OPTIMAL CONTROL OF INTEGRATED HUMAN THERMAL SYSTEM BY RESPONSE SURFACE METHODOLOGY

by 6408

HEMANT N. OZARKAR

B.E. (Mechanical) (1965)
Poona University, India

A MASTER'S REPORT

submitted in partial fulfillment of the

requirement for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1971

Approved by:

Ching Lai Hwang
Major Professor

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Many industrial and management problems involve maximization or minimization of functions of several variables. The values of the variables which give a maximum or minimum value of a function of several variables are called the optimum values and are of vital importance in practice. A wide variety of efficient search techniques has been developed for finding mathematically the maximum or minimum of such a function. However, in order to use these methods, a mathematical model to express the functional relationship in terms of the variables is required. Very often, for a practical problem, this relationship is quite complex and it is not possible or feasible to build a rigorous mathematical model for it. Such a relationship is sometimes obtainable only from actual experimentation.

A situation like this can best be handled by a powerful technique known as a Response Surface Methodology (RSM), developed by Box and Wilson [1,2,3,4,9]. In this technique, an optimum point is found by experimentation, which in general is an iterative procedure. An experimental iteration consists of a postulation of a mathematical model, selection of an experimental design and analysis of data. Very possibly the analysis of an initial set of data suggests the need for either further experimentation or a modification of the current model or both. Thus a cycle is initiated which is repeated as often as necessary to reach a satisfactory conclusion.

In practice, however, experimentation is often costly and time-consuming. RSM uses a sequential experimentation and requires a number of experiments. Thus, on the whole, process of finding optimum values of variables may involve excessive cost and time.

This process of whole experimentation, however, can be simulated beforehand to minimize the number of required experiments and to provide a guide-line for the actual experimentation. This simulation can be carried out with the help of high-speed digital computers with much ease and comparatively at much less cost. This helps in obtaining information about the number of experiments to be carried out, and time and money involved etc.

The purpose of this report is twofold. First is to develop a computer program to obtain an optimum point which minimizes (maximizes) a given function of multi-dimensional variables using Response Surface Methodology. Second is to use the method in carrying out a computer simulation of optimal control of the integrated human thermal system.

This report first describes the response surface methodology. The method consists of two phases. Starting from any point in the experimental region, phase one brings the point within a "striking distance" of the optimum point, while phase two further leads to the actual optimum point. In doing so, in general, an efficient design of experiment is needed. An efficient design of experiment not only minimizes the number of experiments, but also provides the required information with maximum precision.

A simple two-dimensional production scheduling problem is then solved to illustrate the use of the method. This problem has been solved by other methods such as the Hooke and Jeeves pattern search [7], sequential simplex pattern search [5], and conjugate gradient [8]. The results compare well with those by all the above methods.

The report then deals with the optimization of the integrated human thermal system. When the environment is too hot and it is not feasible to cool the environment, the next best alternative is to cool the man. This cooling can be done by circulating coolant in the network of tubes which are held in contact with the surface of skin, and conducting heat away from the body. The cooling device is to maintain the human body in a state of thermoneutrality by properly controlling its operating variables (the coolant temperature and the coolant flow rates). The operating efforts of controling the cooling devices should be minimized.

A study on modeling, simulation, and optimal control of an integrated human thermal system has been carried out by Hsu [6]. The integrated human thermal system is formulated by incorporating an external thermal regulation device into a human thermal system. In Hsu's work [6] a mathematical model representing the integrated system, and an optimal control problem have been formulated. The well-developed linear programming technique has been employed for obtaining the optimal control variables of the cooling devices. Hsu's optimal control problem can be solved experimentally at the KSU-ASHRAE test facility. In the present study the optimal control problem is solved by numerical experimentation (simulation) using the response surface methodology. Mathematical models developed in Hsu's work [6] are employed in the present work.

REFERENCES

1. Box, G.E.P., "The Exploration and Exploitation of Response Surfaces", Biometrics, 10, 16-60 (1954).

2. Box, G.E.P., and J.S. Hunter, "Experimental Designs for the Exploration and Exploitation of Response Surfaces", In a book edited by V. Chew, Experimental Designs in Industry, John Wiley and Sons, Inc., New York, 1958.

3. Box, G.E.P., and K.B. Wilson, "On the Experimental Attainment of Optimum Conditions," J. Roy. Statist., Ser. B, 13, 1-45 (1951).

4. Davies, O. L., The Design and Analysis of Industrial Experiments, Hafner, New York, Chapter 11, 495-578, 1954.

5. Fan, L. T., C. L. Hwang, and F. A. Tillman, "A Sequential Simplex Pattern Search Solution to Production Planning Problems", AIIE Trans. 1 (3), 267-273 (1969).

6. Hsu, F. T., "Modeling, Simulation, and Optimal Control of the Human Thermal System", Ph.D. Dissertation, Kansas State University, 1971.

7. Hwang, C. L., L. T. Fan, and S. Kumar, "Hooke and Jeeves Pattern Search Solution to Optimal Production Planning Problems" Report No. 18, Institute for Systems Design and Optimization, Kansas State University, 1969.

8. Hwang, C. L., L. T. Fan, and N. C. Pereira, "Conjugate Gradients Searching Techniques for Production Planning", Institute for Systems Design and Optimization, Kansas State University, 1969.

9. Read, D. R., "Design of Chemical Experiments", Biometrics, 10, 1-15 (1954).

CHAPTER 2

RESPONSE SURFACE METHODOLOGY

2.1 STATEMENT OF THE PROBLEM

If, in the k-dimensional space, a function S of k variables $x_1, x_2, \ldots, x_k$ is given by

$$S = \phi\ (x_1, x_2, \ldots, x_k), \tag{1}$$

then the problem is to find, in the minimum number of experimental iterations, the point $(x_{1s}, x_{2s}, \ldots, x_{ks})$ within the experimental region at which S is maximum or minimum.

2.2 DESCRIPTION OF THE METHOD

In this method, an optimum point is sought by sequential experimentation. The iterative procedure of the experimentation is started from any point chosen in the region under consideration. In the neighborhood of this point enough experiments are performed. This enables one to fit, by the method of least squares, a polynomial approximation of sufficient order to provide a local representation of the surface. This knowledge of the "local geography" of the region is used to proceed to a further region at which higher order responses are expected. Further experiments are performed in this region, and the whole process is repeated until no further gain is achieved [2].

The starting point is usually not near the optimum point. To start with, therefore, a polynomial approximation of the first order is employed to represent the local surface passing through this point. This provides not only the simplicity but also the economy in experimentation in the sense that comparatively less number of points are required to fit a first degree curve. This assumption would be abandonded and a second *order approximation* adopted only when the first order approximating function had proved inadequate [2].

Based on the order of approximate function, the method can be divided into two phases. Phase one employs a *linear approximation* whereas phase two employs a quadratic one. Phase one, by linear approximation, provides a rapid progress from the starting base point, which is usually far from the optimum, to a point within "striking distance" of it [3], while phase two, by a quadratic approximation of surface, leads the further progress to the actual optimum point.

2.3. PHASE ONE

Phase one can be started from any point in the region of the k-dimensional space. A *suitable design of experiments* is employed, trials are carried out and data are obtained in order to know the local nature of the surface. For most cases, the best design among the first order designs is provided by 2^k factorial design, where k denotes the number of variables [see Appendix A]. A polynomial of the first degree is then fitted to this data by the method of least squares. The surface is thus represented locally by a plane

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i \tag{2}$$

where y is the value of function S, called response, at a point $(x_1, x_2, \ldots, x_k)$ in the space. A considerable advance "up (down) the hill" can be made by following the calculated path of steepest ascent (descent) which corresponds to altering each variable x_i in proportion to its estimated first derivative β_i [3]. A search is carried out along this direction starting from the base point at the centre of the design till a point is reached where no further improvement is possible. However, since the calculated path is not likely to pass through or very near the optimum, it is probable that still considerable further progress can be made by another iteration. The provisional optimum point obtained in the iteration becomes a base point for the following iteration and the whole process is repeated. Eventually a point is reached which is near the optimum. The surface contours near the optimum become non-linear. Hence, at this point, the approximation of a surface as a plane does not hold good any longer. Phase one ends at this stage. The region obtained at the end of phase one is called "near-optimum" region.

The steps to be carried out in phase one can be summarized as follows:

1. Select a starting point $(x_1, x_2, \ldots, x_k)$ in the experimental region.
2. Construct a factorial design of 2^k type with the starting point as the base.
3. Evaluate the objective function S at the design points by equation (1).
4. Fit a first degree curve

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i$$

to the data at design points by the method of least squares. This curve represents the equation of the plane passing through the base point.

5. Determine the direction of steepest ascent (descent) from the equation of the plane. The gradient in x_i direction is proportional to β_i.
6. Proceed up (down) in the direction of steepest ascent (descent), i.e., vary each variable x_i in proportion to first order derivative β_i.
7. Evaluate objective function S at the trial points till a point is reached where no further improvement in the objective function is possible.
8. Choose a base point of maximum (minimum) response from the trials performed in step (7). Construct a new 2^k factorial design at this base point.
9. Repeat steps (3) through (7) till no further progress is possible over an iteration. At this point an approximation of the first degree of polynomial does not hold good and hence no further progress is possible. This region is called a "near-stationary" region and phase one ends at this stage.

2.4 EXPLANATION OF PHASE ONE (THEORETICAL BACKGROUND)

In order to know the "local geography" of the region around any point, information in the close neighborhood of the point is obtained, this information being the responses at the points. A close neighborhood around a point can be defined by a set of points lying on a hypersphere with the center at the point and radius as a sufficiently small distance chosen at will. Then the direction at that point on which points of higher (lower) response is given by joining the point to another point on the hypersphere at which the response is maximum (minimum). This direction is called the direction of steepest ascent (descent).

Thus, if 0 is the point at which the direction of steepest ascent (descent) is required, and P is the point of maximum (minimum) response among all the points lying on the hypersphere of center 0 and a sufficiently small radius r, then the line OP, joining points 0 and P gives the required direction of steepest ascent.

It can be shown that the point P is one of the points at which the hypersphere touches a response contour [1]. In other words, if 0 is assumed as the origin, then the co-ordinates of P are proportional to the first order derivatives at P, assumed not all zero. [see Appendix B].

However, in general, the derivatives at P are unknown. But these derivatives can be expressed by their Taylor's series expansion about the origin.

$$\frac{\partial\phi(P)}{\partial x_t} = \frac{\partial\phi(0)}{\partial x_t} + \frac{\partial}{\partial x_t}\left(\frac{\partial\phi(0)}{\partial x_1}\right) x_1 + \frac{\partial}{\partial x_t}\left(\frac{\partial\phi(0)}{\partial x_2}\right) x_2$$

$$+ \cdots + \frac{1}{2!}\left[\frac{\partial}{\partial x_t}\left(\frac{\partial^2\phi(0)}{\partial x_1^{\,2}}\right) x_1^2 + \frac{\partial}{\partial x_t}\left(\frac{\partial^2\phi(0)}{\partial x_2^{\,2}}\right) x_2^2\right.$$

$$\left. + \cdots + \frac{\partial}{\partial x_t}\left(\frac{\partial^2\phi(0)}{\partial x_1 \partial x_2}\right) x_1\, x_2\right] + \cdots, \qquad t = 1, 2, \ldots, k \qquad (3)$$

or in the form

$$\phi_t(P) = \left[D_t\left\{\sum_{s=0}^{\infty} (\sum_{t=1}^{k} D_t\ x_t)^s\ /\ s!\right\}\right]\phi\ (0) \tag{4}$$

where ϕ_t and D_t stand for derivatives with respect to variable x_t. The term on the right hand side of equation (4) is obtained by expanding the expression in the square bracket, operating on ϕ and evaluating at 0. If only the first order terms are considered and the second and higher order terms are neglected, then equation (4) can be written as

$$\phi_t(P) = \phi_t(0), \qquad t = 1, 2, \ldots, k \tag{5}$$

This is equivalent to saying that the surface contour $\phi(0)$ passing through 0 is a plane. The direction of steepest ascent (descent) is then given by the first order derivatives at 0, $\phi_t(0)$.

Thus, in order to know the direction of steepest ascent (descent), it is necessary to determine the equation of the plane and the values of the first order derivatives at 0. This is achieved by obtaining an equation by the method of least squares [see Appendix C]. $\phi(0)$, the surface passing through the point 0, then can be expressed by the regression equation

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i\ x_i \tag{6}$$

The β's in this equation represent the required first order derivatives of the response function at the origin.

Equation (5) can now be written as

$$\phi_t(P) = \beta_t, \qquad t = 1, 2, \ldots, k \tag{7}$$

In the close neighborhood of 0, defined by a hypersphere of sufficiently small radius r and center 0, the maximum (minimum) response will then be obtained at the point whose co-ordinates are proportional to β's. This point lies on the direction of steepest ascent (descent).

By successive application of equation (7), it is possible to find a point of higher (lower) response, as long as the surfaces at trial points can be represented by equation (6). A new equation is necessary where this equation does not hold good to represent a surface at the point under consideration.

When a point near optimum is reached, the non-linear response surface around such a point can no longer be approximated by a first order equation. Hence phase one ends at this stage.

2.5 PHASE TWO

Phase two starts from the point where phase one ends. Comparatively more detailed experiments are required in this phase than in the previous one because of approximation of a surface by a higher degree (second degree) polynomial.

The maximum (minimum) point attained at the end of phase one becomes the base point for the first iteration of phase two. A suitable design of experiment is chosen and a number of points around the base point are selected. For most cases, among the second-order designs, composite designs provide the best designs [see Appendix D]. Objective functions are then evaluated at these experimental points and the following second degree curve is fitted to these data.

$$y = \beta_0 + \sum \beta_i x_i + \sum\sum \beta_{ij} x_i x_j \qquad (8)$$

where y is the value of function S at a point $(x_1, x_2, \ldots, x_k)$. This represents a response surface passing through the base point.

It may take a variety of forms such as spheroid, ellipsoid, hyperboid or paraboid. It is usually quite impossible to appreciate the nature of the fitted surface by inspection of the values of co-efficients β_i and β_{ij}. The nature of system is, however, made readily apparent if conic form is reduced to canonical form.

This consists essentially of shifting the origin to the center of the curve and rotating the co-ordinate axes so that they correspond to the axes of the conic [see Appendix E]. When reduced to a canonical form, equation (8) appears as

$$Y - Y_s = \lambda_1 X_1^2 + \lambda_2 X_2^2 + \cdots + \lambda_k X_k^2$$

or

$$Y - Y_s = \sum_{i=1}^{k} \lambda_i X_i^2 \tag{9}$$

where Y_s = value of Y at the centre

λ_i = co-efficients

X_i = principal axes of the conic

This canonical form does the same function for phase two as does the direction of steepest ascent (descent) for phase one. Equation (9) shows the loss (or gain) of response on moving from the centre point. Thus if all λ_i are negative (or positive), the centre point becomes the maximum (minimum) point. Whereas if one or more λ's are positive (negative), surface is elliptic hyperboloid and would possess a col instead of a true maximum (minimum) and if one or more λ's are or approach to zero, surface would become elliptic or hyperbolic cylinders and possess a ridge.

The steps to be carried out in this phase can be summarized as follows:

1. Construct a suitable composite design around the point attained at the end of phase one.
2. Evaluate objective function S at the design points.
3. Fit a second degree curve

$$y = \beta_0 + \sum\beta_i x_i + \sum\sum\beta_{ij} x_{ij} x_j$$

   to the data of step (2), by the method of least squares.
4. Reduce the above conical form to a canonical form

$$Y - Y_s = \sum\lambda_i X_i^2$$

5. Observe the signs of all λ's. If all are negative, the stationary point of the curve represents the minimum point. If some or all λ's are positive or zero, no optimum point exists.

2.6 COMPUTER PROGRAM

A FORTRAN computer program is developed for this method.

The user is required to provide the following values in the main program -

K = No. of variables

SVAL = Starting values for variables

STEP = Step sizes for variables

FSTEP = Final step sizes for variable.

In additional to this, a user has to provide a subroutine OBJECT (S,X). The detailed instructions regarding the use of computer program are included in the program itself. The logical flow chart is given in Appendix F, and FORTRAN statements in Appendix G.

REFERENCES

1. Box, G.E.P., and K.B. Wilson, "On the Experimental Attainment of Optimam Conditions," J. Roy. Statist., Ser. B, 13, 1-45 (1951).

2. Box, G.E.P., and J.S. Hunter, "Experimental Designs for the Exploration and Exploitation of Response Surfaces," In a book edited by V. Chew, Experimental Designs in Industry, John Wiley and Sons, Inc., New York, 1958.

3. Read, D.R., "Design of Chemical Experiments", Biometrics, 10, 1-15 (1954).

CHAPTER 3

OPTIMAL PRODUCTION SCHEDULING PROBLEM

3.1 INTRODUCTION

To illustrate the response surface methodology, a two-dimensional production scheduling problem [1,2,3] is considered here. The problem and its solution are described in details.

3.2 PRODUCTION SCHEDULING AND INVENTORY CONTROL PROBLEM

To illustrate the method a simple production scheduling problem has been considered. This problem is a multi-periods production scheduling problem in which the objective is to minimize the operating cost for the planning period. The total cost is composed of the production cost and the inventory cost. The costs for changing the production level and for carrying inventory are given by

$C(x_n - x_{n-1})^2$ = cost due to change in production level from the (n-1)th period to the n-th period,

$D(E - I_n)^2$ = inventory cost at the n-th period,

where C, D and E are positive constants, and x_n and I_n are the production level and the inventory level at the n-th period respectively.

The problem is to find x_n, n = 1, 2, ..., N which minimize the total cost f defined by

$$f = \sum_{n=1}^{N} [C(x_n - x_{n-1})^2 + D(E - I_n)^2] \qquad (1)$$

where

$$I_n = I_{n-1} + x_n - Q_n, \qquad n = 1, 2, \ldots, N$$

provided that x_0, I_0 and Q_n, n = 1, 2, ..., N are given. x_0, I_0, and Q_n are the production at the initial period, the inventory at the initial period, and the sales at the n-th period, respectively.

A two period system is considered in a numerical illustration. The two decision variables x_1 and x_2 have been determined so that the following cost function is minimized.

$$f(x_1,x_2) = C(x_1-x_0)^2 + D(E-I_1)^2 + C(x_2-x_1)^2 + D(E-I_2)^2 \qquad (2)$$

where

$$I_n = I_{n-1} + x_n - Q_n, \qquad n = 1, 2 \qquad (3)$$

The constants C, D, and E, the demand Q_n, n = 1, 2, the initial production level x_0, and the initial inventory level I_0, are as follows:

$$C = 100, \qquad D = 20, \qquad E = 10,$$
$$Q_1 = 30, \qquad I_0 = 12,$$
$$Q_2 = 10, \qquad x_0 = 15.$$

Using equation (3) and the values given above, we have

$$f(x_1,x_2) = 100(x_1-15)^2 + 20(28-x_1)^2 + 100(x_2-x_1)^2 + 20(38-x_1-x_2)^2 \qquad (4)$$

3.3 SOLUTION BY RESPONSE SURFACE METHODOLOGY

To illustrate the procedure, contour lines for equal values of the total cost given by equation (4) are shown in Fig. 1. Also presented in the figure are the steps of the response surface methodology described in the preceeding chapter. Details of the output of solution from computer are presented in Appendix H.

The starting base point is (5,10) with a 2^2 factorial design. The step size for the factorial design is (2,2) and the final step size is (0.1,0.1). The objective function S is evaluated at these design points as well as the center point (base point). The co-ordinates of the points and the functional value at these points are

Point No.	Co-ordinates (x_1, x_2)	Objective Function S
1	(3, 8)	43,980
2	(3, 12)	45,580
3	(7, 8)	25,900
4	(7, 12)	24,940
5	(5, 10)	33,660

From these data a first degree curve is fitted as

$$y = 58{,}212 - 4{,}840\ x_1 + 80\ x_2$$

This represents the equation of a response surface (a plane) passing through x^5 (5, 10). The gradient-components of steepest descent-in the direction of x_1 and x_2 are in proportion of -4,840 to 80. The negative sign of the coefficient of x_1 shows a decrease in y with increase in x_1

whereas the positive sign of the coefficient of x_2 shows an increase in y with increase in x_2. The step size of the searching in the direction of the steepest descent is then (+2.000, -0.033). The step size is in the same proportion of +4,840 to -80, the reversal of signs is due to minimization. The next trial point is x^6 (7.00, 9.97) and its functional value is 24,948. The trial point x^6 (7.00, 9.97) is better than the base point x^5 (5, 10). The searching is, therefore, continued in the same direction with the same step size of searching. The new trial point becomes x^7 (9.0, 9.93). The procedure is continued till a point is found where no further improvement is possible. The results of the procedure are summarized below.

Point No.	Co-ordinates (x_1, x_2)	Objective Function	Remarks
5	(5.00, 10.00)	33,660.00	Base point
6	(7.00, 9.97)	24,948.00	Successful
7	(9.00, 9.93)	18,177.53	Successful
8	(11.00, 9.90)	13,348.44	Successful
9	(13.00, 9.87)	10,460.77	Successful
10	(15.00, 9.83)	9,514.50	Successful
11	(17.00, 9.80)	10,509.66	Failed.

The functional value at point x^{11} (17.00, 9.80) is no better than that at the previous trial point x^{10} (15.00, 9.83), hence experiment 1 ends providing x^{10} (15.00, 9.83) as a new base point for the next experiment.

In the second experiment, again a 2^2 factorial design is constructed around the base point, x^{10} (15.00, 9.83). Functional values (data) are evaluated at these design points as well as at the base point (trial numbers 12 through 16). An equation of the plane fitted to these data is

$$y = 26,207 - 13.54\ x_1 - 1,559.60\ x_2.$$

From the coefficients of the variables, the searching step size is determined as (0.017, 2.00). It is in proportion of 13.54 to 1559.60. Searching in the steepest descent direction is carried out using this searching step size, (.017, 2.00). Trials 17, 18 and 19 show improvement in the functional value, whereas trial 20 yields no improvement. Then experiment 2 ends and the trial point x^{19} (15.05, 15.83) becomes the base point for experiment 3. Again, a 2^2 factorial design is constructed around the base point x^{19} (15.05, 15.83). The objective functions are evaluated, an equation of a plane is fitted to the data and the step size for search is calculated. The trial point x^{26} (17.05, 16.10) yields better results, but the next trial point x^{27} (19.05, 16.37) fails. The x^{26} may become a base point for the next experiment, however, one of the factorial design points x^{24} (17.05, 17.83) is found to yield better results than the point x^{26}. Hence x^{24} becomes a base point for experiment 4 where trials 28 through 32 determine the equation of the plane passing through the base point, $x^{24} = x^{32}$ (17.05, 17.83). The step size for search is calculated as (2.00, -0.207). The first trial point x^{33} (19.05, 17.63), however, does not yield a better functional value than that at the base point x^{32}. This is an indication that the searching step size is too large, hence the new searching step size becomes (1.00, -0.104). With this step size, the next trial point x^{34} (18.05, 17.73) yields a better result than that at the base point. With the failure of the following trial point, x^{35} (19.05, 17.63), eventually x^{34} becomes a new base point for the following experiment. Experiment 5 starts with $x^{34} = x^{40}$ (18.05, 17.73) as the base point. The step size for the factorial design is kept unaltered as (2,2); however, the step size for searching is a half of the original step size. This gives the searching step size as (-1.000, 0.813). The first trial point x^{41} (17.05, 18.54) does not yield a better result than the base point $x^{34} = x^{40}$ (18.05, 17.73), indicating that the step size for searching is too large. The new searching step size, therefore, becomes (-0.500, 0.407). This yields a better point x^{42} (17.55, 18.14) which eventually becomes the base point for experiment 6. Experiment 6 further reduces the searching step size from (0.5, 0.5) to (0.25, 0.25) and experiment 7 from (0.25, 0.25) to (0.125, 0.125). This size combined with the direction of steepest descent makes searching step size as (0.125, -0.039) in the following experiment 8. In the first trial, no better point than the base x^{64} of the experiment is found. Reducing the step size to a half would have made its value 0.0625 which is less than 0.1, the final step size. At this stage, the locally best point is x^{64} (17.75, 18.21) with a functional value of 2961.85. Phase one ends providing this point x^{64} (17.75, 18.21) as the base point for phase two.

For phase two, a central composite design with $\alpha = 2$ is chosen. This requires a total of 9 points including the base point. The objective functions are evaluated at these points, x^{66} through x^{74}. A second degree curve fitted to these data is

$$y = 68{,}624 - 5{,}724\ x_1 - 1{,}612\ x_2$$
$$+ 241\ x_1^2 - 158\ x_1 x_2 + 122\ x_2^2$$

This represents a second degree curve (one of the curves from family of conics such as circle, ellipse, parabola or hyperbola) around the base point x^{70} (17.75, 18.21). The center of this curve is at x^{75} (17.79, 18.16) with functional value of 2961.03. This is better than the base point. The base point for experiment 2 is then taken as the above center point, x^{75}. A central composite design with $\alpha = 2$ is again constructed around this base point. Objective functions are evaluated at these points x^{76} through x^{84} and a second degree curve fitted. The equation of the curve is

$$y = 102{,}144 - 7.648\ x_1 - 3.472\ x_2$$
$$+ 271\ x_1^2 - 108\ x_1 x_2 + 152\ x_2^2.$$

The center of this curve is found as x^{85} (17.80, 18.18) with a functional value of 2960.89, which is better than the base point $x^{80} = x^{75}$. The third experiment is performed with this center, x^{85} (17.80, 18.18) as the base. A central composite design with $\alpha = 2$ is constructed around this base point. Objective functions are evaluated and a second degree curve fitted. The center of this curve is found as x^{95} (17.79, 18.16) with functional value as 2961.04 which is no better than the base point $x^{85} = x^{90}$ (17.80, 18.18). This failure ends the phase two.

The solution can be summarized as

Starting point	=	(5.00, 10.00)
Optimal point	=	(17.80, 18.18)
Optimal value	=	2960.89
No. of functional value evaluated	=	95

Table 1 shows and compares the optimal results of two-dimensional production scheduling problem obtained by Hooke and Jeeve's pattern search method [2], the sequential pattern search method [1], and the method of conjugate gradients [3]. The optimal values of the production rates differ slightly, although the minimum costs are almost identical.

REFERENCES

1. Fan, L.T., C.L. Hwang, and F.A. Tillman, "A Sequential Simplex Pattern Search Solution to Production Planning Problems", AIIE Trans. Vol. 1, No. 3, pp. 267-273 (1969).

2. Hwang, C.L., L.T. Fan, and S. Kumar, "Hooke and Jeeves Pattern Search Solution to Optimal Production Planning Problems", Report No. 18, Institute for Systems Design and Optimization, Kansas State University, Manhattan, 1969.

3. Hwang, C.L., L.T. Fan, and N.C. Pereira, "Conjugate Gradients Searching Techniques for Production Planning", Institute for Systems Design and Optimization, Kansas State University, Manhattan, 1970.

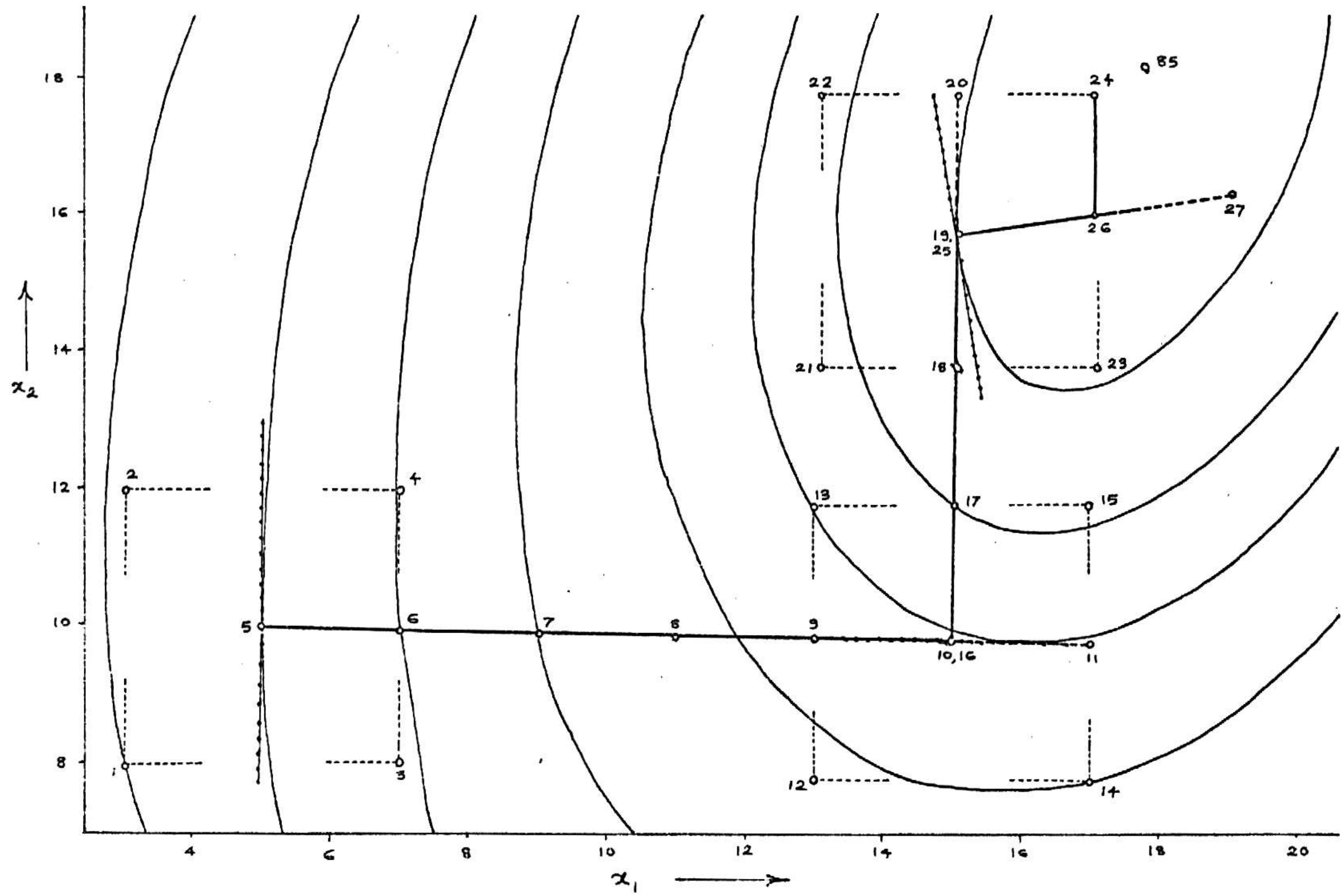


Fig. 1 Response Surface Methodology Applied to Production Scheduling Problem Involving Two Decision Variables (continued)

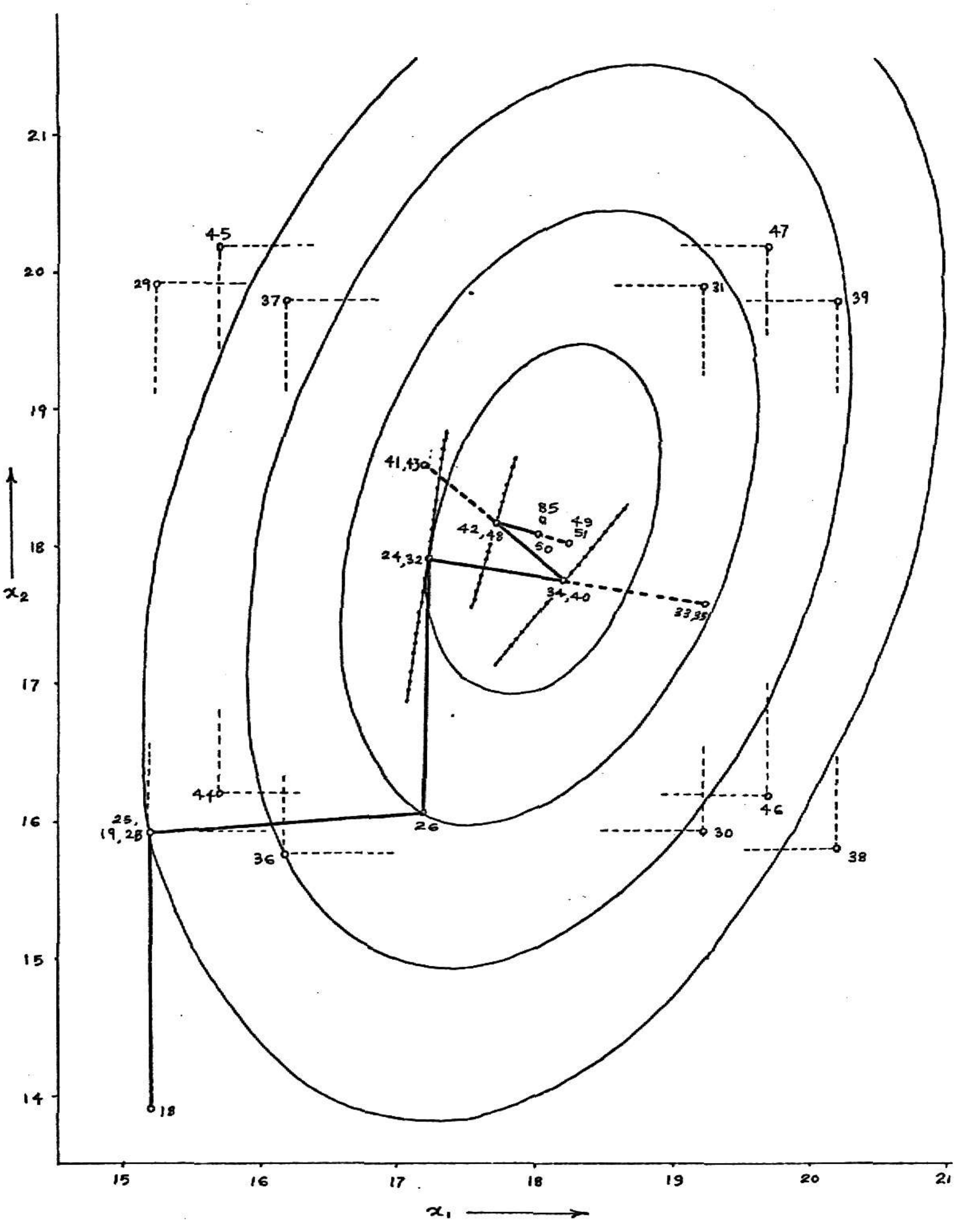


Fig. 1 Response Surface Methodology Applied to Production Scheduling Problem Involving Two Decision Variables (continued).

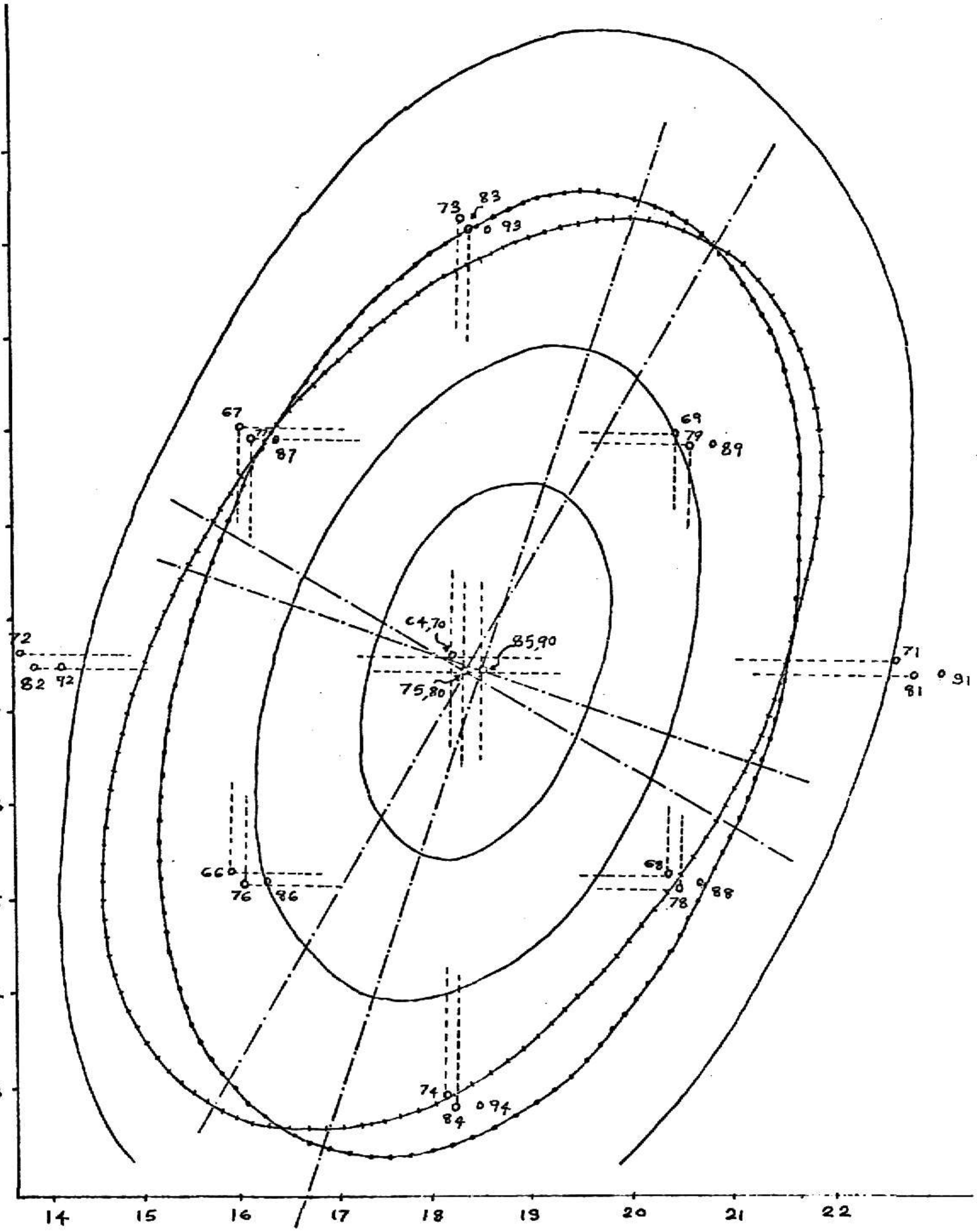


Fig. 1. Response Surface Methodology Applied to Production Scheduling Problem Involving Two Decision Variables

TABLE 1

COMPARISON OF RESULTS

Method	Minimum Point	Value of S at Minimum
1. Hooke and Jeeves [2]	(17.81, 18.21)	2960.740
2. Sequential Simplex Pattern [1]	(17.8 , 18.2)	2961.00
3. Conjugate Gradients [3]		
a) Fletcher and Powell	(17.82, 18.21)	2960.713
b) Fletcher and Reeves	(17.82, 18.21)	2960.713
4. RSM (present work)	(17.80, 18.18)	2960.89

CHAPTER 4

OPTIMAL CONTROL OF AN INTEGRATED HUMAN THERMAL SYSTEM

4.1 INTRODUCTION

Persons exposed to a thermally hostile environment can usually be protected individually by wearing properly designed external thermal regulation devices to control the body temperature. A space suit, for example, is used to protect a person who is involved in space exploration.

Different types of external thermal regulation devices have different levels of capability in protecting an individual from thermal exhaustion. Webb [24], Bitterly [1, 2], and others [6, 8] have developed the external thermal regulation devices for an entire human body. Konz and his associates [16, 18] have examined the effectiveness of the regulation device on the head. The external thermal regulation devices on the head and the torso are considered in this work. The arms and legs are assumed to be insulated from the outside environment.

The external thermal regulation device is controlled properly so that the temperature of the human body is maintained in thermal comfort (thermoneutrality) and that the possible control effort imposed on the operation of the regulation device is minimized. The regulation device consists of a network of tubes which is held in contact with the surface of the skin. A liquid coolant is assumed to be flowing constantly inside the tubes. The liquid coolant temperature and the mass flow rates are the operating variables of the device.

A study of modeling, simulation, and optimal control on an integrated human thermal system has been carried out by Hsu [14]. An integrated human thermal system is formulated by incorporating an external thermal regulation device into a human thermal system. In his work a mathematical model representing the integrated system is formulated and a nonlinear objective function are linearized. The optimal control problem is formulated into a standard linear programming form and is solved by the well-developed linear programming technique.

In the present study the response surface metholology (RSM) [4, 5, 7, 10] is employed to obtain optimal control policies for the steady-state control of coolant inlet temperature and the coolant mass flow rates for the external thermal regulation devices on the head and torso. The optimal policy is obtained for the nonlinear objective function. Also obtained is the optimal policy for the linearized objective function which was treated by Hsu [14].

The present study may be considered as a computer simulation (a numerical experiment) of the optimal control of the integrated human thermal system. It may become a guide for an actual experiment of searching for the optimal control or operation of the regulation cooling devices.

4.2 STATEMENT OF THE PROBLEM

An approach to maintain a comfortable and efficient working condition for a man working under a thermal stress is to cool the individual man [11]. The cooling system considered in this study is an external conduction thermal regulation device. It consists of a network of tubes which is held in contact with the surface of the skin and is assumed to be insulated from the outside environment. The main function of the device is to remove a portion of or the total metabolic heat generated in the body. The control parameters of the external thermal regulation device are the inlet coolant temperature and its mass flow rate.

The objectives of the controlling the regulation device are to maintain the temperature of the human body in thermal comfort (thermo-neutrality) and to minimize the possible effort imposed on the operation of the device. Specifically, the purpose of the control is to minimize the sum of (1) the deviation of temperatures at the various locations of the body from their desired comfort temperatures, and (2) the control efforts imposed on the external thermal regulation device. The problem is to find the optimal inlet coolant temperature and the mass flow rate which minimize the objective function.

It is generally accepted that the sensitive thermal receptors in hypothalamus [12, 13, 18] and in the skin [20, 22] play important roles in thermoregulation [18]. Similarly, there is a general agreement that sensitive thermal receptors exist in the spinal cord, muscle, and respiratory tract [3, 20]. Crosbie et al. [9] proposed a model of thermo-regulator which regulates the physiological parameters. The regulation is proportional to the deviation of the core temperature from a set point.

The model proposed by Stolwijk and Hardy [23] assumes that the sensitive thermal receptors are located in the core of the head, in the surface of the skin, and in the muscle. It also assumes that each of these sensitive thermal receptors has zero output when a local temperature corresponds to a set-point temperature for the core of the head, the skin, and the muscle. The set-point temperatures of the core of the head, the skin, and the muscle are supposed to determine whether the thermal response of the body is to increase heat loss by sweating and vasodilation or to increase heat storage by shivering and vasoconstriction. The "switching" action of the physiological thermostats is to prevent overheating and overcooling of the body. The following set points have been suggested by Stolwijk and Hardy [23].

T_{HC} = 36.6°C (97.9°F) for the core of the head

T_S = 34.1°C (93.4°F) for the skin

T_M = 35.9°C (96.6°F) for the muscle

As long as the set-point temperatures are maintained at appropriate locations of the body, the natural thermoregulatory functions of shivering, sweating, and vascular adjustment can be minimized.

The set-point temperatures suggested by Stolwijk and hardy [23] are included in the mathematical expression of the objective function. The expression includes the deviation of the local temperatures from their set-point temperatures and the efforts imposed on the operation of the external thermal regulation device as shown below:

$$\begin{aligned} S = {} & W_1 \left| T_{Brain} - 36.6 \right| + W_2 \left| T_{Skin} - 34.1 \right| \\ & + W_3 \left| T_{Muscle} - 35.9 \right| + W_4 \left| T_{in} - 15.6 \right| \\ & + W_5 \left| \dot{m}' - 4.539 \right| + W_6 \left| \dot{m}'' - 4.539 \right| \end{aligned} \tag{1}$$

where W_i, i = 1, 2, ..., 6, are weighting factors. The terms on the right-hand side of equation (1) represent, in order, the deviation of the brain temperature, the mean skin temperature, and the muscle temperature from each set-point temperature, an effort needed to cool or warm the liquid coolant which has the tap water temperature (15.6^{o}C), as well as an effort needed to increase the mass flow rate of the external thermal regulation device of the head and that of the torso from 4.539 Kg/hr (10 lbs/hr). The values of the weighting factors are selected as $W_1 = 2.0$, $W_2 = W_3 = 1.0$, and $W_4 = W_5 = W_6 = 0.1$. The weighting factor of the brain, W_1, is twice as large as those of the skin and the muscle, W_2 and W_3, because the function of the brain is considered to be more important than those of the skin and the muscle. The weighting factors of the inlet coolant temperature and the mass flow rates are so selected that the magnitude of the contribution of each term on the right-hand side of equation (1) to the objective function is approximately of the same order as those of first three terms.

Also considered in the present study is the objective function of the following form.

$$S = W_1 \left| T_{Brain} - 36.6 \right| + W_2 \left| T_{Skin} - 34.1 \right| + W_3 \left| T_{Muscle} - 35.9 \right|$$

$$+ W_4 \left| T_{IN} - 15.6 \right| + W_5 \left| \dot{M}' - 0.2203 \right| + W_6 \left| \dot{M}'' - 0.2203 \right| \quad (2)$$

To apply well-developed linear programming technique for obtaining optimal control policies Hsu [14] defined the objective function in the form of equation (2). Physical meaning of each term of equation (2) is the same as in equation (1). However, the values of the weighting factors are $W_1 = 2.0$, $W_2 = W_3 = W_4 = 1.0$, and $W_5 = W_6 = 100.0$. These are so selected that the magnitudes of each of the three terms of the first group will be of the same order. Similarly each of the three terms of the second group will be of the same order. The magnitude of the first group is, however, one-tenth of that of the second group. In equation (2) $\dot{M}'$ and $\dot{M}''$ are, respectively, the reciprocal of $\dot{m}'$ and $\dot{m}''$ ($1/\dot{m}'$ and $1/\dot{m}''$), and the constant (0.2203 hr/Kg) is the reciprocal of the mass flow rate of 4.539 (Kg/hr) [14].

4.3 MATHEMATICAL MODEL OF THE INTEGRATED HUMAN THERMAL SYSTEM

A model of an integrated human thermal system can be formulated by incorporating the model of an external thermal regulation device into that of a human thermal system. The mathematical model presented here is closely after that of Hsu [14]. The steady-state mathematical model of the human thermal system used here is based on one of Wissler's models [26]. The present model assumes the existence of an arterial pool and a venous pool in the torso and considers the heat exchange of the torso with adjacent elements only through the pools. Wissler's model considers the heat exchange of the torso with adjacent elements through pulmonary capillaries.

The model considers the following important factors: (1) local generation of heat by metabolic reactions, (2) conduction of heat due to thermal gradients, (3) convection of heat by circulating blood, (4) geometry of the human body, (5) existence of an insulating layer of fat and skin, (6) counter-current heat exchange between adjacent large arteries and veins, (7) sweating, and (8) condition of the environment, including its temperature, velocity, and relative humidity.

Radiation can also be considered.

The geometry of the human body on which the system equations are based is shown in Fig. 1. It consists of six cylindrical elements representing the arms, legs, torso, and head. Each element, consisting of tissue, fat and skin, has a vascular system which can be divided into three subsystems representing the arteries, the veins, and the capillaries.

The heat which is generated in an element by metabolic reactions is: (a) stored in the element, (b) carried away by circulating blood to other elements, or (c) conducted to the surface where it is generally transferred to the environment. If the environmental temperature is higher than the skin temperature, the direction of heat flow is reversed and the heat flows into the element. The heat flow can be expressed mathematically as the differential heat balance equation for the ith element as follows:

$$\frac{1}{r}\frac{d}{dr}\left(K_i\, r\, \frac{dT_i}{dr}\right) + h_{mi} + q_{ci}\,(T_{ai} - T_i) + h_{ai}\,(T_{ai} - T_i) + h_{vi}\,(T_{vi} - T_i) = 0 \tag{3}$$

The effect of heat conduction along the axis is neglected in equation (3) [19].

An assumption is also made that the temperature of the blood having the capillary beds is equal to the temperature of the neighboring tissue. This assumption is acceptable because the capillaries have very small diameters which range from 10μ to 20μ [25]. In the large arteries and veins it is necessary to assume that the rate of heat transfer from the blood in the larger vessels to the neighboring tissue is proportional to the temperature difference between the blood and tissue. The proportionality constant in the ith element is expressed by h_{ai} for the arteries and h_{vi} for the veins.

It is known that the human blood temperature in various locations of the body is different [11, 15, 17]. Therefore, two additional equations which represent the overall thermal energy balances in arteries and veins are required. In deriving such equations, it is assumed that the blood in the large arteries and veins of the ith element has uniform temperatures T_{ai} and T_{vi} respectively, as shown in Fig. 2. The resulting equations are

$$Q_{ai}\,(T_{am} - T_{ai}) + 2\pi L_i \int_0^{a_i} h_{ai}\,(T_i - T_{ai})\, r\, dr$$
$$+ H_{avi}\,(T_{vi} - T_{ai})$$
$$= 0 \tag{4}$$

$$Q_{vi}\,(T_{vn} - T_{vi}) + 2\pi L_i \int_0^{a_i} (q_{ci} + h_{vi})\,(T_i - T_{vi})\, r\, dr$$
$$+ H_{avi}\,(T_{ai} - T_{vi})$$
$$= 0 \tag{5}$$

The boundary condition which represents the heat transfer from the surface of the skin to the external thermal regulation device takes the following form:

$$-\left[K_i \frac{dT_i}{dr}\right]_{r=a_i} = H_i\left[T_i\ (a_i) - T_{ei}\right] \tag{6}$$

Due to the radial symmetry of each element the following condition also exists

$$\left(\frac{dT_i}{dr}\right)_{r=0} = 0 \tag{7}$$

The mathematical model given here is for the steady-state condition of the human thermal system. The model gives rise to the temperature distribution of the human body which is exposed to a specific environmental condition. The mathematical model of the external thermal regulation device will specify the next-to-skin environmental condition. The external thermal regulation device can be considered as a substitutive thermoregulatory device since, if the device has sufficient capability and is properly controlled, the human body can be maintained in a state of thermoneutrality. The main purpose of the external thermal regulation device is to minimize the natural thermoregulatory mechanisms of shivering and sweating by varying the skin temperature as a function of the body metabolism, and to maintain the human body in a state of thermoneutrality. The three essential operative characteristics of the external thermal regulation device are (a) the liquid coolant temperature, (b) the mass flow rate and (c) the design of the device. The skin temperatures are usually regulated by changing the liquid coolant temperature or the mass flow rate.

The schematic diagram of the external thermal regulation device of the head or the torso is shown in Fig. 3. The cylinder which represents the head or the torso is assumed to be surrounded by a network of tubes which is held in contact with the surface of the skin. The top or bottom end of the cylinder is either perfectly insulated from its environment or connected to the other cylinder through the circulation of the blood. The network of tubes is assumed to be perfectly insulated from its outside environment. Also recall that the longitudinal heat conduction is neglected in this model of the human thermal system. The liquid coolant enters the tube from one end and flows from the other end. The external thermal regulation device considered is based on the device presented by Buchberg and Harrah [6]. An assumption is made that the spacing of the tubes is so small that the skin temperature is uniform throughout the element. For example, the space suit developed by the Royal Aircraft Establishment had a center to center spacing of 3/4 inches [6].

The environmental temperature, T_{ei}, in equation (6) can be approximated by the arithmetic mean of the inlet coolant temperature, T_{in}, and the outlet coolant temperature, T_{out}, as

$$T_{ei} = \frac{1}{2}(T_{in} + T_{out}) \tag{8}$$

The heat removed, q, by the device is given by

$$q = \dot{m}\, C_p (T_{out} - T_{in})$$

where $\dot{m}$ is the mass flow rate of the coolant inside the tube, and C_p is the specific heat of the coolant. The outlet coolant temperature can be expressed by

$$T_{out} = T_{in} + \frac{q}{\dot{m}\, C_p}$$

Substituting T_{out} given by the above equation into equation (8) yields

$$T_{ei} = T_{in} + \frac{q}{2\dot{m}\, C_p} = T_{in} + \frac{q}{2\, C_p}\,\dot{M} \qquad (9)$$

where $\dot{M} = 1/\dot{m}$. T_{ei} in equation (9) can be determined for any given inlet coolant temperature, amount of heat to be removed, and mass flow rate. The generated heat by body metabolism is removed by the cooling devices on hood and torso. Arms and legs are insulated from the surrounding environment. The amount of heat removed from the cooling devices on the head and torso can be taken in proportion of 1:3. This means that the cooling hood has removed 25% of the metabolic heat, and jacket on torso 75%. The inlet coolant temperature of both hood and jacket is assumed to be same. Then equation (9) for hood and jacket becomes

$$T_{e1} = T_{in} + \frac{0.75q}{2C_p\dot{m}'} \qquad (10)$$

$$T_{e2} = T_{in} + \frac{0.25q}{2C_p\dot{m}''} \qquad (11)$$

Since arms and legs are insulated

$$H_3 = H_4 = 0 \tag{12}$$

The integrated human thermal system can be formulated by employing equations (10) through (12) with the systems equations of the human thermal system equations (3) through (7).

4.4 FINITE-DIFFERENCE APPROXIMATION OF THE MODEL

The finite-difference technique which is employed in this section enables one to consider the variation of physiological properties at various positions of the body. According to this technique, the independent variables are discretized. Each of the cylindrical elements is divided into a series of concentric cylinders and appropriate values are assigned to the physiological parameters of each concentric cylinder.

The explicit forward finite-difference technique is employed to approximate equation (3) [21]. Each term in equation (3) can be integrated from $r = r_j - (\Delta\ell_-/2)$ to $r = r_j + (\Delta\ell_+/2)$ where $\Delta\ell_-$ represents the space increment to the interior of r_j and $\Delta\ell_+$ represents the space increment to the exterior of r_j. The resulting expression is

$$K_{i+} \, (r_j + \frac{\Delta\ell_+}{2}) \, \frac{T_{i(j+1)} - T_{ij}}{\Delta\ell_+} - K_{i-} \, (r_j - \frac{\Delta\ell_-}{2}) \, \frac{T_{ij} - T_{i(j-1)}}{\Delta\ell_-}$$

$$+ \{\frac{\Delta\ell_-}{2} \, (r_j - \frac{\Delta\ell_-}{4}) \, h_{mi-} + \frac{\Delta\ell_+}{2} \, (r_j + \frac{\Delta\ell_+}{4}) \, h_{mi+}\}$$

$$+ \{\frac{\Delta\ell_-}{2} \, (r_j - \frac{\Delta\ell_-}{4}) \, [(q_{ci-} + h_{ai-})(T_{ai} - T_{ij}) + h_{vi-} \, (T_{vi} - T_{ij})]$$

$$+ \frac{\Delta\ell_+}{2} (r_j + \frac{\Delta\ell_+}{4}) [(q_{ci+} + h_{ai+})(T_{ai} - T_{ij}) + h_{vi+} (T_{vi} - T_{ij})]\}$$

$$= 0 \tag{13}$$

where T_{ij} represents the tissue temperature of the ith element at the jth radial point. The quantity with the negative subscript (-) represents the physiological parameters at the interior of r_j, whereas the positive subscript (+) represents the same properties at the exterior of r_j.

Equation (44) can be written as

$$X_{ij} T_{i(j-1)} + Y_{ij} T_{ij} + Z_{ij} T_{i(j+1)} + A_{ij} T_{ai} + V_{ij} T_{vi} = C_{ij} \tag{14}$$

where

$$X_{ij} = \frac{K_{i-}}{\Delta\ell_-} (r_j - \frac{\Delta\ell_-}{2})$$

$$Y_{ij} = \frac{K_{i+}}{\Delta\ell_+} (r_j + \frac{\Delta\ell_+}{2}) - \frac{K_{i-}}{\Delta\ell_-} (r_j - \frac{\Delta\ell_-}{2}) - \frac{\Delta\ell_-}{2} (r_j - \frac{\Delta\ell_-}{4})(q_{ci-} + h_{ai-})$$

$$- \frac{\Delta\ell_-}{2} (r_j - \frac{\Delta\ell_-}{4}) h_{vi-} - \frac{\Delta\ell_+}{2} (r_j + \frac{\Delta\ell_+}{4})(q_{ci+} + h_{ai+})$$

$$- \frac{\Delta\ell_+}{2} (r_j + \frac{\Delta\ell_+}{4}) h_{vi+}$$

$$Z_{ij} = \frac{K_{i+}}{\Delta\ell_+} (r_j + \frac{\Delta\ell_+}{2})$$

$$A_{ij} = \frac{\Delta \ell_-}{2} (r_j - \frac{\Delta \ell_-}{4})(q_{ci-} + h_{ai-}) + \frac{\Delta \ell_+}{2} (r_j + \frac{\Delta \ell_+}{4})(q_{ci+} + h_{ai+})$$

$$V_{ij} = \frac{\Delta \ell_-}{2} (r_j - \frac{\Delta \ell_-}{4}) h_{vi-} + \frac{\Delta \ell_+}{2} (r_j + \frac{\Delta \ell_+}{4}) h_{vi+}$$

$$C_{ij} = - \frac{\Delta \ell_-}{2} (r_j - \frac{\Delta \ell_-}{4}) h_{mi-} - \frac{\Delta \ell_+}{2} (r_j + \frac{\Delta \ell_+}{4}) h_{mi+}$$

It can be seen that X_{ij}, Y_{ij}, Z_{ij}, A_{ij}, V_{ij}, and C_{ij} are all functions of the physiological parameters and mesh size. The metabolic heat generations, h_{mi-} and h_{mi+}, are considered only at certain layer of the body. Assumptions have been made that heat generation by metabolic reations in the layer of fat and skin is zero and that basal metabolism is distributed uniformly throughout the element except the layer of fat and skin. Any additional heat generated by metabolic reactions due to body exercise is considered to occur in the muscle layers.

At the boundary where $r = r_J$, the integration of equation (3) from $r = r_J - (\Delta \ell_-/2)$ to $r = r_J$ can be approximated by

$$K_{i+} (r_J) [\frac{dT}{dr}]_{r=r_J} - K_{i-} (r_J - \frac{\Delta \ell_-}{2}) \frac{T_{iJ} - T_{i(J-1)}}{\Delta \ell_-}$$

$$+ \frac{\Delta \ell_-}{2} (r_J - \frac{\Delta \ell_-}{4}) [(q_{ci-} + h_{ai-})(T_{ai} - T_{iJ}) + h_{vi-} (T_{vi} - T_{iJ})]$$

$$= 0 \qquad (15)$$

The boundary condition given by equation (6) is employed to evaluate the value of $[K_{i+} (\frac{dT}{dr})]$ at $r = r_J$ in equation (15). This yields

$$- (r_J)\, H_i [T_{iJ} - T_{ei}] - K_{i-} (r_J - \frac{\Delta\ell_-}{2}) \frac{T_{iJ} - T_{i(J-1)}}{\Delta\ell_-}$$

$$+ \frac{\Delta\ell_-}{2} (r_J - \frac{\Delta\ell_-}{4}) [(q_{ci-} + h_{ai-})(T_{ai} - T_{iJ}) + h_{vi-} (T_{vi} - T_{iJ})]$$

$$= 0$$

Recall that the value of metabolic heat generation, h_{mi}, is assumed to be zero in the layer of fat and skin. The above equation can be simplified as

$$X_{iJ} T_{i(J-1)} + Y_{iJ} T_{iJ} + A_{iJ} T_{ai} + V_{iJ} T_{vi} = C_{iJ} \qquad (16)$$

where

$$X_{iJ} = \frac{K_{i-}}{\Delta\ell_-} (r_J - \frac{\Delta\ell_-}{2})$$

$$Y_{iJ} = -r_J H_i - \frac{K_{i-}}{\Delta\ell_-} (r_J - \frac{\Delta\ell_-}{2}) - \frac{\Delta\ell_-}{2} (r_J - \frac{\Delta\ell_-}{4})(q_{ci-} + h_{ai-})$$

$$- \frac{\Delta\ell_-}{2} (r_J - \frac{\Delta\ell_-}{4}) h_{vi-}$$

$$A_{iJ} = \frac{\Delta\ell_-}{2} (r_J - \frac{\Delta\ell_-}{4})(q_{ci-} + h_{ai-})$$

$$V_{iJ} = \frac{\Delta\ell_-}{2} (r_J - \frac{\Delta\ell_-}{4}) h_{vi-}$$

$$C_{iJ} = -r_J H_i T_{ei}$$

Similarly, the finite-difference approximation for equation (3) at the center of each cylindrical element can be obtained by integrating equation (3) from r = 0 to $r = \frac{\Delta\ell_+}{2}$.

$$K_{i+} (\frac{\Delta\ell_+}{2}) \frac{T_{i2} - T_{i1}}{\Delta\ell_+} + \{\frac{\Delta\ell_+}{2} (\frac{\Delta\ell_+}{4}) h_{mi+}\} + \frac{\Delta\ell_+}{2} (\frac{\Delta\ell_+}{4}) [(q_{ci+} + h_{ai+})$$

$$(T_{ai} - T_{i1}) + h_{vi+} (T_{vi} - T_{i1})]$$

$$= 0$$

or

$$Y_{i1}T_{i1} + Z_{i1}T_{i2} + A_{i1}T_{ai} + V_{i1}T_{vi} = C_{i1} \tag{17}$$

where

$$Y_{i1} = -\frac{K_{i+}}{2} - \frac{\Delta\ell_+}{2} (\frac{\Delta\ell_+}{4}) (q_{ci+} + h_{ai+}) - \frac{\Delta\ell_+}{2} (\frac{\Delta\ell_+}{4}) h_{vi+}$$

$$Z_{i1} = \frac{K_{i+}}{2}$$

$$A_{i1} = \frac{\Delta\ell_+}{2} (\frac{\Delta\ell_+}{4}) (q_{ci+} + h_{ai+})$$

$$V_{i1} = \frac{\Delta \ell_+}{2} \left(\frac{\Delta \ell_+}{4}\right) h_{vi+}$$

$$C_{i1} = - \frac{\Delta \ell_+}{2} \left(\frac{\Delta \ell_+}{4}\right) h_{mi+}$$

The coefficients X_{iJ}, Y_{iJ}, A_{iJ}, V_{iJ}, and C_{iJ} in equation (16) and the coefficients Y_{i1}, Z_{i1}, A_{i1}, V_{i1}, and C_{i1} in equation (17) are also functions of the physiological parameters and mesh size.

By using Simpson's rule [21] to carry out the integration, equation (4) becomes

$$Q_{ai} (T_{am} - T_{ai}) - \pi L_i h_{ai} T_{ai} (a_i)^2 + 2\pi L_i h_{ai} \sum_{j=1}^{J} C'_{ij} T_{ij} r_j + H_{avi} (T_{vi} - T_{ai}) = 0 \tag{18}$$

The radius of the cylinder is divided into three equally spaced intervals. The outer third is then divided into two halves. The grid points are designated as r_1, r_2, r_3, r_4, and r_5 (the center point is r_1, r_2 is 1/3 out, r_3 is 2/3 out, r_4 is 5/6 out and r_5 is the surface).

The values of C'_{ij} can be given as follows:

$$C'_{i1} = \frac{1}{3} \left(\frac{a_i}{3}\right)$$

$$C'_{i2} = \frac{4}{3} \left(\frac{a_i}{3}\right)$$

$$C'_{i3} = \frac{1}{3}\left(\frac{a_i}{3}\right) + \frac{1}{3}\left(\frac{a_i}{6}\right)$$

$$C'_{i4} = \frac{4}{3}\left(\frac{a_i}{6}\right)$$

$$C'_{i5} = \frac{1}{3}\left(\frac{a_i}{6}\right)$$

Substituting the above values of C'_{ij} into equation (18), one obtains

$$Q_{ai}(T_{am} - T_{ai}) - \pi L_i h_{ai} T_{ai}\,(a_i)^2 + 2\pi L_i h_{ai}\,\{[\frac{1}{3}\,(\frac{a_i}{3})]\,T_{i1} r_1$$

$$+ [\frac{4}{3}\,(\frac{a_i}{3})]\,T_{i2} r_2 + [\frac{1}{3}\,(\frac{a_i}{3}) + \frac{1}{3}\,(\frac{a_i}{6})]\,T_{i3} r_3$$

$$+ [\frac{4}{3}\,(\frac{a_i}{6})]\,T_{i4} r_4 + [\frac{1}{3}\,(\frac{a_i}{6})]\,T_{i5} r_5\} + H_{avi}\,(T_{vi} - T_{ai})$$

$$= 0$$

This equation can be simplified as

$$P_{i1}T_{i1} + P_{i2}T_{i2} + P_{i3}T_{i3} + P_{i4}T_{i4} + P_{i5}T_{i5} + A_{i6}T_{ai}$$

$$+ V_{i6}T_{vi} + E_{i6}T_{am}$$

$$= 0 \qquad (19)$$

where

$$P_{i1} = 2\pi L_i h_{ai} \left[\frac{1}{3}\left(\frac{a_i}{3}\right)\right] r_1$$

$$P_{i2} = 2\pi L_i h_{ai} \left[\frac{4}{3}\left(\frac{a_i}{3}\right)\right] r_2$$

$$P_{i3} = 2\pi L_i h_{ai} \left[\frac{1}{3}\left(\frac{a_i}{3}\right) + \frac{1}{3}\left(\frac{a_i}{6}\right)\right] r_3$$

$$P_{i4} = 2\pi L_i h_{ai} \left[\frac{4}{3}\left(\frac{a_i}{6}\right)\right] r_4$$

$$P_{i5} = 2\pi L_i h_{ai} \left[\frac{1}{3}\left(\frac{a_i}{6}\right)\right] r_5$$

$$A_{i6} = -Q_{ai} - \pi L_i h_{ai} (a_i)^2 - H_{avi}$$

$$V_{i6} = H_{avi}$$

$$E_{i6} = Q_{ai}$$

Equation (5) can be similarly approximated as

$$Q_{i1}T_{i1} + Q_{i2}T_{i2} + Q_{i3}T_{i3} + Q_{i4}T_{i4} + Q_{i5}T_{i5} + T_{i7}T_{ai}$$

$$+ V_{i7}T_{vi} + E_{i7}\ T_{vn}$$

$$= 0 \qquad (20)$$

where

$$Q_{i1} = 2\pi L_i \, (q_{ci} + h_{vi}) \, [\frac{1}{3} \, (\frac{a_i}{3})] r_1$$

$$Q_{i2} = 2\pi L_i \, (q_{ci} + h_{vi}) \, [\frac{4}{3} \, (\frac{a_i}{3})] r_2$$

$$Q_{i3} = 2\pi L_i \, (q_{ci} + h_{vi}) \, [\frac{1}{3} \, (\frac{a_i}{3}) + \frac{1}{3} \, (\frac{a_i}{6})] r_3$$

$$Q_{i4} = 2\pi L_i \, (q_{ci} + h_{vi}) \, [\frac{4}{3} \, (\frac{a_i}{6})] r_4$$

$$Q_{i5} = 2\pi L_i \, (q_{ci} + h_{vi}) \, [\frac{1}{3} \, (\frac{a_i}{6})] r_5$$

$$A_{i7} = H_{avi}$$

$$V_{i7} = -Q_{vi} - \pi L_i \, (q_{ci} + h_{vi})(a_i)^2 - H_{avi}$$

$$E_{i7} = -Q_{vi}$$

The system equations, equations (3), (4), and (5), with the boundary conditions, equations (6) and (7), for the ith element can now be replaced by a set of linear algebraic simultaneous equations represented by equations (14), (16), (17), (19) and (20). In summary

$$
\begin{aligned}
&Y_{i1}T_{i1} + Z_{i1}T_{i2} && + A_{i1}T_{ai} + V_{i1}T_{vi} = C_{i1} \\
&X_{i2}T_{i1} + Y_{i2}T_{i2} + Z_{i2}T_{i3} && + A_{i2}T_{ai} + V_{i2}T_{vi} = C_{i2} \\
&\qquad X_{i3}T_{i2} + Y_{i3}T_{i3} + Z_{i3}T_{i4} && + A_{i3}T_{ai} + V_{i3}T_{vi} = C_{i3} \\
&\qquad\qquad X_{i4}T_{i3} + Y_{i4}T_{i4} + Z_{i4}T_{i5} && + A_{i4}T_{ai} + V_{i4}T_{vi} = C_{i4} \qquad (21) \\
&\qquad\qquad\qquad X_{i5}T_{i4} + Y_{i5}T_{i5} && + A_{i5}T_{ai} + V_{i5}T_{vi} = C_{i5} \\
&P_{i1}T_{i1} + P_{i2}T_{i2} + P_{i3}T_{i3} + P_{i4}T_{i4} + P_{i5}T_{i5} && + A_{i6}T_{ai} + V_{i6}T_{vi} = -E_{i6}T_{am} \\
&Q_{i1}T_{i1} + Q_{i2}T_{i2} + Q_{i3}T_{i3} + Q_{i4}T_{i4} + Q_{i5}T_{i5} && + A_{i7}T_{ai} + V_{i7}T_{vi} = -E_{i7}T_{vn}
\end{aligned}
$$

The number of difference equations obtained from the technique employed in this work depends on the number of grid points used to discretize the independent variable. A set of simultaneous linear algebraic equations included in equation (21) are obtained by using five grid points to discretize the radial distance of the ith element. With seven simultaneous linear algebraic equations representing the thermal characteristics of each element, a total of twenty-eight simultaneous linear algebraic equations (seven equations each for head, torso, arm, and leg) is required to represent the human thermal system. The synthesized system equations are illustrated in Table 1. These linearized simultaneous equations of the human thermal system cna be employed to generate the temperature distribution in various elements of the body under a specified environmental condition.

The integrated human thermal system can be represented by the above 28 equations modified by equations (10) through (12). The environmental temperatures T_{ei} appears in C_{i5}, i = 1, ..., 4. These can be replaced by the expressions (10), (11) and (8) along with (12). The modified form will appear as

$$\left.\begin{aligned} C_{15} &= - a_1H_1 \left(T_{in} + \frac{0.75q}{2C_p} \dot{m}'\right) \\ C_{25} &= - a_2H_2 \left(T_{in} + \frac{0.25q}{2C_p} \dot{m}''\right) \\ C_{35} &= 0 \\ C_{45} &= 0 \end{aligned}\right\} \qquad (22)$$

Once the temperature distribution at each element is known, the temperatures of brain, skin and muscle are given as

$$\begin{aligned} T_{Brain} &= T_{21} \\ T_{Skin} &= \frac{1}{2} (T_{15} + T_{25}) \qquad (23) \\ T_{Muscle} &= \frac{1}{4} (T_{33} + T_{34} + T_{43} + T_{44}) \end{aligned}$$

4.5 EVALUATION OF OBJECTIVE FUNCTION

Thus, for any given set of control variables, objective function can be evaluated as follows:

1. Select a set of values for control variables, i.e., the coolant inlet temperature and the coolant mass flow rates for hood and jacket.
2. Substitute these values in equations (10) and (11).
3. Substitute the physiological constants given by Table 2 in equations of Table 1.
4. Solve the system of 28 simultaneous linear equations. Obtain the various temperatures

$$[T_{ij}\ (j = 1, \ldots, 5),\ T_{ai},\ T_{vi}], \qquad i = 1, \ldots, 4$$

5. Calculate the temperatures at brain, skin and muscle.
6. Substitute these temperatures and values of control variables to evaluate the objective function S given by either equation (1) or (2).

4.6 RESULTS AND DISCUSSION

Four problems of an integrated human thermal system are solved by using the present method. Two forms of the objective function given by equations (1) and (2) and two rates of metabolic heat generation are considered. The metabolic heat generation rates are chosen as 300 BTU/hr and 3000 BTU/hr. The results are presented in Tables 3 through 6.

Different points are chosen arbitrarily as the starting point in each case. Table 3 shows the results of the optimal control for a metabolic heat generation rate of 300 BTU/hr. The objective function is that given by equation (1). Test 1 starts with a coolant temperature of $70^{o}F$ and coolant mass flow rates of 15 lb/hr. and 15 lb/hr for hood and jacket respectively. The optimal control is found to correspond to a coolant temperature of 63.0°F and coolant mass flow rates of 6 lb/hr. and 10 lb/hr for hood and jacket respectively. The temperatures of brain, skin and muscle are 97.45°F, 94.81°F, and 97.98°F respectively. Test 2 starts with a combination of control values of 60°F, 15 lb/hr and 5 lb/hr. The optimal results, however, are almost the same as those of test 1. Table 4 shows the results of the optimal control for a metabolic heat generation rate of 3000 BTU/hr. The objective function is that given by equation (1). Again, two different starting points are chosen. The values of optimal control variables are, however, almost the same. Tables 5 and 6 show results of optimal control for the form of objective function given by equation (2) and for metabolic heat generation rates of 300 BTU/hr and 3000 BTU/hr. The step by step results of a problem which has the form of the objective function given by equation (1) are given in Appendix I.

Table 7 presents a comparison of the results obtained by the present method with those by Hsu [14]. Hsu has used a technique of Linear Programming for obtaining optimal values. This allows inclusion of constraints on the variables. The constraints are concerned with the capacity of the cooling device in terms of the values of the control variables, and the limitations on the temperature of brain, skin and muscle beyond their set temperatures for safe working condition. These constraints are as follows:

Coolant inlet temperature	$T_{in} \geq 15.6^{\circ}C$
Mass flow rate for jacket	$\dot{m}' \geq 10$ lbs/hr

Mass flow rate for hood $\dot{m}'' \geq 10$ lbs/hr

Brain temperature $\pm 1^{\circ}C$ of set temp.

Skin temperature $\pm 10^{\circ}C$ of set temp.

Muscle temperature $\pm 15^{\circ}C$ of set temp.

These are not taken into consideration initially for the working of the present method. However, it will be seen that the vital constraints regarding the limitations on brain, skin and muscle temperatures are not violated. The less vital constraints regarding the mass flow rates for the jacket and hood are violated, but the violation is of little practical consequence.

The temperatures of the brain, skin and muscle obtained by both methods are almost the same. However, a mass flow rate for the jacket is considerably lower by the present method than by Hsu's. The present method gives this rate as 9.81 lbs/hr. whereas Hsu has obtained a value of 29.5 lbs/hr. This difference of 19.7 causes a considerable difference in the objective function. The temperature distribution along the body is almost the same as the pattern shown in Fig. 4. It will be seen that the temperature distributions along the four elements - head, torso, arms and legs - are fairly similar in the both cases. The temperatures obtained by the present method, however, are lower by about $0.5^{\circ}F$ than those by Hsu. However, the temperature of the outer surfaces of head and torso determined by the two methods are considerably different. They are $93.2^{\circ}F$ by Hsu's method and $90.86^{\circ}F$ by the present method for the head, and $88.4^{\circ}F$ by Hsu's method and $79.74^{\circ}F$ by the present method for the torso.

A further comparison is made between the results for heat removal rates of 300 BTU/hr and 3000 BTU/hr. It is given in Table 8. It will be seen that the temperatures of brain, skin and muscle are almost the same. They are also within allowable limits. The inlet coolant temperature has decreased from 20^{o}C to 14^{o}C, whereas the mass flow rates are increased considerable from 10 lb/hr to 53 lb/hr for a higher heat removal rate. The increase in the mass flow rate for hood is, however, not considerable.

Finally a comparison is made between the results using two different forms of the objective function. It is given in Table 9. Again the temperatures of brain, skin and muscle are almost the same and within allowable limits. The mass flow rate for the jacket is unaltered, but the coolant inlet temperature and mass flow rate for the hood are lower for the objective function of equation (1) than those for the objective function of equation (1) than those for the objective function equation (2).

NOMENCLATURE

a_i = radius of the ith element

C_p = specific heat

H_{avi} = proportionality constant of direct heat transfer between large arteries and veins

H_i = heat transfer coefficient at the surface of the ith element

h_{ai} = proportionality constant of heat transfer between the arteries and tissue per unit volume

h_{mi} = metabolic heat generation per unit volume

h_{vi} = proportionality constant of heat transfer between the veins and tissue per unit volume

$K_i(r)$ = thermal conductivity of tissue, bone, fat, or skin

L_i = length of the ith element

$\dot{M}'$ = reciprocal of $\dot{m}'$

$\dot{M}''$ = reciprocal of $\dot{m}''$

$\dot{m}$ = mass flow rate of coolant inside the tube

$\dot{m}'$ = mass flow rate of coolant for jacket

$\dot{m}''$ = mass flow rate of coolant for hood

Q_{ai} = product of the mass flow rate and specific heat for blood entering the large arteries of the ith element from the mth element

Q_{vi} = product of the mass flow rate and specific heat for venous blood entering the large veins of the ith element from the nth element

q = amount of heat to be removed

q_{ci} = product of the mass flow rate and specific heat of blood entering the capillary beds per unit volume

r = distance of tissue, bone, fat or skin from the axis of element

T_{ai} = temperature of the arterial blood

$T_{am}(t)$ = temperature of the blood entering the large arteries of the blood entering the large arteries of the ith element from the mth element

T_{Brain} = temperature of the brain

T_{ei} = effective environmental temperature at the surface of the element

T_{HC} = set temperature for the core of the head

T_{in} = inlet coolant temperature

$T_i(r)$ = temperature of the tissue, bone, fat, or skin at a distance r from the axis of the ith element

T_M = set temperature for the muscle

T_{Muscle} = mean temperature of muscle

T_{out} = outlet coolant temperature

T_s = set temperature for the skin

T_{skin} = mean temperature of skin

T_{vi} = temperature of the venous blood entering the large veins of the ith element from the nth element

W_i = weighting factor

$\Delta\ell_-$ = space increment to the left of r_j

$\Delta\ell_+$ = space increment to the right of r_j

ρ = density of blood in tissue

REFERENCES

1. Bitterly, J. G., "Evaporative Cooling Garment System (ECGS)," Douglas Missile and Space System Division, 1968.

2. Bitterly, J. G., "ECGS High-Performance Liquid-Phase-Change Space Suit Garment Cooling System Development," Proceedings of the Symposium on Individual Cooling, S. Konz, Editor, Kansas State University, Manhattan, Kansas, 1969.

3. Bligh, J., "Localization of the Thermal Stimulus to Polypnoea," J. Physiol. (London) 135, 14 (1956).

4. Box, G. E. P. and K. B. Wilson, "On the Experimental Attainment of Optimum Conditions," J. of the Royal Statistical Society, Series B, vol. 13, pp. 1 - 45 (1951).

5. Box, G. E. P. "The Exploration and Exploitation of Response Surfaces: Some General Considerations and Examples," Biometrics, vol. 10, pp. 16 - 60 (1954).

6. Buchberg, H., and C. B. Harrah, "Conduction Cooling of the Human Body--A Biothermal Analysis," Thermal Problems in Biotechnology, ASME, 1968.

7. Cochran, W. G., and G. M. Cox, Experimental Designs, Wiley, New York, 1957.

8. Crocker, J. J., D. C. Jennings, and P. Webb, "Metabolic Heat Balances in Working Men Wearing Liquid-Cooled Sealed Clothing," Proceedings of the Third Manned Space Flight Meeting, American Institute of Aeronautic and Astronautics. pp. 111-117, 1954.

9. Crosbie, R. J., J. D. Hardy, and E. Fessenden, "Electrical Analog Simulation of Temperature Regulation in Man," In: Temperature, Its Measurement and Control in Science and Industry, Part III, Chapt. 55, p. 625. Reinhold, New York, (1963).

10. Davies, O. L., The Design and Analysis of Industrial Experiments, Hafner, New York, Chapter 11, 495-578, 1954.

11. Guyton, A. C., Text Book of Medical Physiology, 3rd Edition, Saunders, Philadelphia, 1966.

12. Hammel, H. T., D. C. Jackson, J. A. J. Stolwijk, J. D. Hardy, and S. B. Stromme, "Temperature Regulation by Hypothalamic Proportional Control with Adjustable Set Point," J. Appl. Physiol. 18, 1146-1154 (1963).

13. Hardy, J. D., R. F. Hellon, and K. Sutherland, "Temperature Sensitive Neurons in the Dog's Hypothalamus," J. Physiol. (London) 175, 242-253 (1964).

14. Hsu, F. T., "Modeling, Simulation, and Optimal Control of the Human Thermal System", Ph.D. Thesis, Kansas State Univeristy, 1971.

15. King, B. G. and M. J. Showers, Human Anatomy and Physiology, 6th Edition, Saunders, Philadelphia, 1969.

16. Konz, S. and J. Duncan, "Cooling With a Water Cooled Hood," Proceedings of the Symposium on Individual Cooling, S. Konz, Editor Kansas State University, 1969.

17. Licht, S., Medical Climatology, The eighth volume of Physical Medicine Library, Waverly Press, Baltimore, 1964.

18. Nakayama, T. H., J. S. Eiseman, and J. D. Hardy, "Single Unit Activity of Anterior Hypothalamus During Local Heating," Science 134, 560-561 (1961).

19. Pennes, H. H., "Analysis of Tissue and Arterial Blood Temperatures in the Resting Human Forearm," J. Appl. Physiol., 1, 93-112 (1948).

20. Robinson, S., F. R. Meyer, J. L. Newton, C. H. Ts'ao, and L. O. Holgersen, "Relations Between Sweating, Cutaneous Blood Flow, and Body Temperature in Work," J. Appl. Physiol., 20, 575 (1965).

21. Salvadori, M. G. and M. L. Baron, Numerical Methods in Engineering, 2nd Edition, Prentice-Hall, N. J., 1961.

22. Stolwijk, J. A. J., and J. D. Hardy, "Partitional Calorimetric Studies of Response of Man to Thermal Transients," J. Appl. Physiol., 21, 3, 967 (1966).

23. Stolwijk, J. A. J., and J. D. Hardy, "Temperature Regulations in Man--A Theoretical Study," Pflugers Archieve, 291, 129-162 (1966).

24. Webb, P., "Automatic Cooling: Strategies, Designs, and Evaluations," NASA SP-234, Portable Life Support Systems, 1969.

25. Wissler, E. H., "Steady-State Temperature Distribution in Man," J. Appl. Physiol, 16, (4), 734-740 (1961).

26. Wissler, E. H., "A Mathematical Model of the Human Thermal System," Bulletin of Mathematical Biophysics, 26, 147-166 (1964).

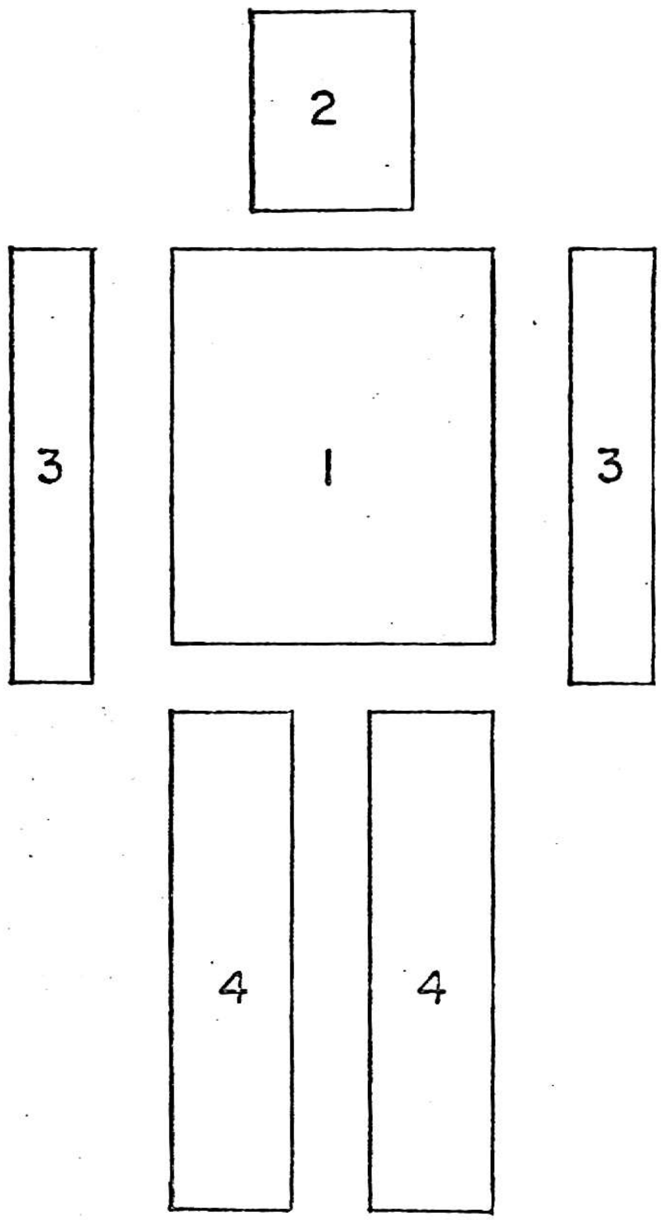


Fig. 1. A schematic diagram of the human body [14]

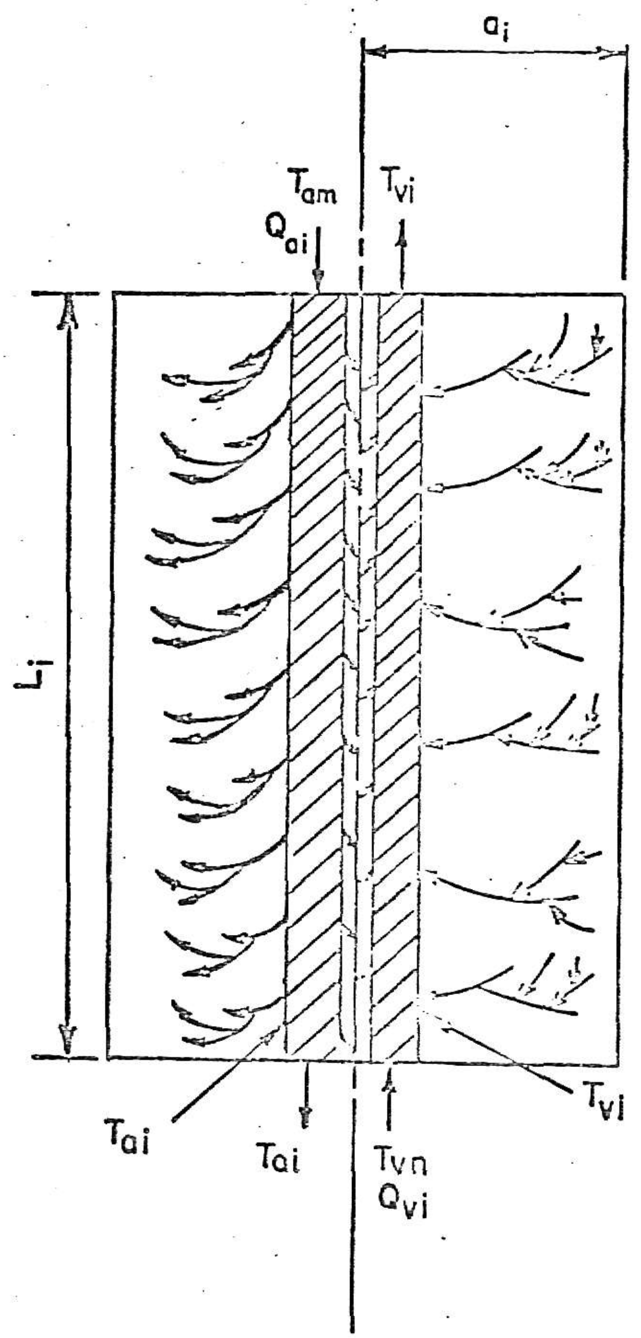


Fig. 2. Vascular system of the ith element.[14]

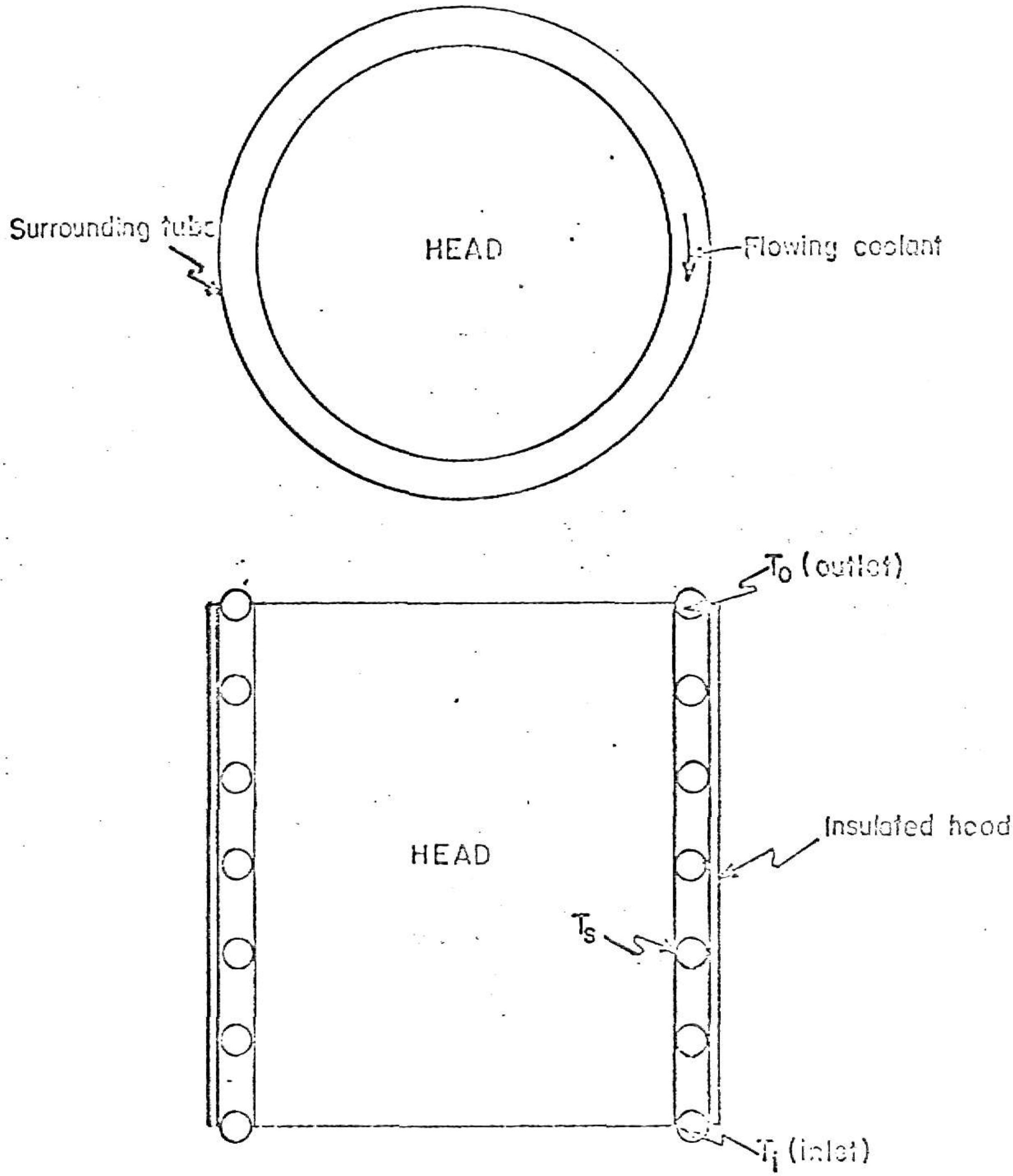


Fig. 3 Schematic diagram of conduction cooling. [14]

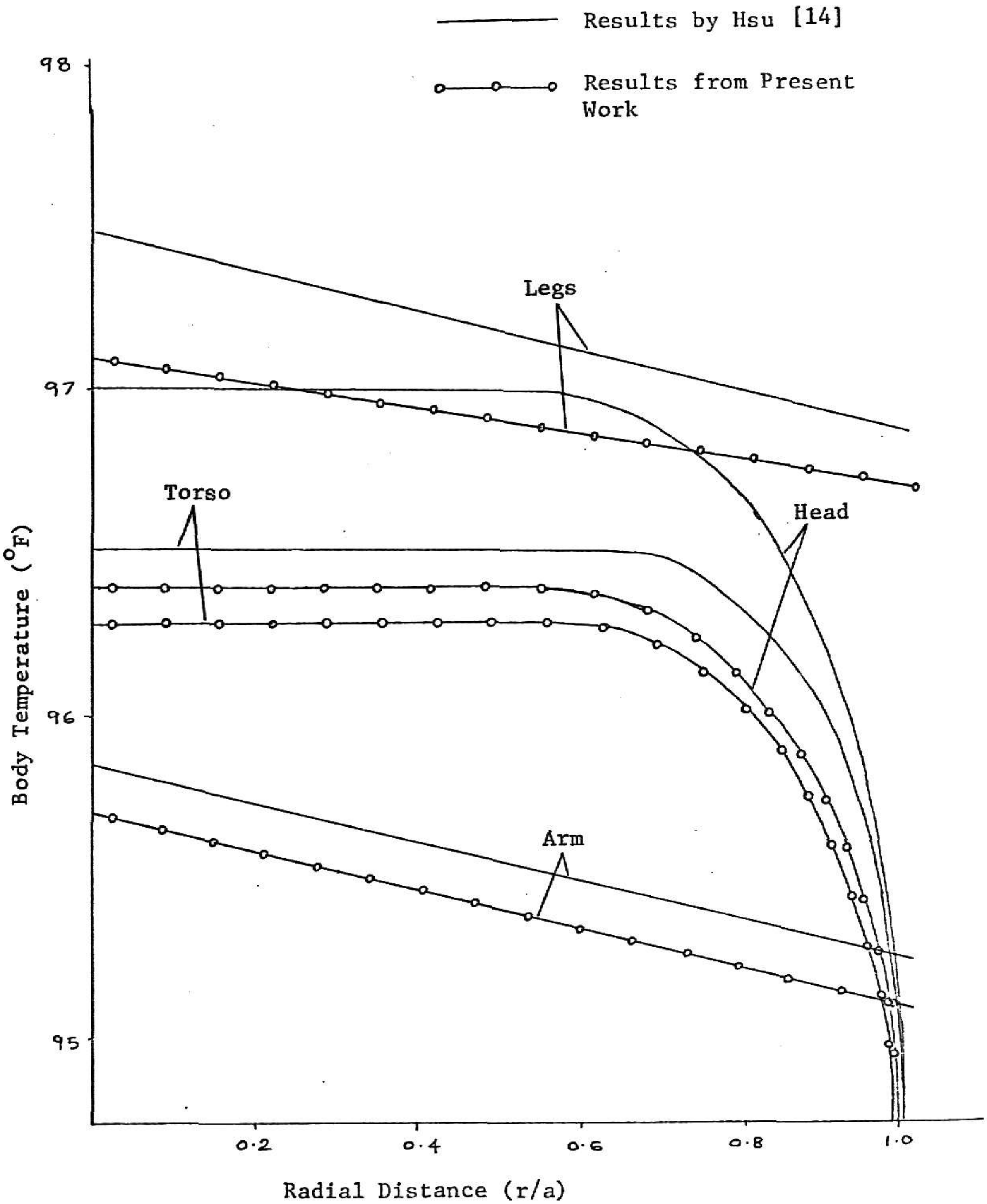


Fig. 4 Temperature Profiles in Various Elements of the Body at the Optimal Condition with Cooling Hood and Jacket

Table 1. Linear Algebraic Equations for the Human Thermal System. [14]

(Schematic representation of the form of the linear simultaneous equations)

$$Y_{21}T_{21} + Z_{21}T_{22} + A_{21}T_{a2} + V_{21}T_{v2} = C_{21}$$
$$X_{22}T_{21} + Y_{22}T_{22} + Z_{22}T_{23} + A_{22}T_{a2} + V_{22}T_{v2} = C_{22}$$
$$X_{23}T_{22} + Y_{23}T_{23} + Z_{23}T_{24} + A_{23}T_{a2} + V_{23}T_{v2} = C_{23}$$
$$X_{24}T_{23} + Y_{24}T_{24} + Z_{24}T_{25} + A_{24}T_{a2} + V_{24}T_{v2} = C_{24}$$
$$X_{25}T_{24} + Y_{25}T_{25} + A_{25}T_{a2} + V_{25}T_{v2} = C_{25}$$
$$P_{21}T_{21} + P_{22}T_{22} + P_{23}T_{23} + P_{24}T_{24} + P_{25}T_{25} + A_{26}T_{a2} + V_{26}T_{v2} + \boxed{E_{26}T_{a1}} = C_{26}$$
$$Q_{21}T_{21} + Q_{22}T_{2} + Q_{23}T_{23} + Q_{24}T_{24} + Q_{25}T_{25} + A_{27}T_{a2} + V_{27}T_{v2} = C_{27}$$
$$A_{11}T_{a1} + V_{11}T_{v1} + Y_{11}T_{11} + Z_{11}T_{12} = C_{11}$$
$$A_{12}T_{a1} + V_{12}T_{v1} + X_{12}T_{11} + Y_{12}T_{12} + Z_{12}T_{13} = C_{12}$$
$$A_{13}T_{a1} + V_{13}T_{v1} + X_{13}T_{12} + Y_{13}T_{13} + Z_{13}T_{14} = C_{13}$$
$$A_{14}T_{a1} + V_{14}T_{v1} + X_{14}T_{13} + Y_{14}T_{14} + Z_{14}T_{15} = C_{14}$$
$$A_{15}T_{a1} + V_{15}T_{v1} + X_{15}T_{14} + Y_{15}T_{15} = C_{15}$$
$$A_{16}T_{a1} + V_{16}T_{v1} + P_{11}T_{11} + P_{12}T_{12} + P_{13}T_{13} + P_{14}T_{14} + P_{15}T_{15} = C_{16}$$
$$\boxed{E_{17}T_{v2} +} A_{17}T_{a1} + V_{17}T_{v1} + Q_{11}P_{11} + Q_{12}T_{12} + Q_{13}T_{13} + Q_{14}T_{14} + Q_{15}T_{15} + \boxed{E_{17}T_{v3}{*}2} + \boxed{E_{17}T_{v4}{*}2} = C_{17}$$
$$V_{31}T_{v3} + Y_{31}T_{31} + Z_{31}T_{32} + A_{31}T_{a3} = C_{31}$$
$$V_{32}T_{v3} + X_{32}T_{31} + Y_{32}T_{32} + Z_{32}T_{33} + A_{32}T_{a3} = C_{32}$$
$$V_{33}T_{v3} + X_{33}T_{32} + Y_{33}T_{33} + Z_{33}T_{34} + A_{33}T_{a3} = C_{33}$$
$$V_{34}T_{v3} + X_{34}T_{33} + Y_{34}T_{34} + Z_{34}T_{35} + A_{34}T_{a3} = C_{34}$$
$$V_{35}T_{v3} + X_{35}T_{34} + Y_{35}T_{35} + A_{35}T_{a3} = C_{35}$$
$$\boxed{E_{36}T_{a1}} \quad V_{36}T_{v3} + P_{31}T_{31} + P_{32}T_{32} + P_{33}T_{33} + P_{34}T_{34} + P_{35}T_{35} + A_{36}T_{a3} = C_{36}$$
$$V_{37}T_{v3} + Q_{31}T_{31} + Q_{32}T_{32} + Q_{33}T_{33} + Q_{34}T_{34} + Q_{35}T_{35} + A_{37}T_{a3} = C_{37}$$
$$V_{41}T_{v4} + Y_{41}T_{41} + Z_{41}T_{42} + A_{41}T_{a4} = C_{41}$$
$$V_{42}T_{v4} + X_{42}T_{41} + Y_{42}T_{42} + Z_{42}T_{43} + A_{42}T_{a4} = C_{42}$$
$$V_{43}T_{v4} + X_{43}T_{42} + Y_{43}T_{43} + Z_{43}T_{44} + A_{43}T_{a4} = C_{43}$$
$$V_{44}T_{v4} + X_{44}T_{43} + Y_{44}T_{44} + Z_{44}T_{45} + A_{44}T_{a4} = C_{44}$$
$$V_{45}T_{v4} + X_{45}T_{44} + Y_{45}T_{45} + A_{45}T_{a4} = C_{45}$$
$$\boxed{E_{46}T_{a1}} \quad V_{46}T_{v4} + P_{41}T_{41} + P_{42}T_{42} + P_{43}T_{43} + P_{44}T_{44} + P_{45}T_{45} + A_{46}T_{a4} = C_{46}$$
$$V_{47}T_{v4} + Q_{41}T_{41} + Q_{42}T_{42} + Q_{43}T_{43} + Q_{44}T_{44} + Q_{45}T_{45} + A_{47}T_{a4} = C_{47}$$

Table 2. Numerical Data for Basal Man [11,15,17,25]

Cylindrical Element	Radius A_i(cm) [ft]	Length L_i(cm) [ft]	q_{ci}, h_{ai}, h_{vi} $\frac{cal}{cm^3 \times sec \times {}^\circ C}$ $[\frac{Btu}{ft^3 \times hr \times {}^\circ F}]$	$\dot{H}_{avi}$ $\frac{cal}{sec \times {}^\circ C}$ $[\frac{Btu}{hr \times {}^\circ F}]$	h_{mi} $\frac{cal}{cm^3 \times sec}$ $[\frac{Btu}{ft^3 \times hr}]$	Q_{ai} $\frac{cal}{sec \times {}^\circ C}$ $[\frac{Btu}{hr \times {}^\circ F}]$	Q_{vi} $\frac{cal}{sec \times {}^\circ C}$ $[\frac{Btu}{hr \times {}^\circ F}]$	Remarks
1 (torso)	14.0 [0.459]	74 [2.428]	0.00118 [0.26519]	0.00118 [0.00936]	0.000319 [129.04]	0 [0]	23.25 [184.37]	
2 (head)	8.9 [0.292]	23 [0.755]	0.00118 [0.26519]	0.00118 [0.00936]	0.000727 [294.09]	11.25 [89.21]	0 [0]	
3 (arm)	4.5 [0.148]	60 [1.969]	0.00030 [0.06742]	0.00030 [0.00238]	0.000133 [53.80]	2.4 [19.03]	0 [0]	
4 (leg)	7.0 [0.230]	77 [2.526]	0.00020 [0.04495]	0.00020 [0.00159]	0.000133 [53.80]	3.6 [28.55]	0 [0]	

TABLE 3. OPTIMAL CONTROL OF AN INTEGRATED HUMAN THERMAL SYSTEM

Form of objective function : As given by equation (1)

Heat removal rate
(Metabolic heat generation rate: 300 BTU/HR

	Test 1			Test 2		
	T_{in} ^oC [^oF]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]	T_{in} ^oC [^oF]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]
Starting Point	21.10 [70.00]	6.54 [15.00]	6.54 [15.00]	15.55 [60.00]	6.54 [15.00]	2.18 [5.00]
Optimal Point	17.22 [63.00]	2.61 [6.00]	4.36 [10.00]	16.55 [61.80]	2.63 [6.02]	4.40 [10.09]
	T_{Brain} ^oC [^oF]	T_{Skin} ^oC [^oF]	T_{Muscle} ^oC [^oF]	T_{Brain} ^oC [^oF]	T_{Skin} ^oC [^oF]	T_{Muscle} ^oC [^oF]
Set Value	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]
Actual Value	36.35 [97.45]	34.90 [94.81]	36.65 [97.98]	35.65 [96.18]	34.20 [93.54]	35.10 [96.71]
Optimal Value of Objective Function	2.566			2.580		

TABLE 4. OPTIMAL CONTROL OF AN INTEGRATED HUMAN THERMAL SYSTEM

Form of objection function : As given by equation (1)

Heat removal rate
(Metabolic heat generation rate: 3000 BTU/HR

	Test 1			Test 2		
	T_{in} °C [°F]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]	T_{in} °C [°F]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]
Starting Point	15.55 [60.00]	21.80 [50.00]	13.08 [30.00]	21.10 [70.00]	8.72 [20.00]	21.80 [50.00]
Optimal Point	14.70 [58.50]	21.58 [49.50]	10.25 [23.50]	14.60 [58.27]	21.58 [49.50]	10.25 [23.52]
	T_{Brain} °C [°F]	T_{Skin} °C [°F]	T_{Muscle} °C [°F]	T_{Brain} °C [°F]	T_{Skin} °C [°F]	T_{Muscle} °C [°F]
Set Value	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]
Actual Value	36.60 [97.90]	35.30 [95.55]	36.80 [98.24]	36.50 [97.66]	35.10 [95.20]	36.80 [98.21]
Optimal Value of Objective Function	3.266			3.271		

TABLE 5. OPTIMAL CONTROL OF AN INTEGRATED HUMAN THERMAL SYSTEM

Form of objective function : As given by equation (2)

Heat removal rate
(Metabolic heat generation rate: 300 BTU/HR

	Test 1			Test 2			Test 3		
	T_{in} °C [°F]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]	T_{in} °C [°F]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]	T_{in} °C [°F]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]
Starting Point	10.00 [50.00]	8.72 [20.00]	8.72 [20.00]	15.55 [60.00]	4.36 [10.00]	6.54 [15.00]	26.55 [80.00]	2.18 [5.00]	2.18 [5.00]
Optimal Point	20.30 [68.54]	4.32 [9.91]	4.28 [9.81]	19.90 [67.91]	4.32 [9.91]	4.45 [10.21]	20.00 [68.79]	4.39 [10.08]	4.26 [9.77]
	T_{Brain} °C [°F]	T_{Skin} °C [°F]	T_{Muscle} °C [°F]	T_{Brain} °C [°F]	T_{Skin} °C [°F]	T_{Muscle} °C [°F]	T_{Brain} °C [°F]	T_{Skin} °C [°F]	T_{Muscle} °C [°F]
Set Value	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]
Actual Value	35.81 [96.44]	34.50 [94.10]	36.02 [96.80]	35.65 [96.17]	34.14 [93.47]	35.66 [96.19]	35.85 [96.53]	34.55 [94.19]	35.66 [96.19]
Optimal Value of Objective Function	7.421			7.752			7.643		

TABLE 6. OPTIMAL CONTROL OF AN INTEGRATED HUMAN THERMAL SYSTEM

Form of objective function : As given by equation (2)

Heat removal rate
(Metabolic heat generation rate: 3000 BTU/HR

	Test 1			Test 2			Test 3		
	T_{in} °C [°F]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]	T_{in} °C [°F]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]	T_{in} °C [°F]	$\dot{m}'$ Kg/hr [lb/hr]	$\dot{m}''$ Kg/hr [lb/hr]
Starting Point	15.55 [60.00]	21.80 [50.00]	8.72 [20.00]	10.00 [50.00]	17.44 [40.00]	2.18 [5.00]	21.10 [70.00]	17.44 [40.00]	6.54 [15.00]
Optimal Point	14.03 [57.24]	4.36 [10.00]	23.25 [53.24	11.40 [52.59	4.36 [10.00]	18.75 [43.01]	13.50 [56.00	4.30 [9.86]	21.87 [50.15]
	T_{Brain} °C [°F]	T_{Skin} °C [°F]	T_{Muscle} °C [°F]	T_{Brain} °C [°F]	T_{Skin} °C [°F]	T_{Muscle} °C [°F]	T_{Brain} °C [°F]	T_{Skin} °C [°F]	T_{Muscle} °C [°F]
Set Value	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]	36.60 [97.90]	34.10 [93.40]	35.90 [96.60]
Actual Value	36.45 [97.64]	35.65 [96.21]	36.40 [97.56]	36.35 [97.48]	35.45 [95.85]	36.35 [97.49]	36.46 [97.66]	35.65 [96.19]	36.44 [97.59]
Optimal Value of Objective Function	21.833			23.371			22.566		

TABLE 7. COMPARISON OF RESULTS BETWEEN THE PRESENT METHOD AND THE HSU'S FOR 300 BTU/HR METABOLIC HEAT GENERATION RATE.

VARIABLE	SET POINT	RESULTS: PRESENT	RESULTS: HSU'S [14]
Brain Temperature	$36.6^\circ C$ ($97.9^\circ F$)	$35.81^\circ C$ ($96.46^\circ F$)	$36.10^\circ C$ ($96.98^\circ F$)
Skin Temperature	$34.1^\circ C$ ($93.4^\circ F$)	$34.50^\circ C$ ($94.10^\circ F$)	$34.90^\circ C$ ($94.82^\circ F$)
Muscle Temperature	$35.9^\circ C$ ($96.6^\circ F$)	$36.02^\circ C$ ($96.84^\circ F$)	$35.70^\circ C$ ($96.26^\circ F$)
Coolant Temperature	$15.6^\circ C$ ($60.08^\circ F$)	$20.30^\circ C$ ($68.54^\circ F$)	$24.50^\circ C$ ($76.1^\circ F$)
Mass Flow Rate for Hood	4.36 Kg/hr (10 lb/hr)	4.32 Kg/hr (9.91 lb/hr)	4.36 Kg/hr (10.00 lb/hr)
Mass Flow Rate for Jacket	4.36 Kg/hr (10 lb/hr)	4.28 Kg/hr (9.81 lb/hr)	12.86 Kg/hr (29.50 lb/hr)
Objective Function		7.421	29.990

TABLE 8. COMPARISON OF RESULTS FOR METABOLIC HEAT GENERATION RATES OF 300 BTU/HR AND 3000 BTU/HR.

VARIABLE	SET POINT	RESULTS 300 BTU/HR	RESULTS 3000 BTU/HR
Brain Temperature	36.6°C (97.9°F)	35.81°C (96.46°F)	36.47°C (97.64°F)
Skin Temperature	34.1°C (93.4°F)	34.50°C (94.10°F)	35.67°C (96.21°F)
Muscle Temperature	35.90°C (96.6°F)	36.02°C (96.84°F)	36.42°C (97.56°F)
Coolant Inlet Temperature	15.6°C (60.08°F)	20.30°C (68.54°F)	14.02°C (57.56°F)
Mass Flow Rate for Hood	4.36 Kg/hr (10 lb/hr)	4.32 Kg/hr (9.91 lb/hr)	4.36 Kg/hr (10.00 lb/hr)
Mass Flow Rate for Jacket	4.36 Kg/hr (10 lb/hr)	4.28 Kg/hr (9.81 lb/hr)	23.26 Kg/hr (53.34 lb/hr)
Objective Function		7.421	21.833

TABLE 9. COMPARISON OF RESULTS USING OBJECTIVE FUNCTIONS OF EQUATIONS (1) AND (2)

VARIABLE	SET POINT	RESULTS EQUATION (2)	EQUATION (1)
Brain Temperature	36.6°C (97.9°F)	35.81°C (96.44°F)	36.36°C (97.45°F)
Skin Temperature	34.1°C (93.4°F)	34.50°C (94.10°F)	34.89°C (94.81°F)
Muscle Temperature	34.9°C (96.6°F)	36.02°C (96.84°F)	36.65°C (97.98°F)
Coolant Inlet Temperature	15.6°C (60.08°F)	20.30°C (68.54°F)	17.22°C (63.00°F)
Mass Flow Rate for Hood	4.36 Kg/hr (10.00 lb/hr)	4.32 Kg/hr (9.91 lb/hr)	2.62 Kg/hr (6.00 lb/hr)
Mass Flow Rate for Jacket	4.36 Kg/hr (10.00 lb/hr)	4.28 Kg/hr (9.81 lb/hr)	4.36 Kg/hr (10.00 lb/hr)
Objective Function		7.421	2.566

CHAPTER 5

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

The response surface methodology is basically an experimental method for obtaining an optimal value of a function of multi-dimensional variables. This report presents a scheme for carrying out a prior computer simulation of the experimental search for optimum values by means of response surface methodology. Computer simulation being less costly and more flexible provides a very good guide line for actual experimentation.

A computer program to obtain an optimum point by means of response surface methodology is developed. To demonstrate the method, the solution of a two-dimensional production scheduling problem is presented in details.

The report also presents results of computer simulations of the optimal control of an integrated human thermal system by the method. A study of modeling, simulation and optimal control of an integrated human thermal system has been carried out by Hsu. He has determined the optimal control policy by employing a well-known technique of linear programming. His results can be verified experimentally at the KSU-ASHRAE test facility. The present study provides a numerical experimentation (simulation) of the optimal control problem using the response surface methodology. A comparison is made between the results obtained and those by Hsu. Apparantly a better set of conditions is obtained. In additon to the form of objective function used by Hsu, which is required for linear programming, a different and more rigorous form is used to obtain a set of optimal conditions. This set is also better than that obtained by Hsu in the sense that it requires less efforts - physiological and operating the device - and still maintains the body in thermoneutrality.

Suggestions for further improvement in the computer program are

1. The computer program developed is a basic program for an experimental method. The designs provided in the present program are
   a. factorial designs of 2^n type
   b. central composite designs
   c. non-central composite designs.

   However, when the number of variables (factors) is more than five, the size of the experiment becomes too large. A fractional factorial designs - one-half, one-fourth, one-eighth etc. - are then required to keep the number of points in the practical limit. Subroutines are necessary to handle this type of designs depending on the number of variables.
2. The present computer program does not provide a means of conversion of the second degree equation from a conical to a canonical form. A computer program is necessary to develop to include this feature which will help in analyzing the characteristics of the fitted curves.

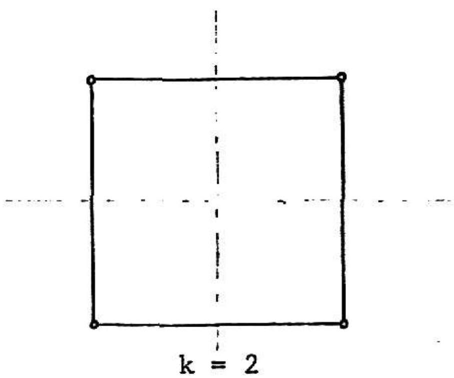
APPENDIX A

FIRST ORDER DESIGNS

These designs are suitable for determining the first order differential coefficients. Since only the first degree terms are involved in the polynomial, two levels of factors are sufficient for a unique fit. Each factor of the set, x_1, x_2, ..., x_k, is varied only at two levels. These levels are taken, for convenience, to be -1 and +1. The 'zero' level of a factor indicates the normal condition or the absence of any modification. The point at which all factors are at the zero level serves as the base point or the center point of the design. '-1' level refers to the lower level and '+1' level refers to the higher level, with respect to the centre point.

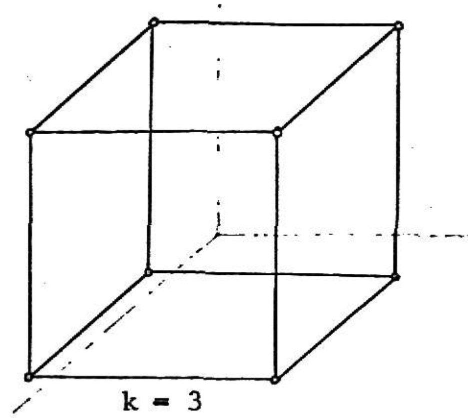
A. Factorial Designs

These are called 2^n factorial designs. A complete factorial design of this type consists of 2^k points in space at the vertices of a k-dimensional hypercube of side 2 units with its centre at the point (0, 0, ..., 0). The design points for k = 2 and k = 3 are shown below.



k = 2

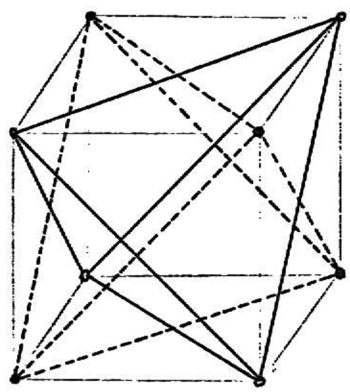
No. of points = $2^2 = 4$



k = 3

No. of points = $2^3 = 8$.

If k, the number of factors is five or more, the complete factorial design may become prohibitatively large. However, the number of trials can be reduced considerably, with sacrifice of certain unimportant information, but still retaining important information, by a technique called confounding in fractional factorial designs. These designs are a half, a quarter, a one-eighth, a one-sixteenth etc. replicate of a complete factorial design. A typical one-half-replicate designs of 2^3 factorial are shown below.



One design consists of solid points and another of hollow points. It requires only one-half the number of trials of complete factorial designs. The design points are at vertices of a regular tetrahydron.

B. First order designs of type A

Designs of type A of order d can be defined as those which give unbiased estimates of all derivatives of order 1 to d, providing that the assumption is strictly true that all terms of higher degree may be ignored. In this, the number of trials N can be as small as M, the number of constants to be determined. First order designs of this type are provided by the multifactorial designs of Plackett and Burman.

Designs for $k = 3, 7, 11, \ldots, 4m-1, 99$ factors in $N = 4, 8, 12, \ldots, 4m, \ldots, 100$ experiments are developed. For intermediate values of k, the next higher design must be used.

C. First order designs of type B

Designs of type B of order d can be defined as those which give unbiased estimates of all derivatives of order 1 to d, even though terms of order d + 1 exist. In this, the number of trials N, must be larger than M, the number of constants to be determined.

These designs can always be obtained by duplicating the appropriate first order design of A with reversed sign. Thus for type B designs, for k factors, 2N experiments are necessary. Since for each factor the levels +1 and -1 occur equal number of times, none of the first order estimates are biased by quadratic effects. Thus, if curvature of a surface, however great, could be expressed in terms of quadratic effects alone, the estimates of the first order effects would remain completely unaffected. On the other hand, the existance of mixed second order derivatives, corresponding to the two-factor interactions, may bias these estimates.

APPENDIX B

THE CO-ORDINATES OF P ARE PROPORTIONAL TO THE FIRST ORDER DERIVATIVES AT P

It is required to locate a point P, at a distance of r from the origin 0, in the k-dimensional space, such that the response at P is maximum (minimum).

Let the point P be $(x_1, x_2, \ldots, x_k)$. Then the response function at point P is given by

$$\phi(P) = \phi(x_1, x_2, x_3, \ldots, x_k) \tag{B-1}$$

This is required to be maximum (minimum), subject to

$$OP = r$$

or

$$r^2 = \sum_{i=1}^{k} x_i^2 \tag{B-2}$$

Using Lagrange's method of undetermined multipliers, a new function ψ can be defined as

$$\psi = \phi(P) + \lambda \sum_{i=1}^{k} x_i^2 \tag{B-3}$$

where λ is Lagrange's multiplier. This can be maximized (minimized) by putting all first order partial derivatives $\frac{\partial \psi}{\partial x_i}$, $i = 1, 2, \ldots, k$

equal to zero. This gives

$$\frac{\partial \phi(P)}{\partial x_i} + 2\lambda\ x_i = 0$$

or

$$x_i = -\frac{1}{2\lambda} \cdot \frac{\partial \phi(P)}{\partial x_i} \qquad \text{(B-4)}$$

λ can be determined as

$$\lambda = -\frac{1}{2} \cdot \frac{1}{x_i} \cdot \frac{\partial \phi(P)}{\partial x_i}\ , \qquad i = 1, 2, \ldots, k$$

Therefore,

$$\lambda = -\frac{1}{2} \cdot \frac{\phi_i(P)}{x_i}$$

$$= -\frac{1}{2} \sqrt{\sum_{j=1}^{k} [\phi_j(P)]^2} \Big/ \sqrt{\sum_{j=1}^{k} x_j^2}$$

$$= -\frac{1}{2} \ \frac{\sqrt{\sum [\phi_j(P)]^2}}{r} \qquad \text{(B-5)}$$

(where $\phi_i(P) = \frac{\partial \phi(P)}{\partial x_i}$)

Substituting λ back in equation (B-4) yields

$$x_i = \frac{-\ \phi_i(P)}{2\lambda}$$

$$= \frac{+\ \phi_i(P)}{\overline{\sum[\phi_j(P)]^2}/r}$$

$$= \frac{r}{\overline{\sum[\phi_j(P)]^2}}\ \phi_i(P)$$

$$= \mu \cdot \phi_i(P) \tag{B-6}$$

where

$$\mu = \frac{r}{\overline{\sum[\phi_j(P)]^2}}$$

$$= \text{constant.}$$

Therefore, x_i, co-ordinates of the point P, at a distance of r from 0, at which the response is maximum (minimum) are proportional to the first order partial derivatives of a response function at P, $\frac{\partial\phi(P)}{\partial x_i}$.

APPENDIX C

DETERMINATION OF CONSTANTS IN POLYNOMIAL BY REGRESSION

Suppose that, within a given region of k-dimensional space, the response function is represented by a model given by a polynomial of a degree d. The model will then include all the terms of degree d and less and can be represented by

$$\eta = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \cdots + \beta_k X_k$$

$$+ \beta_{11} X_1^2 + \beta_{22} X_2^2 + \cdots + \beta_{12} x_1 x_2$$

$$+ \beta_{111} X_1^3 + \cdots \qquad \text{(C-1)}$$

where η is the true response at the points $(x_1, x_2, \ldots, x_k)$ in the region concerned. Further if the model involves L terms then it can be written as

$$\eta = \sum_{i=0}^{L-1} \beta_i X_i \qquad \text{(C-2)}$$

where X is called an independent variable and consists of powers and cross products of the co-ordinates of the points such as

$$X_0 = 1$$

$$X_i = x_i \qquad i = 1, 2, \ldots, k$$

$$X_{k+i} = x_i^2 \qquad i = 1, 2, \ldots, k$$

$$X_{2k+i} = x_1 x_i \qquad i = 2, 3, \ldots, k \qquad \text{etc.}$$

When (C-2) is expressed in the matrix form

$$\eta = X\beta, \tag{C-3}$$

η is called a matrix of true responses, X a matrix of independent variables, and β a matrix of constants.

In order to determine the model completely the constants β's should be evaluated. This can be done by observing the response at a sufficient number of points placed suitably in the region concerned and fitting a regression curve.

If N points (x_1, x_2, ..., x_k) are chosen for this, these points constitute what is known as experimental design. A trial is carried out at each of these N points in turn. The co-ordinates of an experimental point are the levels of k factors at that trial. The N x k matrix D whose elements are the co-ordinates of the N experimental points is called a design matrix. The observations at these points form a vector Y of observed responses. Y is assumed to be distributed normally with mean η and variance σ^2, i.e. $E(y) = \eta$. Then the model can be written as

$$Y = X\beta + \varepsilon \tag{C-4}$$

where ε is the matrix of experimental errors, which are normally distributed with mean zero and variance σ^2. The method of least squares requires $\varepsilon'\varepsilon$ to be minimized.

$$\varepsilon = Y - X\beta$$

$$\therefore \quad \varepsilon'\varepsilon = (Y - X\beta)'(Y - X\beta)$$

$$= Y'Y - \beta'X'Y - Y'X\beta + \beta'X'X\beta$$

$$= Y'Y - 2\beta'X'Y + \beta'X'X\beta \qquad \text{(C-5)}$$

since $\beta'X'Y = Y'X\beta$ both being scalar quantities.

$\varepsilon'\varepsilon$ will be minimimum if $\frac{\partial(\varepsilon'\varepsilon)}{\partial\beta} = 0$

i.e.

$$- 2X'Y + 2X'XB = 0$$

or

$$B = (X'X)^{-1} (X'Y).$$

Thus if B is the vector of estimates, it is given by

$$B = [(X'X)^{-1}X']Y \qquad \text{(C-6)}$$

The least square estimate B is an unbiased estimate of β in the model i.e. $E(B) = \beta$.

The variances and co-variances of these estimates are given by $C^{-1}\sigma^2$ where the LxL matrix C is given by $C = X'X$ and consists of sums of squares and products of the independent variables. C^{-1} is called a precision matrix.

In practice, it is desired that the estimates b_0, b_1, b_2 etc. (forming a matrix B) should be unbiased and should have minimum variance. The assumption of unbiased estimates is true, however, only if the postulated model is the correct model to consider. If it is not the correct model, then the estimates are biased i.e., $E(B) \neq \beta$. The extent of the biases depends not only on the postulated and correct model, but also on the values of X, the independent variables entering the regression calculations.

In Response Surface Methodology, the correct model is seldom known or used. Generally an approximation of a lower degree of polynomial is used as a model. Hence bias in the estimates is unevitable. A good choice of an experimental design provides the estimates with less bias even if a model other than the correct one is postulated and fitted.

APPENDIX D

SECOND ORDER DESIGNS

These designs are suitable for determining the first and second order differential co-efficients. For a unique fit of the second degree polynomial, at least three levels are required. Hence three-level factorial designs - called 3^n designs - may be employed. In this type of design, if k is the member of factors, then 3^k number of trials are required. Unfortunately when k is greater than 2, the number of trials required often greatly exceeds the number of constants to be estimated. Consequently, when maximum economy in experiment is essential, 3^n designs are unsatisfactory. With four factors, for example, the number of trials required is $3^4 = 81$ whereas number of constants to be determined is only 15. The number of trials can be reduced by fractional replication. This method, however, is much less effective.

For these reasons, alternative designs have been sought which will give all necessary estimates without necessitating a number of trials greatly in excess of the number of constants to be determined. These are called Composite Designs.

These can be built up from two-level factorials or fractional factorials. The procedure is first to choose a two-level design so that all effects of the first order and all interaction effects of second order can be estimated. This design is then supplimented with further points which allow the estimation of the quadratic effects.

The inherent advantage of the composite designs is that they allow the work to proceed in stages. In the first stage, a first order design can be completed, and if it is found to be necessary to estimate second order effects, extra points can be added. Thus the work carried out is not wasted, and addition of extra points placed strategically gives a fair estimation of the second order effects.

There are two types of composite designs - central and non-central composite designs. When it appears that the region of interest lies in the centre of the first order design, then a central composite design is used. Whereas when it appears to lie towards one corner of the first order design, then a non-central composite design is used.

APPENDIX E

CONVERSION FROM A CONICAL TO A CANONICAL FORM

The equation of a curve in a canical form is

$$\eta = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{j=1}^{k} \beta_{ij} x_i x_j$$

This is to be converted in a cononical form as

$$Y - Y_s = \sum_{i=1}^{k} \lambda_i X_i^2$$

where Y is response, Y_s is response at s, stationary point, λ_1, λ_2, ..., λ_k are coefficients and X_1, X_2, ..., X_k are principal axis. The procedure for conversion is to

i) calculate the position of s, centre of the system of contours and the value. Also, calculate Y_s, the response at s.

ii) calculate the value of λ_1, λ_2, ..., λ_k

iii) calculate the directions of X_1, X_2, ..., X_k.

i) Calculate s and Y_s

s is the point at which the response is stationary, i.e.,

$\frac{\partial n}{\partial x_1}$, $\frac{\partial n}{\partial x_2}$, ... are zero. Differentiating η with respect to x_1, x_2, ..., x_k in turn, k linear simultaneous equations are obtained. On solving these simultaneous equations for the co-ordinates of s, $(x_{1s}, x_{2s}, \ldots, x_{ks})$, the stationary point on the fitted surface is obtained. On substituting these values, Y_s, the predicted response at s is obtained.

ii) Determining coefficients λ_1, λ_2, ... etc.

determinant Q = 0 (E-1)

is called a characteristic equation, where the Determinant Q is such that

$$\left.\begin{array}{lll} \|Q_{ii}\| = \beta_{ii} - \lambda & & i = 1, \ldots, k \\ \text{and} & & \\ \|Q_{ij}\| = \frac{1}{2}\,\beta_{ij} & i \neq j & \begin{array}{l} i = 1, \ldots, k \\ j = 1, \ldots, k \end{array} \end{array}\right\} \qquad \text{(E-2)}$$

where

β_{ii} are the quadratic effects of factor i and

β_{ij} are the two-factor interaction effects between factors i and j.

The roots of this characteristic equation give the required coefficients λ_1, λ_2, ..., etc.

iii) Determining the direction of axes

The following k equations should first be obtained

$$\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_k \end{pmatrix} = M \cdot \begin{pmatrix} x_1 - x_{1s} \\ x_2 - x_{2s} \\ \vdots \\ x_k - x_{ks} \end{pmatrix} \qquad \text{(E-3)}$$

where

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & \cdots & m_{1k} \\ m_{21} & m_{22} & m_{23} & \cdots & m_{2k} \\ \vdots & & & & \\ m_{k1} & m_{k2} & m_{k3} & \cdots & m_{kk} \end{pmatrix}$$

The restrictions on M are -

i) The sum of squares of the elements in any row or column should be equal to 1 i.e.

$$\sum_{j=1}^{k} (m_{ij})^2 = 1 \text{ or } \sum_{i=1}^{k} (m_{ij})^2 = 1 \qquad i = 1, \ldots, k \tag{E-4}$$

ii) The sum of products of the elements in any row or column should be equal to zero. i.e.

$$\left.\begin{array}{ll} \sum_{i=1}^{k} m_{ij} \cdot m_{i\ell} = 0 & \begin{array}{l} j = 1, \ldots, k \\ \ell = 1, \ldots, k \\ j \neq \ell \end{array} \\ \text{or} & \\ \sum_{j=1}^{k} m_{ij} \cdot m_{\ell j} = 0 & \begin{array}{l} i = 1, \ldots, k \\ \ell = 1, \ldots, k \\ i \neq \ell \end{array} \end{array}\right\} \tag{E-5}$$

Then the equation of X_t - axis will be given by solving k-1 equations

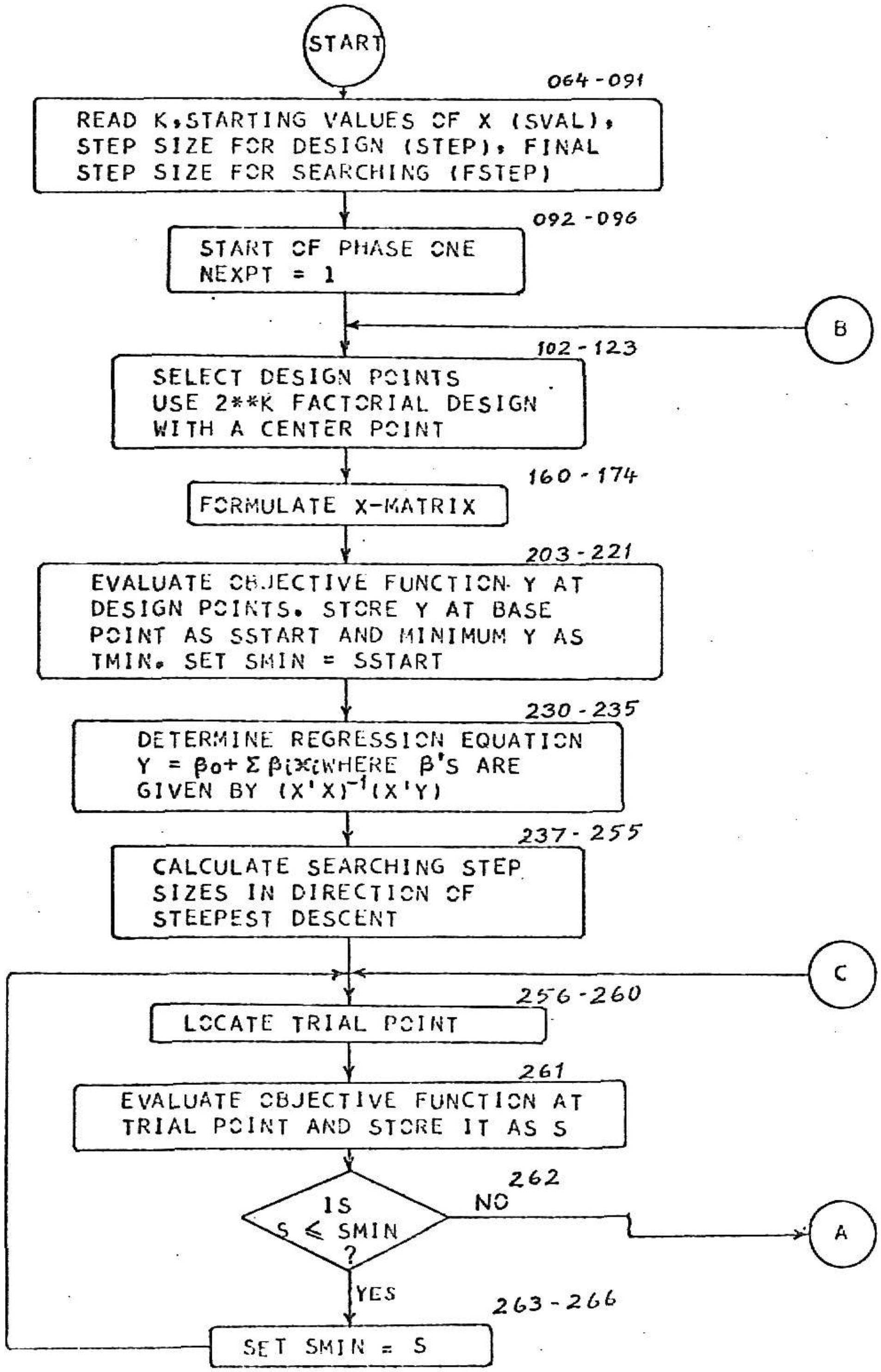
$$X_1 = 0, \quad X_2 = 0, \ldots, X_{t-1} = 0, \quad X_{t+1} = 0, \ldots, X_k = 0$$

Also if the equation of a plane through the X_s and X_t axes is required, it can be obtained by solving k - 2 equations

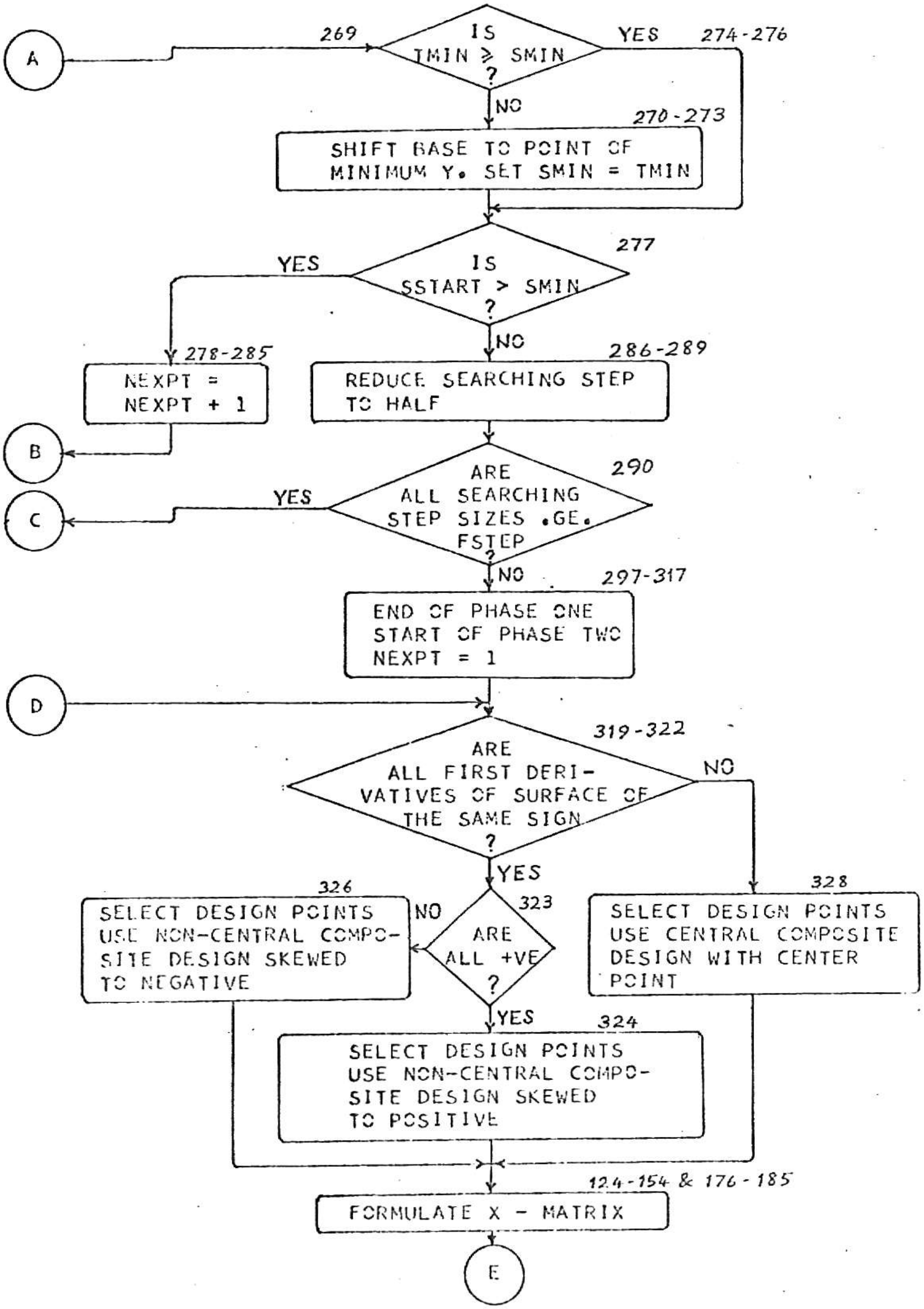
$X_1 = 0, \ldots, X_k = 0$ where $X_s = 0$ & $X_t = 0$ are omitted.

APPENDIX F

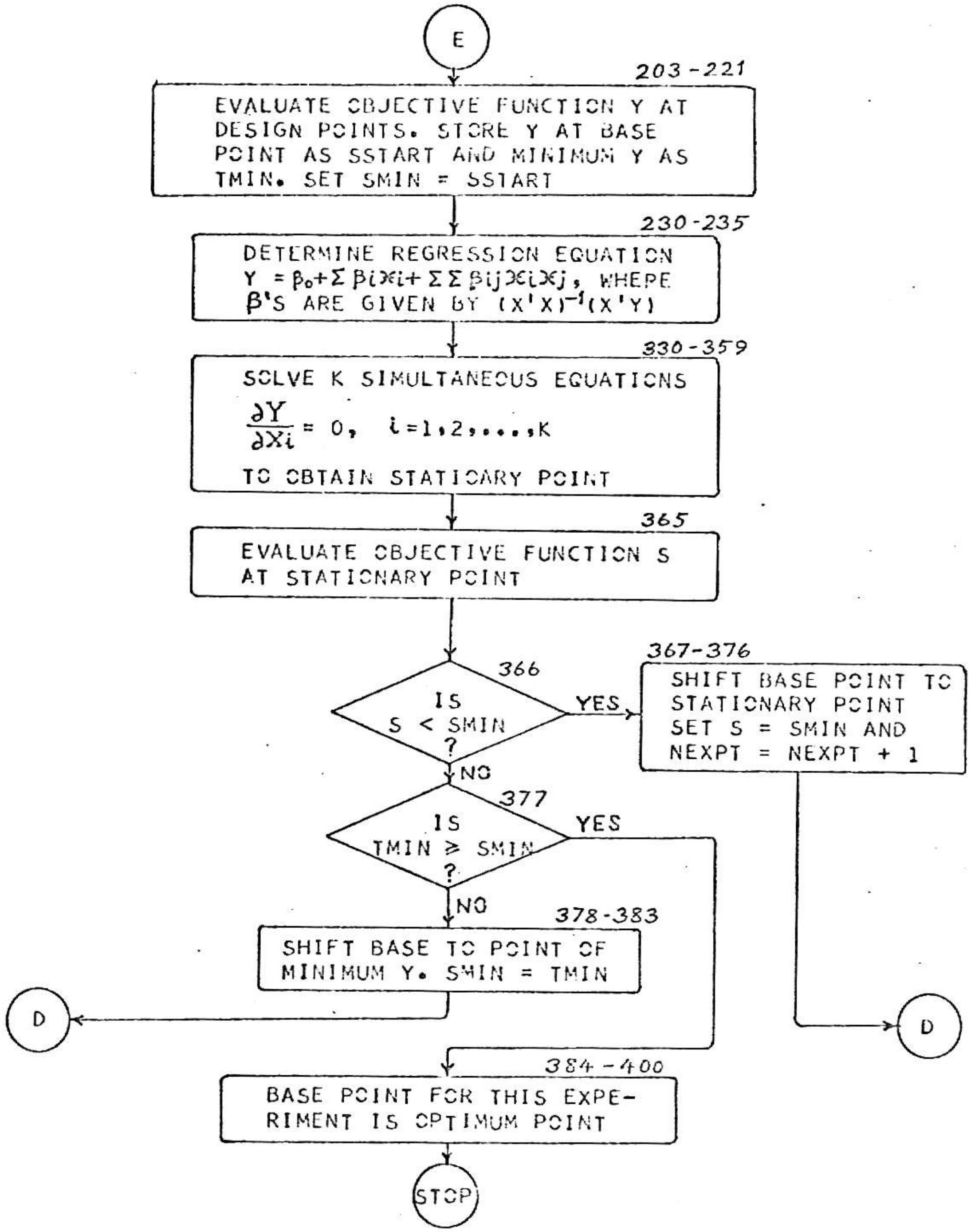
FLOW CHART FOR MAIN COMPUTER PROGRAM



Flow Chart (continued)



Flow Chart (continued)


E
203 - 221
EVALUATE OBJECTIVE FUNCTION Y AT DESIGN POINTS. STORE Y AT BASE POINT AS SSTART AND MINIMUM Y AS TMIN. SET SMIN = SSTART
230 - 235
DETERMINE REGRESSION EQUATION $Y = \beta_0 + \Sigma \beta_i X_i + \Sigma\Sigma \beta_{ij} X_i X_j$, WHERE β'S ARE GIVEN BY $(X'X)^{-1}(X'Y)$
330 - 359
SOLVE K SIMULTANEOUS EQUATIONS $\frac{\partial Y}{\partial X_i} = 0$, $i = 1, 2, \ldots, K$ TO OBTAIN STATIONARY POINT
365
EVALUATE OBJECTIVE FUNCTION S AT STATIONARY POINT
366
IS S < SMIN ?
YES
NO
367-376
SHIFT BASE POINT TO STATIONARY POINT SET S = SMIN AND NEXPT = NEXPT + 1
377
IS TMIN ≥ SMIN ?
YES
NO
378-383
SHIFT BASE TO POINT OF MINIMUM Y. SMIN = TMIN
D
D
384 - 400
BASE POINT FOR THIS EXPERIMENT IS OPTIMUM POINT
STOP

APPENDIX G

COMPUTER PROGRAM


```
C
C***************************************************************************
C
C        THIS IS A PROGRAM FOR OPTIMIZATION OF A MULTIVARIABLE FUNCTION
C        BY THE RESPONSE SURFACE METHODOLOGY (RSM). THOUGH THIS PROGRAM
C        CAN HANDLE ANY NUMBER OF VARIABLES, A FUNCTION OF MORE THAN
C        FIVE VARIABLES IS NOT RECOMMENDED.
C
C        *** DATA TO BE FURNISHED BY THE USER ***
C
C        NPROB = PROBELEM NUMBER
C            K = NUMBER OF VARIABLES
C         SVAL = STARTING VALUE
C         STEP = STEP SIZE
C        FSTEP = FINAL STEP SIZE
C
C        *** PRINTING FORMATS TO BE PROVIDED BY THE USER ***
C
C        FORMATS FOR PRINTING NAME OF THE PROBLEM AND MATHEMATICAL
C        EXPRESSION OF THE OBJECTIVE FUNCTION. THESE SHOULD BE OF THE
C        FOLLOWING TYPE :
C
C        FORMAT 2001 : THIS PROVIDES NAME OF THE PROBLEM
C        2001 FORMAT(25X,'   -NAME OF PROBLEM -    ')
C
C        FORMAT 2002 : THIS PROVIDES OBJECTIVE FUNCTION
C        2002 FORMAT(21X,'   -OBJECTIVE FUNCTION -    ')
C
C        *** SUBROUTINES TO BE PROVIDED BY THE USER ***
C
C        SUBROUTINE FOR EVALUATING THE OBJECTIVE FUNCTION S GIVEN
C        VALUES OF VARIABLES X(I). THIS SHOULD BE OF THE FOLLOWING TYPE:
C
C        SUBROUTINE OBJECT (S,X)
C        DIMENSION X(10)
C        S = F(X)    GIVE FUNCTIONAL RELATIONSHIP HERE
C        RETURN
C        END
C
C        *** OTHER VARIABLES USED IN THE PROGRAM ***
C
C         INDEX = INDEX FOR DENOTING TYPE OF DESIGN
C               =  0 IF DESIGN IS CENTRAL COMPOSITE DESIGN
C               = -1 IF DESIGN IS NON-CENTRAL COMPOSITE DESIGN SKEWED
C                    TO NEGATIVE
C               = +1 IF DESIGN IS NON-CENTRAL COMPOSITE DESIGN SKEWED
C                    TO POSITIVE
C         NFACT = NUMBER OF POINTS FOR 2**K FACTORIAL DESIGN
C        NCFACT = NUMBER OF POINTS FOR 2**K FACTORIAL DESIGN WITH
C                 CENTER POINT
C        NCCOMP = NUMBER OF POINTS FOR CENTRAL COMPOSITE DESIGN WITH
C                 CENTER POINT
C        NSCOMP = NUMBER OF POINTS FOR NON-CENTRAL COMPOSITE DESIGN
C                 SKEWED TO EITHER NEGATIVE OR POSITIVE
C          TMOD = SEARCHING STEP SIZE
C         NCOF  = NUMBER OF CONSTANTS IN POLYNOMIAL
C        SSTART = VALUE OF THE OBJECTIVE FUNCTION AT THE STARTING POINT
C        PSTART = VALUE OF OBJECTIVE FUNCTION AT THE START OF PHASE
```

```
            C          SMIN = MINIMUM VALUE OF OBJECTIVE FUNCTION
            C          TMIN = MINIMUM VALUE OF OBJECTIVE FUNCTION EVALUATED AT
            C                 THE DESIGN POINTS
            C         PSVAL = VALUES OF VARIABLES AT THE START OF PHASE
            C
            C*****************************************************************************
            C
0001              DIMENSION SVAL(25),STEP(25),FSTEP(25),X(25),XI(25),XTRIAL(25)
0002              DIMENSION XS(25),XX(25,100),Y(25),T(25),TMOD(25),B(25),BETA(25)
0003              DIMENSION XPX(900),XPY(900),LM(100),ML(100)
0004              DIMENSION R(25,25),PP(25),QQ(25),PR(25,100),PSVAL(25)
            C
            C*****   PROVIDE PROPER FORMATS 2001 AND 2002
            C
0005         2001 FORMAT('      ')
0006         2002 FORMAT('    ')
            C
0007         1001 FORMAT(/)
0008         1002 FORMAT(//)
0009         1003 FORMAT(///)
0010         1004 FORMAT(////)
0011         1005 FORMAT('1')
0012         1010 FORMAT(16I5)
0013         1030 FORMAT(6F12.3)
0014          999 FORMAT(24X,'*** RESPONSE SURFACE METHODOLOGY ***')
0015          998 FORMAT (9X,'PROBLEM NO.',I4)
0016          996 FORMAT(17X,'NUMBER OF VARIABLES -',I4)
0017          993 FORMAT(17X,'VARIABLE',18X,2('X(',I1,')',5X),/)
0018          992 FORMAT(17X,'STARTING VALUE', 10X,3(F7.2,2X))
0019          991 FORMAT(17X,'STEP SIZE FOR INCREMENT ',3(F7.2,2X))
0020          990 FORMAT(17X,'FINAL STEP SIZE',9X,3(F7.2,2X))
0021          989 FORMAT(17X,'OBJECTIVE FUNCTION :')
0022          987 FORMAT(17X,'THE ABOVE OBJECTIVE FUNCTION IS TO BE MINIMIZED')
0023          986 FORMAT(17X,'METHOD USED FOR MINIMIZATION - MODIFIED VERSION OF RES
                 1PONSE SURFACE',/,43X,'METHODOLOGY BY BOX AND WILSON(1951)')
0024          985 FORMAT(7X,'PHASE ONE : RESPONSE SURFACE IS APPROXIMATED AS A PLANE
                 1')
0025          984 FORMAT(///,7X,'EXPERIMENT NO.',I3,//)
0026          983 FORMAT(7X,'STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT T
                 1HE BASE POINT',//)
0027          982 FORMAT(7X,'A. BASE POINT  : X(1) =',F7.2,'   X(2) =',F7.2/)
0028          981 FORMAT(7X,'B. DESIGN USED : 2**2 FACTORIAL DESIGN WITH CENTER POIN
                 1T',/)
0029          980 FORMAT(7X,'B. DESIGN USED : CENTRAL COMPOSITE DESIGN WITH ALPHA =
                 12',/)
0030          979 FORMAT( 7X,'B. DESIGN USED : NON-CENTRAL COMPOSITE DESIGN SKEWED T
                 1OWARDS ',/,22X,'NEGATIVE WITH ALPHA = 2',/)
0031          978 FORMAT( 7X,'B. DESIGN USED : NON-CENTRAL COMPOSITE DESIGN SKEWED T
                 1OWARDS ',/,22X,'POSITIVE WITH ALPHA = 2',/)
0032          977 FORMAT ( 7X,'C. NUMBER OF TRIALS REQUIRED :',I3,/)
0033          976 FORMAT(7X,'D. RESULTS OF TRIALS',/)
0034          973 FORMAT(10X,'TRIAL',9X,'TRIAL POINT',12X,'OBJECTIVE',11X,'REMARKS',
                 1/,11X,'NO.',11X,'X(1)'4X,'X(2)',11X,'FUNCTION',/)
0035          972 FORMAT( 11X,I3,7X,2(F8.2), 7X,F12.2)
0036          971 FORMAT(/,7X,'E. EQUATION OF A SURFACE AT THE BASE POINT'/,12X,'Y =
                 1 ',F12.2,'+',F12.2,'X1 +',F12.2,'X2 +',F12.2,'X3')
0037          970 FORMAT( /,7X,'E. EQUATION OF A SURFACE AT THE BASE POINT',/,12X,
                 1'Y =',F12.2,'+',F12.2,'X1 +',  F12.2,'X2 +',F12.2,'X1.X1',
```

```
                  2/    ,15X,'+',F12.2,'X1.X2 +',F12.2,'X2.X2')
0038          969 FORMAT(///,7X,'STEP II : APPLICATION OF THE STEEPEST ASCENT PROCED
                 1URE',/)
0039          968 FORMAT( 7X,'A. MODIFIED STEP SIZES',/,13X,'VARIABLE',8X,'STEP SIZE
                 1')
0040          967 FORMAT(15X,'X(',I1,')',11X,F7.3)
0041          966 FORMAT( /,7X,'B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF S
                 1TEEPEST ASCENT',/ )
0042          963 FORMAT (//,7X,'STEP III : SUMMARY OF THE EXPERIMENT',/ ,7X,
                 1'A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) =',F7.2,5X,
                 2'X(2) =',F7.2,/,33X,'OBJECTIVE FUNCTION =',F12.3,//,7X,
                 3'B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) =',F7.2,5X,
                 4'X(2) =',F7.2,/,33X,'OBJECTIVE FUNCTION =',F12.3,//,7X,'C. REMARKS
                 5 :' )
0043          962 FORMAT(15X,'END OF THE EXPERIMENT.')
0044          961 FORMAT ( 15X,'THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION.
                 1EXPERIMENT IS SUCCESSFUL.',/15X,'BASE POINT AT THE END OF THE EXPE
                 2RIMENT BECOMES THE STARTING POINT ',/15X,'OF THE SUBSEQUENT EXPERI
                 3MENT.')
0045          960 FORMAT( 15X,'THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION.',/
                 1          15X,'REDUCE STEP SIZE BY HALF. REPEAT STEP II.'/)
0046          959 FORMAT( 15X,'STEP SIZE CANNOT BE FURTHER REDUCED.END OF THE EXPERI
                 1MENT.')
0047          958 FORMAT(15X,'THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXP
                 1ERIMENT HAS FAILED.',/,15X,'APPROXIMATION OF A SURFACE AS A PLANE
                 2NO LONGER HOLDS GOOD. END OF THE PHASE.')
0048          956 FORMAT(20X,'X(1) =',F7.2,5X,'X(2) =',F7.2)
0049          955 FORMAT(20X,'OBJECTIVE FUNCTION =',F9.3)
0050          954 FORMAT(11X,I3,7X,2(F8.2), 7X,F12.2,11X,'SUCCESSFUL')
0051          953 FORMAT(11X,I3,7X,2(F8.2),7X,F12.2,11X,'FAILED')
0052          952 FORMAT(21X,2(F8.2),7X,F12.2,11X,'BASE POINT')
0053          930 FORMAT(28X,'*** SUMMARY OF PHASE ONE ***')
0054          929 FORMAT(14X,'BASE POINT AT THE START OF PHASE ONE')
0055          928 FORMAT(14X,'OPTIMUM POINT (SO FAR) AT THE END OF PHASE ONE')
0056          926 FORMAT( 7X,'PHASE TWO : RESPONSE SURFACE IS APPROXIMATED AS A SECO
                 1ND DEGREE CURVE')
0057          925 FORMAT(///,7X,'STEP II : DETERMINATION OF THE CO-ORDINATES OF AND
                 1OBJECTIVE FUNCTION',/,17X,'AT THE CENTER OF THE SURFACE',/)
0058          924 FORMAT( 7X,'A. CO-ORDINATES OF THE CENTER')
0059          923 FORMAT(10X,'X(',I2,') = ',F7.2)
0060          922 FORMAT(/,7X,'B. TRIAL AT THE CENTER POINT')
0061          920 FORMAT(28X,'*** SUMMARY OF PHASE TWO ***')
0062          919 FORMAT(14X,'BASE POINT AT THE START OF PHASE TWO')
0063          918 FORMAT(14X,'OPTIMUM POINT AT THE END OF PHASE TWO')
              C
              C          INITIALIZE
              C
0064                TMIN = 1.0E40
0065                INDEX = 0
0066                PRINT 1005
0067                PRINT 1002
0068                PRINT 999
0069                READ 1010,NPROB
0070                PRINT 1002
0071                PRINT 998,NPROB
0072                PRINT 1002
0073                PRINT 2001
0074                READ1010,K
```

```
      PRINT 1001
      PRINT 996,K
      PRINT 1001
      PRINT 993,(I,I=1,K)
      READ 1030,(SVAL(I),I=1,K)
      PRINT 992,(SVAL(I),I=1,K)
      READ 1030,(STEP(I),I=1,K)
      PRINT 991,(STEP(I),I=1,K)
      READ 1030,(FSTEP(I),I=1,K)
      PRINT 990,(FSTEP(I),I=1,K)
      PRINT 1001
      PRINT 989
      PRINT 2002
      PRINT 1001
      PRINT 987
      PRINT 1001
      PRINT 986
C
      NTRIAL = 0
      IPHASE = 1
      PRINT 1005
      PRINT 1001
      PRINT 985
C
C        DETERMINE NUMBER OF DESIGN POINTS
C
      NFACT = 2**K
      NCFACT = NFACT + 1
      NCOMP = NFACT + 2*K
      NCCOMP = NCOMP + 1
      NSCOMP = NFACT + K
C
C        DETERMINE CO-ORDINATE OF DESIGN POINTS REQUIRED FOR PHASE ONE.
C        DESIGN USED IS 2**K FACTORIAL DESIGN WITH CENTER POINT
C
      DO 23 I = 1,K
      DO 24 J = 1,NCCOMP
      XX(I,J) = 0.0
   24 CONTINUE
   23 CONTINUE
  101 N = NCFACT
      DO 2 I=1,K
      II = 2.0**(K-I)
      JJ = NFACT/II
      DO 3 J = 1,NFACT
      DO 4 M=1,JJ,2
      MM = M + 1
      IF(J-M*II) 105,105,110
  110 IF(J-MM*II)115,115,4
  115 XX(I,J) = 1.0
      GO TO 3
  105 XX(I,J) = -1.0
      GO TO 3
    4 CONTINUE
    3 CONTINUE
    2 CONTINUE
       GO TO 159
C
```

```
              C        DETERMINE CO-ORDINATES OF ADDITIONAL DESIGN POINTS REQUIRED
              C        FOR PHASE TWO. DESIGN USED IS EITHER CENTRAL OR NON-CENTRAL
              C        COMPOSITE DESIGN WITH ALPHA = 2
              C
 0124             102 IF(INDEX.NE.0) GO TO 475
 0125                 N = NCCOMP
 0126                 J = NCFACT
 0127                 MM = 2
 0128                 XM = 0.0
 0129                 GO TO 490
 0130             475 N = NSCOMP
 0131                 DO 485 I = 1,K
 0132                 DO 486 J = NCFACT,N
 0133                 XX(I,J) = 1.0
 0134             486 CONTINUE
 0135             485 CONTINUE
 0136                 J = NFACT
 0137                 MM = 1
 0138                 XM = 1.0
 0139             490 ALPHA = 2.0
 0140                 DO 71 I = 1,K
 0141                 DO 72 M = 1,MM
 0142             254 J = J+1
 0143                 GO TO (251,252),M
 0144             251 XX(I,J) = ALPHA + XM
 0145                 GO TO 72
 0146             252 XX(I,J) = -ALPHA-XM
 0147              72 CONTINUE
 0148              71 CONTINUE
 0149                 IF(INDEX .EQ. 0) GO TO 159
 0150                 DO 505 I = 1,K
 0151                 DO 506 J = NCFACT,N
 0152                 XX(I,J) = XX(I,J)*INDEX
 0153             506 CONTINUE
 0154             505 CONTINUE
              C
 0155             159 CONTINUE
 0156                 DO 425 I = 1,K
 0157                 PSVAL(I) = SVAL(I)
 0158             425 CONTINUE
 0159                 IF(IPHASE.EQ.1) NEXPT =0
              C
              C        FORMULATE X-MATRIX, MATRIX OF INDEPENDENT VARIABLES
              C
 0160                 DO 1 I=1,N
 0161                 X(I) = 1.0
 0162               1 CONTINUE
              C
 0163             160 DO 6 I=1,K
 0164                 T(I) = STEP(I)
 0165                 B(I) = SVAL(I)
 0166               6 CONTINUE
              C
 0167             161 NEXPT = NEXPT + 1
 0168                 DO 21 I = 1,K
 0169                 DO 5 J=1,N
 0170                 L = J+I*N
 0171                 X(L) = XX(I,J)*T(I) + B(I)
```

```
       5 CONTINUE
      21 CONTINUE
         NCOF = K+1
C
         GO TO (199,198),IPHASE
     198 L = (K+1)*N
         DO 16 I=1,K
         DO 17 J=I,K
         DO 18 M=1,N
         L = L + 1
         X(L) = X(I*N+M)*X(J*N+M)
      18 CONTINUE
      17 CONTINUE
      16 CONTINUE
         NCOF = K + 1 + (K*K + K)/2.0
C
     199 CONTINUE
         PRINT 984,NEXPT
         PRINT 983
         PRINT 982,(SVAL(I),I=1,K   )
         IF(IPHASE.EQ.2) GO TO 350
         PRINT 981
         GO TO 654
     350 CONTINUE
         IF(INDEX) 651,652,653
     651 PRINT 979
         GO TO 654
     652 PRINT 980
         GO TO 654
     653 PRINT 978
     654 CONTINUE
         PRINT 977,N
         PRINT 976
C
C           PHASE ONE AND TWO. STEP I.
C           EVALUATE OBJECTIVE FUCTION AT THE DESIGN POINTS.
C           FORMULATE Y-MATRIX, MATRIX OF OBSERVATIONS.
C
         DO 7 I = 1,N
         DO 8 J=1,K
         XI(J) = X(J*N + I)
         PR(I,J) = XI(J)
       8 CONTINUE
         NTRIAL = NTRIAL + 1
         CALL OBJECT  (S,XI)
         PR(I,K+1) = S
         IF(TMIN.LT.S) GO TO 600
         TMIN = S
         MIN = I
     600 Y(I) = S
       7 CONTINUE
         PRINT 973
         NTRIAL = NTRIAL-N
         DO 420 I = 1,N
         NTRIAL = NTRIAL + 1
         PRINT 972,NTRIAL,(PR(I,J),J=1,3)
     420 CONTINUE
         IF(INDEX) 616,617,616
```

```
0223            616 SSTART = SMIN
0224                GO TO 618
0225            617 SSTART = PR(5,K+1)
0226            618 CONTINUE
0227                SMIN = SSTART
0228                IF(IPHASE.EQ.2) GO TO 205
0229                IF(NEXPT.EQ.1)PSTART = SSTART
          C
          C             EVALUATE CO-EFFICIENTS OF POLYNOMIAL BY METHOD OF LEAST
          C             SQUARES.
          C
0230            205 CALL TPRD(X,X,XPX,N,NCOF,0,0,NCOF)
0231                CALL TPRD(X,Y,XPY,N,NCOF,0,0,1)
0232                CALL MINV(XPX,NCOF,D,LM,ML)
0233                CALL MPRD(XPX,XPY,BETA,NCOF,NCOF,0,0,1)
0234                IF(IPHASE.EQ.2) GO TO 435
          C
          C             PHASE ONE. STEP II.
          C
0235                PRINT 971,(BETA(I),I=1,NCOF)
0236                PRINT 969
0237            140 DO 9 I=2,NCOF
0238                BETA(I) = -BETA(I)
0239              9 CONTINUE
0240            141 CONTINUE
0241                PRINT 968
0242                BMAX = ABS(BETA(2))
0243                K1 = K+1
0244                DO 615 I = 2,K1
0245                IF(BMAX.GE.ABS(BETA(I))) GO TO 615
0246                BMAX = ABS(BETA(I))
0247            615 CONTINUE
0248                DO 61 I = 2,NCOF
0249                TMOD(I) = BETA(I)*T(I-1)/BMAX
0250                L = I-1
0251                PRINT 967,L,TMOD(I)
0252             61 CONTINUE
0253                PRINT 966
0254                PRINT 973
0255                PRINT 952,(B(I),I=1,K),SMIN
          C
          C             TRIALS ALONG THE DIRECTION OF STEEPEST DESCENT
          C
0256            130 CONTINUE
0257                NTRIAL = NTRIAL + 1
0258                DO 10 I = 1,K
0259                XTRIAL (I) = B(I) + TMOD(I+1)
0260             10 CONTINUE
0261                CALL OBJECT(S,XTRIAL)
0262                IF (SMIN - S) 125,120,120
          C
          C             TRIAL SUCCESSFUL. CONTINUE SEARCH.
          C
0263            120 SMIN = S
0264                PRINT 954,NTRIAL,(XTRIAL(I),I=1,K),S
0265                DO 11 I = 1,K
0266                B(I) = XTRIAL(I)
0267             11 CONTINUE
```

```
0268                GO TO 130
              C
              C        TRIAL FAILED.
              C
0269            125 IF(TMIN.GE.SMIN)GO TO 601
0270                SMIN = TMIN
0271                DO 602 I = 1,K
0272                B(I) = PR(MIN,I)
0273            602 CONTINUE
0274            601 CONTINUE
0275                PRINT 953,NTRIAL,(XTRIAL(I),I=1,K),S
0276            603 CONTINUE
0277                IF(SSTART - SMIN )155,155,150
              C
              C        EXPERIMENT IS SUCCESSFUL. STEP III.
              C
0278            150 CONTINUE
0279                PRINT 963,(SVAL(I),I=1,K),SSTART,(B(I),I=1,K),SMIN
0280                SSTART =SMIN
0281                PRINT961
0282                DO 15 I=1,K
0283                SVAL(I) = B(I)
0284             15 CONTINUE
0285                GO TO 160
              C
              C        NO IMPROVEMENT OVER FIRST TRIAL. SEARCHING STEP SIZE TOO LARGE.
              C        REDUCE BY HALF.
              C
0286            155 CONTINUE
0287                IND = 0
0288                DO 630 I = 1,K
0289                 T(I) = T(I)/2.0
0290                IF(T(I)-FSTEP(I)) 631,630,630
0291            631 T(I) = FSTEP(I)
0292                IND = IND+1
0293                IF(IND-K) 630,503,503
0294            630 CONTINUE
0295                PRINT 960
0296                GO TO 141
              C
              C        SEARCHING STEP SIZE CANNOT BE REDUCED FURTHER. END OF PHASE ONE.
              C
0297            503 IPHASE = 2
0298                NEXPT=0
              C
              C        SUMMARY OF PHASE ONE.
              C
0299                PRINT 959
0300                PRINT 963,(SVAL(I),I=1,K),SSTART,(B(I),I=1,K),SMIN
0301                PRINT 958
0302                PRINT 1003
0303                PRINT 930
0304                PRINT 1002
0305                PRINT 929
0306                PRINT 956,(PSVAL(I),I=1,K)
0307                PRINT 955, PSTART
0308                PRINT 1002
0309                PRINT 928
```

```
                PRINT 956,(B(I),I=1,K)
                PRINT 955,SMIN
                DO 510 I=1,K
                PSVAL(I) =B(I)
            510 CONTINUE
                PSTART =SMIN
                PRINT 1001
                PRINT 926
      C
      C            DETERMINE THE TYPE OF DESIGN TO BE USED FOR PHASE TWO.
      C
            612 CONTINUE
                DO 450 I =2,K
                W = BETA(I)*BETA(I+1)
                IF(W.LT.0)GO TO 465
            450 CONTINUE
                IF(BETA(2).LT.0) GO TO 455
                INDEX = 1
                GO TO 102
            455 INDEX = -1
                GO TO 102
            465 INDEX = 0
                GO TO 102
      C
      C            PHASE TWO. STEP II.
      C
            435 PRINT 970,(BETA(I),I=1,NCOF)
                PRINT 925
                PRINT 924
            210 DO41 J=1,K
                DO 32 I=1,K
                IF(J-I)301,302,305
            301 R(I,J) = BETA(LL+1)
                L1 = L1+1
                GO TO 34
            302 LL = 1 + K
                DO 33 L=1,J
                IF(L-1) 303,33,303
            303 LL = LL + K-L+2
             33 CONTINUE
                LL = LL + 1
                R(I,J) = 2.0*BETA(LL)
                L1 = LL
                GO TO 34
            305 R(I,J) = R(J,I)
             34 M = I+(J-1)*K
                PP(M) = R(I,J)
             32 CONTINUE
             41 CONTINUE
                CALL MINV (PP,K,D,LM,ML)
                DO 35 I = 1,K
                CQ(I) = -BETA(I+1)
             35 CONTINUE
                CALL MPRD(PP,CQ,XS,K,K,0,0,1)
                DO 440 I = 1,K
                PRINT 923,I,XS(I)
            440 CONTINUE
                PRINT 922
```

```
            NTRIAL = NTRIAL + 1
            PRINT 973
            PRINT 952,(B(I),I=1,K),SMIN
            CALL OBJECT(S,XS)
            IF(SMIN-S)225,220,220
C
C              EXPERIMENT IS SUCCESSFUL.
C
        220 CONTINUE
            PRINT 954,NTRIAL,(XS(I),I=1,K),S
            PRINT 963,(B(I),I=1,K),SMIN,(XS(I),I=1,K),S
            PRINT 961
            SMIN = S
            DO 31 I=1,K
            B(I) = XS(I)
            SVAL(I) = B(I)
         31 CONTINUE
            GO TO 161
C
C              EXPERIMENT HAS FAILED.
C
        225 IF(TMIN.GE.SMIN) GO TO 610
            SMIN = TMIN
            DO 611 I = 1,K
            SVAL(I) = PR(MIN,I)
        611 CONTINUE
            PRINT 953,NTRIAL,(XS(I),I=1,K),S
            GO TO 612
        610 CONTINUE
            PRINT 953,NTRIAL,(XS(I),I=1,K),S
            PRINT 963,(B(I),I=1,K),SMIN,(XS(I),I=1,K),S
C
C              SUMMARY OF PHASE TWO.
C
            PRINT 958
            PRINT 1003
            PRINT 920
            PRINT 1003
            PRINT 919
            PRINT 956,(PSVAL(I),I=1,K)
            PRINT 955,PSTART
            PRINT 1002
            PRINT 918
            PRINT 956,(B(I),I=1,K)
            PRINT 955,SMIN
        245 CONTINUE
            STOP
            END
```

APPENDIX H

COMPUTER OUTPUT OF OPTIMAL PRODUCTION SCHEDULING PROBLEM

PHASE ONE : RESPONSE SURFACE IS APPROXIMATED AS A PLANE

EXPERIMENT NO. 1

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 5.00 X(2) = 10.00

B. DESIGN USED : 2**2 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED : 5

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
1	3.00	8.00	43980.00	
2	3.00	12.00	45580.00	
3	7.00	8.00	25900.00	
4	7.00	12.00	24940.00	
5	5.00	10.00	33660.00	

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = 58212.00+ -4840.11X1 + 80.00X2 +

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	2.000
X(2)	-0.033

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	5.00	10.00	33660.00	BASE POINT
6	7.00	9.97	24948.05	SUCCESSFUL
7	9.00	9.93	18177.53	SUCCESSFUL
8	11.00	9.90	13348.44	SUCCESSFUL
9	13.00	9.87	10460.77	SUCCESSFUL
10	15.00	9.83	9514.51	SUCCESSFUL
11	17.00	9.80	10509.66	FAILED

STEP III : SUMMARY OF THE EXPERIMENT
A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 5.00 X(2) = 10.00
OBJECTIVE FUNCTION = 33660.000

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 15.00 X(2) = 9.83
OBJECTIVE FUNCTION = 9514.508

C. REMARKS :
THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL.
BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT
OF THE SUBSECUENT EXPERIMENT.

EXPERIMENT NO. 2

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 15.00 X(2) = 9.83

B. DESIGN USED : 2**2 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED : 5

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
12	13.00	7.83	13460.95	
13	13.00	11.83	8502.28	
14	17.00	7.83	14686.73	
15	17.00	11.83	7168.06	
16	15.00	9.83	9514.51	

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = 26207.00+ -13.54X1 + -1559.61X2 +

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	0.017
X(2)	2.000

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	15.00	9.83	9514.51	BASE POINT
17	15.02	11.83	6869.45	SUCCESSFUL
18	15.03	13.83	5173.43	SUCCESSFUL
19	15.05	15.83	4426.43	SUCCESSFUL
20	15.07	17.83	4628.46	FAILED

STEP III : SUMMARY OF THE EXPERIMENT
A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 15.00 X(2) = 9.83
OBJECTIVE FUNCTION = 9514.508

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 15.05 X(2) = 15.83
OBJECTIVE FUNCTION = 4426.434

C. REMARKS :
THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL.
BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT
OF THE SUBSEQUENT EXPERIMENT.

EXPERIMENT NO. 3

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 15.05 X(2) = 15.83

B. DESIGN USED : 2**2 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED : 5

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
21	13.05	13.83	7379.54	
22	13.05	17.83	8147.52	
23	17.05	13.83	4865.34	
24	17.05	17.83	3073.33	
25	15.05	15.83	4426.43	

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = 21663.38+ -948.62X1 + -128.00X2 +

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	2.000
X(2)	0.270

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	15.05	15.83	4426.43	BASE POINT
26	17.05	16.10	3377.18	SUCCESSFUL
27	19.05	16.37	4092.70	FAILED

STEP III : SUMMARY OF THE EXPERIMENT
A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 15.05 X(2) = 15.83
OBJECTIVE FUNCTION = 4426.434

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 17.05 X(2) = 17.83
OBJECTIVE FUNCTION = 3073.330

C. REMARKS :
THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL.
BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT
OF THE SUBSEQUENT EXPERIMENT.

EXPERIMENT NO. 4

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 17.05 X(2) = 17.83

B. DESIGN USED : 2**2 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED : 5

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
28	15.05	15.83	4426.44	
29	15.05	19.83	5834.41	
30	19.05	15.83	4472.24	
31	19.05	19.83	3320.22	
32	17.05	17.83	3073.33	

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = 8916.81+ -308.58X1 + 31.98X2 +

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	2.000
X(2)	-0.207

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	17.05	17.83	3073.33	BASE POINT
33	19.05	17.63	3481.08	FAILED

THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION.
REDUCE STEP SIZE BY HALF. REPEAT STEP II.

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	1.000
X(2)	-0.104

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	17.05	17.83	3073.33	BASE POINT
34	18.05	17.73	3019.34	SUCCESSFUL
35	19.05	17.63	3481.09	FAILED

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 17.05 X(2) = 17.83
OBJECTIVE FUNCTION = 3073.330

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 18.05 X(2) = 17.73
OBJECTIVE FUNCTION = 3019.337

C. REMARKS :
THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL. BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT OF THE SUBSECUENT EXPERIMENT.

EXPERIMENT NO. 5

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 18.05 X(2) = 17.73

B. DESIGN USED : 2**2 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED : 5

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
36	16.05	15.73	3749.03	
37	16.05	19.73	4417.50	
38	20.05	15.73	5781.16	
39	20.05	19.73	3889.65	
40	18.05	17.73	3019.34	

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = 3488.13+ 188.00X1 + -152.90X2 +

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	-1.000
X(2)	0.813

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	18.05	17.73	3019.34	BASE POINT
41	17.05	18.54	3156.47	FAILED

THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION.
REDUCE STEP SIZE BY HALF. REPEAT STEP II.

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	-0.500
X(2)	0.407

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	18.05	17.73	3019.34	BASE POINT
42	17.55	18.14	2975.53	SUCCESSFUL
43	17.05	18.54	3156.47	FAILED

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 18.05 X(2) = 17.73
OBJECTIVE FUNCTION = 3019.337

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 17.55 X(2) = 18.14
OBJECTIVE FUNCTION = 2975.527

C. REMARKS :
THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL
BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT
OF THE SUBSEQUENT EXPERIMENT.

EXPERIMENT NO. 6

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 17.55 X(2) = 18.14

B. DESIGN USED : 2**2 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED : 5

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
44	15.55	16.14	3960.15	
45	15.55	20.14	5339.01	
46	19.55	16.14	4772.04	
47	19.55	20.14	3590.90	
48	17.55	18.14	2975.53	

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = 5734.06+ -117.36X1 + 24.70X2 +

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	0.500
X(2)	-0.105

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	17.55	18.14	2975.53	BASE POINT
49	18.05	18.03	2984.18	FAILED

THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION.
REDUCE STEP SIZE BY HALF. REPEAT STEP II.

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	0.250
X(2)	-0.053

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	17.55	18.14	2975.53	BASE POINT
50	17.80	18.08	2962.41	SUCCESSFUL
51	18.05	18.03	2984.18	FAILED

STEP III : SUMMARY OF THE EXPERIMENT
A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 17.55 X(2) = 18.14
OBJECTIVE FUNCTION = 2975.527

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 17.80 X(2) = 18.08
OBJECTIVE FUNCTION = 2962.410

C. REMARKS :
THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL.
BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT
OF THE SUBSEQUENT EXPERIMENT.

EXPERIMENT NO. 7

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 17.80 X(2) = 18.08

B. DESIGN USED : 2**2 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED : 5

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
52	15.80	16.08	3795.49	
53	15.80	20.08	4963.71	
54	19.80	16.08	5121.10	
55	19.80	20.08	3729.33	
56	17.80	18.08	2962.41	

E. EQUATION OF A SURFACE AT THE BASE POINT

Y = 4417.13+ 11.38X1 + -27.95X2 +

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	-0.102
X(2)	0.250

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	17.80	18.08	2962.41	BASE POINT
57	17.70	18.33	2968.32	FAILED

THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION.
REDUCE STEP SIZE BY HALF. REPEAT STEP II.

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	-0.051
X(2)	0.125

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	17.80	18.08	2962.41	BASE POINT
58	17.75	18.21	2961.85	SUCCESSFUL
59	17.70	18.33	2968.32	FAILED

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 17.80 X(2) = 18.08
OBJECTIVE FUNCTION = 2962.410

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 17.75 X(2) = 18.21
OBJECTIVE FUNCTION = 2961.651

C. REMARKS :

THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL. BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT OF THE SUBSEQUENT EXPERIMENT.

EXPERIMENT NO. 8

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 17.75 X(2) = 18.21

B. DESIGN USED : 2**2 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED : 5

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
60	15.75	16.21	3807.49	
61	15.75	20.21	5128.27	
62	19.75	16.21	4955.42	
63	19.75	20.21	3716.22	
64	17.75	18.21	2961.85	

E. EQUATION OF A SURFACE AT THE BASE POINT

Y = 4514.25+ -33.03X1 + 10.20X2 +

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	0.125
X(2)	-0.039

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	17.75	18.21	2961.85	BASE POINT
65	17.88	18.17	2962.03	FAILED

STEP SIZE CANNOT BE FURTHER REDUCED.END OF THE EXPERIMENT.

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 17.75 X(2) = 18.21
OBJECTIVE FUNCTION = 2961.851

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 17.75 X(2) = 18.21
OBJECTIVE FUNCTION = 2961.851

C. REMARKS :

THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT HAS FAILED.

APPROXIMATION OF A SURFACE AS A PLANE NO LONGER HOLDS GOOD. END OF THE PHASE.

*** SUMMARY OF PHASE ONE ***

BASE POINT AT THE START OF PHASE ONE
X(1) = 5.00 X(2) = 10.00
OBJECTIVE FUNCTION =33660.000

OPTIMUM POINT (SO FAR) AT THE END OF PHASE ONE
X(1) = 17.75 X(2) = 18.21
OBJECTIVE FUNCTION = 2961.851

PHASE TWO : RESPONSE SURFACE IS APPROXIMATED AS A SECOND DEGREE CURVE

EXPERIMENT NO. 1

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 17.75 X(2) = 18.21

B. DESIGN USED : CENTRAL COMPOSITE DESIGN WITH ALPHA = 2

C. NUMBER OF TRIALS REQUIRED : 9

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
66	15.75	16.21	3807.49	
67	15.75	20.21	5126.27	
68	19.75	16.21	4955.42	
69	19.75	20.21	3716.22	
70	17.75	18.21	2961.85	
71	21.75	18.21	6669.78	
72	13.75	18.21	6933.91	
73	17.75	22.21	4922.64	
74	17.75	14.21	4841.05	

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = 68624.00+ -5724.00X1 + -1612.00X2 + 241.37X1.X1
+ -157.78X1.X2 + 121.69X2.X2

STEP II : DETERMINATION OF THE CO-ORDINATES OF AND OBJECTIVE FUNCTION
AT THE CENTER OF THE SURFACE

A. CO-ORDINATES OF THE CENTER
X(1) = 17.7)
X(2) = 18.16

B. TRIAL AT THE CENTER POINT

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
	17.75	18.21	2961.85	BASE POINT
75	17.79	18.16	2961.03	SUCCESSFUL

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 17.75 X(2) = 18.21
OBJECTIVE FUNCTION = 2961.851

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 17.79 X(2) = 18.16
OBJECTIVE FUNCTION = 2961.029

C. REMARKS :
THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL.
BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT
OF THE SUBSECUENT EXPERIMENT.

EXPERIMENT NO. 2

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 17.79 X(2) = 18.16

B. DESIGN USED : CENTRAL COMPOSITE DESIGN WITH ALPHA = 2

C. NUMBER OF TRIALS REQUIRED : 9

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	X(2)	OBJECTIVE FUNCTION	REMARKS
76	15.79	16.16	3788.43	
77	15.79	20.16	5033.38	
78	19.79	16.16	5048.66	
79	19.79	20.16	3733.63	
80	17.79	18.16	2961.03	
81	21.79	18.16	6781.26	
82	13.79	18.16	6820.78	
83	17.79	22.16	4845.98	
84	17.79	14.16	4916.06	

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = 68272.00+ -5705.00X1 + -1586.00X2 + 241.06X1.X1
+ -158.17X1.X2 + 121.06X2.X2

STEP II : DETERMINATION OF THE CO-ORDINATES OF AND OBJECTIVE FUNCTION

AT THE CENTER OF THE SURFACE

A. CO-ORDINATES OF THE CENTER
X(1) = 17.80
X(2) = 18.18

B. TRIAL AT THE CENTER POINT

TRIAL NO.	TRIAL POINT X(1)	TRIAL POINT X(2)	OBJECTIVE FUNCTION	REMARKS
	17.79	18.16	2961.03	BASE POINT
85	17.80	18.18	2960.89	SUCCESSFUL

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 17.79 X(2) = 18.16
OBJECTIVE FUNCTION = 2961.029

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 17.80 X(2) = 18.18
OBJECTIVE FUNCTION = 2960.890

C. REMARKS :
THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL. BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT OF THE SUBSEQUENT EXPERIMENT.

EXPERIMENT NO. 3

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : X(1) = 17.80 X(2) = 18.18

B. DESIGN USED : CENTRAL COMPOSITE DESIGN WITH ALPHA = 2

C. NUMBER OF TRIALS REQUIRED : 9

D. RESULTS OF TRIALS

TRIAL NO.	TRIAL POINT X(1)	TRIAL POINT X(2)	OBJECTIVE FUNCTION	REMARKS
86	15.80	16.18	3783.23	
87	15.80	20.18	5042.38	
88	19.80	16.18	5039.39	
89	19.80	20.18	3738.55	
90	17.80	18.18	2960.89	
91	21.80	18.18	6777.04	
92	13.80	18.18	6824.72	
93	17.80	22.18	4860.04	
94	17.80	14.18	4901.73	

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = 68768.00+ -5736.00X1 + -1615.00X2 + 241.51X1.X1
\+ -157.21X1.X2 + 121.44X2.X2

STEP II : DETERMINATION OF THE CO-ORDINATES OF AND OBJECTIVE FUNCTION
AT THE CENTER OF THE SURFACE

A. CO-ORDINATES OF THE CENTER
X(1) = 17.79
X(2) = 18.16

B. TRIAL AT THE CENTER POINT

TRIAL NO.	TRIAL POINT X(1)	TRIAL POINT X(2)	OBJECTIVE FUNCTION	REMARKS
	17.80	18.18	2960.89	BASE POINT
95	17.79	18.16	2961.04	FAILED

STEP III : SUMMARY OF THE EXPERIMENT
A. BASE POINT AT THE START OF THE EXPERIMENT : X(1) = 17.80 X(2) = 18.18
OBJECTIVE FUNCTION = 2960.890

B. BASE POINT AT THE END OF THE EXPERIMENT : X(1) = 17.79 X(2) = 18.16
OBJECTIVE FUNCTION = 2961.035

C. REMARKS :
THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT HAS FAILED.
APPROXIMATION OF A SURFACE AS A PLANE NO LONGER HOLDS GOOD. END OF THE PHASE.

*** SUMMARY OF PHASE TWO ***

BASE POINT AT THE START OF PHASE TWO
X(1) = 17.75 X(2) = 18.21
OBJECTIVE FUNCTION = 2961.851

OPTIMUM POINT AT THE END OF PHASE TWO
X(1) = 17.80 X(2) = 18.18
OBJECTIVE FUNCTION = 2960.890

APPENDIX I

SOLUTION AND COMPUTER OUTPUT OF A PROBLEM OF OPTIMUM INTEGRATED HUMAN THERMAL SYSTEM

Step by step results of a problem which has the form of the objective function given by equation (1) are given here. Metabolic heat generation rate is 300 BTU/hr. The results are summarized in Table 5 under Test 1.

The starting base point is (50, 20, 20) with a 2^3 factorial design. The three variables are coolant inlet temperature (^oF), coolant mass flow rate for hood (lb/hr) and for jacket (lb/hr). The step size of the factorial design is (2, 2, 2) and the final step size is (0.5, 0.5, 0.5). At each design point, using the values of control variables the temperature distribution over whole body and therefrom the temperatures at brain, skin and muscle are obtained. These are used to evaluate the objective function. A first degree curve is fitted to these data (points x^1 to x^9). The equation of the plane passing through x^9 (50, 20, 20) is obtained as

$$y = 186.305 - 2.778x_1 + 1.112x_2 + 0.582x_3$$

The gradients in the direction of x_1, x_2 and x_3 are proportional to the coefficients of respective variables. The components of the step size for searching (2.00, -0.80, -0.42) is proportional to these coefficients. The next trial point is x^{10}(52, 19.20, 19.58) and objective function at this point is 74.90 which is better than that at the base point x^9, 81.62. The searching is, therefore, continued in the same direction with the same step size of searching. The new trial point becomes x^{11} (54, 18.4, 19.16). The procedure is continued till a point is found where no further improvement is possible. The objective function at the point x^{20} (72, 11.19, 15.39), 20.396, is no better than the previous trial point x^{19} (70, 12.0, 15.81), 19.660, hence experiment 1 ends providing x^{19} (70, 12.0, 15.81) as a new base point for the next experiment.

In experiment 2, again a 2^3 factorial design is constructed around the base point x^{19} (70, 12.0, 15.81). Objective functions (data) are evaluated at these design points as well as base point (trial numbers 21 through 29). An equation of a plane fitted to these data is

$$y = - 21.156 + 0.083x_1 + 1.864x_2 + 0.905x_3.$$

It is observed that the values of the coefficients of variables are reduced and hence less improvement is expected than in experiment 1. The searching step size is determined as (-0.09, -2.00, -0.97). The trial 30 is successful using this step size in the direction of steepest descent. The trial 31, however, is a failure. The x^{30} may become a base point for the next experiment, however, one of the design points x^{21} (68.00, 10.00, 13.81) is found to yield better results than the point x^{30}. Hence x^{21} becomes a base point for experiment 3.

Experiment 3 is repeated in the same way as in trials 32 through 40, and x^{42} (67.54, 9.91, 9.81) is obtained as a base point for experiment 4. Trials 44 through 52 determine the equation of the plane passing through the base point $x^{42} = x^{52}$ (67.54, 9.91, 9.81). The step size for search is calculated as (-0.086, -0.171, 2.000). The first trial point x^{53} (67.46, 9.74, 11.81), however, does not yield a better value of objective function than that at the base point x^{52}. This is an indication that the searching step size is too large, hence the new searching step size becomes (-0.043, -0.086, 1.00). In further trial 54, this step size is reduced further. This can continue till the stopping criteria of the final step size is satisfied. In the next trial 55 such reduction in search step size is applied and finally the step size becomes less than the final step size. Phase one ends providing point x^{52}(67.54, 9.91, 9.81) as a base for phase two.

For phase two, a central composite design with $\alpha = 2$ is chosen. This requires total 15 points including the base point. The objective functions are evaluated at these points (x^{56} through x^{70}). A second degree curve is fitted to these data yields

$$\begin{aligned} y = 488.000 &+ 1.049x_1 - 80.00x_2 - 19.813x_3 \\ &+ 0.064x_1^2 - 0.319x_1x_2 - 0.797x_1x_3 \\ &+ 4.14x_2^2 + 1.516x_2x_3 + 2.867x_3^2 \end{aligned}$$

This is an ellipsoid passing through the base point x^{52} (67.54, 9.91, 9.81). Its center is x^{71} (20.45, 9.44, 2.10). A trial at this point reveals that this point is not better than the base point. In design points, however, a better point x^{65} (68.54, 9.91, 9.81) is found. This becomes a base point for the next experiment. A central composite design with $\alpha = 2$ is again constructed around this point. Objective functions are evaluated at these points 72 through 86, and a second degree curve fitted. The equation of the curve is

$$y = 757.000 - 1.781x_1 - 96.750x_2 - 43.938x_3$$
$$+ 0.065x_1^2 - 0.348x_1x_2 - 0.372x_1x_3 + 5.324x_2^2$$
$$+ 1.386x_2x_3 + 2.766x_3^2$$

The center of this curve is found as x^{87} (66.06, 10.00, 9.60) with the objective function as 10.993, which is no better than the base point x^{80}. No other better point is also found and therefore this ends the phase two.

The solution can be summarized as

starting point (50.00, 20.00, 20.00)

optimal point (68.54, 9.91, 9.81)

optimal value = 7.421

No. of times objective function is evaluated = 87

APPENDIX I (CONTINUED)

```
*** RESPONSE SURFACE METHODOLOGY ***


PROBLEM NO.    1


          OPTIMAL CONTROL OF THE INTEGRATED
               HUMAN THERMAL SYSTEM


NUMBER OF VARIABLES -    3

VARIABLE NAMES
    X(1) - INLET TEMPERATURE OF THE COOLANT
           IN THEJACKET AND HOOD - DEGREES FAHRENHEIT
    X(2) - MASS FLOW RATEOF THE COOLANT
           IN THE JACKET - LBS PER HOUR
    X(3) - MASS FLOW RATE OF THE COOLANT
           IN THE HOOD - LBS PER HOUR

VARIABLE                        X( 1)     X( 2)     X( 3)


STARTING VALUE                  50.00     20.00     20.00
STEP SIZE FOR INCREMENT          2.00      2.00      2.00
FINAL STEP SIZE                  0.50      0.50      0.50

OBJECTIVE FUNCTION :
    S = 2.0*ABS(TB - 36.6) + ABS(TS-34.1) + ABS(TM-35.9)
        + ABS(TIN-15.6) + 100.0*ABS(2.203/WFLOW1-0.2203)
        + 100.0*ABS(2.203/WFLOW2-0.2203)

THE ABOVE OBJECTIVE FUNCTION IS TO BE MINIMIZED

METHOD USED FOR MINIMIZATION - MODIFIED VERSION OF RESPONSE SURFACE
                               METHODOLOGY BY BOX AND WILSON(1951)
```

```
PHASE ONE : RESPONSE SURFACE IS APPROXIMATED AS A PLANE


EXPERIMENT NO.  1

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT  : INLET TEMP. OF COOLANT    -  50.00 DEG. F
                 MASS FLOW RATE FOR JACKET-  20.00 LBS PER HOUR
                 MASS FLOW RATE FOR HOOD  -  20.00 LBS PER HOUR
                 METABOLIC RATE           - 300.00 BTU PER HOUR

B. DESIGN USED : 2**3 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED :  9

D. RESULTS OF TRIALS

   ** INTERMEDIATE RESULTS **

   TRIAL NO.    1: X(1) =  48.00      X(2) =  18.00      X(3) =  18.00

                   BRAIN  TEMP.  =    71.20 DEG. F
                   SKIN   TEMP.  =    68.97 DEG. F
                   MUSCLE TEMP.  =    71.51 DEG. F
                   OBJECTIVE FUNCTION =    83.446

   TRIAL NO.    2: X(1) =  48.00      X(2) =  18.00      X(3) =  22.00

                   BRAIN  TEMP.  =    71.16 DEG. F
                   SKIN   TEMP.  =    68.92 DEG. F
                   MUSCLE TEMP.  =    71.48 DEG. F
                   OBJECTIVE FUNCTION =    85.772

   TRIAL NO.    3: X(1) =  48.00      X(2) =  22.00      X(3) =  18.00

                   BRAIN  TEMP.  =    70.19 DEG. F
                   SKIN   TEMP.  =    68.01 DEG. F
                   MUSCLE TEMP.  =    70.49 DEG. F
                   OBJECTIVE FUNCTION =    87.894

   TRIAL NO.    4: X(1) =  48.00      X(2) =  22.00      X(3) =  22.00

                   BRAIN  TEMP.  =    70.15 DEG. F
                   SKIN   TEMP.  =    67.95 DEG. F
                   MUSCLE TEMP.  =    70.45 DEG. F
                   OBJECTIVE FUNCTION =    90.220
```

```
TRIAL NO.   5: X(1) =   52.00      X(2) =   18.00      X(3) =   18.00

             BRAIN  TEMP.   =    75.20 DEG. F
             SKIN   TEMP.   =    72.97 DEG. F
             MUSCLE TEMP.   =    75.51 DEG. F
             OBJECTIVE FUNCTION =     72.335

TRIAL NO.   6: X(1) =   52.00      X(2) =   18.00      X(3) =   22.00

             BRAIN  TEMP.   =    75.16 DEG. F
             SKIN   TEMP.   =    72.92 DEG. F
             MUSCLE TEMP.   =    75.48 DEG. F
             OBJECTIVE FUNCTION =     74.661

TRIAL NO.   7: X(1) =   52.00      X(2) =   22.00      X(3) =   18.00

             BRAIN  TEMP.   =    74.19 DEG. F
             SKIN   TEMP.   =    72.01 DEG. F
             MUSCLE TEMP.   =    74.49 DEG. F
             OBJECTIVE FUNCTION =     76.783

TRIAL NO.   8: X(1) =   52.00      X(2) =   22.00      X(3) =   22.00

             BRAIN  TEMP.   =    74.15 DEG. F
             SKIN   TEMP.   =    71.95 DEG. F
             MUSCLE TEMP.   =    74.45 DEG. F
             OBJECTIVE FUNCTION =     79.109

TRIAL NO.   9: X(1) =   50.00      X(2) =   20.00      X(3) =   20.00

             BRAIN  TEMP.   =    72.62 DEG. F
             SKIN   TEMP.   =    70.41 DEG. F
             MUSCLE TEMP.   =    72.93 DEG. F
             OBJECTIVE FUNCTION =     81.616

** END OF INTERMEDIATE RESULTS **

SUMMARY OF RESULTS
```

TRIAL NO.	TRIAL POINT X(1)	X(2)	X(3)	OBJECTIVE FUNCTION
1	48.00	18.00	18.00	83.45
2	48.00	18.00	22.00	85.77
3	48.00	22.00	18.00	87.89
4	48.00	22.00	22.00	90.22
5	52.00	18.00	18.00	72.33
6	52.00	18.00	22.00	74.66
7	52.00	22.00	18.00	76.78
8	52.00	22.00	22.00	79.11
9	50.00	20.00	20.00	81.62

```
E. EQUATION OF A SURFACE AT THE BASE POINT
   Y =    186.305+ -2.778X(1) +  1.112X(2) +  0.582X(3)
```

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE	STEP SIZE
X(1)	2.000
X(2)	-0.800
X(3)	-0.419

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

BASE POINT : X(1) = 50.00 X(2) = 20.00 X(3) = 20.00
OBJECTIVE FUNCTION = 81.616

TRIAL NO. 10: X(1) = 52.00 X(2) = 19.20 X(3) = 19.58

BRAIN TEMP. = 74.83 DEG. F
SKIN TEMP. = 72.62 DEG. F
MUSCLE TEMP. = 75.15 DEG. F
OBJECTIVE FUNCTION = 74.896

THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

TRIAL NO. 11: X(1) = 54.00 X(2) = 18.40 X(3) = 19.16

BRAIN TEMP. = 77.06 DEG. F
SKIN TEMP. = 74.84 DEG. F
MUSCLE TEMP. = 77.38 DEG. F
OBJECTIVE FUNCTION = 68.086

THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

TRIAL NO. 12: X(1) = 56.00 X(2) = 17.60 X(3) = 18.74

BRAIN TEMP. = 79.32 DEG. F
SKIN TEMP. = 77.08 DEG. F
MUSCLE TEMP. = 79.64 DEG. F
OBJECTIVE FUNCTION = 61.174

THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

TRIAL NO. 13: X(1) = 58.00 X(2) = 16.80 X(3) = 18.32

BRAIN TEMP. = 81.59 DEG. F
SKIN TEMP. = 79.35 DEG. F
MUSCLE TEMP. = 81.91 DEG. F
OBJECTIVE FUNCTION = 54.146

THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

TRIAL NO. 14: X(1) = 60.00 X(2) = 16.00 X(3) = 17.91

```
                BRAIN  TEMP.   =    83.89 DEG. F
                SKIN   TEMP.   =    81.64 DEG. F
                MUSCLE TEMP.   =    84.22 DEG. F
                OBJECTIVE FUNCTION =     46.985

          THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
          TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

   TRIAL NO.  15: X(1) =  62.00      X(2) =  15.20      X(3) =  17.49

                BRAIN  TEMP.   =    86.23 DEG. F
                SKIN   TEMP.   =    83.96 DEG. F
                MUSCLE TEMP.   =    86.56 DEG. F
                OBJECTIVE FUNCTION =     41.805

          THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
          TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

   TRIAL NO.  16: X(1) =  64.00      X(2) =  14.40      X(3) =  17.07

                BRAIN  TEMP.   =    88.60 DEG. F
                SKIN   TEMP.   =    86.31 DEG. F
                MUSCLE TEMP.   =    88.94 DEG. F
                OBJECTIVE FUNCTION =     36.537

          THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
          TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

   TRIAL NO.  17: X(1) =  66.00      X(2) =  13.60      X(3) =  16.65

                BRAIN  TEMP.   =    91.01 DEG. F
                SKIN   TEMP.   =    88.71 DEG. F
                MUSCLE TEMP.   =    91.36 DEG. F
                OBJECTIVE FUNCTION =     31.064

          THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
          TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

   TRIAL NO.  18: X(1) =  68.00      X(2) =  12.80      X(3) =  16.23

                BRAIN  TEMP.   =    93.47 DEG. F
                SKIN   TEMP.   =    91.16 DEG. F
                MUSCLE TEMP.   =    93.83 DEG. F
                OBJECTIVE FUNCTION =     25.348

          THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
          TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

   TRIAL NO.  19: X(1) =  70.00      X(2) =  12.00      X(3) =  15.81

                BRAIN  TEMP.   =    96.00 DEG. F
                SKIN   TEMP.   =    93.67 DEG. F
                MUSCLE TEMP.   =    96.37 DEG. F
                OBJECTIVE FUNCTION =     19.660

          THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
          TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

   TRIAL NO.  20: X(1) =  72.00      X(2) =  11.19      X(3) =  15.39
```

```
BRAIN  TEMP.  =   98.60 DEG. F
SKIN   TEMP.  =   96.24 DEG. F
MUSCLE TEMP.  =   98.98 DEG. F
OBJECTIVE FUNCTION =    20.396
```

THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL HAS FAILED.
THE TRIAL POINT YIELDING MINIMUM OBJECTIVE FUCTION BECOMES THE BASE POINT.

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT :

```
X(1) =  50.00     X(2) =  20.00      X(3) =  20.00
OBJECTIVE FUNCTION    19.660
```

B. BASE POINT AT THE END OF THE EXPERIMENT :

```
X(1) =  70.00     X(2) =  12.00      X(3) =  15.81
OBJECTIVE FUNCTION =    19.660
```

C. REMARKS :

THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT IS SUCCESSFUL.
BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT
OF THE SUBSEQUENT EXPERIMENT.

```
EXPERIMENT NO.  2

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT  : INLET TEMP. OF COOLANT    -  70.00 DEG. F
                 MASS FLOW RATE FOR JACKET-  12.00 LBS PER HOUR
                 MASS FLOW RATE FOR HOOD   -  15.81 LBS PER HOUR
                 METABOLIC RATE            - 300.00 BTU PER HOUR

B. DESIGN USED : 2**3 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED :  9

D. RESULTS OF TRIALS

   ** INTERMEDIATE RESULTS **

   TRIAL NO.  21: X(1) =   68.00      X(2) =   10.00      X(3) =   13.81

                  BRAIN  TEMP.  =    95.70 DEG. F
                  SKIN   TEMP.  =    93.31 DEG. F
                  MUSCLE TEMP.  =    96.10 DEG. F
                  OBJECTIVE FUNCTION =     13.243

   TRIAL NO.  22: X(1) =   68.00      X(2) =   10.00      X(3) =   17.81

                  BRAIN  TEMP.  =    95.63 DEG. F
                  SKIN   TEMP.  =    93.21 DEG. F
                  MUSCLE TEMP.  =    96.04 DEG. F
                  OBJECTIVE FUNCTION =     16.987

   TRIAL NO.  23: X(1) =   68.00      X(2) =   14.00      X(3) =   13.81

                  BRAIN  TEMP.  =    92.85 DEG. F
                  SKIN   TEMP.  =    90.58 DEG. F
                  MUSCLE TEMP.  =    93.19 DEG. F
                  OBJECTIVE FUNCTION =     25.813

   TRIAL NO.  24: X(1) =   68.00      X(2) =   14.00      X(3) =   17.81

                  BRAIN  TEMP.  =    92.78 DEG. F
                  SKIN   TEMP.  =    90.49 DEG. F
                  MUSCLE TEMP.  =    93.13 DEG. F
                  OBJECTIVE FUNCTION =     29.557

   TRIAL NO.  25: X(1) =   72.00      X(2) =   10.00      X(3) =   13.81

                  BRAIN  TEMP.  =    99.70 DEG. F
                  SKIN   TEMP.  =    97.31 DEG. F
                  MUSCLE TEMP.  =   100.10 DEG. F
                  OBJECTIVE FUNCTION =     18.850
```

TRIAL NO. 26: X(1) = 72.00 X(2) = 10.00 X(3) = 17.81

BRAIN TEMP. = 95.63 DEG. F
SKIN TEMP. = 97.21 DEG. F
MUSCLE TEMP. = 100.04 DEG. F
OBJECTIVE FUNCTION = 22.270

TRIAL NO. 27: X(1) = 72.00 X(2) = 14.00 X(3) = 13.81

BRAIN TEMP. = 96.85 DEG. F
SKIN TEMP. = 94.58 DEG. F
MUSCLE TEMP. = 97.19 DEG. F
OBJECTIVE FUNCTION = 21.119

TRIAL NO. 28: X(1) = 72.00 X(2) = 14.00 X(3) = 17.81

BRAIN TEMP. = 96.78 DEG. F
SKIN TEMP. = 94.49 DEG. F
MUSCLE TEMP. = 97.13 DEG. F
OBJECTIVE FUNCTION = 24.694

TRIAL NO. 29: X(1) = 70.00 X(2) = 12.00 X(3) = 15.81

BRAIN TEMP. = 96.00 DEG. F
SKIN TEMP. = 93.67 DEG. F
MUSCLE TEMP. = 96.37 DEG. F
OBJECTIVE FUNCTION = 19.660

** END OF INTERMEDIATE RESULTS **

SUMMARY OF RESULTS

TRIAL NO.	TRIAL POINT X(1)	X(2)	X(3)	OBJECTIVE FUNCTION
21	68.00	10.00	13.81	13.24
22	68.00	10.00	17.81	16.99
23	68.00	14.00	13.81	25.81
24	68.00	14.00	17.81	29.56
25	72.00	10.00	13.81	18.85
26	72.00	10.00	17.81	22.27
27	72.00	14.00	13.81	21.12
28	72.00	14.00	17.81	24.69
29	70.00	12.00	15.81	19.66

E. EQUATION OF A SURFACE AT THE BASE POINT
Y = -21.156+ 0.083X(1) + 1.864X(2) + 0.905X(3)

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES
VARIABLE STEP SIZE

```
        X(1)              -0.090
        X(2)              -2.000
        X(3)              -0.971

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

   BASE POINT    : X(1) =   70.00      X(2) =   12.00      X(3) =   15.81
                   OBJECTIVE FUNCTION =      19.660

   TRIAL NO.  30: X(1) =   69.91      X(2) =   10.00      X(3) =   14.84

                   BRAIN  TEMP.  =    97.59 DEG. F
                   SKIN   TEMP.  =    95.19 DEG. F
                   MUSCLE TEMP.  =    97.99 DEG. F
                   OBJECTIVE FUNCTION =      14.750

        THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
        TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

   TRIAL NO.  31: X(1) =   69.82      X(2) =    8.00      X(3) =   13.87

                   BRAIN  TEMP.  =   100.01 DEG. F
                   SKIN   TEMP.  =    97.51 DEG. F
                   MUSCLE TEMP.  =   100.47 DEG. F
                   OBJECTIVE FUNCTION =      23.883

        THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL HAS FAILED.
        THE TRIAL POINT YIELDING MINIMUM OBJECTIVE FUCTION BECOMES THE BASE POINT.

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT :

                   X(1) =   70.00      X(2) =   12.00      X(3) =   15.81
                   OBJECTIVE FUNCTION     13.243

B. BASE POINT AT THE END OF THE EXPERIMENT :

                   X(1) =   68.00      X(2) =   10.00      X(3) =   13.81
                   OBJECTIVE FUNCTION =      13.243

C. REMARKS :

        THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION.  EXPERIMENT IS SUCCESSFUL.
        BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT
        OF THE SUBSECUENT EXPERIMENT.
```

EXPERIMENT NO. 3

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT : INLET TEMP. OF COOLANT - 68.00 DEG. F
MASS FLOW RATE FOR JACKET- 10.00 LBS PER HOUR
MASS FLOW RATE FOR HCOD - 13.81 LBS PER HOUR
METABOLIC RATE - 300.00 BTU PER HOUR

B. DESIGN USED : 2**3 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED : 9

D. RESULTS OF TRIALS

** INTERMEDIATE RESULTS **

TRIAL NO. 32: X(1) = 66.00 X(2) = 8.00 X(3) = 11.81

BRAIN TEMP. = 96.24 DEG. F
SKIN TEMP. = 93.76 DEG. F
MUSCLE TEMP. = 96.69 DEG. F
OBJECTIVE FUNCTION = 14.262

TRIAL NO. 33: X(1) = 66.00 X(2) = 8.00 X(3) = 15.81

BRAIN TEMP. = 96.15 DEG. F
SKIN TEMP. = 93.64 DEG. F
MUSCLE TEMP. = 96.61 DEG. F
OBJECTIVE FUNCTION = 18.977

TRIAL NO. 34: X(1) = 66.00 X(2) = 12.00 X(3) = 11.81

BRAIN TEMP. = 92.09 DEG. F
SKIN TEMP. = 89.79 DEG. F
MUSCLE TEMP. = 92.45 DEG. F
OBJECTIVE FUNCTION = 21.077

TRIAL NO. 35: X(1) = 66.00 X(2) = 12.00 X(3) = 15.81

BRAIN TEMP. = 92.00 DEG. F
SKIN TEMP. = 89.67 DEG. F
MUSCLE TEMP. = 92.37 DEG. F
OBJECTIVE FUNCTION = 26.009

TRIAL NO. 36: X(1) = 70.00 X(2) = 8.00 X(3) = 11.81

BRAIN TEMP. = 100.24 DEG. F
SKIN TEMP. = 97.76 DEG. F
MUSCLE TEMP. = 100.69 DEG. F
OBJECTIVE FUNCTION = 21.738

```
TRIAL NO.  37: X(1) =   70.00      X(2) =    8.00      X(3) =  15.81

               BRAIN  TEMP.  =  100.15 DEG. F
               SKIN   TEMP.  =   97.64 DEG. F
               MUSCLE TEMP.  =  100.61 DEG. F
               OBJECTIVE FUNCTION =    26.242

TRIAL NO.  38: X(1) =   70.00      X(2) =   12.00      X(3) =  11.81

               BRAIN  TEMP.  =   96.09 DEG. F
               SKIN   TEMP.  =   93.79 DEG. F
               MUSCLE TEMP.  =   96.45 DEG. F
               OBJECTIVE FUNCTION =    14.865

TRIAL NO.  39: X(1) =   70.00      X(2) =   12.00      X(3) =  15.81

               BRAIN  TEMP.  =   96.00 DEG. F
               SKIN   TEMP.  =   93.67 DEG. F
               MUSCLE TEMP.  =   96.37 DEG. F
               OBJECTIVE FUNCTION =    19.660

TRIAL NO.  40: X(1) =   68.00      X(2) =   10.00      X(3) =  13.81

               BRAIN  TEMP.  =   95.70 DEG. F
               SKIN   TEMP.  =   93.31 DEG. F
               MUSCLE TEMP.  =   96.10 DEG. F
               OBJECTIVE FUNCTION =    13.243

** END OF INTERMEDIATE RESULTS **

SUMMARY OF RESULTS
```

TRIAL NO.	TRIAL POINT X(1)	X(2)	X(3)	OBJECTIVE FUNCTION
32	66.00	8.00	11.81	14.26
33	66.00	8.00	15.81	18.98
34	66.00	12.00	11.81	21.08
35	66.00	12.00	15.81	26.01
36	70.00	8.00	11.81	21.74
37	70.00	8.00	15.81	26.24
38	70.00	12.00	11.81	14.86
39	70.00	12.00	15.81	19.66
40	68.00	10.00	13.81	13.24

```
E. EQUATION OF A SURFACE AT THE BASE POINT
     Y =     -6.247+  0.136X(1) +  0.024X(2) +  1.184X(3)


STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE


A. MODIFIED STEP SIZES
      VARIABLE          STEP SIZE
```

```
        X(1)            -0.229
        X(2)            -0.041
        X(3)            -2.000

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

   BASE POINT    : X(1) =  68.00      X(2) =  10.00      X(3) =  13.81
                   OBJECTIVE FUNCTION =     13.243

   TRIAL NO.  41: X(1) =  67.77      X(2) =   9.95      X(3) =  11.81

                   BRAIN  TEMP.  =    95.56 DEG. F
                   SKIN   TEMP.  =    93.19 DEG. F
                   MUSCLE TEMP.  =    95.96 DEG. F
                   OBJECTIVE FUNCTION =     10.802

        THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
        TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

   TRIAL NO.  42: X(1) =  67.54      X(2) =   9.91      X(3) =   9.81

                   BRAIN  TEMP.  =    95.45 DEG. F
                   SKIN   TEMP.  =    93.10 DEG. F
                   MUSCLE TEMP.  =    95.83 DEG. F
                   OBJECTIVE FUNCTION =      8.053

        THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL IS SUCCESSFUL.
        TRIAL POINT BECOMES THE BASE POINT FOR THE SUBSEQUENT TRIAL.

   TRIAL NO.  43: X(1) =  67.31      X(2) =   9.87      X(3) =   7.81

                   BRAIN  TEMP.  =    95.38 DEG. F
                   SKIN   TEMP.  =    93.06 DEG. F
                   MUSCLE TEMP.  =    95.74 DEG. F
                   OBJECTIVE FUNCTION =     13.922

        THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL HAS FAILED.
        THE TRIAL POINT YIELDING MINIMUM OBJECTIVE FUCTION BECOMES THE BASE POINT.

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT :

                   X(1) =  68.00      X(2) =  10.00      X(3) =  13.81
                   OBJECTIVE FUNCTION      8.053

B. BASE POINT AT THE END OF THE EXPERIMENT :

                   X(1) =  67.54      X(2) =   9.91      X(3) =   9.81
                   OBJECTIVE FUNCTION =     8.053

C. REMARKS :

        THERE IS AN IMPROVEMENT IN THE OBJECTIVE FUNCTION.  EXPERIMENT IS SUCCESSFUL.
        BASE POINT AT THE END OF THE EXPERIMENT BECOMES THE STARTING POINT
        OF THE SUBSEQUENT EXPERIMENT.
```

```
EXPERIMENT NO.  4

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT  : INLET TEMP. OF COOLANT    -  67.54 DEG. F
                 MASS FLOW RATE FOR JACKET-   9.91 LBS PER HOUR
                 MASS FLOW RATE FOR HOOD  -   9.81 LBS PER HOUR
                 METABOLIC RATE           - 300.00 BTU PER HOUR

B. DESIGN USED : 2**3 FACTORIAL DESIGN WITH CENTER POINT

C. NUMBER OF TRIALS REQUIRED :  9

D. RESULTS OF TRIALS

   ** INTERMEDIATE RESULTS **

   TRIAL NO.  44: X(1) =  65.54     X(2) =   7.91     X(3) =   7.81

                  BRAIN  TEMP.  =   96.10 DEG. F
                  SKIN   TEMP.  =   93.68 DEG. F
                  MUSCLE TEMP.  =   96.52 DEG. F
                  OBJECTIVE FUNCTION =    17.209

   TRIAL NO.  45: X(1) =  65.54     X(2) =   7.91     X(3) =  11.81

                  BRAIN  TEMP.  =   95.92 DEG. F
                  SKIN   TEMP.  =   93.43 DEG. F
                  MUSCLE TEMP.  =   96.37 DEG. F
                  OBJECTIVE FUNCTION =    14.576

   TRIAL NO.  46: X(1) =  65.54     X(2) =  11.91     X(3) =   7.81

                  BRAIN  TEMP.  =   91.88 DEG. F
                  SKIN   TEMP.  =   89.63 DEG. F
                  MUSCLE TEMP.  =   92.20 DEG. F
                  OBJECTIVE FUNCTION =    23.943

   TRIAL NO.  47: X(1) =  65.54     X(2) =  11.91     X(3) =  11.81

                  BRAIN  TEMP.  =   91.69 DEG. F
                  SKIN   TEMP.  =   89.39 DEG. F
                  MUSCLE TEMP.  =   92.05 DEG. F
                  OBJECTIVE FUNCTION =    21.587

   TRIAL NO.  48: X(1) =  69.54     X(2) =   7.91     X(3) =   7.81

                  BRAIN  TEMP.  =  100.10 DEG. F
                  SKIN   TEMP.  =   97.68 DEG. F
                  MUSCLE TEMP.  =  100.52 DEG. F
                  OBJECTIVE FUNCTION =    24.261
```

```
TRIAL NO.  49: X(1) =  69.54      X(2) =   7.91      X(3) =  11.81

               BRAIN  TEMP.   =    99.92 DEG. F
               SKIN   TEMP.   =    97.43 DEG. F
               MUSCLE TEMP.   =   100.37 DEG. F
               OBJECTIVE FUNCTION =     21.040

TRIAL NO.  50: X(1) =  69.54      X(2) =  11.91      X(3) =   7.81

               BRAIN  TEMP.   =    95.88 DEG. F
               SKIN   TEMP.   =    93.63 DEG. F
               MUSCLE TEMP.   =    96.20 DEG. F
               OBJECTIVE FUNCTION =     17.559

TRIAL NO.  51: X(1) =  69.54      X(2) =  11.91      X(3) =  11.81

               BRAIN  TEMP.   =    95.69 DEG. F
               SKIN   TEMP.   =    93.39 DEG. F
               MUSCLE TEMP.   =    96.05 DEG. F
               OBJECTIVE FUNCTION =     14.926

TRIAL NO.  52: X(1) =  67.54      X(2) =   9.91      X(3) =   9.81

               BRAIN  TEMP.   =    95.45 DEG. F
               SKIN   TEMP.   =    93.10 DEG. F
               MUSCLE TEMP.   =    95.83 DEG. F
               OBJECTIVE FUNCTION =      8.053
```

** END OF INTERMEDIATE RESULTS **

SUMMARY OF RESULTS

TRIAL NO.	TRIAL POINT X(1)	X(2)	X(3)	OBJECTIVE FUNCTION
44	65.54	7.91	7.81	17.21
45	65.54	7.91	11.81	14.58
46	65.54	11.91	7.81	23.94
47	65.54	11.91	11.81	21.59
48	69.54	7.91	7.81	24.26
49	69.54	7.91	11.81	21.04
50	69.54	11.91	7.81	17.56
51	69.54	11.91	11.81	14.93
52	67.54	9.91	9.81	8.05

E. EQUATION OF A SURFACE AT THE BASE POINT

Y =   22.219+ 0.029X(1) + 0.058X(2) + -0.678X(3)

STEP II : APPLICATION OF THE STEEPEST ASCENT PROCEDURE

A. MODIFIED STEP SIZES

VARIABLE   STEP SIZE

```
        X(1)              -0.086
        X(2)              -0.171
        X(3)               2.000

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

   BASE POINT    : X(1) =  67.54      X(2) =    9.91      X(3) =    9.81
                   OBJECTIVE FUNCTION =      8.053

   TRIAL NO.  53: X(1) =  67.46      X(2) =    9.74      X(3) =  11.81

                   BRAIN  TEMP.  =    95.47 DEG. F
                   SKIN   TEMP.  =    93.08 DEG. F
                   MUSCLE TEMP.  =    95.87 DEG. F
                   OBJECTIVE FUNCTION =     11.329

        THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL HAS FAILED.
        THE TRIAL POINT YIELDING MINIMUM OBJECTIVE FUCTION BECOMES THE BASE POINT.
        REDUCE STEP SIZE BY HALF. REPEAT STEP II.

A. MODIFIED STEP SIZES
     VARIABLE          STEP SIZE
        X(1)              -0.043
        X(2)              -0.086
        X(3)               1.000

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

   BASE POINT    : X(1) =  67.54      X(2) =    9.91      X(3) =    9.81
                   OBJECTIVE FUNCTION =      8.053

   TRIAL NO.  54: X(1) =  67.50      X(2) =    9.83      X(3) =  10.81

                   BRAIN  TEMP.  =    95.46 DEG. F
                   SKIN   TEMP.  =    93.08 DEG. F
                   MUSCLE TEMP.  =    95.85 DEG. F
                   OBJECTIVE FUNCTION =      9.453

        THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL HAS FAILED.
        THE TRIAL POINT YIELDING MINIMUM OBJECTIVE FUCTION BECOMES THE BASE POINT.
        REDUCE STEP SIZE BY HALF. REPEAT STEP II.

A. MODIFIED STEP SIZES
     VARIABLE          STEP SIZE
        X(1)              -0.022
        X(2)              -0.043
        X(3)               0.500

B. RESULTS OF TRIALS AT POINTS IN THE DIRECTION OF SEEPEST ASCENT

   BASE POINT    : X(1) =  67.54      X(2) =    9.91      X(3) =    9.81
                   OBJECTIVE FUNCTION =      8.053

   TRIAL NO.  55: X(1) =  67.52      X(2) =    9.87      X(3) =  10.31
```

```
BRAIN  TEMP.  =    95.45 DEG. F
SKIN   TEMP.  =    93.09 DEG. F
MUSCLE TEMP.  =    95.84 DEG. F
OBJECTIVE FUNCTION =    8.385
```

THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL HAS FAILED.
THE TRIAL POINT YIELDING MINIMUM OBJECTIVE FUCTION BECOMES THE BASE POINT.
STEP SIZE CANNOT BE FURTHER REDUCED.END OF THE EXPERIMENT.

STEP III : SUMMARY OF THE EXPERIMENT

A. BASE POINT AT THE START OF THE EXPERIMENT :

```
X(1) =  67.54     X(2) =   9.91     X(3) =   9.81
OBJECTIVE FUNCTION     8.053
```

B. BASE POINT AT THE END OF THE EXPERIMENT :

```
X(1) =  67.54     X(2) =   9.91     X(3) =   9.81
OBJECTIVE FUNCTION =    8.053
```

C. REMARKS :

THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. EXPERIMENT HAS FAILED.
APPROXIMATION OF A SURFACE AS A PLANE NO LONGER HOLDS GOOD. END OF THE PHASE.

*** SUMMARY OF PHASE ONE ***

```
BASE POINT AT THE START OF PHASE ONE
      INLET TEMPERATURE OF COOLANT     -  50.00  DEGREES FAHRENHEIT
      MASS FLOW RATE FOR JACKET        -  20.00  LBS PER HOUR
      MASS FLOW RATE FOR HOOD          -  20.00  LBS PER HOUR
      METABOLIC HEAT GENERATION RATE - 300.00  BTU PER HOUR

      OBJECTIVE FUNCTION =   81.616


OPTIMUM POINT (SO FAR) AT THE END OF PHASE ONE
      INLET TEMPERATURE OF COOLANT     -  67.54  DEGREES FAHRENHEIT
      MASS FLOW RATE FOR JACKET        -   9.91  LBS PER HOUR
      MASS FLOW RATE FOR HOOD          -   9.81  LBS PER HOUR
      METABOLIC HEAT GENERATION RATE - 300.00  BTU PER HOUR

      OBJECTIVE FUNCTION =    8.053
```

```
PHASE TWO : RESPONSE SURFACE IS APPROXIMATED AS A SECOND DEGREE CURVE
EXPERIMENT NO.  1

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

A. BASE POINT  : INLET TEMP. OF COOLANT   -  67.54 DEG. F
                 MASS FLOW RATE FOR JACKET-   9.91 LBS PER HOUR
                 MASS FLOW RATE FOR HOOD  -   9.81 LBS PER HOUR
                 METABOLIC RATE           - 300.00 BTU PER HOUR

B. DESIGN USED : CENTRAL COMPOSITE DESIGN WITH ALPHA = 2

C. NUMBER OF TRIALS REQUIRED : 15

D. RESULTS OF TRIALS

   ** INTERMEDIATE RESULTS **

   TRIAL NO.  56: X(1) =  67.04     X(2) =   9.41     X(3) =   9.31

                  BRAIN  TEMP.  =   95.51 DEG. F
                  SKIN   TEMP.  =   93.14 DEG. F
                  MUSCLE TEMP.  =   95.90 DEG. F
                  OBJECTIVE FUNCTION =    10.039

   TRIAL NO.  57: X(1) =  67.04     X(2) =   9.41     X(3) =  10.31

                  BRAIN  TEMP.  =   95.46 DEG. F
                  SKIN   TEMP.  =   93.08 DEG. F
                  MUSCLE TEMP.  =   95.86 DEG. F
                  OBJECTIVE FUNCTION =     9.183

   TRIAL NO.  58: X(1) =  67.04     X(2) =  10.41     X(3) =   9.31

                  BRAIN  TEMP.  =   94.49 DEG. F
                  SKIN   TEMP.  =   92.17 DEG. F
                  MUSCLE TEMP.  =   94.86 DEG. F
                  OBJECTIVE FUNCTION =    11.781

   TRIAL NO.  59: X(1) =  67.04     X(2) =  10.41     X(3) =  10.31

                  BRAIN  TEMP.  =   94.45 DEG. F
                  SKIN   TEMP.  =   92.11 DEG. F
                  MUSCLE TEMP.  =   94.82 DEG. F
                  OBJECTIVE FUNCTION =    10.926

   TRIAL NO.  60: X(1) =  68.04     X(2) =   9.41     X(3) =   9.31
```

```
                  BRAIN  TEMP.   =    96.51 DEG. F
                  SKIN   TEMP.   =    94.14 DEG. F
                  MUSCLE TEMP.   =    96.90 DEG. F
                  OBJECTIVE FUNCTION =      9.525

TRIAL NO.   61: X(1) =   68.04      X(2) =    9.41      X(3) =  10.31

                  BRAIN  TEMP.   =    96.46 DEG. F
                  SKIN   TEMP.   =    94.08 DEG. F
                  MUSCLE TEMP.   =    96.86 DEG. F
                  OBJECTIVE FUNCTION =      8.561

TRIAL NO.   62: X(1) =   68.04      X(2) =   10.41      X(3) =   9.31

                  BRAIN  TEMP.   =    95.49 DEG. F
                  SKIN   TEMP.   =    93.17 DEG. F
                  MUSCLE TEMP.   =    95.86 DEG. F
                  OBJECTIVE FUNCTION =     10.115

TRIAL NO.   63: X(1) =   68.04      X(2) =   10.41      X(3) =  10.31

                  BRAIN  TEMP.   =    95.45 DEG. F
                  SKIN   TEMP.   =    93.11 DEG. F
                  MUSCLE TEMP.   =    95.82 DEG. F
                  OBJECTIVE FUNCTION =      9.259

TRIAL NO.   64: X(1) =   67.54      X(2) =    9.91      X(3) =   9.81

                  BRAIN  TEMP.   =    95.45 DEG. F
                  SKIN   TEMP.   =    93.10 DEG. F
                  MUSCLE TEMP.   =    95.83 DEG. F
                  OBJECTIVE FUNCTION =      8.053

TRIAL NO.   65: X(1) =   68.54      X(2) =    9.91      X(3) =   9.81

                  BRAIN  TEMP.   =    96.45 DEG. F
                  SKIN   TEMP.   =    94.10 DEG. F
                  MUSCLE TEMP.   =    96.83 DEG. F
                  OBJECTIVE FUNCTION =      7.421

TRIAL NO.   66: X(1) =   66.54      X(2) =    9.91      X(3) =   9.81

                  BRAIN  TEMP.   =    94.45 DEG. F
                  SKIN   TEMP.   =    92.10 DEG. F
                  MUSCLE TEMP.   =    94.83 DEG. F
                  OBJECTIVE FUNCTION =      9.720

TRIAL NO.   67: X(1) =   67.54      X(2) =   10.91      X(3) =   9.81

                  BRAIN  TEMP.   =    94.53 DEG. F
                  SKIN   TEMP.   =    92.22 DEG. F
                  MUSCLE TEMP.   =    94.89 DEG. F
```

```
                OBJECTIVE FUNCTION =      11.735

TRIAL NO.  68: X(1) =   67.54      X(2) =    8.91      X(3) =    9.81

                BRAIN  TEMP.  =   96.58 DEG. F
                SKIN   TEMP.  =   94.18 DEG. F
                MUSCLE TEMP.  =   96.98 DEG. F
                OBJECTIVE FUNCTION =      9.345

TRIAL NO.  69: X(1) =   67.54      X(2) =    9.91      X(3) =   10.81

                BRAIN  TEMP.  =   95.41 DEG. F
                SKIN   TEMP.  =   93.04 DEG. F
                MUSCLE TEMP.  =   95.80 DEG. F
                OBJECTIVE FUNCTION =      9.381

TRIAL NO.  70: X(1) =   67.54      X(2) =    9.91      X(3) =    8.81

                BRAIN  TEMP.  =   95.50 DEG. F
                SKIN   TEMP.  =   93.16 DEG. F
                MUSCLE TEMP.  =   95.87 DEG. F
                OBJECTIVE FUNCTION =     10.485
```

** END OF INTERMEDIATE RESULTS **

SUMMARY OF RESULTS

TRIAL NO.	TRIAL POINT X(1)	X(2)	X(3)	OBJECTIVE FUNCTION
56	67.04	9.41	9.31	10.04
57	67.04	9.41	10.31	9.18
58	67.04	10.41	9.31	11.78
59	67.04	10.41	10.31	10.93
60	68.04	9.41	9.31	9.52
61	68.04	9.41	10.31	8.56
62	68.04	10.41	9.31	10.11
63	68.04	10.41	10.31	9.26
64	67.54	9.91	9.81	8.05
65	68.54	9.91	9.81	7.42
66	66.54	9.91	9.81	9.72
67	67.54	10.91	9.81	11.73
68	67.54	8.91	9.81	9.35
69	67.54	9.91	10.81	9.38
70	67.54	9.91	8.81	10.49

E. EQUATION OF A SURFACE AT THE BASE POINT

```
  Y=488.000+  1.049X1 +-80.000X2 +-19.813X3 +  0.064X1.X1
    + -0.319X1.X2 + -0.797X1.X3 +  4.414X2.X2 +  1.516X2.X3 +  2.867X3.X3
```

STEP II : DETERMINATION OF THE CO-ORDINATES OF AND OBJECTIVE FUNCTION
AT THE CENTER OF THE SURFACE

A. CO-ORDINATES OF THE CENTER

B. TRIAL AT THE CENTER POINT

```
TRIAL NO.  71: X(1) = 162.67      X(2) =  10.96      X(3) =  23.16

               BRAIN  TEMP.  =  189.37 DEG. F
               SKIN   TEMP.  =  186.97 DEG. F
               MUSCLE TEMP.  =  189.77 DEG. F
               OBJECTIVE FUNCTION =   276.841

     THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL HAS FAILED.
     THE TRIAL POINT YIELDING MINIMUM OBJECTIVE FUCTION BECOMES THE BASE POINT.
```

EXPERIMENT NO. 2

STEP I : DETERMINATION OF AN EQUATION OF A SURFACE AT THE BASE POINT

```
A. BASE POINT  : INLET TEMP. OF COOLANT    -  68.54 DEG. F
                 MASS FLOW RATE FOR JACKET-   9.91 LBS PER HOUR
                 MASS FLOW RATE FOR HOOD   -   9.81 LBS PER HOUR
                 METABOLIC RATE            - 300.00 BTU PER HOUR
```

B. DESIGN USED : CENTRAL COMPOSITE DESIGN WITH ALPHA = 2

C. NUMBER OF TRIALS REQUIRED : 15

D. RESULTS OF TRIALS

** INTERMEDIATE RESULTS **

```
TRIAL NO.  72: X(1) =  68.04      X(2) =   9.41      X(3) =   9.31

               BRAIN  TEMP.  =   96.51 DEG. F
               SKIN   TEMP.  =   94.14 DEG. F
               MUSCLE TEMP.  =   96.90 DEG. F
               OBJECTIVE FUNCTION =      9.525

TRIAL NO.  73: X(1) =  68.04      X(2) =   9.41      X(3) =  10.31

               BRAIN  TEMP.  =   96.46 DEG. F
               SKIN   TEMP.  =   94.08 DEG. F
               MUSCLE TEMP.  =   96.86 DEG. F
               OBJECTIVE FUNCTION =      8.561

TRIAL NO.  74: X(1) =  68.04      X(2) =  10.41      X(3) =   9.31

               BRAIN  TEMP.  =   95.49 DEG. F
               SKIN   TEMP.  =   93.17 DEG. F
               MUSCLE TEMP.  =   95.86 DEG. F
               OBJECTIVE FUNCTION =     10.115

TRIAL NO.  75: X(1) =  68.04      X(2) =  10.41      X(3) =  10.31

               BRAIN  TEMP.  =   95.45 DEG. F
               SKIN   TEMP.  =   93.11 DEG. F
               MUSCLE TEMP.  =   95.82 DEG. F
               OBJECTIVE FUNCTION =      9.259

TRIAL NO.  76: X(1) =  69.04      X(2) =   9.41      X(3) =   9.31

               BRAIN  TEMP.  =   97.51 DEG. F
               SKIN   TEMP.  =   95.14 DEG. F
               MUSCLE TEMP.  =   97.90 DEG. F
               OBJECTIVE FUNCTION =     10.080
```

```
TRIAL NO.  77: X(1) =  69.04      X(2) =   9.41      X(3) =  10.31

               BRAIN  TEMP.  =   97.46 DEG. F
               SKIN   TEMP.  =   95.08 DEG. F
               MUSCLE TEMP.  =   97.86 DEG. F
               OBJECTIVE FUNCTION =     9.117

TRIAL NO.  78: X(1) =  69.04      X(2) =  10.41      X(3) =   9.31

               BRAIN  TEMP.  =   96.49 DEG. F
               SKIN   TEMP.  =   94.17 DEG. F
               MUSCLE TEMP.  =   96.86 DEG. F
               OBJECTIVE FUNCTION =     9.590

TRIAL NO.  79: X(1) =  69.04      X(2) =  10.41      X(3) =  10.31

               BRAIN  TEMP.  =   96.45 DEG. F
               SKIN   TEMP.  =   94.11 DEG. F
               MUSCLE TEMP.  =   96.82 DEG. F
               OBJECTIVE FUNCTION =     8.627

TRIAL NO.  80: X(1) =  68.54      X(2) =   9.91      X(3) =   9.81

               BRAIN  TEMP.  =   96.45 DEG. F
               SKIN   TEMP.  =   94.10 DEG. F
               MUSCLE TEMP.  =   96.83 DEG. F
               OBJECTIVE FUNCTION =     7.421

TRIAL NO.  81: X(1) =  69.54      X(2) =   9.91      X(3) =   9.81

               BRAIN  TEMP.  =   97.45 DEG. F
               SKIN   TEMP.  =   95.10 DEG. F
               MUSCLE TEMP.  =   97.83 DEG. F
               OBJECTIVE FUNCTION =     7.976

TRIAL NO.  82: X(1) =  67.54      X(2) =   9.91      X(3) =   9.81

               BRAIN  TEMP.  =   95.45 DEG. F
               SKIN   TEMP.  =   93.10 DEG. F
               MUSCLE TEMP.  =   95.83 DEG. F
               OBJECTIVE FUNCTION =     8.053

TRIAL NO.  83: X(1) =  68.54      X(2) =  10.91      X(3) =   9.81

               BRAIN  TEMP.  =   95.53 DEG. F
               SKIN   TEMP.  =   93.22 DEG. F
               MUSCLE TEMP.  =   95.89 DEG. F
               OBJECTIVE FUNCTION =    10.068

TRIAL NO.  84: X(1) =  68.54      X(2) =   8.91      X(3) =   9.81

               BRAIN  TEMP.  =   97.58 DEG. F
```

```
            SKIN   TEMP.  =   95.18 DEG. F
            MUSCLE TEMP.  =   97.98 DEG. F
            OBJECTIVE FUNCTION =    9.901

TRIAL NO.  85: X(1) =  68.54     X(2) =   9.91     X(3) =  10.81

            BRAIN  TEMP.  =   96.41 DEG. F
            SKIN   TEMP.  =   94.04 DEG. F
            MUSCLE TEMP.  =   96.80 DEG. F
            OBJECTIVE FUNCTION =    8.651

TRIAL NO.  86: X(1) =  68.54     X(2) =   9.91     X(3) =   8.81

            BRAIN  TEMP.  =   96.50 DEG. F
            SKIN   TEMP.  =   94.16 DEG. F
            MUSCLE TEMP.  =   96.87 DEG. F
            OBJECTIVE FUNCTION =    9.973

** END OF INTERMEDIATE RESULTS **

SUMMARY OF RESULTS
```

TRIAL NO.	TRIAL POINT X(1)	X(2)	X(3)	OBJECTIVE FUNCTION
72	68.04	9.41	9.31	9.52
73	68.04	9.41	10.31	8.56
74	68.04	10.41	9.31	10.11
75	68.04	10.41	10.31	9.26
76	69.04	9.41	9.31	10.08
77	69.04	9.41	10.31	9.12
78	69.04	10.41	9.31	9.59
79	69.04	10.41	10.31	8.63
80	68.54	9.91	9.81	7.42
81	69.54	9.91	9.81	7.98
82	67.54	9.91	9.81	8.05
83	68.54	10.91	9.81	10.07
84	68.54	8.91	9.81	9.90
85	68.54	9.91	10.81	8.65
86	68.54	9.91	8.81	9.97

```
E. EQUATION OF A SURFACE AT THE BASE POINT
   Y=757.000+ -1.781X1 +-96.750X2 +-43.938X3 +  0.065X1.X1
     + -0.348X1.X2 + -0.372X1.X3 +  5.324X2.X2 +  1.386X2.X3 +  2.766X3.X3

STEP II : DETERMINATION OF THE CO-ORDINATES OF AND OBJECTIVE FUNCTION
          AT THE CENTER OF THE SURFACE

A. CO-ORDINATES OF THE CENTER

B. TRIAL AT THE CENTER POINT

TRIAL NO.  87: X(1) =  69.29     X(2) =  10.04     X(3) =  10.09

            BRAIN  TEMP.  =   97.06 DEG. F
```

```
SKIN   TEMP.  =   94.71 DEG. F
MUSCLE TEMP.  =   97.44 DEG. F
OBJECTIVE FUNCTION =     7.496

THERE IS NO IMPROVEMENT IN THE OBJECTIVE FUNCTION. TRIAL HAS FAILED.
THE TRIAL POINT YIELDING MINIMUM OBJECTIVE FUCTION BECOMES THE BASE POINT.
END OF THE EXPERIMENT.


          *** SUMMARY OF PHASE TWO ***


BASE POINT AT THE START OF PHASE TWO
      INLET TEMPERATURE OF COOLANT     -  67.54  DEGREES FAHRENHEIT
      MASS FLOW RATE FOR JACKET       -   9.91  LBS PER HOUR
      MASS FLOW RATE FOR HOOD         -   9.81  LBS PER HOUR
      METABOLIC HEAT GENERATION RATE - 300.00  BTU PER HOUR

      OBJECTIVE FUNCTION =    8.053


OPTIMUM POINT AT THE END OF PHASE TWO
      INLET TEMPERATURE OF COOLANT     -  68.54  DEGREES FAHRENHEIT
      MASS FLOW RATE FOR JACKET       -   9.91  LBS PER HOUR
      MASS FLOW RATE FOR HOOD         -   9.81  LBS PER HOUR
      METABOLIC HEAT GENERATION RATE - 300.00  BTU PER HOUR

      OBJECTIVE FUNCTION =    7.421
```

ACKNOWLEDGEMENTS

The author thanks Dr. C. L. Hwang, and Dr. L. T. Fan for their valuable guidance and encouragement in the preparation of this report. Thanks are also due to Mrs. M. Jirak for typing a neat report.

This study was partly supported by NASA Grant No. NGR-17-001-034.

OPTIMAL CONTROL OF INTEGRATED HUMAN

THERMAL SYSTEM BY RESPONSE SURFACE METHODOLOGY

by

HEMANT N. OZARKAR

B.E. (Mechanical) (1965)

Poona University, India

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1971

ABSTRACT

The purpose of this report is twofold-to develop a computer program to obtain an optimum point which minimizes (maximizes) a given function of multi-dimensional variables using Response Surface Methodology, and to carry out a computer simulation of optimal control of the integrated human thermal system by this method.

The response surface methodology is described first. An optimum point is sought by sequential experimentation. The iterative procedure of the experimentation is started from any point chosen in the region under consideration. In the neighborhood of this point enough experiments are performed which enables one to fit, by the method of least squares, a polynomial approximation of sufficient order to provide a local representation of the surface. The method is divided into two phases depending upon the order of approximate function. Phase one employes a linear approximation whereas phase two employs a quadratic one. Phase one, by linear approximation, provides a rapid progress from the starting base point, which is usually far from the optimum, to a point within "striking distance" of it, while phase two, by a quadratic approximation of surface, leads the further progress to the actual optimum point. In doing so an efficient design of experiment is needed. An efficient design of experiment not only minimizes the number of experiments, but also provides the required information with maximum precision.

A simple two-dimensional production scheduling problem is then solved to illustrate the use of method. This problem has been solved by other methods. Results from the present method compare very well with those by the other methods.

The report then presents the results of computer simulations of optimal control of integrated human thermal system. The control parameters of the external thermal regulation device are the inlet coolant temperature and its mass flow rate. The objectives of the controlling the regulation device are to maintain the temperature of the human body in thermal comfort (thermoneutrality) and to minimize the possible effort imposed on the operation of the device. A study on modeling, simulation and optimal control of an integrated human thermal system has been carried out by Hsu. He has obtained the optimal control policies by employing a well-known technique of linear programming. His optimal control problem can be carried out experimentally at KSU-ASHRAE test facility. The present study provides a numerical experimentation (simulation) of the optimal control problem using response surface methodology. A comparison is made between the present results obtained and those by Hsu. In addition to the form of objective function used by Hsu, which was necessary for linear programming, a different and more rigorous form is used to obtain a set of optimal conditions.