THE CONTROL OF MASS FLOW RATE IN A BLOWDOWN TYPE WIND TUNNEL

by 680

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NOMENCLATURE

AV_i	ith actuator and valve (i = 1, 2 and 3)
c_p	Specific heat of air at constant pressure (BTU/lbm/°R)
C_s	Constant
c_v	Specific heat of air at constant volume (BTU/lbm/°R)
C_v	Water capacity of valve for 1 psi drop (gallons/minute)
k	c_p/c_v
K_s, K_v	Proportionality constants (functions of area of valve opening and coefficient of discharge)
M_r	Mass of air at any instant in low pressure tank (lbm)
M_s	Mass of air at any instant in high pressure tanks (lbm)
P_2, P_3	Pressure settings of second and third pressure switches (psia)
P_a	Pressure signal from controller (psia)
P_r	Pressure at any instant in low pressure tank (psia)
P_s	Pressure at any instant in high pressure tanks (psia)
P_{set}	Set pressure of controller (psia)
R	Gas constant for air (ft lbf/lbm/°R)
S_i	ith pressure switch (i = 1, 2 and 3)
t	time (seconds)
T_2	Temperature of air at any instant entering low pressure tank (°R)
T_s	Temperature of air at any instant in high pressure tank (°R)
T_r	Temperature at any instant in low pressure tank (°R)
V_r	Volume of low pressure tank (ft^3)
V_s	Volume of high pressure tank (ft^3)
W	Mass flow rate of air out at any instant from low pressure tank (lbm/second)

W_1 Mass flow rate of air out at any instant from high pressure tanks (lbm/second)

X_i Valve travel at any instant of ith valve (ft.)

X_{max} Maximum valve travel of all three valves (ft.)

CHAPTER I

INTRODUCTION

In the missile site situated near Wamego, Kansas, which is now being changed into a laboratory by the Mechanical Engineering Department of Kansas State University, there is a system which was used for filling the missiles with liquid oxygen. Now this system is to be used for blowing air at a constant supersonic speed in a wind tunnel.

The existing system is shown schematically in Figure 1, page 2. Initially the high pressure and low pressure tanks are to be filled with air at 1600 and 150 psia respectively and the temperature in both allowed to stabilize at approximately $530^{\circ}R$. The problem is to maintain a constant pressure of 150 psia in the low pressure tank in order to supply the wind tunnel with a constant mass flow rate.

As indicated in Figure 1, S_1, S_2 and S_3 are three pressure switches and AV_1, AV_2 and AV_3 are three actuator valves. The opening and closing of these switches determines whether or not the controller signal is being fed to the corresponding actuator-valve. For example, if switch S_1 is open and switches S_2 and S_3 are closed then the controller signal is being fed only to the first actuator-valve and the others do not receive any signal, i.e. they are not in operation and are closed.

The control action takes place in the following manner. Suppose the mass flow rate into the low pressure tank increases then the pressure in this tank will tend to increase. This increase in pressure will be sensed

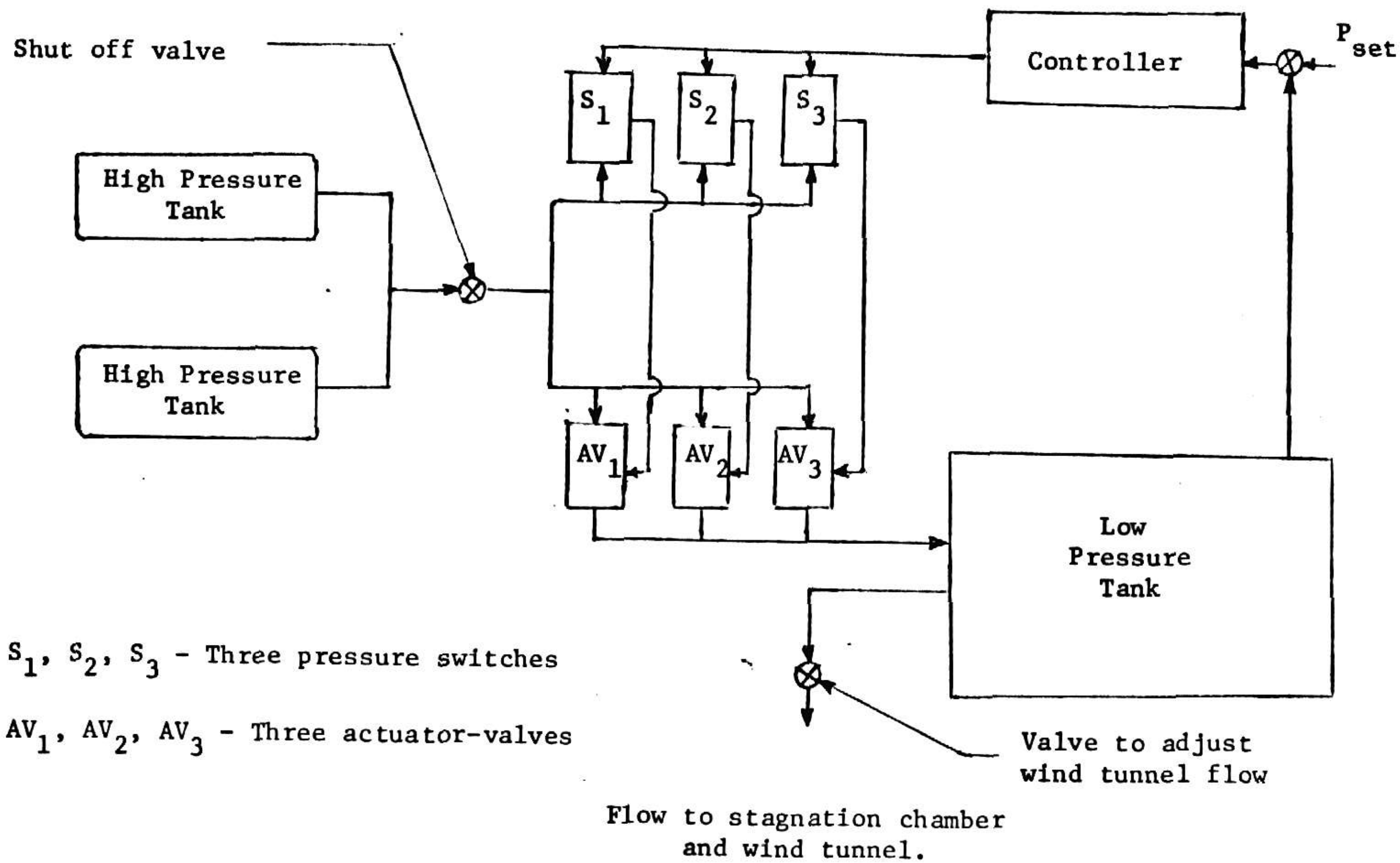

Figure 1. Schematic Diagram of the Existing System.

by the controller and it will send a signal to the actuator-valves* so that the total valve opening will be reduced. The control action is reversed when the mass flow rate into the low pressure tank decreases.

In the beginning, only one pressure switch, S_1, is open which means that the air is flowing through valve 1 only. As the pressure in the high pressure tank decreases it reaches a value at which switch S_2 is turned on automatically. Switch S_3 is automatically turned on in a similar manner when a certain lower pressure is reached.

As the air in the high pressure tanks expands its temperature decreases, therefore the mass flow rate from the high pressure tank to the low pressure tank will not stay constant. Actually the temperature will decrease most of the time. Therefore the mass flow rate out of the low pressure tank can not be kept exactly constant just by keeping the pressure, in this tank, constant.

In this report an effort has been made to determine the following:

1. How long can the mass flow rate from the low pressure tank be kept constant for various mass flow rates to the wind tunnel as given in Table 1, page 4?
2. What is the 'best' setting of the pressure switches S_2 and S_3 to keep the mass flow rate from the low pressure tank as constant as possible?
3. The overall stability of the system?

The answer to the questions asked can only be obtained by deriving and solving the equations which define the system mathematically.

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* The setting of the pressure switches will determine, to which actuator-valves the signal is being sent.

TABLE I

DESIRED FLOW AND PRESSURES FOR WIND TUNNEL

Test Section (Mach No.)	Stagnation Chamber Pressure (psia)	Mass flow rate (lbm/second)
1	30	25.2
1.5	30	19
2	35	17.5
2.5	56	17
3	85	16.9
3.5	117	16
4	153	13.7
4.5	185	9.4

CHAPTER II

DERIVATION OF EQUATIONS

A summary of the assumptions used to derive the system equations:

1. Kinetic energy of air is negligible compared to other energies at the inlet, outlet and inside of all tanks.
2. There is no significant change in the potential energy throughout the system.
3. The expansion of air in the high pressure tanks is reversible and adiabatic.
4. There is no heat transfer from the surroundings to the air or vice versa.
5. The pressure drop in the pipelines is negligible.
6. The expansion of the air in the valves is adiabatic.
7. The mixing of air in the low pressure tank is perfect and adiabatic.

A. High Pressure Tank

Applying the law of conservation of mass and energy at stations (1) and (2) (see Figure 2, page 7) and treating the expansion as reversible and adiabatic the following equation is obtained

$$\frac{(144\ P_s)^{\frac{k-1}{k}}}{T_s} = C_s \tag{1}$$

Since the kinetic energy at both stations is negligible and there is no change in potential energy, then

$$W_1\ c_p\ T_s + \frac{d}{dt}\left(M_s\ c_v\ T_s\right) = 0 \tag{2}$$

But

$$M_s\ T_s = \frac{144\ P_s\ V_s}{R}$$

Substituting this in equation (2) and solving for $\dot{P}_s$

$$\dot{P}_s = -\frac{W_1\ k\ T_s\ R}{144\ V_s} \tag{3}$$

Solving for T_s in equation (1) and substituting in equation (3)

$$\dot{P}_s = -\frac{W_1\ k\ (144\ P_s)^{\frac{k-1}{k}}\ R}{144\ C_s\ V_s} \tag{4}$$

Given that

$V_s = 2490\ ft^3$

$k = 1.4$

$R = 53.35\ ft.\ lbf/lbm/^{o}R$

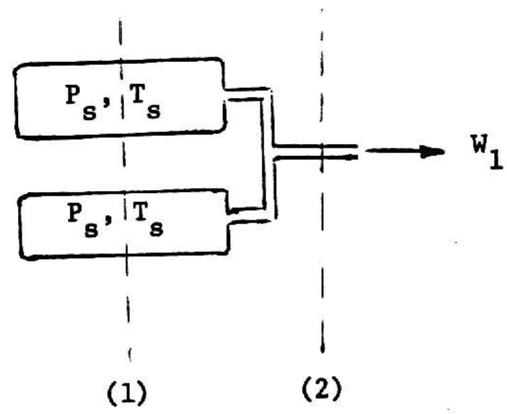

Figure 2. High Pressure Tanks.

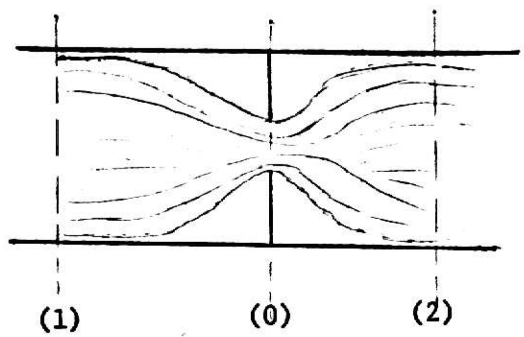

Figure 3. Idealized Flow Model of Valve.

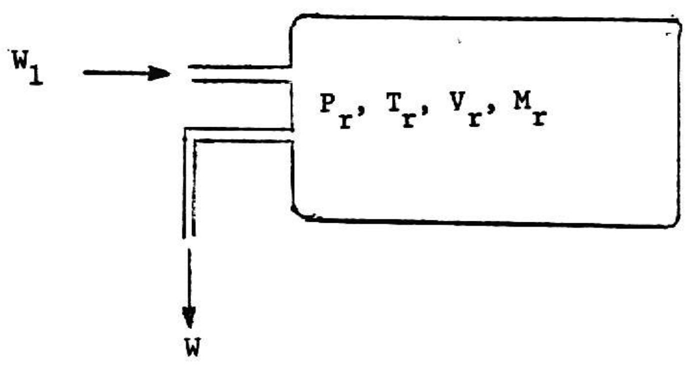

Figure 4. Low Pressure Tank.

and initially

$$P_s = 1600 \text{ psia}$$

$$T_s = 530^oR$$

equation (4) can be simplified as

$$\dot{P}_s = - 0.01345\, P_s^{0.286}\, W_1 \tag{5}$$

because

$$C_s = \frac{(144 \times 1600)^{\frac{1.4-1}{1.4}}}{530}$$

$$= 0.064$$

B. Flow Through Valves

The mass flow rate of a compressible fluid through any of the three valves is given by:

1. For critical flow ($P_r/P_s \leq 0.528$) (1) *

$$W_1 = K_s \frac{P_s}{\sqrt{T_s}} \tag{6a}$$

where K_s is a constant which is a function of the coefficient of discharge, the area of the valve opening and other constants.

2. For subcritical flow ($P_r/P_s \geq 0.528$) (2)

$$W_1 = 0.0204\, C_v \sqrt{\frac{(P_s + P_r)(P_s - P_r)}{T_s}} \tag{6b}$$

* Numbers in parentheses refer to list of references on page 26.

where

C_v is the water capacity of the valve for 1 psi drop in gallons/minute. When $P_r/P_s = 0.528$ the two expressions for W_1 are equal and it is possible to get an expression for K_s.

$$K_s = 0.01737\ C_v$$

The manufacturer of the valve, A. W. Cash Co., Decatur, Ill., was contacted to determine the value of C_v. They indicated that C_v is proportional to the valve travel and its maximum value is 19.5 for a maximum valve travel of 1.125 inches.

Therefore

$$K_s = 3.62\ (X_1 + X_2 + X_3) \tag{7}$$

for all three valves.

Since $C_s = 0.064$ equation (1) becomes

$$T_s = 64.4\ P_s^{0.286}$$

Substituting this and equation (7) in equation (6a) gives

$$W_1 = 0.45\ (X_1 + X_2 + X_3)\ P_s^{0.857} \tag{8}$$

If the law of conservation of energy is applied to stations far upstream and downstream of the valves it can be shown that (see Figure 3, page 7)

$$c_p\ T_1 = c_p\ T_2$$

or

$$T_1 = T_2$$

where

$$T_1 = T_s$$

and

T_2 = temperature of air entering low pressure tank, which implies that the overall flow of air through the valves is essentially a constant temperature process. That is to say that at any instant the temperatures of air at stations far upstream and far downstream are equal. This can be reasoned as follows. Stations (1) and (2) are situated far upstream and downstream respectively from the valves and station (0) is at the valve orifice. The flow of air from station (1) to (0) is reversible and adiabatic but the overall expansion process from station (1) to station (2) is a irreversible adiabatic and constant temperature process, because whatever kinetic energy is gained at the expense of enthalpy from station (1) to (0) is again converted back to enthalpy during the process (0) to (2).

C. Low Pressure Tank

Applying the law of conservation of mass to the low pressure tank (see Figure 4, page 7)

$$\dot{M}_r = W_1 - W$$

or

$$\frac{d}{dt}\left(\frac{144\ P_r\ V_r}{R\ T_r}\right) = W_1 - W$$

Since

$$V_r = 3476\ ft^3$$

and

$$R = 53.35 \text{ ft lbf/lbm/}^{o}\text{R}$$

Therefore

$$\dot{T}_r = \frac{T_r}{P_r}\left[\dot{P}_r - \frac{T_r\,(W_1 - W)}{9360}\right] \tag{9}$$

Applying the law of conservation of energy

$$W_1\, c_p\, T_s = W\, c_p\, T_r + \frac{d}{dt}(M_r\, c_v\, T_r)$$

or

$$k\,(W_1\, T_s - W\, T_r) = \frac{144\, V_r}{R}\,\dot{P}_r$$

or

$$\dot{P}_r = 0.0096\, W_1\, P_s^{0.286} - 0.000149\, W\, T_r \tag{10}$$

D. Controller

The controller for the system is of the proportional plus integral type. The output signal of the controller can be defined as

$$P_a = K_1\,(P_{set} - P_r) + K_2\int_0^t (P_{set} - P_r)dt + K_3$$

where

K_1 = 5 psi/psi

K_2 = 0.167 repeats/second

K_3 = 17.7 psia

P_{set} = 150 psia

or

$$P_a = 5\ (150 - P_r) + 0.167 \int_0^t (150 - P_r)dt + 17.7$$

or

$$\dot{P}_a = -5\ \dot{P}_r + 0.167\ (150 - P_r) \tag{11}$$

E. Actuator, Valve and Valve Positioner

This system is shown in block diagram by Figure 5.

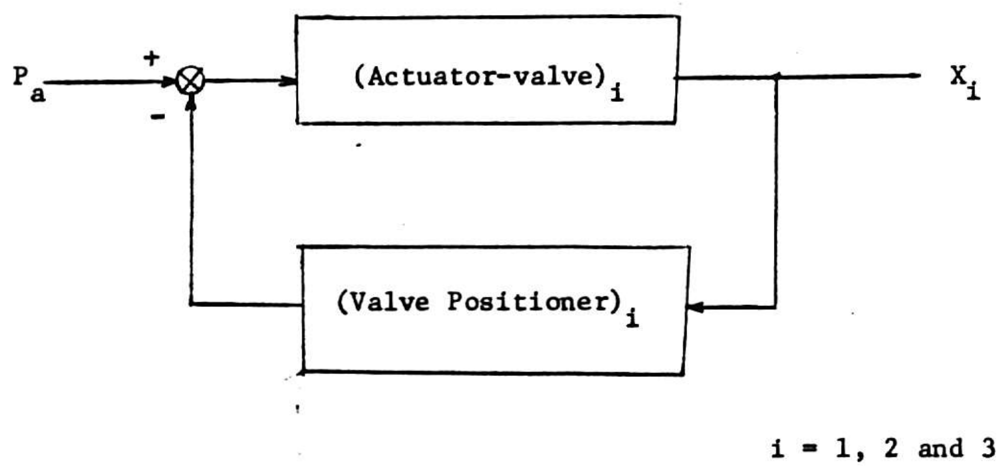

Figure 5. Schematic diagram of valve, actuator and valve positioner.

The dynamic characteristics of an actuator-valve and valve positioner system can ordinarily be approximated by a second order linear differential equation (3). To determine the parameter values of this system a step input was applied to the actual system and the response curves recorded. It was clear from the response curves that the steady state was reached much faster when the valve was opening than when closing. A second order linear approximation was made for each direction of travel and the following equations obtained.

$$P_a = A_i \ddot{X}_i + 35.7\sqrt{A_i}\,\dot{X}_i + 318\, X_i + 14.7 \qquad (12)$$

$$i = 1,\ 2 \text{ and } 3.$$

where

$$A_i = 564 \text{ when } X_i > 0$$

and

$$A_i = 4.5 \text{ when } \dot{X}_i < 0$$

and

$$X_2 = 0 \text{ when } P_2 < P_s < 1600, \text{ otherwise given by equation (12)}$$

$$X_3 = 0 \text{ when } P_3 < P_s < 1600, \text{ otherwise given by equation (12)}$$

where

P_2 and P_3 are the pressure settings of pressure switches S_2 and S_3 respectively.

F. Flow Through Valve Downstream of Low Pressure Tank

If the flow through this valve is critical then it is independent of the downstream pressure and

$$W = K_v \frac{P_r}{\sqrt{T_r}} \tag{13}$$

Since the opening of this valve is constant throughout the operating time a mean value of the constant K_v should be used which can be obtained from the following equation.

$$K_v = \frac{W_{mean} \sqrt{T_{r\ mean}}}{P_{r\ mean}} \tag{14}$$

("mean" in the subscripts means the average value of the corresponding variable during the time of operation.)

CHAPTER III

SOLUTION OF DIFFERENTIAL EQUATIONS ON DIGITAL COMPUTER

The Runge Kutta integration formulas were used to solve the system of differential equations derived in chapter II (4). A brief explanation of this technique is given below. To start with, all of the differential equations must be expressed in the following form.

$$\dot{Z}_j = f_j(Z_1, Z_2, \ldots, Z_n, t) \tag{15}$$

$$j = 1, 2, 3 \ldots, n.$$

with initial conditions

$$Z_j(0) = Z_{j0}$$

where t is the independent variable and Z_j are dependent variables. The Runge Kutta formula for this system of equations is

$$Z_j(t + \Delta t) = Z_j(t) + (A_j + 2\,B_j + 2\,C_j + D_j)/6 \tag{16}$$

where

$$\begin{aligned}
A_j &= f_j(Z_1, Z_2, \ldots, Z_n, t)\Delta t \\
B_j &= f_j\left[(Z_1 + \tfrac{1}{2}A_1), (Z_2 + \tfrac{1}{2}A_2), \ldots, (Z_n + \tfrac{1}{2}A_n), (t + \tfrac{1}{2}\Delta t)\right]\Delta t \\
C_j &= f_j\left[(Z_1 + \tfrac{1}{2}B_1), (Z_2 + \tfrac{1}{2}B_2), \ldots, (Z_n + \tfrac{1}{2}B_n), (t + \tfrac{1}{2}\Delta t)\right]\Delta t \\
D_j &= f_j\left[(Z_1 + C_1), (Z_2 + C_2), \ldots, (Z_n + C_n), (t + \Delta t)\right]\Delta t
\end{aligned} \tag{17}$$

and Δt is the step size.

Since the initial conditions are known the value of each variable after an interval Δt can be obtained. These values are used to calculate the values of the variables after another Δt interval and this process is repeated until the desired final conditions are reached.

In this report calculations have been made for only one desired wind tunnel operating condition, i.e. a stagnation chamber pressure of 35 psia and a mass flow rate of 17.5 lbm/second, (see Table I, page 4). The reason for this is because other operating conditions will have similar results and because the time required to calculate one set of results is of the order of 30 minutes.

As the wind tunnel stagnation chamber pressure is less than one half of the low pressure tank pressure the condition of critical flow is satisfied. As the temperature in the low pressure tank is varying, decreasing most of the time, and there is no advance knowledge of how much it is going to decrease, a temperature drop of $50^{o}F$ was assumed for the low pressure tank to start calculations. This gives a mean temperature of $505^{o}R$. But the $P_{r\ mean}$ and W_{mean} are 150 psia and 17.5 lbm/second respectively, therefore from equation (14)

$$K_v = 2.66$$

Substituting this back in equation (13) gives

$$W = 2.66 \frac{P_r}{\sqrt{T_r}} \qquad (13a)$$

The 'actual' value of the mean temperature and pressure can then be calculated from the results obtained using this value of K_v. A new value of K_v can then be determined from the mean temperature and pressure thus obtained.

Now the new value of K_v should be used to compute future results on the computer to give a corrected mass flow rate W.

Now substituting equations (8) and (13a) into equations (5), (9) and (10) and substituting $\dot{P}_r$ from equation (10) into equations (9) and (11) the following equations are obtained.

$$\dot{P}_s = 0.00605\ (X_1 + X_2 + X_3)\ P_s^{1.142} \tag{18}$$

$$\dot{P}_r = 0.00432\ (X_1 + X_2 + X_3)\ P_s^{1.142} - 0.000396\ P_r\sqrt{T_r} \tag{19}$$

$$\dot{T}_r = \frac{T_r}{P_r}\left[0.00432\ (X_1 + X_2 + X_3)\ P_s^{1.142} - 0.000396\ P_r\sqrt{T_r} - 0.00048\ T_r\ (X_1 + X_2 + X_3)\ P_s^{0.857} - 0.00284\frac{P_r}{\sqrt{T_r}}\right] \tag{20}$$

$$\dot{P}_a = -\ 0.0216\ (X_1 + X_2 + X_3)\ P_s^{1.142} + 0.00198\ P_r\sqrt{T_r} + 0.167\ (150 - P_r) \tag{21}$$

To change equation (12) into a first order differential equations for i = 1, 2 and 3 let

$$\dot{X}_1 = Y_1 \tag{22}$$

$$\dot{X}_2 = Y_2 \tag{23}$$

$$\dot{X}_3 = Y_3 \tag{24}$$

Substituting equations (23), (24) and (25) into equations (12)

$$\dot{Y}_1 = (P_a - 35.7\sqrt{A_1}\ Y_1 - 318\ X_1 - 14.7)/A_1 \tag{25}$$

$$\dot{Y}_2 = (P_a - 35.7\sqrt{A_2}\ Y_2 - 318\ X_2 - 14.7)/A_2 \tag{26}$$

$$\dot{Y}_3 = (P_a - 35.7\sqrt{A_3}\ Y_3 - 318\ X_3 - 14.7)/A_3 \tag{27}$$

The initial conditions for equations (18) through (27) are

$P_s(0) = 1600$ psia, $P_r(0) = 150$ psia, $T_r(0) = 530^oR$, $P_a(0) = 17.7$ psia,

$Y_1(0) = Y_2(0) = Y_3(0) = 0$ feet/second, $X_1(0) = X_2(0) = X_3(0) = 0$ feet.

The constraints for equations (25), (26) and (27) are

$A_i = 564$ if $Y_i > 0$

$A_i = 4.5$ if $Y_i < 0$

$i = 1, 2$ and 3

and

$X_2 = 0$ when $P_2 < P_s < 1600$

otherwise given by equations (22) through (27).

$X_3 = 0$ when $P_3 < P_s < 1600$, $P_3 < P_2$

otherwise given by equations (22) through (27).

P_2 and P_3, the settings of pressure switches S_2 and S_3, were so chosen that the next valve will start to open before the previous one had opened completely. By calculation these values were determined to be approximately 1400 and 800 psia respectively. Since interest was restricted to conditions of critical flow through the control valves the process was terminated when the pressure in the high pressure tank reached 300 psia. For pressures lower than this the flow through the valves AV_1, AV_2 and AV_3 would be sub-critical and equation (6b) have to be used for the flow.

The equations (18) through (27) together with the initial conditions and constraints were readily solved on the IBM 360 digital computer using the Runge Kutta technique.

The computer program is listed in Appendix A.

CHAPTER IV

DISCUSSION OF RESULTS

The curves in Figures 6 and 7, pages 21 and 22, show how T_r, P_r, P_s, T_s and W vary with time. It is clear from the curves that the mass flow rate has not been affected much at the points where valves AV_2 and AV_3 start opening. However the temperature in the low pressure tank is decreasing all the time, except for a little while when valves AV_1 and AV_2 have just opened from their closed positions.

It is clear from the curves that the low pressure tank temperature decreases rather slowly in the beginning but very fast in the end. The rate of temperature drop is increasing as the pressure and temperature in the high pressure tanks are decreasing and at the end the temperature in the low pressure tank has decreased so much that even the decrease in pressure in this tank can not decrease the mass flow rate very much. The reason that the pressure in the low pressure tank decreases is because all the valves reach a completely open position when the pressure in the high pressure tank reaches 416.59 psia and then remain open until the end of process. Since the valves can not be opened wider and the high pressure tanks pressure is decreasing the mass flow rate into the low pressure tank also decreases which results in the drop in pressure in the low pressure tank and hence forth this pressure can not be controlled. But at this instant the temperature in this tank is falling so rapidly that the mass flow rate increases for a little while even though the pressure is decreasing. But the drop in temperature can not cope with the drop in pressure, and the mass flow rate

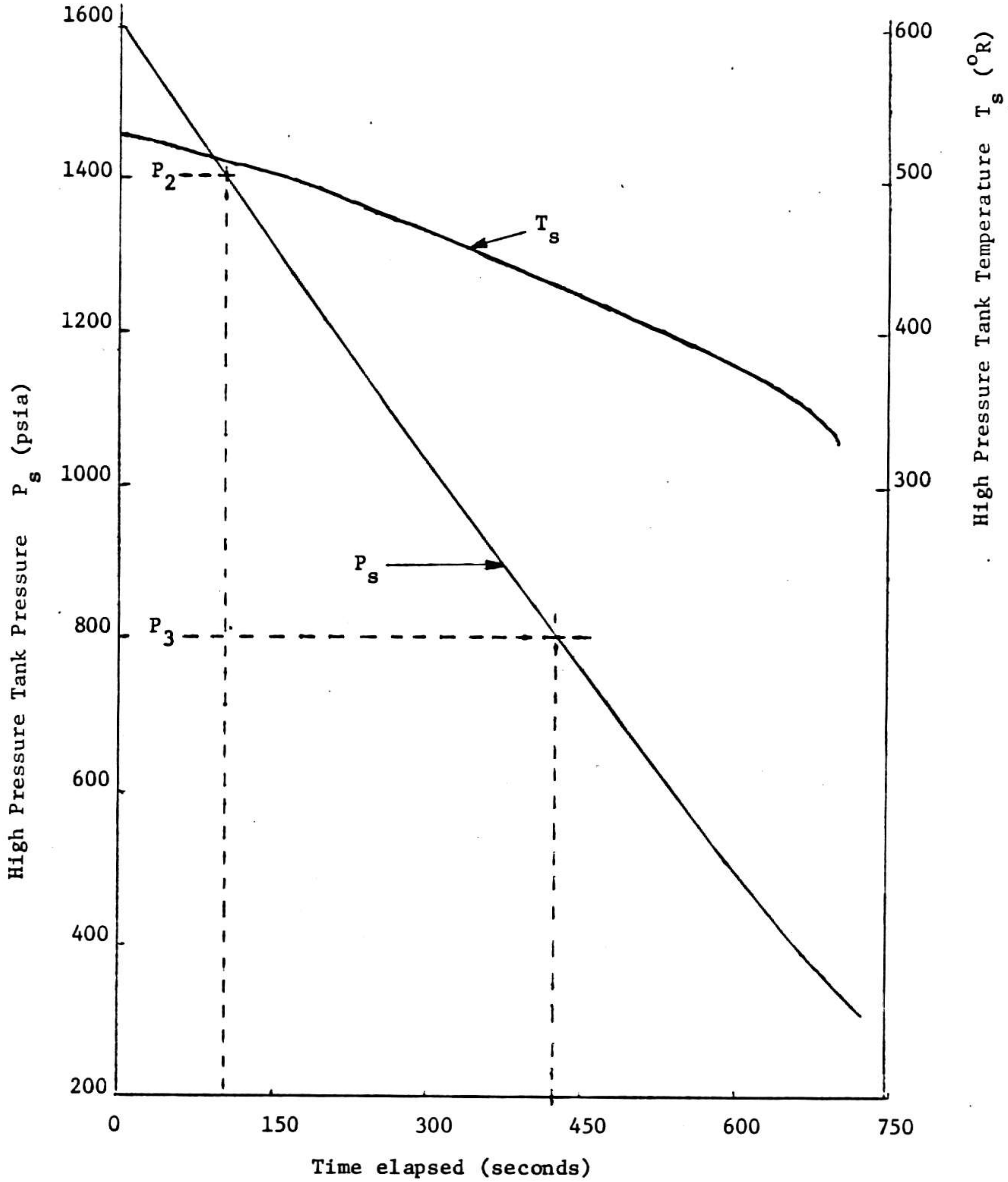

Figure 6. Variation of temperature and pressure of high pressure tank with time elapsed.

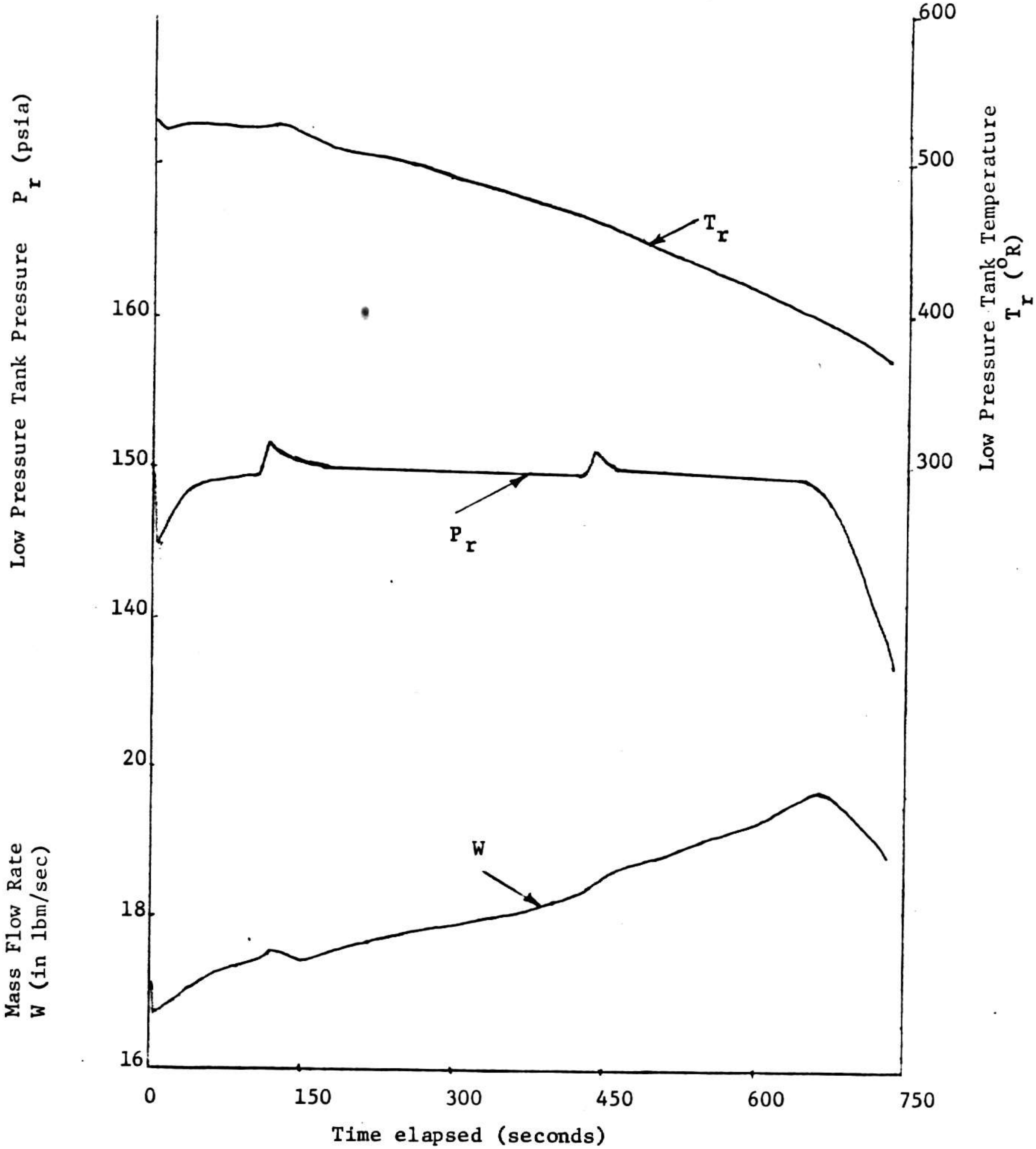

Figure 7. Variation of low pressure tank pressure, temperature and mass flow rate with time elapsed.

starts decreasing gradually.

The value of K_v used in the computation of these results was based on an estimate of $T_{r\ mean}$ of 505°R arrived at by assuming a low pressure tank temperature drop of 50°F. The calculated temperature drop is 155°F which is far more than the 50°F drop which was assumed for these calculations. For future calculations a mean temperature of 455°R should be used giving a K_v value of 2.48 for a mass flow rate of 17.5 lbm/second. However, based on $K_v = 2.66$, the average mass flow rate during 732 seconds of operating time is 18.35 lbm/second with a variation of ± 7.7%.

As the results obtained by the computer have been computed at a discrete interval of time these results are approximate and the error in each step is of the order of 0.00001. But this error is a compensating error, i.e. some times it is positive and sometimes negative. But since the formulation of the problem itself is based on several assumptions the computational errors are of secondary importance. Experimental results are needed to verify the validity of the original assumptions and to establish the overall accuracy of the results.

CHAPTER V

CONCLUSION AND RECOMMENDATIONS

In this analysis all processes have been assumed to be adiabatic. This is not true in the actual case. There will always be some heat transfer from the surroundings to the air in both tanks and its rate would be proportional to the temperature difference between the surroundings and the air. As a matter of fact this heat transfer will help in keeping the mass flow rate more nearly constant. Therefore in the actual case the drop in low pressure tank temperature would not be as much as in the assumed adiabatic case and the mass flow rate would not change as much as the calculated value.

It is clear from the curves that the settings of the pressure switches is not very critical because there has not been any appreciable change in the mass flow rate at the time when a valve starts to open from its completely closed position. But one thing should be kept in mind; any valve should start to open before any other valves already open reaches a completely open position.

In this process the pressure in the high pressure tank drops from 1600 psia to 300 psia in 732 seconds and at the end the mass flow rate is still 18.82 lbm/second. Thus it can be observed from Figure 7 that the mass flow rate can still be kept constant within ± 7.7% even if the pressure in the high pressure tanks is allowed to drop below 300 psia. Thus it can be said that an average mass flow rate of 18.35 lbm/second can be maintained within ± 7.7% for longer than 732 seconds.

It is obvious from the curves that the longer the duration of operation the greater the drop in temperatures of both the high and low pressure tanks.

If the system is allowed to operate for shorter times and the temperature is allowed to stabilize at atmospheric temperature before each operation, the mass flow rate can be kept even more constant than ± 7.7% of a given mean value by simply controlling the pressure in the low pressure tank.

As the volume of the low pressure tank is very large, all the disturbances die very soon and there are no oscillations of the pressure in this tank. The system behaves as a stable system.

The computer time, to solve these differential equations, can be reduced if KBM RKGS Subroutine is used. This Subroutine automatically fixes the step size depending on how fast the variables are changing.

In order to keep the mass flow rate more nearly constant it is recommended that one of the following control schemes be considered for incorporation into the system.

1. A temperature control loop, with a heat exchanger, could be added in order to maintain a constant temperature in the low pressure tank.
2. The pressure set point of the controller could be made a function of the temperature in the low pressure tank so that the pressure in this tank be varied in order to keep the mass flow rate out constant.

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1. Class Notes of the Course "Fluid Control System," ME 735, taught by Dr. R. O. Turnquist.

2. "Test Procedures for the Evaluation of Control Valve Flow Performance," The American Society of Mechanical Engineers Paper No. 55-A-152.

3. Frequency response characteristics of Type 3560-657/30 Positioner-Actuator dated September 24, 1962, Fisher Governor Company, Marshalltown, Iowa.

4. Class Notes of the course "Quantitative Techniques in Industrial Engineering" taught by Dr. E. S. Lee.

APPENDIX A

APPENDIX A. COMPUTER PROGRAM

```
201 FORMAT(4H PA= F15.4)
202 FORMAT(4H X1= F16.8)
203 FORMAT(4H X2= F16.8)
204 FORMAT(4H X3= F16.8)
100 FORMAT(1H ,4F15.4,4F16.8)
    DT=.1
    S1=.00605*DT
    S2=.00432*DT
    S3=1.142
    S4=.000396*DT
    S5=.0001 *DT
    S6=.00198*DT
    S7=.0216 *DT
    S8=.857
    S9=2.84
    P1=1400.
    P2=800.
    PS=1600.
    X1=0.
    X2=0.
    X3=0.
    PR=150.
    TR=530.
    PA=17.7
    Y1=0.
    Y2=0.
    Y3=0.
    ZM=2.66*PR/TR**.5
    PRINT 100    ,PS,PR,TR,PA,X1,X2,X3,ZM
 43 IF(P1-PS) 41,42,42
 41 A1=-S1*X1*PS**S3
    B1= S2 *X1*PS**S3-    S4*PR*TR**0.5
    C1= TR*(B1-S5*TR*(.48*X1*PS**S8-S9*PR/TR**.5))/PR
    D1= -S7*X1*PS**S3+S6*PR*TR**.5+.167*(150.-PR) *DT
    E1= Y1*DT
    IF(Y1) 51,52,52
 51 A= 4.5
    GO TO 53
 52 A= 564
 53 H1= (PA- 35.7*A**.5*Y1-318.*X1-14.7)/A *DT
```

```
      A2=-S1*(X1+E1/2.)*(PS+A1/2.)**S3
      B2=S2*(X1+E1/2.)*(PS+A1/2.)**S3-S4*(PR+B1/2.)*(TR+C1/2.)**.5
      C21= .48*(X1+E1/2.)*(PS+A1/2.)**S8-S9*(PR+B1/2.)/(TR+C1/2.)**.5
      C2= (TR+C1/2.)*(B2-S5*(TR+C1/2.)*C21)/(PR+B1/2.)
      D21=-S7*(X1+E1/2.)*(PS+ A1/2.)**S3+.167*(150.-(PR+B1/2.)) *DT
      D2= D21+S6*(PR+B1/2.)*(TR+C1/2.)**.5
      E2=(Y1+H1/2. )*DT
      H2=(PA+D1/2.-35.7*A**.5*(Y1+H1/2.)-318.*(X1+E1/2.)-14.7)/A *DT
      A3=-S1*(X1+E2/2.)*(PS+A2/2.)**S3
      B3=S2*(X1+E2/2.)*(PS+A2/2.)**S3-S4*(PR+B2/2.)*(TR+C2/2.)**.5
      C31=.48*(X1+E2/2.)*(PS+A2/2.)**S8-S9*(PR+B2/2.)/(TR+C2/2.)**.5
      C3=(TR+C2/2.)*(B3-S5*(TR+C2/2.)*C31)/(PR+B2/2.)
      D31=-S7*(X1+E2/2.)*(PS+A2/2.)**S3+.167*(150.-PR-B2/2.) *DT
      D3=D31+S6*(PR+B2/2.)*(TR+C2/2.)**.5
      E3=(Y1+H2/2.)*DT
      H3=(PA+D2/2.-35.7*A**.5*(Y1+H2/2.)-318.*(X1+E2/2.)-14.7)/A *DT
      A4=-S1*(X1+E3)*(PS+A3)**S3
      B4=S2*(X1+E3)*(PS+A3)**S3-S4*(PR+B3)*(TR+C3)**.5
      C41=.48*(X1+E3)*(PS+A3)**S8-S9*(PR+B3)/(TR+C3)**.5
      C4=(TR+C3)*(B4-S5*(TR+C3)*C31)/(PR+B3)
      D41=-S7*(X1+E3)*(PS+A3)**S3+.167*(150.-PR-B3) *DT
      D4=D41+S6*(PR+B3)*(TR+C3)**.5
      E4=(Y1+H3)*DT
      H4=(PA+D3-35.7*A**.5*(Y1+H3)-318.*(X1+E3)-14.7)/A*DT
      PS=PS+(A1+2.*A2+2.*A3+A4)/6.
      PR=PR+(B1+2.*B2+2.*B3+B4)/6.
      TR=TR+(C1+2.*C2+2.*C3+C4)/6.
      PA=PA+(D1+2.*D2+2.*D3+D4)/6.
      X1=X1+(E1+2.*E2+2.*E3+E4)/6.
      Y1=Y1+(H1+2.*H2+2.*H3+H4)/6.
      ZM=2.66*PR/TR**.5
      IF (PA) 81,82,82
   81 PRINT 201,PA
      PA=0.
   82 IF (X1) 83,84,84
   83 PRINT 202,X1
      X1=0.
      Y1=0.
      GO TO 111
   84 IF(.0937-X1) 110,111,111
  110 PRINT 202,X1
      X1=.0937
      Y1=0.
  111 PRINT 100     ,PS,PR,TR,PA,X1,X2,X3,ZM
      GO TO 43
   42 IF(P2-PS) 44,45,45
   44 A1=-S1*(X1+X2)*PS**S3
      B1=S2*(X1+X2)*PS**S3-S4*PR*TR**.5
      C1=TR*(B1-S5*TR*(.48*(X1+X2)*PS**S8-S9*PR/TR**.5))/PR
      D1=-S7*(X1+X2)*PS**S3+S6*PR*TR**.5+.167*(150.-PR)  *DT
```

```
      E1=Y1*DT
      F1=Y2   *DT
      IF(Y1) 54,55,55
   54 AL=4.5
      GO TO 56
   55 AL=564.0
   56 H1=(PA-35.7*AL**.5*Y1-318.*X1-14.7)/AL *DT
      IF (Y2) 57,58,58
   57 AM=4.5
      GO TO 59
   58 AM=564.
   59 Q1=(PA-35.7*AM**.5*Y2-318.*X2-14.7)/AM *DT
      A2=-S1*(X1+E1/2.+X2+F1/2.)*(PS+A1/2.)**S3
      B21=S2*(X1+E1/2.+X2+F1/2.)*(PS+A1/2.)**S3
      B22=S4*(PR+B1/2.)*(TR+C1/2.)**.5
      B2=B21-B22
      C21=.48*(X1+E1/2.+X2+F1/2.)*(PS+A1/2.)**S8
      C22=S9*(PR+B1/2.)/(TR+C1/2.)**.5
      C2=(TR+C1/2.)*(B2-S5*(TR+C1/2.)*(C21-C22))/(PR+B1/2.)
      D21=-S7*(X1+E1/2.+X2+F1/2.)*(PS+A1/2.)**S3
      D22=+.167*(150.-(PR+B1/2.)) *DT
      D2=D21+D22+S6*(PR+B1/2.)*(TR+C1/2.)**.5
      E2=(Y1+H1/2.)*DT
      F2=(Y2+Q1/2.)*DT
      H2=(PA+D1/2.-35.7*AL**.5*(Y1+H1/2.)-318.*(X1+E1/2.)-14.7)/AL *DT
      Q2=(PA+D1/2.-35.7*AM**.5*(Y2+Q1/2.)-318.*(X2+F1/2.)-14.7)/AM *DT
      A3=-S1*(X1+E2/2.+X2+F2/2.)*(PS+A2/2.)**S3
      B31=S2*(X1+E2/2.+X2+F2/2.)*(PS+A2/2.)**S3
      B32=S4*(PR+B2/2.)*(TR+C2/2.)**.5
      B3=B31-B32
      C31=.48*(X1+E2/2.+X2+F2/2.)*(PS+A2/2.)**S8
      C32=S9*(PR+B2/2.)/(TR+C2/2.)**.5
      C3=(TR+C2/2.)*(B3-S5*(TR+C2/2.)*(C31-C32))/(PR+B2/2.)
      D31=-S7*(X1+E2/2.+X2+F2/2.)*(PS+A2/2.)**S3
      D32=+.167*(150.-(PR+B2/2.)) *DT
      D3=D31+D32+S6*(PR+B2/2.)*(TR+C2/2.)**.5
      E3=(Y1+H2/2.)*DT
      F3=(Y2+Q2/2.)*DT
      H3=(PA+D2/2.-35.7*AL**.5*(Y1+H2/2.)-318.*(X1+E2/2.)-14.7)/AL *DT
      Q3=(PA+D2/2.-35.7*AM**.5*(Y2+Q2/2.)-318.*(X2+F2/2.)-14.7)/AM *DT
      A4=-S1*(X1+E3+X2+F3)*(PS+A3)**S3
      B41=S2*(X1+E3+X2+F3)*(PS+A3)**S3
      B42=S4*(PR+B3)*(TR+C3)**.5
      B4=B41-B42
      C41=.48*(X1+E3+X2+F3)*(PS+A3)**S8
      C42=S9*(PR+B3)/(TR+C3)**.5
      C4=(TR+C3)*(B4-S5*(TR+C3)*(C41-C42))/(PR+B3)
      D41=-S7*(X1+E3+X2+F3)*(PS+A3)**S3
      D42=+.167*(150.-(PR+B3)) *DT
      D4=D41+D42+S6*(PR+B3)*(TR+C3)**.5
```

```
      E4=(Y1+H3)*DT
      F4=(Y2+Q3)*DT
      H4=(PA+D3-35.7*AL**.5*(Y1+H3)-318.*(X1+E3)-14.7)/AL*DT
      Q4=(PA+D3-35.7*AM**.5*(Y2+Q3)-318.*(X2+F3)-14.7)/AM   *DT
      PS  =  PS+(A1+2.*A2+2.*A3+A4)/6.
      PR  =  PR+(B1+2.*B2+2.*B3+B4)/6.
      TR  =  TR+(C1+2.*C2+2.*C3+C4)/6.
      PA  =  PA+(D1+2.*D2+2.*D3+D4)/6.
      X1  =  X1+(E1+2.*E2+2.*E3+E4)/6.
      X2  =  X2+(F1+2.*F2+2.*F3+F4)/6.
      Y1  =  Y1+(H1+2.*H2+2.*H3+H4)/6.
      Y2  =  Y2+(Q1+2.*Q2+2.*Q3+Q4)/6.
      ZM=2.66*PR/TR**.5
      IF (PA) 85,86,86
   85 PRINT 201, PA
      PA=0.
   86 IF (X1) 87,88,88
   87 PRINT 202,X1
      X1=0.
      Y1=0.
      GO TO 302
   88 IF (.0937-X1) 301,302,302
  301 PRINT 202,X1
      X1=0.0937
      Y1=0.
  302 IF (X2) 89,90,90
   89 PRINT 203,X2
      X2=0.
      Y2=0.
      GO TO 306
   90 IF(.0937-X2) 305,306,306
  305 PRINT 203,X2
      X2=0.0937
      Y2=0.
  306 PRINT 100,PS,PR,TR,PA,X1,X2,X3,ZM
      GO TO 42
   45 A1=-S1*(X1+X2+X3)*PS**S3
      B1=S2*(X1+X2+X3)*PS**S3-S4*PR*TR**.5
      C1=TR*(B1-S5*TR*(.48*(X1+X2+X3)*PS**S8-S9*PR/TR**.5))/PR
      D1=-S7*(X1+X2+X3)*PS**S3+S6*TR**.5*PR+.167*(150.-PR) *DT
      E1=Y1 *DT
      F1=Y2  *DT
      G1=Y3     *DT
      IF(Y1)61,62,62
   61 AL = 4.5
      GO TO 63
   62 AL  = 564.
   63 H1  = (PA-35.7*AL**.5*Y1-318.*X1-14.7)/AL *DT
      IF(Y2)64,65,65
   64 AM = 4.5
```

```
      GO TO 66
   65 AM = 564.
   66 Q1 = (PA-35.7*AM**.5*Y2-318.*X2-14.7)/AM *DT
      IF(Y3)67,68,68
   67 AN = 4.5
      GO TO 69
   68 AN = 564.
   69 R1 = (PA-35.7*AN**.5*Y3-318.*X3-14.7)/AN *DT
      A2 = -S1*(X1+X2+X3+(E1+F1+G1)/2.)*(PS+A1/2.)**S3
      B21 = S2*(X1+X2+X3+(E1+F1+G1)/2.)*(PS+A1/2.)**S3
      B22 = S4*(PR+B1/2.)* (TR+C1/2.)**.5
      B2 = B21-B22
      C21 = .48*(X1+X2+X3+(E1+F1+G1)/2.)*(PS+A1/2.)**S8
      C22 = S9*(PR+B1/2.)/(TR+C1/2.)**.5
      C2 = (TR+C1/2.)*(B2-S5*(TR+C1/2.)*(C21-C22))/(PR+B1/2.)
      D21 = -S7*(X1+X2+X3+(E1+F1+G1)/2.)*(PS+A1/2.)**S3
      D22 =  .167*(150.-(PR+B1/2.)) *DT
      D2 = D21+D22+S6*(PR+B1/2.)*(TR+C1/2.)**.5
      E2 =(Y1+H1/2.)*DT
      F2 =(Y2+Q1/2.)*DT
      G2 =(Y3+R1/2. )*DT
      H2=(PA+D1/2.-35.7*AL**.5*(Y1+H1/2.)- 318.*(X1+E1/2.)-14.7)/AL*DT
      Q2=(PA+D1/2.-35.7*AM**.5*(Y2+Q1/2.)-318.*(X2+F1/2.)-14.7)/AM *DT
      R2=(PA+D1/2.-35.7*AN**.5*(Y3+R1/2.)-318.*(X3+G1/2.)-14.7)/AN *DT
      A3=-S1*(X1+X2+X3+(E2+F2+G2)/2.)*(PS+A2/2.)**S3
      B31=S2*(X1+X2+X3+(E2+F2+G2)/2.)*(PS+A2/2.)**S3
      B32=S4*(PR+B2/2.)*(TR+C2/2.)**.5
      B3=B31-B32
      C31= .48*(X1+X2+X3+(E2+F2+G2)/2.)*(PS+A2/2.)**S8
      C32=S9*(PR+B2/2.)/(TR+C2/2.)**.5
      C3=(TR+C2/2.)*(B3-S5*(TR+C2/2.)*(C31-C32))/(PR+B2/2.)
      D31=-S7*(X1+X2+X3+(E2+F2+G2)/2.)*(PS+A2/2.)**S3
      D32= .167*(       150.-(PR+B2/2.)) *DT
      D3=D31+D32+S6*(PR+B2/2.)*(TR+C2/2.)**.5
      E3=(Y1+H2/2.)*DT
      F3=(Y2+Q2/2.)*DT
      G3=(Y3+R2/2.)*DT
      H3=(PA+D2/2.-35.7*AL**.5*(Y1+H2/2.)-318.*(X1+E2/2.)    -14.7)/AL*DT
      Q3=(PA+D2/2.-35.7*AM**.5*(Y2+Q2/2.)-318.*(X2+F2/2.)-14.7)/AM *DT
      R3=(PA+D2/2.-35.7*AN**.5*(Y3+R2/2.)-318.*(X3+G2/2.)-14.7)/AN *DT
      A4=-S1*(X1+X2+X3+E3+F3+G3)*(PS+A3)**S3
      B41=S2*(X1+X2+X3+E3+F3+G3)*(PS+A3)**S3
      B42=S4*(PR+B3)*(TR+C3)**.5
      B4=B41-B42
      C41=.48*(X1+X2+X3+E3+F3+G3)*(PS+A3)**S8
      C42=S9*(PR      +B3)/(TR+C3)**.5
      C4=(TR+C3)*(B4-S5*(TR+C3)*(C41-C42))/(PR+B3)
      D41=-S7*(X1+X2+X3+E3+F3+G3)*(PS+A3)**S3
      D42= .167*(150.-(PR+B3)) *DT
      D4=D41+D42+S6*(PR+B3)*(TR+C3)**.5
```

```
      E4 =(Y1+H3       )*DT
      F4 =(Y2+Q3       )*DT
      G4 =(Y3+R3        )*DT
      H4=(PA+D3-35.7*AL**.5*(Y1+H3)-318.*(X1+E3)-14.7)/AL     *DT
      Q4 = (PA+D3-35.7*AM**.5*(Y2+Q3)-318.*(X2+F3)-14.7)/AM   *DT
      R4 = (PA+D3-35.7*AN**.5*(Y3+R3)-318.*(X3+G3)-14.7)/AN     *DT
      PS = PS+(A1+2.*A2+2.*A3+A4)/6.
      PR = PR+(B1+2.*B2+2.*B3+B4)/6.
      TR = TR+(C1+2.*C2+2.*C3+C4)/6.
      PA = PA+(D1+2.*D2+2.*D3+D4)/6.
      X1 = X1+(E1+2.*E2+2.*E3+E4)/6.
      X2 = X2+(F1+2.*F2+2.*F3+F4)/6.
      X3 = X3+(G1+2.*G2+2.*G3+G4)/6.
      Y1 = Y1+(H1+2.*H2+2.*H3+H4)/6.
      Y2 = Y2+(Q1+2.*Q2+2.*Q3+Q4)/6.
      Y3 = Y3+(R1+2.*R2+2.*R3+R4)/6.
      ZM = 2.66*PR/TR**.5
      IF (PA) 91,92,92
   91 PRINT 201,PA
      PA=0.
   92 IF(X1) 93,94,94
   93 PRINT 202,X1
      X1=0.
      Y1=0.
      GO TO 96
   94 IF(.0937-X1) 95,96,96
   95 PRINT 202,X1
      X1=.0937
      Y1=0.
   96 IF(X2) 97,98,98
   97 PRINT 203,X2
      X2=0.
      Y2=0.
      GO TO 402
   98 IF(.0937-X2) 401,402,402
  401 PRINT 203,X2
      X2=.0937
      Y2=0.
  402 IF(X3) 403,404,404
  403 PRINT 204,X3
      X3=0.
      Y3=0.
      GO TO 406
  404 IF(.0937-X3) 405,406,406
  405 PRINT 204,X3
      X3=.0937
      Y3=0.
  406 PRINT 100,PS,PR,TR,PA,X1,X2,X3,ZM
      IF(PS-300)71,71,45
   71 STOP
      END
```

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THE CONTROL OF MASS FLOW RATE IN A BLOWDOWN TYPE WIND TUNNEL

by

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AN ABSTRACT OF A MASTER'S REPORT

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MASTER OF SCIENCE

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This report presents an analysis of an existing pressure control system consisting of two high pressure tanks, three control valves, a low pressure tank, three pressure switches, a pneumatic controller and a fixed valve. The object of this pressure control system is to keep the pressure of air approximately constant in order to keep the mass flow rate out of the low pressure tank to a wind tunnel as constant as possible.

All linear and nonlinear differential equations relating different variables have been derived for this system. These equations were solved on a digital computer using the Runge Kutta technique to obtain values of system temperature, pressure and mass flow rate as functions of time.

The results obtained show that it is possible to maintain a wind-tunnel mass flow rate of 18.82 lbm/second (with ± 7.7% accuracy) for a time interval of 732 seconds with the existing pressure control system.