REACTIVITY CALCULATIONS FOR THE KANSAS STATE UNIVERSITY TRIGA MARK II NUCLEAR REACTOR

by 6202

LELAND WAYNE RANEY

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Approved by:

J. K. Shultis

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1.0 INTRODUCTION

The concept of reactivity is of fundamental importance in the control and operation of nuclear reactors. The operating specifications for a reactor usually contain limits on the total amount of reactivity available in the core, the maximum reactivity worth of an experiment inserted in the core, the reactivity insertion rate due to control rod withdrawal, the minumum shut-down margin to maintain the reactor in a subcritical condition, as well as many other restrictions that concern the safety and reliability of the plant. Reactivity is commonly used to determine the deviation of the reactor system from its steady-state critical condition. Generally, the larger the absolute value of the reactivity the faster the neutron flux will change, and hence the faster the reactor power will change. The reactivity is made positive to increase the reactor power and made negative to decrease the reactor power.

Although the concept of reactivity is widely used, the exact calculation of reactivity in any given reactor is usually quite difficult. The methods of reactivity determination almost always employ some approximations in the calculations. To recognize the limitations of these various approximations used in reactivity calculations, the definition of reactivity and the commonly used approximations to it need to be understood.

The concept of reactivity usually arises in the description of the time behavior of the reactor power by the well known point reactor kinetic equations (PRKE). Many different expressions for reactivity have been obtained by reducing various kinetic equations to the standard PRKE form. Ideally, the spatial-time neutron behavior in a reactor should be described by the transport equation. It is possible to reduce the transport equation exactly to the PRKE form and obtain an "exact" expression for reactivity.[1,2] However,

this exact definition involves the solution of the original transport equation. Even numerical solutions to the transport equation are exceedingly difficult to obtain, and thus various approximations of the transport equation are commonly employed. The point reactor kinetic equations and reactivity are commonly developed using one of the simplest of these approximations, the one-speed diffusion model.[3-9] However, another approximation of the transport equation, namely the energy-dependent diffusion kinetic equations, is almost always used for practical applications in nuclear engineering. Even these energy-dependent diffusion kinetic equations are often quite difficult to solve, and as a result, the multigroup approximation of the diffusion kinetic equations has come into wide use. Therefore, one of the purposes of this thesis is to derive exactly, from the energy-dependent and multigroup diffusion kinetic equations, the point reactor kinetic equations. In the process of this derivation the following kinetic parameters will be defined: mean prompt generation time, effective delayed neutron fraction, effective delayed neutron precursor density, effective neutron source, and, of course, reactivity. Three different approximations are applied to the definition of reactivity to obtain three approximate expressions for determining reactivity. The first approximation yields an expression which is identical to that obtained by first-order perturbation theory[7]. The second approximation gives the widely used "inhour" equation[8], while the third approximation produces an expression involving only the effective multiplication factor.

A second objective of this thesis is to try to develop a simplified means of predicting reactivity for the Kansas State University Triga Mark II nuclear reactor. Two different approaches were employed in the search for a simplified means of predicting reactivity. The first approach used the "one-speed" diffusion model of first-order perturbation theory. This first technique was

chosen because it was the easiest to develop and use. The second approach employed an expression involving the effective multiplication factor and "two-group" theory. This second method was more difficult to analyze, and as a result, a computer program was written which was used for reactivity predictions. To serve as a comparison with the predicted reactivities and hence as a check on the method of prediction, reactivities were experimentally measured in the Triga reactor. These experimental measurements include aluminum to water, fuel to water, and graphite to water reactivity differences.

2.0 THEORETICAL DEVELOPMENT

2.1 The Diffusion Kinetic Equations

Any study that is concerned with the behavior of neutrons in scattering, absorbing, and multiplying media uses some model to describe the neutron population in the system. The energy-dependent diffusion kinetic equations are commonly employed in nuclear engineering to describe this time, energy, and position dependent behavior of neutrons. These energy-dependent diffusion kinetic equations, similar to those given by Glasstone and Edlund[4] and Meghreblian and Holmes[7], can be written as

$$\frac{\partial\phi(\underline{r},E,t)}{v(E)\ \partial t} = \nabla D(\underline{r},E,t)\cdot\nabla\phi(\underline{r},E,t) - \Sigma_t(\underline{r},E,t)\phi(\underline{r},E,t) + S(\underline{r},E,t)$$
$$+ \sum_j \chi_p^j(E)\int_o^\infty dE'\ (1-\beta^j)\nu^j(E')\Sigma_f^j(\underline{r},E',t)\phi(\underline{r},E',t)$$
$$+ \sum_{i=1}^{6} \chi_d^i(E)\lambda_i C_i(\underline{r},t) + \int_o^\infty dE'\ \alpha(E'\to E)\Sigma_s(\underline{r},E',t)\phi(\underline{r},E',t) \quad (2.1.1)$$

and

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = \sum_j \int_o^\infty dE'\ \beta_i^j \nu^j(E')\Sigma_f^j(\underline{r},E',t)\phi(\underline{r},E',t) - \lambda_i C_i(\underline{r},t)\ ,$$
$$i = 1,\cdots,6\ , \quad (2.1.2)$$

where it has been assumed there are six delayed neutron groups. The summation over j is a summation over all fissile species. The quantities found in Eqs. (2.1.1) and (2.1.2) are defined below.

1. $\phi(\underline{r},E,t)$ = the neutron flux distribution which is a function of position, energy, and time.
2. Σ_f, Σ_s, Σ_t = respectively the macroscopic cross-sections for fissions, scattering, and total interactions.
3. D = the diffusion coefficient.
4. $\nu^j(E)$ = the mean number of neutrons released in a fission event which was induced by a neutron of energy E in the j-th type of fissile material.

5. C_i = the delayed neutron precursor density for the i-th group of delayed neutrons.

6. λ_i = the decay constant of the i-th kind of delayed neutron precursor.

7. β = the ratio of the delayed neutrons to the total number of fission neutrons (delayed neutron fraction).

8. β_i = the fraction of all fission neutrons emitted by the i-th group of delayed neutron precursors.

9. S = the neutron production from some external neutron source.

10. v = the speed of neutrons of energy E, which is related to the neutron density, n, and the neutron flux by

$$\phi(\underline{r},E,t) = n(\underline{r},E,t)v(E) \ . \tag{2.1.3}$$

11. $\alpha(E' \to E)dE$ = the fraction of neutrons scattered from an energy E' into energy dE about E, such that

$$\int_o^\infty dE \ \alpha(E'\to E) = 1 \ . \tag{2.1.4}$$

12. χ_p = the energy distribution of the prompt fission neutrons, with

$$\int_o^\infty dE \ \chi_p(E) = 1 \ . \tag{2.1.5}$$

13. χ_d^i = the energy distribution of the delayed neutrons from the i-th group with the following normalization:

$$\int_o^\infty dE \ \chi_d^i(E) = 1 \ . \tag{2.1.6}$$

The general diffusion kinetic equations, (2.1.1) and (2.1.2), may be written more concisely by introducing the following operators:

$$M \equiv v(E)[\nabla D(\underline{r},E,t)\cdot\nabla - \Sigma_t(\underline{r},E,t) + \int_o^\infty dE' \ \alpha(E'\to E)\Sigma_s(\underline{r},E,t)] \ , \tag{2.1.7}$$

$$N_o \equiv v(E)\textstyle\sum_j \chi_p^j(E)\int_o^\infty dE'\nu^j(E')(1-\beta^j)\Sigma_f^j(\underline{r},E',t) \ , \tag{2.1.8}$$

$$N_i \equiv \textstyle\sum_j \int_o^\infty dE' \ \beta_i^j \nu^j(E')\Sigma_f^j(\underline{r},E',t) \ , \tag{2.1.9}$$

and

$$B \equiv M + N_o \ . \tag{2.1.10}$$

The operator M describes the leakage, absorption, and scattering of neutrons in the media. The prompt neutron production operator and the i-th group delayed neutron precursor production operator are denoted respectively by N_o and N_i. The Boltzmann operator, B, describes neutrons in a scattering, absorbing, and multiplying medium neglecting the delayed neutrons. With these operator definitions the general diffusion kinetic equations may be rewritten as

$$\frac{\partial \phi}{\partial t} = B\phi + v \sum_{i=1}^{6} \chi_d^i \lambda_i C_i + vS \tag{2.1.11}$$

and

$$\frac{\partial C_i}{\partial t} = N_i \phi - \lambda_i C_i \ , \quad i = 1,\cdots,6 \ . \tag{2.1.12}$$

Generally, to solve these kinetic equations, (2.1.11) and (2.1.12), several conditions must be described. First, from physical arguments the neutron flux, ϕ, must be continuous, positive, and finite throughout the volume of the reactor. In addition to these physical conditions on the neutron flux, the following initial and boundary conditions must be specified in order to uniquely determine a solution to the kinetic equations for $t > 0$:[6]

1. the initial values of the neutron flux and precursor densities, $\phi(\underline{r},E,0)$ and $C_i(\underline{r},0)$
2. the conditions on the neutron flux at the outer boundaries of the medium (a widely used boundary condition requires the neutron flux to go to zero at the system's outer boundaries).

To determine the form of the diffusion kinetic equations for a critical reactor, let the external source go to zero and let all terms of the kinetic equations (2.1.11) and 2.1.12) be time independent. The diffusion kinetic equations for a critical reactor are

$$B_o\phi_o = -v \sum_{i=1}^{6} \chi_d^i \lambda_i C_{io} \tag{2.1.13}$$

and

$$N_{io}\phi_o = \lambda_i C_{io} \quad , \quad i = 1,\cdots,6 \quad , \tag{2.1.14}$$

where the subscript "o" denotes the critical steady-state condition. Substitution of Eq. (2.1.14) into Eq. (2.1.13) gives the following equation for the critical flux distribution.

$$B_o\phi_o = -v \sum_{i=1}^{6} \chi_d^i N_{io}\phi_o \tag{2.1.15}$$

Again define an operator R_o,

$$R_o \equiv B_o + v \sum_{i=1}^{6} \chi_d^i N_{io} \quad , \tag{2.1.16}$$

which is the steady-state operator describing a critical reactor. This critical steady-state operator completely describes the physical composition of the reactor system. The critical reactor system can then be described by the equation

$$R_o\phi_o = 0 \quad . \tag{2.1.17}$$

Generally, the only solution to Eq. (2.1.17) is the trivial solution ($\phi_o \equiv 0$). However, for certain "critical" conditions on R_o there exist non-trivial solutions ($\phi_o \neq 0$) to Eq. (2.1.17). These critical conditions on R_o are established when the neutron flux, ϕ_o, is non-negative throughout the entire

reactor volume. To help clarify this point, let the operator R_o be described in terms of its eigenvalues, ω_i, and its eigenfunctions, ϕ_i, namely

$$R_o\phi_i = \omega_i\phi_i \quad . \tag{2.1.18}$$

The critical conditions on R_o now exist when one of the eigenfunctions, ϕ_o, is non-negative in the reactor volume and when the associated eigenvalue, ω_o, is equal to zero. Under these conditions, the reactor described by R_o is just critical.

2.2 The Neutron Importance

A useful concept to describe the importance of various physical changes in the reactor is given by the neutron importance.[7] Consider the injection of a unit of neutron flux at $t = 0$, $E = E'$, and $\underline{r} = \underline{r}'$ into a critical reactor which was void of neutrons for all time prior to $t = 0$. The neutron importance is defined as the total fission rate in this critical reactor at long times ($t \to \infty$) after the injection.[1] Generally, the neutron importance at the outer edge of the reactor is less than it is near the center of the core. This difference in importance is due primarily to the higher probability of leakage of the neutrons from the system at the reactor boundaries. The neutron importance also tends to be lower near a strong neutron absorber due to the higher probability of neutrons being removed from the system by absorption.

To determine the neutron importance of a critical reactor, consider the general diffusion kinetic equations (2.1.11) and (2.1.12). These equations can be written in matrix form as

$$\frac{\partial \underline{\Psi}(\underline{r},E,t)}{\partial t} = \underline{\underline{A}}\underline{\Psi}(\underline{r},E,t) \ , \tag{2.2.1}$$

where

$$\underline{\Psi}(\underline{r},E,t) = \text{col}(\ \phi(\underline{r},E,t),\ C_1(\underline{r},t),\cdots,\ C_6(\underline{r},t)\ ) \tag{2.2.2}$$

and

$$\underline{\underline{A}} = \begin{pmatrix} B & v\lambda_1\chi_d^1 & v\lambda_2\chi_d^2 & \cdots & v\lambda_6\chi_d^6 \\ N_1 & -\lambda_1 & 0 & \cdots & 0 \\ N_2 & 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ N_6 & 0 & 0 & & -\lambda_6 \end{pmatrix} . \tag{2.2.3}$$

From the initial condition of the problem, namely a unit of neutron flux at E', $\underline{r}'$, and $t = 0$ with no flux for $t < 0$, the initial values for the flux and delayed neutron precursors are

$$\phi(\underline{r},E,0) = \delta(\underline{r} - \underline{r}')\ \delta(E - E') \tag{2.2.4}$$

and

$$C_i(\underline{r},0) = 0\ ,\quad i = 1,\cdots,6\ , \tag{2.2.5}$$

or equivalently

$$\underline{\Psi}(r,E,0) = \text{col}(\ \delta(\underline{r} - \underline{r}')\ \delta(E - E')\ ,\ 0,\ 0,\cdots,0\ )\ , \tag{2.2.6}$$

where $\delta(\underline{r} - \underline{r}')$ and $\delta(E - E')$ are the Dirac delta functions. To obtain the neutron importance one must determine the asymptotic solution of Eq. (2.2.1) subject to the initial conditions of Eq. (2.2.6).

To solve this neutron importance problem the concept of adjoint operators and functions must be introduced. Consider a scalar operator O operating on some function $y(\underline{r},E)$. An operator $O^\dagger$ is defined to be the adjoint of the

operator O if

$$\int_V d^3r \int_0^\infty dE\ y^\dagger(\underline{r},E)\cdot Oy(\underline{r},E) \equiv \int_V d^3r \int_0^\infty dE\ O^\dagger y^\dagger(\underline{r},E)\cdot y(\underline{r},E)\ , \qquad (2.2.7)$$

or equivalently in scalar product notation

$$\langle y^\dagger | Oy\rangle \equiv \langle O^\dagger y^\dagger | y\rangle\ , \qquad (2.2.8)$$

holds for any function y satisfying the regular boundary conditions, and for any function $y^\dagger$ satisfying the so-called adjoint boundary conditions.[1] The adjoint boundary condition is the boundary condition that the function $y^\dagger$ must satisfy such that Eq. (2.2.8) is satisfied.

In the present study the only operators of any concern are the $\nabla D\cdot\nabla$ and the integral operators. To illustrate the method of finding the adjoint of an operator consider these two operators.

Example 1: To treat the operator, $\nabla\cdot D\nabla$, the following identities are introduced:

$$\int_0^\infty dE \int_V d^3r\ y^\dagger \nabla D\cdot\nabla y \equiv \int_0^\infty dE \int_V d^3r \nabla(y^\dagger D\cdot\nabla y) - \int_0^\infty dE \int_V d^3r\ (\nabla y^\dagger)\cdot(D\cdot\nabla y) \qquad (2.2.9)$$

and

$$\int_0^\infty dE \int_V d^3r\ y\nabla D\cdot\nabla y^\dagger \equiv \int_0^\infty dE \int_V d^3r\ \nabla(yD\cdot\nabla y^\dagger) - \int_0^\infty dE \int_V d^3r\ (\nabla y)\cdot(D\cdot\nabla y^\dagger)\ . \qquad (2.2.10)$$

If the functions in the first integral on the right-hand side of Eqs. (2.2.9) and (2.2.10) are non-singular and continuous, Gauss' theorem may be used, namely

$$\int_0^\infty dE \int_V d^3r\ \nabla(y^\dagger D\cdot\nabla y) = \int_0^\infty dE \int_S d^2r\ (y^\dagger D\cdot\nabla y) \qquad (2.2.11)$$

and

$$\int_0^\infty dE \int_V d^3r\ \nabla(yD\cdot\nabla y^\dagger) = \int_0^\infty dE \int_S d^2r\ (yD\cdot\nabla y^\dagger)\ , \qquad (2.2.12)$$

where s is the surface of the reactor and d^2r is a differential area on that surface. A commonly used boundary condition on the neutron flux, in this case y, requires the flux to vanish at the reactor boundaries, and if the adjoint neutron flux, $y^\dagger$, is also required to vanish at the reactor surface, the integrals of Eqs. (2.2.11) and (2.2.12) vanish. Thus the first integral on the right-hand side of Eqs. (2.2.9) and (2.2.10) also disappears.

Since the order of scalar multiplication is immaterial, the second integral on the right-hand side of Eqs. (2.2.9) and (2.2.10) is equal to each other. Therefore, the operators, O and $O^\dagger$, in Eq. (2.2.7) are both equal to $\nabla D\cdot\nabla$. When an operator and its adjoint are equal, they are said to be "self-adjoint". Scalar quantities such as Σ_f, Σ_s, and Σ_t are also considered to be self-adjoint.

Example 2: The adjoint integral operator is obtained by interchanging the order of integration.[1] If the scalar product $\langle y^\dagger|Oy\rangle$ is given by

$$\langle y^\dagger|Oy\rangle = \int_V d^3r \int_0^\infty dE\ y^\dagger(\underline{r},E) \int_0^\infty dE'\ \alpha(E' \to E)\Sigma_s(\underline{r},E')y(\underline{r},E') \ , \qquad (2.2.13)$$

then interchanging the order of integration will yield the scalar product $\langle O^\dagger y^\dagger|y\rangle$, namely

$$\int_V d^3r \int_0^\infty dE\ \Sigma_s(\underline{r},E)y(r,E) \int_0^\infty dE'\ \alpha(E \to E')y^\dagger(\underline{r},E') = \langle O^\dagger y^\dagger|y\rangle \ . \qquad (2.2.14)$$

The concept of an adjoint can be readily extended to matrix operators. The adjoint of the matrix $\underline{\underline{O}}$ is denoted by $\underline{\underline{O}}^\dagger$, such that

$$\langle \underline{y}^\dagger|\underline{\underline{O}}\underline{y}\rangle \equiv \langle \underline{\underline{O}}^\dagger \underline{y}^\dagger|\underline{y}\rangle \ , \qquad (2.2.15)$$

where the quantity $\langle \underline{y}^\dagger|\underline{\underline{O}}\underline{y}\rangle$ is given by

$$\langle \underline{y}^\dagger|\underline{\underline{O}}\underline{y}\rangle = \int_V d^3r \int_0^\infty dE\ (y_1^\dagger, y_2^\dagger, \cdots, y_n^\dagger) \begin{pmatrix} o_{11} & o_{12} & \cdots & o_{1n} \\ o_{21} & o_{22} & \cdots & o_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ o_{n1} & o_{n2} & \cdots & o_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \ . \qquad (2.2.16)$$

It can be shown that $\underline{\underline{O}}^{\dagger}$ is obtained by taking the adjoint of each element of the transpose of matrix $\underline{\underline{O}}$.[1,4]

Since the concept of the adjoint has been introduced, one may proceed in the determination of the neutron importance. To continue the development of the neutron importance, the following assumptions are needed:

1. The eigenvalues, ω_n and $\omega_n^{\dagger}$, and the associated eigenvectors, $\underline{\psi}_n$ and $\underline{\psi}_n^{\dagger}$, of the matrix operators $\underline{\underline{A}}$ and $\underline{\underline{A}}^{\dagger}$ are assumed to be known, such that

$$\underline{\underline{A}}\underline{\psi}_n = \omega_n \underline{\psi}_n \tag{2.2.17}$$

and

$$\underline{\underline{A}}^{\dagger}\underline{\psi}_n^{\dagger} = \omega_n^{\dagger}\underline{\psi}_n^{\dagger} . \tag{2.2.18}$$

2. The eigenvectors $\underline{\psi}_n$ and $\underline{\psi}_n^{\dagger}$ are complete, *id est*, any vector $\phi(\underline{r},E)$ defined in the medium can be expanded in terms of the eigenvectors $\underline{\psi}_n$ as

$$\underline{\phi}(\underline{r},E) = \sum_{n=0}^{\infty} a_n \underline{\psi}_n(\underline{r},E) , \tag{2.2.19}$$

where the quantity a_n is unique.[1]

3. The eigenvectors $\underline{\psi}_n$ and $\underline{\psi}_n^{\dagger}$ form a biorthonormal set, namely

$$\langle \underline{\psi}_m^{\dagger} | \underline{\psi}_n \rangle = \delta_{mn} , \tag{2.2.20}$$

where δ_{mn} is the Kroenecker delta.

The existence of the eigenvalues, eigenvectors, and their completeness properties generally cannot be proven. However, for particular cases (e.g., one-speed, one-dimension diffusion theory) the eigenvectors and

eigenvalues can be explicitly determined and the above completeness properties can be demonstrated.

Since it has been assumed that the eigenvectors form a complete set, the neutron flux and the delayed neutron precursor density distribution vector $\underline{\Psi}$ can be expanded in terms of the eigenvectors $\underline{\psi}_n$ as

$$\underline{\Psi}(\underline{r},E,t) = \sum_{n=0}^{\infty} a_n(t)\underline{\psi}_n(\underline{r},E) \ . \tag{2.2.21}$$

There are, in general, an infinite number of eigenvectors because the matrix $\underline{\underline{A}}$ is actually a matrix of continuous operators which are operating on a vector of continuous functions.

To determine the time behavior of $\underline{\Psi}$, substitute Eq. (2.2.21) into equation (2.2.1) to yield

$$\sum_{n=0}^{\infty} \frac{\partial a_n(t)}{\partial t} \underline{\psi}_n(r,E) = \sum_{n=0}^{\infty} a_n(t)\underline{\underline{A}}\underline{\psi}_n(\underline{r},E) \ , \tag{2.2.22}$$

or from Eq. (2.2.17)

$$\sum_{n=0}^{\infty} \frac{\partial a_n(t)}{\partial t} \underline{\psi}_n(\underline{r},E) = \sum_{n=0}^{\infty} a_n(t)\omega_n\underline{\psi}_n(\underline{r},E) \ . \tag{2.2.23}$$

Multiply Eq. (2.2.23) by $\underline{\psi}_m^{\dagger}$, integrate over all energy and the reactor volume, and apply the orthonormal condition to obtain

$$\frac{\partial a_m(t)}{\partial t} = a_m(t)\omega_m \ . \tag{2.2.24}$$

Finally, integrating equation (2.2.24) with respect to time and applying the initial condition yields

$$a_m(t) = a_m(0)e^{\omega_m t} \ . \tag{2.2.25}$$

The coefficient $a_m(0)$ is determined by taking the scalar product of equation (2.2.21) with $\underline{\psi}_m^{\dagger}(\underline{r},E)$, namely

$$a_m(0) = \langle \underline{\psi}_m^{\dagger}(\underline{r},E) | \underline{\Psi}(\underline{r}',E',0) \rangle \ . \tag{2.2.26}$$

The vector $\underline{\Psi}(\underline{r}',E',0)$ has all terms equal to zero except for the term for the neutron flux distribution which is equal to $\delta(\underline{r} - \underline{r}')\delta(E - E')$. From these initial conditions equation (2.2.26) yields

$$a_m(0) = \langle \underline{\psi}_m^{\dagger}(\underline{r},E) | \delta(\underline{r} - \underline{r}')\delta(E - E') \rangle = \psi_{m1}^{\dagger}(\underline{r}',E') \ , \tag{2.2.27}$$

where $\psi_{m1}^{\dagger}$, is the first component of the vector $\underline{\psi}_m^{\dagger}$. Finally, from Eqs. (2.2.21), (2.2.25), and (2.2.27), the solution $\underline{\Psi}(\underline{r},E,t)$ is given by

$$\underline{\Psi}(\underline{r},E,t) = \sum_{n=0}^{\infty} \psi_{n1}^{\dagger}(\underline{r}',E')e^{\omega_n t}\underline{\psi}_n(\underline{r},E) \ . \tag{2.2.28}$$

Since the reactor is assumed to be critical, the reactor power cannot change with time indefinitely, but will approach some steady-state level. Physically, this requires all of the eigenvalues, ω_n, to be less than or equal to zero. From this physical requirement, one of the eigenvalues, say ω_o, must be zero and all other eigenvalues must have negative real parts. Thus, the coefficients of the remaining eigenvalues decay exponentially in time leaving the eigenvector $\underline{\psi}_o(\underline{r},E)$ to describe the asymptotic steady-state values for the neutron flux and delayed neutron precursor densities. By considering only the portion of the eigenvector, $\underline{\psi}_o$, that deals with the neutron flux distribution, namely ψ_{o1}, the flux distribution after a long period of time will be given by

$$\lim_{t\to\infty} \phi(\underline{r},E,t) = \psi_{o1}^{\dagger}(\underline{r}',E')\psi_{o1}(\underline{r},E) \ . \tag{2.2.29}$$

Finally, multiply both sides of Eq. (2.2.29) by the macroscopic fission cross-section, Σ_f, (which is assumed constant in time) and integrate over all energy and the reactor volume, to obtain

$$\lim_{t\to\infty} \langle \Sigma_f(r,E) | \phi(\underline{r},E,t) \rangle = \psi_{o1}^{\dagger}(\underline{r}',E') \langle \Sigma_f(r,E) | \psi_{o1}(\underline{r},E) \rangle \ . \qquad (2.2.30)$$

Since this is the asymptotic total fission rate, the right-hand side of Eq. (2.2.30) is simply the neutron importance. Thus, it is seen that the importance of the injection of a unit of neutron flux at $\underline{r} = \underline{r}'$ and $E = E'$ is directly proportional to the adjoint neutron flux, $\psi_{o1}^{\dagger}(\underline{r}',E')$.

2.3 Development of the Point Reactor Kinetic Equations

In this section it is shown how the well known point reactor kinetic equations are derived exactly from the general diffusion kinetic equations. The reason such a derivation is important is that through such a derivation the various kinetic parameters (ρ, $\overline{\beta}$, ℓ, $\overline{C}_i$) can be related exactly to the physical properties of a nuclear reactor.

Since one is interested in how far from criticality any given reactor system is, a source free critical reference reactor is introduced. This fictitious reference reactor has the same geometry and "similar" nuclear properties as the real reactor. There are, of course, many such choices, any or all of the nuclear properties (D, Σ_a, Σ_f, Σ_s, etc.) may differ. If the actual reactor is critical, then the reference reactor can be chosen as the actual reactor. Since the reference reactor is critical, it can be completely described by the steady-state operator R_o. For this reference reactor the steady-state flux $\psi_o(\underline{r},E)$, is given by

$$R_o \psi_o(\underline{r},E) = 0 \ , \qquad (2.3.1)$$

and the adjoint steady-state flux, $\psi_o^{\dagger}(\underline{r},E)$, is given by

$$R_o^{\dagger} \psi_o^{\dagger}(\underline{r},E) = 0 \ . \qquad (2.3.2)$$

The calculation of ψ_o and $\psi_o^\dagger$ for equations (2.3.1) and (2.3.2) is a problem in neutron statics. Although for realistic situations such calculations are far from being trivial, it will be assumed that these steady-state fluxes are known.

With the concept of the critical reference reactor, the point reactor kinetic equations for the real reactor can be derived from the diffusion kinetic equations. Let the neutron flux be written as the product of two functions, namely

$$\phi(\underline{r},E,t) = \Phi(\underline{r},E,t)P(t) \ , \tag{2.3.3}$$

where Φ is the shape of the neutron flux and the factor P is the time dependent magnitude of the flux. Such a decomposition is not unique nor is it restrictive. Substitution of Eq. (2.3.3) into the diffusion kinetic equations (2.1.11) and (2.1.12) yields

$$\Phi\frac{\partial P}{\partial t} + P\frac{\partial \Phi}{\partial t} = PB\Phi + v\sum_{i=1}^{6}\chi_d^i\lambda_i C_i + Sv \tag{2.3.4}$$

and

$$\frac{\partial C_i}{\partial t} = PN_i\Phi - \lambda_i C_i \ , \quad i = 1,\cdots,6 \ . \tag{2.3.5}$$

The operators B and N_i are assumed to be explicit rather than implicit functions of time (this implies all feedback effects are neglected). Further, we assume the reactor geometry is constant in time. Multiply equation (2.3.4) by $\psi_o^\dagger(\underline{r},E)$ and equation (2.3.5) by $\psi_o^\dagger(\underline{r},E)$ and $v\chi_d^i(E)$, then integrate both equations over the volume of the reactor and all energy, to obtain

$$\langle\psi_o^\dagger|\Phi\rangle\frac{\partial P}{\partial t} + P\frac{\partial}{\partial t}\langle\psi_o^\dagger|\Phi\rangle = P\langle\psi_o^\dagger|B\Phi\rangle + \sum_{i=1}^{6}\lambda_i\langle\psi_o^\dagger|v\chi_d^i C_i\rangle + \langle\psi_o^\dagger|Sv\rangle \qquad (2.3.6)$$

and

$$\frac{\partial}{\partial t}\langle\psi_o^\dagger|v\chi_d^i C_i\rangle = P\langle\psi_o^\dagger|v\chi_d^i N_i\Phi\rangle - \lambda_i\langle\psi_o^\dagger|v\chi_d^i C_i\rangle \ , \quad i = 1,\cdots,6 \ . \qquad (2.3.7)$$

It is now assumed that the decomposition (2.3.3) is such that the following normalization condition holds:

$$\frac{d}{dt}\langle\psi_o^\dagger|\Phi\rangle \equiv 0 \ . \qquad (2.3.8)$$

Physically this normalization condition infers that the total neutron importance must remain constant in time for the critical reference reactor which has a distribution function of $\Phi(\underline{r},E,t)$. According to Eq. (2.3.8), the shape function $\psi_o^\dagger(\underline{r},E)$ must be chosen such that the total neutron importance of the reference reactor will remain constant in time even though locally $\Phi(\underline{r},E,t)$ itself may vary slowly in time.* The application of the normalization condition reduces the kinetic equations to the equations

$$\langle\psi_o^\dagger|\Phi\rangle\frac{\partial P}{\partial t} = P\langle\psi_o^\dagger|B\Phi\rangle + \sum_{i=1}^{6}\lambda_i\langle\psi_o^\dagger|v\chi_d^i C_i\rangle + \langle\psi_o^\dagger|Sv\rangle \qquad (2.3.9)$$

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* An alternative interpretation of the normalization condition can be obtained by expanding Φ in the orthonormal eigenfunctions, ψ_j and $\psi_j^\dagger$, of R_o, namely

$$\Phi = \langle\psi_o^\dagger|\Phi\rangle\psi_o + \sum_{j=1}^{\infty}\langle\psi_j^\dagger|\Phi\rangle\psi_j \ .$$

The normalization condition, Eq. (2.3.8), requires that the coefficient of the fundamental mode be independent of time, however, the remaining expansion coefficients may indeed by time dependent. It is also clear that the normalization does not imply that the shape function be independent of time locally.

and

$$\frac{\partial}{\partial t}\langle\psi_o^\dagger|v\chi_d^i C_i\rangle = P\langle\psi_o^\dagger|v\chi_d^i N_i\Phi\rangle - \lambda_i\langle\psi_o^\dagger|v\chi_d^i C_i\rangle \ , \quad i = 1,\cdots,6 \ . \tag{2.3.10}$$

To determine the deviations of the reactor parameters of the actual reactor from those of the critical reference reactor, a perturbation operator δB is defined as

$$\delta B(t) \equiv B(t) + v\sum_{i=1}^{6}\chi_d^i N_i(t) - R_o \ , \tag{2.3.11}$$

or more explicitly,

$$\delta B(t) \equiv v\nabla\delta D\cdot\nabla - v\delta\Sigma_t + v\int_o^\infty dE' \ \delta[\alpha\Sigma_s + \textstyle\sum_j \overline{\chi}^j\nu^j\Sigma_f^j] \ , \tag{2.3.12}$$

where $\delta\Sigma_i$ is the deviation from the critical state of the i-th type (i = s, f, t) macroscopic cross-section (i.e., $\delta\Sigma_i \equiv \Sigma_i - \Sigma_{io}$). With this definition the first term on the right-hand side of Eq. (2.3.9) can be expressed as

$$\langle\psi_o^\dagger|B\Phi\rangle = \langle\psi_o^\dagger|\delta B\Phi\rangle - \sum_{i=1}^{6}\langle\psi_o^\dagger|v\chi_d^i N_i\Phi\rangle + \langle\psi_o^\dagger|R_o\Phi\rangle \ . \tag{2.3.13}$$

From the definition of the adjoint of an operator, Eq. (2.2.7), and the relationship of Eq. (2.3.2), clearly $\langle\psi_o^\dagger|R_o\Phi\rangle = 0$, hence Eq. (2.3.13) reduces to

$$\langle\psi_o^\dagger|\delta B\Phi\rangle = \langle\psi_o^\dagger|B\Phi\rangle + \sum_{i=1}^{6}\langle\psi_o^\dagger|v\chi_d^i N_i\Phi\rangle \ . \tag{2.3.14}$$

Substitution of Eq. (2.3.14) into Eqs. (2.3.9) and (2.3.10) and use of the normalization condition (2.3.8) produces the following form of the kinetic equations:

$$\langle\psi_o^\dagger|\Phi\rangle\frac{\partial P}{\partial t} = P[\langle\psi_o^\dagger|\delta B\Phi\rangle - \sum_{i=1}^{6}\langle\psi_o^\dagger|v\chi_d^i N_i\Phi\rangle] + \sum_{i=1}^{6}\langle\psi_o^\dagger|v\chi_d^i C_i\rangle + \langle\psi_o^\dagger|Sv\rangle \tag{2.3.15}$$

and

$$\frac{\partial}{\partial t}\langle\psi_o^\dagger|\nu\chi_d^i C_i\rangle = P\langle\psi_o^\dagger|\nu\chi_d^i N_i\Phi\rangle - \lambda_i\langle\psi_o^\dagger|\nu\chi_d^i C_i\rangle \ , \quad i = 1,\cdots,6 \ . \tag{2.3.16}$$

Equations (2.3.15) and (2.3.16) can be cast into the standard form of the point reactor kinetic equations (PRKE), namely

$$\frac{dP}{dt} = \frac{\rho - \bar{\beta}}{\ell} P + \sum_{i=1}^{6} \lambda_i \bar{C}_i + \bar{S} \tag{2.3.17}$$

and

$$\frac{d\bar{C}_i}{dt} = \frac{\bar{\beta}_i}{\ell} P - \lambda_i \bar{C}_i \ , \quad i = 1,\cdots,6 \ , \tag{2.3.18}$$

where the terms ℓ, $\bar{\beta}_i$, $\bar{\beta}$, $\bar{C}_i$, $\bar{S}$, and ρ are defined below:

1. The mean prompt generation time,

$$\ell \equiv \frac{1}{F}\langle\psi_o^\dagger|\Phi\rangle \ , \tag{2.3.19}$$

is proportional to the total importance of all of the neutrons in the reactor.

2. The effective delayed neutron fraction,

$$\bar{\beta}_i \equiv \frac{1}{F}\langle\psi_o^\dagger|\nu\chi_d^i N_i\Phi\rangle \ , \tag{2.3.20}$$

is proportional to the importance of all of the delayed neutrons of the i-th group emitted per second in the reactor. The importance of the delayed neutrons is greater than that of prompt neutrons since the delayed neutrons are emitted at lower energies and thus avoid some of the absorption and leakage during the slowing down or thermalization process.

3. The total effective delayed neutron fraction,

$$\bar{\beta} \equiv \sum_{i=1}^{6} \bar{\beta}_i \ , \tag{2.3.21}$$

is the sum of the effective delayed neutron fractions for all six delayed neutron groups.

4. The effective concentration of the delayed neutron precursor of the i-th group is

$$\overline{C}_i \equiv \frac{1}{F\ell} \langle \psi_o^\dagger | \nu \chi_d^i C_i \rangle \ . \tag{2.3.22}$$

5. The effective neutron source,

$$\overline{S} \equiv \frac{1}{F\ell} \langle \psi_o^\dagger | Sv \rangle \ , \tag{2.3.23}$$

is proportional to the importance of all neutrons introduced per second in the entire reactor by the external neutron source.

6. The reactivity,

$$\rho \equiv \frac{1}{F} \langle \psi_o^\dagger | \delta B \Phi \rangle \ , \tag{2.3.24}$$

is a measure of the deviation of the reactor from the critical reference reactor.

It should be noted that the point reactor kinetic equations are exact and are simply a weighted average (in space and energy) form of the original diffusion kinetic equations. The normalization factor, F, found in the above definitions is completely <u>arbitrary</u>. This implies that none of the above PRKE quantities can be measured absolutely, but rather only the ratios of these quantities may be experimentally measured. In many texts[6-9] reactivity is given by

$$\rho \equiv (k_{eff} - 1) \ / \ k_{eff} \ , \tag{2.3.25}$$

where k_{eff} is described as the effective multiplication factor. This effective multiplication factor is the ratio of the number of fission neutrons emitted

in the preceding cycle.[1,11] In a following section the exact definition of reactivity, Eq. (2.3.24), is shown to reduce to this simple reactor statics result by using various approximations and by choosing the following definition for F.

$$F \equiv \langle \psi_o^\dagger | N\Phi \rangle \tag{2.3.26}$$

The quantity N is the total neutron production operator, namely

$$N \equiv N_o + \nu \sum_{i=1}^{6} \chi_d^i N_i \ . \tag{2.3.27}$$

Although reactivity is a dimensionless number, it is commonly expressed in terms of some unit.[9] There are several units associated with reactivity, they differ only by a constant. A widely used unit is the dollar, which is the amount of reactivity that will cause the reactor to become critical on prompt neutrons only, namely[4]

$$\rho = \overline{\beta} \ . \tag{2.3.28}$$

When the reactor is prompt critical the delayed neutrons are not necessary to sustain the neutron population, thus the reactor power can change very rapidly. The reactivity measured in dollars and cents is quite common due to the fact that the same amount of reactivity measured in dollars will cause similar dynamic response in two dissimilar reactors. Another important feature of the dollar unit of reactivity is that the reactivity of a reactor can be measured directly in dollars ($\rho/\overline{\beta}$). This direct measurement of reactivity can be obtained because the measurement is independent of the arbitrary normalization factor F.

2.4 Approximate Calculational Expressions for Reactivity

In the process of deriving the point reactor kinetic equations, the term reactivity has been defined as

$$\rho \equiv \langle \psi_o^\dagger | \delta B \Phi \rangle \; / \; \langle \psi_o^\dagger | N \Phi \rangle \; . \tag{2.4.1}$$

This definition of reactivity contains the unknown shape function, Φ, which can only be obtained from the original diffusion kinetic equations. Obviously, if one knew Φ, there would be no need to solve the point reactor kinetic equations! However, the diffusion kinetic equations are exceedingly difficult to solve, thus to use the PRKE one must employ various approximations for the shape function. In the following sections three different approximations for the shape function are introduced — the constant shape approximation, the asymptotic period approximation, and the adiabatic approximation.

2.4.1 The Constant Shape Approximation

In the constant shape approximation it is assumed that the shape function, Φ, is proportional to the steady-state neutron flux distribution, ψ_o, of the reference reactor. Therefore the neutron flux distribution can be approximated by

$$\phi(\underline{r},E,t) \simeq P(t)\psi_o(\underline{r},E) \; . \tag{2.4.2}$$

To understand the meaning of the factor $P(t)$, multiply both sides of (2.4.2) by the factor $\Sigma_f(\underline{r},E)$ and integrate over all energy and the volume of the reactor to give

$$\langle \Sigma_f(\underline{r},E) | \phi(\underline{r},E,t) \rangle \simeq P(t) \langle \Sigma_f(\underline{r},E) | \psi_o(\underline{r},E) \rangle \; . \tag{2.4.3}$$

The term $\langle \Sigma_f(\underline{r},E) | \phi(\underline{r},E,t) \rangle$ is proportional to the power of the reactor. If the macroscopic fission cross-section, Σ_f, is time independent, the factor $\langle \Sigma_f(\underline{r},E) | \psi_o(\underline{r},E,) \rangle$ is independent of time and hence the factor P(t) can be interpreted as the time variation of the reactor's power.

The reactivity using the constant shape approximation in Eq. (2.4.1) becomes

$$\rho(t) = \langle \psi_o^\dagger(\underline{r},E) | \delta B(t) \psi_o(\underline{r},E) \rangle / \langle \psi_o^\dagger(\underline{r},E) | N(t) \psi_o(\underline{r},E) \rangle \ . \qquad (2.4.4)$$

This same result can be obtained by applying first order perturbation theory to the diffusion equations.[7] This is the simplest and perhaps most common approximation used to calculate reactivity changes. We would expect the constant shape approximation or the first order perturbation theory form of reactivity to be a fair description if the reactor is only slightly removed from its critical state. If the perturbation causes a significant change in the shape function of the reactor, this approximation is no longer valid. However, this is the only approximation which allows any significant analytical analysis to be performed on the kinetics of a reactor system.

2.4.2 The Asymptotic Period Approximation

A second method for the calculation of reactivity is obtained by analyzing the time behavior of the reactor in terms of the so-called "ω-modes". In this technique one looks for solutions of the general diffusion kinetic equations in the form

$$\phi(\underline{r},E,t) = \phi_{\omega_j}(\underline{r},E) e^{\omega_j t} \qquad (2.4.5)$$

and

$$C_i(\underline{r},t) = C_{i\omega_j}(\underline{r}) e^{\omega_j t} \ , \qquad (2.4.6)$$

where ω_j is the time constant of the j-th ω-mode. There are, in general, an infinite number of solutions and hence an infinite number of values for the quantities $\phi_{\omega j}$, $C_{i\omega j}$, and ω_j.

The factor $C_i(\underline{r},t)$ in the kinetic equations may be eliminated by the substitution of Eq. (2.4.6) into Eq. (2.1.12) to obtain

$$C_{i\omega j}e^{\omega_j t} = N_i\phi_{\omega j}e^{\omega_j t} \,/\, (\omega_j + \lambda_i) \;, \quad i = 1,\cdots,6 \;. \tag{2.4.7}$$

The substitution of this result into Eq. (2.1.11), neglecting the external source, gives

$$B\phi_{\omega j} = [\omega_j - v\sum_{i=1}^{6}\frac{\lambda_i\chi_d^i N_i}{\omega_j + \lambda_i}]\phi_{\omega j} \;. \tag{2.4.8}$$

This is an eigenvalue problem where $\phi_{\omega j}$ and ω_j are the eigenfunctions and eigenvalues respectively. Let the eigenvalues be ordered such that $\mathrm{Re}\,\{\omega_o\} > \mathrm{Re}\,\{\omega_1\} > \cdots$. This ordering of the terms allows the fundamental mode (j = 0) to dominate the remaining terms as time increases. Physically, if the reactor is just critical ($\mathrm{Re}\,\{\omega_o\} = 0$), the fundamental mode will remain constant and the remaining modes will decay exponentially with time. If the reactor is subcritical ($\mathrm{Re}\,\{\omega_o\} < 0$), the fundamental mode will decay exponentially with time, however, the remaining modes will decay even faster. If the reactor is supercritical ($\mathrm{Re}\,\{\omega_o\} > 0$), the fundamental mode will have the largest growth rate, the remaining terms may either increase or decrease exponentially in time. Thus after long times ($t \to \infty$) the neutron flux distribution is determined by the fundamental or asymptotic mode.

The asymptotic period approximation assumes that the neutron flux distribution is given by the instantaneous fundamental mode, namely

$$\phi(\underline{r},E,t) = \phi_{\omega o}(\underline{r},E)e^{\omega_o t} \;. \tag{2.4.9}$$

The fundamental mode responds instantly to any change in the reactor system. Generally, the quantities $\phi_{\omega o}$ and ω_o are functions of time because the operators B and N_i are time dependent. This asymptotic period approximation then assumes the reactor system responds instantaneously by the characteristics of the fundamental or asymptotic mode. The instantaneous values of the shape function, $\phi_{\omega o}$, and the time constant, ω_o, may in principle be obtained by solving the eigenvalue problem of Eq. (2.4.8). The fundamental mode time constant may also be determined experimentally in a reactor by letting the reactor approach some asymptotic period, $1/\omega_o$, following the insertion of reactivity into the reactor.

The application of the asymptotic period approximation will yield another expression for the calculation of reactivity. Replacing the shape function, Φ, in Eq. (2.4.1) by the asymptotic period approximation of the shape function, $\phi_{\omega o}$, will give

$$\rho(t) = \langle\psi_o^\dagger(\underline{r},E)|\delta B(t)\phi_{\omega o}(\underline{r},E,t)\rangle \;/\; \langle\psi_o^\dagger(\underline{r},E)|N(t)\phi_{\omega o}(\underline{r},E,t)\rangle \; . \qquad (2.4.10)$$

Substitution of Eq. (2.3.11), the definition of the perturbation operator δB, into Eq. (2.4.10) yields

$$\rho = [\langle\psi_o^\dagger|B\phi_{\omega o}\rangle + \langle\psi_o^\dagger|v\sum_{i=1}^{6}\chi_d^i N_i\phi_{\omega o}\rangle - \langle\psi_o^\dagger|R_o\phi_{\omega o}\rangle] \;/\; \langle\psi_o^\dagger|N\phi_{\omega o}\rangle \; . \qquad (2.4.11)$$

The term $\langle\psi_o^\dagger|R_o\phi_{\omega o}\rangle$ is equal to $\langle R_o^\dagger\psi_o^\dagger|\phi_{\omega o}\rangle$, which by virtue of Eq. (2.3.2) is equal to zero. From Eq. (2.4.8) the reactivity then becomes

$$\rho = [\langle\psi_o^\dagger|\omega_o\phi_{\omega o}\rangle + \langle\psi_o^\dagger|v\sum_{i=1}^{6}\frac{\chi_d^i\omega_o N_i}{\omega_o + \lambda_i}\phi_{\omega o}\rangle] \;/\; \langle\psi_o^\dagger|N\phi_{\omega o}\rangle \; , \qquad (2.4.12)$$

or since

$$\ell = \langle\psi_o^\dagger|\phi_{\omega o}\rangle \;/\; \langle\psi_o^\dagger|N\phi_{\omega o}\rangle \qquad (2.4.13)$$

and

$$\bar{\beta}_i = \langle \psi_o^\dagger | \nu \chi_d^i N_i \phi_{\omega o} \rangle \; / \; \langle \psi_o^\dagger | N \phi_{\omega o} \rangle \; , \tag{2.4.14}$$

then

$$\rho = \omega_o [\ell + \sum_{i=1}^{6} \frac{\bar{\beta}_i}{\omega_o + \lambda_i}] \; . \tag{2.4.15}$$

This result is the well known "inhour equation" which relates the reactivity to the asymptotic period ($1/\omega_o$). If the operators in (2.4.8) vary in time, the shape function, $\phi_{\omega o}$, and the time constant, ω_o, will be functions of time, and thus the quantities ℓ, $\bar{\beta}_i$, and ρ will in general be time dependent. However, in practical applications, such as the reactivity calibration of control rods, the quantities ℓ and $\bar{\beta}_i$ are assumed to be constants. These parameters, previously determined, are utilized in the "inhour equation", (2.4.15), together with the reactor time behavior to calculate the inserted reactivity. The reactivity determined from the "inhour equation", or the asymptotic period approximation, is commonly known as the "dynamic reactivity".

2.4.3 The Adiabatic Approximation

For a third method of determining reactivity, consider a particular reference reactor which has the same geometry and "similar" nuclear properties as the real reactor. The reference reactor, in this case, is chosen to be identical to the real reactor in all respects except that the average number of neutrons released per fission is ν^j/k_{eff}. The real number k_{eff} is chosen such that the fictitious reference reactor is just critical. If the real reactor is critical then the reference reactor is identical to the real reactor and k_{eff} is equal to unity. The neutron flux, ϕ_k, for this critical reference reactor is obtained by combining Eqs. (2.1.10) and (2.1.15) and replacing ν^j by ν^j/k_{eff}, namely

$$M(t)\phi_k(\underline{r},E,t) = \frac{-1}{k_{eff}(t)} \left[N_o(t) + \sum_{i=1}^{6} \chi_d^i(E)N_i(t)\right] \phi_k(\underline{r},E,t) \ . \qquad (2.4.16)$$

The function $\phi_k(\underline{r},E,t)$ and $k_{eff}(t)$ may vary with time because the operators $M(t)$, $N_o(t)$, and $N_i(t)$ may be time dependent. The solution to Eq. (2.4.16) is an eigenvalue problem, i.e. there are any number of eigenvalues ($1/k_{eff}$) and eigenfunctions (ϕ_k) which are solutions. However, only one of these eigenfunctions, ϕ_o, will remain positive throughout the reactor volume. This one eigenfunction is the desired shape function and it corresponds to the largest value for k_{eff}.[1,2,7]

The factor k_{eff} as used in Eq. (2.4.16) is strictly a mathematical concept used in this analysis, however it can be shown that it is the actual effective multiplication factor of the real reactor.[1,10] To show this physical interpretation, consider only the fundamental mode neutron flux distribution, ϕ_o, of the fictitious reference reactor. At some time t = 0, inject this neutron flux as a source into the real non-critical reactor which has no neutron flux for all time prior to t = 0. Let the time behavior of this neutron flux injected into the real reactor be described by the function $x(\underline{r},E,t)$. This function is subject to the following conditions:

1. $x(\underline{r},E,0) = \phi_o$ (2.4.17)

2. $\lim_{t\to\infty} x(\underline{r},E,t) = 0$. (2.4.18)

The quantity x can then be described by

$$\frac{\partial x(\underline{r},E,t)}{\partial t} = Mx(\underline{r},E,t) \ , \qquad (2.4.19)$$

where the operator M describes the removal of the neutrons by any means be it absorption or leakage. Integration of Eq. (2.4.19) over time yields

$$x(\underline{r},E,t)\Big|_o^\infty = \int_o^\infty dt\ Mx(\underline{r},E,t) \ , \tag{2.4.20}$$

which if M is assumed to be time independent becomes

$$\phi_o = x(\underline{r},E,0) = -M\int_o^\infty dt\ x(\underline{r},E,t) \ . \tag{2.4.21}$$

Some of these original neutrons are lost due to absorption in the fuel and some of these absorbed neutrons cause fissioning in the fuel. In the fission process a second generation of neutrons is produced. Let the second generation neutron flux distribution in the real reactor be denoted by $z(\underline{r},E,t)$. The production of this flux is given by

$$\frac{\partial z}{\partial t} = [N_o + \sum_{i=1}^{6} \chi_d^i N_i]\ x \ . \tag{2.4.22}$$

Integration of Eq. (2.4.22) over time gives

$$z(\underline{r},E,t)\Big|_o^\infty = \int_o^\infty dt\ [N_o + \sum_{i=1}^{6} \chi_d^i N_i]\ x(\underline{r},E,t) \ , \tag{2.4.23}$$

which, if the operators N_o and N_i are stationary, becomes

$$z(\underline{r},E,t)\Big|_o^\infty = [N_o + \sum_{i=1}^{6} \chi_d^i N_i]\int_o^\infty dt\ x(\underline{r},E,t) \ . \tag{2.4.24}$$

The total second generation neutron flux is $\lim_{t\to\infty} z(\underline{r},E,t)$. Since the right-hand side of Eq. (2.4.24) is equal to $-k_{eff}M\int_o^\infty dt\ x(\underline{r},E,t)$, by virtue of Eq. (2.4.16), the ratio between the first and second generation neutron fluxes is given by

$$\frac{\lim_{t\to\infty} z(\underline{r},E,t)}{\phi_o} = \frac{-k_{eff}\ M\int_o^\infty dt\ x(\underline{r},E,t)}{-M\int_o^\infty dt\ x(r,E,t)} = k_{eff} \ . \tag{2.4.25}$$

This is a common definition of the effective multiplication factor.[5,9]

To develop the third method of calculating reactivity, the "adiabatic approximation" is applied to the exact definition of reactivity. This

adiabatic approximation assumes the shape function, ϕ, may be replaced by ϕ_o. The reactivity is then given by

$$\rho(t) = \langle\psi_o^\dagger(\underline{r},E)|\delta B(t)\phi_o(\underline{r},E,t)\rangle \; / \; \langle\psi_o^\dagger(\underline{r},E)|N(t)\phi_o(\underline{r},E,t)\rangle \; , \qquad (2.4.26)$$

where ϕ_o is the desired solution of Eq. (2.4.16). The perturbation operator, δB, as defined in Eq. (2.3.11) may be written as

$$\delta B(t) = M(t) + N(t) - R_o \; . \qquad (2.4.27)$$

Substitution of Eq. (2.4.27) into Eq. (2.4.26) then gives the reactivity as

$$\rho = [\langle\psi_o^\dagger|M\phi_o\rangle + \langle\psi_o^\dagger|N\phi_o\rangle - \langle\psi_o^\dagger|R_o\phi_o\rangle] \; / \; \langle\psi_o^\dagger|N\phi_o\rangle \; . \qquad (2.4.28)$$

The term $\langle\psi_o^\dagger|R_o\phi_o\rangle$ is equal to zero as was previously shown for Eq. (2.4.11). The term $\langle\psi_o^\dagger|M\phi_o\rangle$ is equal to $-\langle\psi_o^\dagger|N\phi_o\rangle/k_{eff}$ by virtue of Eq. (2.4.16), and thus the reactivity is given by

$$\rho = [(-\langle\psi_o^\dagger|N\phi_o\rangle/k_{eff}) + \langle\psi_o^\dagger|N\phi_o\rangle] \; / \; \langle\psi_o^\dagger|N\phi_o\rangle \; , \qquad (2.4.29)$$

or more simply

$$\rho(t) = [k_{eff}(t) - 1] \; / \; k_{eff}(t) \; . \qquad (2.4.30)$$

This is a familiar definition of reactivity encountered in reactor statics and is known as the "static reactivity".

The "static reactivity" as calculated from Eqs. (2.4.16) and (2.4.30) is commonly used to predict reactivities and is useful over a large range of reactivities. For reactivities up to prompt critical ($\rho = \bar{\beta}$), the numerical values of the "static" and "dynamic" reactivities should be virtually identical.[1,2]

It is commonly assumed in the industry that reactivity is additive (i.e., if δB_1 gives ρ_1 and δB_2 yields ρ_2, then $\delta B_1 + \delta B_2$ gives $\rho_1 + \rho_2$). However, the additive property is only valid for the constant shape approximation. In the adiabatic and asymptotic period approximations the shape function varies with the perturbation and thus the calculated reactivities fail to be truly additive.

2.5 Extension to Multigroup Equations

In all of the previous work the nuclear properties, fluxes, shape functions, and various operators have been assumed to be continuous functions of position, energy, and time. However, these position, energy, and time dependent variables may be very complicated and are exceedingly difficult to determine. A technique often used to obtain numerical solutions is known as the multigroup approximation. The multigroup approximation partitions the independent variable (or variables) into discrete regions, and then averages the dependent variables as well as the other equation parameters over these discrete regions. The result of this averaging process is a set of equations with fewer independent variables. One of the most common approximations to the diffusion equations in reactor analysis is to break up the energy variable into several regions and average the flux and cross-sections over each region. The resulting equations are known as the "multigroup (in energy) diffusion equations".

The purpose of this section is to show how the results of the previous sections can be applied directly to these multigroup (in energy) diffusion equations. The multigroup diffusion kinetic equations describing the k-th energy group are

$$\frac{1}{v_k}\frac{\partial \phi_k(\underline{r},t)}{\partial t} = \nabla D_k(\underline{r},t)\cdot\nabla\phi_k(\underline{r},t) - \Sigma_{tk}(\underline{r},t)\phi_k(\underline{r},t) + \sum_{k'=1}^{K} \alpha_{k'\to k}\Sigma_{sk'}(\underline{r},t)\phi_{k'}(\underline{r},t)$$

$$+ \sum_j \chi^j_{pk} \sum_{k'=1}^{K} (1-\beta^j)\nu^j_{k'}\Sigma^j_{fk'}(\underline{r},t)\phi_{k'}(\underline{r},t) + \sum_{i=1}^{6} \chi^i_{dk}\lambda_i C_i(\underline{r},t) + S_k(\underline{r},t) \quad (2.5.1)$$

and

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = \sum_j \sum_{k'=1}^{K} \beta^j_i \nu^j_{k'}\Sigma^j_{fk'}(\underline{r},t)\phi_{k'}(\underline{r},t) - \lambda_i C_i(\underline{r},t) \ , \quad i = 1,\cdots,6 \ , \quad (2.5.2)$$

where the summation of j includes all fissile materials. The entire energy spectrum is partitioned into K distinct energy groups. The variables and parameters found in Eqs. (2.5.1) and (2.5.2) are the weighted energy averages of the corresponding energy dependent quantities. These k-th group quantities are defined below.

1. ϕ_k = the k-th group neutron flux.
2. v_k = the average neutron speed.
3. D_k = the diffusion coefficient.
4. Σ_{fk}, Σ_{sk}, Σ_{tk} = macroscopic cross-sections for fission, scattering, and total interactions respectively.
5. $\nu_{k'}$ = the average number of neutrons released per fission, caused by a neutron of the k'-th energy group.
6. χ_{pk} = the fraction of prompt neutrons that are emitted into the k-th energy group.
7. χ^i_{dk} = the fraction of i-th group delayed neutrons emitted into the k-th group.
8. $\alpha_{k'\to k}$ = the fraction of neutrons scattered from the k'-th energy group into the k-th group.
9. S_k = the external neutron source for the k-th energy group.
10. The remaining quantities C_i, λ_i, β_i, and β are the same as in the energy dependent case, Eqs. (2.1.1) and (2.1.2).

A method by which these weight averaged quantities may be determined is given by Meghreblian and Holmes[7] in the section on multigroup equations.

The multigroup kinetic equations, (2.5.1) and (2.5.2), may be written more concisely be introducing the operators:

$$O_k \equiv v_k \nabla D_k(\underline{r},t)\cdot\nabla - v_k \Sigma_{tk}(\underline{r},t) \ , \tag{2.5.3}$$

$$Q_{k'\to k} \equiv v_k \alpha_{k'\to k} \Sigma_{sk'}(\underline{r},t) + v_k \textstyle\sum_j \chi^j_{pk}(1-\beta^j)\nu^j_{k'}\Sigma^j_{fk'}(\underline{r},t) \ , \tag{2.5.4}$$

and

$$T_{k'\to i} \equiv \textstyle\sum_j \beta^j_i \nu^j_{k'} \Sigma^j_{fk'}(\underline{r},t) \ . \tag{2.5.5}$$

These operators allow the multigroup diffusion kinetic equations (2.5.1) and (2.5.2) to be rewritten as

$$\frac{\partial \phi_k}{\partial t} = O_k \phi_k + \sum_{k'=1}^{K} Q_{k'\to k}\phi_{k'} + v_k \sum_{i=1}^{6} \chi^i_{dk}\lambda_i C_i + v_k S_k \tag{2.5.6}$$

and

$$\frac{\partial C_i}{\partial t} = \sum_{k'=1}^{K} T_{k'\to i}\phi_{k'} - \lambda_i C_i \ , \quad i = 1,\cdots,6 \ , \tag{2.5.7}$$

where k can take on any value from one to K. There are K different neutron flux distributions and K different sets of equations similar to Eqs. (2.5.6) and (2.5.7). These kinetic equations, (2.5.6) and (2.5.7), can also be cast in a matrix equation as

$$\frac{\partial \underline{\Psi}(\underline{r},t)}{\partial t} = \underline{\underline{A}}\underline{\Psi}(\underline{r},t) \ , \tag{2.5.8}$$

where the external neutron source is neglected and where

$$\underline{\Psi}(\underline{r},t) = \mathrm{col}(\phi_1(\underline{r},t), \phi_2(\underline{r},t),\cdots, \phi_K(\underline{r},t), C_1(\underline{r},t),\cdots, C_6(\underline{r},t)) \tag{2.5.9}$$

and

$$\underline{\underline{A}} = \begin{pmatrix} O_1+Q_{1\to1} & Q_{2\to1} & \cdots & Q_{K\to1} & \nu_1\lambda_1\chi_{d1}^1 & \nu_1\lambda_2\chi_{d1}^2 & \cdots & \nu_1\lambda_6\chi_{d1}^6 \\ Q_{1\to2} & O_2+Q_{2\to2} & \cdots & Q_{K\to2} & \nu_2\lambda_1\chi_{d2}^1 & \nu_2\lambda_2\chi_{d2}^2 & \cdots & \nu_2\lambda_6\chi_{d2}^6 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Q_{1\to K} & Q_{2\to K} & \cdots & O_K+Q_{K\to K} & \nu_K\lambda_1\chi_{dK}^1 & \nu_K\lambda_2\chi_{dK}^2 & \cdots & \nu_K\lambda_6\chi_{dK}^6 \\ T_{1\to1} & T_{2\to1} & \cdots & T_{K\to1} & -\lambda_1 & 0 & \cdots & 0 \\ T_{1\to2} & T_{2\to2} & \cdots & T_{K\to2} & 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ T_{1\to6} & T_{2\to6} & \cdots & T_{K\to6} & 0 & 0 & \cdots & -\lambda_6 \end{pmatrix} . \qquad (2.5.10)$$

Matrix and vector operators can be found which correspond to the previously defined operators: M, N_o, N_i, N, B, R_o, and δB. The search for the matrix and vector operators is desirable because the derivations and arguments presented in the previous sections can be used directly to give the multigroup form of all of the equations. The matrix and vector operators may be found by analyzing the matrix operator $\underline{\underline{A}}$, the operators given in Eqs. (2.5.3) through (2.5.5), and the kinetic equations (2.5.6) and (2.5.7). The Boltzmann operator, B, given by Eq. (2.1.12), now becomes the K by K submatrix situated in the upper left-hand corner of matrix $\underline{\underline{A}}$, namely

$$\underline{\underline{B}} \equiv \begin{pmatrix} O_1+Q_{1\to1} & Q_{2\to1} & \cdots & Q_{K\to1} \\ Q_{1\to2} & O_2+Q_{2\to2} & \cdots & Q_{K\to2} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{1\to K} & Q_{2\to K} & \cdots & O_K+Q_{K\to K} \end{pmatrix} . \qquad (2.5.11)$$

The operator M becomes the K by K matrix

$$\underline{\underline{M}} \equiv \begin{pmatrix} {}^{0}_{1}+\nu_1\alpha_{1\to1}\Sigma_{s1} & \nu_1\alpha_{2\to1}\Sigma_{s2} & \cdots & \nu_1\alpha_{K\to1}\Sigma_{sK} \\ \nu_2\alpha_{1\to2}\Sigma_{s1} & {}^{0}_{2}+\nu_2\alpha_{2\to2}\Sigma_{s2} & \cdots & \nu_2\alpha_{K\to2}\Sigma_{sK} \\ \vdots & \vdots & \ddots & \vdots \\ \nu_K\alpha_{1\to K}\Sigma_{s1} & \nu_K\alpha_{2\to K}\Sigma_{s2} & \cdots & {}^{0}_{K}+\nu_K\alpha_{K\to K}\Sigma_{sK} \end{pmatrix} , \tag{2.5.12}$$

which describes the absorption, leakage, and thermalization processes. The prompt neutron production operator, $\underline{\underline{N}}_o$, is now cast into the following K by K matrix,

$$\underline{\underline{N}}_o \equiv \begin{pmatrix} n_{o1\to1} & n_{o2\to1} & \cdots & n_{oK\to1} \\ n_{o1\to2} & n_{o2\to2} & \cdots & n_{oK\to2} \\ \vdots & \vdots & \ddots & \vdots \\ n_{o1\to K} & n_{o2\to K} & \cdots & n_{oK\to K} \end{pmatrix} , \tag{2.5.13}$$

where

$$n_{ok'\to k} \equiv \nu_k \sum_j \chi^j_{pk}(1-\beta^j)\nu^j_{k'}\Sigma^j_{fk'} . \tag{2.5.14}$$

The i-th group delayed neutron precursor production operator, $\underline{N}_i$, as given by Eq. (2.1.9), is now given by the row vector

$$\underline{N}_i \equiv (T_{1\to i}, T_{2\to i}, \cdots, T_{K\to i}) . \tag{2.5.15}$$

If the quantity $\nu\chi^i_d\lambda_i$ is the column vector

$$\lambda_i \underline{\nu\chi}^i_d \equiv \lambda_i \ \mathrm{col}(\nu_1\chi^i_{d1}, \nu_2\chi^i_{d2}, \cdots, \nu_K\chi^i_{dK}) , \tag{2.5.16}$$

then the total neutron production operator, N, becomes

$$\underline{\underline{N}} \equiv \underline{\underline{N}}_o + \sum_{i=1}^{6} \underline{\nu\chi}^i_d \, \underline{N}_i . \tag{2.5.17}$$

Following through with the same technique as given in Section 2.1, the steady-state operator, R_o, defined in Eq. (2.1.16), now becomes a K by K matrix operator,

$$\underline{\underline{R}}_o \equiv \underline{\underline{B}}_o + \sum_{i=1}^{6} \nu \underline{\chi}_d^i \underline{N}_{io} \ . \tag{2.5.18}$$

The neutron flux distribution, $\underline{\phi}$, is also a vector, namely

$$\underline{\phi} = \text{col}(\phi_1, \phi_2, \cdots, \phi_K) \ . \tag{2.5.19}$$

These matrix and vector forms of the operators allow the point reactor kinetic equations and the reactivity to be developed using matrix equations.

To derive the point reactor kinetic equations in the multigroup form, the steady-state flux vector, $\underline{\psi}_o$, and its adjoint, $\underline{\psi}_o^\dagger$, are assumed to be known, such that

$$\underline{\underline{R}}_o \underline{\psi}_o = \underline{0} \tag{2.5.20}$$

and

$$\underline{\underline{R}}_o^\dagger \underline{\psi}_o^\dagger = \underline{0} \ , \tag{2.5.21}$$

where $\underline{0}$ is the null vector. The neutron flux distribution, $\underline{\phi}$, may be given as the product of two quantities, the shape vector, Φ, and the magnitude, P, namely

$$\underline{\phi}(\underline{r},t) = \underline{\Phi}(\underline{r},t)P(t) \ , \tag{2.5.22}$$

or more explicitly

$$\phi(\underline{r},t) = P(t) \ \text{col}(\Phi_1(\underline{r},t), \Phi_2(\underline{r},t), \cdots, \Phi_K(\underline{r},t)) \ . \tag{2.5.23}$$

The perturbation matrix operator, δB, can be written as

$$\underline{\underline{\delta B}} \equiv \underline{\underline{B}} + \sum_{i=1}^{6} \underline{\nu\chi_d^i}\ \underline{N}_i - \underline{\underline{R}}_o \ . \tag{2.5.24}$$

By using these matrices and vectors in the equations of the previous energy-dependent derivations, the multigroup form of the point reactor kinetic equations may be developed. However, the integration over all energy in the energy dependent case is now described by the dot-products of the various multigroup variables and parameters. If, in the integration over the reactor volume, the product of two vectors is considered to be a dot-product, the PRKE take on the standard form as

$$\frac{dP}{dt} = \frac{\rho - \bar{\beta}}{\ell} P + \sum_{i=1}^{6} \lambda_i \bar{C}_i + \bar{S} \tag{2.5.25}$$

and

$$\frac{d\bar{C}_i}{dt} = \frac{\bar{\beta}_i}{\ell} P - \lambda_i \bar{C}_i \ , \quad i = 1,\cdots,6 \ , \tag{2.5.26}$$

where the kinetic parameters are defined as:

1. $$\ell \equiv \frac{1}{F}\int_v d^3r\ \underline{\psi}_o^\dagger \cdot \underline{\Phi} \equiv \frac{1}{F} \langle \underline{\psi}_o^\dagger | \underline{\Phi} \rangle \tag{2.5.27}$$

2. $$\bar{\beta}_i \equiv \frac{1}{F}\int_v d^3r\ \underline{\psi}_o^\dagger \cdot (\underline{\nu\chi_d^i}\ \underline{N}_i)\underline{\Phi} \equiv \frac{1}{F} \langle \underline{\psi}_o^\dagger | \underline{\nu\chi_d^i}\ \underline{N}_i \underline{\Phi} \rangle \tag{2.5.28}$$

3. $$\bar{\beta} \equiv \sum_{i=1}^{6} \bar{\beta}_i \tag{2.5.29}$$

4. $$\bar{C}_i \equiv \frac{1}{F\ell}\int_v d^3r\ \underline{\psi}_o^\dagger \cdot \underline{\nu\chi_d^i}\ C_i \equiv \frac{1}{F\ell} \langle \underline{\psi}_o^\dagger | \underline{\nu\chi_d^i}\ C_i \rangle \tag{2.5.30}$$

5. $$\bar{S} \equiv \frac{1}{F\ell}\int_v d^3r\ \underline{\psi}_o^\dagger \cdot \underline{Sv} \equiv \frac{1}{F\ell} \langle \underline{\psi}_o^\dagger | \underline{Sv} \rangle \tag{2.5.31}$$

6. $$\rho \equiv \frac{1}{F}\int_v d^3r\ \underline{\psi}_o^\dagger | \underline{\underline{\delta B}}\Phi \equiv \frac{1}{F} \langle \underline{\psi}_o^\dagger | \underline{\underline{\delta B}}\Phi \rangle \ . \tag{2.5.32}$$

As before, F is arbitrary, but following the discussion in Section 2.3, we

choose F as

$$F \equiv \langle \underline{\psi}_o^\dagger | \underline{\underline{N}} \underline{\Phi} \rangle \ , \tag{2.5.33}$$

which yields the following form for reactivity

$$\rho \equiv \langle \underline{\psi}_o^\dagger | \underline{\underline{\delta B}} \ \underline{\Phi} \rangle \ / \ \langle \underline{\psi}_o^\dagger | \underline{\underline{N}} \underline{\Phi} \rangle \ . \tag{2.5.34}$$

The three commonly used calculational expressions for determining reactivity follow directly by applying the multigroup approximations of the shape vector, $\underline{\Phi}$. These three calculational expressions of reactivity are:

$$\rho = \langle \underline{\psi}_o^\dagger | \underline{\underline{\delta B}} \underline{\psi}_o \rangle \ / \ \langle \underline{\psi}_o^\dagger | \underline{\underline{N}} \underline{\psi}_o \rangle \ . \tag{2.5.35}$$

for the constant shape approximation,

$$\rho = \omega_o [\ell + \sum_{i=1}^{6} \frac{\bar{\beta}_i}{\omega_o + \lambda_i}] \tag{2.5.36}$$

for the asymptotic period approximation, and

$$\rho = (k_{eff} - 1) \ / \ k_{eff} \tag{2.5.37}$$

where k_{eff} is the largest eigenvalue of the adiabatic approximation.

3.0 EXPERIMENTAL MEASUREMENTS IN THE TRIGA REACTOR

One of the objectives of this thesis was to find a simplified method of predicting the reactivity of the Kansas State University Triga Mark II (KSUTMII) nuclear reactor. As a check on the predicted reactivities, an experiment was performed to obtain results which could be used as a comparison. The experiment consisted of measuring the aluminum to water, fuel to water, and graphite to water reactivity differences from the change in control rod positions and the control rod calibration curves.

To determine a control rod calibration curve, the control rod to be calibrated is situated initially at a given position. The remaining control rods are withdrawn until the reactor is critical. The rod to be calibrated is then withdrawn to another position causing the reactor power to increase. The reactor power will eventually approach some asymptotic period $1/\omega_o$, where ω_o is the fundamental eigenvalue of Eq. (2.4.8). This asymptotic period $(1/\omega_o)$ is then used in the "inhour equation" (2.4.15) to ascertain the "dynamic" reactivity worth of that control rod section. The control rod to be calibrated is left at the new position and the other control rods are inserted until the reactor is again critical. The reactor power is again caused to rise by withdrawing the rod to be calibrated. From the new asymptotic period and the "inhour equation", the reactivity worth of the second section of the control rod is determined. This procedure is repeated until the entire control rod is calibrated. To obtain the control rod calibration curve, reactivity is assumed to be additive and the reactivity sums are plotted versus the control rod positions. Usually a control rod calibration curve will remain valid unless the neutron flux distribution near the control rod is altered by changing the reactor system. In the following reactivity

experiments it was assumed that the calibration curves did not vary while the nuclear properties of the reactor were being changed.

Once the control rod calibration curves are known, the reactivity of some change in the reactor properties is readily measured by noting the critical positions of the control rods before and after the change. For example, to determine the fuel to water reactivity difference, the reactor system was first established in the "core-eleven" configuration (shown in Figure 1). The "pulse-safety" rod (core position D-10) and the "regulating" rod (core position E-21) were withdrawn to their upper limit switches. The "shim" rod (core position C- 3) was then withdrawn until the reactor became critical. From the shim rod calibration curve (Figure 2) the "excess" reactivity left in the reactor was ascertained. The excess reactivity is the reactivity worth of that portion of the control rod remaining in the reactor core. After the critical rod position for core-eleven was determined, the shim rod was fully inserted to insure the reactor would remain subcritical during any changes in the core configuration. One of the fuel elements was then removed and the shim rod was again withdrawn until the reactor was critical. The excess reactivity left in the shim was obtained, then the shim rod was inserted into the reactor. The removed fuel element was then reinserted to return to the original core-eleven configuration and the shim rod was again withdrawn until the reactor was critical. This second measurement of core-eleven was to verify that the core-eleven excess reactivity was unchanged, and hence served as a check that the fuel element was properly seated in the core. This experimental procedure was continued for each of the remaining sixty-two fuel element positions.[11] The fuel to water reactivity difference was then calculated as the difference between the excess reactivities obtained with the fuel element inserted and with the fuel element removed. The results of these

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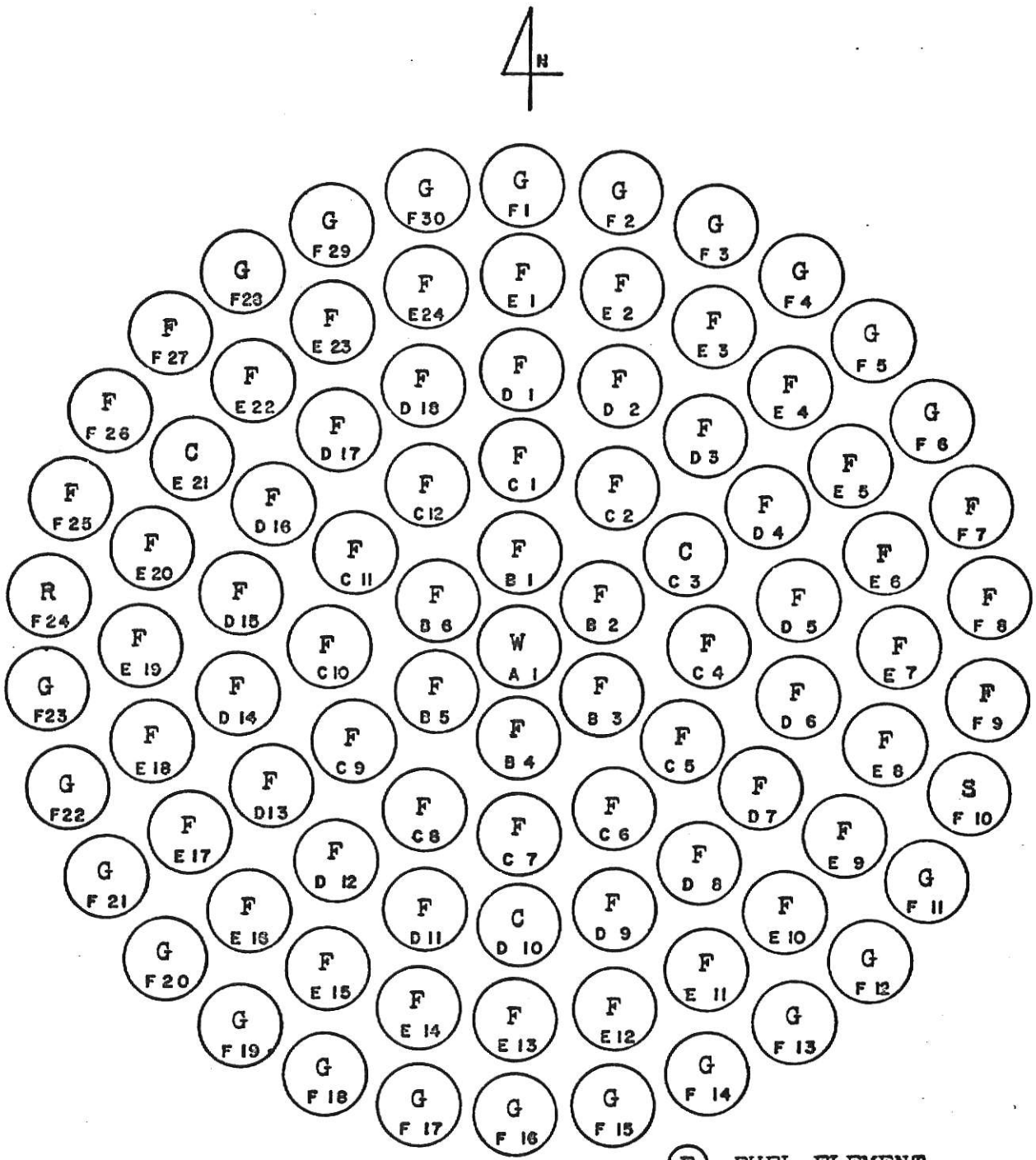

Figure 1. Core-eleven loading of the Kansas State University Triga Mark II nuclear reactor.

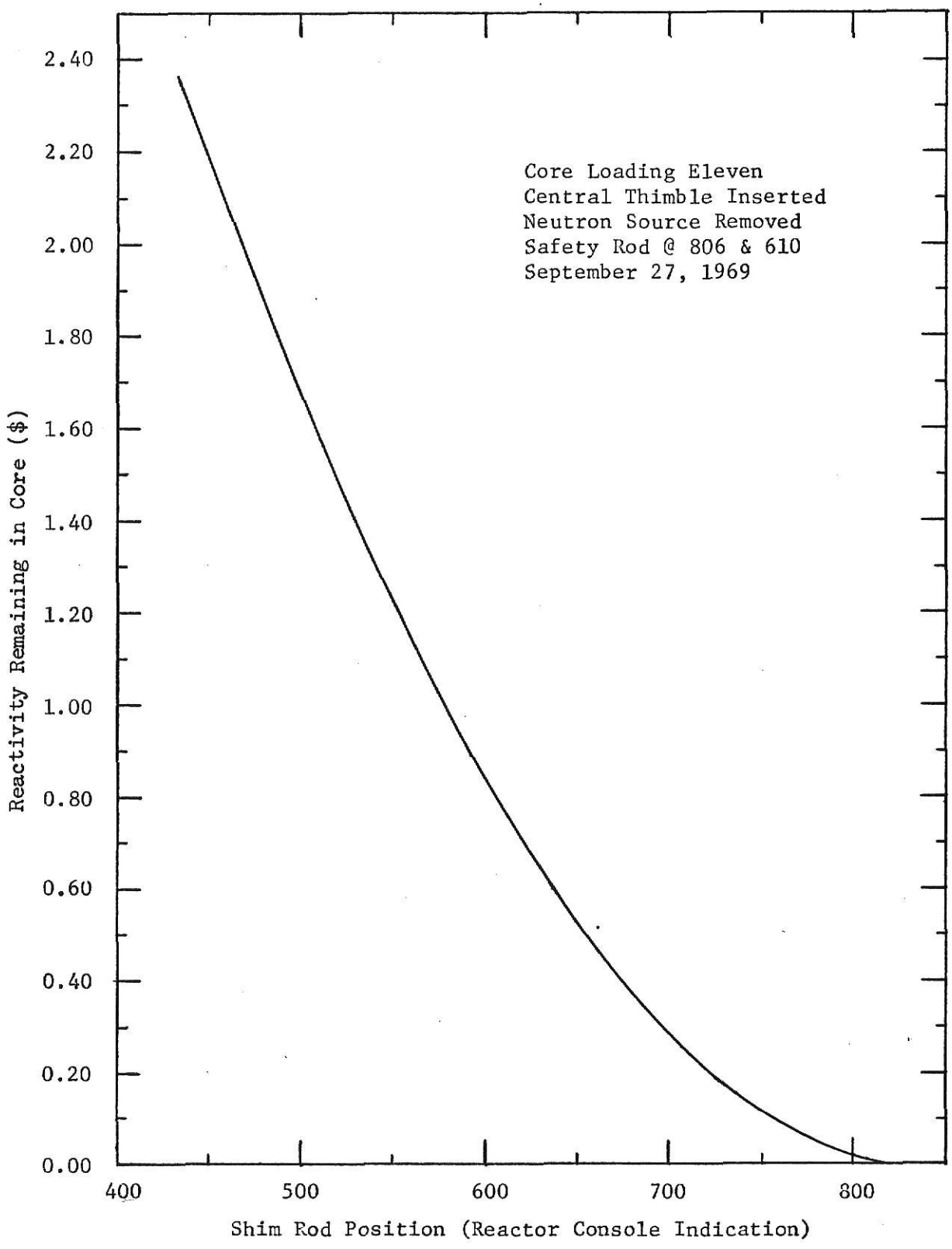

Figure 2. Shim rod calibration curve.

measured fuel to water reactivity differences are given in Table I.

Table I lists the reactor conditions and the measurements obtained during the fuel to water reactivity difference measurements. There are no definite units associated with the rod positions listed in Table I. The shim rod positions were read from a digital indicator on the reactor console. However, the arbitrary scale presents no difficulties in reactivity determination because the shim rod calibration curve also used the reactor console rod position readings.

From Table I it can be seen that, generally, the fuel near the center of the reactor was worth more than the fuel near the reactor edge, and the worth of the elements within any given ring varied directly with the mass of fuel (U-235) in the elements. The fuel to water reactivity measurements also tended to be larger for the positions near the three control rods and near the neutron source. This increase in fuel element worth near the control rods is reasonable because the three control rod positions (C- 3, D-10, and E-21) contained water (i.e., the control rods were either partially or completely withdrawn during the experiment) and hence, there was a local peaking of the thermal neutron flux. Similarly, the reactivity measurements for the fuel elements near the neutron source position (F-10) were also peaked, because this position also contained water during the experiment.

To determine the aluminum to water and graphite to water reactivity differences, an experimental procedure similar to that utilized for the fuel to water measurements, was employed. However, for these measurements the water was replaced respectively by an aluminum element and by a graphite element. These measurements were made in only eight of the sixty-three fuel positions, namely: B- 2, C- 2, C- 7, C- 8, C- 9, D-13, E-17, and F- 8.[11] These eight positions were selected by choosing the positions that covered the

Table I. Fuel to Water Reactivity Measurements*

Date	Position	Element (Grams U-235)	Water Rod Position (Excess Reactivity)	Core 11 Rod Position (Excess Reactivity)	Fuel-Water Worth
10-14-69				447 ($2.21)	
	E- 1	2740 (36.32)	514 ($1.57)	447 (2.21)	$ .64
	E- 2	2719 (36.30)	509 (1.61)	447 (2.21)	.60
	E- 3	2751 (36.30)	508 (1.62)	446 (2.22)	.60
	E- 4	2730 (36.29)	506 (1.64)	447 (2.21)	.57
	E- 5	2725 (36.26)	503 (1.67)	447 (2.21)	.54
10-15-69				446 ($2.22)	
	E- 6	2720 (36.24)	491 ($1.79)	446 (2.22)	$ .44
	E- 7	2735 (36.18)	494 (1.76)	445 (2.23)	.46
	E- 8	2728 (36.16)	512 (1.58)	445 (2.23)	.65
	E- 9	2781 (36.09)	513 (1.57)	444 (2.24)	.66
10-16-69				444 ($2.24)	
	F- 7	2764 (35.01)	486 ($1.83)	444 (2.24)	$ .41
	F- 8	2736 (32.70)	481 (1.88)	443 (2.25)	.36
	F- 9	2760 (35.19)	495 (1.75	443 (2.25)	.50
				450 (2.19)	
	E-10	2733 (36.03)	520 (1.50)	448 (2.21)	.70
	E-11	2753 (35.99)	518 (1.52)	448 (2.21)	.69
	E-12	2748 (35.98)	521 (1.49)	449 (2.20)	.71
				445 (2.23)	
	E-13	2746 (35.97)	534 (1.37)	445 (2.23)	.86
	E-14	2776 (35.95)	518 (1.52)	442 (2.26)	.72
10-29-69				445 ($2.23)	
	E-15	2770 (35.92)	514 ($1.57)	445 (2.23)	$ .66
	E-16	2742 (35.92)	515 (1.56)	445 (2.23)	.67
	E-17	2754 (35.87)	512 (1.59)	445 (2.23)	.64
	E-18	2766 (35.84)	520 (1.50)	445 (2.23)	.73
	E-19	2773 (35.81)	519 (1.51)	445 (2.23)	.72
	E-20	2772 (35.80)	524 (1.47)	444 (2.24)	.76
	F-25	2768 (35.64)	503 (1.67)	445 (2.23)	.56
11-20-69				449 ($2.20)	
	F-26	2137 (35.53)	510 ($1.60)	449 (2.20)	$ .60
	F-27	2727 (35.23)	508 (1.62)	450 (2.19)	.58
	E-22	2771 (35.73)	530 (1.41)	449 (2.20)	.78
1-30-70				443 ($2.25)	
	E-23	2738 (35.72)	519 ($1.51)	442 (2.26)	$ .75
	E-24	2783 (35.70)	514 (1.57)	442 (2.26)	.69
	D- 1	2784 (35.16)	507 (1.63)	442 (2.26)	.63
1-30-70	D- 2	2752 (36.92)	517 ($1.53)	442 ($2.26)	$ .73
	D- 3	2723 (36.87)	521 (1.49)	442 (2.26)	.77
				442 (2.26)	
	D- 4	2721 (36.84)	543 (1.30)	440 (2.28)	.97
	D- 5	2750 (36.82)	524 (1.47)	440 (2.28)	.81
	D- 6	2778 (36.77)	512 (1.58)	440 (2.28)	.70
	D- 7	2722 (36.75)	508 (1.62)	440 (2.28)	.66
	D- 8	2775 (36.58)	513 (1.57)	440 (2.28)	.71
	D- 9	2763 (36.55)	538 (1.34)	439 (2.29)	.95
	D-11	2755 (36.55)	540 (1.33)	440 (2.28)	.95
	D-12	2767 (36.51)	517 (1.54)	440 (2.28)	.74
2- 6-70				443 ($2.25)	
	D-13	2782 (36.51)	518 ($1.52)	442 (2.26)	$ .74
	D-14	2780 (36.48)	527 (1.44)	443 (2.25)	.81
	D-15	2743 (36.45)	530 (1.41)	443 (2.25)	.84
	D-16	2737 (36.40)	554 (1.20)	443 (2.25)	1.05
	D-17	2765 (36.38)	533 (1.38)	443 (2.25)	.87
				441 (2.27)	
	D-18	2726 (36.36)	525 (1.45)	442 (2.26)	.81
2- 9-70				439 ($2.29)	
	C- 1	2779 (37.41)	540 ($1.32)	441 (2.27)	$ .96
	C- 2	2757 (37.24)	604 (.81)	437 (2.31)	1.47
	C- 4	2739 (37.22)	585 (.95)	438 (2.30)	1.35
	C- 5	2761 (37.15)	531 (1.40)	437 (2.31)	.91
4- 9-70				447 ($2.21)	
	B- 5	2789TC(35.28)	612 ($ .75)	449 (2.20)	$1.45
	B- 3	2788TC(38.12)	605 (.80)	449 (2.20)	1.40
	C- 6	2747 (37.14)	568 (1.08)	450 (2.19)	1.11
	C- 7	2759 (37.11)	585 (.91)	448 (2.21)	1.29
	C- 8	2731 (37.10)	566 (1.10)	448 (2.21)	1.11
	C- 9	2769 (36.98)	551 (1.23)	448 (2.21)	.98
	C-10	2762 (36.97)	561 (1.14)	448 (2.21)	1.07
	C-11	2734 (36.94)	557 (1.17)	448 (2.21)	1.04
	C-12	2745 (36.93)	566 (1.10)	449 (2.20)	1.10
	B- 1	2777 (37.56)	612 (.75)	449 (2.20)	1.45
	B- 4	2744 (38.11)	616 (.73)	450 (2.19)	1.46
	B- 6	2724 (37.49)	606 (.79)	449 (2.20)	1.41
5- 5-70				447 ($2.22)	
	B- 2	2732 (38.03)	677 ($ .38)	447 (2.22)	$1.84

* Reactor Conditions: Core Loading- No. 11, Power- 0.1 Watt, Source- Removed, Pulse Rod- Upper Limit Switch, Regulating Rod- Upper Limit Switch, Central Thimble- Installed.

range of the previously measured fuel to water reactivity differences. The fuel element in B- 2 was worth the most and the fuel element in F- 8 was worth the least, the remaining positions had fuel to water worths somewhere in between these two. At the same time as the aluminum to water and graphite to water experiments were being made, another fuel element (serial no. 2749) was also inserted and measured to obtain another fuel to water measurement. These measurements actually compare the normal fuel for each position with aluminum, graphite, and the fuel element (2749). The fuel to graphite measurements are shown in Table II. Also listed in Table II are fuel to water reactivity measurements for these eight positions, which together with Table I give an indication of the reproducibility of the reactivity measurements. The fuel to aluminum and the fuel to fuel (2749) measurements are listed in Table III. Aluminum to water and graphite to water reactivity difference measurements were also taken in the central thimble. These measurements are shown in Table IV.

To obtain the aluminum to water, graphite to water, and fuel (2749) to water reactivity differences, the values for fuel to aluminum, fuel to graphite, and fuel to fuel (2749) were subtracted from the associated fuel to water reactivity measurements. These calculated reactivity differences — aluminum to water, graphite to water, and fuel (2749) to water for positions B- 2, C- 2, C- 7, C- 8, C- 9, D-13, E-17, and F- 8 plus aluminum to water and graphite to water in the central thimble — are tabulated in Table V.

As a check on the reproducibility of the measurements, the fuel to water experiment was carried out several times. It was found that the measurements for each position all agree within plus or minus two cents.

Table II. Fuel to Water and Fuel to Graphite Reactivity Measurements*

Position	Water Rod Position (Excess Reactivity)	Fuel-Water Worth	Graphite Rod Position (Excess Reactivity)	Fuel-Graph. Worth	Core 11 Rod Position (Excess Reactivity)
5- 5-70					448 ($2.21)
F- 8	487 ($1.83)	$ .38	478 ($1.91)	$ .30	448 (2.21)
C- 2	614 (.74)	1.47	561 (1.14)	1.07	448 (2.21)
C- 8	564 (1.12)	1.10	539 (1.33)	.89	447 (2.22)
5-28-70					456 ($2.12)
C- 9	560 ($1.15)	$ .97	547 ($1.26)	$ .86	456 (2.12)
D-13	531 (1.40)	.72	525 (1.45)	.67	457 (2.11)
E-17	524 (1.47)	.65	506 (1.64)	.48	456 (2.12)
C- 7	595 (.88)	1.24	556 (1.18)	.94	456 (2.12)
B- 2	700 (.28)	1.84	607 (.79)	1.33	456 (2.12)

Table III. Fuel to Aluminum and Fuel to Fuel (2749) Reactivity Measurements*

Position	Aluminum Rod Position (Excess Reactivity)	Fuel-Alum. Worth	Fuel (2749) 36.93 grams U-235 Rod Position (Excess Reactivity)	Fuel-2749 Worth	Core 11 Rod Position (Excess Reactivity)
8- 3-70					459 ($2.10)
E-17	537 ($1.35)	$ .75	452 ($2.16)	$-.06	459 (2.10)
D-13	570 (1.07)	1.03	456 (2.12)	-.02	458 (2.10)
C- 9	607 (.79)	1.31	455 (2.13)	.03	459 (2.10)
C- 8	612 (.75)	1.35	451 (2.17)	-.07	459 (2.10)
C- 7	616 (.73)	1.37	456 (2.12)	-.02	459 (2.10)
F- 8	503 (1.67)	.43	456 (2.12)	-.01	457 (2.11)
C- 2	630 (.64)	1.49	455 (2.13)	0.00	454 (2.15)
B- 2	701 (.27)	1.89	453 (2.15)	.01	453 (2.16)

* Reactor Conditions: Core Loading- No. 11, Power- 0.1 Watt, Source- Removed, Pulse Rod- Upper Limit Switch, Regulating Rod- Upper Limit Switch, Central Thimble- Installed.

Table IV. Graphite to Water and Aluminum to Water Reactivity Measurements in the Central Thimble*

Configuration	Rod Position (Excess Reactivity)		
Core 11	450	($2.19)	
F- 9: Removed	501	(1.69)	
Central Thimble: Removed	492	(1.78)	
Graphite: Inserted	510	(1.60)	$-.18 Graph.-Water Worth
Central Thimble: Removed	492	(1.78)	
Aluminum: Inserted	571	(1.06)	$-.72 Alum.-Water Worth
Central Thimble: Removed	492	(1.78)	
Core 11	441	(2.28)	
Central Thimble: Inserted	450	(2.19)	

* Reactor Conditions: Power- 0.1 Watt, Source- Removed, Pulse Rod- Upper Limit Switch, Regulating Rod- Upper Limit Switch, Position- Central Thimble, Date- 8-17-70.

Table V. Aluminum to Water, Graphite to Water, and Fuel (2749) to Water Reactivity Differences

Position	Aluminum-Water Reactivity Difference	Graphite-Water Reactivity Difference	Fuel (2749)-Water Reactivity Difference
B- 2	$-.05	$ .51	$1.84
C- 2	-.02	.40	1.47
C- 7	-.13	.30	1.26
C- 8	-.25	.21	1.17
C- 9	-.34	.11	1.00
D-13	-.31	.05	.74
E-17	-.10	.17	.71
F- 8	-.05	.08	.39
Central Thimble	-.72	-.18	

4.0 REACTIVITY CALCULATIONS

4.1 The Constant Shape Approximation Form of Calculating Reactivity

The first attempt in the search for a simplified means of predicting reactivities for the KSUTMII used one of the simplest methods for reactivity calculations. This method employs the one-speed diffusion theory and the constant shape approximation (or the first-order perturbation theory) of Section 2.4.1. The energy dependent constant shape approximation form of reactivity is given by Eq. (2.4.4), namely

$$\rho(t) = \frac{\langle \phi_o^\dagger(\underline{r},E) \,|\, \delta B(t) \phi_o(\underline{r},E) \rangle}{\langle \phi_o^\dagger(\underline{r},E) \,|\, N(t) \phi_o(\underline{r},E) \rangle} .$$

For one-speed diffusion theory this result can be simplified considerably. There will be no energy dependence, and more importantly, the diffusion equation operators become self-adjoint so the adjoint steady-state flux, $\phi_o^\dagger$, is equal to the ordinary steady-state flux, ϕ_o. Application of the one-speed diffusion theory and substitution of the explicit expressions for the operators B and N gives another expression,

$$\rho = \frac{\int_v d^3r \; \phi_o(\underline{r})[\nabla \delta D \cdot \nabla - \delta\Sigma_a + \delta\nu\Sigma_f]\phi_o(\underline{r})}{\int_v d^3r \; \phi_o(r)\nu\Sigma_f\phi_o(\underline{r})} \qquad (4.1.1)$$

for reactivity. The quantities δD, $\delta\Sigma_a$, and $\delta\nu\Sigma_f$ are equal respectively to $D - D_o$, $\Sigma_a - \Sigma_{ao}$, and $\nu\Sigma_f - \nu\Sigma_{fo}$, where the subscript "o" denotes the respective nuclear property of the unperturbed system. Unless there is a change in the macroscopic fission cross-section during the perturbation, the quantity Σ_f in the denominator has the unperturbed value Σ_{fo}. However, it is consistent with first-order perturbation theory to assume that this macroscopic fission cross-section always takes on the unperturbed value Σ_{fo} even if Σ_f does change.

If the neutron flux, ϕ_o, is required to go to zero at the reactor boundaries, then the application of Gauss' theorem to the first term on the right-hand side of the identity

$$\int_v d^3r\ \phi_o \nabla\delta D\cdot\nabla\phi_o \equiv \int_v d^3r\ \nabla(\phi_o \delta D\cdot\nabla\phi_o) - \int_v d^3r\ (\nabla\phi_o)(\delta D\cdot\nabla\phi_o) \qquad (4.1.2)$$

will yield

$$\int_v d^3r\ \phi_o \nabla\delta D\cdot\nabla\phi_o = -\int_v d^3r\ (\nabla\phi_o)(\delta D\cdot\nabla\phi_o)\ . \qquad (4.1.3)$$

Substitution of this result into Eq. (4.1.1) will yield

$$\rho = \frac{\int_v d^3r[\delta(\nu\Sigma_f - \Sigma_a)\phi_o^2 - \delta D(\nabla\phi_o)^2}{\int_v d^3r \nu\Sigma_{fo}\phi_o^2}\ . \qquad (4.1.4)$$

Some assumptions can be made to simplify the calculations using Eq. (4.1.4). The integral in the denominator does not vary with the perturbation, therefore it can be assumed to be a constant, K. If the perturbation is assumed to be uniformly distributed over the volume of the change, then the changes in the macroscopic cross-sections and the diffusion coefficient are position independent within the perturbed region. Since the integral in the numerator has a non-zero value only over the volume of the change and since the changes in the macroscopic cross-sections and the diffusion coefficient are position independent in that region, the reactivity equation (4.1.4) becomes

$$\rho = \frac{\delta(\nu\Sigma_f - \Sigma_a)\int_v d^3r\phi_o^2 - \delta D\int_v d^3r\ (\nabla\phi_o)^2}{K}\ . \qquad (4.1.5)$$

By introducing the quantities

$$A^2 \equiv \int_v d^3r\ \phi_o^2/K \qquad (4.1.6)$$

and

$$B^2 \equiv \int_v d^3r \ (\nabla\phi_o)^2/K \quad , \tag{4.1.7}$$

the reactivity may be rewritten as

$$\rho = \delta(\nu\Sigma_f - \Sigma_a)A^2 - \delta DB^2 \ . \tag{4.1.8}$$

The quantities A^2 and B^2 can be determined one of two ways. The neutron flux distribution can be obtained from steady-state reactor theory or from experimental measurements, then the quantities A^2 and B^2 can be evaluated using Eqs. (4.1.6) and (4.1.7). However, this method is quite sensitive to any errors in the flux shape because the quantities for the magnitude and the slope of the flux distribution are squared in Eqs. (4.1.6) and (4.1.7).

A second method and the one used in this analysis was to use the results of two reactivity measurements and Eq. (4.1.8) to give two simultaneous equations. These two simultaneous equations for each position are sufficient to determine the values of A^2 and B^2 for that position. As a check on these values of A^2 and B^2 and hence as a check on the validity of using the one-speed diffusion, a third reactivity quantity is calculated and compared with its measured value. In this work the aluminum to water and graphite to water measurements were used to determine A^2 and B^2 then the fuel to water reactivity was calculated and compared with its measured value.

All cross-sections (Σ_a and $\nu\Sigma_f$) and diffusion coefficients (D) used in these calculations are the "homogenized" form of the thermal one-group constants obtained from Table 9.1 of reference 12. To obtain the "homogenized" or averaged nuclear properties of a region, the nuclear properties of each constituent is weighted proportionally to its volume fraction[8], namely

$$D = \sum_j 1/(v_j/D_j) \qquad (4.1.9)$$

and

$$\Sigma_i = \sum_j v_j \Sigma_{ij} \ , \quad i = a,f \ . \qquad (4.1.10)$$

The summation over j is a summation over all constituents included in the homogenization process. The quantities v_j and D_j refer respectively to the volume fraction and the diffusion coefficient of each material. The j-th constituent macroscopic fission and absorption cross-sections are given respectively by Σ_f and Σ_a. These homogenized values for the quantities $\nu\Sigma_f - \Sigma_a$ and D for the aluminum, graphite, water and for each of the fuel elements in positions B- 2, C- 2, C- 7, C- 8, C- 9, D-13, E-17, and F- 8 are listed in Table VI. The calculated and measured fuel to water reactivity differences are also listed in Table VI.

Table VI. Measured and Calculated Fuel to Water Reactivity Differences

Element	$\nu\Sigma_f - \Sigma_a$	D	Calculated Reactivity	Measured Reactivity
Fuel (B- 2)	.157 cm^{-}	.340 cm	$5.01	$1.84
Fuel (C- 2)	.154	.348	3.74	1.47
Fuel (C- 7)	.154	.349	3.04	1.24
Fuel (C- 8)	.154	.349	2.50	1.10
Fuel (C- 9)	.153	.350	1.78	.98
Fuel (D-13)	.151	.355	1.12	.74
Fuel (E-17)	.148	.361	1.73	.64
Fuel (F- 8)	.135	.395	.75	.39
Aluminum	-.014	3.456		
Graphite	-.001	.977		
Water	-.022	.164		

The results tabulated in Table VI indicate the one-speed diffusion model of first-order perturbation theory is a very poor method to predict reactivities for the Triga reactor. There are probably two large sources of error for this method. The first possible source of error is the perturbations were not small thus the constant shape approximation as used in the development of first-order perturbation theory form of reactivity was not valid. Another possible source of error is the energy dependence of the neutrons is neglected in one-speed diffusion theory and the Triga reactor contains a significant neutron flux in the higher energy ranges.[13]

4.2 The Adiabatic Approximation Form of Calculating Reactivity

A second approach to finding a means of predicting reactivity was to use the adiabatic approximation to two-group diffusion theory. This method was chosen because it provided some representation of the energy dependence of the neutrons and also because it allows the neutron flux distribution to vary with the perturbation.

To simplify the use of this two-group adiabatic approximation method of calculating reactivity for the Triga reactor, a model of the Triga was developed. This model approximates some of the physical characteristics of the reactor. The first approximation of the Triga assumes the reactor consists of seven concentric homogeneous regions or rings. The Triga reactor is made up of cylindrical fuel elements and water arranged in concentric rings to form a cylindrical core (see Figure 1). This approximation simplifies the calculations by removing all angular dependence in the material properties and in the neutron flux distribution. The seven concentric rings were considered to have the following dimensions.

1. The A-ring or central thimble was considered to extend from the center (r = 0) to a radius of 2.19 centimeters.
2. The B-ring extended from a radius of 2.19 centimeters to a radius of 6.11 centimeters.
3. The C-ring extended from 6.11 to 10.08 centimeters.
4. The D-ring was the region from 10.08 to 14.05 centimeters.
5. The E-ring was from 14.05 to 18.02 centimeters.
6. The F-ring extended from 18.02 to 22.86 centimeters.
7. The graphite reflector included everything beyond 22.86 centimeters.

Except for the graphite reflector, these dimensions are simply the physical dimensions of the Triga reactor core. The graphite reflector in the model of the Triga is considered to have an outer boundary that approaches infinity. To homogenize each of the seven regions, the average nuclear properties of each region are obtained by using Eqs. (4.1.9) and (4.1.10).

The second approximation used in the establishment of a geometrical model of the reactor assumes the reactor does not have a reflector in the axial dimension. Even though the Triga reactor contains a graphite reflector in the axial as well as the radial dimensions, this assumption was needed due to the analytical difficulty involved in determining the neutron flux distribution in a totally reflected reactor. Partial compensation for the lack of the axial reflector was provided by using an extrapolated dimension, H, in the axial direction.

If we assume that the perturbation is a step change at t = 0 and that the material properties remain constant following the perturbation, then the equations describing the reactor system are time independent for $t \neq 0$. To calculate the reactivity effect of the perturbation all we need to know is the ϕ_o and k_{eff} of Section 2.4.3 before and after the perturbation. Therefore,

consider the two-group time independent version of Eq. (2.4.16) for the description of a homogeneous region in a reactor system, namely

$$D_1\nabla^2\phi_1(\underline{r}) - \Sigma_{a1}\phi_1(\underline{r}) - \Sigma_{s1}\phi_1(\underline{r}) = \frac{-1}{k_{eff}}[\nu_1\Sigma_{f1}\phi_1(\underline{r}) + \nu_2\Sigma_{f2}\phi_2(\underline{r})] \quad (4.2.1)$$

and

$$D_2\nabla^2\phi_2(\underline{r}) - \Sigma_{a2}\phi_2(\underline{r}) = -\Sigma_{s1}\phi_1(\underline{r}) \ . \quad (4.2.2)$$

The subscripts "1" and "2" refer respectively to the fast and thermal energy groups. In equations (4.2.1) and (4.2.2) all fission neutrons both prompt and delayed are considered to be emitted in the fast energy group. The term $\Sigma_{s1}\phi_1$ is a pseudo-scattering rate describing the effective rate at which neutrons are down-scattered from the fast to the thermal energy group.

The solutions to Eqs. (4.2.1) and (4.2.2) other than the trivial zero solutions may be found by requiring the neutron fluxes to satisfy the conditions

$$\nabla^2\phi_i(\underline{r}) + \gamma^2\phi_i(\underline{r}) = 0 \ , \quad i = 1,2 \ , \quad (4.2.3)$$

where γ^2 is a constant to be determined.[4,7,8] Substitution of this requirement into Eqs. (4.2.1) and (4.2.2) will give

$$D_1\gamma^2\phi_1 + \Sigma_{s1}\phi_1 + \Sigma_{a1}\phi_1 - \frac{\nu_1\Sigma_{f1}\phi_1}{k_{eff}} = \frac{\nu_2\Sigma_{f2}\phi_2}{k_{eff}} \quad (4.2.4)$$

and

$$D_2\gamma^2\phi_2 + \Sigma_{a2}\phi_2 = \Sigma_{s1}\phi_1 \ . \quad (4.2.5)$$

By introducing the quantities

$$L_1^2 \equiv (\Sigma_{a1} + \Sigma_{s1} - \frac{\nu_1\Sigma_{f1}}{k_{eff}})/D_1 \quad (4.2.6)$$

and

$$L_2^2 \equiv \Sigma_{a2}/D_2 \ , \tag{4.2.7}$$

the ratio between the fluxes in Eqs. (4.2.4) and (4.2.5) can be expressed as

$$\frac{\phi_1}{\phi_2} = \frac{\nu_2\Sigma_{f2}/(k_{eff}D_1)}{\gamma^2 + L_1^2} = \frac{\gamma^2 + L_2^2}{\Sigma_{s1}/D_1} \ . \tag{4.2.8}$$

From this result, the parameter γ^2 must satisfy

$$\gamma^4 + \gamma^2(L_1^2 + L_2^2) + L_1L_2 - \frac{\nu_2\Sigma_{f2}\Sigma_{s1}}{k_{eff}D_1D_2} = 0 \ . \tag{4.2.9}$$

The solutions of this equation for γ^2 are

$$\gamma_a^2 = 0.5\{-(L_1^2 + L_2^2) + \left[(L_1^2 - L_2^2)^2 + \frac{4\nu_2\Sigma_{f2}\Sigma_{s1}}{k_{eff}D_1D_2}\right]^{0.5}\} \tag{4.2.10}$$

and

$$\gamma_b^2 = 0.5\{-(L_1^2 + L_2^2) - \left[(L_1^2 - L_2^2)^2 + \frac{4\nu_2\Sigma_{f2}\Sigma_{s1}}{k_{eff}D_1D_2}\right]^{0.5}\} < 0 \ . \tag{4.2.11}$$

Notice, γ_a^2 and γ_b^2 are functions of material properties only.

Now consider the Triga reactor model and let the origin of the cylindrical coordinate system be in the center of this model. For the present cylindrical reactor, unreflected in the axial direction, the axial variation of the flux is[4,7,8]

$$\phi_i(r,z) \propto \mathrm{Cos}\left(\frac{\pi z}{H}\right) \ , \quad i = 1,2 \ . \tag{4.2.12}$$

Thus we look for solutions to Eq. (4.2.3) of the form

$$\phi_i(r,z) = C_iR(r)\mathrm{Cos}\left(\frac{\pi z}{H}\right) \ , \quad i = 1,2 \ . \tag{4.2.13}$$

Substitution of Eq. (4.2.13) into Eq. (4.2.3) yields

$$\nabla^2[C_i R(r)\mathrm{Cos}\left(\frac{\pi z}{H}\right)] + \gamma_j^2 C_i R(r)\mathrm{Cos}\left(\frac{\pi z}{H}\right) = 0 \ , \quad i = 1,2, \quad j = a,b, \qquad (4.2.14)$$

or equivalently

$$\frac{d^2R(r)}{dr^2} + \frac{dR(r)}{r\,dr} + [\gamma_j^2 - \left(\frac{\pi}{H}\right)^2]R(r) = 0 \ , \quad j = a,b. \qquad (4.2.15)$$

By introducing a new quantity, α_j^2, as

$$\alpha_j^2 \equiv [\gamma_j^2 - \left(\frac{\pi}{H}\right)^2] \ , \quad j = a,b, \qquad (4.2.16)$$

Eq. (4.2.15) becomes

$$\frac{d^2R(r)}{dr^2} + \frac{dR(r)}{r\,dr} + \alpha_j^2 R(r) = 0 \ , \quad j = a,b. \qquad (4.2.17)$$

The quantity α_b^2 will always remain negative and the quantity α_a^2 may be either positive, negative, or zero.

The solution to Eq. (4.2.17) (with $\alpha_j \equiv |[\alpha_j^2]^{0.5}|$) is

$$R(r) = AJ_o(\alpha_j r) + BY_o(\alpha_j r) \ , \quad j = a,b, \qquad (4.2.18)$$

where J_o and Y_o are the zeroth order Bessel functions of the first and second kind respectively. The quantities A and B are constants which need to be determined. If the quantities α_a^2 and α_b^2 are negative then the solution given in Eq. (4.2.18) is

$$R(r) = AI_o(\alpha_j r) + BK_o(\alpha_j r) \ , \quad j = a,b. \qquad (4.2.19)$$

Thus the total radial neutron flux is given by

$$R(r) = A_a J_o(\alpha_a r) + B_a Y_o(\alpha_a r) + A_b I_o(\alpha_b r) + B_b K_o(\alpha_b r) \qquad (4.2.20)$$

if α_a^2 is positive and

$$R(r) = A_a I_o(\alpha_a r) + B_a K_o(\alpha_a r) + A_b I_o(\alpha_b r) + B_b K_o(\alpha_b r) \qquad (4.2.21)$$

if α_a^2 is negative. If these are the thermal radial neutron flux distributions (i.e. $C_2 = 1$) then the fast radial neutron flux distributions are given by

$$R(r) = C_{1a}A_a J_o(\alpha_a r) + C_{1a}B_a Y_o(\alpha_a r) + C_{1b}A_b I_o(\alpha_b r) + C_{1b}B_b K_o(\alpha_b r) \quad (4.2.22)$$

and

$$R(r) = C_{1a}A_a I_o(\alpha_a r) + C_{1a}B_a K_o(\alpha_a r) + C_{1b}A_b I_o(\alpha_b r) + C_{1b}B_b K_o(\alpha_b r) \quad (4.2.23)$$

respectively, where C_{1a} and C_{1b} are determined from Eq. (4.2.8) as

$$C_{ij} = \frac{\gamma_j^2 + L_2^2}{\Sigma_{s1}/D_2}, \quad j = a,b. \quad (4.2.24)$$

Notice, C_{1a} and C_{1b} are functions of material properties only.

The above solutions are for a multiplying medium, to obtain the radial neutron flux distributions for a non-multiplying medium (Σ_{f1} and $\Sigma_{f2} = 0$), a special case of the above solutions is employed. For this case both Σ_{f1} and Σ_{f2} are set equal to zero. The values of γ_a^2 and γ_b^2 become $-L_1^2$ and $-L_2^2$ respectively, where L_1^2 is now equal to $(\Sigma_{a1} + \Sigma_{s1})/D_1$. The radial fast and thermal neutron flux distributions are given by Eqs. (4.2.23) and (4.2.21) respectively, where

$$\alpha_a = [L_1^2 + \left(\frac{\pi}{H}\right)^2]^{0.5} \quad (4.2.25)$$

and

$$\alpha_b = [L_2^2 + \left(\frac{\pi}{H}\right)^2]^{0.5} . \quad (4.2.26)$$

The constants C_{1a} and C_{1b} are

$$C_{1a} = \frac{\gamma_a^2 + L_2^2}{\Sigma_{s1}/D_2} = \frac{L_2^2 - L_1^2}{\Sigma_{s1}/D_2} \quad (4.2.27)$$

and

$$C_{1b} = \frac{\gamma_b^2 + L_2^2}{\Sigma_{s1}/D_2} = 0 \ . \qquad (4.2.28)$$

The radial fast and thermal neutron flux distributions are thus given as

$$R(r) = C_{1a}A_a I_o(\alpha_a r) + C_{1a}B_a K_o(\alpha_a r) \qquad (4.2.29)$$

and

$$R(r) = A_a I_o(\alpha_a r) + B_a K_o(\alpha_a r) + A_b I_o(\alpha_b r) + B_b K_o(\alpha_b r) \qquad (4.2.30)$$

respectively.

There are four coefficients, namely A_a, B_a, A_b, and B_b, that need to be determined for each of the seven homogeneous regions of the Triga model (i.e., 28 unknown constants). These unknown constants are determined by applying the boundary conditions to the neutron flux solutions of each region. Four of these unknown constants may be determined by relying upon the physical characteristics of a neutron flux, namely the flux must remain real, non-negative, and bounded. Since the Bessel functions Y_o and K_o become unbounded for small arguments, the coefficients of the terms containing these functions must be equal to zero for the region that contains the origin ($r = 0$). Similarly, since the Bessel function I_o becomes unbounded for large arguments, the coefficients of the terms containing the Bessel function I_o must be equal to zero for the region whose outer limits are at large distances ($r \to \infty$), i.e., for the reflector in the present model of the Triga. Relationships between the twenty-four remaining constants are obtained by equating the fast and thermal fluxes and the fast and thermal neutron currents at each of the six interfaces. If we denote the fluxes of region j by ϕ_i^j, $i = 1, 2$, and R_j as the interface between region j and region $j + 1$, the above conditions become

$$(\phi_i^j - \phi_i^{j+1})\big|_{r = R_j} = 0 \ , \quad i = 1,2 \tag{4.2.31}$$

and

$$(D_i^j \nabla\phi_i^j - D_i^{j+1} \nabla\phi_i^{j+1})\big|_{r = R_j} = 0 \ , \quad i = 1,2 \ . \tag{4.2.32}$$

Generally there will only be the trivial zero solution to these twenty-four simultaneous equations for A_a, B_a, A_b, and B_b. Only when the system is critical will there be a non-trivial solution. For this special case the determinant of the matrix form of Eqs. (4.2.31) and (4.2.32) must be equal to zero. This determinant which involves only material and geometric properties is called the critical determinant. This critical determinant will not equal zero unless the material properties and the geometry factors both conform to produce a critical reactor. In other words, if the reactor is not critical either the material properties or the geometry or both must be changed until the determinant equals zero, then this value of k_{eff} is used in Eq. (2.4.30), $\rho = (k_{eff} - 1)/k_{eff}$, to calculate the reactivity.

Since the two-group adiabatic approximation method of calculating reactivity contains operations that are quite time consuming, a computer program was written. This computer program, as described in the Appendix, sets up the critical determinant, adjusts the value of k_{eff}, and calculates the reactivity for the Triga reactor model, or for any similar multiregion, radially reflected, cylindrical reactor.

4.3 Results for the Two-Group Adiabatic Approximation

With this Triga reactor model, reactivity calculations could be attempted. The first step in these reactivity calculations is the establishment of a basic loading of this reactor model. This basic loading is

used as a common comparator and as a starting point for all changes in core configuration used in the reactivity predictions. This basic loading of the reactor model (Figure 3) consists of a fuel element in every position of the B, C, D, and E-rings, water in the central thimble, and graphite reflector elements in the entire F-ring.

During any change in this basic core configuration it is assumed that the change is made over the entire length of every position within that ring. This assumption maintains the angular and axial independence of the neutron flux distribution. The reactivity calculations then give the reactivity for a change in the entire ring. In order to determine the reactivity of a change in a single position within a ring, the reactivities are assumed to be additive and the worth of each single element is assumed to be equal to the worth of the entire ring divided by the total number of elements within that ring.

To illustrate this point, suppose we want to determine the aluminum to water reactivity difference for any one of the positions in the C-ring, say C- 9. First the values for k_{eff} and reactivity are determined for the basic reactor system with the C-ring completely filled with water. Next the same quantities are determined for all twelve positions in the C-ring filled with aluminum. The aluminum to water worth for the entire ring is then the difference in the calculated reactivities ($\rho_a - \rho_w$). For a single position in the C-ring the aluminum to water worth is equal to one twelfth that of the entire ring. A similar procedure may be used to obtain the aluminum to water, fuel to water, and graphite to water reactivity differences for the central thimble, B, C, D, E, and F-ring positions. The nuclear properties (obtained from reference 14) and the volume fractions of each constituent used in these reactivity calculations are listed in Table VII. The calculated reactivity differences are shown in Table VIII along with the measured reactivities.

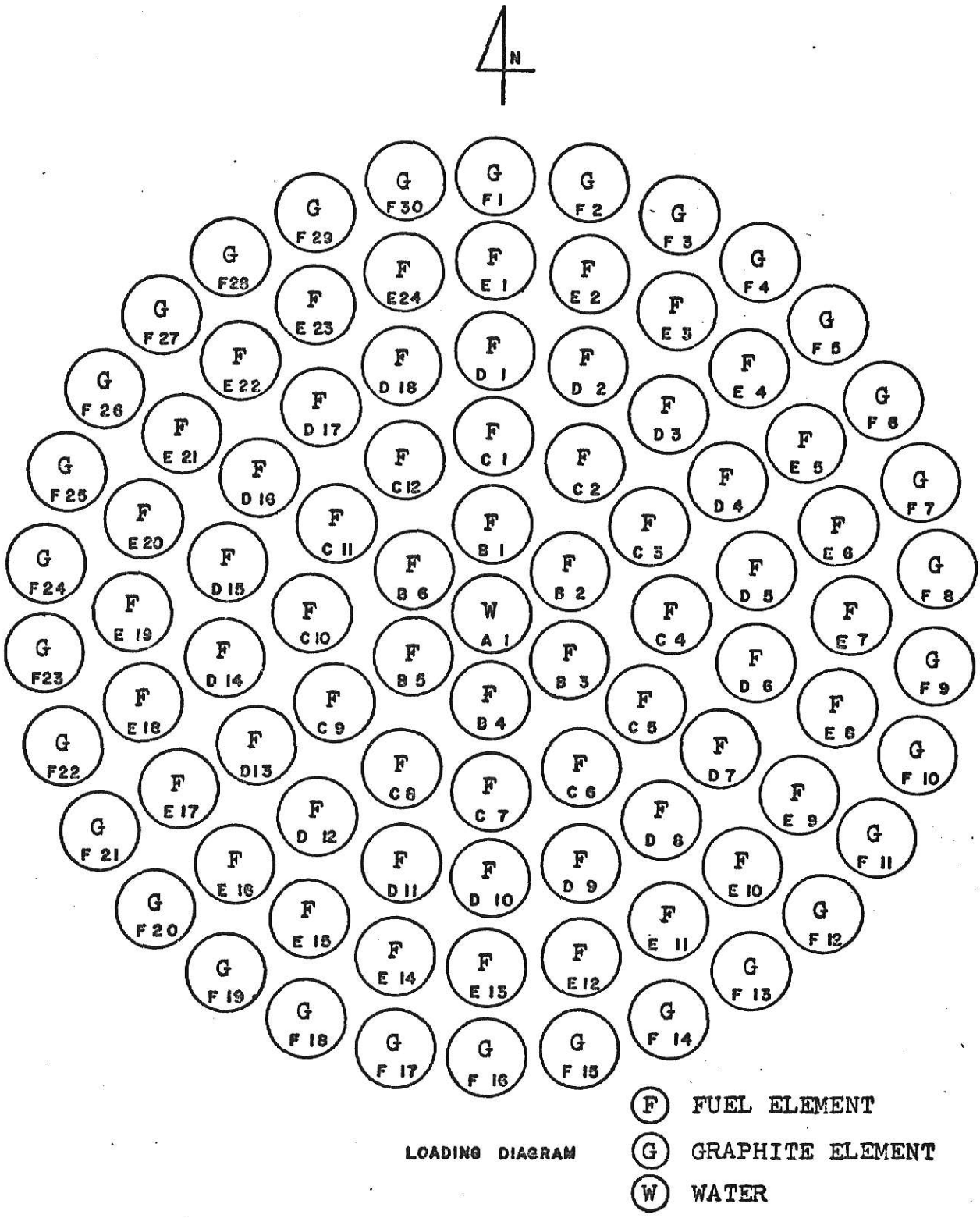

Figure 3. Basic loading of Triga reactor model.

An attempt was made to determine the worth of a single element by making the change in a single position, then homogenizing everything in that ring. The results from this single element attempt are also shown in Table VIII. All reactivity values shown in Table VIII are listed in units of dollars. To obtain the dollar value of reactivity the $\Delta k/k$ calculated value is divided by $\overline{\beta}$ (.0079)[15]. In these reactivity calculations the value of the extrapolated height, H, was selected to be 44.9 centimeters. This particular value was chosen because it best fit the axial flux distribution of the Triga reactor, which was experimentally measured by Bouchey[16]. It has been observed that the reactivity predictions are somewhat sensitive to the value selected for H. For example, a B-ring fuel to water reactivity prediction is \$1.53 for an H of 76.2 centimeters and for an H of 36.34 centimeters the reactivity prediction was \$2.08.

A comparison of the measured and calculated values for the reactivity differences (Table VIII) indicates that the method in which one element is changed is exceedingly poor. These poor results are probably due to the fact that this technique does not allow the perturbation and hence the neutron flux distribution to change locally to the extent of the real case. When the change is made uniformly in the entire ring then each local change is nearer that of the real case. The results from this second method of using the two-group adiabatic approximation indicate that this method slightly over predicts for the regions near the center of the core and slightly under predicts for the outer regions. This slight over and under prediction could be caused by any number of reasons. First in the Triga reactor model there are no control rods, however, in the experiments the shim rod was always partially in the core. The location of the shim rod would tend to depress the neutron flux near the reactor center and would tend to increase it near the outer edges.

This is probably the major contributor to the slight differences. Other possible sources of error include the fact that compensation for a reflector cannot be complete by using an extrapolated boundary. The energy spectrum of the reactor neutron flux may not be the same as the spectrum used to determine the two-group constants. Another source of error is in the comparison of the measured and predicted reactivities. The measured reactivity differences used the asymptotic period approximation method of calculating reactivity, where the predicted values used the adiabatic approximation method of calculating reactivity. Also both methods assumed reactivity to be additive, a property which is not valid in theory. The computer program neglects any angular dependence in the perturbations and in the neutron flux distributions, however, the reactivity measurements indicate that there is a significant variation within each ring. A more sophisticated calculational scheme would obviously have to allow for such noncylindrically symmetric effects. With all of these possible sources of error, the computer program seemed to be able to predict the reactivity differences fairly well.

Table VII. Two-Group Nuclear Properties

Element	Volume Fraction*	D_1^a (cm)	D_2^a (cm)	Σ_{a1}^a (cm^{-1})	Σ_{a2}^a (cm^{-1})	Σ_{s1}^a (cm^{-1})	$\nu\Sigma_{f1}^a$ (cm^{-1})	$\nu\Sigma_{f2}^a$ (cm^{-1})
Fuel (C. Th.)	.7288	9.750[-1]	3.344[-1]	1.592[-3]	1.642[-1]	2.364[-2]	8.194[-4]	2.974[-1]
Fuel (B-ring)	.6416	9.519[-1]	3.264[-1]	1.630[-3]	1.682[-1]	2.423[-2]	8.397[-4]	3.048[-1]
Fuel (C-ring)	.6514	9.600[-1]	3.292[-1]	1.617[-3]	1.668[-1]	2.402[-2]	8.324[-4]	4.182[-1]
Fuel (D-ring)	.6550	9.750[-1]	3.344[-1]	1.592[-3]	1.642[-1]	2.364[-2]	8.193[-4]	2.974[-1]
Fuel (E-ring)	.6566	9.885[-1]	3.391[-1]	1.570[-3]	1.619[-1]	2.331[-2]	8.079[-4]	2.932[-1]
Fuel (F-ring)	.5286	9.750[-1]	3.344[-1]	1.592[-3]	1.642[-1]	2.364[-2]	8.194[-4]	2.974[-1]
Reflector	1.0000	1.192[0]	9.404[-1]	2.147[-5]	2.279[-4]	2.821[-3]		
Aluminum		3.858[0]	3.366[0]	4.613[-4]	1.230[-2]	6.081[-5]		
Graphite		1.262[0]	9.979[-1]	5.664[-5]	1.193[-3]	2.600[-3]		
Water		1.423[0]	1.469[-1]	1.188[-3]	1.954[-2]	4.288[-2]		

* The remaining volume fraction of each ring is comprised of water.

a Each number in this column is multiplied by ten which has been raised to the power within the brackets (i.e., 9.60[-1] = .960).

Table VIII. Measured and Calculated Reactivity Differences

Position	Reactivity	Measured Values ($)	Prediction by Changing Entire Ring ($)	Prediction by Changing Single Element ($)
B- 2	Aluminum-Water	-.05	-.01	
	Graphite-Water	.51	.56	
	Fuel-Water	1.84	1.86	.56
B-min, ave, max	Fuel-Water	1.40, 1.50, 1.84	1.86	.56
C- 2, 7, 8, 9	Aluminum-Water	-.02, -.13, -.25, -.34	-.42	
	Graphite-Water	.40, .30, .21, .11	.21	
	Fuel-Water	1.47, 1.24, 1.10, .97	1.23	.28
C-min, ave, max	Fuel-Water	.91, 1.13, 1.47	1.23	.28
D-13	Aluminum-Water	-.31	-.41	
	Graphite-Water	.05	.14	
	Fuel-Water	.74	.87	.19
D-min, ave, max	Fuel-Water	.63, .81, 1.05	.87	.19
E-17	Aluminum-Water	-.10	-.15	
	Graphite-Water	.17	.18	
	Fuel-Water	.64	.68	.25
E-min, ave, max	Fuel-Water	.44, .66, .86	.68	.25
F- 8	Aluminum-Water	-.05	-.04	
	Graphite-Water	.08	.09	
	Fuel-Water	.36	.36	.16
F-min, ave, max	Fuel-Water	.36, .50, .60	.36	.16
Central Thimble	Aluminum-Water	-.73	-.78	
	Graphite-Water	-.19	-.06	
	Fuel-Water	not measured	.92	.92

5.0 SUMMARY

In this work the point reactor kinetic equations have been derived from the general energy-dependent diffusion kinetic equations. From this development, by introducing the concept of neutron importance, reactivity was defined exactly in terms of the solution to the original diffusion equations. The expression for reactivity was further developed to show how it could be approximately calculated without first finding the complete diffusion equation solution. Three different approximations were applied to the exact definition of reactivity to obtain three different simplified calculational expressions for reactivity. These three calculational expressions for reactivity are: an expression similar to that obtained from first-order perturbation theory, an expression known as the "inhour" equation, and an expression involving only the effective multiplication factor. It was noted that reactivity was not truly additive except in the case of the first-order perturbation theory approximation.

The multigroup diffusion kinetic equations were stated and it was shown how the point reactor kinetic equations could also be derived from these multigroup equations. For the multigroup derivations, various operators, defined previously for the energy-dependent kinetic equations, were described in matrix and vector notation so that prior derivations and arguments could be reapplied. The result of these multigroup derivations was the multigroup version of reactivity and its three calculational expressions.

Two different calculational expressions for reactivity were employed in two different attempts at predicting the reactivity for the Kansas State University Triga Mark II nuclear reactor. The first attempt at predicting reactivity utilized the one-speed diffusion model of the first-order

perturbation theory expression. The results from this first attempt, as shown in Table VI, were completely unsatisfactory. The second attempt employed the two-group version of the expression involving the effective multiplication factor. For this second attempt at calculating reactivities a simplified computer program was written. The program used a homogenized and cylindrically symmetric model of the Triga reactor to calculate the effective multiplication factor for each core configuration. The difference in the reactivities for different core configurations gave the reactivity difference for that configuration change. The aluminum to water, fuel to water, and graphite to water reactivity differences for single fuel element positions were calculated and tabulated in Table VIII.

An experiment was performed to determine how well the computer program could predict reactivity changes for the KSUTMII reactor. The experiment measured the fuel to water reactivity difference for all sixty-three fuel element positions and the aluminum to water and graphite to water reactivity differences for a few selected positions. These measurements, as listed in Tables I and V, indicate that the simplified program was able to predict the reactivity differences fairly accurately.

6.0 SUGGESTIONS FOR FURTHER STUDY

For further work in the area of predicting reactivity, serious consideration should be given to the elimination of some of the dubious assumptions made in the numerical analysis of this work. Two of the more questionable assumptions neglected the axial reflectors and the angular dependence of the reactor materials. To obtain better reactivity predictions some means of replacing these assumptions need to be utilized. A promising technique for obtaining better reactivity predictions would be the employment of some of the ideas advanced in the treatment of completely reflected reactors as described by Meghreblian and Holmes[7]. Another approach to obtain better reactivity predictions would be to describe the reactor system in at least two-dimensions, namely R-Θ. This second approach would describe the angular and radial material variations. Serious consideration could also be given to three dimensional solutions which would account for material variations in all three dimensions (R-Θ-Z). However, the application of these extensions will not be simple — for example, the calculations for a two or three dimensional analysis requires a fairly sophisticated computer code. It may be possible to combine some existing multi-dimensional, multigroup codes (e.g. Equipoise-3 or Exterminator-2)[17] with the totally reflected reactor analysis. Finally, consideration should also be given to using a better energy representation of the neutron flux by using three or more energy groups.

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THANK YOU

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9.0 APPENDIX

9.1 Description of the Reactivity Computer Program

This computer program for predicting reactivities will set up the critical determinant for the Triga reactor model or any similar multiregion, radially reflected, cylindrically symmetric reactor with axial and angular independent material properties. The critical determinant is then made to approach zero by changing the value for the factor k_{eff}. The value of k_{eff} that causes the determinant to vanish is determined by an iterative procedure. Mathematically, this is an eigenvalue problem and thus many eigenvalues, k_{eff}, will cause the determinant to vanish. The physical value for k_{eff} is the largest such eigenvalue. To find this physical or largest eigenvalue, a starting value for k_{eff} is determined such that it is larger than the actual k_{eff} for the reactor, then the value for k_{eff} is reduced until the determinant vanishes.

To illustrate the procedure used to determine the effective multiplication factor, k_{eff}, of the reactor, the critical determinant is set up for the reactor using the starting value for k_{eff} and then the value of the determinant is calculated. The critical determinant is again set up and solved but the value of k_{eff} has been reduced by a small fraction (.001). A linear interpolation or extrapolation technique using the two previous values for both k_{eff} and the determinant is used to obtain a better approximation of k_{eff} for the next iteration. The determinant is again set up and solved and a better value for k_{eff} is determined from the last two sets of values for k_{eff} and the determinant. This iteration technique continues until the value of the determinant is less than 10^{-8} and the value for reactivity is changing by less than 10^{-4} percent. After each iteration the values for k_{eff}, the reactivity (RHO) and the determinant (DET) are printed out. During the last iteration the latest values for

k_{eff}, the reactivity, and the last original and reduced determinants are printed out.

The method used to calculate the value of the critical determinant and to obtain the reduced determinant is similar to that of the Gauss-Jordan reduction[18]. This technique sweeps out (i.e., makes equal to zero) the elements of the original determinant that lie above or below the main diagonal except for the last column. Then all but one of the elements of the main diagonal are made equal to unity. The value of this non-unity diagonal element is simply the value of the determinant.

The last column of the reduced determinant then contains non-zero values, these values are proportional to the values for the unknown coefficients. The actual number of coefficients employed for a multiregion reactor is four times the number of regions minus one [4(n-1)]. Due to the fact that the equations described in the critical determinant are homogeneous, one of the coefficients must be arbitrary and the other coefficients are solved in terms of this arbitrary coefficient. The arbitrary coefficient may then be selected so that one of the fluxes will have, say, a certain maximum value or the total area beneath one of the flux curves is equal to some number or any other desired normalization condition. In this computer program the arbitrary coefficient is assumed to be equal to unity.

The program that was used in this work is listed on the following pages, together with a sample input and output.

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

9.2 Computer Program

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(36,36),CONT(10),ENTS(10),RMIN(10),RMAX(10),D1(10),
     1 D2(10),SIGA1(10),SIGA2(10),SIGS1(10),ANUF1(10),ANUF2(10)
C
C    PROGRAM SETS UP AND REDUCES TWO GROUP CRITICAL DETERMINANT FOR A
C    CYLINDRICAL REACTOR.  THE REACTOR IS ASSUMED TO BE BARE ON THE TOP
C    AND BOTTOM, OR MULTIREGION MAKE UP IN THE RADIAL DIMENSION, AND HAS
C    NO ANGULAR DEPENDENCE IN CONSTRUCTION.  PROGRAM ALSO CALCULATES THE
C    EFFECTIVE MULTIPLICATION CONSTANT(KEFF) AND THE REACTIVITY(RHO).
C
C    NREGNS = NUMBER OF REGIONS  (OUTSIDE REGION SHOULD BE NON-MULTIPLYING
C    LIKE WATER, GRAPHITE, AIR, ETC.)
C
C    H = EXTRAPOLATED SIZE IN VERTICAL DIMENSION
C
C    NREG = REGION NUMBER
C
C    RMIN = INSIDE RADIUS OF REGION
C
C    RMAX = OUTSIDE RADIUS OF REGION
C
C    CONT,ENTS = 8 LETTER DESCRIPTION OF REGION
C
C    D1 = FAST DIFFUSION COEFFICIENT OF REGION
C
C    D2 = THERMAL DIFFUSION COEFFICIENT OF REGION
C
C    SIGA1 = FAST MACROSCOPIC ABSORPTION CROSS-SECTION OF REGION
C
C    SIGA2 = THERMAL MACROSCOPIC ABSORPTION CROSS-SECTION OF REGION
C
C    SIGS1 = FAST MACROSCOPIC SCATTERING CROSS-SECTION(FROM FAST TO THERMAL)
C
C    ANUF1 = FAST MACROSCOPIC FISSION CROSS-SECTION TIMES NU(NEUTS/FISSION)
C
C    ANUF2 = THERMAL MACROSCOPIC FISSION CROSS-SECTION TIMES NU(NEUTS/FISSION)
C
  100 READ(1,1) NREGNS, H
    1 FORMAT (I2, D18.9)
      IF(NREGNS.EQ.0) CALL EXIT
      WRITE (3,12)
   12 FORMAT(1H1)
      AKEFF = 100.
      DO 65 I = 1,NREGNS
      READ(1,7) NREG, RMIN(NREG), RMAX(NREG), CONT(NREG), ENTS(NREG)
    7 FORMAT(I2,8X, 2D10.4, 2A4)
      J = NREG
      READ(1,2)D1(J),D2(J),SIGA1(J),SIGA2(J),SIGS1(J),ANUF1(J),ANUF2(J)
    2 FORMAT (8D10.4)
      ANUF = ANUF1(J) + ANUF2(J)
      IF(ANUF.EQ.0.)  GO TO 65
C
C    MAKES AN INITIAL GUESS FOR KEFF
C
      AK1 = D1(J)/(SIGA1(J)+SIGS1(J)-(ANUF1(J)/AKEFF))
```

```
      AL2 = D2(J)/SIGA2(J)
      BKEFF = ANUF2(J)*SIGS1(J)/(D1(J)*D2(J)*(.25*((((((3.141592654/H)
     1 **2)*2.C)+((1./AK1)+(1./AL2))**2)-(((1./AK1)+(1./AL2))**2))
     2 +1./(AK1*AL2)))
      IF(BKEFF.GT.AKEFF) GO TO 65
      AKEFF = BKEFF
   65 CONTINUE
      CKEFF = AKEFF
      ITER = 0
      R4 = .999
      DET4 = .CC01
      ABSDET = 1.0
      M = 4 *(NREGNS-1)
      N = 4 *(NREGNS-1)
  150 ITER = ITER + 1
C
C   LOOP CLEARS OUT MATRIX (ALL ELEMENTS EQUAL ZERO).
C
      DO 10 I = 1,M
      DO 10 J = 1,N
      A(I,J) = 0.0
   10 CONTINUE
      IF(ITER.EQ.2) AKEFF = CKEFF * 0.999
C
C   LOOP CALCULATES AND ARRANGES ELEMENTS OF CRITICAL DETERMINANT
C
      DO 85 I = 1,NREGNS
      K = ((I-2)*4)+1
      L = ((I-2)*4)+3
      ANUF = ANUF1(I) + ANUF2(I)
      IF(ANUF.EQ.0.)  GO TO 170
      AK1M2 = D1(I) / (SIGA1(I) + SIGS1(I) - ( ANUF1(I)/AKEFF))
      AL2M2 = D2(I) / SIGA2(I)
      SQR = ((((1./AK1M2)+(1./AL2M2))**2)-4.*((1./(AK1M2*AL2M2))-
     1 ((ANUF2(I)*SIGS1(I))/(D2(I)*D1(I)*AKEFF))))**.5
      AMU2 = (-((1./AK1M2)+(1./AL2M2))+SQR)/2.
      AMU = (AMU2 - ((3.141592654/H)**2))**.5
      ANU2 = (((1./AK1M2)+(1./AL2M2))+SQR)/2.
      ANU = (ANU2 + ((3.141592654/H)**2))**.5
      B = SIGS1(I)/(SIGA2(I)+(D2(I)*AMU2))
      C = SIGS1(I)/(SIGA2(I)-(D2(I)*ANU2))
      W = AMU * RMIN(I)
      X = ANU * RMIN(I)
      Y = AMU * RMAX(I)
      Z = ANU * RMAX(I)
      IF(I.EQ.1) GO TO 160
      CALL DBESI(X,0,BICNR1,IER)
      CALL DBESI(X,1,BI1NR1,IER)
      CALL DBESJ(W,0,BJCMR1,1.CD-08,IER)
      CALL DBESJ(W,1,BJ1MR1,1.CD-08,IER)
      CALL DBESK(X,0,BKCNR1,IER)
      CALL DBESK(X,1,BK1NR1,IER)
      CALL DBESY(W,0,BYCMR1,IER)
      CALL DBESY(W,1,BY1MR1,IER)
      CALL DBESK(Z,0,BKCNR2,IER)
      CALL DBESK(Z,1,BK1NR2,IER)
      CALL DBESY(Y,0,BYCMR2,IER)
      CALL DBESY(Y,1,BY1MR2,IER)
      A(K+0,L+0) = -BKCNR1
      A(K+0,L+1) = -BY0MR1
```

```
      A(K+0,L+2) = -BIONR1
      A(K+0,L+3) = -BJOMR1
      A(K+1,L+0) = -C * BKONR1
      A(K+1,L+1) = -B * BYOMR1
      A(K+1,L+2) = -C * BIONR1
      A(K+1,L+3) = -B * BJOMR1
      A(K+2,L+0) = D1(I) * ANU * BK1NR1
      A(K+2,L+1) = D1(I) * AMU * BY1MR1
      A(K+2,L+2) = -D1(I) * ANU * BI1NR1
      A(K+2,L+3) = D1(I) * AMU * BJ1MR1
      A(K+3,L+0) = C * D2(I) * ANU * BK1NR1
      A(K+3,L+1) = B * D2(I) * AMU * BY1MR1
      A(K+3,L+2) = -C * D2(I) * ANU * BI1NR1
      A(K+3,L+3) = B * D2(I) * AMU * BJ1MR1
      A(K+4,L+0) = BKONR2
      A(K+4,L+1) = BYOMR2
      A(K+5,L+0) = C * BKONR2
      A(K+5,L+1) = B* BYOMR2
      A(K+6,L+0) = -D1(I) * ANU * BK1NR2
      A(K+6,L+1) = -D1(I) * AMU * BY1MR2
      A(K+7,L+0) = -C * D2(I) * ANU * BK1NR2
      A(K+7,L+1) = -B * D2(I) * AMU * BY1MR2
  160 CONTINUE
      CALL DBESI(Z,0,BIONR2,IER)
      CALL DBESI(Z,1,BI1NR2,IER)
      CALL DBESJ(Y,0,BJOMR2,1.0D-08,IER)
      CALL DBESJ(Y,1,BJ1MR2,1.0D-08,IER)
      A(K+4,L+2) = BIONR2
      A(K+4,L+3) = BJOMR2
      A(K+5,L+2) = C * BIONR2
      A(K+5,L+3) = B * BJOMR2
      A(K+6,L+2) = D1(I) * ANU * BI1NR2
      A(K+6,L+3) = -D1(I) * AMU * BJ1MR2
      A(K+7,L+2) = C * D2(I) * ANU * BI1NR2
      A(K+7,L+3) = -B * D2(I) * AMU * BJ1MR2
      GO TO 85
  170 AK1N2 = (SIGA1(I)+SIGS1(I))/D1(I)
      AK2N2 = SIGA2(I)/D2(I)
      AK1N = (AK1N2 + (3.141592654/H)**2)**.5
      AK2N = (AK2N2 + (3.141592654/H)**2)**.5
      E = SIGS1(I) / (D2(I) * (AK2N2 - AK1N2))
      W = AK1N * RMIN(I)
      X = AK2N * RMIN(I)
      Y = AK1N * RMAX(I)
      Z = AK2N * RMAX(I)
      IF(I.EQ.1) GO TO 180
      CALL DBESK(W,0,BKOK11,IER)
      CALL DBESK(W,1,BK1K11,IER)
      CALL DBESK(X,0,BKOK21,IER)
      CALL DBESK(X,1,BK1K21,IER)
      A(K+0,L+0) = -BKOK11
      A(K+1,L+0) = -E * BKOK11
      A(K+1,L+1) = -BKOK21
      A(K+2,L+0) = D1(I) * AK1N * BK1K11
      A(K+3,L+0) = E * D2(I) * AK1N * BK1K11
      A(K+3,L+1) = D2(I) * AK2N * BK1K21
      IF(I.EQ.NREGNS) GO TO 85
      CALL DBESI(W,0,BIOK11,IER)
      CALL DBESI(W,1,BI1K11,IER)
      CALL DBESI(X,0,BIOK21,IER)
```

```
      CALL DBESI(X,1,BI1K21,IER)
      CALL DBESK(Y,0,BKCK12,IER)
      CALL DBESK(Y,1,BK1K12,IER)
      CALL DBESK(Z,0,BKCK22,IER)
      CALL DBESK(Z,1,BK1K22,IER)
      A(K+0,L+2) = -BIOK11
      A(K+1,L+2) = -E * BIOK11
      A(K+1,L+3) = -BIOK21
      A(K+2,L+2) = -D1(I) * AK1N * BI1K11
      A(K+3,L+2) = -E * D2(I) * AK1N * BI1K11
      A(K+3,L+3) = -D2(I) * AK2N * BI1K21
      A(K+4,L+0) = BKOK12
      A(K+5,L+0) = E * BKOK12
      A(K+5,L+1) = BKOK22
      A(K+6,L+0) = -D1(I) * AK1N * BK1K12
      A(K+7,L+0) = -E * D2(I) * AK1N * BK1K12
      A(K+7,L+1) = -D2(I) * AK2N * BK1K22
  180 CONTINUE
      CALL DBESI(Y,0,BICK12,IER)
      CALL DBESI(Y,1,BI1K12,IER)
      CALL DBESI(Z,0,BICK22,IER)
      CALL DBESI(Z,1,BI1K22,IER)
      A(K+4,L+2) = BIOK12
      A(K+5,L+2) = E * BIOK12
      A(K+5,L+3) = BIOK22
      A(K+6,L+2) = D1(I) * AK1N * BI1K12
      A(K+7,L+2) = E * D2(I) * AK1N * BI1K12
      A(K+7,L+3) = D2(I) * AK2N * BI1K22
   85 CONTINUE
      RHO = (AKEFF - 1.0)/AKEFF
      IF(ITER.LE.4) GO TO 200
      IF(ABSDET.GT.1.0D-08) GO TO 200
      CHGRHO = DABS((RHO1-RHO)/RHO)
      IF(CHGRHO.GT.1.0D-06) GO TO 200
      WRITE(3,8)
    8 FORMAT (////,'  CONTENTS      RMIN',8X,'RMAX',9X,'D1',10X,'D2',
     1 8X,'SIGA1',7X,'SIGA2',7X,'SIGS1       NUSIGF1      NUSIGF2',/)
      DO 75 I = 1,NREGNS
      WRITE(3,9) CONT(I),ENTS(I),RMIN(I),RMAX(I),D1(I),D2(I),SIGA1(I),
     1 SIGA2(I),SIGS1(I),ANUF1(I),ANUF2(I)
    9 FORMAT(2X,2A4, 9(1PD12.4))
   75 CONTINUE
      WRITE(3,5) AKEFF
    5 FORMAT(////,2X,'ORIGINAL MATRIX',50X,'KEFF = ',1PD15.9,/)
C
C    WRITES OUT ORIGINAL MATRIX
C
      DO 20 I = 1,M
      WRITE (3,3) (A(I,J), J=1,N)
    3 FORMAT (/,10(1PD12.4))
   20 CONTINUE
C
C    MAKES EVERY ELEMENT NON-ZERO
C
  200 DO 90 J = 1,N
      DO 95 I = 2,M
      A(1,J) = A(1,J) + A(I,J)
   95 CONTINUE
      DO 90 I = 2,M
      A(I,J) = A(I,J) + A(1,J)
```

```
   90 CONTINUE
      IF (N.LT.M) M = N
      MIN = N -1
      IF(N.GT.M) MIN = M
C
C    LOOP REDUCES MATRIX
C
      DO 30 I = 1,MIN
      IF (A(I,I).EQ.0.) GO TO 70
      GO TO 130
   70 DO 110 K = I,M
      IF (A(K,I).EQ.0.) GO TO 110
      DO 120 J = I,N
      A(I,J) = A(I,J) + A(K,J)
  120 CONTINUE
      GO TO 130
  110 CONTINUE
      GO TO 140
  130 DENOM = A(I,I)
      DO 40 J = I,N
      A(I,J) = A(I,J)/DENOM
   40 CONTINUE
      DO 50 K = 1,M
      IF (K.EQ.I) GO TO 50
      IF (A(K,I).EQ.0.) GO TO 50
      DEN = A(K,I)
      DO 60 L = 1,N
      A(K,L) = ((A(K,L)/DEN) - A(I,L)) * DEN
   60 CONTINUE
   50 CONTINUE
      DO 30 J = I,N
      A(M,J) = A(M,J) * DENOM
   30 CONTINUE
      IF(ITER.LE.4) GO TO 190
      IF(ABSDET.GT.1.0D-08) GO TO 190
      IF(CHGRHO.GT.1.0D-06) GO TO 190
  140 WRITE (3,6) RHO
    6 FORMAT(////,2X,'REDUCED MATRIX',51X,'RHO =',1PD17.9,' DK/K',/)
C
C    WRITES OUT REDUCED MATRIX
C
C    THE LOWER ELEMENT OF THE MAIN DIAGONAL OF THE REDUCED MATRIX IS THE VALUE OF
C    THE DETERMINANT.  EACH ROW IS THE REDUCED EQUATION SOLVING FOR THE
C    COEFFICIENTS OF THE ORIGINAL EQUATIONS.
C
      DO 80 I = 1,M
      WRITE (3,4) (A(I,J), J=1,N)
    4 FORMAT (/,10(1PD12.4))
   80 CONTINUE
      GO TO 100
  190 ABSDET = DABS(A(M,M))
      WRITE(3,11) AKEFF, RHO, A(M,M)
   11 FORMAT (2X,'KEFF =',1PD16.9,10X,'RHO = ',1PD16.9,' DK/K',10X,
     1 'DET =',1PD12.4)
C
C    USES LINEAR INTERPOLATION OF PAST TWO VALUES TO GET A BETTER APPROXIMATION
C    OF AKEFF TO BE USED IN NEXT CALCULATION
C
      DET3 = A(M,M)
      R3 = AKEFF
```

```
AKEFF = R3 - (DET3/((DET4-DET3)/(R4-R3)))
DET4 = DET3
R4 = R3
RHO1 = RHO
GO TO 150
END
```

```
      SUBROUTINE DBESI(X,N, BI,IER)
C                                                                       BESI 001
C     ..................................................................BESI 002
C                                                                       BESI 003
C        SUBROUTINE DBESI                                               BESI 004
C                                                                       BESI 005
C        PURPOSE                                                        BESI 006
C           COMPUTE THE I BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDERBESI 007
C                                                                       BESI 008
C        USAGE                                                          BESI 009
C           CALL DBESI(X,N,BI,IER)                                      BESI 010
C                                                                       BESI 011
C        DESCRIPTION OF PARAMETERS                                      BESI 012
C           X  -THE ARGUMENT OF THE I BESSEL FUNCTION DESIRED           BESI 013
C           N  -THE ORDER OF THE I BESSEL FUNCTION DESIRED              BESI 014
C           BI -THE RESULTANT I BESSEL FUNCTION                         BESI 015
C           IER-RESULTANT ERROR CODE WHERE                              BESI 016
C              IER=0 NO ERROR                                           BESI 017
C              IER=1 N IS NEGATIVE                                      BESI 018
C              IER=2 X IS NEGATIVE                                      BESI 019
C              IER=3 UNDERFLOW, BI .LT. 1.E-69, BI SET TO 0.0           BESI M01
C              IER=4 OVERFLOW, X .GT. 170 WHERE X .GT. N                BESI M02
C                                                                       BESI 020
C        REMARKS                                                        BESI 021
C           N AND X MUST BE .GE. ZERO                                   BESI M03
C                                                                       BESI 025
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  BESI 026
C           NONE                                                        BESI 027
C                                                                       BESI 028
C        METHOD                                                         BESI 029
C           COMPUTES I BESSEL FUNCTION USING SERIES OR ASYMPTOTIC       BESI M04
C           APPROXIMATION DEPENDING ON RANGE OF ARGUMENTS.              BESI M05
C                                                                       BESI 033
C     ..................................................................BESI 034
C                                                                       BESI 035
      IMPLICIT REAL*8(A-H,O-Z)                                          BESI 036
C                                                                       BESI 037
C     CHECK FOR ERRORS IN N AND X AND EXIT IF ANY ARE PRESENT           BESI 038
C                                                                       BESI 039
      IER=0                                                             BESI 040
      BI=1.0                                                            BESI 041
      IF(N)150,15,10                                                    BESI 042
   10 IF(X)160,20,20                                                    BESI 043
   15 IF(X)160,17,20                                                    BESI 044
   17 RETURN                                                            BESI 045
C                                                                       BESI 046
C     DEFINE TOLERANCE                                                  BESI 047
C                                                                       BESI 048
   20 TOL=1.E-6                                                         BESI 049
C                                                                       BESI 050
C     IF ARGUMENT GT 12 AND GT N, USE ASYMPTOTIC FORM                   BESI 051
C                                                                       BESI 052
      IF(X-12.)40,40,30                                                 BESI 053
   30 IF(X-DFLOAT(N))40,40,110                                          BESI 054
C                                                                       BESI 055
C     COMPUTE FIRST TERM OF SERIES AND SET INITIAL VALUE OF THE SUM     BESI 056
C                                                                       BESI 057
   40 XX=X/2.                                                           BESI 058
```

```
   50 TERM=1.C                                                          BESI M06
      IF(N)70,70,55                                                     BESI M07
   55 CO 60 I=1,N                                                       BESI M08
      FI=I                                                              BESI M09
      IF(DABS(TERM)-1.E-68)56,60,60                                      BESI M1
   56 IER=3                                                             BESI M11
      BI=0.C                                                            BESI M12
      RETURN                                                            BESI M13
   60 TERM=TERM*XX/FI                                                   BESI M14
   70 BI=TERM                                                           BESI M15
      XX=XX*XX                                                          BESI 066
C                                                                       BESI 067
C     COMPUTE TERMS, STOPPING WHEN ABS(TERM) LE ABS(SUM OF TERMS)       BESI M16
C     TIMES TOLERANCE                                                   BESI 069
C                                                                       BESI 070
      DO 90 K=1,1000                                                    BESI M17
      IF(DABS(TERM)-DABS(BI*TOL))100,100,80                              BESI 0
   80 FK=K*(N+K)                                                        BESI 073
      TERM=TERM*(XX/FK)                                                 BESI 074
   90 BI=BI+TERM                                                        BESI 075
C                                                                       BESI 076
C     RETURN BI AS ANSWER                                               BESI 077
C                                                                       BESI 078
  100 RETURN                                                            BESI 079
C                                                                       BESI 080
C     X GT 12 AND X GT N, SO USE ASYMPTOTIC APPROXIMATION               BESI 081
C                                                                       BESI 082
  110 FN=4*N*N                                                          BESI 083
      IF(X-170.0)115,111,111                                            BESI M18
  111 IER=4                                                             BESI M19
      RETURN                                                            BESI M20
  115 XX=1./(8.*X)                                                      BESI M21
      TERM=1.                                                           BESI 085
      BI=1.                                                             BESI 086
      DO 130 K=1,30                                                     BESI 087
      IF(DABS(TERM)-DABS(TOL*BI))140,140,120                             BESI 0
  120 FK=(2*K-1)**2                                                     BESI 089
      TERM=TERM*XX*(FK-FN)/DFLOAT(K)                                    BESI 090
  130 BI=BI+TERM                                                        BESI 091
C                                                                       BESI M22
C     SIGNIFICANCE LOST AFTER 30 TERMS, TRY SERIES                      BESI M23
C                                                                       BESI M24
      GO TO 40                                                          BESI M25
  140 PI=3.141592653                                                    BESI 092
      BI=BI*DEXP(X)/DSQRT(2.*PI*X)
      GO TO 100                                                         BESI 094
  150 IER=1                                                             BESI 095
      GO TO 100                                                         BESI 096
  160 IER=2                                                             BESI 097
      GO TO 100                                                         BESI 098
      END                                                               BESI 099
```

```
      SUBROUTINE DBESJ(X,N,BJ,D,IER)
C                                                                       BESJ 001
C     .................................................................BESJ 002
C                                                                       BESJ 003
C        SUBROUTINE DBESJ
C                                                                       BESJ 005
C        PURPOSE                                                        BESJ 006
C           COMPUTE THE J BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDERBESJ 007
C                                                                       BESJ 008
C        USAGE                                                          BESJ 009
C           CALL DBESJ(X,N,BJ,D,IER)
C                                                                       BESJ 011
C        DESCRIPTION OF PARAMETERS                                      BESJ 012
C           X  -THE ARGUMENT OF THE J BESSEL FUNCTION DESIRED           BESJ 013
C           N  -THE ORDER OF THE J BESSEL FUNCTION DESIRED              BESJ 014
C           BJ -THE RESULTANT J BESSEL FUNCTION                         BESJ 015
C           D  -REQUIRED ACCURACY                                       BESJ 016
C           IER-RESULTANT ERROR CODE WHERE                              BESJ 017
C              IER=0  NO ERROR                                          BESJ 018
C              IER=1  N IS NEGATIVE                                     BESJ 019
C              IER=2  X IS NEGATIVE OR ZERO                             BESJ 020
C              IER=3  REQUIRED ACCURACY NOT OBTAINED                    BESJ 021
C              IER=4  RANGE OF N COMPARED TO X NOT CORRECT (SEE REMARKS)BESJ 022
C                                                                       BESJ 023
C        REMARKS                                                        BESJ 024
C           N MUST BE GREATER THAN OR EQUAL TO ZERO, BUT IT MUST BE     BESJ 025
C           LESS THAN                                                   BESJ 026
C              20+10*X-X** 2/3   FOR X LESS THAN OR EQUAL TO 15         BESJ 027
C              90+X/2            FOR X GREATER THAN 15                  BESJ 028
C                                                                       BESJ 029
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  BESJ 030
C           NONE                                                        BESJ 031
C                                                                       BESJ 032
C        METHOD                                                         BESJ 033
C           RECURRENCE RELATION TECHNIQUE DESCRIBED BY H. GOLDSTEIN AND BESJ 034
C           R.M. THALER,-RECURRENCE TECHNIQUES FOR THE CALCULATION OF   BESJ 035
C           BESSEL FUNCTIONS-,M.T.A.C.,V.13,PP.102-108 AND I.A. STEGUN  BESJ 036
C           AND M. ABRAMOWITZ,-GENERATION OF BESSEL FUNCTIONS ON HIGH   BESJ 037
C           SPEED COMPUTERS-,M.T.A.C.,V.11,1957,PP.255-257              BESJ 038
C                                                                       BESJ 039
C     .................................................................BESJ 040
C                                                                       BESJ 041
      IMPLICIT REAL*8(A-H,O-Z)                                          BESJ 042
C                                                                       BESJ 043
      BJ=.0                                                             BESJ 044
      IF(N)10,20,20                                                     BESJ 045
   10 IER=1                                                             BESJ 046
      RETURN                                                            BESJ 047
   20 IF(N.NE.0 ) GO TO 220
      IF(X.NE.0.) GO TO 220
      BJ=1.
      RETURN
  220 IF(X)30,30,31
   30 IER=2                                                             BESJ 049
      RETURN                                                            BESJ 050
   31 IF(X-15.)32,32,34                                                 BESJ 051
   32 NTEST=20.+10.*X-X** 2/3                                           BESJ 052
      GO TO 36                                                          BESJ 053
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```
   34 NTEST=90.+X/2.                                                BESJ 054
   36 IF(N-NTEST)40,38,38                                           BESJ 055
   38 IER=4                                                         BESJ 056
      RETURN                                                        BESJ 057
   40 IER=0                                                         BESJ 058
      N1=N+1                                                        BESJ 059
      BPREV=.0                                                      BESJ 060
C                                                                   BESJ 061
C     COMPUTE STARTING VALUE OF M                                   BESJ 062
C                                                                   BESJ 063
      IF(X-5.)50,60,60                                              BESJ 064
   50 MA=X+6.                                                       BESJ 065
      GO TO 70                                                      BESJ 066
   60 MA=1.4*X+60./X                                                BESJ 067
   70 MB=N+ICINT(X)/4+2
      MZERO=MAXC(MA,MB)                                             BESJ 069
C                                                                   BESJ 070
C     SET UPPER LIMIT OF M                                          BESJ 071
C                                                                   BESJ 072
      MMAX=NTEST                                                    BESJ 073
  100 DO 190 M=MZERO,MMAX,3                                         BESJ 074
C                                                                   BESJ 075
C     SET F(M),F(M-1)                                               BESJ 076
C                                                                   BESJ 077
      FM1=1.0E-28                                                   BESJ 078
      FM=.0                                                         BESJ 079
      ALPHA=.0                                                      BESJ 080
      IF(M-(M/2)*2)120,110,120                                      BESJ 081
  110 JT=-1                                                         BESJ 082
      GO TO 130                                                     BESJ 083
  120 JT=1                                                          BESJ 084
  130 M2=M-2                                                        BESJ 085
      DO 160 K=1,M2                                                 BESJ 086
      MK=M-K                                                        BESJ 087
      BMK=2.*DFLOAT(MK)*FM1/X-FM                                    BESJ 088
      FM=FM1                                                        BESJ 089
      FM1=BMK                                                       BESJ 090
      IF(MK-N-1)150,140,150                                         BESJ 091
  140 BJ=BMK                                                        BESJ 092
  150 JT=-JT                                                        BESJ 093
      S=1+JT                                                        BESJ 094
  160 ALPHA=ALPHA+BMK*S                                             BESJ 095
      BMK=2.*FM1/X-FM                                               BESJ 096
      IF(N)180,170,180                                              BESJ 097
  170 BJ=BMK                                                        BESJ 098
  180 ALPHA=ALPHA+BMK                                               BESJ 099
      BJ=BJ/ALPHA                                                   BESJ 100
      IF(DABS(BJ-BPREV)-DABS(C*BJ))200,200,190
  190 BPREV=BJ                                                      BESJ 102
      IER=3                                                         BESJ 103
  200 RETURN                                                        BESJ 104
      END                                                           BESJ 105
```

```
      SUBROUTINE DBESK(X,N,BK,IER)
C                                                                       BESK 001
C     .....................................................................BESK 002
C                                                                       BESK 003
C        SUBROUTINE DBESK                                               BESK 004
C                                                                       BESK 005
C           COMPUTE THE K BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDERBESK 006
C                                                                       BESK 007
C        USAGE                                                          BESK 008
C           CALL DBESK(X,N,BK,IER)                                      BESK 009
C                                                                       BESK 010
C        DESCRIPTION OF PARAMETERS                                      BESK 011
C           X  -THE ARGUMENT OF THE K BESSEL FUNCTION DESIRED           BESK 012
C           N  -THE ORDER OF THE K BESSEL FUNCTION DESIRED              BESK 013
C           BK -THE RESULTANT K BESSEL FUNCTION                         BESK 014
C           IER-RESULTANT ERROR CODE WHERE                              BESK 015
C              IER=0  NO ERROR                                          BESK 016
C              IER=1  N IS NEGATIVE                                     BESK 017
C              IER=2  X IS ZERO OR NEGATIVE                             BESK 018
C              IER=3  X .GT. 170, MACHINE RANGE EXCEEDED                BESK M01
C              IER=4  BK .GT. 10**70                                    BESK M02
C                                                                       BESK 019
C        REMARKS                                                        BESK 020
C           N MUST BE GREATER THAN OR EQUAL TO ZERO                     BESK 021
C           X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE OF    BESK 022
C           THE MACHINE                                                 BESK 023
C                                                                       BESK 024
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  BESK 025
C           NONE                                                        BESK 026
C                                                                       BESK 027
C        METHOD                                                         BESK 028
C           COMPUTES ZERO ORDER AND FIRST ORDER BESSEL FUNCTIONS USING  BESK 029
C           SERIES APPROXIMATIONS AND THEN COMPUTES N TH ORDER FUNCTION BESK 030
C           USING RECURRENCE RELATION.                                  BESK 031
C           RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE  BESK 032
C           AS DESCRIBED BY A.J.M.HITCHCOCK,-POLYNOMIAL APPROXIMATIONS  BESK 033
C           TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED    BESK 034
C           FUNCTIONS-, M.T.A.C., V.11,1957,PP.86-88, AND G.N. WATSON,  BESK 035
C           -A TREATISE ON THE THEORY OF BESSEL FUNCTIONS-, CAMBRIDGE   BESK 036
C           UNIVERSITY PRESS, 1958, P. 62                               BESK 037
C                                                                       BESK 038
C     .....................................................................BESK 039
C                                                                       BESK 040
      IMPLICIT REAL*8(A-H,O-Z)                                          BESK 041
      DIMENSION T(12)                                                   BESK 042
      BK=.0                                                             BESK 043
      IF(N)10,11,11                                                     BESK 044
   10 IER=1                                                             BESK 045
      RETURN                                                            BESK 046
   11 IF(X)12,12,20                                                     BESK 047
   12 IER=2                                                             BESK 048
      RETURN                                                            BESK 049
   20 IF(X-170.0)22,22,21                                               BESK M03
   21 IER=3                                                             BESK M04
      RETURN                                                            BESK M05
   22 IER=0                                                             BESK M06
      IF(X-1.)36,36,25                                                  BESK 051
   25 A=DEXP(-X)
```

```
      B=1./X                                                          BESK 053
      C=DSQRT(B)
      T(1)=B                                                          BESK 055
      DO 26 L=2,12                                                    BESK 056
   26 T(L)=T(L-1)*B                                                   BESK 057
      IF(N-1)27,29,27                                                 BESK 058
C                                                                     BESK 059
C     COMPUTE KO USING POLYNOMIAL APPROXIMATION                       BESK 060
C                                                                     BESK 061
   27 GO=A*(1.25331414-.15666418*T(1)+.088111278*T(2)-.091390954*T(3) BESK 062
     2+.13445962*T(4)-.22S985C3*T(5)+.37924C97*T(6)-.52472773*T(7)    BESK 063
     3+.55753684*T(8)-.4262629*T(9)+.21845181*T(10)-.066809767*T(11)  BESK 064
     4+.C0918938S*T(12))*C                                           BESK 065
      IF(N)20,28,29                                                   BESK 066
   28 BK=GO                                                           BESK 067
      RETURN                                                          BESK 068
C                                                                     BESK 069
C     COMPUTE K1 USING POLYNOMIAL APPROXIMATION                       BESK 070
C                                                                     BESK 071
   29 G1=A*(1.2533141+.4699270*T(1)-.14685830*T(2)+.12804266*T(3)     BESK 072
     2-.17364316*T(4)+.2847618I*T(5)-.45943421*T(6)+.62833807*T(7)    BESK 073
     3-.66322954*T(8)+.50502386*T(9)-.25813C38*T(10)+.07880CC12*T(11) BESK 074
     4-.C1C824177*T(12))*C                                           BESK 075
      IF(N-1)20,30,31                                                 BESK 076
   30 BK=G1                                                           BESK 077
      RETURN                                                          BESK 078
C                                                                     BESK 079
C     FROM KO,K1 COMPUTE KN USING RECURRENCE RELATION                 BESK 080
C                                                                     BESK 081
   31 DO 35 J=2,N                                                     BESK 082
      GJ=2.*(DFLOAT(J)-1.)*G1/X+GO                                    BESK 083
      IF(GJ-1.0E70)33,33,32                                           BESK M07
   32 IER=4                                                           BESK M08
      GO TO 34                                                        BESK M09
   33 GO=G1                                                           BESK M10
   35 G1=GJ                                                           BESK 085
   34 BK=GJ                                                           BESK M11
      RETURN                                                          BESK 087
   36 B=X/2.                                                          BESK 088
      A=.57721566+ DLOG(B)
      C=B*B                                                           BESK 090
      IF(N-1)37,43,37                                                 BESK 091
C                                                                     BESK 092
C     COMPUTE KO USING SERIES EXPANSION                               BESK 093
C                                                                     BESK 094
   37 GO=-A                                                           BESK 095
      X2J=1.                                                          BESK 096
      FACT=1.                                                         BESK 097
      HJ=.0                                                           BESK 098
      DO 40 J=1,6                                                     BESK 099
      RJ=1./DFLOAT(J)                                                 BESK 100
      X2J=X2J*C                                                       BESK 101
      FACT=FACT*RJ*RJ                                                 BESK 102
      HJ=HJ+RJ                                                        BESK 103
   40 GC=GO+X2J*FACT*(HJ-A)                                           BESK 104
      IF(N)43,42,43                                                   BESK 105
   42 BK=GO                                                           BESK 106
      RETURN                                                          BESK 107
C                                                                     BESK 108
C     COMPUTE K1 USING SERIES EXPANSION                               BESK 109
```

```
C                                                                       BESK 110
   43 X2J=B                                                             BESK 111
      FACT=1.                                                           BESK 112
      HJ=1.                                                             BESK 113
      G1=1./X+X2J*(.5+A-HJ)                                             BESK 114
      DO 50 J=2,8                                                       BESK 115
      X2J=X2J*C                                                         BESK 116
      RJ=1./DFLOAT(J)                                                   BESK 117
      FACT=FACT*RJ*RJ                                                   BESK 118
      HJ=HJ+RJ                                                          BESK 119
   50 G1=G1+X2J*FACT*(.5+(A-HJ)*DFLOAT(J))                              BESK 120
      IF(N-1)31,52,31                                                   BESK 121
   52 BK=G1                                                             BESK 122
      RETURN                                                            BESK 123
      END                                                               BESK 124
```

```
      SUBROUTINE DBESY(X,N,BY,IER)
C                                                                       BESY 001
C     .................................................................BESY 002
C                                                                       BESY 003
C        SUBROUTINE DBESY
C                                                                       BESY 005
C        PURPOSE                                                        BESY 006
C           COMPUTE THE Y BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDERBESY 007
C                                                                       BESY 008
C        USAGE                                                          BESY 009
C           CALL DBESY(X,N,BY,IER)
C                                                                       BESY 011
C        DESCRIPTION OF PARAMETERS                                      BESY 012
C           X  -THE ARGUMENT OF THE Y BESSEL FUNCTION DESIRED           BESY 013
C           N  -THE ORDER OF THE Y BESSEL FUNCTION DESIRED              BESY 014
C           BY -THE RESULTANT Y BESSEL FUNCTION                         BESY 015
C           IER-RESULTANT ERROR CODE WHERE                              BESY 016
C              IER=0  NO ERROR                                          BESY 017
C              IER=1  N IS NEGATIVE                                     BESY 018
C              IER=2  X IS NEGATIVE OR ZERO                             BESY 019
C              IER=3  BY HAS EXCEEDED MAGNITUDE OF 10**70               BESY M01
C                                                                       BESY 020
C        REMARKS                                                        BESY 021
C           VERY SMALL VALUES OF X MAY CAUSE THE RANGE OF THE LIBRARY   BESY 022
C           FUNCTION ALOG TO BE EXCEEDED                                BESY 023
C           X MUST BE GREATER THAN ZERO                                 BESY 024
C           N MUST BE GREATER THAN OR EQUAL TO ZERO                     BESY 025
C                                                                       BESY 026
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  BESY 027
C           NONE                                                        BESY 028
C                                                                       BESY 029
C        METHOD                                                         BESY 030
C           RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE  BESY 031
C           AS DESCRIBED BY A.J.M.HITCHCOCK,-POLYNOMIAL APPROXIMATIONS  BESY 032
C           TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED    BESY 033
C           FUNCTIONS-, M.T.A.C., V.11,1957,PP.86-88, AND G.N. WATSON,  BESY 034
C           -A TREATISE ON THE THEORY OF BESSEL FUNCTIONS-, CAMBRIDGE   BESY 035
C           UNIVERSITY PRESS, 1958, P. 62                               BESY 036
C                                                                       BESY 037
C     .................................................................BESY 038
C                                                                       BESY 039
      IMPLICIT REAL*8(A-H,O-Z)                                          BESY 040
C                                                                       BESY 041
C     CHECK FOR ERRORS IN N AND X                                       BESY 042
      IF(X)190,190,20                                                   BESY 046
C                                                                       BESY 048
C     BRANCH IF X LESS THAN OR EQUAL 4                                  BESY 049
C                                                                       BESY 050
   20 IF(X-4.0)40,40,30                                                 BESY M06
C                                                                       BESY 052
C       COMPUTE YO AND Y1 FOR X GREATER THAN 4                          BESY 053
C                                                                       BESY 054
   30 T1=4.0/X                                                          BESY M07
      T2=T1*T1                                                          BESY M08
      PO=((((-.CCC0037043*T2+.CCC0173565)*T2-.0000487613)*T2            BESY M09
     1  +.00017343)*T2-.001753062)*T2+.3989423                         BESY M10
      QO=((((.CCC0032312*T2-.CCC0142078)*T2+.0000342468)*T2             BESY M11
     1  -.0000869791)*T2+.CCC4564324)*T2-.01246694                     BESY M12
```

```
      P1=((((.00C0042414*T2-.CC0020C920)*T2+.0CC0580759)*T2                BESY M13
     1  -.00022320C)*T2+.302921826)*T2+.3989423                           BESY M14
      Q1=((((-.CCC0036594*T2+.CCC01622)*T2-.0C00398708)*T2                 BESY M15
     1  +.0001C64741)*T2-.0006390400)*T2+.03740084                        BEST M16
      A=2.C/DSQRT(X)
      B=A*T1                                                             BESY M18
      C=X-.7853982                                                       BESY M19
      YC=A*PC*DSIN(C)+B*QC*DCCS(C)
      Y1=-A*P1*DCOS(C)+B*Q1*DSIN(C)
      GO TO 90                                                           BESY 096
C                                                                        BESY 097
C       COMPUTE YO AND Y1 FCR X LESS THAN OR EQUAL TO 4                  BESY 098
C                                                                        BESY 099
   40 XX=X/2.                                                            BESY 100
      X2=XX*XX                                                           BESY 101
      T= DLOG(XX)+.5772157
      SUM=0.                                                             BESY 103
      TERM=T                                                             BESY 104
      YO=T                                                               BESY 105
      DO 70 L=1,15                                                       BESY 106
      IF(L-1)50,60,50                                                    BESY 107
   5C SUM=SUM+1./DFLOAT(L-1)                                             BESY 108
   60 FL=L                                                               BESY 109
      TS=T-SUM                                                           BESY 110
      TERM=(TERM*(-X2)/FL**2)*(1.-1./(FL*TS))                            BESY 111
   70 YC=YO+TERM                                                         BESY 112
      TERM = XX*(T-.5)                                                   BESY 113
      SUM=0.                                                             BESY 114
      Y1=TERM                                                            BESY 115
      DO 8C L=2,16                                                       BESY 116
      SUM=SUM+1./DFLOAT(L-1)                                             BESY 117
      FL=L                                                               BESY 118
      FL1=FL-1.                                                          BESY 119
      TS=T-SUM                                                           BESY 120
      TERM=(TERM*(-X2)/(FL1*FL))*((TS-.5/FL)/(TS+.5/FL1))                BESY 121
   80 Y1=Y1+TERM                                                         BESY 122
      PI2=.6366198                                                       BESY M23
      YO=PI2*YC                                                          BESY 124
      Y1=-PI2/X+PI2*Y1                                                   BESY 125
C                                                                        BESY 126
C     CHECK IF ONLY YO CR Y1 IS DESIRED                                  BESY 127
C                                                                        BESY 128
   90 IF(N-1)1CC,100,13C                                                 BESY 129
C                                                                        BESY 130
C     RETURN EITHER YO CR Y1 AS REQUIRED                                 BESY 131
C                                                                        BESY 132
  100 IF(N)180,120,110                                                   BESY 133
  110 BY=Y1                                                              BESY 134
      GC TO 170                                                          BESY 135
  120 BY=YO                                                              BESY 136
      GO TO 170                                                          BESY 137
C                                                                        BESY 138
C    PERFORM RECURRENCE OPERATIONS TO FIND YN(X)                         BESY 139
C                                                                        BESY 140
  130 YA=YO                                                              BESY 141
      YB=Y1                                                              BESY 142
      K=1                                                                BESY 143
  140 T=DFLOAT(2*K)/X                                                    BESY 144
      YC=T*YB-YA                                                         BESY 145
      IF(DABS(YC)-1.0E7C)145,145,141
```

```
141 IER=3                                                          BESY M03
    RETURN                                                         BESY M04
145 K=K+1                                                          BESY M05
    K=K+1                                                          BESY 146
    IF(K-N)150,160,150                                             BESY 147
150 YA=YB                                                          BESY 148
    YB=YC                                                          BESY 149
    GO TO 140                                                      BESY 150
160 BY=YC                                                          BESY 151
170 RETURN                                                         BESY 152
180 IER=1                                                          BESY 153
    RETURN                                                         BESY 154
190 IER=2                                                          BESY 155
    RETURN                                                         BESY 156
    END                                                            BESY 157
```

SAMPLE DATA

```
 7  44.9
 1          0.0          2.1869       WATER
1.423       .1469        .001188      .01954       .04288
 2          2.1869       6.1138       FUEL
1.0800      .22697       .0014719     .11494       .030915      .00053870  .19553
 3          6.1138       10.0787      FUEL
1.0828      .22979       .0014673     .11547       .030594      .00054228  .19683
 4          10.0787      14.0487      FUEL
1.0939      .23216       .0014525     .11427       .030279      .00053662  .19478
 5          14.0487      18.0213      FUEL
1.1043      .23396       .0014390     .11302       .030033      .00053046  .19254
 6          18.0213      22.86        GRAPHITE
1.3329      .26746       .00058999    .0098424     .021589
 7          22.86        100.         GRAPHITE
1.192       .9404        .00002147    .0002279     .002821
```

```
KEFF = 1.3927755130 00        RHO =  2.7681681490-01 DK/K        DET =  1.8279D-06
KEFF = 1.3813926730 00        RHO =  2.7609286740-01 DK/K        DET =  1.8070D-06
KEFF = 1.2617676760 00        RHO =  2.0746107330-01 DK/K        DET =  5.7961D-07
KEFF = 1.2052767340 00        RHO =  1.7031505650-01 DK/K        DET =  2.8809D-07
KEFF = 1.1494516150 00        RHO =  1.3001992660-01 DK/K        DET =  1.1176D-07
KEFF = 1.1140660040 00        RHO =  1.0238711520-01 DK/K        DET =  4.1166D-08
KEFF = 1.0934325500 00        RHO =  9.5448846690-02 DK/K        DET =  1.1195D-08
KEFF = 1.0857258220 00        RHO =  7.8957155020-02 DK/K        DET =  1.7923D-09
KEFF = 1.0842567280 00        RHO =  7.7709205050-02 DK/K        DET =  1.0213D-10
KEFF = 1.0841675540 00        RHO =  7.7633685400-02 DK/K        DET =  1.0219D-12
KEFF = 1.0841670570 00        RHO =  7.7632922060-02 DK/K        DET =  5.9203D-16
```

CONTENTS	RMIN	RMAX	D1	D2	SIGA1	SIGA2	SIGS1	NUSIGF1	NUSIGF2
WATER	0.0	2.1869D 00	1.4230D 00	1.4690D-01	1.1880D-03	1.9540D-02	4.2880D-02	0.0	0.0
FUEL	2.1869D 00	6.1138D 00	1.0800D 00	2.2697D-01	1.4719D-03	1.1494D-01	3.0915D-02	5.3870D-04	1.0955D-01
FUEL	6.1138D 00	1.0079D 01	1.0828D 00	2.2979D-01	1.4673D-03	1.1547D-01	3.0594D-02	5.4228D-04	1.0968D-01
FUEL	1.0079D 01	1.4049D 01	1.0939D 00	2.3216D-01	1.4525D-03	1.1427D-01	3.0279D-02	5.3662D-04	1.0947D-01
FUEL	1.4049D 01	1.8021D 01	1.1043D 00	2.3396D-01	1.4390D-03	1.1302D-01	3.0033D-02	5.3046D-04	1.0925D-01
GRAPHITE	1.8021D 01	2.2860D 01	1.3329D 00	2.6746D-01	5.8999D-04	9.8424D-03	2.1589D-02	0.0	0.0
GRAPHITE	2.2860D 01	1.0000D 02	1.1920D 00	9.4004D-01	2.1470D-05	2.2790D-04	2.8210D-03	0.0	0.0

ORIGINAL MATRIX KEFF = 1.0841670560 00

```
1.0433D 00  0.0          -1.8107D-01  1.0469D 00 -1.7821D 00 -9.8895D-01  0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0         0.0         0.0          0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0

2.9844D 00  1.1718D 00    5.6446D-01  2.7391D-01  5.5556D 00 -2.5876D-01  0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0         0.0         0.0          0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0

5.7009D-02  0.0           1.8584D-01 -3.2943D-01 -8.9830D-01  1.0886D-02  0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0         0.0         0.0          0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0

1.6834D-02  2.4030D-02   -1.2175D-01 -1.8115D-02  5.6851D-01  5.9859D-04  0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0         0.0         0.0          0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0

0.0         0.0           6.0245D-03 -3.2297D-01  1.8344D 01  9.1524D-01 -6.1458D-03  3.2700D-01 -1.8055D 01 -9.1620D-01
0.0         0.0           0.0         0.0         0.0         0.0          0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0

0.0         0.0          -1.8781D-02 -8.4503D-02 -5.7185D 01  2.3947D-01  1.8935D-02  8.4281D-02  5.5625D 01 -2.3612D-01
0.0         0.0           0.0         0.0         0.0         0.0          0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0

0.0         0.0          -5.3550D-03  1.3321D-01  1.3016D 01 -2.9296D-02  5.4573D-03 -1.3345D-01 -1.2786D 01  2.9073D-02
0.0         0.0           0.0         0.0         0.0         0.0          0.0          0.0          0.0          0.0
0.0         0.0           0.0         0.0
```

```
 0.0          0.0          3.5083D-03  7.3249D-03 -8.5276D 00 -1.6109D-03 -3.5681D-03 -7.2993D-03  8.3597D 00  1.5902D-03
 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          2.5489D-04  6.0515D-02  2.6290D 02  7.8031D-01
-2.7647D-04 -4.3027D-02 -2.4489D 02 -7.8682D-01  0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0         -7.8529D-04  1.5597D-02 -8.0996D 02  2.0112D-01
 8.5175D-04 -1.2380D-02  7.5446D 02 -2.0281D-01  0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0         -2.1808D-04  8.4203D-02  1.9656D 02 -4.4436D-02
 2.3668D-04 -8.5153D-02 -1.8293D 02  4.3649D-02  0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          1.4259D-04  4.6056D-03 -1.2852D 02 -2.4305D-03
-1.5475D-04 -4.6583D-03  1.1961D 02  2.3878D-03  0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
 1.2737D-05  2.5955D-01  3.8089D 03  6.0744D-01 -1.4071D-05 -2.8889D-01 -3.4801D 03 -6.1783D-01  0.0         0.0
 0.0          0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
-3.9241D-05  7.7212D-02 -1.1735D 04  1.5658D-01  4.3449D-05 -7.4678D-02  1.0746D 04 -1.5971D-01  0.0         0.0
 0.0          0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
-1.0718D-05  5.4802D-02  2.9079D 03 -5.4352D-02  1.1848D-05 -5.5925D-02 -2.6561D 03  5.3617D-02  0.0         0.0
 0.0          0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
 7.0082D-06  2.9980D-03 -1.9013D 03 -2.9734D-03 -7.7510D-06 -3.0628D-03  1.7376D 03  2.9364D-03  0.0         0.0
 0.0          0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0         6.9159D-07  4.4415D-01  5.5170D 04  4.1349D-01 -5.2565D-02  0.0
-3.6787D 00   0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0        -2.1355D-06  1.1481D-01 -1.7036D 05  1.0689D-01 -2.1047D-01 -1.5970D-02
-1.4729D 01 -8.5901D 00   0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0        -5.7657D-07  3.0905D-02  4.2609D 04 -5.9060D-02  1.2087D-01  0.0
-5.6115D-01   0.0          0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0         3.7720D-07  1.6925D-03 -2.7875D 04 -3.2345D-03  9.7114D-03  9.8546D-04
-4.5084D-01 -3.9875D-01   0.0         0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          2.3122D-02  0.0
 6.5330D 00   0.0         -1.2105D-01  0.0

 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          0.0         0.0
 0.0          0.0          0.0         0.0         0.0         0.0          0.0          0.0          9.2577D-02  5.2171D-03
 2.6158D 01   2.0273D 01   1.6950D-01 -1.7891D-01
```

```
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          -5.1570D-03  0.0
1.0646D 00   0.0          1.5185D-02   0.0

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          -4.1433D-03  -3.2010D-04
8.5537D-01   9.7995D-01   -1.6775D-02  1.5367D-02
```

REDUCED MATRIX RHO = 7.763292161D-02 DK/K

```
1.0000D 00   0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          -3.4634D-01

0.0          1.0000D 00   0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          7.4905D-01

0.0          0.0          1.0000D 00   0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          1.0037D-01

0.0          0.0          0.0          1.0000D 00   0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          -1.6536D-02

0.0          0.0          0.0          0.0          1.0000D 00   0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          -1.2609D-05

0.0          0.0          0.0          0.0          0.0          1.0000D 00   0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          -4.0125D-01

0.0          0.0          0.0          0.0          0.0          0.0          1.0000D 00   0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          1.2991D-01

0.0          0.0          0.0          0.0          0.0          0.0          0.0          1.0000D 00   0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          -1.5769D-02

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          1.0000D 00   0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          1.6472D-07

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          1.0000D 00
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          -4.0113D-01

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
1.0000D 00   0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          1.1532D-01

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          1.0000D 00   0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
```

```
0.0          0.0          0.0          -1.0486D-02

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          1.0000D 00   0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          1.7241D-07

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          1.0000D 00   0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          -3.9839D-01

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          1.0000D 00   0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          -1.8444D 00

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          1.0000D 00   0.0          0.0          0.0          0.0
0.0          0.0          0.0          -2.3663D-03

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          1.0000D 00   0.0          0.0          0.0
0.0          0.0          0.0          1.8065D-07

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          1.0000D 00   0.0          0.0
0.0          0.0          0.0          -3.9558D-01

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          1.0000D 00   0.0
0.0          0.0          0.0          -2.7146D 00

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          1.0000D 00
0.0          0.0          0.0          3.1254D 01

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
1.0000D 00   0.0          0.0          -3.2510D-03

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          1.0000D 00   0.0          5.3709D-03

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          1.0000D 00   -6.9399D-01

0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          2.7598D-21
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REACTIVITY CALCULATIONS FOR THE KANSAS STATE UNIVERSITY TRIGA MARK II NUCLEAR REACTOR

by

LELAND WAYNE RANEY

B. S., Kansas State University, 1966

AN ABSTRACT OF A MASTER'S THESIS

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Manhattan, Kansas

1971

ABSTRACT

The purpose of this work was to derive rigorously the point reactor kinetic equations from both the energy-dependent diffusion kinetic equations and the multigroup diffusion kinetic equations. From such a derivation a precise definition for the concept of reactivity was obtained. Three different approximate calculational expressions for determining reactivity were developed. The first approximation yielded a very simple result similar to that obtained from first-order perturbation theory. By using better approximations, expressions for reactivity were obtained in terms of the effective multiplication factor and the "inhour" equation. A simplified computer program has been written that predicts reactivity changes for the Kansas State University Triga Mark II nuclear reactor. To test the results of the program, reactivities were experimentally measured within the KSU Triga Mark II nuclear reactor. These experimentally measured reactivities (fuel to water, aluminum to water, and graphite to water reactivity differences) are compared to the computer predicted values. It was found that the computer program was able to predict the reactivity differences fairly accurately.