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A REVIEW OF RESEARCH AND LITERATURE' RELATED TO
VERBAL PROBLEM SOLVING' AND APPLICATIONS TO
IMPROVEMENT 'OF STUDENT ABILITIES

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Chapter 1

INTRODUCTION

A major objective of any mathematics teacher today is developing the ability of students to solve verbal problems. There are numerous situations in the classroom in which the problem solving ability is necessary, and as the student leaves school he will be faced with opportunities to use this ability.

A verbal, or word, problem may be defined as a written question raised for inquiry, consideration, discussion, decision or solution. In each verbal problem a situation is described that involves a quantitative question for which the individual has no ready answer.

Developing good problem solving techniques should be a major part of a good mathematics curriculum for several reasons:

1. Learning to solve textbook word problems is important in the study of mathematics because they give the student an exposure to problem situations well beyond those possible through real life experience. At the same time the student is learning to deal with problems in a form he will encounter throughout his education.

2. Verbal problems provide practice in computational skill in a more interesting manner than a list of exercises.

3. Competence in living and working in an age with so great an emphasis on applied science is dependent upon a person's understanding and skill in using problem solving methods in personal, social, and

economic situations.

4. The verbal problem may demonstrate the practical part of mathematics and thus motivate a student to learn the skills necessary for the course he is taking and to retain these skills for further use.

The effectiveness of any education program must be judged by its success in developing the student's ability to function as a responsible citizen. This is also true of the mathematics program, it can be effective only if it develops the ability of the student to solve problems he will encounter in life activities.

The purpose of this report was to examine research studies and literature written concerning verbal problem solving and to relate them to improvement of student abilities.

Three areas of research and literature were examined: (1) student problem solving abilities--examining the relationship between a student's success in problem solving and other characteristics of his thinking and personality, (2) problem content--how the content of the problem is related to a student's ability to solve it, and (3) problem solving processes--studies of the processes used by students to solve verbal problems and how these processes can be improved.

After a review of the research and literature, student difficulties in verbal problem solving were examined to see how they might be corrected using the implications of the research and literature.

Chapter 2

A REVIEW OF RELATED RESEARCH AND LITERATURE

Many research studies have been conducted concerning the ability of students to solve verbal problems. In this report only those studies concerned with problem solving ability, problem content, and problem solving processes were examined. Much research found on the same topic did not agree as some articles completely refuted what another had concluded. Also, many experimenters found no statistical difference between their control and experimental groups. The sampling of research cited in this report is given for its direct application to the mathematics student having difficulty in verbal problem solving.

STUDENT PROBLEM SOLVING ABILITIES

Many factors make up a student's ability in solving verbal problems. The understanding of these factors and how they relate to problem solving can lead a teacher in finding the best method to help students improve their problem solving abilities.

Stevens

In a study conducted by B. A. Stevens¹ the relationships of ability in silent reading, power in the fundamental operations of arithmetic, power in solving reasoning problems in arithmetic, and

¹B. A. Stevens, "Problem Solving in Arithmetic," Journal of Educational Research, 25:253-260, April/May, 1932.

"general intelligence" test scores were compared. A comparison of scores was made from a wide sampling of students in the fourth, fifth, sixth, and seventh grades in the different areas given above. The study used the correlation method in the comparison of the scores. Each score for a particular ability was obtained by a standardized test given to the pupils.

The experiment results showed that ability in fundamental operations in arithmetic was more closely correlated with ability in problem solving than was general reading ability. Test of problem analysis seemed to have higher correlation with the test of problem solving than did tests of general reading ability or of fundamental operations. Although in this test this correlation between problem analysis and problem solving did exist, other studies seem to indicate that special training in problem analysis does not result in higher achievement for normal pupils in problem solving. Stevens felt that this study and others suggest that a large proportion of our pupils may be solving reasoning problems in arithmetic by means of type solutions rather than by means of vital reasoning processes.

Tate and Stanier

A study by Merle W. Tate and Barbra Stanier was concerned with the kinds of errors made by good and poor problem solvers in judgements as measured by critical thinking and practical-judgement tests.² This study used data from a previous study in which 117 good problem solvers

²Merle W. Tate and Barbra Stanier, "Errors in Judgement of Good and Poor Problem Solvers," Journal of Experimental Education, 32:371-376, Summer, 1964.

and 117 poor problem solvers were selected. The median IQ of the good problem solvers was 114.8; that of the poor problem solvers was 113.6. The median chronological age of the good problem solvers was 156 months; that of the poor problem solvers was 154 months.

The test to examine the errors in critical thinking consisted of fourteen paragraphs dealing with science and social studies topics. Each paragraph was followed by a number of statements, 84 statements in all. The subjects were to mark each statement as true, probably true, not enough facts, probably false, and false on the basis of the information contained in the paragraph. They were to use the response not enough facts if there was not enough information in the paragraph to determine the truth or falsity of the statement.

As the authors expected, the test was relatively hard. The mean score of good problem solvers was 38.2 on the 84 items. The mean score of poor problem solvers was 29.9. The difference was significant, the t ratio being 9.01 with 209 degrees of freedom.

Test results showed that, in general, both groups checked as true or probably true statements frequently heard or commonly believed as true, even if there was no evidence to support the statement presented in the paragraph. Results of the test also showed that poor problem solvers tend to judge statements as true or false and avoid the other alternatives more frequently than the good problem solver. The good problem solvers seem to make better use of the information they are given.

The differences in responses of the good and poor problem solvers can perhaps be explained in terms of temperamental rather than cognitive differences. The good problem solver responded with not enough facts

when he did not know the answer rather than the extreme true or false. On the other hand, the poor problem solver would reject not enough facts and respond with the true or false because of impulsiveness or intolerance of ambiguities.

The errors in practical judgement test consisted of 36 questions of the following type:

1. Why is a traffic officer superior to a traffic light?
 - ☐ He can help people cross the street.
 - ☐ He can think.
 - ☐ He can control traffic.
 - ☐ He can arrest a driver who does something wrong.

7. Why are skyscrapers built in large cities?
 - ☐ To save space.
 - ☐ To beautify the city.
 - ☐ To draw tourists.
 - ☐ Because land is expensive.

Subjects were to write 1 before the answer they considered the best and 2 before the next best answer. In scoring, the answers were ranked from 1 (best) to 4 (poorest) on a key. The differences between a subject's number and the key's number were summed and subtracted from 100. The mean score of the good problem solvers was 55.3 and the poor problem solvers was 45.0.

In addition to practical information and evaluation of past experience, success on the practical judgement test seemed to require the ability to delay a response until all of the answers had been read and impartially analyzed. It seemed probable that the differences between the errors of good and poor problem solvers were due to differences in susceptibility to affective answers. The poor problem solvers preformed better on 11 of the 36 questions; actually only 18 questions having answers involving beliefs or feelings of the students elicited definitely better performance from the good problem solvers.

Alexander

Vincent E. Alexander analyzed characteristic differences between high and low achievers in problem solving.³ For the study, pairs of high and low achievers in seventh grade problem solving were matched according to sex, IQ, and mental age in months. These pairs were compared with respect to certain factors and abilities. The measure of problem solving ability was determined by the pupil's score on a standardized test. Only the general conclusions for this study were given. All differences were statistically significant at or above the 5% level of confidence.

The results of the study are given in the following outline:

- A. Characteristics of high achievers
 - 1. Specific mental abilities
 - a. General reasoning ability
 - b. Ability to understand verbal concepts
 - 2. Quantitative skills
 - a. Understanding mathematical terms and concepts
 - b. Skill in computation
 - 3. General reading skills
 - a. Comprehension of reading material
 - b. Understanding words in context
 - 4. Problem solving reading skills
 - a. Comprehension of statements in problems
 - b. Selection of relevant details in problems
 - c. Selection of correct procedures to solve problems
 - 5. Interpretation of quantitative materials
 - a. Finding data from graphs, tables, charts, and maps
 - b. Perception of relationships involving comparison of data
 - c. Recognition of limitations of given data
- B. No significant difference
 - 1. Specific mental abilities
 - a. Ability to use words easily
 - b. Ability to visualize objects in two or three dimension
 - 2. Socio-economic status
 - 3. Quantitative skills
 - a. Timed addition of whole numbers
- C. Characteristics of low achievers

³Vincent E. Alexander, "Seventh Graders Ability to Solve Problems," School Science and Mathematics, 60:603-606, November, 1960.

1. Interpretation of data errors
 - a. Tendency to require more information than necessary to judge data
 - b. Going beyond the data given (finding an answer even when there is not sufficient information)
 - c. Inaccuracy due to carelessness, reading difficulties, or inability to see relationships

From this study, Alexander set up guides for the teacher in planning instruction in problem solving:

A differentiated program--Instruction must be adapted to the needs and abilities of students. Some of the assigned problems should be of low enough difficulty to be solved by all of the students. There should also be some problems difficult enough to challenge the high achiever. "Diagnosis of difficulties and activities to improve problem solving skills, instructional materials, learning experiences, and goals to be achieved should be selected and organized to meet individual variations in problem solving ability."

Selection of printed materials--Materials are to be selected so that the vocabulary and language structure are appropriate to the reading level of the pupils who will be using them.

Improvement of general reading ability--General reading can be improved through specific reading instruction.

Development of reading skills related to problem solving--The pupils are to have instruction in using the meaning of items and statements in verbal problems as clues to computation processes. The pupils need to have an understanding of basic and enriched meanings of mathematical terms and concepts.

Development of mathematical concepts--The pupils need to learn the relationships between quantities and processes.

Skill in fundamental operations--The pupils need the ability to

employ processes with understanding.

Interpretation of quantitative materials--Pupils should be taught to interpret and visualize facts and relationships in charts, tables, graphs, and maps. They should be able to compare data, recognize limitations of given data, and discriminate between relevant and irrelevant data.

Kennedy, Eliot, and Krulee

A test was developed by George Kennedy, John Eliot, and Gilbert Krulee to find the error patterns in the problem solving processes of students. Their experiment had two purposes: (1) to determine if students differ with respect to their solution patterns for algebraic word problems; and (2) to determine how students use the information given to them in problem statements.⁴

Kennedy, Eliot, and Krulee gave the following five steps which students use in solving word problems:

1. The student reads the problem and forms a rough hypothesis.
2. The student looks for information requiring translating into mathematical symbols.
3. The student decides what type of relationships are needed to form an appropriate equation.
4. The student acertains whether he has identified the physical or logical inferences needed to solve the problem.
5. The student is ready to solve the equation and obtain a

⁴George Kennedy, John Eliot, and Gilbert Krulee, "Error Patterns in Problem Solving Formulations," Psychology in the School, 7:98-99, January, 1970.

solution.

The first purpose of the study, to determine if students differ with respect to their solution patterns for algebraic word problems, was divided into two parts: (a) that the difference between less and more able students is a function of their ability to recognize the relationships needed to form an appropriate equation, and (b) that the difference between less and more able students is a function of their ability to add or to identify the logical or physical inferences needed to solve the problem.

The subjects of the study consisted of 28 high school juniors from Evanston Township High School in Evanston, Illinois. All subjects were from the same teacher's average and advanced mathematics classes. The subjects were selected in order to have an equal number of average and advanced students and an equal number of boys and girls. All backgrounds of the students indicated that the majority were from families with professional backgrounds. All subjects took the College Entrance Examination and plan to go to college. Most of the students were planning to enter a professional field. The subjects reflected the upper to middle class families.

The test given consisted of the following six problems:

1. $\frac{3y - 4}{8} = \frac{4y + 8}{4}, \quad y = ?$
2. A man is three times as old as his son. Eleven years from now he will be only twice as old as his son. How old is the son at present?
3. $B(X - B) = X - (2 - B), \quad X = ?$
4. Mary can wash the dishes in a half hour, and Tracy can wash them in 25 minutes. After Mary has worked for 10 minutes, Tracy begins to help her. How long will it take both girls to finish the dishes?

$$\begin{aligned}
 5. \quad A - X^2 &= 4X - 21 \\
 -X - 56 + 9X &= 4X \\
 X &= ?
 \end{aligned}$$

6. An automobile radiator contains 4 gallons of a mixture of water and antifreeze. If the mixture in now is 20% antifreeze, how much of the mixture must be drawn off and replaced by pure antifreeze to get a mixture containing 30% antifreeze?

Each student was taken alone into a soundproof recording room, and he was told that they were interested in the way in which people go about solving algebra problems. He was asked to solve the six problems, putting all his work on paper and saying anything which came to mind no matter how trivial. He was reminded that the interest was in how he worked the problem, not his answer.

After the tests were scored, it was found that the numerical problems offered little difficulty for any subjects. The word problems were more difficult for the less able student. Of problems 4 and 6, there were 15 of 28 correct solutions of honor students but only 5 of 28 for average students.

The first part of the first hypothesis (difference is a function of ability to recognize the relationships needed to form an appropriate equation) was not supported because both groups recognized the relationships needed for equations equally well. However, the second part (difference is a function of ability to add or to identify the logical or physical inferences needed to solve the problem) was supported. When comparing the cumulative frequency for each step of the problem solutions, the greatest discrepancy occurred at the step requiring students to identify logical or physical inferences in the problem statement.

The hypothesis that less able students use facts in the order that they appeared in the problem was partially supported as only some

of the less able students did formulate word problems sequentially as facts appeared.

The results of the study indicate that "teachers should be less concerned with teaching students to define the relationships between problem elements and more concerned with helping them to identify the logical and physical assumptions made in the problem statement." Although the sample size of this study was too small to be reliable, the results can be used to help some students that are having difficulty solving verbal problems.

The preceding research studies were all on topics concerned with some phase of the relation of a student's problem solving ability to the different factors which make up his ability in problem solving. These studies can be very valuable if they are used to correct the difficulties of the less able student in problem solving.

PROBLEM CONTENT

An examination of problem solving must include research studies concerning the content of the problem itself. This would include the vocabulary of the problem, the relation of the problem to the experiences of the student, and the preferences of the student as to the type of problem he likes to solve. The research given was to determine the relation between a problem's content and the student's ability to solve the problem.

Lyda and Church

A study was done by Wesley J. Lyda and Ruby Summers Church to determine the potency of direct, practical mathematical experiences, as

distinguished from those activities encountered only in the classroom, as a factor in solving realistic verbal "reasoning" problems in arithmetic.⁵

The subjects for the experiment were Mrs. Church's fifth grade class of a Negro school in Fort Valley, Georgia. Students were divided into three groups: average, above average, and below average. The average students had an IQ of 90 to 109 on two Group Intelligence Tests, Arithmetical Computation Grade Equivalent of 5.0 to 5.5, and Reading Grade Equivalent of 5.0 to 5.5. The above average students had an IQ of 110 and above, Arithmetical Computation Grade Equivalent of 5.6 and above, and Reading Grade Equivalent of 5.6 and above. The below average students had an IQ of 70 to 89, Arithmetical Computation Grade Equivalent of 3.0 to 5.0, and Reading Grade Equivalent of 3.0 to 5.0. There were 16 average students, 5 above average students, and 9 below average students.

This study was designed in terms of three basic assumptions:

1. "Intelligence, arithmetic computation, reading, and direct, practical experiences in arithmetic, as distinguished from classroom activities, are presumably conditioning factors in success in solving realistic verbal 'reasoning' problems."
2. "Most of the pupils in schools are average in intelligence."
3. "Textbooks are to be written primarily for the average student."

⁵ Wesley J. Lyda and Ruby Summers Church, "Direct Practical Arithmetical Experiences and Success in Solving Realistic Verbal 'Reasoning' Problems in Arithmetic," Journal of Educational Research, 57:530-533, July-August, 1964.

To select the realistic verbal "reasoning" problems to be used in the experiment, the authors' used Brueckner and Grossnickle's Standards for Evaluating Problems and considered verbal "reasoning" problems in each of 11 units of the Row-Peterson text. The researchers selected a random sample of 150 of the most realistic verbal "reasoning" problems. The problems were then submitted to a panel consisting of a college instructor in mathematics who was very interested in education, an instructional supervisor, and a fifth grade teacher. The panel also used Brueckner and Grossnickle's Standards for Evaluating Problems and rated the problems as most realistic, realistic, and least realistic. For a problem to be included in the test it had to be rated most realistic by two of the three judges. The 25 most realistic verbal "reasoning" problems were selected in this way.

An arithmetical experience check list was designed to determine pupil experience based upon and parallel to the situations involved in the group of 25 problems. The pupils were to tell if they had had an experience often--three times or more, seldom--once or twice, or never--not at all. Some examples of the types of experiences are:

1. Selling tickets for a school play
2. Buying milk in the classroom or cafeteria
3. Planning a trip to the museum

The 25 problems were divided into series of five tests with five problems in each test. These tests were given daily for five days; a maximum time of thirty minutes was allowed for each test. Some examples of the problems that relate to the above experiences are:

1. The fifth grade girls sold 235 tickets for their play. The boys sold 309. How many more tickets did the boys sell than the girls? How many tickets did they sell together?

2. The pupils in our room paid \$1.68 for milk today. If the pupils pay 8¢ for each bottle of milk, how many bottles of milk did they buy?
3. The 75 fifth grade pupils are planning a trip to the museum. It will cost \$18.75 to hire a bus to take them. If each pupil pays an equal share, how much should each one pay?

In analyzing the percentage of average, above average, and below average pupils who had certain arithmetical experiences, the experimenters analyzed the data in terms of experiences in which 75% or more of the pupils indicated that they had a given experience often, seldom, or never. Table 1 shows the percent of arithmetical experiences for each group that were marked often, seldom, or never. For example, the table shows that 75% of the above average students marked 40% of the experiences often, 12% of the experiences seldom, and 12% of the experiences never.

Table 1

Arithmetical Experience

Group	Percent marked often	Percent marked seldom	Percent marked never
Above average	40	12	12
Average	28	16	12
Below average	20	12	16

Table 2 shows the data analyzed on the incidence of arithmetical experiences and success in problem solving. Again the data is for 75% or more of the pupils which marked a given experience as often, seldom, or never. The table shows the number of problems marked often, seldom, or never and the percent of students that worked those problems

Table 2
Arithmetical Experiences and Success
in Solving the Problems

Group	Problem experiences marked often			Problem experiences marked seldom			Problem experiences marked never		
	Number	Percent of students working problems correctly from those marked		Number	Percent of students working problems correctly from those marked		Number	Percent of students working problems correctly from those marked	
Above average	10	90% - 7 80% - 1 75% - 3		3	75% - 3		3	100% - 1 75% - 1 60% - 1	
Average	7	90% - 5 79% - 1 75% - 1		4	100% - 4		3	87% - 2 50% - 1	
Below average	5	100% - 3 88% - 1 78% - 1		3	100% - 1 0% - 2		4	43% - 1 0% - 3	

correctly.

In terms of the number of problems solved satisfactorily, it was found that 90% or more of the average group worked 10 or 40% of the problems correctly, 90% or more of the above average group worked 9 or 36% of the problems correctly, and 90% or more of the below average group worked 5 or 20% of the problems correctly.

In terms of the pupils of the study, the authors based the following conclusions on uniformities that occurred throughout the study:

1. All of the students had certain arithmetical experiences in common.
2. Regardless of the group, there were pupils who had never had certain arithmetical experiences.
3. Direct, practical arithmetical experience seems to be a more potent conditioning factor in success in solving realistic verbal "reasoning" problems for those pupils in the average and below average groups than for those pupils in the above average group.
4. The probability of working satisfactorily realistic verbal "reasoning" problems in which the three groups had not had the corresponding direct, practical arithmetical experiences was greater for the average group than for the below average group; likewise, it was greater for the above average than for the average and below average groups respectively.
5. The below average's difficulties are greatly increased when they have not had the direct, practical arithmetical experiences corresponding to realistic verbal "reasoning" problems.

A study was done by Herbert Lloyd Bowman to determine the relation of general intelligence to types of problems best liked and most successfully performed by pupils of the junior high school level.⁶ The test used consisted of 50 problems arranged in two forms of 25 problems each. The test was created to measure the preferences of a student and his performance. The problems constructed consisted of five types: (1) problems based upon adult life activities, (2) problems based upon child life activities, (3) problems whose nature are to be found in the field of science, (4) problems so stated so as to take on the nature of a puzzle, and (5) problems of pure computation only, in which the directions for each procedure were given. There were five problems on each page with one of each type and each problem on a page was of the same difficulty. The student was to solve the five problems of a group and then indicate the one of a group that he liked best.

Three groups were used for the study. Group I of 100 students was made up of the upper quartile of IQ. Group II of 203 students consisted of the middle 50% of IQ. Group III of 110 students was of the lower quartile of IQ. A comparison of percentages of preference and performance of all three groups were made and the following conclusions were reached:

1. There is less variation in both preference and performance by pupils of higher intelligence with respect to types of problems than with lower intelligence.
2. Pupils of lower intelligence prefer problems involving few

⁶ Herbert Lloyd Bowman, "Reported Preference and Performance in Problem Solving According to Intelligence Groups," Journal of Educational Research, 25:295-299, April-May, 1932.

or no complex situations and they perform better on these problems.

3. Pupils of lower intelligence show little preference or ability on problems of a science or puzzle type. This would suggest that such problems should be constructed on a lower ability level when presented to pupils of this kind.

4. Pupils of high ability in problem solving and in intelligence show a comparatively high degree of preference and performance with respect to problems of scientific type. For this level of student, more problems of this type might be used to maintain interest.

5. Problems dealing with child life activities seem to be highly preferred and successfully performed by all pupils of junior high school grades. If arithmetic problems through their solutions offer to children means of meeting real needs they become important factors in the education of the child.

6. Building interest in problem solving is best accomplished by having children work problems which are not too difficult and which represent genuine childhood situations. This is primarily true for those students who are of average or below average ability.

Travers

Kenneth Travers studied the nature of preferences for problem solving situations--under what circumstances they exist and how they relate to problem solving success.⁷ He did this study because he thought it might provide much needed information concerning the nature of pupils'

⁷ Kenneth Travers, "A Test of Pupil Preference for Problem Solving Situations in Junior High School Mathematics," The Journal of Experimental Education, 35:9-18, Summer, 1967.

thought processes as they attempt to solve mathematical problems.

The Test of Choice Behavior in Number Situations was devised in an attempt to identify pupil preferences for problem solving situations. The test involved three situations: (1) mechanical-scientific, (2) social-economic, and (3) abstract. The test was designed to yield a preference score from highest to lowest for each type of situation; a no preference score was possible. Fifteen pairs of word problems based on 5 topics in junior high school mathematics were used. Each pair of problems consisted of two of the three problem situations and all possible pairings of the three situations were made for each of the five topics. The pair of problems involved the same numerical combinations, used similar style of phrasing, employed the same verbal clues, and had comparative levels of difficulty of vocabulary. Each pair of problems had the same answer.

The student was to read each problem and then cross out the one he did not want to work. He then worked the remaining problem giving his solution. His test was scored to determine his preference; his choices were rated 1, 2, and 3 with 1 being his preference. If he had no preference, each situation was given a 2.

The test was also scored on the basis of number of correctly answered problems. A problem was correct if the correct numerical answer was given, or if it was evident from the pupils work that the appropriate mathematical operation had been applied to the right numbers. After the test was scored, two ratios were formed. One ratio was the number of correct answers of the first preference to the number attempted. The other ratio was the number of correct responses of the other two situations to the number attempted.

The experimental sample consisted of 120 male ninth grade mathematics students from each of two high schools in two central Illinois cities. One school was an all boy Catholic school and the other was a public coed school. Students were taken in equal numbers from college preparatory and non-college (general) mathematics classes. Each student was given the Kuder Preference Record-Vocational (KPR-V) and on the basis of his highest score was placed in one of six interest groups: (1) outdoors, mechanical; (2) computational; (3) scientific; (4) clerical; (5) persuasive, social service; and (6) artistic, literary, musical. There were 40 students in each group.

The researcher found that 119 of 240 ranked social-economic first in preference, only 9 of 240 ranked abstract problems first, and 77 of 240 had no preference (2/3 of this group was in college preparatory). These data suggest that highest preference for either the mechanical-scientific or social-economic situation related to some extent to the highest KPR-V interest. The results showed that more abstract problems were chosen by high achievers than low achievers and more by public school students than by parochial school students. More preference to types of problems was shown by the low achiever than by the high achiever.

The problem solving success was analyzed and the number of students with success ratios greater in the preferred situations than in the non-preferred situations did not differ from that which would be expected by chance.

The study showed that low achievers seemed restricted in the types of problem situations they felt they could achieve in. High level students seemed much less restricted. The results from the study implied

that the high achiever could more readily deal with different situations of the same mathematical structure. They were perhaps less dependent than the low achievers upon familiar concrete interpretations of the problems, and better able to use basic mathematical relationships, properties, and rules of inference.

The study showed some preference for problems in which the student has an interest. The test also points to the fact that the problem materials used in the mathematics exercises did make a difference.

The public school teachers were using the UICSM program in algebra and these teachers were trained in the program methods. Even non-UICSM students were taught by these teachers. The parochial school was more traditional. These differences were detected somewhat in the problem preferences of the students.

Scott and Lighthall

Scott and Lighthall developed a study to determine the relationship between the content of the verbal problem and the degree of disadvantage of the pupil.⁸ The main purpose of the study was to explore the possible relationships between high need (HN) and low need (LN) contents on arithmetic problem solving. For the purposes of the experiment LN was defined as food and shelter, HN was defined as love and belongingness, mastery, education, exploration, and travel. Arithmetic problems for the experiment were selected to test the influence of need level content on degree of success in problem solving.

⁸Ralph Scott and Frederick F. Lighthall, "Relationship Between Content, Sex, Grade, and Degree of Disadvantaged in Arithmetic Problem Solving," Journal of School Psychology, 6:61-67, Fall, 1967.

The experimenters predicted that skill in arithmetic problem solving would vary with content level and degree of disadvantage. Advantaged would score better than disadvantaged but this difference would lessen on problems with LN content. Advantaged would do better on HN content problems than on LN content problems and the reverse would be true of the disadvantaged.

The subjects for the experiment were chosen from two Evanston, Illinois schools. There were 132 students and they were divided into three groups. The three groups were Caucasian advantaged (CA), Negro advantaged (NA), and Negro disadvantaged (ND). NA and ND attended the same school. School personnel determined the degree of disadvantage of each pupil. For the students in the CA and NA group food and shelter were consistently assured. ND were not certain that they would be provided with food and clothing. Subjects were selected randomly after the degree of disadvantage was determined. IQ ranges and means for each group are as follows: CA, 84-131, 113.8; NA, 75-133, 100.7; ND, 71-115, 87.8. IQ was not considered in the selection of subjects, but since the IQ's have such a wide range the results cannot be totally reliable.

The authors assumed that the NA group were generally less advantaged than the CA group. All three groups contained equal numbers of boys and girls, and of third and fourth graders. There were eleven students in each of the twelve categories.

The test consisted of 15 items from the New Stanford Arithmetic Test. They had a maximum grade equivalence of 6.6. Each of the 15 items was expanded into a cluster of 4 problems. In each cluster, 2 problems were HN content and 2 were LN content. In HN and LN problems, digits and languages were equated as much as practicable. The test was

divided into 2 forms, A and B, each test consisted of 30 problems, two from each cluster.

The format of the test is illustrated by Cluster 5 with its scoring.

5. Jim has 3 marbles, Bob has 8 and Bill has 9. If they put them all together, how many will there be? (HN)
6. Tom has 4 pieces of candy, Larry has 6 and Roosevelt has 7. If they put them all together, how many will there be? (LN)
35. Jack has 3 candy bars, Bill has 8 and Bob has 9. If they put them all together, how many will there be? (LN)
36. Jim has 4 model airplanes, Melvin has 6 and Joe has 7. If they put them all together, how many will there be? (HN)

The test was given in April of 1965 to groups of approximately 20 students at a time. The test was given by two white, female, college-trained administrators. Form A was given, then a five minute rest, and then form B was given. The subjects were allowed to ask questions if they needed help.

The performance of the three groups was compared by the number of correct responses on LN content items, number of correct responses on HN content items, and the number of total correct responses. The scores of the tests were compared and as predicted the importance of need level progressively declined with the increase in the degree of advantage. These differences, however, were statistically insignificant.

From the results of the test, the researchers concluded that "age-grade effects, general reasoning ability as measured by IQ tests, and/or degree of disadvantage are more important in arithmetic problem solving than the need level represented in item content. A principal component analysis of the data suggested that factors associated with the difficulty and the mathematical content of the items, rather than

the need content, accounted for differences in performance."

The research concerning problem content showed that there is not much relation between the content of a problem and the better student's ability to solve the problem. However, this is not always true of the average and below average student. An effective teacher can help a poorer student to achieve success by giving him problems to solve that he can relate to.

PROBLEM SOLVING PROCESSES

The processes a student uses to solve verbal problems can give the teacher much information on helping the student to improve his abilities. The research studies in this section examine some processes students use and how they relate to the student's problem solving ability and the improvement of his ability.

Russell and Holmes

A study was conducted by Russell and Holmes to compare the effects of practice in various types of reading in algebra with the effects of spending the same amount of time actually solving verbal problems.⁹ The purpose of the experiment was divided into three parts: (1) to compare the effects of more practice in solving problems with the effects of instruction in reading to interpret a problem, reading to recognize algebraic symbolism, algebraic vocabulary, and general mathematical reading; (2) to find the improvement in general reading ability

⁹ David H. Russell and F. Melville Holmes, "An Experimental Comparison of Algebraic Reading Practice and the Solving of Additional Verbal Problems in Tenth Grade Algebra," Mathematics Teacher, 34:347-352, December, 1941.

and algebraic reading ability as a result of the instruction given; and (3) to find out which sex group benefits more in the reading instruction in regard to both algebra ability and reading ability.

The subjects for the experiment were four of the five tenth grade classes of Bedford Road Collegiate, Saskatoon, Saskatchewan. All classes were taught by the same teacher for 24 lesson periods of 35 minutes each during January and February 1940. Materials consisted of solution of verbal problems using simultaneous linear equations. The experimental and control groups were matched on the basis of four tests: (1) mental ages from Laylock Test of Mental Ability, (2) an Algebra Problems Test constructed by the authors, (3) reading scores on the Thorndike-McCall Reading Test, and (4) an Algebra Reading Test. Ninety-nine subjects were used in the experiment.

The daily lesson for both groups was 35 minutes. In the control group, the entire period was used for the solution of verbal problems; no change in classroom procedure was made. In the experimental group, the first 15 minutes was spent using a mimeographed sheet on reading exercises. The first 8 minutes of the period was used to read and answer the exercises. Seven minutes was then allowed for correction and discussion of the exercises. The remaining 20 minutes was spent doing the same problems as the control group insofar as time permitted. No work was allowed outside of school by either group.

A few examples of the exercises and their directions are given.

Reading Exercise 6

Many of the mistakes made in solving algebraic problems are due to the fact that symbols and words commonly used in problems are not understood, or are not properly interpreted in the problem. In each of the following problems the answer(s) to the question is to be underlined. Each question involves some word or symbol which you

should know in order to read problems successfully.

1. In the following list of algebraic terms underline two which are like terms and circle two which are unlike terms.

$$3ab^2, \quad 3ab, \quad 7a, \quad 5ab, \quad 7ab^2$$

2. Which of the following expressions is the "quotient of three times a number and the product of two other numbers"?

$$3a + bc, \quad 3a/bc, \quad 3a(bc)$$

Reading Exercise 22

Following each of the given problems are four answers to the problem, one of which is correct. In each case circle the answer which you would estimate to be most correct. Carefully study the problem before making your estimate but do not solve the problem. Simply mark the answer which seems most reasonable.

- A. How many years must \$850 be invested at 5% simple interest in order to yield \$255?

(1) 20 years (2) 5 years (3) 2 years (4) 12 years

- B. Find two consecutive even numbers whose squares differ by 52.

(1) 52, 54 (2) 5, 8 (3) 100, 102 (4) 16, 14

The procedure was the same for both groups and was one of individual practice. Students were allowed to discuss difficulties among themselves and common difficulties were presented for class discussion. Each type of problem was introduced with an explanation of general methods of attack. The types of problems were: business, number and digit, mixture, measurement, age, and work.

The initial and final tests were the same consisting of 16 problems. The experimental group showed more gain than the control group on algebra reading. The test results showed that there was no reliable difference between reading practice and solving more problems on ability to solve verbal problems. An unreliably small but consistent difference was shown in favor of the group which did more problems. This was as

true of complete solutions as of deriving equations only. A comparison of boy's and girl's scores, showed no reliable differences in them in either group. However, the girls seemed to gain slightly more than the boys in the control group (1/2 question in 16) and boys gained slightly more than girls in the experimental group (1 question in 16).

The authors felt that the "only reliable result is that instruction in the reading of algebra is reliable in itself. This test implies that some more important factor or factors, other than reading ability, exist which influence the ability to solve problems."

The progress of both groups was satisfactory. The interest of students and enjoyment of work seemed greater for the experimental group. Time was controlled in this study, in actual practice extra help in how to read could be given in study periods or at home and then time would not be used in class which could be given to practice in solving verbal problems.

Lueck

An experiment to determine if an emphasis on the fundamental concepts in teaching the writing of equations is worthwhile was done by William R. Lueck.¹⁰

The subjects for the experiment were from four classes of elementary algebra ranging in size from 15 to 25. There were two classes for each group. The experimental group was taught the fundamental concepts for writing equations. After studying algebra for a given time, all students were given a test containing items in algebraic repre-

¹⁰William R. Lueck, "An Experiment in Writing Algebraic Equations," Journal of Educational Research, 39:292-295, October, 1948.

sentation and forming equations from problems. The scores from this test served as a basis for matching the control and experimental group. At the conclusion of the instructional period, all pupils were given a final test. The initial and final scores were then compared and changes noted.

The usual achievement test could not be used for the initial test because they contained too much material that was not directly concerned with forming equations. The experimenter developed a test which was to measure the pupils ability to write equations for given problems. There were 20 items in the test. Before the initial test was given, the pupils were taught eight weeks or until they reached a pre-determined page. This instruction was necessary so they would understand what was meant by writing an equation, and given examples of how to write them. The test was arranged in two parts of approximately equal difficulty and was administered on two separate days. At no time were two students sitting at adjacent desks working the same test. The total testing time was 46 minutes.

The investigator attempted to keep instruction uniform in all classes prior to the test. Class time was divided in half for all classes; first half for instruction and second half for working on assignments with teacher supervision.

After the initial test, the procedure was observed with the control group, but with the two experimental classes the teaching procedure was changed every time a new series of problems was encountered. To this group the instructor explained that "in the statement of an algebra problem is embodied a fundamental fact in varying degrees of obscurity which must be discovered by the pupil before an equation could be written."

Examples were given to show the use of this concept in forming equations. After this instruction, the pupils in the experimental group were required to write a short statement of this fundamental fact at the beginning of each solution to a problem. On a number of occasions the pupils were shown that the same basic idea was contained in several apparently different problems. The instructor occasionally found it necessary to assist the slower students in finding the basic ideas.

The instructional procedure continued until 10 days before the close of the school year. Both groups went through the usual year end review for both problem solving and in the mechanical phases of algebra. In the problem solving phase, 45 problems were given to be solved and discussed that covered the years work.

The final test consisted of 52 problems for which the pupil was only to write the equation. The test questions were of 26 different types found in recently published textbooks with 2 problems of each type. The test was given in 2 parts taking 2 days with a total of 45 minutes testing time. The student was asked to write only the equation so that the test would measure only his ability to write equations rather than his manipulative skills.

The arithmetical mean on the final test for the experimental group was 17.72 and that of the control group was 14.84. Fifty-five percent of the experimental group liked to find and write fundamental ideas. Forty-five percent of this group did not like to write the fundamental ideas or declined to answer. Ninety-five percent of the experimental group said the method of finding and discussing the idea made the writing of equations and solving easier. Superior students in the experimental group showed a greater gain over their match than did the

inferior.

The following conclusions were given by the experimenter:

1. "In teaching the forming of equations in elementary algebra an emphasis on the fundamental concepts required in the process yielded better results than a lack of such emphasis."
2. "The method under consideration proved considerably more effective with the more capable students."
3. "In the opinion of the pupils the procedure used with the experimental group was helpful in solving problems."
4. "Because of the limited size of the samples used in this study, generalizations are hazardous. The differences in performance between the experimental and control groups could be accounted for by chance factors. However, considerable confidence may be placed in the assertion that a real difference exists."

Hawkins

George E. Hawkins made a study to determine the effectiveness of considerable practice devised to help develop problem solving abilities.¹¹ With pupils in one section of an algebra class exercises giving specific practice in translating the English expressions into algebraic symbols and in analyzing problems were used.

Two groups were used for the study and on the basis of standardized tests given no group seemed to have an advantage over the other. The students had had practically no experience in solving verbal problems algebraically other than putting information in formulas. The students

¹¹ George E. Hawkins, "Teaching Verbal Problems in First Year Algebra," School Science and Mathematics, 32:655-660, June, 1932.

had had considerable practice in putting English phrases into algebraic symbols. The students were working on verbal problems in chapters on simple, simultaneous, and quadratic equations.

The control group had only instruction from the teacher and then did assigned problems getting additional help from illustrated examples in the text. The experimental group were assigned the same problems but were given additional practice material especially prepared to give practice in symbolic notation and aid in analyzing the problems. About 10 minutes per day was given on these exercises, 5 minutes for writing answers and 5 minutes for discussion. These pupils seemed to have less difficulty with the assigned problems in the text and they were able to do the same amount of work in the same length of time.

A test was given before each group began work on simple equations and again after they had completed the unit on quadratic equations. This test consisted of 12 verbal problems in which the students were asked to form the equation needed in the solution of the problem, but they were not to solve the equation. On the pretest the average and median scores of both groups were the same, but on the final test the average and median scores of the experimental group were each higher than those of the control group. From this it seemed that the additional practice given one group had a favorable influence on their improvement in ability to form equations in the solution of verbal problems.

Also a test of 17 exercises in translating English expressions into algebraic symbols was given at the end of work on verbal problems. On this test the average and median scores were the same for both groups. Since both groups had sufficient practice to put English into algebraic symbols their scores were the same on this test. The experimental group

did better on the 12 exercises since they had had more practice in analyzing and were specifically trained for this.

"The study as a whole," stated Hawkins, "seems to confirm the belief that considerable practice in problem analysis is an aid to the pupil in solving verbal problems. Exercises in which problems are analyzed or partially analyzed for the pupil distributed through the lists of verbal problems make progress less difficult for the pupil, furnish him with a technique of procedure, are an aid in helping him to attack other problems, and as a result, enable him to solve verbal problems more successfully after the assigned work is completed."

In the preceding three research studies some type of extra practice was given to the students in an effort to increase their problem solving abilities. For some students this practice was not greatly beneficial, however for some students it can be of great help. The teacher must use skill to determine which students should receive extra practice and in what manner. Extra practice given incorrectly could seem more of a punishment than a help.

SUMMARY

This chapter was a review of some of the research that has been done concerning the abilities of students to solve verbal problems. The first section of the chapter was research on problem solving abilities of students. A study by Stevens was reviewed which showed that for some students the ability to analyze problems correctly was a large factor in their ability to solve verbal problems correctly. A study by Tate and Stanier showed that good problem solvers are able to look at verbal problems more objectively than poor problem solvers. The poor problem

solver is more apt to make decisions on the basis of what he believes is true rather than on the implications presented in the problem. A study by Alexander was given which analyzed the characteristic differences between high and low achievers in verbal problem solving. From these differences, Alexander gave suggestions that can be used by teachers as guides in planning problem solving instruction. The last study given in the section was by Kennedy, Eliot, and Krulee. The study examined the processes used by students in solving verbal problems. They found that the student's greatest difficulty was in defining logical and physical assumptions made in the problem.

The second type of research studied was on the content of the verbal problem. Lyda and Church found in their study that some average and below average students were more likely to have success on problems containing experiences they have had. The study by Bowman showed that the poorer student preferred problems involving few or no complex situations. On the other hand, pupils of higher intelligence preferred problems of the scientific type. The study of Travers that was reviewed was also concerned with the pupil's performance and his preference on types of verbal problems. This study showed that poor problem solvers felt restricted in the types of problems they could solve while the better problem solvers were less dependent upon familiar concrete interpretations of the problems. The study reviewed by Scott and Lighthall showed that factors associated with the difficulty and mathematical content of the exercise was more of a factor in the ability to solve problems than was the need content of the problem.

The last type of research reviewed was on the problem solving processes of the student. The study by Russell and Holmes showed that

for some students practice in solving verbal problems was as beneficial as practice in various types of reading in algebra and solving less problems. The study reviewed by Lueck found that having students find the fundamental idea contained in a problem helped them to solve the problem. The last study reviewed was by Hawkins. He concluded that considerable practice in problem analysis does aid the student in solving verbal problems.

Chapter 3

METHODS OF TREATING STUDENT DIFFICULTIES

Student difficulties in verbal problem solving can be classified in three broad categories: (1) inability to determine the meaning of the problem, (2) inability to determine the necessary procedure for solving the problem, and (3) inability to do the computations required to solve the problem. For each student who is having difficulties in solving verbal problems, the causes of his difficulties must be determined and effective techniques be applied to improve the pupil's ability.

Just as no one difficulty is common for all students, no one remedy is right for the same difficulty. Each pupil is an individual; therefore, methods of help may vary for each pupil. Notice must be taken of the extreme variation in abilities, motives, interests, and home environment. After a pupil's difficulty is determined, a method of remedy should be given. If this remedy fails then another should be given. This process must be continued until the student has overcome his particular difficulty. The teacher should never decide a student is "hopeless"; the student needs success.

DETERMINING DIFFICULTIES

Before a teacher can help a student overcome his difficulties, he must know what difficulties the student is having. The student himself might know where his trouble is, but otherwise the teacher must take steps to determine the difficulty.

A method of analyzing difficulties is to have pupils follow a planned sequence of steps in solving verbal problems. His work for each step should be carefully put down on paper. In this way, the teacher can see where the student's trouble first begins on each problem. If the student is consistently having trouble at any particular step, this step would be a good place to start on improving the student's ability.

Another method would be to meet with each student alone to discuss the verbal problems and his reasons for the answers he obtained. Through this conference, if pertinent questions are asked, the teacher can determine the student's difficulty.

PROBLEM MEANING

For some students, the inability to understand the meaning of the verbal problem may be his basic difficulty in solving these problems. The inability to understand the meaning of verbal problems may be due to an inability to read well in any subject. In such cases a planned program of remedial reading should be used. Meanwhile, the student still needs to learn to solve verbal problems. A method of bypassing the reading difficulty is to record the problems to be worked on tape. A type of recorder which the student can easily stop and go back to any problem and one with headsets usually works best. If possible, each student with a reading problem should be able to use the tape alone since a group of students will not all work at the same rate.

Many students who are excellent readers have trouble understanding the meaning of verbal problems. Perhaps this difficulty is due to the pressure of other teachers on students to read faster and learn to skim material. Reading in mathematics is one time skimming is

to be forbidden. Verbal problems are written concisely and each individual word usually has its place and is not to be skimmed over. The meaning of a verbal problem can be obtained only with careful reading. For the student who reads well but still does not understand the meaning, the teacher must impress upon him the necessity for slow and careful reading.

Students who can read most problems with understanding may find a problem for which they can not understand the meaning. This difficulty might be caused by the terminology or vocabulary in the problem. Consider the following verbal problem:

A submarine left a surface ship and cruised due south at a constant rate of 28 knots. If the surface ship started off at the same time on a course due north at a constant rate of 22 knots,¹ in how many hours will the ships be 125 nautical miles apart?

This problem might give a student no difficulty if it were two cars going opposite directions in rates given in miles per hour and a distance apart given in miles, but a student can be so confused over the meaning of knots and nautical miles that he can not understand the problem. The teacher must review each problem in advance to determine if there will be vocabulary problems. Any words which could cause difficulty should be discussed before the students begin their work on an assignment.

Although reading ability is important, the study by Stevens² showed that general reading ability is not as closely correlated to problem solving ability as are other factors. However, a student may

¹Mary P. Dolciani and William Wooton, Modern Algebra, Structure and Method, Book One (rev. ed.; Boston: Houghton Mifflin Co., 1970), p. 188.

²Stevens, loc. cit.

have a good general reading ability but a poor mathematical reading ability. The research by Russell and Holmes³ on the effects of practice in reading in algebra shows that the students as a whole did not solve problems better as a result of their algebra reading practice. The study did show that the practice did improve their ability to read algebraic problems. A student cannot solve a problem unless he is first able to read it with understanding. For the student whose difficulty lies in his inability to read verbal problems, practice in mathematical reading would be most beneficial. Students are practicing verbal problem reading if problems are read orally and discussed in class. The day that a new section of verbal problems are to be assigned, part of the class period could be spent reading the problems and discussing them. If the teacher feels that all students do not need this reading practice, the class could be divided in groups where part are working on their assignment individually and others are discussing problem meanings.

If oral practice is not given in class, when a student asks the usual question "I just don't get this", the teacher should first have the student read the problem aloud. If this reading does not help, the student should then try to restate the problem in his own words. If the student is unable to restate the problem or still cannot grasp the meaning, the teacher should ask leading questions until the student thoroughly understands the situation. Under no circumstances should the teacher tell the student the meaning of the problem. The student must learn to discover processes to find meanings in order to cope with problems outside his mathematics class.

³Russell and Holmes, loc. cit.

For the student who needs extra practice in reading verbal problems, exercises can be developed which do not ask the pupil to solve problems but read and make decisions concerning their reading. One example of this type of exercise would be the exercises developed by Russell and Holmes⁴ for use in their experiment. Another method of mathematical reading practice is to give the student a passage to read that contains quantitative materials. After this passage has been read, the student will then answer a group of questions. Following is an example of this type of exercise:

The propeller of a modern air transport travelling 300 m.p.h. revolves at approximately 2,150 r.p.m. And on a trip between San Francisco and New York the propeller will revolve 1,109,000 times; but before the propeller had reached one billion revolutions 900-odd trips between these two cities would be required.

To revolve 72 billion times, or the equivalent number of dollars in the new (federal) budget, the airplane would have to circle the earth approximately 6,500 times.

Perhaps the example would be even more forceful in the calculation that this same airplane in traveling 300 m.p.h. would require 63 years of constant flying to equal 72 billion revolutions of the propeller.⁵

Pupils answered the following questions by reference to the article:

1. About how many revolutions does the propeller on a modern air transport make per minute?
2. About how fast do these airplanes travel?
3. The number 1,109,000 is the number of times that _____.
4. How many-trips between San Francisco and New York would the airplane make before its propeller revolved 1 billion times?
5. In circling the earth 6,500 times, the propeller would revolve 72 billion times, which is equal to the number of dollars _____.

⁴Russell and Holmes, op. cit., pp. 349-350.

⁵John L. Marks, C. Richard Purdy, and Lucien B. Kinney, Teaching Arithmetic for Understanding (New York: McGraw-Hill Book Co., Inc., 1958), p. 317.

6. How many years would it take the plane to circle the earth 6,500 times?

Marks states that "... solution of any problem depends on mastery of appropriate vocabulary and knowledge of the elements of the situation."⁷ The experiments conducted by Scott and Lighthall,⁸ Lyda and Church,⁹ and Alexander¹⁰ support this statement for some students (especially those who are considered average and below in ability). This view gives insight into the content of problems to be assigned or to the programs devised to prepare a student for certain types of problems.

For a slow student, the teacher must be careful of the difficulty of problems that are assigned. A student who is faced with a task of difficult problems and does not have the ability for this has no chance even if he tries. This student would gain more if given problems which he could successfully solve; if he gains confidence in himself he will be ready to try a harder set of problems.

If a student is having difficulty working problems on a topic, problems which relate to experience which the student has had or to interests he has had may encourage him to try harder to solve them. Problems of this type will have "meaning" to the student. This would be especially true for a general mathematics class. Problems can be found to illustrate the same concept and yet fit the experiences and interests of many different students.

Bowman¹¹ and Travers¹² found in their studies that some students

⁶Ibid.

⁸Scott and Lighthall, loc. cit.

¹⁰Alexander, loc. cit.

¹²Travers, loc. cit.

⁷Ibid., p. 311.

⁹Lyda and Church, loc. cit.

¹¹Bowman, loc. cit.

perform better on those problems for which they have a preference. Perhaps these students do better on those tasks that they like, but perhaps they perform better because they can understand the meaning of those problems which they like. Students cannot go through life doing only what they like, but maybe it wouldn't hurt them to get their choice every now and then. One way of letting them choose the problems they prefer would be to give a large group of verbal problems and allow the students to choose problems they want to work. The group of problems could be concerned with the same topic so even though the students were choosing their own problems they would not lack practice in any concept.

Many students have the ability to understand verbal problems but don't care to try. For these students the method of correction must be a motivational force. Letting the students choose their problems to work would be one possible type of motivation. It was discussed earlier that many students do better when problems concern their experiences; however, many students may be bored with this type of exercise. For these students problems must be put in some other form. They might be more motivated to work problems of a puzzle type or problems written in a humorous form.

Field trips, where applicable, can be a good motivational force. Many topics in general mathematics can be more interesting if the students could see first hand the development of some of the problems they are working. The teacher must be careful in planning field trips to be sure that they intensify the material to be studied.

PROBLEM SOLVING PROCEDURES

After a student understands the problem he is working, he must be

able to apply some type of procedure in working the problem. Perhaps the best way for most students to begin is to have a systematic approach for analyzing and solving a verbal problem.

One series of steps that can be used in verbal problem solving is given by Lucien B. Kinney:¹³

1. Carefully read the problem to understand its meaning
2. Decide what is needed to solve the problem
3. Look for any hidden questions
4. Select a process to use
5. Make an estimate to decide on a reasonable answer
6. Perform computations
7. Check the final answer for its reasonableness by comparing to step 5

Research has shown that no one sequence of steps is any better than others; so, if a sequence is given in the textbook the student is using, it would be practical to use that sequence. A series such as this may not always be necessary; but, for a student who has no idea where to begin, a series can lead him through a problem to a solution. Each student should be encouraged to modify a series of steps for his own personal use. Students should not be forced to follow any given pattern; an intuitive pupil may be stifled in this way.

For many students steps 2 and 3 in the given series may be their first downfall. Some problems give more information than is needed to solve the problem; the student must learn to choose the data that is necessary for solving the problem. In other problems all the needed

¹³Lucien B. Kinney, "Developing Ability to Solve Problems," Mathematics Teacher, 52:290-294, April, 1959.

information is not specifically given; the student must find the hidden question to decide how some of the data can be combined to solve the problem as it is stated. The study by Tate and Stanier¹⁴ showed that poor students may tend to find an answer with the data given even if there is not enough or all the data is not necessary. For pupils that have difficulties on these steps the teacher should give supplementary exercises which concentrate on extra data and insufficient data. Exercises can be written which give numbers that are not needed to solve the problem and the student could be asked to find the unnecessary numbers. Likewise, exercises could be written which do not give all the data that would be required to solve the problem. The student would be asked to tell what must be known in order to solve the problem. With practice, the student can develop skill in omitting extraneous material and deciding what data must be found.

Perhaps the greatest difficulty students have in problem solving is to select the correct process to use in solving verbal problems. Students must be taught to recognize that certain words imply the operation to be used; for example, more or greater imply that addition is to be used. The students must also be taught to recognize the logical or physical inferences needed to solve the problem.¹⁵ The student must have practice in using such relations as $\text{rate} \times \text{time} = \text{distance}$ and $\text{length} \times \text{width} = \text{area of a rectangle}$. If the student does not have this background knowledge he will be unable to find the correct procedure to use in solving a verbal problem involving logical or physical inferences.

¹⁴Tate and Stanier, loc. cit.

¹⁵Kennedy, Eliot, and Krulec, loc. cit.

Inability to find a process to use in solving a verbal problem might be caused by a student's inability to put the English sentences into mathematical sentences. Both Hawkins¹⁶ and Lueck¹⁷ found evidence that practice in writing algebraic equations could help the student solve verbal problems.

One method of practice in translating English words to algebraic notation is to give exercises that deal only with this phase of problem solving. Practice translating algebraic symbols to words also gives practice in the reverse. Students could be given an equation such as $2X + 3 = 15$ and be asked to write a problem to fit it. Katherine O'Brien¹⁸ suggests to aid a student in writing an equation by having him draw a vertical line down his paper and have him write phrases on the left and their algebraic equivalent on the right. When all phrases have been written the student should be able to combine them into an equation. A student can be encouraged to find the correct equation for a verbal problem if he is given credit for writing a correct equation even if his numerical answer is incorrect.

Another aid in finding the correct process is for students to make a table or diagram describing the problem. Consider the following example:

Mr. Jones left Elmsville at 8:00 A.M. one morning and drove to Bond City at a constant rate of 50 miles per hour. At 10:00 A.M. the same day, Sgt. Holliday of the Highway Patrol left Elmsville. Following the same route, he arrived in Bond City at the same time as Mr. Jones. If both men arrived in Bond City at 3:00 P.M., at

¹⁶ Hawkins, loc. cit.

¹⁷ Lueck, loc. cit.

¹⁸ Katherine E. O'Brien, "Problem-Solving," Mathematics Teacher, 49:79-86, February, 1956.

what constant rate had Sgt. Holliday traveled?¹⁹

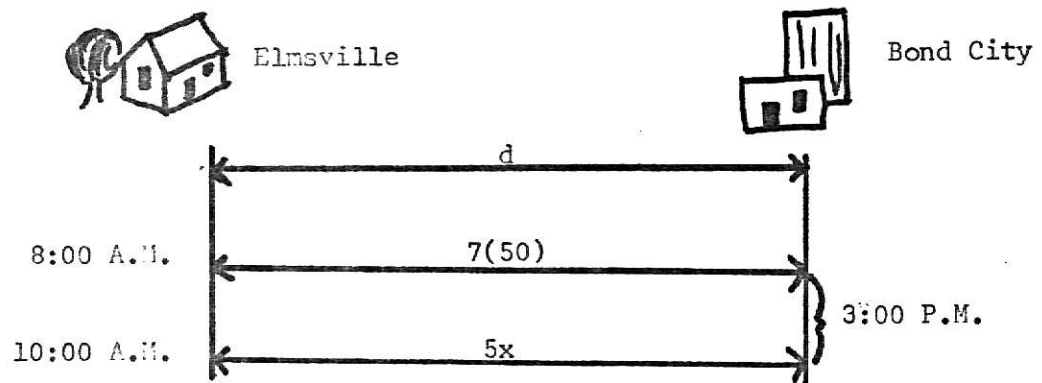


Figure 1

Diagram of a Problem

	r	t	d
Mr. Jones	50	7	350
Sgt. Holliday	x	5	5x

Distance of = Distance of
Mr. Jones Sgt. Holliday

$$350 = 5x$$

Figure 2

Table Illustrating a Problem

Through a graphic scheme of the types illustrated the student can identify the elements of the problem and formulate an explicit statement of their relations to each other. They must be cautioned that there are problems that have information that will not fit in a box. The student must not try to arrange the data to "fit".

¹⁹Dolciani, *op. cit.*, p. 189.

For some students who find verbal problem solving difficult because they cannot define the procedure to use, "how would you find" type questions can help them determine procedures without doing computations. Following is one example of this type of question:

John can ride his bicycle at a constant rate of 3 miles per hour. If he rides to Bill's house which is 9 miles away, how would you find how long it takes John to get to Bill's house?

After reading the problem, the student would describe the process he would use to answer the question.

In some cases when a student is having difficulty determining the procedure to use for a problem that has large numbers or difficult terminology, the problem could be restated using smaller numbers or a simpler problem could be given that uses the same relationships.

No matter what method is used to aid a student in finding the correct procedure, he must not be allowed to rely completely on one method. When he has a problem that does not follow the usual method he should be able to use an alternate method to solve the problem.

Another factor which may help students solve verbal problems is adequate time. Problem solving cannot be rushed. Time must be allowed for the student to develop his abilities in problem solving. Also a teacher must not expect the same amount of work from all students. The above average students should be required to do more problems than the slower student.

The abilities of students to solve verbal problems can be increased if students are allowed to discuss the problems among themselves. A student will understand material better if he can explain it to another student.

COMPUTATION

Although the major emphasis in problem solving is to correctly analyze a problem and write an equation to solve it, no problem is totally correct unless the numerical answer is correct. The problems a student encounters in life outside of school will not want processes but answers. If the difficulty in problem solving of a student is in estimation or computation, then he needs drill problems in these areas. This report will not cite examples for correcting these difficulties since many books can be found which offer exercises on estimating and computation.

SUMMARY

In this chapter the review of research and literature was applied to improvement of student abilities. Methods of determining student difficulties in order to apply proper methods of treatment were given. Methods were discussed which could help the student better understand the meaning of the problem. These methods were for improvement of mathematical reading, content of the problem, and student motivation.

The second type of improvement methods discussed were those involving the procedures used by the students in solving verbal problems. These methods began with the use of a systematic series of steps to be used by the student in analyzing the problem. Methods were then discussed to aid the student in improving his abilities for any step of the series on which he was having difficulty.

Computation was given as a major difficulty of students in solving verbal problems, but no methods of improvement were discussed.

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A REVIEW OF RESEARCH AND LITERATURE RELATED TO
VERBAL PROBLEM SOLVING AND APPLICATIONS TO
IMPROVEMENT OF STUDENT ABILITIES

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AN ABSTRACT OF A MASTER'S REPORT

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Improving the ability of students to solve verbal problems is a major emphasis of mathematics teachers today. This emphasis is necessary for the student's abilities in the mathematics classroom and other classrooms as well as for the life situations he will encounter outside of school. The purpose of this report was to examine research and literature concerning verbal problem solving and to relate them to the improvement of student abilities.

Three areas of research and literature were examined: (1) student problem solving abilities--examining the relationship between a student's success in problem solving and other characteristics of his thinking and personality, (2) problem content--how the content of the problem is related to a student's ability to solve it, and (3) problem solving processes--studies of the processes used by students to solve verbal problems and how these processes could be improved.

The research and literature were used to develop methods for the treatment of student difficulties in three areas. Those areas were: (1) inability to determine the meaning of the problem, (2) inability to determine the necessary procedure for solving the problem, and (3) inability to do the computations required to solve the problem. For each area, difficulties were defined and methods of improvement were discussed.