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LAMINAR FLOW THROUGH AN ANNULUS

WITH POROUS WALLS

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Diploma, Taipei Institute of Technology
Republic of China, 1964

9589

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

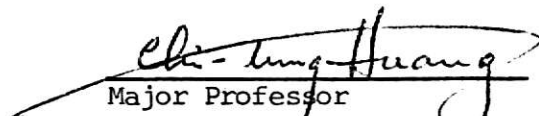
MASTER OF SCIENCE

Department of Applied Mechanics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1972

Approved by:


Major Professor

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NOMENCLATURE

(r, θ, z)	Cylindrical coordinates used to indicate the directions
(q_r, q_θ, q_z)	The velocity components in the directions of r, θ , and z respectively
q_r^*, q_z^*	The non-dimensional velocity components in the r and z directions respectively
a, b	The inner and outer radii of the annulus respectively
p	The pressure
Q_a, Q_b	The constant wall velocities at the inner and outer walls of the annulus, respectively
Q	Typical velocity based on the velocities of suction or injection at the wall ($bQ = bQ_b + aQ_a$)
$U(0)$	The axial velocity occurring at the entrance for the annulus
$f(r), h(r)$	The arbitrary non-dimensional functions of r
$F(\eta), H(\eta)$	The arbitrary non-dimensional functions of η
R	Reynolds number for the cross flow in the annulus $R = \frac{bQ}{2\nu}$
N	The Reynolds number for the longitudinal flow in the annulus $N = \frac{bU(0)}{\nu}$
C_{fi}, C_{fo}	The inner and outer walls skin frictional coefficients respectively
η	A dimensionless distance parameter equal to $(r/b)^2$ for the annulus
η_0	The ratio of inner to outer radius $(a/b)^2$
ν	The fluid kinematic viscosity ($\tilde{\mu}/\rho$)
$\tilde{\mu}, \hat{\lambda}$	The first and second coefficients of viscosity
δ_{ij}	Kronecker delta
\mathcal{E}_{ij}	The rate of linear strain tensor $\mathcal{E}_{ij} = 1/2 (q_{i,j} + q_{j,i})$
σ_{ij}	The stress tensor

σ_i	The stress vector
f_i	Body forces
q_i	The velocity vector
n_i	The unit normal vector
$\frac{D}{Dt}$	The comoving derivatives
$F_{ij} \dots$	The total flow entity

I. INTRODUCTION

The problems of normal fluid injection and suction through a porous annulus or a channel play a vital role in many kinds of engineering applications. It is of practical interest in connection with the transpiration or sweat-cooling of heated surfaces such as turbine blades, rocket walls, or wing surface in high-speed flight. In recent time, the problem of flow through an annulus with porous walls has gained considerable importance in view of its technological and aeronautical applications. If fluid is withdrawn through porous walls, thickness of boundary layer decreases and the flow become more stable. Thus, transition from laminar to turbulent flow is delayed. The study of this phenomena by way of mathematical analyses and the solutions solved by various approximate methods for obtaining solutions valid for different restricted ranges of fluid suction and injection at the porous walls of different kind of flows have appeared in literature.

In 1953, Berman (1) initiated the study of the laminar flow in a uniformly porous channel by applying a perturbation method. Under his assumptions that a solution for the flow between porous parallel plates with constant and equal permeability at both walls, the velocity component normal to the wall was independent of the distance along the channel for the case of very low suction and injection. Sellars(2) and Yuan(3) extended the investigation to higher cross flow Reynolds numbers, in which the reciprocal of the Reynolds number R is used as a perturbation parameter.

Yuan and Finkelstein (4) treated the problem for flow through a circular tube. He solved and obtained a solution of the ordinary differential equation by perturbation procedure for small values of normal fluid velocity at the wall. In addition, however, there an asymptotic solution to the reciprocal

of the normal velocity Reynolds number, also valid for higher injection and suction rates, was developed. Mordochow (5) solved the problems on laminar flow through a channel and tube with injection by using a method of averages in conjunction with auxiliary boundary conditions derived from the governing ordinary differential equation. It is valid for small suction (or injection) and also for large suction (or injection) and has obtained good agreement with the results of Berman (1) and Yuan (3).

In 1958, Berman (6) again derived and solved the problem for flow through a porous annulus. The solution is the simple case where the amount of fluid entering through the outer wall is equal to the amount of fluid leaving through the inner wall. He simplified the Navier-Stokes equation and obtained an exact solution directly by integrations. Terrill (7) reduced the problem of laminar flow through a porous annulus with constant velocity of suction at the walls and with swirl to a solution of four sets of non-linear differential equations. He solved the particular case of the problem by series perturbation method.

In recent years, electronic digital computers have expanded the application of numerical techniques. Using these devices, it is a straightforward matter to provide the numerical solution of several thousand simultaneous linear and non-linear differential equations subject to a complete set of initial conditions. This kind of computational speed has led to radically new ideas and concepts for solving large system of equations. In this report, the quasilinearization technique is used in obtaining numerical solutions of the velocity profiles by the influence of the cross flow Reynolds numbers R , the effect of the pressure in the annulus walls, and the coefficients of skin friction at the wall. Results of these efforts are graphically illustrated.

II. GOVERNING EQUATIONS FOR LAMINAR

INCOMPRESSIBLE FLOW

The differential equations which give a complete solution of the motion of fluid are the equation of continuity (conservation of mass) and the equation of motion. The equation of motion is often referred to as the Navier-Stokes equation.

2.1. Conservation of Mass

Consider a surface S enclosing a fixed region of space V , through which a continuum flows as shown in Fig.(1). Let the outer unit normal vector to a differential surface element dS be $n_i(r)$: Let $\rho = \rho(r, t)$ be the mass density at the point, at time t , and let $q_i(r, t)$ be the velocity of mass. It is obvious that the concept of conservation of mass, i.e. the rate of increase of mass in region V is exactly equal to the amount of mass flowing into the region per unit of time, The rate of increase of mass within any fixed volume is

$$\frac{D}{Dt} \int_V \rho \, dV = \int_V \frac{D\rho}{Dt} \, dV \quad (2.1)$$

Mass emerges from the volume, passing through the boundary surface at the rate $\rho q_i n_i dS$. Thus, if the mass is to be conserved, it is necessary that

$$\int_V \frac{\partial \rho}{\partial t} \, dV = - \int_S \rho q_i n_i \, dS \quad (2.2)$$

or

$$\int_V \frac{\partial \rho}{\partial t} \, dV + \int_S \rho q_i n_i \, dS = 0 \quad (2.3)$$

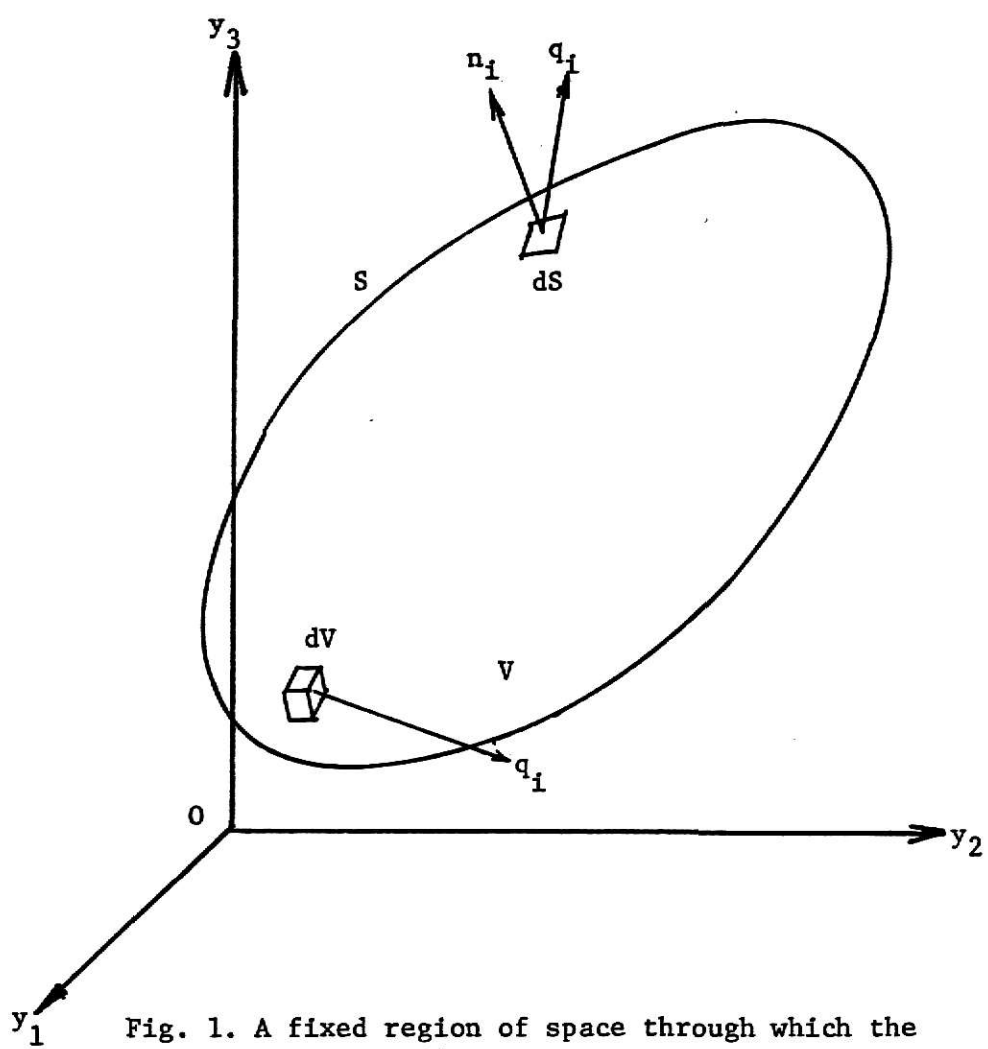


Fig. 1. A fixed region of space through which the continuum flows

Applying Gauss' theorem to convert the surface integral to a volume integral. Equation (2.3) becomes

$$\int_V \left[\frac{\partial \rho}{\partial t} + (\rho q_i)_{,i} \right] dV = 0 \quad (2.4)$$

Since this equation is true for any volume V , and since the function and q_i and their derivatives are continuous

$$\frac{\partial \rho}{\partial t} + (\rho q_i)_{,i} = 0 \quad (2.5)$$

Equation (2.5) is the conventional equation of continuity. For the steady state, the equation (2.5) becomes

$$(\rho q_i)_{,i} = 0 \quad (2.6)$$

For an incompressible fluid, the equation (2.6) becomes

$$q_i{}_{,i} = 0 \quad (2.7)$$

2.2. The Momentum Equation

The principle of conservation of momentum is given by Newton's second law of motion. Choose an arbitrary region of space, V , in which the continuum moves as shown in Fig.(2).

Newton's second law states that the total force acting on a particle dV is equal to the mass dV times the acceleration dq_i/dt . The total force acting on a particle is the sum of the body forces and surface forces. These arise from the pressures acting on the control volume surface. These pressures

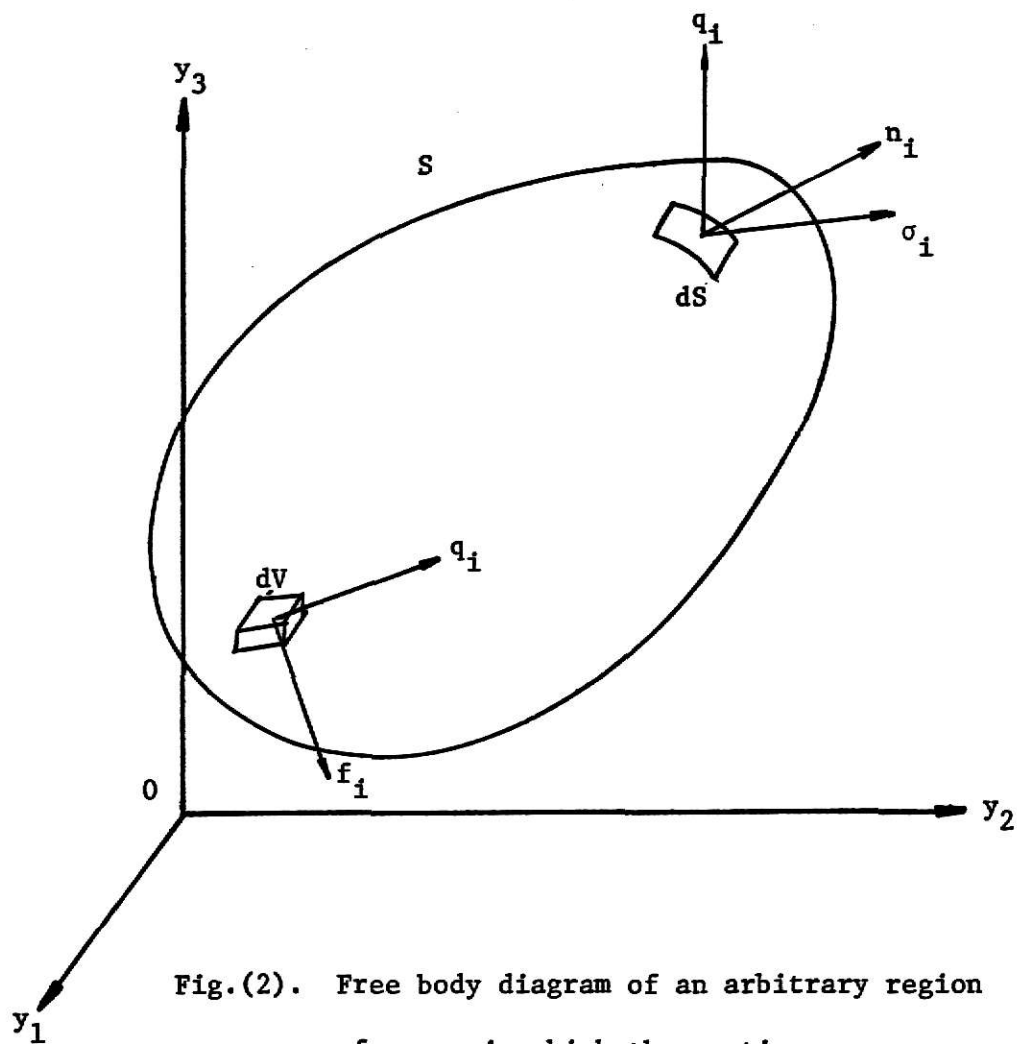


Fig.(2). Free body diagram of an arbitrary region of space in which the continuum moves

are due to forces fields such as gravitational, electric, and magnetic fields. It is more convenient here to consider the same amount of fluid. The Newton's second law can be written as

$$\int_V \rho \frac{Dq_i}{Dt} dV = \int_S \tau_i dS + \int_V \rho f_i dV \quad (2.8)$$

Converting the left side of equation (2.8) into the sum of surface integral and volume integral by using the Reynolds transport theorem,

$$\begin{aligned} \frac{D}{Dt} \int_V \rho F_{ij} \dots dV &= \int_V \rho \frac{DF_{ij} \dots}{Dt} dV \\ &= \int_V \frac{\partial}{\partial t} (\rho F_{ij} \dots) dV + \int_S \rho q_r n_r F_{ij} \dots dS \end{aligned} \quad (2.9)$$

Thus

$$\int_S \tau_i dS + \int_V \rho f_i dV = \int_V \frac{\partial (\rho q_i)}{\partial t} dV + \int_S \rho q_i q_j n_j dS \quad (2.10)$$

where $n_j(y)$ is the unit normal vector to the surface element dS on S . by replacing τ_i in terms of the stress tensor τ_{ij} .

$$\int_S \tau_i dS = \int_V \tau_{ji} n_j dV \quad (2.11)$$

and transforming the surface integral in eqn.(2.10) into volume integral by means of Gauss' theorem, eqn.(2.10) can be rewritten as

$$\int_V \left\{ \sigma_{ji,j} + \rho f_i - \frac{d(\rho q_i)}{dt} - (\rho q_i q_j)_{,j} \right\} dV \quad (2.12)$$

Since this holds true for any volume, and since all functions as well as their derivatives are assumed continuous, then

$$\frac{d(\rho q_i)}{dt} + (\rho q_i q_j)_{,j} = \rho f_i + \sigma_{ji,j} \quad (2.13)$$

or

$$\frac{Dq_i}{Dt} = f_i + \frac{1}{\rho} \sigma_{ji,j} \quad (2.14)$$

In considering the surface forces, if it is assumed that the fluid is isotropic, homogeneous, and Newtonian. The surface forces, are expressed by the stress tensor with a scalar viscosity as

$$\sigma_{ij} = - (p + \tilde{\lambda} \varepsilon_{kk}) \delta_{ij} + 2 \tilde{\mu} \varepsilon_{ij} \quad (2.15)$$

By utilizing the relationship

$$\varepsilon_{ij} = \frac{1}{2} (q_{i,j} + q_{j,i}) \quad (2.16)$$

hence

$$\frac{Dq_i}{Dt} = f_i - \frac{1}{\rho} p_{,i} + \frac{\tilde{\lambda} + \tilde{\mu}}{\rho} q_{j,ji} + \frac{\tilde{\mu}}{\rho} q_{i,jj} \quad (2.17)$$

where $\tilde{\lambda}, \tilde{\mu}$ are constants

If the flow is incompressible then the eqn.(2.17) for the momentum equation (Navier-Stokes) becomes

$$\frac{Dq_i}{Dt} = f_i - \frac{1}{\rho} p_{,i} + \frac{\tilde{\mu}}{\rho} q_{i,jj} \quad (2.18)$$

III. REDUCTION OF THE FLOW EQUATIONS

3.1. Simplified Differential Equations

Consider the fluid flow with suction and injection through the porous walls. The following assumptions are made:

1. Steady flow ($\frac{\partial}{\partial t} = 0$),
2. The fluid is incompressible ($\frac{D\rho}{Dt} = 0$),
3. Body forces (gravity field) are neglected ($f_i = 0$),
4. The flow is laminar,
5. The fluid flowing through the porous walls is uniform throughout,
6. No swirl velocity.

With the assumptions, (1) through (6), the governing equations (2.7) and (2.18) are reduced to the forms

$$q_j q_{i,j} = - \frac{1}{\rho} p_{,i} + \nu q_{i,jj} \quad i, j = 1, 2, 3 \quad (3.1)$$

$$\text{and} \quad q_{i,i} = 0 \quad i = 1, 2, 3$$

3.2. Flow through a Porous Annulus

Choose a cylindrical polar coordinate system (r, θ, z) where the axis $o-z$ lies along the center of the annulus. The velocity vector at point (r, θ, z) is

$$q_i \implies (q_1, q_2, q_3) \implies (q_r, q_\theta, q_z)$$

If the resultant flow is assumed to be independent of θ , and $(\quad)_{,\theta} = 0$, because of axially symmetric flow, the Navier-Stokes and continuity equations became

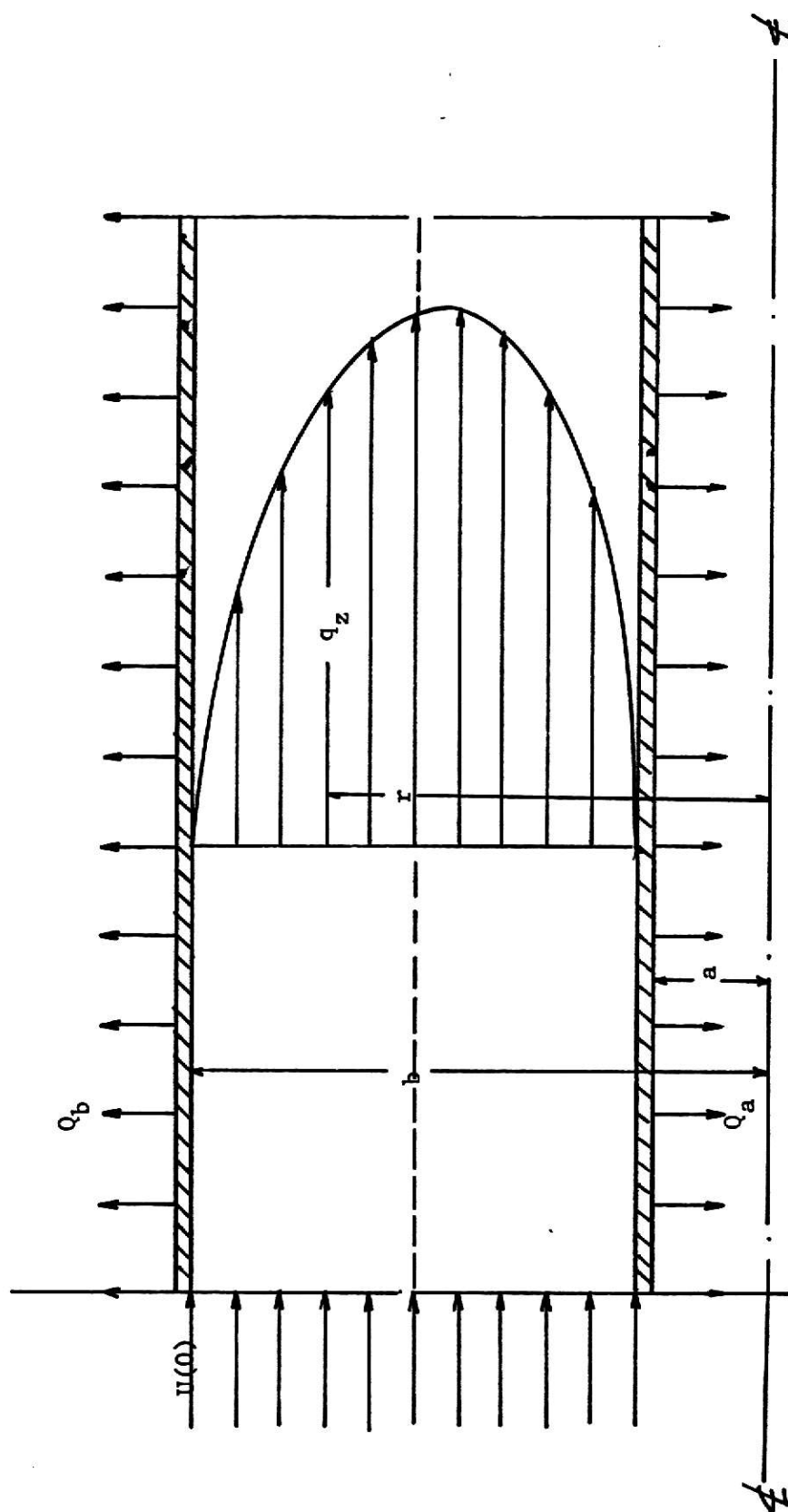


Fig.3. Flow through an annulus pipe with porous walls

$$q_z q_{z,z} + q_r q_{z,r} = - \frac{1}{\rho} p_{,z} + \nu (q_{z,rr} + \frac{1}{r} q_{z,r} + q_{z,zz}) \quad (3.2a)$$

$$q_z q_{r,z} + q_r q_{r,r} = - \frac{1}{\rho} p_{,r} + \nu (q_{r,zz} + q_{r,rr} + \frac{1}{r} q_{r,r} - \frac{q_r}{r^2}) \quad (3.2b)$$

$$\text{and} \quad (rq_z)_{,z} + (rq_r)_{,r} = 0 \quad (3.2c)$$

The above equations will be used to investigate the fluid flow in an annulus pipe with uniform permeability for outer and inner walls. The boundary conditions are:

$$\begin{array}{lll} \text{at } r = a & q_r = -Q_a & q_z = 0 \\ \text{at } r = b & q_r = Q_b & q_z = 0 \end{array} \quad (3.3)$$

The above boundary conditions imply that fluid is being withdrawn through the annulus walls with constant velocity Q_a at $r = a$ and Q_b at $r = b$. For obtaining a solution of equations (3.2) with boundary conditions (3.3), the radial velocity component can be assumed as a function of r alone

$$q_r = bQ \frac{f(r)}{r} \quad (3.4)$$

where $f(r)$ is an arbitrary non-dimensional function of r , Q is a typical wall velocity based on the suction or injection velocity Q_a or Q_b . The equation of continuity yields

$$q_z = -bQ \frac{f'(r)}{r} z + Qh(r) \quad (3.5)$$

where $h(r)$ is an arbitrary non-dimensional function of r and $f'(r)$ is the first derivative of $f(r)$ with respect to r . If equation (3.4) and (3.5) are substituted into the equation (3.2a) and (3.2b), then

$$\begin{aligned} \frac{1}{\rho} p_{,z} = & - \left[\frac{b^2 Q^2}{r^3} (-rff'' + ff' + rf'^2) + \frac{\gamma bQ}{r^3} (r^2 f''' + rf'' - f') \right] z \\ & - \left[\frac{bQ}{r} (fh' - f'h) + \frac{\gamma}{r} (rh'' + h') \right] \end{aligned} \quad (3.6a)$$

$$\frac{1}{\rho} p_{,r} = - \frac{b^2 Q^2}{r^3} (rff' - f^2) + \frac{bQ}{r^2} (rf'' - f') \quad (3.6b)$$

The right hand side of (3.6b) is seen to be a function of r only, hence differentiation with respect to z , and yields

$$p_{,rz} = 0 \quad (3.7)$$

Also by differentiating (3.6a) with respect to r and equating the coefficient of z to zero, it yields

$$\begin{aligned} r^2 (f'f'' - ff''') + r (3ff'' + f'^2) - 3ff' \\ - \frac{\gamma}{bQ} (-r^3 f^{iv} + 2r^2 f''' - 3rf'' + 3f') = 0 \end{aligned} \quad (3.8a)$$

and

$$r (fh'' - f''h) + (f'h - fh') - \frac{\gamma}{bQ} (r^2 h''' + rh'' - h') = 0 \quad (3.8b)$$

Eqn.(3.8) can be considerably simplified by introducing the non-dimensional independent variable defined by

$$\eta = (r/b)^2 \quad (3.9)$$

and defining the functions occurring in the velocity components as

$$f(r) = F(\eta) \quad h(r) = H(\eta) \quad (3.10)$$

hence the equation (3.8) reduces to

$$\eta F^{iv} + 2F''' + R(F'F'' - FF''') = 0 \quad (3.11)$$

and

$$\eta H''' + 2H'' + R(HF'' - FH'') = 0 \quad (3.12)$$

where $R = \frac{bQ}{2\nu}$ is a cross flow Reynolds number.

BOUNDARY CONDITIONS

In order to complete the formulation of the problems the governing equations (3.11) and (3.12) must be accompanied by a set of boundary conditions.

The boundary conditions of $F(\eta)$ and $H(\eta)$ are obtained from equations (3.4), through (3.5) and (3.10), thus

$$F(\eta_0) = -\frac{Q_a}{Q} \eta_0^{1/2} = -\sqrt{\beta} \quad F(1) = \frac{Q_b}{Q} = \beta \quad (3.13a)$$

$$F'(\eta_0) = 0 \quad F'(1) = 0 \quad (3.13b)$$

$$\text{and} \quad H(\eta_0) = 0 \quad H(1) = 0 \quad (3.13c)$$

in which

$$\eta_0 = (a/b)^2$$

The equation (3.11) does not involve $H(\eta)$, the fourth-order, non-linear differential equation together with four associate boundary conditions

(3.13a) and (3.13b) constitute an unique solution of the equations of motion and continuity as formulated.

In regard to eqn.(3.12), it needs an extra boundary condition to solve the third-order, non-linear differential equation. Compare equations (3.12) with (3.11) subjected to the boundary condition (3.13c), gives

$$H(\eta) \propto F'(\eta)$$

or

$$H(\eta) = \frac{U(0)}{Q} F'(\eta) \quad (3.14)$$

where $U(0)$ is an arbitrary constant. If the suction or injection begins from $z = 0$, the $U(0)F'(\eta)$ will be the velocity profile at $z = 0$. Prior to obtaining some solutions for $F(\eta)$, the choice of typical velocity has to be considered.

For simplicity, it is generally assumed that the typical velocity is the magnitude of the total walls injection or suction.

$$bQ = bQ_b + aQ_a \quad (3.15)$$

Integrating eqn.(3.11), one obtains the form,

$$\eta F''' + F'' + R(F'^2 - FF'') = k \quad (3.16)$$

where k is the constant of integration. The general boundary conditions for the flow through porous walls with suction or injection can be written as

$$F(\eta_0) = -\alpha \quad F(1) = \beta \quad (3.17)$$

$$F'(\eta_0) = 0 \quad F'(1) = 0$$

where α and β are constants.

3.3. Representations of the Velocity Profiles

From eqn.(3.9) the velocity components in the axial and radial directions are given in terms of the dimensionless distance parameter η . The radial velocity component is

$$q_r = Q \frac{F(\eta)}{\sqrt{\eta}} \quad (3.18)$$

and the axial velocity component in the annulus is

$$\begin{aligned} q_z &= U(0) \left[1 - \frac{2Q}{U(0)} \left(\frac{z}{b} \right) \right] F'(\eta) \\ &= U(0) \left[1 - \frac{4R}{N} \left(\frac{z}{b} \right) \right] F'(\eta) \end{aligned} \quad (3.19)$$

where $R = \frac{bQ}{2\gamma}$, and $N = \frac{bU(0)}{\gamma}$.

3.4. The Effect of Porous Walls on Annulus Flow

a. The Pressure Drop

The pressure distribution in the annulus can be obtained after integrating eqns.(3.6a) and (3.6b) and by (3.9) and (3.10). One obtains the pressure drop in dimensionless form after some rearrangement

$$\begin{aligned} p_r^* &= \frac{p(z,1) - p(z,\eta)}{1/2 Q^2} \\ &= \frac{2\tilde{M}Q}{b} \left[F' - \frac{R}{2} \left(\frac{F^2}{\eta} \right) \right]_{\eta}^1 \end{aligned} \quad (3.20)$$

for the radial pressure variation and

$$p_z^* = \frac{p(0,\eta) - p(z,\eta)}{1/2 U(0)^2}$$

$$= \frac{8k}{N} \left(\frac{z}{b} \right) \left[1 - \frac{2R}{N} \left(\frac{z}{b} \right) \right] \quad (3.21)$$

for the axial pressure drop

b. The Skin Friction at the Wall

The coefficient of skin friction at the inner and outer walls of the annulus are indicated as C_{fi} and C_{fo} , respectively. The shear stresses acting on the inner wall and outer wall are defined as τ_i and τ_o , respectively.

$$\begin{aligned} \tau_i &= - (q_{z,r})_{\eta = \eta_o} \\ &= - \frac{2\tilde{\mu}U(0)}{b} \left[1 - \frac{4R}{N} \left(\frac{z}{b} \right) \right] \sqrt{\eta_o} F''(\eta_o) \end{aligned} \quad (3.22a)$$

$$\begin{aligned} \tau_o &= - (q_{z,r})_{\eta = 1} \\ &= - \frac{2\tilde{\mu}U(0)}{b} \left[1 - \frac{4R}{N} \left(\frac{z}{b} \right) \right] F''(1) \end{aligned} \quad (3.22b)$$

the coefficients of skin friction are then

$$\begin{aligned} C_{fi} &= \frac{2 |\tau_i|}{U(0)^2} \\ &= \frac{4\sqrt{\eta_o}}{N} \left| \left[1 - \frac{4R}{N} \left(\frac{z}{b} \right) \right] F''(\eta_o) \right| \end{aligned} \quad (3.23a)$$

$$\begin{aligned} C_{fo} &= \frac{2 |\tau_o|}{U(0)^2} \\ &= \frac{4}{N} \left| \left[1 - \frac{4R}{N} \left(\frac{z}{b} \right) \right] F''(1) \right| \end{aligned} \quad (3.23b)$$

IV. APPROXIMATE ANALYSIS

4.1. Quasilinearization Method

Quasilinearization may be viewed as an extension of Newton's method for the solution of algebraic equations to the solution of differential equations. The method can be applied to a system of linear and non-linear first-order ordinary differential equations.

Consider the vector equation

$$\frac{d\bar{x}}{d\eta} = f(\bar{x}) \quad (4.1)$$

with m final conditions

$$x_j(\eta_1) = x_{j\eta_1} \quad j = 1, 2, \dots, m \quad (4.2)$$

and $n - m$ ($m \leq \frac{n}{2}$) initial conditions

$$x_k(\eta_0) = x_{k\eta_0} \quad k = m+1, m+2, \dots, n \quad (4.3)$$

where \bar{x} is a vector composed of n -dependent variables in the system of equations, η the independent variable, and $f(\bar{x})$ a vector function that gives the derivatives of the dependent variable. If $f(\bar{x}) = f_i(x_1, x_2, \dots, x_n, \eta)$ is linear, equation (4.1) can be solved by using a step by step integration method. If $f(\bar{x}) = f_i(x_1, x_2, \dots, x_n, \eta)$ is non-linear and all of the boundary conditions use initial-value type, the Runge-Kutta-Gill method can be employed directly. Since $f_i(x_1, x_2, \dots, x_n, \eta)$ is non-linear and has a two point boundary type, posed by (3.11), the vector form of equation (4.1) is first linearized around point $(x_1, x_2, \dots, x_n, \eta)$ as follow:

$$\frac{d\bar{x}}{d\eta}^{(k+1)} = f(\bar{x}_k, \eta) + J(\bar{x}_k) (\bar{x}_{k+1} - \bar{x}_k) \quad (4.4)$$

where \bar{x} and f are in vector form and represent the vectors (x_1, x_2, \dots, x_n) and (f_1, f_2, \dots, f_n) respectively. The Jacobian matrix $J(\bar{x}_k)$ is defined by

$$J(\bar{x}_k) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}_k$$

The $(k+1)$ approximation, \bar{x}_{k+1} , to the solution is obtained by expanding the right-hand side about the k^{th} approximation \bar{x}_k and retaining the linear term as a recurrent form equation (4.4) can also be considered.

This equation is linear and one may use the usual way for solving the system of linear ordinary differential equations by starting with assumptions of the initial conditions.

A particular solution may be obtained by integrating the whole equation with the initial conditions

$$x_i(\eta_0) = P_i(\eta_0) = 0 \quad i = 1, 2, \dots, n \quad (4.5)$$

The homogeneous solutions are then made for each of n boundary conditions B_p with the initial conditions

$$x_i(\eta_0) = H_{ij}(\eta_0) = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad i, j = 1, 2, \dots, n \quad (4.6)$$

where δ_{ij} is the Kronecker delta

The general solution of eqn.(4.1) will be

$$x_i^{(k+1)}(\eta) = P_i(\eta) + \sum_{j=1}^n C_j H_{ij}(\eta) \quad i = 1, 2, \dots, n \quad (4.7)$$

If the particular solution is chosen to satisfy the $n-m$ known initial conditions, then there are only m remaining boundary conditions to satisfy, and only m homogeneous solutions are required, thus the general solution is reduced to

$$x_i^{(k+1)}(\eta) = P_i(\eta) + \sum_{j=1}^m C_j H_{ij}(\eta) \quad i = 1, 2, \dots, n \quad (4.8)$$

The constants of integration, C_j , can be determined from the m final conditions in each iteration

$$x_j^{(k+1)}(\eta_1) = x_j \eta_1 \quad j = 1, 2, \dots, m \quad (4.9)$$

Since the $n-m$ given initial conditions have been used in selecting the appropriate initial conditions of the particular solutions P_i , $i = m+1, m+2, \dots, n$. The number of equations to be integrated is obviously reduced from $(1+n)$ sets (one set of particular solution and n sets of homogeneous

solutions) to $(1+m)$ sets or less. Since for the case of $m \leq \frac{n}{2}$, one can treat the problem in the reverse direction, i.e. integrating from terminal state to save the computation. The approximation $x^{(k+1)}(\eta)$ is fed back into the next iteration to obtain the approximation $x^{(k+2)}(\eta)$, and the process is repeated until satisfactory convergence is secured. With a sufficiently good initial approximation, the solution of eqn.(4.8) converges quadratically and monotonically to the solution equation (4.1).

4.2. Solution for the Flow through a Porous Annulus

For convenience, the eqn.(3.11) can be converted to a system of simultaneous first-order ordinary differential equations. If

$$\begin{aligned} F &= x_1 \\ F' &= x_2 \\ F'' &= x_3 \\ F''' &= x_4 \\ F^{iv} &= x_5 \end{aligned} \tag{4.10}$$

the eqn.(3.11) becomes

$$\begin{aligned} \frac{dx_1}{d\eta} &= x_2 \\ \frac{dx_2}{d\eta} &= x_3 \\ \frac{dx_3}{d\eta} &= x_4 \\ \frac{dx_4}{d\eta} &= -\frac{1}{\eta} \left[2x_4 + R(x_2x_3 - x_1x_4) \right] \end{aligned} \tag{4.11}$$

The corresponding general boundary conditions are

$$\begin{aligned}
 x_1(\eta_0) &= -\alpha & x_1(1) &= \beta \\
 x_2(\eta_0) &= 0 & x_2(1) &= 0
 \end{aligned}
 \tag{4.12}$$

The recurrent relation for the system of equations as the eqn.(4.4) after some arrangement, can be written in matrix form

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix}_{k+1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{R}{\eta}x_4 & -\frac{R}{\eta}x_3 & -\frac{R}{\eta}x_2 & \frac{1}{\eta}(-2+Rx_1) \end{bmatrix}_k \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{k+1} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{R}{\eta}(x_2x_3 - x_1x_4) \end{bmatrix}_k$$

with the boundary conditions

$$\begin{aligned}
 x_1^{k+1}(\eta_0) &= -\alpha & x_1^{k+1}(1) &= \beta \\
 x_2^{k+1}(\eta_0) &= 0 & x_2^{k+1}(1) &= 0
 \end{aligned}$$

There are two initial conditions and two terminal conditions, if the particular solution $P_i(\eta_0)$ is selected as

$$P_i(\eta_0) = [-\alpha, 0, 0, 0]^T$$

Then, there are only two remaining homogeneous solutions

$$x_i^{(k+1)}(\eta) = P_i(\eta) + C_3 H_{i3}(\eta) + C_4 H_{i4}(\eta) \quad i = 1, 2, \dots, 4$$

where $H_{i3}(\eta_0)$ and $H_{i4}(\eta_0)$ are assumed as (4.6)

$$H_{i3}(\gamma) = [0, 0, 1, 0]^T$$

$$H_{i4}(\gamma) = [0, 0, 0, 1]^T$$

Two constants C_3 and C_4 can be determined from the final conditions in each iteration.

$$x_1^{(k+1)}(1) = \beta$$

$$x_2^{(k+1)}(1) = 0$$

V. NUMERICAL RESULTS AND DISCUSSION

The numerical results are graphically illustrated in Fig.(5) through (12) in dimensionless form.

Fig.(5). through (8) are examples of the axial velocity profiles for the flow through a porous annulus with different suction and injection rates, at the position $z/b = 10$ and the axial flow Reynolds number $N = 1000$ for the cases as shown in Fig.(4):

- i). Outer porous wall with inner impermeable tube

$$\alpha' = 0$$

$$\beta = 1$$

- ii). Inner porous wall with outer impermeable tube

$$\alpha' = 1$$

$$\beta = 0$$

- iii). Both inner and outer walls have equal permeable rates

$$\alpha' = 0.5$$

$$\beta = 0.5$$

Fig.(5) and Fig.(6) show the effect on the flow caused by suction, and Fig.(7) and Fig.(8) show the effect of the flow caused by the injection in an annulus.

The mean axial velocity \bar{q}_z is defined as

$$\begin{aligned} \bar{q}_z &= \frac{\int_a^b r q_z dr}{\int_a^b r dr} = \frac{1}{1 - \eta_o} \int_{\eta_o}^1 q_z d\eta \\ &= \frac{(\alpha' + \beta)U(0)}{1 - \eta_o} \left[1 - \frac{4R}{N} \left(\frac{z}{b} \right) \right] \end{aligned}$$

Hence, the axial velocity component is given in terms of the mean velocity, thus

$$q_z^* = q_z / \bar{q}_z = \frac{1 - \eta_0}{\alpha + \beta} F'(\eta) \quad (\alpha + \beta) \neq 0$$

The velocity profile as shown here are independent of the axial flow Reynolds number N and the location of the annulus.

The effect of porous walls on the pressure drop in the axial flow direction and the coefficient of skin friction are shown in Fig.(9) and (10), respectively.

The previous effort provided a quasilinearization technique for solving the flow equations and indicated the application of the technique to the flow through a porous annulus. For further analysis, it is necessary to make some assumptions about the ratio of the inner and outer walls of the annulus. This would involve either letting $\eta_0 \rightarrow 0$ and obtaining a solution as a flow through a porous pipe, or letting $\eta_0 \rightarrow 1$ representing the flow between porous parallel plates.

The flow in an annulus is particularly interesting in that, as the radius ratio tends to one, i.e., $\eta_0 \rightarrow 1$ the properties of the flow approach those for the fluid between two porous plates. On the other hand, for small values of the radius ratio and an impermeable inner wall the flow behaves like the flow through a porous pipe. This conflicts with the boundary conditions, $F'(\eta_0) = 0$, since the boundary conditions for flow through porous pipe are

at $\eta = 0$	$\lim_{\eta \rightarrow 0} \frac{F(\eta)}{\sqrt{\eta}} = 0$	$\lim_{\eta \rightarrow 0} \sqrt{\eta} F''(\eta) = 0$
at $\eta = 1$	$F(1) = 1$	$F'(1) = 0$

The equations of motion and continuity are associated with the boundary conditions including two initial conditions of limiting forms and two terminal conditions for the porous pipe. To try to obtain numerical solutions by using the quasilinearization technique would be extremely complicated due to the limiting forms appeared in the two initial boundary conditions. An analytical study of this topic has been obtained by many classical methods. Yuan and Finkelstein (4) indicated the correct separation of variables for a uniformly porous tube and gave perturbation solution valid for small values of the cross flow Reynolds number R , based on the normal velocity at the walls. Also, an asymptotic solution to the first power of the reciprocal of the cross flow R was employed, and is valid for large values of the injection velocity at the wall.

The computations for the flow through a porous annulus were programmed in Fortran IV and the solutions were carried out on an IBM 360/50 computer (see Appendix). The results for the small cross-flow Reynolds number $R = 1$ to $R = 2$ are in excellent agreement with Terrill's (7). The discrepancy becomes greater as the value of R increases up to ± 5 , (see Fig.11). Convergence of the quasilinearization process was tested by comparing the values of x_i^k and x_i^{k+1} at the point of F and F' in the problem of a porous annulus, is shown in Fig.(12). The initial approximation is certainly uninspired. The second approximation and succeeding approximations converge rapidly from that side. No instability is shown by the final profiles, except that the flow itself is unstable.

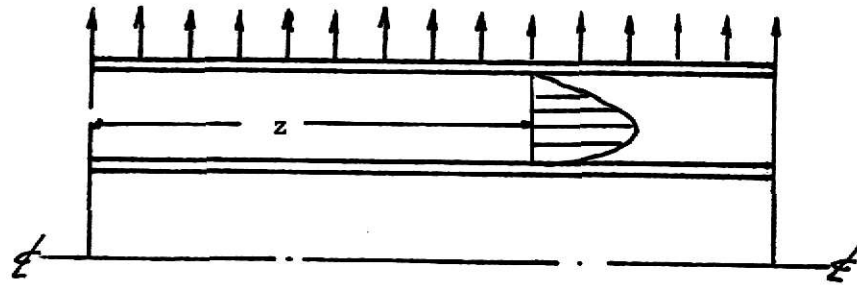


Fig.4a. Case (i)

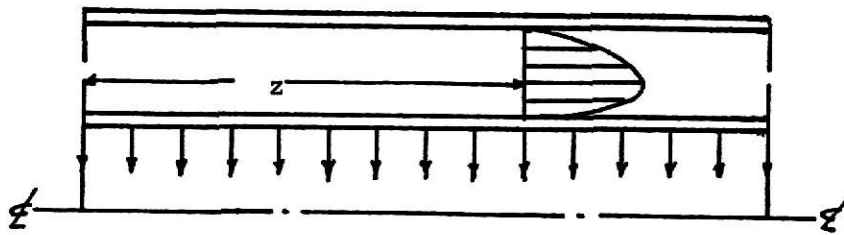


Fig.4b. Case (ii)

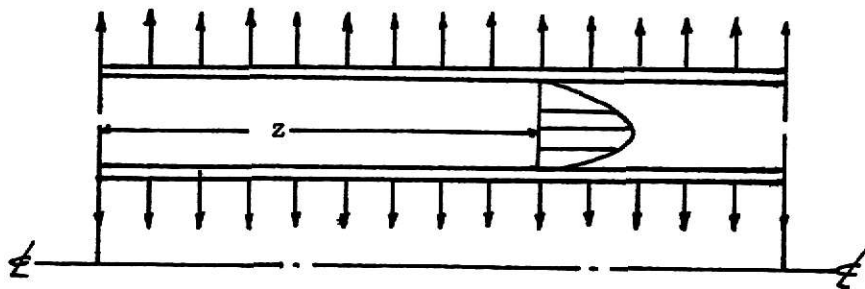


Fig.4c. Case (iii)

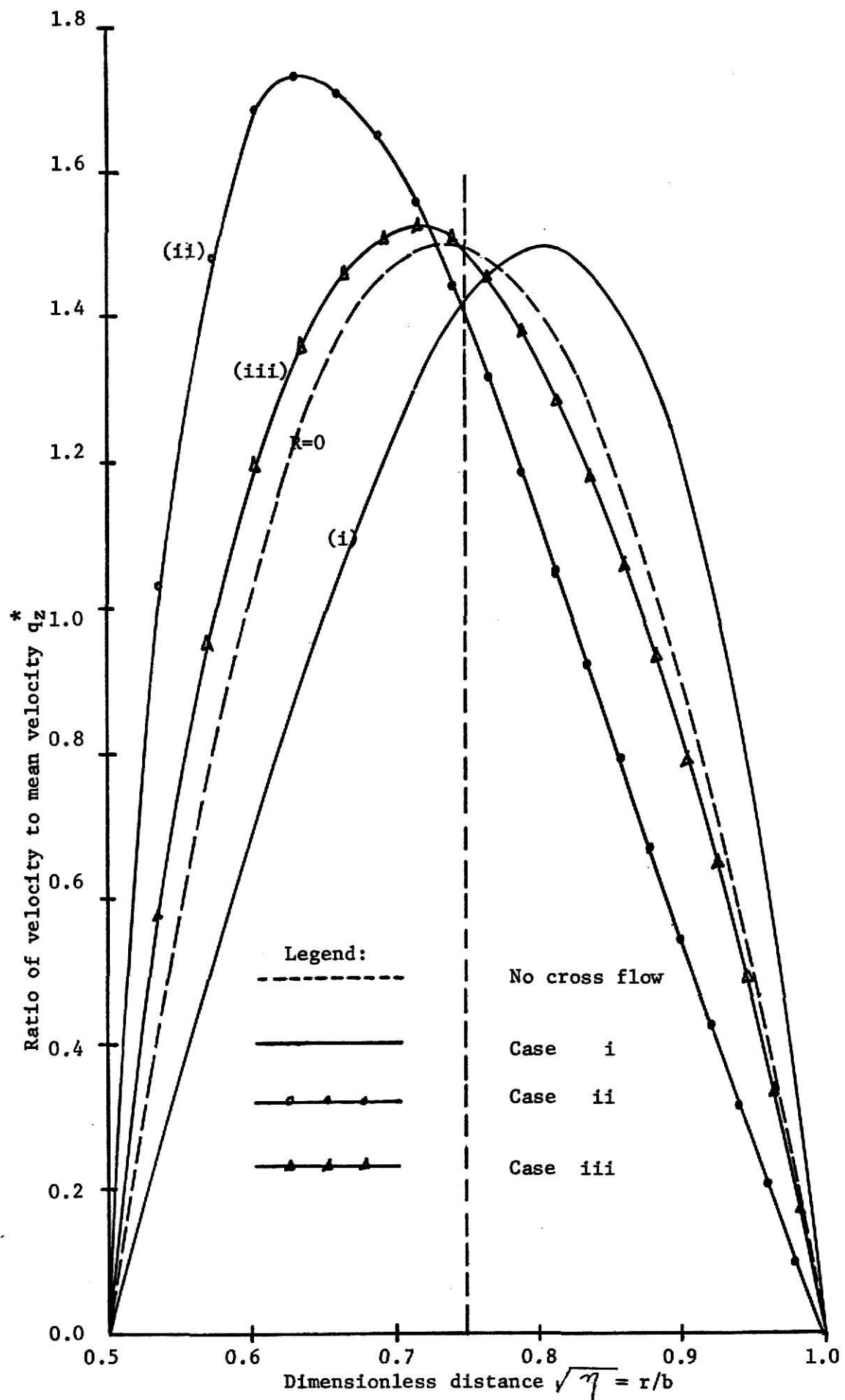
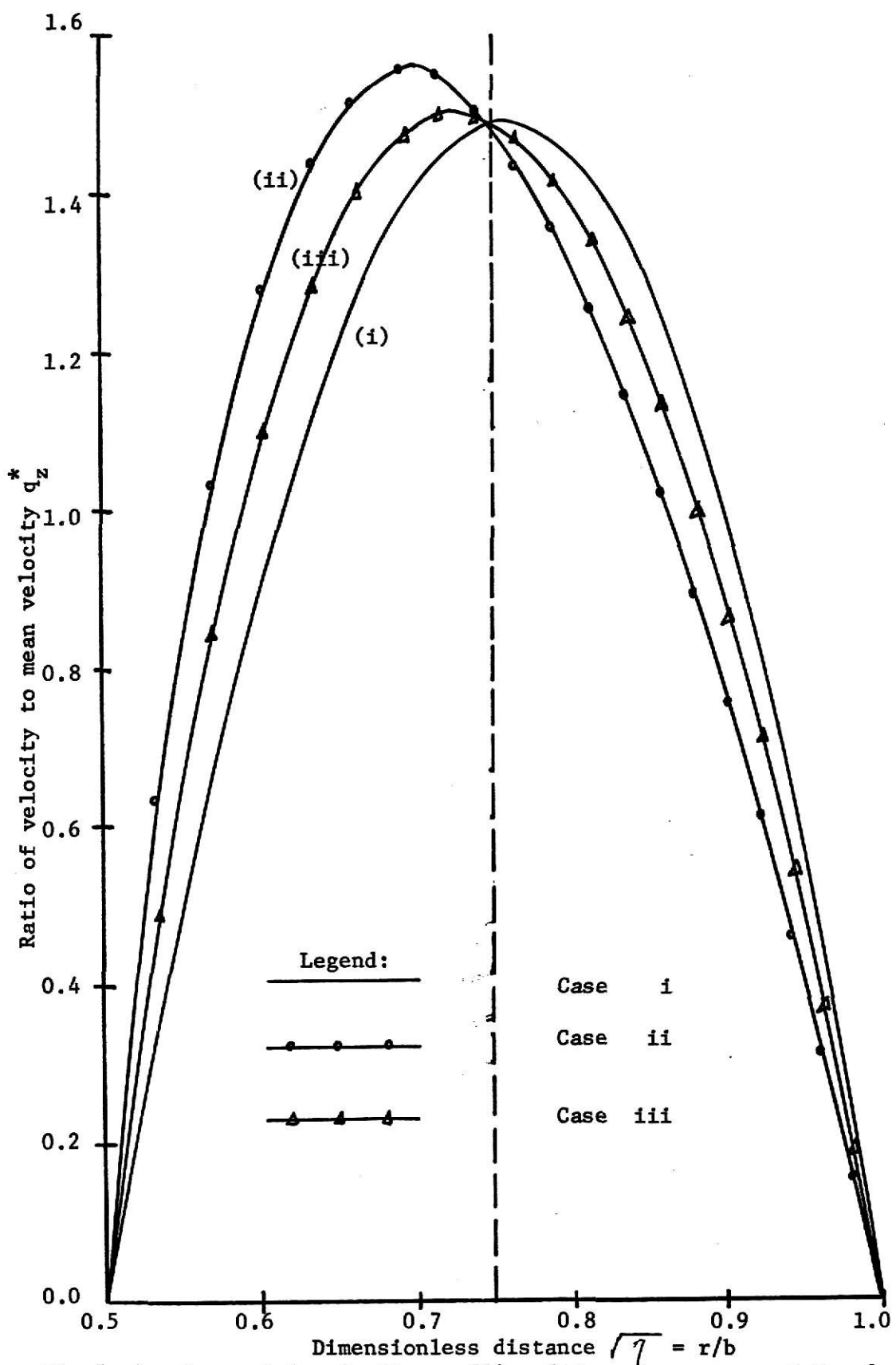
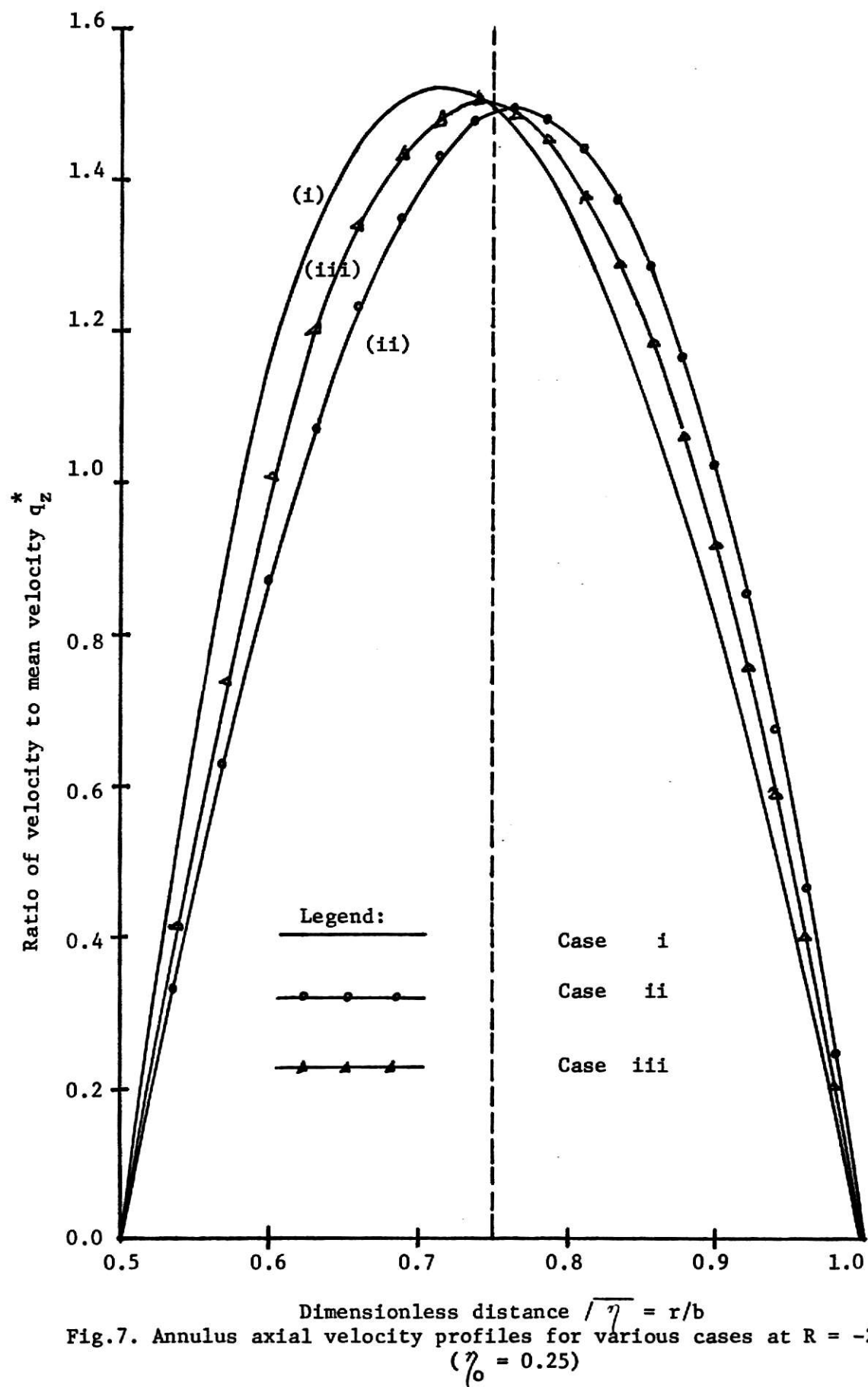


Fig.6. Annulus axial velocity profiles for various cases at $R = 5$
 $(\eta_0 = 0.25)$





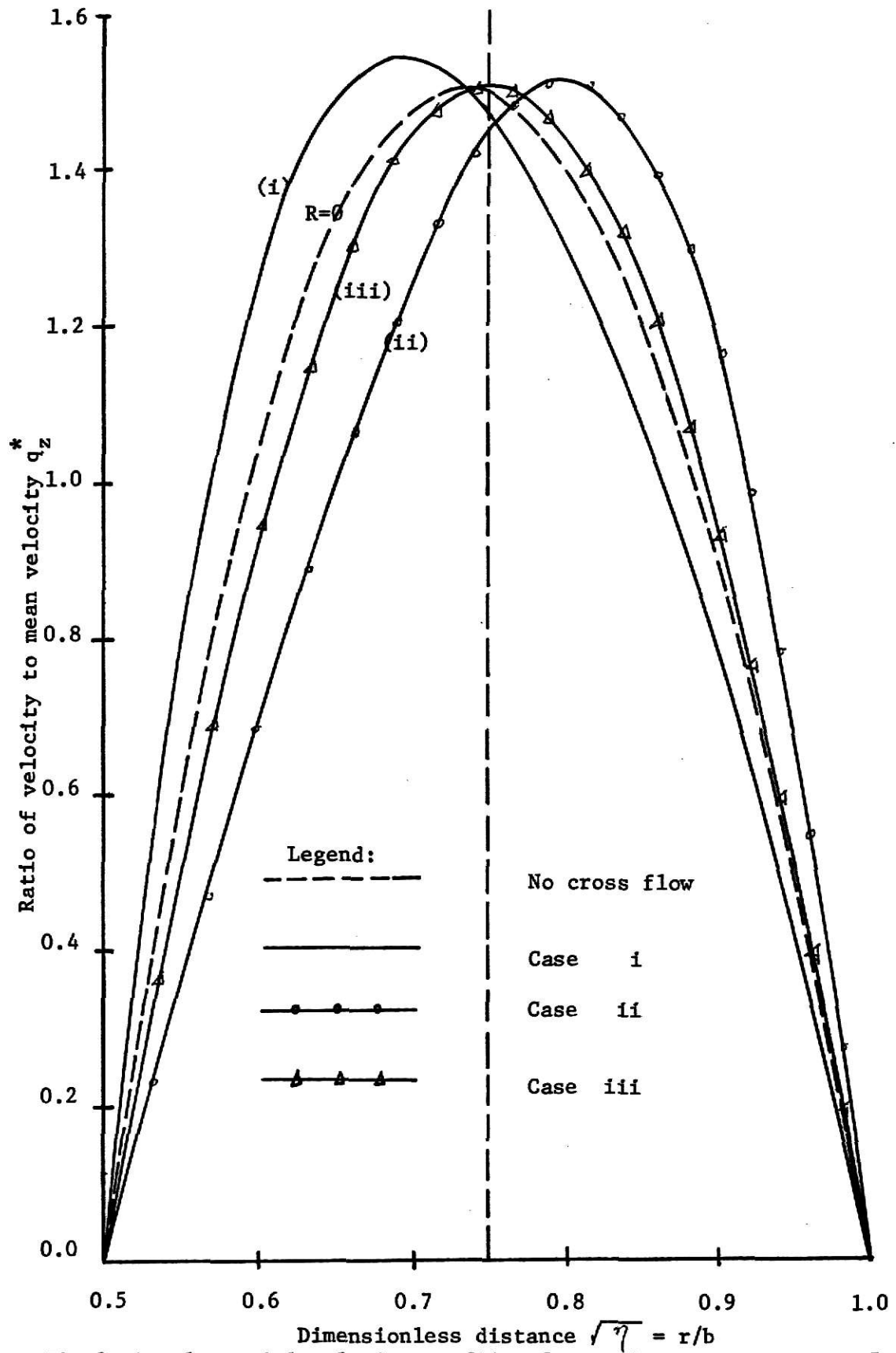


Fig.8. Annulus axial velocity profiles for various cases at $R = -5$
 $(\eta_0 = 0.25)$

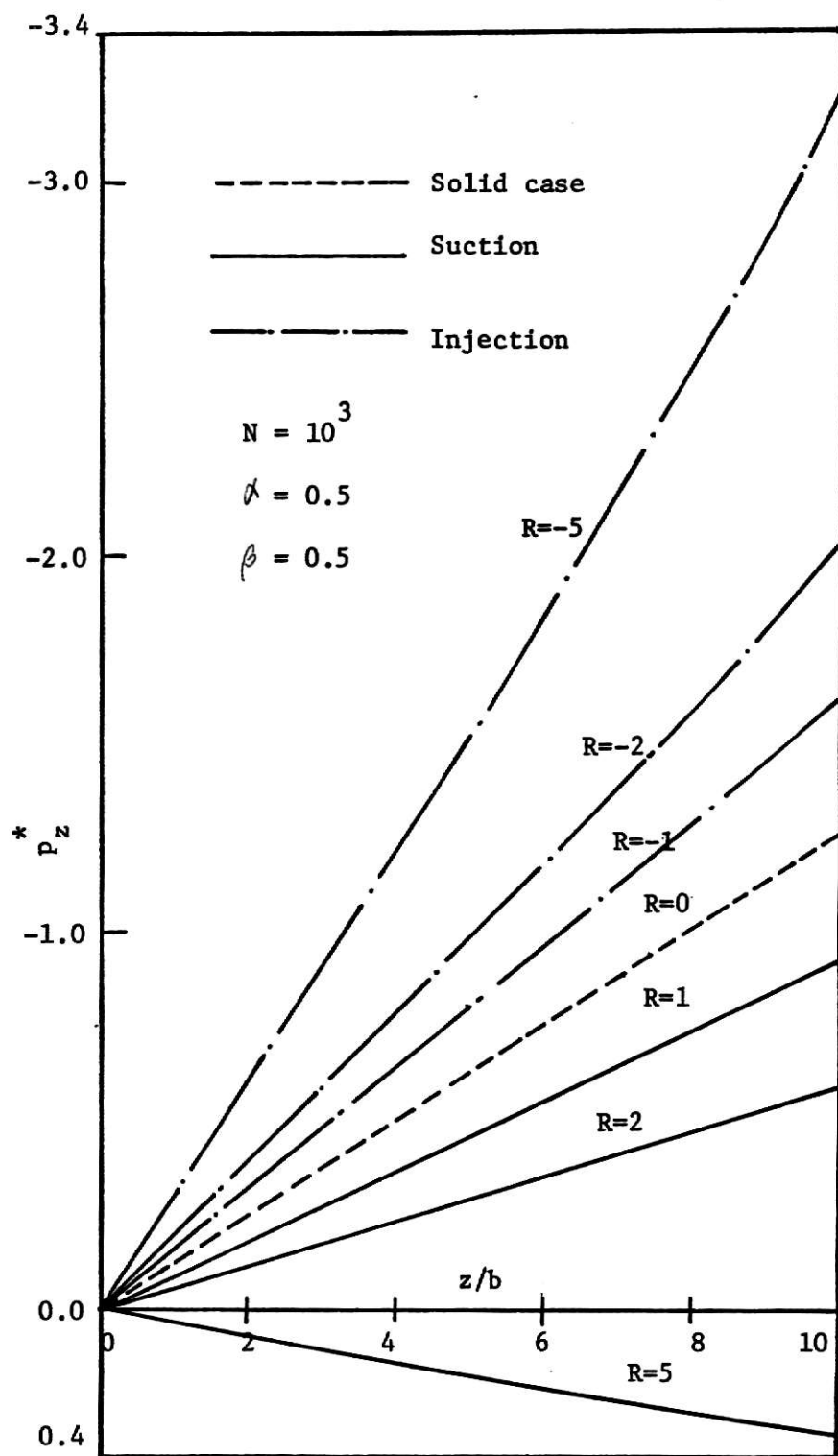
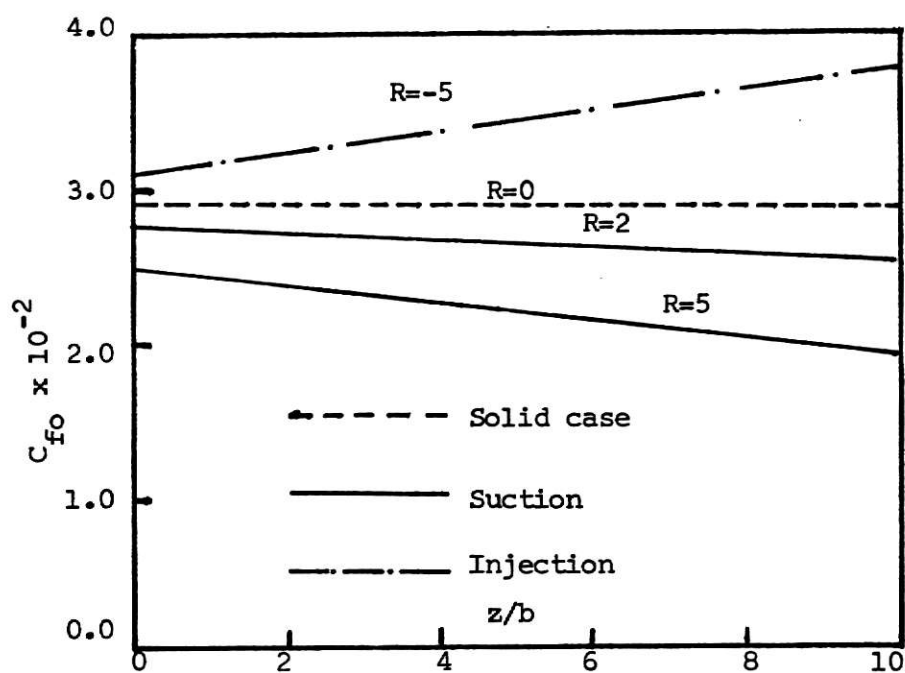
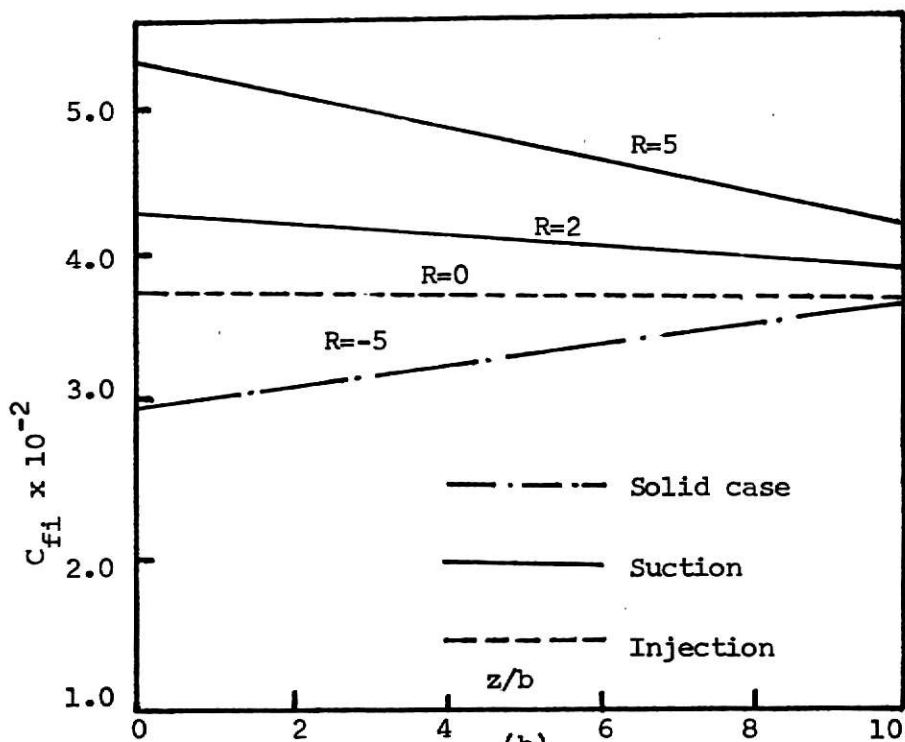


Fig.9. Axial pressure drop vs length in flow direction for various R



(a)



(b)

Fig.(10). Variation of wall frictional coefficient in flow direction for various R ($N = 10^3$, $\alpha = 0.5$, $\beta = 0.5$)

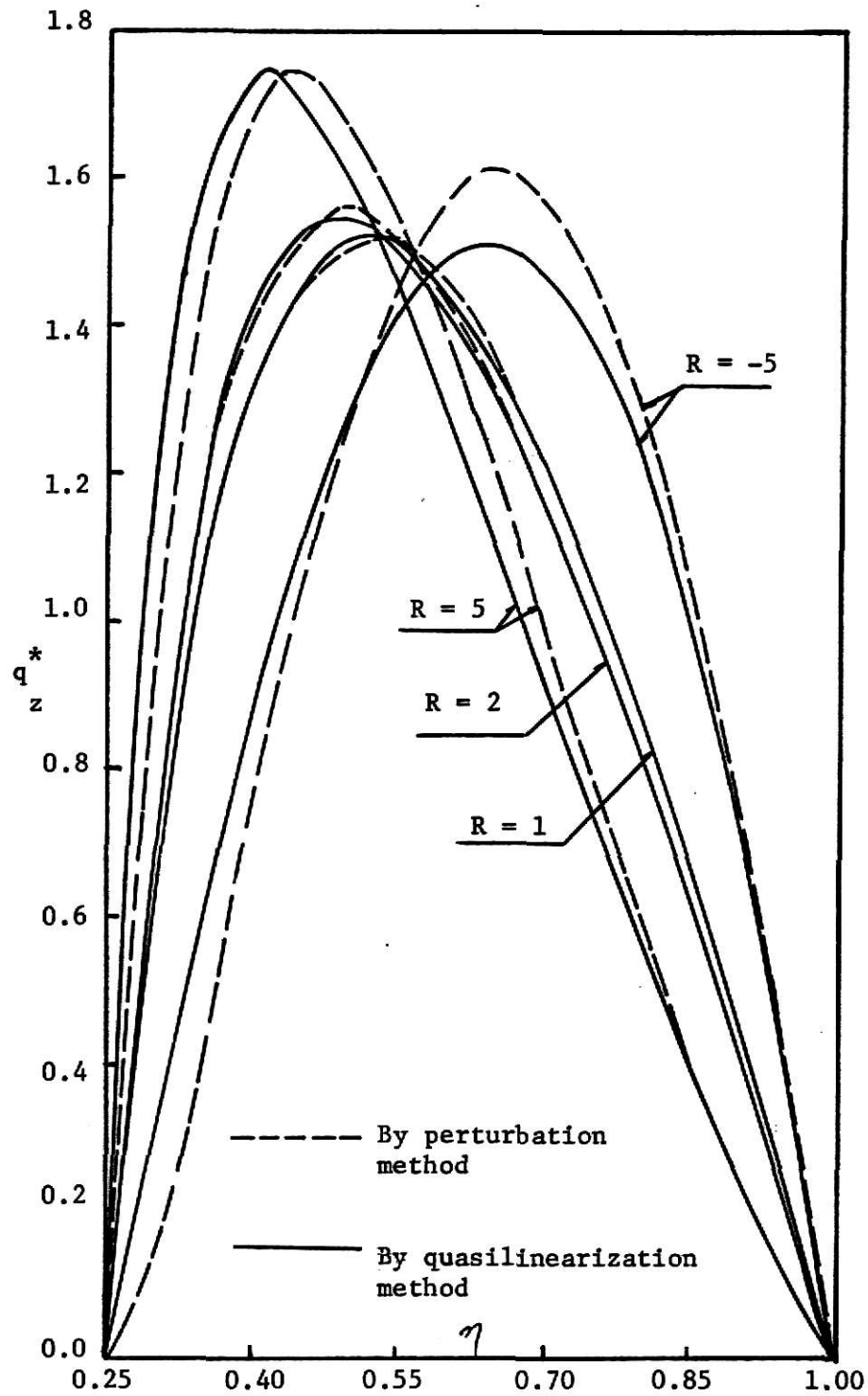


Fig.11. The comparison of velocity profiles for the case of $\eta_0 = 0.25$, $\alpha = 1.0$ and $\beta = 0.0$ by using the method of quasilinearization and the perturbation

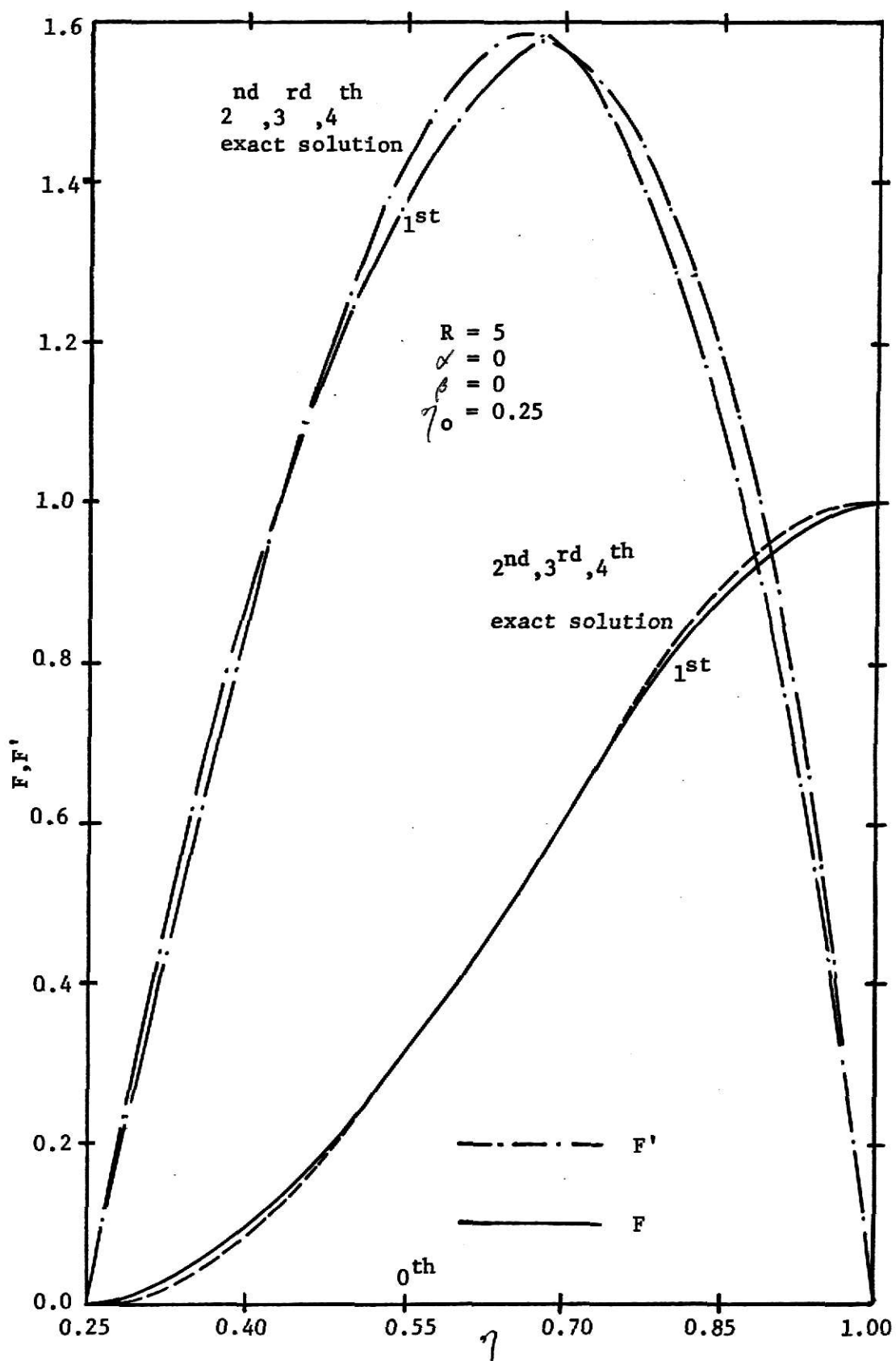


Fig.12. Profiles of F , F' for flow through a porous annulus

VI. CONCLUSION AND REMARKS

With the assumptions regarding the flow conditions listed in III, a solution of Navier-Stokes equations has been obtained for the case of the steady state flow of an incompressible fluid in an annulus with porous walls. The non-linear problem, which is so complicated that the solutions are difficult to obtain analytically, the approach of quasilinearization is introduced.

The velocity distribution in the main flow direction at any arbitrary cross-section of the annulus can be calculated from eqn.(3.19). In the case of the solid wall annulus, the axial velocity is not symmetrical and the maximum velocity occurs closer to the inner wall of the annulus.

When fluid flows through an annulus with porous outer wall, the maximum velocity is closer to the outer wall when the suction ($R > 0$) occurs and closer to the inner wall when the injection occurs. For higher injection rate, the velocity q_z^* is increased slightly and maximum axial velocity occurs at a point closer to the inner annulus wall. In this case, the flow appears stable.

In the case of flow through an annulus with porous inner annulus wall, the profiles of the axial component of velocity appear stable at the conditions of moderate injection ($R < 0$) and the lower rate of suction ($R > 0$). The axial velocity profiles increase rapidly while the higher suction occurs. The point of the maximum velocity was closer to the inner annulus wall, and the velocity profile soon developed an inflection point suggesting flow instability.

For the case of injection and suction at both walls, it is interesting to note that the velocity profiles might be considered as the combination of cases (i) and (ii) for the small rates of suction and injection. But it

cannot always hold true as the cross flow Reynolds number increased.

The cross flow Reynolds number required to produce the inflection point in the profiles varies as the radius ratio becomes greater. The difference of the axial velocity component for the above cases becomes more exaggerated as the radius ratio of the annulus is decreased.

The pressure distributions in the main flow direction are shown in Fig.(9). Considered the flow through the solid wall annulus, $R = 0$. It was found that the pressure drop in the axial direction is a linear case with the variation of (z/b) and the pressure drop became appreciably larger, even for very small fluid injection at the walls of the annulus, than that in the case of flow through the solid wall annulus. The pressure distributions are also greatly affected by the ratio of the radii η_0 . When the ratio of the radii η_0 is small (or large) the change of the corresponding pressure is small (or large), even in the case of solid wall annulus.

The values assigned to (z/b) are limited by the fact that at some value of z/b in the annulus, all the fluid has been removed through the annulus walls. From eqn.(3.21), it can be shown that the range of allowable values for z/b is

$$0 \leq \frac{z}{b} \leq \frac{N}{2R}$$

One of the most practical applications is the skin friction coefficient at the wall. In solid annulus pipe flow, the skin friction at the inner wall and the outer wall are defined as C_{fi} and C_{fo} , respectively and both of them have the constant value of $\frac{4/\eta_0}{N} |F''(\eta_0)|$ and $\frac{4}{N} |F''(1)|$. The wall frictional coefficients as calculated from eqn.(3.23). It is indicated that the effect of injection ($R < 0$) in an annulus flow is to increase the wall frictional coefficients, and the suction ($R > 0$) to decrease the wall frictional coefficients.

Finally, in an attempt to solve the problems of the flow through the annulus with porous wall, the quasilinearization method has been widely used in solving the non-linear differential equations for any arbitrary cross flow R , except in a few limiting cases that have been stated previously. The method used in this report provides the following advantages.

1. The procedure of calculation is simple and usually attacked by numerical integration using the digital computer.

2. The method gives promise of producing rapid convergence to solution of unstable flow problems from arbitrary initial guesses. Iterative solution of the resulting linear differential equation usually converges quadratically to the solution of the original equation. The proof was given by Bellman and Kalaba (8).

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ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to those who have assisted and encouraged him during the course of this work. He wishes to thank his parents Mr. and Mrs. Chien, and his brothers for their guidance, support, and encouragement, and his wife, Molly, for her patience, understanding and constant encouragement from the land ten thousand miles away from here.

Finally, the author wishes to thank Dr. C. L. Huang for bringing this problem to his attention, for guiding and directing him during the course of this work, and for assisting him, and to the members of his committee and also to Dr. K. K. Hu for their guidance and constructive criticism during the preparation of this report. Thanks are also extended to Miss G. E. Givins for reviewing the manuscript. Dr. Huang's constant enthusiasm and dedication provide an inspiration for which this author is indeed grateful.

APPENDIX
COMPUTER PROGRAM


```

C      FLOW THROUGH POROUS ANNULLS BY QUASI-LINEAR METHOD
C      MAIN PROGRAM
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      DIMENSION PSTAR(3),CF1(3),CF2(3)
      DIMENSION Q(4),Y(101,4),YH1(4),YH2(4),YP(4),S(101),D1(101,4)
      DIMENSION VZ(101,3),RA(3),RB(3)
      DIMENSION RR(7),AK(101,4)
      DIMENSION D2(101,4),E(101,4),A(2,4),B(4),G(3),C(2),ER(101),DY(4)
1  FORMAT(////,10X,'ETA=',F10.5,/)
2  FORMAT(10X,'REYNOLDS NUMBER=',F10.5,/)
3  FORMAT(20X,'AXIAL VELOCITY PROFILE U/UAVE',/,27X,'AT Z/B=10 N=1000'
1',/)
4  FORMAT(12X,'CASE I',12X,'CASE II',11X,'CASE III',/)
5  FORMAT(6X,3D18.8)
6  FORMAT(//,20X,'CONSTANT OF INTEGRATION',/)
7  FORMAT(//,20X,'PRESSURE DROP IN AXIAL DIRECTION',/)
8  FORMAT(F10.5)
9  FORMAT(2X,2F10.5)
21 FORMAT(//,20X,'SKIN FRICTION',/)
22 FORMAT(6X,3D18.8,/)
23 FORMAT(6X,3D18.8,/////////)
      RT=10.0
      REN=1000.0
      LL=2
      L=41
      NN=7
      DIV=L-1
      READ(5,8)(RR(K),K=1,NN)
      DO 59 I=1,3
      READ(5,9) RA(I),RB(I)
59  CONTINUE
      DO 1005 JX=1,3
      READ(5,9) ETA,AINIT
      H=(1.0-ETA)/DIV
      DO 1000 K=1,NN
      RE=RR(K)
C      INITIAL CONDITIONS
      DO 80 J=1,L
      DO 82 I=1,4
82  Y(J,I)=AINIT
80  CONTINUE
      DO 1001 JC=1,3
      ALPHA=RA(JC)
      BELTA=RB(JC)
      NO=C
81  NO=NO+1
      DO 86 I=1,L
86  S(I)=Y(I,1)
C      RUNGE-KUTTA-GILL INTEGRATION
C      INITIAL APPROXIMATION
      DO 83 I=1,4
83  YH1(I)=0.0

```



```

      YH1(3)=1.0
      DO 84 I=1,4
84    YH2(I)=0.0
      YH2(4)=1.0
      DO 85 I=1,4
85    YP(I)=0.0
      YP(1)=ALPHA
88    DO 11 I=1,4
11    Q(I)=0.0
      DO 91 I=1,4
91    D1(1,I)=YH1(I)
      X=ETA
      DO 92 I=2,L
      II=I-1
      DO 111 J=1,4
111   AK(II,J)=Y(II,J)
      CALL RKG(X,H,YH1,Q,DY,AK,II,RE,1)
      DO 93 M=1,4
93    D1(I,M)=YH1(M)
92    CONTINUE
      DO 94 I=1,4
94    D2(1,I)=YH2(I)
      DO 12 I=1,4
12    Q(I)=0.0
      X=ETA
      DO 95 I=2,L
      II=I-1
      DO 112 J=1,4
112   AK(II,J)=Y(II,J)
      CALL RKG(X,H,YH2,Q,DY,AK,II,RE,1)
      DO 96 M=1,4
96    D2(I,M)=YH2(M)
95    CONTINUE
C
C    PART SOLM
      DO 201 I=1,4
201   E(1,I)=YP(I)
      DO 13 I=1,4
13    Q(I)=0.0
      X=ETA
      DO 202 I=2,L
      II=I-1
      DO 113 J=1,4
113   AK(II,J)=Y(II,J)
      CALL RKG(X,H,YP,Q,DY,AK,II,RE,2)
      DO 203 M=1,4
203   E(1,M)=YP(M)
202   CONTINUE
301   A(1,1)=D1(L,1)
      A(1,2)=D2(L,1)
      A(2,1)=D1(L,2)
      A(2,2)=D2(L,2)

```



```

302 B1=BELTA-E(L,1)
    B2=-E(L,2)
305 DET=A(1,1)*A(2,2)-A(1,2)*A(2,1)
    C(1)=(A(2,2)*B1-A(1,2)*B2)/DET
    C(2)=(-A(2,1)*B1+A(1,1)*B2)/DET
    DO 210 I=1,L
    DO 208 J=1,4
208 Y(I,J)=E(I,J)+C(1)*D1(I,J)+C(2)*D2(I,J)
210 CONTINUE
    P=C.0001
    DO 211 I=1,L
    ER(I)=CABS(Y(I,1)-S(I))
    IF(NO.EQ.15)GC TO 1003
    IF (ER(I).GT.P) GO TO 81
211 CONTINUE
    X=ETA
    DO 212 I=1,L,LL
350 VZ(I,JC)=(1.0-ETA)*Y(I,2)
403 X=X+LL*H
212 CONTINUE
    G(JC)=Y(L,4)+Y(L,3)+RE*(Y(L,2)**2-Y(L,1)*Y(L,3))
    PSTAR(JC)=8.0*G(JC)*RT*(1.0-2.0*RE*RT/REN)/REN
    CF1(JC)=4.0*DSQRT(ETA)*(1.0-4.0*RE*RT/REN)*Y(1,3)/REN
    CF2(JC)=4.0*(1.0-4.0*RE*RT/REN)*Y(L,3)/REN
1001 CONTINUE
1003 WRITE(6,1)ETA
    WRITE(6,2) RE
    WRITE(6,3)
    WRITE(6,4)
    DO 213 I=1,L,LL
    WRITE(6,5)(VZ(I,JC),JC=1,3)
213 CONTINUE
    WRITE(6,6)
    WRITE(6,5)(G(JC),JC=1,3)
    WRITE(6,7)
    WRITE(6,5)(PSTAR(JC),JC=1,3)
    WRITE(6,21)
    WRITE(6,22)(CF1(JC),JC=1,3)
    WRITE(6,23)(CF2(JC),JC=1,3)
1000 CONTINUE
1005 CONTINUE
    STOP
    END

```

C
C

```

SUBROUTINE DERIV(X,H,Y,DY,K,K,RE,JC)
IMPLICIT REAL*8(A-H,U-Z),INTEGER(I-N)
DIMENSION Y(4),DY(4),W(10,4)
DO 31 I=1,3
31 DY(I)=Y(I+1)
35 DY(4)=(RE*(Y(1)*W(K,4)-Y(2)*W(K,3)-Y(3)*W(K,2)+Y(4)*W(K,1))
    1-2.0*Y(4))/X

```



```

      IF (JC.NE.2) GO TO 37
36  DY(4)=(RE*(Y(1)*W(K,4)-Y(2)*W(K,3)-Y(3)*W(K,2)+Y(4)*W(K,1))
      1-2.0*Y(4)+RE*(W(K,2)*W(K,3)-W(K,1)*W(K,4)))/X
37  RETURN
      END
      SUBROUTINE RKG(X,H,Y,Q,DY,W,K,RE,JC)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      DIMENSION Y(4),DY(4),Q(4),A(2),W(101,4)
      A(1)=0.2928321881345
      A(2)=1.7071067811865
5    H2=0.5*H
      CALL DERIV(X,H,Y,DY,W,K,RE,JC)
      DO 10 I=1,4
      R=H2*DY(I)-Q(I)
      Y(I)=Y(I)+R
10   Q(I)=Q(I)+3.0*R-H2*DY(I)
      X=X+H2
      DO 20 J=1,2
      CALL DERIV(X,H,Y,DY,W,K,RE,JC)
      DO 30 I=1,4
      R=A(J)*(H*DY(I)-Q(I))
      Y(I)=Y(I)+R
30   Q(I)=Q(I)+3.0*R-A(J)*H*DY(I)
20  CONTINUE
      X=X+H2
      CALL DERIV(X,H,Y,DY,W,K,RE,JC)
      DO 40 I=1,4
      R=(H*DY(I)-2.0*Q(I))/6.0
      Y(I)=Y(I)+R
40   Q(I)=Q(I)+3.0*R-H2*DY(I)
      RETURN
      END

```


LAMINAR FLOW THROUGH AN ANNULUS
WITH POROUS WALLS

by

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Diploma, Taipei Institute of Technology
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AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Applied Mechanics

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1972

The purpose of this study is to present an analysis of a steady, laminar incompressible flow through an annulus with suction and injection velocity at the walls. The exact solution of the Navier-Stokes equations reduced to fourth-order non-linear differential equations with appropriate boundary conditions are obtained. A quasilinearization method was used to solve the latter equations for an arbitrary flows through the porous walls.

The development of essential equations of the incompressible flow is presented first. A flow through porous annulus is then studied. The velocity components, the pressure distributions, and the coefficients of wall friction are expressed as functions of the ratio of velocity through the walls, on position coordinates, annulus dimensions, and fluid properties.