Using dynamic geometry software to examine its effects on student progression through the van Hiele levels

by

Andrew Lehman

B.S., Bluffton University, 2009
M.S., Ball State University, 2019

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Curriculum and Instruction
College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

2024

Abstract

Geometry is a strand of mathematics that can benefit individuals in many areas of life. Test results, such as the TIMSS results, indicate that geometry is an area for growth among American students. With a need for students to better understand geometry, several academic standards have been created to direct geometry instruction in the United States. Aside from this growing need, advances in technology have been introduced to classrooms. Dynamic geometry software, such as GeoGebra, is an example of this technology. Does GeoGebra help students develop their geometric reasoning skills? This study used a mixed methods design to investigate the effects that GeoGebra has in the progression of fifth-grade students through the first three levels of the van Hiele theory.

Keywords: van Hiele levels, geometry, dynamic geometry software

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Approved by:

Major Professor
Dr. David Allen

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Dedication

This dissertation is dedicated first and foremost to the best daughter a dad could have, Claire Elizabeth. The countless hours she gave up spending with me so I could study and pursue my doctoral degree certainly did not go unnoticed. Claire has a burning desire to be a teacher one day. Whether she continues to have that passion or not, I hope my time and success in this program will always remind her of how important education is.

Secondly, this dissertation is dedicated to all my family, especially my parents; to all my friends, especially Tab and Brent; and to my coworkers, especially Kristie and Kristin. These individuals believed in me, supported me, and kept me going when I was unsure I could. They prayed prayer after prayer for me and shared many words of affirmation. All of these things helped me to stay motivated and positive.

I also dedicate this dissertation to every teacher and professor who mentored me along the way from Kindergarten through this doctoral degree. I have been blessed to have many great educators who helped me become a confident math student. I am grateful to my advisor, Dr. David Allen, and my committee for their countless hours meeting with me and working alongside me on this dissertation journey.

Thank you to Claire and everyone else who has supported me along the way. I am forever grateful for each and every one of you.

Chapter 1 - Introduction

Overview

Geometry is an essential strand of mathematics. Students can use geometry to better understand facts about our world, as geometric thinking does not just apply to shapes (Erdogan et al., 2009). The skills required for geometric thinking help with deductive reasoning and critical thinking, among many other skills. The National Council for Teachers of Mathematics (NCTM) has placed geometry as one of its core standards. Moreover, NCTM has promoted the van Hiele theory as a means for the development of geometric reasoning (Choi-Koh, 1999). The van Hiele model comprises five hierarchical levels of understanding geometric thought.

Educators have noted for quite some time that students should learn geometry, yet American students tend to lack strength with geometry topics. The 2019 Trends in International Mathematics and Science Study (TIMSS), an international examination given every four years to students in grades 4 and 8 to compare mathematics and science achievement of students in the United States to students elsewhere, revealed that students in grade 4 ranked 25th in geometry out of the 55 countries that participated. Out of the 36 countries that participated, students in grade 8 ranked 17th in geometry (Mullis et al., 2020). This data suggests that students in the United States have room to improve in geometry. By combining this acknowledgment with recommendations from NCTM, American educators can close the gap for students.

Technology is quite commonplace in the lives of students. NCTM (2014) has called for mathematics classrooms to reflect this reality by making technology an integral part of instruction. Many students are familiar with calculators and spreadsheets as forms of technology that can be used in the classroom. Dynamic Geometry Software (DGS) is another type of useful technology. Students who are familiar with technology in their personal lives and at school may adapt well to using DGS as an essential technological tool for learning geometric concepts.

NCTM (2014) states that tools and technology must be indispensable classroom features for meaningful mathematics learning. Applets and DGS, such as GeoGebra and Geometer's Sketchpad, may constitute technology. DGS helps students explore geometric conjectures since dragging the objects preserves the underlying relationships among them while constructing diagrams (National Council of Teachers of Mathematics [NCTM], 2014).

Technology can provide many affordances to the learning development of students. This happens when students use technology to visualize and understand important mathematical concepts, which in turn leads to mathematical reasoning (NCTM, 2014). Furthermore, NCTM (2014) argues that technology can aid students in mathematical investigations that may otherwise be too time-consuming.

Researchers have conducted several studies regarding the validity of the van Hiele theory and its framework for geometric reasoning (Abdullah & Zakaria, 2013; Burger & Shaughnessy, 1986; Clements & Battista, 1992; Usiskin, 1982; Yalley et al., 2021). After reviewing research that used the van Hiele theory, the researcher could not find a study that looked specifically at how fifth-grade students in the United States used DGS to further their understanding of geometric thought as they advanced through the levels of geometric reasoning, as proposed by van Hiele and his wife in 1957. This study sought to fill this gap by exploring how technology, specifically GeoGebra, may be used to help students advance through the van Hiele levels.

Statement of the Problem

The usage of DGS in the classroom is becoming commonplace in many classrooms today. DGS is especially growing in popularity in advanced countries like the United States

(Adelabu et al., 2019). This software could support geometric learning for fifth-grade students. With the importance of being able to reason geometrically having been validated by several research studies, such as Usiskin (1982) and Burger and Shaughnessy (1986), the researcher found it beneficial to examine what the effects of using a DGS could have on the progression of the van Hiele levels for fifth-grade students.

Research Purpose

This study explores the usage of GeoGebra and its effect on the progression through the first three levels of the van Hiele model of geometric thinking for fifth-grade students at an elementary school in the Midwest.

Quantitative Questions

1. What effect does utilizing GeoGebra during geometry instruction have on fifth-grade students' van Hiele level of geometric thinking about polygons, as measured by a pre- and posttest?
    a. Are there differences based on gender?
    b. Are there any differences based on prior achievement?

Qualitative Questions

2. What are students' experiences using GeoGebra during geometry instruction?
    a. How do students experience using tools available in GeoGebra?
    b. When students are observed using (or not using) tools available in GeoGebra, what is their level of geometric thinking?

Mixed Methods Question

3. What is the impact of using GeoGebra to support fifth-grade students' learning about polygons?

Design of the Study

During this research study, I used a convergent mixed-methods design and used triangulation to help ensure the results and findings were reliable and accurate (Creswell et al., 2003; Creswell and Plano Clark, 2007). I collected quantitative data from the van Hiele Geometry Test (Usiskin, 1982). I used four different instruments in a case-study design to collect the qualitative data. The instruments used were journal entries collected from the students, screenshots and screen recordings from the study activities, screen recordings of the end of unit GeoGebra tasks, which were modeled after tasks implemented in studies conducted by Burger and Shaughnessy (1986) and Wang and Kinzel (2014), and responses to the survey.

Limitations and Delimitations of the Study

Limitations

Like every research study, this study had limitations. The design of the study was one such limitation. The case study approach utilized in this study limited the results to the context in which the study was conducted; therefore, the data may not be generalizable to other fifth-grade students.

Other limitations came from the student demographics and familiarity with GeoGebra. Students from lower SES families and those from the Amish community, a religious group that does not use technology, may have had less familiarity with technology than their peers due to a lack of access to iPads and the internet. In general, the students in the sample, even if they were familiar with technology, may have lacked a familiarity with and an ability to use GeoGebra effectively. This study did not analyze the ability to use GeoGebra but instead sought to determine how it is utilized with geometric reasoning. Because DGS is not used frequently in elementary classrooms, students may have had a difficult time using the software versus participating in traditional methods of geometry instruction.

Additional limitations with the study may have existed. For example, average school attendance rates may have limited the data due to a loss in instructional time throughout the geometry unit. Additionally, the researcher's knowledge of GeoGebra may have been a limitation. Finally, due to the sample being conveniently selected from the researcher's classes, limitations may have naturally occurred as the researcher already had a rapport with the students. Additionally, the researcher used materials created by the researcher.

Delimitations

The researcher can control some factors in every research study, such as how participants are selected. The convenience sampling may have limited the generalizability of the results to other students who belong to demographic categories not represented by the homogenous groups of students at the Midwestern elementary school where the research study occurred.

Positionality

As a child, I was genuinely curious about the world around me. My curiosity ranged from nature to athletics to academic concepts, especially mathematical concepts. Numbers, coins, and shapes were at the top of my list of interests. While I was intrigued by many mathematical concepts, geometry was the mathematical topic that intrigued me the most. This curiosity for geometry, especially about different polygons, developed even more as I grew from a small child to an upper elementary student.

I remember sitting in church with my family and drawing as many shapes as possible. As I got older, I would develop and create irregular polygons. I would try to make polygons with as many sides as I could fit on one paper, I would try to duplicate polygons, and lastly, I would attempt to see similarities between the polygons I constructed each Sunday. While this curiosity, fostered by my inability to focus on the sermon, helped me to develop a passion for mathematics and geometry, I struggled with geometry topics in elementary and middle school.

Beginning in late elementary school, my math teachers often showed us Venn Diagrams and other diagrams to explain the hierarchy of polygons. I struggled to understand the diagrams and see how the polygons were related. The difficulty I experienced with geometry continued in my high school and undergraduate courses, as well. It was not until I became a fifth-grade teacher that I understood the diagrams my teachers often used when I was a student.

In addition to having a passion for math and geometry, I have always enjoyed technology. While in elementary school, I became interested in using computers as a learning tool, especially through my experiences playing Oregon Trail. As technology became more prevalent in my classrooms, I began leveraging technology to help me better understand mathematics.

As an adult, I am still interested in plane geometry and in using technology. As a current fifth-grade math teacher, one of my favorite topics to teach is a unit on polygons. I work in a 1:1 school setting and enjoy implementing technology into my lessons. While my childhood passions for geometry and using technology might not currently look the same as they did thirty years ago, my passions for geometry and technology have been transformed into what they are today, hence I am interested in researching how the usage of a dynamic geometry software impacts the development of geometric reasoning within students.

Definition of Terms

Constructions- the creation of different polygons using dynamic geometry software

Deduction- a type of reasoning in which students make sense of new topics using what they know

Dynamic Geometry Software- interactive online software that allows students to manipulate geometric constructions in a two-dimensional environment

GeoGebra- a specific dynamic geometry software program

Geometry- the type of math referring to plane geometry or the geometry of two-dimensional shapes

Manipulations- the movement and modification of objects using dynamic geometry software

Properties of polygons- the characteristics of polygons, such as parallel, perpendicular, and congruent segments

van Hiele Model- a theory for geometric reasoning; in this study, the modified levels will be used. The first three levels of the model considered in this study are level 1 through level 3

Summary

In this chapter, I presented the background and rationale for the study, which included a summary of the current literature on this topic and how this research could fill in gaps within the literature. The research purpose and questions addressed through this research were outlined. This research examined the effects the usage of GeoGebra has on student progression through levels of geometric reasoning as developed through the van Hiele theory. Finally, the limitations of the study were mentioned. Moving forward, chapter two presents an in-depth review of the literature about using dynamic geometry software in the classroom as well as current literature discussing the instruction and assessment of van Hiele levels in the classroom. Additionally, chapter two describes the theoretical frameworks, the van Hiele theory and symbolic interactionism, that will be used in this study.

Chapter 2 - Literature Review

Introduction

Geometry is an early form of mathematics that still plays an integral role in mathematics in the 21st century. Euclid composed the first geometry textbook around 300 BCE. American schools have not emphasized geometry as strongly as other strands of mathematics, despite its integral role in mathematics and the real world for several hundred years. Lappan (1999) reported that international counterparts outperform students in the US in geometry, according to recent TIMSS assessments. Strutchens & Blume (1997) and Mullis et al. (2000) have validated that students underperform at high rates in the geometry classroom. In recent years, academic standards for learning, such as the NCTM Standards and the Common Core State Standards for Mathematics (CCSSM), as well as organizations like the National Council of Teachers of Mathematics (NCTM), have placed more emphasis on geometry instruction than before. The purpose of this research is to consider how the use of GeoGebra, a dynamic geometry software, impacts the ability of students to advance through the levels of geometric understanding, as provided through the framework of the van Hiele theory.

This literature review contains several components. The literature review begins with the history of geometry instruction in the United States and the current emphasis on geometry instruction per the NCTM Standards, the CCSSM, and state-level standards for mathematics. Next, I describe the van Hiele theory. The literature discusses the implementation and usage of the van Hiele theory in the United States, including studies that support the framework. Then, I describe symbolic interactionism as it pairs well with the van Hiele theory for this study. Finally, I discuss the implementation of technology in mathematics instruction, especially the usage of dynamic geometry software, and how it can be used in correlation with the van Hiele theory for instructional purposes by using the framework of symbolic interactionism. The arguments made throughout the literature review all relate to the initial research questions as each component of the literature review builds on each other so that data and conclusions can be made to help fill in the current gap in research surrounding the usage of GeoGebra in the fifth-grade classroom while implementing the van Hiele theory for geometry instruction.

Shifts in Geometry Instruction in the United States

Geometry Instruction Before the van Hiele Theory

For over 200 years, many attempts have been made to create informal geometry programs for American students in elementary school (Matthews, 2004). The elementary math curriculum usually did not include geometry content beyond shape recognition and the arithmetic of measurement. William R. Longley (1930) shared the sentiment of educators of his time regarding the need to place a greater emphasis on geometry instruction by stating:

> Geometry is one of the oldest educational disciplines. More than any other, it has retained its essential character for centuries. Why, then, should this most stable subject of our curriculum be questioned at the present time? (p. 29)

NCTM included an argument in its 1966 publication of *The Fifth Yearbook: The Teaching of Geometry* for the instruction of informal geometry, which includes intuitive and experimental geometry, prior to high school geometry courses (Matthews, 2004). Individuals like Longley and organizations like NCTM paved the way for developing formal geometry programs in elementary schools.

A shift in American education began in 1955 when educators developed a new emphasis on geometry instruction. Most of this shift occurred in the 1960s (Trafton & LeBlanc, 1973). Educators emphasized informal geometry instruction from 1955 to 1970. Trafton & LeBlanc

(1973) stated, “The inclusion of substantial among informal geometry has been identified as one major characteristic of the ‘modern mathematics’ movement at the elementary school level” (p. 11). They continued to state:

> Within a decade, the area of geometry has become accepted by curriculum planners and, increasingly, by classroom teachers as a necessary and integral dimension of the elementary school mathematics curriculum. Although the idea of including geometry is well accepted, a wide divergence in thinking exists regarding the particulars of what constitutes an appropriate program for boys and girls. (p. 11)

Carl Allendoerfer (1969) presented problems with the geometry curricula being used and offered four solutions: quit teaching geometry, change nothing, proceed slowly with small changes, or consider more drastic long-range changes, adding to the conversation of the time. His top recommendation was to implement small changes over time. The shift began in 1955 and led to a new emphasis on informal geometry being taught in elementary schools.

Changes to Geometry Instruction After the Development of the van Hiele Theory

Educators use academic standards for learning to guide what content they need to teach their students and provide guidance for students regarding what they are expected to know after a particular grade level. While there was a shift during the 1960s and 1970s in the emphasis on geometry instruction in the United States, the next wave of change was initiated in 1989 by NCTM. In 1989, NCTM developed its *Curriculum and Evaluation Standards for School Mathematics*, which included academic geometry standards (Van de Walle, 2004). These geometry standards indicated that students in grades 5 through 8 should discover relationships of figures “by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying” (NCTM, 1989, p. 112). While NCTM developed its learning standards over a decade before its 2000 publication, *Principles and Standards for School Mathematics*, it was not until that publication that NCTM aligned its geometry standards more closely with the van Hiele levels (Matthews, 2004). The publication recommended that students in grades Kindergarten through second grade should achieve a Level 0 and be progressing towards a Level 1; students in Grades 3-5 should have mastered a Level 1 and be progressing towards a Level 2; and that students in grades sixth through eight should have achieved a Level 2 and be ready for Level 3 (NCTM, 2000). Following this progression, NCTM argued that, by high school, students would be ready to perform at Level 3. NCTM has played a significant role in transforming geometry instruction over the past three decades by developing and refining geometry standards for learning.

The 1989 NCTM standards and the 2002 No Child Left Behind Act (NCLB) greatly impacted the development and revision of state-level mathematics standards (Dingman et al., 2013). According to Long (2003), by 1997, the NCTM standards had influenced the academic standards found in many states. These standards were typically broad statements of learning that did not specify the exact grade level in which specific instruction and learning should occur; instead, the standards were typically written using grade bands (Dingman et al., 2013). The creators of the 2002 NCLB Act revised state standards by requiring them to be grade-level specific for grades 3-8 (Dingman et al., 2013; NCLB, 2002). The influence of NCTM's standards and the NCLB Act on state-level math standards are still recognized today.

In 2009, the National Governors Association and the Council of Chief State School Officers created the Common Core State Standards Initiative (CCSSI) to develop a set of standards that would provide consistency in the academic standards taught in the United States (Dingman et al., 2013). Their work developed a new set of standards known as the Common

Core State Standards for Mathematics (CCSSM). The CCSSM once again increased the emphasis on geometry instruction by realigning when certain geometric concepts should be taught and by recommending additional geometry concepts to be taught. The CCSSM geometry standards represented a significant shift in geometry instruction (Teuscher et al., 2015). According to Teuscher et al. (2015), this shift has been evident through research studies such as Confrey & Krupa (2010), Dingman et al., (2013), and Reys et al., (2012). The development of the CCSSM presented a new era of geometry instruction; however, the CCSSM were influenced by the van Hiele model (as discussed in future sections of this literature review), which had been developed decades earlier. The CCSSM developed rigorous standards for everyone who chose to adopt them.

Researchers have completed several studies comparing the rigor of the CCSSM to the previous state-level standards. Jill Newton (2011) determined that 47% of the state-level standards in the 42 states she examined contained geometry standards at a van Hiele Level 1 (using the renumbered Levels), and 49% were at a van Hiele Level 2. Just 4% of the state-level standards were at a van Hiele Level 3. Likewise, she determined that 20% of the CCSSM standards were at a van Hiele Level 1, 50% were at a van Hiele Level 2, and 30% were at a van Hiele Level 3. Her study showed that the greatest relative frequency of these standards occurred in grades 3-5 (Dingman et al., 2013). These findings indicate the shift in the geometry standards and the greater emphasis the CCSSM geometry standards place on higher van Hiele levels, as seen by the increase in the number of standards at Level 3.

A 2022 study by Zhou et al. analyzed geometry standards in the CCSSM by using what they referred to as Descriptive Learning Expectations (DLE). In their study, each focus of a standard was a DLE. For example, if a standard had two foci, they considered it to have two

DLEs. Using the renumbered system of van Hiele levels, they determined that 32% of the DLEs for grades Kindergarten through third were Level 1, 32% were Level 2, and 37% were Level 3. They determined that 0% of the DLEs for grades four to six were Level 1, 33% were Level 2, and 67% were Level 3. Finally, for grades 7 and 8 and high school, they determined that 0% of the DLEs were Level 1, 5% were Level 2, 75% were Level 3, and 20% were Level 4. Through the analysis of the CCSSM geometry standards, they determined that the CCSSM place more emphasis on informal deduction in multiple ways, the CCSSM promote the usage of geometry software, and the CCSSM provide an opportunity for students to share their thought processes.

The research shows that there has been a shift in the academic standards and expectations for geometry instruction. With a greater emphasis on geometry standards, and specifically on standards at higher van Hiele levels, teachers need to find ways to support students in their geometric reasoning skills. Teachers can do this by aligning their instructional methods with the van Hiele framework.

van Hiele Theory and Framework for Geometric Thinking

While Americans were making shifts in their geometry instruction during the 1950s, developments of new ideas of geometry instruction were taking shape across the Atlantic Ocean in the Netherlands. The ideas belonged to Pierre van Hiele and Dina van Hiele-Geldof.

The van Hiele Theory is one existing theory that pertains to geometric reasoning. It was developed by husband-and-wife duo Pierre van Hiele and Dina van Hiele-Geldof in 1957. The van Hieles had been high school geometry teachers in the Netherlands. As a high school geometry teacher, at a Montessori school and later at a traditional school, Pierre made efforts to help facilitate his students' geometric reasoning skills (Mathews, 2004). As part of their dissertation studies at the University of Utrecht, they used their previous experiences as geometry teachers to research the process of learning geometry and the struggle many students experience (Connolly, 2010). Through his experiences, he realized that the mere learning of facts, often delivered by the teacher even if students were not ready to understand such facts, did not develop insight (van Hiele, 1986). This realization led Pierre and his wife to develop their theory of geometric reasoning.

Development of the Theory

The doctoral work of Pierre and Dina was separate, but they both focused on the same topic to develop a new theory behind the learning and reasoning of geometry. Pierre's work focused on developing five discrete levels that students must progress through to gain reasoning skills in geometry. Dina's work focused on five phases of instruction that students must go through within each level (as proposed by Pierre) to advance to the next level of geometric reasoning (Fuys et al., 1988). Both the levels and phases are the two critical components of the van Hiele theory.

High school and upper-level geometry courses require students to prove geometric theorems. The van Hieles believed that for students to be successful in these courses, they need to go through the levels and phases of learning to achieve the ability to prove the geometric theorems. In 1959, Pierre van Hiele said:

> The student learns by rote to operate with [mathematical] relations that he does not understand, and of which he has not seen the origin...therefore, the system of relations is an independent construction having no rapport with other experiences of the child. This means that the student knows only what has been taught to him and what has been deduced from it. He has not learned to establish connections

between the system and the sensory world. You will not know how to apply what he has learned in a new situation. (p. 62)

The five levels of the van Hiele theory are visualization, analysis, informal deduction, formal deduction, and rigor. The modified levels are as follows:

- *Level 1 (Basic Level): Visualization.* At this level, students have a visual understanding of shapes. Students are aware of space and do not use properties to identify a shape; instead, a geometric figure is viewed as a particular shape. Students at this level can learn vocabulary, reproduce figures, and identify specified shapes. For example, a student will recognize a square because it looks like a square, not because it has specific geometric properties.
- *Level 2: Analysis.* At this level, students begin to use analysis to identify figures by their geometric properties, not just their overall appearance. The analyses happen through the process of observation and experimentation. While students at this level can see different properties, they do not see the relationships between them. As provided by students at this level, definitions of figures contain more than the minimum essential information. For example, a student will recognize that a square has all congruent sides, four right angles, or two pairs of parallel sides. Another example is that a student will be able to recognize that opposite angles of a parallelogram are equal.
- *Level 3: Informal Deduction.* At this level, students recognize the importance of properties. Students can order properties and see the relationships between them, such as when a student sees that opposite sides are parallel, they also know that the opposite angles must be congruent. Students can use class inclusion to identify figures and make informal arguments using definitions. For example, a student will be able to apply the

definition of a rhombus to different figures to determine which figures are rhombi and which are not. The student will recognize and justify a square as a rhombus through this process.

- *Level 4: Formal Deduction.* At this level, students understand the significance of deduction as a means for developing geometric theory within an axiomatic system. Students understand the interrelationships and the role of definitions, terms, axioms, postulates, and theorems. They can see proof. Because of these understandings, they can reason logically and create proofs through meaning. For example, a student can construct a proof, not just recite a memorized proof. The student can also recognize that they can develop the proof in more than one way, understand the interaction between necessary and sufficient conditions, and see differences between statements and their converse. A student can prove that a square is a rhombus through these processes. Most high school geometry courses are taught at this level.
- *Level 5: Rigor.* At this level, students see geometry as being abstract. They can analyze and prove theorems in different postulational systems. Students can now study Non-Euclidean geometry. For example, students can use parallel postulates to make geometric determinations (Burger & Shaughnessy, 1986; Crowley, 1987; van Hiele, 1986).

According to Susan Connolly (2010), “Each level has its own set of terminology, symbols, concepts, and reasoning strategies” (p. 9). Due to each level being distinct, it is hard for someone at a lower level to understand the processes and language of a higher level. Likewise, it is hard for someone at a higher level to recall the thinking that occurs at a lower level (Usiskin, 1982). According to Usiskin (1982), what is intrinsic at one level is extrinsic at the next; hence, individuals at different levels will be unable to understand each other. Students, as proposed by the van Hieles, progress sequentially through the discrete levels by going through the five instructional phases. When Pierre van Hiele first introduced this concept, he said:

> You can say somebody has attained a higher level of thinking when a new order of thinking enables him, with regard to certain operations, to apply these operations on new objects. The attainment of the new level cannot be affected by teaching, but still, by a suitable choice of exercises, the teacher can create a situation for the pupil favorable to the attainment of the higher level of thinking. (van Hiele, 1955, p. 289 as cited by van Hiele, 1986, p. 39)

The five identified instructional phases a student must pass through to advance from one level to the next are inquiry/information, directed orientation, explication, free orientation, and integration. Each phase provides students with appropriate language and thought processes for the instruction to be meaningful (Fuys et al., 1988). The instructional phases of the van Hiele theory are as follows:

- *Inquiry/Information*. During this phase, the teacher uses discussion with the students to determine what prior knowledge the students have regarding the concept. Using this information, the teacher will develop instructional activities for the student's current level. The teacher will also introduce the concepts that will be studied and will use level-specific vocabulary. For example, a teacher may ask, "What is a square? What is a rhombus? How are they alike? How are they different? Could a square be a rhombus? Could a rhombus be a square?"
- *Directed Orientation.* During this phase, the teacher engages students in carefully sequenced activities at their level to build awareness in the students of the concepts being taught. Through the progression of activities, students will see the structures

characteristic of their level. Students must use materials and geometric figures as part of the activities. Through the manipulations of the materials and geometric figures, students can develop a deeper understanding of the concepts. For example, a teacher may tell students to use a virtual geoboard to construct a parallelogram, then make a second construction by creating a similar parallelogram to the original but a larger one, and a third construction by creating a similar parallelogram to the other two but smaller in size.

- *Explication*. During this phase, the students become self-reliant. They express their observations through words from the manipulations of the materials and geometric figures. During this phase, teachers introduce the technical terms and vocabulary appropriate for the student's level, but otherwise, there is minimal input from the teacher. For example, the students will share with their peers and the teacher what properties and figures came from manipulating the parallelogram in the above example.
- *Free Orientation*. During this phase, teachers give students tasks that can be approached through various methods, open-ended tasks, and tasks that require multiple steps to be solved. Through the tasks, students orient themselves with geometric figures. Students are focused on something other than real problem-solving. During this phase, they focus on ordering the manipulations required to accomplish the task. The student's focus leads to them explicitly recognizing relationships between the objects. During this process, the teacher is attentive to the student's creativity. For example, students may be required to fold a piece of paper into quarters and then visualize what shape may be made if they cut off the corner that adjoins all four quarters. Students would need to justify their response and then think critically about what type of figure may be created depending on the angle of said cut. The students might then have to think critically about the intersection of the

diagonals. They may be asked to make a conjecture as to why the area of a rhombus can be determined by taking one-half of the product of the diagonals.

- *Integration*. During this phase, students summarize what they learned during the lesson. The teacher may summarize the students' actions to spark conversation, but the teacher should refrain from presenting new information. Students can understand their manipulations and develop an insight into the operation. For example, students will be able to state the properties and origins of a parallelogram or rhombus that emerged during the phases of learning (Crowley, 1987; van Hiele-Geldof, 1957/1984).

Modification to the Theory

Since its creation in 1957, the van Hiele theory has undergone one modification. According to Pegg & Davey (1998), the levels that van Hiele numbered 0 to 4 did not account for individuals whose reasoning skills were below those defined by Level 0. van Hiele believed that everyone was at least at a Level 0 (Senk, 1989). Van Hiele’s error in this conjecture was that the students he studied were all in high school; he did not study younger students. It was discovered that some individuals, especially younger students, fell below Level 0. To account for this discovery, van Hiele and other researchers renumbered the levels from 1 to 5 (Pusey, 2003). Anyone not at the current Level 1 is now assigned a Level 0, which is considered the prerecognition level. Battista (2002) described this new level, the prerecognition level, as “students are unable to identify many common shapes because they do not adequately attend to all of a shape’s visual characteristics” (p. 334).

van Hiele Theory in the United States

The United States began to take an interest in restructuring geometry education, using the van Hiele theory, in the 1980s. In 1980, the National Science Foundation supported a process to translate the publications and projects written by the van Hieles (Fuys et al., 1988). During this time period, the Cognitive Development Achievement in Secondary School Geometry (CDASSG) Project looked at how they could transform the van Hiele model from just a theory into a practical and predictive assessment (Connolly, 2010). The developers and implementers of the assessment tool gained insight into the usefulness of the van Hiele theory. One such insight was that "in geometry classes that study proof, the fall van Hiele levels of over half the students are too low to afford even a 2 in 5 chance of success at proof" (Usiskin, 1982, p. 84). In 1986, Burger and Shaughnessy conducted a study by interviewing students in various grade levels to determine the usefulness of the theory in describing student geometrical thought processes (Connolly, 2010). They concluded that the van Hiele levels are useful in describing students' thinking processes on geometry tasks, that the levels can be characterized operationally by student behavior, and that an interview procedure can be developed to reveal predominate levels of reasoning on specific geometry tasks (Burger & Shaughnessy, 1986). In 1988, a project led by Fuys et al. focused on the instructional aspects of the van Hiele theory (Fuys et al., 1988). The findings of these studies in the 1980s caused the United States to take notice of the van Hiele theory.

The study by Burger and Shaughnessy (1986) did find some differences with the original model. While the van Hieles believed the levels to be discrete, Burger and Shaughnessy's (1986) results indicated that the levels might be more continuous than the van Hieles thought, as it was hard to classify and pinpoint an exact level in some instances as students oscillated between levels; sometimes they even oscillated on the same questions. They concluded that this behavior makes sense for students transitioning between levels, especially between Levels 1 and 2 (Pusey, 2003). Other researchers have supported the notion that the levels may be more continuous than the van Hieles had thought, as can be seen by Clements et al. (2001), Fuys (1985), Fuys et al. (1988), and Usiskin (1982). In addition to Burger and Shaughnessy's (1986) findings that students could be in transition stages between levels, they also found that some students do function at the formal deduction level (Pusey, 2003), which was also supported by Mayberry (1983) and Usiskin (1982). They also discovered that students may be at different levels for geometric topics and tasks. This discovery was also supported by Mayberry's (1983) study. Lastly, their studies had evidence that the levels are hierarchical, which aligns with the findings of Mayberry (1983) and Fuys (1985). Other research studies supported the differences that Burger and Shaughnessy (1986) discovered.

Other research studies have been conducted to test the discreteness of the levels, as described by the van Hiele theory. One research study conducted by Gutierrez, Jaime, and Fortuny (1991) found that students could be at more than one level simultaneously. They argued:

> The way of evaluating thinking levels explained in this article allows for the possibility that a student can develop two consecutive levels of reasoning at the same time, although what usually happens is that the acquisition of the lower level is more complete than the acquisition of the upper level. (Gutierrez et al., 1991a, p. 250)

A research study conducted by Gutierrez, Jaime, Burger, and Shaughnessy (1991) also found that the levels were more discrete than initially proposed. While these research studies found varying results concerning the discreteness of the levels, these same studies found the van Hiele model to be effective for geometry instruction.

Additional Research involving the van Hiele Theory

Researchers have conducted several research studies over time to test the effectiveness and validity of the theory. The first major study to be conducted with the van Hiele model was conducted in 1982 by Usiskin (Fuys, 1985). Using a multiple-choice test, Usiskin found a moderately strong relationship (r = 0.64) between students' van Hiele level and their geometry grades. He also found that most of the high school sophomores in the study were at Level 0 or 1 (using the original levels) (Pusey, 2003).

Sharon Senk (1989) conducted a study to see if a student's van Hiele level could be used as a predictor of success in writing geometric proofs. Senk concluded that her results indicated that a student's van Hiele level did impact the student's success in high school geometry. Her study implies a need for strong geometry instruction, with several opportunities for students to engage with geometry in the lower levels of the van Hiele model and for this instruction to take place prior to formal geometry courses in high school (Pusey, 2003).

In addition to Usiskin and Senk, other researchers have conducted many additional studies. Another study by Mayberry (1983) determined that the van Hiele levels are hierarchical, which was supported by the research conducted by Fuys et al. (1988). Mayberry (1983) also concluded that students could be on different van Hiele levels for different geometric concepts, which contradicts Pierre van Hiele's beliefs. The research conducted by Usiskin (1982) found that students can be assigned a van Hiele level, as proposed by Pierre van Hiele, but that students may be in transition from one level to the next, and it is those students who may not reliably be assigned to one level or another. Researchers have conducted many research studies to examine the effectiveness of the van Hiele theory and have collected data that both supports and extends the work of van Hiele.

Use of van Hiele Theory in Elementary Schools

While researchers have conducted several studies to test the effectiveness and validity of the van Hiele theory, some studies have specifically analyzed the implementation of the theory in elementary and middle schools. Research by Carroll (1998) and Mistretta (2000) has provided a voice to this piece of the conversation. Carroll (1998) found that when a shift in fifth and sixth-grade geometry moved the emphasis from isolated properties of shapes to a focus on the relationships among the properties, the students made significant gains in van Hiele levels compared to the students who were not in the study. Mistretta (2000) found that using open discussion and hands-on experiences with eighth-grade students helped them significantly improve the van Hiele levels. What are the implications of these studies for elementary and middle school geometry instruction? They show that when students are actively engaged in the learning process, the focus of the learning is turned to the relationships among the properties of the figures, which allows students to develop an adequate understanding of geometric relationships as outlined by the van Hiele theory. Research shows that the van Hiele theory is practical and can be implemented in elementary and middle schools. As a result of these and other research studies, the van Hiele theory was used to significantly influence the creation of the NCTM standards and CCSSM (Vojkuvkova, 2012).

NCTM has argued in favor of using the van Hiele theory as a framework for geometry instruction. NCTM's 1987 Yearbook was dedicated to geometry, as it was titled *Learning and Teaching Geometry, K-12.* In the first chapter, which was entitled *The van Hiele Model of the Development of Thought*, Crowley stated:

> The model of geometric thought and the phases of learning developed by the van Hieles propose a means for identifying a student's level of geometric maturity and suggest ways

> to help students to progress through the levels. Instruction rather than maturation is highlighted as the most significant factor contributing to this development…The need now is for classroom teachers and researchers to refine the phases of learning, develop van Hiele-based materials, and implement those materials and philosophies in the classroom setting. Geometric thinking can be accessible to everyone. (p. 15)

The van Hiele model provides a framework for geometry instruction but allows teachers creativity when designing their instruction. Teachers may use a variety of activities, manipulatives, and learning aides as they provide instruction using the five phases of instruction that are central to the van Hiele theory. Researchers and mathematicians have provided examples of teachers' instructional geometry activities that align with the van Hiele model.

Jenni Way (2011, Revised 2019) provided an activity in her article “The Development of Spatial and Geometric Thinking: the Importance of Instruction” that she wrote for the University of Cambridge’s NRICH website, which provides free resources from the University of Cambridge math faculty to K-12 educators. The activity that she suggested is for students around the ages of 7 to 10. Students would be given a typical seven-piece tangram set. During Phase 1, students would explore the pieces and discover some of their properties and structures. During Phase 2, the teacher could encourage students to see if all the larger pieces could be made by combining two of the smaller pieces. For Phase 3, students would learn the names and terminology to describe the individual pieces. Students would then be asked descriptive questions about the pieces, such as which have right angles, parallel sides, and so on. During Phase 4, students may be required to compose a square using the pieces in as many ways as possible. For Phase 5, the students would share with each other what they learned, or they may be asked to write everything they know about a given shape. This lesson plan progresses students from recognizing shapes to being able to discuss their specific geometric properties and find relationships between the shapes.

Researchers have developed and published other lessons and activities. For example, Show et al. (1995) developed concept diagrams to promote students' understanding of geometric figures and their relationships. Naylor (2000) suggested that students use dot arrays to draw quadrilaterals and sort them by their attributes. Barkley and Cruz (2001) used the phases of learning in the van Hiele theory to create Ute Indian beadwork designs for a fifth-grade geometry unit. Demsey (2002) developed the idea for Kindergarten students to use stamps to realize that the shape on the stamp is always the same, no matter how it is oriented. Woleck (2003) used her "Tricky Triangles" activity with her first-grade students to help them transition from Level 0 to Level 1. The triangles in her activity were not equilateral and were oriented differently.

Crowley (1987) offered examples of activities that promote learning and geometric reasoning using the van Hiele framework. She said:

> Teachers in the early elementary years can provide basic-level exploratory experiences through cutouts, geoboards, paper folding, D-sticks, straws, grid work, tessellations, tangrams, and geometric puzzles. Middle school and junior high school experiences, roughly at Levels 1 and 2, can include working with grids, collections of shapes, "property cards," "family trees," and "what's my name" games. (p. 6)

Crowley listed relatively common items in her recommendations for activities and materials. Teachers can easily access the materials she recommended and use them in learning activities for students to learn geometric reasoning skills through the van Hiele framework.

In addition to choosing appropriate instructional methods, learning tools and materials, and activities, educators need to be aware of the alignment of van Hiele levels in the many different math curricula that are present. According to van Hiele (as cited in Fuys et al., 1988), "The Brooklyn College Project investigation made clear that in the geometry materials in grades K-8 textbooks, the van Hiele levels are mixed up—not sequenced—and because of this, the higher levels are rarely reached" (p. iii). Whitman et al. (1997) showed ten years later that only 29% of lessons in grade 6 were at a van Hiele Level 1 (the percentage is much less for each lower grade level), and only 1% of the lessons in sixth-grade textbooks in the United States were at a van Hiele Level 2. The study also found that American textbooks for grades 1 through 5 contained 0 lessons at a van Hiele Level 2. What are the implications of these studies? Teachers and schools need to be sure they choose math curricula that incorporate lessons at higher van Hiele levels and that they contain lessons that are sequenced with the van Hiele levels if students are going to be prepared to enter high school geometry classrooms, in which most of the content is taught at a van Hiele Level 3, using the original numbering system.

Symbolic Interactionism

Many researchers have contributed to and shaped the theory of learning known as symbolic interactionism. Herbert Blumer, a student of the well-known sociologist and psychologist George Herbert Meade, is credited with developing symbolic interactionism. (Aksan et al., 2009; Carter & Fuller, 2015). Blumer extended Mead's work on constructionism by stating, "because individuals make meaning of objects based on their own understanding of the objects, any acts towards objects, things, events, and interactions are informed by the meaning an individual makes" (Bhattacharya, 2017, p. 60). While Meade and Blumer are the most common names associated with symbolic interactionism, many other researchers have also contributed to the symbolic interactionism theory. These researchers include John Dewey, Robert Park, Charles Cooley, Manford Kuhn, and Sheldon Stryker (Aksan et al., 2009; Carter &

Fuller, 2015). Each researcher who contributed to symbolic interactionism held to its central theme, though they each held their own perspective of symbolic interactionism.

Symbols play an integral role in this theory. They cultivate direct facts. Researchers theorize that people live in two different environments: natural and symbolic (Aksan et al., 2009). According to Aksan et al. (2009) and Bhattacharya (2017), symbolism is entirely contingent upon what each person determines to be symbolic, tangible, or anything else that the individual interacted with and derived meaning from. Examples of symbols include language, dialogues, and policies. Everyone determines what is a symbol and therefore determines what has significant meaning for them.

As the name of the theory denotes, in addition to the symbols themselves, there is an interactive component to the theory. Blumer also stated that since people use an interpretive process while making meaning of objects, they constantly adjust their understanding, considering their continued interactions (Blumer, 1986). Blumer believed that meaning is derived from objects, events, and phenomena. He also believed that interaction with others creates the condition of meaning rather than the symbol being an intrinsic feature (Aksan et al., 2009).

In short, symbolic interactionism explains how people interact with tangible and intangible symbols that create meaning for the individuals in their lived experiences (Bhattacharya, 2017). Humans build meaning from their experiences, and as they have more experiences, their meanings may change through an interpretative process (Aksan et al., 2009; Carter & Fuller, 2015). Blumer assumed that “humans act toward things on the basis of meanings these things have for them” (Bhattacharya, 2017, p. 60). The tenets of symbolic interactionism that relate directly to this research study are that “individuals act based on the meanings objects have for them" and "meanings are continuously created and recreated through interpreting processes during interaction with others" (Carter & Fuller, 2015, p.2).

Symbolic interactionism and the van Hiele theory have many similarities. Both are theories of learning that involve symbols. The notion that symbols lead to a student's learning and understanding is central to both theories. Regarding mathematics, both theories intersect neatly. Dynamic geometry software can utilize symbols from which students can interact and make meaning. DGS can be used in different ways to promote student engagement with geometry and, in effect, with the various levels of the van Hiele theory. In summary, the theoretical framework of symbolic interactionism and the van Hiele theory align well with this research study's purpose, methodology, and methods.

Technology in the Mathematics Classroom

NCTM argues in its 2000 publication of *Principles to Standards* and its 2014 publication of *Principles to Actions* that technology should be used in the classroom. For instance, NCTM argues that "an excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking (NCTM, 2014, p. 78). Within the same publication, NCTM states, "Computers, tablets….make available a range of applications that support students in exploring mathematics as well as in making sense of concepts and procedures and engaging in mathematical reasoning" (NCTM, 2014). It is evident through *Principles to Standards* (2000) and *Principles to Actions* (2014) that NCTM views technology usage as important in the mathematics classroom.

Additional researchers and publications have discussed the importance of using technology to teach geometry. de Villiers (1998) found that using computers in the math classroom makes experimentation feasible in many areas of mathematics, including geometry. NCTM geometry standards indicate a need for students to engage with the appropriate tools during geometry instruction (Matthews, 2004; NCTM, 2000). Finally, the CCSSM encourage technology to be used in the teaching and learning of mathematics through tools such as dynamic geometry software (Zhou, 2022).

The Role of Dynamic Geometry Software

Dynamic geometry software is a specific technology that has become more prevalent in America's mathematics classrooms over the past several years. DGS, like Geometer's Sketchpad and GeoGebra, can provide experiential learning for students by enabling them to see geometric patterns as they emerge (Connolly, 2010: NCTM, 2000; NCTM, 2014). According to Connolly (2010), instead of being constrained by the available static examples, students can encounter an "endless continuum of cases within moments" (p. 24). From these encounters, students can make constructions, modify their constructions, or work with previously developed content to make discoveries. The discoveries lead to conjectures, and depending on the feedback that students receive from the DGS, students can refine their conjectures. According to Sher (2002), the DGS affords students the ability for them to focus on their visual observation skills more so than when they engage with static examples. DGS has provided students with many affordances.

According to Olive (1998), DGS provides a new avenue for learning geometry. Unlike traditional approaches that build up geometry through axioms, definitions, and theorems, DGS allows students to engage with geometry concepts in real-time, allowing students to explore different phenomena. Through this exploration, observation, and conjecturing, students can use inductive reasoning to build mathematical relationships and meanings for themselves. The affordance of real-time learning provides students with new levels of engagement with geometric concepts.

DGS can be implemented in several different ways in the classroom. As mentioned earlier, students can make their own constructions or work with previously created content. This content can include drawings in which students can explore lengths and angles and make conjectures (Connolly, 2010). Figures can be created ahead of time for students to explore theorems. Students can examine the relationships of geometric figures. The possibilities of DGS are unmatched in comparison to static examples. Goldenburg and Cuoco (1998) described this comparison by stating:

> In fact, except for what students do in their heads, paper-and-pencil geometry also 'involves only action and does not require a description.' Part of what students learn in geometry is, as Poincaré put it, the art of applying good reasoning to bad drawings—adding the descriptions that specify which features or relationships in the drawings are intended, which are incidental, and which are to be totally ignored as they attempt to draw inferences about the figure depicted. (p. 365)

Technology can also be a tool used in geometry in accordance with the van Hiele model. Several studies have looked at this relationship. A 1998 study by Parsons et al. examined if computer-assisted instruction could improve the van Hiele levels in students up to Level 2. Using a pretest-posttest design with the van Hiele test created by Usiskin (1982), they found that the computer-assisted instruction did help students progress to Level 2. A 2001 study by Clements et al. used the Logo programming language to examine how elementary students learn geometric concepts. They found that students who used the software exemplified higher gains in describing properties of shapes at Level 1 than those who did not learn using it (Pusey, 2003).

Students can use dynamic geometry software to explore relationships. The transition to seeing the relationship between properties and figures shows a mastery of van Hiele Level 2. The studies by Parsons et al. (1998) and Clements et al. (2001) are two studies that show how technology can be used to help students progress through the van Hiele levels.

Chapter Summary

Geometry is a form of mathematics that has been around for centuries and continues to play an important role today. Former NCTM President J. Michael Shaughnessy said, “Geometry provides an incredibly rich medium for both mathematical content and mathematical reasoning” (Shaughnessy, 2011, para. 4). There have been many shifts in mathematics education in the United States, including the emphasis placed on geometry instruction. It was not until the latter half of the twentieth century that Pierre van Hiele and Dina van Hiele-Geldof developed their theory of geometric reasoning. The van Hiele theory played a prominent role in transforming and emphasizing geometry instruction in the United States. At the start of the twenty-first century, the emphasis on geometry instruction was once again realigned through the development of more stringent standards for geometry, standards that used the van Hiele model as their framework, in American schools as developed by organizations like NCTM, the CCSSI, and by departments of education in individual states.

Since its inception, several studies have been conducted to test the effectiveness of the van Hiele model. While there is some disagreement regarding the discreteness of the levels, researchers tend to support the van Hiele model, believing that its levels and phases lead to effective geometry instruction and hence is effective in developing geometric reasoning skills in students. Considering current research regarding the effectiveness of the van Hiele framework and the understanding that geometry places a crucial role in many facets of life (not just in the high school geometry classroom), curriculum and instruction in elementary and middle school needs to support the growth of and progression of students through the van Hiele levels in order for students to be successful in high school geometry courses and beyond.

NCTM and the CCSSM encourage the use of technology for the instruction of mathematics. Technology has become more prevalent in America's classrooms, especially since NCTM publicized *Principles to Standards* (2000) and *Principles to Actions* (2014) and since the CCSSM were developed. Another reason technology has become more prevalent in the classroom is that technology has become normalized in our society. Lastly, technology has become more prevalent in classrooms due to the COVID-19 pandemic, which forced schools to adapt to provide instruction using technology during periods of the lockdown. Specifically, DGS is a technology that can aid with geometry instruction. Research studies have shown that DGS can help students develop their geometric reasoning skills as they progress through the van Hiele levels of geometric reasoning.

Through a review of the current literature, it is evident that, as a whole, geometry instruction in the United States has yet to be fruitful in helping students to grow in their geometric reasoning skills. The van Hiele theory is one theory of learning that could be used more prevalently as a framework for geometry instruction in the United States. We must find ways to provide more effective geometry instruction, as learning geometry is essential in aiding students with visualization, manipulation, image segmentation, correction, and object representation. It also aids in the development of deductive reasoning skills. One of our nation's leading math education organizations, NCTM, has stated that students in grades three through five should have mastered Level 1 and progress towards Level 2 by the time they enter grade 6. Symbolic Interactionism provides a framework for which dynamic geometry software can be used as an instructional tool to help students progress through the van Hiele levels of geometric reasoning. The progression can happen as students manipulate objects and make conjectures about the relationships between the objects. Combining the van Hiele framework with the usage of dynamic geometry software may be a solution to providing more effective geometry instruction to fifth-grade students in the United States. This study contributed to the current literature by studying the use of dynamic geometry software and its effect on fifth-grade student progression through the first three levels of the van Hiele model of geometric reasoning.

Chapter 3 - Methodology

Purpose of Study

This study explored the usage of GeoGebra and its effect on the progression through the first three levels of the van Hiele model of geometric reasoning for fifth-grade students at an elementary school in the Midwest. The following sections describe this study's design and methodology, including a rationale for the instruments used to measure the effects of GeoGebra on the progression of the van Hiele levels of geometric thinking of fifth-grade students. Additionally, the study's research questions are found below.

Quantitative Questions

1. What effect does utilizing GeoGebra during geometry instruction have on fifth-grade students' van Hiele level of geometric thinking about polygons, as measured by a pre- and posttest?
    a. Are there differences based on gender?
    b. Are there any differences based on prior achievement?

Qualitative Questions

2. What are students' experiences using GeoGebra during geometry instruction?
    a. How do students experience using tools available in GeoGebra?
    b. When students are observed using (or not using) tools available in GeoGebra, what is their level of geometric thinking?

Mixed Methods Question

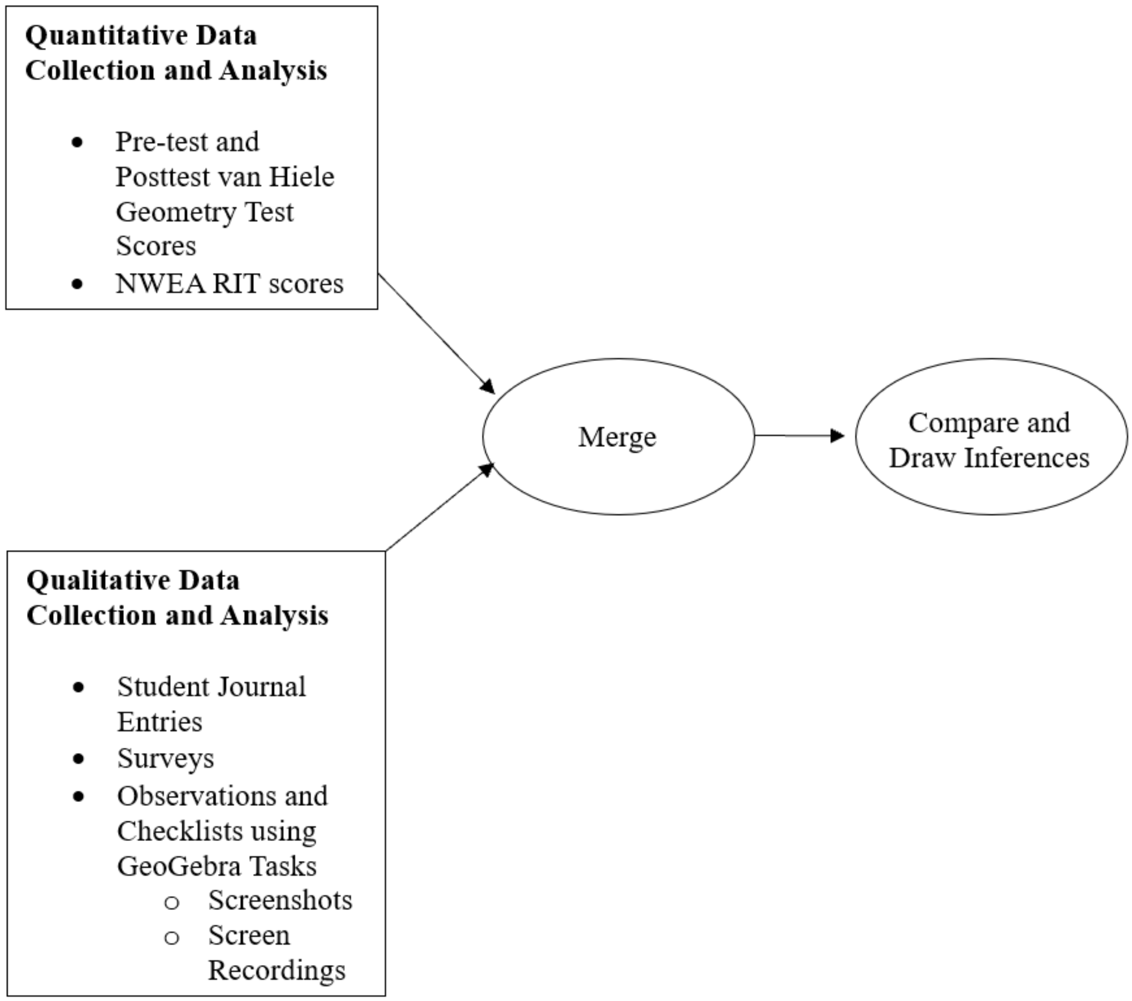
3. What is the impact of using GeoGebra to support fifth-grade students' learning about polygons?

Research Design and Methods

Research Design

The purpose of this study was to investigate the effect the usage of GeoGebra has on the progression of fifth-grade students through the first three van Hiele levels. This mixed-method study used a triangulation design, which analyzes multiple sources of data to gain a comprehensive understanding of the research questions and the phenomena that surrounds them (Creswell et al., 2003). According to Morse (1991), a triangulation design is used “to obtain different but complementary data on the same topic” (p. 122). More specifically, this research study used a convergent mixed methods design (Creswell, 2022). Creswell and Plano Clark (2007) stated, “The use of quantitative and qualitative approaches in combination provides a better understanding of research problems than either approach alone” (p.5). In other words, neither approach alone would have allowed me to investigate the research questions fully; however, by implementing both approaches, I capitalized on the strengths that are inherent to each approach in order to study the research questions more fully. As shown in Figure 3.1, using a convergent mixed methods design allowed me to collect and analyze the quantitative data separately from the qualitative data. Finally, I merged the two findings to make comparisons and draw inferences.

Figure 3.1. Convergent Mixed Methods Design



Adapted from Creswell (2022).

The quantitative design associated with this research study was quasi-experimental, utilizing a pretest-posttest design. This design allowed me to see if there was a change in the geometric reasoning levels over the course of the geometry unit for each of the participants. The qualitative design used an observational case study as the methodological framework. According to Bhattacharya (2017), the case study approach allows a researcher to "uncover contributing reasons for cause-and-effect relationships" (p. 109). The case study methodology allowed me to look for an in-depth understanding of how using GeoGebra may affect students' ability to progress through the van Hiele levels. The qualitative approach through the case study design was imperative to this study as it allowed me to analyze the actions and mathematical thought processes of each of the participants as they sorted and constructed polygons using GeoGebra.

Robert E. Stake and Sharan Merriam are two seminal authors contributing to case study research. Both researchers use interpretivist/constructivist approaches to their ideas of case study. Stake believes that constructivism and existentialism should be applied as the epistemologies researchers use to design their case studies (Stake, 1995). Like Stake, Merriam also argues that constructivism should be the epistemological orientation used in case studies since it is "the key philosophical assumption upon which all types of qualitative research are based is the view that reality is constructed by individuals interacting with their social worlds" (Merriam, 1998, p. 6). Stake's views of case study contribute to the idea that researchers are interpreters and gatherers of interpretations. Like Stake, Merriam believes that the construction of knowledge is a layered process (Yazan, 2015). Merriam and Stake present constructivist paradigms through their epistemological orientations of case study research, which pair well with symbolic interactionism and the van Hiele theory, the two theories used for this research.

This study used a mixed methods, triangulation design to accurately analyze the effect the usage of GeoGebra has on the progression of the van Hiele levels of fifth-grade students. The pretest-posttest design was used to conduct statistical analyses to determine the effect of using GeoGebra on the progression of geometric reasoning and thinking. Observations and checklists were used within the case study design to see which GeoGebra tools students used as part of their learning process and the development of geometric reasoning skills. They were also used to help code the indicators, as Burger and Shaughnessy (1986) proposed, that students demonstrated for each van Hiele level while using GeoGebra. In addition to using the level indicators as checklists, I used a survey and the students' journal entries to gather data.

Methods

Three methods were used as part of the research design: quantitative, qualitative, and mixed methdods. The quantitative method used a pretest-posttest design. This design was appropriate because it allowed the effects of using GeoGebra on student progression through the first three van Hiele levels to be analyzed. This design also allowed for comparing two groups to determine GeoGebra's effect on the progression through the first three van Hiele levels. The qualitative methods used were journal entries that students composed after completing each study activity, a survey that students completed regarding their experiences with using GeoGebra, and a list of level indicators used for observations and checklists, as observed through screen recordings and screenshots, through the case study design. This design was appropriate as it allowed me to analyze the geometric reasoning levels for the students by looking for specific indicators for each van Hiele level, which could not be measured with a pretest-posttest design. The qualitative methods also allowed me to see which, if any, affordances provided by GeoGebra were used as students completed the tasks to demonstrate geometric reasoning. The mixed methods allowed me to converge the quantitative and qualitative data together to gain a fuller understanding of GeoGebra's effects on students' geometric reasoning abilities.

Participants

The participants in this study were fifth-grade students at an elementary school located in the Midwest of the United States. The nature of this study required fifth-grade students who had access to iPads and GeoGebra. Because I teach fifth-grade students who met this criterion, each of the participants (*n* = 38) came from my math classes. Of the 38 participants, 18 were female and 20 were male. The students took a pretest before geometry instruction and then a posttest after geometry instruction. Throughout the research study, students wrote journal entries after each study activity in order to record their thoughts, feelings, and experiences with GeoGebra and the geometry content associated with the study activities. Students completed five geometry tasks using GeoGebra at the conclusion of the unit, as well as a survey, which allowed me to gain perspectives from the students. I gathered consent from the parents and the participants' assent to use the participants' test scores and data collected during instruction and from the surveys and geometry tasks as part of the data analysis.

For the qualitative data analysis, 14 students were selected from the group of 38. Hennink and Kaiser (2022) provided a description of saturation for qualitative research. I determined that 14 students met the saturation level for qualitative research for this particular study. I chose to use stratified sampling to ensure that students from each of the van Hiele levels, as determined by the quantitative results, were selected. After completing the quantitative data analysis and assigning students levels, I selected both students who were assigned a Level 0. The remaining students were chosen at random within each strata or van Hiele level. I choose four students from each of the remaining three van Hiele levels. This helped to ensure that I had a good representation and spread of geometric reasoning abilities in the qualitative study.

The population was selected from a rural public school in Northeast Indiana that serves pre-kindergarten through fifth-grade students. During the 2023-2024 school year, the elementary school had an enrollment of 647 students. The population was predominantly white, as these students comprised 87.3% of the student population. Hispanic students comprised 9.3% of the enrollment, multiracial students comprised 1.7%, and combined, American Indian, Asian, Black, and Native Hawaiian or Other Pacific Islander students comprised 1.7% of the student population. The English language learner percentage was 13.2%, and those receiving special education services comprised 26.4% of the student population. Economically disadvantaged students comprised 45.3% of the student population. The state's Department of Education awarded the elementary school an "A" for the 2020-2021 school year. The school rating has not changed since the 2020-2021 school year due to changing state assessments and the COVID-19 pandemic.

Study Activities

I collected the necessary data over 16 days from all students who had parent permission and had given their assent to participate in the study. Before any geometry instruction, I administered the pretest to all class participants. Participants took the posttest upon completing the polygon unit instead of at the end of instruction on all geometry standards. The students completed the pretest and posttest using paper and pencil, allowing them to annotate if they wished. Students used iPads to complete the GeoGebra tasks. Finally, they completed a survey that included both Likert and open-ended questions.

The unit consisted of 11 lessons spread over the duration of the geometry unit, with some of the lessons taking multiple days (Appendix H). Each lesson included time given to students to complete the daily assignments. I believed that giving students adequate time to complete their assignments at school was necessary so that the participants could not receive additional help outside the classroom. This helped to ensure that each student's understanding of the lesson was shaped by the GeoGebra activities and not by outside resources.

The unit's 11 lessons included assessments and instruction. Lesson 1 was the van Hiele Geometry Test pretest, Lesson 9 was the van Hiele Geometry Test posttest, and Lessons 10 and

11 were the GeoGebra tasks. Lessons 2 through 8 were instructional lessons when the study activities were carried out. Lesson 2 introduced GeoGebra, while Lessons 3 through 8 pertained to the geometry standards. In Lesson 3, students learned about the classification of polygons. Lessons 4 and 5 centered on classifying triangles by their angles and sides. Lesson 6 focused on classifying quadrilaterals. Lesson 7 was a review lesson of all the geometry content that was taught to that point in time as well as a review of how to construct polygons in GeoGebra. The lesson ended with students creating pictures using only polygons. Finally, Lesson 8 centered on the hierarchy of quadrilaterals.

Data Sources and Collection

The design of this study utilized a mixed-method approach. Therefore, both quantitative and qualitative data sources were used. This study employed a total of eight unique data sources. One of these sources was quantitative: the van Hiele Geometry Test developed by Usiskin in 1982 was used to collect information regarding each participant's van Hiele level of geometric reasoning. The qualitative data was collected through seven different sources. Five of these data sources were the end-of-unit GeoGebra tasks. Data was also collected from the students' journal entries and from a survey.

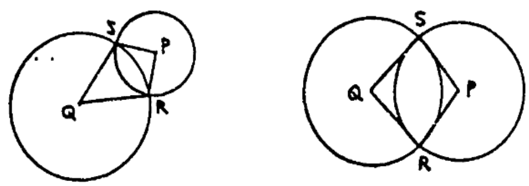
Quantitative Data

van Hiele Geometry Test

The quantitative data was collected using a pretest-posttest design. I used the van Hiele Geometry Test, designed by Usiskin (1982), for both the pretest and the posttest (see Appendix A). Usiskin's van Hiele Geometry Test included a question about the geometry of circles. I removed question 10 from the original test and replaced it with a Level 2 van Hiele question about triangles since the Common Core standards do not require fifth-grade students to know the geometry content associated with circles. Figure 3.2 shows the original question 10, and Figure 3.3 shows the modified question.

Figure 3.2. Question 10 on Usiskin's van Hiele Geometry Test

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(D) is <u>not</u> always true?

(A) PRQS will have two pairs of sides of equal length.

(B) PRQS will have at least two angles of equal measure.

(C) The lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ will be perpendicular.

(D) Angles P and Q will have the same measure.

(E) All of (A)-(D) are true.

Figure 3.3. Question 10 on the van Hiele Geometry Test used by the Researcher

10. An equiangular triangle is a triangle with three angles of the same measure. Look at the example.

Which of the following is true in every equiangular triangle?

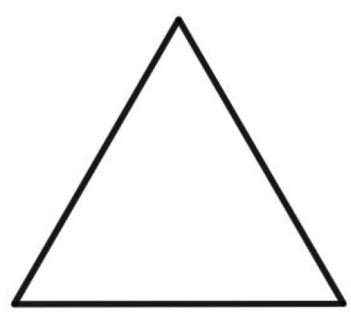
Ⓐ Only two of the sides will have the same length.

Ⓑ One angle has to be a right angle.

Ⓒ All three angles must be obtuse.

Ⓓ All three sides must be the same length.

Ⓔ None of the above (A-D) are true in every equiangular triangle.



The van Hiele Geometry Test created by Usiskin contained 25 questions, five per level. Since this study concerned the first three van Hiele levels, I removed the last ten questions. The van Hiele Geometry Test given to the participants in this study consisted of 15 questions, five from each level. Table 3.1 shows the test's design by indicating the relationship between the van Hiele levels and the questions on the test.

Table 3.1. Design of the van Hiele Geometry Test

van Hiele Levels of Geometric Reasoning	Question Number
Level 1: Visualization	1-5
Level 2: Analysis	6-10
Level 3: Informal Deduction	11-15

Northwest Evaluation Association Measures of Academic Progress

The Northwest Evaluation Association Measures of Academic Progress (NWEA MAP test) was used to compare geometry achievement prior to instruction. The NWEA MAP test assigns students RIT (Rasch Unit) scores, which represent a student's academic achievement level on a continuous scale. Because the RIT scores are equal interval scores, the difference between scores is the same (NWEA, nd). In addition, these scores are said to be stable, which allows for a student's scores to be directly compared, which can be beneficial for measuring growth over time.

Participants at the school where the research was conducted take the NWEA MAP test three times per academic year. For the pretest score, I used the RIT scores specific to the skill "Identification and Classification of 2-D Shapes" from the Winter 2022 NWEA MAP Test mathematics assessment. The posttest score was found using the RIT score for "Identification and Classification of 2-D Shapes" from the Winter 2023 NWEA MAP Test mathematics assessment, which students completed prior to receiving this research study's geometry instruction.

Qualitative Data

GeoGebra Tasks

Using GeoGebra, participants completed five tasks. I modeled the five tasks after those used by Burger and Shaughnessy (1986) and Wang and Kinzel (2014). The five GeoGebra tasks comprised three construction tasks and two sorting tasks. Table 3.2 describes the tasks.

Table 3.2. Descriptions of the GeoGebra Tasks

Tasks	Description	Appendix
Rectangle Construction	Students constructed as many rectangles as possible that were not similar or congruent.	B
Triangle Construction	Students constructed an equilateral and an isosceles triangle. Next, they constructed as many triangles as possible that were not similar or congruent to the others.	C
Rhombus Construction	Student constructed as many rhombuses as possible that were not similar or congruent.	D
Quadrilateral Sort	Students sorted 15 quadrilaterals into predetermined categories of squares, rectangles, parallelograms, and rhombuses. Some quadrilaterals did not fit into the categories. Students then justified their sorting and determined if they could have made a sort with fewer categories.	E
Polygon Sort	Students sorted 18 polygons as they wished; they were allowed to use subgroups. They explained their sorting and described which, if any, GeoGebra tools helped them with the sorting. Finally, students completed the task again, but they were required to sort the same polygons in a different way.	F

Construction Tasks. The construction tasks were designed to test the participants' knowledge of fundamental geometric concepts about rectangles, triangles, and rhombuses. While not the main focus of these tasks, students had to demonstrate their understanding of the terms "congruent" and "similar" as they were required to construct as many polygons as they could that were not congruent or similar to each other. The complexity of each task required students to apply their knowledge of the geometric properties to construct the polygons accurately.

Sorting Tasks. The sorting tasks were designed as open-ended tasks, allowing students to sort the polygons in various ways. While the quadrilateral sorting task had predefined categories, students could sort the quadrilaterals into the given categories as they saw fit. The polygon sort allowed students to develop their own classification methods for sorting and grouping the polygons. While completing these tasks, students took screen recordings of their actions using the iPad's built-in screen recording feature. Throughout the screen recordings, students narrated their mathematical thoughts and processes and described the features of GeoGebra that they utilized.

Survey. I created a survey (see Appendix G) to assess the impact of using GeoGebra on the participants' learning experiences and understanding of polygons. Specifically, it aimed to gather information on several aspects of their perception of GeoGebra, such as how GeoGebra did or did not enhance their level of understanding, how GeoGebra helped them to visualize and manipulate the polygons, what challenges and benefits the participants experienced while using GeoGebra, and lastly the impact that the participants believed GeoGebra had on their geometry content knowledge and their confidence with geometry content. Table 3.3 describes the survey questions.

Table 3.3. Descriptions of Survey Questions

Question	Focus	Description
1	Enhancement of Understanding	This question explored how the use of GeoGebra contributed to an improvement in the participants' comprehension of polygon properties and characteristics. It encouraged them to provide specific examples or share experiences that highlighted the effectiveness of GeoGebra in enhancing their understanding.
2	Visualization and Manipulation	This question focused on the ways in which GeoGebra facilitates the visualization and manipulation of different types of polygons. It sought insights into any moments of realization or discovery that students may have had while using the software, emphasizing the tool's role in making these aspects more accessible.
3	Challenges and Support	This question asked respondents to reflect on their experiences in creating and analyzing polygons using GeoGebra. It explored any challenges they faced and investigated how the tools supported them in overcoming those challenges. Additionally, it sought information on specific features of GeoGebra that were particularly helpful in this context.
4	Impact on Geometry Skills and Confidence	This question considered the broader connections between using GeoGebra and the students' overall understanding of geometry. It explored whether the software influenced their ability to solve geometric problems involving polygons and whether they felt more confident in their geometry skills due to using GeoGebra.

Journal Entries. After each study activity, students were encouraged to create journal entries. I did not provide prompts to the students, so the students were able to journal authentically. Students wrote about their experiences utilizing GeoGebra and their understanding of the geometric concepts explored in the study activities. The students wrote honestly about their successes with GeoGebra and their frustrations and questions after completing each lesson. The journal entries provided insights into students' daily experiences with the study activities.

Data Analysis

Utilizing a convergent mixed methods design to analyze the data, I analyzed the quantitative data separately from the qualitative data to answer the quantitative and qualitative research questions. Lastly, I used the quantitative and qualitative data analysis to triangulate the data to answer the mixed methods research question. Table 3.4 shows the alignment between the data sources and their corresponding research questions.

Table 3.4. Alignment of Research Questions, Data Sources, and Data Analysis

Research Question	Sub Questions	Data Sources	Data Analysis
RQ1: What effect does utilizing GeoGebra during geometry instruction have on fifth-grade students' van Hiele level of geometric thinking about polygons, as measured by a pre- and posttest?		Pre- and post- van Hiele Geometry Test	Wilcoxon Signed-Rank Test
	Are there any differences based on gender?		Wilcoxon Rank-Sum Test
	Are there any differences based on prior achievement?	NWEA RIT scores from Winter 2022 and Winter 2023	
RQ2: What are students' experiences using GeoGebra during geometry instruction and how do these experiences align with students' levels of geometric thinking?		Journal entries, survey results, screenshots	Coding
	How do students experience using tools available in GeoGebra?		
	When students are observed using (or not using) tools available in GeoGebra, what is their level of geometric thinking?	Screen recordings of students working on the GeoGebra Tasks	Coding using Level Indicators
RQ 3: What is the impact of using GeoGebra to support fifth-grade students' learning about polygons?		All sources mentioned above	Table and scatterplot

Quantitative Data Analysis

Research Question 1: What effect does utilizing GeoGebra during geometry instruction have on fifth-grade students' van Hiele level of geometric thinking about polygons, as measured by a pre- and posttest?

I scored the van Hiele Geometry Test using the grading methods from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project (Usiskin, 1982). Participants were awarded a score when they met the criterion of correctly answering 3 of the 5 questions for a van Hiele level when the 3 out of 5 criterion was applied, and they were awarded a score when they met the criterion of correctly answering 4 of the 5 questions for a van Hiele level when the 4 out of 5 criterion was applied. Scores were assigned to participants for meeting the criterion for each level. Table 3.5 shows how the scoring was completed.

Table 3.5. Scores Assigned for Meeting the Specific Criterion

van Hiele Criteria	Question Number	Score
Level 1: Visualization	1-5	1
Level 2: Analysis	6-10	2
Level 3: Informal Deduction	11-15	4

The weighted sums were determined by adding the scores for each level. The weighted sum was used to help me determine each participant's performance on the test and each student's van Hiele level. Because the levels are believed to be sequential and cannot be skipped, the weighted sums helped highlight each participant's performance on the test. For example, a student may have answered 3 of the 5 questions correctly for Level 1, 2 of the 5 questions for Level 2, and 3 of the 5 questions correctly for Level 3. Following the theory guidelines, this student was assigned to Level 1 since he or she did not meet the criterion for Level 2. By using weighted sums, each student could be placed into a category while at the same time, allowing me to identify which students answered questions accurately at levels above their assigned levels. Table 3.6 shows the weighted sums that were used to assign levels to each of the participants. Once the scoring was completed, I used R to analyze the data. I conducted a Shapiro-Wilk test to determine normality of the data. The p-value for each of the Shapiro-Wilk tests yielded results in which the p-value was less than 0.05.

Table 3.6. Weighted Sums Assigned to Each Level

van Hiele Criteria	Weighted Sums
Level 0: Prerecognition	0, 2, 4, 6
Level 1: Visualization	1, 5
Level 2: Analysis	3
Level 3: Informal Deduction	7

Because the results of the Shapiro-Wilk tests indicated that the data was not parametric for each instance the test was conducted, I used the Wilcoxon Signed-Rank Test to complete the analysis. The Wilcoxon Signed-Rank test compares dependent samples when the data does not exhibit normal distribution. The data was analyzed using both the 3 out of 5 criterion and the 4 out of 5 criterion using a significance level set at $p < 0.05$. Lastly, I compared each student's assigned van Hiele level using both criteria.

Research Question 1a: Are there differences based on gender?

The scored data used to answer the main question, Research Question 1, was used to answer this sub-question. Once the data was sorted by gender, I conducted a Shapiro-Wilk test in R to check for normality. The results indicated that the data was nonparametric. Since the data was not parametric and the data were independent of each other, I conducted a Wilcoxon Rank-Sum test. The Wilcoxon Rank Sum Test is used to compare independent samples when the data does not exhibit normal distribution. The data was analyzed using both the 3 out of 5 criterion and the 4 out of 5 criterion using a significance level set at $p < 0.05$.

Research Question 1b: Are there any differences based on prior achievement?

The RIT scores for the skill of "Identification and Classification of 2-D Shapes" were collected from the Winter 2022 and Winter 2023 NWEA MAP Test mathematics assessments. Because the scores are listed as RIT scores, which are ranges, I modified the ranges to single scores while maintaining an equal interval to make analyzing the data easier. This process was completed using the first two numerals of each RIT score. Table 3.7 shows the score adjustment.

Table 3.7. Score Conversion From RIT to a Single Score

RIT Score	Adjusted Score
191-200	19
201-210	20
211-220	21
221-230	22
232-240	23
241-250	24
251-260	25
261-270	26
271-280	27

Once the scores were adjusted, I looked at the students' overall scores to determine the percentage of students whose scores decreased, stayed the same, and increased. I then found the percentage of students who decreased in van Hiele levels, the percentage of those who stayed the same, and the percentage of those who increased. Then, I compared the two percentages. Lastly, I looked at each student, noted if they increased or decreased their NWEA RIT Score, and compared that with any changes in the van Hiele score.

Qualitative Data Analysis

Throughout the qualitative data analysis process, I utilized a peer debriefer. The reason for this is that the peer debriefing process helps researchers identify their biases and gaps in their data analysis, which enhances the validity, credibility, and quality of the research (QDAcity, n.d.). According to Lincoln and Guba (1985), peer debriefing supports the credibility of the data in qualitative research while providing a means to establish the overall trustworthiness of qualitative findings. "Peer debriefing contributes to confirming that the findings and the interpretations are worthy, honest, and believable" (Spall, 1998, para. 3). The peer debriefer selected for this study is a PhD student at Kansas State University and a current high school math teacher. The peer debriefer had previously received training on peer debriefing for another study. Prior to this study, I met with the peer debriefer to make sure she understood the codes and level indicators and to confirm that she understood the coding process that was used with this study. Throughout the qualitative data analysis, the peer debriefer remained impartial to the study.

The qualitative data was analyzed through a coding process, which took place in three phases, one phase for each question and sub question. Per Creswell (2008), before beginning either phase of coding, I read through the data first to get a general sense of the gathered data. I looked for tone of the ideas and recorded general thoughts about the data. The coding process was conducted by me, the classroom teacher, and the previously mentioned peer debriefer. Each phase uncovered insights and patterns within the data to help answer the qualitative research questions.

Research Question 2: What are students' experiences using GeoGebra during geometry instruction?

Three data sources were used to answer this research question. I used the survey, student journal entries, and screenshots to gain insights into the students' experiences. The coding of this data took place in two rounds of theming. After reading through the data to gain a general sense of the tones surrounding student experiences with GeoGeobra, I determined that three themes surrounding the overall feelings of the students regarding their experience using GeoGebra existed. I then used a priori coding to code the three sentiments. Upon identifying three themes or sentiments that summarized the overall feelings of the students regarding their experiences, the second round of coding took place. During this coding round, I used open coding to further break down the three themes into six different themes that described the rationale for the main three themes. The peer reviewer coded the data in the same manner as I did to confirm the validity of my analysis.

Research Question 2a: How do students experience using tools available in GeoGebra?

The second coding phase took place over two rounds. The first round of coding required the usage of GeoGebra tools to be identified in the screenshots and screen recordings. Using a priori coding, I looked for are located on Table 8. It also required that all mentions of GeoGebra tools be identified in all student responses on the survey or the questions associated with each GeoGebra task. I also looked for the mention of GeoGebra tools in the journal entries that students submitted. After identifying the mention and usage of GeoGebra tools through the various data sources, a second round of coding took place. I used an inductive approach to identify themes surrounding the usage of the tools to gather evidence as to how students experience using tools in GeoGebra.

Table 3.8. GeoGebra Tools Used by the Participants and Their Purpose

Tool	Purpose
Angle	Find the interior or exterior measure of an angle
Angle with Given Size	Create an interior angle with a specific measure
Delete	Delete an object
Distance of Length	Find the length of a segment or side of a polygon
Midpoint or Center	Find the midpoint of a segment or polygon
Parallel Line	Create a line that is parallel to a given line or segment
Perpendicular Line	Create a line that is perpendicular to a given line or segment
Point	Add a point on an object, line, or segment or use to create a line or segment
Polygon	Create any polygon free-handed
Regular Polygon	Create a regular polygon with *n* sides
Segment	Create a segment of any length
Segment with Given Length	Create a segment with a specific length
Show/Hide Object	Removes an object from being shown or allows object to reappear
Text	Create labels and other notations
Zoom In	Zoom in on objects
Zoom Out	Zoom out to see more of the GeoGebra screen

Research Question 2b: When students are observed using (or not using) tools available in GeoGebra, what is their level of geometric thinking?

A third phase of coding took place to answer this question. The screen recordings students submitted while completing the five end-of-unit GeoGebra tasks were analyzed to answer this question. I used the a priori coding to look for the level indicators (see Appendix H) that Burger and Shaughnessy (1986) used in their study to code the videos. The level indicators have been proven to be reliable and valid as they were derived directly from the work of Pierre and Dina van Hiele. After I coded the videos using the level indicators, I met with the peer reviewer to ensure that my coding process was grounded in accuracy. The peer review process helped to ensure that my interpretations of the data were honest, unbiased, believable, and trustworthy.

Mixed Methods Data Analysis

Research Question 3: What is the impact of using GeoGebra to support fifth-grade students' learning about polygons?

I used a triangulation approach to answer this question by first converging the qualitative data together and then by converging the qualitative data with the quantitative data. I compared the data side by side to determine GeoGebra's impact on the students' geometric reasoning. Specifically, I looked at the changes in the students' van Hiele levels, as measured by the results of the pretest and posttest, and compared those changes with the qualitative findings, specifically the van Hiele level behaviors that students exhibited, as indicated by the level indicators used by Burger and Shaughnessy (1986).

Reliability and Trustworthiness

Previous research has validated the van Hiele Geometry Test and the level indicators. The studies that have been used to validate the van Hiele Geometry Test include Burger and Shaughnessy (1986), Mayberry (1983), Senk (1989), and Gutierrez et al. (1991a). The methods for analyzing, including the observations and checklist of level indicators for the GeoGebra tasks, have been validated by prior research, such as Burger and Shaughnessy (1986) and Wang and Kinzel (2014). Using instruments used in previous studies and a survey and student journal entries, which allowed students to provide honest responses to open-ended questions, the data collected is considered reliable and can be considered trustworthy.

Summary

This research study was conducted using the frameworks of symbolic interactionism and the van Hiele theory. As the researcher, I used a case study approach in which the participants were conveniently chosen rather than randomly assigned. I used four different instruments using these approaches. The van Hiele Geometry Test was used to collect quantitative data. Observations and checklists were completed while watching and analyzing screen recordings and screenshots of students completing GeoGebra tasks; these, along with a survey and student journal entries, were used to collect the qualitative data. The results of each instrument were then triangulated to answer the mixed methods research question.

Chapter 4 - Results

The purpose of this convergent mixed methods research study was to consider how the use of GeoGebra, a dynamic geometry software, impacts the ability of students to advance through the first three levels of geometric reasoning, as provided through the framework of the van Hiele theory. The quantitative data was collected using a pretest-posttest design utilizing the van Hiele Geometry Test (Usiskin, 1982). Additionally, scores from the NWEA assessment were used to determine if differences existed based on prior achievement. The qualitative data was collected through GeoGebra tasks, a survey, and student journal entries. The quantitative findings are described first and are followed by the qualitative findings. Finally, the findings from the mixed methods data analysis are presented. The data that is presented in this chapter was analyzed to answer the following research questions:

Quantitative Questions

1. What effect does utilizing GeoGebra during geometry instruction have on fifth-grade students' van Hiele level of geometric thinking about polygons, as measured by a pre- and posttest?
    a. Are there differences based on gender?
    b. Are there any differences based on prior achievement?

Qualitative Questions

2. What are students' experiences using GeoGebra during geometry instruction?
    c. How do students experience using tools available in GeoGebra?
    d. When students are observed using (or not using) tools available in GeoGebra, what is their level of geometric thinking?

Mixed Methods Question

3. What is the impact of using GeoGebra to support fifth-grade students' learning about polygons?

Quantitative Results

Results for van Hiele Levels

Research Question 1: What effect does utilizing GeoGebra during geometry instruction have on fifth-grade students' van Hiele level of geometric thinking about polygons, as measured by a pre- and posttest?

Thirty-eight participants took the van Hiele Geometry Test before geometry instruction occurred. The distribution of answer choices was spread differently for the questions. Table 4.1 shows the distribution of answers. The bolded numbers on the table indicate the correct answers.

The pretest allowed me to determine each participant's initial van Hiele level. I discovered that when applying the 3 out of 5 criterion, which states that to obtain a van Hiele level, the student must have answered at least three of the five questions correctly for that given level, seven students (18.42%) were assigned Level 0, 16 students were assigned to Level 1 (42.11%), 10 students were assigned to Level 2 (26.32%), and five students were assigned to Level 3 (13.16%). When I applied the 4 out of 5 criterion, which states that to obtain a van Hiele level, the student must have answered at least four of the five questions correctly for that given level, 24 students (63.15%) were assigned Level 0, 10 students were assigned to Level 1 (26.32%), two students were assigned to Level 2 (5.26%), and two students were assigned to Level 3 (5.26%). Table 4.2 shows the distribution of students by level as assigned on the pretest using the 3 out of 5 criterion. Table 4.3 shows the distribution of students by level as assigned on the pretest using the 4 out of 5 criterion. Using the 3 out of 5 criterion, 39.48% of the students were classified at a Level 2 or 3, and 10.52% of the students were classified at a Level 2 or 3 using the 4 out of 5 criterion.

Table 4.1. Distribution of the Pretest Answers

Item Number	A	B	C	D	E	No Response	Percent Correct
1	0	**18**	0	19	1	0	47.4
2	1	0	7	**29**	1	0	76.3
3	1	0	**37**	0	0	0	97.4
4	2	**22**	7	1	6	0	57.9
5	4	4	7	3	**19**	1	50.0
6	11	**7**	12	8	0	0	18.4
7	3	1	3	8	**23**	0	60.5
8	**5**	4	8	6	7	8	13.2
9	2	2	**20**	2	8	4	52.6
10	2	1	1	**28**	3	3	73.7
11	1	10	**12**	0	3	12	31.6
12	4	**11**	3	2	9	9	28.9
13	**15**	2	0	0	20	1	39.5
14	**9**	11	5	3	5	5	23.7
15	7	**3**	7	5	9	7	7.9

Table 4.2. Distribution of Students Using the 3 out of 5 Criterion

van Hiele Level	Number of Students	Percentage
Level 0	7	18.42%
Level 1	16	42.11%
Level 2	10	26.32%
Level 3	5	13.16%

Table 4.3. Distribution of Students Using the 4 out of 5 Criterion

van Hiele Level	Number of Students	Percentage
Level 0	24	63.15%
Level 1	10	26.32%
Level 2	2	5.26%
Level 3	2	5.26%

The same 38 participants took the van Hiele Geometry Test after the geometry instruction. The distribution of answer choices was spread differently for the questions. Table 4.4 shows the distribution of answers. The bolded numbers on the table indicate the correct answers. Except for question 12, the posttest results showed that the questions were answered with higher accuracy than on the pretest.

The posttest allowed me to determine each participant's ending van Hiele levels. I discovered that when applying the 3 out of 5 criterion, two students (5.26%) were assigned Level 0, seven students were assigned to Level 1 (18.42%), 14 students were assigned to Level 2 (36.84%), and 15 students were assigned to Level 3 (39.47%). When I applied the 4 out of 5 criterion, 12 students (31.58%) were assigned Level 0, nine students were assigned to Level 1 (23.68%), 10 students were assigned to Level 2 (26.32%), and seven students were assigned to Level 3 (18.42%). Table 4.5 shows the distribution of students by level as assigned on the posttest using the 3 out of 5 criterion. Table 4.6 shows the distribution of students by level as assigned on the posttest using the 4 out of 5 criterion.

Table 4.4. Distribution of the Posttest Answers

Item Number	A	B	C	D	E	No Response	Percent Correct
1	0	**26**	1	11	0	0	68.4
2	0	0	0	**37**	1	0	97.4
3	0	0	**38**	0	0	0	100.0
4	0	**25**	10	0	3	0	65.8
5	1	2	7	1	**27**	0	71.1
6	0	**24**	7	7	0	0	63.2
7	2	0	7	2	**27**	0	71.1
8	**16**	9	3	0	10	0	42.1
9	0	0	**36**	0	2	0	94.7
10	2	0	0	**35**	1	0	92.1
11	0	3	**28**	0	7	0	73.7
12	9	**10**	3	1	15	0	26.3
13	**34**	0	0	0	4	0	89.5
14	**11**	9	6	6	6	0	28.9
15	3	**15**	1	6	13	0	39.5

Table 4.5. Distribution of Students Using the 3 out of 5 Criterion

van Hiele Level	Number of Students	Percentage
Level 0	2	5.26%
Level 1	7	18.42%
Level 2	14	36.84%
Level 3	15	39.47%

Table 4.6. Distribution of Students Using the 4 out of 5 Criterion

van Hiele Level	Number of Students	Percentage
Level 0	12	31.58%
Level 1	9	23.68%
Level 2	10	26.32%
Level 3	7	18.42%

Using the 3 out of 5 criterion, 76.31% of the students were classified at a Level 2 or 3, and 44.74% were classified at a Level 2 or 3 using the 4 out of 5 criterion. This is an increase of 36.81 percentage points for the 3 out of 5 criterion and an increase of 34.24 percentage points for the 4 out of 5 criterion.

A Shapiro-Wilk test was completed using R to check the normality for the distribution of the assigned levels. The Shapiro-Wilk tests for the pretest levels using both the 3 out of 5 and 4 out of 5 criteria produced p-values < 0.001. Because the p-values for both tests were smaller than the alpha of 0.05, the results indicated that the data was not parametric.

Whenever one data set is considered nonparametric, a t-test cannot be used. Therefore, I used a Wilcoxon Signed Rank test to compare the medians of the dependent (pretest and posttest) data. For the 3 out of 5 criterion, the Wilcoxon Signed Rank test produced a test statistic of V = 50 and a p-value < 0.001. Because the p-value was less than the alpha of 0.05, the test indicated a statistically significant difference in the level assignments of the participants. For the 4 out of 5 criterion, the Wilcoxon Signed Rank test produced a test statistic of V = 82 and a p-value = 0.002. The p-value was less than the alpha of 0.05, which indicated a statistically significant difference in the level assignments for the students.

Using the quantile information and box plots, it was determined that this change was in the positive direction. Figure 4.1 shows the box plot for the 3 out of 5 criterion and Figure 4.2 shows the box plot for the 4 out of 5 criterion.

Figure 4.1. Box Plot Using the 3 out of 5 Criterion

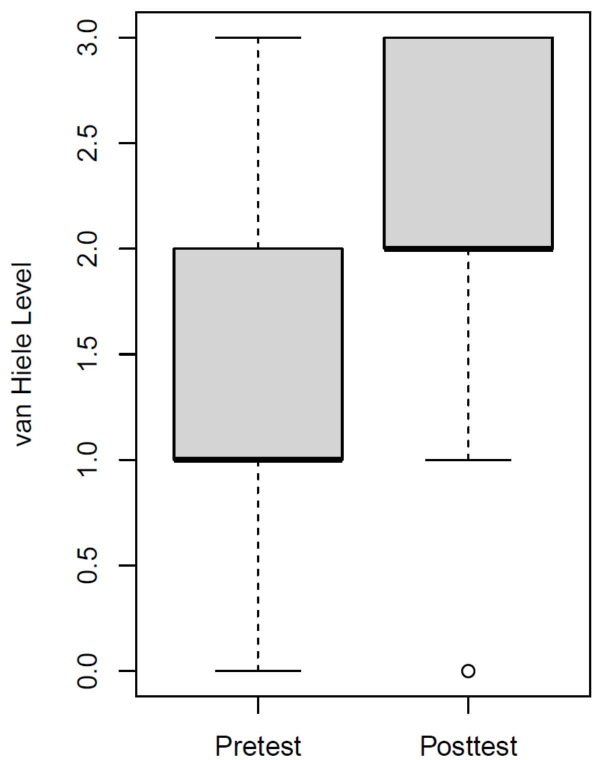
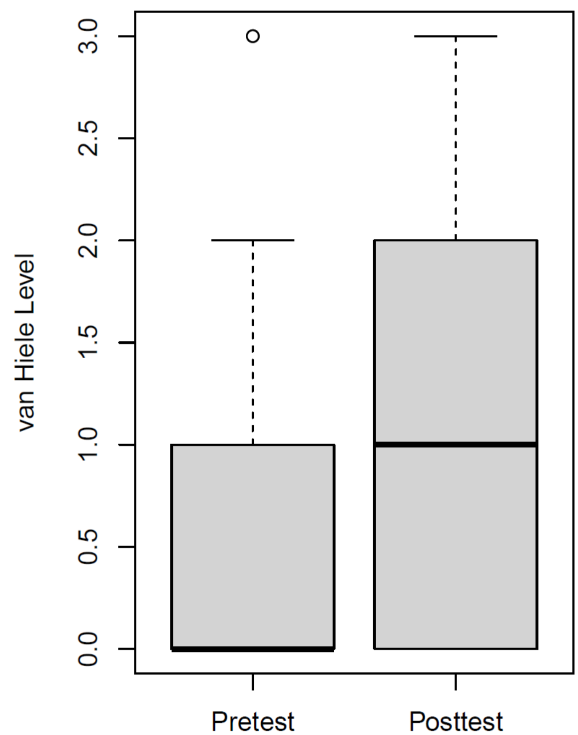


Figure 4.2 Box Plot Using the 4 out of 5 Criterion



After completing the statistical analysis, I compared each student's assigned level using both criteria. I found that 17 students were assigned the same level using both criteria. Using the 3 out of 5 criterion, 14 of the students were assigned a van Hiele level that was one level higher than what they were assigned using the 4 out of 5 criterion. Using the 3 out of 5 criterion, seven students were assigned a van Hiele level that was two or three levels higher than their assigned level using the 4 out of 5 criterion.

As noted previously, students answered questions more accurately on the posttest than on the pretest. Table 4.7 shows the percentage of questions that students answered accurately for each level on the pretest and the posttest.

Table 4.7. Percent of Questions Correctly Answered by Level

Level	Pretest	Posttest	Percent Change
1	65.8	80.5	22.4
2	43.7	72.6	66.3
3	26.3	51.6	96.0

Research Question 1a: Are there differences based on gender?

The same data used to answer Research Question 1 was again analyzed to answer Research Question 1a. To answer this question, the data was separated into two categories by gender: female and male.

Due to the nonparametric data, an independent samples t-test could not be used. I used a Wilcoxon Rank Sum test to compare the median levels for females and males. This test was conducted using both the pretest and posttest data, as well as both criterion, to see if any differences in the median scores existed on either the pretest or posttest. Using the 3 out of 5 criterion, the test produced a test statistic of W = 176.5 and a p-value = 0.85 for the pretest. The test produced a test statistic of W = 140.5 and a p-value of 0.35 for the posttest. Using the 4 out of 5 criterion, the test produced a test statistic of W = 164 and a p-value = 0.85 for the pretest data and a test statistic of W = 164 and a p-value of 0.64 for the posttest data. In every case, the p-value was larger than the alpha of 0.05; therefore, the tests did not indicate a significant difference in the test scores for females and males.

Table 4.8 shows the percentage of questions that both genders answered accurately on the pretest and the posttest.

Table 4.8. Percent of Questions Correctly Answered by Level

Level	Pretest	Posttest	Percent Change
Female	44.8	69.3	54.5
Male	46.3	67.0	44.6

Using the quantile information and box plots, it was determined that the data had no statistical difference. Figure 4.3 shows the box plot for the pretest using the 3 out of 5 criterion, Figure 4.4 shows the box plot for the posttest using the 3 out of 5 criterion, Figure 4.5 shows the box plot for the pretest using the 4 out of 5 criterion, and Figure 4.6 shows the box plot for the posttest using the 4 out of 5 criterion.

Figure 4.3. Comparison of van Hiele Pretest Levels Using the 3 out of 5 Criterion

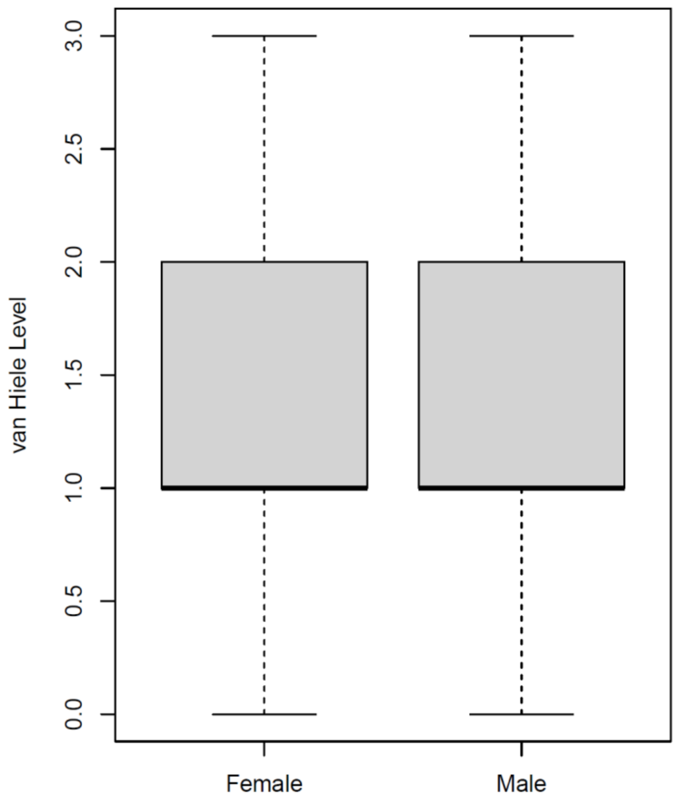


Figure 4.4. Comparison of van Hiele Posttest Levels Using the 3 out of 5 Criterion

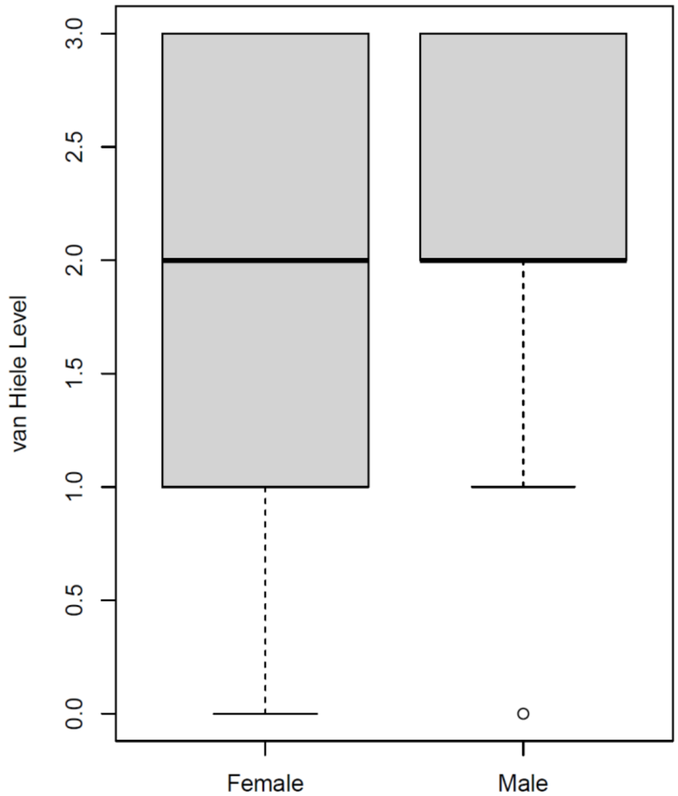


Figure 4.5. Comparison of van Hiele Pretest Levels Using the 4 out of 5 Criterion

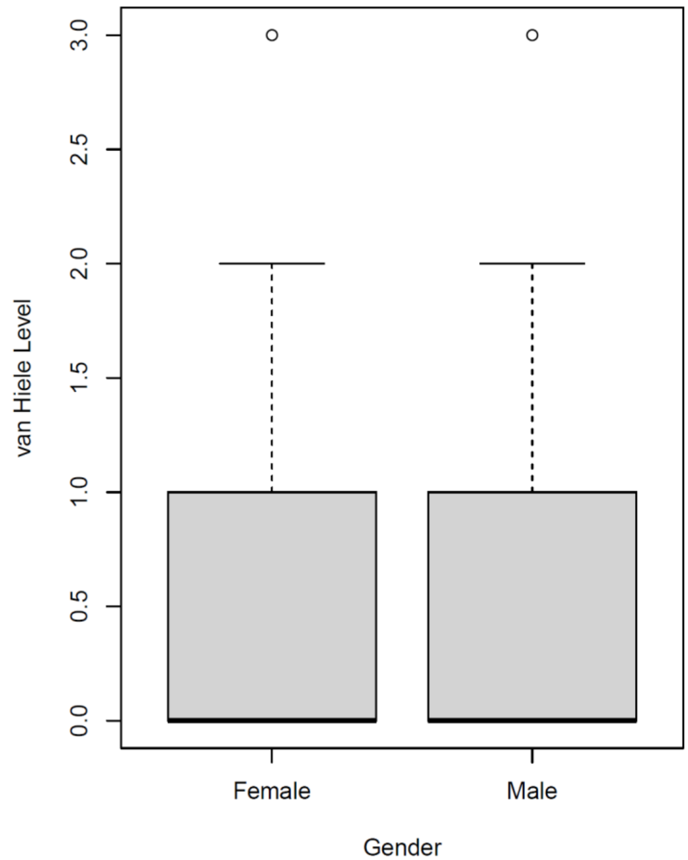
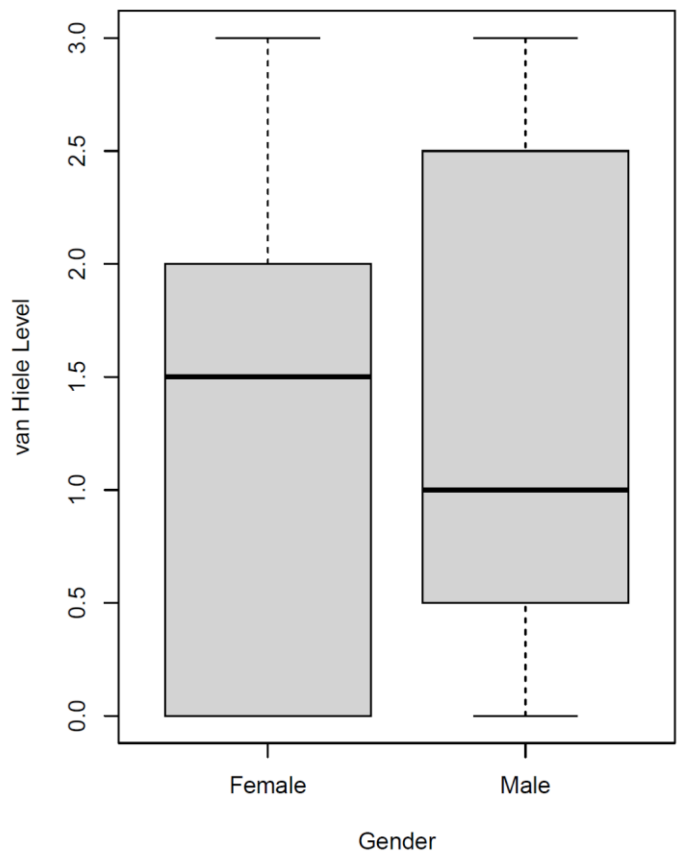


Figure 4.6. Comparison of van Hiele Posttest Levels Using the 4 out of 5 Criterion

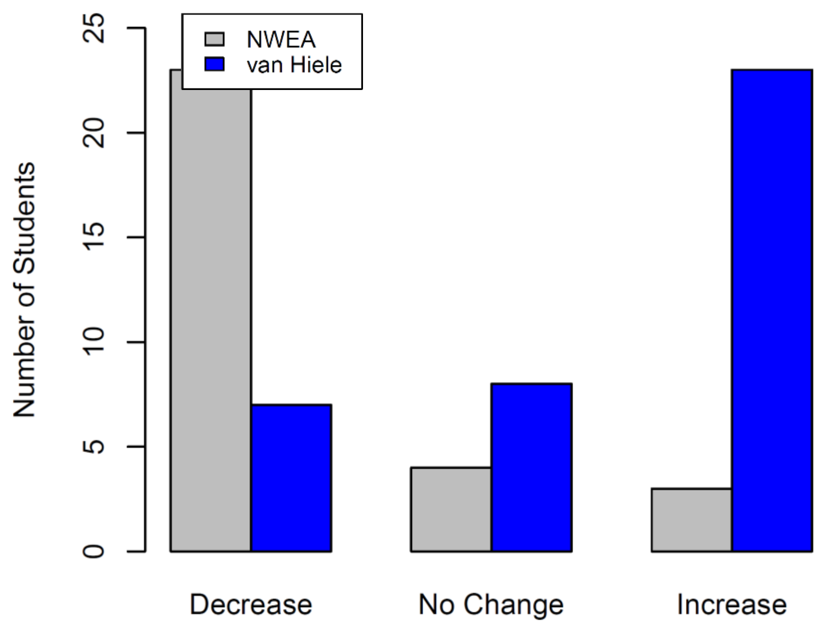


Research Question 1b: Are there any differences based on prior achievement?

To answer this question, I removed the results for four of the 38 students who participated in the research study while analyzing the data to answer this question. This is because those four students had been categorized at a van Hiele level 3 on both the pretest and posttest. While these students did not show any decreases in van Hiele levels, they were unable to show any potential increases as the van Hiele Geometry Test used in this study only tested up to a van Hiele level 3.

After removing the data for four of the students in the study, I analyzed the data for the remaining 34 students. To measure prior achievement, I looked at the NWEA results from Winter 2022 to Winter 2023. Of these 34 students, 23 students showed a decrease in scores, three showed no change in scores, and eight showed an increase in scores. In comparison to the changes in van Hiele levels for the 34 students from the pretest to the posttest, four students showed a decrease in scores, seven showed no change in scores, and 23 showed an increase in scores. Figure 4.7 shows these changes.

Figure 4.7. Comparison of Changes in NWEA Scores and van Hiele Levels



After looking at the data collectively, I looked at each student's data individually. Of the 34 students, four students showed decreases in both their NWEA results and van Hiele levels, two students showed no change in their NWEA results or van Hiele levels, and eight students showed increases in both their NWEA results and van Hiele levels. The remaining 20 students had different changes in their NWEA results compared to changes in their van Hiele levels. Of the 20 remaining students, 15 showed a decrease in NWEA scores while showing an increase in van Hiele levels. One student scored the same on both NWEA tests while increasing their van Hiele level, and four students showed a decrease in the NWEA scores while showing no change in their van Hiele levels.

Results for Weighted Sums

Research Question 1: What effect does utilizing GeoGebra during geometry instruction have on fifth-grade students van Hiele level of geometric thinking about polygons, as measured by a pre- and posttest?

I determined the total weighted sum for each test using both criteria. Table 4.9 shows the total weighted sums for each test and criterion.

Table 4.9. Total Weighted Sum For All Participants For Each Test and Criterion

Criterion	Test	Total Weighted Sum
3 out of 5	Pretest	95
3 out of 5	Posttest	168
4 out of 5	Pretest	40
4 out of 5	Posttest	107

A Shapiro-Wilk test was conducted for each test and criterion to determine if the weighted sums were normally distributed. The Shapiro-Wilk test yielded a test statistic $W = 0.83$ and a p-value < 0.001 for the pretest using the 3 out of 5 criterion and a test statistic of $W = 0.61$ and a p-value < 0.001 for the pretest using the 4 out of 5 criterion.

Since the p-value in both instances was less than the alpha of 0.05, a Wilcoxon Signed Rank test was used for both criteria. The Wilcoxon Signed Rank test for the 3 out of 5 criterion produced a test statistic of $V = 56$ and a p-value < 0.001 and a test statistic of $V = 75$ and a p-value < 0.001 for the 4 out of 5 criterion. In both cases, the p-value was less than the alpha of 0.05, indicating a significant difference in the weighted sums from the pretest to the posttest for both criteria.

Using the quantile information and box plots, it was determined that there is a statistical difference in the data for both criteria. Figure 4.8 shows the box plot for the 3 out of 5 criterion and Figure 4.9 shows the box plot for the 4 out of 5 criterion.

Figure 4.8. Comparisons of Weighted Sums Using 3 out of 5 Criterion

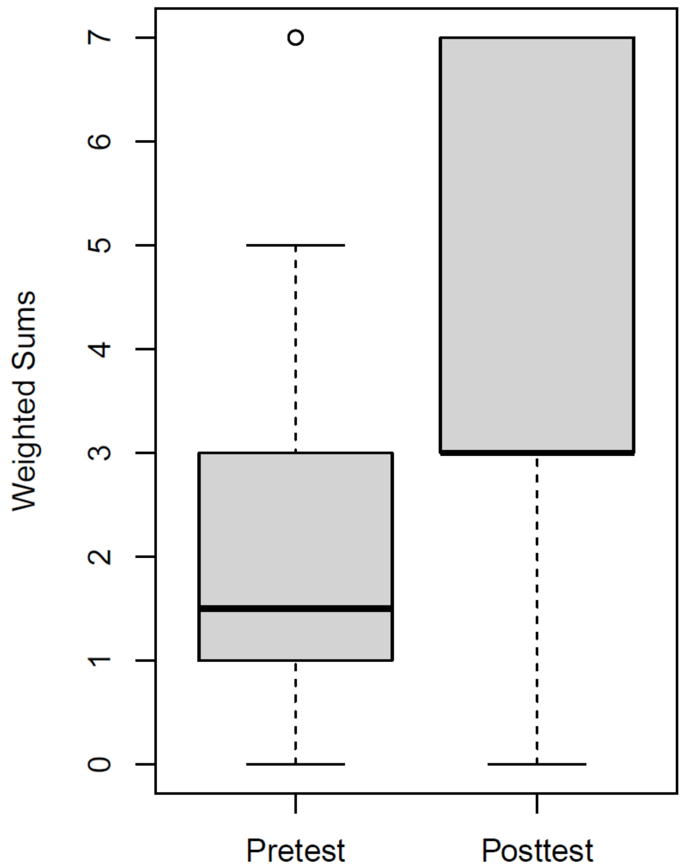
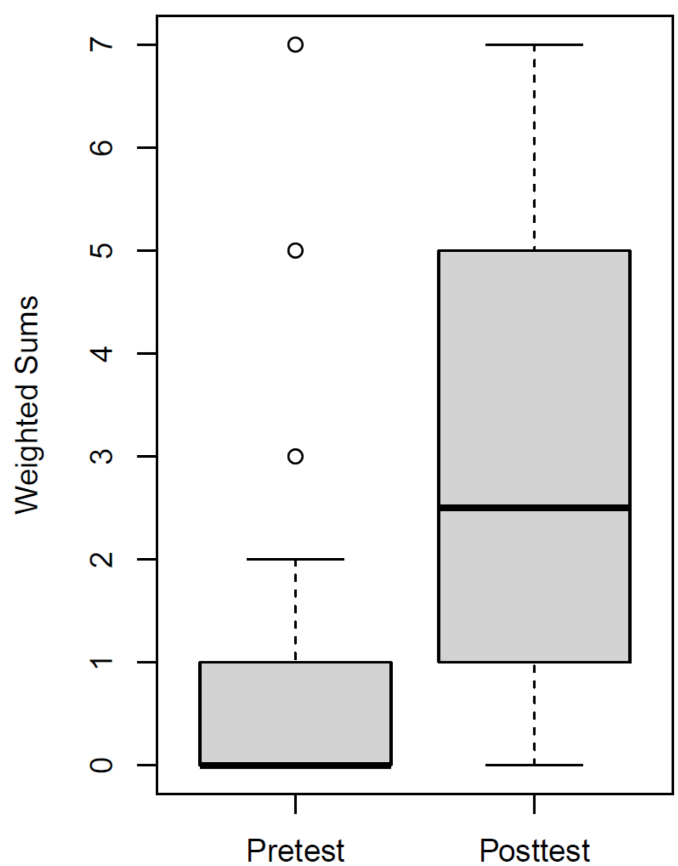


Figure 4.9. Comparisons of Weighted Sums Using 4 out of 5 Criterion



Qualitative Findings

The qualitative data coding process was conducted in three different phases, with each phase uncovering insights and patterns within the data to help answer the qualitative research questions. Each phase and round of coding process is described below. By detailing the progression through these phases, a comprehensive understanding of the data analysis process was developed.

Research Question 2: What are students' experiences using GeoGebra during geometry instruction?

Student surveys and journal entries were analyzed to answer this research question. The first round of coding the surveys and journal entries was conducted using an a priori approach to locate the feelings and sentiments students had regarding their experiences with GeoGebra Through this process, three themes were coded, which revealed varying perspectives on GeoGebra. These perspectives ranged from positive experiences to challenges the participants experienced while using GeoGebra. The responses were categorized into three main themes: positive experiences, confusing experiences, and negative experiences. While these themes may vary, the participants indicated insights they gained through using GeoGebra, regardless of their individual experiences.

Positive Experiences. Students expressed enthusiasm and appreciation for GeoGebra. They found it engaging, enjoyable, and a valuable learning tool. Students highlighted features such as the ability to explore shapes, measure angles, and visualize geometric properties. Many appreciated the convenience of digital tools over traditional paper and pencil methods. Moreover, about one-third of the students noted how GeoGebra helped them understand geometry concepts better and faster.

The survey asked students if they would want to continue to use GeoGebra to learn geometry. All but three respondents said they would like to continue to use GeoGebra. The students who indicated they would continue to use GeoGebra were self-aware of how using GeoGebra benefited them. One student said, "Yes, 100%! I would use it in 6th grade because I love how the tools help you understand and have fun while you are learning." Another student stated, "Yes, because it helped me learn a lot about geometry. I improved a lot from GeoGebra."

These and other similar responses helped to confirm that students had positive experiences with GeoGebra.

Confusing Experiences. While many students found GeoGebra beneficial, some comments indicated that students encountered challenges or experienced moments of confusion while using GeoGebra. Common issues included difficulties navigating the software, accidental actions that lead to unexpected results, and occasional technical glitches. Some students found specific tools confusing and expressed frustration when GeoGebra did not meet their expectations. However, it is important to note that in each entry in which a response was coded as confusing, another response within the same entry was coded as a positive experience, which indicated that the experiences students found to be confusing were mitigated with further GeoGebra usage and were positive overall. For example, one student stated in their math journal, “It took me a little while to figure out how to use them [the tools], but once I figured it out, it was easy.” Another student wrote in her journal, “GeoGebra helps me, but it is confusing sometimes if it doesn’t work, but it is still fun.”

Negative Experiences. Six participants indicated negative experiences with GeoGebra. The experiences pertained to the usability of GeoGebra’s tools and features. In some instances, the students referred to their own knowledge, or lack thereof, with GeoGebra and being able to accomplish what they wanted to accomplish while constructing polygons. Two experiences, coded as negative experiences, appeared more than once. First, some students had difficulties plotting points where they wanted to, and they experienced plotting more points than intended. A student noted, “One downside is sometimes it plots points when you don’t want it to.” The second experience mentioned more than once was the struggle to measure the angles of polygons. When students used the angle tool to measure an angle, GeoGebra would sometimes show the measure of the exterior angle, due to user error, rather than the measure of the interior angle. For example, one student wrote, “I hate how much it [GeoGebra] likes exterior angles. It always puts exterior angles instead of interior angles. It is kind of a distraction.”

Second Round of Coding. After the initial round of coding, a second round of coding took place. This round of coding used an inductive approach. Through this approach, six different themes were developed. These themes surrounded tools, efficiency, ease of use, fun, symbolic interactionism, and benefits of usage. These themes were found within all three types of experiences, as identified during the first round of coding. Upon a second reading using the six codes, I found similarities between the codes and determined some instances required more than one code. When confirming this action with the peer debriefer, the peer debriefer said several feelings and experiences could be coded in multiple ways. Eventually, the codes were combined into three pairs to code similar feelings and experiences: tools and efficiency, benefits and symbolic interactionism, and ease of use and fun.

Tools and Efficiency. During the study activities and end-of-unit GeoGebra tasks, the students actively engaged with GeoGebra software through various interactions, as captured in their journal entries, survey responses, screenshots, and screen recordings. These documents not only revealed a pattern of positive feedback concerning the intuitive design and versatility of GeoGebra’s tools, but they also frequently highlighted how these tools simplified what they saw as complex procedures for measuring and constructing angles and sides for the polygons. More so, students described what the tools meant to them personally. Through these descriptions, students often concluded how they provided a level of efficiency that physical tools, paper, and pencils could not provide.

Regarding GeoGebra's tools, one student expressed appreciation for the specific functionalities of the tools, saying:

> The tools helped me a lot. I really liked the angle with given size tool because it makes it easy to make acute, right, and obtuse angles. I am glad there are tools. And there is a tool for almost everything. I like that the tools also help you learn. Because if they just gave you the answer, you would not have to do any work.

This statement highlights the perceived educational value of the tools, which not only aid in constructing polygons but also facilitate a deeper understanding of the concepts by requiring student interaction and problem-solving.

Further exploration of the tool's impact revealed students' preference for the polygon tool. One student remarked, "The tool I liked the best was probably the polygon tool because it makes making shapes so easy. You just tap as many points as you want, then you put in a point, and BOOM! A shape!" Another student echoed this sentiment: "I like that there is a polygon tool to make shapes instead of just making segments to connect points." The polygon tool was used by students frequently during the study activities.

Additionally, students noted the practical applications of various other tools in GeoGebra. One student detailed their use of the tools for specific tasks:

> It helped me because there were tools to use. Like when I was looking at rectangles, I could use the distance or length tool to measure the sides or angle tools to measure the angles or sometimes I could show the diagonals. Then I just had to observe and figure out the geometric properties.

Another student highlighted the software's advantages, stating they could "visualize and manipulate the polygons better with the tools on GeoGebra" and appreciated being able to "make shapes using the gridlines." These remarks collectively demonstrate the students' perceived utility of GeoGebra's tools in facilitating their learning.

As previously mentioned, students not only identified how the tools benefited them and their overall appreciation for the tools, but they also noted how the tools provided a level of efficiency that they thought would not be achieved through other means of learning geometry, such as traditional methods of drawing and measuring. One student noted the ease of creating precise angles and measurements with the software:

> You can go and put a 90-degree angle in all four angles, and it does it for you, and if you give a length it also does it, and if we were to do it on paper, we would have to use a ruler and protractor and find the measurements ourselves.

This sentiment was echoed by another student who appreciated the simplicity of constructing polygons digitally. The student said, "I feel like rectangles, squares, and all triangles are easier to construct on a device than paper and pencil."

Further insights from the participants revealed that GeoGebra's tools not only made learning geometry more accessible but also more engaging. As one student explained, "GeoGebra showed me the measurements, and then I just had to observe and figure out the geometric properties." As noted by some students, the ability to manipulate and directly observe the properties of shapes enhanced their learning process. Another student described his experience by saying, "We could easily move a shape to figure out all of the geometric properties." The convenience of the software was frequently cited, with students appreciating not having to resort to manual drawing. One student remarked, "GeoGebra is very convenient to not have to draw shapes. So, it did make it easier to visualize and modify different types of polygons."

Students were particularly vocal about their preference for GeoGebra over traditional tools, noting significant advantages in terms of time and effort. Statements such as, "It's so much easier and takes so much less time than just using a ruler and protractor and having to measure all those lines" and "It is faster this way than by hand" illustrated the perceived efficiency of the software. Additionally, the ease of learning and continued engagement with geometry using GeoGebra was highlighted as one student stated, "I would use GeoGebra to continue learning geometry because it is easier than pencil and paper." These reflections demonstrate the impactful role the students believed GeoGebra's tools played in enhancing their educational experiences by simplifying complex procedures and allowing for more dynamic interaction with the material.

Benefits and Symbolic Interactionism. During the study, students' interactions with the GeoGebra software were documented extensively through journal entries, survey responses, screenshots, and screen recordings. The documented interactions not only provided a rich data set that highlighted the practical benefits of GeoGebra's tools, such as their role in simplifying complex geometric constructions, but they also illuminated the deeper, symbolic interactions between the students and the geometric concepts facilitated by the software.

Throughout the study, students repeatedly expressed how the tools enhanced their understanding of geometric properties. One student noted, "GeoGebra helped me because it helped me learn the angles, lengths, sides, and names. It did this by allowing me to explore and make different shapes and then it would tell me about the shape." Another student commented that "GeoGebra made it easier to see the shapes in front of me. The tools made it easier to help me learn the properties of the shapes." The perceived benefits were tied to the meanings each student developed from interactions with the shapes in the GeoGebra software. A student commented, “Since GeoGebra makes it easy to move a shape, it was easy to visualize and learn about the shapes.”

Many students highlighted the software's role in deepening their comprehension of specific geometric concepts. For instance, a student described her experience by saying:

> I really didn’t know about the different properties of shapes before using GeoGebra. Like a parallelogram, I didn’t know it had opposite sides parallel, opposite sides congruent, and opposite angles congruent. I know of all the properties for all quadrilaterals because of GeoGebra.

Another student noted, “By using the tools and modifying the quadrilaterals, I was able to learn about diagonals.” Furthermore, a student commented, “I just learned that all triangles have 180 degrees inside. Even if all the triangles are different, they will still have 180 degrees!” The theme of symbolic interactionism was apparent in these statements. Moreover, it was apparent when one student said, “GeoGebra is easy and faster and helps me better because I get to make and create real life things and get to use creativity and my imagination to learn but also have fun with math and geometry.”

The tool's ability to simplify the visualization and modification of polygons was noted by the students. "GeoGebra made it easy to visualize and modify different types of polygons," one student commented. Another student stated, “Thanks to GeoGebra, I can visualize most of the triangles and quadrilaterals. When we could move a shape to figure out all the geometric properties, that helped a lot too.” One student shared a specific learning moment:

> It helped me to know, visualize, and modify polygons when we used the distance and length or the move tools because we could make the polygons smaller and bigger with their lengths, and I noticed that the angles would stay the same.

Lastly, a student remarked:

> GeoGebra helped me to visualize the diagonals of different polygons. Here’s an example: when we went into this session, I didn’t even know what geometric properties triangles and quadrilaterals had. I also forgot that parallelograms were a thing. I knew nothing about trapezoids. I guess I just assumed they were like every other shape. Before using GeoGebra, diagonals were not even existing to me.

Furthermore, GeoGebra's tools were praised for reducing the stress associated with learning geometry and for promoting a hands-on approach to exploring mathematical concepts. A student reflected, "GeoGebra helped me be less stressed. It let me personally explore more than I thought I could." Another student echoed this sentiment: "Since GeoGebra makes it easy to move a shape, it was easy to visualize and learn about the shapes." Moreover, a student commented on how GeoGebra benefited her by allowing her to test her conjectures. She stated:

> It helped me know the properties because you could construct a shape and if you didn’t know the shape, you could go and think about what you already know and then you could look at what you have and memorize the other properties you see.

Lastly, the impact of GeoGebra on students' ability to recall and apply geometric properties was evident. "I could construct a shape, and then I could use the tools to find the geometric properties of many polygons. It showed me what the shapes have in common," mentioned one student. Another student elaborated on their ongoing relationship with GeoGebra by stating, "I would continue to use GeoGebra to learn geometry because it helped me remember how to classify the shapes by proving what shape it is by using geometric properties."

Ease of Use and Fun. Students' journal entries, survey responses, and screen recordings consistently highlighted how students considered the tools to be user-friendly and engaging. As indicated by the data, the tools transformed the students' learning experiences into fun and interactive processes. This section presents the data pertaining to the specific aspects of GeoGebra that students found enjoyable and easy to use.

Student feedback consistently emphasized the ease of constructing geometric shapes using GeoGebra. One student remarked, "I feel like rectangles, squares, and all triangles are easy to construct on GeoGebra." Another student further highlighted this ease, noting the simplicity of manipulating shapes: "Since GeoGebra makes it easy to move your shapes, the shapes were easy to visualize."

The students also frequently mentioned the fun aspect of using GeoGebra in their geometry lessons. Statements such as "I love how GeoGebra allows you to have fun while you're learning" and "GeoGebra is pretty fun!" were common. Another student expressed a newfound enjoyment in geometry, despite previous disinterest: "All in all, it's very fun and I really like it and honestly I don't like geometry that much but GeoGebra makes it fun."

GeoGebra's tools were praised for their ability to facilitate an intuitive understanding of geometry. "The tools were easy to use which made it easier to understand the geometry," one student explained. This was supported by another's experience: "GeoGebra made it easy to make shapes. If I had to draw a square, the shape might have the wrong measurements so geometric properties would be much harder to see than on GeoGebra where the shapes are 99.99% accurate." Such feedback highlights GeoGebra's precision and reliability.

The flexibility and creativity afforded by GeoGebra were also noted as significant benefits. "GeoGebra allows you to make many shapes and draw with different colors. It is very fun to do," shared one student. Another student reflected on the potential for exploration, saying, "Currently, I think GeoGebra is really great. The creativity in my little brain is exploding with ideas. This whole thing makes me have SO MANY questions. I love GeoGebra." This sense of creative freedom and the ability to explore complex ideas in an accessible way were seen as key advantages of the software, as noted by the students.

Lastly, the students' responses strongly highlighted the potential for continued use of GeoGebra. As previously mentioned, one student said, “Yes, 100% I would want to use GeoGebra in 6th grade! Because I love how the tools help you understand the geometry, and you have fun while you are learning.” Another student added, "I would use GeoGebra again because it is easier than pencil and paper, and there are lots of useful tools. It was just more fun to be able to move shapes around." These sentiments underline the strong preference for GeoGebra over traditional methods, pointing to its perceived effectiveness as a learning tool.

Research Question 2a: How do students experience using tools available in GeoGebra?

To answer this sub-question, a second phase of coding was conducted, which used two rounds of coding. The first coding round looked at the survey results (specifically question number four), the video transcripts from the study activities and GeoGebra tasks, the screen recordings of the GeoGebra tasks, journal entries created by the participants, and screenshots. During this round, I used a priori coding to identify each tool the students mentioned. I also identified where GeoGebra tools were used in the screen recordings. After identifying the mention of and usage of tools in these data collection sources, I used an inductive approach to look for themes surrounding the mentioned tools.

Four themes became present in the data. The first theme that emerged was that students found the tools beneficial. Within that broad theme, I identified three more themes that pertained to how students perceived those benefits by describing the various ways that students used the GeoGebra tools. The three themes within this context are to construct, to gain insights, and to prove. The data indicated that the participants tended to use a combination of these three themes as their purposes for using the GeoGebra tools. The collected data showed that every student used tools for multiple purposes, as described by these three themes.

Beneficial. The overall feelings of the participants were positive regarding using the GeoGebra tools. The survey and the journal entries revealed a widespread positive sentiment regarding using the GeoGebra tools. In response to question number four on the survey, which asked the participants to share which tools they found useful and to provide examples, a consensus emerged regarding the perceived benefits of the tools. Across the responses, participants consistently expressed appreciation for the GeoGebra tools that aided their exploration and creation of the various polygons. Phrases such as “they helped me” and “I found them useful” were quite common, indicating a general satisfaction with the utility of the tools within the various study activities and tasks.

A significant proportion of the students were generally satisfied with the GeoGebra tools. One participant's statement encapsulates this satisfaction, stating, “I found all of the tools to be really useful.” This participant then justified their sentiment, highlighting how the tools were used. Another participant shared a similar sentiment and stated, “All of the tools were useful. They were hard to use at first but they got really easy to work with. They all really helped out a lot!” This statement indicates that there may have been an initial learning curve for the students but they were able to adjust and find the tools easy to use. This student then justified the response by attributing the usage of the tools to their understanding of different geometric properties and how to construct different polygons.

The journal entries revealed that participants’ perceptions of the GeoGebra tools evolved through the geometry unit. Initial responses generally indicated that participants believed the tools were difficult or cumbersome to use. In subsequent journal entries, participants indicated an improvement in their comfort levels with using the tools and acknowledged their proficiency. For example, one student wrote, “At first, it is kind of hard to understand, but once you get the hang of it, it is pretty easy.” This trajectory, from the initial apprehension to confidently utilizing the tools to construct polygons, discover insights and geometric properties of given polygons, and prove the names of polygons based on their geometric properties, highlighted the students’ perceived benefits of the GeoGebra tools.

To construct. The participants had to construct several polygons throughout the study activities and in the final GeoGebra tasks. While using some GeoGebra tools allowed the participants to construct polygons freehandedly, other tools provided affordances that eliminated the possibility of human error while constructing the polygons to fidelity. The tools and functionalities of GeoGebra that helped students make constructions included using gridlines, the angle tool, the distance or length tool, the parallel lines tool, and the perpendicular lines tool.

GeoGebra makes three types of gridlines available. During this study, by student choice, students used only two kinds of gridlines: isometric and traditional. While using the traditional gridlines, the students mostly used the major gridlines and did not use the minor gridlines. The third type of gridlines that GeoGebra includes, which are polar gridlines, were not used by any participants during the study. Figure 4.10 shows an example of triangles constructed using isometric gridlines. Students who used the isometric gridlines only did so when constructing triangles; the isometric gridlines were not used to construct any other polygons. Figure 4.11 shows an example of a rhombus constructed using the major gridlines. When students used gridlines to make constructions, they used them to either trace the constructed polygons they drew freehandedly, or used the gridlines to help them visually measure the lengths of sides or the measures of particular angles.

Students typically utilized more than one tool while making constructions. In addition to using major gridlines, Figure 4.10 provides an example of a student's construction of a rhombus in which the student also used the angle tool to construct the rhombus. The student to whom this work belonged used the segment tool to construct the sides of the rhombus. Using the angle tool to measure the angles, the student adjusted the position of the endpoints to construct a rhombus that met the criteria of having congruent opposite sides. Figure 4.12 is a screenshot of a student's work in which the student used both the angle tool and the distance or length tool. Additionally, this individual used the segment tool to construct triangles. The angle tool and the distance or length tool were then used to help determine the placement of the segments and their lengths to construct different types of triangles.

Figure 4.10. Use of Isometric Gridlines to Construct Triangles

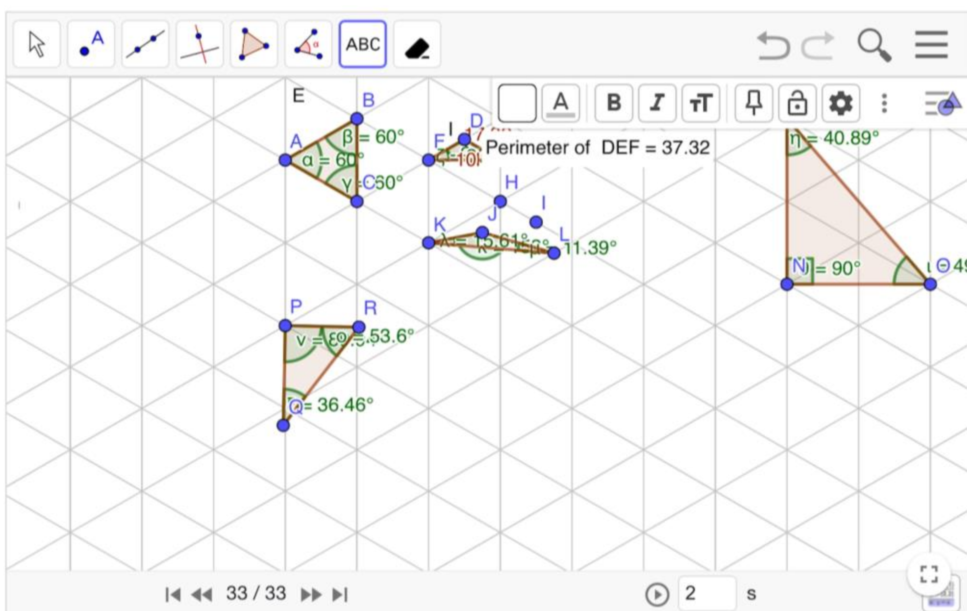
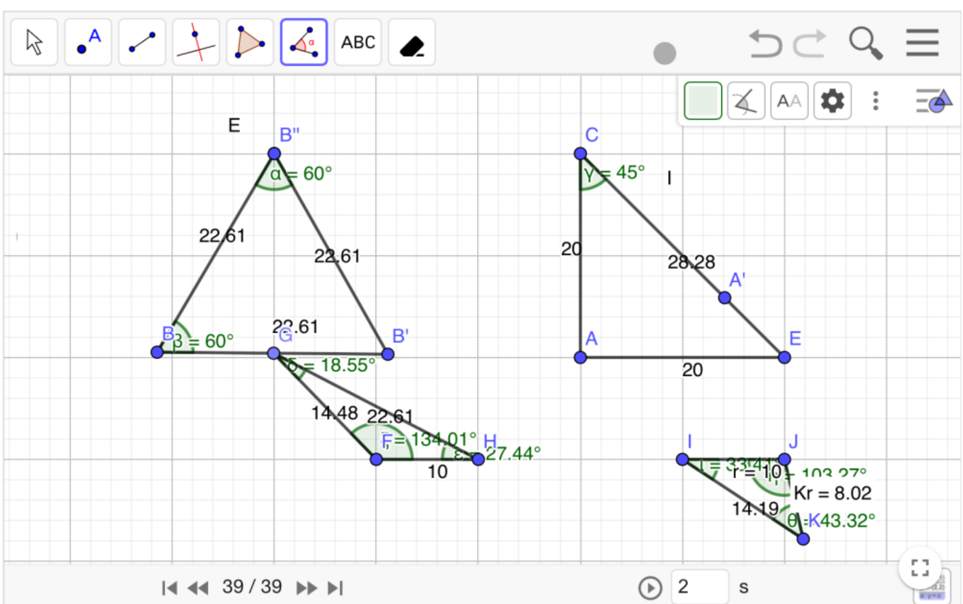


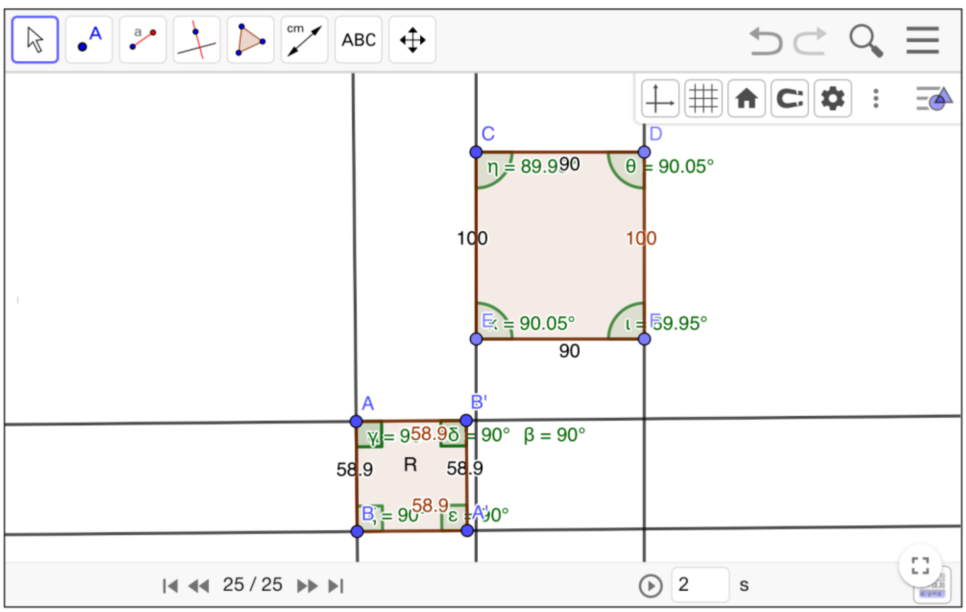
Figure 4.11. Use of Major Gridlines and Angle Tool to Construct a Rhombus

Figure 4.12. Use of the Angle and Distance or Length Tools to Construct Triangles



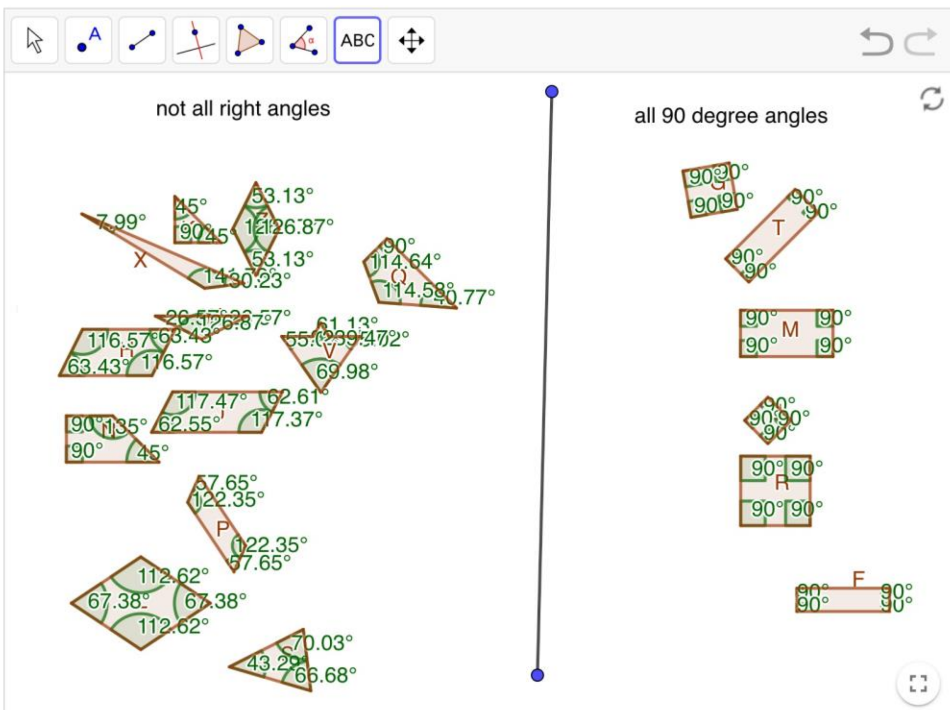
While the gridlines and angle tool were used most frequently in conjunction with the segment and polygon tools, some students chose to use either the parallel lines or perpendicular lines tool or even both to help them construct polygons. Students tended to use these tools when constructing rectangles and rhombuses. Figure 4.13 provides an example of a student's work in which the student used the parallel lines and perpendicular lines tools to construct rectangles. This student then used the polygon tool to construct the rectangles. In this example, the angle tool was not used in the construction of the rectangles; rather, it was used to confirm the angles.

Figure 4.13. Use of Parallel Lines and Perpendicular Lines Tools to Construct Rectangles



To gain insights. The second theme that developed while looking inductively at how students utilized the GeoGebra tools was that students used them to gain insights. They utilized the GeoGebra tools, such as the distance or length tool and the angle tool. When utilizing the tools in this manner, students used the tools to determine the geometric properties of the polygons they were working with. The tools were used throughout the study activities and the final GeoGebra tasks. These tools were used in this manner to help identify polygons and sort them. Figure 4.14 is a screenshot of one student's polygon sort. In this particular example, the student used the angle tool to determine the angle measurements for each polygon and then used those insights to sort the polygons into two groups: one group for polygons that contained all right angles and one group for the polygons that contained angles that were not all right angles.

Figure 4.14. Use of Angle Tool to Discover Geometric Properties



One study activity required students to determine the name of a given polygon. This activity had the students look at the polygon that was constructed in GeoGebra and to properly name the polygon. After looking at the polygon, the students had to name it without using any GeoGebra tools. Once the students made an educated guess about the classification of the polygon, they used the GeoGebra tools to confirm their guess. All but two students initially thought the polygon was a rectangle. Students were free to use whichever tools they wanted to confirm their conjecture. Students tended to use both the angle tool and the distance or length tool. Figure 4.15 provides an example of one student's work. This student traced the constructed polygon with the polygon tool and then used the angle tool to measure the angles. The student also used the distance or length tool to measure the lengths of each side. Another student used the lines tool. This student used the lines tool by overlaying a line on the top segment and one on the bottom to see if the segments were parallel. Figure 4.16 shows a screenshot of this student's work. The student was able to infer that the top and bottom segments were not parallel simply by looking at the gap between the constructed lines. Eventually, the student was able to scroll over enough to see that the lines intersected.

Figure 4.15. Tracing of Free-Handed Polygon With Polygon Tool

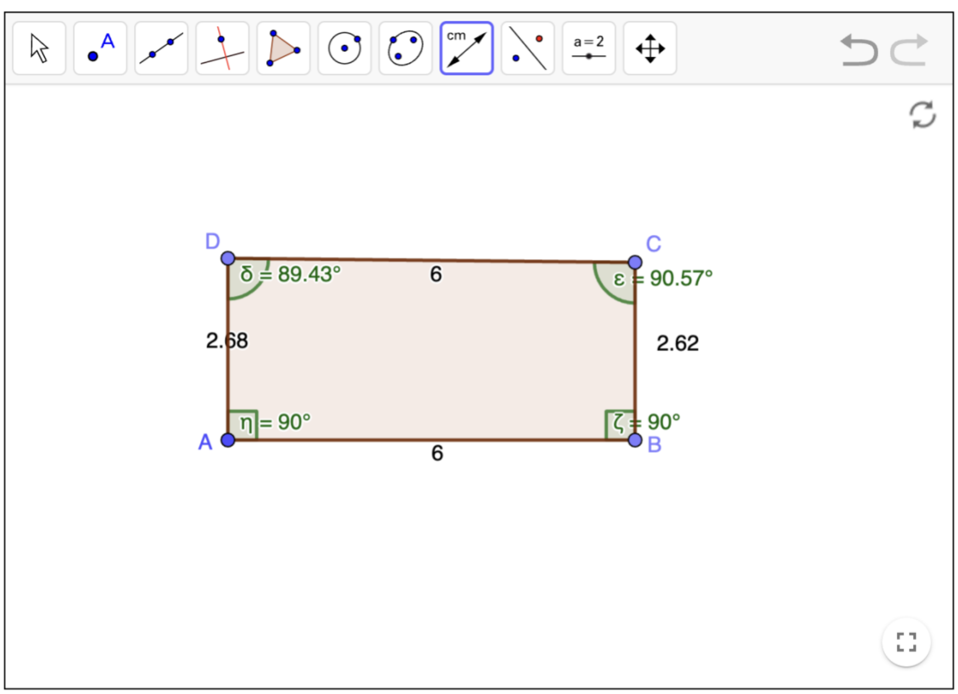
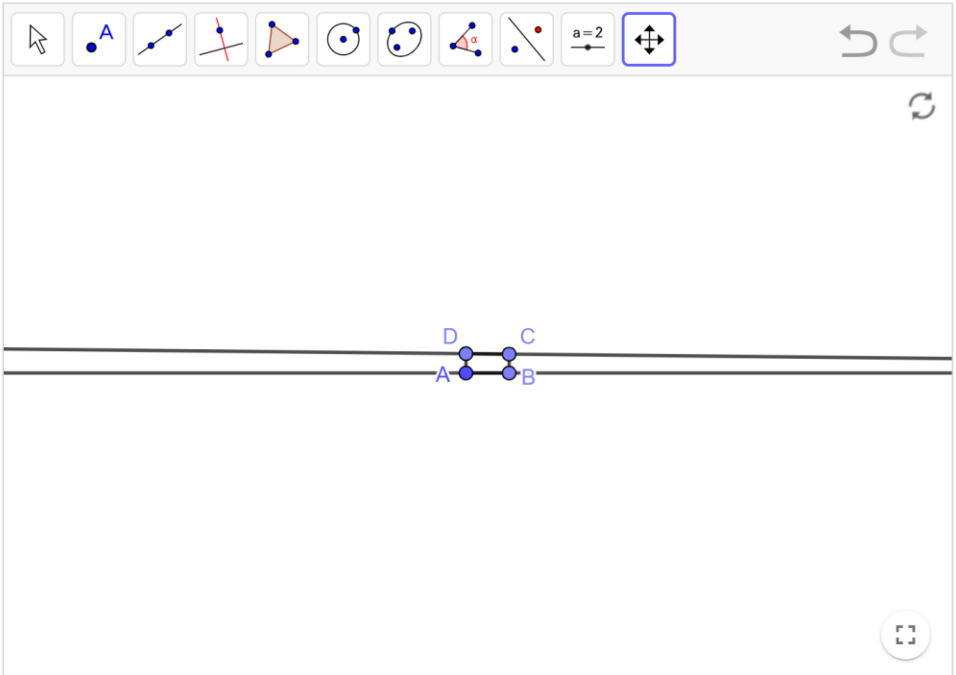


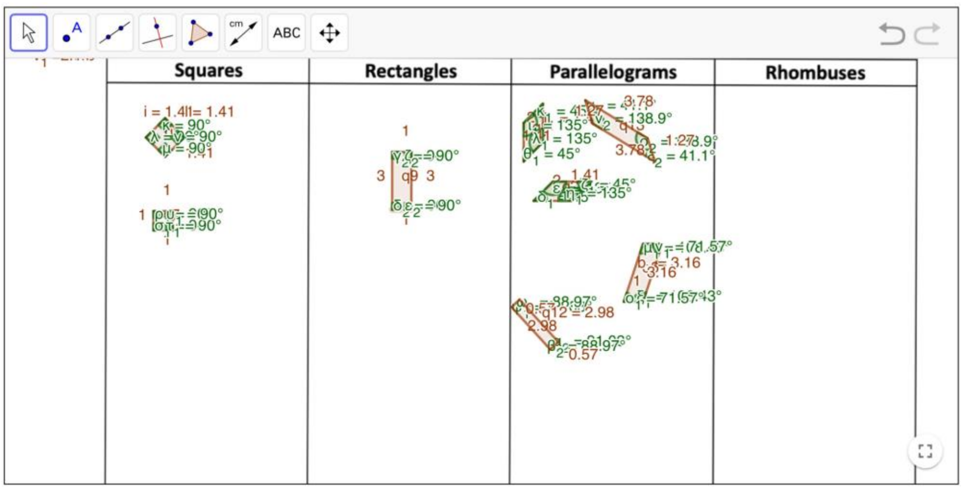
Figure 4.16. Use of the Lines Tool to Determine if Sides Are Parallel



After using the GeoGebra tools, students deduced that the constructed polygon was not a rectangle. Students were able to gain insights about the geometric properties of the polygon by using the GeoGebra tools. Students had to share if their guess was correct. Students realized they were incorrect as not all angles were right angles, nor were the top and bottom segments parallel. Many students then correctly identified the polygon as a trapezoid based on the insights they gleaned from using the GeoGebra tools.

To prove. The third theme that developed while analyzing the data is that once students knew the geometric properties of the polygons, they used the tools to prove the classifications and names of constructed polygons. This was especially apparent while looking at data from the quadrilateral sort. Figure 4.17 shows an example of how the GeoGebra tools were used to prove the classifications and names of the given quadrilaterals. The student to whom the example belongs used the distance or length tool and the angle tool to justify why each quadrilateral was placed, or not placed, into a certain column. Looking back at Figure 4.15 on page 92, we can see that the student also proved the constructed polygon was not a rectangle as the polygon did not contain four right angles.

Figure 4.17. Use of Tools to Prove Quadrilateral Classification



Research Question 2b: When students are observed using (or not using) tools available in GeoGebra, what is their level of geometric thinking?

I used a third phase of coding, with two rounds, to answer this sub-question. I examined the screen recordings and screenshots to identify instances where students demonstrated geometric thinking. This phase involved a detailed analysis of visual data to pinpoint specific moments of geometric reasoning. Timestamps were recorded for each moment the students showed evidence of geometric thinking. Using a priori coding, each identified instance was then categorized according to the level indicators used by Burger and Shaughnessy (1986), as detailed in Appendix H. By applying these indicators consistently, I systematically classified the students' geometric thinking into distinct levels. This systematic approach ensured that all relevant data points were accurately captured and coded.

I identified tool usage within the recorded instances of geometric thinking for the second round of coding using a priori coding. I carefully reviewed each instance of geometric thinking to determine whether the GeoGebra tools were utilized. This involved a detailed analysis of the screen recordings and the student responses to the questions asked within each GeoGebra task, as seen on the screenshots. Through this process, I tracked the presence and application of specific GeoGebra tools. By coding these aspects, I looked at how the students interacted with the GeoGebra tools and how those interactions intersected with their geometric thinking.

To ensure the reliability of my coding process, I collaborated closely with the peer reviewer. We thoroughly reviewed the coded data points together, verifying the accuracy and consistency of the coding process. This collaborative verification process was not just a formality but an essential part of my research, as it provided an additional layer of scrutiny to confirm that the coding process was rigorous and systematic and that the findings were free from bias. Additionally, this approach ensured that the data was reliable and valid, and our collective effort was reflected in the overall quality of my research.

The following two sections demonstrate some instances when students used tools and when they did not use tools when showing evidence of geometric thinking and reasoning.

Instances of Using Tools

As students engaged with the GeoGebra tasks, they frequently utilized various tools within the software to aid their geometric reasoning and thinking. One student highlighted the utility of the angle tool and the angle with a given size tool, explaining:

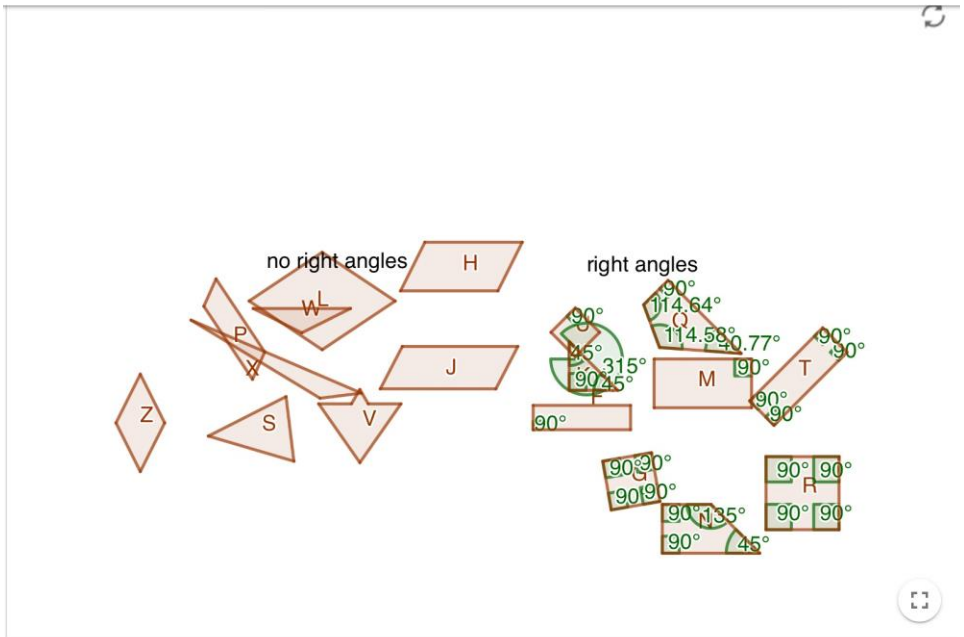
> The angle tool and the angle with a given size tool helped me measure the angles and sides so I could know the difference between isosceles, equilateral, and scalene triangles. Here's an example, if a triangle that I thought was equilateral had an angle measure 59.9 degrees I would be wrong. It also helped me add up the angles to see if they added up to 180 degrees.

This illustrates how the student leveraged these tools to accurately identify and classify different types of triangles based on their internal angles and side measurements.

Another student described using the angle tool to verify the presence of right angles within various shapes. They stated, "The angle tool helped me to see if any of the shapes had right angles or exactly 90 degrees measured." This tool's usage demonstrates a methodical approach to determining whether a given shape contains right angles, which is a critical step in classifying and understanding the properties of different geometric figures. As can be seen in Figure 4.18, the student ensured precision in their geometric analysis by measuring and confirming the exact angle measurements.

A different participant echoed the importance of precision when dealing with angles close to 90 degrees. She shared, "I used the angle tool in case the polygons had like a 90.02 degree angle. I would know so I won't make that mistake. A right angle needs to be EXACTLY 90 degrees." The emphasis on exactness underscores the student's awareness of the strict definitions in geometry, particularly the requirement for right angles to be precisely 90 degrees. The tool enabled the student to avoid errors in her reasoning by providing exact measurements.

Figure 4.18. Screenshot of Student's Work Showing How the Angle Tool Helped



One student faced challenges sorting quadrilaterals based on their angle measurements. This student stated:

> I didn't place all of them into a column because they weren't exactly the measurements of any quadrilateral I know of. Here's an example, q10 was so super close to being a parallelogram, except their angles were just a smidge off. To be a parallelogram, opposite sides have to be congruent, but they weren't. The measurements are on one angle 69.09 and the opposite 68.85. They are so close but not exact, so that's why I didn't put some quadrilaterals into a column.

This account demonstrates how the student used precise angle measurements to determine whether a shape met the criteria for classification as a specific quadrilateral type. The slight discrepancies in angle measurements led to the decision not to classify the shape as a parallelogram, reflecting the student's commitment to accuracy and the detailed analysis facilitated by the GeoGebra tools.

No Tools

As students tackled the GeoGebra tasks, they demonstrated their geometric reasoning skills through both the use and non-use of GeoGebra’s tools. While comparing different shapes, one student provided a detailed verbal analysis of his geometric reasoning without using GeoGebra’s tools. He explained:

> They are alike because H is a parallelogram and Z is a rhombus, but if you look at their geometric properties a parallelogram needs to have opposite sides parallel, opposite sides congruent, and opposite angles congruent which a rhombus has. The geometric properties for a rhombus are all sides congruent, opposite sides are angles congruent, they are both parallelograms for all those reasons. They have two obtuse and two acute angles.

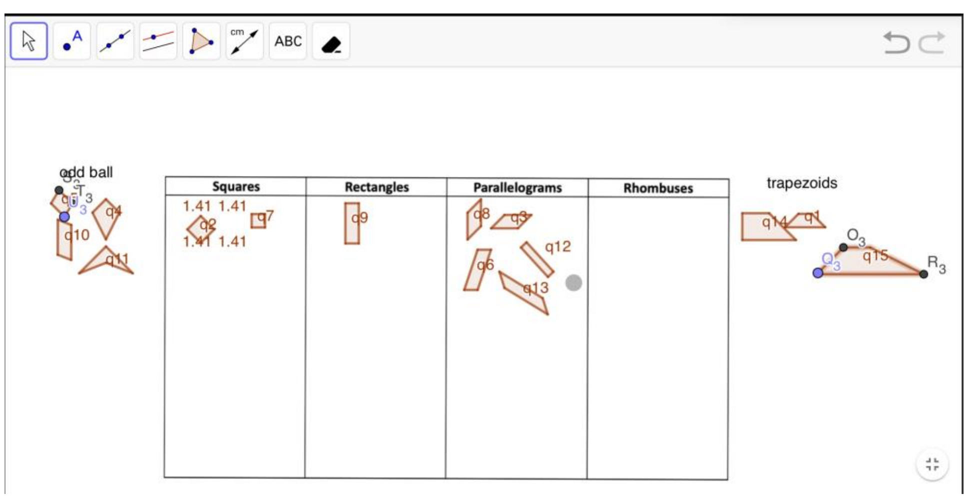
This explanation highlights the student's understanding of geometric properties and his ability to articulate the similarities between a parallelogram and a rhombus based on these properties without using any of the tools to help deduce the geometric properties of the given polygons.

In another task, a student assessed the classification of a triangle based on its angle properties without resorting to GeoGebra tools. He observed, “I can see that this triangle doesn't have two angles that are congruent, so it can't be isosceles. I need to adjust one of these angles to make it the same as another.” This approach showcases the student's grasp of the criteria for identifying an isosceles triangle, emphasizing the need for congruent angles to meet this classification.

Further, students expressed their understanding of the properties of rectangles and squares through verbal reasoning. One student remarked, “Rectangles are not always squares, but squares are always rectangles,” demonstrating their awareness of the hierarchical relationship between these shapes. Another student discussed constructing a rectangle, noting, “If you want to construct a rectangle and it doesn't have all 90-degree angles, you can fix the angles using a tool.” This comment reflects the student's knowledge of the defining characteristics of rectangles and the potential use of tools to achieve precise geometric constructions.

Lastly, the statement “All sides are congruent” was used by several students to describe a specific geometric property, especially in the context of identifying or verifying a shape such as a square or rhombus. This simple yet precise observation underscores the student's ability to recognize and articulate key geometric properties without using GeoGebra’s tools. As seen in Figure 4.19, one student looked visually at the sides of each quadrilateral to determine the placement of the quadrilaterals within the columns and which quadrilaterals did not belong in a column on the quadrilateral sort.

Figure 4.19. Screenshot of Geometric Reasoning Without the Usage of GeoGebra’s Tools



Assigning van Hiele Levels

To assign van Hiele levels to the students, I began the data analysis by coding each screen recording and its corresponding screenshot for each of the five end-of-unit GeoGebra tasks for the 14 students whose work was collected for qualitative analysis. The objective was to identify students' behaviors and verbalizations that could be categorized within a specific van Hiele level of geometric thinking. This initial coding revealed that it was feasible to assign a van Hiele level to each task performed by the students.

After coding the individual tasks, I assigned a van Hiele level to each task based on the observed behaviors and verbalizations. This process involved carefully examining each instance where a student demonstrated a specific level of geometric thinking, as defined by the van Hiele model. Students typically demonstrated behaviors across the three van Hiele levels that were looked at in this study. To make a final determination for each task, I looked at the overall performance of the students, based on the indicators, to assign a level based on their overall behaviors and verbalizations. Subsequently, each student was assigned an overall van Hiele level by finding the median van Hiele level of all five tasks.

To ensure the validity of the coding and analysis, the peer reviewer independently coded and analyzed 10 of the videos and their accompanying screenshots. The videos that were analyzed by the peer reviewer were selected so that videos from students across the four van Hiele levels, as assigned by the quantitative data analysis, would be analyzed. During our discussions, we found that our analyses were consistent 90% of the time. The timestamps where we noted geometric reasoning generally matched, indicating a high level of agreement in our observations. Occasionally, we encountered discrepancies in our coding. For example, one of us might have coded a second instance of the same indicator for the same event, particularly when the action, behavior, or verbalization spanned several minutes, which led to differences in the number of instances coded. Despite these minor differences, our overall task-level assignments were largely aligned. Of the ten tasks reviewed by the peer reviewer, we differed in our van Hiele level assignment for only one task.

In the task where our coding differed, I initially assigned a Level 3 while the peer reviewer assigned a Level 2. The peer reviewer had noted more instances of the student comparing shapes explicitly by means of properties of their components, a Level 2 indicator, than I did. After re-evaluating the screen recording and screenshots, I adjusted the assigned level for this task to Level 2, aligning with the peer reviewer's analysis.

Following the task-level analysis, I assigned each student an overall van Hiele level. To determine an overall van Hiele level, I found the median van Hiele level for all five tasks. I then compared this value to the levels for each task. Except for one student, the median level was the same as the mode for each student, which helped to confirm that the median level assignment was an appropriate method for determining each student's overall van Hiele level.

Uncovering Gaps. This coding process allowed me to see the gaps in students' ability to reason geometrically. By watching the screen recordings and the corresponding screenshots, I analyzed each student's behavior using the level indicators. This process allowed me to gain insights into each student's knowledge of the various geometric topics in the unit. Moreover, I could see misunderstandings or a lack of complete geometrical understanding and reasoning as it pertained to the topics of squares and rectangles, trapezoids, unknown shapes, and the number of construction possibilities.

Squares and rectangles. While the students seemed to understand the geometric properties of squares and rectangles, I quickly noticed a gap in the understanding of some of the students. It became apparent that some students struggled to understand the hierarchical relationship between squares and rectangles. This gap became apparent when analyzing the rectangle construction and the sorting quadrilaterals tasks. One student said, "Rectangles are not squares, but squares are rectangles." This statement indicates that the student neglected to realize that rectangles can have four congruent sides, making them squares. Another student identified a rectangle as a square since it had four right angles. This indicated that the student believed any shape with four right angles must be a square and neglected to consider the other geometric properties that comprise squares and rectangles.

Trapezoids. While the students seemed to have grasped the concepts of the various quadrilaterals we worked with, trapezoids seemed to be an issue for some students. This gap was apparent on the quadrilateral sort. The quadrilateral sort had predefined categories for students to place the shapes into. Not every quadrilateral fit into the predefined categories. One student correctly identified the quadrilaterals that did not fit into the predefined categories but incorrectly identified all those quadrilaterals as trapezoids. This alludes to the notion that the student assumes every quadrilateral that is not a parallelogram is a trapezoid, which is not true, as quadrilaterals can be kites.

On the polygon sort task, a student sorted many of the polygons correctly into various groups; however, she had a misconception of what a trapezoid is. This student identified all polygons she didn't place into a column as a trapezoid. A gap in this student's understanding was that she believed every quadrilateral that is not a parallelogram must be a trapezoid. Other students also mimicked this behavior when completing the quadrilateral and polygon sorts.

Anything that didn't fit. While analyzing the quadrilateral and polygon sorts, I realized that students struggled with shapes they didn't know where to place or group. In the polygon sort, some students put these shapes in a miscellaneous group, while others did not even attempt to put them into a group or provide any labels. On the quadrilateral sort, several students could explain why certain quadrilaterals did not belong in the columns, though other students did not know what to do with those quadrilaterals. Some of those students made a trapezoid column, and others ignored them. I noticed that many students believed the quadrilaterals should belong somewhere, but they did not know where or how to group or label them, thus causing some frustration.

Infinite amount of shapes. The students had a hard time imagining or wrapping their minds around the notion that there is an infinite amount of rectangles, triangles, and rhombuses. This was apparent in all the construction tasks. A Level 1 indicator states, "Inability to conceive of an infinite variety of types of shapes." Each construction task included this question: If class time was unlimited, how many different [name of polygon] do you think you could construct? The lowest number that a student said was 10. The student who replied 10 responded to the number of triangles he could construct. He followed his statement by arguing that "there are so many different variations of isosceles triangles" that he could probably draw more. This response did not indicate the number of scalene or equilateral triangles the student thought he could construct. Other students made statements such as "a lot." One student replied 100 in response to the number of triangles she believed she could construct. She justified her response by stating, "because there are many possibilities." This same student indicated that she could draw millions of rectangles. While some statements alluded to the fact that students believed several different constructions could be made, not one student indicated that an infinite number of constructions could be created.

Mixed Methods Results

Research Question 3: What is the impact of using GeoGebra to support fifth-grade students' learning about polygons?

The convergent mixed methods approach is widely recognized and frequently employed in educational research. This approach is pertinent in studies where integrating quantitative and qualitative data provides a more comprehensive understanding of the research question. By converging these two data types, researchers can explore different dimensions of a phenomenon, allowing for a deeper and more refined analysis. As previously stated, in this study, the use of a convergent mixed methods approach was crucial to understanding the impact of GeoGebra on students' van Hiele levels of geometric thinking. This methodological choice enabled me to examine how GeoGebra influenced the development of geometric reasoning skills, as measured through task-based assessments, and how these results compare with the results from the van Hiele Geometry Test, a traditional assessment.

Given that this study utilized a convergent mixed methods design, equal weight was given to both the quantitative and qualitative data. This balanced approach allowed for thoroughly exploring this research question, ensuring neither data type was considered superior. By treating both data sources equally, I identified similarities and discrepancies, which gave me a more comprehensive understanding of how GeoGebra impacted students' geometric thinking.

An Excel table was created to organize and analyze the data, serving to organize both the quantitative and qualitative data collected from the 14 students in which qualitative data was collected. Table 4.10 highlights the students' geometric thinking from quantitative and qualitative data. The quantitative data, derived from the van Hiele Geometry Test, provided a traditional measure of each student's geometric understanding. In contrast, the qualitative data from the GeoGebra tasks offered insights into how students applied their knowledge in a dynamic and interactive environment. The table's layout facilitated direct comparisons between these two data sources, making identifying consistencies and discrepancies in the van Hiele Level assignments easier.

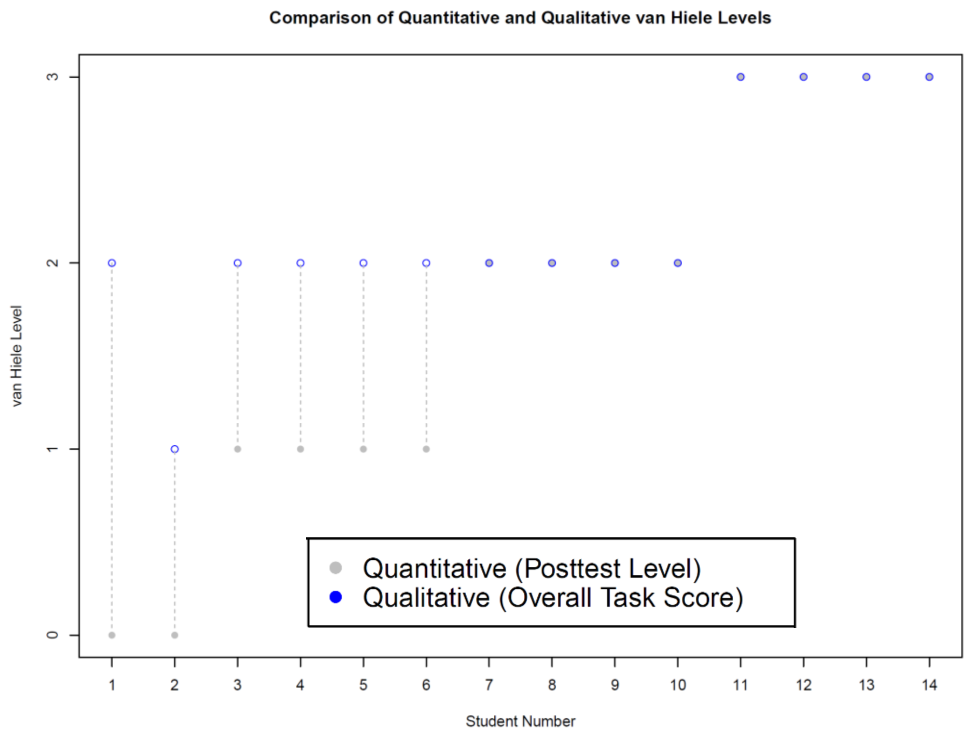
I chose to include the weighted sums on the table, as the sums can indicate if a student met the level criterion and followed the model sequentially as described or if the student was able to successfully answer questions correctly at higher levels than their assigned level. For example, the weighted sum can indicate when a student could not meet the criterion for level n but could do so for $n + 1$. In particular, I looked at the three students who had the weighted sums of 0, 5, and 6, which indicated they either met zero criteria or criteria for levels higher than they were assigned. In all three instances, these individuals were assigned a higher overall van Hiele Level on the GeoGebra tasks than they were assigned from the van Hiele Geometry Test. This data indicates a point for further study.

The table's organization proved invaluable during the data analysis process, as it provided a clear and concise way to compare the quantitative and qualitative van Hiele levels. This comparison was essential for determining the alignment between the two data sources and for identifying any potential discrepancies. Using the data in the table, I created Figure 4.20, which is a scatter plot that visually represents the relationship between the quantitative and qualitative van Hiele levels. The scatter plot was a critical tool in the analysis, as it provided a visual representation of the data, making it easier to see patterns and trends as the results were highlighted.

Table 4.10. van Hiele Level Assignments for Quantitative and Qualitative Data.

Student Number	3 out of 5 Posttest Level	Weighted Sum	Constructing Triangles Level	Constructing Rhombuses Level	Constructing Rectangles Level	Sorting Quadrilaterals Level	Sorting Polygons Level	Overall Task Score
31104	0	6	2	1	2	1	2	2
31072	0	0	2	3	1	1	1	1
30013	1	5	3	3	2	2	2	2
31030	1	1	2	3	2	2	2	2
31084	1	1	2	2	2	1	2	2
31027	1	1	2	1	2	1	2	2
30014	2	3	3	3	2	2	2	2
30015	2	3	2	1	2	1	2	2
31098	2	3	2	2	2	1	2	2
31062	2	3	3	2	3	1	2	2
31122	3	7	3	3	2	2	3	3
31078	3	7	3	3	2	3	3	3
31044	3	7	3	3	3	3	3	3
31093	3	7	3	3	2	2	3	3

Figure 4.20. **Scatterplot Showing van Hiele Level Assignments**



In Figure 4.20, the gray dots represent the van Hiele levels assigned through the quantitative data, while the open blue dots represent the levels assigned through the qualitative data. This color coding allowed for a clear distinction between the two data sources. In cases where there was no difference between the quantitative and qualitative levels, the gray dots were enclosed by the blue dots, indicating alignment. However, in cases where the levels differed, the two dots were connected by a dashed line, visually representing the discrepancy between the level assignments. This visual representation was crucial for identifying students whose geometric reasoning appeared to be more advanced in the context of the GeoGebra tasks than in the traditional test.

The analysis revealed that eight of the 14 students in the dataset had matching quantitative and qualitative van Hiele levels. However, for the six students whose van Hiele levels differed between the two data analyses, the qualitative levels assigned through the GeoGebra tasks were higher than those assigned through the van Hiele Geometry Test. It is important to note that there were no cases in which the quantitative levels were higher than the qualitative levels. In particular, students who were assigned a van Hiele Level 0 or 1 on the van Hiele Geometry Test were all assigned higher levels on the GeoGebra tasks.

The scatter plot in Figure 4.20 on page 107 provided a clear visual representation of these findings. It made it easy to see that most students' van Hiele levels were consistent across both data types. However, the scatter plot highlighted discrepancies where the qualitative assessment assigned higher levels than the quantitative analysis.

As noted, of the 14 students included in the mixed methods section of the study, six students were assigned different van Hiele levels when comparing the quantitative and qualitative results. For the six students whose levels were different, five had a discrepancy of one van Hiele level when using both criteria during the quantitative analysis. The remaining eight students were assigned the same van Hiele level for both criteria during the quantitative analysis, which also aligned with the same van Hiele level that was assigned during the qualitative analysis.

The quantitative and qualitative data were further analyzed using R. Analysis revealed that the mean difference between the quantitative and qualitative van Hiele levels was -0.5, with a standard deviation of 0.65. This result indicates a slight tendency for the qualitative levels to be higher than the quantitative levels, and the standard deviation indicates some variability in the differences between the two assessments.

Summary

In this chapter, I examined the impact of using GeoGebra on the geometric reasoning levels of fifth-grade students, as guided by the van Hiele theory of geometric reasoning. The study utilized a mixed methods approach by combining quantitative and qualitative data to explore how GeoGebra influences the possible advancement in van Hiele levels for fifth-grade students. The results of the quantitative, qualitative, and mixed methods research questions were discussed in detail.

The quantitative data, collected from the pre- and posttest scores collected from each student's van Hiele Geometry Tests, revealed a significant improvement in students' geometric reasoning skills by the conclusion of the geometry unit. At the start of instruction, the pretests showed that most students were classified at lower van Hiele levels. Overall, the posttest scores indicated a statistically significant increase in the van Hiele levels of the students, with a much higher number of students at Level 2 or 3. Next, gender differences were analyzed, but I did not find any statistically significant differences in the growth that students of both genders experienced throughout the study. Finally, I discovered that there was some correlation with the changes in van Hiele levels compared to each student's prior achievement, as determined by NWEA scores.

The qualitative data gathered through surveys, student journal entries, and GeoGebra tasks provided more profound insights into students' ability to reason geometrically. Students expressed positive experiences of using GeoGebra as a learning tool. They highlighted the usefulness of GeoGebra's various tools in their explorations. Some students mentioned challenges and confusion, particularly as they relate to the usability of the software itself. Still, the students later indicated that these challenges and confusion were often mitigated as they used the software more. The analysis identified three key themes: the benefits of GeoGebra's tools in constructing shapes, gaining insights into geometric properties, and proving geometric classifications. Throughout the unit, it appeared that the students' use of GeoGebra was tied to their level of thinking, as indicated by instances of students using higher-order thinking as they utilized GeoGebra's tools effectively.

The mixed methods analysis revealed that the qualitative data from GeoGebra tasks indicated higher levels of geometric thinking than the quantitative data for six of the students. It helped to show that the level assignments from the quantitative and qualitative data matched for eight of the students.

This chapter demonstrates that GeoGebra positively impacted students' geometric reasoning, as evidenced by improvements in the students' final van Hiele levels, according to the quantitative and qualitative data. The convergent mixed methods design of this study helped to see the overall increases in students' van Hiele levels. The next chapter discusses the results in more detail and provides a rationale behind the results.

Chapter 5 - Discussion

Overview of the Study

This research study looked at the effects of using GeoGebra, a dynamic geometry software, on fifth-grade students' progression through the first three levels of the van Hiele Theory, a theory that serves as a model for geometric reasoning. The study employed a mixed methods approach, which combined quantitative and qualitative data to explore these effects thoroughly.

The study was conducted in a rural Midwestern elementary school, where the participants were members of my math classes. Quantitative data was collected from 38 students. Of those 38 students, qualitative data was collected from 14. The van Hiele theory, which outlines five hierarchal levels of geometric reasoning and symbolic interactionism, served as the theoretical framework through which the study was conducted. This study looked at the first three levels of the van Hiele theory: visualization, analysis, and informal deduction.

The quantitative data was collected using a pretest-posttest approach, with the van Hiele Geometry Test serving as the method for data collection. Journal entries, surveys, and GeoGebra tasks provided the qualitative data. Together, the quantitative and qualitative data were used independently and collectively to answer the research questions. The purpose of the research questions was to gain insights into the overall effect that the usage of GeoGebra has on students' geometric reasoning; moreover, within the larger group, the research questions sought to see if differences existed between genders and students with varying levels of prior achievement. Lastly, the questions provided an understanding into the student's experiences using GeoGebra. The mixed methods design helped to ensure that both the quantitative and qualitative data captured any changes in the geometric reasoning levels of the students, which helped to develop a comprehensive understanding of the study's impact.

By triangulating the data from various sources, which included journal entries, screen recordings and screenshots, and a survey, I was able to gain insights into the student interactions with GeoGebra and its tools. These allowed me to explore if the tools supported or hindered their learning, as they perceived it. The overwhelming positive responses from the students, indicated that they saw GeoGebra has having a positive effect on developing their geometric reasoning skills.

Discussion of the Findings

Quantitative Discussion

The quantitative analysis of this study indicated that the integration of GeoGebra into geometry instruction positively impacted the geometric reasoning levels of fifth-grade students, as measured by their van Hiele levels. Specifically, when using the 3 out of 5 criterion, the percentage of students at Levels 2 and 3 increased from 39.5% on the pretest to 76.3% on the posttest, and when using the 4 out of 5 criterion, the percentage of students at Levels 2 and 3 increased from 10.5% on the pretest to 44.7% on the posttest. The Wilcoxon Signed Rank Tests for both criterion showed the results to be statistically significant as the p-values for both tests were below 0.05, which was the set alpha.

The data and subsequent statistical tests showed that these improvements were observed for both male and female students and did not indicate statistically significant differences in the van Hiele Level increases between the two genders. The Wilcoxon Rank Sum tests yielded p-values higher than the set alpha of 0.05 for both the pretest and posttest data for both sets of criterion. This further suggests that GeoGebra was an effective method for geometry instruction for all students.

Using NWEA scores to determine prior achievement, the data showed that this achievement paralleled the changes in students' van Hiele levels in some instances and not in others. For example, some students had decreased NWEA scores and van Hiele levels, as measured by the pretest and posttest scores. However, data for most students showed increases for both metrics. Some students did have a decrease in their NWEA scores and an increase in their van Hiele levels. This suggests that while standardized test scores provide some insight into student achievement, other metrics, such as GeoGebra, may help to provide a fuller picture or better capture gains in geometric reasoning.

The analysis of the weighted sums further supported the notion that students demonstrated significant gains in their geometric reasoning. The Wilcoxon Signed Rank Test for both the 3 out of 5 and 4 out of 5 criterion showed that the scores from the pretests to the posttests were statistically significant, as the p-values were below the alpha of 0.05. The weighted sums were indicators of some students' ability to reason geometrically above their assigned level.

These findings suggest that GeoGebra positively impacted students' geometric thinking, with significant gains in van Hiele levels observed.

Qualitative Discussion

The qualitative analysis of students' experiences using GeoGebra during instruction showed an intricate interaction between the learners and GeoGebra's tools. Data was collected from students through surveys, journal entries, and observations of screen recordings and screenshots. The gathered data highlighted the benefits and challenges the students found with the tools. The study uncovered generally positive thoughts from the students, with many expressing GeoGebra's ability to enhance their understanding of geometric concepts. This was especially true regarding the visualization and manipulation of the various polygons the students explored. It is important to note that some students shared confusing and/or negative experiences with GeoGebra.

Several key findings developed through the data analysis showed the roles GeoGebra's tools had in developing students' geometric reasoning skills. One finding from the study is that the GeoGebra tools played an integral role in helping students gain deeper insights into geometric properties. The tools allowed the students to explore, construct, and analyze shapes precisely. For example, students often mentioned the angle tool and distance or length tool as key tools for determining the specific properties of polygons. Secondly, students used the tools to help them construct polygons, using the geometric properties they knew for each polygon. Lastly, the tools were used to help them prove the classification of polygons by analyzing the geometric properties of the given polygons. Once students found they could use the tools to help them make constructions, gain insights, and prove the classification of polygons, they were able to use this information to explore polygons more collectively. This was especially true when they explored the hierarchy of quadrilaterals.

As noted earlier, the data also revealed some challenges the students faced when using GeoGebra. For example, some students indicated that they initially experienced confusion when first using the tools. This learning curve was apparent in the journal entries collected from the beginning of the geometry unit. As the unit progressed, students mentioned in their journal entries that their proficiency with using the tools effectively and efficiently had improved with additional usage.

The data also shows that individual student performances on the various GeoGebra tasks were varied. Students generally performed better on the construction tasks, which had students construct triangles, rhombuses, and rectangles, than they did on the sorting tasks, which required students to sort quadrilaterals and to sort polygons. Specifically, the median van Hiele level for the sorting quadrilaterals tasks was 1.5, and the median level for the sorting polygons task was 2. In contrast, the median levels for the constructing tasks were 2.5, 3, and 2, respectively. This indicated that the sorting tasks may have been more complex than the constructing tasks. Thus, they may have required higher-order thinking skills for students to complete versus the required cognitive demand of the construction tasks focused on solitary polygons.

These findings point to the use of GeoGebra significantly impacting the students' learning experiences. GeoGebra's tools were generally well-received and played a crucial role in helping students understand and apply geometric concepts. However, the study did highlight some initial confusion and negative experiences, which suggests that additional instructional support may be necessary to help ensure the students can use GeoGebra's tools from the first lesson effectively.

Mixed methods Discussion

The mixed methods analysis conducted in this study provided compelling evidence of GeoGebra's impact on fifth-grade students' geometric reasoning levels. By utilizing a convergent mixed methods approach, I integrated quantitative data collected from the van Hiele Geometry Test with the qualitative data from GeoGebra tasks to understand how GeoGebra influences geometric reasoning as it pertains to polygons.

The qualitative data indicated higher levels of geometric reasoning than the quantitative data for six of the 14 students included in the mixed methods part of the study. This discrepancy happened only for those who were assigned a Level 0 or 1 from the quantitative data analysis. For five of these students, the assigned van Hiele level from the qualitative data was one level higher than the assigned level from the van Hiele Geometry Test. The qualitative data for the remaining student assigned the student to a van Hiele level two levels higher than the quantitative assignment. This suggests that GeoGebra's interactive and dynamic features may have allowed the students to express and develop a more advanced understanding of geometric concepts than what was captured by the van Hiele Geometry Test, a traditional paper-pencil assessment.

The discrepancy in van Hiele level assignments for six of the 14 students is important to look at further. Usiskin (1982) found that students whose assigned levels differ, when using both criteria, may be in transition from the lower level to the next. Five students out of the six who had discrepancies, had a discrepancy of only one van Hiele level. The qualitative data for four of these students placed them one level higher that their assigned level from the quantitative data. So, for these four students, it is possible that they may be transitioning from one van Hiele level to the next. The qualitative data gathered from GeoGebra, may suggest that these students are beginning to grasp more advanced geometric concepts that were not fully reflected in their quantitative scores. The consistency observed in the van Hiele levels of the remaining eight students, across both criteria and types of data, indicates that these students have likely been placed into the van Hiele level that accurately reflects their ability.

The findings also suggest that GeoGebra might be especially beneficial for students who struggle with traditional forms of assessment. For instance, 12 students missed question number one on the van Hiele Geometry Test, which required students to find all the squares. All 12 students who missed the problem identified the rectangle as a square. Still, these same 12 students correctly demonstrated their understanding of rectangles and squares, as evidenced by their actions on the quadrilateral sort and rhombus construction tasks. This indicates that the van Hiele Geometry Test may have underestimated the students' geometric reasoning levels. One rationale for this inference is that GeoGebra engaged students differently than the the van Hiele Geometry Test. The students were able to move quickly through the multiple choice van Hiele test without giving much thought to the questions, but the GeoGebra tasks were more time-consuming and thus the students spent more time processing through their understanding of the geometric concepts as they interacted with the polygons using the software. This provides crucial insight as it demonstrates the potential GeoGebra has to support and enhance the learning of students and help them to demonstrate their learning and understanding.

The weighted sums analysis revealed that two students did not meet the criterion for a certain van Hiele level on the van Hiele Geometry Test but could answer questions correctly at a higher level. The qualitative data showed that these two students could reason geometrically at a higher level than the quantitative data indicated. This could indicate a correlation between the weighted sums and a student's ability to reason geometrically that is not typically associated with the weighted sums when using only the van Hiele Geometry Test as a metric for one's van Hiele level.

Moreover, the qualitative data provided valuable insights into how students interacted with GeoGebra. Students frequently mentioned how GeoGebra's tools helped them to visualize and manipulate shapes, which led to a deeper understanding of geometric properties. Eight students had van Hiele level assignments from the quantitative data that matched their level assignments from the qualitative data, which indicates a good alignment between the two assessment methods for these individuals. For the other six students, the higher van Hiele level assignments from the qualitative data may offer a fuller measure of their geometric reasoning ability, especially if they are students who do not typically perform well with traditional assessments.

These findings indicate that GeoGebra positively impacts students' geometric reasoning, as it provides students with opportunities to demonstrate higher levels of geometric reasoning that the van Hiele Geometry Test may not have captured.

Educational Implications

The purpose of this study was to explore the usage of GeoGebra and its effect on the progression through the first three levels of the van Hiele model of geometric thinking for fifth-grade students. The study used a mixed methods approach to gather various types of data to better explore the purpose. The results of this study have both theoretical and practical implications.

Theoretical Implications

This study's results aligned with the van Hiele Theory of geometric reasoning. The theory emphasizes instructional phases to help students increase their geometric reasoning skills. The data revealed that study activities completed using GeoGebra helped students advance in their van Hiele levels. While the van Hieles believed the levels to not only be discrete but also that individuals could reason at one level and not another until undergoing the instructional phases, this research challenges the notion of the discreteness of the levels. Some students demonstrated behaviors indicative of multiple levels simultaneously, as seen in the qualitative data. This was especially apparent when students exhibited behaviors on the same GeoGebra task for more than one level and when assigned differing levels for the GeoGebra tasks. The literature shows that other researchers have had similar findings in their research. Burger and Shaughnessy (1986), Gutierrez et al. (1991b), and Mayberry (1983) all found that the levels may not be as discrete as the van Hieles first suggested. This study's results contribute to this discussion and demonstrate that students may reason at more than one level.

Discrepancies in the quantitative and qualitative data for some students indicated that one form of assessment may not always accurately evaluate one’s understanding. In this study, it appears that the usage of GeoGebra allowed students to demonstrate their geometric reasoning abilities at the same level or better than they were able to using the van Hiele Geometry Test, which is a traditional form of assessment. This study raises the question as to the validity of relying solely on one form of assessment, especially traditional assessments, to measure the geometric reasoning abilities of students and indicates that teachers should incorporate other types of assessment into the classroom, such as interactive, technology-based assessments.

The results of this study highlight the importance of symbolic interactionism in the learning and understanding of geometry. The students in the study interacted with various polygons in GeoGebra. As students explored, manipulated, and constructed polygons, they gained a deeper understanding of geometric properties. Aksan et al.’s (2009), Blumer’s (1986), and Bhattacharya’s (2017) views on the role of symbols and interaction in learning support the facilitation of learning that GeoGebra provided for the students. This study extends their views to how dynamic geometry software, like GeoGebra, can enhance symbolic interaction, which allows students to engage with and internalize geometric concepts through a dynamic approach.

Practical Implications

The literature suggests that students in American schools cannot reason geometrically at levels that would make them successful in high school geometry courses (Lappan, 1999; Mullis et al., 2000; Strutchens & Blume, 1997). As a result of this startling realization, organizations like NCTM and initiatives like the CCSSM have encouraged a deeper emphasis on the teaching and learning of geometry in the elementary grades. This deeper emphasis on geometry instruction has realigned academic standards. Research by Newton (2011) and Zhou et al. (2022) shows that new academic standards emphasize higher van Hiele levels in elementary grades, such as fifth grade. While emphasizing more geometry instruction is a step toward helping students grow in their geometric reasoning abilities, finding effective ways to provide that instruction is important. This study garnered strong evidence that GeoGebra is an effective tool for enhancing geometry instruction for fifth-grade students. The noted improvements in scores from the pretest to the posttest suggest that using GeoGebra as a method for teaching and learning can lead to positive gains in geometric reasoning levels for students.

The results of the study's findings make a strong case for implementing technology, especially dynamic geometry software, into the classrooms of fifth-grade students. This aligns with NCTM's (2000) and CCSSM's (2010) emphasis on using technology in the classroom. The results indicated curriculum development should include integrating dynamic geometry software, such as GeoGebra, into geometry units. The data also shows that fifth-grade students will be served well by using GeoGebra, regardless of gender or prior achievement. In addition to an emphasis on using dynamic geometry software in the new curricula, professional development needs to be provided for educators so that they will be able to help students maximize the benefits of the software. The study highlighted some confusion that students had, which suggests that teacher guidance and proper instruction will help to implement GeoGebra into the classrooms in an accessible and engaging way for all students.

These implications demonstrate GeoGebra's significant potential for transforming geometry education for fifth-grade students. As the theoretical and practical implications show, this study contributes to a deeper understanding of how dynamic geometry software positively affects students' ability to reason geometrically. Moreover, it positively impacts student progression through the van Hiele levels.

Recommendations for Future Research

The methodology I utilized with this study played an instrumental role in revealing the full scope of GeoGebra’s effects on students’ van Hiele levels. It helped highlight both GeoGebra's strengths and potential areas for further investigation. While the results of the study indicated that GeoGebra could significantly increase students’ geometric reasoning, especially by advancing their van Hiele levels, it also exposed discrepancies between the quantitative and qualitative results, which used two different forms of assessment: the van Hiele Geometry Test, which is a paper and pencil test, and interactive tasks that were completed using GeoGebra. The discrepancies suggest that GeoGebra may be a more accurate metric for determining the van Hiele level of a student. Future research could analyze these differences and explore if GeoGebra actually presents as a more effective metric for determining one’s van Hiele level than the traditional van Hiele Geometry Test.

A second area for further exploration is the initial learning curve of using dynamic geometry software. Although the students in this study seemed to become proficient using GeoGebra, some noted initial challenges with the software that provided them with confusing and negative learning experiences. Future studies could focus on ways to provide better instruction for students initially using dynamic geometry software before beginning to implement it as the main learning tool for geometry instruction, as these initial experiences could have affected the learning outcomes for the students who had these experiences.

Two different types of GeoGebra tasks were used at the end of the study and evaluated to determine students' van Hiele levels. The data collected from the two types of tasks, constructing and sorting, provided some insights that warrant further study. The analysis suggested that students performed better on the construction tasks. This difference raises questions about how different types of tasks might differently impact students' geometric reasoning development and, subsequently, their assigned van Hiele level. Future research could explore how the two designs influence the learning outcomes and whether one type of design is more effective than the other at advancing students through the van Hiele levels and/or assigning levels.

The study also found that different van Hiele levels were assigned for various GeoGebra tasks. This finding parallels the current literature, which indicates students may be at different levels for different geometric shapes. For future studies, it would be of keen interest to examine whether GeoGebra's effectiveness in increasing van Hiele levels varies across different geometric concepts. This could involve a more detailed analysis of how students interact with specific shapes and properties within GeoGebra and whether these interactions lead to different levels of understanding.

Another area for further study involves the analysis of weighted sums. The analysis completed in this study showed that some students could reason at higher levels than the level for which they met the criterion. This finding indicates that the van Hiele Geometry Test may not fully capture students' geometric reasoning skills. Future research could look at different test modifications to see if the results better parallel with the findings from other assessment forms, such as GeoGebra tasks. For example, would more than five questions per van Hiele level help to provide a fuller picture? Would a test that contained five or fewer questions for each level help to provide a fuller picture if the questions all pertained to the same type of polygon (i.e., all triangles)?

Longitudinal research is also necessary to help confirm whether the positive effects of GeoGebra on students' geometric reasoning were short-lived or if they could be sustained over time. While this study showed significant short-term gains, it is unknown whether these gains will be sustained as students move from fifth grade to their high school geometry courses.

Lastly, this study could be conducted again to see if replication yields the same results. It would also be important to see if the study would produce similar results if conducted in other educational settings and with a different demographic of students. The study's results did not show a significant difference between genders, but further investigation could examine additional student demographics such as socioeconomic status or cultural background. Moreover, this future research could look at ways to make geometry instruction that utilizes dynamic geometry software more equitable for all students.

Conclusion

Geometry is a strand of mathematics that is integral to the American education system and beyond. With advancements in technology and a further emphasis on geometry in elementary schools, determining the impact of using GeoGebra on students' van Hiele levels is significant. The van Hiele model is a foundational framework in mathematics education, categorizing students' geometric reasoning into hierarchical levels. Understanding how dynamic geometry software, like GeoGebra, affects these levels is vital for educators aiming to enhance geometric reasoning in their students. In this study, using GeoGebra was central to the instructional process. I sought to assess its effectiveness through rigorous quantitative and qualitative data analysis. By assigning van Hiele levels through both the van Hiele Geometry

Test and a series of GeoGebra-based tasks, I was able to gain a holistic view of each student's geometric thinking and thus determine the effects that GeoGebra had on progressing the fifth-grade students through the van Hiele levels.

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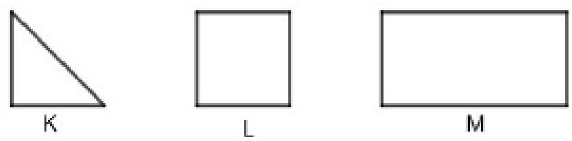
Appendix A - van Hiele Geometry Test

Usiskin van Hiele Geometry Test

ID ____________________________

Date ____________________

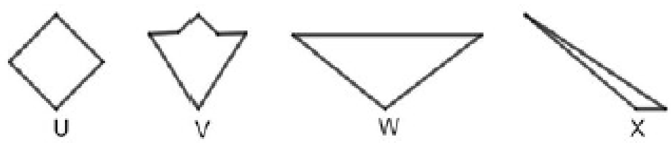
1.



Which of these are squares?

(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares.

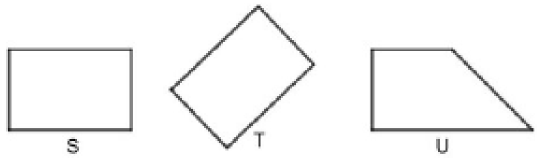
2.



Which of these are triangles?

(A) None of these are triangles.
(B) V only
(C) W only
(D) W and X only
(E) V and W only

3.



Which of these are rectangles?

(A) S only

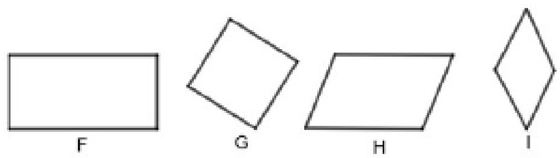
(B) T only

(C) S and T only

(D) S and U only

(E) All are rectangles.

4.



Which of these are squares?

(A) None of these are squares.

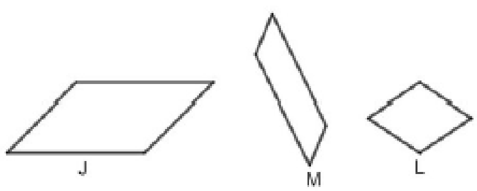
(B) G only

(C) F and G only

(D) G and I only

(E) All are squares.

5.



Which of these are parallelograms?

(A) J only

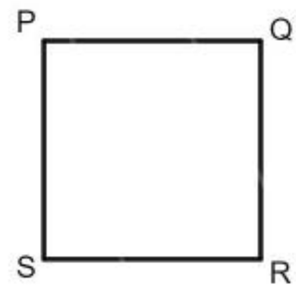
(B) L only

(C) J and M only

(D) None of these are parallelograms.

(E) All are parallelograms.

6. PQRS is a square.



Which relationship is true in all squares?

(A) Segment PR and segment RS have the same length.

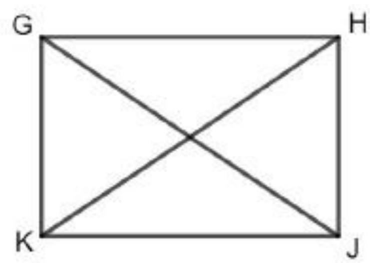
(B) Segment QS and segment PR are perpendicular.

(C) Segment PS and segment QR are perpendicular.

(D) Segment PS and segment QS have the same length.

(E) Angle Q is larger than angle R.

7. In a rectangle GHJK, segment GJ and segment HK are the diagonals.



Which option below is **not** true of **every** rectangle?

(A) There are four right angles.

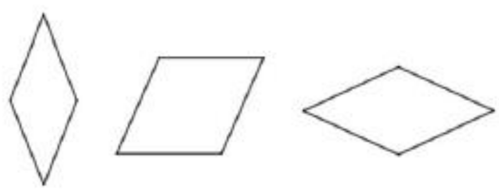
(B) There are four sides.

(C) The diagonals have the same length.

(D) The opposite sides have the same length.

(E) All of the above (A-D) are true in every rectangle.

8.



A rhombus is a 4-sided figure with all sides of the same length. Look at the three examples. Which of the following is **not** true in every rhombus?

(A) The two diagonals have the same length.

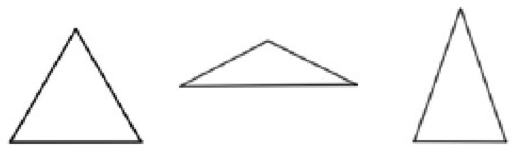
(B) Each diagonal bisects two angles of the rhombus.

(C) The two diagonals are perpendicular.

(D) The opposite angles have the same measure.

(E) All of the above (A-D) are true in every rhombus.

9.



An isosceles triangle is a triangle with two sides of equal length. Look at the examples.

Which of the following is true in every isosceles triangle?

A The three sides must have the same length.

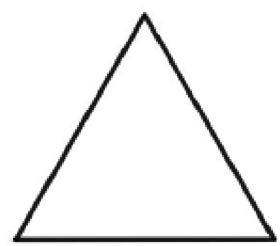
B One side must have twice the length of another side.

C There must be at least two angles with the same measure.

D The three angles must have the same measure.

E None of the above (A-D) are true in every isosceles triangle.

10. An equiangular triangle is a triangle with three angles of the same measure. Look at the example.



Which of the following is true in every equiangular triangle?

A Only two of the sides will have the same length.

B One angle has to be a right angle.

C All three angles must be obtuse.

D All three sides must be the same length.

E None of the above (A-D) are true in every equiangular triangle.

11. Here are two statements:

Statement 1: Figure F is a rectangle.
Statement 2: Figure F is a triangle.

A If statement 1 is true, then statement 2 is true.

B If statement 1 is false, then statement 2 is true.

C Statement 1 and statement 2 cannot both be true.

D Statement 1 and statement 2 cannot both be false.

E None of the above (A-D) are correct.

12. Here are two statements:

Statement S: Triangle ABC has three sides of the same length.
Statement T: In triangle ABC, angle B and angle C have the same measure.

(A) Statement S and statement T cannot both be true.

(B) If statement S is true, then statement T is true.

(C) If statement T is true, then statement S is true.

(D) If statement S is false, then statement T is false.

(E) None of the above (A-D) are correct.

13. Which of these can be called rectangles?

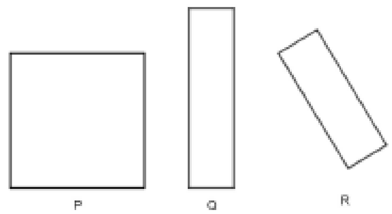
(A) All can.

(B) Q only

(C) R only

(D) P and Q only

(E) Q and R only



14. Which is true?

(A) All properties of rectangles are properties of all squares.

(B) All properties of squares are properties of all rectangles.

(C) All properties of rectangles are properties of parallelograms.

(D) All properties of squares are properties of parallelograms.

(E) None of the above (A-D) are true.

15. What do all rectangles have that some parallelograms do not have?

(A) Opposite sides equal

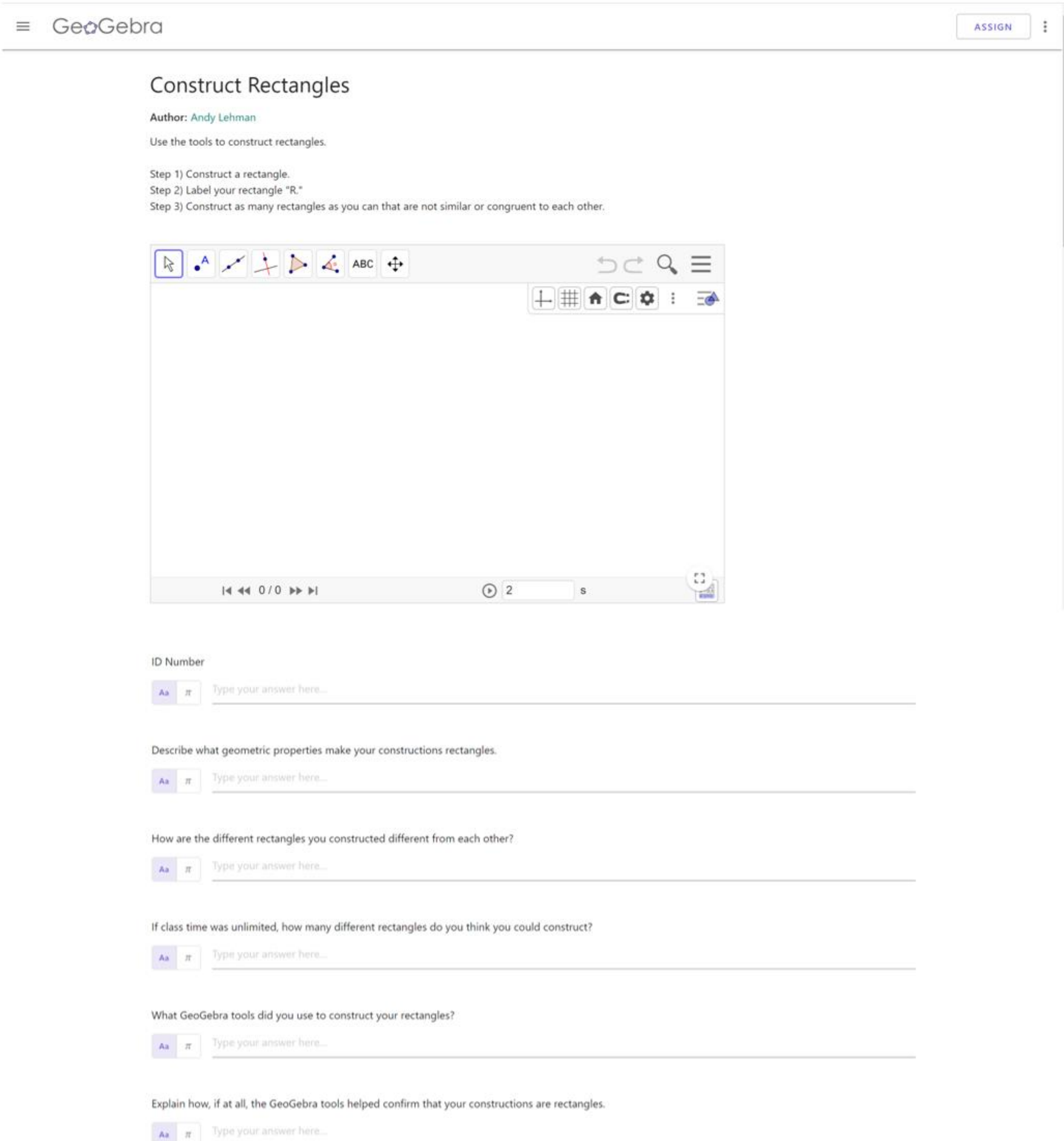
(B) Diagonals equal

(C) Opposite sides parallel

(D) Opposite angles equal

(E) None of the above (A-D).

Appendix B - GeoGebra Construct Rectangles Task


GeoGebra

ASSIGN

Construct Rectangles

Author: Andy Lehman

Use the tools to construct rectangles.

Step 1) Construct a rectangle.
Step 2) Label your rectangle "R."
Step 3) Construct as many rectangles as you can that are not similar or congruent to each other.

ABC

0 / 0

2 s

ID Number

Type your answer here...

Describe what geometric properties make your constructions rectangles.

Type your answer here...

How are the different rectangles you constructed different from each other?

Type your answer here...

If class time was unlimited, how many different rectangles do you think you could construct?

Type your answer here...

What GeoGebra tools did you use to construct your rectangles?

Type your answer here...

Explain how, if at all, the GeoGebra tools helped confirm that your constructions are rectangles.

Type your answer here...

Appendix C - GeoGebra Construct Triangles Task

GeoGebra

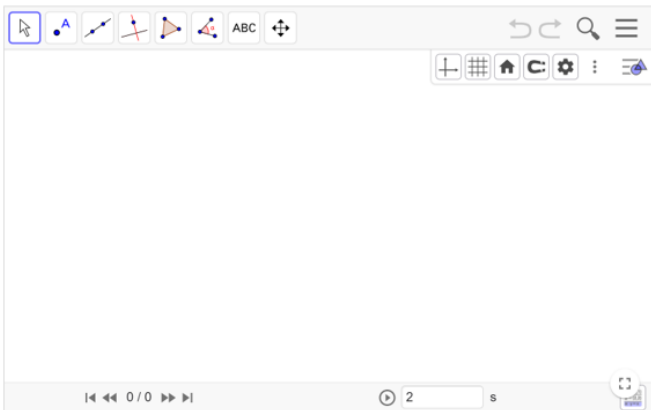
ASSIGN

Construct Triangles

Author: Andy Lehman

Use the GeoGebra tools to construct the triangles. Use the text tool to label your constructions.

Step 1) Construct an equilateral triangle.
Step 2) Label the equilateral triangle "E."
Step 3) Construct an isosceles triangle.
Step 4) Label the isosceles triangle "I."
Step 5) Construct as many triangles as you can that are not similar or congruent to each other.



ID Number

Type your answer here...

Describe what geometric properties make your constructions triangles.

Type your answer here...

What geometric properties make triangle E an equilateral triangle?

Type your answer here...

What geometric properties make triangle I an isosceles triangle?

Type your answer here...

How are the different triangles you constructed different from each other?

Type your answer here...

If class time was unlimited, how many different triangles do you think you could construct?

Type your answer here...

What GeoGebra tools did you use to construct your triangles?

Type your answer here...

Explain how, if at all, the GeoGebra tools helped confirm that your constructions are triangles.

Type your answer here...

Appendix D - GeoGebra Construct Rhombuses Task

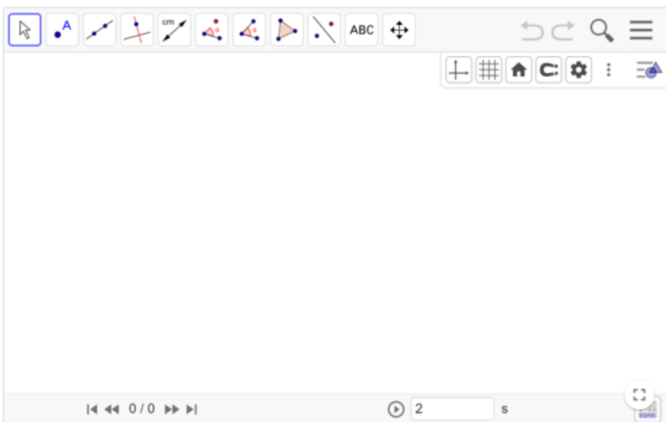
GeoGebra ASSIGN

Construct Rhombuses

Author: Andy Lehman

Use the tools to construct rhombuses.

Step 1) Construct a rhombus that does not contain right angles.
Step 2) Label your rhombus "R."
Step 3) Construct as many rhombuses as you can that are not similar or congruent to each other.



ID Number

Aa π Type your answer here...

Describe what geometric properties make your constructions rhombuses.

Aa π Type your answer here...

How are the different rhombuses you constructed different from each other?

Aa π Type your answer here...

If class time was unlimited, how many different rhombuses do you think you could construct?

Aa π Type your answer here...

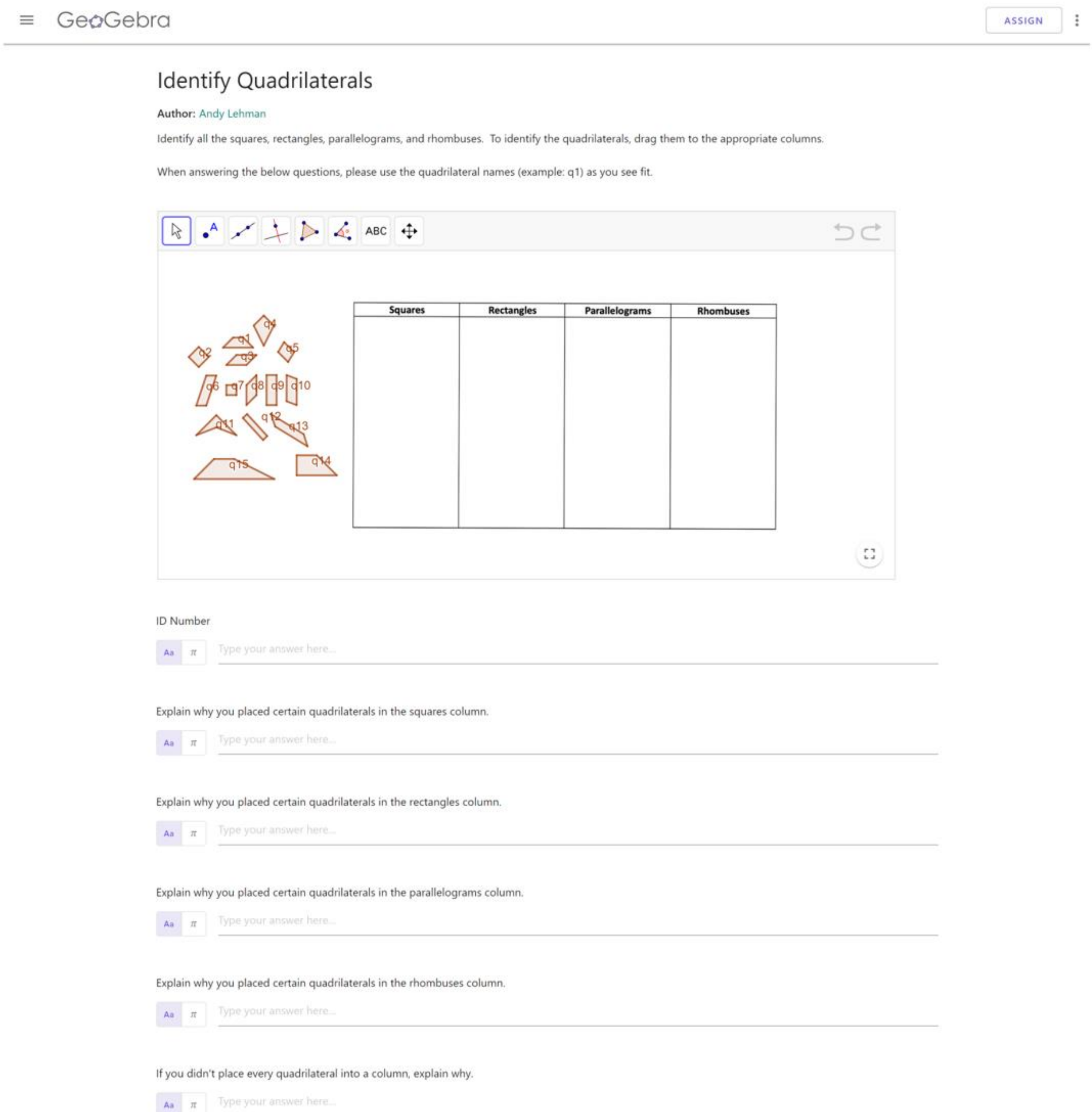
What GeoGebra tools did you use to construct your rhombuses?

Aa π Type your answer here...

Explain how, if at all, the GeoGebra tools helped confirm that your constructions are rhombuses.

Aa π Type your answer here...

Appendix E - GeoGebra Identify Quadrilaterals Task


GeoGebra

ASSIGN

Identify Quadrilaterals

Author: Andy Lehman

Identify all the squares, rectangles, parallelograms, and rhombuses. To identify the quadrilaterals, drag them to the appropriate columns.

When answering the below questions, please use the quadrilateral names (example: q1) as you see fit.

ID Number

Type your answer here...

Explain why you placed certain quadrilaterals in the squares column.

Type your answer here...

Explain why you placed certain quadrilaterals in the rectangles column.

Type your answer here...

Explain why you placed certain quadrilaterals in the parallelograms column.

Type your answer here...

Explain why you placed certain quadrilaterals in the rhombuses column.

Type your answer here...

If you didn't place every quadrilateral into a column, explain why.

Type your answer here...

What would you tell someone to look for in order to pick out all the squares?

Aa π Type your answer here...

What would you tell someone to look for in order to pick out all the rectangles?

Aa π Type your answer here...

What would you tell someone to look for in order to pick out all the parallelograms?

Aa π Type your answer here...

What would you tell someone to look for in order to pick out all the rhombuses?

Aa π Type your answer here...

Could you make a shorter list? In other words, could you have sorted these polygons using fewer classifications? If yes, how could you have sorted the quadrilaterals using fewer classifications? (For example, is q2 a rectangle? Is q9 a parallelogram?)

Aa π Type your answer here...

Appendix F - GeoGebra Sort Polygons Task

≡ GeoGebra ASSIGN

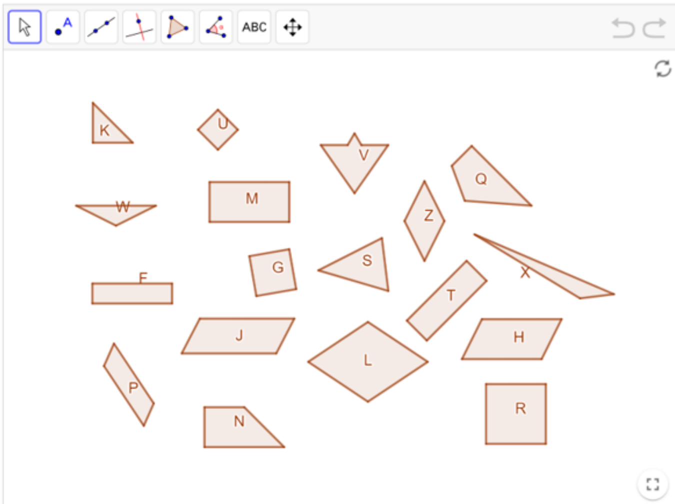
Sorting Polygons

Author: Andy Lehman

ID Number

Aa π Type your answer here...

Sort the following 18 polygons as you wish. Subgroups may be created if necessary. You may use the Text tool to label your groups.



Answer the following questions based on your sorting above.

Explain how you sorted the polygons.

Aa π Type your answer here...

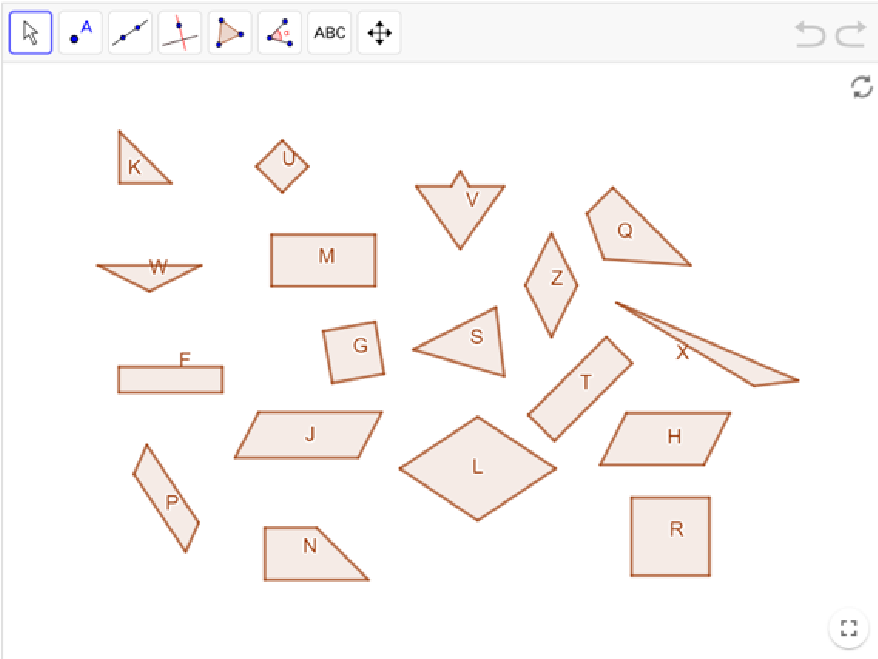
Which GeoGebra tools, if any, helped confirm that you sorted the polygons correctly?

Aa π Type your answer here...

If you used the GeoGebra tools, explain how they helped confirm that you sorted the polygons correctly.

Aa π Type your answer here...

Now create a different grouping of the same polygons. Subgroups may be created if necessary.



Answer the following questions for the second sort you completed.

Explain how you sorted the polygons.

Type your answer here...

Which GeoGebra tools, if any, helped confirm that you sorted the polygons correctly?

Type your answer here...

If you used the GeoGebra tools, explain how they helped confirm that you sorted the polygons correctly.

Type your answer here...

Use the polygons above to answer the following questions.

How are polygons H and Z alike? List as many similarities as you can.

Type your answer here...

How are polygons R, F, and K alike? List as many similarities as you can.

Type your answer here...

Appendix G - Survey

ID Number ________________

Date __________________

Beliefs About GeoGebra Survey

Thank you for taking the time to share your experiences using GeoGebra to explore polygons. Your insights are valuable in understanding how this digital tool affected your understanding of geometry. Please feel free to provide as much detail as you wish and remember that you are not required to answer all the questions. Your honest and thoughtful responses will help Mr. Lehman gain a deeper understanding of your learning journey with GeoGebra.

1. How did using GeoGebra to explore polygons enhance your understanding of their properties and characteristics? Please provide specific examples or experiences that stood out to you.

2. Please circle your response to each statement.

GeoGebra helped me visualize polygons.	😩 Strongly Disagree	😦 Disagree	😐 Neutral	😃 Agree	😁 Strongly Agree
GeoGebra helped me manipulate polygons.	😩 Strongly Disagree	😦 Disagree	😐 Neutral	😃 Agree	😁 Strongly Agree

In what ways, if any, did GeoGebra make it easier for you to visualize and modify different types of polygons?

Can you share any "aha" moments or discoveries you made while using the software?

3. How did GeoGebra help you learn more about the properties and polygons as a whole?

4. Which GeoGebra tools did you find useful? Please provide examples.

5. Please circle your response to the statement.

GeoGebra helped me become more confident in geometry.	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Would you want to continue to use GeoGebra to learn geometry? If "yes," why? If "no," why not?

Appendix H - GeoGebra Tasks Checklist and Level Indicators

GeoGebra Tasks (Burger & Shaughnessy, 1986)

1. Given different quadrilaterals, students will need to identify all the squares, rectangles, parallelograms, and rhombuses. (Evaluates level 1)
   a. Justify labels by using GeoGebra tools

2. Given 18 polygons, sort them and justify grouping. Then create and justify a second grouping. Subgroups can be created. (Wang & Kinzel, 2014).
   a. Justify by using GeoGebra tools. (Evaluates level 3)

3. Construct an equilateral triangle and an isosceles triangle. Describe what attributes make them triangles. Construct as many triangles as you can that are not similar or congruent to each other. (Evaluates levels 2 and 3)
   a. Three sides
   b. Three angles
   c. The sum of interior angles is 180 degrees
   d. Three vertices

4. Construct a rhombus that does not contain right angles. What attributes make your figure a rhombus? Construct as many rhombuses as you can that are not similar or congruent to each other. (Evaluates levels 2 and 3)
   a. All sides congruent
   b. Opposite angles congruent
   c. Diagonals bisect at right angles
   d. Diagonals divide rhombus into four congruent triangles
   e. Two lines of symmetry are the two diagonals
   f. Rotational symmetry with 180-degree rotation

5. Construct a rectangle. What attributes make your figure a rectangle? Construct as many rectangles as you can that are not similar or congruent to each other. (Evaluates levels 2 and 3)
   a. Opposite sides are congruent
   b. Opposite sides are parallel
   c. Opposite angles are congruent
   d. All angles are 90 degrees
   e. The diagonals bisect each other
   f. Has two lines of symmetry

Level Indicators

Level 1

1. Use of imprecise properties (qualities) to compare drawings and to identify, characterize, and sort shapes.
2. References to visual prototypes to characterize shapes.
3. Inclusion of irrelevant attributes when identifying and describing shapes, such as orientation of the figure on the page.
4. Inability to conceive of an infinite variety of types of shapes
5. Inconsistent sortings; that is, sortings by properties not shared by the sorted shapes.
6. Inability to use properties as necessary conditions to determine a shape

Level 2

1. Comparing shapes explicitly by means of properties of their components.
2. Prohibiting class inclusions among general types of shapes, such as quadrilaterals. *(Don't consider a shape to be classified in more than one way)*
3. Sorting by single attributes, such as properties of sides, while neglecting angles, symmetry, and so forth.
4. Application of a litany of necessary properties instead of determining sufficient properties when identifying shapes and explaining identifications
5. Descriptions of types of shapes by explicit use of their properties, rather than by type names, even if known. For example, instead of rectangle, the shape would be referred to as a four-sided figure with all right angles.
6. Explicit rejection of textbook definitions of shapes in favor of personal characterizations.
7. Treating geometry as physics when testing the validity of a proposition; for example, relying on a variety of drawings and making observations about them.
8. Explicit lack of understanding of mathematical proof.

Level 3

1. Formation of complete definitions of types of shapes.
2. Ability to modify definitions and immediately accept and use definitions of new concepts
3. Explicit references to definitions.
4. Ability to accept equivalent forms of definitions.
5. Acceptance of logical partial ordering among types of shapes, including class inclusions.
6. Ability to sort shapes according to a variety of mathematically precise attributes.
7. Explicit use of "if, then" statements.
8. Ability to form correct informal deductive arguments, implicitly using such logical forms as the chain rule (if p implies q and q implies r, then p implies r) and the law of detachment (modus ponens).
9. Confusion between the roles of axiom and theorem.

Appendix I - Daily Lesson Plans

5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

5.G.4. Classify two-dimensional figures in a hierarchy based on properties.

Block 2 -- Instruction via GeoGebra

Lesson	Date	Topic	Objective	Key Points	Vocabulary	Materials	Assessment
1	2/1/24	Take VHGT pretest				Paper copy of VHGT	VHGT
2	2/21/24	Introduction to GeoGebra	Students will familiarize themselves with the various tools that GeoGebra has to offer	GeoGebra is an interactive software that can be used in geometry	GeoGebra	iPads	
	2/22/24					Tutorial 1	
						Tutorial 2	
3	2/22/24	Classify polygons	Students will identify the attributes belonging to a category of two-dimensional figures	Polygons are flat, closed shapes, with straight edges	Polygon	Polygon or Not	Classifying Polygons worksheet
	2/23/0234		Students will classify polygons based on their number of sides	Polygons are classifed by the number of their sides (look at 3-10 sided polygons)	Congruent	Classifying Polygons	
				Polygons can be regular or irregular	Similar	Classifying Polygons	
					Two-Demensional shape		
					Triangle		
					Qudadrilateral		
					Pentagon		
					Hexagon		
					Heptagon		
					Octagon		
					Nonagon		
					Decagon		
					Point		
					Line		
					Segment		
4	2/23/24	Classify triangles by angles	Students will classify triangles by angle measure	Each triangle is classfied by the type of interior angles it has	Interior angles	Angle Types	Classifying Triangles by Angle Measure worksheet
	2/26/24				Acute angle	Classifying Triangles by Angles	
	2/27/24				Equiangular		
					Right angle	Angles of Triangles	
					Obtuse angle	Constructing Triangles	
5	2/27/24	Classify triangles by angles and sides	Students will classify triangles by angle measure and side length	Every triangle has exactly two names as determined by their angles and sides	Equilateral triangle	Classifying Triangle by Sides	Classifying Triangles by Angles and by Sides worksheet
	2/28/24			Lines and arcs can be used to indicate congruent sides or angles	Isosceles triangle	Exploring Types of Triangles	
				The name for the angle classification is not related to the name of the side classification	Scalene triangle	Drawing Triangles	
						Classifying Triangles	
						Constructing Triangles	
6	2/29/24	Classifying Quadrilaterals	Students will classify quadrilaterals based on geometric properties	There are five shapes within the quadrilateral family (rectangle, square, parallelogram, rhombus, and trapezoid) that we will use for this unit	Rectangle	Parallel and perpendicular lines	Intro to Quadrilaterals: The Family Tree worksheet
	3/1/24			Names of quadrilaterals are justified by the properties (sides and angles)	Square		
					Parallelogram		
					Rhombus	Parallelogram Template	
					Trapezoid (exclusive)	Rectangle Template	
					Lines of symmetry	Square Template	
					Parallel	Rhombus Template	
					Perpendicular	Isosceles Trapezoid Template	
					Diagonals	Quadrilateral Investigation	
					Bisect	Quadrilateral Creation Templates	
						Classifying Quadrilaterals	
						Constructing Quadrilaterals	
7	3/5/24	Make a picture using only polygons	Identifty the names and quantities of polygons used			GeoGebra Geometry blank applet	Uses only polygons in drawing
						Polygon Recording Sheet	Drawing with accurate list of names and quantities of the polygons used
8	3/6/24	Hierarchy of Quaidrilaterals	Students will use their knowledge of geometric properties to give quadrilaterals multiple names	Each geometric property gives a shape a different name	Hierarchy	Hierarchy of Quadrilaterals	Hierarchy of Quadrilaterals worksheet
	3/7/24			Shapes may have more than one name			
				The mathematical properties define the names of the shapes, not how they look (i.e. rhombi do not have to look the same)			
9	3/8/24	Take VHGT posttest				Paper copy of VHGT	VHGT
10	3/11/24	Complete Survey and GeoGebra Tasks				Geometry Survey	
	3/12/24					Identify Quadrilaterals	
11	3/14	Complete GeoGebra Tasks				Construct Rectangles	
						Construct Rhombuses	
						Construct Triangles	
						Sorting Polygons	

Appendix J - Links to GeoGebra Activities

Lesson 2

Tutorial 1–https://www.geogebra.org/m/u6wtzmmg

Tutorial 2–https://www.geogebra.org/m/wan5atfs

Lesson 3

Polygon or Not–https://www.geogebra.org/m/udvbwnjj

Classifying Polygons–https://www.geogebra.org/m/ajxuksbm

Classifying Polygons–https://www.geogebra.org/m/tacbfwhu

Lesson 4

Angle Types–https://www.geogebra.org/m/xwvhqzxg

Classifying Triangles by Angles–https://www.geogebra.org/m/wjdyzr2f

Angles of Triangles–https://www.geogebra.org/m/vvztyyvf

Constructing Triangles-- https://www.geogebra.org/m/ggmqnuvc

Lesson 5

Classifying Triangle by Sides–https://www.geogebra.org/m/dnvw8rds

Exploring Types of Triangles–https://www.geogebra.org/m/tswcyscq

Drawing Triangles–https://www.geogebra.org/m/vzn8ncnu

Classifying Triangles–https://www.geogebra.org/m/ckq4mbxp

Constructing Triangles–https://www.geogebra.org/m/epyrk95w

Lesson 6

Parallel and perpendicular lines–https://www.geogebra.org/m/ggvcue6u

Parallelogram Template–https://www.geogebra.org/m/h6nsudhc

Rectangle Template–https://www.geogebra.org/m/mdukfyvv

Square Template–https://www.geogebra.org/m/uxqnktrt

Rhombus Template–https://www.geogebra.org/m/ddsusd83

Isosceles Trapezoid Template–https://www.geogebra.org/m/yd6sutdj

Quadrilateral Investigation–https://www.geogebra.org/m/qvffvy7d

Quadrilateral Creation Templates–https://www.geogebra.org/m/ev7p55gm

Classifying Quadrilaterals–https://www.geogebra.org/m/vyqswufb

Constructing Quadrilaterals–https://www.geogebra.org/m/pqukwn9u

Lesson 8

Hierarchy of Quadrilaterals–https://www.geogebra.org/m/eu6jht2g

End of Unit Tasks

Identify Quadrilaterals–https://www.geogebra.org/m/mdfubu5s

Construct Rectangles–https://www.geogebra.org/m/nc2euhnk

Construct Rhombuses–https://www.geogebra.org/m/k7cmhs8c

Construct Triangles–https://www.geogebra.org/m/ewy84h9n

Sorting Polygons–https://www.geogebra.org/m/tke3bx34

Appendix K - IRB Letter of Exemption

Kansas State University | University Research Compliance Office

TO: David Allen
Curriculum and Instruction
Manhattan, KS 66506

Proposal Number: IRB-11977

FROM: Lisa Rubin, Chair
Committee on Research Involving Human Subjects

DATE: 12/19/2023

RE: Proposal Entitled, "Using Dynamic Geometry Software to Examine Its Effects On Student Progression Through The Van Hiele Levels."

The Committee on Research Involving Human Subjects / Institutional Review Board (IRB) for Kansas State University has reviewed the proposal identified above and has determined that it is EXEMPT from further IRB review. This exemption applies only to the proposal - as written – and currently on file with the IRB. Any change potentially affecting human subjects must be approved by the IRB prior to implementation and may disqualify the proposal from exemption.

Based upon information provided to the IRB, this activity is exempt under the criteria set forth in the Federal Policy for the Protection of Human Subjects, **45 CFR §104(d), category:Exempt Category 1.**

Certain research is exempt from the requirements of HHS/OHRP regulations. A determination that research is exempt does not imply that investigators have no ethical responsibilities to subjects in such research; it means only that the regulatory requirements related to IRB review, informed consent, and assurance of compliance do not apply to the research.

Any unanticipated problems involving risk to subjects or to others must be reported immediately to the Chair of the Committee on Research Involving Human Subjects, the University Research Compliance Office, and if the subjects are KSU students, to the Director of the Student Health Center.

Electronically signed by Lisa Rubin on 12/19/2023 11:36 PM ET

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