TWO-PHASE PRESSURE DROP
A LITERATURE SURVEY AND CORRELATION ANALYSIS

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INTRODUCTION

Any combination — solid-liquid, solid-gas, or liquid-gas — in motion con-
stitutes two-phase flow. Recently, considerable effort has been expended on
liquid-gas flow, since this type of mixture is vital to the analysis and design of
refrigeration cycles, power plants, compressors, condensers, evaporators,
boilers, and more recently, to nuclear reactor research and space rocket technol-
ogy. In these investigations, two-phase pressure drop calculations are significantly
important. Unfortunately, these calculations can be quite difficult and complex for
the following reasons: (1) external and internal bounding surfaces are continuously
interacting; (2) forces and thermal interactions at the interfaces affect changes
in the fields of flow velocities, pressures, temperatures, and thermal and diffusion
fluxes; and (3) twice as many flow and property variables in comparison with
single-phase flow theoretically complicates even the simplest physical flow models.

Because simple approximations have recently proven inadequate for most
practical applications, emphasis has been placed on methods utilizing physical
models correlated with experimental data. These models attempt to describe the
phenomena without becoming so mathematically complex as to render the method
of computation used impracticable. Unfortunately, the methods presently avail-
able still are too complicated and inaccurate for consistent application in standard
engineering equipment design.

This report summarizes the present methods of two-phase pressure drop prediction, gives enough essential background material to provide a useful physical picture, and, hopefully, provides enough details to stimulate additional study and interest over the field. Additional references and excellent reviews have been published by Isbin et al. (28); Jens and Leppert (30); Tong (48); Bennett (9); Scott (43); Anderson and Russell (3); and Gresham et al. (20).

LITERATURE SURVEY

A true scientific basis for analyzing experimental data on single-phase fluid flow did not exist until 1883, when Osborne Reynolds developed from dimensional analysis the equations for dynamic similarity[1] and dimensional homogeneity.[2] Earlier experimenters, beginning with Couplet in 1732, attempted to develop empirical equations to fit their experimental data. These equations were, however, limited in that they applied only for particular pipe sizes within given velocity ranges and only for specific fluids. Following Reynolds, in 1914, Stanton and Pannell (47) investigated single-phase fluid flow through smooth pipes, applying a dimensional analysis to the fluid friction problems appearing in engineering practice. Their experiments on compressible and incompressible flow confirmed the feasibility of using a dimensional approach.

In 1934, serious interest was taken in applying the theory of similarity to the motion of liquid-gas mixtures. The following excerpt from a paper by Schmidt (42) published in that year defined the situation at that time:

"The motion of liquids by means of rising steam or gas bubbles, such as takes place in steam-boiler tubes and evaporator apparatus, is so complicated that its analytical calculation is impossible. Evan an attempt to establish the necessary differential equations meets with insuperable difficulties. On the other hand, resort to the point of view of similarity offers the possibility of reducing the variables to a certain minimum number of dimensionless magnitudes. The solution of the problem is then found in the form

[1]Similarity analysis involves dimensionless numbers with selected independent variables used to describe a system i. e., Reynolds, Weber, and Euler numbers.

[2]Dimensional homogeneity — an equation is dimensionally homogeneous if the dimensions of each term of the equation are the same.

of a relation between these magnitudes, which usually have to be determined experimentally."

By 1944, it was generally realized that because of the large number of variables involved, a single universal equation would be inadequate to correlate pressure losses over the entire range of two-component flow. Martinelli et al. (35) at the University of California were the first to investigate the problem, using Schmidt's suggested similarity and experimental correlation analysis. Their studies involved isothermal pressure drops for two-phase two-component flow, without mass exchange and in a horizontal pipe. Eliminating the use of flow patterns, they obtained a fairly satisfactory method of solution by relating the actual pressure drop (ΔP_{TP}) to its single-phase counterpart ($\Delta P_{\ell}, \Delta P_{g}$) flowing through the same pipe. These studies were based on turbulent flow experimental data only. In 1946, Martinelli et al. (37) extended their studies to the viscous region. Later Lockhart and Martinelli (33) further extended the method to include all flow regimes.[3] Various investigators have since attempted to apply the equations developed by Martinelli and his co-workers to systems with and without mass transfer and for known flow patterns[4] in vertical and horizontal pipes. In 1948, Martinelli and Nelson (36) adapted the two-component correlation of Lockhart and Martinelli to single-component systems by a simple correction to account for changes in the axial component of momentum. The two-component systems of Lockhart and Martinelli did not involve appreciable mass transfer between the phases, whereas the single-component systems of Martinelli and Nelson did. Martinelli and Nelson originally did not possess sufficient experimental data to verify their approach. Martinelli's

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[3]Hereafter referred to as the Lockhart-Martinelli correlation. See page 43 for this method and example problem.

[4]Methods of flow classification are discussed beginning on page 8.

studies provided the first acceptable methods to predict two-phase, two-component or one-component flow with or without mass exchange.

In 1951, Abou-Sabe (1) at the University of California suggested that the Martinelli correlation (33, 36) should be modified to compensate for the effects of flow pattern and roughness existing at the gas-liquid interface. Baker (7) later developed equations to calculate the pressure losses of gas-oil mixtures flowing in pipelines. His equations essentially modified the Lockhart-Martinelli correlations for annular, slug, plug, and bubble flow. Prior to this, in 1946, Armand (4), and Armand and Tretchev (5) in Russia, had investigated the flow of air-water mixtures in pipes. They developed three equations for correlating pressure losses for annular mist, annular flow, and stratified bubble flow. Later, studies by Gazley and Bergelin (17), Jenkins (29), and Holden (23) at the University of Delaware showed that the Lockhart-Martinelli correlation was not applicable to stratified flow.

The best discussion of the Homogeneous method in which a two-phase friction factor (analogous to the Fanning single-phase friction factor) is used has been presented in a paper by Owens (40).[5]

Harvey and Faust (22)[6] later combined the Homogeneous and Martinelli approaches by selecting assumptions from both. Rogers (41) then utilized this mixed-model method to predict the behavior of two-phase hydrogen flows in well insulated transfer systems. A large number of computations were required to obtain numerical results from the equations of Harvey and Faust; Rogers used a high-speed computer.

Dukler et al. (14) from his "Comparison of Existing Correlations"

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[5] See page 39 for the Homogeneous method.

[6] Mixed models are discussed on page 53.

states that over 25 correlations now exist. Dukler's own correlation[7] involves a two-phase Reynolds number, the Koo equation (31), as well as other significant terms to compensate for elevation, including an iterative procedure to evaluate the acceleration correction factor. This method predicts the two-phase pressure drop independent of flow pattern. Additional methods by Dukler utilizing similarity analysis involved the following hypothesis (1) assuming homogeneous flow, applicable to the mist or spray-flow region, and (2) calculating a two-phase function factor and Reynold's number in terms of liquid properties. These criteria are applicable especially to flow patterns with high liquid-to-gas ratios such as plug and bubble flow.

Chenoweth and Martin (11)[8] have introduced an empirical correlation for horizontal flow with gas and liquid superficial Reynolds numbers greater than 2000. This correlation used larger pipe diameters and greater pressures than did the correlation of Lockhart and Martinelli (33). It empirically correlated the ratio $(\Delta P_{TP}/\Delta P_{\ell *})$ of the two-phase pressure drop and pressure drop of the mixture if only the liquid were present, to the liquid-volume fraction of the feed (R). A parameter that is a function of single-phase friction factors obtained from a Moody chart appears in the correlation.

Hughmark (25)[9] empirically correlated the ratio of the two-phase friction factor $(f_{TP}/f_{\ell *})$ as a function of the liquid-volume fraction in the feed stream, using a two-phase Reynolds number as a parameter.[10] The single-phase friction factor is then obtained from the usual single-phase correlations, using average properties for the gas-liquid mixture in the pipe. Hughmark's correlation

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[7]Dukler's method and example problem may be found on page 33.

[8]See page 30 for Chenoweth and Martin correlation.

[9]See page 31 Hughmark's correlation.

[10]$Re_{TP} = \dfrac{DG}{R_\ell \mu_\ell + R_g \mu_g}$ (Eq. 25).

can be used either for horizontal or vertical flow. Levy's method[11] proposed in 1963 (32) was based on a mixing-length model. This model treated the two-phase system as a continuous medium, and applied to it methods and assumptions widely accepted in single-phase turbulent flow. In 1962, Govier and Omer (18) summarized the state of knowledge concerning the calculation of pressure drop and other related quantities in two-phase flow:

"The principal flow patterns are understood in a qualitative way and the effect on flow pattern of the major variables, mainly the mass velocities of the phases, is recognized. In nearly all cases, the real significance of the viscosities of the phases, the possible separate roles of the densities of the phases, and the influence of the diameter of the pipe is not known.

"Extensive experimental data confirm the fact that the pressure drop for a two-phase system is influenced by the flow pattern, and indicate the need either for separate pressure drop correlations for each flow pattern, or for the incorporation into a master pressure drop correlation of these variables which in fact define the flow pattern.

"A large number of correlations are available for the prediction of two-phase pressure drop in terms of the major variables affecting it. Each of these has its own shortcomings, being restricted to certain flow patterns, to gases of specified densities, to certain diameters of pipe, or the like. There is as yet no generally satisfactory universal correlation. "

Excellent charts, tables, reviews, and comparisons have been published in the literature, namely by Bennett (9), Tong (48), Scott (43), Isbin et al. (27), and Gresham et al. (20). Experimental data (containing over 9000 experimental measurements) may be obtained from the University of Houston's Data Bank, compiled by Dukler et al. (15).

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[11] Levy's approximation theory may be found on page 54.

ANALYSIS (TWO-PHASE FLOW)

Before analyzing two-phase pressure-drop correlations, it will be necessary to describe the various flow classifications which are most generally accepted in the literature. These classifications provide the means whereby mathematical expressions may be applied to the energy and momentum balances. Two-phase flow is classified by system, appearance, and type.

System Classifications

Classification by <u>system</u> involves either being single component (a pure liquid and its vapor), or two or more components with any one component present in either or both of its phases. Systems may have mass transfer between phases (i.e., vaporization), or otherwise. They may be isothermal, adiabatic, or have intermediate temperature behaviors.

Appearance Classification

A <u>flow pattern</u> refers to the visual appearance of a particular flow condition. Alves (2) illustrated seven flow patterns for horizontal flow (Fig. 1).
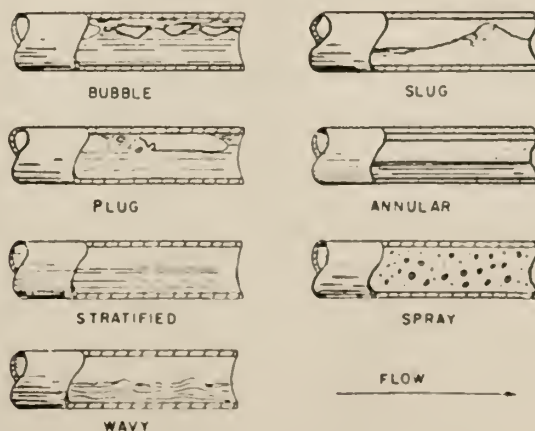
Fig. 1. Flow patterns in horizontal flow.

These flow patterns are:

Bubble flow. Discrete gas bubbles move along the upper surface of the pipe at approximately the same velocity as the liquid. At high liquid rates, bubbles may be dispersed throughout the liquid, a pattern often referred to as froth flow.

Plug flow. The gas bubbles tend to coalesce as gas flow rate increases, to form gas plugs which may fill a large part of the cross-sectional area of smaller tubes.

Stratified flow. Complete stratification of gas and liquid, with the gas occupying a constant fraction of the cross-sectional area in the upper portion of pipe, over a smooth liquid-gas interface. It occurs at lower liquid rates than bubble or plug flow, and more readily in larger tubes.

Wavy flow. Increasing gas rate produces waves of increasing amplitude at the stratified gas-liquid interface, because of the higher gas velocity.

Slug flow. Wave amplitudes increase to seal the tube, and the liquid wave is picked up by the rapidly moving gas to form a frothy slug which passes through the pipe at a much greater velocity than the average liquid velocity. Slug flow is also formed from plug flow as the gas flow rate is increased at constant liquid rate.

Annular flow. Gravitational forces become less important than inter-phase forces, and the liquid is mainly carried as a thin film along the tube wall. The gas moves at a high velocity in the core of the tube and carries with it some of the liquid as a spray. Film flow is a name also applied to this pattern.

Mist or spray flow. More and more liquid is carried in the gaseous core at the expense of the annular film, until nearly all of the liquid is entrained in the gas. The pattern has also been called dispersed flow or fog flow.

Upward vertical flow patterns are more complex than horizontal flow patterns because as the gas velocity increases at a constant rate, a dispersed type of flow will be reached at lower gas velocities due to the influence of gravity on the liquid. Vertical flow patterns tend toward radial symmetry. This is not the case in horizontal flow. A classification of vertical flow patterns based largely on air-water mixtures has been given by Nicklin and Davidson (39) (see Fig. 2).   (No literature on downward flow was found.)

BUBBLE   SLUG   FROTH   ANNULAR   MIST

— INCREASING GAS FLOW —→
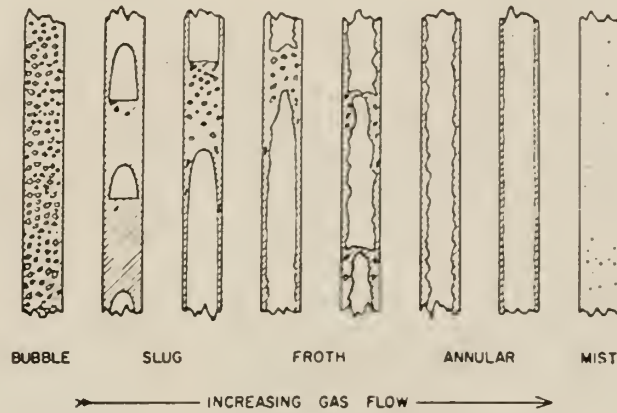
Fig. 2. Flow patterns in vertical flow.

Bubble flow. Gas is dispersed in the upward flowing liquid in the form of individual bubbles of various sizes. As gas flow increases, the bubbles increase steadily in numbers and size.

Slug (or plug) flow. The gas bubbles coalesce to form larger bullet shaped slugs having a parabolic outline at the head. These slugs increase in length and diameter, and their upward velocity increases as the gas rate increases. The slugs are separated by liquid plugs which contain gas bubble inclusions. As the gas slug moves along the tube, liquid flows down through the thin liquid annulus surrounding it into the bubble-filled liquid plug beneath.

Froth flow.   When back flow of the liquid around the slugs nearly stops the slug becomes unstable, and gas slugs seem to merge with the liquid into a patternless turbulent mixture having the general nature of a coarse emulsion. The elements of this structure are in a continual process of collapse and re-formation.

Annular (or climbing film) flow.   The gas travels up the core of the tube at a high velocity and the liquid forms an annular film around the tube walls. Initially, this film may be fairly thick and have long waves on which are superimposed a pattern of fine capillary waves.   As gas flow rate increases, the film becomes thinner and the amount of the liquid entrained as droplets in the central gaseous core increases.

Mist (spray, fog, fully dispersed) flow.   At very high gas rates the amount of liquid entrainment increases until apparently all the liquid is carried up the tube as a mist or fog.   Although a thin liquid film may exist on the wall, its presence is not obvious in this region.

Distinguishing the various possible flow patterns has usually been approached experimentally by visual observation where the results represent the space relationships of the two phases.   Photographs, moving pictures, and stroboscopic techniques have been utilized to more clearly define transitions between the various flow patterns.   So far these transitions have not been acceptably defined.

Type Classification

Because of the inherent difficulties present in classifying and applying flow patterns to a correlation model, Martinelli et al. (35) classified two-phase flow on the basis of turbulence, or lack of it, which exists in each phase.

This classification results in four possible combinations hereafter referred to as <u>flow types.</u> They are:

1. Viscous liquid flow — viscous gas flow (v-v).

2. Viscous liquid flow — turbulent gas flow (v-t).

3. Turbulent liquid flow — viscous gas flow (t-v).

4. Turbulent liquid flow — turbulent gas flow (t-t).

Lockhart and Martinelli (33) used the Reynold's number to determine the turbulence or non-turbulence of each phase. (Re > 2000 - turbulent; Re < 1000 - viscous, where 1000 < Re < 2000 indicates transitional instability.)

Other methods have been proposed to predict flow types from flow conditions, fluid properties and pipe geometries. These have been presented by Baker (7), Griffith and Wallis (21), Govier et al. (19) among others. From these charts approximate determinations can be made as to the type of flow which should occur for a specific flow problem. This information may then be used in one of two ways: either to facilitate physical comprehension of the fluid motion in the equipment, or to allow the selection of the best pressure-drop correlation within the various flow conditions. A sample problem is included in Appendix A which demonstrates Baker's method. Anderson and Russell (3) present an excellent review of this subject.

Recently, Smissaert (45) presented a paper in which he stated, "Flow pattern effects were found to be conveniently described by means of a variable exponent of the Froude number. This exponent was observed to be a linearly decreasing function of the logarithm of the ratio of the volumetric flow rates."

This experimental study applies only to vapor-liquid, low-circulation rates, but it is indicative of the type of research and conclusions that are being presented in an effort to comprehend the mechanisms and phenomena of the two-phase flow.

Special Terms

In conjunction with the various classifications of two-phase flow previously presented, it is useful to define and derive some terms pertinent to the study of two-phase flow and two-phase pressure drop. (For description of nomenclature see Appendix D.)

Flow pattern. Flow classification based on the visual appearance of the flow.

Flow type. Distinguishing between the various flow mechanisms on the basis of turbulence or non-turbulence, using Reynold's number as the criterion.

Flow model. A theoretical combination of flow patterns and types which enables utilization of simplified mathematical techniques to solve for unknown flow parameters.

Mixture quality. Ratio of the mass flow rate of the gas to the total mass flow rate of the mixture:

$$\text{Vapor mass fraction} = \text{Quality} = X = w_g/w \tag{1}$$

$$\text{where } w = w_g + w_\ell \tag{2}$$

Void fraction. Ratio of the volume occupied by the gas to the total volume of a channel section which brackets the cross section of interest. The void fraction is also given by the ratio of the cross section through which the gas is flowing to the total cross section of the channel:

$$\text{Void fraction} = \alpha = v_g/v = A_g/A_p \tag{3}$$

Slip ratio. The slip ratio at a particular cross section is defined as the ratio of the actual gas velocity and the actual liquid velocity at the cross-section:

$$\text{Slip ratio} = u_g/u_\ell \tag{4}$$

Volume fraction. Fraction of pipe occupied by a single phase of a mixture.

expressed. The following summary will be helpful. The gas and liquid velocities are defined to be:

$$u_g = w_g / A_g \rho_g \tag{9}$$

and

$$u_\ell = w_\ell / A_\ell \rho_\ell \tag{10}$$

where

$$A_g = \alpha A_p \tag{11}$$

and

$$A_\ell = (1 - \alpha) A_p . \tag{12}$$

The slip ratio is, therefore, equal to

$$u_g / u_\ell = \frac{w_g}{w_\ell} \frac{1 - \alpha}{\alpha} \frac{\rho_\ell}{\rho_g} . \tag{13}$$

The ratio of the mass flow rates can also be expressed as

$$w_g / w_\ell = \frac{w_g}{w - w_g} = \frac{w_g / w}{1 - (w_g / w)} = \frac{X}{1 - X} \tag{14}$$

and the slip ratio is then determined by

$$\frac{u_g}{u_\ell} = \frac{X}{1 - X} \frac{1 - \alpha}{\alpha} \frac{\rho_\ell}{\rho_g} . \tag{15}$$

Subtraction of Eq. (10) from Eq. (9) and substitution of Eqs. (11) and (12) give the relative velocity

$$u_g - u_\ell = \frac{w_g}{\alpha A_p \rho_g} - \frac{w_\ell}{(1 - \alpha) A_p \rho_\ell} \tag{16}$$

or

$$u_g - u_\ell = w \left[\frac{w_g}{w} \frac{1}{\alpha A_p \rho_g} - \frac{w_\ell}{w} \frac{1}{(1 - \alpha) A_p \rho_\ell} \right] . \tag{17}$$

Substitution of Eq. (14) and simplification gives

$$u_g - u_\ell = \frac{w}{A_p \rho_\ell} \left[\left(\frac{X}{\alpha} \right) \left(\frac{\rho_\ell}{\rho_g} \right) - \left(\frac{1 - X}{1 - \alpha} \right) \right] . \tag{18}$$

Substitution of Eq. (7) yields

$$u_g - u_\ell = u_{sm} \left[\left(\frac{X}{\alpha} \right) \left(\frac{\rho_\ell}{\rho_g} \right) - \left(\frac{1 - X}{1 - \alpha} \right) \right] . \tag{19}$$

Physical Relationships Among Flow Parameters

To acquaint the reader with a physical picture of two-phase flow inter-
action, the following description given by Smissaert (45) outlines the funda-
mental relationships among the various flow parameters:

1. Influence of Flow Conditions

"As a general rule, the slip ratio and the relative velocity increase
with increasing quality. However, the rate of change of the slip as a
function of quality is larger for low qualities than for high qualities. The
influence of quality on the slip ratio decreases significantly with increasing
pressure and also decreases with increasing circulation rate, but to a lesser exten

"Slip ratios decrease and relative velocities increase with increas-
ing superficial liquid velocity. The magnitude of the velocity effect
appears to decrease with decreasing quality and increasing pressure.
The rate of change of the slip as a function of quality also decreases
with increasing circulation rate.

"Pressure has a dual influence on two-phase flow phenomena. The
increase in gas density, which is a direct effect of the increase in
pressure, will reduce the buoyant force which the liquid is exerting on
the gaseous phase. As a result, the slip ratio will decrease with increas-
ing pressure. A second effect is the decrease of the volume of gas
which is present in the two-phase mixture. This effect results in a
change of flow pattern, which has a substantial influence when a churn
or semi-annular flow pattern is effective.

"Geometry includes such items as the form of the channel cross-
section (rectangular, circular, etc.) and the magnitude of the equiv-
alent diameters smaller than 2 in. The effect of geometry is more

significant the smaller the equivalent diameter. Flow patterns are believed to be responsible for this phenomenon."

2. Influence of the Fluid Properties

"Although previous studies have been successful in establishing an acceptable theory on the effects of flow conditions, the same cannot be said about the understanding of the influence of fluid properties. Although these effects have been studied in the past, there still exist numerous controversies about their relative importance. Fluid properties (density not included) are considered to have little or no influence on slippage. It should be noted, however, that the experimental study of the influence of fluid properties is inherently difficult. There is virtually no way to change one property of a fluid and keep the others constant at the same time. Moreover, in order to investigate a sufficiently wide range, one is compelled to employ different fluids. As a result, the influence of one property is generally masked by that of the other properties.

"It is an easily accepted and experimentally verified fact that the slip ratios increase with increasing liquid density. From a previous study by Moore and Wilde (38) it seems that slippage is to some extent dependent upon the surface tension of the liquid. The influence of viscosity has repeatedly been reported as negligible."

ANALYSIS (TWO-PHASE PRESSURE DROP)

Generalized Analysis

By interaction between phases, two-phase pressure drops are generally greater than those experienced in single-phase flows with comparable mass flow rates. For any tube, two-phase pressure drop consists of the summation of losses due to friction, acceleration, and elevation. Frictional losses (the amount of energy that the medium releases in overcoming friction between the tube wall and the fluid) are always present and occur for any orientation of the flow channel and for adiabatic or heated conditions. These losses are the most difficult to analyze. Acceleration losses (change of kinetic energy caused when the medium accelerates or changes momentum) which for horizontal pipes and no friction usually are evaluated simply as $A_p \Delta P_a$ (momentum of inlet streams minus the momentum of outlet streams). Momentum losses frequently occur in forced-circulation boiling when vaporization causes the mixture density to vary along the tube. Losses in pressure due to elevation are associated with the weight of the medium whenever it changes its vertical elevation and are present both for vertical and inclined flow systems. Allowances for hydrostatic head also are usually made separately by $\Delta P_z = \frac{g}{g_c} \int_1^2 dz/v$ where

$$1/v = \rho_g R_g + \rho_\ell (1 - R_g) \tag{20}$$

To predict acceleration losses and elevation losses, knowledge of the mean two-phase mixture density is necessary. In vapor-liquid flow, the mean velocities of the two phases (dependent on the cross-sectional area of each) generally are not equal. Because of this "slip," the true fraction of the pipe

cross-section occupied by either phase differs from the ratio of gas and liquid volumes entering the tube. As a result, the mean mixture density cannot be calculated on the basis of quality (vapor mass fraction) alone, but requires a knowledge of the void fraction (fraction of the channel cross section occupied by vapor). Void fraction, then, is an important parameter used to predict the hydraulics of many two-phase flow systems, since without it, the accelerative and hydrostatic contributions to the pressure drop cannot be evaluated.

Holdup correlations, to predict the volume fraction of liquid and gas at a specified cross section in the pipe for known quantities of liquid and gas entering the pipe, are essential for most frictional pressure-drop calculations. Holdup differs from the volume fraction determined from the densities and mass flow rate because of slip and density changes caused by the pressure gradient in the pipe.

Early methods to determine holdup suggested by Lockhart and Martinelli (33), Levy (32), Martinelli and Nelson (36), and Hughmark (26) consider the extreme flow models of homogeneous and annular flow. Bankoff (8) in 1960 suggested a holdup correlation which did not assume these extreme conditions of flow. He considered a model in which the mixture flows as a suspension of bubbles in the liquid, and where radial gradients exist in the concentration of bubbles.

By assuming a power law distribution for the velocity and void fraction

$$u/u_m = S^{1/m} \tag{21}$$

$$\alpha/\alpha_m = S^{1/n} \tag{22}$$

Bankoff (8) derives the following equation for holdup:

$$\frac{1}{X} = 1 - \frac{\rho_\ell}{\rho_g}\left(1 - \frac{K''}{R_g}\right) \tag{23}$$

The flow parameter K'' is a function of m and n, and was calculated from the empirical equation

$$K'' = 0.71 + 0.0001\ P \tag{24}$$

K'' applies only for steam-water (atmospheric – 2000 lb/in.^{2}) systems. This equation does not apply to air-liquid, two-phase flow.

Hughmark (25) correlated a more general method for determining K'' in Eq. 23. His flow parameter (designated by K) is obtained from the dimensionless chart shown in Fig. 3 which relates the following three variables:

LEGEND

Z	K
1.3	0.185
1.5	0.225
2.0	0.325
3.0	0.49
4.0	0.605
5.0	0.675
6.0	0.72
8.0	0.767
10	0.78
15	0.808
20	0.83
40	0.088
70	0.93
130	0.98

$$Z = (Re_{TP})^{1/6}(Fr_m)^{1/8}/\lambda^{1/4}$$
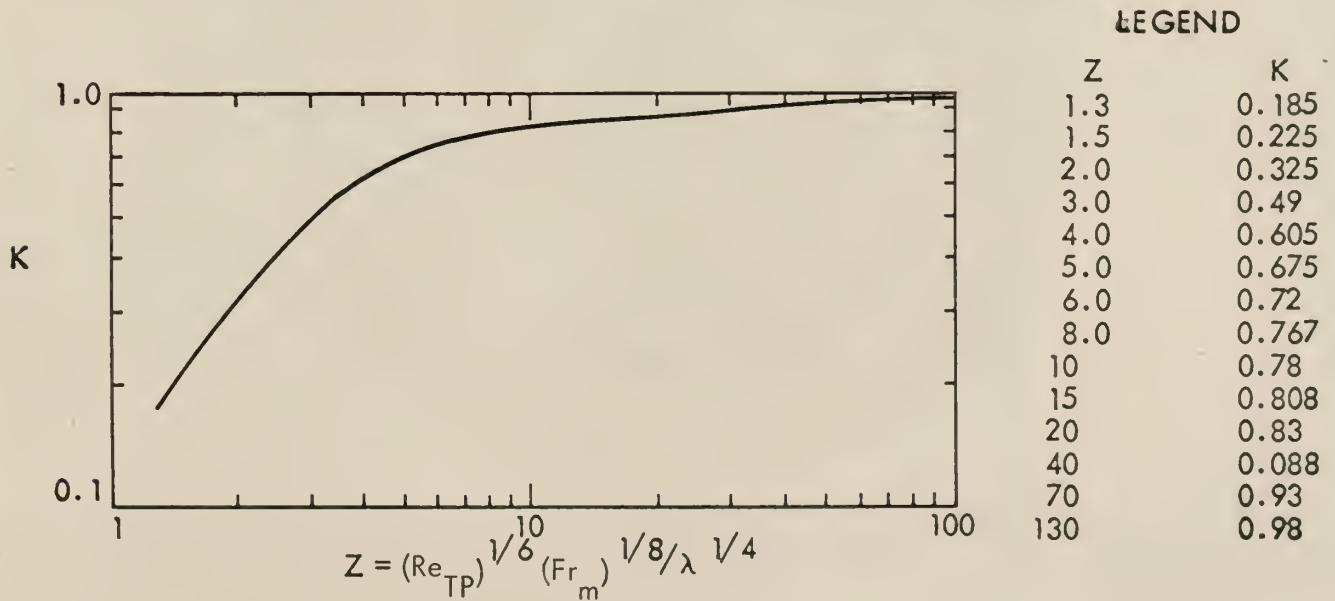
Fig. 3. Hughmark's correlation for flow parameter K from Z.

1. Reynold's Number

$$\text{Re}_{TP} = \frac{DG}{(R_\ell \mu_\ell + R_g \mu_g)} \tag{25}$$

where $(R_\ell \mu_\ell + R_g \mu_g)$ is a two-phase viscosity term.

2. Froude Number

$$\text{Fr}_m = \frac{u_{NS}^2}{gD} \tag{26}$$

(the velocity is defined for the two phases traveling at the same velocity, i.e., no slip between phases).

3. Liquid volume fraction

$$\lambda = \frac{w_\ell v_\ell}{w_\ell v_\ell + w_g v_g}, \tag{27}$$

by the equation

$$Z = \frac{(\text{Re}_{TP})^{1/6} \, (\text{Fr}_m)^{1/8}}{(\lambda)^{1/4}}. \tag{28}$$

To determine the volume fraction (holdup), an example problem is included as a part of Dukler's method to predict pressure drop.[12]

The experimental techniques currently employed to measure pressure drop in two-phase flow involve the use of piezometer rings for vertical flow, and single-pressure taps for horizontal flow. Normally, phase separators adjacent to the pressure tap ensure single-phase communication with the pressure measuring device. Essentially, the techniques involved are strongly dependent on the flow pattern under investigation. Special problems occur

[12]See page 33 for Dukler's method and example problem.

when flow patterns of the plug and slug type are considered. The mean pressure at any cross section fluctuates considerably, and the determination and significance of a mean pressure with time poses difficult problems. For steady horizontal flow (i. e. stratified flow) another difficulty arises due to the variation in pressure around any vertical diagnostic tube. Because it is only in annular and dispersed flow that these difficulties are reduced to a minimum, annular flow has received the most experimental and theoretical study.

A comprehensive listing of recent experimental steam-water investigations has been given by Tong (48).[13] A similar table has been presented by Hughmark (25), including different two-phase components.

<div align="center">Energy and Momentum Equations</div>

Energy and momentum equations are fundamental to the analysis, derivation, and discussion of any two-phase pressure drop calculation method. In 1962, Vohr (49) published a paper in which the proper form of the energy equation (no mass transfer) is discussed. White and Lamb (50) in the same journal reviewed the use of momentum and energy equations in two-phase flow. Scott (43) presented an excellent review in which the limits and common conditions encountered experimentally are briefly summarized. This paper includes a review of a paper by Nicklin and Davidson (39) in which the irreversible terms of the equation are discussed. These articles furnish ample background for applying the equations of energy and momentum to the pressure-drop correlations presented in the literature.

The energy equation, mechanical energy equation, and momentum equation essential for the analysis, derivation and discussion of two-phase pressure

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[13]See Appendix B.

drop correlation methods are derived in a manner dependent upon whether the flow pattern being considered is continuous (annular) or discontinuous (plug flow). Continuous flow patterns are analyzed microscopically.[*] Discontinuous flow patterns are analyzed with sufficient accuracy by macroscopic[*] balances. In either case, assumptions are usually made to simplify the mathematics as much as possible. Since most two-phase flow investigations are carried out with one dimension in the steady state and with constant flow rates, these assumptions don't significantly affect the accuracy of the resulting equations. For two-phase flow systems, with or without mass transfer between phases, and assuming no shaft work, Eqs. 29-32 result.

Macroscopic[*] Balance

Macroscopically, the usual approach assumes pressure constant for any given cross section of the pipe. Momentum and energy equations are then written separately for each phase with the constraint that the static pressure drop is identical for both phases over the same increment of flow length, then added to give overall expressions. Unfortunately, the resulting overall two-phase balances do not have simple relationships to each other as exist in single-phase flow. It becomes necessary to weight the momentum terms by their respective velocities in order to form correct energy quantities. By adding these weighted equations which are applicable to each phase, the results are:

a. Momentum Equation

$$- A dP = -\left(A_\ell dP_{W_\ell} + A_g dP_{W_g} \right) = \frac{d(w_\ell \bar{u}_\ell)}{g_c} + \frac{d(w_g \bar{u}_g)}{g_c} + (\rho_\ell A_\ell + \rho_g A_g) \frac{g}{g_c} \, dz$$

$$+ \left(\gamma_{W_g} C'_{W_g} + \gamma_{W_\ell} C'_{W_\ell} \right) dz \qquad (29)$$

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[*]See footnote, p. 24.

b. Mechanical Energy Equation

$$\frac{d(w_g \bar{u}_g^2) + d\left(w_\ell \bar{u}_\ell^2\right)}{2g_c} + (A_\ell \bar{u}_\ell + A_g \bar{u}_g)\, dP + (\rho_\ell A_\ell \bar{u}_\ell + \rho_g A_g \bar{u}_g)\, \frac{g}{g_c}\, dz$$

$$= -dE_g - dE_\ell = -dE_{g\ell} \qquad (30)$$

c. Energy Equation (no shaft work, adiabatic)

$$\frac{d}{dz}\left[\rho_g \bar{u}_g A_g \left(\frac{P}{\rho_g} + U_g + \frac{\bar{u}_g^2}{2g_c} + \frac{g}{g_c} z\right) + \rho_\ell \bar{u}_\ell A_\ell \left(\frac{P}{\rho_\ell} + U_\ell + \frac{\bar{u}_\ell^2}{2g_c} + \frac{g}{g_c} z\right)\right] = 0 \quad (31)$$

After performing the differentiation (noting that $z_g = z_\ell$, $d\rho_\ell = 0$, and $dw_g = -dw_\ell$), there results:

$$\left(\frac{w_g}{\rho_g} + \frac{w_\ell}{\rho_\ell}\right) dP - \left(A_g \bar{u}_g P\right) \frac{d\rho_g}{\rho_g} + w_g\, dU_g + w_\ell\, dU_\ell + \frac{w_g \bar{u}_g\, d\bar{u}_g}{g_c} + \frac{w_\ell \bar{u}_\ell\, d\bar{u}_\ell}{g_c}$$

$$+ (w_g + w_\ell)\, \frac{g}{g_c}\, dz + \left(\frac{1}{\rho_g} - \frac{1}{\rho_\ell}\right) P\, dw_g + \left(U_g - U_\ell\right) dw_g + \left(\frac{\bar{u}_g^2}{2g_c} - \frac{\bar{u}_\ell^2}{2g_c}\right) dw_g = 0$$

$$(32)$$

Microscopic* Balance

Microscopically, continuous flow may be analyzed by differential equations simplified according to the various simplifying assumptions applicable to the specific problem. The following example follows the unpublished notes of Mr. G. W. Patraw:

For a system having constant area flow with phase change, friction, heat addition, and considering a pipe element dL, the inlet parameters to a fluid element of unit width and length dL are given in Table 1.

---

* Microscopic balance refers to stratified, annular, and spray horizontal flow patterns (Fig. 1) as well as annular and mist vertical flow patterns (Fig. 2). Macroscopic balances refer to the remainder of the flow patterns.

Table 1. Inlet parameters to a fluid element.

Property	Vapor	Liquid	Total
Velocity	u_g	u_ℓ	
Mass Flow	w_g	w_ℓ	w
Enthalpy (static)	$w_g h_g$	$w_\ell h_\ell$	$w_g h_g + w_\ell h_\ell$
Pressure (static)	P	P	
Specific Volume	v_g	v_ℓ	

Thermal equilibrium between the two phases is assumed.

1. Energy Balance Across the Element

    a. Differential change in enthalpy

$$dH = d\,(w_\ell h_\ell) + d\,(w_g h_g)$$

$$dH = h_\ell\,dw_\ell + w_\ell\,dh_\ell + h_g\,dw_g + w_g\,dh_g \tag{33}$$

    b. Differential change in kinetic energy

$$dKE = \frac{1}{2g_c J}\,d\left(w_\ell u_\ell^{\,2} + w_g u_g^{\,2}\right)$$

$$dKE = \frac{1}{2g_c J}\left(u_\ell^{\,2}\,dw_\ell + 2\,w_\ell u_\ell\,du_\ell + u_g^{\,2}\,dw_g + 2\,w_g\,du_g\right) \tag{34}$$

    c. Heat added to system

$$dQ' = q\,dL \tag{35}$$

There are no potential energy changes or shaft work so

$$dH + dKE = dQ' \tag{36}$$

Introducing continuity and quality $\left(X = \dfrac{w_g}{w}\right)$ (1)

$$w_g = Xw \qquad\qquad dw_g = w\,dX \tag{1a}$$

$$w_\ell = (1 - X)w \qquad\qquad dw_\ell = -w\,dX \tag{1b}$$

and letting $h_{g\ell} = h_g - h_\ell$, the heat of vaporization, the result is: (37)

$$w \left[h_{g\ell} + \frac{1}{2g_c J} \left(u_g^2 - u_\ell^2 \right) \right] dX + w(1 - X) \, dh_\ell + wX \, dh_g$$

$$+ \frac{w}{g_c J} \left[(1 - X) \, u_\ell \, du_\ell + X \, u_g \, du_g \right] - q \, dL = 0 \quad (38)$$

For most flows of interest, the kinetic energy terms are very small compared to the thermal energy terms, so the energy equation becomes

$$w \, h_{g\ell} \, dX + w(1 - X) \, dh_\ell + wX \, dh_g - q \, dL = 0 \quad (39)$$

The change in enthalpy terms (dh) are also small, so we have approximately on integration

$$Q' = w h_{g\ell} \left(X_{final} - X_{initial} \right) \quad (40)$$

which is the usual approximation made in a heat flow balance. For this particular analysis, the energy equation need no longer be considered, other than remembering it shows that the quality is linear with distance for constant heat input $\left(\frac{dX}{dL} = constant \right)$.

2. Momentum Balance Across the Element

The change in pressure is equal to the change in pressure due to friction plus the change in pressure due to the change in momentum of the stream. (dP_f is always negative.)

$$dP_{TP} = dP_f + dP_a \quad (41)$$

The dP_f term will be discussed later. The momentum term is:

$$- dP_a = \frac{1}{g_c A_p} \left[(1 - X) \, w \, du_\ell + Xw \, du_g + (u_g - u_\ell) \, wdX \right] \quad (42)$$

The liquid and gas phases may flow at different velocities. From continuity

$$u_\ell = \frac{(1 - X) \, v_\ell \, w}{A_\ell} \qquad u_g = \frac{X v_g w}{A_g} \quad (43, 44)$$

Substituting Eq. (11) into Eqs. 43, 44 we obtain

$$u_\ell = \frac{(1 - X)\, v_\ell\, w}{(1 - \alpha)\, A_p} \qquad\qquad u_g = \frac{X\, v_g\, w}{\alpha\, A_p} \tag{45, 46}$$

$$du_\ell = \frac{(1 - X)\, v_\ell\, w}{(1 - \alpha)\, A_p} \left(\frac{dv_\ell}{v_\ell} - \frac{dX}{1 - X} + \frac{d\alpha}{1 - \alpha} \right) \tag{47}$$

$$du_g = \frac{v_g\, w X}{\alpha\, A_p} \left(\frac{dv_g}{v_g} + \frac{dX}{X} - \frac{d\alpha}{\alpha} \right) \tag{48}$$

$$-dP_a = \frac{w^2}{g_c A_p^{\,2}} \left[\frac{(1 - X)^2\, v_\ell}{(1 - \alpha)} \left(\frac{dv_\ell}{v_\ell} - \frac{dX}{1 - X} + \frac{d\alpha}{1 - \alpha} \right) \right.$$

$$\left. + \frac{X^2\, v_g}{\alpha} \left(\frac{dv_g}{\alpha} + \frac{dX}{X} - \frac{d\alpha}{\alpha} \right) + \left(\frac{X v_g}{\alpha} - \frac{(1 - X)\, v_\ell}{(1 - \alpha)} \right) dX \right] \tag{49}$$

Since the compressibility of the liquid is generally small, the pressure-momentum equation becomes

$$dP = dP_f - \frac{w^2}{g_c A_p^{\,2}} \left[\frac{(1 - X)^2\, v_\ell}{(1 - \alpha)} \left(\frac{d\alpha}{(1 - \alpha)} - \frac{dX}{(1 - X)} \right) + \frac{X^2\, v_g}{\alpha} \left(\frac{dv_g}{v_g} + \frac{dX}{X} - \frac{d\alpha}{\alpha} \right) \right.$$

$$\left. + \left(\frac{X v_g}{\alpha} - \frac{(1 - X)\, v_\ell}{(1 - \alpha)} \right) dX \right] \tag{50}$$

letting

$$\frac{dv_g}{dL} = \frac{dv_g}{dP} \frac{dP}{dL_{TP}} \tag{51}$$

and converting to derivative form

$$\frac{dP}{dL_{TP}} = \frac{dP_f}{dL} - \frac{w^2}{g_c A_p^{\,2}} \left[\frac{(1 - X)^2\, v_\ell}{(1 - \alpha)} \left(\frac{1}{1 - \alpha} \frac{d\alpha}{dL} - \frac{1}{1 - X} \frac{dX}{dL} \right) \right.$$

$$\left. + \frac{X^2\, v_g}{\alpha} \left(\frac{1}{v_g} \frac{dv_g}{dP} \frac{dP}{dL_{TP}} + \frac{1}{X} \frac{dX}{dL} - \frac{1}{\alpha} \frac{d\alpha}{dL} \right) + \left(\frac{X v_g}{\alpha} - \frac{(1 - X) v_\ell}{(1 - \alpha)} \right) \frac{dX}{dL} \right] \tag{52}$$

thus

$$\left(1 + \frac{w^2}{g_c A_p^2} \frac{X^2}{\alpha} \frac{dv_g}{dP}\right) \frac{dP}{dL}_{TP} = \frac{dP_f}{dL} - \frac{w^2}{g_c A_p^2} \left[\frac{d\alpha}{dL} \left(\frac{(1 - X)^2 \, v_\ell}{(1 - \alpha)^2} - \frac{X^2}{\alpha^2} \, v_g \right) \right.$$

$$\left. + \, 2 \, \frac{dX}{dL} \left(\frac{X v_g}{\alpha} - \frac{(1 - X) \, v_\ell}{(1 - \alpha)} \right) \right] \tag{53}$$

This is a form of the momentum equation which may be applied to the Homogeneous model presented on page 40 of this report.

CORRELATION METHODS

General

For every problem which has resisted definitive solution over a long period of time, the methods of approach are usually sophisticated by steps in which: (1) imperfect correlations are presented based on limited data; (2) these correlations are improved as the problem receives further study and more sophisticated tools are used for its analysis; and (3) ultimately, physical limits upon the problem render various solutions more satisfactory than others, depending upon the magnitude of their complexity and application.

In the interim, five classical patterns may be separately identified:

1. Empirical correlations;

2. Correlations utilizing dimensional analysis;

3. Correlations utilizing similarity analysis and model theory;

4. Mathematical analysis of a simplified physical model and development of equations relating the variables;

5. Approximation theory(solutions to the energy, momentum, and conservation equations with empirical expressions used for the turbulent transport terms, approximations to the boundary conditions, and assumptions involving the relative magnitude of various terms in the equations. Usually the resulting relationship among variables is obtained by numerical solutions for the complex equations).

So far, two-phase pressure drop correlations have been published which involve four of the above five types. The fifth appears unlikely. All five approaches are discussed. Examples have been shown which demonstrate the most important methods.

Empirical Approach

Many empirical correlations have appeared. Most of these can be used with poor reliability beyond the range of the data from which they were constructed, as shown in a paper by Dukler et al. (14). Empirical correlations for horizontal flow appear in references by Chenoweth and Martin (11), Hoogendorn (24), Isbin et al. (27) and Sobocinski and Huntington (46) Correlations for vertical conduits are given in references by Calvert (10), Govier and Omer (18) and Hughmark (25). Excellent summaries may be found in Tong (48) and Scott (43).

Chenoweth and Martin Correlation

This correlation applies for turbulent-turbulent horizontal flow in pipes and was intended to improve upon previous methods especially suited for predicting high pressures in large-diameter pipes. Entirely empirical, it yields $\Delta P_{TP}/\Delta P_{\ell}^{*}$ as a function of liquid-volume fraction of the feed with a quantity $\psi_g^{*} \rho_{\ell}/\psi_{\ell} \rho_g$ as a parameter. For this correlation ΔP_{ℓ}^{*} is evaluated as the pressure drop based on the total mass flow, using the liquid phase properties. The parameter $\psi_g^{*} \rho_{\ell}/\psi_{\ell} \rho_g$ is defined as

$$\frac{\psi_g^{*} \rho_{\ell}}{\psi_{\ell} \rho_g} = \frac{\Delta P_g^{*}}{\Delta P_{\ell}^{*}} = \frac{\left[f_g^{*} \, z/D + \Sigma K' \right] \rho_{\ell}}{\left[f_{\ell}^{*} \, z/D + \Sigma K' \right] \rho_g} \tag{54}$$

where the f^{*} are evaluated at the total mass flow from a Moody chart, and the term $\Sigma K'$ is an allowance for valves and fittings.

The originators claimed an agreement with the experimental pressure drops of ±35% for this correlation. Additional evaluations were made by Collier and Hewitt (12) who applied the method to air-water, vertical-annular flow. Aziz and Govier (6) found poor agreement for low liquid volume fractions, and

Isbin et al. (27) investigated steam-water flows at pressures from 400-1000 psia, and concluded errors over 100% occurred over much of the liquid-volume fraction range (predictions being low at low volume-fractions, and high at high volume fractions). The net result of these later investigations indicated that this correlation gives results somewhat worse than the Lockhart-Martinelli methods which are presented on page 43.

Hughmark Correlation

Hughmark (25) proposed a similar correlation which plots the ratio of the friction factors for two-phase flow and the flow of a fictitious single-phase fluid against the liquid volume fraction entering but uses a slightly different parameter, that is, (Re_{TP-m}) (P_g/P_ℓ) $(0.085/D)$ as a third quantity. This choice of parameters amounts to using a fluid viscosity weighted for the quantities of the two fluids rather than a ratio of viscosities as in Chenoweth and Martin's work, and introducing a diameter, inasmuch as Hughmark's work was based on 1-in. pipe. The correlation is claimed to be generally applicable for horizontal flow, but in comparison with Chenoweth and Martin's work, the improvement shown is slight.

Dimensional Method

The general use of dimensionless groups for empirical correlations is widespread; however, two-phase pressure drop correlations based primarily on dimensional analysis have not yet appeared in the literature. This is due to the number of variables which exist in a two-phase flow. Among the independent variables involved are:

1. diameter,

2,3. mass flow rate of each phase,

4,5. density of each phase,

6,7. viscosities of each phase,

8. interfacial tension, and

9. acceleration of gravity.

For the above variables, 6 dimensionless ratios may be derived. Neglecting gas viscosity and interfacial tension, 4 dimensionless ratios still exist.

When heat transfer is present the thermal properties of each phase, such as thermal conductivity and specific heat, as well as the temperature profiles of the system, must be considered. When mass transfer is taking place, additional factors will include the diffusion coefficient in each phase for each component being transferred, the concentration of each component in each phase, and the equilibrium relationships between phases for each component. This very large number of factors explains why a universal or general correlation for any transport process would be very complex, and why dimensional analysis is at a disadvantage in comparison with other experimental correlation methods.

Similarity Analysis (Dukler Method)

Prior to a paper presented by Dukler et al. (15) in 1964, no solution in which the principles of similarity in a formal manner were used appeared in the literature. An excellent review of Dukler's method has been presented by Anderson and Russell (3).

Essentially, Dukler relates the parameters involving two-phase flow by the Euler and Reynolds numbers familiarly used in single-phase flow. Similarity theory states that two single-phase flow systems are dynamically similar if the Reynolds numbers and the Euler numbers are equal respectively. In the case of single-phase flow, the Euler number is twice the Fanning friction factor. This condition does not provide enough information with which to find the Reynolds number or two-phase friction factor. However, once relationships are found from experimental data for one system (the model), the condition of dynamic similarity requires that these same relationships apply to all similar systems. In Dukler's original paper, similarity relationships for single- and two-phase flow are developed in parallel, clearly demonstrating this analogy.

Once similarity analysis has been used, correctly defining the parameters, experimental data may then be utilized to develop further relationships between these parameters. Thus, data from the model may be used to expand data taken at other conditions and in other systems. Dukler distinguishes between the various flow patterns by selecting constants and eliminating particular terms in his original equations which hold for the special cases – no slip (homogeneous flow), and special slip conditions. He then statistically analyzes the deviation between theory and experimental values and compares these results to the method of Lockhart and Martinelli (33). Dependent upon improving methods to predict holdup, Dukler's method gives better agreement with experimental data than do earlier correlations by Martinelli et al. (35).

After the volume fraction has been determined, the system of Dukler et al. (15) may be used to calculate pressure drop. The following steps appear:

1.   A two-phase Reynolds number based on flow conditions, as well as the conditions in the pipe, must be calculated.

2.   A friction factor is then calculated from the two-phase Reynolds number in the Koo equation (31).

3.   The frictional pressure drop is calculated from an equation which relates the friction factor, total mass flow rate, and physical properties to the frictional pressure drop.

4.   The effect of elevation is calculated by using an average mixture density.

5.   An iterative procedure is used to evaluate the acceleration correction factor.   For the case in which the gas and liquid volume fractions and the gas density do not change much along the length of the pipe, the acceleration correction term is negligible.   This happens when the pressure drop is small.

6.   The individual contributions of friction, momentum change, and elevation are then added to give the total pressure drop.

An example problem, presented by Anderson and Russell (3), demonstrates Dukler's method:

"Given: a 350-ft section of horizontal 1-in. smooth pipe.   The flow rates are as follows: 1000 lb/hr of water and 15 lb/hr of air.   The discharge end of the pipe is at a pressure of 1 atm and the system is isothermal at a temperature of 68°F.

"Find: The predicted pressure drop for the test section by using the general correlation of Dukler et al. (14).

I. With the method of Hughmark (26) used to calculate the volume fraction

(holdup) of gas in the pipeline:

"To obtain average physical properties and to obtain an initial correction

for the acceleration losses, it is necessary to assume a total pressure drop

and calculate an entrance pressure and an average pressure. A pressure drop

of 4.6 psi will be assumed, giving an entrance pressure of 19.3 psia and an

average pressure of 17 psia. The calculations will be done with these

assumptions and they must be validated at the end. The physical properties

at the assumed average value for the pressure of 17 psia are as follows:

ρ_ℓ = 62.4 lb/ft^3, ρ_g = 0.0870 lb/ft^3, μ_ℓ = 1 cP, and μ_g = 0.018 cP.

"Assume the volume fractions in the pipe to be R_g = 0.75 and R_ℓ = 0.25.

Calculation of these fractions involves an iterative procedure because of the

dependence of Hughmark's parameter Z on the pipeline volume fractions. A

reasonable value for the first assumption, for the liquid-volume fraction of the

fluid flowing in the pipe, is the liquid-volume fraction of the mixture entering

the pipe. The calculation for the final iteration is as follows: Calculation of

the total mass velocity G:

$$G = G_\ell + G_g \tag{55}$$
$$G = 1000/(0.25\pi \times 0.0833^2) + 15/(0.25\pi \times 0.0833^2)$$
$$G = 186,000 \text{ lb/hr-ft}^2$$

"Calculation of a two-phase Reynolds number:

$$Re_{TP} = DG/(R_\ell \mu_\ell + R_g \mu_g) \tag{25}$$
$$Re_{TP} = \frac{(0.0833)\ (186,000)}{(0.25)\ (1)\ (2.42) + (0.75)\ (0.018)\ (2.42)}$$
$$Re_{TP} = 35,000$$

"Calculation of u_{NS}, the velocity defined for no slip between the phases:

$$u_{NS} = (Q_\ell + Q_g)/A_p \tag{7}$$

$$Q_\ell = 1000/(62.4 \times 3600) \tag{9}$$

$$Q_\ell = 0.00445 \text{ ft}^3/\text{sec}$$

$$Q_g = 15/(0.087 \times 3600) \tag{10}$$

$$Q_g = 0.0479 \text{ ft}^3/\text{sec}$$

$$u_{NS} = (Q_\ell + Q_g)/A_p$$

$$u_{NS} = (0.00445 + 0.0479)/(0.25\pi \times 0.0833^2)$$

$$u_{NS} = 9.60 \text{ ft/sec}$$

"Calculation of Fr_m, the mixture Froude number based on velocity, which is in turn calculated assuming no slip between the phases:

$$Fr_m = u_{NS}^2/gD \tag{26}$$

$$Fr_m = 9.60^2/(32.2 \times 0.0833)$$

$$Fr_m = 34.3$$

Calculation of λ, the liquid volume fraction of the fluid flowing in the pipe evaluated at the average pressure:

$$\lambda = Q_\ell/(Q_g + Q_\ell) \tag{27}$$

$$\lambda = 0.00445/(0.00445 + 0.0479)$$

$$\lambda = 0.085$$

"Calculation of Z:

$$Z = (Re_{TP})^{1/6} (Fr_m)^{1/8}/(\lambda_\ell)^{1/4} \tag{28}$$

$$Z = (35,000)^{1/6} (34.3)^{1/8}/(0.085)^{1/4}$$

$$Z = 16.4$$

"From Fig. 3 or Table 1, K is found to be 0.814.

"Calculation of R_g:

$$1/X = \left(1 - \frac{\rho_\ell}{\rho_g}\right)(1 - K/R_g) \tag{23}$$

$$X = 15/1015 = 0.0148$$

$$1/0.0148 = (1 - 62.4/0.087)(1 - 0.814/R_g)$$

$$R_g = 0.745$$

"Calculation of R_ℓ:

$$R_\ell = 1 - R_g \tag{56}$$

$$R_\ell = 1 - 0.745$$

$$R_\ell = 0.255$$

From the values obtained for R_g and R_ℓ, it is seen that the values assumed for these parameters in the calculation of the two-phase Reynolds number were valid.

"II. With the method of Dukler et al. (15) used to calculate the frictional pressure drop:

"Calculation of ρ_{NS}:

$$\rho_{NS} = \rho_\ell \lambda + \rho_g (1 - \lambda) \tag{57}$$

$$\rho_{NS} = (62.4)(0.085) + (0.087)(0.915)$$

$$\rho_{NS} = 5.38 \text{ lb/ft}^3$$

"Calculation of μ_{NS}:

$$\mu_{NS} = \mu_\ell \lambda + \mu_g (1 - \lambda) \tag{58}$$

$$\mu_{NS} = (1)(0.085) + (0.018)(0.915)$$

$$\mu_{NS} = 0.102 \text{ cP}$$

"Calculation of β:

$$\beta = (\rho_\ell/\rho_{NS})(\lambda^2/R_\ell) + (\rho_g/\rho_{NS})(1 - \lambda)^2/R_g \tag{59}$$

$$\beta = (62.4/5.38)(0.085^2/0.255) + (0.087/5.38)(0.915^2/0.745)$$

$$\beta = 0.360$$

Calculation of Re_m, a mixture Reynolds number:

$$Re_m = (DG/\mu_{NS})\beta \qquad (25)$$

$$Re_m = [\,(0.0833 \times 186,000)/(0.102 \times 2.42)\,]\,[\,0.360\,]$$

$$Re_m = 32,600$$

Calculation of friction factor f_o using the Koo equation (31):

$$f_o = 0.00140 + 0.125/(Re_m)^{0.32} \qquad (60)$$

$$f_o = 0.00140 + 0.125/(32,600)^{0.32}$$

$$f_o = 0.0059$$

Calculation of $F(\lambda) = f_0/f_{0-Re-m}$, an empirical ratio given by Dukler (15):

$$F(\lambda) = 1.0 + \gamma/(1.281 - 0.478\gamma + 0.444\gamma^2 - 0.044\gamma^3 + 0.00843\gamma^4) \qquad (61)$$

where $\gamma = -\ln(\lambda) = -\ln(0.085) = 2.47$ $\qquad (62)$

$$F(\lambda) = 2.45$$

Calculation of $(\Delta P/\Delta L)_f$, the frictional pressure drop:

$$(\Delta P/\Delta L)_f = \left(2G^2 f_o/g_c D\rho_{NS}\right)F(\lambda)\beta \qquad (63)$$

$$(\Delta P/\Delta L)_f = \frac{(2 \times 186,000^2 \times 0.0059 \times 2.45 \times 0.360)}{(32.2 \times 3600^2 \times 0.0833 \times 5.38)}$$

$$(\Delta P/\Delta L)_f = 1.91 \; lb/ft^2 \text{ per ft of pipe}$$

$$(\Delta P/\Delta L)_f = 0.0133 \; lb/in.^2 \text{ per ft of pipe}$$

Calculation of acceleration correction term:

$$(\Delta P/\Delta L)_a = \left[1/g_c A_p^2 \Delta L\right]\left[w_g^2 \; \Delta\left(1/\rho_g \overline{R}_g\right) + w_\ell^2/\rho_\ell \; \Delta(1/\overline{R}_\ell)\right] \qquad (64)$$

Since the assumed pressure drop is small, the changes R_ℓ and R_g will be small, and R_g and R_ℓ can be taken as average values of 17 psia. This statement will only be true if the value assumed for the pressure drop is the correct one. With average values for R_ℓ and R_g are used, Eq. (64) becomes the following:

$$(\Delta P/\Delta L)_a = \left[1/g_c A_p^{\,2}\Delta L\right]\left[W_g^{\,2}/\overline{R}_g \quad \Delta (1/\rho_g)\right] \tag{65}$$

$$(\Delta P/\Delta L)_a = \left[15^2/(32.2)(3600)^2(0.25\pi)^2(0.0833)^4(350)(0.745)\right.$$
$$\left.\left[1/0.751 - 1/0.990\right]\right]$$

$$(\Delta P/\Delta L)_a = 0.0000212 \text{ lb/ft}^2 \text{ per ft of pipe}$$

This term is negligible compared to the frictional pressure drop.

Calculation of $(\Delta P/\Delta L)$:

$$(\Delta P/\Delta L) = (\Delta P/\Delta L)_a + (\Delta P/\Delta L)_f \tag{66}$$

$$(\Delta P/\Delta L) = 0.0133 \text{ psi/ft of pipe}$$

Calculation of the actual pressure drop:

$$(\Delta P) = (\Delta P/\Delta L)\Delta L \tag{67}$$

$$(\Delta P) = (0.0133)(350)$$

$$(\Delta P) = 4.65 \text{ psi}$$

The original assumption of 4.6 psi for the pressure drop is valid. If it were
not, a new value would be assumed and the iterative procedure would be repeated
(including a new holdup calculation) until a satisfactory agreement were obtained.

Mathematical Analysis Involving Simplified Physical Models

Homogeneous Model

Correlations which have received widest acclaim and popularity have been
those based on a simplified physical model. The simplest mathematical model
considers the extreme flow condition whereby the liquid and vapor phases are
completely and finely dispersed. This condition is approximated in the fully
dispersed or "fog" flow pattern. Due to its simplicity, this method has been
applied to steam-water flow often without regard to flow pattern. The usual
assumptions made are: (a) equal vapor and liquid velocities; (b) attainment
of thermodynamic or physical equilibrium between phases; (c) use of a single-

phase friction factor in the P_f term and substitution of mean specific volume values; and (d) use of a weighted viscosity in the evaluation of Reynolds number for determination of the single-phase friction factor. The mean viscosity is usually determined by the relationship

$$\frac{1}{\mu_m} = \frac{(1 - X)}{\mu_\ell} + \frac{X}{\mu_g} \quad . \tag{68}$$

Although most investigators would suspect that this condition is seldom fulfilled, many useful results have been derived. Generally, the difficulties of utilizing this method involve the proper selection of the single-phase friction factor, as well as the determination of physical equilibrium between phases. Investigators presently evaluate the friction factor from standard correlations for liquid flow, using Reynolds number based upon the liquid viscosity mentioned above. For forced convection flow, Owens (40) shows that the two-phase friction factor may be assumed to be the same as that for the liquid. The assumption of physical equilibrium is undesirable but necessary to effect results.

The most complete expression for $\left(\frac{\Delta P}{\Delta L}\right)_{TP}$ may be derived by using the momentum equation (Eq. 53) and the following expressions for α and $d\alpha$:

$$\alpha = \frac{X v_g}{(1 - X) v_\ell + X v_g} \tag{69}$$

$$d\alpha = \frac{v_\ell v_g \, dX + X (1 - X) v_\ell \, dv_g}{\left[(1 - X) v_\ell + X v_g\right]^2} \tag{70}$$

This derivation, however, is quite algebraically involved. A more managable approach by Owens (40) uses a mean specific volume written from the continuity equation (assuming the velocities of the two phases are equal).

$$\bar{v} = \frac{w_g v_g + w_\ell v_\ell}{w} \tag{71}$$

$$= v_\ell + X (v_g - v_\ell) \tag{72}$$

Owens' components of Eq. 41 are:

$$\left(\frac{dP}{dL}\right)_f = \frac{f_{TP} \, G^2 \, \bar{v}}{d \, D \, g_c}$$

(73)

(Both Owens (40) and Dukler (14) discuss the problems of selecting a proper two phase friction factor.)

$$\left(\frac{dP}{dL}\right)_a = \frac{G^2}{g_c} \left[\frac{\partial \bar{v}}{\partial X} \frac{dX}{dL} + \frac{\partial \bar{v}}{\partial v_g} \frac{dv_g}{dP} \left(\frac{dP}{dL}\right)_{TP} \right]$$

(74)

Using the differential of Eq. 72 and substituting into Eq's. (73) and (74), Owens obtained for a final expression for two phase pressure drop:

$$-\left(\frac{dP}{dL}\right)_{TP} = \frac{\dfrac{f_{TP} \, G^2 \, v_\ell}{2 \, D \, g_c} \left[1 + X\left(\dfrac{v_g}{v_\ell} - 1\right) \right] + \dfrac{G^2 \, v_\ell}{g_c} \left(\dfrac{v_g}{v_\ell} - 1\right)\dfrac{dX}{dL}}{1 + \dfrac{XG^2}{g_c} \dfrac{dv_g}{dP}}$$

(75)

To consider elevation, a $\left(\dfrac{dP}{dL}\right)_z$ term must be added to the numerator of the above equation.

$$\left(\frac{dP}{dL}\right)_z = \frac{\sin\theta}{v_\ell \left[1 + X\left(\dfrac{v_g}{v_\ell} - 1\right) \right]}$$

(76)

Stepwise integration over the desired interval may now be performed to obtain a final answer which will be dependent on how X varies with L, and the validity of f_{TP}. For the case where $\dfrac{v_g}{v_\ell} - 1$, and $\dfrac{dv_g}{dP}$ are essentially constant, and for a linear variation of X, Eq. 75 becomes:

$$\Delta P = -\frac{1}{B} \, \mathrm{Ln}\left(\frac{1 + X_2 B}{1 + X_1 B}\right) \left[\frac{\Delta L}{\Delta X}\left(\frac{dP}{dL}\right)_{\ell_1}\left(1 - \frac{A}{B}\right) - AC\right] \tag{77}$$

$$-\frac{\Delta L A}{B}\left(\frac{dP}{dL}\right)_{\ell_1}$$

where

$$A = \frac{v_g}{v_\ell} - 1, \quad B = \frac{dv_g}{dP}\,\frac{G^2}{g_c}, \quad \text{and } C = \frac{G^2}{g}\, v_\ell.$$

Other "friction factor" methods (derived by analogy from single-phase flow in which the two-phase pressure drop can be expressed by the conventional Fanning equation) have been discussed in a paper by Govier and Omer (18). In general, "friction-factor" methods do not require the assumptions made in the special case of the homogeneous model, and can be applied equally well to vertical or horizontal flows; such methods of "correlation by analogy" give good results even though the choice of correlating parameters varies widely. Excellent papers reviewing this correlation have been presented by Owens (40), Bennett (9), Isbin et al. (27), and Tong (48).

Annular Flow Models

Annular flow occurs when the gas travels up the tube at a high velocity and the liquid forms an annular film around the tube walls. Methods which predict pressure drop using this model have been widely discussed due to the popularity of Martinelli's et al. (35) early correlations based on the annular flow model. For a complete listing of papers presented, reviews by Isbin et al.

(27), Scott (43), Bennett (9), Jens and Leppert (30), Tong (48), and Gresham et al. (20) provide excellent summaries and comparisons.

Briefly, the Martinelli method originated from a series of studies of isothermal two-phase pressure drop in horizontal pipes. The investigation was confined to frictional pressure drop alone, as the use of horizontal pipes eliminated hydrostatic head terms and isothermal conditions allowed accelera-tion pressure drop to be considered negligible. The isothermal pressure drop correlation was then extended by a process of integration to two-phase flow with vaporization by application to the forced circulation of water. A number of other correlations between frictional pressure drop and properties of the system have been proposed for isothermal gas liquid flow, but only the Martinelli method has been extended to two-phase flow with vaporization. Modifications by Levy (32), Marchaterre (34), and Davis and David (13) based on a momentum-exchange model are summarized in Scott (43).

Lockhart-Martinelli correlation

This derivation, based on limiting assumptions, has been applied to all regions of two-phase flow both by the originators and by many other investi-gators. Briefly, the original assumptions as applied to isothermal two-phase pressure drop in horizontal pipes were: (a) the static pressure drop for the gas phase is equal to that of the liquid phase, regardless of flow pattern; (b) the volume occupied by the gas plus that occupied by liquid at any instant equals the pipe volume; and (c) the frictional gas pressure drop equals the frictional liquid pressure drop which equals the static pressure drop, i.e., the momentum and hydrostatic pressure drops are negligible compared with the frictional pressure drop; (d) two-phase flow can be divided into four types

which are combinations of the two phases with either phase in viscous or turbulent flow. It was assumed that laminar and turbulent flow could be defined for each phase as the usual function of Reynolds number based on the particular phase flowing in the pipe alone.

Martinelli et al. (35) originally introduced correlations for turbulent-turbulent and viscous-turbulent (liquid-gas) flow. Martinelli et al. (37) later extended their correlation to viscous-viscous flow. Finally, all four com-binations, as well as for void fractions, were presented by Lockhart and Martinelli (33).

Essentially, the correlation presents a plot of an empirical function, ϕ, against a parameter, X, with one curve representing each of the four flow regimes. The correlating quantities are defined as

$$\phi_g^2 = \frac{(\Delta P/\Delta L)_{TP}}{(\Delta P/\Delta L)_g} \tag{78}$$

or

$$\phi_\ell^2 = \frac{(\Delta P/\Delta L)_{TP}}{(\Delta P/\Delta L)_\ell} \tag{79}$$

and

$$\chi^2 = \frac{(\Delta P/\Delta L)_\ell}{(\Delta P/\Delta L)_g} \tag{80}$$

where χ^2 represents the ratio of the pressure drop functionals of each phase, i.e.,

$$\chi^2 = \left(\frac{w_\ell}{w_g}\right)^{1.8} \left(\frac{v_\ell}{v_g}\right)\left(\frac{\mu_\ell}{\mu_g}\right)^{0.2} = \left(\frac{1-X}{X}\right)^{1.8} \left(\frac{v_\ell}{v_g}\right)\left(\frac{\mu_\ell}{\mu_g}\right)^{0.2} \tag{81}$$

for the turbulent situation.

In Eqs. (78) and (79), the quantities $(\Delta P/\Delta L)_\ell$ or $(\Delta P/\Delta L)_g$ are calculated from conventional single-phase correlations on the basis that the liquid or gas

is flowing in the pipe alone at the same individual mass-flow rate as the two-phase case. Complete expressions for x^2, and the relationships between ϕ_g, ϕ_ℓ, R_g, and R_ℓ are given by Lockhart and Martinelli (33). The relationships are shown graphically in Fig. 4.
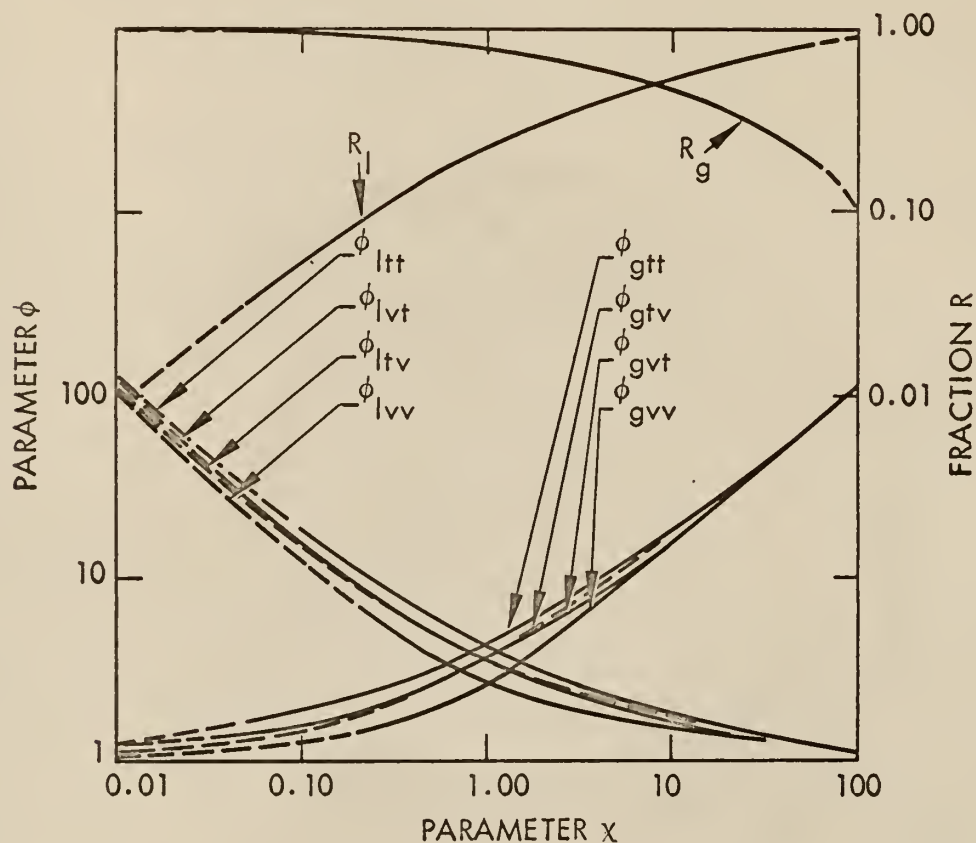
Fig. 4. Lockhart and Martinelli's faired curves showing relation among ϕ_ℓ, ϕ_g, R_ℓ, and R_g for all flow mechanisms.

Originally, the validity of the correlation was questioned for high and low values of the parameter X, and for two-phase mixtures near the critical point. Later investigations revealed that the calculation of pressure drops by the Lockhart-Martinelli method appear to be reasonably useful only for the turbulent-turbulent regions. Although it can be applied to all flow patterns, accuracy of prediction will be poor for other cases. Perhaps it is best considered as a partial correlation which requires modification in individual

cases to achieve good accuracy. Certainly there seems to be no clear reason why there should be a simple general relationship between the two-phase frictional pressure-drop and fictitious single-phase drops. At the same value of X in the same system, it is possible to have two different flow patterns with two-phase pressure drops which differ by over 100%. The Lockhart-Martinelli correlation is a rather gross smoothing of the actual relationships.

To illustrate the application of the Lockhart-Martinelli correlation, an example problem adapted from Lockhart-Martinelli's original paper (36) is given.[14]

"It is desired to estimate the pressure drop per foot of tube for the flow of oil and air in a capillary tube 0.00488 ft in diameter. The properties of the oil and air and the flow rates are given below:

$$w_g = 2.55 \times 10^{-5} \text{ lb/sec}$$
$$\rho_g = 0.075 \text{ lb/ft}^3 \text{ (at 1 atm, 70°F)}$$
$$\mu_g = 1.22 \times 10^{-5} \text{ lb/sec ft}$$
$$w_\ell = 3.07 \times 10^{-5} \text{ lb/sec}$$
$$\rho_\ell = 54.3 \text{ lb/ft}^3$$
$$\mu_\ell = 4.25 \times 10^{-3} \text{ lb/sec ft}$$

"Let us assume, for the moment, that the flow is viscous-viscous, items 1 through 4, below, are necessary to check this assumption.

1) Calculation of X_{vv}:

From Table 2, first row, first column, it follows:

$$X_{vv}^2 = \frac{w_\ell \rho_g \mu_\ell}{w_\ell \rho_\ell \mu_g} = \frac{2.07 \times 0.075 \times 4.25 \times 10^2}{2.55 \times 54.3 \times 1.22} = 0.392 \tag{82}$$

$$\therefore X_{vv} = 0.629$$

[14]Arranged from the unpublished notes of Mr. F. S. Martino, Kansas State University Graduate School.

Table 2. Lockhart and Martinelli's summary of algebraic relations among the various parameters.

Note: For any flow mechanism $R_\ell + R_g = 1$ and $\phi_g = X\phi_\ell$.

Parameter	Flow	tt	vt	tv	vv
$\chi^2 = \dfrac{\left(\frac{\Delta P}{\Delta L}\right)_\ell}{\left(\frac{\Delta P}{\Delta L}\right)_g}$		$\left(\dfrac{w_\ell}{w_g}\right)^{1.8}\dfrac{\rho_g}{\rho_\ell}\left(\dfrac{\mu_\ell}{\mu_g}\right)^{0.2}$	$\dfrac{C_\ell}{C_g}Re_{gp}^{-0.8}\dfrac{w_\ell}{w_g}\dfrac{\rho_g}{\rho_\ell}\dfrac{\mu_\ell}{\mu_g}$	$\dfrac{C_\ell}{C_g}Re_{\ell p}^{0.8}\dfrac{w_\ell}{w_g}\dfrac{\rho_g}{\rho_\ell}\dfrac{\mu_\ell}{\mu_g}$	$\dfrac{w_\ell}{w_g}\dfrac{\rho_g}{\rho_\ell}\dfrac{\mu_\ell}{\mu_g}$
$\phi_\ell^2 = \dfrac{\left(\frac{\Delta P}{\Delta L}\right)_{TP}}{\left(\frac{\Delta P}{\Delta L}\right)_\ell}$		$\dfrac{\phi_{gtt}^2}{\chi_{tt}^2}$	$\dfrac{\phi_{gvt}^2}{\chi_{vt}^2}$	$\dfrac{\phi_{gtv}^2}{\chi_{tv}^2}$	$\dfrac{\phi_{gvv}^2}{\chi_{vv}^2}$
$\phi_g^2 = \dfrac{\left(\frac{\Delta P}{\Delta L}\right)_{TP}}{\left(\frac{\Delta P}{\Delta L}\right)_g}$		$\phi_{ttt}^2\,\chi_{tt}^2$	$\phi_{\ell vt}^2\,\chi_{vt}^2$	$\phi_{ttv}^2\,\chi_{tv}^2$	$\phi_{tvv}^2\,\chi_{vv}^2$
$\alpha' = R_\ell\left(\dfrac{D_p}{D_\ell}\right)^2$		$R_\ell^4\,\phi_{\ell tt}^{3.33}$	$R_\ell^2\,\phi_{\ell vt}^2$	$R_\ell^4\,\phi_{\ell tv}^{3.33}$	$R_\ell^2\,\phi_{\ell vv}^2$
$\beta' = R_g\left(\dfrac{D_p}{D_g}\right)^2$		$R_g^4\,\phi_{gtt}^{3.33}$	$R_g^4\,\phi_{gvt}^{3.33}$	$R_g^2\,\phi_{gtv}^2$	$R_g^2\,\phi_{gvv}^2$
$\left(\dfrac{D_p}{D_\ell}\right)^2 = \dfrac{\alpha'}{R_\ell}$		$R_\ell^3\,\phi_{\ell tt}^{3.33}$	$R_\ell\,\phi_{\ell vt}^2$	$R_\ell^3\,\phi_{\ell tv}^{3.33}$	$R_\ell\,\phi_{\ell vv}^2$
$\left(\dfrac{D_p}{D_g}\right)^2 = \dfrac{\beta'}{R_g}$		$R_g^3\,\phi_{gtt}^{3.33}$	$R_g^3\,\phi_{gvt}^{3.33}$	$R_g\,\phi_{gtv}^2$	$R_g\,\phi_{gvv}^2$

2) "Calculation of ϕ_{gvv}, $\phi_{\ell vv}$, R_ℓ and R_g:

From Fig. 4, with $\chi_{vv} = 0.629$:

$$\phi_{gvv} = 1.98$$

$$R_\ell = 0.1782 \text{ and } R_g = 0.8218$$

From Table 2:

$$\phi_{\ell vv} = \frac{\phi_{gvv}}{\chi_{vv}} = \frac{1.98}{0.629} = 3.15 \tag{83}$$

3) "Calculation of D_ℓ and D_g:

From Table 2, last column, last two rows:

$$\left(\frac{D_p}{D_\ell}\right)^2 = R_\ell \, \phi_{\ell vv}^2 = 0.1782 \,(3.15)^2 = 1.77 \tag{84}$$

$$\left(\frac{D_p}{D_g}\right)^2 = R_g \, \phi_{gvv}^2 = 0.8218 \,(1.98)^2 = 3.22 \tag{85}$$

Therefore;

$$D_\ell = 0.00367 \text{ ft}$$

and

$$D_g = 0.00272 \text{ ft}$$

4) "Calculation of Re_ℓ and Re_g:

$$f_g = \frac{C_g}{Re_g^m} = \frac{C_g}{\left(\dfrac{4}{\pi} \dfrac{w_g}{\beta' \, D_g \mu_g}\right)^m} \tag{86}$$

$$f_\ell = \frac{C_\ell}{Re_\ell^n} = \frac{C_\ell}{\left(\dfrac{4}{\pi} \dfrac{w_\ell}{\alpha' \, D_\ell \mu_\ell}\right)^n} \tag{87}$$

From Eqs. (86) and (87), with $n = m = 1$,

$$Re_g = \frac{4w_g}{\pi D_g \mu_g \beta'} \tag{86a}$$

$$Re_\ell = \frac{4w_\ell}{\pi \alpha' D_\ell \mu_\ell} \tag{87a}$$

But from Table 2,

$$\alpha'_{vv} = R_\ell^2 \phi_{\ell vv}^2 = 0.1782 \times 1.77 = 0.314 \tag{88}$$

$$\beta'_{vv} = R_g^2 \phi_{gvv}^2 = 0.8218 \times 3.22 = 2.64 \tag{89}$$

Thus

$$Re_g = \frac{4 \times 2.55 \times 10^{-5}}{3.14 \times 2.64 \times 2.72 \times 10^{-3} \times 1.22 \times 10^{-5}} = 370$$

Similarly,

$$Re_\ell = 5.38.$$

With these magnitudes of the Reynolds numbers, assume viscous-viscous flow. (Viscous flow - $R_e < 1000$, p. 12.)

5) "Calculation of the pressure drop:

Once the viscous-viscous character of the flow has been established, the pressure drop is readily calculated.

From

$$\left(\frac{\Delta P}{\Delta L}\right)_\ell = \frac{2 \left(\frac{4}{\pi}\right)^{2-n} C_\ell' \mu_\ell^n w_\ell^{2-n}}{D_p^{5-n} \rho_\ell g_c} \tag{90}$$

with $n = 1$ and $C_\ell = C_\ell' = 16$,

$$\left(\frac{\Delta P}{\Delta L}\right)_\ell = \frac{8 \times 16 \mu_\ell w_\ell}{\pi D_p^4 \rho_\ell g_c} = 0.025 \text{ psi/ft}$$

From Table 2

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP} = \phi^2_{\ell vv} \left(\frac{\Delta P}{\Delta L}\right)_{\ell} \tag{91}$$

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP} = \phi^2_{\ell vv} \left(\frac{\Delta P}{\Delta L}\right)_{\ell} = (3.15)^2 \, (0.025) = 0.248 \text{ psi/ft}$$

The measured pressure drop was 0.295 psi/ft, and the deviation is of the order of 6%."

Martinelli-Nelson correlation

According to Jens and Leppert (30) in 1955, the following quotation provides an interesting opinion:

"It is the opinion of the authors that the most generally useful method available at the present time for predicting pressure drop during forced-circulation boiling in tubes is the one proposed by Martinelli and Nelson (36). This statement is made in spite of the fact that the method represents essentially an extrapolation; not of experimental data for boiling flow, but of data for adiabatic flow of gas-liquid mixtures (air and water, for example). Furthermore, the prediction is for maximum and minimum limits of pressure drops which, in many practical cases, are separated by a factor of two or more."

These empirical correlations were originally based on data obtained for isothermal horizontal flow at pressures close to atmospheric (to 50 psi), normal temperatures, and pipe diameters to 1 inch, using air and eight different liquids. Essentially, the paper extends the Lockhart-Martinelli correlation to include conditions in which the vapor flow rate increased and the liquid flow rate decreased directly with flow length, as in the forced circulation boiling of water. The basic assumptions made were as follows:

a) at point conditions in the boiling tube, thermodynamic equilibrium exists;

b) the flow type was turbulent-turbulent and the original correlation between $\phi_{\ell tt}$ and X_{tt} could be used as a basis for correlation;

c) on the basis of a) and b), point conditions in the tube could be described by the relationship

$$\left(\frac{dP}{dL}\right)_{TPf} = \left(\frac{dP}{dL}\right)_{\ell} \phi^2_{\ell tt} \tag{92}$$

d) a linear relationship exists between steam quality and tube length.

The Lockhart-Martinelli correlation was then integrated with pressure level as a parameter. The resulting values did not agree with experiments at intermediate pressures, or with requirements at the critical point, so corrections were applied to the calculations. In effect, an empirical dependence was enforced on the factor ϕ^2. Similarly, given values for R_g or R_ℓ as a function of X^2 at atmospheric pressure and at critical conditions $(R_g = R_\ell)$, values for other pressures were arbitrarily interpolated. The resulting set of curves given by Martinelli and Nelson (36) allow calculation of frictional pressure-drops in the steam-water system (Fig. 5).

The momentum pressure drop can often be neglected, particulary when no mass transfer occurs in a system. Three equations, all approximate, are given below for calculation of these acceleration pressure losses, ΔP_a, between two sections, 1 and 2, with the mean velocities of gas and liquid,

$$\Delta P_a = \frac{G_m^2}{g_c}\left[\frac{X_2^2}{\rho_{g2} R_{g2}} - \frac{X_1^2}{\rho_{g1} R_{g1}} + \frac{(1-X_2)^2}{\rho_{\ell 2}(1-R_{g2})} - \frac{(1-X_1)^2}{\rho_{\ell 1}(1-R_{g1})}\right] \tag{93}$$

For an inlet quality, $X_1 = 0$, (Martinelli-Nelson form)

$$\Delta P_a = \frac{G_m^2}{g_c} \left[\frac{X_2^2}{R_{g2}\rho_{g2}} + \frac{(1-X_2)^2}{(1-R_{g2})\rho_\ell} - \frac{1}{\rho_\ell} \right] = \frac{r_2 G_m^2}{g_c} \tag{94}$$

For an assumption of non-slip or homogeneous flow,

$$\Delta P_a = \frac{G_m^2}{g_c} \left[\frac{X_2}{\rho_{g2}} - \frac{X_1}{\rho_{g1}} + \frac{X_2}{\rho_{\ell2}} - \frac{X_1}{\rho_{\ell1}} \right] \tag{95}$$

Graphs giving the quantity, r_2, as a function of quality and pressure have been given by Martinelli and Nelson, based on their empirical correlations (see Fig. 6).
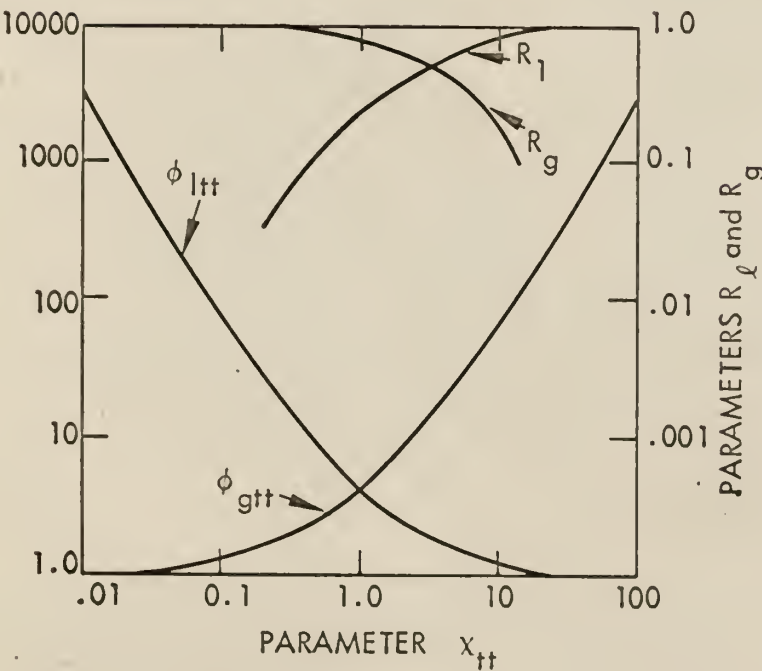
Fig. 5. Martinelli and Nelson's plot of $\phi_{\ell tt}$, ϕ_{gtt}, R_ℓ and R_g, versus $\sqrt{X_{tt}}$.
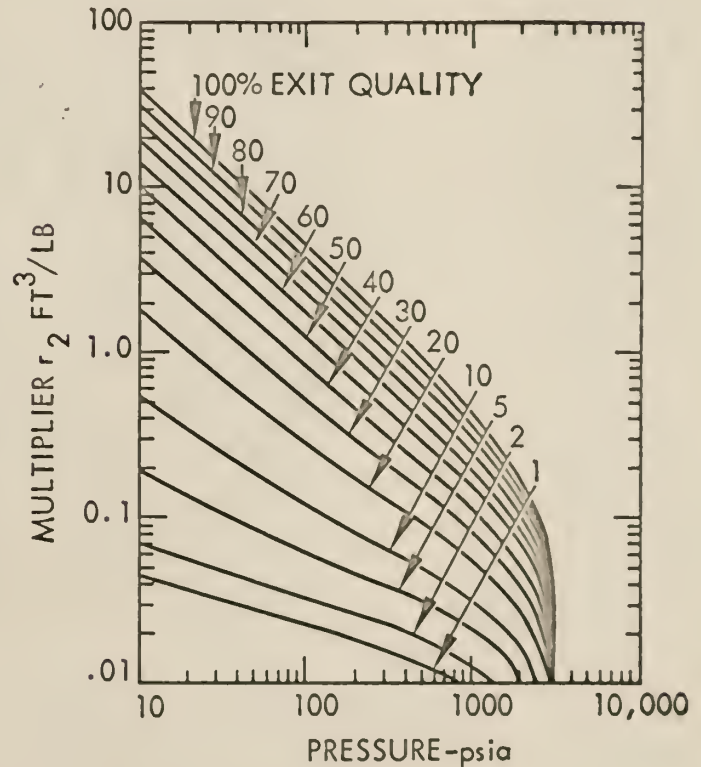
Fig. 6. Martinelli and Nelson's multiplier r_2 versus absolute pressure for various exit qualities [the pressure drop due to acceleration of fluid during evaporation equals $r_2 (G^2/g)$.]

For no mass transfer between phases, the acceleration loss is approximately

$$\Delta P_a = G_\ell^2 \left[\frac{1}{\rho_{\ell 2} R_{\ell 2}} - \frac{1}{\rho_{\ell 1} R_{\ell 1}} \right] + G_g^2 \left[\frac{1}{\rho_{g2} R_{g2}} - \frac{1}{\rho_{g1} R_{g1}} \right] \qquad (96)$$

A review by Sher (44) discusses the Martinelli prediction of two-phase densities. An excellent summary of both Martinelli methods was presented by Lockhart and Martinelli (33) in 1949.

As a further aid to assist in using the Martinelli method to predict two-phase pressure drop, a sample computer code (from the unpublished notes of G. W. Patraw) is presented in Appendix B.

Mixed Models

Mixed models are characterized as having features based upon both the Martinelli and Homogeneous models, even though the assumptions for both are contradictory. Homogeneous flow is implied by the assumption of no radial variation in velocity and density; however, the Martinelli correlation is used for two-phase pressure drops. In some cases, the choice of conditions is based upon convenience, or is made in an attempt to bracket flow conditions. From the very nature of the premises made, however, it is expected that mixed models can serve only a very limited application.

The Harvey-Faust model (22) applied to steam-water flow, and one which was also adopted by Rogers (41) to the flow of vapor-liquid hydrogen, represents an extreme choice of assumption. When velocity and density are assumed to be homogeneous throughout the mixture for the Homogeneous method, by using the Martinelli correlation for two-phase pressure drop, the results, though useful, still should not be expected to replace exact correlations which apply to specific problems.

<center>Approximation Theory[15] (Levy)</center>

Recently an approach through the solution of the equations of motion was presented. Using Prandtl mixing-length theory and the equations for potential flow to develop an equation for the density distribution, Levy (32) solved the equations of motion by numerical means. The resulting relationships between shear stress and the operational variables showed agreement only with very restricted data.

Of the available analytical solutions, the mixing length model is most akin to the one proposed by Bankoff (8). Bankoff treated the two-phase flow as a continuous medium and postulated velocity and void profiles of the power type. Levy's (32) solution confirmed Bankoff's basic assumption about the void and velocity distribution. Forms of the mixing-length theory differ from Bankoff's in that the mixing-length solutions were found to vary with flow conditions and geometry.

Generally speaking, Levy's model reduces the two-phase system to a single-phase system, and the numerous analytical methods previously developed for single-phase flow are then applied to solve corresponding two-phase problems. Because of the rigorous numerical and theoretical calculations involved, the real value of Levy's model lies not in its application to two-phase pressure drop but in its applicability to several other two-phase problems which to date have escaped solution (i. e., entrainment of liquid by gas streams, rough channels, entry length in two-phase flow, heat transfer in two-phase flow, and flow with a gas core where the density at the center of the channel ρ_m falls below the gas density ρ_g).

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[15]Approximation theory involves the procedures by which complex equations may be simplified to give an approximate solution.

DISCUSSION

A number of correlation comparisons have been presented in the literature. Various techniques have been used to evaluate these correlations. The most recent is presented in a paper by Dukler et al. (15) of the University of Houston. A computer recorded experimental data taken from published and unpublished sources. Approxiately 15,000 data points were collected. A coding system was then developed to permit rapid location covering any specified range of conditions. The data were grouped according to test section characteristics, run conditions, and experimental results. The resulting deck, designated the "data bank," is used to compare the data resulting from relatively consistent experimental observation with the results calculated from the various methods available. By statistical calculations, each method may then be compared for accuracy by utilizing three statistical parameters: $\bar{d}$, the average percent deviation of the calculated values from the measured values; σ, the variance of individual percent deviations; and Ψ, which is the width of the band around $\bar{d}$ that contains 68% of the calculated $\bar{d}$'s. Five correlations were compared by Dukler, who used line size, viscosity, pressure range, and flow pattern as variables (Tables 3, 4, 5). The overall comparison indicated the Lockhart-Martinelli correlation was better than the other four except for two special cases. Regression analysis, including a comparison of scatters between Martinelli's correlation and the experimental data, verified that the method needs still more refinement.

Most investigators have chosen to compare their methods to the original Lockhart-Martinelli correlation or the Homogeneous method. Reviews by Isbin et al. (28) and Gresham et al. (20) compare each correlations's theoretical and physical limits. Anderson and Russell (3) using

the same method as Dukler (Tables 6, 7), furnish additional correlation comparisons. Hughmark's comparisons (26) are presented in Table 8.

Brief mention should be made concerning the present status of investigation. Journals published monthly by ASME and AIChE provide good sources of information and periodically list special conferences held to discuss the status of current or recent projects. A review of the proceedings are frequently published in professional journals. Since improvements in two-phase systems depend considerably upon break-throughs in other areas, studies relating to mass transfer, heat transfer, chemical reaction, flow stability, critical flow, and flow over restrictions are especially important to investigators working on new pressure-drop correlations. Complete discussions of these areas may be found in Tong (48), Scott (43), Anderson and Russell (3), and Isbin et al. (28).

Table 3. Dukler's comparisons of pressure drop correlations for various pressure ranges (one-component system).

Pressure Range (psia)	D (in.)	Baker			Bankoff			Chenoweth Martin			Lockhart Martinelli			Yagi			n
		d̄	σ	Ψ	d̄	σ	Ψ	d̄	σ	Ψ	d̄	σ	Ψ	d̄	σ	Ψ	
25-100	1	234	238	—	4,852	3,964	—	53.5	85.9	67.5	39.5	66.6	55.0	-38.8	65.2	45.0*	93
400-800	1/2	632	368	—	377	206	—	55.9	88.2	65.0	63.6	57.0	52.5*	-87.7	8.8	7.5	103
1000-1400	1/2	358	177	—	31.9	73.0	52.5	63.6	145	112	-17.5	21.1	17.5*	-85.6	16.2	10.0	131
All Data Points		414	310					6.6	128		23.5	60.7	*	-72.6	42.6		327

* Correlation seemed to predict best for each row.

Table 4. Dukler's comparisons of pressure drop correlations[†] showing effects of line size and liquid viscosity (two-component systems).

D (in.)	μ_L (cp)	Baker			Bankoff			Chenoweth Martin			Lockhart Martinelli			Yagi			n	σ_D	Ψ_D
		d̄	σ	Ψ	d̄	σ	Ψ	d̄	σ	Ψ	d̄	σ	Ψ	d̄	σ	Ψ			
1	1	64.2	40.0	45.0	2080	980	—	-8.5	17.8	15.0	-6.6	10.1	10.0*	40.9	29.1	30.0	224	6.0	5.0
	3	77.4	335	87.5	1172	2220	—	11.2	55.6	30.0	3.8	29.1	20.0*	183	123	—	230	22.0	17.0
	20	30.7	89.5	40.0	737	1384	—	42.5	94.2	65.0	-5.5	24.7	20.0*	481	192	—	156	18.0	14.0
2	1	-13.6	60.3	65.0	1178	2910	—	-2.7	24.8	20.0*	9.2	37.7	25.0	162	228	—	320	16.0	7.0
	3	19.3	79.0	82.5	4810	4654	—	8.4	45.3	45.0	-4.7	22.9	25.0*	62.3	74.5	80.0	398	16.0	9.0
	20	73.0	159	90.0	2804	4893	—	95.4	268	—	13.2	52.9	30.0*	271	325	—	401	24.0	8.0
3-1/2	1	11.5	79.2	82.5	2176	3072	—	15.0	40.2	30.0*	31.0	50.2	47.5	27.6	104	97.5	109	—	—
	3	7.1	60.1	72.5	4720	5000	—	27.8	62.0	45.0	16.3	39.3	22.5*	84.5	86.1	—	67	18.0	12.0
	20	31.8	60.6	47.5	2432	3561	—	51.0	91.8	57.5	-0.4	26.2	22.5*	147	83.4	—	111	27.0	13.0
5-1/2	1	-70.5	11.6	10.0	254	213	—	51.2	23.7	30.0	38.3	12.2	12.5*	93.3	22.3	22.5	24	—	—
	3	-0.5	44.6	45.0	2096	3704	—	20.0	57.5	45.0	11.6	41.5	37.5*	106	80.8	—	131	19.0	13.0
	20	7.6	47.8	50.0	2692	5263	—	37.1	79.4	47.5	-1.0	24.8	25.0*	120	69.0	—	122	21.0	7.0
All Data Points		26.2	159	65.0				27.6	89.5	42.5	4.2	36.0	25.0*	155	184		2293		

† d̄, σ, and Ψ expressed as underlined percentages.

* Correlation seemed to predict best for each row.

Table 5. Dukler's comparison of pressure drop correlations showing effects of flow pattern.

Observed flow pattern	Baker				Chenoweth Martin			Lockhart Martinelli			$n^\dagger$
	$\bar{d}$	σ	Ψ	n	$\bar{d}$	σ	Ψ	$\bar{d}$	σ	Ψ	
Bubble	15.6	168	30.0	960	—	—	—	—	—	—	0
Plug	116	92.9	100	69	-6.5	19.8	10.0*	9.4	36.3	20.0	270
Stratified	—	—	—	—	122	103	—	23.3	33.0	22.5*	34
Wave	-91.0	2.5	2.5	98	138	301	135	38.4	85.7	42.5*	287
Slug	61.0	218	135	251	12.8	92.1	27.5	2.9	31.2	17.5*	974
Annular	68.7	81.2	100	430	22.2	81.1	65.0	-12.8	35.6	30.0*	265
Dispersed	16.9	35.0	30.0	133	84.4	81.1	85.0	18.0	34.1	25.0*	111

† Except for Baker's Correlation, which is based on the number of points calculated (14) to be in each flow regime, all correlations are tested on the same points.

* Correlation seemed to predict best for each row.

Table 6. Anderson and Russell's values of $\bar{d}$, σ, and Ψ for pressure drop correlations for different line sizes and liquid viscosities (using Dukler's method).

Line Size in.	Liquid Viscosity cP.	Dukler et al. (general)			Lockhart-Martinelli			Chenoweth-Martin			Baker		
		$\bar{d}$	σ	Ψ	$\bar{d}$	σ	Ψ	$\bar{d}$	σ	Ψ	$\bar{d}$	σ	Ψ
1	1	-25.2	10.2	13.5	-6.6	10.1	10.0	-8.5	17.8	15.0	64.2	40.0	45.0
	3	8.6	24.8	12.0	3.8	29.1	20.0	11.2	55.6	30.0	77.4	335	87.5
	20	6.7	24.4	18.6	-5.5	24.7	20.0	42.5	94.2	65.0	30.7	89.5	40.0
2	1	2.4	18.4	15.5	9.2	37.7	25.0	-2.7	248	20.0	-13.6	60.3	65.0
	3	1.6	19.7	16.0	-4.7	22.9	25.0	8.4	45.3	45.0	19.3	79.0	82.5
	20	10.3	27.2	20.0	13.2	52.9	30.0	95.4	268	—	73.0	159	90.0
3-1/2	1	-0.3	26.8	26.2	31.0	50.2	47.5	15.0	40.2	30.0	11.5	79.2	82.5
	3	9.3	24.9	25.0	16.3	39.3	22.5	27.8	62.0	45.0	7.1	60.1	72.5
	20	10.6	24.5	18.6	-0.4	26.2	22.5	51.0	91.8	57.5	31.8	60.6	47.5
5-1/2	1	50.6	18.8	19.3	38.3	12.2	12.5	51.2	23.7	30.0	-70.5	11.6	10.0
	3	11.2	19.2	16.0	11.6	41.5	37.5	20.0	57.5	45.0	-0.5	44.6	45.0
5*	20	7.3	22.7	14.0	-1.0	24.8	25.0	37.1	79.4	47.5	9.6	47.8	50.0

*Size used by Dukler et al.

Table 7. Anderson and Russell's values of $\bar{d}$, σ, and Ψ for pressure drop correlations for different flow patterns (using Dukler's method).

Flow type	Dukler et al. (general)			Lockhart-Martinelli			Chenoweth-Martin			Baker		
	$\bar{d}$	σ	Ψ	$\bar{d}$	σ	Ψ	$\bar{d}$	σ	Ψ	$\bar{d}$	σ	Ψ
Plug	9.5	18.0	15.1	9.4	36.3	20.0	-6.5	19.8	10.0	116	92.9	100
Stratified	13.4	30.3	17.5	23.3	33.0	22.5	122	103	—	—	—	—
Wave	11.5	22.6	20.5	33.4	85.7	42.5	138	301	135	-91.0	2.5	2.5
Slug	9.5	21.7	15.6	2.9	31.2	17.5	12.8	92.2	27.5	61.0	218	135
Annular	-11.2	26.0	19.0	-12.8	35.6	30.0	222	81.1	65.0	68.7	81.2	100
Dispersed	14.8	16.9	17.1	18.0	34.1	35.0	84.4	81.1	85.0	16.9	35.0	30.0

Table 8.   Hughmark's comparison of experimental horizontal flow data with correlations.

Hughmark's series no.	Average absolute deviation, ΔP_{TP}, %				No. of tests for Hughmark method
	Lockhart-Martinelli	Chenoweth-Martin	Poettman[16]	Hughmark	
1	19.2	14.9	68	15.5	59
2	38.6	19.9	32.0	14.3	77
3	21.9	23.2	72	21.6	61
4	75.3	18.5	35.2	24.3	46
5	18.6	20.1	30	26.4	22
6	21.7	15.6	45	28.8	33
7	19.4	—	19.3	25.3	59
8	18.7	—	22.3	12.3	33
9	31	—	25.5	12.2	20
10	17.2	22.3	43	26.7	50
11	18.2	25.2	20.7	20.8	18
12	29.6	—	14.3	27.9	22
13	25.7	20.8	17.5	21.6	28
14	28	26.7	55	27.5	25
15	24.1	49	83	18.9	228
16	13.0	6.3	30.5	11.7	15
17	19.7	8.0	25.5	6.0	18
18	25.2	4.8	23.5	9.2	18
19	42.3	19.6	49.5	24.5	29
20	22.7	22.6	27.4	10.1	40
21	32.6	—	46.5	16.9	31
22	90.5	43	55	42.5	15
23	63	45	54	38	8
24	8.2	12.7	35.2	6.9	57
25	31.3	37.8	35.4	24	30
26	28.4	34.8	14.4	24.6	25
27	18.5	18.1	85.5	23	23
28	22.9	17.8	69.5	18.2	26
29	24.7	22.4	47	15.3	27
30	60.2	26.6	32.1	19.5	48
31	63.1	20.1	24.9	13.3	31
32	92	24.7	26.6	11.4	33
33	45	15.3	24	11.6	52
34	61.6	20.8	35.2	10.6	30
35	69	15.2	22.9	8.3	20
36	67	23.4	45.5	11.2	22
Av.	36.3	22.4	38.4	18.9	Total 1379

[16] Poettman, F. H., et al., J. Petrol. Technol. 8:17 (1956).

SUMMARY

This paper presents a literature review of selected two-phase pressure drop correlations, with enough background material to indicate the major source of difficulties being encountered currently in such an analysis. From this review, the conclusions are:

1. Two-phase flow, as classified according to system, appearance and type, involves so many variables that the possibility for obtaining a universal solution to the problem appears realistically unfeasible.

2. Two-phase pressure drop consists of the summation of pressure losses by friction, momentum exchange between phases, and elevation phenomena. The components due to acceleration and elevation are much easier to calculate than the components due to friction. Energy, momentum, and continuity equations are fundamental to the analysis of all three. This paper reviews these equations and includes the physical significance of various simplifying assumptions which permit mathematical solution.

3. Correlations and example methods have been presented which may be categorized into the following four approaches: empirical, similarity, mathematical analysis of the simplified physical models, and approximation theory. The mathematical analysis of simplified physical models in the forms of the Homogeneous and Martinelli methods have received greatest attention and application of the newer approaches, Dukler's method appears most promising. Depending on the specifics of the problem, deviation from experimental data (20% to 80%) must be expected. The major inability of the simplified physical model approach to correlate with experimental data may be traced to its neglect of important variables and, in particular, to its failure to deal with the flow structure. The hydrodynamics of two-phase flow will require a great deal more study before more exact results may be expected.

RECOMMENDATIONS

It is suggested that this paper be utilized for preliminary analysis of two-phase pressure drop by investigators who are unfamiliar with the variety and limitations of the correlations presented in the literature.

Pertinent recommendations of the author are:

1. Greater emphasis should be placed on the evaluation of correlation methods currently found in the literature. It is now the responsibility of the investigator to reevaluate these correlations, analyze their errors and/or limitations, and construct better correlations therefrom.

2. Though short-term research analyzing methods and modification of existing methods certainly will be very useful, emphasis should be placed upon longer range programs which will, in time, significantly diminish the inherent complexities and experimental difficulties presently encountered.

ACKNOWLEDGMENTS

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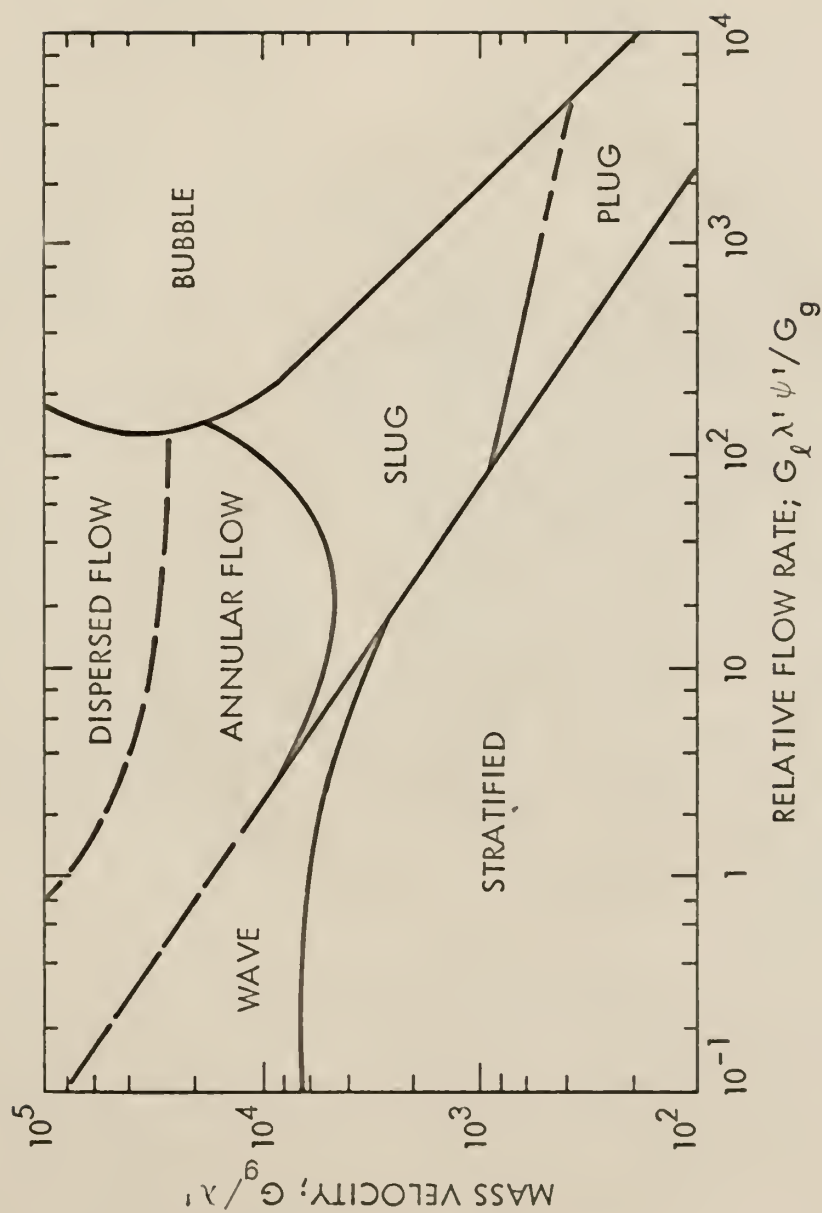   Chem. Eng. J. 8: 281. May 1962.

APPENDICES

Plate I. Baker's chart for two-phase flow regimes.

$$\left\{ \lambda' = \left[\left(\frac{\rho_v}{0.075} \right) \left(\frac{\rho_\ell}{62.3} \right) \right]^{1/2} \right., \quad \psi' = \left(\frac{73}{\sigma_\ell} \right) \left[\mu_\ell \left(\frac{62.3}{\rho_\ell} \right)^2 \right]^{1/3} \right\}.$$

Baker's Flow-Prediction
Example Problem

This example will demonstrate the use of the flow-prediction chart by Baker.

Given: A 1-in. horizontal pipe, a water-flow rate of 2180 lb/hr, an air-flow rate of 10.9 lb/hr, and flow conditions such that the average physical properties of the air and water are estimated to be as follows: μ_ℓ = 1 cP, ρ_ℓ = 62.3 lb/ft^3, γ_ℓ = 73 dynes/cm, ρ_g = 0.075 lb/ft^3.

Find the expected flow pattern by using Plate I and the method of Baker:

Calculation of λ' :

$$\lambda' = \left[(\rho_g/ 0.075) \, (\rho_1/ 62.3) \right]^{1/2}$$

$$\lambda' = \left[(0.075/ 0.075) \, (62.3/ 62.3) \right]^{1/2} \tag{97}$$

$$\lambda' = 1.0$$

Calculation of ψ' :

$$\psi' = \left[73/\gamma L \right] \left[\mu_1 (62.3/ \rho L) \right]^{1/3}$$

$$\psi' = \left[73/ 73 \right] \left[1 (62.3/ 62.3) \right]^{1/3} \tag{98}$$

$$\psi' = 1.0$$

Calculation of G_ℓ:

$$G_\ell = \omega_\ell A_p$$

$$G_\ell = 2180/ (0.25 \, \pi \times 0.0833)^2 \tag{10}$$

$$G_\ell = 400,000 \text{ lb/hr-ft}^2$$

Calculation of G_g:

$$G_g = w_g A_p \tag{9}$$

$$G_g = 10.9/(0.25 \, \pi \times 0.0833)^2$$

$$G_g = 2000 \quad \text{lb/hr-ft}^2$$

Calculation of $G_\ell \lambda' \psi' / G_g$:

$$G_\ell \lambda' \psi' / G_g = (400,000 \times 1 \times 1)/2,000$$

$$G_\ell \lambda' \psi' / G_g = 200$$

Calculation of G_g/λ':

$$G_g/\lambda' = 2000/1$$

$$G_g/\lambda' = 2000$$

From Plate I, the pattern is slug flow.

APPENDIX B

TONG'S SUMMARY OF RECENT EXPERIMENTAL STUDIES FOR PRESSURE DROP IN STEAM-WATER MIXTURES

Source[*]	Description of test	Data Range		
		Pressure (psia)	Mass velocity (10^6 lb/hr ft^2)	Quality (% by wt)
Weiss (W13) (1952)	Electrically heated vertical stainless steel tube 0.174 in. I.D. × 24 in. long	20-1400	0.18-1.8	0-100
Isbin et al. (16) (1958)	Unheated horizontal tubes 0.484-1.062 in. I.D., 3 ft 8½ in. to 8 ft long	25-1415	0.30-1.40	3-98
Lester (L12) (1958)	Unheated horizontal steel pipe lines 4.06 and 6.06 in. I.D. × 40 ft	30-100	0.04-0.54	10-85
Schrock and Grossman (S7) (1959)	Electrically heated vertical stainless steel tubes 0.118 in. I.D. × 15, 30, and 40 in. long, 0.237 in. I.D. × 15 and 30 in. long, 0.432 in. I.D. × 30 in. long	42-505	0.18-3.28	5-57
Becker et al. (B17) (1962)	Electrically heated vertical stainless steel tubes 0.305 in. I.D. and 0.391 in. I.D. × 10.25 ft	85-600	0.29-3.80	0.80
Perroud et al. (P3) (1960)	Electrically heated and unheated stainless steel vertical tubes 0.197 in. I.D. and 0.394 in. I.D., 15.75 and 31.5 in. long, respectively	140-940	0.24-6.05	0-100
Hoglund et al. (H19) (1961)	Electrically heated vertical stainless steel rectangular channel 2× 0.25 × 60 in.	150-600	0.6	0.9-6.5
Haywood et al. (H8)(1961)	1 × 1½ in. I.D. tubes heated and unheated oriented both horizontally and vertically: heated lengths 16 and 24 ft	250-3000	0.54-1.26	0-57

TONG'S SUMMARY OF RECENT EXPERIMENTAL STUDIES
FOR PRESSURE DROP IN STEAM-WATER MIXTURES

Source[*]	Description of test	Pressure (psia)	Mass velocity (10^6 lb/hr ft^2)	Quality (% by wt)
Davidson et al. (D1)(1943)	Straight or flat spirally coiled tubes; inside diameters from 0.5-1.75 in. I.D., 10 to 15 ft long; all sections were heated	290-3300	0.07-3.89	0-100
Cicchitti et al. (C12) (1960)	Electrically heated and unheated vertical stainless steel tube 0.202 in. I.D. × 15.8 in. long	510-770	1.60-2.90	24-70
Armand and Tretchev (A9) (1947)	Horizontal stainless steel tubes, heated and unheated, 2.2 in. I.D. × 13.1 ft	510-1280	——	0-90
Bertoletti et al. (B36) (1961)	Electrically heated and unheated stainless steel vertical tubes and annuli	600-1200	0.74-2.95	20-85
	Tubes 0.204 in. I.D.× 3.9, 8.3, 16.2, and 31.4 in. long 0.126, 0.158, 0.248, and 0.323 in. I.D. × 31.5 in. long.			
	Annuli 0.394 in. O.D.× 0.197 in. I.D.× 23.2 in. 0.394 in. O.D.× 0.152 in. I.D.× 23.2 in.			
Sher et al. (S13)(1962)	Vertical heated rod cluster assembly 16 rods 0.416 O.D. × 36 in. long 4×4 array on 0.570 in. centers (lower half) 0.367 in. O.D.× 36 in. long on 0.570 in. centers (top half) including spacers in box 2.399 in. square	615	1.0-2.5	0-5

TONG'S SUMMARY OF RECENT EXPERIMENTAL STUDIES
FOR PRESSURE DROP IN STEAM-WATER MIXTURES

Source[*]	Description of test	Data Range		
		Pressure (psia)	Mass velocity (10^6 lb/hr ft^2)	Quality (% by wt)
Janssen and Kervinen (J7) (1961)	Vertical unheated stainless steel rod cluster assembly 16 rods 0.423 in. O.D.×40 in. long 4×4 array on 0.76 in. centers in box 2.8×2.8 in. (and including various orifice fittings, wire spacer, etc.)	1000	0.1-1.0	2-26
Columbia University Engineering Research Laboratories (C21)(1961-62)	Various vertical 7-rod and 19-rod cluster assemblies both with and without wire wraps	1000	0.5-1.6	0-40
Quinn (Q4) (1961)	Vertical unheated rod cluster assembly 25 rods 0.363 in. O.D. ×29.7 in. long 5×5 array on 0.615 in. centers in box 3.04×3.04 in. (both with and without wire spiral spacers)	1000	0.7-1.2	0-15
Sher (S12) (1957)	Electrically heated stainless steel vertical rectangular channels 1.0 x 0.097×27 in. and 1.0×0.050×27 in.	1100-2000	0.7-5.0	0-40

[*]References listed in the column are from Tong (48).

APPENDIX C

TWO-PHASE PRESSURE DROP COMPUTER CODE

(Homogeneous and Lockhart-Martinelli-Nelson Methods)

High-speed digital computers, by reducing computation time, now allow
the engineer to solve complex design problems. Among the numerous types
of problems which can now be solved efficiently are trial and error, or iter-
ative optimization designs. Problems of this type normally are approached
through the use of a general problem code that relies upon and calls for
specialized subroutines. These subroutines provide answers to problems
whose methods of approach are likely to be continually updated or varied.
Calculation methods of this type are especially flexible in that, while better
subroutines are being developed, the general program remains operational.

According to its importance, numerous two-phase pressure drop
subroutines have been written to handle a variety of flow conditions. As an
example of how the various computation methods have been programmed, the
Homogeneous and Lockhart-Martinelli-Nelson subroutines, taken from the per-
sonal notes of G. W. Patraw, are presented. Tong (48) provided the prin-
cipal reference for these methods. Both of the codes have been written for
a 6600/3600 CDC computer in FORTRAN. The author considers only the method
of approach important. No attempt has been made to develop or refine
these subroutines more thoroughly.

Homogeneous Subroutine

```
*       CARDS COLUMN
*       LIST P
*       FORTRAN               (      )
        SUBROUTINE HOMOG
        COMMON /DROP/ D,Y,W,PO,XIO,KON,MESH
        COMMON /PROP/ CP,CONP,TP,VISL,VISG,VP,VL
        COMMON/HOM/DELP
        DATA (PI=3.14159),(GL=32.17),(PCVT=2120.)
        A = PI*D*D/4.
        A1 = .092*(4.*W/PI)**1.8*VISL**.2/GL/D**4.8
        A2 = (W/A)**2/GL
        UN =KON
        DXIDL = OU/Y
        AMESH =MESH
        DL =Y/AMESH
        XI = XIO-.5*DXIDL*DL
        P = PO*PCVT
        DO 99 I=1,MESH
        X1 = XI+DXIDL*DL
        XI1 = 1.-XI
        VG = VP/P
        DVDP = -VG/P
        T = TP+CONP/LOGF(CP*PCVT/P)
        REG = 4.*XI*W/(PI*D*VISG)
        REL = 4.*XI1*W/(PI*D*VISL)
        DPFDL =-A1*(VL+XI*VG)/GL**.2
        C1O = A2*VG*DXIDL
        TOP = DPFDL-C1O
        C8 = XI*A2*DVDP
        BOT = 1.+C8
        DP =TOP/BOT*DL
        P = P+DP
        U = W/A*(VL+XI*VG)
        DELPF = DPFDL*DL
        DELPM =  DP-DELPF
        WRITE OUTPUT TAPE 3,1001,XI,VG,T,REG,REL,DPFDL,C1O,C8,DP,U,DELPF,
       X DELPM
   1001 FORMAT(7E12.3/5E12.3)
        IF (P) 999,999,99
     99 CONTINUE
        DELP = 1.-P/(PO*PCVT)
        GO TO 555
    999 DELP = 0.
    555 RETURN
        END
```

```
*       CARDS COLUMN
*       LIST ↓
*       FORTRAN              LMN
        SUBROUTINE LMN
        COMMON /DROP/ D,Y,W,PO,XIC,KON,MESH
        COMMON /PROP/ CP,CONP,TP,VISL,VISG,VP,VL
        COMMON /LOCK/DELP
        DATA (PI=3.14159),(GE=32.17),(PCVT=212.)
        A = PI*D*D/4.
        A1 = .092*(4.*W/PI)**1.8*VISG**.2/GE/D**4.8
        A2 = (W/A)**2/GE
        A3 = SQRTF(VL)*(VISL/VISG)**.1
        ON =KON
        DXIDL = ON/Y
        AMESH =MESH
        DL=Y/AMESH
        XI = XIC-.5*DXIDL*DL
        P = PO*PCVT
        DO 99 I=1,MESH
        XI = XI+DXIDL*DL
        XI1 = 1.-XI
        VG = VP/P
        DVDP = -VG/P
        XT = (XI1/XI)**.9/SQRTF(VG)*A3
        RXT = SQRTF(XT)
        PHI = EXPF(1.16*RXT)/XI**(.21*RXT)+XT
        AO = XI1*VL+XI*VG
        T = TP+CONP/LOGF(CP*PCVT/P)
        DPFDL = A1*VG*XI**1.8
        DPFDL =-PHI**2*DPFDL
        C1    = XI1**2*VL/ALF1
        C2 = XI**2*VG/ALF
        C3 = -1./XI1+DALDX*DXTDX/ALF1
        C4 = 1./XI-DALDX*DXTDX/ALF
        TOP = DPFDL-A2*DXIDL*(C1*C3+C2*C4)
        C5 = C1/ALF1-C2/ALF
        C6 = C5*DALDX*DXTDV
        C7 = XI*XI/ALF
        C8 = A2*DVDP*(C6+C7)
        BOT = 1.+C8
        DP =TOP/BOT*DL
        P = P+DP
        UG = XI*VG*W/(ALF*A)
        UL = XI1*VL*W/(ALF1*A)
        DELPF = DPFDL*DL
        DELPM =  DP-DELPF
        REG = 4.*XI*W/(PI*D*VISG)
        REL = 4.*XI1*W/(PI*D*VISL)
        F = .046/REG**.2
        WRITE OUTPUT TAPE 3,1001,XI,VG,XT,PHI,DALDX,ALF,DXTDX,DXTDV,T,
       X DPFDL,C1,C2,C3,C4,C5,C6,C7,C8,DP,UG,UL,DELPF,DELPM,REG,REL,F
   1001 FORMAT(7F12.3/6E12.3/7E12.3/6E12.3)
        IF (P) 999,999,99
     99 CONTINUE
        DELP = 1.-P/(PO*PCVT)
        GO TO 555
    999 DELP = C.
    555 RETURN
        END
```

APPENDIX D

NOMENCLATURE

A   Area, L^2

C   Constant defined in Blasius Equation (33)

C'   Wetted perimeter, L

D   Pipe inner diameter, L

d   Differential

$\bar{d}$ Average percent deviation of the calculated values from measured values.

E   Mechanical energy dissipated, LF/M

f   Friction factor

f*   Chenoweth and Martin fictitious friction factor based on Fanning equation

F   Function

Fr   Froude number

g   Acceleration due to gravity, L/T^2

g_c 32.17 LM/FT^2

G   Mass velocity, M/L^2T

h   Specific enthalpy, LF/M

H   Enthalpy, LF

J   778 ft-lb_f/B

K   Hughmark flow parameter. See Fig. 3.

K'   Chenoweth and Martin allowance for valves and fittings. See Eq. 55.

K"   Bankoff's flow parameter. See Eq. 24.

KE   Kinetic energy, LF

L   Length, L

P   Pressure, F/L^2

Q   Volumetric flow rate, L^3/T

Q'   Total heat flow, LF/T

q   Specific heat flow, LF/L

r   Martinelli-Nelson multiplier, L^3/M, (36)

R   Local volume fraction (holdup)

$\overline{R}$ Average volume fraction

Re   Reynolds number

S   Dimensionless distance from the wall

u   Velocity, L/T

u'   Relative velocity, L/T

U   Thermodynamic internal energy, LF/M

v   Specific volume, L^3/M

$\overline{v}$ Mean specific volume, L^3/M

w   Mass flow rate, M/T

We   Weber number

X   Quality (vapor mass fraction)

z   Elevation distance, L

Z   Hughmark dimensionless correlating factor, see Eq. 29

Subscripts and Superscripts

a   Acceleration term

f   Friction

g   Gas

ℓ Liquid

m   Mixture

m   Exponent of Reynolds modulus in the Blasius expression for the friction factor for the gas phase.

n   Exponent of Reynolds modulus in the Blasius expression for the friction factor in the liquid phase.

NS   No slip

o   Single phase flow

p   Pipe

s   Superficial

t   Turbulent

tt   Turbulent liquid, turbulent gas flow

TP   Two phase

v   Viscous

vv   Viscous liquid, viscous gas flow

W   Wall

z   Elevation

1,2   Position 1, position 2

*   Fictitious

Greek Letters

α Void fraction

α' Flow modulus for gas (L-M model)

β Dimensionless group defined by Eq. 59

β' Flow type modulus for liquid (L-M model)

γ Shearing stress $- F/L^2$

Δ Small change

λ Ratio of the volumetric flow rate of liquid to the total volumetric flow rate at average pressure.

λ' Baker flow parameter. See Eq. 97.

μ Viscosity, F/TL

π 3.1416

θ Angle of elevation

ρ Density, M/L^3

σ Variance of individual percent deviation

ϕ Function of χ utilized in calculating two-phase pressure drop

χ Dimensionless parameter (Lockhart-Martinelli-Nelson models)

Ψ Width of band around $\bar{d}$ that contains 68% of the calculated $\bar{d}$'s

ψ Dimensionless group

ψ^* Chenoweth and Martin fictitious dimensionless group

ψ' Baker's dimensionless group. See Eq. 98.

TWO-PHASE PRESSURE DROP
A LITERATURE SURVEY AND CORRELATION ANALYSIS

by

RAYMOND D. CAUGHRON

B. S., Kansas State University, 1966

———————

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1967

ABSTRACT

Methods for predicting pressure drops in two-phase flow systems have been investigated. A literature survey covers the historical development and current status of the subject. Flow classification, technical terms, important equations, and interrelationships among variables pertinent to the study of two-phase flow have been reviewed. A general description of two-phase pressure drop has been presented including the energy and momentum equations. Prediction methods have been categorized according to method of approach (empirical, dimensional, similarity, mathematical analysis applied to simplified physical models, and approximation theory). Numerical examples have been included to demonstrate the Dukler and Lockhart-Martinelli approaches. Comparison methods by Dukler, Anderson and Russell, and Hughmark with tables have been included. Baker's chart and method of predicting two-phase flow regimes, experimental steam-water data, and computer sub-routines for the Homogeneous and Lockhart-Martinelli methods are given in the Appendices.