ADVERTISING IN INDUSTRIAL SYSTEMS-

AN INDUSTRIAL DYNAMICS APPROACH

by

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A MASTER'S REPORT

submitted in partial fulfillment of the

requirement for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1971

Approved by

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ACKNOWLEDGEMENT

The author wishes to thank his major professor, Dr. C. L. Hwang, for his helpful remarks and valuable guidance throughout this work. He is also grateful to his committee members, Dr. F. A. Tillman and Dr. L. T. Fan, for their comments on the work.

Thanks also go to Mrs. Marie Jirak for her excellent and careful typing of the manuscript.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

This report is a study of Industrial Dynamics, and its application to some advertising practices. Industrial dynamics is the study of the time-varying behavior of systems within the framework of their feedback-loop structures. It was invented on the belief that the behavior of systems (physical, social, biological, and so on) can be investigated through the study of the cause-and-effect relationships between their components. The principles of feedback control, which are extensively used in engineering systems, might as well be applied to social, economic, and biological systems. Much, if not all, of the behavior of such complicated systems could be explained using the principles of feedback theory.

Chapter 2 of this report is a review of the principles of industrial dynamics. The four processes that make the foundation for industrial dynamics, i.e., theory of information-feedback systems, decision-making processes, simulation techniques, and availability of low-cost high-speed computers, are described. The last part of the chapter is devoted to defining the rules and guidelines that help to identify and represent the feedback characteristics of such systems, and finally build their dynamic models.

Since its invention during the period 1956-61 at Massachusetts Institute of Technology, industrial dynamics has found both pros and cons. Some (mostly the followers of J. W. Forrester, who has done the pioneering works on industrial dynamics) have praised it as a theory of structure of systems. On the contrary, some have considered industrial dynamics merely as a new simulation technique, specifically because of the DYNAMO-compiler, which is designed to handle the industrial dynamics models. In Chapter 3, a review of the literature on industrial dynamics, both for and against, is presented.

Chapter 4 is an attempt to apply the industrial dynamics principles to a simple industrial system , and try to trace the behavior of the system through its feedback structure. The organization of the system, which deals with production and distribution is defined, the cause-and-effect interactions between system components are investigated, and the dynamic model of the system is presented. The model is later used to study the time-varying behavior of the system, and explore the possible ways of improvement. It should be pointed out that the content of Chapter 4 is essentially based on the analysis given in Chapters 2 and 15 of [13].

The second objective of this report is the application of industrial dynamics principles to some advertising models. The first part of Chapter 5, is the description of a simple advertising model proposed by Forrester [13]. The model assumes a direct relationship between advertising budget and production level at factory, at every moment of time. The effect of such an advertising practice on the production-distribution of Chapter 4 is investigated.

In the second part of Chapter 5, three other advertising models are introduced. In recent years, Operations Research has found some applications in the field of marketing. A study by Vidale and Wolfe [49,50], based on extensive experimentations, has proposed a mathematical model of sales response to advertising; however, it does not investigate the behavior of the system under the proposed model, which is of more interest as far as this report is concerned. It is basically a study of the possible effects of advertising on retail sales. Here, the model is used, within the context of industrial dynamics, to investigate the behavior of the whole production-distribution system, and not only the retail sales, under three different types of advertising practices:

1 - Protracted advertising campaigns;

2 - Short-time, intensive advertising campaigns;

3 - Impulse-type advertising campaigns.

The last chapter offers a comparison between the results of the system simulation under three advertising models of Chapter 5. In doing so, certain criteria have been taken into consideration, such as total sales generated as a result of advertising, variations and fluctuations in the system parameters such as production rate, inventories, and so on, tendency of the system to fluctuate, etc.

Use of industrial dynamics models is closely associated with DYNAMO-compiler. This special-purpose language provides a fast and reliable means of simulation, although it is not the only one.

Throughout this report, DYNAMO is used to simulate the system behavior.

CHAPTER 2

GENERAL CONCEPTS OF INDUSTRIAL DYNAMICS

2.1. INTRODUCTION

"Industrial Dynamics" is the study of the time-varying behavior of the systems, with a view at their information feedback characteristics. As such, it proposes that the character of any organization (system) is determined by the dynamic interactions among its interconnected feedback networks (loops).

Forrester [13] defines "industrial dynamics" as:

> "....... The study of the information-feedback characteristics of industrial activity to show how organizational structure, amplification in policies, and time delays (in decisions and actions) interact to influence the success of the enterprise. It treats the interactions between the flows of information, money, orders, materials, personnel, and capital equipment in a company, an industry, or a national economy.
>
> Industrial dynamics provides a single framework for integrating the functional areas of management - marketing, production, accounting, research and development, and capital investment. It is a quantitative and experimental approach for relating organizational structure and corporate policy to industrial growth and stability."

As was pointed out, industrial dynamics focuses on the behavior of systems and their components within the framework of their information feedback characteristics. Therefore it is primarily based on the theory of information-feedback system; however, there are three other foundations for industrial dynamics. They are:

- Decision-making Processes;
- Simulation Techniques;
- Availability of Low-cost Digital Computers.

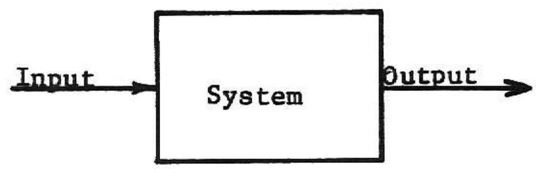
2.2. THEORY OF INFORMATION-FEEDBACK SYSTEMS

The concept of servomechanisms (information-feedback systems) was evolved during and after World War II.

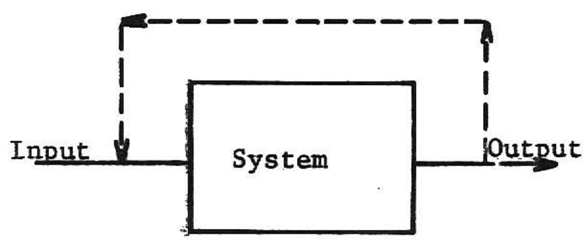
A system, as used here, is identified by a grouping of parts (physical as well as people) which function together for a common goal. This is a broad definition which encompasses almost any organization (physical, social, economic, etc.) with which we are familiar.

The first step toward understanding the concepts and principles of systems would require an orderly structure whereby the information about the system could be organized. This structure would serve as a basis for unifying the diversities of the physiological, industrial, and economic systems, just as the laws of physics provide a structure for today's technology [11,13].

Systems are classified as either "open" or "closed, or feedback" systems. In an open system, the output is affected by the input, but has no effect upon the input. Another words, the future action is not controlled by the past action (Fig. 2.1a). On the other hand, in a closed or feedback system, the information about the output of the system is brought back to control and maintain the future action. This implies a closed-loop structure as shown in Fig. 2.1b [4].



(a) Open System



(b) Closed System

Fig. 2.1. Open and Closed Systems

Definition of Feedback Control System. American Institute for Electrical Engineers (AIEE) defines the feedback control systems as follows [37]: "A feedback control system is a control system which tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the difference as means of control."

Hammond [28] defines such systems in the following way: "A feedback system comprises one or more distinguishable elements which react on each other in a predetermined manner and are arranged so that a closed ring or loop of dependencies is formed."

Kuo's definition of feedback control systems is as follows [32]: "Systems comprising one or more feedback loops which compare the controlled signal c with the command signal r; the difference (e = r - c) is used to drive c into correspondence with r."

Forrester, within the context of industrial dynamics, states that [13]: "An information-feedback system exists whenever the environment leads to a decision that results in action which affects the environment and thereby influences future decision."

The structure of an information feedback loop is illustrated in Fig. 2.2.

There are two classes of feedback systems:

1 - Negative feedback systems, where a goal is sought, and the response of the system is in such a direction as to eliminate any discrepancy between the system state and the goal.

2 - Positive feedback systems, in which the result of the action generates still greater action i.e., the system generates a growth process.

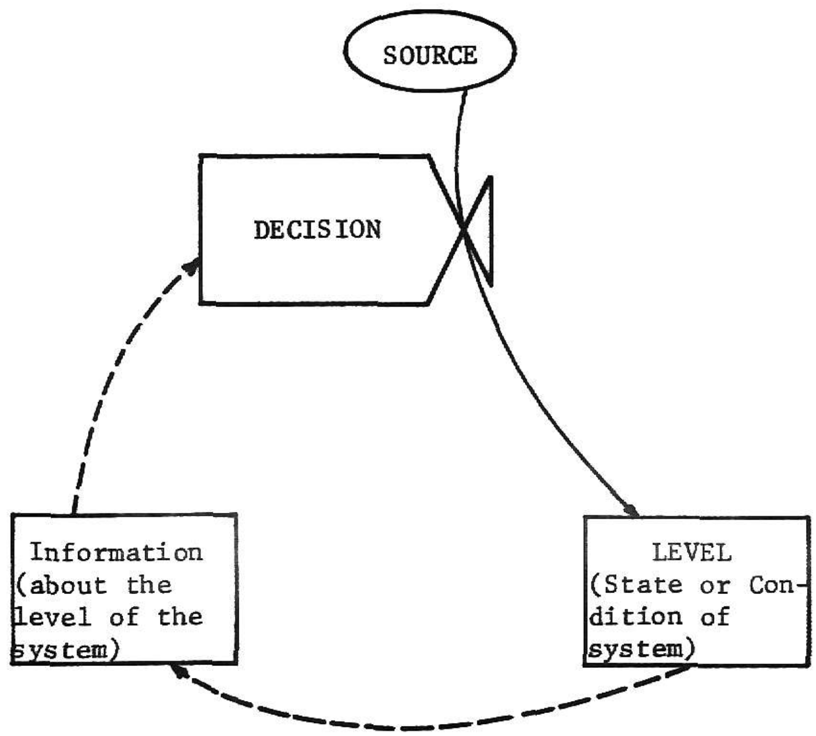


Fig. 2.2. Structure of a Feedback Loop

The concept of information-feedback systems, whether positive or negative, provides a basis for understanding and interpreting the behavior of social systems. The feedback theory has been successfully applied to mechanical and electrical systems over the past decades, but due to the complexity of the social organizations, it has been only during the last decade that the principles of feedback systems have been developed far enough to be applied to social systems as well.

In the study of information-feedback system, there are three important characteristics which determine the behavior of such systems - structure, delays, and amplifications. Structure of a system determines how the components are related to one another. Delays always exist in the transfer of information, in decision-making processes, and in the actions that follow the decisions. Amplifications manifest themselves whenever an action is more forceful than what is thought to be, considering the information that is the basis for decision-making. There are other concepts in the theory of feedback systems which are equally applied in social systems, such as noise spectrum, bandwidth, natural frequencies, filter distortion, gain, and transient response [11].

These general concepts provide the essential framework within which the behavior of social systems is analyzed. It is along this line that an organization can operate effectively if the control system structure of the organization is recognized, and the dynamic interactions between its components are understood [35].

2.3. DECISION-MAKING PROCESSES

The second foundation for industrial dynamics is the theory of decison-making, which evolved during the 1950's out of the process of automatizing military tactical operations [13]. This was due to the necessity for formalizing the decision-making policy, which, in turn, was a result of the quicker pace of warfare, to which human organizations no longer could respond. Consequently it became more and more necessary to set the formal rules for decision-making, and divert from "tactical judgement and experience."

> "A decade of time and thousands of people were involved in this interpretation of military decision processes and automatizing the general policies that are the basis for tactical military decision making. It has been amply demonstrated that carefully selected formal rules can lead to short-term tactical decisions, that excel those made by human judgement under the pressure of time, or with men having insufficient experience and practice, or in the rigidity of large organizations." [13]

The same rules apply to social systems as well. The complexity of these systems is now growing beyond the ability of human decision-making processes, and therefore attention is shifted toward making use of formal rules of decision-making in industrial and economic systems. The management of such systems is increasingly concerned with setting up the formal rules governing the decisions, and interpreting the effects of such policies on the systems.

2.4. SIMULATION TECHNIQUES VS. ANALYTICAL APPROACHES

Simulation techniques have long been known, and used. An example is the navigation tables which were used in the past.

Simulation is an experimental approach to representing the behavior of systems, whenever the mathematical analysis is not powerful enough to obtain an analytical solution, and wherever the cost is not a limiting factor.

In general, it is the process of substituting an experiment or a model for a system, because that experiment or model is easier to study and experiment with, than the system itself. Today, however, simulation techniques make use of mathematical models, used on a computer, to represent a physical or social system [11].

A mathematical model is a set of precise equations describing how the system varies, and how the conditions of system at one time could be used to determine the state of the system at a brief time later. But it cannot tell how to determine the condition of the system in some distant future, directly and without going through the step-by-step process of computing the intermediate states of the system. Whereas the analytical solution of the system determines its state at any time; however, it should be noted that the present social and industrial systems are of such degrees of complexity that the present-day mathematics is not adequate to find analytical solutions to these systems, nor is it powerful enough to develop, in some instances, the set of differential equations describing the behavior of the systems. Simulation techniques come to help whenever such difficulties arise.

In the beginning, the use of simulation was limited by the time and cost involved, and it was not until the invention of high-speed computers that these techniques were vastly applied to industrial as well as business systems. Now the time and cost are not limiting factors any more, and with the invention of new special-purpose simulation languages, the use of this methods becomes more and more easy. However, identifying the operational aspects of a system, determining the assumptions, making the model, and finally analyzing the results of the simulation, need a great deal of skill and expertise.

This, of course, is part of the responsibilities of today's management.

2.5. LOW-COST COMPUTERS

Electronic digital computers provide the fourth foundation for industrial dynamics. Rapid development in this field was accomplished during 1950's, and technical performance of such computers constantly has increased, by a factor of 10 per year, over the same decade [13]. The progress has not stopped, however.

The low-cost, high-speed computers have opened new possibilities that are far beyond the current applications; computers are mostly used in the role of a system component, and very seldom as tools for the design of industrial and social systems. As active system components, they are widely used for data processing and information handling, but even in these cases, major areas of application for formal data processing are still open; probably 98% of the information flows that are important in determining the characteristics of today's social systems, lie outside of the formal data processing channels; some of these flows are among the most crucials in affecting the behavior of the system [11].

On the whole, the advent of computers:

".....was a technological change greater than that effected in going from chemical to atomic explosives. Society cannot absorb so big a change in a mere ten years. We have a tremendous untapped backlog of potential devices and applications. It is now to be expected that machine progress will stay ahead of conceptual progress in industrial and economic dynamics. Computing machines are now so widely available, and the cost of computation and machine programming is so low relative to other costs, that the former difficulties in activating a simulation model need no longer determine our rate of progress in understanding system dynamics." [13]

2.6. STRUCTURE OF SYSTEMS

The importance of an orderly structure as a first step toward understanding the behavior of systems was already pointed out. Here the essentials of such structure are outlined.

A structure is necessary in order to interpret the observations. Without a structure any observation may, at first, seem meaningless, but if it can be determined which category it falls in, it can be identified and interpreted in a proper manner.

The structure of a system should be determined within a closed boundary; within a closed boundary, the feedback loops, i.e., the basic elements of system structure, are identified; levels and rates are the essential variables within a loop; and within the rate variable, goal, apparent condition, discrepancy, and action are the important constituents (Fig. 2.3) [21].

Closed Boundary. In formulating the structure of a system, the boundary of the system should be recognized; it would contain the smallest number of components within which the dynamic behavior of the system is to be observed.

The concept of closed boundary is important, because the interactions within the system, as viewed by industrial dynamics, are the causes of

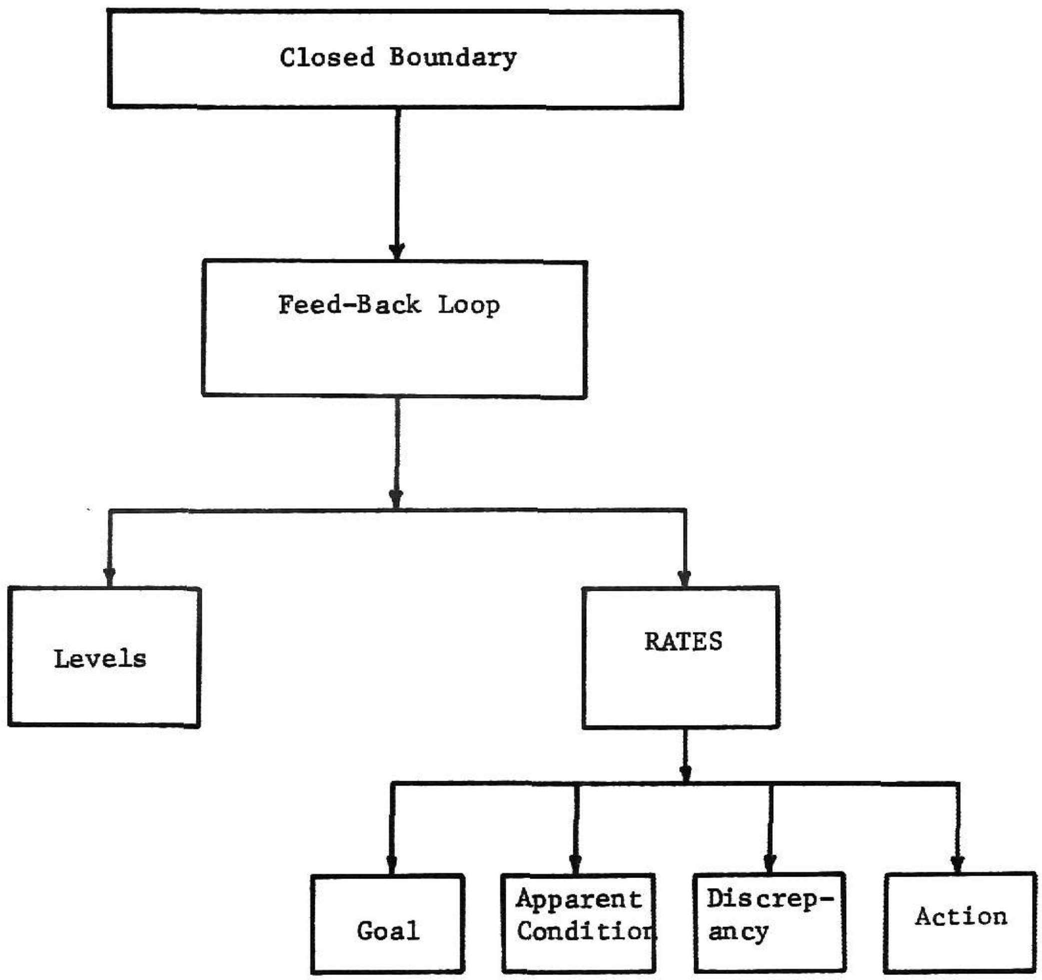


Fig. 2.3. Hierarchy of System Structure

dynamic behavior. The effect of exogenous variables on the system is of no interest at this stage. Therefore the closed boundary of the system should include in it all the interactions that are essential to the behavior of the system.

Feedback Loops. Feedback loop is the basic structural element within the closed boundary (Fig. 2.2). The feedback loop connects the decision to action, to state (or level) of the system, to information about the level, and back to decision process.

A system usually consists of a number of loops, and the interactions between the loops affects the dynamic behavior of the system.

The term "decision", as used in the loop, applies to a broader definition of the term than the simple human decision. It may be purely a human decision, or the one made by mechanical or electrical devices. Whatever the nature of such decision, it always takes place in a feedback loop.

Levels and Rates. A feedback loop, at a lower level of hierarchy, has its own substructure. There are two fundamental components within a feedback loop, i.e., the level variables and the rate variables.

Levels describe the state of the system at any particular time. They are accumulations of the results of actions. Inventories, number of orders on hand, amounts of information, bank balances, and goods in transit are just a few examples of level variables. Levels can represent "information" variables as well as "physical" ones; they may also represent such intangible and vague variables such as awareness of a product among people, feelings, satisfaction, reputation of a company. Mathematically speaking, level equations are equivalent to the process of integration.

Rate variables, on the other hand, determine the slope of the level variables, that is, they determine how the levels change with time. They are the activities, the decision functions of a system, and may represent such activities as movement of goods, sales, expenditure of money, hiring rate of people, and so on.

Because the rates act only by affecting the levels, they cannot interact directly. This also implies that the rates are dependent on levels, and constants, and not on any other rates.

Accordingly, the values of levels depend on their previous values, and the inflow and outflow rates. They do not depend on previous values of other levels.

Confusion may arise in distinguishing between levels and rates, especially in cases where they are both measured with the same units. An appropriate way of checking this is to imagine that all activity in the system is brought to rest; the levels will still exist, whereas the rates will stop flowing.

Goals, Apparent Condition, Discrepancy, and Action. Although the rate and level variables are the substructures of a feedback loop, it is possible to look for sub-substructures within these substructures. However, the structure of a level variable is straightforward, that is, the new value of a level variable is obtained only by adding the change in the level to its previous value; therefore, it is not useful to breakdown the structure of the level equations.

Rate equation, on the other hand, has a different structure and meaning; as was stated earlier, a rate equation is a decision function, or a policy statement. As such, it tells how the available information is utilized to make the decision. The action follows the decision, but if there is any delay between decision and action, it would involve the presence of intermediate levels in the model. The rate equation is an algebraic equation, and free of delays and time-dependent changes. Such changes are only created by level equations. [21]

Within a policy statement (rate equation) these components are essential:

a. Goal;

b. Observed state of the system at any time;

c. Discrepancy between the observed state of the system, and the goal;

d. Action, based on the discrepancy.

Figure 2.4, shows the relation between these four components. More precisely, the above mentioned relation can be modified as in Fig. 2.5.

The rate equation sets a goal, makes a comparison between the apparent system condition (which is not necessarily the true system condition) and the goal, detects any possible discrepancy, and uses the information about the discrepancy to guide the next action.

In positive-feedback loop, "goal" is not the same as in a negative loop. In the latter case, goal is that state of the system toward which the policy is aimed, while in the former, goal is that state from which the system departs, and the discrepancy between apparent system condition and the goal will lead to further increase in discrepancy.

2.7. EQUATIONS, COMPUTATION, AND FLOW DIAGRAMS

In simulating the behavior of systems by using mathematical models, there must be a set of conventions, in the form of equations, in order to

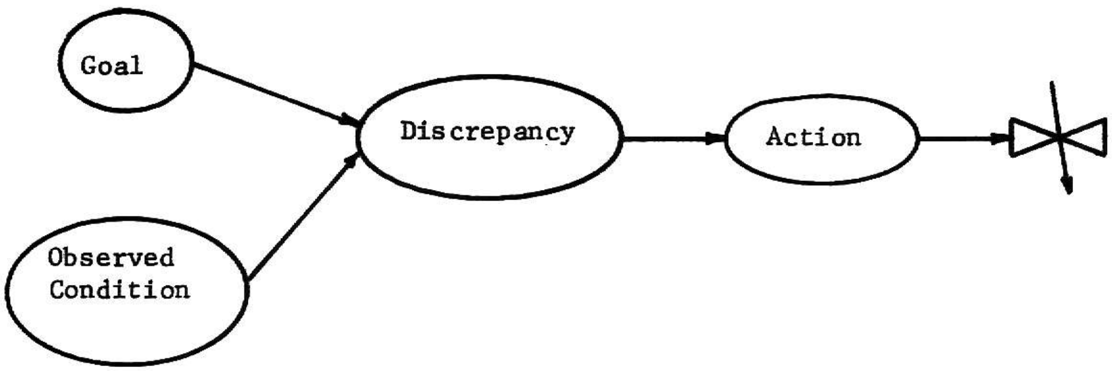


Fig. 2.4. Components of a Rate Equation

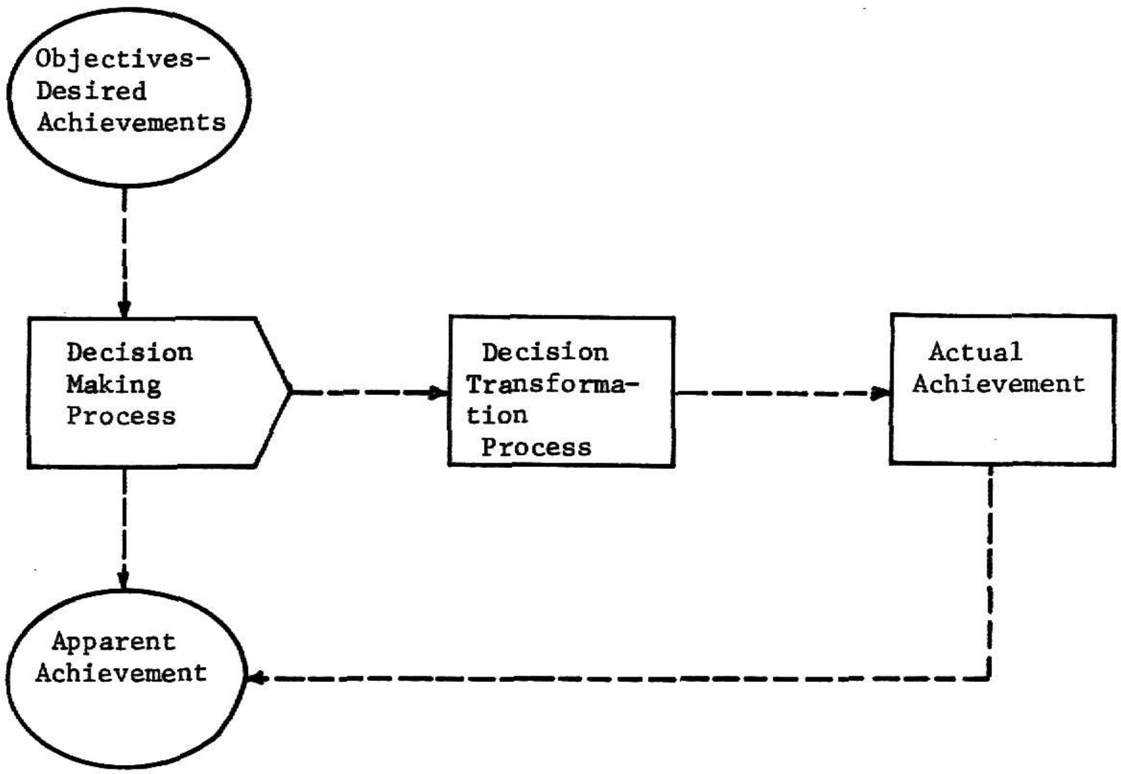


Fig. 2.5. Control System Structure

convey the structure of the model. These conventions should be capable of describing the situation, concepts, interactions, and decision processes in the system [13].

Simulation is essentially a step-by-step computation based on the system of equations describing the situation. This implies the need for a computing sequence, and the related symbols representing the model. In industrial dynamics simulation models we encounter two basic types of equations: level and rate equations; besides, there are other types of equations such as auxiliary and supplementary equations; constants and initial values are other parts of a model. Finally, the time step, i.e, the time interval between two consequent computations is of importance.

Equations, although necessary to describe the system, would be more meaningful if accompanied by some pictorial representation of the model, which shows the different kinds of movements among the system components. From here, arises the need for flow diagrams and related symbols.

Computing Sequence. Computing sequence determines how the computation of the system equations proceeds through time. The convention used in industrial dynamics is as shown in Fig. 2.6. Current time (present) is designated by 'K', past by 'J', and future by 'L'. 'J' is one DT behind 'K', and 'L' one DT ahead. 'DT' is the time step, as defined above, or the solution interval.

Starting at time'K', the levels at time 'J', and the rates, existed over the time interval 'JK' are avialable from previous computations; now the levels at time 'K' could be determined because they are only dependent on their previous values, at time 'J', and the rates over the time interval 'JK'. After the levels are computed at time 'K', the rates that would

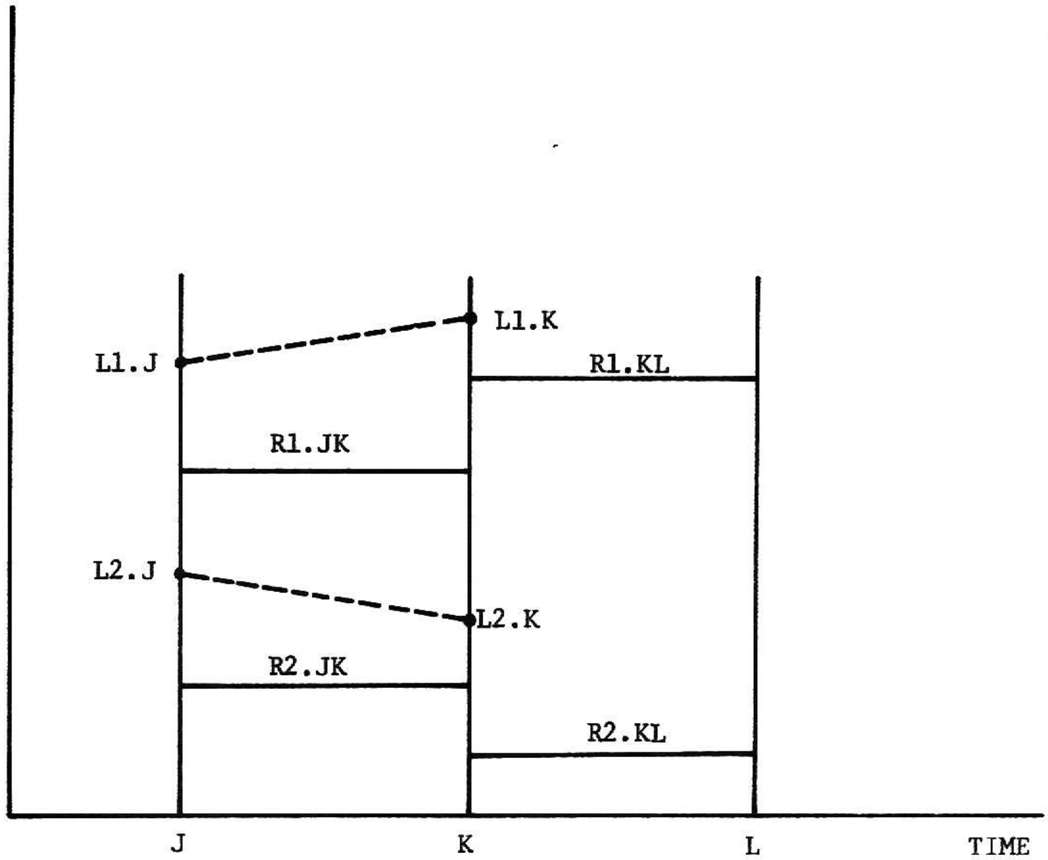


Fig. 2.6. Computation Sequence

exist over time interval 'KL' can be determined. The state of the system is now as shown in Fig. 2.6.

The sequence of computations now repeats for the next point in time; this will be done by advancing the time indicators, J, K, and L, one DT ahead, as in Fig. 2.7. The new 'J' is the old 'K', the new 'K' is the old 'L', and new 'L' would be one DT ahead of new 'K'.

The computations are exactly repeated in the same way. The previous rates over time interval 'KL' are now the rates over the time interval 'JK'.

Symbols and Time Notations in Equations. Variables and constants of the model should be represented by some symbols. Moreover, time notations, should specify the moment of time at which a variable applies. The time notations should also be specified such that they could be represented on computer printers. The following conventions(*) have arbitrarily been adopted in industrial dynamics:

- A symbol representing a variable or constant will consist of at most six characters, the first of which is always alphabetic.
- Level variables alway carry the single letter 'J' or 'K', separated from the variable name by a period, indicating their values at that time. Examples:

A.J	EMPLOY.J
LEVEL.K	INVEN.K
LE57.K	CASH.J

- Rates exist over a time interval, therefore the corresponding symbols carry two-letter time notations such as 'JK' or 'KL', separated from the variable name by a period; examples are:

*These are in accordance with the specifications of DYNAMO compiler, to be discussed in Appendix I.

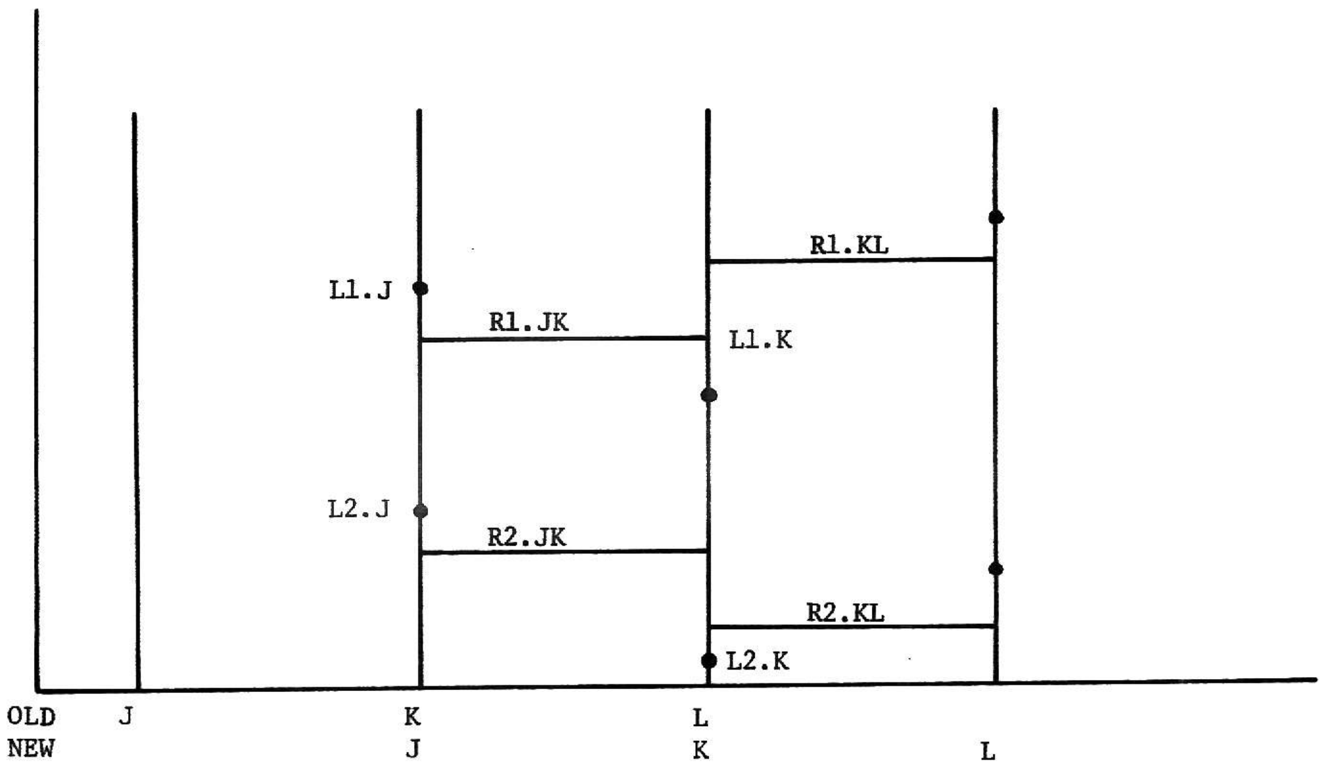


Fig. 2.7. Computation Sequence

RATE1.JK	B.KL
RATE2.KL	EMHR.JK
FLOW.JK	FEED.KL

- Constants carry no time notations, as their values are not changed with time. Examples:

CONST	MASH
ABCD	CASH
JEF	SALE

Level and Rate Equations. Levels represent reservoirs whose contents are varying by inflow and outflow rates. The value of a level depends on its previous value, and the change in its content during the time interval. The general form of a level equation would be:

LEVEL.K = LEVEL.J + (DT)(IR.JK-OR.JK) 2-1,L

LEVEL.K = Present value of the level, designated by 'LEVEL';(units)

LEVEL.J = Value of level from previous time; (units)

DT = Solution interval between 'J' and 'K'; (time units)

IR = Incoming rate to level 'LEVEL'; (units/time)

IR.JK = Value of incoming rate over the time interval 'JK';

OR = Outgoing rate from level 'LEVEL'; (units/time)

OR.JK = Value of outgoing rate over the time interval 'JK';

(2-1,L) represents a number assigned to the equation for reference, (it can be any number), and 'L' represents the type of equation, i.e., a Level equation.

A level equation is equivalent to the process of integration. Another words, the above equation can be written in integral form, as:

$$LEVEL = LEVEL_0 + \int_0^t (IR-OR)dt \qquad 2\text{-}2$$

in which $LEVEL_0$ is the initial value of the level variable LEVEL. The time interval 'dt' corresponds to 'DT' in previous equation. 'DT', therefore, is only appeared in level equations and not in any other type.

Rate equations represent the flow rates between levels of system, therefore the input to such equations are only levels and constants. The general form of a rate equation is:

$$RATE.KL = f(\text{Levels \& Constants}) \qquad 2\text{-}3,R$$

RATE.KL = Value of rate 'RATE' over the time interval KL; (units/time)

f = any function, or relationship, of levels and constants

Equation is assigned a number (2-3), and its type is designed by letter 'R'.

<u>Auxiliary and Supplementary Equations</u>. Often, the actual formulation of a rate equation may become complicated, and it would be more clear if the equation could be written in terms of its algebraic subdivisions, or parts. These subdivisions or parts are called "auxiliary equations." The following example will show the concept of auxiliary equations [13]:

$$SSR.KL = \frac{UOR.K}{DFR.K} \qquad 2\text{-}4,R$$

$$DFR.K = DHR + DUR \frac{IDR.K}{IAR.K} \qquad 2\text{-}5,A$$

$$IDR.K = (AIR)(RSR.K) \qquad 2\text{-}6,A$$

In this example, the value of the rate 'SSR' depends on the values of two levels, 'UOR' and 'DFR'. The value of 'DFR', however, depends on the constants 'DHR' and 'DUR', and levels 'IDR' and 'IAR'. The value of 'IDR' itself depends on the values of constant 'AIR' and level 'RSR'.

Writing equation (2-4,R) in this way, helps clarify its structure. It does not alter anything in the rate equation; it still depends on levels and constants, as can be seen by substituting (2-6,A) into (2-5,A) and the result into (2-4,R), to obtain:

$$SSR.K = \frac{UOR.K}{DHR + DUR \dfrac{(AIR)(RSR.K)}{IAR.K}} \qquad 2\text{-}7,R$$

which shows the dependence of 'SSR' on constants and levels.

The appearance of letter 'A' indicates the type of the equation, i.e., "Auxiliary."

Supplementary equations define variables which are not actual variables in the system, but they contain information which is desired to be printed or plotted. As such, these variables are only used in printing or plotting instruction.

Supplementary equations will be denoted by letter 'S'.

Initial-Value Equations and Constants. All levels should be given initial values before the start of the simulation process. These initial values for levels are necessary to determine the flow rates over the first time interval (0 - DT). Initial-value equations would be used to serve this purpose.

The initial-value equations are also used to determine the values of some constants from other constants. These equations are designated by letter 'N'.

Constant values of the model are designated by letter 'C'. They should be given actual numerical values.

Flow Diagrams. A pictorial representation of an equation system will simplify the formulation. To many people, a flow diagram is more understandable than a set of equations.

The Industrial Dynamics Research Group at the M.I.T. Sloan School of Management has developed a set of standard symbols for flow diagrams of dynamic models [13]. This set is capable of representing the interrelationships in the system; it distinguishes between levels and rates. It clearly illustrates the six flow systems (flow of information, material, orders, money, personnel, and equipment); it shows the factors affecting the decision functions. However, it does not represent the functional relationships within such functions, but the equation number describing the functional relationship is specified.

Flows - The six types of flow systems are represented in Fig. 2.8.

Levels - levels are represented by a rectangle, with the name of the level inside the rectangle, and inflow and outflow rates as shown in Fig. 2.9. The flow system need not be specified, because the representation of flow lines will determine its kind.

Rates - Decision functions act as valves in the flow channels, and hence their representation (Fig. 2.10). The constants and the levels affecting the rates are as shown. The name and the equation number defining the decision function are specified inside the symbol.

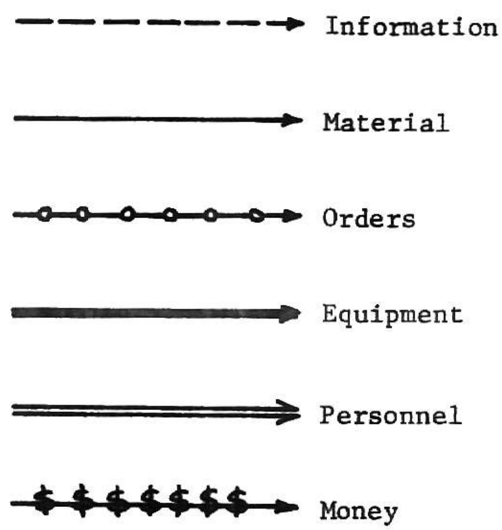


Fig. 2.8. Flow Symbols

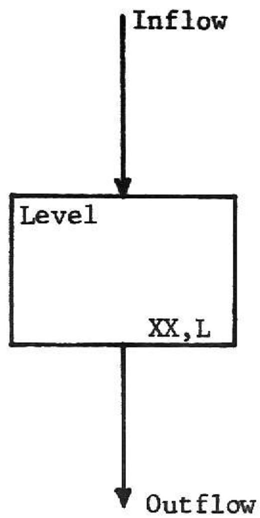


Fig. 2.9 Levels

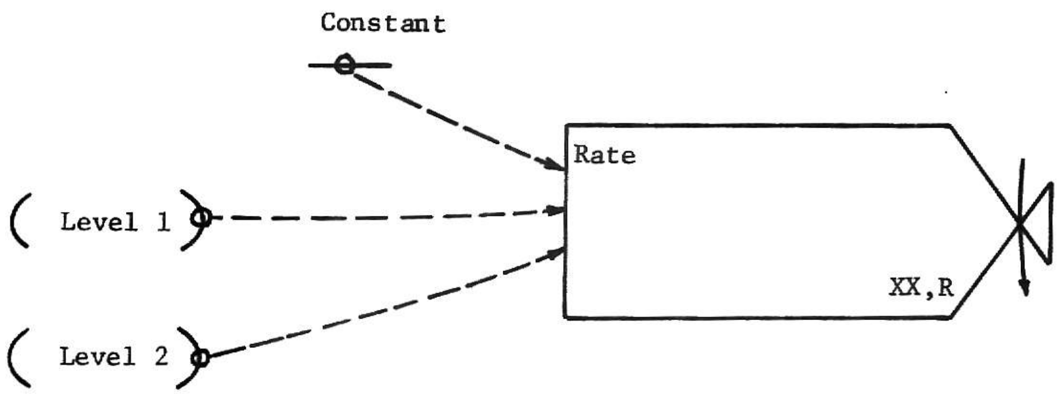


Fig. 2.10 Rates

Information Take-Off - Information flows are an essential part of any system. The take-off of information about a variable does not affect the variable. It is shown by a small circle at the information source, and the information flow line (Fig. 2.11).
In some instances, the level at which the information take-off takes place, is itself an information level, in which case the take-off might affect the level.

Auxiliary Variables - These variables have been subdivided out of the decision functions, because of their independent meanings. They are shown by a circle, with the name of the variable and its equation number inside (Fig. 2.12). The parameters and levels on which the auxiliary variable depends, are shown as usual.

Parameters (Constants) - The values of parameters will not change during a simulation run. However, they might change between successive runs. They are shown by a line above or below the parameter symbol, and an information take-off (Fig. 2.13).

Sources and Sinks - In many cases, the source of the flow has no influence on the system, and it is assumed that the flow is coming from an 'infinite' source. Such a source will not be exhausted, and can yield any rate of flow demanded by the system. Similarly, a flow might terminate into a 'sink', which has no significant dynamic characteristics. They are shown in Fig. 2.14.

Variables on Other Diagram - In cases where the flow diagram of the system is divided into sections, the flow lines and information

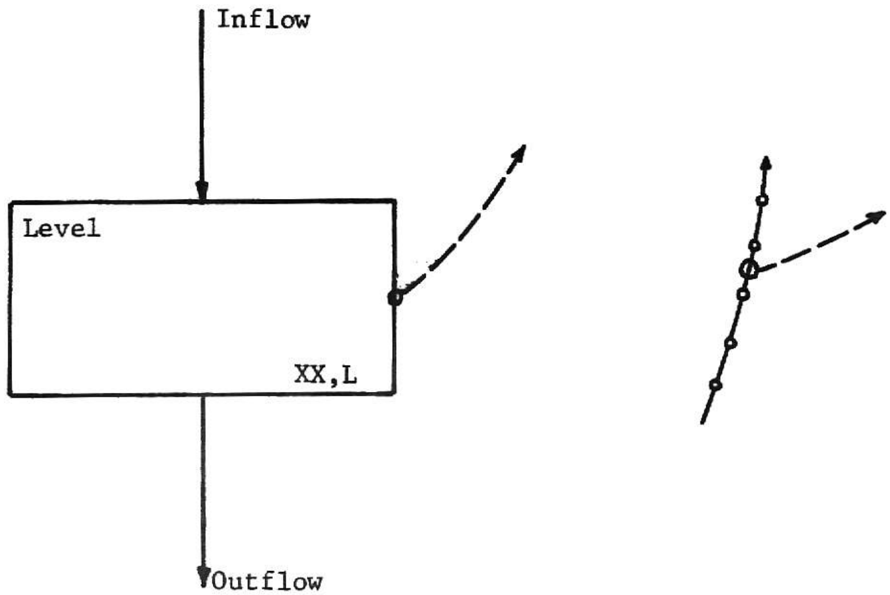


Fig. 2.11 Information Take-Off

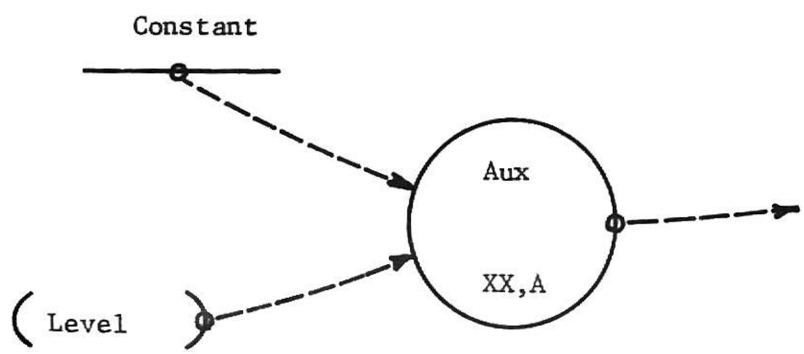


Fig. 2.12. Auxiliary Variables

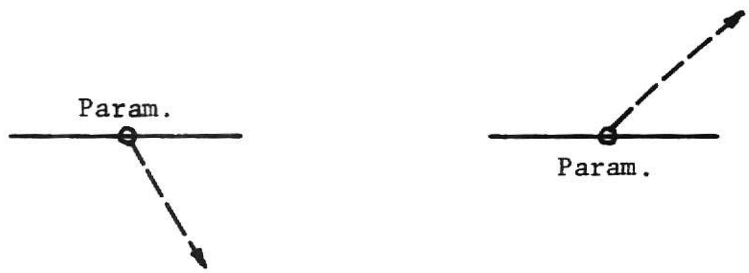


Fig. 2.13 Constants

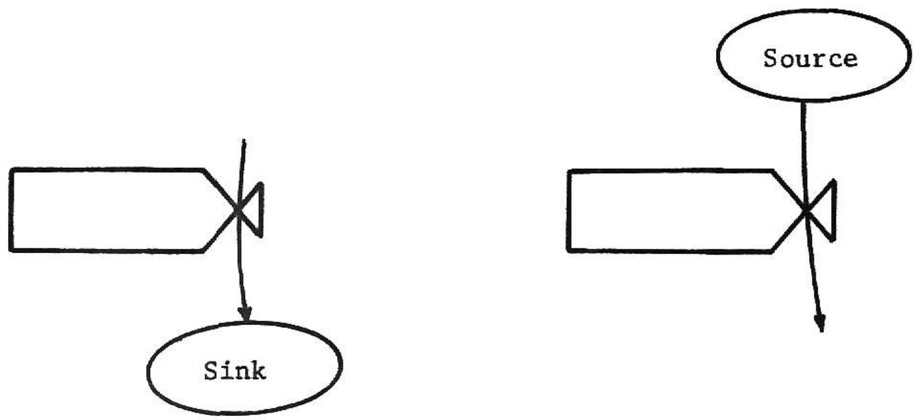


Fig. 2.14 Sources & Sinks

transfers that cross from one section to another, will be identified as in Fig. 2.15. The origin or destination, name, abbreviation, equation number, and the type of the equation are specified.

Delays - Exponential delays (to be discussed in next section), can be represented by a combination of levels and rates. But an abbreviated symbol is needed, because the delays are frequently encountered. This symbol is as in Fig. 2.16.

Time Interval, DT. The solution interval, DT, is not a characteristic of the system, but rather, a parameter of the model representing the system.

The process of simulation of the system, implies that the flow rates, during the time interval DT, are considered constant, and therefore the actual system behavior will be replaced by a piece-wise linear curve, as shown in Figs. 2.17a and 2.17b. But the length of DT should be small enough so that this piece-wise linearization of the system performance is close enough to the real performance. In (a) the real performance is not far from the simulated performance, whereas in (b) the two performances are quite apart. This has been only due to choosing a larger DT. But a smaller DT requires much more computer time, as the process of simulation is repeated each DT time. Therefore, there is a trade-off between a closer approximation to the system by choosing a smaller DT, and a greater computer time and cost, as a consequence.

As a rule of thumb, the solution interval, DT, should be at most half, or less, of the shortest first-order delay (to be discussed in next section) in the system. In case of 3rd-order delays, it should be 1/6, or less, of the shortest 3rd-order delay in the system.

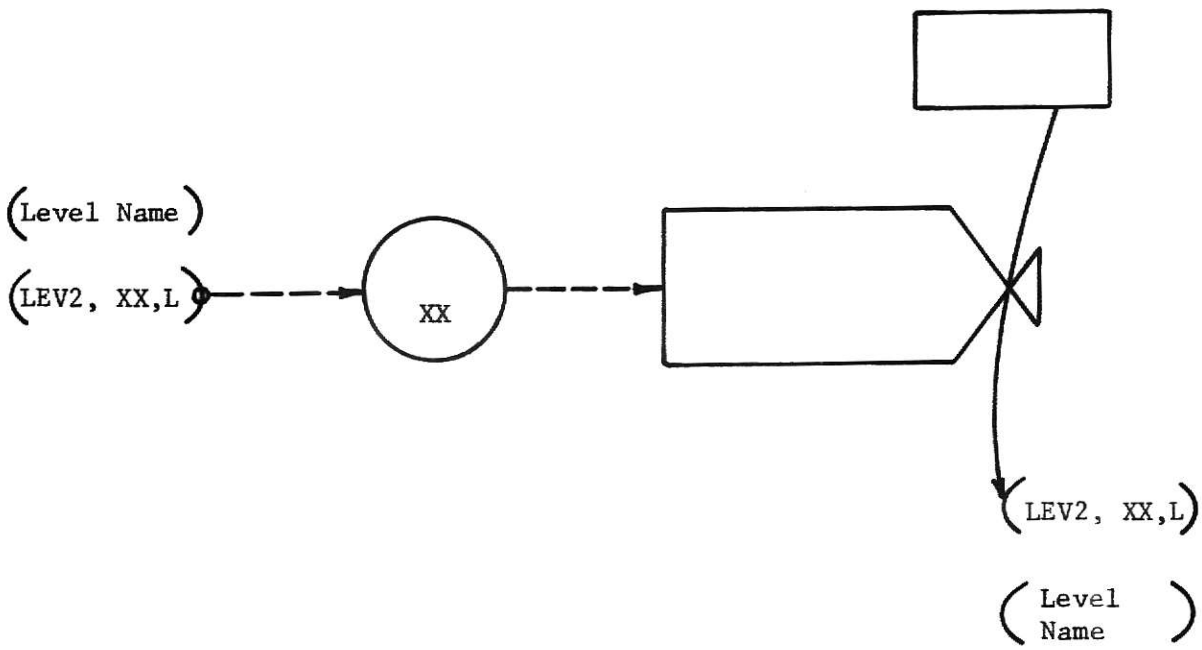
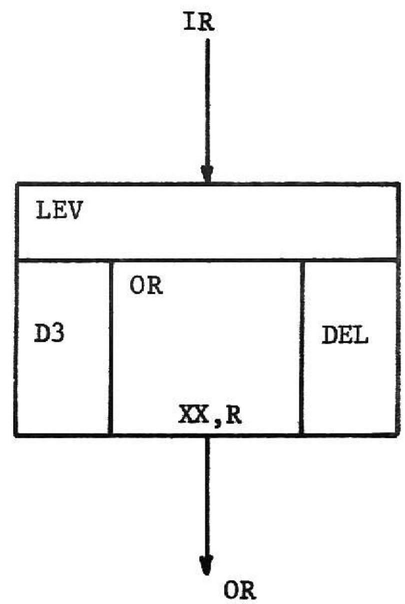
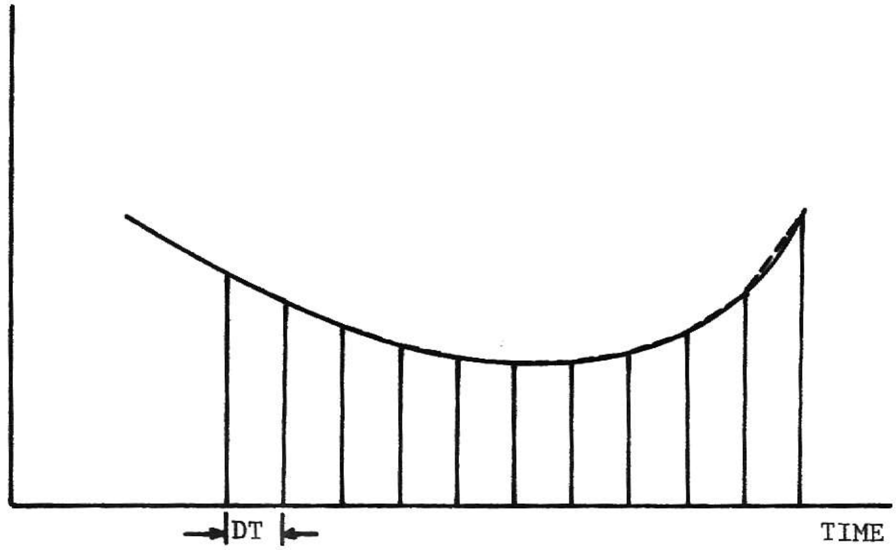


Fig. 2.15 Variables on Other Diagrams

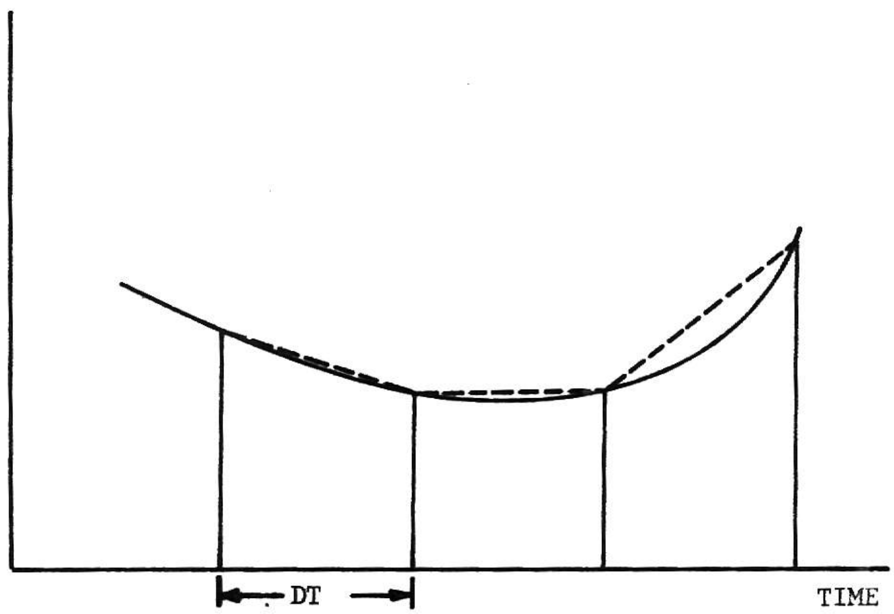


IR - Input Rate
OR - Output Rate
LEV - Level in Transit
XX,L - Eq. No. and Type, for Level "LEV".
D3 - Order of Delays
DEL - Average Delay
XX,R - Eq. No. and Type for Rate "OR".

Fig. 2.16 Delays



(a)



(b)

Fig. 2.17 Effect of Solution Interval, DT

2.8. DELAYS

The effect of time delays on the behavior of dynamic systems is important, because delays are found in almost all flow channels of the system, and they determine many aspect of the system behavior [35].

Delays exist in both physical flows(materials, people, money, etc.), and information flows. But to avoid too many details in the system model, which arise from consideration of all delays, two simplifications are suggested [13]: first, shorter delays, compared to longer and more effective ones, are eliminated, and second, delays arising from separate processes coming one after another, can be represented, in many cases, by only one delay.

Definitions and Characteristics. A delay is defined as "a conversion process that accepts a given inflow rate, and delivers a resulting flow rate at the output. The outflow may differ instant by instant from the inflow rate under dynamic circumstances where the rates are changing in value. This necessarily implies that the delay contains a variable amount of the quantity in transit." [13]

Often, a delay can be considered as a special case of level variables, in which the outflow rate is controlled by the internal level stored in the delay (and, of course, a number of constants). Whereas, in the general concept of level, there is no restriction on the number of factors controlling the outflow.

There are two important characteristics for a delay: the "average delay," and the "transient response."

- Average delay determines the steady-state behavior of the delay. At steady state, the outflow rate, inflow rate, and the level in-between are constant.

- Transient response determines how the time-shape of the outflow rate is affected by the time-shape of the inflow rate, while the latter changes with time.

  Although the average delay may be the same for different delays, their transient responses are not necessarily the same.

Exponential Delays. Delays of various kinds, can be created by various computational processes. Here a special class of delays - exponential delays - are considered, because of their simplicity in form, and their adequacy to represent many intuitive aspects of the social systems.

A "first-order" exponential delay consists of an inflow rate, an outflow rate, and a level between them, the value of outflow being dependent on the level and the average delay. Inflow rate is the output of some other part of the system. Such a delay is illustrated in Fig. 2.18.

In above figure, (a) is the "long-hand" representation of the delay, and (b) the "short-hand" form, as used in dynamic flow diagrams.

The outflow rate, in a first-order delay, is equal to the level divided by the average delay:

$$OR.KL = \frac{LEV.K}{DEL} \qquad \text{2-8,R}$$

The above decision function results from the state of the system, and not from managerial functions. The level stored in the delay, LEV, should be now determined by a level equation.

$$LEV.K = LEV.J + (DT)(IR.JK - OR.JK) \qquad \text{2-9,L}$$

The above two equations are sufficient to define a first-order exponential delay.

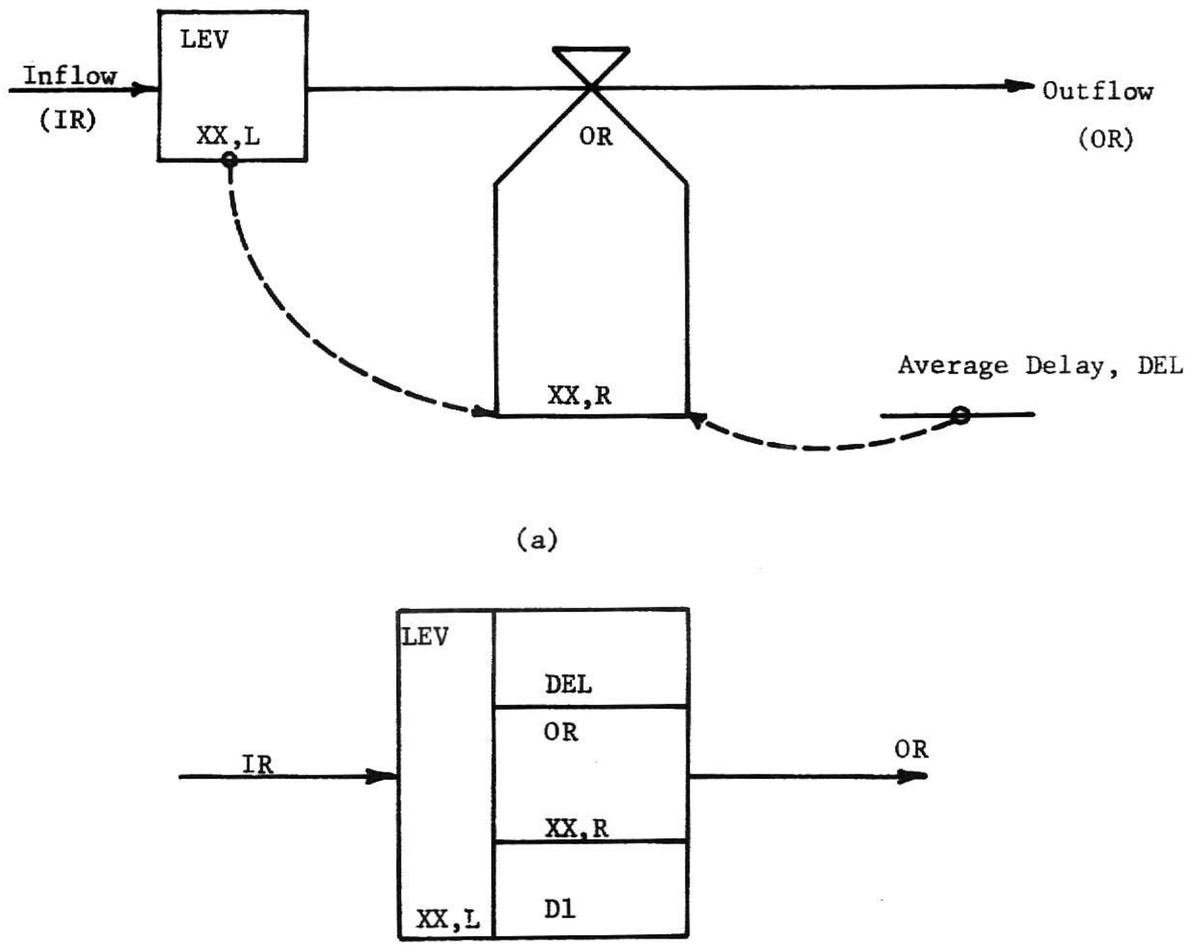


Fig. 2.18 - First-Order Exponential Delay

Higher-order exponential delays are obtained by cascading first-order delays, as in Fig. 2.19, which shows a third-order delay, and its "short-hand" representation. In this case, the total average delay, DEL, will be divided equally between the three stages.

The total average delay of a higher-order delay may be equal to that of a first order, but again, their transient responses will be different.

The equation system for the third-order delay is as follows:

$$OR1.KL = \frac{LEV1.K}{\frac{1}{3}\,DEL} \qquad 2\text{-}10,R$$

$$LEV1.K = LEV1.J + (DT)(IR.JK - OR1.JK) \qquad 2\text{-}11,L$$

$$OR2.KL = \frac{LEV2.K}{\frac{1}{3}\,DEL} \qquad 2\text{-}12,R$$

$$LEV2.K = LEV2.J + (DT)(OR1.JK-OR2.JK) \qquad 2\text{-}13,L$$

$$OR.KL = \frac{LEV3.K}{\frac{1}{3}\,DEL} \qquad 2\text{-}14,R$$

$$LEV3.K = LEV3.J + (DT)(OR2.JK - OR.JK) \qquad 2\text{-}15,L$$

and the total quantity stored in the delay, at any time:

$$LEV.K = LEV1.K + LEV2.K + LEV2.K \qquad 2\text{-}16,\ A$$

Substituting equations (2-11), (2-13), and (2-15) into (2-16) gives:

$$LEV.K = LEV.J + (DT)(IR.JK - OR.JK) \qquad 2\text{-}17,L$$

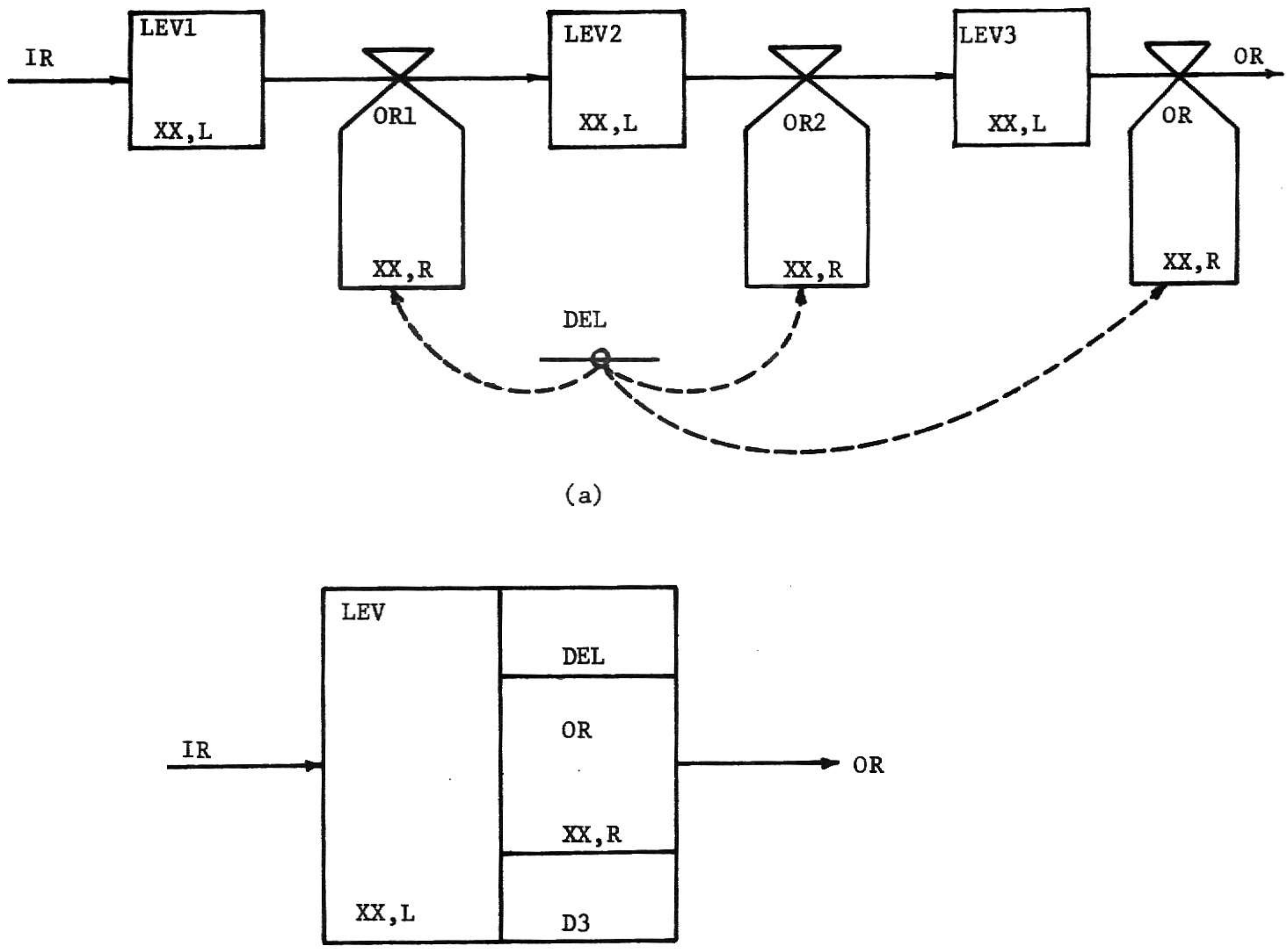


Fig. 2.19 Third-Order Exponential Delay

Equations (2-10) through (2-15) are concisely abbreviated by the following functional notation:

OR.KL = DELAY3(IR.JK, DEL)   2-18,R

Equations 2-17 and 2-18 are sufficient to describe a third-order delay.

Time-Response of Exponential Delays. The mathematical forms of the delays were investigated. Here the transient response of these delays will be considered. The inputs to the delay would be an "impulse function," and a "step function".

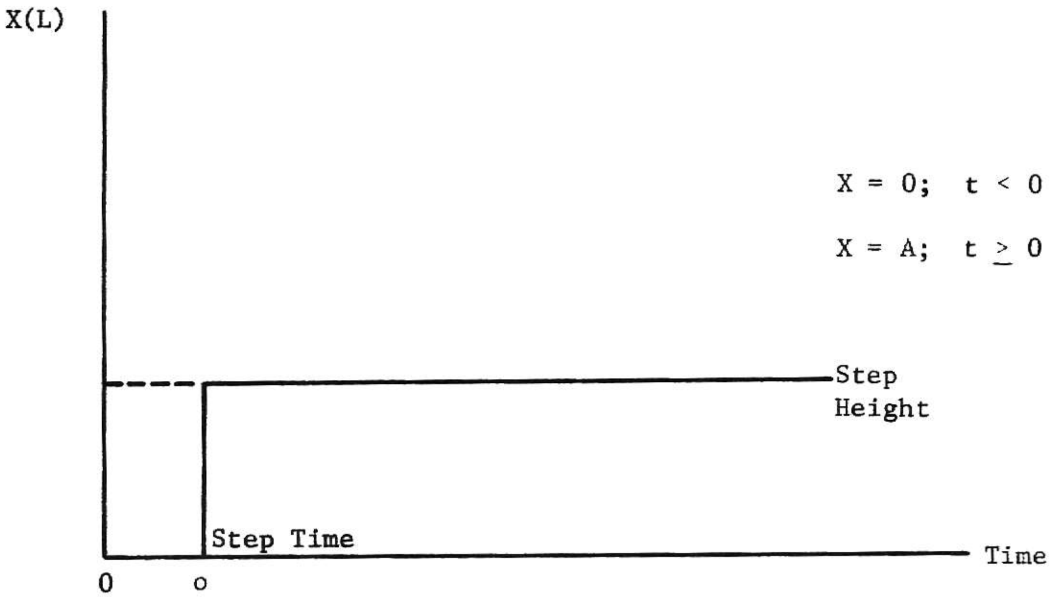
The impulse and step functions and their mathematical representations are shown in Fig. 2.20.

The response of different types of delays to step & impulse functions are outlined in Figs. 2.21 through 2.24.

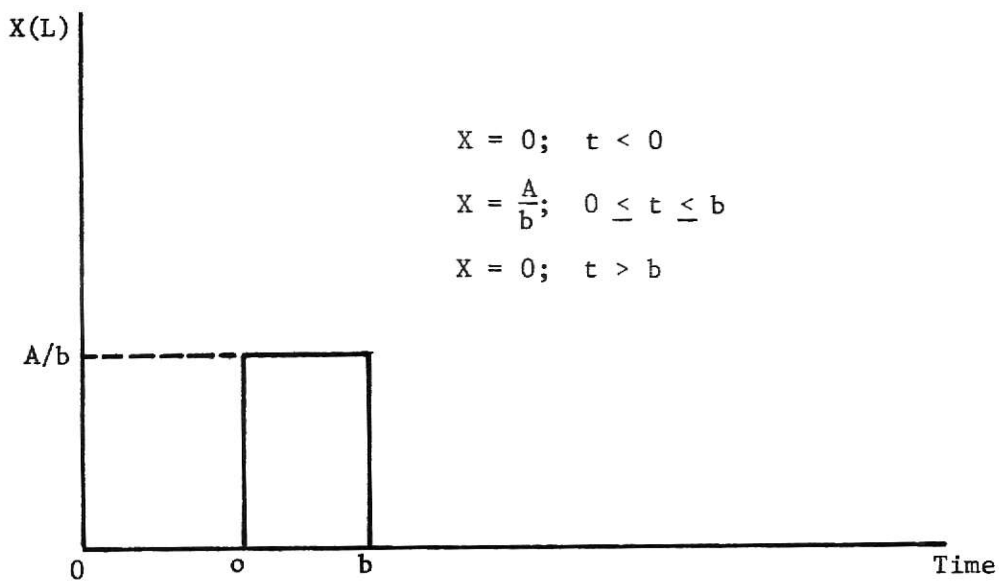
In Fig. 2.21, for a first-order delay, and for an impulse input, the rate of output is maximum at the time the input is applied, and then it exponentially decreases. With a step function as input (Fig. 2.21b), it can be shown [13] that the slope of the curve at the origin is maximum, and it decreases thereafter. The output rises to 63.2% of the final value after a time equivalent to the average delay, DEL.

For a second-order delay (Fig. 2.22) the initial output of the system, in response to an impulse input, is zero, and the slope of the output curve is maximum at the origin (Fig. 2.22a). The response of this delay to a step input will be delayed for a short time at first, and then it gradually rises to approach the final value.

A third-order delay responds to an impulse input function only after a time lag has elapsed. The output rate is zero at the beginning, and is rising thereafter (Fig. 2.23a). Its response to a step input is similar to that of a second-order delay, but with a longer delay at the beginning (Fig. 2.23b).

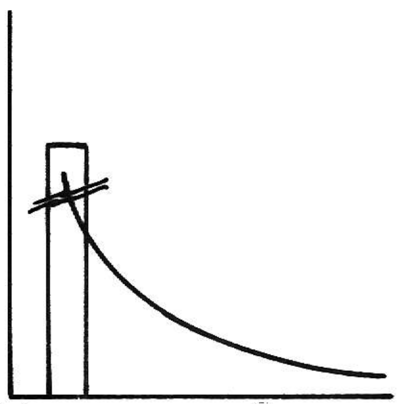


(a) Step Function

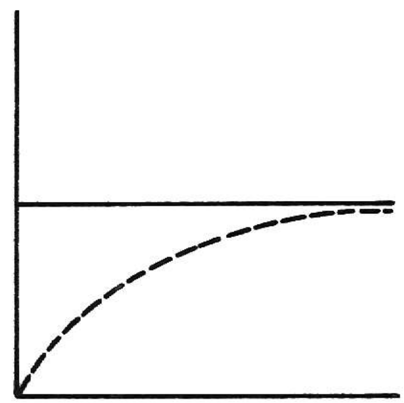


(b) Impulse Function

Fig. 2.20 Impulse and Step Functions

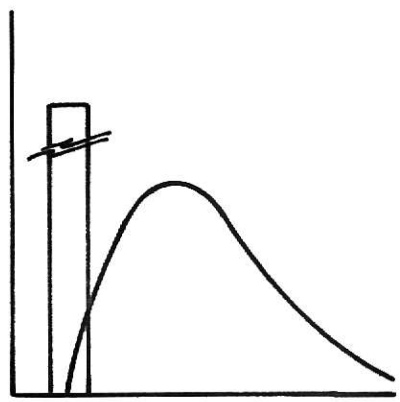


(a) Response to Impulse Input

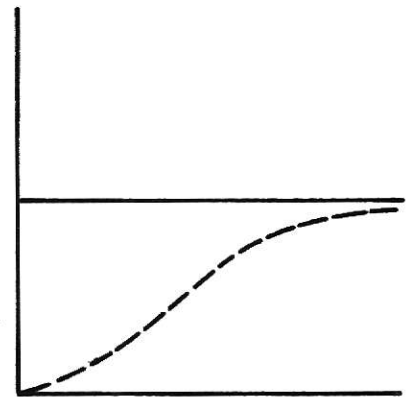


(b) Response to Step Input

Fig. 2.21 First-Order Delay's Responses

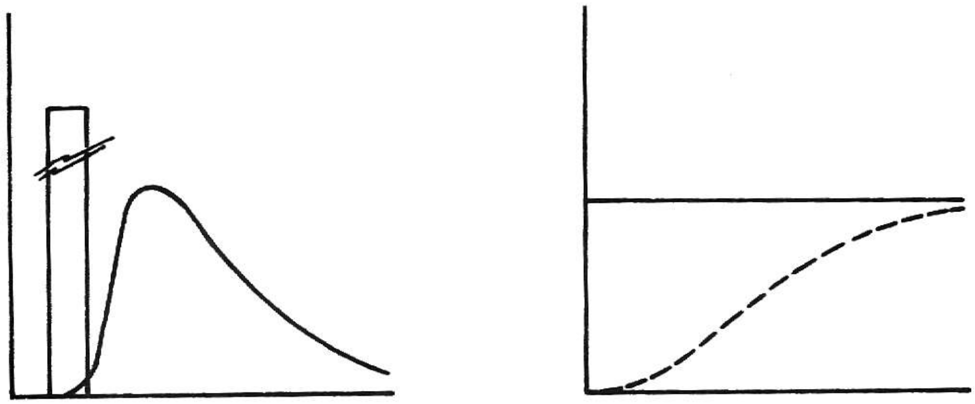


(a) Response to Impulse Input



(b) Response to Step Input

Fig. 2.22 Second-Order Delay's Responses



(a) Response to Impulse Input (b) Response to Step Input

Fig. 2.23 Third-Order Delay's Responses

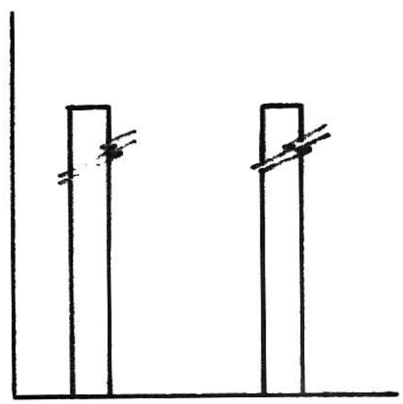
First-order delays could be cascaded into any number, until the final time-shape of the output fits the intuititve concepts of a certain situation to be represented. An example is coupling an infinite number of first-order delays one after another. This is called a discrete, or pipeline, or infinite-order delay. Its response to the input is different for the others' in that nothing happens until the average delay time, DEL, is passed, at which time input is reproduced exactly (Fig. 2.24a and b).

A third-order delay, in most cases, satisfies many of the intuitive concepts of the behavior of social systems. However, if a different time-shape of output is desired, it can be obtained, approximately if not exactly, by cascading first-order delays one after another. For example, the initial delay in the output of the delay, in response to the input, can be increased, or the maximum output of the delay, in response to an impulse input, can be postponed to a later time.

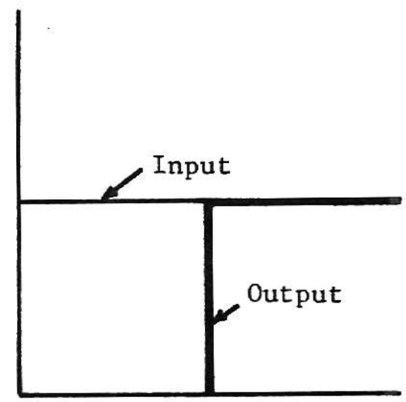
In Figs. 2.25 and 2.26 are shown the accurately plotted output responses of 1st, 2nd, 3rd, 6th, and infinite-order delays to an impulse input and a step input, respectively. The horizontal axis represents the dimensionless quantity "time/DEL", and the vertical axis, the dimensionless quantity "Output Rate / $\left(\frac{\text{Initial Rate of Input}}{\text{DEL}}\right)$", in order to normalize all the curves.

2.9. MODELING

A model is a substitute for a real object or a system. It can be physical or abstract, representing a set of rules and relationships that describe the system. In simulating the behavior of systems, we are dealing with a special class of abstract models, namely, mathematical models. Abstract models, "include mental images, literary descriptions, behavior



(a) Response to Impulse Input



(b) Response to Step Input

Fig. 2.24 Infinite-Order (Discrete) Delay's Responses

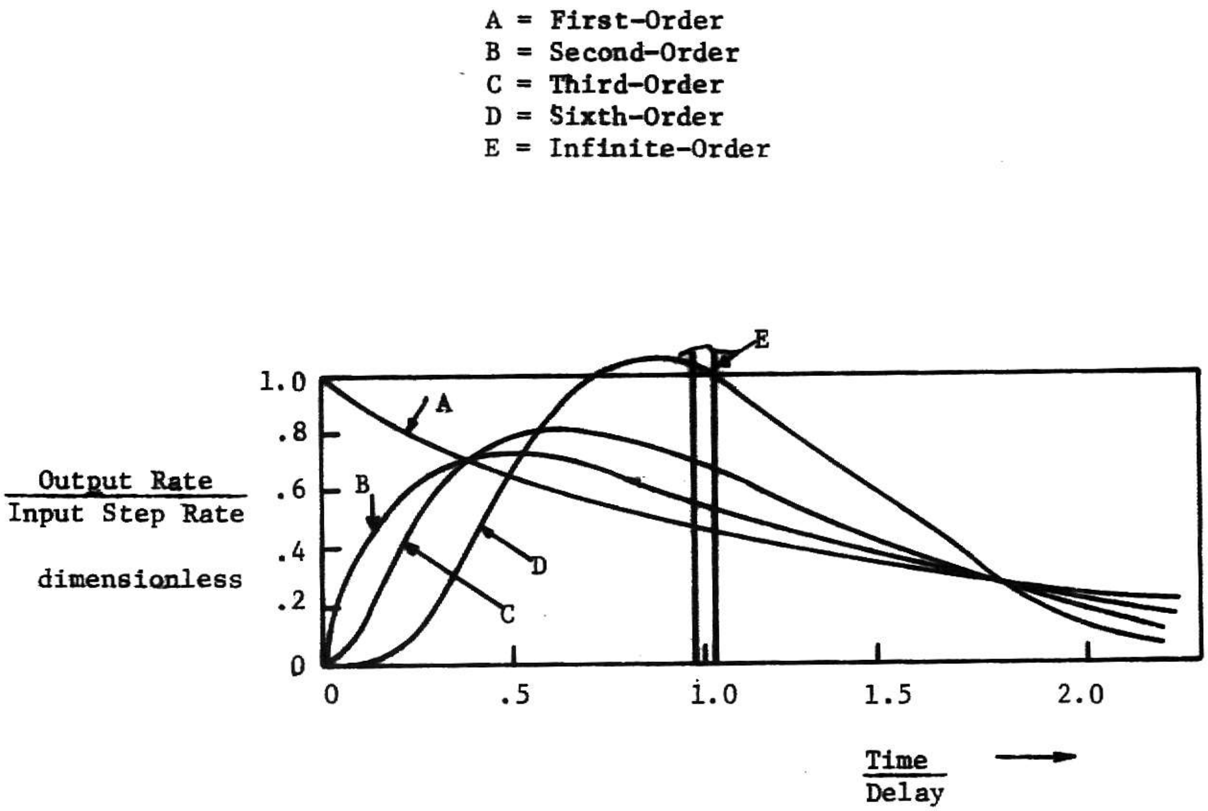


Fig. 2.25 Exponential-Delay Responses to a Unit Impulse

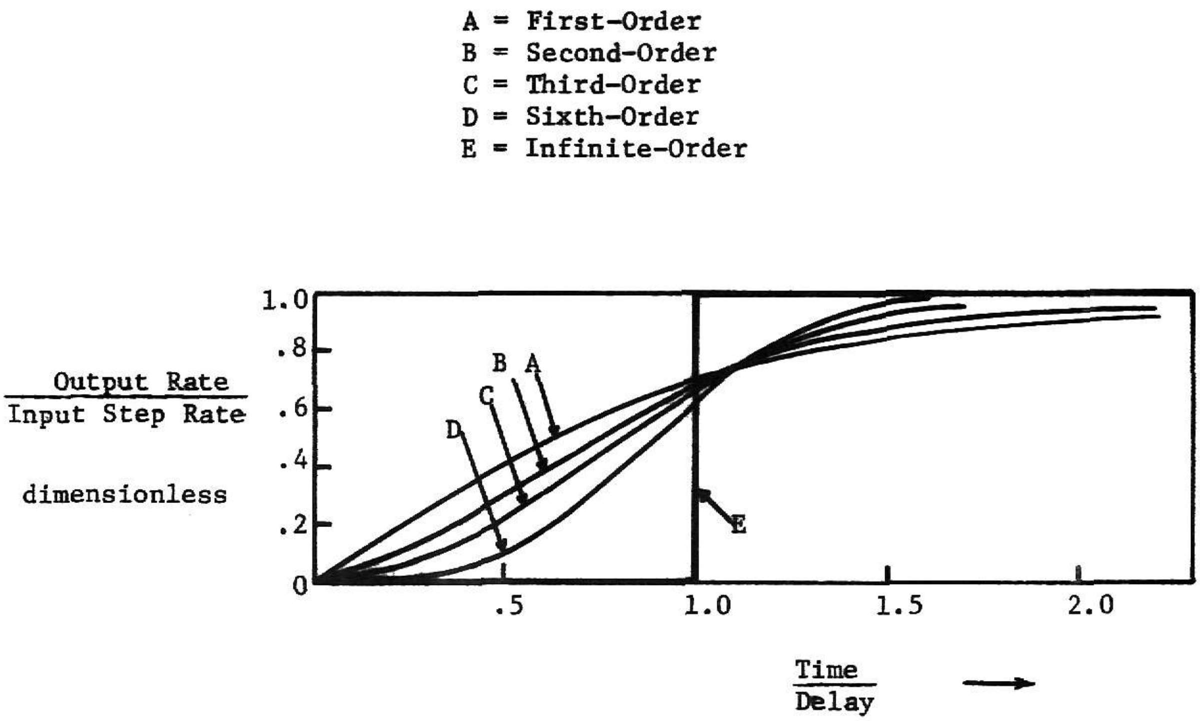


Fig. 2.26 Exponential-Delay Responses to a Unit-Step Rate

rules for games, and legal codes." [21]

Mental models, although good at manipulating situations that associate words and ideas, are not adequate to describe the time-varying behavior of modern complex systems. Their defects are [21]:

1 - They are ill-defined; their structure does not keep up with changes in assumptions and real life observations.

2 - Assumptions about such models are often not identified clearly.

3 - Mental models are quite difficult to communicate to others.

4 - Mental models cannot be manipulated easily and effectively; wrong conclusions about the system behavior are highly likely to occur even if the model is correct, because of the inability of the human mind to manage all aspects of a complex system, and therefore its tendency to fragmentation in such systems.

Basis of Model Usefulness. Usefulness or validity of a model of dynamic systems, should not be judged by its ability to predict a specific action in the future, or against an imaginary measure of perfection, but rather on the basis of its advantages over the mental and descriptive models, and its ability to reproduce the behavior characteristics of the system [21, 35].

A model cannot be, and is not, an exact representation of "reality", and as such, it is successful if it gives a better insight into the system it represents, than what can be obtained otherwise. They should be approved of, if they improve the accuracy of the representation of the system.

Formulation of Dynamic System Models. In this section, the factors to be considered in model-building of systems are represented. These factors are directly related to the purpose the model should serve.

Therefore, a system model is built on the bases of the questions about the system, and its purposes. It will be refined and extended over the time as the knowledge about the system is increased. This implies that an all-inclusive model is very difficult and unlikely to achieve. [13]

In questioning the different aspects of the system, those factors should be identified that have important influences on answers sought. There must not be any restrictions on the inclusion of any factor that is relevant to the system behavior. Nor should there be any limitations to a specific class of factors. In a model, "technical, legal, managerial, economic, psychological, organizational, monetary, and historical factors" might very well be considered in their proper places. [13]

Social and industrial systems are, as previously stated, closed-loop, information-feedback systems. Their models, therefore, should identify the feedback loops and their interactions. The decision processes within the system, the actions that follow the decisions, the resulting changes in different levels of the system, and information flows about new conditions, that will provide new basis for the decision processes, are the essential elements in these loops.

Within information-feedback systems, the time delays, amplifications, and information distortions should be carefully investigated, as they are important elements to affect the loop characteristics.

Time delay. Time delays exist in almost every stage of the system performance, such as delays in decision making, in gathering information, in transportation, and so on. Although delays are an important factor in our systems, they do not always have bad effects on system behavior. As will be investigated later, in many cases, delays have little effect on the overall performance of the system, and in other cases, introduction of delays in proper places in the system will improve its character.

Amplification. Amplifications occur whenever the response of the system, at some point, is greater than what might be expected from the cause of the response. An example is the fluctuations in the rate of factory production, as a result of small changes in retail sales. The policies that determine decisions, which control flow rates, are the sources of amplification [13], and hence, their careful examination is an essential part of the system modeling.

Information Distortions. Information can be distorted not only by delays and amplifications, but also by "errors, random noise, and unknown perturbations from external sources." [21] Since decision processes are based on information, any distortions in information will influence the actions following the decisions. This points out the concept of "information quality," at each point in system - as determined by the amount of bias, error, and distortion in it. Information, as the basis for decisions, and its source should be given enough attention in the process of model-building.

It should be emphasized, finally, that model variables should sufficiently correspond to the real-system variables, and this will be achieved only by "constant alertness and re-examination of whether the decision functions adequately represent the concepts, social pressures, and sources of information that control the actual decisions." [13]

CHAPTER 3

LITERATURE SURVEY ON INDUSTRIAL DYNAMICS

This chapter presents a brief survey of the literature on industrial dynamics. The DYNAMO compiler is also described.

3.1. LITERATURE ON INDUSTRIAL DYNAMICS

Industrial dynamics is a recent development in the area of management science, and is claimed to be "evolving toward a theory of structure in systems as well as being an approach to corporate policy design" [15].

It originated at M.I.T.'s Sloan School of Management; J. W. Forrester, with his extensive background in theory of feedback control, computer science, and management practice, started the pioneering works on industrial dynamics, during the time period 1956-61. His first book on the subject, "Industrial Dynamics" [13], was published in 1961. Since then, he has done most of the work in this area, although his followers are also to be credited for their works on interpretation of the principles of industrial dynamics, and its application in actual industrial systems.

To date, the only books dealing directly with industrial dynamics and its principles are the two written by Forrester, i.e., "Industrial Dynamics," and "Principles of Systems"* [21]. The first one gives a detailed account of various aspects of the subject, its foundations, model building of dynamic systems, mathematical representation of models, problems related to model validation, etc. It further gives examples of how to

*It has not yet been published in the final form, and is only available in a preliminary edition.

apply those principles to investigate the behavior of some complex industrial situations.

The second source, although not in the final form at this time, is an attempt toward building a comprehensive theory of structure of systems. It embarks on the assumption that all systems (physical, social, economic, biological, and so on) have in common what can be termed as "feedback behavior." All systems, therefore, can be studied by applying the principles of information - feedback control.

Besides the two major works on the subject, there are several papers published, both by Forrester and his followers, with the purpose of explaining and implementing the industrial dynamics aspects. Carlson [3] has reviewed the essential ideas of industrial dynamics. Forrester [10,11,12,14,16,18,19,20,22,23] over the last decade has tried to resolve the confusions on industrial dynamics, explain its foundations and its relations to other fields of science, and describe the difficulties encountered when attempting to apply industrial dynamics to real world situations. Roberts [41,42], by far one of the most active associates of Forrester, besides his own extensive works on the management of research and development projects, which will be discussed in next section, has also tried to clarify the industrial dynamics concepts, and specifically explained how it can be used as a "control system" for management problems.

It should be pointed out, however, that there are not enough literature and educational material available on the subject to be of help to the beginner to achieve the necessary understanding of the systems. Forrester himself has pointed to this shortcoming:

> "...The industrial dynamics literature suggests the promise of advantages which may accrue from a better understanding of systems, but it does not adequately convey the essential mathematics of the field, nor expose the principles which should guide judgement in modeling of systems, nor does it provide an adequate number of examples to be used as guides in system structuring" [15].

He further suggests that the existing gaps should be filled by "an appropriate treatment of the mathematics of system dynamics," having "enough examples of the system structure" to explain the behavior of corporate and economic systems, and "bridging articles to show how system concepts can be applied in the functional areas" of management practices.

3.2. LITERATURE ON APPLICATIONS OF INDUSTRIAL DYNAMICS

Since its development, industrial dynamics has found applications in areas ranging from production-distribution, inventory control, advertising, and so on, to management of research and development projects, production cycles, and even such highly complicated systems as urban and world organizations.

Among the first applications of industrial dynamics to real world situations, is the case study of a customer-producer-employment practice in an electronic components company, done in large part by Fey [8 ,13]. It is an investigation of the causes for a fluctuating employment level with a variation factor of 2 to 1, and peak-to-peak intervals of about 2 years; the study explores the possible ways of improvement, besides analyzing the situation.

Roberts [45] has applied the principles of industrial dynamics to research and development projects, to investigate the underlying causes of their failure or success, such as overexpenditure, estimation of cost and effort, evaluation of the project progress, and so on. He has also explained some specific aspects of R & D management elsewhere [43,44,46].

Katz [31] has shown how management of R & D projects can use industrial dynamics approach to solve the related problems.

Schlager [10] has done three studies on three types of production problems, approaching some R & D aspects in one of them.

Forrester [9,13,24] has studied the advertising problem in industrial systems, and proposed a dynamic model for advertising practices. Weymar [52] has also approached the problem of interactions between firm and market, using industrial dynamics principles.

Forrester [12] has analyzed the dynamic processes which lead to corporate growth, and Nord [36] has studied the effects of capacity - acquisition on the growth of a product.

Meadows [34] has studied the variations in both price and production rate of commodities. He has proposed a "general dynamic commodity model" to predict the production cycles of the commodities. To test the model, he has further used three sets of parameters representing the U.S. hog system, chicken system, and beef production. The model has predicted, in all three cases, the corresponding cycles observed in those systems. The study concludes that "entirely new approaches" should be devised in order to effectively stabilize the commodity systems.

Use of industrial dynamics has not been confined to industrial management problems. As it explores the general principles of systems, it can be applied to a variety of organizations. There are two studies of this kind to date; in "Urban Dynamics" [25], Forrester examines the life cycle of an urban area within the context of feedback structure of its components. It investigates the interactions between factors such as industries, housing, and people, in a newly developed area, and shows how interplay between these factors causes the urban structure to develop in the beginning, and then stagnate.

It is mainly a method of analysis, rather than a policy-making study; however, it gives valuable insights into urban problems.

In the second study, "World Dynamics" [26], Forrester considers the whole world as a system, and shows how interactions between demographic, industrial, and agricultural subsystems of the world organization determine its course of behavior. The computer model of the world system predicts how response to policies in guiding population growth, natural resource usage, pollution control, agriculture, and technology creates the "great sense of futility" now exhibited everywhere, and how different alternatives result from different policies.

3.3. CRITICISMS OF INDUSTRIAL DYNAMICS

It may not be inappropriate to point out, very briefly, the criticisms of industrial dynamics. The reactions to this new approach and its claims have not been all enthusiastic. Some have criticized the whole approach, some have seen it merely as a simulation technique, and some have expressed doubts about its claims.

As was mentioned in the last chapter, industrial dynamics has been defined as "...The science of feedback behavior in social systems" [15].

Among the first criticisms of industrial dynamics is the one by Wagner [51] who has challenged this claim.

> "Does Industrial Dynamics represent a truly scientific approach? Or does it represent the judgmental approach of a particular scientist? Are there principles for applying industrial dynamics as a diagnostic tool which are not tied to the personality traits of a particular practitioner?"

The critics have found little evidence so far to back this claim. They tend to regard industrial dynamics merely as a new simulation technique, which, like other ones, has the same difficulties of model validation. By model validation are meant two specific features:

1 - A model should predict the "characteristic behavior" of the system.

2 - Changes in the <u>model</u> which improve the <u>simulated</u> behavior of the system, must do so when applied to the real system.

Forrester's view about models is somehow different:

> "...A useful model of a real system should be able to represent the <u>nature</u> of the system. But ... quantitative prediction of <u>specific events</u> at <u>particular future times</u> has not been included in the objectives of a model" [13].

Holt [45] has disagreed with this idea, and commented that "The problem of validating the model here is extremely critical in the effective use of this sort of approach."

Still as a simulation technique, industrial dynamics has been subject to criticisms because of its structure and model representation. Ansoff and Slevin [1] have called this a "highly stylized structure and a comparably stylized system of notation" which serve the purpose of making "the model compatible with "DYNAMO compiler.

The insistence of industrial dynamics on feedback behavior of systems, which is consequently and necessarily reflected in model building of such systems, has been questioned by some critics. Holt [45] has suggested that "It will not always be wise to limit ourselves to models based on circular causality in spite of the computational simplicity that may be obtainable by this approach;" and Ansoff and Slevin [1] have commented that this insistence on feedback characteristics of systems, and the specific format of the DYNAMO compiler, are reasons that "Instead of dealing with limitations, one can address himself to identifying those problems of the firm and of other industrial organizations which are most likely to fit the model format;" moreover, it is not necessary to believe "That all aspects of the firm are best studied by means of information feedback systems."

The special format of the model structure, and of the DYNAMO compiler, the critics say, require a complete quantification of all system aspects, which has raised serious objections.

> "One gets the suspicion of a man being driven by a computer. Behind the protestation of validity of quantification one senses the spectre of DYNAMO, the investment it represents, and its appetite for data in a very particular data format" [1].

These are the major criticisms of industrial dynamics, to which Forrester, of course, has answered properly [17]. He has insisted on the initial belief in the feedback behavior of systems, and has argued that industrial dynamics meets the requirements and definitions of a theory of structure of systems. Many of the shortcomings, however, have been attributed to lack of enough work, literature, and educational material, which is a matter of time.

3.4. DYNAMO COMPILER*

DYNAMO, standing for DYNAmic MOdels, is a computer program which compiles and executes the simulation models of dynamic systems. It was specifically designed to make the computations of industrial dynamic models easy. As such, it is a special-purpose simulation language, which like other special-purpose languages, has its own advantages and disadvantages.

It was originated in 1958 at M.I.T., and was later modified and further developed, by a group of digital computer experts, over a period of 3 years [13]. It has been improved over the years to cover various desireable features, and to fit the new computers as they appeared.

As of this time, the latest version of the language, called DYNAMO II, is available [39].

The specific featues of this computer program are as follow [38,39]:

1) It is easily understandable;
2) It has a short compilation time, because unlike other languages which have both a "source" and an "object" language, it has only one.

*A brief description of DYNAMO rules and specifications appear in Appendix I.

3) It gives the output in the form of printed tables and/or plotted graphs, which are of great help to users. There are flexibilities in the form of each of the above outputs.
4) There is no required order of equation writing. It sets the order of computation itself.
5) It creates some of the "implied" initial conditions.
6) It has an extensive error checking facility to help the user to correct the model.

The main disadvantage with DYNAMO, is lack of enough literature to enable the user to fully utilize its various features. The present literature do not give enough information on intricate aspects of DYNAMO.

CHAPTER 4

A PRODUCTION-DISTRIBUTION SYSTEM AS VIEWED BY INDUSTRIAL DYNAMICS

4.1. INTRODUCTION

The concepts of industrial dynamics were discussed in Chapter 2. In this chapter, it is attempted to apply these concepts to a simple industrial system, dealing only with production and distribution, identify the feedback characteristics of such system, build its dynamic model, and study the behavior of the system under different test conditions, as revealed by the simulation results. What follows, is essentially based on the discussions about industrial systems in Chapters 2 and 15 of Reference [13].

It is a recurring phenomenon in production-distribution systems to have fluctuations in the production rate due to changes in retail sales. Therefore, the problem facing such systems is to match the production rate with the consumers sales rate, to the extent possible.

The objective of this study is to investigate this phenomenon in the context of industrial dynamics principles, and try to trace the cause-and-effect structure of the organizational policies which lead to such fluctuations in the production rate.

There are a few considerations and assumptions about the system:

1 - In a production-distribution system, there are six essential flow systems, namely, flows of orders, materials, information, personnel, money, and capital equipment. Of these, three are omitted; they are: flows of personnel, money, and capital equipment. This is because consideration of all flows is not, at this stage, necessary, unless the

system is operating at such a point where personnel, money, and equipment are difficult to acquire; otherwise, in a "normal" situation, these flows usually don't affect the behavior of the system

2 - Interactions between system and market are not considered, i.e., factory does not promote its productions.

3 - In any production-distribution system, the ordering policy between levels of the system (i.e., from retailers to distributers, and from distributers to factory warehouse) is based on some prediction of the future sales, such as extrapolating the present level, forecasting techniques, and so on; in the present model, it is assumed that the sales will continue at the present level.

4.2. ORGANIZATION OF THE PRODUCTION-DISTRIBUTION SYSTEM

An industrial system, in its simplest form, consists of production and distribution levels. There are, of course, subdivisions within each level, and also interactions between the system and market. However, since the scope of this report is limited to defining the basic principles of I.D. and applying these principles to some simple examples, we will avoid the complications which arise from considering the whole structure of the organization, and which make it difficult, at the first step, to understand those principles.

The organization of the production-distribution system is shown in Fig. 4.1. The system consists of 3 levels: factory and factory warehouse, distributers, and retailers. Materials flow downward from each level to the next, and orders flow upward between the levels; these rates (materials & orders) would necessarily pass through levels, here the inventories and order backlogs at each level.

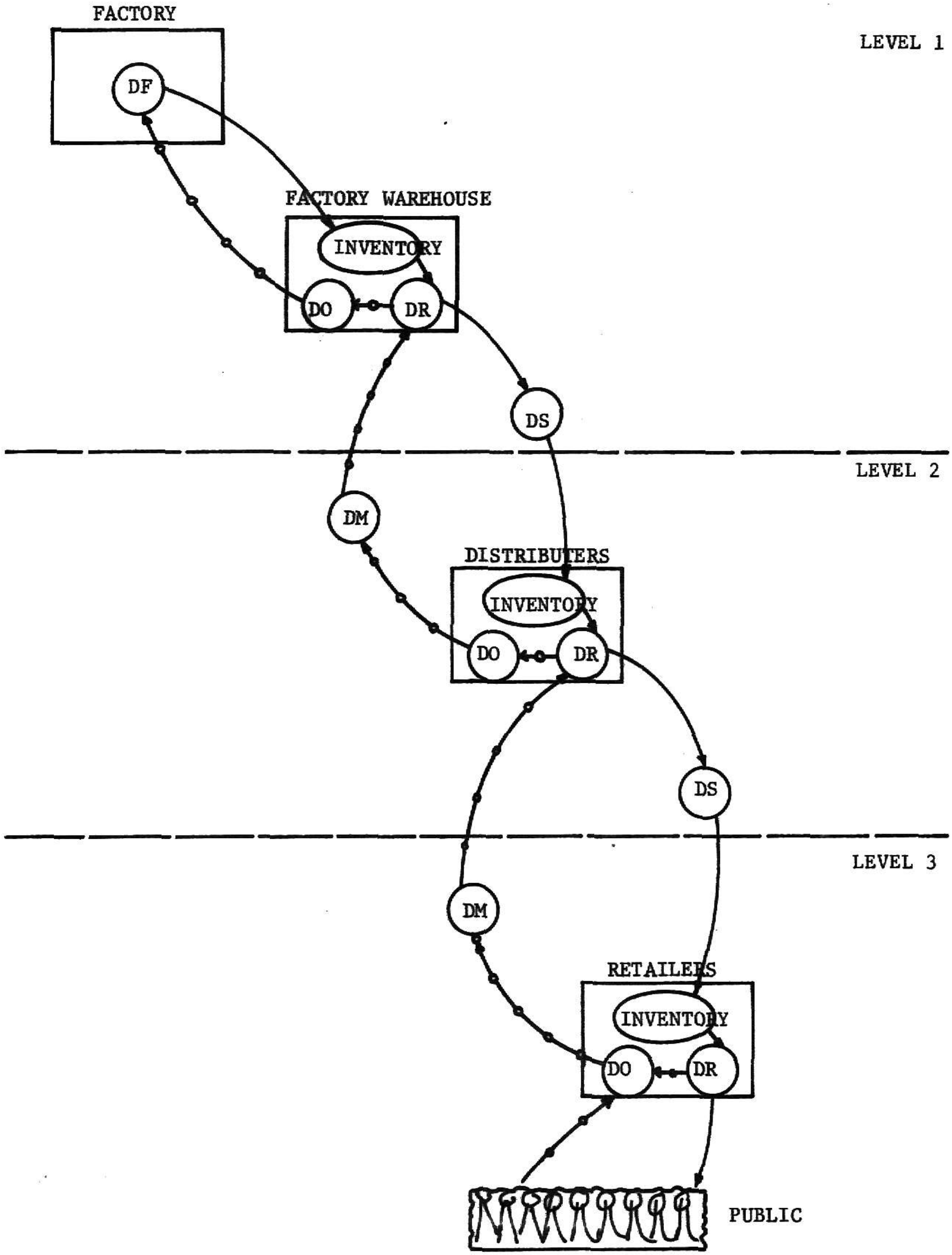


Fig. 4.1. Organization of a Production-Distribution System

There are delays in the flow channels of the system; they are as listed below:

a) Factory lead-time delay, DF.

b) Shipping delays between levels, DS.

c) Order-filling delays at all three levels, DR.

d) Delays in making decision about the orders to the next higher level, DO.

e) Mailing delays of orders between any level and the next higher one, DM.

Since the three levels are similar in function, the level and rate variables will also be similar at each level. Considering the three flow channels at work in this system, the level and rate variables can be enumerated as follow:

A) Level Variables. The following level variables are identified at each system level:

1 - Backlog of unfilled orders received;

2 - Inventory of items in stock;

3 - Average sales level, which is the basis for the ordering policy from one level to another.

B) Rate Variables. At each system level, the rate variables are:

1 - Rate of outgoing orders to the next higher level;

2 - Rate of incoming materials from the next higher level (shipping rate);

3 - Rate of outgoing materials to the next lower level (shipping rate);

4 - Rate of incoming orders from the next lower level.

The above delays and variables are the essential factors in the structure of the system, and constitute the bases for the system model, to be discussed in next sections.

4.3. MATHEMATICAL REPRESENTATION OF THE SYSTEM

In the last section, the structure of the production-distribution system was outlined, and level and rate variables, as well as delays at each level, were identified. In this section, the mathematical relationships between the system variables and parameters are presented; however, because of the similarities between the functions of the three levels, the mathematical relationships can be stated in a general form, so that at each level they would be redefined to represent that specific level.

At each level, there is ordinarily a backlog of unfilled orders; this level variable is represented as the following integral:

$$X_1(t) = X_1(0) + \int_0^t [Z_1(t) - Z_2(t)]\, dt \tag{4.1}$$

where:

$X_1(t)$ = Unfilled orders at time t;

$X_1(0)$ = Initial value of unfilled orders;

$Z_1(t)$ = Orders received at time t;

$Z_2(t)$ = Shipments sent out at time t.

The second level variable is the inventory of goods at each level, which takes a similar form:

$$X_2(t) = X_2(0) + \int_0^t [Z_3(t) - Z_2(t)]dt \tag{4.2}$$

where:

$X_2(t)$ = Actual Inventory at time t;

$X_2(0)$ = Initial value of actual inventory;

$Z_3(t)$ = Shipments received at time t;

At this point, the rate of shipments sent out, $Z_2(t)$, would be determined; this rate is directly proportional to the level of unfilled orders, $X_1(t)$, and inversely to delay in filling orders, therefore:

$$Z_2(t) = \frac{X_1(t)}{Y_1(t)} \tag{4.3}$$

where:

$Y_1(t)$ = Time delay in filling orders, at time t.

The time delay in filling orders is not constant, and varies as the actual level of inventory deviates from desired level. It can be represented as:

$$Y_1(t) = C_1 + C_2 \; [X_{2d}(t) \; / \; X_2(t)] \tag{4.4}$$

where:

C_1 = A constant, representing the minimum delay in handling orders;

C_2 = A constant, representing the average delay in unfilled orders caused by out-of-stock item at "normal level" of inventory;

$X_{2d}(t)$ = Desired level of inventory at time t.

The desired inventory itself is usually represented as a percentage of average sales:

$$X_{2d}(t) = C_3 \cdot X_3(t) \tag{4.4a}$$

where:

C_3 = proportionality coefficient;

$X_3(t)$ = Average sales (sales smoothed) at time t.

The third level variable, that is the average sales at each level (or the smoothed sales at each level), is the basis for ordering from the next higher level, and here is represented by a "first-order exponential smoothing," in the form of the following difference equation:

$$\Delta X_3(t) = X_3(t + DT) - X_3(t) = \frac{DT}{d_1} (Z_1(t) - X_3(t)) \tag{4.5}$$

where:

d_1 = Smoothing time constant;

DT = Delta time, or the difference time.

One of the rate variables of each level is the purchasing rate decision to be made*. It must include not only the orders received, but also the discrepancies between desired and actual levels of inventory, between desired and actual levels of pipeline orders, and between actual and normal level of unfilled orders; therefore:

$$Z_4(t) = Z_1(t) + \frac{1}{d_2} [\{X_{2d}(t) - X_2(t)\} + \{X_{4d}(t) - X_4(t)\} + \{X_1(t) - X_{1n}(t)\}] \tag{4-6}$$

*At factory level, this variable becomes the manufacturing rate.

where:

d_2 = Delay in inventory adjustment;

$X_4(t)$ = Actual level of orders in "pipelines" at time t;

$X_{4d}(t)$ = Desired level of orders in "pipelines" at time t;

$X_{1n}(t)$ = Normal level of unfilled orders at time t.

The variables $X_{4d}(t)$, $X_4(t)$, and $X_{1n}(t)$ are defined as follows:

$$X_{4d}(t) = X_3(t) \cdot [C_4 + C_5 + Y_{1h}(t) + C_6] \tag{4-7}$$

where:

C_4 = A constant, representing clerical delays in order processing;

C_5 = A constant, representing mailing delays;

C_6 = A constant, representing transportation delay;

$Y_{1h}(t)$ = Delay in filling orders, at the next higher level, at time t.

$$X_4(t) = Y_2(t) + Y_3(t) + X_{1h}(t) + Y_4(t) \tag{4-8}$$

where:

$Y_2(t)$ = Clerical in-process orders at time t;

$Y_3(t)$ = Purchase orders in mail at time t;

$Y_4(t)$ = Materials in transit to the level, at time t;

$X_{1h}(t)$ = Unfilled orders at the next higher level, at time t;

Finally:

$$X_{1n}(t) = X_3(t) \cdot [C_1 + C_2] \tag{4.9}$$

In equation (4-8), there are three variables representing the delays at a level, that is, $Y_2(t)$, $Y_3(t)$, and $Y_4(t)$. Here it is assumed that third-order delays are adequately representing the processes of order handling, mailing, and shipping. Therefore:

$$Y_2(t) = Y_2(0) + \int_0^t [Z_4(t) - Z_5(t)]dt \tag{4.10}$$

$Z_5(t)$ = purchase orders sent out, at time t.

$Z_5(t)$ is represented by the following set of equations, describing a third-order delay:

$$\begin{cases} \frac{d}{dt} L_1(t) = Z_4(t) - Z_{51}(t); \qquad Z_{51}(t) = \dfrac{L_1(t)}{\frac{1}{3} d_c} & (4.10a) \\ \\ \frac{d}{dt} L_2(t) = Z_{51}(t) - Z_{52}(t); \qquad Z_{52}(t) = \dfrac{L_2(t)}{\frac{1}{3} d_c} & (4.10b) \\ \\ \frac{d}{dt} L_3(t) = Z_{52}(t) - Z_5(t); \qquad Z_5(t) = \dfrac{L_3(t)}{\frac{1}{3} d_c} & (4.10c) \end{cases}$$

where:

$L_1(t)$, $L_2(t)$, $L_3(t)$ = The three corresponding levels in a third-order delay;

$Z_{51}(t)$, $Z_{52}(t)$ = Rates of flow between each two adjacent levels;

d_c = Delay in clerical order placing.

Similarly:

$$Y_3(t) = Y_3(0) + \int_0^t [Z_5(t) - Z_{1h}(t)]dt \tag{4.11}$$

where:

$Z_{1h}(t)$ = Orders received at the next higher level,

and is represented by the following third-order delay:

$$\left\{ \begin{aligned} &\frac{d}{dt} L_1(t) = Z_5(t) - R_1(t); \qquad && R_1(t) = \frac{L_1(t)}{\frac{1}{3} d_m} && \text{(4.11a)} \\[2ex] &\frac{d}{dt} L_2(t) = R_1(t) - R_2(t); && R_2(t) = \frac{L_2(t)}{\frac{1}{3} d_m} && \text{(4.11d)} \\[2ex] &\frac{d}{dt} L_3(t) = R_2(t) - Z_{1h}(t); && Z_{1h}(t) = \frac{L_3(t)}{\frac{1}{3} d_m} && \text{(4.11c)} \end{aligned} \right.$$

where:

d_m = Mailing delay between two levels.

And finally:

$$Y_4(t) = Y_4(0) + \int_0^t [Z_{2h}(t) - Z_3(t)]dt \tag{4.12}$$

where:

Z_{2h} = Shipments sent from the next higher level to corresponding level, and is represented by the following third-order delay:

$$\frac{d}{dt} L_1(t) = Z_{2h}(t) - R_1(t); \qquad R_1(t) = \frac{L_1(t)}{\frac{1}{3} d_t} \tag{4.12a}$$

$$\frac{d}{dt} L_2(t) = R_1(t) - R_2(t); \qquad R_2(t) = \frac{L_2(t)}{\frac{1}{3} d_t} \tag{4.12b}$$

$$\frac{d}{dt} L_3(t) = R_2(t) - Z_3(t), \qquad Z_3(t) = \frac{L_3(t)}{\frac{1}{3} d_t}$$

where:

d_t = Delay in transportation of goods to the corresponding level.

4.4. DEVELOPMENT OF THE SYSTEM MODEL

Figure 4.2. shows the very basic feedback structure of the system, at retail section only; the functions of the three levels are the same, and hence their flow diagrams will be similar.

The rate and level variables, as mentioned already, are illustrated; the purchase rate at customers is the independent input to the system. As it increases, the average sales rate will increase, which will influence the order rate to the distributers. Increased retail sales also increase the backlog of unfilled orders, which will affect: 1) the order rate to distributers in the direction of more orders, and 2) the shipment of goods to customers in the direction of increased shipments. This is basically a positive feedback loop, representing the growth of sales in

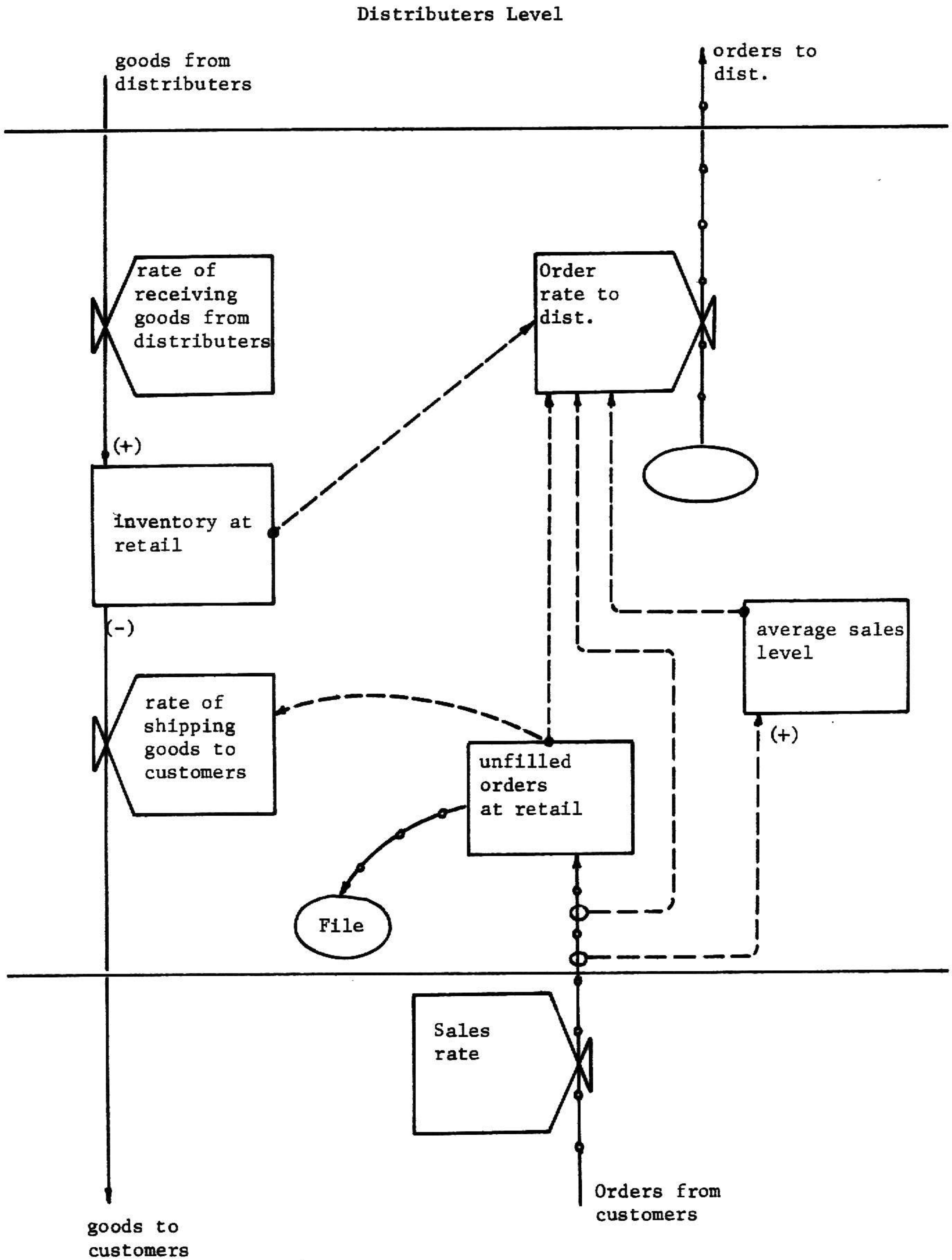


Fig. 4.2. Basic Feedback Structure of Retail Section

the beginning. However, a positive feedback loop cannot go uncontrolled; there are always factors to control the growth process in a positive loop, and here, the decreasing number of potential buyers is the controlling factor.

As the retail sales level off and stabilize at the new "normal" level, the outgoing rate of goods to customers will decrease, which will increase the level of inventory. If the level of inventory is to be kept at a desired value, the discrepancy between incoming and outgoing rates at the inventory should promptly be reflected in the order rate to the distributers. Here, the loop takes up the function of a negative one, seeking the goal of adjusting to the new level of sales. However, the goal may not be achieved smoothly and without over- and under-shoots, specially if the system has a tendency to oscillate.

In Fig. 4.3 the more detailed flow diagram of the retail section is shown. For a complete description of this and other levels' flow diagrams, as well as their respective equation sets, the interested reader is referred to chapter 15 of Reference [13].

The considerations that have led to build the system model are beyond the scope and limits of this report. It suffices, for our purpose, to point out that the system model consists of 73 equations, based on the mathematical representation of the system variables. These equations describe the relationships between variables at each level, interactions between levels, and the initial conditions.

In this report, the system model is used to investigate a few of the characteristics of the production-distribution system. The listing of the equations, initial values, and parameter values is given in Appendix II.

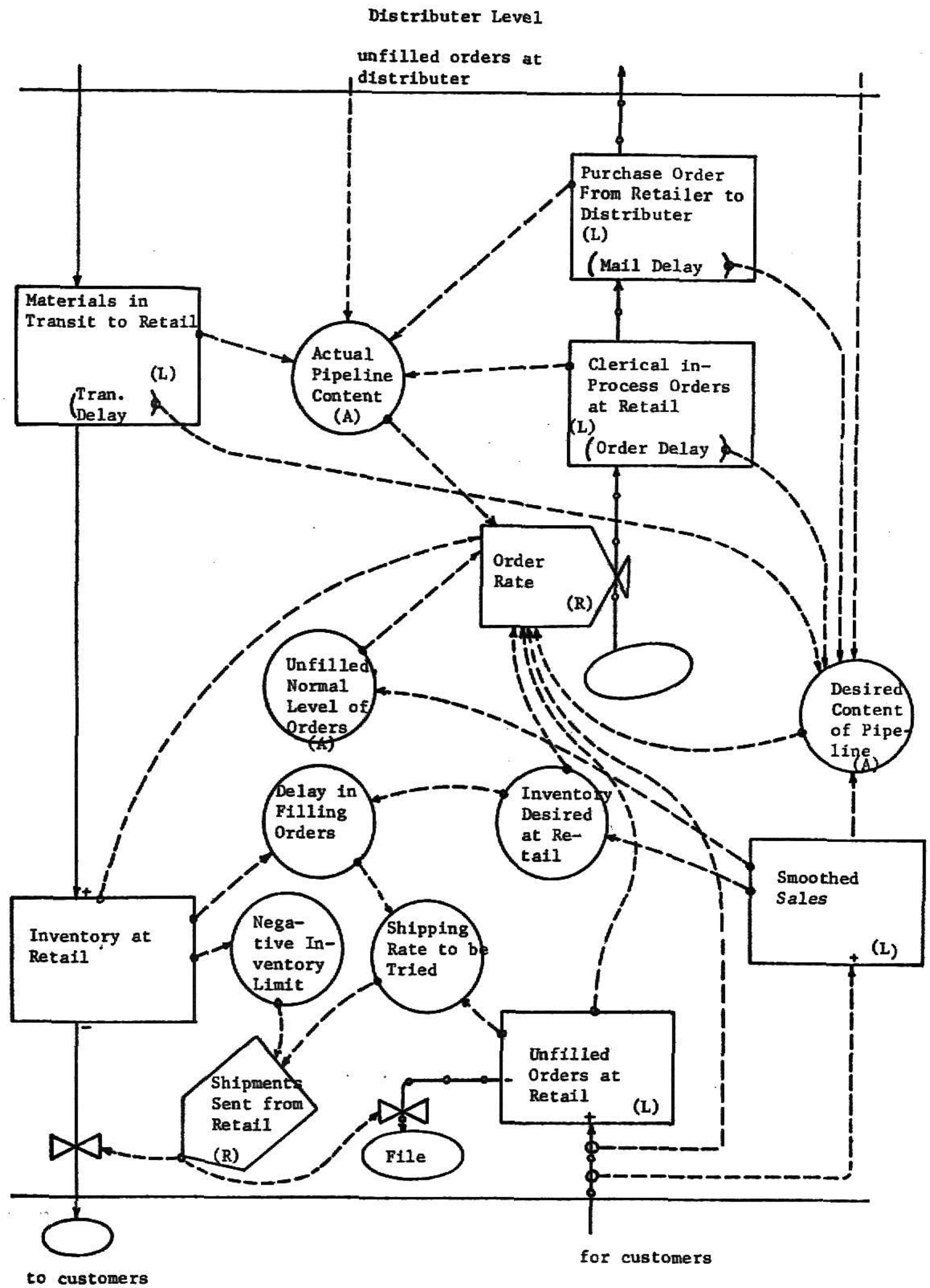


Fig. 4.3 Flow Diagram of Retail Section

4.5. TEST RUNS OF THE MODEL

4.5.1. Introduction. Having the system model developed, it becomes possible to study the behavior of the system under various conditions. What would be more realistic, is to use a set of actual time series of sales in the past, and compare the "simulated" response of the system with the "real" one. However, this is another complication which is not in accordance with the simplicity in approach, necessary at the preliminary stages.

A variety of test inputs could be used to disturb the state of the system, and study its response. Such inputs include a step increase in retail sales, a periodic change of some specific duration, and more realistically, a series of random fluctuations at retail. A reasonable assumption, which may also be tried, is to limit the factory expansion capacity to some upper bound.

In this section, two examples are tried. One is a step increase of retail sales, equal to 15% above the steady-state level of sales, and after 4 weeks from the simulation starting point. Another example will investigate the fact that in a system such as the one under consideration not all delays are bad. We would observe the improvement in the system behavior as the value of a particular delay is increased.

4.5.2. Step Input. A step input is the simplest test function. It represents a sudden increase in a system parameter caused by an external disturbance. It contains an infinite band of frequencies, and if the system to which it is applied, has a tendency toward a specific mode of response, it will be "excited" by the step input.

The model of the system, given previously, is accompanied by the following equations, representing a step function:

RRR.KL = RRI + RCR.K 74,R

RCR.K = STEP (150, 4) 75,A

RRR = Requisitions Received at Retail, units/week

RR1 = Retail Requisitions, Initial Rate, equal to 1000 units/week

RCR = Requisition Change at Retail, units/week

STEP = Functional notation for a step increase equal to 150 units at time 4.

In Fig. 4.4 the output of the computer simulation of above model is shown. System is in the steady-state up to the 4th week, when the disturbance, created by the step change in retail sales, occurs. It can be seen that the disturbance is gradually progressed toward the factory, as Table 4.1 shows the peak values of the system variables, and their corresponding times.

It has to be noted that the retail sales, as the test input to the system, are assumed to be independent of it; this is not a reasonable assumption, since retail sales are affected by the product availability, and advertising, factors that are not considered at this stage.

The orders from retailers to distributers increase, because of the new sales level at retail, and reaches the 15% level after about 2 months, because of the delays indicated in the system organization. However, they still continue to rise, because at retail level, additional orders are

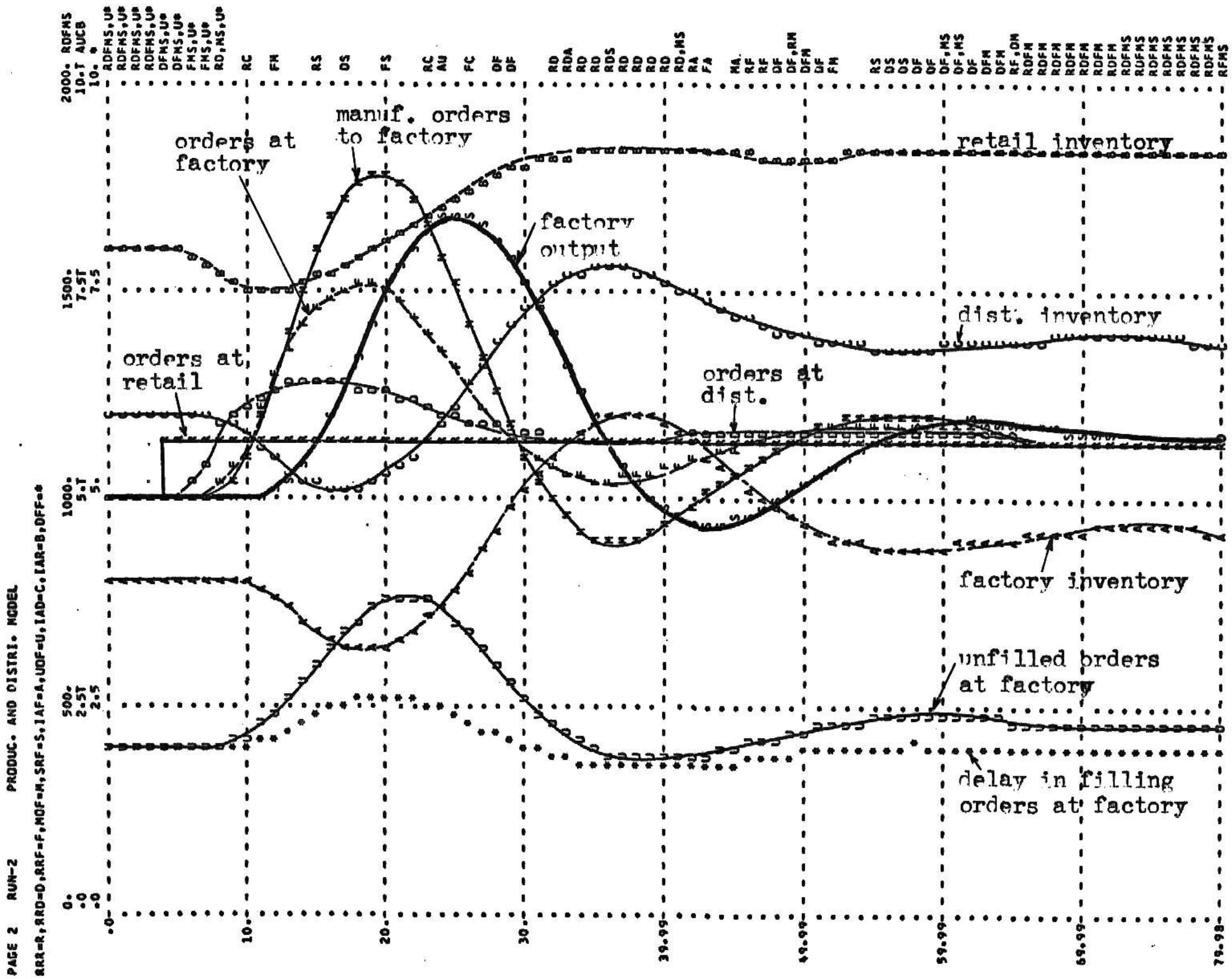


Fig. 4.4. Production-Distribution System

Table 4.1. Peak Values of System Variables and Their Times, for a Step Increase of 15%

Variable	Peak Value of Change (%)	Time of Occurence (week)
Retail Sales	+ 15	4, Constant Thereafter
Dist. Orders from Retail	+ 25	14
Fac. Orders from Dist.	+ 55	19
Manufac. Orders to Fac.	+ 79	19
Fac. Production Output	+ 70	25

placed for the following reasons:

1 - To increase the inventories correspondingly;

2 - To increase the level of orders and goods in system pipelines, by 15%, to match the 15% increase in sales.

After the additional orders are supplied, distributers orders from retailers gradually reach the 15% increase (after 35 weeks).

Distributers, in turn, place orders to the factory warehouse, not only to cope with the increased order rate at retailers, which remains above retail sales for about 27 weeks, but also to increase their own inventories, and order and goods in system pipelines between factory warehouse and distributers level. As a result, the initial disturbance at retail shows a greater swing at the distributers level; it reaches a peak value of 55% above the previous level, at the 19th week.

Similarly, the manufacturing orders to the factory are increase, partly because of the increasing sales to the distributers, and partly because of falling inventory, which drops by 18% at the 19th week. Manufacturing orders reach a peak of 70% at the same time. Factory production follows a similar pattern, and 6 weeks after the manufacturing orders reach their peak, factory production reaches a peak of 70% at the 25th week; this 6-week period is the factory lead time.

As the inventory requirements are satisfied, the order rates decrease and the reversible effects take place. At the 36th week, distributers orders to the factory fall by 10% below the retail sales level, and at the 37th week, manufacturing orders to the factory drops to a value which is 14% below that level. Factor output, therefore, after 6 weeks, at the 43rd week reaches its minimum value which is 10% below the initial value, and 25% below the current sales level at retail.

As can be seen, it takes about 80 weeks (1 1/2 years) before the system variables stabilize to their new values corresponding to the 15% increase at retail sales.

The system shows an oscillatory mode; the variables swing in both directions until they stabilize. Factory production peaks occur at 25th and 63rd weeks; the 38 weeks between the successive peaks, correspond to the "natural frequency" of the system, and therefore, the system is sensitive to any periodic disturbance with a duration of approximately 38 weeks.

4.5.3. Changes In Inventory Adjustment

In the system model, ordering policies at each level are defined by equations 9, 27, and 45 (Appendix II). The constants DIR, DID, and DIF are factors which determine how fast an inventory deficit is being acted on, at each level, respectively. More precisely, the constant DIR at retailers indicates what fraction of the discrepancy between current inventory and pipeline level and desired level of inventory at retail, is ordered in the following week in order to adjust the inventory; a value of DIR = 16 specifies that $\frac{1}{16}$ of the above mentioned discrepancy will be adjusted per week.

To illustrate the effect of this parameter, the values of DIR, DID, and DIF, which were all equal to 4 weeks in the last example, are assumed to be equal to 1, 2, 4, 8, 16, and 32 weeks, in different runs. The results are shown in Fig. 4.5, with a periodic change in retail sales as the test input. In this figure, all curves are the factory output. The upper curve corresponds to:

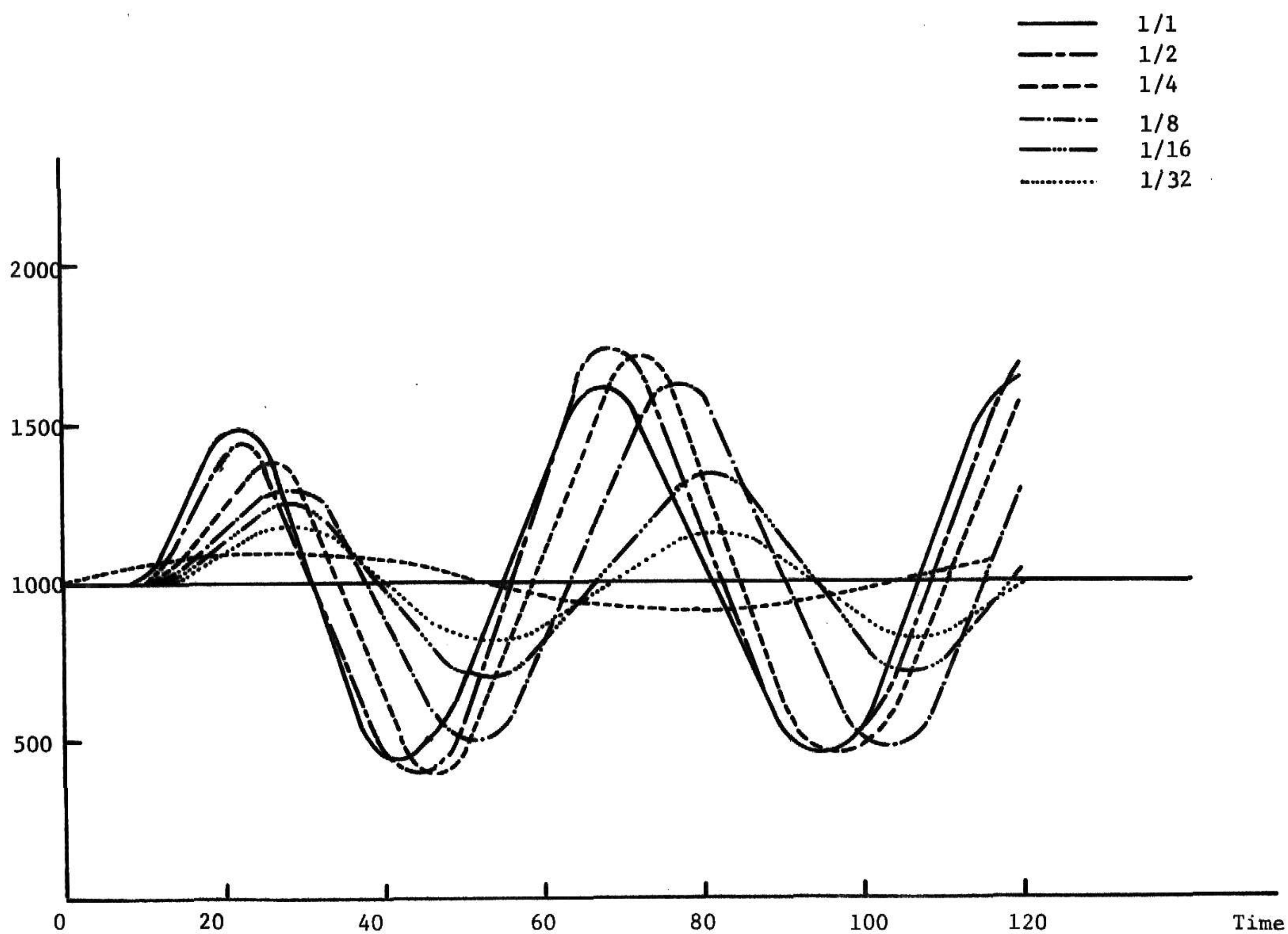


Fig. 4.5. Effect of Change in Inventory Adjustment on Fac. Production

DIR = DID = DIF = 1 week

and the lower one to:

DIR = DID = DIF = 32 weeks

With a value of DIR = DID = DIF = 1 week, i.e., with the whole discrepancy in inventory being reflected in orders in the following week, the factory output reaches a 48% above, 58% below, 60% above, and 54% below the normal level. As the delay in inventory adjustment increases, the peaks and valleys become less conspicuous. For a delay equal to 32 weeks, the peaks and valleys are: 38% above, 40% below, 40% above, and 40% below normal level.

It can be seen that increasing the values of parameters DIR, DID, and DID, which represents a change in ordering policy, leads to less fluctuations in factory output.

CHAPTER 5

ADVERTISING IN INDUSTRIAL SYSTEMS

In Chapter 4, the model of industrial system was developed, with the assumption that there were no interactions between the system and market; that is, the promotional efforts by the system to influence the purchasing habit of the customers were not taken into account. This, of course, would introduce some simplicity into the model, which is helpful at the first step, but as the progress toward system improvement is achieved, this factor has to be considered.

In this chapter, the system model will be refined to include an advertising section, and the responses of the system to different advertising procedures will be investigated.

5.1. ADVERTISING AND ITS NATURE

Advertising has been defined in many ways. The American Marketing Association Committee on Definitions defines advertising as: "Any paid form of non-personal presentation and promotion of ideas, goods, or services by an identified sponsor" [40].

Some authors have criticized this definition as being concerned with the "construction" of advertisements and the selection and use of media [48]. Dirksen and Kroeger [7] define advertising as being: "... A dynamic force. It is communicating on a mass basis information about the qualities and features of product, services, and ideas of significance to the customer, with the intention of persuading sufficient buyers to warrant the expenditure involved, and thus to be profitable or worthwhile from the standpoint of the designated sponsor."

Crawford's definition runs as [5]: "Advertising is the art of persuading people to do with frequency and in large numbers something you want them to do. People invented advertising in order to accomplish this - to create a channel of communication from seller to buyer.... Advertising is not words alone, or pictures, or magazines, or outdoor billboards, or television, or psychological research into human behavior. Advertising is people using these tools of communication to get other people to do something about products or services or ideas."

Whatever the definition of advertising, it always begins with the definition of an objective, which is what the audience of advertising is expected to do. This objective, in general, has been defined as "information and persuation," [5] that is, the goal of an advertising campaign is to inform the people, and consequently persuade them to do something.

There have been discussions about the "information" that advertising is supposed to give, and this is where most of the criticisms of advertising is directed at. The critics question the "truthfullness" of advertisers in giving information to people. They suggest that the function of advertising should be "interpreting the want-satisfying qualities of services, products, or ideas in terms of the needs and desires of consumers." [48]

We shall not further discuss the question of the nature of advertising, since it is beyond the scope of this report.

5.2. DEVELOPING THE ADVERTISING MODEL

This section is based on the advertising model developed in Chapter 16 of Reference [13]. For details of the model, the reader is referred to that book.

The behavior of the production-distribution system of last Chapter would not be meaningful unless the interactions between the system and the market are considered. In the study of the production-distribution system, it is assumed that the retail sales are the independent input to the system, whereas they are greatly influenced by the promotional efforts, which are now part of any such systems.

In this section the behavior of the system under advertising is investigated.

Problem. The problem to be investigated is the possible effects of different advertising models on system behavior, and specifically on the production level at the factory.

Assumptions. The assumptions under which the model is built are as follows:

1 - There is an "infinite source" of "prospective customers;" prospective customer is any one who is aware of his need of the production, although he may not be able to buy the product immediately.

2 - The prospective customers exist independent of any sales efforts; This is because they need the product anyway.

3 - Advertising, in general, seeks two goals: shifting the purchase time to an earlier date, and increasing the market share. Here the first of the two is considered.

4 - The above assumption also implies that intercompany competition has no significant effect.

5 - Advertising expenditures are proportional to the sales level; as the sales increase, advertising budget rises, and vice versa.

The above assumptions lead to the conclusion that the independent input to the system is not the retail sales, but the flow of prospective customers as they are affected by advertising practices.

System Model. To add the market-interaction to the system, it is assumed that the production level at the factory constitutes the information on which the advertising budget is based. This information is not readily available, because of the time delays necessary to smooth the average rate of manufacturing.

Decisions regarding the advertising expenditures are usually made after a time delay, and advertising agencies and media would introduce another delay before presenting the actual advertisement to public.

The effect of advertising is not immediate; there is evidence that awareness about the product being advertised gradually builds up [13,49,50]. People do not react to advertisements at once, and hence there is another delay before the effect of advertising can be taken into consideration.

The system model, in this form, is basically a positive feedback loop; as the retail sales increase, the production level at the factory will eventually increase; a higher production level means a greater advertising budget, and at final analysis, a higher level of awareness among the public. If advertising awareness has any effect, it would make the prospective customers purchase sooner than they would otherwise; this increases the retail sales. The growth within the positive loop continues, but it is subject to the existence of prospective customers. The growth only stops when the pool of prospective purchasers depletes. The structure of the system with advertising is depicted in the flow diagram of Fig. 5.1.

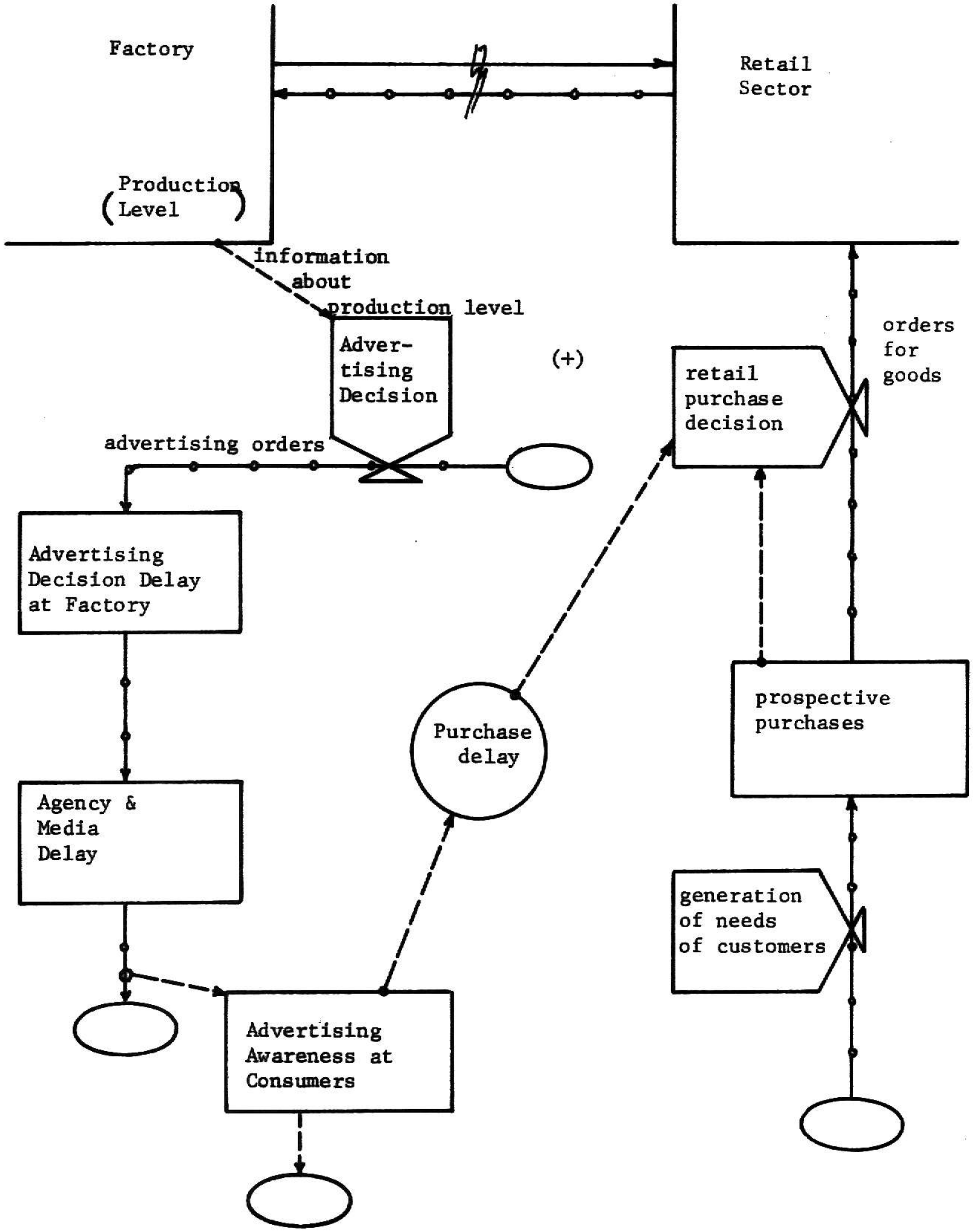


Fig. 5.1. Flow Diagram of the Advertising Section

The equations describing the advertising section, to be added to the production-distribution system of last chapter, appear in Appendix III.

5.3. SYSTEM'S BEHAVIOR WITH ADVERTISING

Figure 5.2. illustrates the results of a 2-year simulation run of the system with advertising section added. It has been assumed that the factory production rate is limited to 40% above the initial sales rate, and a step increase of 10% above the initial level has occured in the "generation of need" at consumers.

Retail sales are taking place randomly; factory output is relatively unchanged for more than 2 months in the beginning, as the orders placed at distributers and eventually at factory do not change considerably. During the same time, factory warehouse inventory is also constant, but at retail, inventory fluctuates, and shows a declining trend. As a result, orders are placed at distributers level, and finally at factory. Factory warehouse inventory starts decreasing, and shortly after, production rate at factory starts increasing. It reaches a maximum of 135% at 41st week, and then gradually declines. Inventories at retail and at factory warehouse start to increase, almost at the same time, as a result of higher production rate at factory, and consequently a higher rate of shipment to each level.

As the retail sales continue fluctuating, they cause another peak in orders received at factory, and therefore another peak in factory output at 62nd week. Similarly, inventories at retail and at factory warehouse reach a peak at 74th and 80th weeks, respectively.

The highly fluctuating rate of production at factory is not only

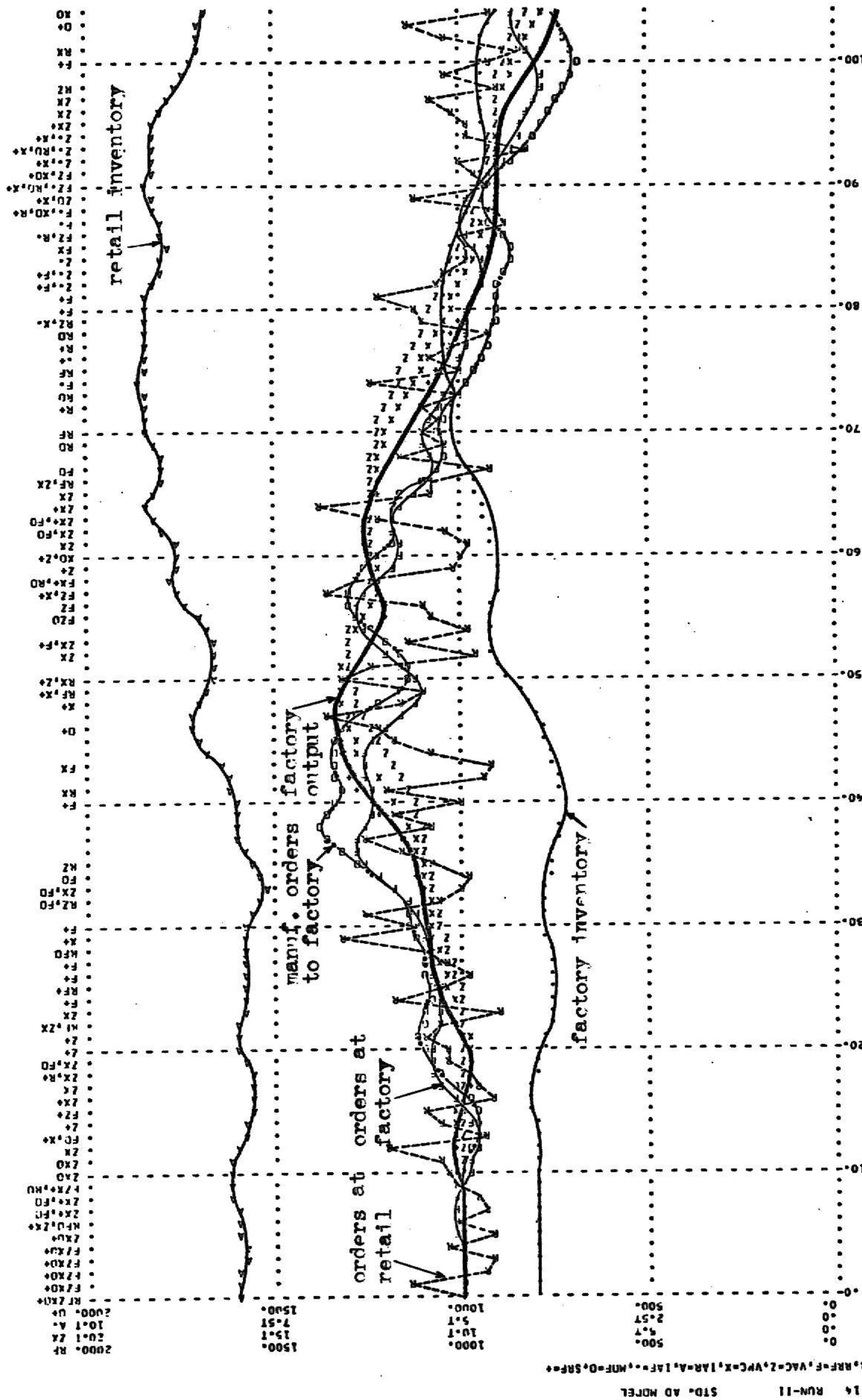


Fig. 5.2. Advertising Model with Advertising Budget Proportional to Sales

because of the system structure, but also because of the structure of advertising model. A longer simulation run (for example for 7 years) shows even wider fluctuations at factory output, higher peaks, valleys, and period of full-load operation, which are highly undesireable.

5.4. OPERATIONS RESEARCH MODELS OF ADVERTISING

In the discussion of the previous model, which was used as a starting point for what follows, there were some assumptions regarding the response of customers to advertising. These assumptions are essential to any model-building process.

In recent years, Operations Research has been extensively applied to industrial situations; however, it has not been as much used in the field of advertising, due to the difficulties in estimating the response of the people to promotional efforts. Although it is not a promising field for OR studies, some quantitative aspects of advertising have drawn attention recently, such as evaluation of advertising effectiveness, allocation of advertising budgets among products and media, determining the advertising budget, and so on.

Measuring advertising effectiveness, however, is the most difficult part of any model building, since there is not any clear and agreed-upon definition of effectiveness, and accordingly there are various techniuqes for measuring the effectiveness of advertising campaigns [33]. In general, effectiveness "Is used to denote the degree to which advertising can change people's external or internal behavior with respect to an item-product, service, or idea-advertised, and in the direction desired by advertiser" [47].

A study by Vidale and Wolfe [49 ,50], based on extensive experimentations over large portions of U.S. market, has investigated the problem of advertising effectiveness, and proposed a mathematical model of sales response to advertising, based on their experiments.

Description of the Model. The model is based on three parameters, affecting the sales response to advertising. They are:

1 - The Sales Decay Constant (SDC)

2 - The Saturation Level (SL)

3 - The Response Constant (RC)

a) The Sales Decay Constant (SDC). The first parameter determines how the sales rate decreases in the absence of advertising. It has been observed that when advertising stops, the sales fall down; this is because of the product obsolescence, competition, and so on.

The rate of decline, called sales decay constant, has been found to be generally constant for a product, meaning that a constant portion of sales is lost each year, provided market conditions are relatively constant, and allowances are made for seasonal changes and similar random factors.

The sales decay rate is proportional to the sales rate, therefore:

If $S(t)$ = rate of sales at time t

and λ = sales decay constant

Then, the change in sales rate would be:

$$\frac{dS(t)}{dt} = - \lambda . S(t) \tag{5-1}$$

The solution to this differential equation is:

$$S(t) = S(0) \;.\; e^{-\lambda . t} \qquad (5\text{-}2)$$

where S(0) is the initial sales rate.

The sales decay constant varies from product to product; it has a large value for items that are highly seasonal or items in highly competitive situations, and a small value for items that are well-established, and that face little or no competition.

b) The Saturation Level (SL). Saturation Level is the practical limit of sales that an item might acquire. Its value depends on the product itself, as well as on the advertising medium used.

Saturation level can be thought of as the upper limit of sales that can be generated regardless of what is spent on advertising thereafter. Another words, when the saturation level is reached, advertising loses its effect, and there is no change in sales rate per dollar increase in advertising expenditure. Mathematically speaking, when:

$$S(t) = SL$$

the change in sales rate:

$$\frac{dS(t)}{dt} = 0$$

c) The Response Constant. The third parameter, the response constant, or more accurately, zero-sale response constant, r, is the sales generated per dollar of advertising, when sales are zero.

The concept of response constant is better understood by noticing that as the sales reach the saturation level, the effect of advertising decreases, because the number of potential customers decreases. The authors have assumed a linear decrease in advertising effectiveness, i.e, a constant value for response constant. Therefore, if:

$$S(t) = 0 \qquad \frac{dS(t)}{dt} = r$$

and when

$$S(t) = SL \qquad \frac{dS(t)}{dt} = 0$$

Figure 5.3. shows the linear relationship between $S(t)$, $\frac{dS(t)}{dt}$, and r.

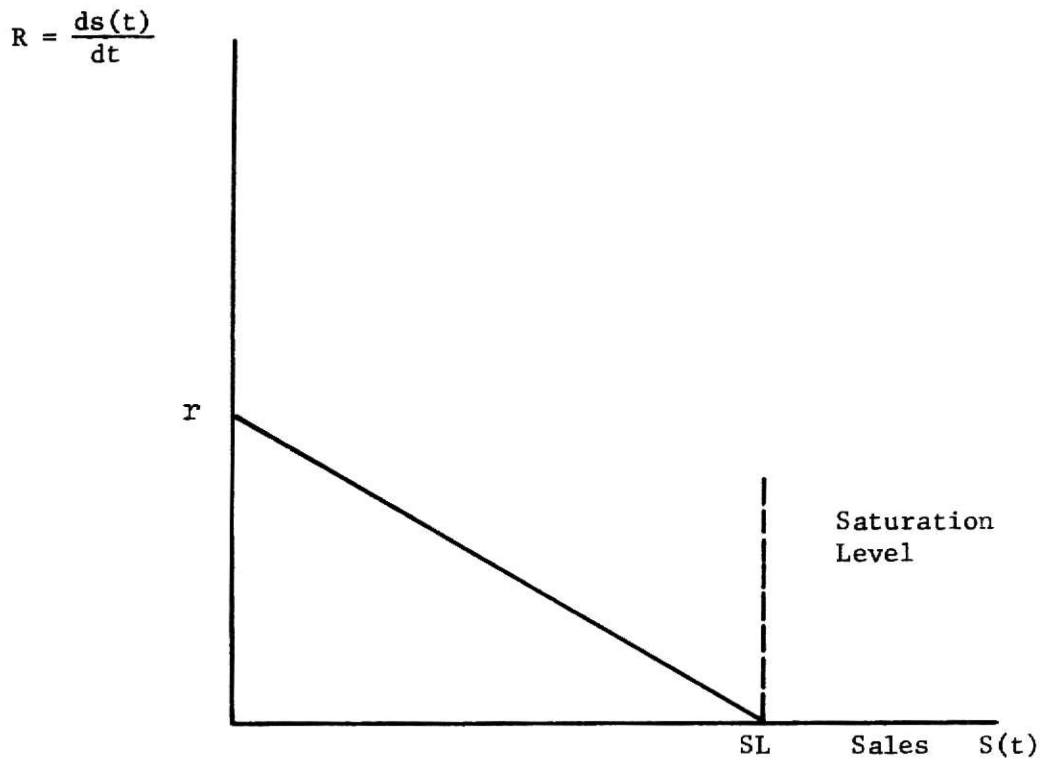
In many cases sales are not zero when advertising campaign begins; it is, therefore, directed at that portion of the market that is not affected yet. If the sales rate before advertising starts is S_0, and saturation level is SL, the affected portion of the market is $\frac{S_0}{SL}$, and $(1 - \frac{S_0}{SL})$ represents the unaffected part. In that case, sales generated per advertising dollar are given by:

$$r.[1 - \frac{S_0}{SL}] \ .$$

Mathematical Representation of Model. The mathematical model of sales response to advertising states that "the change in sales rate equals the sales generated in the "unaffected portion" of the market, minus the sales lost due to sales decay function;" in mathematical notations:

$$\frac{dS(t)}{dt} = r.A(t) \ . \ [1 - \frac{S_0}{SL}] - \lambda.S(t) \tag{5-3}$$

in which:



$$\frac{R}{r} + \frac{S(t)}{SL} = 1$$

or

$$R = \frac{ds(t)}{dt} = r \, \frac{SL - S(t)}{SL} = r \, [1 - \frac{S(t)}{SL}]$$

Fig. 5.3. Response Constant vs. Sales Rate

$\frac{dS(t)}{dt}$ = change in sales rate at time t; (\$/time)/time

r = Response constant; 1/time

A(t) = Advertising rate; \$/time

S_0 = Initial rate of sales, before advertising starts; \$/time

SL = Saturation level; \$/time

λ = Sales decay constant; 1/time

S(t) = Sales rate at time t; \$/time

Different types of advertising campaigns, based on equation (5-3), along with their industrial dynamics representations, are discussed in next sections, and the results of the simulation of the system behavior are investigated.

In all the examples that follow, the values of the parameters of the system model are as listed in Table 5.1.

5.5. CONSTANT ADVERTISING OVER A PERIOD

In some campaigns, the amount spent on advertising is constant for a period of time, T, and it is zero afterward. The analytical solution to equation (5-3), for a constant rate of advertising over a time period T, is [2]:

$$S(t) = \left(\frac{SL}{1 + \frac{\lambda . SL}{r . A}}\right)\left(1 - e^{-\left(\frac{r.A}{SL} + \lambda\right)t}\right) + S_0 \cdot e^{-\left(\frac{r.A}{SL} + \lambda\right)t} \qquad (t \leq T) \tag{5-4}$$

After advertising stops at time T, sales decrease exponentially according to equation (5-2):

Table 5.1 - Parameter Values for System Model

Parameter	Value
Sales Decay Constant	.005/week
Saturation Level	15000 $/week
Response Constant	0.5/week

$$S(t) = S(T) \cdot e^{-\lambda(t-T)} \qquad (t > T) \qquad (5\text{-}5)$$

For an advertising campaign of this kind, sales rate will have a general form as in Fig. 5.4.

To simulate the behavior of the system under advertising for a constant time, it has been assumed that a campaign equal to 10% of the initial sales volume was decided upon at the factory to be run for 25 weeks (or, approximately 6 months). After a time delay, representing the decision process at the factory, and the delay at advertising agencies and media, the campaign is actually presented to public for a period of 25 weeks.

The above procedure was programmed in DYNAMO, and later added to the production-distribution system model of Chapter 4 with little modifications to fit the situation. The model listing appears in Appendix III.

Figure 5.5 illustrates the simulation behavior of the system for 2 years. The initial response to advertising campaign is slow; as the advertising "awareness" or "effectiveness" gradually builds up, the sales rate picks up, and the inventory at retail starts falling down, because there has not been any preparations in advance, and also because the orders for goods placed at higher level would encounter a delay, and would not arrive immediately. It can be observed that the orders received at factory are constant for about 2 months after the advertising starts, indicating the sales level before the campaign. As a result, inventory at factory warehouse remains constant for more than two months.

Retail sales continue to rise as the awareness increases, and 6 weeks after the advertising stops, they reach a peak (at 36th week), and thereafter decline exponentially according to sales decay function.

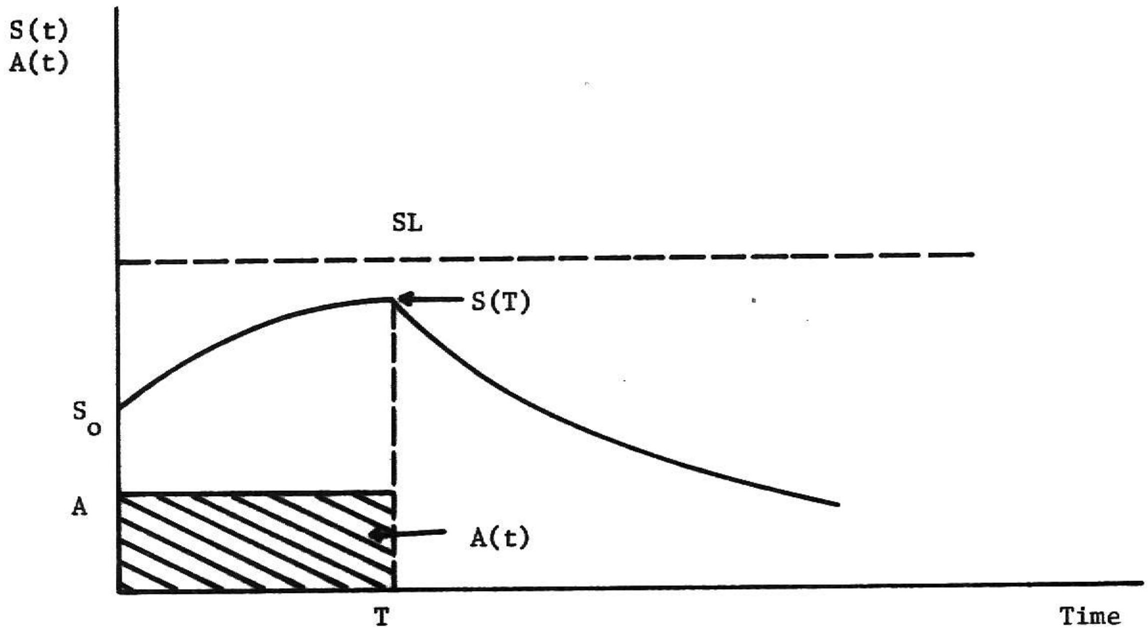


Fig. 5.4. Sales Response to Constant Advertising of Duration T.

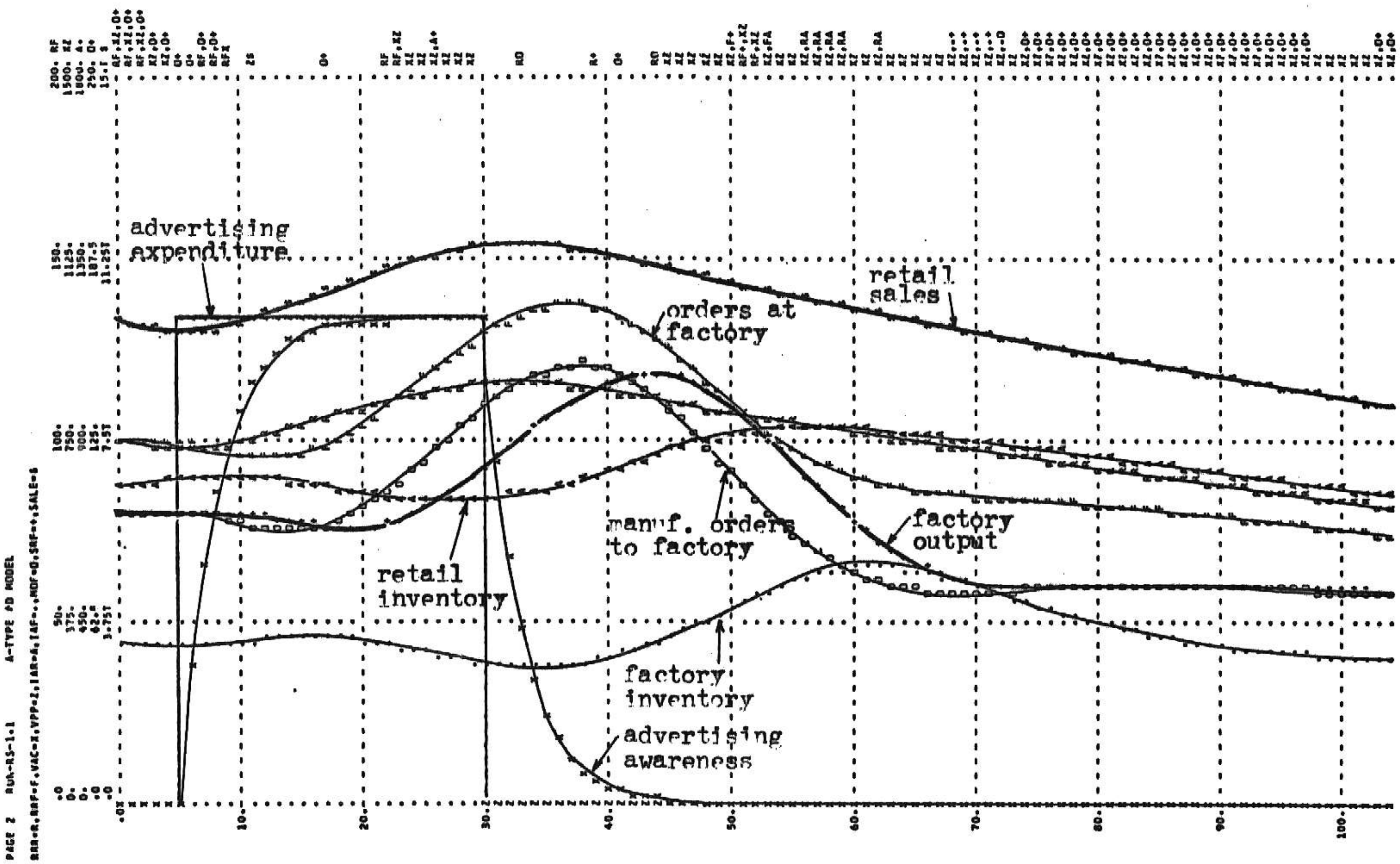


Fig. 5.5. Constant Advertising for a Period of Time

Because of the delays in the system structure, the rising rate of retail sales is not immediately reflected at factory, and orders to factory warehouse start rising only 7 weeks after the retail sales have started increasing. However, once started, the orders received at factory warehouse will increase faster than the sales rate (amplification), and as a result, they reach a maximum almost at the same time as the retail sales rate.

Manufacturing orders to the factory, from factory warehouse, follow a similar pattern, and 6 weeks after they are at maximum, production rate at factory reaches its maximum (44th week). The 6-week delay corresponds to the factory lead-time.

Inventories at both retailers and factory warehouse show the same behavior. In the beginning, they are almost unchanged, but as soon as the advertising campaign starts, and as soon as the retail sales go up, both inventories start falling down; however, inventory at retail reaches a minimum sooner than it does at factory-warehouse, because of the delays in the flow channels of the system. Retail inventory is at minimum at 25th week, while at factory warehouse, inventory reaches its lowest level at 30th week.

At 30th week, advertising stops, and loses its effectiveness gradually. At a point where its effectiveness is not great enough to encourage any more sales, retail purchases start falling down, at an exponential rate.

With retail sales going down, manufacturing orders at factory will gradually decrease, and so will the factory output. According to previous discussions about amplifications in the system, here it can be observed that although retail sales fall down at a small rate (SDC = .005 or .5% per week), the rate of change is amplified through the system levels, and will result in a greater rate of change at factory output (2.8% per week). It takes a relatively long time (more than 20 weeks) before orders placed at factory, and consequently factory output, can keep up correspondingly with the falling retail sales.

Although advertising stops at 30th week, and retail sales start to fall down at 36th week, inventories at retail and at factory warehouse are still increasing. This is because the steady increase in retail sales from 6th week to 36th week is easily interpreted as the new sales trend, and therefore, orders are placed to increase inventory and pipeline contents correspondingly. It takes more than 5 months before falling retail sales affect the inventories; they only start decreasing after 60th week.

In this example, it was assumed that the factory production, which is about 100 units/week in the beginning, can be expanded to meet the requirements. As a result, it reaches a maximum of 150 units/week at 44th week, an increase of about 50%. It then levels off to about 70 units/week at 70th week. In a more realistic situation, there might be a limitation on the factory production expansion.

5.6. SHORT-TIME, INTENSIVE ADVERTISING

Advertising campaigns are not always protracted over time; some advertisers prefer to spend the whole advertising budget on a single advertisement, and in a short time. This is called a "single-pulse campaign."

In equation (5-3), if the time T becomes negligible, the solution to the equation is [2]:

$$S(t) = SL.e^{-\lambda t} = (SL - S_0) \; . \; e^{-(\frac{r.a}{SL} + \lambda)t} \qquad (5\text{-}6)$$

where

a = Total money spent on advertising.

The general form of the sales response to advertising pulse is shown in Fig. 5.6. Sales rate increases in a short time, and then exponentially declines as the sales decay function takes effect.

For dynamic model of this advertising campaign, it has been assumed that the total advertising budget, which in the previous example was spent over a time period of 25 weeks, is now being spent in a short time, say one week; the same considerations regarding the time delays at the factory and at agencies and media are also applied. This procedure was programmed in DYNAMO, and the model appears in Appendix III.

In Fig. 5.7 are shown the results of the system simulation for 2 years. Advertising campaign is shown as a single pulse occuring at the end of 6th week. The sales that have been declining previous to advertising campaign, pick up after a very short time, and continue rising up to the 15th week; this is mainly because of sales response function which is at work throughout this period. After the sales rate reaches a maximum at 15th week, the sales decay function takes effect, because advertising effectiveness is now at such a low level that it cannot create any more sales. The sales rate starts to decrease after 15th week.

The orders received at factory warehouse are almost constant in the beginning, and at the 11th week, they start to increase, and a week after that, manufacturing orders to factory encounters a similar trend. Factory output has always a 6-week time lag, and therefore it reaches a maximum

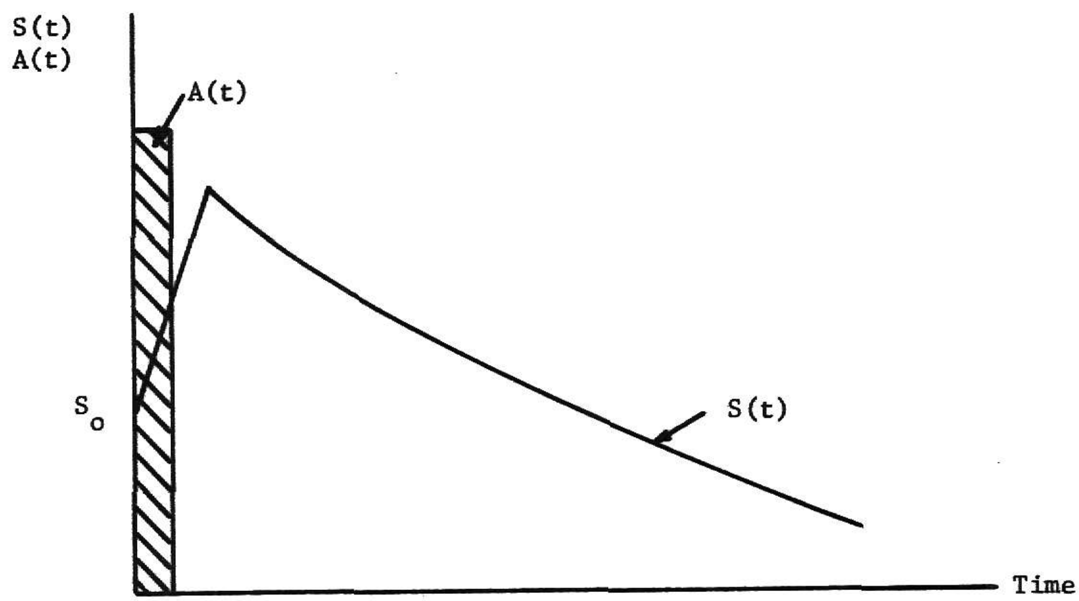


Fig. 5.6 Sales Response to Short-Time, Intensive Advertising

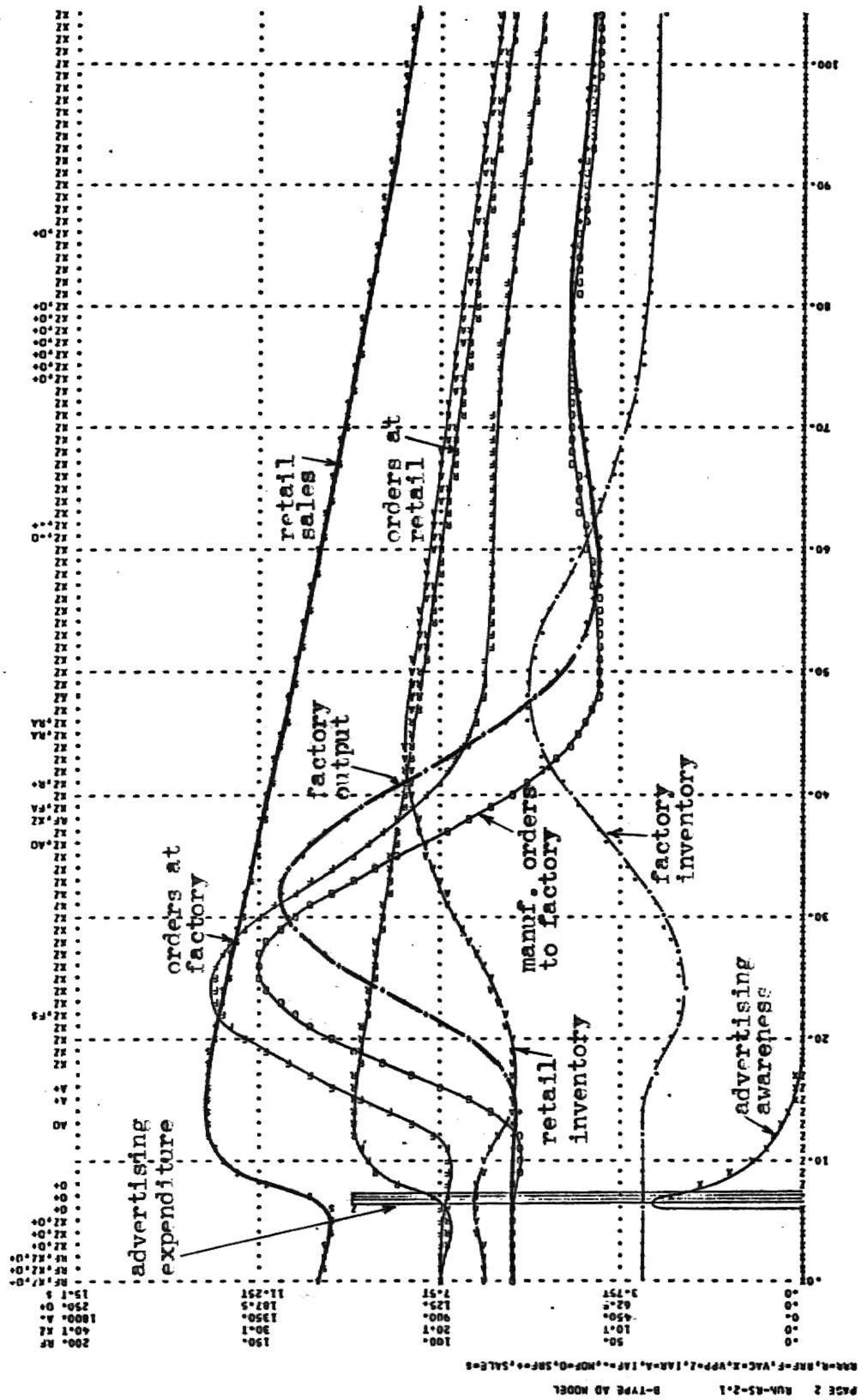


Fig. 5.7. Short-Time, Intensive Advertising

at 32nd week, while manufacturing orders were at peak at 26th week.

The same kind of behavior for inventories, as in the previous example, is observed. Both retail inventory and factory warehouse inventory decline in the beginning, and only after a long time delay (18 weeks at retail, and 24 weeks at factory warehouse) they start increasing. Although sales at retail are declining at this time, inventories keep rising, partly because of the need to replace the items, and partly because sales level is higher than it was in the beginning, and therefore it requires a higher level of inventory. However, because of the delays and amplifications, the declining sales rate is reflected in inventory levels after the 44th week at retail, and after the 48th week at factory warehouse. From then on, they decrease in correspondence with retail sales.

Unlike the previous case, here the factory output has two peaks, one at the 32nd week and about 80% <u>above</u> the initial level, and another at 82nd week and about 18% <u>below</u> the initial level; the peaks are almost a year apart. However, the first peak is much more conspicuous than the second one.

5.7. IMPULSE-TYPE ADVERTISING

In the previous example, advertising campaign is carried out only once. It was observed that, following the campaign, sales rate started to rise, but after a short time it followed a decreasing trend, as the advertising campaign lost its "effectiveness." The problem of fast-decreasing effectiveness of any advertising campaign can be overcome by carrying out another one at a later time, when the effectivenss of, or awareness about the previous one is at a very low level, such that it is not capable of increasing the sales any more. This type of advertising campaign is called "impulse-type" advertising, and it is basically consisted of a series of "short-time intensive" ones, carried out at intervals of time, as in Fig. 5.8. Neither the amount spent on advertising each time, nor the interval between the consequent "impulses" need be constant.

The analytical solution to equation (5-3), in case of impulse type advertising with constant interval and budget, would be a series of analytical solution to short-time, intensive kind of advertising, separated from each other by intervals equal to T, that is, sales rates vs. time are:

$$\left.\begin{array}{ll} S_1(t) = SL.e^{-\lambda.t} - (SL - S_0) \cdot e^{-(\frac{r.a}{SL} + \lambda)t} & 0 < t \leq T \\ S_2(t) = SL.e^{-\lambda.(t-T)} - (SL-S_1(T)).e^{-(\frac{r.a}{SL} + \lambda)(t-T)} & T < t \leq 2T \\ S_3(t) = SL.e^{-\lambda(t-2T)} - (SL-S_2(2T)).e^{-(\frac{r.a}{SL} + \lambda)(t-2T)} & 2T < t \leq 3T \\ \vdots & \end{array}\right\} \quad (5\text{-}7)$$

where a is the amount spent each time, and:

$$A = n.a \qquad (5\text{-}8)$$

where:

A = Total advertising budget

n = Number of campaigns.

In this case, sales rate will have a general form, over time, as in Fig. 5.9.

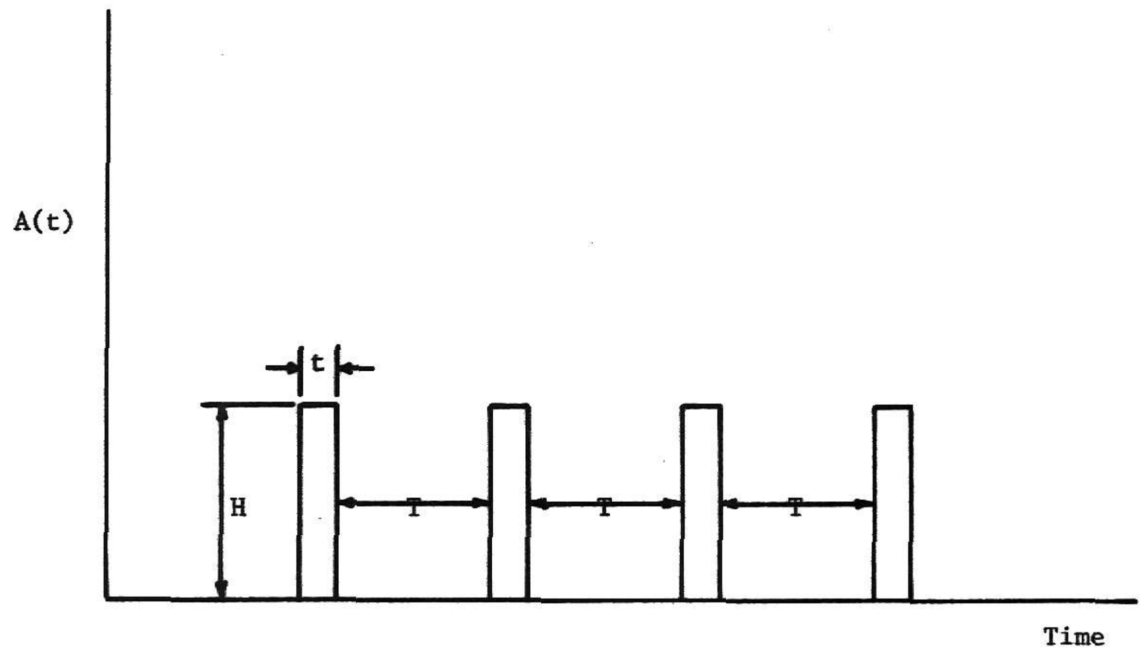


Fig. 5.8. Impulse-Type Advertising vs. Time

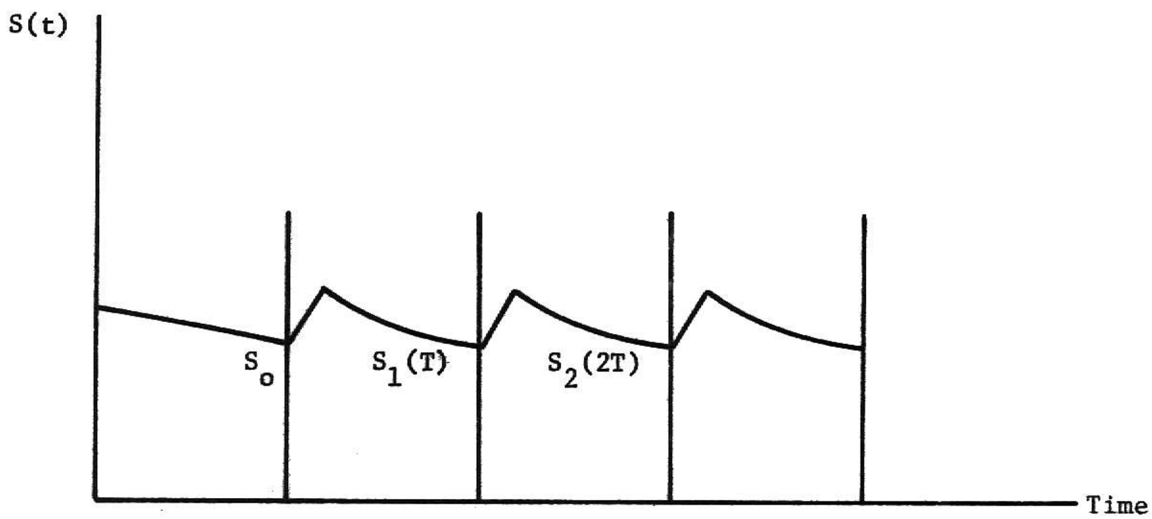


Fig. 5.9 Sales Response to Impulse - Type Advertising

In modelling the above procedure in DYNAMO, it was assumed that the total advertising budget of the example of Section 5.5, which was spent constantly over a time period of 25 weeks, is now being spent equally on 14 "impulse" campaigns, separated from each other by an interval of 7 weeks. The DYNAMO model of the above example appears in Appendix III.

Figure 5.10 shows the simulated behavior of the system under impulse-type advertising, for a period of 2 years. The first campaign is carried out at the 5th week, representing the delays at factory and at agencies and media.

Before the first campaign at time 5, sales have decreased slightly. A short time after the campaign, they start increasing at a small pace, and then remain constant for a period afterward. They start decreasing again, until the next "pulse" is applied, when the same pattern, with slight variations, happens again. It can be observed that, in general, sales are decreasing but at a very small pace; at the end of the 2-year period sales rate is at 9,500 $/week, a decrease of 5% over two years, or 2.5% per year. This is, of course, due to the low level of advertising budget.

Factory output shows slight variations; it has a minimum at the 29th week, which is about 8% below the initial level. At the 45th week, it reaches a level which is about 6% below the initial level, and remain constant thereafter, for a period of 32 weeks; it further decreases by 1% after the 77th week, and holds the new level up to the end of simulation run.

Inventories at retail and at factory warehouse show the same kind of behavior; their fluctuations are very slight. At the end of simulation run, inventories are down about 4% at retail, and about 5% at factory warehouse.

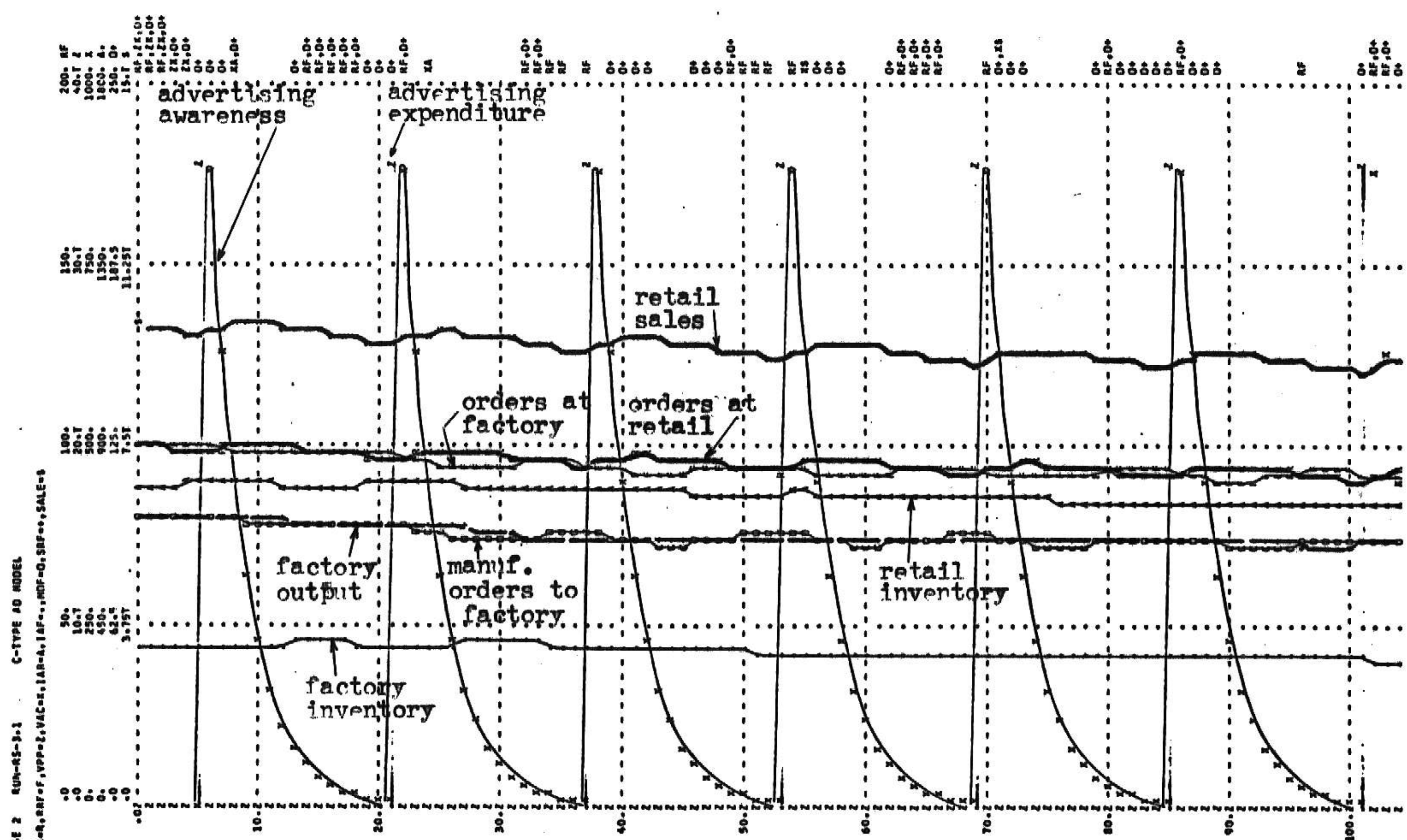


Fig. 5.10. Impulse-Type Advertising

5.8. RANDOM FLUCTUATIONS IN RETAIL SALES

In the last three models of sales response to advertising, it was assumed that the sales changed, over the time, in a deterministic way. However, there are many factors that affect a purchase in a practical situation, and that cannot be taken into account individually; instead, their effect can be approximated by introducing a "noise-band" into the sales rate at retail.

The effect of random fluctuations on retail sales can be observed in Figs. 5.11, 5.12, and 5.13 which are the same as Figs. 5.5, 5.7, and 5.10, respectively, but a noise-band is added to the sales rate to obtain the "actual" sales rate. The random deviations are sampled from a "random number generator", which generates random numbers with a mean of 0.0, and standard deviation of .05.

In Figs. 5.11, 5.12, and 5.13 are shown the results of the system simulation, with random noise, as described above, added to the sales rate at retail*. Random fluctuations are superimposed on the "deterministic" sales curve, but the same general trend as before can be observed in the "actual" sales.

*The model listings of above procedure, written in DYNAMO, appear in Appendix III.

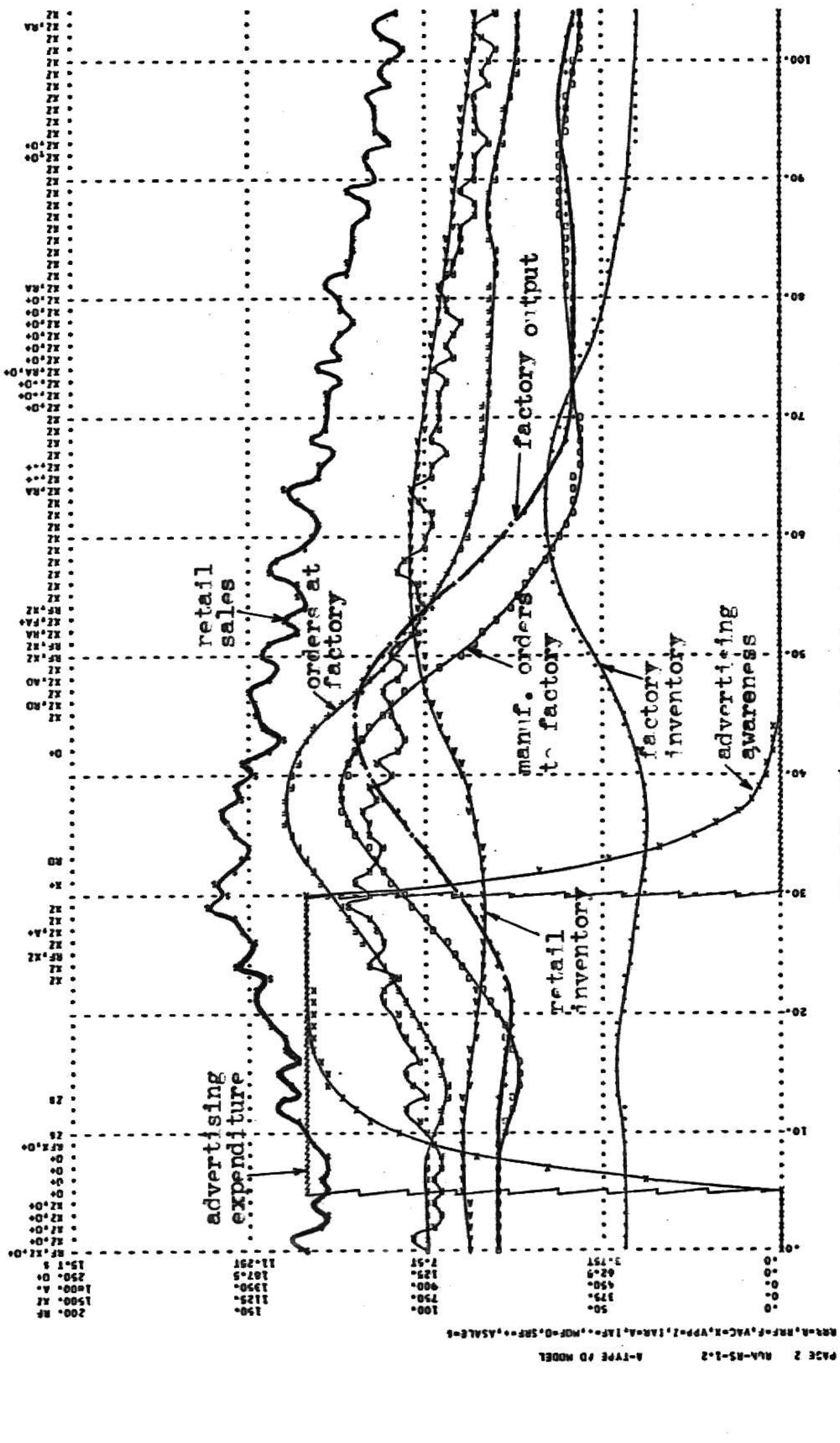


Fig. 5.11. Constant Advertising for a Period of Time; Random Fluctuations in Sales

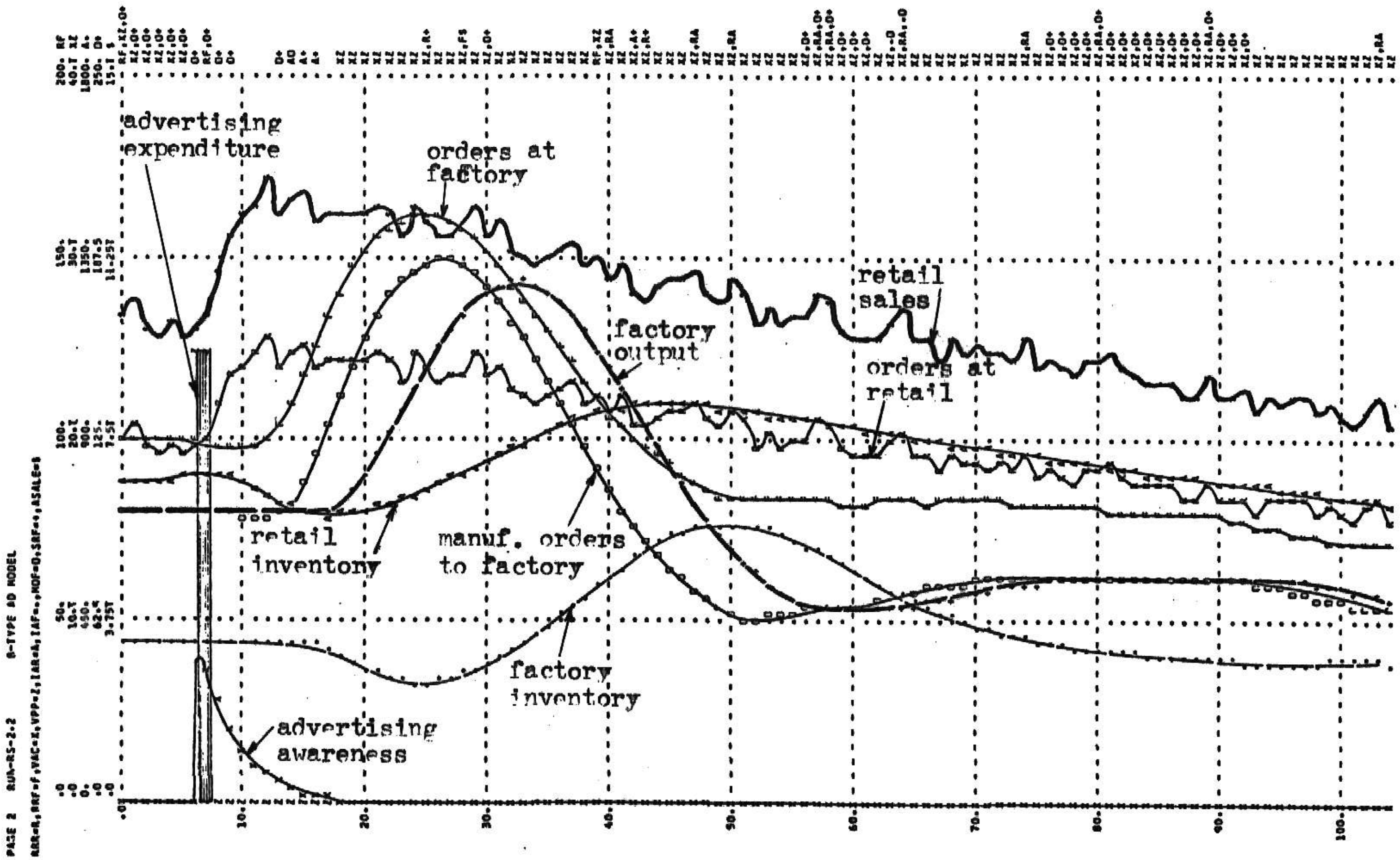


Fig. 5.12. Short-Time, Intensive Advertising; Random Fluctuations in Sales

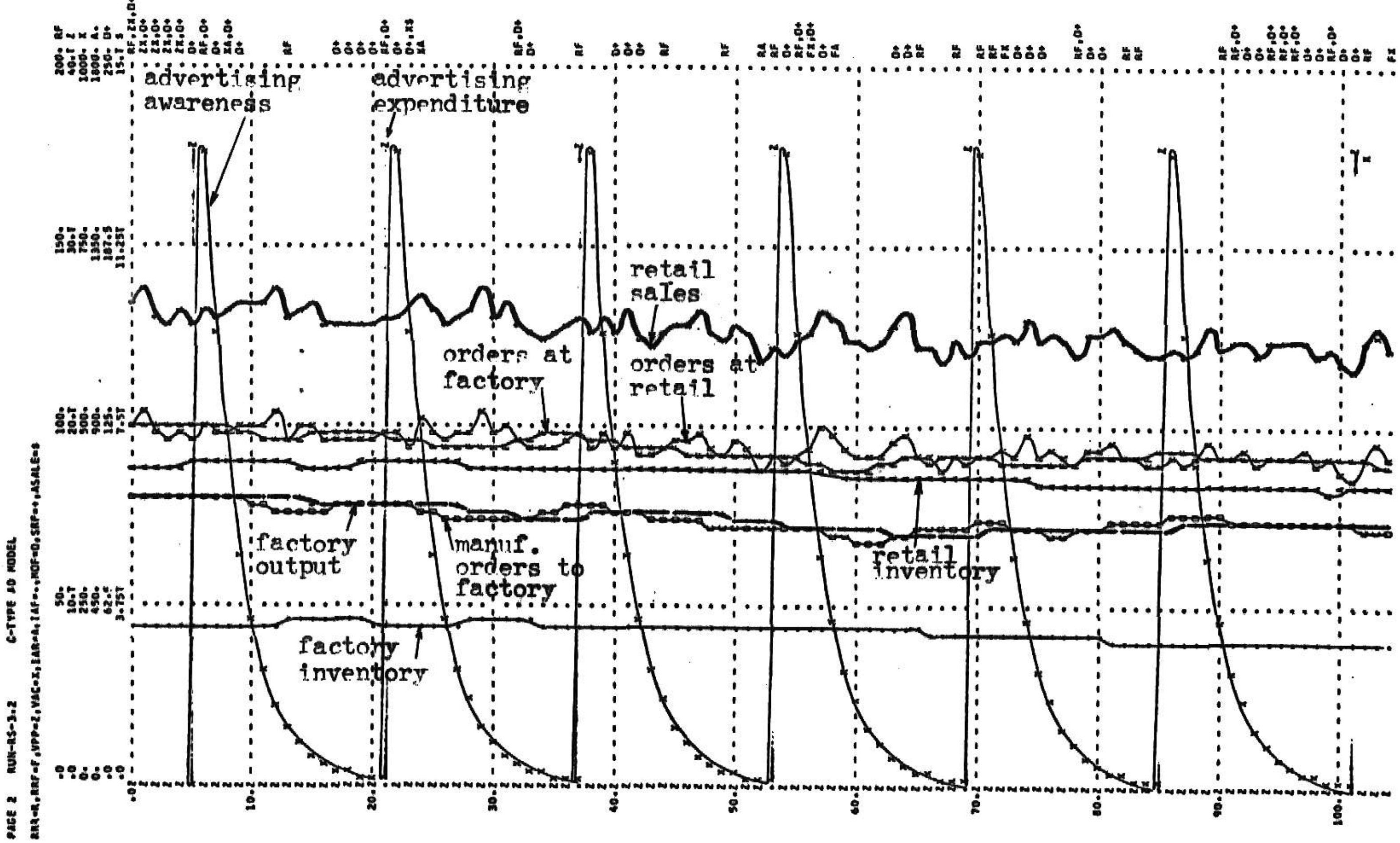


Fig. 5.13. Impulse-Type Advertising; Random Fluctuations in Sales

CHAPTER 6

DISCUSSION AND CONCLUSION

The results of the three different advertising practices* were examined in the last chapter. It was observed that:

1) For an A-type campaign, the retail sales rate grows in the beginning, reaches a maximum, and then declines exponentially.

2) For a B-type campaign, sales rate immediately starts rising, reaches a maximum value in a relatively short time, and then exponentially drops.

3) For a C-type campaign, the behavior described in (2) above, takes place within each interval, and on the whole, sales are declining.

Not only the moment-by-moment sales at retail are important, but also the total sales generated as the result of advertising campaigns could be used as one criterion to compare the three practices. Figure 6.1 and Table 6.1 show the total sales, after the advertising campaign starts and upto the end of simulation run, for the three models. The values are taken from the simulation results of Chapter 5.

The total sales for A-type and B-type advertisings are almost the same, and practically equal. However, there is a substantial difference of about $57000 between the C-type advertising and the other two; C-type advertising generates less sales.

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*For convenience of reference, the three advertising practices are coded as follow:

- Protracted advertising: A-Type
- Short-time, intensive advertising: B-Type
- Impulse-type advertising: C-Type

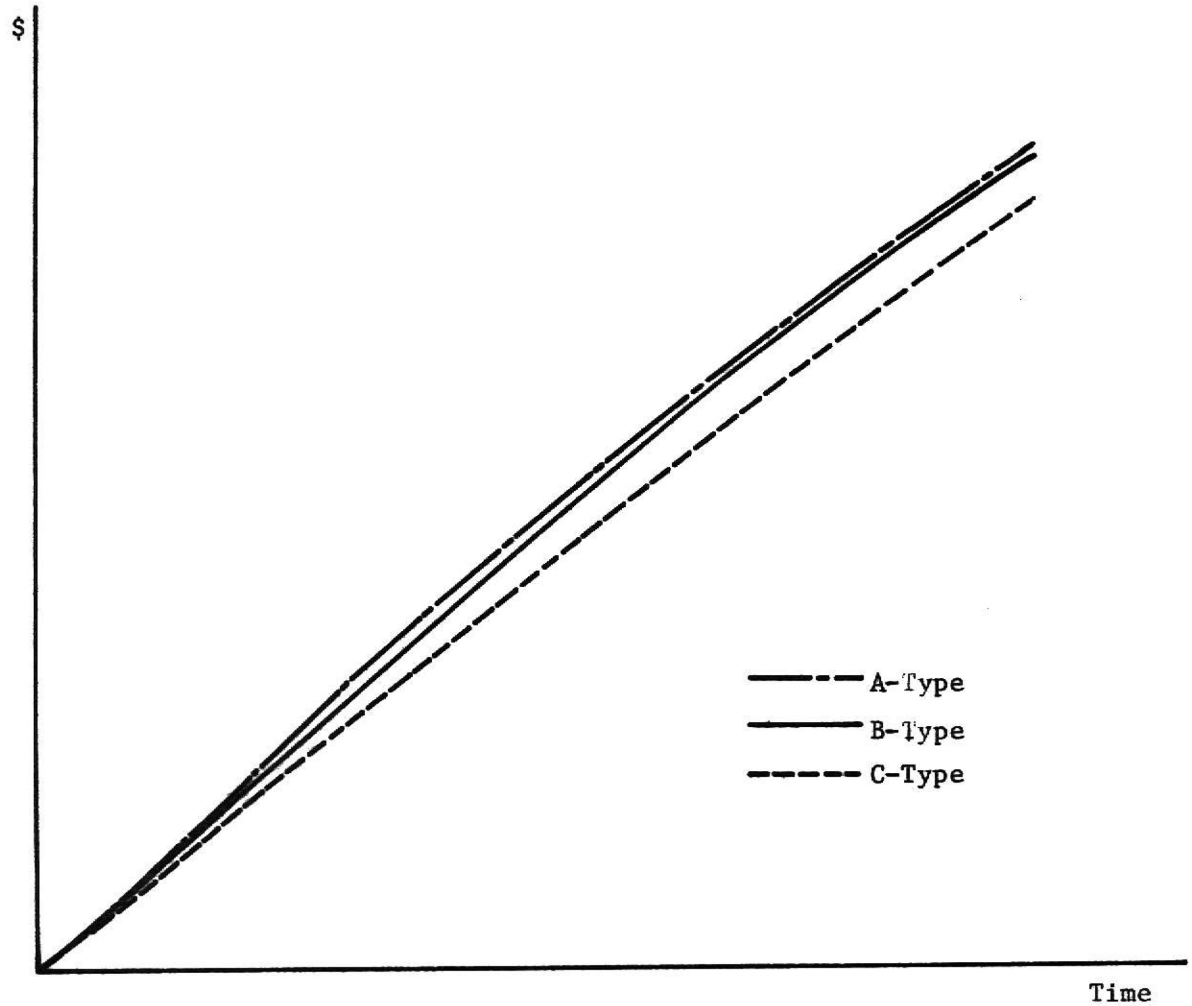

Fig. 6.1. Total Sales Generated After Advertising, for 3 Models

Table 6.1. Total Sales Generated After Advertising, for Different Advertising Campaigns

Advertising Type	Total Sales Generated After Advertising, 1000$
A = Protracted	1000
B = Short-Time Intensive	1004
C = Impulse	943

The second factor to be taken into account is the magnitude of fluctuations in production rate at factory, and in inventories. There are costs associated with changing the production rate and inventory level. Therefore smooth rates of production and inventory levels should be considered, in some occasions, in addition to total sales.

Table 6.2 shows how factory output and factory warehouse inventory change with respect to their initial values. In case of factory output, A-type advertising requires an increase of up to 48% in production facilities in order to meet the requirements; the lowest level of production is 26.6% below the initial value. For B-type advertising, these figures are 80% and 29.4%, respectively. However, for C-type advertising, factory output remains at about the initial rate; the lowest level of production is about 9.1% below the initial rate.

Considering the inventory level at factory warehouse, A-type advertising requires expanding the inventory facilities up to 47.5% above the initial level. The lowest level of inventory is about 12.5% below the initial capacity. For B-type advertising, these figures are 70% above and 29.5% below the initial level. However, in case of C-type advertising, an increase of 5% in inventory facilities is required, and the lowest level would be 7.5% below the initial capacity.

A third factor, closely associated with above analysis, is the costs incurred when production rate and inventory level change to new values. The cost function may be represented by [Holt, et. al., 30]:

$$S = \sum_{n=1}^{N} [C(\theta_n - \theta_{n-1})^2 + D(E - I_n)^2] \tag{6-1}$$

where:

Table 6.2. Variations of Factory Output, and Inventory, with Respect to Their Initial Values

Advertising Type	Factory Output (Units/Week) Initial Value	Max. (%)	Min. (%)	Factory Warehouse Inventory (Units) Initial Value	Max. (%)	Min. (%)
A	100	148.28 (48.3%)	73.41 (-26.6%)	400	590 (47.5%)	350 (-12.5%)
B	100	179.80 (79.8%)	70.60 (-29.4%)	400	680 (70%)	283 (-29.3%)
C	100	100 (0%)	90.89 (-9.11%)	400	420 (5%)	370 (-7.5%)

where:

S = Total cost of changing production rate and inventory level;

θ_n = Production rate during the nth period;

I_n = Inventory level at the end of the nth period;

E = Desired inventory level

C,D = Cost coefficients, constant;

N = Number of periods

Here, it is assumed that, for a period of 2 years, 7 periods each of 15 weeks duration, are to be considered. The mean production rate and inventory level at factory, during each period, are shown in Tables 6.3 - 6.5, for A-, B-, and C-type advertisings, respectively.

In Table 6.6 are shown the estimated costs, S, according to Eq. (6-1), for the three advertising campaigns, and for different values of constants C and D.

Since the values of C and D depend on the specifications of the product, production techniques, and inventory practices, it is up to the advertiser to decide what kind of advertising would be feasible. If C and D are relatively large (as in the first row of Table 6.6), the associated costs in case of A-, and B-type advertisings are quite high relative to the sales margin between C-type and A-, or B-type advertisings ($57,000). Therefore, it would be feasible to choose C-type advertising.

On the other hand, if values of C and D are small (as in the third row of Table 6.6), A-type advertising seems to be more feasible, because the difference between total sales and the cost, is greater in this case.

In cases where C and D have intermediate values (second row of Table 6.1), still C-type advertising is likely to be more feasible; the

Table 6.3. Mean Production Rate and Inventory Level at each Period, for Protracted Advertising Campaign (A-Type)

Period	Mean Production Rate θ_n	$\theta_n - \theta_{n-1}$	Mean Inventory Level J_n	$E-I_n$ (E=400)
1	100		400	0
		7		
2	107		335	65
		28		
3	135		365	35
		13		
4	122		475	75
		35		
5	87		520	120
		10		
6	77		440	40
		2		
7	75		375	25

Table 6.4. Mean Production Rate and Inventory Level at each Period, for Short-Time, Intensive Advertising Campaign (B-Type)

Period	Mean Production Rate θ_n	$\theta_n \quad \theta_{n-1}$	Mean Inventory Level J_n	$E-I_n$ (E=400)
1	100		400	0
		38		
2	137.5		310	90
		2		
3	140		520	120
		60		
4	80		610	210
		15		
5	75		400	0
		6		
6	81		375	25
		6		
7	75		360	40

Table 6.5. Mean Production Rate and Inventory Level at each Period, for Impulse-Type Advertising Campaign (C-Type)

Period	Mean Production Rate θ_n	θ_n θ_{n-1}	Mean Inventory Level J_n	$E-1_n$ (E=400)
1	100.0		400	0
		5		
2	95.0		400	0
		2		
3	93.0		398	2
		1.5		
4	91.5		378	22
		.5		
5	91.0		380	20
		.5		
6	90.5		375	25
		.5		
7	91.0		370	30

Table 6.6. Total Cost of Changing Production Rate and Inventory Level, for Different Advertising Practices, and for Different Values of Parameters

Advertising Type / Parameter Values	Protracted Type (A)	Short-Time, Intensive Type (8)	Impulse Type (c)
C = 100 D = 20	$787,100	$1,891,000	$51,460
C = 10 D = 2	78,710	189,100	5,146
C = 1 D = .20	7,871	18,910	514.6

difference between the total sales generated, and the cost is greater for C-type than for other two.

B-type advertising is not feasible in any case. While it generates almost the same total sales as in A-type, the associated cost of production planning and inventory control is twice as much as that of A-type, for all values of C and D.

REFERENCES

1. Ansoff, H. I, and D. P. Slevin, "An Appreciation of Industrial Dynamics," Management Science, Vol. 14, No. 7, March 1968.

2. Bass, F. M., et. al., Editors, Mathematical Models and Methods in Marketing, Richard D. Irwine, Inc., Homewood, Ill., 1961.

3. Carlson, B. R., "Industrial Dynamics," Management Services, May-June 1964.

4. Coughanowr, D. R., and L. B. Koppel, Process Systems Analysis and Control, McGraw-Hill Book Co., New York, N.Y., 1965.

5. Crawford, J. W., Advertising, Allyn and Bacon, Inc., Boston, Mass., 1965.

6. D'Azzo, J. J., and C. H. Houpis, Feedback Control System Analysis and Synthesis, McGraw-Hill Book Co., New York, N.Y., 1960.

7. Dirksen, C. J., and A. Kroeger, Advertising Principles and Problems, Richard D. Irwine, Inc., Homewood, Ill., 1968.

8. Fey, W. R., An Industrial Dynamics Study of an Electronic Components Manufacturer, Transactions of the Fifth Annual Conference of the American Production and Inventory Control Society, Sept. 1962.

9. Forrester, J. W., "Advertising: A Problem in Industrial Dynamics," Harvard Business Review, Vol. 37, No. 2, March-April 1959.

10. _________, "A New Avenue to Management," Technology Review, January 1964.

11. _________, "Common Foundations Underlying Engineering and Management," IEEE Spectrum, Vol. 1, No. 9, Sept. 1964.

12. _________, "Dynamics of Corporate Growth," Proceedings of M.I.T. Conference on Management Strategy for Corporate Growth in New England, Nov. 1963.

13. _________, Industrial Dynamics, The M.I.T. Press, Cambridge, Mass., 1961.

14. _________, "Industrial Dynamics," in Carl Heyel, editor, The Encyclopedia of Management, Reinhold Publishing Co., New York, N.Y., 1963.

15. _________, "Industrial Dynamics - After the First Decade," Management Science, Vol. 14, No. 7, March 1968.

16. _________, "Industrial Dynamics: A Major Breakthrough for Decision Makers," Harvard Business Review, Vol. 36, No. 4, July-August 1958.

17. _________, "Industrial Dynamics - A Response to Ansoff and Slevin," Management Science, Vol. 14, No. 9, May 1968.

18. _________, "Management Science - Its Impact," Technology Review, January 1962.

19. _________, "New Frontiers," Proceedings of the Eastern Joint Computer Conference, Dec. 1958.

20. _________, "New Opportunities for Instrumentation and Control in Management Systems," Journal of Engineering Education, Vol. 53, No. 53, No. 10, June 1963.

21. _________, Principles of Systems, Wright-Allen Press, CAmbridge, Mass., 1968.

22. _________, "Simulative Approaches for Improving Knowledge of Business Processes and Environments," Proceedings of the 13th CIOS Conference, 1963.

23. _________, "The Impact of Feedback Control Concepts on the Management Sciences," 1960 F.I.E.R. Distinguished Lecture, Foundation for Instrumentation Education and Research, Oct. 1960.

24. _________, "The Relationship of Advertising to Corporate Management," Proceedings of the Fourth Annual Conference of the Advertising Research Foundation, Oct. 1958.

25. _________, Urban Dynamics, The M.I.T. Press, Cambridge, Mass., 1969.

26. _________, World Dynamics, Wright-Allen Press, Cambridge, Mass., 1971.

27. Greenberger, M. (Editor), Computers and the World of Future, The M.I.T. Press, Cambridge, Mass., 1962.

28. Hammond, P. M., Feedback Theory and its Applications, The McMillan Co., New York, N.Y., 1958.

29. Holt, C. C., F. Modigliani, J. F. Muth, and H. A. Simon, Planning Production, Inventories, and Work Force, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1960.

30. Hwang, C. L., L. T. Fan, F. A. Tillman, and R. Sharma, "Optimal Production Planning and Inventory Control," The International Journal of Production Research, Vol. 8, No. 1, 1969.

31. Katz, A., "An Industrial Dynamics Approach to the Management of Research and Development," I.R.E. Transactions on Engineering Management, Sept. 1959.

32. Kuo, B. C., Automatic Control Systems, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1962.

33. Lucas, D. B., and S. H. Britt, Measuring Advertising Effectiveness, McGraw-Hill Book Co., New York, N. Y., 1963.

34. Meadows, D. L., Dynamics of Commodity Production Cycles, Wright-Allen Press, Cambridge, Mass., 1970.

35. Meier, R. C., W. T. Newell, and H. L. Pazer, Simulation in Business and Economics, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1969.

36. Nord, O. C., Growth of a Product: Effects of Capacity-Acquision Policies, The M.I.T. Press, Cambridge, Mass., 1963.

37. "Proposed Symbols and Terms for Feedback Control Systems," AIIE Committee Reports, Electrical Engineering, Vol. 70, 1951.

38. Pugh, III, A. L., DYNAMO User's Manual, The M.I.T. Press, Cambridge, Mass., 1961.

39. _________, DYNAMO II User's Manual, The M.I.T. Press, Cambridge, Mass., 1961.

40. Report of the Definition Committee, Journal of Marketing, Vol. 13, No. 2, 1948.

41. Roberts, E. B., "Industrial Dynamics and the Design of Management Control Systems," Management Technology, Dec. 1963.

42. _________, "New Directions in Industrial Dynamics," Industrial Management Review, Vol. 6, No. 1, Fall 1964.

43. _________, "Research and Development Policy-Making," Technology Review, June 1964.

44. _________, "Simulation Techniques for Understanding R and D Management," 1959 I.R.E. National Convention Record, Part 10, March 1959.

45. _________, The Dynamics of Research and Development, Harper and Row Publishers, Inc., New York, N. Y., 1964.

46. _________, "Toward a new Theory for Research and Development," Industrial Management Review, Vol. 4, No. 1, Fall 1962.

47. Robinson, P. J. (Editor), Advertising Measurement and Decision Making, Allyn and Bacon, Inc., Boston, Mass., 1968.

48. Sandage, C. H., and V. Fryburger, Advertising Theory and Practice, Richard D. Irwine, Inc., Homewood, Ill., 1960.

49. Vidale, M. L., and H. B. Wolfe, "An Operations Research Study of Sales Response to Advertising," Operations Research, Vol. 5, No. 3, June 1957.

50. ________, "Response of Sales to Advertising, in: Mudrick, P. G., Mathematical Models in Marketing, Intext Publishers, Scranton, Pa., 1971.

51. Wagner, H. M., "Industrial Dynamics" (book review), Management Science, Vol. 10, No. 1, Oct. 1963.

52. Weymar, F. H., "Industrial Dynamics: Interaction between the Firm and its Market," in: Alderson, W., and S. Shapiro, Marketing and the Computer, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963.

APPENDIX I

A SHORT GUIDE TO DYNAMO

A SHORT GUIDE TO DYNAMO

1. VARIABLE NAMES

Any variable name may be up to 5 characters, the first of which is always alphabetic. Example:

LEVEL	L5
RATE2	R1250

2. EQUATION TYPES

As discussed in Chapter 2, there are two main variables in a dynamic model: level variable and rate variable. Accordingly, the equations defining these variables are called level and rate equations. However, there are other types of equations, such as axiliary, supplementary, and initial-value equations. When writing a model, the type of equation should be specified in the first column of the punched card. Example:

```
12345....
L   L1.K = L1.J + DT * (INFL.JK - OTFL.JK)
R   INFL.KL = LEV.K/DEL
```

Other equation types are specified as follow:

A	Auxiliary
S	Supplementary
N	Initial Condition or Initial-Value

Constants. Constants are designated by the letter C in the first column. Several constants may be specified on a card. Example:

```
12345....
C   CA = 5
C   A = 5, CA = 12.00, D = 7.00
```

Equation Writing. Following symbols are used for the ordinary mathematical operations:

+ Addition

- Subtraction

* Multiplication

/ Division

The symbol *, however, may be omitted in many cases. Example:

A = (B + C) * (D + E)

and

A = (B + C)(D + E)

are equivalent.

The order of operations within a statement is the same as other languages (such as FORTRAN, for example.)

Subscripts. Level variables are always single-subscripted, like:

L1.K, LEVEL.J, INVEN.J

The same is true for auxiliary and supplementary variables, whose single subscript is always K.

Rate variables are double-subscripted, such as:

R1.JK, FLOW2.KL, OTFL3.JK

Table A.1 should help in identifying the subscripts [38,39].

Card Punching. Equation or quantity type is to be punched in column 1. There should at least be one blank after that, and before equation starts. No blanks are allowed within the equation or statement; if there is any blank, the compiler ignores what comes after the blank. The statement

TABLE A.L. SUBSCRIPT TABLE

Quantity Type on Left of Equation		Subscript on Left	Subscripts on Quantities on Right if Quantity is:					
			L	A	R	S	C	N
L	Level	K	J	J	JK	np	none	none
A	Auxiliary	K	K	K	JK	np	none	none
R	Rate	KL	K	K	JK	np	none	none
S	Supplementary	K	K	K	JK	K	none	none
C	Constant	none	np	np	np	np	np	np
N	Initial Value or Computed Constant	none	none	none	none	none	none	none

np = not permitted

on a card should not extend beyond column 72, but continuation cards are allowed in any number. A continuation card starts with letter X in column 1. Example:

```
12345....                              7
                                       2
A   L1.K = LL1,K + LL2.K + ....    + LA.K
X   + LB.K/CAT
```

Numerical Computations. In DYNAMO, there is no provision for fixed-point arithmetic. All computations are carried out in floating-point form. The numerical values, however, may be specified as fixed-point values, for simplicity; the compiler converts them into floating-point values.

3. DIRECTION CARDS

Direction cards are used to specify the length of the simulation run, time interval, printing and plotting instructions, and quantitites to be printed or plotted. They are as follow.

Identification Card. This is the first card of any model, which provides a title for the model. It begins with an asterisk (*) in the first column; the title cannot be more than 40 character, nor can any word be more than 8 letters. Example:

```
12345

*  MODEL OF WORLD SYSTEM
```

RUN Card. Each run and rerun is assigned a run number, by which it is filed. Run card is the last card of any model, and run number can have up to 8 characters. Example:

```
123456....
RUN  NO.1
RUN  NEW X
```

SPEC Card. This card provides values for the following four basic data necessary for any simulation:

DT	Time interval, in weeks;
LENGTH	Length of simulation run, in weeks;
PRTPER	Printing interval, in weeks;
PLTPER	Plotting interval, in weeks.

These values are specified in the following manner:

```
1234567....
SPEC  DT = .1/LENGTH = 104/PLTPER = 1/PRTPER = 5
```

which means:

1) Time interval of computation is .1 week;
2) Simulation is carried out for 104 weeks;
3) Values are plotted every week;
4) Values are printed every 5 weeks.

Any of the above values, except DT, can be zero.

PRINT Card. This card specifies what values to be printed and in what form. In any table of printed values, TIME is automatically supplied by compiler. A PRINT card such as:

```
1234567....
PRINT ABC,DEF,L1,L2,L3
```

is equivalent to:

```
PRINT ABC/DEF/L1/L2/L3
```

and will result in a printed output such as the following:

```
TIME  ABC    DEF    L1    L2    L3
```

- - - - - - -

But a PRINT statement such as:

```
1234567....
PRINT ABC,DEF/L1,L2/L3
```

will print the results in an output like:

```
TIME    ABC    L1    L3
        DEF    L2
```

- - - - - - -

In printing, DYNAMO gives only up to 5 significant digits. If values are very big or very small, they are scaled by some power of 10 by the compiler; the scaling factor will appear in the title of the tabulated output. Example:

```
TIME    ABC    L1    L3
        DEF    L2
- - - - - - -
        E+03   E+04  E+00
E+00
        E-06   E+05
- - - - - - -
```

This indicates that values of TIME and L3 are not scaled; values of ABC are divided by 10^3, L1 by 10^4, L2 by 10^5, and DEF by 10^{-6} before being printed.

The scaling factor can be specified by user. For example:

```
1234567....
PRINT A(4.2)/B(3.4)/C(-6.3)
```

will print the values of A after dividing them by 10^4, and choosing 2 significant decimal places after decimal point. The values of B are divided by 10^{+3}, and 4 decimal places are chosen. Finally the values of C are divided by 10^{-6}, and 3 decimal places are chosen.

Up to 14 quantities can be printed in a table output.

PLOT Cards. DYNAMO has an automatic plotting feature which enables the user to plot up to 10 quantities on a single graph. The scales may be chosen by the user or by the compiler. Any quantity should be assigned a character by which that quantity is represented in the graph. Example:

```
1234567....
PLOT  L1 = */L2 = +/LEVEL = L
```

will plot quantities L1, L2, and LEVEL, represented by character *, +, and L respectively.

If a group of quantities are to be plotted to the same scale, they should be separated by slashes:

```
1234567....
PLOT  RAT = R, CAT = C/MIR = M, MILL = L/XYZ = X
```

will plot RAT and CAT variables to the same scale, MIR, and MILL to the same scale, and XYZ to another scale.

Scales may be specified by the user, or else it will be chosen by the compiler. It is possible to choose only the upper limit, or the lower limit of the scale; computer will choose the other one. The unspecified limit is given as an asterisk (*). Example:

```
1234567...
PLOT  RAT=R,CAT=C(0,200)/MIR=M,MILL=L(0,*)/XYZ=X(*,10000)
```

will plot variables RAT and CAT to a scale from 0 to 200, MIR and MILL to a scale from 0 to an upper limit to be determined by computer, and XYZ to a scale with an upper limit of 10000, and a lower limit to be chosen by DYNAMO.

4. COMMENT CARDS

Comments are allowed on NOTE cards. Example:

```
1234567....
NOTE  THE MODEL OF THE SYSTEM FOLLOWS:
```

Comments may also be added to a statement after leaving a blank. Example:

```
1234567....
L     L1.K = L1.J + DT * (OR.JK - IR.JK)  INVENTORY LEVEL
```

5. RERUNS

A model may be rerun several times, for different parameter values. Only constant values, table values, and direction cards may be changed in each run.

Each rerun should also be assigned a run number.

6. FUNCTIONS

DYNAMO has a number of built-in functions, and also has provisions to accept user-written functions. The built-in functions are described below.

Common Functions. Common functions are:

EXP(A)	$A \leq 174$	Exponential
LOGN(A)	$A > 0$	Natural Logarithm
SQRT(A)	$A \geq 0$	Square Root
SIN(A)	$A < 823,000$	Sine Function
COS(A)	$A < 823,000$	Cosine Function

Random Number Generator. DYNAMO has two random number generators:

1) NORMRN (MEAN, SDEV) generates random numbers normally distributed with mean equal to MEAN, and standard deviation equal to SDEV.

2) NOISE( ) generate random numbers uniformly distributed between -.5 and +.5. Note the parentheses but no arguments.

Third-Order Delays. DYNAMO has two third-order delay functions: a material delay, and an information delay.

DELAY3(IN, DEL)	is a material delay;
DLINF3(IN, DEL)	is an information delay;

where:

IN = Input to the delay

DEL = Magnitude of the delay

PULSE Function.

PULSE(HGHT,FRST,INTVL)

provides a pulse train of height HGHT, with a width of DT. The first pulse appears at time FRST, and thereafter at regular intervals of length INTVL. Neither HGHT nor INTVL need be constant.

RAMP Function.

RAMP(SLP, STRT)

in equivalent to (Fig. A.1):

$$\text{RAMP} = 0 \qquad \text{if TIME} \leq \text{STRT}$$

$$\text{RAMP} = \sum_{\text{STRT}}^{\text{TIME}} \text{SLP} * \text{DT} \qquad \text{if TIME} > \text{STRT}$$

SAMPLE Function.

SAMPLE(X, INTVL, ISAM)

sets SAMPLE equl to X at sample times separated by intervals of length INTVL, and holds the value until the next sampling time. ISAM is the initial value of SAMPLE.

STEP Function.

STEP(HGHT, STTM)

is equivalent to (Fig. A.2):

$$\text{STEP} = 0 \qquad \text{if TIME} < \text{STTM}$$

$$\text{STEP} = \text{HGHT} \qquad \text{if TIME} > \text{STTM}$$

Both HGHT and STTM may be variables.

MAX and MIN Functions.

MAX(P,Q)

sets:

$$\text{MAX} = P \qquad \text{if} \quad P \geq Q$$

$$\text{MAX} = Q \qquad \text{if} \quad P < Q$$

Similarly:

MIN(P,Q)

sets:

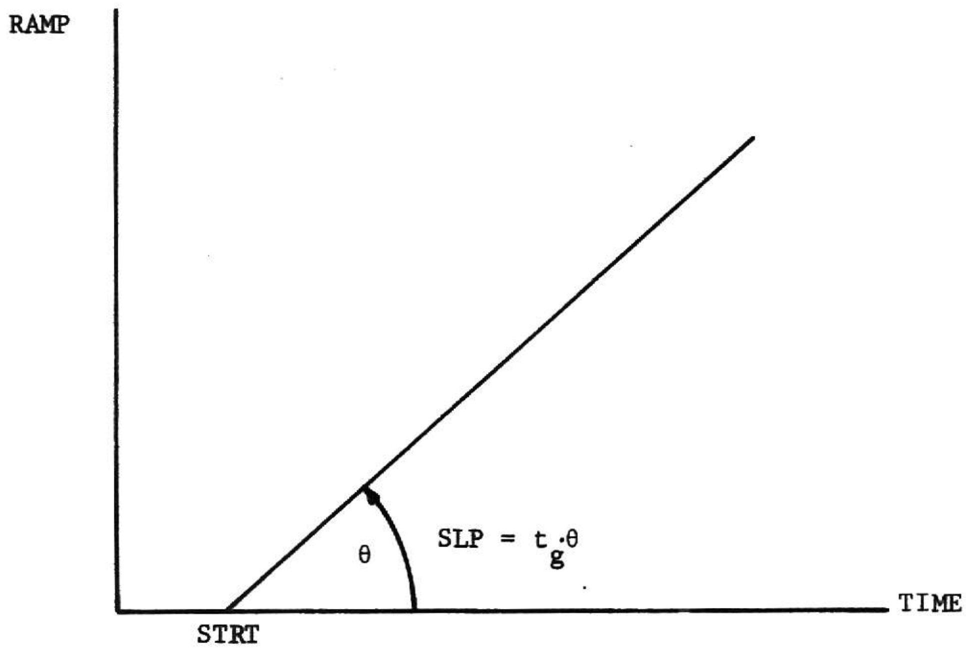

Fig. A.1. RAMP Function

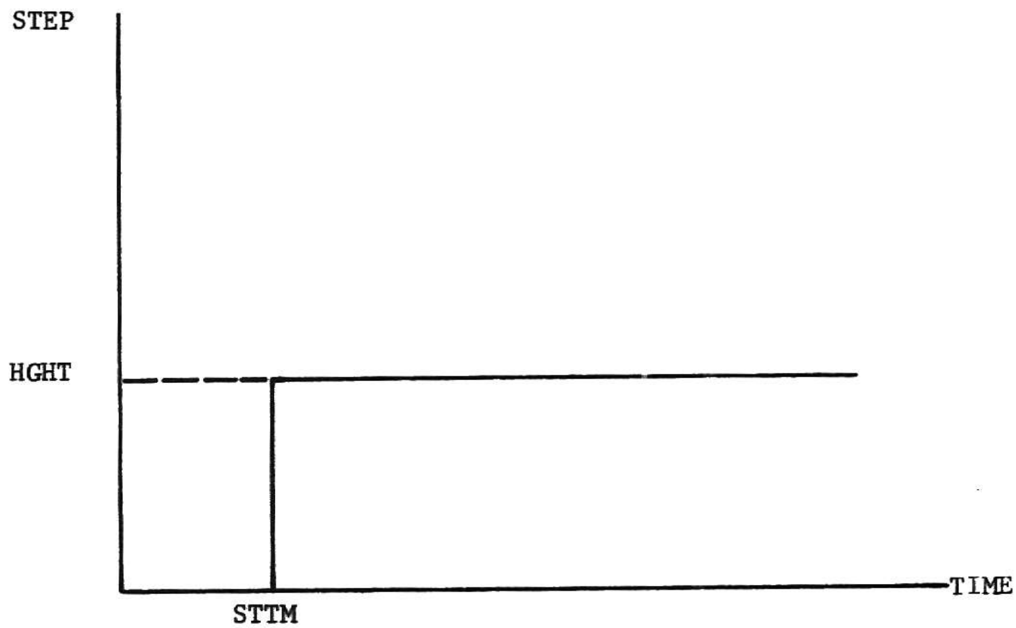

Fig. A.2 STEP Function

$$MIN = P \quad \text{if} \quad P \leq Q$$

$$MIN = Q \quad \text{if} \quad P > Q$$

CLIP Function.

CLIP (P,Q,R,S)

sets:

$$CLIP = P \quad \text{if} \quad R \geq S$$

$$CLIP = Q \quad \quad R < S$$

SWITCH Function.

SWITHC (P,Q,R)

sets:

$$SWITCH = P \quad \text{if} \quad R = 0$$

$$SWITCH = Q \quad \text{if} \quad R \neq 0$$

TABLE Function. It may be desireable to express the values of a variable in terms of the values of another one, in the form of a table. The form of a table look-up function is:

TABLE (TNAME, X, XLOW, XHIGH, XINCR)

TNAME - name of the table

X - independent variable

XLOW - lowest value of range of independent variable

XHIGH - highest value of range of independent variable

XINCR - increment of independent variable.

Suppose that the values of auxiliary variable Y are given as:

X	Y
1	45
2	50
3	60
4	68
5	76
6	80

It can then be represented as the following TABLE function:

```
1234567....
A     Y.K = TABLE (TNAME, X.K, 1.0, 6.0, 1)
T     TNAME = 45/50/60/68/76/80
```

Note the letter T in the first column denoting the TABLE function.

The method of interpolation is used by DYNAMO to compute the values that are not in the table.

In TABLE functions, the values of X should not exceed the specified range, otherwise there will be an error message. The TABHL function removes this deficiency by extending the extreme values of the dependent variable if the independent variable exceeds its range. In above example, Y will have a value of 80 for all values of X > 6, and a value of 45 for all values of X < 1, if TABHL is used.

<u>SMOOTH Function</u>. This function exponentially smooths a quantity, and has the following form:

SMOOTH(IN, DEL)

IN = Input to be smoothed;

DEL = Smoothing constant or delay.

APPENDIX II

DYNAMO MODEL OF PRODUCTION-DISTRIBUTION SYSTEM

```
*       PRODUC. AND DISTRI. MODEL
NOTE    THIS SYSTEM DEALS ONLY WITH PRODUCTION AND DISTRIBUTION
NOTE
NOTE    FOR THE COMPLETE DESCRIPTION OF THE PRODUCTION-
NOTE    DISTRIBUTION MODEL SEE:
NOTE         J.W. FORRESTER:
NOTE         "INDUSTRIAL DYNAMICS," THE M.I.T. PRESS,
NOTE         CAMBRIDGE,MASS., 1961; CHAPTERS 2 & 15
NOTE    ****************************************************
NOTE    EQUATIONS FOR RETAIL SECTOR
NOTE    ****************************************************
L       UOR.K=UOR.J+(DT)(RRR.JK-SSR.JK)
L       IAR.K=IAR.J+(DT)(SRR.JK-SSR.JK)
A       STR.K=(UOR.K)/(DFR.K)
A       NIR.K=(IAR.K)/DT
R       SSR.KL=CLIP(STR.K,NIR.K,NIR.K,STR.K)
A       DFR.K=DHR+MNR.K/IAR.K
A       MNR.K=(DUR)(IDR.K)
A       IDR.K=(AIR)(RSR.K)
L       RSR.K=RSR.J+(DT/DRR)(RRR.JK-RSR.J)
R       PDR.KL=RRR.JK+(1/DIR)(A1.K+B1.K+C1.K)
A       A1.K=IDR.K-IAR.K
A       B1.K=LDR.K-LAR.K
A       C1.K=UOR.K-UNR.K
A       LDR.K=(RSR.K)(DCR+DMR+DFD.K+DTR)
A       LAR.K=CPR.K+PMR.K+UOD.K+MTR.K
A       UNR.K=(RSR.K)(DHR+DUR)
L       CPR.K=CPR.J+(DT)(PDR.JK-PSR.JK)
L       PMR.K=PMR.J+(DT)(PSR.JK-RRD.JK)
R       PSR.KL=DELAY3(PDR.JK,DCR)
R       RRD.KL=DELAY3(PSR.JK,DMR)
L       MTR.K=MTR.J+(DT)(SSD.JK-SRR.JK)
R       SRR.KL=DELAY3(SSD.JK,DTR)
NOTE    ****************************************************
NOTE    EQUATIONS FOR THE DISTRIBUTION SECTION
NOTE    ****************************************************
L       UOD.K=UOD.J+(DT)(RRD.JK-SSD.JK)
L       IAD.K=IAD.J+(DT)(SRD.JK-SSD.JK)
A       STD.K=(UOD.K)/(DFD.K)
A       NID.K=(IAD.K)/DT
R       SSD.KL=CLIP(STD.K,NID.K,NID.K,STD.K)
A       DFD.K=DHD+MOD.K/IAD.K
A       MOD.K=(DUD)(IDD.K)
A       IDD.K=(AID)(RSD.K)
L       RSD.K=RSD.J+(DT/DRD)(RRD.JK-RSD.J)
R       PDD.KL=RRD.JK+(1/DID)(A2.K+B2.K+C2.K)
A       A2.K=IDD.K-IAD.K
A       B2.K=LDD.K-LAD.K
A       C2.K=UOD.K-UND.K
A       LDD.K=(RSD.K)(DCD+DMD+DFF.K+DTD)
```

```
A       LAD.K=CPD.K+PMD.K+UCF.K+MTD.K
A       UND.K=(RSD.K)(DHD+DLD)
L       CPD.K=CPD.J+(DT)(PDD.JK-PSD.JK)
R       PSD.KL=DELAY3(PDD.JK,DCD)
L       PMD.K=PMD.J+(DT)(PSD.JK-RRF.JK)
R       RRF.KL=DELAY3(PSD.JK,DMD)
L       MTD.K=MTD.J+(DT)(SSF.JK-SRD.JK)
R       SRD.KL=DELAY3(SSF.JK,DTD)
NOTE    *********************************************
NOTE    EQUATIONS FOR THE FACTORY SECTOR
NOTE    *********************************************
L       LCF.K=LCF.J+(DT)(RRF.JK-SSF.JK)
L       IAF.K=IAF.J+(DT)(SRF.JK-SSF.JK)
A       STF.K=(LCF.K)/(DFF.K)
A       NIF.K=(IAF.K)/DT
R       SSF.KL=CLIP(STF.K,NIF.K,NIF.K,STF.K)
A       DFF.K=DHF+MPF.K/IAF.K
A       MPF.K=(DUF)(IDF.K)
A       IDF.K=(AIF)(RSF.K)
L       RSF.K=RSF.J+(DT/DRF)(RRF.JK-RSF.J)
A       MWF.K=RRF.JK+(1/DIF)(A3.K+B3.K+C3.K)
A       A3.K=IDF.K-IAF.K
A       B3.K=LDF.K-LAF.K
A       C3.K=LCF.K-UNF.K
R       MDF.KL=CLIP(MWF.K,ALF,ALF,MWF.K)
A       LDF.K=(RSF.K)(DCF+DPF)
A       LAF.K=CPF.K+CFF.K
A       UNF.K=(RSF.K)(DHF+DUF)
L       CPF.K=CPF.J+(DT)(MDF.JK-MCF.JK)
R       MCF.KL=DELAY3(MDF.JK,DCF)
L       CPF.K=CPF.J+(DT)(MOF.JK-SRF.JK)
R       SRF.KL=DELAY3(MCF.JK,DPF)
NOTE    *********************************************
NOTE    INITIAL CONDITIONS
NOTE    *********************************************
N       RRR=RRI
N       LCR=(RSR)(DHR+DLR)
N       IAR=(AIR)(RSR)
N       RSR=RRR
N       CPR=(DCR)(RRR)
N       PMR=(DMR)(RRR)
N       MTR=(DTR)(RRR)
N       RRD=RRR
N       LCD=(RSD)(DHD+DLD)
N       IAD=(AID)(RSD)
N       RSD=RRD
N       CPD=(DCD)(RRD)
N       PMD=(DMD)(RRD)
N       MTD=(DTD)(RRD)
N       RRF=RRR
```

```
N      UCF=(RSF)(CHF+CUF)
N      IAF=(AIF)(RSF)
N      RSF=RRF
N      CPF=(DCF)(RRF)
N      CPF=(DPF)(RRF)
NOTE   ***************************************************
NOTE   PARAMETERS(CONSTANTS) OF THE SYSTEM
NOTE   ***************************************************
C      AID=6 WEEKS
C      AIF=4 WEEKS
C      AIR=8 WEEKS
C      ALF=1000000
C      DCD=2 WEEKS
C      DCF=1 WEEK
C      DCR=3 WEEKS
C      DHD=1.0 WEEK
C      DHF=1.0 WEEK
C      DHR=1.0 WEEK
C      DID=4 WEEKS
C      DIF=4 WEEKS
C      DIR=4 WEEKS
C      DMD=0.5 WEEK
C      DMR=0.5 WEEK
C      DRD=8 WEEKS
C      DRF=8 WEEKS
C      DRR=8 WEEKS
C      DPF=6.0 WEEKS
C      DTD=2.0 WEEKS
C      DTR=1.0 WEEK
C      DUD=0.6 WEEK
C      DUF=1.0 WEEK
C      DUR=0.4 WEEK
NOTE   INPUT TEST FUNCTIONS
NOTE   ***************************************************
NOTE   ***************************************************
1R     RRR.KL=RRI+RCR.K
2A     RCR.K=STEP(150,4)
C      RRI=1000
NOTE   ***************************************************
NOTE   SUPPLEMENTARY EQUATIONS
NOTE   ***************************************************
S      TIS.K=IAR.K+IAD.K+IAF.K
NOTE   ***************************************************
NOTE   PRINTING & PLOTTING INSTRUCTIONS
NOTE   ***************************************************
PRINT  1)RRR/2)RRD/3)RRF/4)MOF/5)UCF/6)IAF/7)IAR/8)IAD/9)SRF/10)DFF/11)TI
X1     S
PLOT   RRR=R,RRD=D,RRF=F,MOF=M,SRF=S(0,2000)/IAF=A,UCF=U,IAD=D,IAR=B(0,10
X1     000)/DFF=*(0,10)
NOTE   ***************************************************
```

```
SPEC  CT=C.C5/LENGTH=EC/PLTPER=1/PRTPER=C
RUN   1
```

APPENDIX III

DYNAMO MODELS OF ADVERTISING

```
*       STD. AD MOCEL
NCTE    FCR THE COMPLETE DESCRIPTION OF THE ACVERTISING MODEL
NOTE    AND PRCDUCTICN-CISTRIBUTICN SYSTEM SEE:
NCTE         J.W. FORRESTER:
NOTE         "INDUSTRIAL DYNAMICS," THE M.I.T. PRESS,
NOTE         CAMBRIDGE,MASS., 1961; CHAPTERS 2, 15, & 16
NCTE
NCTE    ******************************************************
NCTE    ECUATICNS FOR RETAIL SECTOR
NOTE    ******************************************************
L       UCR.K=UOR.J+(CT)(RRR.JK-SSR.JK)
L       IAR.K=IAR.J+(CT)(SRR.JK-SSR.JK)
A       STR.K=(UOR.K)/(CFR.K)
A       NIR.K=(IAR.K)/CT
R       SSR.KL=CLIP(STR.K,NIR.K,NIR.K,STR.K)
A       CFR.K=CHR+MNR.K/IAR.K
A       MNR.K=(DUR)(ICR.K)
A       ICR.K=(AIR)(RSR.K)
L       RSR.K=RSR.J+(CT/DRR)(RRR.JK-RSR.J)
R       PCR.KL=RRR.JK+(1/DIR)(A1.K+B1.K+C1.K)
A       A1.K=ICR.K-IAR.K
A       B1.K=LDR.K-LAR.K
A       C1.K=LCR.K-UNR.K
A       LCR.K=(RSR.K)(CCR+DFD.K+DTR)
A       LAR.K=CPR.K+UCC.K+MTR.K
A       UNR.K=(RSR.K)(CHR+DUR)
L       CPR.K=CPR.J+(CT)(PDR.JK-RRD.JK)
R       RRD.KL=DELAY3(PCR.JK,CCR)
L       MTR.K=MTR.J+(CT)(SSD.JK-SRR.JK)
R       SRR.KL=DELAY3(SSD.JK,DTR)
NCTE    ******************************************************
NOTE    ECUATIONS FOR THE DISTRIBUTICN SECTICN
NOTE    ******************************************************
L       UCD.K=UOD.J+(CT)(RRD.JK-SSD.JK)
L       IAD.K=IAD.J+(CT)(SRD.JK-SSD.JK)
A       STD.K=(UOD.K)/(CFD.K)
A       NID.K=(IAD.K)/CT
R       SSD.KL=CLIP(STC.K,NID.K,NID.K,STD.K)
A       DFD.K=DHD+MOD.K/IAD.K
A       MCD.K=(DUD)(ICC.K)
A       ICC.K=(AID)(RSC.K)
L       RSD.K=RSD.J+(CT/DRD)(RRD.JK-RSD.J)
R       PCD.KL=RRD.JK+(1/DID)(A2.K+B2.K+C2.K)
A       A2.K=ICC.K-IAC.K
A       B2.K=LDD.K-LAC.K
A       C2.K=LOD.K-UNC.K
A       LCD.K=(RSD.K)(CCD+DFF.K+DTD)
A       LAD.K=CPD.K+UCF.K+MTD.K
A       UND.K=(RSC.K)(CHC+DUD)
L       CPD.K=CPD.J+(DT)(PDD.JK-RRF.JK)
```

```
R       RRF.KL=DELAY3(PCD.JK,DCD)
L       MTD.K=MTD.J+(DT)(SSF.JK-SRD.JK)
R       SRD.KL=DELAY3(SSF.JK,DTD)
NOTE    ******************************************************
NOTE    EQUATIONS FOR THE FACTORY SECTOR
NOTE    ******************************************************
L       UOF.K=UOF.J+(DT)(RRF.JK-SSF.JK)
L       IAF.K=IAF.J+(DT)(SRF.JK-SSF.JK)
A       STF.K=(UOF.K)/(DFF.K)
A       NIF.K=(IAF.K)/DT
R       SSF.KL=CLIP(STF.K,NIF.K,NIF.K,STF.K)
A       DFF.K=DHF+MPF.K/IAF.K
A       MPF.K=(DUF)(IDF.K)
A       IDF.K=(AIF)(RSF.K)
L       RSF.K=RSF.J+(DT/DRF)(RRF.JK-RSF.J)
A       MWF.K=RRF.JK+(1/DIF)(A3.K+B3.K+C3.K)
A       A3.K=IDF.K-IAF.K
A       B3.K=LDF.K-LAF.K
A       C3.K=LOF.K-UNF.K
R       MDF.KL=CLIP(MWF.K,ALF,ALF,MWF.K)
A       LDF.K=(RSF.K)(DCF+DPF)
A       LAF.K=CPF.K+OPF.K
A       UNF.K=(RSF.K)(DHF+DUF)
L       CPF.K=CPF.J+(DT)(MDF.JK-MOF.JK)
R       MOF.KL=DELAY3(MDF.JK,DCF)
L       OPF.K=OPF.J+(DT)(MOF.JK-SRF.JK)
R       SRF.KL=DELAY3(MDF.JK,DPF)
NOTE    ******************************************************
NOTE    INITIAL CONDITIONS
NOTE    ******************************************************
N       UOR=(RSR)(DHR+DLR)
N       IAR=(AIR)(RSR)
N       RSR=RRR
N       CPR=(DCR)(RRI)
N       PDR=RRR
N       MTR=(DTR)(RRI)
N       SSD=RRD
N       UOD=(RSD)(DHD+DLD)
N       IAD=(AID)(RSD)
N       RSD=RRD
N       CPD=(DCD)(RRI)
N       PCD=RRD
N       MTD=(DTD)(RRI)
N       SSF=RRF
N       MOF=RRF
N       UOF=(RSF)(DHF+DLF)
N       IAF=(AIF)(RSF)
N       RSF=RRF
N       CPF=(DCF)(RRI)
N       OPF=(DPF)(RRI)
```

```
NOTE   ****************************************************
NOTE   PARAMETERS(CONSTANTS) OF THE SYSTEM
NOTE   ****************************************************
C      AID=6 WEEKS
C      AIF=4 WEEKS
C      AIR=8 WEEKS
C      ALF=1400
C      DCD=2.5
C      DCF=1 WEEK
C      DCR=3.5
C      DHD=1.0 WEEK
C      DHF=1.0 WEEK
C      DHR=1.0 WEEK
C      DID=8 WEEKS
C      DIF=8 WEEKS
C      DIR=8 WEEKS
C      DRD=8 WEEKS
C      DRF=8 WEEKS
C      DRR=8 WEEKS
C      DTD=2.0 WEEKS
C      DPF=6.0 WEEKS
C      DTR=1.0 WEEK
C      DUD=0.6 WEEK
C      DUF=1.0 WEEK
C      DUR=0.4 WEEK
NOTE   ****************************************************
NOTE   DYNAMIC MODEL OF INDUSTRIAL SYSTEM WHEN ADVERTISING
NOTE   IS INTRODUCED
NOTE   ****************************************************
1R     VDF.KL=(MAF.K)(LPF)(AVS)
2L     MAF.K=MAF.J+(DT/DMS)(MOF.JK-MAF.J)
3R     VCF.KL=DELAY3(VDF.JK,DVF)
4R     VMC.KL=DELAY3(VCF.JK,DVA)
5L     VAC.K=VAC.J+(DT/DVC)(VMC.JK-VAC.J)
6A     DPC.K=DSV+(DZV-DSV)(EXP(-VAC.K/ASL))
7R     RRR.KL=(PPC.K/DPC.K)(1+NPR.K)
8L     PPC.K=PPC.J+(DT)(GNC.JK-RRR.JK)
9R     GNC.KL=GNI+CNC.K
L      TOTS.K=TOTS.J+DT*SALE.J
A      SALE.K=UPF*RRR.JK
NOTE   ****************************************************
NOTE   INPUT TEST FUNCTIONS
NOTE   ****************************************************
12A    CNC.K=STEP(STH,STT)
10A    NPR.K=SAMPLE(NNR.K,ANL,0.0)
11A    NNR.K=NORMRN(0.0,ADN)
NOTE   ****************************************************
NOTE   INITIAL CONDITIONS
NOTE   ****************************************************
15N    MAF=RRF
```

```
16N    VCF=(MAF)(UPF)(AVS)
17N    VAC=VMC
18N    ASL=VAC
19N    DPC=DSV+(DZV-CSV)(EXP(-VAC/ASL))
2CN    PPC=(RRR)(DPC)
21N    RRR=GNI
N      RRI=GNI
N      TOTS=SLZRO
NCTE   **************************************************
NCTE   SUPPLEMENTARY ECUATIONS
NOTE   **************************************************
S      PLT1.K=IAF.K-ICF.K
S      TIS.K=IAR.K+IAC.K+IAF.K
NOTE   **************************************************
NCTE   PARAMETERS(CCNSTANTS) CF ADVERTISING SECTICN
NCTE   **************************************************
C      UPF=1CC $,UNIT FRICE AT FACTCRY
C      AVS=0.1C DIMENSIONLESS,% CF SALES GOING TC ACVERTISING
C      DMS=4 WEEKS
C      DSV=15 WEEKS
C      DVA=2 WEEKS
C      CVC=3 WEEKS
C      CVF=3 WEEKS
C      DZV=7C WEEKS
C      GNI=1CCO UNITS/WEEKS
C      SLZRO=0.0 INITIAL ACCUMULATICN OF SALES
NCTE   INPUT-TEST-FUNCTION CONSTANTS
C      ACN=0.1
C      STH=1CO
C      STT=4
C      ANL=1 WEEK
NOTE   **************************************************
NCTE   PRINTING & PLCTTING INSTRUCTICNS
NCTE   **************************************************
PRINT  1)SALE/2)TCTS
PLOT   RRR=R,RRF=F(C,2CCO)/VAC=Z,VMC=X(0,20CCO)/IAR=A,IAF=.(0,10000)/MOF=
X1     O,SRF=+(0,20CC)
PLOT   MAF=G,GNC=I(C,15CO)/VMC=X(0,20000)/RRR=R,RRF=F(0,2000)
PLOT   SALE=$/TOTS=T
NOTE   **************************************************
SPEC   DT=0.10/LENGTH=104/PLTPER=1/PRTPER=1
RUN    STD.
```

```
*       A-TYPE AD MODEL
NOTE         THE FOLLOWING CODES ARE USED TO DESCRIBE THE
NOTE         DIFFERENT ADVERTISING MODELS:
NOTE         A-TYPE : PROTRACTED ADVERTISING
NOTE         B-TYPE : SHORT-TIME, INTENSIVE ADVERTISING
NOTE         C-TYPE : IMPULSE-TYPE ADVERTISING
NOTE    FOR THE MATHEMATICAL DESCRIPTION OF THE MODEL SEE:
NOTE         CHAPTER 5 OF THIS REPORT, AND ALSO:
NOTE         VIDALE,M.L., AND H.B. WOLFE:
NOTE         "AN OPERATIONS REASEARCH STUDY OF SALES RESPONSE
NOTE         TO ADVERTISING, "OPERATIONS RESEARCH,  VOL.15,
NOTE         NO.3, JUNE 1957.
NOTE
NOTE    FOR THE COMPLETE DESCRIPTION OF THE PRODUCTION-
NOTE    DISTRIBUTION MODEL SEE:
NOTE         J.W. FORRESTER:
NOTE         "INDUSTRIAL DYNAMICS," THE M.I.T. PRESS,
NOTE         CAMBRIDGE,MASS., 1961; CHAPTERS 2 & 15
NOTE    ***************************************************
NOTE    EQUATIONS FOR RETAIL SECTOR
NOTE    ***************************************************
L       UOR.K=UOR.J+(DT)(RRR.JK-SSR.JK)
L       IAR.K=IAR.J+(DT)(SRR.JK-SSR.JK)
A       STR.K=(UOR.K)/(DFR.K)
A       NIR.K=(IAR.K)/DT
R       SSR.KL=CLIP(STR.K,NIR.K,NIR.K,STR.K)
A       DFR.K=DHR+MNR.K/IAR.K
A       MNR.K=(DUR)(IDR.K)
A       IDR.K=(AIR)(RSR.K)
L       RSR.K=RSR.J+(DT/DRR)(RRR.JK-RSR.J)
R       PDR.KL=RRR.JK+(1/DIR)(A1.K+B1.K+C1.K)
A       A1.K=IDR.K-IAR.K
A       B1.K=LDR.K-LAR.K
A       C1.K=UOR.K-UNR.K
A       LDR.K=(RSR.K)(DCR+DFD.K+DTR)
A       LAR.K=CPR.K+UOD.K+MTR.K
A       UNR.K=(RSR.K)(DHR+DUR)
L       CPR.K=CPR.J+(DT)(PDR.JK-RRC.JK)
R       RRC.KL=DELAY3(PDR.JK,DCR)
L       MTR.K=MTR.J+(DT)(SSD.JK-SRR.JK)
R       SRR.KL=DELAY3(SSD.JK,DTR)
NOTE    ***************************************************
NOTE    EQUATIONS FOR THE DISTRIBUTION SECTION
NOTE    ***************************************************
L       UOD.K=UOD.J+(DT)(RRD.JK-SSD.JK)
L       IAD.K=IAD.J+(DT)(SRD.JK-SSD.JK)
A       STD.K=(UOD.K)/(DFD.K)
A       NID.K=(IAD.K)/DT
R       SSD.KL=CLIP(STD.K,NID.K,NID.K,STD.K)
A       DFD.K=DHD+MOD.K/IAD.K
```

```
A       MDD.K=(DUD)(IDD.K)
A       IDD.K=(AID)(RSD.K)
L       RSD.K=RSD.J+(DT/DRD)(RRD.JK-RSD.J)
R       PDD.KL=RRD.JK+(1/DID)(A2.K+B2.K+C2.K)
A       A2.K=IDD.K-IAD.K
A       B2.K=LDD.K-LAD.K
A       C2.K=LDD.K-UND.K
A       LDD.K=(RSD.K)(DCD+DFF.K+DTD)
A       LAD.K=CPD.K+LDF.K+MTD.K
A       UND.K=(RSD.K)(DHD+DUD)
L       CPD.K=CPD.J+(DT)(PDD.JK-RRF.JK)
R       RRF.KL=DELAY3(PDD.JK,DCD)
L       MTD.K=MTD.J+(DT)(SSF.JK-SRD.JK)
R       SRD.KL=DELAY3(SSF.JK,DTD)
NOTE    **************************************************
NOTE    EQUATIONS FOR THE FACTORY SECTOR
NOTE    **************************************************
L       LDF.K=LDF.J+(DT)(RRF.JK-SSF.JK)
L       IAF.K=IAF.J+(DT)(SRF.JK-SSF.JK)
A       STF.K=(LDF.K)/(DFF.K)
A       NIF.K=(IAF.K)/DT
R       SSF.KL=CLIP(STF.K,NIF.K,NIF.K,STF.K)
A       DFF.K=DHF+MPF.K/IAF.K
A       MPF.K=(DUF)(IDF.K)
A       IDF.K=(AIF)(RSF.K)
L       RSF.K=RSF.J+(DT/DRF)(RRF.JK-RSF.J)
A       MWF.K=RRF.JK+(1/DIF)(A3.K+B3.K+C3.K)
A       A3.K=IDF.K-IAF.K
A       B3.K=LDF.K-LAF.K
A       C3.K=LDF.K-UNF.K
R       MDF.KL=CLIP(MWF.K,ALF,ALF,MWF.K)
A       LDF.K=(RSF.K)(DCF+DPF)
A       LAF.K=CPF.K+CFF.K
A       UNF.K=(RSF.K)(DHF+DLF)
L       CPF.K=CPF.J+(DT)(MDF.JK-MCF.JK)
R       MCF.KL=DELAY3(MDF.JK,DCF)
L       CPF.K=CPF.J+(DT)(MCF.JK-SRF.JK)
R       SRF.KL=DELAY3(MCF.JK,DPF)
NOTE    **************************************************
NOTE    INITIAL CONDITIONS
NOTE    **************************************************
N       LDR=(RSR)(DHR+DLR)
N       IAR=(AIR)(RSR)
N       RSR=RRR
N       CPR=(DCR)(RRR)
N       PDR=RRR
N       MTR=(DTR)(RRR)
N       SSD=RRD
N       LDD=(RSD)(DHD+DLD)
N       IAD=(AID)(RSD)
```

```
N      RSC=RRC
N      CPC=(CCC)(RRR)
N      FCC=RRC
N      MTD=(CTC)(RRR)
N      SSF=RRF
N      MCF=RRF
N      LCF=(RSF)(CHF+CLF)
N      IAF=(AIF)(RSF)
N      RSF=RRF
N      CPF=(CCF)(RRR)
N      CPF=(CPF)(RRR)
NCTE   ***************************************************
NCTE   PARAMETERS(CCNSTANTS) CF THE SYSTEM
NCTE   ***************************************************
C      AIC=6 WEEKS
C      AIF=4 WEEKS
C      AIR=8 WEEKS
C      ALF=3CCC
C      CCC=2.5
C      CCF=1 WEEK
C      CCR=3.5
C      CHC=1.C WEEK
C      CHF=1.C WEEK
C      CHR=1.C WEEK
C      CIC=8 WEEKS
C      CIF=8 WEEKS
C      CIR=8 WEEKS
C      CRC=8 WEEKS
C      CRF=8 WEEKS
C      CRR=8 WEEKS
C      CTC=2.C WEEKS
C      CPF=6.C WEEKS
C      CTR=1.C WEEK
C      CLC=C.6 WEEK
C      CLF=1.C WEEK
C      CLR=C.4 WEEK
NCTE   ***************************************************
NCTE   MCDEL CF THE A-TYPE ACVERTISING PRACTICE, CISCUSSEC
NCTE   IN CHAPTER 5.
NCTE   ***************************************************
R      VCF.KL=CLIP(FRA,CAR,TIME.K,TT)
3R     VCF.KL=CELAY3(VCF.JK,CVF)
R      VPP.KL=SAMPLE(AC.K,INTVL,INVAL)
A      AC.K=TABHL(TNAM,TIME.K,TLC,THI,TINCR)
T      TNAM=C/1CCC/1CCC/1CCO/1CCC/1CCC/C 1CCC=SALE*AVS
5L     VAC.K=VAC.J+(CT/CVC)(VPP.JK-VAC.J)
L      SALE.K=SALE.J+(CT)((RC)(VAC.J)(1.0-(SALE.J)/SL)-(SCC)(SALE.J))
R      RRR.KL=(SALE.K)/(LPF)
L      TCTS.K=TOTS.J+CT*SALE.J
NCTE   ***************************************************
```

```
NOTE   INITIAL CONDITIONS
NOTE   **************************************************
N      CAR=(SALE)(AVS)
N      VCF=CAR
17N    VAC=VFP
N      RRR=SALE/UPF
N      SALE=10000 $/WEEK
N      TLC=DVA+DVF-TINCR
N      THI=TT+TLC+TINCR
N      INTVL=TINCR
N      TCTS=SLZRO
NOTE   **************************************************
NOTE   SUPPLEMENTARY EQUATIONS
NOTE   **************************************************
S      PLT1.K=IAF.K-IDF.K
S      TIS.K=IAR.K+IAD.K+IAF.K
NOTE   **************************************************
NOTE   PARAMETERS(CONSTANTS) OF ADVERTISING SECTION
NOTE   **************************************************
C      AVS=0.10 DIMENSIONLESS,% OF SALES GOING TO ADVERTISING
C      TINCR=5
C      INVAL=0.0
C      UPF=100 $,UNIT PRICE AT FACTORY
C      DVA=2 WEEKS
C      DVC=3 WEEKS
C      DVF=3 WEEKS
C      RC=0.5 PER WEEK
C      SL=15000 $/WEEK
C      SDC=0.005 PER WEEK, OR 0.26/YEAR
C      FRA=0.0 FINAL RATE OF ADVERTISING
C      TT=25 WEEKS, APPROXIMATELY 6 MONTHS
C      SLZRO=0.0 INITIAL ACCUMULATION OF SALES
NOTE   **************************************************
NOTE   PRINTING & PLOTTING INSTRUCTIONS
NOTE   **************************************************
PRINT  1)RRR/2)RRF/3)TIS/4)SPF/5)SALE/6)TCTS
PLOT   RRR=R,RRF=F(0,200)/VAC=X,VPP=Z(0,1500)/IAR=A,IAF=.(0,1800)/MCF=0,
X1     RF=+(0,250)/SALE=$(0,15000)
PLOT   SALE=$(0,15000)/TCTS=T(0,1200000)
NOTE   **************************************************
SPEC   DT=0.1/LENGTH=104/PRTPER=1/PLTPER=1
RUN    RS-1.1
```

```
*       B-TYPE AD MODEL
NOTE         THE FOLLOWING CODES ARE USED TO DESCRIBE THE
NOTE         DIFFERENT ADVERTISING MODELS:
NOTE         A-TYPE : PROTRACTED ADVERTISING
NOTE         B-TYPE : SHORT-TIME, INTENSIVE ADVERTISING
NOTE         C-TYPE : IMPULSE-TYPE ADVERTISING
NOTE    FOR THE MATHEMATICAL DESCRIPTION OF THE MODEL SEE:
NOTE         CHAPTER 5 OF THIS REPORT, AND ALSO:
NOTE         VIDALE,M.L., AND H.B. WOLFE:
NOTE         "AN OPERATIONS REASEARCH STUDY OF SALES RESPONSE
NOTE         TO ADVERTISING, "OPERATIONS RESEARCH,  VOL.15,
NOTE         NO.3, JUNE 1957.
NOTE
NOTE    FOR THE COMPLETE DESCRIPTION OF THE PRODUCTION-
NOTE    DISTRIBUTION MODEL SEE:
NOTE         J.W. FORRESTER:
NOTE         "INDUSTRIAL DYNAMICS," THE M.I.T. PRESS,
NOTE         CAMBRIDGE,MASS., 1961; CHAPTERS 2 & 15
NOTE    *****************************************************
NOTE    EQUATIONS FOR RETAIL SECTOR
NOTE    *****************************************************
L       UOR.K=UOR.J+(DT)(RRR.JK-SSR.JK)
L       IAR.K=IAR.J+(DT)(SRR.JK-SSR.JK)
A       STR.K=(UOR.K)/(DFR.K)
A       NIR.K=(IAR.K)/DT
R       SSR.KL=CLIP(STR.K,NIR.K,NIR.K,STR.K)
A       DFR.K=DHR+MNR.K/IAR.K
A       MNR.K=(DUR)(IDR.K)
A       IDR.K=(AIR)(RSR.K)
L       RSR.K=RSR.J+(DT/DRR)(RRR.JK-RSR.J)
R       PDR.KL=RRR.JK+(1/DIR)(A1.K+B1.K+C1.K)
A       A1.K=IDR.K-IAR.K
A       B1.K=LDR.K-LAR.K
A       C1.K=UOR.K-UNR.K
A       LDR.K=(RSR.K)(DCR+DFD.K+DTR)
A       LAR.K=CPR.K+UOD.K+MTR.K
A       UNR.K=(RSR.K)(DHR+DUR)
L       CPR.K=CPR.J+(DT)(PDR.JK-RRD.JK)
R       RRD.KL=DELAY3(PDR.JK,DCR)
L       MTR.K=MTR.J+(DT)(SSD.JK-SRR.JK)
R       SRR.KL=DELAY3(SSD.JK,DTR)
NOTE    *****************************************************
NOTE    EQUATIONS FOR THE DISTRIBUTION SECTION
NOTE    *****************************************************
L       UOD.K=UOD.J+(DT)(RRD.JK-SSD.JK)
L       IAD.K=IAD.J+(DT)(SRD.JK-SSD.JK)
A       STD.K=(UOD.K)/(DFD.K)
A       NID.K=(IAD.K)/DT
R       SSD.KL=CLIP(STD.K,NID.K,NID.K,STD.K)
A       DFD.K=DHD+MOD.K/IAD.K
```

```
A      MCD.K=(CUC)(ICC.K)
A      ICC.K=(AIC)(RSC.K)
L      RSC.K=RSC.J+(CT/CRC)(RRD.JK-RSD.J)
R      PCC.KL=RRC.JK+(1/CIC)(A2.K+B2.K+C2.K)
A      A2.K=ICC.K-IAC.K
A      B2.K=LCC.K-LAC.K
A      C2.K=LCC.K-UNC.K
A      LCD.K=(RSD.K)(CCC+DFF.K+CTC)
A      LAC.K=CPC.K+LCF.K+MTC.K
A      UNC.K=(RSC.K)(CHC+CUC)
L      CPC.K=CPC.J+(CT)(PDC.JK-RRF.JK)
R      RRF.KL=DELAY3(PCC.JK,CCD)
L      MTC.K=MTC.J+(CT)(SSF.JK-SRC.JK)
R      SRC.KL=DELAY3(SSF.JK,CTD)
NCTE   ****************************************************
NCTE   ECUATICNS FCR THE FACTCRY SECTCR
NCTE   ****************************************************
L      LCF.K=LCF.J+(CT)(RRF.JK-SSF.JK)
L      IAF.K=IAF.J+(CT)(SRF.JK-SSF.JK)
A      STF.K=(LCF.K)/(CFF.K)
A      NIF.K=(IAF.K)/CT
R      SSF.KL=CLIP(STF.K,NIF.K,NIF.K,STF.K)
A      CFF.K=CHF+MPF.K/IAF.K
A      MPF.K=(CUF)(ICF.K)
A      ICF.K=(AIF)(RSF.K)
L      RSF.K=RSF.J+(CT/CRF)(RRF.JK-RSF.J)
A      MWF.K=RRF.JK+(1/CIF)(A3.K+B3.K+C3.K)
A      A3.K=ICF.K-IAF.K
A      B3.K=LCF.K-LAF.K
A      C3.K=LCF.K-UNF.K
R      MCF.KL=CLIP(MWF.K,ALF,ALF,MWF.K)
A      LCF.K=(RSF.K)(CCF+DPF)
A      LAF.K=CPF.K+CFF.K
A      UNF.K=(RSF.K)(CHF+CUF)
L      CPF.K=CPF.J+(CT)(MDF.JK-MCF.JK)
R      MCF.KL=DELAY3(MCF.JK,CCF)
L      CFF.K=CPF.J+(CT)(MCF.JK-SRF.JK)
R      SRF.KL=DELAY3(MCF.JK,CPF)
NCTE   ****************************************************
NCTE   INITIAL CCNCITICNS
NCTE   ****************************************************
N      LCR=(RSR)(CHR+CLR)
N      IAR=(AIR)(RSR)
N      RSR=RRR
N      CPR=(CCR)(RRR)
N      PCR=RRR
N      MTR=(CTR)(RRR)
N      SSD=RRC
N      LCD=(RSD)(CHC+CLC)
N      IAD=(AIC)(RSC)
```

```
N     RSC=RRC
N     CPC=(CCC)(RRR)
N     PCC=RRC
N     MTC=(CTC)(RRR)
N     SSF=RRF
N     MCF=RRF
N     LCF=(RSF)(CHF+CLF)
N     IAF=(AIF)(RSF)
N     RSF=RRF
N     CPF=(DCF)(RRR)
N     CPF=(CPF)(RRR)
NCTE  ****************************************************
NCTE  PARAMETERS(CONSTANTS) CF THE SYSTEM
NCTE  ****************************************************
C     AIC=6 WEEKS
C     AIF=4 WEEKS
C     AIR=8 WEEKS
C     ALF=3CCC
C     CCC=2.5
C     CCF=1 WEEK
C     CCR=3.5
C     CHC=1.C WEEK
C     CHF=1.C WEEK
C     CHR=1.C WEEK
C     CID=8 WEEKS
C     CIF=8 WEEKS
C     CIR=8 WEEKS
C     CRC=8 WEEKS
C     CRF=8 WEEKS
C     CRR=8 WEEKS
C     CTC=2.C WEEKS
C     CPF=6.C WEEKS
C     CTR=1.C WEEK
C     CUC=C.6 WEEK
C     CLF=1.C WEEK
C     CLR=C.4 WEEK
NCTE  ****************************************************
NCTE  MCDEL CF THE B-TYPE ACVERTISING PRACTICE, CISCUSSEC
NCTE  IN CHAPTER 5.
NCTE  ****************************************************
R     VCF.KL=CLIP(FRA,SAR,TIME.K,T)
3R    VCF.KL=CELAY3(VCF.JK,CVF)
R     VPP.KL=TABHL(TNAME,TIME.K,TLC,THI,TINCR)
T     TNAME=C.CCC/25CCC/25CCC/25CCC/25CCC/250CC/25CCC/25CCC/250CC/25C00
X1    25COC/25CCC/C.CCC  25CCO=SALE*AVS*TT
L     VAC.K=VAC.J+(CT/CVC)(VPP.JK-VAC.J)
L     SALE.K=SALE.J+(CT)((RC)(VAC.J)(1.0-(SALE.J)/SL)-(SCC)(SALE.J))
R     FRR.KL=(SALE.K)/(LPF)
L     TCTS.K=TCTS.J+CT*SALE.J
NCTE  ****************************************************
```

```
NOTE   INITIAL CONDITIONS
NOTE   ****************************************************
N      VCF=SAR
N      SAR=(SALE)(AVS)(TT)
N      VAC=VFP
N      RRR=SALE/UPF
N      SALE=10000 $/WEEK
N      TLC=T+DVF+DVA-DT
N      THI=TLC+T+2*DT
N      TINCR=DT
N      TOTS=SLZRO
NOTE   SUPPLEMENTARY EQUATIONS
NOTE   ****************************************************
NOTE   ****************************************************
S      PLT1.K=IAF.K-IDF.K
S      TIS.K=IAR.K+IAD.K+IAF.K
NOTE   ****************************************************
NOTE   PARAMETERS(CONSTANTS) OF ADVERTISING SECTION
NOTE   ****************************************************
C      AVS=0.10 DIMENSIONLESS,% OF SALES GOING TO ADVERTISING
C      UPF=100 $,UNIT PRICE AT FACTORY
C      DVA=2 WEEKS
C      DVC=3 WEEKS
C      DVF=3 WEEKS
C      RC=0.5 PER WEEK
C      SL=15000 $/WEEK
C      SDC=0.005 PER WEEK, OR 0.26/YEAR
C      FRA=0.0 FINAL RATE OF ADVERTISING
C      T=1 WEEK; STOPPING TIME FOR AD CAMPAIGN
C      TT=25 WEEKS,CAMPAIGN PERIOD IN PREVIOUS EXAMPLE
C      SLZRO=0.0 INITIAL ACCUMULATION OF SALES
NOTE   ****************************************************
NOTE   PRINTING & PLOTTING INSTRUCTIONS
NOTE   ****************************************************
PRINT  1)RRR/2)RRF/3)TIS/4)SRF/5)SALE/6)TOTS
PLOT   RRR=R,RRF=F(0,200)/VAC=X,VPF=Z(0,40000)/IAR=A,IAF=.(0,1800)/MOF=U
X1     SRF=+(0,250)/SALE=$(0,15000)
PLOT   SALE=$(0,15000)/TOTS=T(0,1200000)
NOTE   ****************************************************
SPEC   DT=0.1/LENGTH=104/PRTPER=1/PLTPER=1
RUN    RS-2.1
```

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*       C-TYPE AC MCCEL
NCTE          THE FCLLCWING CCCES ARE USED TC CESCRIBE THE
NCTE          CIFFERENT ACVERTISING MCCELS:
NCTE          A-TYPE : PRCTRACTED ACVERTISING
NCTE          B-TYPE : SHORT-TIME, INTENSIVE ACVERTISING
NCTE          C-TYPE : IMPULSE-TYPE ACVERTISING
NCTE    FCR THE MATHEMATICAL CESCRIPTICN CF THE MCCEL SEE:
NCTE          CHAPTER 5 CF THIS REPORT, ANC ALSC:
NCTE          VICALE,M.L., ANC H.B. WCLFE:
NCTE          "AN CPERATICNS REASEARCH STUCY CF SALES RESPCNSE
NCTE          TC ACVERTISING, "CPERATICNS RESEARCH,  VCL.15,
NCTE          NC.3, JUNE 1957.
NCTE
NCTE    FCR THE CCMPLETE CESCRIPTICN CF THE PRCCUCTICN-
NCTE    CISTRIBUTICN MCCEL SEE:
NCTE          J.W. FORRESTER:
NCTE          "INCUSTRIAL CYNAMICS," THE M.I.T. PRESS,
NCTE          CAMBRICGE,MASS., 1961; CHAPTERS 2 & 15
NCTE    ******************************************************
NCTE    ECUATICNS FCR RETAIL SECTCR
NCTE    ******************************************************
L       UCR.K=UCR.J+(CT)(RRR.JK-SSR.JK)
L       IAR.K=IAR.J+(CT)(SRR.JK-SSR.JK)
A       STR.K=(UCR.K)/(CFR.K)
A       NIR.K=(IAR.K)/CT
R       SSR.KL=CLIP(STR.K,NIR.K,NIR.K,STR.K)
A       CFR.K=CHR+MNR.K/IAR.K
A       MNR.K=(CUR)(ICR.K)
A       ICR.K=(AIR)(RSR.K)
L       RSR.K=RSR.J+(CT/CRR)(RRR.JK-RSR.J)
R       PCR.KL=RRR.JK+(1/CIR)(A1.K+B1.K+C1.K)
A       A1.K=ICR.K-IAR.K
A       B1.K=LCR.K-LAR.K
A       C1.K=UCR.K-UNR.K
A       LCR.K=(RSR.K)(CCR+CFC.K+CTR)
A       LAR.K=CPR.K+UCC.K+MTR.K
A       UNR.K=(RSR.K)(CHR+DLR)
L       CPR.K=CPR.J+(CT)(PCR.JK-RRC.JK)
R       RRC.KL=CELAY3(PCR.JK,CCR)
L       MTR.K=MTR.J+(CT)(SSC.JK-SRR.JK)
R       SRR.KL=CELAY3(SSD.JK,CTR)
NCTE    ******************************************************
NCTE    ECUATICNS FOR THE CISTRIBUTICN SECTICN
NCTE    ******************************************************
L       UCC.K=UCC.J+(CT)(RRC.JK-SSC.JK)
L       IAC.K=IAC.J+(CT)(SRC.JK-SSC.JK)
A       STD.K=(UCC.K)/(CFD.K)
A       NIC.K=(IAC.K)/CT
R       SSC.KL=CLIP(STC.K,NIC.K,NIC.K,STC.K)
A       CFD.K=CHC+MOC.K/IAC.K
```

```
A      MCC.K=(CUC)(ICC.K)
A      ICC.K=(AIC)(RSC.K)
L      RSD.K=RSD.J+(CT/CPC)(RRD.JK-RSC.J)
R      PCC.KL=RRC.JK+(1/DID)(A2.K+B2.K+C2.K)
A      A2.K=ICC.K-IAC.K
A      B2.K=LCC.K-LAC.K
A      C2.K=LCC.K-UNC.K
A      LCC.K=(RSC.K)(CCC+DFF.K+CTC)
A      LAC.K=CPC.K+LCF.K+MTC.K
A      UNC.K=(RSC.K)(CHC+DLC)
L      CPC.K=CPC.J+(CT)(PDC.JK-RRF.JK)
R      RRF.KL=CELAY3(PCC.JK,CCD)
L      MTD.K=MTD.J+(CT)(SSF.JK-SRC.JK)
R      SRC.KL=CELAY3(SSF.JK,CTC)
NCTE   ******************************************************
NCTE   ECUATICNS FCR THE FACTCRY SECTCR
NCTE   ******************************************************
L      UCF.K=UCF.J+(CT)(PRF.JK-SSF.JK)
L      IAF.K=IAF.J+(CT)(SRF.JK-SSF.JK)
A      STF.K=(UCF.K)/(CFF.K)
A      NIF.K=(IAF.K)/CT
R      SSF.KL=CLIP(STF.K,NIF.K,NIF.K,STF.K)
A      CFF.K=CHF+MPF.K/IAF.K
A      MFF.K=(CUF)(ICF.K)
A      ICF.K=(AIF)(RSF.K)
L      RSF.K=RSF.J+(CT/CRF)(RRF.JK-RSF.J)
A      MWF.K=RRF.JK+(1/CIF)(A3.K+B3.K+C3.K)
A      A3.K=ICF.K-IAF.K
A      B3.K=LCF.K-LAF.K
A      C3.K=LCF.K-UNF.K
R      MCF.KL=CLIP(MWF.K,ALF,ALF,MWF.K)
A      LCF.K=(RSF.K)(CCF+DPF)
A      LAF.K=CPF.K+CPF.K
A      UNF.K=(RSF.K)(CHF+CUF)
L      CPF.K=CPF.J+(CT)(MDF.JK-MCF.JK)
R      MCF.KL=CELAY3(MCF.JK,CCF)
L      CPF.K=CPF.J+(CT)(MCF.JK-SRF.JK)
R      SRF.KL=CELAY3(MCF.JK,CPF)
NCTE   ******************************************************
NCTE   INITIAL CCNCITICNS
NCTE   ******************************************************
N      UCR=(RSR)(CHR+CLR)
N      IAR=(AIR)(RSR)
N      RSR=RRR
N      CPR=(CCR)(RRR)
N      PCR=RRR
N      MTR=(CTR)(RRR)
N      SSC=RRC
N      LCD=(RSC)(CHC+CLC)
N      IAC=(AIC)(RSC)
```

```
N      RSD=RRD
N      CPD=(DCD)(RRR)
N      PDD=RRD
N      MTD=(DTD)(RRR)
N      SSF=RRF
N      MDF=RRF
N      LCF=(RSF)(DHF+DLF)
N      IAF=(AIF)(RSF)
N      RSF=RRF
N      CPF=(DCF)(RRR)
N      CPF=(DPF)(RRR)
NOTE   **************************************************
NOTE   PARAMETERS(CONSTANTS) OF THE SYSTEM
NOTE   **************************************************
C      AID=6 WEEKS
C      AIF=4 WEEKS
C      ALF=3000
C      AIR=8 WEEKS
C      DCD=2.5
C      DCF=1 WEEK
C      DCR=2.5
C      DHD=1.0 WEEK
C      DHF=1.0 WEEK
C      DHR=1.0 WEEK
C      DID=8 WEEKS
C      DIF=8 WEEKS
C      DIR=8 WEEKS
C      DRD=8 WEEKS
C      DRF=8 WEEKS
C      DRR=8 WEEKS
C      DTD=2.0 WEEKS
C      DPF=6.0 WEEKS
C      DTR=1.0 WEEK
C      DLD=0.6 WEEK
C      DLF=1.0 WEEK
C      DLR=0.4 WEEK
NOTE   **************************************************
NOTE   MODEL OF THE C-TYPE ADVERTISING PRACTICE, DISCUSSED
NOTE   IN CHAPTER 5.
NOTE   **************************************************
R      VPP.KL=PULSE(MONEY,FIRST,INTVL)
L      VAC.K=VAC.J+(DT/DVC)(VPP.JK-VAC.J)
L      SALE.K=SALE.J+(DT)((RC)(VAC.J)(1.0-(SALE.J)/SL)-(SDC)(SALE.J))
R      RRR.KL=(SALE.K)/(UPF)
L      TOTS.K=TOTS.J+DT*SALE.J
NOTE   **************************************************
NOTE   INITIAL CONDITIONS FOR ADVERTISING SECTION
NOTE   **************************************************
N      VAC=VPP
N      RRR=SALE/UPF
```

```
N      SALE=1CCCO $/WEEK
N      MCNEY=(SALE)(AVS)(TT)/(NC*CT)
N      FIRST=CVA+CVF
N      TCTS=SLZRC
NCTE   ****************************************************
NCTE   SUPPLEMENTARY ECUATICNS
NCTE   ****************************************************
S      PLT1.K=IAF.K-ICF.K
S      TIS.K=IAR.K+IAC.K+IAF.K
NCTE   ****************************************************
NCTE   PARAMETERS(CCNSTANTS) CF ACVERTISING SECTICN
NCTE   ****************************************************
C      AVS=C.1C CIMENSIONLESS,% CF SALES GCING TC ACVERTISING
C      UPF=1CC $,UNIT PRICE AT FACTCRY
C      CVA=2 WEEKS
C      DVC=3 WEEKS
C      CVF=3 WEEKS
C      RC=C.5 PER WEEK
C      SL=15CCO $/WEEK
C      SDC=C.CC5 PER WEEK, CR C.26/YEAR
C      TT=25 CAMPAIGN PERIOD IN CCNSTANT ACVERTISING
C      SLZRC=C.0 INITIAL ACCUMULATICN CF SALES
C      INTVL=16 WEEKS, APPRCXIMATELY 4 MCNTHS
C      NC=7 NC. CF CAMPAIGNS
NCTE   ****************************************************
NCTE   PRINTING & PLCTTING INSTRUCTICNS
NCTE   ****************************************************
PRINT  1)SALE/2)TCTS
PLOT   RPR=R,RRF=F(C,2CO)/VPP=Z(C,4CCCC)/VAC=X(O,1CCO)/IAR=A,IAF=.(C,160
X1     )/MOF=C,SRF=+(C,25C)/SALE=$(C,15CCO)
PLOT   SALE=$(C,15CCC)/TCTS=T(O,12CCCCC)
NCTE   ****************************************************
SPEC   CT=C.1/LENGTH=1C4/PRTPER=1/PLTPER=1
RUN    RS-3.1
```

```
*       A-TYPE AD MODEL
NOTE         THE FOLLOWING CODES ARE USED TO DESCRIBE THE
NOTE         DIFFERENT ADVERTISING MODELS:
NOTE         A-TYPE : PROTRACTED ADVERTISING
NOTE         B-TYPE : SHORT-TIME, INTENSIVE ADVERTISING
NOTE         C-TYPE : IMPULSE-TYPE ADVERTISING
NOTE    THE FOLLOWING MODEL ASSUMES THAT SALES AT RETAIL
NOTE    HAVE A RANDOM FLUCTUATIONS, WITH A MEAN OF 0.0,
NOTE    AND STANDARD DEVIATION OF 0.05.
NOTE    FOR THE MATHEMATICAL DESCRIPTION OF THE MODEL SEE:
NOTE         CHAPTER 5 OF THIS REPORT, AND ALSO:
NOTE         VIDALE,M.L., AND H.B. WOLFE:
NOTE         "AN OPERATIONS REASEARCH STUDY OF SALES RESPONSE
NOTE         TO ADVERTISING, "OPERATIONS RESEARCH,  VOL.15,
NOTE         NO.3, JUNE 1957.
NOTE
NOTE    FOR THE COMPLETE DESCRIPTION OF THE PRODUCTION-
NOTE    DISTRIBUTION MODEL SEE:
NOTE         J.W. FORRESTER:
NOTE         "INDUSTRIAL DYNAMICS," THE M.I.T. PRESS,
NOTE         CAMBRIDGE,MASS., 1961; CHAPTERS 2 & 15
NOTE    ***************************************************
NOTE    EQUATIONS FOR RETAIL SECTOR
NOTE    ***************************************************
L       UOR.K=UOR.J+(DT)(RRR.JK-SSR.JK)
L       IAR.K=IAR.J+(DT)(SRR.JK-SSR.JK)
A       STR.K=(UOR.K)/(DFR.K)
A       NIR.K=(IAR.K)/DT
R       SSR.KL=CLIP(STR.K,NIR.K,NIR.K,STR.K)
A       DFR.K=DHR+MNR.K/IAR.K
A       MNR.K=(DUR)(IDR.K)
A       IDR.K=(AIR)(RSR.K)
L       RSR.K=RSR.J+(DT/DRR)(RRR.JK-RSR.J)
R       PDR.KL=RRR.JK+(1/DIR)(A1.K+B1.K+C1.K)
A       A1.K=IDR.K-IAR.K
A       B1.K=LDR.K-LAR.K
A       C1.K=UOR.K-UNR.K
A       LDR.K=(RSR.K)(DCR+DFD.K+DTR)
A       LAR.K=CPR.K+UOD.K+MTR.K
A       UNR.K=(RSR.K)(DHR+DUR)
L       CPR.K=CPR.J+(DT)(PDR.JK-RRD.JK)
R       RRD.KL=DELAY3(PDR.JK,DCR)
L       MTR.K=MTR.J+(DT)(SSD.JK-SRR.JK)
R       SRR.KL=DELAY3(SSD.JK,DTR)
NOTE    ***************************************************
NOTE    EQUATIONS FOR THE DISTRIBUTION SECTION
NOTE    ***************************************************
L       UOD.K=UOD.J+(DT)(RRD.JK-SSD.JK)
L       IAD.K=IAD.J+(DT)(SRD.JK-SSD.JK)
A       STD.K=(UOD.K)/(DFD.K)
```

```
A      NID.K=(IAD.K)/DT
R      SSD.KL=CLIP(STD.K,NID.K,NID.K,STD.K)
A      DFD.K=DHD+MOD.K/IAD.K
A      MOD.K=(DUD)(IDD.K)
A      IDD.K=(AID)(RSD.K)
L      RSD.K=RSD.J+(DT/DRD)(RRD.JK-RSD.J)
R      PDD.KL=RRD.JK+(1/DID)(A2.K+B2.K+C2.K)
A      A2.K=IDD.K-IAD.K
A      B2.K=LDD.K-LAD.K
A      C2.K=UDD.K-UND.K
A      LDD.K=(RSD.K)(DCD+DFF.K+DTD)
A      LAD.K=CPD.K+UOF.K+MTD.K
A      UND.K=(RSD.K)(DHD+DUD)
L      CPD.K=CPD.J+(DT)(PDD.JK-RRF.JK)
R      RRF.KL=DELAY3(PDD.JK,DCD)
L      MTD.K=MTD.J+(DT)(SSF.JK-SRD.JK)
R      SRD.KL=DELAY3(SSF.JK,DTD)
NCTE   ************************************************************
NCTE   EQUATICNS FCR THE FACTCRY SECTCR
NCTE   ************************************************************
L      UOF.K=UOF.J+(DT)(RRF.JK-SSF.JK)
L      IAF.K=IAF.J+(DT)(SRF.JK-SSF.JK)
A      STF.K=(UOF.K)/(DFF.K)
A      NIF.K=(IAF.K)/DT
R      SSF.KL=CLIP(STF.K,NIF.K,NIF.K,STF.K)
A      DFF.K=DHF+MPF.K/IAF.K
A      MPF.K=(DUF)(IDF.K)
A      IDF.K=(AIF)(RSF.K)
L      RSF.K=RSF.J+(DT/DRF)(RRF.JK-RSF.J)
A      MWF.K=RRF.JK+(1/DIF)(A3.K+B3.K+C3.K)
A      A3.K=IDF.K-IAF.K
A      B3.K=LDF.K-LAF.K
A      C3.K=UOF.K-UNF.K
R      MDF.KL=CLIP(MWF.K,ALF,ALF,MWF.K)
A      LDF.K=(RSF.K)(DCF+DPF)
A      LAF.K=CPF.K+OPF.K
A      UNF.K=(RSF.K)(DHF+DUF)
L      CPF.K=CPF.J+(DT)(MDF.JK-MCF.JK)
R      MCF.KL=DELAY3(MDF.JK,DCF)
L      OPF.K=OPF.J+(DT)(MCF.JK-SRF.JK)
R      SRF.KL=DELAY3(MCF.JK,DPF)
NCTE   ************************************************************
NCTE   INITIAL CCNCITICNS
NCTE   ************************************************************
N      UOR=(RSR)(DHR+DUR)
N      IAR=(AIR)(RSR)
N      RSR=RRR
N      CPR=(DCR)(RRR)
N      PDR=RRR
N      MTR=(DTR)(RRR)
```

```
N      SSD=RRC
N      LCD=(RSD)(DHC+CLD)
N      IAD=(AID)(RSD)
N      RSD=RRC
N      CPD=(DCD)(RRR)
N      PCD=RRC
N      MTD=(CTD)(RRR)
N      SSF=RRF
N      MCF=RRF
N      LCF=(RSF)(DHF+CLF)
N      IAF=(AIF)(RSF)
N      RSF=RRF
N      CPF=(DCF)(RRR)
N      CPF=(CPF)(RRR)
NOTE   ************************************************************
NOTE   PARAMETERS(CONSTANTS) OF THE SYSTEM
NOTE   ************************************************************
C      AID=6 WEEKS
C      AIF=4 WEEKS
C      AIR=8 WEEKS
C      ALF=3000
C      DCD=2.5
C      DCF=1 WEEK
C      DCR=3.5
C      DHD=1.0 WEEK
C      DHF=1.0 WEEK
C      DHR=1.0 WEEK
C      DID=8 WEEKS
C      DIF=8 WEEKS
C      DIR=8 WEEKS
C      DRD=8 WEEKS
C      DRF=8 WEEKS
C      DRR=8 WEEKS
C      DTD=2.0 WEEKS
C      DPF=6.0 WEEKS
C      DTR=1.0 WEEK
C      CLD=0.6 WEEK
C      CLF=1.0 WEEK
C      DLR=0.4 WEEK
NOTE   ************************************************************
NOTE   MODEL OF THE A-TYPE ADVERTISING PRACTICE, DISCUSSED
NOTE   IN CHAPTER 5.
NOTE   ************************************************************
R      VDF.KL=CLIP(FRA,CAR,TIME.K,TT)
3R     VCF.KL=DELAY3(VDF.JK,DVF)
R      VPP.KL=SAMPLE(AC.K,INTVL,INVAL)
A      AC.K=TABHL(TNAM,TIME.K,TLC,THI,TINCR)
T      TNAM=0/1000/1000/1000/1000/1000/0 1000=SALE*AVS
5L     VAC.K=VAC.J+(DT/DVC)(VPP.JK-VAC.J)
L      SALE.K=SALE.J+(DT)((RC)(VAC.J)(1.0-(SALE.J)/SL)-(SDC)(SALE.J))
```

```
S       ASALE.K=(SALE.K)(1+C1*DEV.K)
A       DEV.K=SAMPLE(NN.K,IVL,INL)
A       NN.K=NCRMRN(MEAN,SDEV)
R       RRR.KL=(ASALE.K)/(UPF)
L       TCTS.K=TCTS.J+DT*ASALE.J
NCTE    ***************************************************
NCTE    INITIAL CCNDITICNS
NCTE    ***************************************************
N       CAR=(SALE)(AVS)
N       VCF=CAR
17N     VAC=VFP
N       RRR=SALE/UPF
N       SALE=1CCCC $/WEEK
N       TLO=DVA+DVF-TINCR
N       THI=TT+TLC+TINCR
N       INTVL=TINCR
N       TCTS=SLZRC
NCTE    ***************************************************
NCTE    SUPPLEMENTARY ECUATICNS
NCTE    ***************************************************
S       PLT1.K=IAF.K-ICF.K
S       TIS.K=IAR.K+IAC.K+IAF.K
NCTE    ***************************************************
NCTE    PARAMETERS(CCNSTANTS) CF THE SYSTEM
NCTE    ***************************************************
C       AVS=C.1C CIMENSIONLESS,% CF SALES GCING TC ACVERTISING
C       TINCR=5
C       INVAL=C.C
C       UPF=1CC $,UNIT PRICE AT FACTCRY
C       DVA=2 WEEKS
C       DVC=3 WEEKS
C       DVF=3 WEEKS
C       RC=C.5 PER WEEK
C       SL=15CCO $/WEEK
C       SDC=C.CC5 PER WEEK, CR 0.26/YEAR
C       FRA=C.C FINAL RATE OF ADVERTISING
C       TT=25 WEEKS, APPRCXIMATELY 6 MCNTHS
C       SLZRC=C.0 INITIAL ACCUMULATICN CF SALES
C       C1=C.5
C       MEAN=C
C       SCEV=C.05
C       IVL=1
C       INL=C
NCTE    ***************************************************
NCTE    PRINTING & PLCTTING INSTRUCTIONS
NCTE    ***************************************************
PRINT   1)RRR/2)RRF/3)TIS/4)SRF/5)ASALE/6)TCTS
PLOT    RRR=R,RRF=F(C,2C0)/VAC=X,VPF=Z(0,150C)/IAR=A,IAF=.(C,1800)/MOF=0,
X1      RF=+(C,25C)/ASALE=$(C,15CCC)
PLOT    ASALE=$(C,15CCC)/TOTS=T(C,12CCCCC)
```

```
NOTE    *****************************************************
SPEC    DT=0.1/LENGTH=104/PRTPER=1/PLTPER=1
RUN     RS-1.2
```

```
*       B-TYPE AD MCDEL
NCTE          THE FCLLCWING CCDES ARE USED TC DESCRIBE THE
NCTF          CIFFERENT ACVERTISING MCDELS:
NCTE          A-TYPE : FRCTRACTED ACVERTISING
NCTE          B-TYPE : SHCRT-TIME, INTENSIVE ACVERTISING
NCTE          C-TYPE : IMPULSE-TYPE ACVERTISING
NCTE    THE FCLLCWING MCDEL ASSUMES THAT SALES AT RETAIL
NCTE    HAVE A RANCCM FLUCTUATIONS, WITH A MEAN CF C.0,
NCTE    ANC STANDARC CEVIATICN OF C.C5.
NCTE    FCR THE MATHEMATICAL CESCRIPTICN CF THE MCDEL SEE:
NCTE          CHAPTER 5 CF THIS REPCRT, ANC ALSC:
NCTE          VICALE,M.L., ANC H.B. WCLFE:
NCTE          "AN CPERATICNS REASEARCH STUCY CF SALES RESPCNSE
NCTE          TC ADVERTISING, "CPERATICNS RESEARCH,  VCL.15,
NCTE          NC.3, JUNE 1957.
NCTE
NCTE    FCR THE CCMPLETE DESCRIPTICN CF THE FRCCUCTICN-
NCTE    CISTRIBUTICN MCCEL SEE:
NCTE          J.W. FCRRESTER:
NCTE          "INCUSTRIAL CYNAMICS," THE M.I.T. PRESS,
NCTE          CAMBRIDGE,MASS., 1961; CHAPTERS 2 & 15
NCTE    ****************************************************
NCTE    ECUATICNS FCR RETAIL SECTCR
NCTE    ****************************************************
L       UCR.K=UCR.J+(CT)(RRR.JK-SSR.JK)
L       IAR.K=IAR.J+(CT)(SRR.JK-SSR.JK)
A       STR.K=(UCR.K)/(CFR.K)
A       NIR.K=(IAR.K)/CT
R       SSR.KL=CLIP(STR.K,NIR.K,NIR.K,STR.K)
A       CFR.K=CHR+MNR.K/IAR.K
A       MNR.K=(CUR)(ICR.K)
A       ICR.K=(AIR)(RSR.K)
L       RSR.K=RSR.J+(CT/CRR)(RRR.JK-RSR.J)
R       PCR.KL=RRR.JK+(1/CIR)(A1.K+B1.K+C1.K)
A       A1.K=ICR.K-IAR.K
A       B1.K=LCR.K-LAR.K
A       C1.K=UCR.K-UNR.K
A       LCR.K=(RSR.K)(CCR+CFC.K+CTR)
A       LAR.K=CPR.K+UCC.K+MTR.K
A       UNR.K=(RSR.K)(CHR+CUR)
L       CPR.K=CPR.J+(CT)(PCR.JK-RRC.JK)
R       RRC.KL=CELAY3(PCR.JK,CCR)
L       MTR.K=MTR.J+(CT)(SSD.JK-SRR.JK)
R       SRR.KL=CELAY3(SSC.JK,CTR)
NCTE    ****************************************************
NCTE    ECUATICNS FCR THE CISTRIBUTICN SECTICN
NCTE    ****************************************************
L       UCD.K=UCD.J+(CT)(RRC.JK-SSC.JK)
L       IAC.K=IAC.J+(CT)(SRC.JK-SSC.JK)
A       STC.K=(UCC.K)/(CFC.K)
```

```
A       NID.K=(IAD.K)/CT
R       SSD.KL=CLIP(STD.K,NID.K,NID.K,STD.K)
A       DFD.K=DHD+MOD.K/IAD.K
A       MOD.K=(DUD)(IDD.K)
A       IDD.K=(AID)(RSD.K)
L       RSD.K=RSD.J+(DT/DRD)(RRD.JK-RSD.J)
R       PDD.KL=RRD.JK+(1/DID)(A2.K+B2.K+C2.K)
A       A2.K=IDD.K-IAD.K
A       B2.K=LDD.K-LAD.K
A       C2.K=LDD.K-UND.K
A       LDD.K=(RSD.K)(DCD+DFF.K+DTD)
A       LAD.K=CPD.K+LDF.K+MTD.K
A       UND.K=(RSD.K)(DHD+DUD)
L       CPD.K=CPD.J+(DT)(PDD.JK-RRF.JK)
R       RRF.KL=DELAY3(PDD.JK,DCD)
L       MTD.K=MTD.J+(DT)(SSF.JK-SRD.JK)
R       SRD.KL=DELAY3(SSF.JK,DTD)
NOTE    ****************************************************
NOTE    EQUATIONS FOR THE FACTORY SECTOR
NOTE    ****************************************************
L       LDF.K=LDF.J+(DT)(RRF.JK-SSF.JK)
L       IAF.K=IAF.J+(DT)(SRF.JK-SSF.JK)
A       STF.K=(LDF.K)/(DFF.K)
A       NIF.K=(IAF.K)/DT
R       SSF.KL=CLIP(STF.K,NIF.K,NIF.K,STF.K)
A       DFF.K=DHF+MPF.K/IAF.K
A       MPF.K=(DUF)(IDF.K)
A       IDF.K=(AIF)(RSF.K)
L       RSF.K=RSF.J+(DT/DRF)(RRF.JK-RSF.J)
A       MWF.K=RRF.JK+(1/DIF)(A3.K+B3.K+C3.K)
A       A3.K=IDF.K-IAF.K
A       B3.K=LDF.K-LAF.K
A       C3.K=LDF.K-UNF.K
R       MDF.KL=CLIP(MWF.K,ALF,ALF,MWF.K)
A       LDF.K=(RSF.K)(DCF+DPF)
A       LAF.K=CPF.K+CPF.K
A       UNF.K=(RSF.K)(DHF+DUF)
L       CPF.K=CPF.J+(DT)(MDF.JK-MCF.JK)
R       MCF.KL=DELAY3(MDF.JK,DCF)
L       CPF.K=CPF.J+(DT)(MOF.JK-SRF.JK)
R       SRF.KL=DELAY3(MCF.JK,DPF)
NOTE    ****************************************************
NOTE    INITIAL CONDITIONS
NOTE    ****************************************************
N       LDR=(RSR)(DHR+DUR)
N       IAR=(AIR)(RSR)
N       RSR=RRR
N       CPR=(DCR)(RRR)
N       PDR=RRR
N       MTR=(DTR)(RRR)
```

```
N      SSD=RRD
N      LDD=(RSD)(DHD+DLD)
N      IAD=(AID)(RSD)
N      RSD=RRD
N      CPD=(DCD)(RRR)
N      PDD=RRD
N      MTD=(DTD)(RRR)
N      SSF=RRF
N      MDF=RRF
N      LDF=(RSF)(DHF+DLF)
N      IAF=(AIF)(RSF)
N      RSF=RRF
N      CPF=(DCF)(RRR)
N      CPF=(DPF)(RRR)
NOTE   ***************************************************
NOTE   PARAMETERS(CONSTANTS) OF THE SYSTEM
NOTE   ***************************************************
C      AID=6 WEEKS
C      AIF=4 WEEKS
C      AIR=8 WEEKS
C      ALF=3000
C      DCD=2.5
C      DCF=1 WEEK
C      DCR=3.5
C      DHD=1.0 WEEK
C      DHF=1.0 WEEK
C      DHR=1.0 WEEK
C      DID=8 WEEKS
C      DIF=8 WEEKS
C      DIR=8 WEEKS
C      DRD=8 WEEKS
C      DRF=8 WEEKS
C      DRR=8 WEEKS
C      DTD=2.0 WEEKS
C      DPF=6.0 WEEKS
C      DTR=1.0 WEEK
C      DLD=0.6 WEEK
C      DLF=1.0 WEEK
C      DLR=0.4 WEEK
NOTE   ***************************************************
NOTE   MODEL OF THE B-TYPE ADVERTISING PRACTICE, DISCUSSED
NOTE   IN CHAPTER 5.
NOTE   ***************************************************
R      VDF.KL=CLIP(FRA,SAP,TIME.K,T)
3R     VCF.KL=DELAY3(VDF.JK,DVF)
R      VPP.KL=TABHL(TNAME,TIME.K,TLF,THI,TINCR)
T      TNAME=0.000/25000/25000/25000/25000/25000/25000/25000/25000/25000/
X1     25000/25000/0.000   25000=SALE*AVS*TT
L      VAC.K=VAC.J+(DT/DVC)(VPP.JK-VAC.J)
L      SALE.K=SALE.J+(DT)((RC)(VAC.J)(1.0-(SALE.J)/SL)-(SDC)(SALE.J))
```

```
S      ASALE.K=(SALE.K)(1+C1*DEV.K)
A      DEV.K=SAMPLE(NN.K,IVL,INL)
A      NN.K=NORMRN(MEAN,SDEV)
R      RRR.KL=(ASALE.K)/(UPF)
L      TOTS.K=TOTS.J+DT*ASALE.J
NOTE   **************************************************
NOTE   INITIAL CONDITIONS
NOTE   **************************************************
N      VDF=SAR
N      SAR=(SALE)(AVS)(TT)
N      VAC=VPP
N      RRR=SALE/UPF
N      SALE=10000 $/WEEK
N      TLC=T+DVF+DVA-DT
N      THI=TLC+T+2*DT
N      TINCR=DT
N      TOTS=SLZRO
NOTE   **************************************************
NOTE   SUPPLEMENTARY EQUATIONS
NOTE   **************************************************
S      PLT1.K=IAF.K-IDF.K
S      TIS.K=IAR.K+IAD.K+IAF.K
NOTE   **************************************************
NOTE   PARAMETERS(CONSTANTS) OF ADVERTISING SECTION
NOTE   **************************************************
C      AVS=0.10 DIMENSIONLESS,% OF SALES GOING TO ADVERTISING
C      UPF=100 $,UNIT PRICE AT FACTORY
C      DVA=2 WEEKS
C      DVC=3 WEEKS
C      DVF=3 WEEKS
C      RC=0.5 PER WEEK
C      SL=15000 $/WEEK
C      SDC=0.005 PER WEEK, OR 0.26/YEAR
C      FRA=0.0 FINAL RATE OF ADVERTISING
C      T=1 WEEK; STOPPING TIME FOR AD CAMPAIGN
C      TT=25 WEEKS,CAMPAIGN PERIOD IN PREVIOUS EXAMPLE
C      SLZRO=0.0 INITIAL ACCUMULATION OF SALES
C      C1=0.5
C      MEAN=0
C      SDEV=0.05
C      IVL=1
C      INL=0
NOTE   **************************************************
NOTE   PRINTING & PLOTTING INSTRUCTIONS
NOTE   **************************************************
PRINT  1)RRR/2)RRF/3)TIS/4)SRF/5)ASALE/6)TOTS
PLOT   RRR=R,RRF=F(0,200)/VAC=X,VPP=Z(0,40000)/IAR=A,IAF=.(0,1800)/MCF=0,
X1     SRF=+(0,250)/ASALE=$(0,15000)
PLOT   ASALE=$(0,15000)/TOTS=T(0,1200000)
NOTE   **************************************************
```

```
SPEC   CT=0.1/LENGTH=104/PRTPER=1/PLTPER=1
RUN    RS-2.2
```

```
*       C-TYPE AC MCCEL
NCTE         THE FCLLCWING CCCES ARE USED TC CESCRIBE THE
NCTE         CIFFERENT ACVERTISING MCCELS:
NCTE         A-TYPE : PRCTRACTED ACVERTISING
NCTE         B-TYPE : SHCRT-TIME, INTENSIVE ACVERTISING
NCTE         C-TYPE : IMPULSE-TYPE ACVERTISING
NCTE    THE FCLLCWING MCCEL ASSUMES THAT SALES AT RETAIL
NCTE    HAVE A RANCCM FLUCTUATIONS, WITH A MEAN CF 0.0,
NCTE    ANC STANCARC CEVIATICN OF 0.05.
NCTE    FCR THE MATHEMATICAL CESCRIPTICN CF THE MCCEL SEE:
NCTE         CHAPTER 5 CF THIS REPCRT, ANC ALSC:
NCTE         VICALE,M.L., AND H.B. WCLFE:
NCTE         "AN CPERATIONS REASEARCH STUDY CF SALES RESPCNSE
NCTE         TC ACVERTISING, "CPERATICNS RESEARCH,  VCL.15,
NCTE         NC.3, JUNE 1957.
NCTE
NCTE    FCR THE CCMPLETE DESCRIPTICN CF THE PRCCUCTICN-
NCTE    CISTRIBUTICN MCCEL SEE:
NCTE         J.W. FCRRESTER:
NCTE         "INDUSTRIAL DYNAMICS," THE M.I.T. PRESS,
NCTE         CAMBRICGE,MASS., 1961; CHAPTERS 2 & 15
NCTE    ***************************************************
NCTE    ECUATICNS FCR RETAIL SECTCR
NCTE    ***************************************************
L       UCR.K=UCR.J+(CT)(RRR.JK-SSR.JK)
L       IAR.K=IAR.J+(CT)(SRR.JK-SSR.JK)
A       STR.K=(UCR.K)/(CFR.K)
A       NIR.K=(IAR.K)/CT
R       SSR.KL=CLIP(STR.K,NIR.K,NIR.K,STR.K)
A       CFR.K=CHR+MNR.K/IAR.K
A       MNR.K=(CUR)(ICR.K)
A       ICR.K=(AIR)(RSR.K)
L       RSR.K=RSR.J+(CT/CRR)(RRR.JK-RSR.J)
R       PCR.KL=RRR.JK+(1/DIR)(A1.K+B1.K+C1.K)
A       A1.K=ICR.K-IAR.K
A       B1.K=LCR.K-LAR.K
A       C1.K=UCR.K-UNR.K
A       LCR.K=(RSR.K)(CCR+CFC.K+CTR)
A       LAR.K=CPR.K+UCC.K+MTR.K
A       UNR.K=(RSR.K)(CHR+CUR)
L       CPR.K=CPR.J+(CT)(PCR.JK-RRC.JK)
R       RRC.KL=CELAY3(PCR.JK,CCR)
L       MTR.K=MTR.J+(CT)(SSC.JK-SRR.JK)
R       SRR.KL=CELAY3(SSD.JK,CTR)
NCTE    ***************************************************
NCTE    ECUATICNS FCR THE CISTRIBUTICN SECTICN
NCTE    ***************************************************
L       UCC.K=UCC.J+(CT)(RRC.JK-SSC.JK)
L       IAD.K=IAC.J+(CT)(SRC.JK-SSC.JK)
A       STD.K=(UCD.K)/(CFD.K)
```

```
A      NIC.K=(IAC.K)/CT
R      SSC.KL=CLIP(STC.K,NIC.K,NIC.K,STC.K)
A      CFC.K=CHC+MCC.K/IAD.K
A      MCC.K=(DLD)(ICC.K)
A      ICC.K=(AIC)(RSC.K)
L      RSC.K=RSC.J+(CT/CRC)(RRD.JK-RSC.J)
R      PCD.KL=RRD.JK+(1/DIC)(A2.K+B2.K+C2.K)
A      A2.K=ICC.K-IAC.K
A      B2.K=LCC.K-LAC.K
A      C2.K=LCD.K-UNC.K
A      LCD.K=(RSC.K)(CCC+DFF.K+DTC)
A      LAC.K=CPC.K+LCF.K+MTC.K
A      UNC.K=(RSC.K)(CHC+CUC)
L      CPD.K=CPC.J+(CT)(PDD.JK-RRF.JK)
R      RRF.KL=DELAY3(PCC.JK,CCD)
L      MTD.K=MTD.J+(CT)(SSF.JK-SRC.JK)
R      SRD.KL=DELAY3(SSF.JK,CTD)
NCTE   ****************************************************
NCTE   ECUATICNS FOR THE FACTORY SECTOR
NCTE   ****************************************************
L      LCF.K=LOF.J+(CT)(RRF.JK-SSF.JK)
L      IAF.K=IAF.J+(CT)(SRF.JK-SSF.JK)
A      STF.K=(LCF.K)/(DFF.K)
A      NIF.K=(IAF.K)/CT
R      SSF.KL=CLIP(STF.K,NIF.K,NIF.K,STF.K)
A      CFF.K=CHF+MPF.K/IAF.K
A      MPF.K=(CUF)(ICF.K)
A      ICF.K=(AIF)(RSF.K)
L      RSF.K=RSF.J+(CT/CRF)(RRF.JK-RSF.J)
A      MWF.K=RRF.JK+(1/CIF)(A3.K+B3.K+C3.K)
A      A3.K=ICF.K-IAF.K
A      B3.K=LCF.K-LAF.K
A      C3.K=LCF.K-UNF.K
R      MCF.KL=CLIP(MWF.K,ALF,ALF,MWF.K)
A      LCF.K=(RSF.K)(CCF+DPF)
A      LAF.K=CPF.K+CFF.K
A      UNF.K=(RSF.K)(CHF+CUF)
L      CPF.K=CPF.J+(CT)(MDF.JK-MCF.JK)
R      MCF.KL=DELAY3(MCF.JK,CCF)
L      CFF.K=CPF.J+(CT)(MCF.JK-SRF.JK)
R      SRF.KL=DELAY3(MCF.JK,CPF)
NCTE   ****************************************************
NCTE   INITIAL CCNCITICNS
NCTE   ****************************************************
N      LCR=(RSR)(CHR+CLR)
N      IAR=(AIR)(RSR)
N      RSR=RRR
N      CPR=(CCR)(RRR)
N      PCR=RRR
N      MTR=(CTR)(RRR)
```

```
N       SSD=RRD
N       LCD=(RSD)(CHD+CLD)
N       IAD=(AID)(RSD)
N       RSD=RRD
N       CPD=(DCD)(RRR)
N       PCD=RRD
N       MTD=(DTD)(RRR)
N       SSF=RRF
N       MCF=RRF
N       LCF=(RSF)(CHF+CLF)
N       IAF=(AIF)(RSF)
N       RSF=RRF
N       CPF=(DCF)(RRR)
N       CPF=(DPF)(RRR)
NOTE    ****************************************************
NOTE    PARAMETERS OF THE SYSTEM(CONSTANTS)
NOTE    ****************************************************
C       AID=6 WEEKS
C       AIF=4 WEEKS
C       ALF=3000
C       AIR=8 WEEKS
C       DCD=2.5
C       DCF=1 WEEK
C       DCR=3.5
C       DHD=1.0 WEEK
C       DHF=1.0 WEEK
C       DHR=1.0 WEEK
C       DID=8 WEEKS
C       DIF=8 WEEKS
C       DIR=8 WEEKS
C       DRD=8 WEEKS
C       DRF=8 WEEKS
C       DRR=8 WEEKS
C       DTD=2.0 WEEKS
C       DPF=6.0 WEEKS
C       DTR=1.0 WEEK
C       DLD=0.6 WEEK
C       DLF=1.0 WEEK
C       DLR=0.4 WEEK
NOTE    ****************************************************
NOTE    MODEL OF THE C-TYPE ADVERTISING PRACTICE, DISCUSSED
NOTE    IN CHAPTER 5.
NOTE    ****************************************************
R       VPP.KL=PULSE(MONEY,FIRST,INTVL)
L       VAC.K=VAC.J+(DT/DVC)(VPP.JK-VAC.J)
L       SALE.K=SALE.J+(DT)((RC)(VAC.J)(1.0-(SALE.J)/SL)-(SDC)(SALE.J))
S       ASALE.K=(SALE.K)(1+C1*DEV.K)
A       DEV.K=SAMPLE(NN.K,IVL,INL)
A       NN.K=NORMRN(MEAN,SDEV)
R       PRR.KL=(ASALE.K)/(UPF)
```

```
L     TCTS.K=TOTS.J+CT*ASALE.J
NCTE  *****************************************************
NCTE  INITIAL CCNCITICN FCR ADVERTISING SECTICN
NCTE  *****************************************************
N     VAC=VFP
N     RRR=SALE/LPF
N     SALE=1CCCC $/WEEK
N     MCNEY=(SALE)(AVS)(TT)/(NC*CT)
N     FIRST=CVA+CVF
N     TCTS=SLZRC
NCTE  *****************************************************
NCTE  SUPPLEMENTARY ECLATICNS
NCTE  *****************************************************
S     PLT1.K=IAF.K-ICF.K
S     TIS.K=IAR.K+IAC.K+IAF.K
NCTE  *****************************************************
NCTE  PARAMETERS(CCNSTANTS) CF ACVERTISING SECTICN
NCTE  *****************************************************
C     AVS=C.1C CIMENSIONLESS,% CF SALES GCING TC ACVERTISING
C     LPF=1CC $,UNIT PRICE AT FACTORY
C     CVA=2 WEEKS
C     DVC=3 WEEKS
C     CVF=3 WEEKS
C     RC=C.5 PER WEEK
C     SL=15CCC $/WEEK
C     SCC=C.CC5 PER WEEK, CR 0.26/YEAR
C     TT=25 CAMPAIGN PERIOC IN CCNSTANT ACVERTISING
C     SLZRC=C.C INITIAL ACCUMULATICN CF SALES
C     INTVL=16 WEEKS, APPRCXIMATELY 4 MCNTHS
C     NC=7 NC. CF CAMPAIGNS
C     C1=C.5
C     MEAN=C
C     SCEV=C.C5
C     IVL=1
C     INL=C
NCTE*******************************************************
NCTE  PRINTING & PLCTTING INSTRUCTICNS
NCTE  *****************************************************
PRINT 1)RRR/2)RRF/3)TIS/4)SRF/5)ASALE/6)TCTS
PLOT  RRR=R,RRF=F(C,2CC)/VPP=Z(C,4CCCC)/VAC=X(C,1CCC)/IAR=A,IAF=.(0,1800
X1    )/MCF=C,SRF=+(C,250)/ASALE=$(0,15CCO)
PLOT  ASALE=$(C,15CCC)/TCTS=T(C,12CCCCC)
NCTE  *****************************************************
SPEC  CT=C.1/LENGTH=1C4/PRTPER=1/PLTPER=1
RUN   RS-3.2
```

ADVERTISING IN INDUSTRIAL SYSTEMS-

AN INDUSTRIAL DYNAMICS APPROACH

by

REZA SHOJALASHKARI

B.S.E.E.

University of Tehran, Iran, 1964

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirement for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1971

The purpose of this report is to study the principles of Industrial Dynamics, as a theory of structure of systems, and apply those principles to some common advertising practices.

The essential concepts and fundamentals of industrial dynamics are outlined, and an application of these concepts to the dynamic model of a simple industrial system, dealing only with production and distribution is reviewed.

An advertising model, which assumes a direct relationship between sales level and advertising expenditure, is added to the production — distribution model to study the dynamic behavior of the system under such a promotional practice. The study is further carried out to incorporate some models of advertising into the production-distribution system model. Three different advertising practices are considered: protracted, short-time, intensive, and impulse-type advertisings. Their effects on the dynamic behavior of the system are investigated. The three practices are compared on the basis of several objectives, such as total cost of production and inventory, fluctuations in system variables, total sales generated as the result of advertising, and so on.