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DETERMINING THE HUBBLE CONSTANT FROM HUBBLE PARAMETER MEASUREMENTS

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ABSTRACT

We use 28 measurements of the Hubble parameter, $H(z)$, at intermediate redshifts $0.07 \leqslant z \leqslant 2.3$ to determine the present-day Hubble constant H_0 in four cosmological models. We measure $H_0 = 68.3^{+2.7}_{-2.6}$, $68.4^{+2.9}_{-3.3}$, $65.0^{+6.5}_{-6.6}$, and $67.9^{+2.4}_{-2.4}$ km s^{-1} Mpc^{-1} (1σ errors) in the ΛCDM (spatially flat and non-flat), ωCDM, and ϕCDM models, respectively. These measured H_0 values are more consistent with the lower values determined from recent data on the cosmic microwave background and baryon acoustic oscillations, as well as with the value found from a median statistical analysis of Huchra's compilation of H_0 measurements, but include the higher local measurements of H_0 within the 2σ confidence limits.

Key words: cosmological parameters – dark energy

1. INTRODUCTION

The current value of the cosmological expansion rate, the Hubble constant H_0, is an important cosmological datum. Although one of the most measured cosmological parameters, it was more than seven decades after Hubble's first measurement before a consensus value for H_0 started to emerge. In 2001 Freedman et al. (2001) provided $H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1} (1σ error including systematics) as a reasonable summary of the H_0 value from the *Hubble Space Telescope* Key Project. In the same year Gott et al. (2001) applied median statistics[4] to 331 H_0 estimates tabulated by Huchra[5] and determined $H_0 = 67 \pm 3.5$ km s^{-1} Mpc^{-1}. During the following decade median statistics was applied to larger compilations of H_0 measurements from Huchra, in 2003 to 461 measurements by Chen et al. (2003), who found $H_0 = 68 \pm 3.5$ km s^{-1} Mpc^{-1}, and in 2011 to 553 measurements by Chen & Ratra (2011a), who found $H_0 = 68 \pm 2.8$ km s^{-1} Mpc^{-1}.

Many more recent H_0 determinations are consistent with these results. For instance, the final *Wilkinson Microwave Anisotropy Probe* (*WMAP*) measurement is $H_0 = 70.0 \pm 2.2$ km s^{-1} Mpc^{-1} (Hinshaw et al. 2013), while the Atacama Cosmology Telescope and the *WMAP* seven-year data on the anisotropy of the cosmic microwave background (CMB) give $H_0 = 70.0 \pm 2.4$ km s^{-1} Mpc^{-1} (Sievers et al. 2013); baryon acoustic oscillations (BAO), Type Ia supernovae, and CMB data result in $H_0 = 67.3 \pm 1.1$ km s^{-1} Mpc^{-1} (Aubourg et al. 2015; also see Ross et al. 2015; Bernal et al. 2016; L'Huillier & Shafieloo 2016; Luković et al. 2016), with the *Planck* 2015 CMB data value being $H_0 = 67.8 \pm 0.9$ km s^{-1} Mpc^{-1} (Ade et al. 2015; but see Addison et al. 2016).

While the consistency of these results is encouraging, some recent local estimates of H_0 are larger. Riess et al. (2011) find $H_0 = 73.8 \pm 2.4$ km s^{-1} Mpc^{-1} (but see Efstathiou 2014, who argues that $H_0 = 72.5 \pm 2.5$ km s^{-1} Mpc^{-1} is a better representation), Freedman et al. (2012) find $H_0 = 74.3 \pm 2.1$ km s^{-1} Mpc^{-1}, while Riess et al. (2016) give $H_0 = 73.24 \pm 1.74$ km s^{-1} Mpc^{-1}.

It is important to understand the reasons for this difference. For instance, the value and uncertainty of H_0 affect observational constraints on other cosmological parameters (see, e.g., Samushia et al. 2007; Chen et al. 2016). Given current cosmological data, the standard model of particle physics with three light neutrino species is more compatible with the lower H_0 value and difficult to reconcile with the higher value (see, e.g., Calabrese et al. 2012); and the difference between the local and global H_0 values might be an indication that the ΛCDM model needs to be extended (see, e.g., Di Valentino et al. 2016).

Here we use measurements of the Hubble parameter, $H(z)$ (where z is redshift), to determine the Hubble constant. $H(z)$ data have previously been used to constrain other cosmological parameters (see, e.g., Samushia & Ratra 2006; Chen & Ratra 2011b; Farooq et al. 2013b, 2015; Farooq & Ratra 2013a; Capozziello et al. 2014; Chen et al. 2015; Meng et al. 2015; Alam et al. 2016; Guo & Zhang 2016; Mukherjee 2016; Solà et al. 2016), including measuring the redshift of the cosmological deceleration–acceleration transition between the earlier nonrelativistic matter-dominated epoch and the current dark-energy-dominated epoch (see, e.g., Farooq & Ratra 2013b; Moresco et al. 2016). See Verde et al. (2014) for an early attempt at measuring H_0 from $H(z)$ data. Here we use more data (28 versus 15 measurements) to higher redshift (2.30 versus 1.04) than Verde et al. (2014) used and so find tighter constraints on H_0.

We find that our H_0 values obtained from $H(z)$ are more consistent with the lower values determined using median statistics or from CMB anisotropy or BAO measurements, and with the predictions of the standard model of particle physics with only three light neutrino species and no "dark radiation." Systematic errors affecting $H(z)$ measurements are largely different from those affecting CMB and BAO measurements. In addition, median statistics does not make use of the error

[4] For applications and discussions of median statistics see Podariu et al. (2001), Chen & Ratra (2003), Mamajek & Hillenbrand (2008), Croft & Dailey (2015), Andreon & Hurn (2012), Farooq et al. (2013a), Crandall & Ratra (2014, 2015), Ding et al. (2015), Crandall et al. (2015), and Zheng et al. (2016). Median statistics does not make use of the error bars of the individual measurements.

[5] https://www.cfa.harvard.edu/~dfabricant/huchra/

Table 1
Hubble Parameter vs. Redshift Data

z	$H(z)$ (km s^{-1} Mpc^{-1})	σ_H (km s^{-1} Mpc^{-1})	Reference
0.070	69	19.6	5
0.090	69	12	1
0.120	68.6	26.2	5
0.170	83	8	1
0.179	75	4	3
0.199	75	5	3
0.200	72.9	29.6	5
0.270	77	14	1
0.280	88.8	36.6	5
0.350	76.3	5.6	7
0.352	83	14	3
0.400	95	17	1
0.440	82.6	7.8	6
0.480	97	62	2
0.593	104	13	3
0.600	87.9	6.1	6
0.680	92	8	3
0.730	97.3	7.0	6
0.781	105	12	3
0.875	125	17	3
0.880	90	40	2
0.900	117	23	1
1.037	154	20	3
1.300	168	17	1
1.430	177	18	1
1.530	140	14	1
1.750	202	40	1
2.300	224	8	4

References. (1) Simon et al. (2005), (2) Stern et al. (2010), (3) Moresco et al. (2012), (4) Busca et al. (2013), (5) Zhang et al. (2014), (6) Blake et al. (2012), (7) Chuang & Wang (2013).

bars of the individual measurements. It is significant that all four techniques result in very similar values of H_0.

To determine H_0 we analyze the $H(z)$ data tabulated in Farooq & Ratra (2013b) and reproduced in Table 1 here[6], using two different dark energy models, ΛCDM (Peebles 1984) and ϕCDM (Peebles & Ratra 1988; Ratra & Peebles 1988), as well as an incomplete, but popular, parameterization of dark energy, ωCDM. In all cases we measure H_0 from the one-dimensional likelihood determined by marginalizing over all other parameters. (Limits on other parameters, such as the current nonrelativistic matter density parameter, are quite reasonable.)

In the next section we summarize the models we use, as well as the ωCDM parameterization. In Section 3 we present our H_0 determinations. We conclude in the final section.

2. ΛCDM, ωCDM, AND ϕCDM

The Hubble parameter of the spatially flat ΛCDM model is

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}\,, \tag{1}$$

[6] The error bars of these $H(z)$ measurements include systematic errors. In the analyses here we ignore the correlations between the three points of Blake et al. (2012); these only very slightly affect the results (Farooq et al. 2016).

while in the general (non-flat) ΛCDM model it is

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0} - \Omega_\Lambda)(1+z)^2 + \Omega_\Lambda}\,, \tag{2}$$

where Ω_{m0} is the current value of the nonrelativistic matter density parameter and Ω_Λ is the cosmological constant density parameter.

In the spatially flat ωCDM parameterization we have

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + (1-\Omega_{m0})(1+z)^{3(1+w_X)}}, \tag{3}$$

where ω_X is the constant, negative, equation-of-state parameter relating the (dark energy) X-fluid pressure and energy density through $p_X = \omega_X \rho_X$. The ωCDM parameterization is incomplete and does not consistently describe inhomogeneities. However, ϕCDM, discussed next, is a consistent dynamical dark energy model.

The Friedmann equation of the spatially flat ϕCDM model is

$$H^2(z) = \frac{8\pi}{3m_p^2}(\rho_m + \rho_\phi), \tag{4}$$

where m_p is the Planck mass, ρ_m is the nonrelativistic matter energy density, and the energy density of the scalar field ϕ is

$$\rho_\phi = \frac{m_p^2}{32\pi}(\dot{\phi}^2 + \kappa m_p^2 \phi^{-\alpha}). \tag{5}$$

Here an overdot denotes a time derivative, $\kappa(m_p, \alpha)$ and α are positive constants, and we have picked an inverse-power-law potential energy density of the scalar field $V(\phi) = \kappa m_p^2 \phi^{-\alpha}/2$. The equation of motion of the scalar field is

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{6}$$

where a is the scale factor. These equations are numerically integrated to provide $H(z)$ in the ϕCDM model (Peebles & Ratra 1988; Samushia 2009; Farooq 2013).

3. ANALYSIS AND RESULTS

We constrain cosmological parameters by minimizing χ_H^2:

$$\chi_H^2(\boldsymbol{p}) = \sum_{i=1}^{N}\frac{[H^{\mathrm{th}}(z_i; \boldsymbol{p}) - H^{\mathrm{obs}}(z_i)]^2}{\sigma_{H,i}^2}, \tag{7}$$

for N measured values of $H^{\mathrm{obs}}(z_i)$ with variance $\sigma_{H,i}^2$ at redshift z_i where H^{th} is the predicted value of $H(z)$ in the cosmological model. $\boldsymbol{p}$ represents the free parameters of the cosmological model under consideration, H_0 and Ω_{m0} in all four cases, with one additional parameter in three of the cases: Ω_Λ in non-flat ΛCDM, ω_X in the spatially flat ωCDM parameterization, and α in the spatially flat ϕCDM model. We use the compilation of 28 $H(z)$ data points from Farooq & Ratra (2013b) as reproduced here in Table 1 to constrain the model parameters under consideration by using the Markov chain Monte Carlo method coded in the publicly available package CosmoMC (Lewis & Bridle 2002).

Our results are summarized in Table 2 and Figures 1–5.

The limits on cosmological parameters shown in Table 2 are derived from the corresponding one-dimensional likelihood

Table 2
Mean Values of Free Parameters of Various Models with 1σ and 2σ Error Bars

Parameter	ΛCDM	Non-flat ΛCDM	XCDM	ϕCDM
H_0	$68.3^{+2.7+5.2}_{-2.6-5.1}$	$68.4^{+2.9+5.9}_{-3.3-5.4}$	$65.0^{+6.5+9.4}_{-6.6-9.3}$	$67.9^{+2.4+4.7}_{-2.4-4.7}$
Ω_{m0}	$0.276^{+0.032+0.072}_{-0.039-0.068}$	$0.267^{+0.049+0.010}_{-0.050-0.102}$	$0.308^{+0.048+0.114}_{-0.076-0.102}$	$0.275^{+0.029+0.063}_{-0.035-0.062}$
Ω_Λ	…	$0.708^{+0.101+0.219}_{-0.167-0.208}$	…	…
w_X	…	…	$-0.780^{+0.196+0.460}_{-0.292-0.414}$	…
α	…	…	…	no limits
$\chi^2_{\min}$	17.0	16.9	17.0	17.0

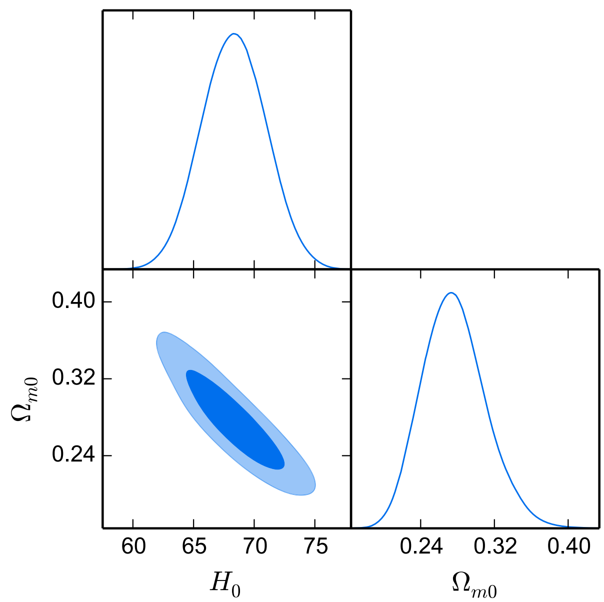

Figure 1. 1σ and 2σ confidence contours of the parameters of the spatially flat ΛCDM model. Marginalized probability distributions of the individual parameters are also displayed.

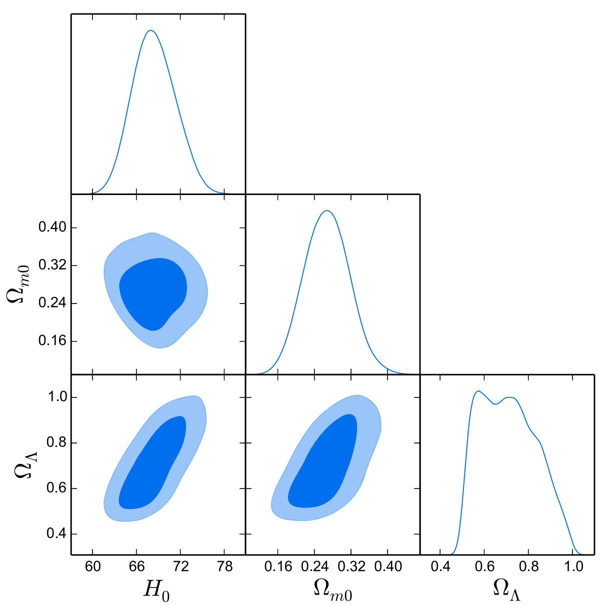

Figure 2. 1σ and 2σ confidence contours of the parameters of the non-flat ΛCDM model. Marginalized probability distributions of the individual parameters are also displayed.

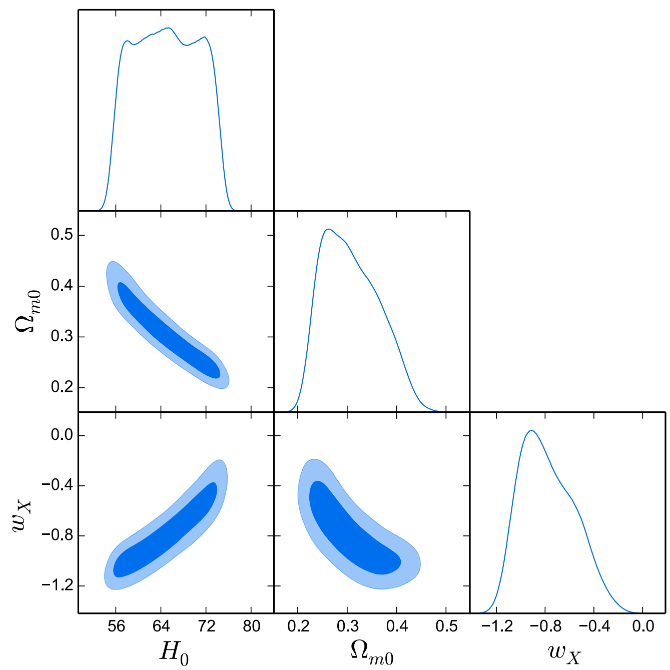

Figure 3. 1σ and 2σ confidence contours of the parameters of the spatially flat ωCDM parameterization. Marginalized probability distributions of the individual parameters are also displayed.

function that results from marginalizing over all of the other parameters. The constraints listed in Table 2 are roughly in line with those now under discussion. The small reduced χ^2 values that follow from the entries in the last line of the table are not unexpected given the results of Farooq et al. (2013a).

The H_0 values listed in Table 2 are in good accord with the lower recent values determined by using median statistics on Huchra's compilation and from CMB and BAO data, as well as with what is expected in the standard model of particle physics with only three light neutrino species and no additional "dark radiation."

There are two high-weight data subsets in our analysis: the cosmic chronometer data from Moresco et al. (2012) and the Lyα data from Busca et al. (2013). Since both of these results are based on relatively new approaches to measuring $H(z)$, it is informative to see an analysis of H_0 when one and then the other of these data sets is omitted from the analysis. When we drop the data of Moresco et al. (2012) from the compilation, we find $H_0 = 67.5^{+3.7+8.0}_{-3.7-8.0}$ km s^{-1} Mpc^{-1}, while by dropping the point of Busca et al. (2013) we obtain $H_0 = 66.9^{+2.8+5.3}_{-2.8-5.5}$ km s^{-1} Mpc^{-1}. Comparing these with the full-data result $H_0 = 68.3^{+2.7+5.2}_{-2.6-5.1}$ km s^{-1} Mpc^{-1}, we observe a minor

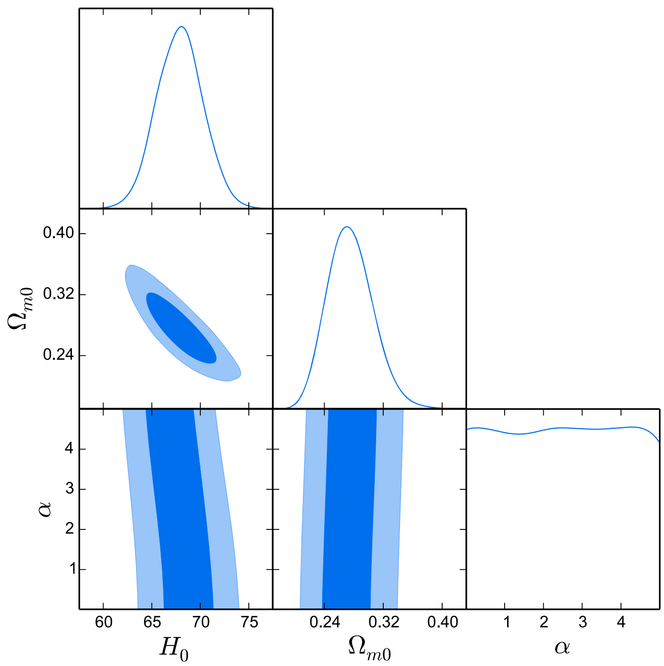

Figure 4. 1σ and 2σ confidence contours of the parameters of the spatially flat ϕCDM model. Marginalized probability distributions of the individual parameters are also displayed.

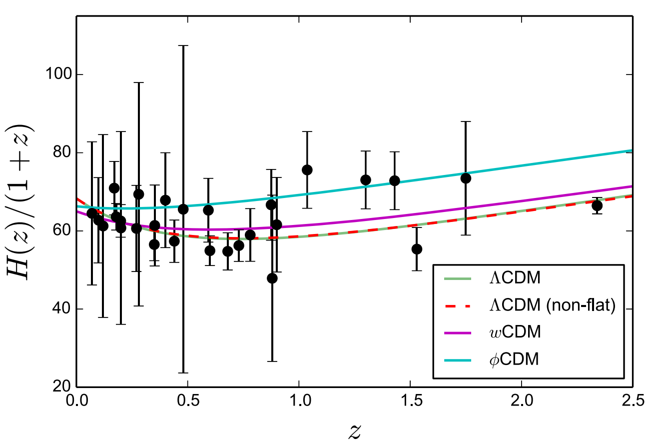

Figure 5. Best-fit model curves and the 28 $H(z)$ data points.

shift in the central values and larger error bars when one or other data subset is omitted from the compilation.

4. CONCLUSIONS

We have used the $H(z)$ data tabulated in Farooq & Ratra (2013b) as reproduced here in Table 1 to measure H_0. The H_0 values we find are more consistent with the lower values determined from the recent CMB and BAO data, as well as with the value found from a median statistics analysis of Huchra's compilation of H_0 measurements.

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