THE RELATIONSHIP BETWEEN STIFFNESS & STRESSES OF MEMBERS OF RIGID FRAMED STRUCTURES

by

HONG KYUN KIM

B. of Science in Engineering, Seoul National University 1955

A MASTER'S THESIS

Submitted in partial fulfillment of the

requirements for the degree

MASTER OF ARCHITECTURAL ENGINEERING

College of Architecture and Design

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1965

Approved by:

Eugene Larson

Major Professor

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I. INTRODUCTION

This study has been made to determine the relationships between stress and variety in size of members in multi-story rigid-framed structures. By analyzing the stresses of many different frames and investigating the results by statistical methods, the varieties of stress are easily observed and reduced to formulas.

The result of the above (the formula) will be very helpful in analyzing the stress of rigid framed structures. Because engineers face the difficulty of choosing the values of moment of inertia (in subsequent paragraphs the symbol I represents moment of inertia) which are determined by the size of members.

In general, stress analysis is processed with trial I values which are chosen from experience. If the size of members are out of the adjustable range of reinforcing steel, re-calculation should be made based on the trial stress analysis. Therefore, it is desirable to avoid the complexities and discrepancies, as mentioned above, in calculating the stress of rigid framed structures. There may be many different ways to investigate this problem based on the load situation and the shape of the structure. The following study is mainly concerned with gravity loading situations of up to three or four story buildings with different story heights and span lengths. In buildings of up to three or four stories, in most cases, wind load does not govern the design since increased allowable unit stresses are used for computation. Therefore, it appears that the slop deflection method is in such cases most suitable for theoretical analysis.

Of the following 15 cases, a typical case (1) has been thoroughly analyzed; in the remaining instances, only the results of moment diagrams are mentioned. As a typical case, a three story, five bay frame, symmetrical at the center line of the mid-bay and which had a perpendicular span length of 20 feet, was chosen. This is shown in Figure 1-1. For loading conditions 30 lb./sq.ft. for snow load, and 60 lb./sq.ft. for live load was selected.

For the purpose of minimization of the error between the trial section and the final section, maximum beam moment is assumed to be $1/10 \ wL^2$. This value was taken from ACI Code Section 904 (C). In the case of a simple beam, maximum bending moment occurs at the center line and the value is $M_o = wL^2/8$. Therefore $1/10 \ wL^2$ is said to be $8/10 \ M_o$.

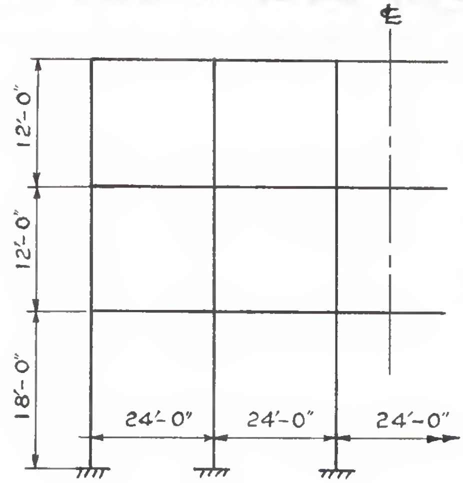


Fig. 1-1

The slab load is distributed at 45 degrees at the corner of intersecting beams and M_o becomes $M_o = w/24\ (3L^2-4a^2)$. Therefore

$$M_{neg.\ max.} = 8/10 \times w/24\ (3L^2-4a^2)$$
$$= \frac{w}{30}\ (3L^2-4a^2)$$

II. PRELIMINARY CALCULATION

A. Floor Area Carried By Beams

In this type of structure, the floor lead carried by a beam is seen in Fig. 2-1. Total load is the sum of the live load and the dead load. The sum is assumed to be 190 lb. per sq. ft. for floor beams and 160 lb. per sq. ft. for roof beams. Is is convenient to convert the dead load of the beam to an equally distributed floor load by dividing the entire floor area which the beam carries.

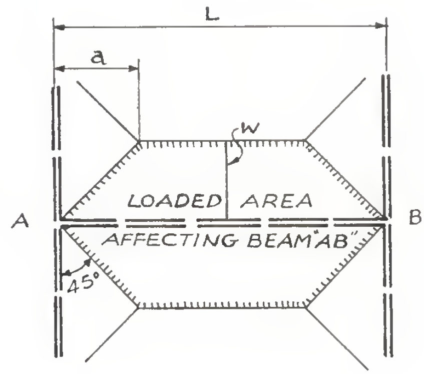


Fig. 2-1

B. Selection of the Size of Beams and Columns

In a practical sense, the size of beams used is consistant through out each floor, the size of columns being determined by the combined axial and flexural stresses. By considering the fact that the interior column is not affected very much by the bending moment induced by gravity load, and the exterior column, which usually has half of the interior gravity load, is much affected by the bending moment, one sees that the size of the column becomes nearly the same through out each floor. Thus, in this study, the size of the interior columns is usually used as the standard for each floor.

1. Size of Beam

$M_{max.} = w/30\ (3L^2 - 4a^2)$

$= 3.6/30\ \ 3(24)^2 - 4(10)^2 = 159.5$ ft.-kips for floor beams

$M_{max.} = 3.0/30\ \ 3(24)^2 - 4(10)^2 = 132.8$ ft.-kips for roof beams

Therefore if we assume b = 14", then

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{159.5 \times 12 \times 1000}{236 \times 14}} = \sqrt{578} = 24.1"$$

including fire-protection and stirrups,

say D = 28" for floor beams.

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{132.8 \times 12 \times 1000}{236 \times 14}} = \sqrt{482} = 21.9$$

say D = 24" for roof beams.

2. Size of Column

Column Load (Interior Col.)

${}_3P_c = (20 \times 24) \times 150 = 72.0$ kips

${}_2P_c = (20 \times 24) \times 180 + 72.0 = 72.0$ kips

${}_1P_c = (20 \times 24) \times 180 + 158.4 = 244.8$ kips

As a trial section for the third floor column, if one chooses 12" x 12" with 4-#9 bars. Then

$A_g = 12" \times 12" = 144$ sq. in.

$P_g = 4/144 = 0.0278$

$A_t = (n-1)\ A_s + A_g = 36 + 144 = 180$ sq. in.

$f_c = P/A_t = 72{,}000/180 = 400$ psi

$P_{all.} = 0.8 \times 144\ (0.225 \times 3000 + 20{,}000 \times 0.0278)$

$= 142$ kips

$F_{c\ all.} = 142/180 = 790$ psi therefore

$f_c/F_c = 400/790 = 0.507$ which is large enough.

C. Moment of Inertia and Stiffness Factors

In preliminary calculations, the effect of reinforcement is not considered in computing the moment of inertia since the exact amount of reinforcing steel is not determined until the stress analysis is completely finished.

It will be helpful to make a table and a diagram of stiffness ratios as shown in Figure 2-2.

TABLE 2-1

	b	D	I	L	K	k
$3^B1,2,3$	14	24	16,150	288	56.1	0.63
$1,2^B1,2,3$	14	28	25,600	288	89.0	1
$3^C1,2,3$	12	12	1,730	144	12.0	0.14
$2^C1,2,3$	14	14	3,200	144	22.2	0.25
$1^C1,2,3$	18	18	8,750	216	40.5	0.46

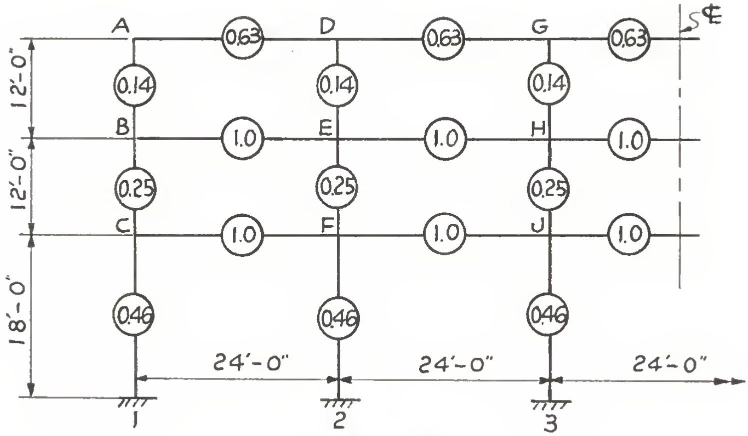


Fig. 2-2

In analyzing the stresses by the slope deflection method, it will be convenient to compute the value of twice the sum of the stiffness ratio at each joint. Special care should be taken in computing the value of 2 k at the joints G, H, and K in Fig. (2-2). For instance at joint G, $\phi G = -\phi G'$ since it is a symmetrical deflection. Therefore, $MGG' = k(2\phi G + \phi G')$ becomes $MGG' = k(2\phi G - \phi G) = k(\phi G) = 1/2\ k(2\phi G)$. Thus 1/2 k is used for 2 kG, 2 kH and 2 kJ.

$$2k_a = 2(0.63 + 0.14) = 1.54$$
$$2k_b = 2(0.14 + 0.25 + 1.0) = 2.78$$
$$2k_c = 2(0.25 + 0.46 + 1.0) = 3.42$$
$$2k_d = 2(0.63 + 0.14 + 0.63) = 2.80$$
$$2k_e = 2(0.14 + 0.25 + 1.0 + 1.0) = 4.78$$
$$2k_f = 2(0.25 + 0.46 + 1.0 + 1.0) = 5.42$$
$$2k_g = 2(0.14 + 0.63 + 0.63/2) = 2.17$$
$$2k_h = 2(0.14 + 0.25 + 1.0 + 1.0/2) = 3.78$$
$$2k_j = 2(0.25 + 0.46 + 1.0 + 1.0/2) = 4.42$$

D. Joint Equations

In any joint, the sum of the bending moment of members at a joint will be zero. Thus following equations are established.

$$M_{ab} + M_{ad} = 0$$
$$M_{ba} + M_{bc} + M_{be} = 0$$
$$M_{cb} + M_{cl} + M_{cf} = 0$$
$$M_{da} + M_{dg} + M_{de} = 0$$
$$M_{eb} + M_{eh} + M_{ef} + M_{ed} = 0$$
$$M_{fc} + M_{fj} + M_{f2} + M_{fe} = 0$$

$M_{gd} + M_{gg'} + M_{gh} = 0$

$M_{he} + M_{hh'} + M_{hg} + M_{hj} = 0$

$M_{jf} + M_{jj'} + M_{jh} + M_{j3} = 0$

E. <u>Mechanical Tabulations</u>

In the slope deflection method, the general form of bending moment at each end of 2 member "A-B" is described as

$$M_{ab} = 2EK\ (2\theta_a + \theta_b - 3R) + M_{ab}^{fix.} \qquad (2\text{-}5\text{-}1)$$

$$M_{ba} = 2EK\ (2\theta_b + \theta_a - 3R) + M_{ba}^{fix.} \qquad (2\text{-}5\text{-}2)$$

If put $\phi = 2EK_o\ (\theta)$

$\psi = 2EK_o\ (-3R)$ then

$$M_{ab} = k\ (2\phi_a + \phi_b + \psi) + M_{ab}^{fix.} \qquad (2\text{-}5\text{-}3)$$

$$M_{ba} = k\ (2\phi_b + \phi_a + \psi) + M_{ba}^{fix.} \qquad (2\text{-}5\text{-}4)$$

K_o denotes a randomly selected standard stiffness factor and k is the stiffness ratio of the member A-B for the standard stiffness factor K_o.

If the joint is fixed and no rotation is allowed, then $R = 0,\ \psi = 0$

$$M_{ab} = k\ (2\phi_a + \phi_b) + M_{ab}^{fix.} \qquad (2\text{-}5\text{-}5)$$

$$M_{ba} = k\ (2\phi_b + \phi_a) + M_{ba}^{fix.} \qquad (2\text{-}5\text{-}6)$$

Each term of joint equation is described on the following page.

$$M_{ab} = 0.14\ (2\phi_a + \phi_b)$$
$$M_{ad} = 0.63\ (2\phi_a + \phi_d) - M_{ad}^{fix.}$$

$$M_{ba} = 0.14\ (2\phi_b + \phi_a)$$
$$M_{be} = 1.0\ (2\phi_b + \phi_e) - M_{be}^{fix.}$$

$$M_{bc} = 0.25\ (2\phi_b + \phi_c)$$

$$M_{cb} = 0.25\ (2\phi_c + \phi_b)$$
$$M_{cf} = 1.0\ (2\phi_c + \phi_f) - M_{cf}^{fix.}$$
$$M_{c1} = 0.46\ (2\phi_c)$$
$$M_{1c} = 0.46\ (\phi_c)$$

$$M_{da} = 0.63\ (2\phi_d + \phi_a) + M_{ad}^{fix.}$$

$$M_{dg} = 0.63\ (2\phi_d + \phi_g) - M_{dg}^{fix.}$$

$$M_{de} = 0.14\ (2\phi_e + \phi_e)$$

$$M_{ed} = 0.14\ (2\phi_e + \phi_d)$$

$$M_{eb} = 1.0\ (2\phi_e + \phi_b) = M_{be}^{fix.}$$

$$M_{eh} = 1.0\ (2\phi_e + \phi_h)$$
$$M_{ef} = 0.25\ (2\phi_e + \phi_f)$$

$$M_{fe} = 0.25\ (2\phi_f + \phi_e)$$
$$M_{fc} = 1.0\ (2\phi_f + \phi_c) + M_{cf}^{fix.}$$

$$M_{fj} = 1.0\ (2\phi_f + \phi_j) - M_{fj}^{fix.}$$
$$M_{f2} = 0.46\ (2\phi_f)$$
$$M_{2f} = 0.46\ (\phi_f)$$

$$M_{gd} = 0.63\ (2\phi_g + \phi_d) + M_{dg}^{fix.}$$
$$M_{gh} = 0.14\ (2\phi_g + \phi_h)$$
$$M_{gg'} = 0.63/2\ (2\phi_g) - M_{gg'}^{fix.}$$

$$M_{hg} = 0.14\ (2\phi_h + \phi_g)$$
$$M_{he} = 1.0\ (2\phi_h + \phi_e) + M_{eh}^{fix.}$$

$$M_{hh'} = 1.0/2(2\phi_h) - M_{hh'}^{fix.}$$
$$M_{hj} = 0.25\ (2\phi_h + \phi_j)$$

$$M_{jh} = 0.25\ (2\phi_j + \phi_h)$$
$$M_{jf} = 1.0\ (2\phi_j + \phi_f) + M_{fj}^{fix.}$$

$$M_{jj'} = 1.0/2\ (2\phi_j - M_{jj'}^{fix.}$$
$$M_{j3} = 0.46\ (2\phi_j)$$
$$M_{3j} = 0.46\ (\phi_j)$$

Unknown factors such as $\emptyset_a$, $\emptyset_b$ .... etc. are dealt with by mechanical tabulation of simultaneous equations as described on page 8.

Fixed end moment $M^{fix.}$ is computed and substituted into the equations on page 8.

1. <u>For Roof Beams</u>

$$M^{fix.} = w/12L \quad (L^3 - 2a^2L + a^3)$$

$$= \frac{(160x20)}{12 \times 24} \quad (24)^3 - 2(10)^2(24)+(10)^3 = 111^{ft.-kipa.}$$

$$M_o = w/24 \cdot (3L^2-4a^2)$$

$$= \frac{160 \times 20}{24} \quad 3(24)^2-4(10)^2 \quad = 178^{ft.-kips}$$

2. <u>For Floor Beams</u>

$$M^{fix.} = \frac{(190 \times 20)}{12x24} \quad (24)^3-2(10)^2(24)+(10)^3 = 132^{ft.-kips.}$$

$$M_o = \frac{190x20}{24} \quad 3(24)^2-4(10)^2 = 212^{ft.-kips.}$$

TABLE 2 - 2

	Øa	Øb	Øc	Ød	Øe	Øf	Øg	Øh	Øj	C
(1)	1.54	0.14		0.63						+ 111
(2)	0.14	2.78	0.25		1.0					+ 132
(3)		0.25	3.42			1.0				+ 132
(4)	0.63			2.80	0.14		0.63			0
(5)		1.0		0.14	4.78	0.25		1.0		0
(6)			1.0		0.25	5.42			1.0	0
(7)				0.63			2.17	0.14		0
(8)					1.0		0.14	3.78	0.25	0
(9)						1.0		0.25	4.42	0
	+72.1	+43.8	+35.4	-16.2	-8.70	-6.12	+4.70	+2.12	+1.26	
	+74.8	+43.8	+37.3	-17.4	-8.78	-6.72	+4.92	+2.05	+1.43	
	+75.3	+43.5	+37.4	-17.6	-8.68	-6.77	+4.98	+2.01	+1.42	
	+75.3	+43.5	+37.4	-17.6	-8.65	-6.77	+5.0	+2.0	+1.42	

$$
\begin{aligned}
M_{ab} &= 0.14\ (2\times 75.3+43.5) = +27.2 \\
M_{ad} &= 0.63\ (2\times 75.3-17.6)-111 = -27.2
\end{aligned}
$$

$$
\begin{aligned}
M_{ba} &= 0.14\ (2\times 43.5+75.3) = 22.7 \\
M_{be} &= 1.0\ (2\times 43.5-8.65)-132 = 53.65 \\
M_{bc} &= 0.25\ (2\times 43.5+37.4) = 31.1
\end{aligned}
$$

$$
\begin{aligned}
M_{cb} &= 0.25\ (2\times 37.4+43.5) = 29.57 \\
M_{cf} &= 1.0\ (2\times 37.4-6.67)-132 = -63.87 \\
M_{c1} &= 0.46\ (2\times 37.4) = 34.3 \\
M_{1c} &= 0.46 \times 37.4 = 17.2
\end{aligned}
$$

$$
\begin{aligned}
M_{da} &= 0.63\ (2\times -17.6=75.3) + 111 = +136.25 \\
M_{dg} &= 0.63\ (2\times -17.6+ 5.0) - 111 = +130.1 \\
M_{de} &= 0.14\ (2\times -17.6-8.65) = -\ 6.15
\end{aligned}
$$

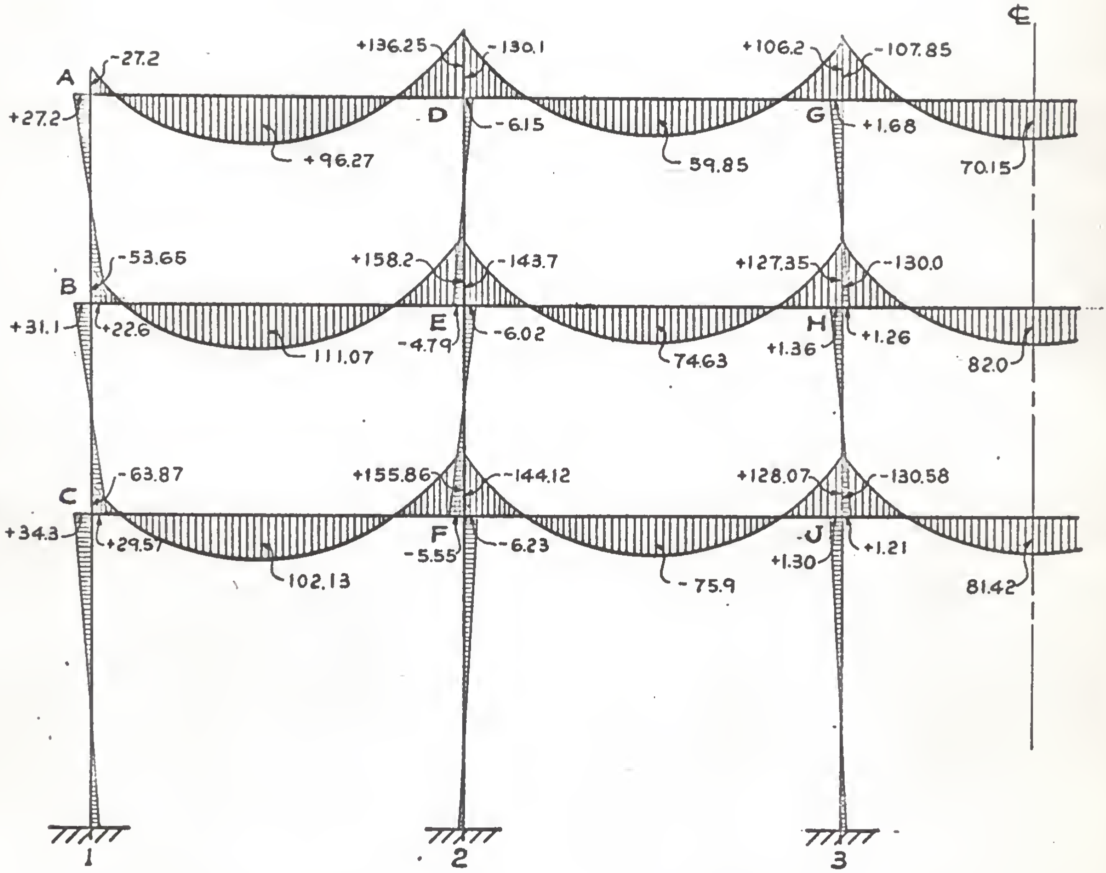
$$
\begin{aligned}
M_{ed} &= 0.14\ (2\times -8.65-17.6) = -4.79 \\
M_{eb} &= 1.0\ (2\times -8.65+43.5) +132 = +\ 158.2 \\
M_{eh} &= 1.0\ (2\times -8.65+2.0) - 132 = -\ 147.3 \\
M_{ef} &= 0.25\ (2\times -8.65-6.77) = -\ 6.02
\end{aligned}
$$

$$
\begin{aligned}
M_{fe} &= 0.25\ (2\times -6.77-8.65) = -5.55 \\
M_{fc} &= 1.0\ (2\times -6.77+37.4) + 132 = +\ 155.86 \\
M_{fj} &= 1.0\ (2\times -6.77+1.42) - 132 = -\ 144.12 \\
M_{f2} &= 0.46\ (2\times -6.77) = -\ 6.23 \\
M_{2f} &= 0.46\ (-6.77) = 3.12
\end{aligned}
$$

$$
\begin{aligned}
M_{gd} &= 0.63\ (2\times 5.0-17.6)+ 111 = +106.2 \\
M_{gh} &= 0.14\ (2\times 5.0+2.0) = +1.68 \\
M_{gg'} &= 0.63/2(2\times 5.0) -111 = -107.85
\end{aligned}
$$

$$
\begin{aligned}
M_{hg} &= 0.14\ (2\times 2.0+5.0) = +1.26 \\
M_{he} &= 1.0\ (2\times 2.0-8.65) +132 = +127.35 \\
M_{hh'} &= 1.0/2\ (2\times 2.0) -132 = -130 \\
M_{hj} &= 0.25\ (2\times 2.0+1.42) = +1.36
\end{aligned}
$$

$$
\begin{aligned}
M_{jh} &= 0.25\ (2\times 1.42+2.0) = +1.21 \\
M_{jf} &= 1.0\ (2\times 1.42-6.77) +132 = +128.07 \\
M_{jj'} &= 1.0/2\ (2\times 1.42) -132 = -130.58 \\
M_{j3} &= 0.46\ (2\times 1.42) = +1.30 \\
M_{3j} &= 0.42(1.42) = +0.65
\end{aligned}
$$



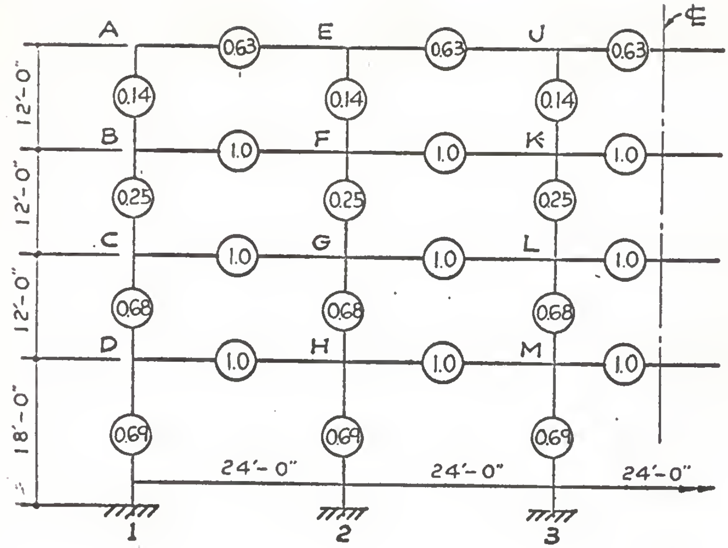
℄
A
-27.2
+27.2
+136.25
-130.1
+106.2
-107.85
D
-6.15
G
+1.68
+96.27
59.85
70.15
B
-53.66
+31.1
+22.6
+158.2
-143.7
+127.35
-130.0
E
-4.79
-6.02
H
+1.36
+1.26
111.07
74.63
82.0
C
-63.87
+34.3
+29.57
+155.86
-144.12
+128.07
-130.58
F
-5.55
-6.23
J
+1.30
+1.21
102.13
-75.9
81.42
1
2
3

Fig. 2-3

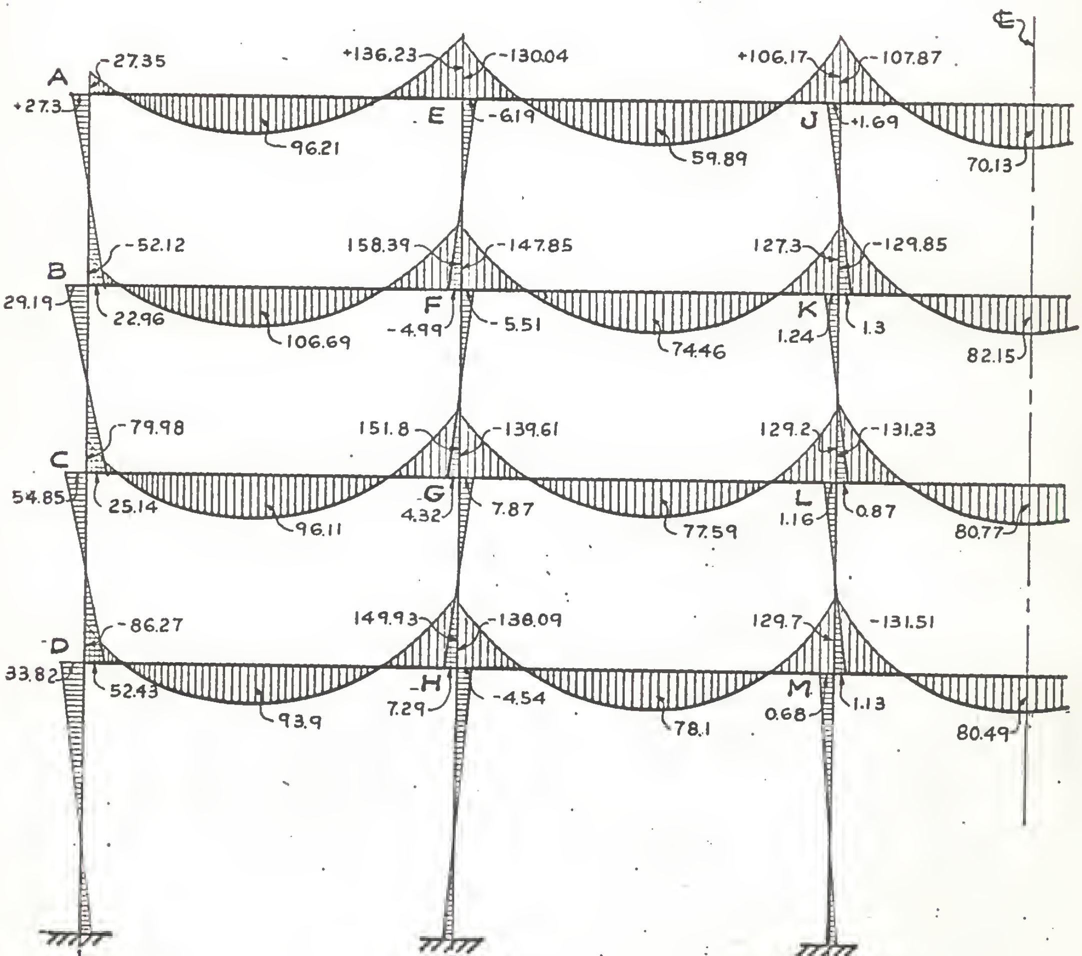
In the following fourteen cases, analysis is done by the same procedure, results are shown on pages 14 - 27.

In these calculations, stiffness ratios of columns are selected after considering the varying axial column loads.

With these results, a relationship will be sought between bending moment, at each joint and at mid-span, with value of simple beam moment in a similar situation.

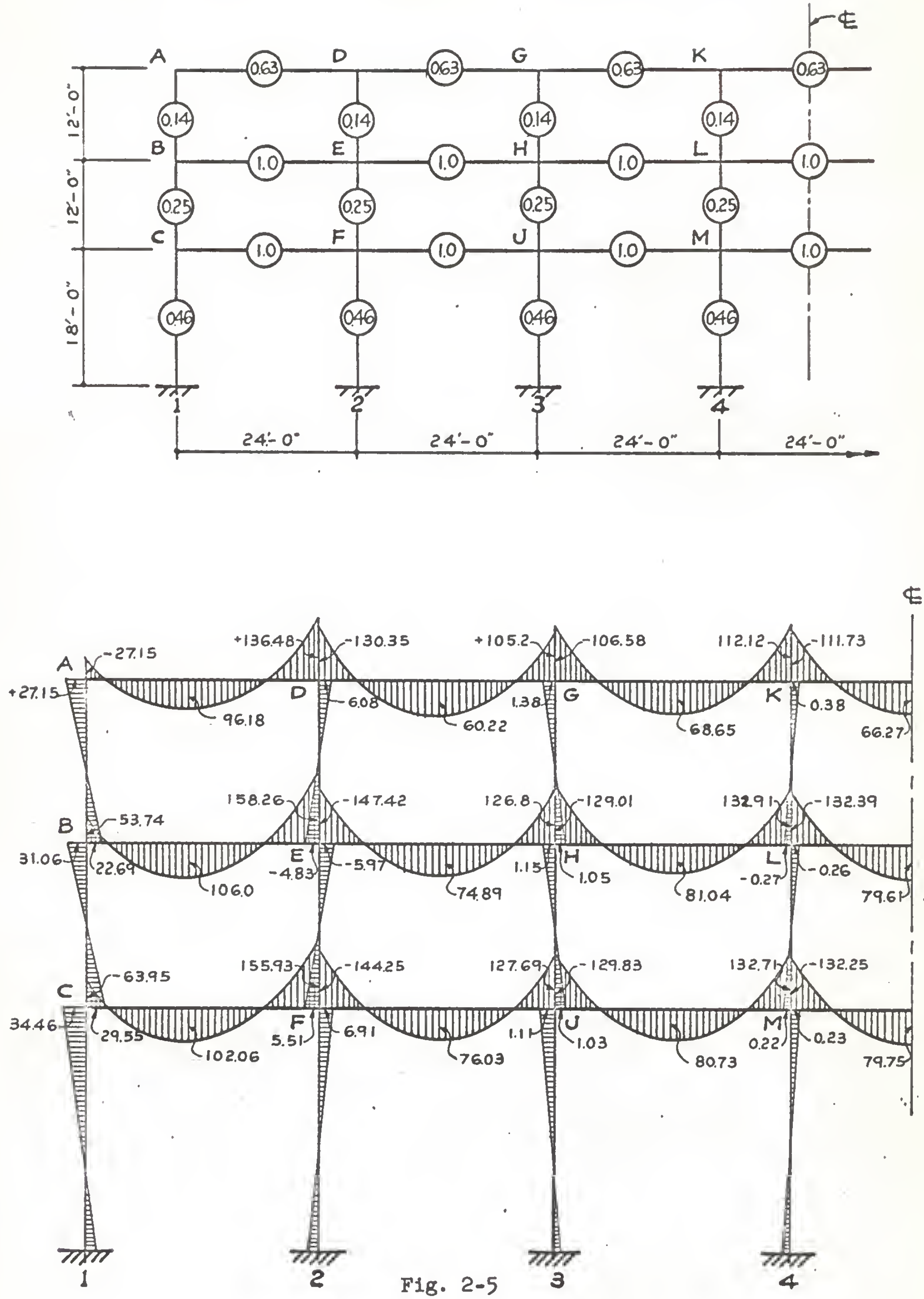


A
E
J
B
F
K
C
G
L
D
H
M
0.63
0.14
1.0
0.25
0.68
0.69
12'-0"
12'-0"
12'-0"
18'-0"
24'-0"
24'-0"
24'-0"
1
2
3



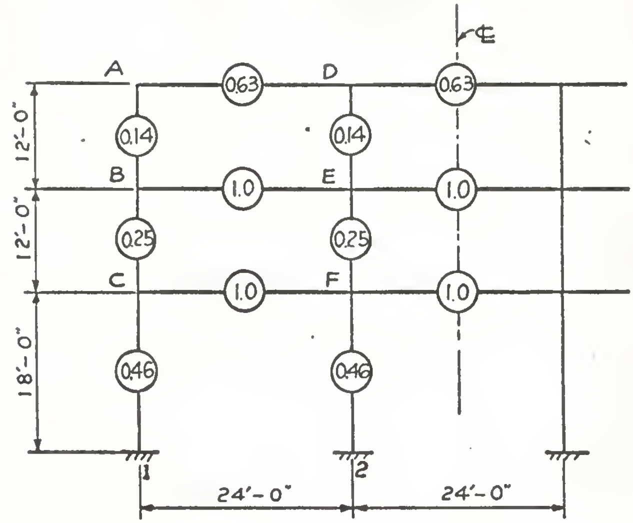
A
+27.3
-27.35
96.21
+136.23
-130.04
E
-6.19
59.89
+106.17
-107.87
J
+1.69
70.13
B
29.19
-52.12
22.96
106.69
158.39
-147.85
F
-4.99
-5.51
74.46
127.3
-129.85
K
1.24
1.3
82.15
C
54.85
-79.98
25.14
96.11
151.8
-139.61
G
4.32
7.87
77.59
129.2
-131.23
L
1.16
0.87
80.77
D
33.82
-86.27
52.43
93.9
149.93
-138.09
H
7.29
-4.54
78.1
129.7
-131.51
M
0.68
1.13
80.49
1
2
3

Fig. 2-4



A
D
G
K
B
E
H
L
C
F
J
M
0.63
0.14
1.0
0.25
0.46
12'-0"
12'-0"
18'-0"
24'-0"
1
2
3
4
+27.15
-27.15
96.18
+136.48
-130.35
6.08
60.22
+105.2
-106.58
1.38
68.65
112.12
-111.73
0.38
66.27
31.06
53.74
22.69
106.0
158.26
-147.42
-4.83
-5.97
74.89
126.8
-129.01
1.15
1.05
81.04
132.91
-132.39
-0.27
-0.26
79.61
34.46
63.95
29.55
102.06
155.93
-144.25
5.51
6.91
76.03
127.69
-129.83
1.11
1.03
80.73
132.71
-132.25
0.22
0.23
79.75

Fig. 2-5


℄
A
D
0.63
0.63
0.14
0.14
B
E
1.0
1.0
0.25
0.25
C
F
1.0
1.0
0.46
0.46
12'-0"
12'-0"
18'-0"
1
2
24'-0"
24'-0"

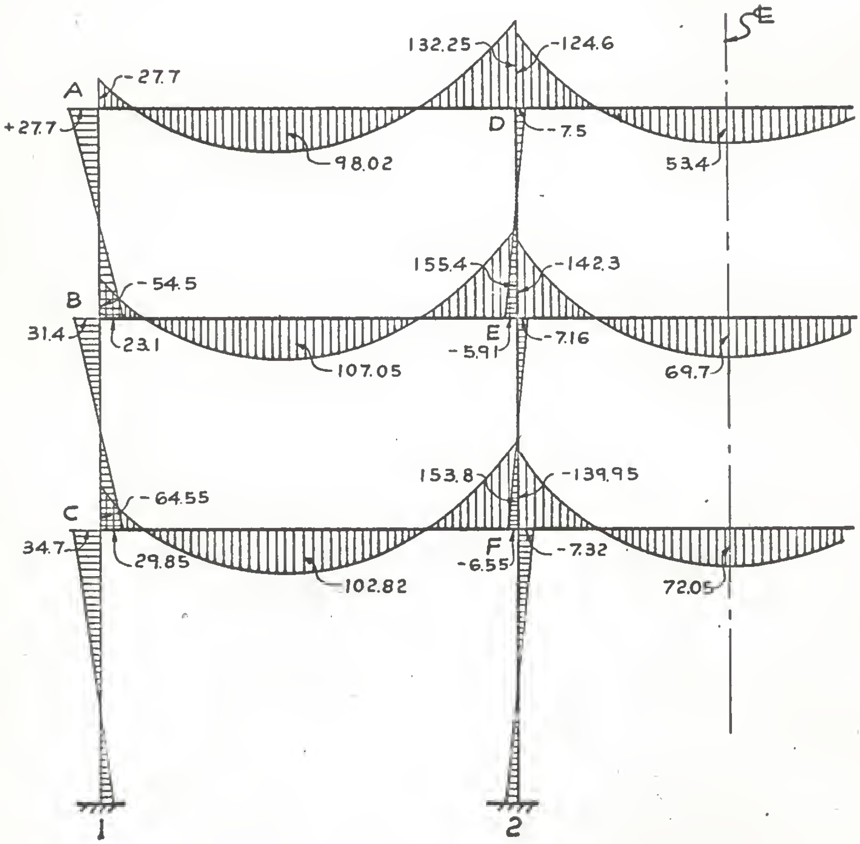
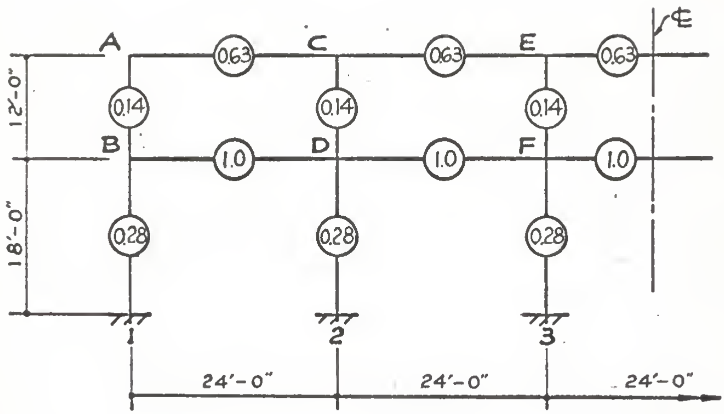

℄
132.25
-124.6
A
-27.7
+27.7
D
-7.5
98.02
53.4
155.4
-142.3
-54.5
B
31.4
23.1
E
-7.16
-5.91
107.05
69.7
153.8
-139.95
-64.55
C
34.7
29.85
F
-7.32
-6.55
102.82
72.05
1
2

Fig. 2-6


A
C
E
B
D
F
℄
0.63
0.63
0.63
0.14
0.14
0.14
1.0
1.0
1.0
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0.28
0.28
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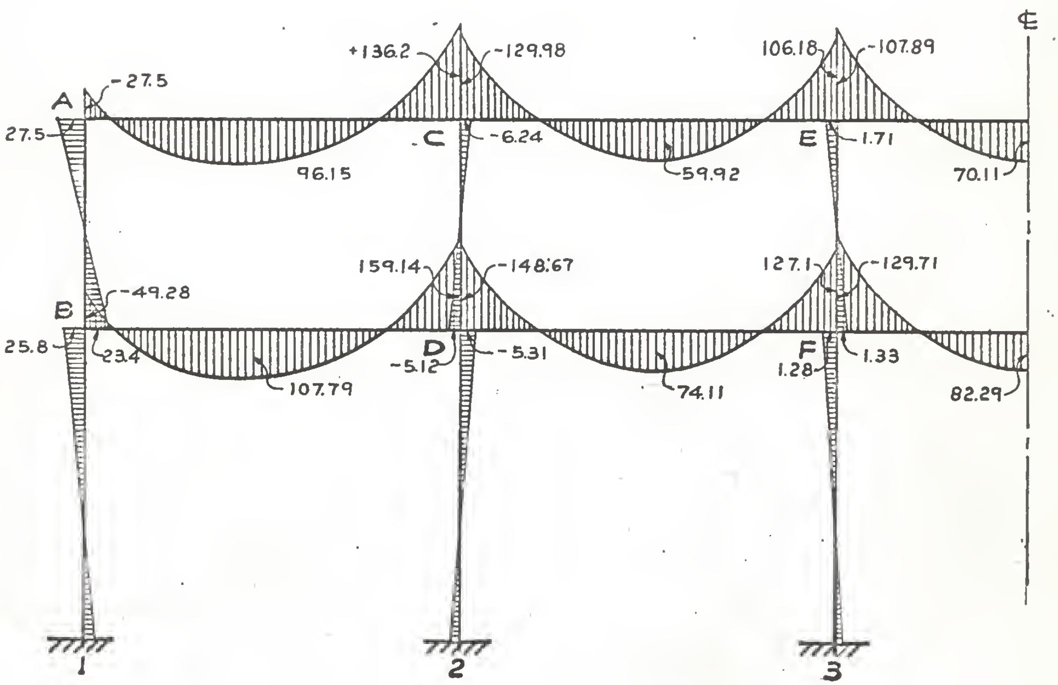
18'-0"
1
2
3
24'-0"
24'-0"
24'-0"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C
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E
1.71
70.11
159.14
-148.67
127.1
-129.71
-49.28
B
25.8
23.4
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D
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-5.31
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F
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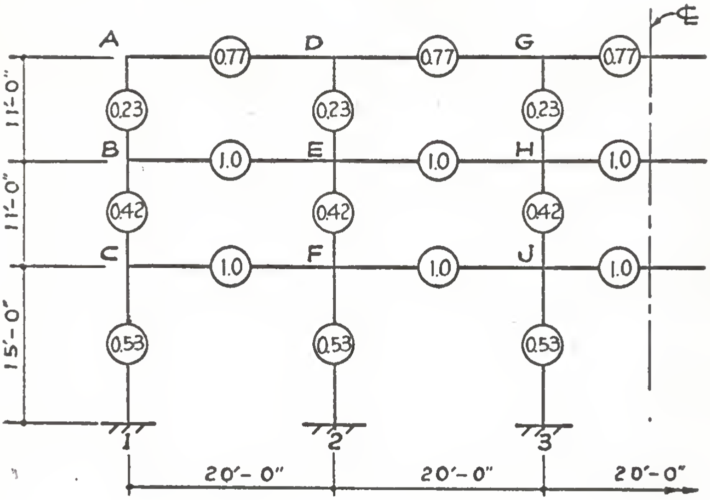
82.29
1
2
3

Fig. 2-7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D
G
℄
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0.23
B
E
H
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0.42
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F
J
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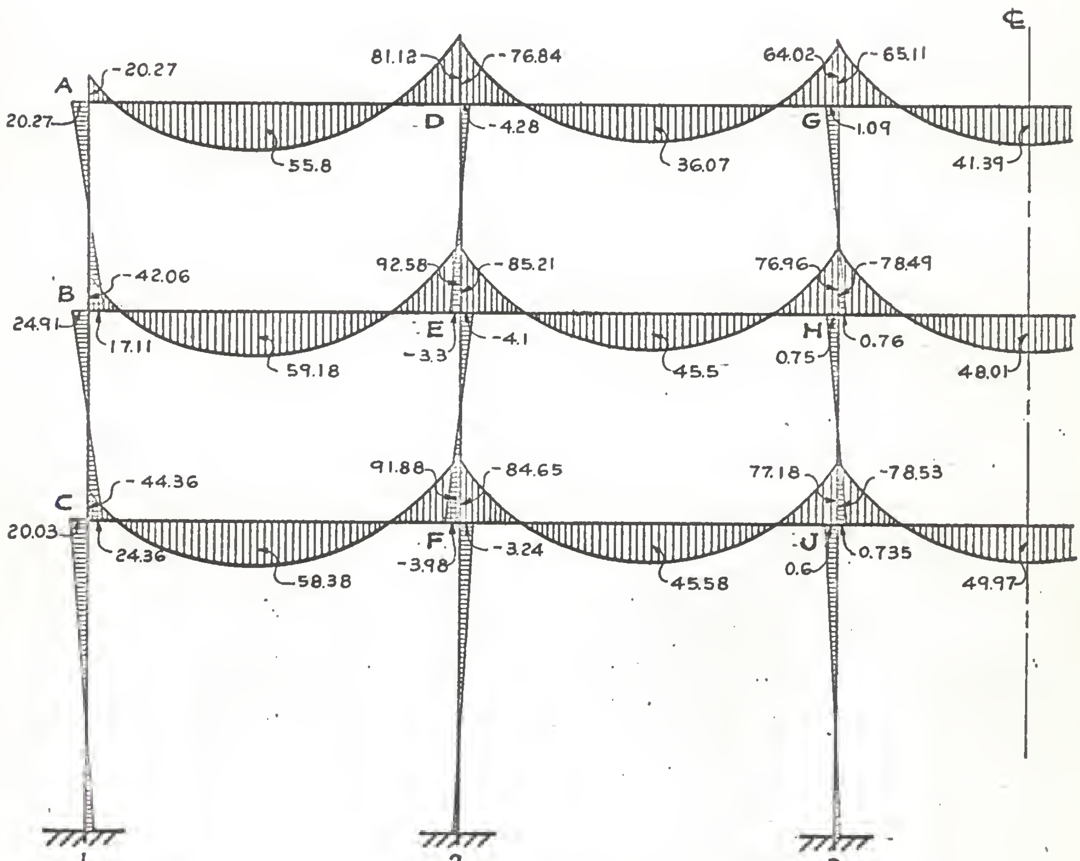
1
2
3
20'-0"
20'-0"
20'-0"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D
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G
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41.39
B
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24.91
17.11
92.58
-85.21
76.96
-78.49
E
-4.1
-3.3
H
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0.75
59.18
45.5
48.01
C
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20.03
24.36
91.88
-84.65
77.18
-78.53
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J
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0.6
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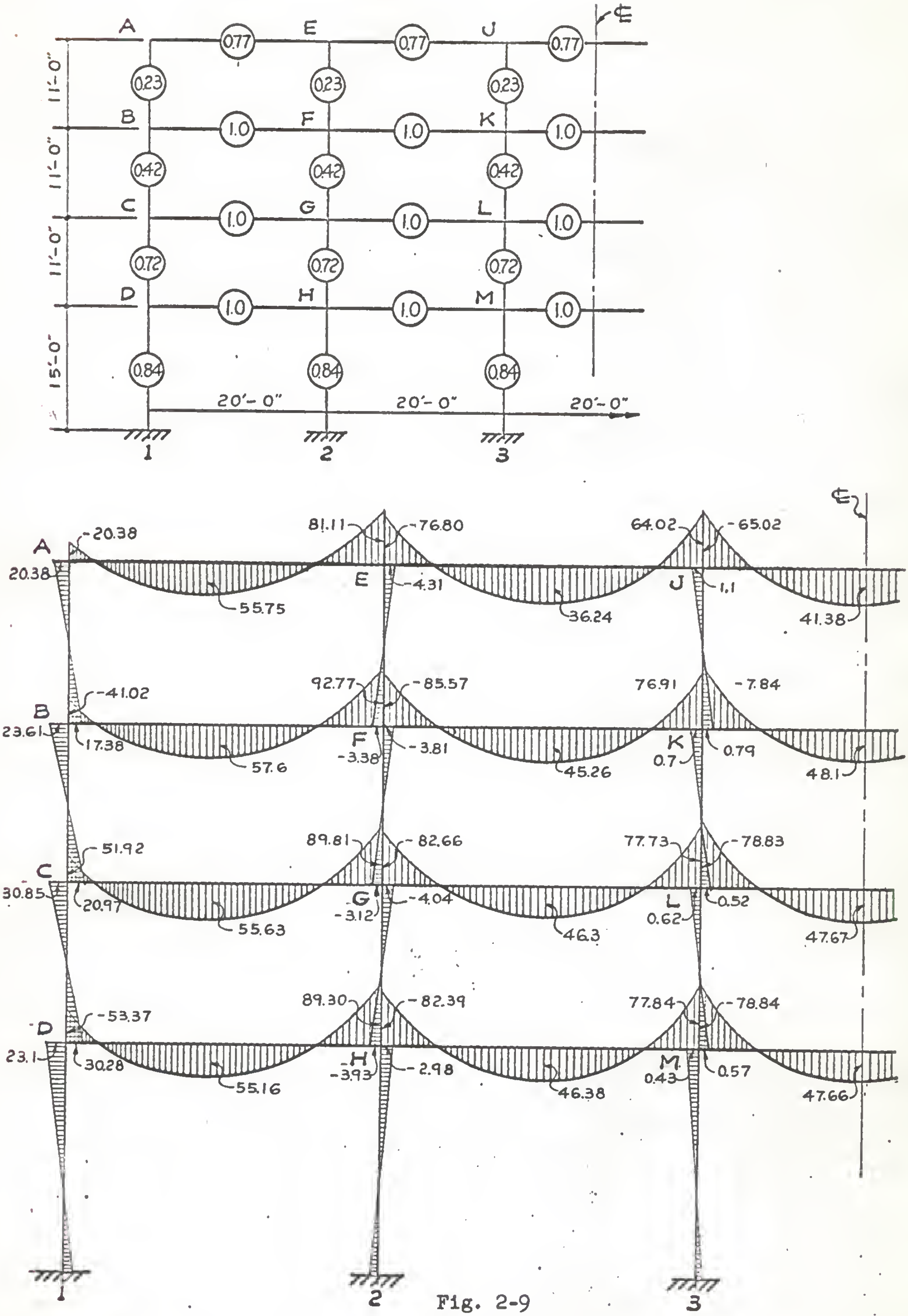
45.58
49.97
1
2
3

Fig. 2-8



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E
J
℄
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0.77
0.23
0.23
0.23
B
F
K
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1.0
1.0
0.42
0.42
0.42
C
G
L
0.72
0.72
0.72
D
H
M
0.84
0.84
0.84
11'-0"
11'-0"
11'-0"
15'-0"
20'-0"
20'-0"
20'-0"
1
2
3
-20.38
20.38
55.75
81.11
-76.80
-4.31
36.24
64.02
-65.02
1.1
41.38
-41.02
23.61
17.38
57.6
92.77
-85.57
-3.38
-3.81
45.26
76.91
-7.84
0.7
0.79
48.1
51.92
30.85
20.97
55.63
89.81
-82.66
-3.12
-4.04
46.3
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0.62
0.52
47.67
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23.1
30.28
55.16
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46.38
77.84
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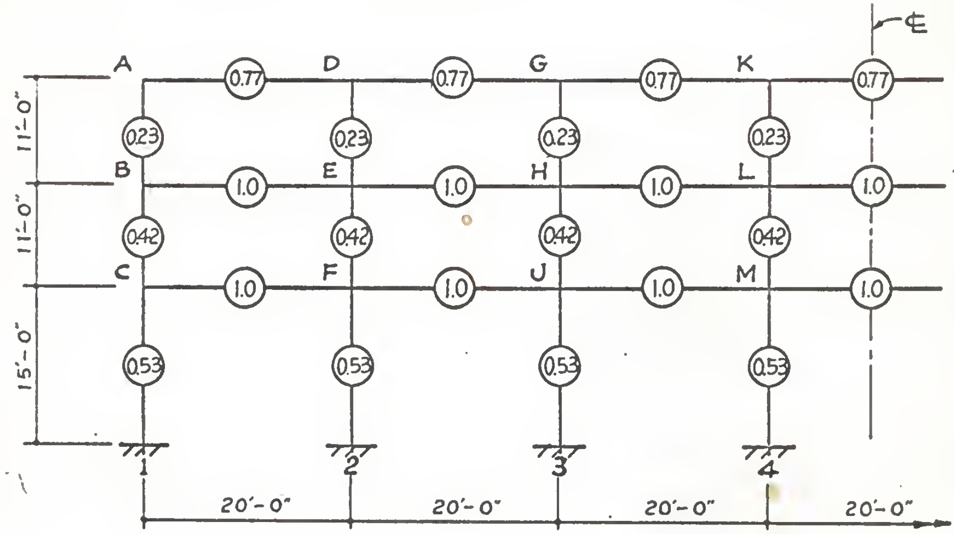
0.43
0.57
47.66

Fig. 2-9



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D
G
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B
E
H
L
C
F
J
M
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1.0
0.42
0.53
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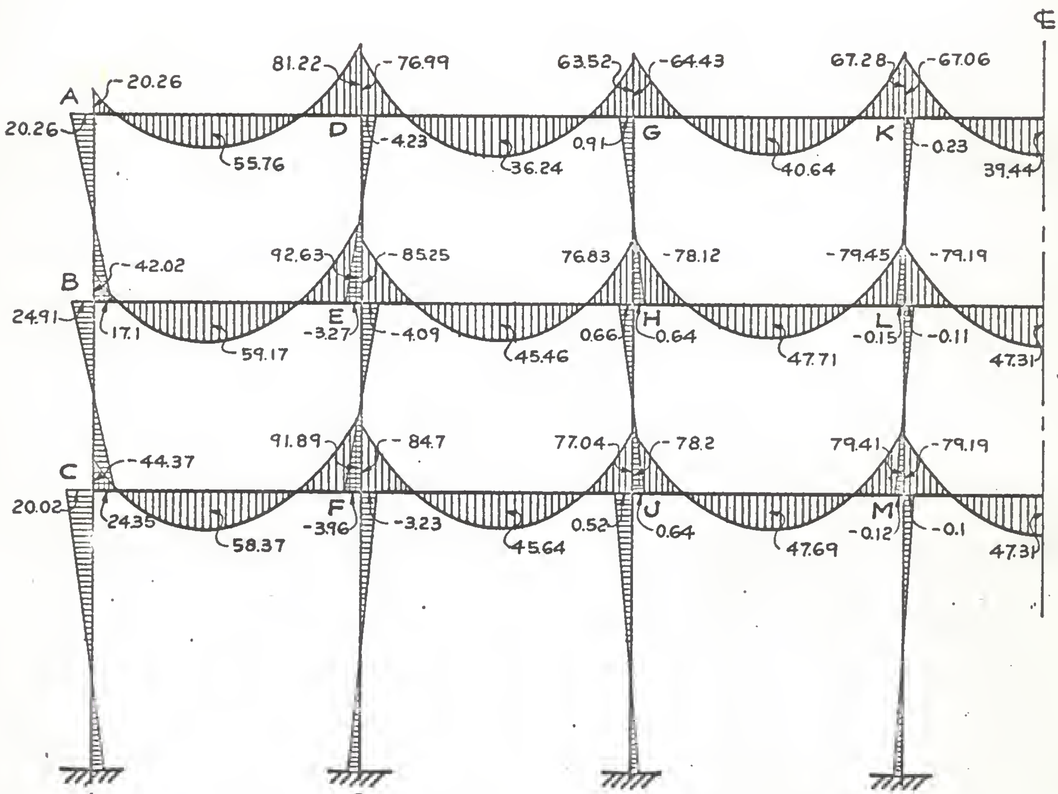
11'-0"
15'-0"
1
2
3
4
20'-0"
20'-0"
20'-0"
20'-0"



℄
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D
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K
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J
M
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-4.23
36.24
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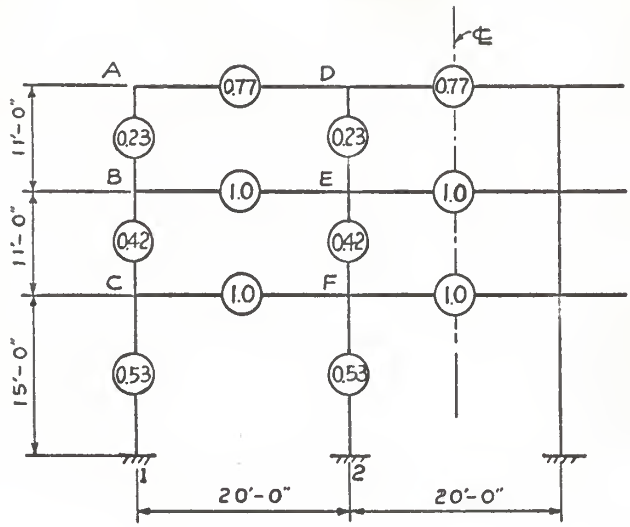
-0.23
39.44
42.02
24.91
17.1
59.17
92.63
-85.25
-3.27
-4.09
45.46
76.83
-78.12
0.66
0.64
47.71
-79.45
-79.19
-0.15
-0.11
47.31
-44.37
20.02
24.35
58.37
91.89
-84.7
-3.96
-3.23
45.64
77.04
-78.2
0.52
0.64
47.69
79.41
-79.19
-0.12
-0.1
47.31
1
2
3
4

Fig. 2-10



℄
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0.77
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0.23
B
E
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1.0
0.42
0.42
C
F
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1.0
0.53
0.53
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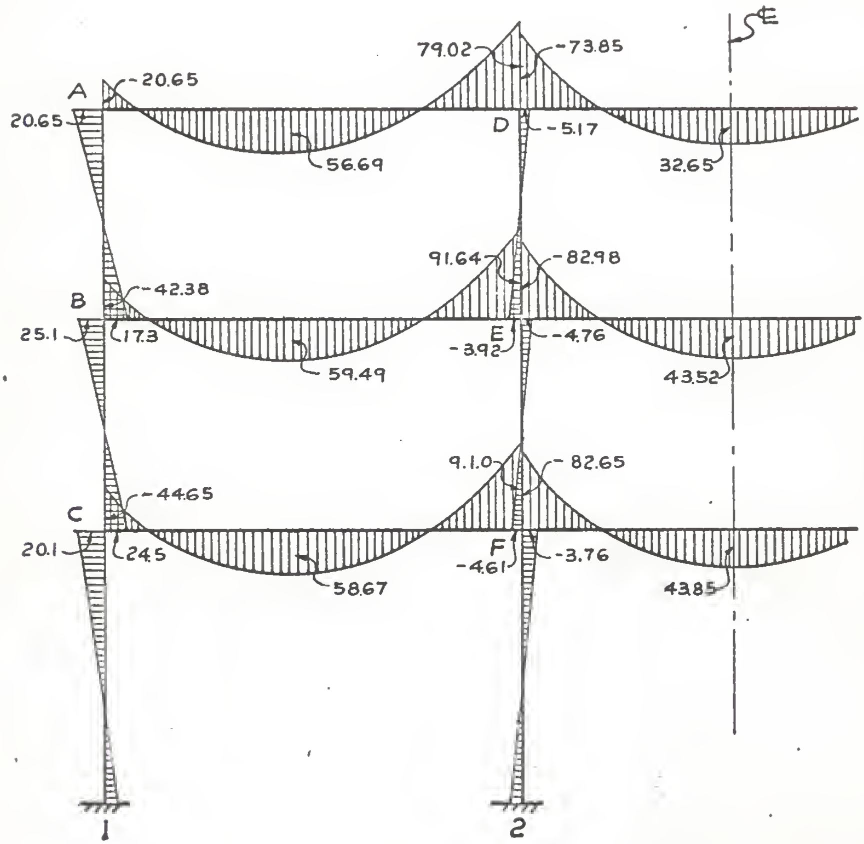
11'-0"
15'-0"
1
2
20'-0"
20'-0"



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D
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56.69
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B
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17.3
E
-4.76
-3.92
59.49
43.52
9.1.0
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C
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24.5
F
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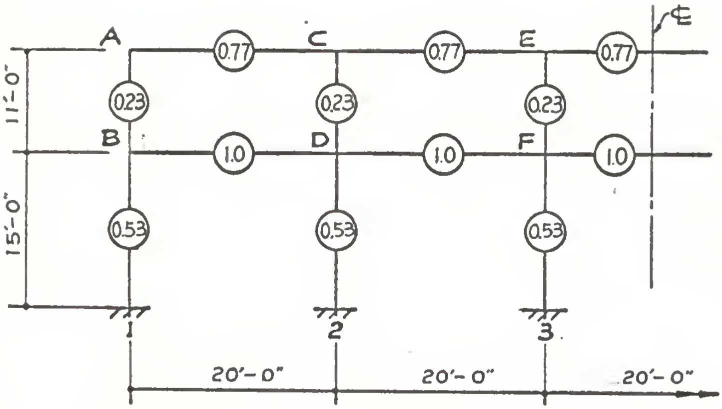
-4.61
58.67
43.85
1
2

Fig. 2-11


A
C
E
℄
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0.77
0.77
11'-0"
0.23
0.23
0.23
B
D
F
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1.0
1.0
15'-0"
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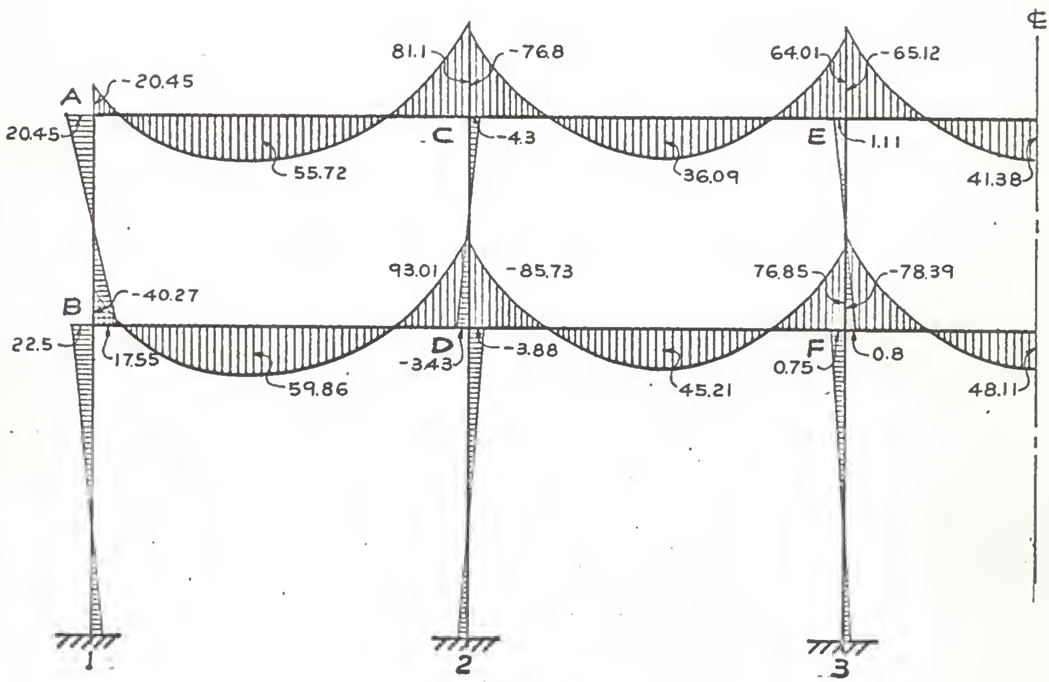
0.53
0.53
1
2
3
20'-0"
20'-0"
20'-0"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20.45
C
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E
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93.01
-85.73
76.85
-78.39
B
-40.27
22.5
17.55
D
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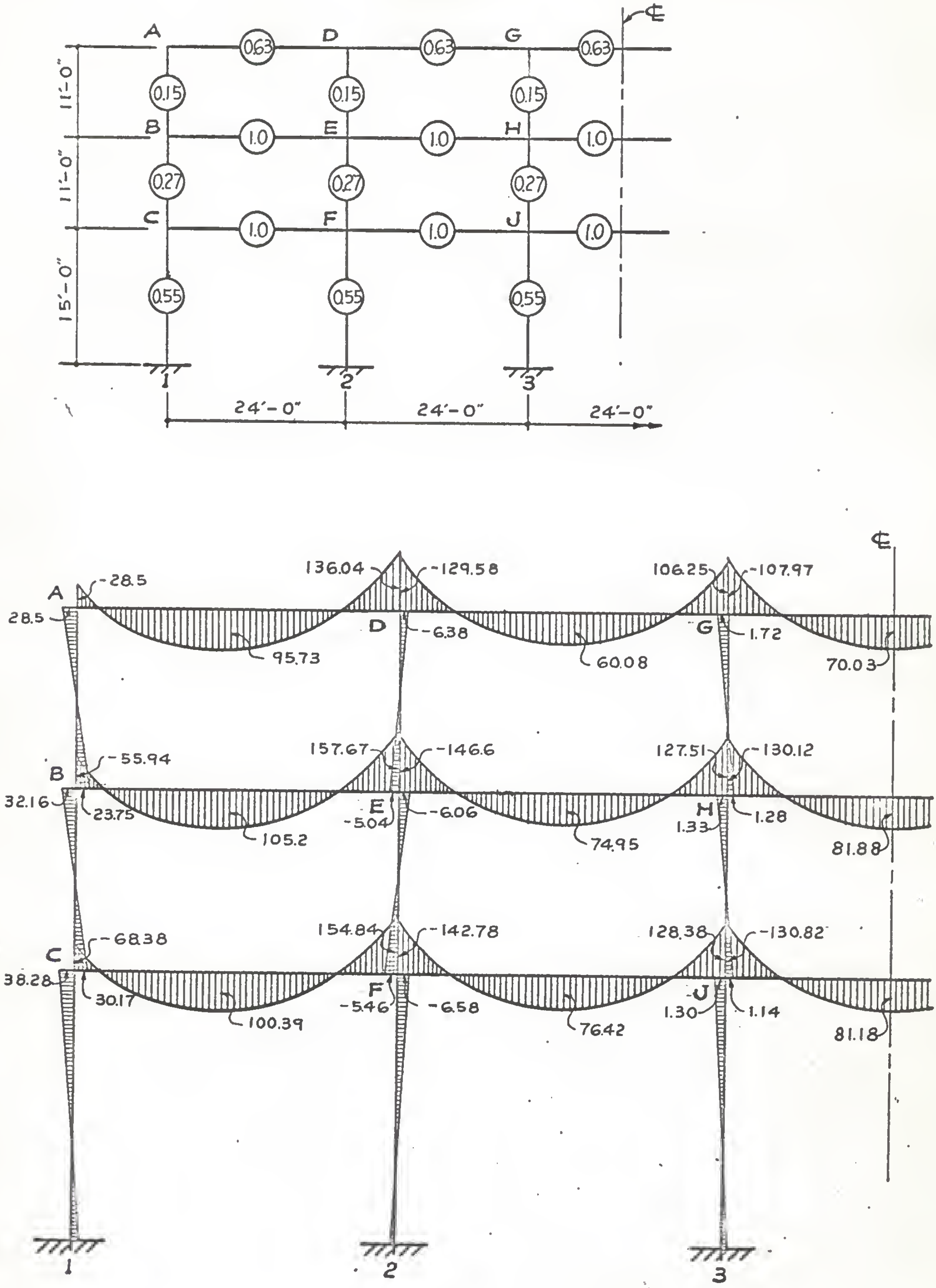
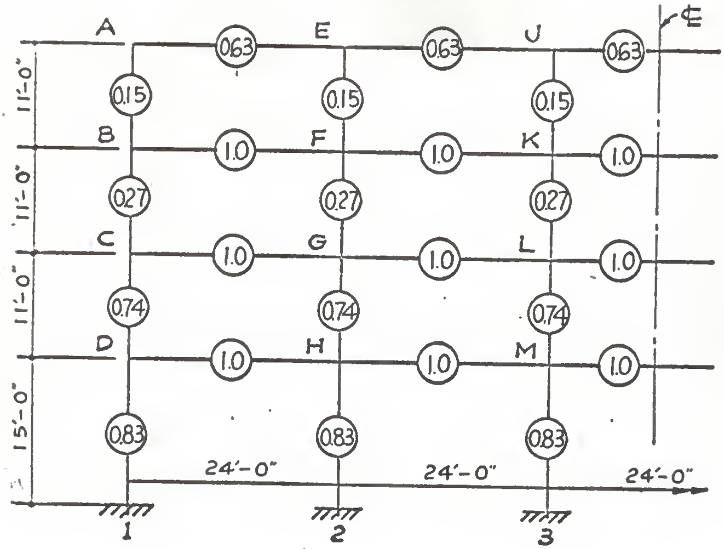
-3.88
F
0.75
0.8
59.86
45.21
48.11
1
2
3

Fig. 2-12



A
D
G
℄
0.63
0.15
B
E
H
1.0
0.27
C
F
J
0.55
11'-0"
11'-0"
15'-0"
1
2
3
24'-0"
24'-0"
24'-0"
-28.5
28.5
136.04
-129.58
106.25
-107.97
-6.38
1.72
95.73
60.08
70.03
-55.94
32.16
23.75
157.67
-146.6
127.51
-130.12
-5.04
-6.06
1.33
1.28
105.2
74.95
81.88
-68.38
38.28
30.17
154.84
-142.78
128.38
-130.82
-5.46
-6.58
1.30
1.14
100.39
76.42
81.18

Fig. 2-13


A
E
J
℄
B
F
K
C
G
L
D
H
M
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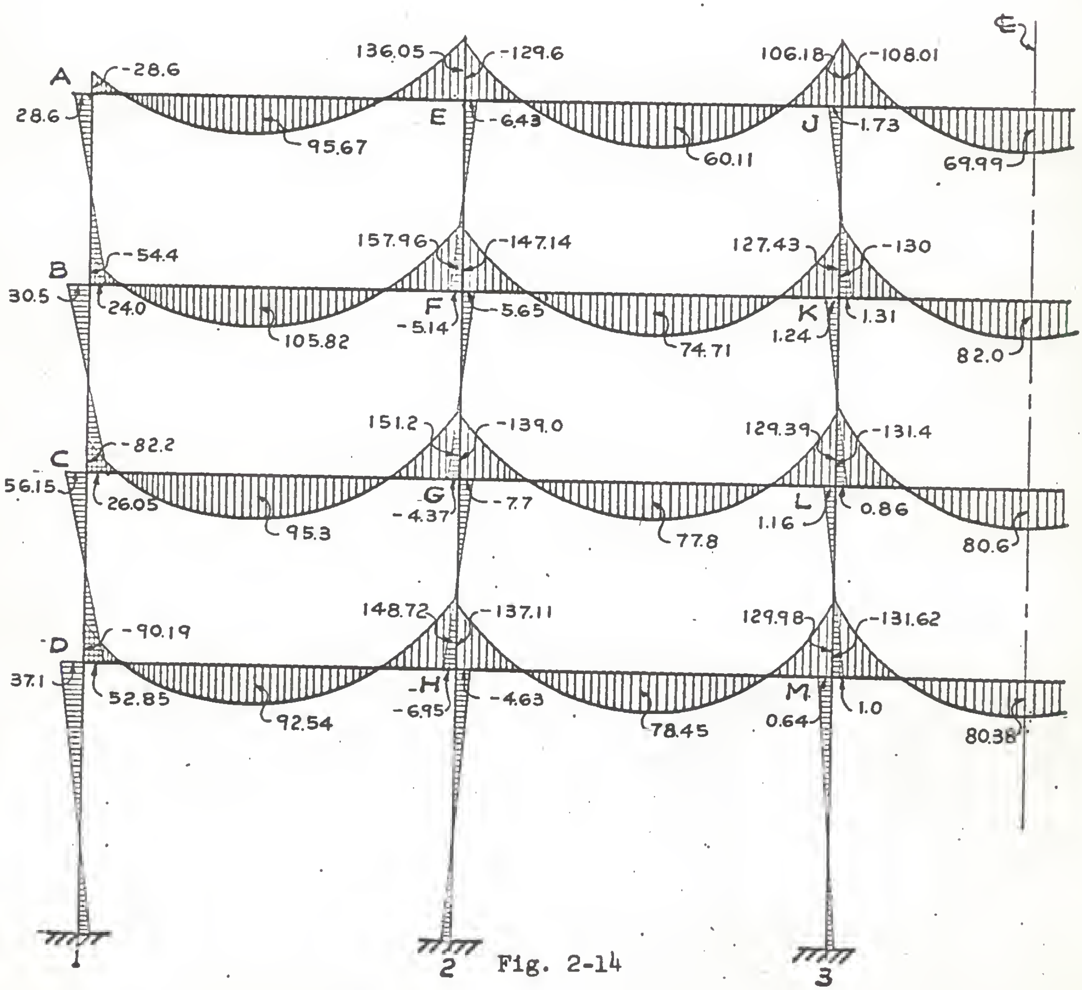
0.15
1.0
0.27
0.74
0.83
11'-0"
15'-0"
24'-0"
1
2
3


A
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28.6
95.67
136.05
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E
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60.11
106.18
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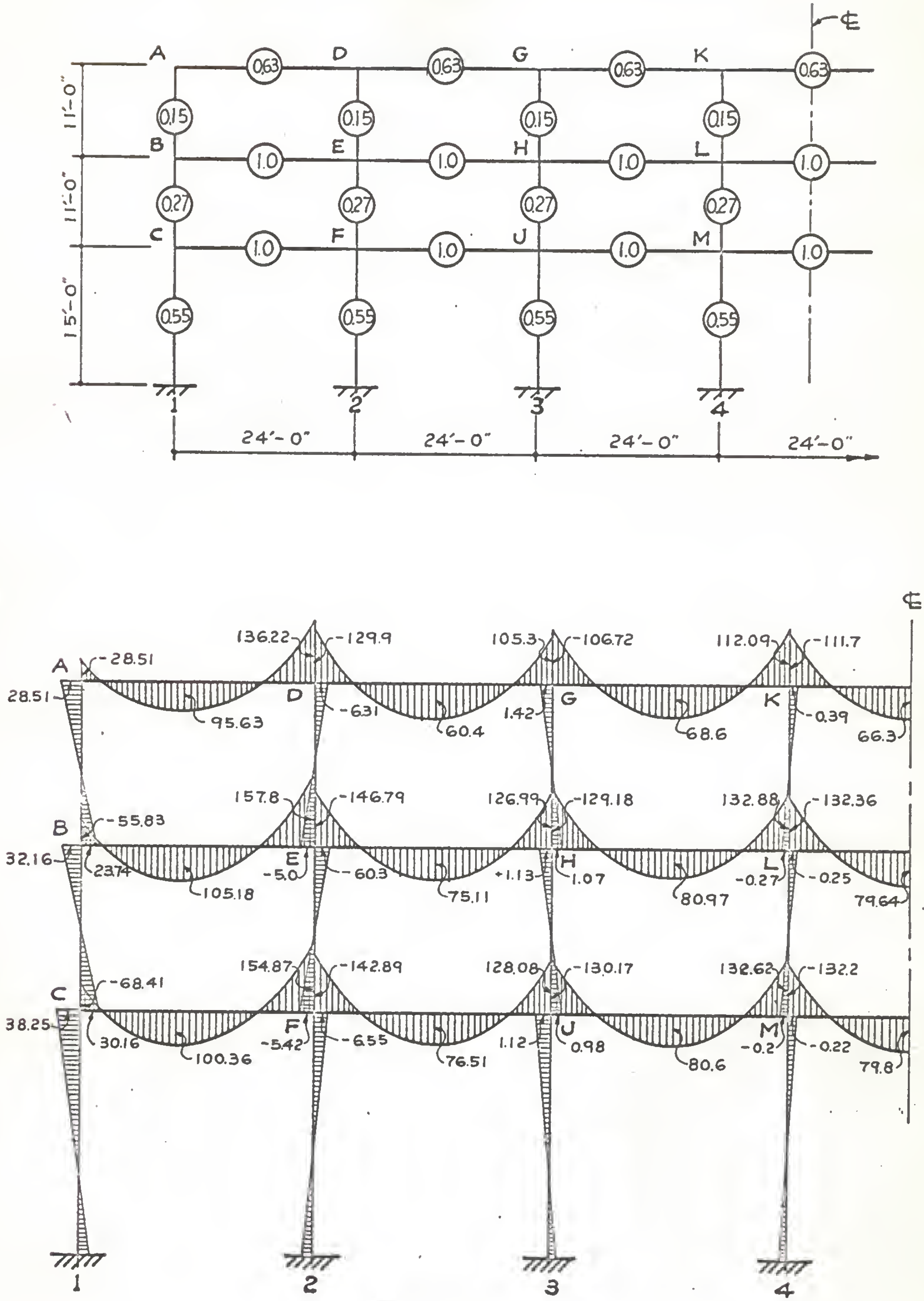
J
1.73
69.99
℄
B
-54.4
30.5
24.0
105.82
157.96
-147.14
F
-5.14
5.65
74.71
127.43
-130
K
1.24
1.31
82.0
C
-82.2
56.15
26.05
95.3
151.2
-139.0
G
-4.37
-7.7
77.8
129.39
-131.4
L
1.16
0.86
80.6
D
-90.19
37.1
52.85
92.54
148.72
-137.11
H
-6.95
-4.63
78.45
129.98
-131.62
M
0.64
1.0
80.38
1
2
3

Fig. 2-14


A
D
G
K
B
E
H
L
C
F
J
M
0.63
0.15
1.0
0.27
0.55
11'-0"
11'-0"
15'-0"
1
2
3
4
24'-0"
℄
136.22
-129.9
105.3
-106.72
112.09
-111.7
-28.51
28.51
95.63
-6.31
60.4
1.42
68.6
-0.39
66.3
157.8
-146.79
126.99
-129.18
132.88
-132.36
-55.83
32.16
23.74
105.18
-5.0
-60.3
75.11
+1.13
1.07
80.97
-0.27
-0.25
79.64
154.87
-142.89
128.08
-130.17
132.62
-132.2
-68.41
38.25
30.16
100.36
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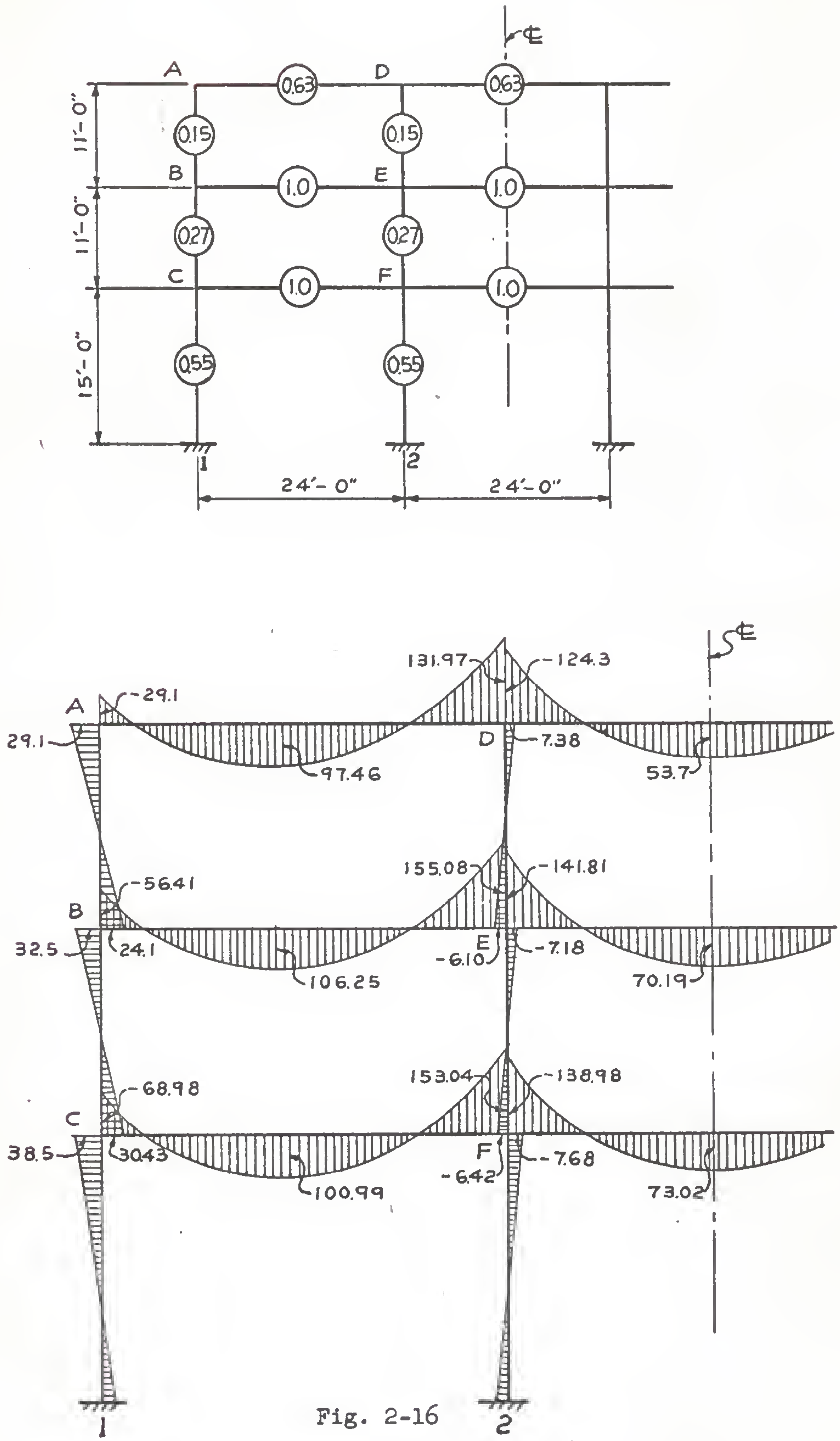
76.51
1.12
0.98
80.6
-0.2
-0.22
79.8

Fig. 2-15



A
D
B
E
C
F
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0.15
1.0
0.27
0.55
11'-0"
15'-0"
24'-0"
1
2
29.1
-29.1
97.46
131.97
-124.3
-7.38
53.7
155.08
-141.81
-56.41
32.5
24.1
106.25
-6.10
-7.18
70.19
153.04
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-68.98
38.5
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100.99
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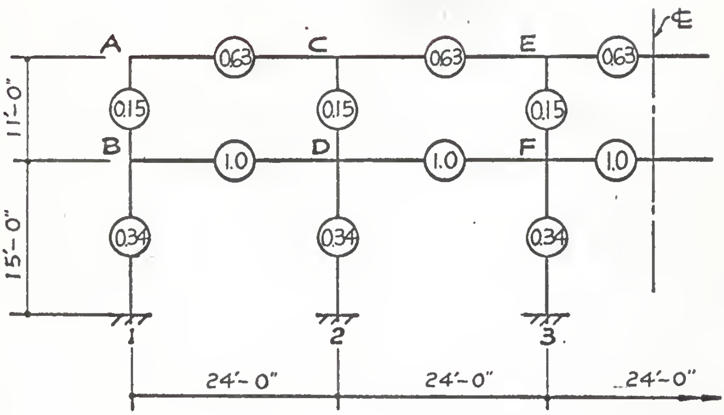
-7.68
73.02

Fig. 2-16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C
E
℄
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0.15
0.15
B
D
F
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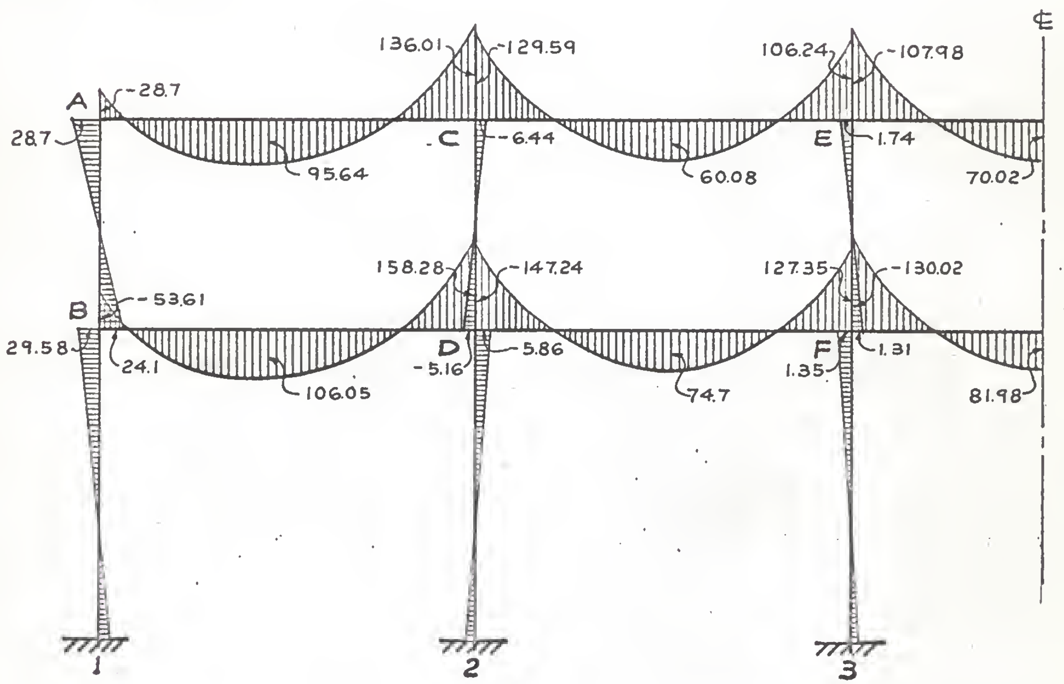
1.0
1.0
0.34
0.34
0.34
11'-0"
15'-0"
1
2
3
24'-0"
24'-0"
24'-0"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127.35
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B
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29.58
24.1
106.05
D
-5.16
-5.86
74.7
F
1.35
1.31
81.98
1
2
3

Fig. 2-17

III. CONSIDERATION FOR THE RESULT OF CALCULATION

A. Negative Moment at Interior Faces of Exterior Beams.

In finding a certain relationship between bending moment and the stiffness ratio with the above result of calculation, the ratio cf K_b (stiffness ratio of an exterior beam) and ΣK (the sum of stiffness ratio of members at the exterior joint of beam) and the ratio of M_b, the negative moment at interior face of exterior beam, and M_o, simple beam moment of the beam, are shown on Table 3-1, and taken as horizontal and vertical component, respectively in the Fig. 3-1.

An almost straight line is shown on Fig.3-1, and it is observed that there exists a rational relationship between the stiffness ratio and the bending moment.

With respect to the ratio $K_b/\Sigma K$, consideration should be given in formulating the above relationship into a formula. For, when the value $K_b/\Sigma K$ is close to zero (in other words K_b becomes smaller), it denotes that both beam ends are fixed, and when the value $K_b/\Sigma k$ is close to one, it denotes a cantilevered beam situation.

Therefore, it is not desirable to pick the above two excessive cases as a standard point in formulating the formula. Error will be minimized by using a reasonably selected $K_b/\Sigma K$ value between zero and one.

Adopted into a general form of linear formula,

$$Y = ax+b$$

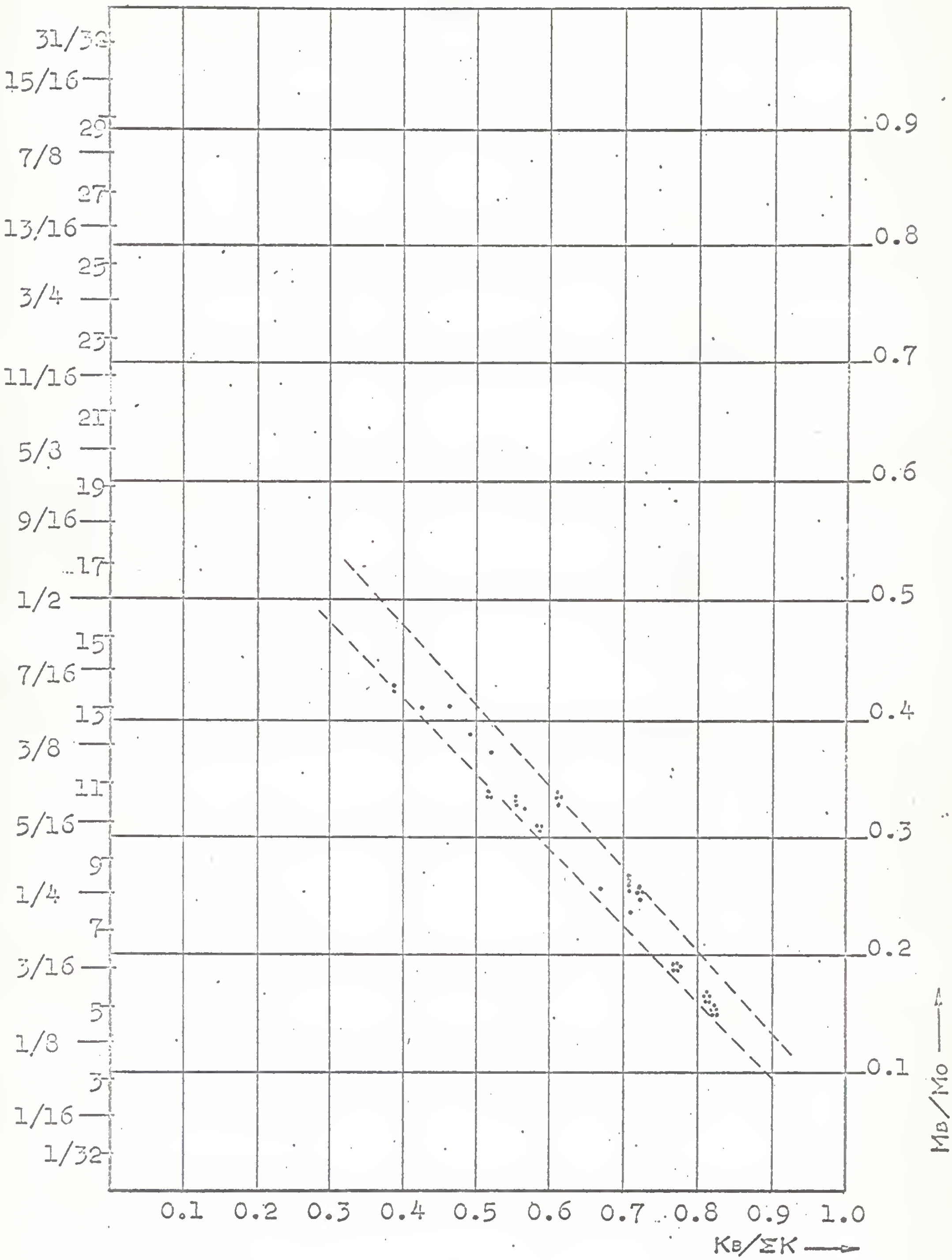
corresponding $K_b/\Sigma K$ values vary from 0.82 to 0.39 while the value

TABLE 3-1

Case 1		Case 2		Case 3		Case 4		Case 5	
Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo
0.82	0.153	0.82	0.154	0.82	0.153	0.82	0.155	0.82	0.154
0.72	0.253	0.72	0.248	0.72	0.254	0.72	0.257	0.705	0.232
0.585	0.305	0.518	0.377	0.585	0.302	0.585	0.305		
		0.422	0.407						

Case 6		Case 7		Case 8		Case 9		Case 10	
Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo
0.77	0.19	0.77	0.191	0.77	0.19	0.77	0.193	0.77	0.192
0.606	0.333	0.606	0.324	0.606	0.333	0.606	0.335	0.568	0.319
0.513	0.35	0.467	0.411	0.513	0.35	0.513	0.353		
		0.391	0.422						

Case 11		Case 12		Case 13		Case 14		Case 15	
Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo	Kb/ΣK	Mb/Mo
0.808	0.16	0808	0.16	0.808	0.16	0.808	0.163	0.808	0.16
0.704	0.264	0.704	0.257	0.704	0.263	0.704	0.266	0.671	0.253
0.55	0.322	0.498	0.387	0.55	0.322	0.55	0.325		
		0.389	0.425						


31/32
15/16
29
7/8
27
13/16
25
3/4
23
11/16
21
5/8
19
9/16
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15
7/16
13
3/8
11
5/16
9
1/4
7
3/16
5
1/8
3
1/16
1/32
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
MB/MO
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
KB/ΣK

Fig. 3-1

of M_b/M_o varies from 0.153 to 0.425, therefore,

$a = 0.153 - 0.425/0.82 - 0.39 = -0.63$

If the standard point is chosen as $K_b/\Sigma K = 0.55$ then $M_b/M_o = 0.322$

thus $M_{neg.} = M_o \quad (K_b/\Sigma K-0.55)\ (-0.63) + 0.322 \quad (3\text{-}1)$

Observing the Fig. 3-1, there would be less error when the value of $K_b/\Sigma K = 1$ is used than when that of $K_b/\Sigma K = 0$ is used. $M_b/M_o = 0.04$ when $K_b/\Sigma K = 1$ thus

$$M_{neg.} = M_o \quad (K_b/\Sigma K-1)\ (-0.63) + 0.04 \qquad (3\text{-}2)$$

As an example, if $K_b/\Sigma K = 0.808$, substitute into the formula (3-1)

$$M_{neg.} = M_o\ (0.808-0.55)\ \ (-0.63) + 0.322$$

$= 0.1595\ M_o$ which is close enough to $0.16\ M_o$ in Table (3-1)

If formula (3-2) is used, and $K_b/\Sigma k = 0.55$, then

$$M_{neg.} = M_o \quad (0.55-1)\ (-0.63) + 0.04 = 0.323\ M_o$$

This value is also close enough to the value $0.322\ M_o$ in Table (3-1).

B. Negative Moment at Interior Faces of Exterior Beams And Its Distribution to Adjacent Columns Above And Below.

As to the exterior column moment, it is somewhat difficult to relate to the stiffness ratio, since the distribution is irregular. As the result of observing Fig. 2-3 - 2-17 it is found that there exist certain relationships between upper and lower moment of a column in each floor.

On the roof floor, upper exterior column moment is same amount as but with opposite sign of the negative moment at the interior faces of the exterior beam.

At each floor, lower column moment is approximately 90% of the upper moment of the same column. Therefore, upper column moment of next lower floor is computed by deducting lower column moment from negative beam moment.

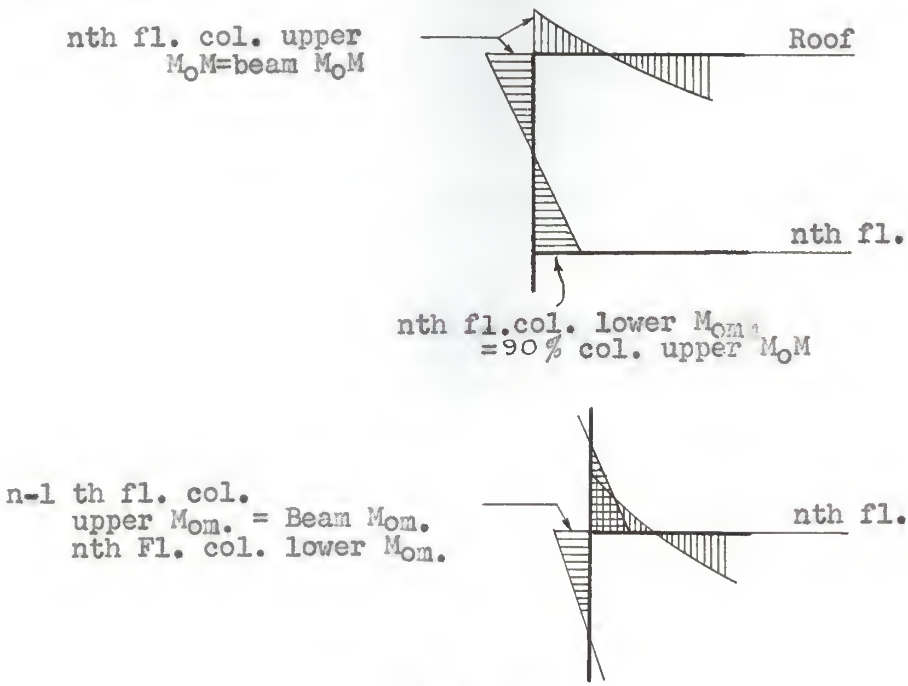

Fig. 3-2

Fig. 3-2 shows the relationship between the negative beam moment and adjacent column moment above and below.

By repeating this process, exterior column moments are easily found. Even this calculation is not accurate enough for precise calculation though is is good enough for approximate analysis, and for determination of suitable member size.

C. Negative Moment at Faces of Interior Spans and Positive Moment at Mid-Span.

According to the ACI code Section 904, negative moment at interior spans of continuous beams are classified by their situation such as exterior faces of first interior supports, other faces of interior supports, etc.

In a practical sense, the size of a beam is determined by the condition which is critical and adjusted by suitable reinforcement. Therefore, for the purpose to assume maximum negative moment, $3/4\ M_o$ may be used, this value is slightly less than that of the continuous beam moment in section 904 (C) of the ACI code.

As to the positive moment, it is generally computed by deducting the average of the negative moments, from M_o which has the maximum simple beam moment.

D. Future Development.

In most cases, the amount of interior column moment is very small and does not much affect design. Thus, negative and positive beam moments and exterior column moment are of more importance in the design. Therefore, this study will be of use not only in approximate stress analysis but also in determining suitable sizes of members for accurate stress analysis.

This article is only concerned with a symmetrical low story building containing a certain live load. It would be of interest and value to investigate more complicate conditions, such as unsymmetrical framed structures, different loading situations, and other situations. By plotting graphs and establishing formulas for these results, contributions to practical usage may be considerable.

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THE RELATIONSHIP BETWEEN STIFFNESS & STRESSES OF MEMBERS OF RIGID FRAMED STRUCTURES

by

HONG KYUN KIM

B. of Science in Engineering, Seoul National University 1955

AN ABSTRACT OF A MASTER'S THESIS

Submitted in partial fulfillment of the

requirements for the degree

MASTER OF ARCHITECTURAL ENGINEERING

College of Architecture and Design

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1965

ABSTRACT

This study is an attempt to describe a suitable method for determining proper member size in rigid framed structures by the slope deflection method.

Beginning with calculations based on a structure of three stories and five bays (symmetrical at the center line). Various combinations of story height, span length and member size were described and analized for suitability.

To this analysis were added comparable analyses of structures of two and four stories in height, and three and seven bays.

Preliminary calculations are for reinforced concrete structures supporting both a dead load (the structure itself) and a live load (here considered to be an office).

In a practical sense, beams and columns of the same size are used throughout each story. Unknown factors such as ØA, ØB ------etc. which denote twice the sum of stiffness ratios multiplied by slope at each joint are solved by mechanical tabulation of simultaneous equations. For all fifteen seperate variations which were analized by the same procedure moment diagrams were drawn.

By observing the above results, the ratio of Kb (the stiffness ratio of an exterior beam) and ΣK (the sum of the stiffness ratio of members at the joint of the beam) and the ratio of Mb (the negative moment at an interior face of an exterior beam) and Mo (the simple beam moment of a member) are shown and plotted

on a graph; then formulated by statistical methods so as to investigate certain relationships between stresses and stiffness. Also distribution of negative moment at an interior face of an exterior beam transmitted to columns above and below the joint is observed and negative moment at all other faces and positive moment at midspan are related to the bending moment of a simple beam in the same situation.