed (1965) discusses such various methods and procedures. The experience obtained through the years after the proposal of the Terzagi theory indicate that for one dimensional consolidation in particular, it gives results of acceptable accuracy in many field cases.

However, laboratory tests on various types of clay indicate that, although the hydrodynamic approach presented by the theory can not be disputed on the whole, there seem to be discrepancies between the theoretical predications and observations of the consolidation behavior.

During the stage of primary consolidation, these apparent discrepancies are largely attributed to secondary (creep) effects and attempts have been made either to modify the assumptions implicit in Terzaghi's theory to agree more closely with observed behavior (Barden (1965) and Schiffman (1964)) or to propose rheologic models which would better suit the observed behavior (Wahls (1962) and Lo (1961)).

In this study, the main point of argument is that, although the existence of secondary effects may not be ignored, most of the discrepancies between the predictions based on Terzagi's theory and observed behavior during laboratory testing may be accounted for by modifying two assumption of constant permeability and the assumption of constant compressibility.

In order to carry the discussion further, a qualitative outline of the Terzaghi theory with its assumptions is accounted for in the following section.

The classical prediction procedure of the rate and amount of consolidation via Terzaghi's theory (1924) includes following assumption (Lambe (1960)):

1. The soil is homogenous (uniform in composition throughout).

2. The soil is fully saturated (zero air voids due to water content being so high).

3. The solid particles and water are incompressible.

4. Compression and flow are one-dimensional (vertical axis being the one of interest).

5. Strains in the soil are relatively small.

6. Darcy's Law is valid for all hydraulic gradients.

7. The coefficient of permeability and the coefficient of volume compressibility remain constant throughout the process.

8. There is a unique relationship, independent of time, between the void ratio and effective stress.

If pore pressure dissipation measurements are also made during consolidation testing; it is possible to estimate the rate of settlement by making use of the dissipation time data. The usual consolidation coefficients calculated by two different procedures usually yield similar results for soft soils (Crawford (1964)).

The conventional Terzaghi theory (1924) proposed for fully saturated soils contains two assumptions which may be criticized from the view point of soil behavior. That is; the supposition of a constant compressibility coefficient a_v and a constant permeability coefficient k. It should be evident that as the consolidation proceeds, (effective stress increase) the soil attains a more compact structure which should inevitably result in a decrease in its overall

compressibility. Evidence of this behavior has been obtained through several studies (Leonards et al. 1964). In addition, it has definitely been established that the permeability is a function of void ratio. It is obvious that the void ratio of a soil sample decreases during consolidation; therefore it is natural to expect a decreasing permeability coefficient during the process. In fact, other researches give experimental as well as theoretical evidence towards the recognition of a variable permeability (Barden (1965), Schiffman (1958), and Schmid (1957)).

The propose of this study is to incorporate these variables in a mathematical treatment of the problem and to demonstrate by proper testing that the inclusion of these two variable factors may in fact account for most of the deviations that repeatedly occur between the predictions based on the Terzaghi theory and the test results (Lo (1961), Crawford (1964), Leonards (1964)).

It should be noted that a similar problem has been treated by Barden and Berry (1965) with a different mathematical approach which results in a non linear partial differential equation whose solution is obtained by a finite difference approach employing a suitable computer program, since a closed form solution can not be obtained and also recently Lekha et al. (2003) studied consolidation of clays for variable permeability and compressibility. In their paper, an analytical closed form solution is obtained for vertical consolidation considering the variation in the compressibility and permeability. In addition, Geng et al (2006) studied non-linear consolidation of soil with variable compressibility and permeability under cyclic loadings. In their paper, a simple semi-analytical method has been developed to solve the one-dimensional non-linear consolidation problems by considering the changes of compressibility and permeability of the soil layer, subjected to complicated time-dependent cyclic loadings at the ground surface.

The line of treatment herein, on the other hand, arrives at the description of the consolidation process by a linear partial differential equation whose closed form solution is obtainable via the theory of linear partial differential equation.

5.2 Mathematical Development

In the mathematical treatment of the problem, the first problem is to decide on the nature of a functional relationship between permeability, compressibility and the main variables governing the process of consolidation. The equation is the manner in which these parameters are to be included into the mathematical model of the consolidation process, while retaining the other assumptions inherent in the classical Terzaghi theory.

Since the dependency of these parameters on void ratio is evident, the most reasonable approach would be to express them as functions of void ratio, or, since ratio is a function of pore water pressure, as functions of pore water pressures. Thus

$$k = k(u) \text{ and } a_v = a_v(u) \tag{5.1}$$

Eq. (5.1) suggests that the properties are functions of both time and space. That is

$$k = k(z,t) \text{ and } a_v = a_v(u,t)$$
 (5.2)

At this stage, a postulate must be made as to the variation of the permeability and compressibility defined by eq. (5.2).

To illustrate the foundations of this postulate, reference is made to Fig. 1a. Prior to loading, the values of permeability and compressibility are constant with depth and may be denoted as k_0 and a_0 , respectively. As soon as the load is applied, consolidation starts and after an infinitesimal time, the excess pore water pressure on the drainage surface z = 0 become zero. Via eq. (5.2) this means that both permeability and compressibility reach their final values and remain constant thereafter at the surface. On the other hand, the values of these properties at any depth vary with time as consolidation proceeds.

Therefore, at mid plane z = H via eq. (5.2), the permeability and compressibility are given respectively by

$$k_m = k(H,t) \text{ and}$$

$$a_m = a_v(H,t)$$
(5.3)

It is possible to describe the variation of these properties with space between drainage surface z = 0 and the mid plane z = H and to define the time dependent functions indicated in eq. (5.3). The variation of these properties with depth may be described by various mathematical functions (Birand (1972)). On the other hand, it is possible to define the "space averages" of permeability and of compressibility as follows:

$$\overline{k} = \frac{\int_{0}^{H} k(z,t)dz}{\int_{0}^{H} dz}$$

$$\overline{a} = \frac{\int_{0}^{H} a_{v}(z,t)dz}{\int_{0}^{H} dz}$$
(5.4)

At this stage, the variation of \overline{k} and \overline{a} with time during the consolidation process needs to be defined.

It seems feasible to define these relationships as functions of decay, i.e.

$$\overline{k} = k_i e^{-\alpha t}$$
 and $\overline{a} = a_i e^{-\beta t}$ (5.5)

with

$$\overline{k}(0) = k_i \qquad \overline{a}(0) = a_i$$

$$\overline{k}(t_f) = k_f \qquad \overline{a}(t_f) = a_f$$

In the expressions above k_i

In the expressions above k_i and a_i are the initial values of \overline{k} and \overline{a} , k_f and a_f are their final values, respectively, after a suitably long time t_f during which the primary consolidation is assumed to be almost complete. It is also possible to define the space variation of k and a_v with suitable functions of depth and carry on with the mathematical treatment by substituting these relationships in eqs. (5.4) (Birand (1972)).

Herein, it is assumed that the time t_f is long enough so that, although its value may be accepted as a finite value mathematically, the excess pore water pressure may be considered to be dissipated at the end of this period for all practical purposes. The final expression governing the dissipation of excess pore pressures by the former approach (Birand (1972)) and by the analysis given herein are found to be substantially the same although the former one may be considered more "exact" by the mathematician. However to the benefit of this exactness, it entails the use of cumbersome mathematical formulations and some necessary simplifying assumptions derived from possible behavior of soils during the consolidation process to facilitate the analysis.

Reference to Fig 5.1b. shows the variation of compressibility via effective stress (or with time for all practical purposes) and represents a general curve usually obtained through consolidation dissipation tests.



Figure 5. 1: (a) Variation of pore water pressure, permeability and compressibility and (b) Variation of Compressibility during consolidation

Eqs. (5.5) mean that both \overline{a} and \overline{k} vary with time as a function of decay and reach their final values at the end of consolidation.

Once the mathematical formulation of permeability \overline{k} and \overline{a} (eq 5.5) are made, it remains to write down the continuity equation of consolidation in the usual manner and substituting these mathematical formulations therein, to obtain the governing equation of consolidation.

The continuity expression is written as follows:

$$\frac{\partial}{\partial z} \left(\bar{k} \frac{\partial u}{\partial z} \right) = \frac{\gamma_w}{1 + e_0} \times \frac{\partial e}{\partial t}$$
(5.7)

On the other hand, the effective stress law gives:

$$\overline{\sigma} = \sigma - u \tag{5.8}$$

where $\overline{\sigma}$ = effective stress, σ = total stress, u = pore pressure.

The compressibility is defined as

$$\overline{a} = -\frac{\partial e}{\partial \overline{\sigma}} = \frac{\partial e}{\partial u}$$
(5.9)

Now, remembering that $\frac{\partial e}{\partial t} = \frac{\partial e}{\partial u} \times \frac{\partial u}{\partial t}$, and substituting eq.(5.9) in eq. (5.7), one obtains:

$$\frac{\partial}{\partial z} \left(\bar{k} \frac{\partial u}{\partial z} \right) = \frac{\gamma_w}{1 + e_0} \times \bar{a} \times \frac{\partial u}{\partial t}$$
(5.10)

In eq (5.10) \overline{k} is independent of space, therefore this equation can be written in the following form:

$$\frac{\partial^2 u}{\partial z^2} = \frac{\gamma_w}{1 + e_0} \times \frac{\overline{a}}{\overline{k}} \times \frac{\partial u}{\partial t}$$
(5.11)

Substituting the values of \overline{a} and \overline{k} from eq (5.5) into this expression; the governing partial differential equation is obtained:

$$\frac{\partial^2 u}{\partial z^2} = \frac{\gamma_w a_i}{(1+e_0)k_i} e^{(\alpha-\beta)} \frac{\partial u}{\partial t}$$
(5.12)

It is possible to express this equation in a dimensionless form by specifying the variables W and Z

$$W = \frac{u}{u_0} \quad , \quad Z = \frac{z}{H} \tag{5.13}$$

where $u_0 =$ initial pore pressure, H = characteristic thickness and a time factor T such that

$$T = \frac{(1+e_0)k_i}{\gamma_w a_i H^2} t$$
(5.14)

Also specifying a constant A,

$$A = \frac{(\alpha - \beta)H^2}{c_v}$$
(5.15)

where
$$c_v = \frac{TH^2}{t}$$
, (5.16)

the usual coefficient of consolidation, eq.(5.12) becomes:

$$\frac{\partial^2 W}{\partial Z^2} = e^{AT} \frac{\partial W}{\partial T}$$
(5.17)

It is obvious that depending on the relative values of α and β (signifying the effects of permeability and compressibility respectively) *A* may assume both positive and negative values, therefore, eq. (5.17) needs to be solved for both positive and negative possible values of *A*. It is of further interest to note that in the case of constant permeability and compressibility during the process, or if the rate of change of both parameters is the same, A becomes equal to zero and the process is governed by the partial differential equation arrived at by Terzaghi (1924).

$$\frac{\partial^2 W}{\partial Z^2} = \frac{\partial W}{\partial T}$$
(5.18)

The solution of eq (5.17), subject to the usual oedemeter boundary conditions, is obtained as:

$$W(Z,T) = \sum_{n=0}^{\infty} \left\{ \frac{4}{(2n+1)\pi} \left(\sin \frac{2n+1}{2} \pi Z \right) \varepsilon^{\frac{-\pi^2 (2n+1)^2}{4} \times \frac{\varepsilon^{\pm AT} - 1}{\pm A}} \right\}$$
(5.19)

where ε is the naperian base of the logarithm and *n* is an integer. Similarly, consolidation is defined by the expression:

$$U = 1 - \frac{\int_{0}^{1} W(Z,T) dZ}{\int_{0}^{1} W(Z,0) dZ} = 1 - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \varepsilon^{-\frac{\pi^2 (2n+1)^2}{4} \times \frac{\varepsilon^{\pm AT} - 1}{\pm A}}$$
(5.20)

and the expression for mid plane pore pressure is given by the expression:

$$W_{m}(Z,T) = \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin \frac{2n+1}{2} \pi \varepsilon^{-\frac{\pi^{2}(2n+1)^{2}}{4} \times \frac{\varepsilon^{4AT}-1}{4}}$$
(5.21)

Eqs. (5.20) and (5.21) are plotted for various positive and negative values of the parameter A as illustrated in Fig 5.2 and Fig 5.3, respectively.

As in the case of the classical theory of consolidation, these curves constitute the bases of the evaluation of rates of settlement via the theory developed in this paper. The use of these curves may often involve a trial and error procedure with regard to the appropriate selection of the value of A



Figure 5. 2: Percent consolidation



Figure 5. 3: Mid-Plane pore pressure dissipation

5.3 Experimental Investigation

The soil used in the study is the Tabriz grey Clay. The properties of these clays are related in general.

The particular soil used showed the following index properties:

LL = 68% PL = 23% PI = 45% SL = 12% with a Casagrande classification of CH

It was intended to study the consolidation of the soil behavior in three distinct conditions these being:

a) Soil sample denoted by S - 1:

Soil in a remolded state at a soft consistency. Duely for this state the first soil sample was prepared at a consistency equaling a water content equal to LL-10%

b) Soil sample denoted by S - 2:

Soil initially in a soft consistency, however, in a preconsolidated state. For this purpose the soil sample in (a) was consolidated up to a certain effective stress, rebound, and then tested to observe its behavior. Thus, the soil was tested in a laboratory induced preconsolidated state.

c) Soil sample denoted by S - 3:

Soil in its natural state, being soaked prior to testing without allowing any change in volume.

This procedure may also be called the soak swell prevented type of test which is used for expansive Tabriz Clay.

Equipment and testing procedure:

The equipment consists of a consolidometer in which base pore pressure could be measured by means of an automatic non flow type of pore pressure apparatus. The layout of the equipment is schematically illustrated in Fig 5.4.

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Figure 5. 4: Pore Pressure Device, Oedometer

Details and the time response characteristics of the pore pressure device have been investigated in earlier research and found to be satisfactory for the purpose of consolidation testing (Birand 1969).

Sample *S*-1 is loaded under the following increments, the compression and pore pressure development dissipation being measured for 24 hours under each increment $(in \frac{kg}{cm^2})$: 0.00-0.25; 0.50; 1.0; 2.0; 4.0; and 8.0.

The pore pressure measurement line is then closed and the sample unloaded to $2.00 \frac{kg}{cm^2}$. This load is kept on the sample for 24 hours. Thus, the new loading stage is made on sample S-2 which is preconsolidated to $8.0 \frac{kg}{cm^2}$. This loading stage consisted of reloading sample S-2 thus prepared in two increments, namely: 2.0-4.0; 4.0-8.0 $\frac{kg}{cm^2}$. For these two increments the compression and pore pressure data are observed as in sample S-1. The sample in its original void ratio, (sample S-3) is flooded without allowing any volume change for 24 hours. Then, it is loaded in the following increments: 1.0; 2.0; 4.0; 8.0 $\frac{kg}{cm^2}$. During loading, the necessary data is obtained as in the previous cases. It should be noted at this stage that the pressure increment ratio used throughout testing is 1.00 to minimize the secondary time effects.

On the other hand, another important factor affecting consolidation behavior is the preconsolidation pressure (Taylor (1948) and Lewis (1950). In this investigation, it is hoped to throw some light on to this controversial point by means of the behavior of test sample S-2, at least for the soil investigated.

5.4 Presentation of Results

In presenting the results, the very first step would be the determination of the amount of primary consolidation. Where pore pressure measurements are made, it is better to use the criterion of zero excess pore empirical procedures of curve fitting. Therefore, as was proposed by Crawford (1964), for each increment, compression amounts were plotted versus the pore

pressure dissipation and the straight line portion of the curve (which is straight up to about 70% consolidation as predicted by means of the mid plane pore pressure data) was extrapolated to zero pore pressures to determine the amount of compression during primary consolidation. This amount is designated as d_{100} . Therefore, if compression of the sample at any time is d_r , the percent consolidation U at that time is found by the expression,

$$U\% = \frac{d_t}{d_{100}} \times 100 \tag{5.22}$$

Using this expression, the percent consolidation values are calculated for each increment and plotted against time in the lower portion in Fig 5.5 to 5.16 inclusive. They are shown by the solid lines marked "experimental".

Subsequently, using the time values corresponding to 50% consolidation on these curves, the coefficient of consolidation of consolidation C_v is obtained via eq (5.16) for the Terzaghi case (Fig 5.2, A = 0) and thus, these curves are fitted through 50% consolidation by the Terzaghi predictions. These predictions are shown in the figures by the dotted lines marked "Terzaghi"

The method proposed herein was then applied as follows:

By inspection a suitable A value is chosen and using the time value corresponding to 50% consolidation once again, a new consolidation coefficient corresponding to this A value is found. Then the fitting procedure related above is carried out using the theoretically developed curves for the chosen A value in Fig 5.2. This trial procedure was repeated until a good agreement between the "experimental" and "predicted" curves was obtained. The curves that fit the actual behavior in the best manner are also shown on the same on the same figure as above. The same fitting procedure for the appropriate A value found as above is applied to the observed pore pressure dissipation values for comparison, this time making use of the theoretically developed curves in Fig 5.3. The results of these comparisons are presented in the top portions of Figs 5.5 to 16 inclusive, solid lines again representing the experimental observations, and the other corresponding to the fitting made via the relevant A value.

5.5 Discussion of Results

An analysis of the curves obtained by the procedures related in the previous section may be related as follows:

In general, it is obvious that for the soil investigated in various states, the Terzaghi theory seems to be still a very powerful tool in predicting the rates of settlement. However, except for the three pressure increments of sample S-1, whose behavior is presented by Figs 5.6, 5.7, 5.8 and for the last pressure increment of sample S-3 presented by Fig 5.16, there exist discrepancies between the actual the actual consolidation behavior and its Terzaghi predictions. These deviations, which are largely attributed to "secondary effects", are seen to be correctable by means of the theory forwarded in this study. This fact very strongly supports the idea that these deviations mostly result from varying compressibility and permeability during consolidation, which is the starting point of the theoretical development in this investigation. The cases exemplified by Figs 5.6, 5.7, 5.8 and 5.16, closely agree with Terzaghi behavior A=0. The actual pore pressure dissipation behavior, where fitted either by the Terzaghi theory A=0 or the theory proposed herein (A = appropriate value) seems to be very closely predictable. This shows once again the predominate character of the hydrodynamic process during consolidation rather than the secondary effects.

It is worth nothing that the parameter A is always positive. Eq (5.15) indicates that in this case, the rate of decrease of permeability is the predominant factor rather than the rate of decrease of compressibility, for the soils tested. It is also interesting to note that for the preconsolidated sample S - 2 this generalized theory is applicable for determining the rates of compression, since the applicability of the Terzaghi theory (rather the hydrodynamic philosophy behind it) has been questioned for preconsolidated soils.

Reviewing the behavior of the sample of soft consistency (sample S-1) it is seen that deviations from Terzaghi theory occur when the first load increment is applied (Fig 5.5) and again when increments of large magnitude are applied (Figs 5.9, 5.10, 5.11). This may be due to the fact that in both cases the soil sample is presumably subjected to larger alteration in its structure and its engineering properties during consolidation. In fact, both compressibility and

permeability should be considered as functions of the magnitude of applied pressure as well as an intrinisic property of the soil depending on its void ratio, structure, degree of saturation, etc. Therefore, it would not be wrong to presume that a soil sample loaded in increments up to a certain pressure would follow a different pressure deformation curve than if the ultimate pressure had been applied all in one step.

In this investigation, note should be made of the fact that that the usual empirical curve fitting methods such as the square root fitting method or the logarithmic fitting method are not employed, mainly due to the fact that the measurement of pore pressure is believed to be a better substitute. In view of the present study on the other hand, a criticism of these methods may be made. Fig 5.17 shows the average consolidation plotted against the square root of the time factor, for the Terzaghi case and for values of A = +1.00 and A = -1.00 it is obvious that the application of the square root fitting method to any soil behavior in any other manner than that of Terzaghi A = 0 would give vastly incorrect results both as to the time of completion of primary consolidation and to the value of the coefficient of consolidation C_v .

Fig 5.18 shows the same curves plotted against the logarithm of time. It seems from these figures that although deviations are apparent, the errors introduced by using the "logarithm of time" fitting method would be smaller. For A = +1.00 this method is seen to yield about 90% primary consolidation instead of 100%. For larger positive A = +1.00 values, the errors become much larger. This observation may in fact account at least partly for the discrepancies that occur between the settlement rates predicted in the laboratory by these empirical rules and those actually taking place in the field.

5.6 Conclusions

As a result of the present study the following conclusions may be reached:

1) The Terzagi theory, in predicting the settlement rates is a very valuable tool.

2) The observed departures from this theory seem to be mostly due to the variation in the compressibility and the permeability of the soil. For the specific soil tested, permeability seems to be the predominant factor.

3) Using the theoretical treatment forwarded in this study, it is possible to eliminate largely the discrepancies and predict the rates of settlement more accurately.

4) As far as the soil used in this study, it is shown that the proposed theory may also account for its apparent departures from the Terzaghi behavior in a preconsolidated state as well.

5) The empirical curve fitting procedures should be applied with caution. Although the logarithmic fitting procedure seems to be more reliable, for important civil engineering estimates of rate of settlement and for research, pore pressure dissipation tests seem to be the best procedure.



Figure 5. 5 : Dissipation Consolidation versus Time



Figure 5. 6: Dissipation Consolidation versus Time



Figure 5. 7: Dissipation Consolidation versus Time



Figure 5. 8: Dissipation Consolidation versus Time



Figure 5. 9: Dissipation Consolidation versus Time



Figure 5. 10: Dissipation Consolidation versus Time



Figure 5. 11: Dissipation Consolidation versus Time



Figure 5. 12: Dissipation Consolidation versus Time



Figure 5. 13: Dissipation Consolidation versus Time



Figure 5. 14: Dissipation Consolidation versus Time



Figure 5. 15: Dissipation Consolidation versus Time



Figure 5. 16: Dissipation Consolidation versus Time


Figure 5. 17: Consolidation (%) versus (Square root scale)



Figure 5. 18: Consolidation versus Logarithm of Time Factor

Notations

- a_{y} Coefficient of compressibility
- c_v Coefficient of consolidation
- e Void ratio
- e_0 Initial void ratio
- f() Function of
- k_{0} Initial coefficient of permeability
- k_{t} Final coefficient of permeability
- n Porosity
- p Stress
- Δp Stress increment
- t Time
- *u* Excess over hydrostatic pressure
- V Velocity
- $V_{x,y,z}$ Directions velocity in x, y, z
- w Water content
- W_{L} Water content at liquid limit
- z Distance
- A Area

C Parameter of $\frac{\Delta U}{\Delta \sigma}$

- D Dissipation of mid-plane pore pressure
- G_s Specific gravity
- H Thickness of layer
- P Load
- S Settlement
- T Dimensionless time factor
- U Degree of consolidation

- V_{v} Volume of voids
- $V_{\rm s}$ Volume of Solids
- V_{w} Volume of water
- W_s Weight of solids
- W_{W} Weight of water
- γ_s Unite weight of solids
- γ_{w} Unite weight of water
- σ Total stress
- $\overline{\sigma}$ Effective stress in soil
- $\Delta\sigma$ Stress increment
- $a_i(T)$ Trial solution (Time function)
- k Permeability
- h Specific permeability
- R Function residual
- u_0 Initial pore pressure
- *u* Pore pressure
- W Pore pressure (Dimensionless)
- Z Space parameter (Dimensionless)
- $\varphi_i(Z)$ Trial solution (Depth function)

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