PRESSURE DROP FOR SINGLE PHASE FLOW THROUGH PACKED BEDS

by

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INTRODUCTION

Knowledge of the factors which contribute to pressure drop in packed beds finds many applications. Motion of ground waters, petroleum, and natural gas through rocks, soil, and sands require a knowledge of the same laws that govern flow through packed beds. Seepage under dams, the permeability of concretes, and surface area or density of many industrial materials are determined by application of these laws. Direct application to chemical engineering is found in filtration, distillation, absorption, fluidized or packed bed catalytic operations, and drying of solid particles.

Several attempts have been made to describe packed beds adequately so that pressure drop could be predicted. These investigations have successfully answered problems of limited scope, but, none have resulted in sufficiently general conclusions to allow the extension of existing information to new packing materials of novel geometrical nature.

This investigation was initiated in order to isolate a series of packed bed variables that could be used as a criterion for predicting pressure drop in any randomly packed bed. Special attention was given to amplifying the effect of variables that had already been recognized as important, isolation of the relevant factors, and a general formulation of these results that would lead to accurate prediction for pressure loss through any packing material.

Variables Which Have Been Considered

The factors which affect pressure drop can be segregated into two groups. The fluid and empty column variables constitute one group. The packed bed variables constitute another.

The role of fluid variables has long been understood sufficiently so that they need not be analysed in the discussion. It is sufficient to select nomenclature for these.

Table 1. Nomenclature for fluid and empty column variables.

g	= gravitational constant converting weight units to force units.
L_t	= depth of the packed zone.
ΔP	= pressure loss due to frictional resistance across L_t. Units of weight divided by area.
U_o	= velocity based upon the empty column.
μ	= absolute viscosity of the fluid.
ρ	= mass density of the fluid.

Table 2 includes basic items which have been given prior consideration. More complex variables are not included since each different investigator has grouped the items to suit his theory or needs. Expression of these complex terms has been avoided for the sake of simplicity of understanding the subject as a whole.

The measurements used to describe packed columns as treated in the literature are given in Table 2.

Table 2. Nomenclature for packing variables needed for discussion of literature.

D_p	= nominal diameter of the packing unit.
D_s	= diameter of a sphere having the same volume as the packing unit.
D_t	= column diameter.
e	= height of an element of surface roughness.
r	= ratio between distance traversed by a fluid and the length of the column.
s_p	= surface of the packing unit.
S_p	= total surface of the packing.
s_s	= surface of a sphere having the same volume as the packing unit.
S_t	= surface area of the column.
V	= total free, or void, volume in the packed zone.
v_p	= volume of the packing unit.
V_p	= total volume occupied by the packing.
V_t	= $V + V_p$ = volume of the empty column.

Some Important Concepts

One very basic concept of the problem of fluid flow has become classic to the representation of pressure drop in packed beds. Dimensional analysis based upon the assumption that pressure drop is a function of fluid density, fluid viscosity, fluid velocity, hydraulic radius or diameter, and surface roughness leads to results that parallel the development of the Fanning equation which is concerned with flow through channels. The result is

$$\frac{dP}{dL} = \delta \frac{\rho U^2}{gD} \phi\left(\frac{D\rho U}{\mu}, \frac{e}{D}\right) \tag{1}$$

where

$\frac{dP}{dL}$ = gradient of frictional pressure loss along the actual path of flow,

δ = constant factor determined from the geometrical nature of the available flow path,

U = mean fluid velocity along the actual path of flow,

D = the equivalent hydraulic diameter or radius of the bed,

other terms are as in Tables 1 or 2.

Early developments did not recognize the influence of δ and $\frac{e}{D}$. The efforts of each individual to express these four variables through consideration of those in Table 2 and the geometrical nature of the packing unit has constituted all theoretical developments concerned with flow through packed beds.

According to the results of this investigation a satisfactory expression for hydraulic radius was perhaps first introduced by Blake (1) in 1922. Blake did not include this development in his paper, but his results were identical to those of Carman (5) who chose to solve for

$$m = \frac{\text{volume of duct}}{\text{surface contacted by the fluid}}$$

This was carried even closer to the concept of hydraulic radius as it was applied to flow through channels by

$$m = \frac{\text{cross-section of duct}}{\text{perimeter of duct}} = \frac{\text{cross-section x length of duct}}{\text{perimeter x length of duct}}$$

$$= \frac{\text{volume of duct}}{\text{surface of duct}}$$

Regardless of the mode of development, it is certainly true that this quantity is a measure of the distance between surfaces that contact the fluid. It has been found convenient to write

$$m = V/S_p \qquad (2)$$

No serious difficulty would be encountered by including column surface with packing surface, a step which appears logical. However, a certain degree of mathematical simplicity is gained by use of equation (2).

Prediction of pressure drop through a wide variety of packing materials and packed bed variables was finally accomplished by introducing a concept which might be termed hydraulic width of the packing unit. In any event, the new term represents the width of the barrier that must be circumvented by the fluid just as "m" represents the width of the path available for flow. A definition was framed so that the following could be approximated mathematically.

$$w = \frac{\text{surface contacting the fluid}}{\text{boundry of obstructing surfaces}}$$

To this end the packing perimeter, C_p, was defined as: the locus of tangent points to the packing that would be generated by a line which moved throughout the packed bed remaining oriented parallel to the column wall. It was then possible to include this mathematical expression for w:

$$w = S_p/C_p \qquad (3)$$

By combining (2) and (3) a very useful measure of the distortion of flow path was obtained and retained in the form

$\frac{w}{m}$ = index to degree of distortion of flow path and other effects of the packing.

It is not clear at present whether w/m, $w(1 + S_t/S_p)/m$, or $w(1 + 0.6S_t/S_p)/m$ will be proved most useful. The term, $(1 + 0.6S_t/S_p)$, was discovered as a very excellent factor for accommodating the confining surface of a circular column; its development will be treated later since it is of secondary importance.

Plate I, Figs. 1, 2, and 3 represent the way in which $\frac{w}{m}$ indexes the degree to which the fluid stream is disturbed. Figure 1 illustrates a bed of spheres, and the arrows suggest the path of flow through the bed. Figure 2 is constructed approximately to scale so that a bed of packing units, which is composed of randomly suspended circular plates, has the same values of "m" and "w" as does the bed of spheres. According to the results of the investigation, the pressure drop through both beds will be identical if U_o, ρ, and μ are identical. Figure 3 represents a bed, similar to the one in Fig. 2, that has the same value for "m" but fewer units of greater "w" are contained. It is predicted that, if U_o, ρ, and μ are the same as before, the pressure drop through this bed will be greater than that for either of the other beds. On the other hand, wire packing producing the same value for "m" should allow very low pressure drop.

The illustration was not tested experimentally, but the principle was repeatedly tested by reference to pressure drop through packed beds of widely differing properties which were approximately equivalent to those in the example.

EXPLANATION OF PLATE I

Fig. 1. Flow through spheres of diameter D_1 producing ΔP_1 for m_1, ρ_1, μ_1, $(U_o)_1$.

Fig. 2. Flow through plates of diameter $2 \times D_1$ should produce ΔP_1 for m_1, ρ_1, μ_1, $(U_o)_1$.

Fig. 3. Flow through plates of diameter $4 \times D_1$ should produce $\Delta P > \Delta P_1$ for m_1, ρ_1, μ_1, $(U_o)_1$.

PLATE I

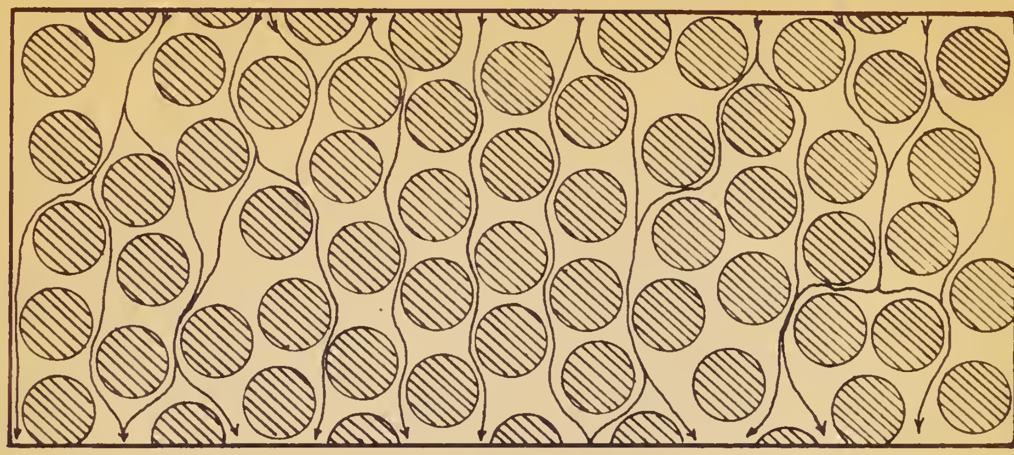

figure 1

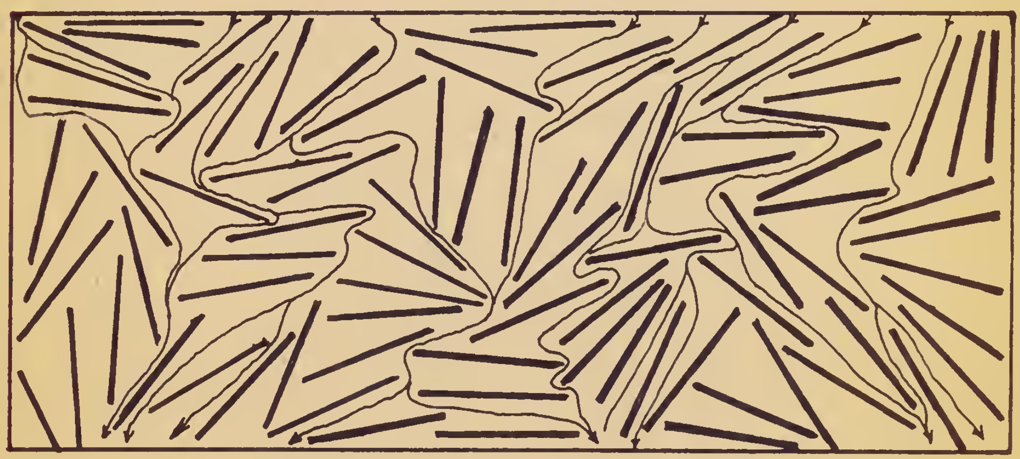

figure 2

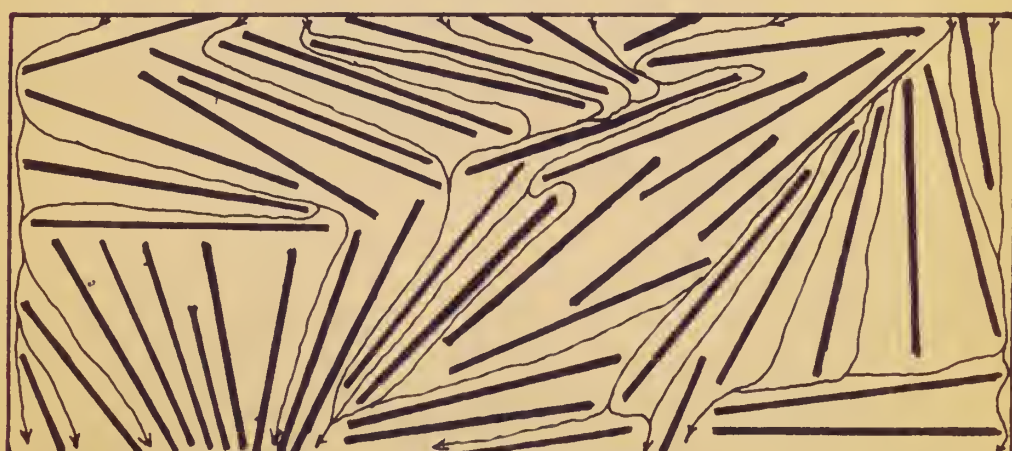

figure 3

LITERATURE REVIEW

The need for an understanding of the factors which contribute to pressure drop in packed beds was realized as early as 1863. The earlier theories utilized the assumptions that a packed bed was comprised of a series of ducts that possessed a total sectional area equal to the area that would be intersected by a plane passing through the bed, and that the surface of the walls of these ducts was equivalent to the surface of the packing material. Investigators who have considered one or two different packing materials have been relatively successful. Those who have considered a large number of packing materials have accepted serious discrepancies or have resorted to extremely complex and unjustifiable empiricisms.

In 1922 Blake (1) analysed flow through several beds of glass and clay Raschig rings. He applied the principles of equation (1) with a relative degree of success. In developing a friction factor and Reynold's number, he assumed that velocity should be in excess of the superficial velocity, U_o, by the ratio V_t/V and that hydraulic radius should be expressed as in equation (2). The ratio, V/V_t, had been previously proven identical to the fraction of column cross-section that was not intersected by the packing material. Blake represented the friction factor by $\Delta PgV^3/L_tU_o{}^2S_pV_t\rho$ and the Reynold's number by $\rho U_oV_t/\mu S_p$. These terms were plotted on logarithmic coordinates. He found that a single line represented results of tests with the glass packing but that lower values of the friction factor were obtained for the clay packing.

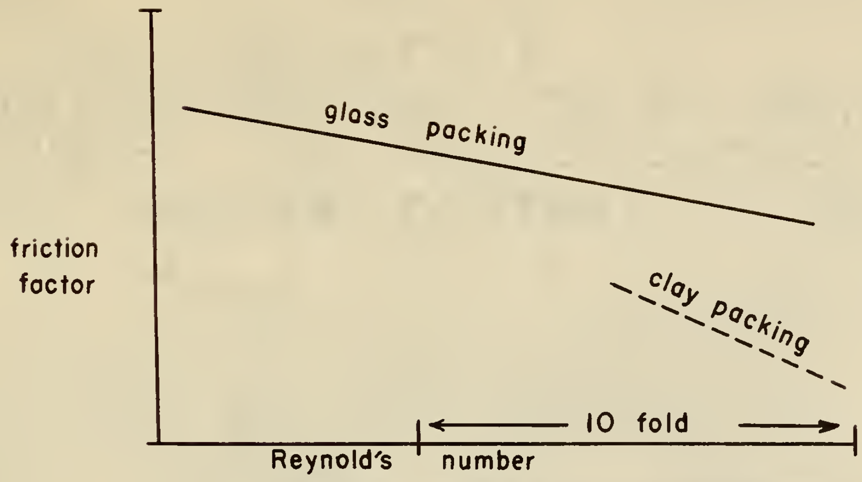

Fig. 4. Blake's (1) representation.

Flow through beds comprised of lead shot was studied by Burke and Plummer (4). They utilized variables equivalent to those of Blake and also represented their results on logarithmic coordinates. These tests illustrated the manner in which ϕ of equation (1) depends upon the Reynold's number representation. The groupings that were used were equivalent to those used by Blake. Figure 5 illustrates these results. The region where $\Delta P/L_t$ is proportional to U_o is laminar flow region. In the turbulent region $\Delta P/L_t$ is proportional to U_o^2. The intermediate zone of transition from laminar to turbulent flow is peculiar to packed beds. A single line represented the results with a satisfactory degree of accuracy.

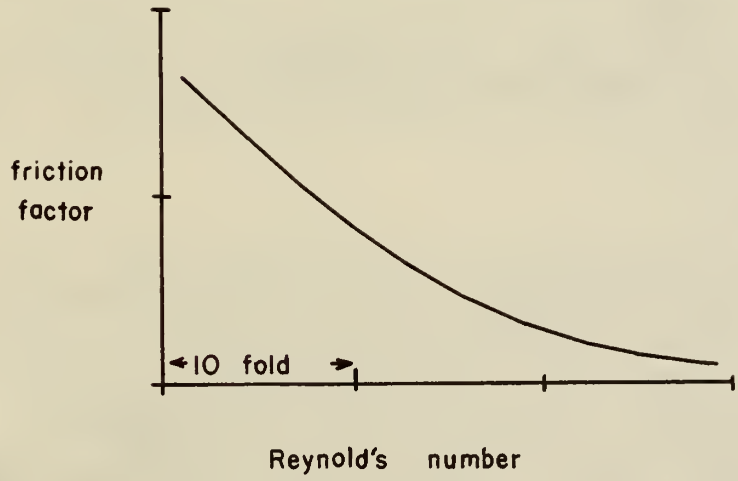

Fig. 5. Results of Burke and Plummer (4).

Literature up to about 1936 was thoroughly surveyed by Carman (5). This survey included much information that did not reach journals in English print. He found that laminar flow could be represented conveniently by the equation

$$\frac{\Delta P}{L_t} \propto \frac{\mu U_o S_p{}^2 V_t}{gV^3}$$

The proportionality factor included considerations of δ in equation (1) from the standpoint of the nature of a sphere and an emperical wall factor correction, $(1 + S_t/S_p)$, which yielded $\Delta P/L_t = \delta'(1 + S_t/S_p)U_oS_p{}^2V_t/gV^3$. Some results of tests by Pirie, given to Carman in a private communication, supply information for beds packed with cubes and prisms. Pirie was quoted to have worked "entirely in the streamline region".

Carman concluded that pressure drop through a variety of packing materials could be represented by

$$\frac{\Delta P}{L_t} = 5\frac{\mu U_o S_p{}^2 V_t}{gV^3} + 0.4 \quad \frac{\rho U_o{}^2 V_t{}^2 S_p}{gV^3}\left(\frac{S_p\mu}{\rho U_o V_t}\right)^{0.1}$$

This equation was formulated from information concerned with flow that was highly laminar, transitional, or highly turbulent. Figure 6 illustrates how this equation represented the information available at that time, the coordinates are identical to those used by Burke and Plummer.

Carman personally conducted a test to ascertain the true length of flow path in a bed of spheres. Unaided visual observations showed that the fluid path was inclined anywhere from 0° to 90° with respect to the container wall, and sometimes the fluid would follow an helical path. The mean inclination to the wall was concluded to be 45°.

A different approach for estimating the hydraulic diameter of a packed bed was being considered in the case of packing materials such as sands or crushed

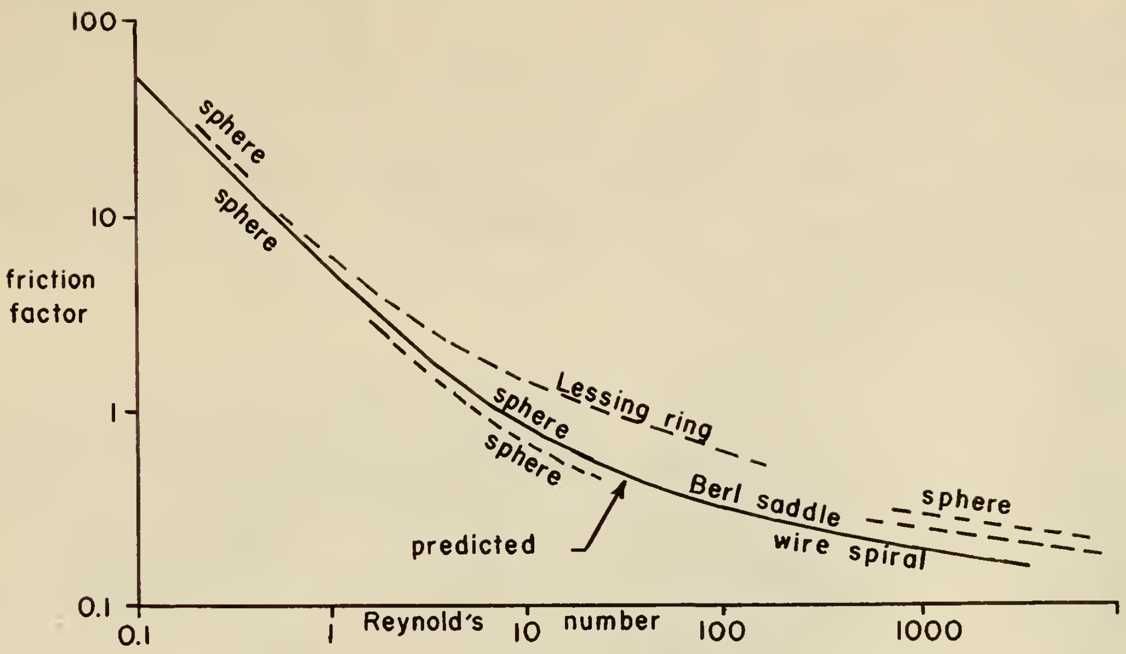

Fig. 6. Conclusions by Carman (5).

stone. These materials frequently possess an indeterminate surface area and the term D_s, as defined in Table 2, was found satisfactory for many empirical correlations. The investigation by Meyer and Work (9) typifies this approach. They found that

$$\frac{\Delta P}{L_t} = \frac{4750 \mu U_{o2} (0.67\ V_t - V)}{g D_v^2 V_t}$$

accomodated their crushed stone very well. D_v was expressed in terms of D_s and V/V_t, thus accommodating wall surface and factors due to variation of free space. Use of D_s was supported by analogy between diameter of a packing unit and its surface according to $S_p/V_p = 6/D_p$ for spheres.

Sullivan and Hertel (14) elaborated on the reasoning that Carman used to predict δ' for laminar flow through beds packed with spheres and textile fibers. They assumed that δ' for packed beds should originate from a basic value of $\delta = 3$ for an arbitrary duct which existed within the packed bed. They assumed that the effect of "r" could be expressed by the mean angle of orientation of surfaces with respect to the overall direction of flow. They expressed this mathematically as $\delta' = 3/(\sin^2\theta)av$. Their reasons for choosing a basic value of $\delta = 3$ can be understood only in the light of the fact that the agreement with experimental results was excellent. An earlier paper by Fowler and Hertel (6) showed that a basic value of $\delta = 3$ exists only in the extreme case of an infinitely wide rectangular duct. Sections that might more logically be present in packed beds are the triangle or square. Both of these possess values for δ which are much less than 3. It is also true that the wide rectangular duct suffers a large depression in δ if the walls converge as often as once every ten times the distance between them.

Table 3. Values of δ for laminar flow through empty ducts (7).

Shape of cross-section	δ
Circular	2.00
Elliptical	
major axis = 2 x minor axis	2.10
major axis = 10 x minor axis	2.42
Rectangular	
square	1.78
length = 2 x width	1.94
length = 10 x width	2.65
infinitely wide	3.00
Triangular	
equilateral triangle	1.67
infinitely high	1.50*

*Included by present author.

The experimental results of Sullivan and Hertel are believed to be the most reliable in the literature. They payed extreme attention to detail and found that

$$\frac{\Delta P}{L_t} = 4.50(1 + 2\,S_t/3\,S_p)^2\,\frac{\mu\,U_o S_p^2 V_t}{gV^3} \pm 0.55 \text{ percent}$$

for their tests with a few beds of spheres. Although this equation was derived from very general considerations, it is applicable only to those few beds of spheres. The wall correction factor, $(1 + 2S_t/3S_p)$, was arrived at by considering mathematical consistency between the circular column, $\delta = 2$, and the value for $\delta' = 4.50$. This correction has been found slightly too large to apply to greater values of S_t/S_p. Similar reasoning, based upon practical consideration of a great variety of packing materials, has led to another value. Sullivan and Hertel maintained $\rho U_o V_t/\mu S_p < 0.014$ thus insuring that totally insignificant transitional effects were encountered.

Wall effects as a function of D_p/D_t has been reviewed by Perry (12). More recent developments along this line have been accomplished in a series of articles by Leva and Grummer (8).

Oman and Watson (11) tested several beds of different packing materials. Flow was almost entirely turbulent. They introduced an accurate picture of the effect of "loose pack" and "close pack" upon pressure drop. It was found that by plotting friction factor $= \dfrac{\Delta P}{L_t} \dfrac{gV^{1.7}}{\rho U_o{}^2 S_p V_t{}^{0.7}}$ vs. Reynold's number $= \dfrac{\rho U_o V_t}{\mu S_p}$ on logarithmic coordinates a best mean line representation was obtained. A factor, (fd/f_1), was used to allow for the effects encountered due to the differences in void space offered by the two methods of packing. One important point is that free volume was included as being raised to the 1.7 power rather than the 3.0 power, a direct contridiction to most previous conclusions. This is in good agreement with results obtained in the present investigation.

Morcom (10) developed an equation for predicting pressure drop in laminar, transition, or turbulent flow regions. His equation agreed well with experimental data for many individual beds and was of the form

$$\frac{\Delta P}{L_t} = k\frac{\mu U_o}{g} + K\frac{\rho U_o^2}{g} \qquad (5)$$

More detail is purposely excluded because reference was made to some very illusive terms such as "normal voids". The most important consideration is that this expression accommodated the various regions of flow well for any one bed. The packing materials used were poorly defined so that results of his individual tests were not useful in a detailed analysis of flow. It is noteworthy that equation (5) is in general agreement with the conclusions of Carman except for absence of the term, $(\mu S_p/\rho U_o V_t)^{0.1}$. An inspection of Fig. 6 shows that pressure drop through spheres would have been better accommodated had this term been absent in Carman's equation.

Illustrations in a text by Rouse (13) typify how the results of studies of flow about suspended objects tend to verify equation (5).

Brownell and Katz (2) introduced several new concepts originating from a comparison of pressure drop in packed beds to pressure loss in conduits. Of primary importance is the discussion of the effect of surface roughness. Figure 7 shows several curves that were illustrated; most of these proposed curves are approximated by equation (5). The different packed bed variables considered by Brownell and Katz are summarized as follows:

$(V_t/V)^m$	= Reynold's number function
$(V_t/V)^n$	= friction factor function
s_s/s_p	= sphericity
e/D_p	= relative surface roughness
$D_p \rho U_o/\mu$	= modified Reynold's number
$(2gD_p \Delta P/L_t \rho U_o^2)$	= modified friction factor

By means of the Reynold's number function and the friction factor function, a representation of equation (1) was moved onto the pipe friction factor curve. Consideration of both sphericity and surface roughness resulted in satisfactory representation for a large series of particles of "primary configuration". Some difficulty was encountered when "splined" rings, or particles of "second configuration", were considered. In the written discussion to the authors, C. E. Lapple expressed doubt as to the possibility that surface roughness actually contributes to pressure drop in packed beds. In answer, the authors agreed that surface roughness was of little importance.

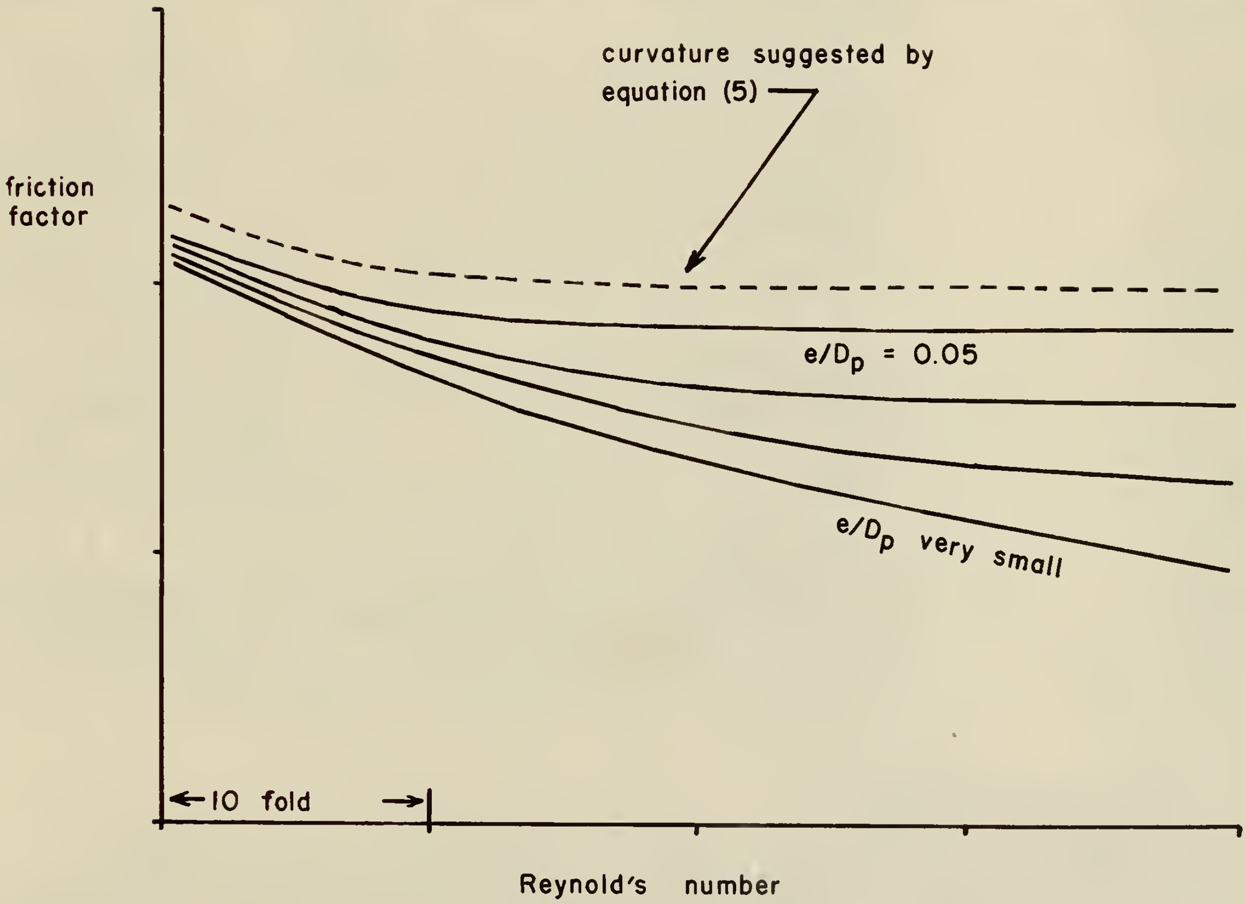

Fig. 7. Effect of surface roughness according to Brownell and Katz (2).

The proposals of Brownell and Katz were elaborated by Brownell, Dombrowski and Dickey (3). They proposed Figs. 8 and 9 to represent the Reynold's number function and the friction factor function. They arrived at no definite conclusions concerning the effect of surface roughness. Intensive tests which were conducted on a few beds tend to verify equation (5).

It is of some interest to note that the Reynold's number function of Fig. 8 will not lend itself to the wire rings tested by the present author. Figure 9 only vaguely suggests a friction factor function for them. The wire rings possessed a sphericity of 0.42 and beds of them possessed 82 to 84 percent void space.

MATERIAL AND METHODS

Pressure drops through several packed beds was observed in order to gain new knowledge as to the effect of tower surface and packing density upon pressure drop. Pressure drop through less conventional packing materials was also sought.

The Packing Materials

Seven packing materials, including four different types of packing units were tested. The volume, surface area, and perimeter of each unit was determined according to a method which was considered to be direct and accurate.

The following outline illustrates the basic measurements, accuracy of

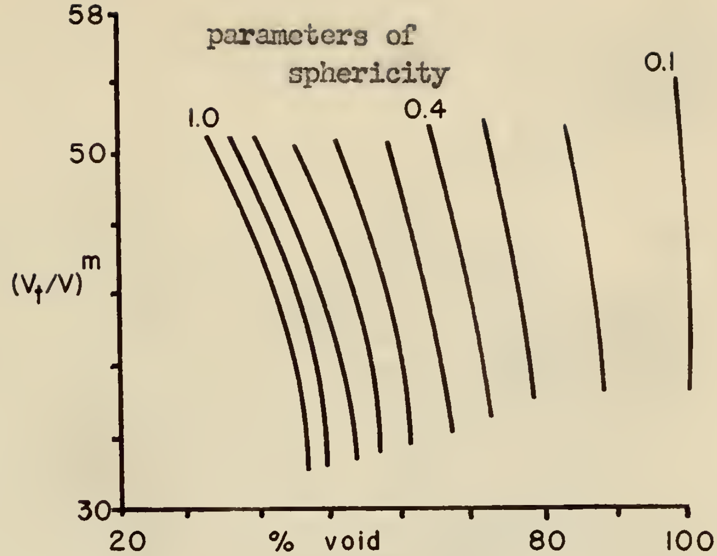

Fig. 8. Brownell's (3) Reynold's number function.

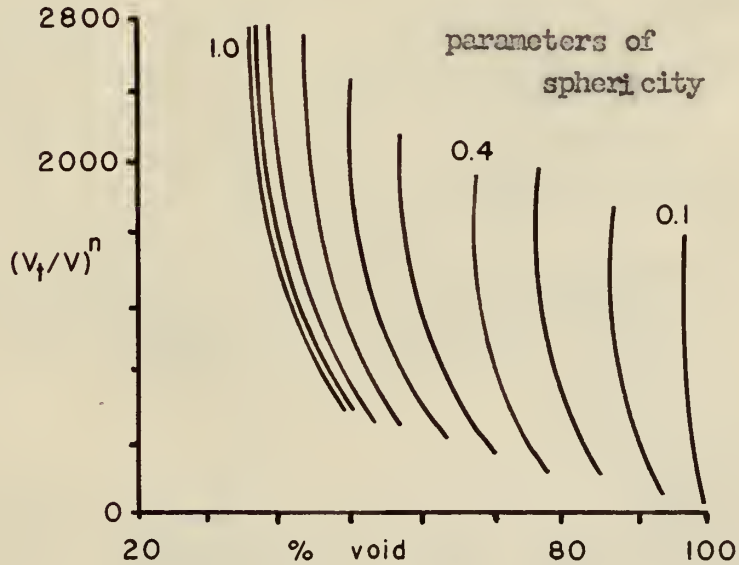

Fig. 9. Brownell's (3) friction factor function.

determination, and the derived information.

Packing #1, wire ring

Diameter of wire by micrometer, 23 arbitrarily selected units: 0.06765 ± 0.00029 inches
Volume by bouyancy in water, 1190 arbitrarily selected units: 3.83×10^{-6} cu. ft.
Volume by water displacement, 25700 arbitrarily selected units: 3.95×10^{-6} cu. ft.
Volume by water displacement, 11750 arbitrarily selected units: 4.04×10^{-6} cu. ft.

Final values:

volume $= 4.00 \times 10^{-6}$ cu. ft.
surface $= 2.89 \times 10^{-3}$ sq. ft.
perimeter $= 0.338$ ft. $= 2L + \pi D$

Packing #2, glass ball

Volume by bouyancy in water, 109 arbitrarily selected units: 1.485×10^{-4} cu. ft.
Volume by water displacement, 500 arbitrarily selected units: 1.506×10^{-4} cu. ft.

Final values:

volume $= 1.502 \times 10^{-4}$ cu. ft.
surface $= 6\ v/D$, $D = 0.0660$ ft.
perimeter $= 0.2074$ ft. $= \pi D$

Packing #3, clay Berl saddle

Width by machinist's rule, 20 arbitrarily selected units: 0.0854 ft.
Width by steel tape, 30 arbitrarily selected units placed edge to edge: 0.0847 ft.
Volume by bouyancy in water, 96 arbitrarily selected units: 1.444×10^{-4} cu. ft.
Volume by water displacement, 275 arbitrarily selected units: 1.374×10^{-4} cu. ft.
Volume by water displacement, 500 arbitrarily selected units: 1.468×10^{-4} cu. ft.

Final values:

volume $= 1.445 \times 10^{-4}$ cu. ft.
surface $= 0.0343$ sq. ft. (from mfg. data*)
perimeter $= 0.538$ ft. $= 2\pi D$

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*Manufactured by the Maurice A. Knight Company, Akron, Ohio.

Packing #4, clay Berl saddle

Width by steel tape, 40 arbitrarily selected units placed edge to edge: 0.0427 ft.
Volume by water displacement, 2400 arbitrarily selected units: 2.54×10^{-5} cu. ft.

Final values:

volume $= 2.54 \times 10^{-5}$ cu. ft.
surface $= 8.80 \times 10^{-3}$ sq. ft. (from mfg. data*)
perimeter $= 0.268$ ft. $= 2\pi D$

Packing #5, clay Raschig ring

Volume by water displacement, 330 arbitrarily selected units: 2.68×10^{-4} cu. ft.
Diameter by steel tape, 58 arbitrarily selected units placed side by side: 0.0860 ft.
Length by steel tape, 95 arbitrarily selected units placed end to end: 0.0873 ft.

Final values:

volume $= 2.68 \times 10^{-4}$ cu. ft.
surface $= 0.0459$ sq. ft.
perimeter $= 0.630$ ft. $= 2\pi D + 2L - 2\pi t$

Packing #6, clay Raschig ring

Volume by water displacement, 2000 arbitrarily selected units: 4.11×10^{-5} cu. ft.
Diameter by steel tape, 94 arbitrarily selected units placed side by side: 0.0435 ft.
Length by steel tape, 167 arbitrarily selected units placed end to end: 0.0444 ft.

Final values:

volume $= 4.11 \times 10^{-5}$ cu. ft.
surface $= 0.01165$ sq. ft.
perimeter $= 0.312$ ft. $= 2\pi D + 2L - 2\pi t$

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*Manufactured by the Maurice A. Knight Company, Akron, Ohio.

Packing #7, metal Raschig ring

Diameter by micrometer, 25 arbitrarily selected units: 1.0115 ± 0.0049 inch
Length by micrometer, 25 arbitrarily selected units: 1.0034 ± 0.0005 inch
Density by Westphal balance, water displacement, 5 arbitrarily selected units: 7.8764 ± 0.0135 gm. per cu. cm.
Mass of total supply, 698 = 8456 ± 6 gm.

Final values:

percent void = 92*
surface = 62.7 sq. ft. per cu. ft.*
perimeter = 0.681 ft. = $2\pi D + 2L - 2\pi t$

The calculations for perimeter are indicated primarily to illustrate how the perimeter is defined for packing materials. Determination of the perimeter of the Berl saddle is considered only an approximate method; all others are exact according to the present definition. Auxiliary measurements, such as thickness of the Raschig rings or length of the wire rings, were observed to agree with the above conclusions.

The Flow System

Three steel pipes of differing diameter were used as columns. Each of these was thirty-six inches in length. The packed zone included the entire length of each pipe while pressure drop measurements were taken from pressure taps located twenty-four inches apart and six inches from the ends.

Plate II illustrates the exact flow system used and the location of the various metering instruments.

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*Manufacturer's information: Metallo Gasket Company.

The pressure taps in the four inch column were different from the piezometer rings used in the three and six inch columns. These were so constructed to facilitate a later study of counter-current flow. No difference in results was noted that could be attributed to the difference in style of the pressure taps.

The diameters of the columns were determined by filling each column with water and noting the amount required for the space between the pressure taps. Column diameters and sectional areas thus determined were as follows:

Column	Diameter, inches	Area, square feet
three inch	3.10	0.05207
four inch	4.06	0.0898
six inch	6.08	0.2019

The porosity of each packed column was determined by the same process used to determine column diameter. The porosity of the bed of metal rings was the exception, its void fraction was ascertained from information published by the manufacturer.

Fluids and Flow Measurement

Three fluids were used: S.A.E. #60 oil, water, and air.

The density of the S.A.E. #60 oil was determined by Westphal balance. The balance was calibrated against water samples at various temperatures. The oil samples of varying temperature were then tested. The balance was found to be very sluggish when measuring the density of the oil; this was overcome by allowing sufficient time for the balance to react. No attempt was made to control temperature closely, temperature being read on the plumet of the balance, since it was not anticipated that serious effects of convection would be existant at the range of temperatures encountered. Results were reproducable to within ±0.0005 gram per cubic centimeter. The results used are as follows:

Temp. °C	Density, lb. per cu. ft.
20	56.28
25	56.11
30	55.95
35	55.75
40	55.56

Viscosity of the oil was determined by the Kansas State Highway Department at different temperatures. Repeated checks on samples that contained possible impurities, such as sludge, emulsified water, or emulsified air, showed that little error resulted from the presence of these impurities. Results of these tests are as follows:

Date	Temp., °F	Viscosity, centistokes	Possible impurities
12-16-50	70	1636.6	dissolved water
	80	1038.5	" "
	100	454.72	" "
	210	25.51	" "
1-4-51	100	445.03	sludge and water
	100	433.68	" " "
2-6-51	100	431.6	" " air
	100	432.2	" " "

The impurities are noted to have affected viscosity very little over a period of two months usage. The average viscosity of samples at 100° F for tests on 12-16-50 and 1-4-51 were used as a basis for calculations. The trend of viscosity with temperature was determined from results of 12-16-50; the logarithm of absolute viscosity was found to vary linearly with the inverse cube of absolute temperature. The validity of the relationship is illustrated thus:

Temp. range, °F	70-80	80-100	100-210
$\Delta \log \mu / \Delta (1000/°R)^3$	0.543	0.5495	0.534

The following viscosity information, being between 80 and 110° F, was derived from the slope determined for 80 to 100° F of 0.261 lb. per ft.-sec.

Temp. °C	30	32	34	36	38	40	42
Visc. rel. to 100° F	1.774	1.520	1.310	1.132	0.985	0.859	0.750
Visc., lb. per ft-sec.	0.463	0.397	0.342	0.295	0.257	0.224	0.196

These viscosities were used for all flow calculations. Flow of the oil was measured by time required for a weighed quantity of oil to flow from the system.

The density and viscosity of the water were taken from the Handbook of Chemistry and Physics (8). The water used was obtained directly from the Manhattan city supply.

The density and viscosity of dry air were obtained from the same source as the information for water. Corrections for moist air were applied as follows:

dens. moist air = dry air (1 - 0.61 abs. hum.)

visc. moist air = visc. dry air - abs. hum. (visc. dry air - visc. wat. vap.)

The estimation for density of moist air is an approximation which is good for low values of absolute humidity. No humidities over one percent were encountered. The estimation for the viscosity of moist air is based on the assumption that viscosities are additive with respect to weight percent. The calculated viscosity was never less than 99 1/2 percent of the viscosity of dry air. The air was obtained from the compressed air supply of the Kansas State College of Agriculture and Applied Science. Humidity was measured with a sling psychrometer and interpreted according to the psychrometric chart from the textbook of Badger and McCabe (17).

Flow of air and water was measured with a flow nozzle made by expanding one end of a short length of brass pipe. The nozzle was 0.0552 ft. in diameter and was mounted in a one inch steel pipe line. Impact and static pressure taps were located at the exit of the nozzle. The nozzle coefficient was found to be constant at $W/\rho\ (\Delta H)^{1/2} = 0.00518$ for the range of flow studied. Calibration was made with water where W = flow, lb. per sec.; ρ = density, lb. per cu. ft.; ΔH = head loss, inches of fluid. This meter was later calibrated with air. The coefficients were found to agree within four percent for the different fluids. The above mentioned coefficient corresponds to a discharge coefficient of about 93 percent. The flow nozzle used is seen in place in Plate I, together with other nozzles of similar construction.

Manometry

Manometers were used to measure pressure differential caused by flow through the nozzles, pressure drop, and pressure. Inverted manometers were used when water or oil was in the system; water filled manometers were used for air flow and pressure drop. Mercury filled manometers were used to measure pressure. All manometers were the "U" tube type and were calibrated in inches of fluid displacement. They are illustrated in Plate II.

Each manometric reading was interpreted so as to include the secondary effects of air as a second fluid and the difference between the density of the fluid in the system and the density of the fluid in the manometer. The latter effect was significant when the temperature of the oil approached 40 degrees centigrade.

Thermometry

Thermometers were located as indicated in Plate II. The column temperature was determined as the median temperature between inlet and outlet points. The temperature change did not exceed two degrees centigrade for any one run. The temperature of the flow meter was assumed to be the same as the temperature indicated by the inlet thermometer. All thermometers were checked against a precision thermometer so that any one temperature reading could be considered accurate within ±0.1 to ±0.2 degrees centigrade. The possibility of a temperature gradient at right angles to the flow path was considered. It was found that a gradient of only one degree centigrade existed when oil in the reservoir was at a temperature of 35 degrees centigrade. Since oil moved far more rapidly through the system than it did through the reservoir, it is assumed that no measurable gradient existed anywhere within the flow system.

Sample Experimental Data

When oil was used in the system, circulation was maintained for one half hour at each different rate to insure equilibrium in the manometers and to insure thermal equilibrium in the system. Two hours were allowed for initial equilibrium for each series of runs. Apparent equilibrium was reached in half of the allowed times. The following information was gathered twice in succession, sometimes three times, in order to determine pressure drop, flow rate, viscosity, and density for each single "run". Since the time pattern for each reading was symmetrical about the flow reading, a direct numerical average of results was made.

Run #87: (first half)

manometer temp. = 23.7° C	
	20 sec.
lower manometer leg = 14.55 inches upper manometer leg = 39.72 inches	nearly simultaneous
	20 sec.
outlet temp. = 37.2° C. flow weight = 1920 less 371 gm flow time = 24.1 sec. time = 08:36 1/2 inlet temp. = 38.2° C	simultaneously
	20 sec.
upper manometer leg = 39.69 inches lower manometer leg = 14.52 inches	nearly simultaneous
	20 sec.
manometer temp. = 24.0° C	

Most of the runs were made during winter months in a large laboratory that opened out of doors, and it was not uncommon for the opening of a door to cause room temperature to suddenly drop one half to one degree centigrade. This would cause the air in the inverted manometer to contract and draw both legs up a fraction of an inch. The differential readings did not vary by more than 0.05 inch in any case. Runs identified by alphabetical symbols were accomplished less systematically; they were noted to yield the same results as the remainder of the tests.

Runs with water were similar to those with oil except that only five or ten minutes were required for equilibrium and that flow was measured with a nozzle. An entire series of readings could be taken within 30 seconds, thus a time schedule was not maintained. Double readings, as below, were usually taken. Sometimes a fine oil ring in the manometer tube facilitated reading so well that one reading was considered sufficient.

Run #79:

col. man. temp. = 21.3° C
lower leg column man. = 18.45 inches
upper leg column man. = 29.20 inches

Run #79 (cont.)

outlet temp. = 26.2° C
inlet temp. = 26.3° C
flow man. temp. = 21.3° C
static flow leg = -11.3 inches
impact flow leg = +15.35 inches, time = 13:24
impact flow leg = +15.35 inches
static flow leg = -11.35 inches
inlet temp. = 26.4° C
outlet temp. = 26.3° C
upper leg column man. = 29.00 inches
lower leg column man. = 18.45 inches, time = 13:25

Runs with air required more information that runs with water. Equilibrium was reached so rapidly that a time schedule was not considered useful. Duplicate readings were made for each run as illustrated below. Some fluctuations in readings were noted, but they were so rapid that a time schedule for making readings would not have been capable of capturing the average reading any better than a fast scanning of all instruments.

Run #105: (first half)

impact leg flow man. = +1.42 inches
static leg flow man. = +2.83 inches
flow man. temp. = 22.8° C
meter side of gage pressure man. for static meter tap = -0.30 inches
atmospheric side of gage pressure man. for static meter tap = +0.16 ins.
inlet temp. = 23.2° C
outlet temp. = 23.0° C
upper leg col. man. = -0.59 inches
lower leg col. man. = +0.81 inches
col. man. temp. = 23.8° C
system side of gage pressure man. for upper column tap = +0.83 inches
atmospheric side of gage pressure man. for upper column tap = +1.09 ins.
time = 16:12

Barometric pressure and humidity of the exit air supplemented this information. Fluctuations in manometer readings never produced discrepancies greater than 2 percent of the manometer displacement for any one run.

Schedule for Each Bed

The beds were packed by introducing about five to ten percent of the required packing material, settling this by rapping the column, and then introducing another five to ten percent of the required packing material. The mixture in bed #14 was introduced in individual portions that represented the simplest subdivision of the mixture. Free space was measured immediately after packing each column, and after all runs were completed. No settling of the packing during runs was noted. Water was the first fluid used, air was next, then oil.

The entire system was flushed with carbon tetrachloride and dried after tests with oil were completed. Then the system was flushed with water. Some oil remained in the system, but never any more than enough to produce a thin oil slick on top of the water.

THEORETICAL DEVELOPMENTS

General Considerations

The first consideration was that of locating a more useful equation than that resulting from dimensional analysis. It was decided that Morcom's (10) representation should adequately determine the relationship between pressure drop and fluid variables.

$$\frac{\Delta P}{L_t} = k \frac{\mu U_o}{g} + K \frac{\rho U_o^2}{g} \qquad (5)$$

Equation (5) has properties such that laminar and turbulent flow may be scrutinized independently. Actually, few experimental data concerning

EXPLANATION OF PLATE II

Sketch of the flow system showing instrument location

C-1 The six inch column.

C-2 The three inch column.

C-4 The four inch column.

E Exit manifold for all three columns.

F Location of the flow meter used to measure the flow rates of water and air.

I Inlet manifold for all three columns.

M-1 Inverted manometer used for measuring the rate of water flow.

M-2 Mercury filled manometer used to measure pressure in the columns.

M-3 Water filled manometer used to measure pressure drop for runs with air.

M-4 Inverted manometer used to measure pressure drop for runs with oil or water.

M-5 Water filled manometer used for measuring the rate of flow of air.

M-6 Mercury filled manometer used for measuring air pressure in the flow meter.

P Piezometer rings and pressure taps.

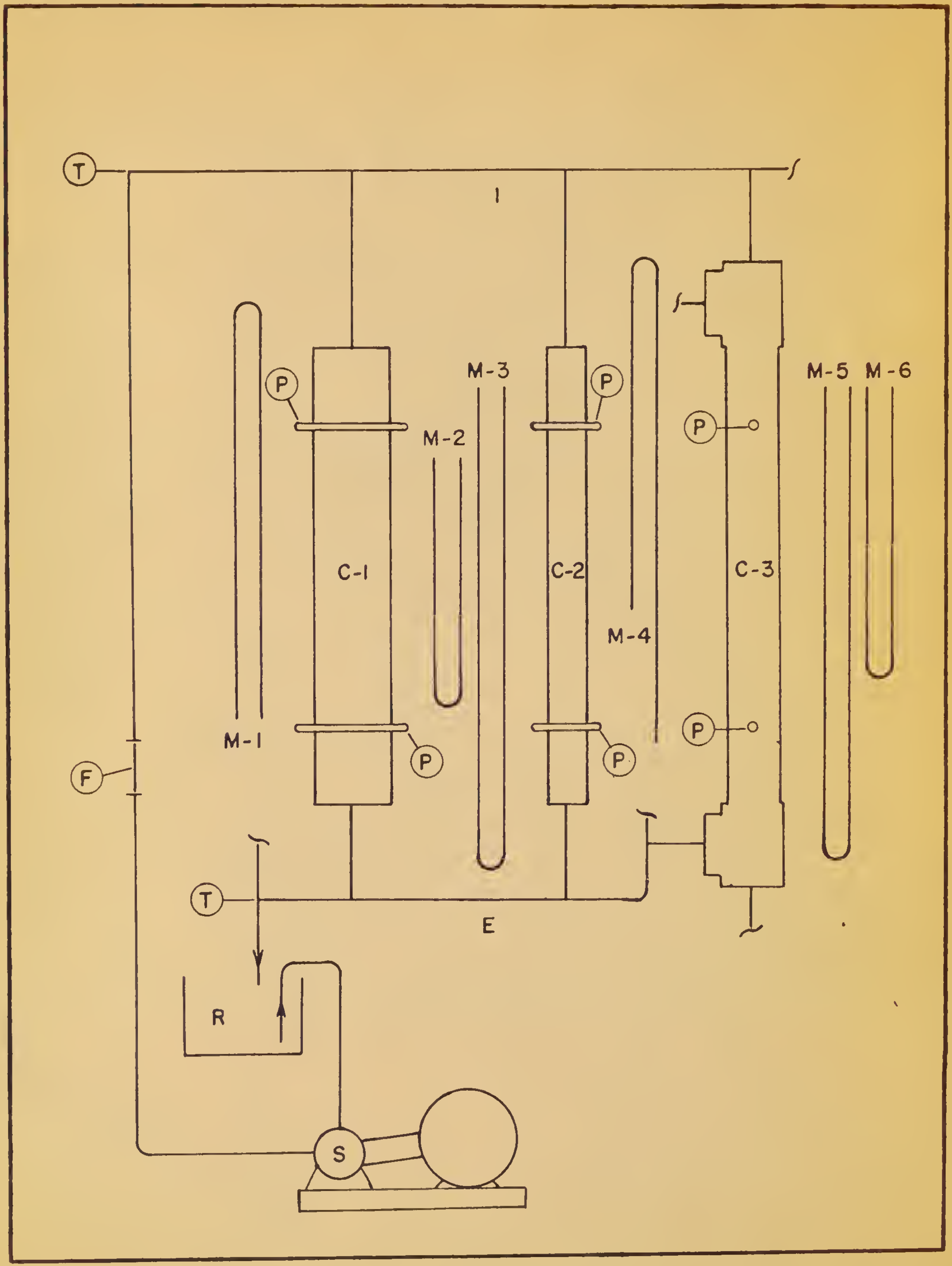
R Reservoir for fluid being circulated.

S Positive displacement pump, eccentric gear type, used to circulate water or oil.

T Thermometers used to measure the temperature of the inlet and outlet streams.

-ʃ Portions of the pipe system that were closed to the circulating fluid.

PLATE II



EXPLANATION OF PLATE III

Photograph of the flow system.

PLATE III

EXPLANATION OF PLATE IV

Photographs of the packing units.

Fig. 10. Upper left, one half inch clay Berl saddle.

Upper right, one inch clay Raschig ring.

Center left, wire ring.

Center right, glass ball.

Lower left, one half inch clay Raschig ring.

Lower right, one inch clay Berl saddle.

Fig. 11. The mixture tested in bed #14.

Fig. 12. The metal Raschig rings.

PLATE IV

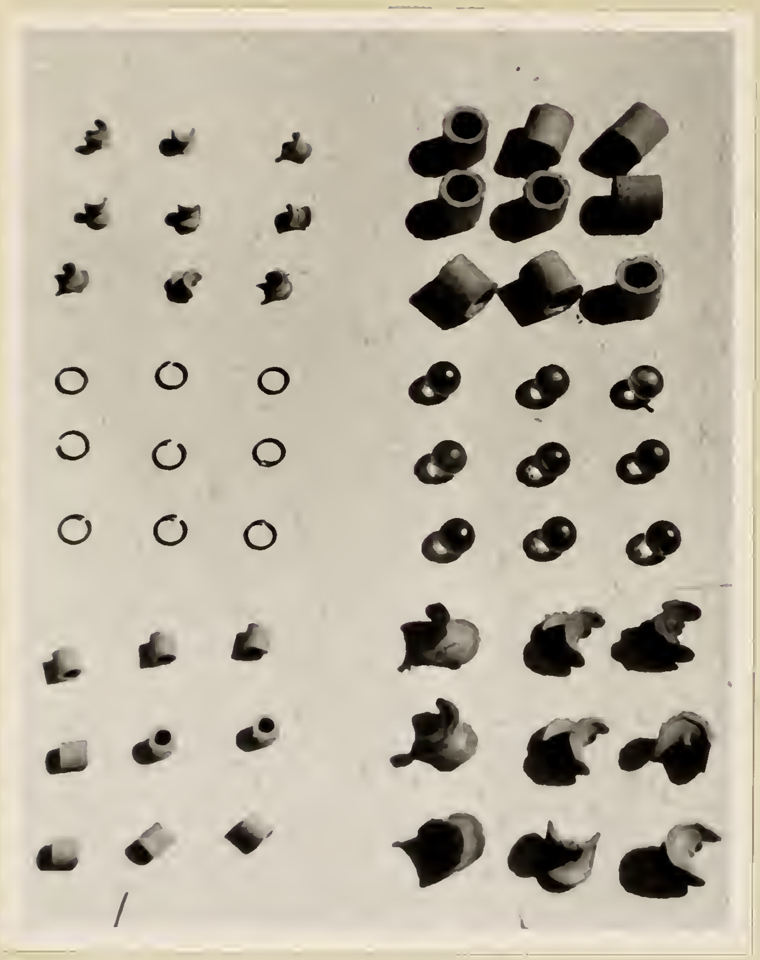


Fig. 10

Fig. 11

Fig. 12

truly turbulent flow were found. This required that laminar flow be studied first. Development of a reliable prediction for an equivalent of "k" made possible direct evaluation of an equivalent for "K" from data obtained for somewhat transitional flow.

Laminar Flow. In order to obtain agreement with equation (1), which summarizes dimensional analysis, it was necessary to approximate certain bed variables.

$$\frac{\Delta P}{L_t} = \delta \frac{\rho U^2}{g^D} \phi \left(\frac{D \rho U}{\mu}, \frac{e}{D} \right) \tag{1}$$

Since fluid and empty column variables, ΔP, L_t, μ, ρ, U_o, were known, a limited number of others was required. Carman's (5) representation for "m" was adopted to replace "D" and was later found to be the proper substitution. This was defined by equation

$$m = V/S_p \tag{2}$$

On first consideration, the classic approximation for area of flow, used by Blake (1) and many other investigators was thought to be useful. This amounted to reducing the column cross-section by the void fraction. Later considerations found this quite valueless.

The final list of bed variables, in addition to the approximation for hydraulic radius was concluded to be:

$$a = \frac{\text{effective area of flow}}{\text{column cross-section}}$$

$$r = \frac{\text{effective length of flow path}}{\text{length of the bed}}$$

$$z = \frac{\text{effective hydraulic radius}}{\text{estimated hydraulic radius}}$$

δ_1 = constant derived from geometrical nature of the flow path

S_t/S_p = ratio between column surface and packing surface

Pressure drop in laminar flow has long been known to be independent of surface roughness.

Turbulent Flow. The same terms that were related to laminar flow, except for δ_1, were considered to be applicable to turbulent flow. This may not have been an exact assumption because for instance, the effective area of flow available to a turbulent stream might be different from the area available to a stream in laminar motion.

The effect of surface roughness was expected to become evident in the turbulent region. Additional terms to be considered included:

δ_2 = constant derived from geometrical nature of the flow path

$e/m = \dfrac{\text{height of surface protrusion}}{\text{estimated hydraulic radius}}$

The Basic Equation. Restatement of equation (5) produced equation (6).

$$\frac{\Delta P}{L_t} = \delta_1 \frac{\mu U_o}{gm^2} \cdot \frac{r}{az^2}\gamma_1\left(\frac{S_t}{S_p}\right) + \delta_2 \frac{\rho U_o^2}{gm} \cdot \frac{r}{a^2 z}\gamma_2\left(\frac{S_t}{S_p}\ \frac{e}{m}\right) \qquad (6)$$

γ was used to represent the arbitrary functions. Examination of equation (6) showed that attempts to solve for all of the bed variables from pressure drop information would be futile. A simplier form was adopted for further analysis of pressure drop.

$$\frac{\Delta P}{L_t} = A' \frac{\mu U_o}{gm^2} + B^* \frac{\rho U_o^2}{gm} \qquad (7)$$

Both A' and B^* could then be quickly evaluated for each bed. It was felt that any randomly packed bed should possess values for δ_1 and δ_2 that depended on the same bed properties that would determine a, r, and z, therefore, all five of these terms would be accountable to some single bed variable.

The Width of a Packing Unit. Satisfactory determination of A' and B* was obtained by introducing the concept of packing width. The width of the packing is a derived property which is not necessarily related to the nominal diameter of the packing unit.

The packing width was defined as the surface area of the packing divided by the perimeter representing boundries which must be circumvented by the fluid. This was expressed mathematically as:

$$w = S_p/C_p \tag{3}$$

In order to facilitate a precise estimation for C_p, this definition was formulated: the packing perimeter consists of the locus of tangent points to the packing that would be generated by a line which moved throughout the packed bed remaining oriented parallel to the column wall.

Comparison of "w" to "m" yielded a variable that uniquely measured the degree to which the fluid path would be distorted and blocked. Scaled illustrations such as Figures 1, 2, and 3 showed that flat packing units might produce a bed containing dead spaces to such a degree that a bed of lesser porosity comprised of somewhat spherical units should produce no more pressure drop. The magnitude of w/m seemed to parallel this effect. As a result of these observations, it was considered feasible that w/m would index basic changes in the bed structure so that the three terms, w/m, S_t/S_p, and e/m could completely describe a packed bed.

Application of the Hypotheses

Preliminary Analysis. The data of several investigators was used to supplement the experimental results of this investigation. Preliminary considerations showed that w/m very decidedly indexed A' and B^* of equation (7). A graphical representation similar to Plate III showed that the limiting value for A' as $w/m \rightarrow 0$ was 50/9. This initial representation also showed that B^* varied directly as did w/m for large columns.

The method of correcting for wall effects that was used by Sullivan and Hertel (14) was adopted. Thus, it was assumed that

$$\frac{\Delta P}{L_t} = A \frac{\mu U_o}{gm^2} (1 + 0.6S_t/S_p)^2 + B^* \frac{\rho U_o^2}{gm} \qquad (8)$$

would accommodate wall effects for laminar flow. That this is true is illustrated in Fig. 15. This equation produced precise correlation for wall effects in the case of spheres, and good correlation for all packing materials for ratios of S_t/S_p from 0.01984 to 0.305. B^* was found more nearly uniformly dependent upon w/D_t than upon S_t/S_p. The therm, $B^* \div w/m$, was found to depend upon w/D_t but no longer upon w/m. The final flow equation took this form:

$$\frac{\Delta P}{L_t} = A \frac{\mu U_o}{gm^2} (1 + 0.6S_t/S_p)^2 + B \frac{\rho U_o^2 w}{gm^2} \qquad (9)$$

with A depending on w/m and
B depending on w/D_t

Contributions of e/m to pressure drop could not be isolated by comparison to values for "e" which were published by Brownell and Katz (2). Thus, it was assumed that normal roughness should not affect pressure drop through randomly packed beds.

Estimations for Perimeter. The contributions to perimeter offered by many packing units, the sphere, the wire or cylinder, the Raschig ring, the prism, and the cube, were noted to be independent of orientation within the bed. The Berl saddle was noted to yield different perimeter with each orientation.

Observing the Berl saddle from various directions showed that the outer edges constituted the perimeter from some views while part of these outer edges ceased to contribute and other elements of perimeter appeared in other views. For this reason, the outer edges were felt to approximate the mean perimeter of the Berl saddle. The perimeter of the "saddle" tested by Brownell and co-workers (3) was approximated by assuming that the units were manufactured from square blanks and that the edges of the square constituted an equivalent of the final perimeter.

Table 4 shows the exact method used to estimate the perimeter of each different packing unit.

Table 4. Perimeter of some packing units.

Unit	Perimeter
Sphere	$C = \pi D$
Wire or cylinder	$C = \pi D + 2L$
Cube	$C = 6D$
Hexagonal prism	$C = 3D + 2L$
Raschig ring	$C = 2\pi D + 2L - 2\pi t$
Berl saddle	$C \cong 2\pi D$
"Saddle" of (3)	$C \cong 4(s_p/2)^{1/2}$

<u>Laminar Flow</u>. Values for A were calculated from information for each individual experimental run. For each bed tested, the logarithmic mean value of A was determined. Table 5 includes these results together with other important information. Detailed lists of the calculated results are included in the appendix.

Table 5. Laminar flow, mean values of A for each bed.

Ref.	Packing	%void	S_t/S_p	w/m	A	Am.l.	$\frac{A}{Am.l.}$
(3)	Glass ball	41.2	0.114	8.56	10.21	10.81	0.945
"	Smooth saddle	93.1	0.041	5.35	11.11	8.48	1.310
"	Rough saddle	93.5	0.044	5.02	7.85	8.26	0.950
"	Berl saddle	72.5	0.113	6.33	6.58	9.16	0.718
"	Raschig ring	70.7	0.121	6.36	7.69	9.18	0.838
(5)	Hexagonal prism	37.7	0.066	9.39	12.32	11.66	1.057
"	" "	42.6	0.065	7.66	9.30	10.17	0.914
"	Cube	34.4	0.075	11.11	14.10	13.36	1.055
"	"	39.7	0.078	9.10	10.80	11.40	0.947
"	"	44.8	0.074	7.40	8.62	9.97	0.865
(14)	Glass ball	39.08	0.01985	9.353	11.58	11.63	0.996
"	" "	39.04	0.01984	9.368	11.63	11.64	0.999
"	" "	39.11	0.09347	9.341	11.61	11.62	0.999
"	" "	39.24	0.09367	9.290	11.64	11.57	1.006
Auth.	Wire ring	81.6	0.059	1.39	6.14	6.20	0.990
"	" "	83.6	0.131	1.21	6.27	6.11	1.026
"	" "	83.2	0.098	1.25	5.96	6.13	0.972
"	Glass ball	42.3	0.225	8.20	10.72	10.62	1.009
"	" "	44.0	0.305	7.64	9.80	10.16	0.965
"	Berl saddle	77.0	0.285	4.54	9.12	7.95	1.147
"	" ".	71.8	0.118	5.94	8.52	8.88	0.959
"	Raschig ring	68.9	0.157	5.63	11.83	8.67	1.364
"	" "	92	0.126	4.43	10.08	7.88	1.297
"	" "	92	0.126	4.43	9.59	7.88	1.217
"	" "	59.5	0.103	7.20	9.74	9.81	0.993
"	Mixture; wire ring, glass ball, Berl saddle, Raschig r.	53.0	0.157	3.57	8.39	7.37	1.140

This rearranged form of equation (8) was used for estimations:

$$A = \frac{\Delta P g m^2}{L_t \mu U_o (1 + 0.6S_t/S_p)^2} - B' N$$

where $N = \rho U_o m/\mu$ and $B' = B^*/(1 + 0.6S_t/S_p)^2$

The values for B^* which were used in this estimation were obtained by initial observation of turbulent flow data for the bed being considered. Since B^* was only approximate, the second term on the right was not allowed to exceed 5 percent of the estimated value for A.

Plate V shows how A varies with w/m. No theoretical considerations to explain the linear relationship on semilogarithmic coordinates were deduced.

The data of Sullivan and Hertel (14) and the intercept for A = 50/9 at w/m = 0 were used to determine this empirical relationship:

$$A = \frac{50}{9} (10)^{0.03430w/m}$$

$$\text{or } \log A = 0.7447 + 0.03430w/m \tag{10}$$

Equation (10) was termed the "mean line" value for A.

<u>Turbulent Flow</u>. Values for B^* were solved by using this modified form of equation (8):

$$B^* = \frac{\Delta P g m}{L_t \rho U_o^2} - A'/N = f - A'/N$$

where $N = m U_o \rho / \mu$ and $A' = A(1 + 0.6S_t/S_p)^2$

B^* was then converted to B according to $B = B^* + w/m$. Values for A' were obtained by use of equation (10). This was done, even when the true value was known, for the sake of maintaining a consistent approach for all information that was at hand.

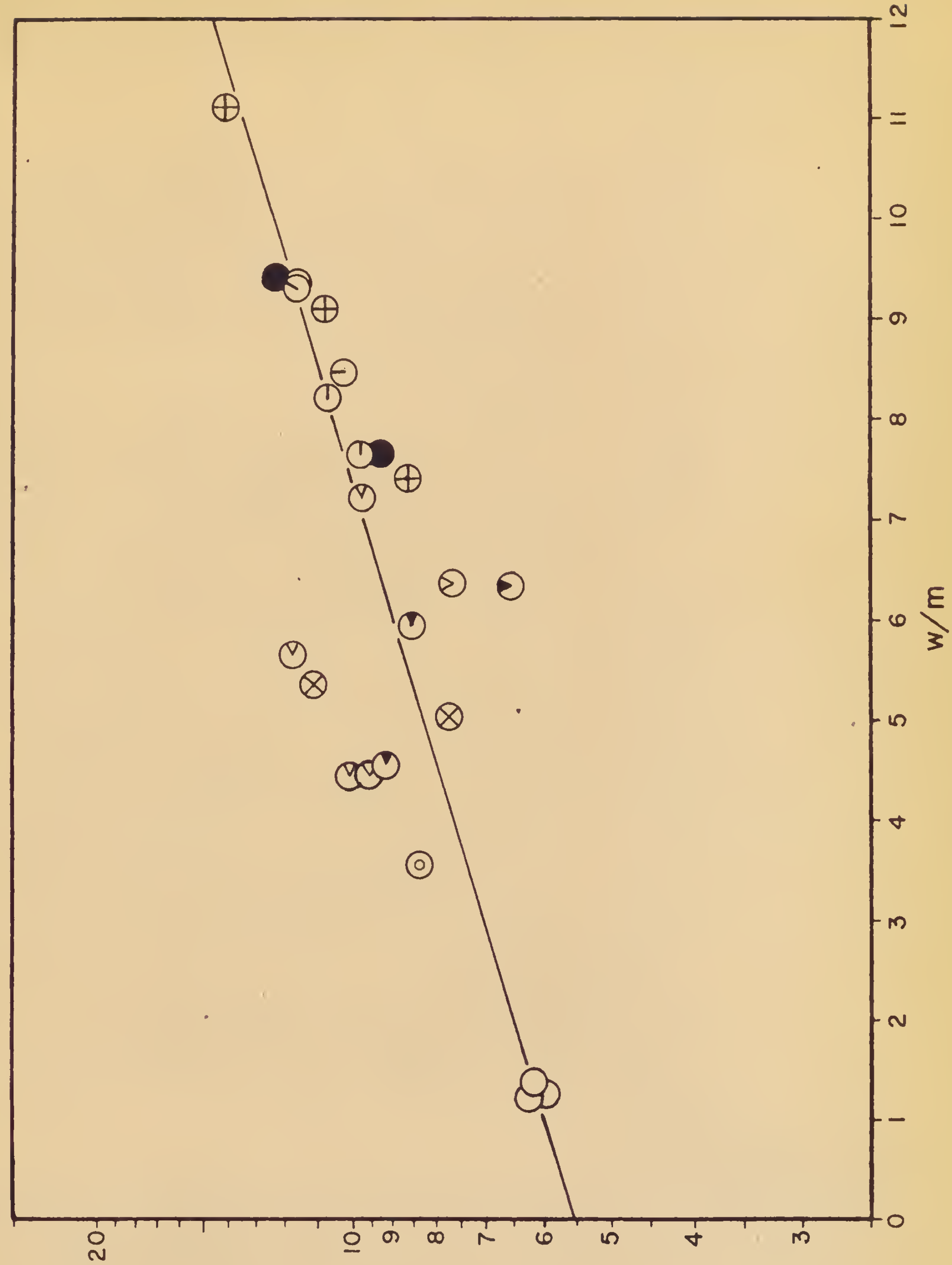
EXPLANATION OF PLATE V

Λ as a function of w/m

Legend:

Packing unit	Investigators (3)	(5)	(14)	Auth.
Cube	—	⊕	—	—
Hex. prism	—	●	—	—
Sphere	⦶	—	◷	⊖
Raschig ring	◴	—	—	◔
Berl saddle	◒	—	—	◕
"Saddle"	⊗	—	—	—
Wire ring	—	—	—	○
Mixture	—	—	—	⊙

PLATE V



The term, A'/N, was allowed to exceed the calculated value for B^* in a few cases, but was gnerally restrained to a maximum value equal to the estimated value for B^*. The logarithmic mean value for B for each bed was determined and placed in Table 6 along with other important information.

Plate VI shows how B varies with w/D_t. Definition of the mean line value for B according to

$$B = 0.25(10)^{-1.766w/D_t} \tag{11}$$

was found quite satisfactory. This was established by first noting that B approached 0.25 as w/D_t approached zero, and by determining the line that would pass through this point presenting the closest approximation to the mass of information.

w/m As a Unique Variable. If w/m were not a unique variable, A and B would show dependence on the other variables. Plate VII shows that the deviations of the constants from their mean line values do not arise from either the percent voids or S_t/S_p. The fact that independence of S_t/S_p existed showed that D_p/D_t would not explain deviations either, D_p/D_t is approximately proportional to S_t/S_p for any given packing material.

The variable, w/m was thus established as the sole criterion for A and w/D_t as the criterion for B. Deviations from the mean line values were attributed to normal experimental errors such as might arise from insufficient column length when large packing units were tested in small diameter columns.

Friction Factor and Reynold's Number. It was desired to represent all experimental information on a single friction factor vs. Reynold's number plot. In order to do this, certain considerations of the variability of A

Table 6. Turbulent flow, log mean values of B for each bed.

Ref.	Packing	% void	w/m	w/Dt	B	Bm.l.	B/Bm.l.
(1)	Glass ring	67	6.53	0.047	0.310	0.206	1.51
"	" "	72	6.32	0.057	0.332	0.198	1.68
"	" "	80	5.24	0.076	0.223	0.184	1.21
"	" "	84.5	4.62	0.090	0.294	0.173	1.70
"	Raschig ring	72	5.35	0.141	0.140	0.141	0.99
(3)	Smooth saddle	93.1	5.35	0.051	0.181	0.203	0.89
"	Rough saddle	93.5	5.02	0.052	0.158	0.202	0.78
"	Glass ball	41.2	8.56	0.101	0.164	0.166	0.99
"	Berl saddle	72.5	6.33	0.130	0.100	0.147	0.68
"	Raschig ring	70.7	6.36	0.122	0.160	0.152	1.05
(4)	Lead shot	37.5	10.00	0.028	0.190	0.223	0.85
"	" "	36.3	10.53	0.029	0.156	0.222	0.70
"	" "	38.0	9.79	0.040	0.178	0.212	0.84
"	" "	37.4	10.03	0.083	0.138	0.179	0.77
"	" "	39.0	9.39	0.058	0.169	0.198	0.85
"	" "	37.0	10.20	0.059	0.164	0.197	0.83
"	" "	39.7	9.10	0.081	0.157	0.180	0.87
"	" "	37.5	10.00	0.082	0.180	0.179	1.01
"	" "	38.3	9.67	0.091	0.198	0.173	1.15
"	" "	42.1	8.26	0.166	0.172	0.118	1.46
(11)	Celite sphere	37.85	9.85	0.054	0.189	0.201	0.94
"	" "	37.90	9.84	"	0.192	"	0.96
"	" "	37.75	9.89	"	0.198	"	0.99
"	" "	37.85	9.85	"	0.207	"	1.03
"	" "	46.90	6.80	"	0.215	"	1.07
"	" "	46.80	6.83	"	0.208	"	1.04
"	" "	46.90	6.80	"	0.206	"	1.02
"	" "	46.40	6.94	"	0.193	"	0.96
"	Celite cylinder	36.1	9.65	0.065	0.194	"	1.01
"	" "	36.5	9.48	"	0.200	"	1.04
"	" "	37.2	9.19	"	0.196	"	1.02
"	" "	45.5	6.55	"	0.162	"	0.84
"	" "	45.7	6.47	"	0.167	"	0.87
"	" "	46.1	6.37	"	0.170	"	0.89
"	Raschig ring	56.3	7.95	0.088	0.186	0.175	1.06
"	" "	55.8	8.12	"	0.185	"	1.06
"	" "	55.55	8.21	"	0.191	"	1.09
"	" "	55.5	8.22	"	0.194	"	1.11
"	" "	55.45	8.23	"	0.198	"	1.13
"	" "	61.35	6.44	"	0.144	"	0.82
"	" "	62.07	6.26	"	0.158	"	0.90
"	" "	62.13	6.25	"	0.147	"	0.84
"	" "	62.3	6.19	"	0.152	"	0.87

Table 6 (cont.).

Ref.	Packing	% void	w/m	w/D_t	B	$B_{m.l.}$	$\frac{B}{B_{m.l.}}$
(11)	Berl saddle	72.05	4.93	0.100	0.177	0.166	1.07
"	" "	71.33	5.10	"	0.179	"	1.08
"	" "	71.05	5.18	"	0.174	"	1.05
"	" "	71.25	5.12	"	0.180	"	1.08
"	" "	76.30	3.95	"	0.167	"	1.01
"	" "	76.35	3.95	"	0.167	"	1.01
"	" "	75.90	4.03	"	0.166	"	1.00
"	" "	76.15	3.97	"	0.171	"	1.03
Auth.	Wire ring	81.6	1.39	0.017	0.237	"	1.02
"	" "	83.6	1.21	0.033	0.251	0.219	1.15
"	" "	83.2	1.25	0.026	0.203	0.225	0.90
"	Glass ball	42.3	8.20	0.195	0.120	0.113	1.06
"	" "	44.0	7.64	0.256	0.140	0.088	1.59
"	" "	38.8	9.47	0.130	0.106	0.147	0.72
"	Berl saddle	77.0	4.54	0.249	0.154	0.091	1.69
"	" "	71.8	5.94	0.121	0.114	0.153	0.75
"	" "	72.7	5.70	0.189	0.067	0.116	0.58
"	Raschig ring	68.9	5.63	0.152	0.213	0.135	1.58
"	" "	74.5	4.28	0.282	0.086	0.079	1.09
"	" "	92	4.43	0.128	0.212	0.148	1.43
"	" "	92	4.43	"	0.213	"	1.44
"	" "	59.5	7.20	0.110	0.177	0.160	1.11
"	Mixture; wire ring, glass ball, Berl saddle, Raschig r.	53.0	3.57	0.076	0.223	0.184	1.21

EXPLANATION OF PLATE VI

B as a function of w/D_t

Legend:

Packing unit	(1)	(3)	(4)	(11)	Auth.
Cylinder	—	—	—	⊙	—
Sphere	—	◷	◶	◴	◵
Raschig ring	◶	◷	—	◴	◵
Berl saddle	—	◕	—	◒	◐
"saddle"	—	⊗	—	—	—
Wire ring	—	—	—	—	○
Mixture	—	—	—	—	◎

PLATE VI

EXPLANATION OF PLATE VII

Illustrations showing that the effects of voids and column surface have been accurately predicted.

Fig. 13. $A/A_{mean\ line}$ versus void fraction.

Fig. 14. $B/B_{mean\ line}$ versus void fraction.

Fig. 15. $A/A_{mean\ line}$ versus ratio of column surface to packing surface.

Legend:

- ⊕ Cube
- ● Hexagonal prism
- ⊙ Cylinder
- ⦶ Sphere
- ⦶ Raschig ring
- ◕ Berl saddle
- ⊗ "saddle" of (3)
- ○ Wire ring
- ◎ Mixture

Conformity with the dotted line illustrates accuracy of prediction.

PLATE VII

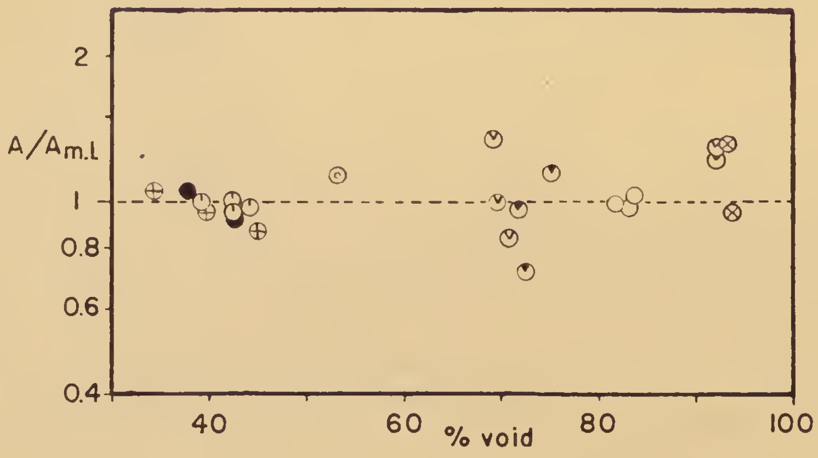


Figure 13.

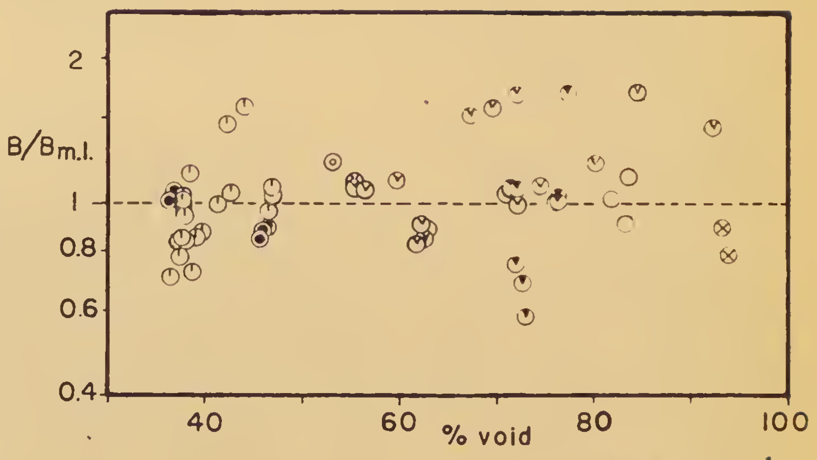


Figure 14.

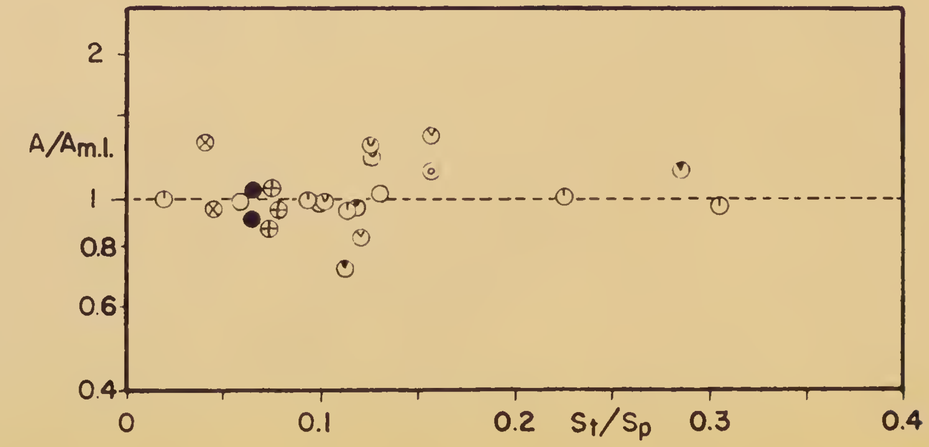


Figure 15.

and B had to be accomplished. The first step was to equate

$$\frac{\Delta P}{L_t} = f \frac{\rho U_o^2}{gm} \tag{12}$$

and, from equation (9),

$$f = A \frac{\mu}{m \rho U_o} (1 + 0.6S_t/S_p)^2 + B \frac{w}{m} \tag{13}$$

Next, a Reynold's number representation which could be used as an abscissa was determined by inspection.

$$Re = \frac{w \rho U_o}{\mu} \quad \frac{B}{A (1 + 0.6S_t/S_p)^2} \tag{14}$$

The residual term was considered to be the friction factor.

$$F = \frac{fm}{Bw} \quad \text{or} \tag{15}$$

$$F = \frac{1}{Re} + 1 \tag{16}$$

Re and F were solved for all of the information that had been used to determine values for A and B. They were also determined for transition flow data that were not used for determining A and B. "f" was determined by equation (12) and Re by use of equation (14). Values for A and B were determined by equations (10) and (11).

Plate VIII compares the actual values for F and Re to the relationship suggested by equation (16). The few beds tested by Brownell and co-workers (3) represented about one third of all data when the individual test runs were counted. Actually, they only tested five beds, or about 7 percent of the number of beds considered. Four out of five of their runs were excluded from

EXPLANATION OF PLATE VIII

Graphical solution to pressure drop through packed beds.

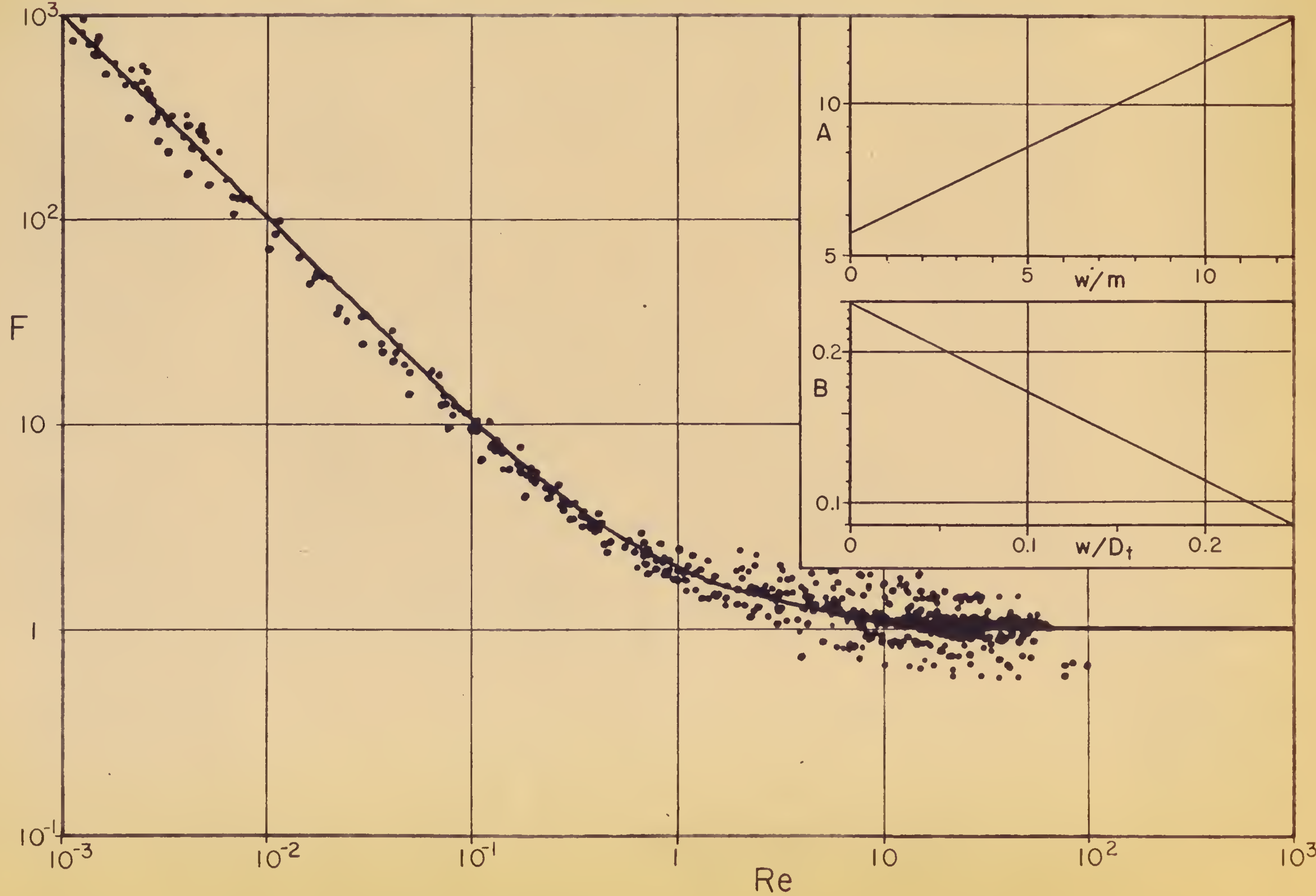
Ordinate: $F = \Delta Pgm/EL_t \rho U_o^2 (w/m)$

Abscissa: $Re = Bw \rho U_o / A \mu (1 + 0.6S_t/S_p)^2$

The following information is shown:

Investigators	Packing materials
(1)	Glass ring and Raschig ring.
(3)	Raschig ring, Berl saddle, glass ball, and "saddle".
(4)	Lead shot.
(11)	Raschig ring, Berl saddle, sphere, and cylinder.
Author	Raschig ring, Berl saddle, glass ball, wire ring, and mixture containing all of these.

PLATE VIII



F
Re
10³
10²
10
1
10⁻¹
10⁻³
10⁻²
10⁻¹
1
10
10²
10³
A
10
5
0
5
w/m
10
B
0.2
0.1
0
0.1
w/D_t
0.2

the graph in order that unjustifiable weight would not be given to them. The actual values of F and Re that were plotted are included in the appendix.

DISCUSSION

Factors of General Importance

This investigation has shown that pressure drop through a wide variety of packing materials can be accurately predicted by reference to three easily determined properties of a packed bed. These properties are: total packing surface, S_p; total packing perimeter, C_p; and free volume of the packed zone, V. Reference to these properties eliminates the necessity for considering highly complex methods of correlation or vague terms such as "norminal particle diameter" or "normal voids".

The scope of packed bed variables which has been studied is summarized in Tables 7 and 8. The accuracy of prediction of pressure drop is also summarized in these tables. The columns headed "average deviation" show how well the pressure drop information for each type of packing material is centered upon the predicted value while the columns headed "root mean square deviation" illustrate the average error involved in predicting pressure drop.

A less obvious advantage of this correlation lies in the fact that coefficient terms do not vary widely for different types of packing materials. The coefficient for laminar flow, A, varies from 5.56 to 13.36 or 2.4 fold. The coefficient for turbulent flow, B, varies from 0.25 to 0.079 or 3.2 fold. These ranges of variation include sparsely packed beds

Table 7. Laminar flow, scope of the investigation and accuracy of the predictions.

Packing	beds obs'd	nom. dia., in.		% voids		S_t/S_p		w/m		log dev*, %	
		small	large	small	large	small	large	small	large	mean	r.m.s.
Cube	3	0.22	0.22	34.4	44.8	0.074	0.078	7.40	11.11	-4.7	9.9
Hexagonal prism	2	0.185	0.185	37.7	42.6	0.065	0.066	7.66	9.39	-1.7	7.7
Sphere	7	0.02734	0.792	39.04	44.0	0.020	0.305	7.64	9.368	-1.2	2.6
Raschig ring	5	0.522	1.048	59.5	92	0.103	0.157	4.43	7.20	12.0	23.7
Berl saddle	3	1.00	1.028	71.8	77.0	0.113	0.285	4.54	6.33	-7.6	23.2
Wire ring	3	0.645	0.645	81.6	83.6	0.059	0.131	1.21	1.39	-0.4	2.3
"saddle" of (3)	2	0.130	0.132	93.1	93.5	0.041	0.044	5.02	5.35	11.6	21.5
Mixt.; wire ring, sphere, Berl sa., Raschig ring	1	0.645, 0.792 0.512, 0.522 respectively		53.0	53.0	0.157	0.157	3.57	3.57	14.0	14.0
All types	26	0.02734	1.048	34.4	93.5	0.020	0.305	1.21	11.11	1.6	14.8

* Log mean deviation refers to the antilogarithm of $n^{-1} \sum \log(A/A_{m.l.})$; root mean square deviation thus refers to the antilogarithm of the absolute value of $\{n^{-1} \sum \log^2(A/A_{m.l.})\}^{\frac{1}{2}}$. This method of averaging lends equal emphasis to deviations in either direction.

Table 8. Turbulent flow, scope of the investigation and accuracy of the predictions.

Packing	beds obs'd	nom. dia., in. small	nom. dia., in. large	% voids small	% voids large	w/m small	w/m large	w/Dt small	w/Dt large	log dev.*, % mean	log dev.*, % r.m.s.
Cylinder	6	0.267	0.267	36.1	46.1	6.37	9.65	0.065	0.065	-6	11
Sphere	22	0.058	0.792	36.3	46.90	6.80	10.53	0.028	0.256	-3	22
Raschig ring	20	0.233	1.048	55.45	92	4.28	8.23	0.047	0.282	15	30
Berl saddle	12	0.5	1.028	71.05	77.0	3.95	6.33	0.100	0.249	-3	30
Wire ring	3	0.645	0.645	81.6	83.6	1.21	1.39	0.017	0.033	0	13
"saddle" of (3)	2	0.130	0.132	93.1	93.5	5.02	5.35	0.051	0.052	-17	21
Mixt.; wire ring, sphere, Berl sa., Raschig ring	1	0.645, 0.512, respectively	0.792 0.522	53.0	53.0	3.57	3.57	0.076	0.076	21	21
All types	66	0.058	1.048	36.1	93.5	1.21	10.53	0.017	0.282	2	25

* Log mean deviation refers to the antilogarithm of $n^{-1} \sum \log(F/F_{m.l.})$; root mean square deviation thus refers to the antilogarithm of the absolute value of $\{n^{-1} \sum \log^2(F/F_{m.l.})\}^{\frac{1}{2}}$. This method of averaging lends equal emphasis to deviations in either direction.

to dense beds and large ratios of column diameter to particle diameter as well as ratios of column diameter to particle diameter as low as 3:1. The recent correlation by Brownell and co-workers (3) involves variation of coefficient terms in the order of magnitude of 10 fold or more and is incapable of predicting pressure drop through wire packing. Most other correlations cannot be compared in this respect because they were not supposed to be general in nature.

The greatest deviations of pressure drop from the predicted value were encountered in the case of information published by Blake (1). Blake measured pressure drop across the entire packed zone. He stated in his paper that the packing support, where units such as Raschig rings usually assume undesirable orientation, may have caused the overall pressure drop to be somewhat higher than the value which should be expected. Estimations for the coefficient for turbulent flow, B, scattered most widely when the ratio of column diameter to particle diameter was small. This is a logical consequence since very few packing units were required for the small columns and the probability of any one such bed producing a representative pressure drop should be small.

The average accuracy of prediction of pressure drop, including questionable results such as those of Blake, was found to be ± 15 percent for laminar flow and ± 25 percent for turbulent flow.

The convergence of this correlation upon those of other persons is best illustrated by the fact that results of other investigators forms the basic body of information upon which the present conclusions are based. The writer performed experimental tests which were primarily designed for illustrating the effect of extremes in packed bed variables. The effect of "loose pack" and "close pack", which was studied by Oman and Watson (11), had been previously treated by empirical corrections. This investigation produced good correlation for such extremes without reference to the method of packing the bed save that randomness should be maintained.

The correlation for B^* as a function of w/m does not show agreement with Stoke's law for freely falling objects in turbulent flow. Stoke's law asserts that B^* should be constant where the objects are highly dispersed. The approximation that has been used, $B^* = B(w/m)$, implies that B^* becomes very small for sparsely packed beds. Pressure drop was not measured for the range where B^* is predicted to be very small; the wire rings constituted the limit in this direction.

Orifice Analogy. An experiment was conducted to determine a reason for the lack of convergence upon Stoke's law. The total energy loss due to flow about a falling object was compared to the total energy loss through an orifice in a pipe line. Freely falling objects, as treated by Stoke's law, are widely dispersed while in a packed column the objects are encountered frequently by the fluid stream. Consequently, loss through widely separated orifices was compared to pressure loss through a series of orifices spaced about one orifice diameter apart. Nine orifices were used. It was found that the coefficient for pressure loss through any one of the widely dispersed orifices was considerably higher than the coefficient for any one of the closely spaced orifices. The results of these tests were as follows:

Case #1, orifices spaced 4.94 orifice diameters apart.

$$B_o^* = \frac{\Delta P_o g}{\rho U^2} = 0.417$$

Case #2, orifices spaced 0.875 orifice diameters apart.

$$B_o^* = \frac{\Delta P_o g}{\rho U^2} = 0.1456$$

In each case, velocity was based on the orifice area. The subscript, o, refers to the fact that an orifice was considered. The experimental procedure and apparatus are described in the appendix.

Case #1 agrees with the head loss coefficient for a single orifice. Case #2 shows that only 34.9 percent as much energy is lost when the orifices are spaced to compare with conditions within the packed bed. This analogy shows that Stoke's law may not be applicable to packed beds. It implies that coefficients in the order of those for turbulent flow through ducts may be approached in packed beds. Ducts offer much less resistance to flow for a given total surface than do suspended objects.

Orientation Near the Column Wall. The coefficient for pressure loss in turbulent flow, B^*, suffers large depressions as the column diameter is decreased. This depression is greater than that suggested by the reduction in w/m which results from the fact that small columns produce less dense beds than do larger columns. Some cross-sections of packed beds were exposed to determine whether the arrangement of packing units was such that the fluid should encounter less resistance near the wall. Photographs of the sections that were damaged least during preparation are shown in Plate IX. All of the sections are shown in the appendix. These sections were prepared by settling the packing into a thin cement slurry, allowing the cement to harden, and then sawing the hardened mass into cross-sections at intervals of about one inch. Inspection of these photographs shows that the beds represent typical degrees of packing density and that the units near

EXPLANATION OF PLATE IX

Cross-sections of packed beds.

Left.	Raschig ring, 1.032 inches in diameter.
Left center.	Raschig ring, 0.522 inches in diameter.
Right center.	Berl saddle, 1.028 inches in diameter.
Right.	Berl saddle, 0.512 inches in diameter.

PLATE IX

the wall are arranged similarly to the units in the interior of the bed. Thus, the reduction in B^* for small columns cannot be attributed to the existance of larger space for passage near the wall. The column wall probably tends to reduce the intensity of turbulence within the bed so that less energy is lost by the fluid stream.

How Well Must a Packed Bed Be Defined

Brownell and Katz (2) felt that porosity of the packed bed should be determined with very delicate precision, their correlation required special knowledge of porosity. Precision certainly does not detract from the validity of results, but it is often difficult to measure certain properties, such as porosity, with a great deal of accuracy. Analysis of the proposed equation shows just how errors in measuring, or predicting, packed bed variables should affect the accuracy of predicting pressure drop. Certain errors may originate from definition of fluid or empty column variables. Lack of randomness within small beds might also contribute errors, these factors are not included in the following discussion. Such factors as these cannot be isolated for the general case, however, the fact that they might exist is sufficient to induce necessary precaution in cases where they may become predominant.

Equation (16) summarizes the method of predicting pressure drop for a range of packed bed variables that includes all of the extremes that might be encountered in its application.

$$\frac{\Delta P}{L_t} = \frac{50}{9}\,(10)^{0.0343w/m}\,\frac{\mu U_o}{gm^2}(1+0.6S_t/S_p)^2$$

$$+\,0.25(10)^{-1.766w/D_t}\,\frac{\rho U_o^2\,w}{gm^2}$$

$$\text{for;}\quad \begin{array}{l} 1 < w/m < 15 \\ 0 < w/D_t < 0.3 \\ 0 < S_t/S_p < 0.35 \\ 0.3 < V/V_t < 1 \end{array} \qquad (16)$$

The range of applicability may be larger, but only the range of certainty is stated. The range of certainty encompasses all of the observations that have been cited.

Equation (17) is identical to equation (16) except that the naperian base, e, has been substituted for the base 10, and "w" and "m" have been resolved into their component factors.

$$\frac{\Delta P}{L_t} = \frac{50}{9}(e)^{0.079S_p^{\,2}/VC_p}\,\frac{\mu U_o S_p^2}{gV^2}(1+0.6S_t/S_p)^2 \qquad (17)$$

$$+\,0.25(e)^{-4.07S_p/D_tC_p}\,\frac{\rho U_o^2 S_p^3}{gV^2C_p}$$

Differentiation of equation (17) produces equations (18) and (19), which are of direct value in estimating the errors which might arise from inaccurate estimation of the different variables. Errors in estimating $\Delta P/L_t$ for highly laminar flow are summarized by equation (18). Fluid and empty column variables are considered subject to no error.

$$\frac{d\,\frac{\Delta P}{L_t}}{\frac{\Delta P}{L_t}} = 0.0790\,\frac{S_p^2}{VC_p}\left(2\frac{dS_p}{S_p} - \frac{dV}{V} - \frac{dC_p}{C_p}\right) + 2\frac{dS_p}{S_p} - 2\frac{dV}{V} - 1.2\frac{S_t}{S_p{}^2}\frac{dS_p}{(1+.6S_t/S_p)}$$

$$= 0.079\frac{w}{m}\left(2\frac{dS_p}{S_p} - \frac{dV}{V} - \frac{dC_p}{C_p}\right) + 2\frac{dS_p}{S_p} - 2\frac{dV}{V} - 1.2\frac{S_t}{S_p}\,\frac{dS_p}{S_p\,(1+.6S_t/S_p)}$$

$$\text{when flow is laminar} \qquad (18)$$

For highly turbulent flow, error is summarized by equation (19).

$$\frac{d\frac{\Delta P}{L_t}}{\frac{\Delta P}{L_t}} = -4.07\frac{S_p}{D_t C_p}\left(\frac{dS_p}{S_p} - \frac{dC_p}{C_p}\right) + \frac{3dS_p}{S_p} - \frac{2dV}{V} - \frac{dC_p}{C_p}$$

$$= -4.07\frac{w}{D_t}\left(\frac{dS_p}{S_p} - \frac{dC_p}{C_p}\right) + \frac{3dS_p}{S_p} - \frac{2dV}{V} - \frac{dC_p}{C_p} \qquad (19)$$

when flow is turbulent

The error contributed by each variable, E, is summarized in Table 9.

Table 9. Contribution of incorrect evaluation of packed bed variables to error in predicting pressure drop.

Variable	Laminar flow	Turbulent flow
V	$E_v = -(2 + 0.079\frac{w}{m})\frac{dV}{V}$	$E_v = -2\,\frac{dV}{V}$
S_p	$E_s = \left\{2 + 0.158\frac{w}{m} - \frac{1.2S_t}{S_p(1 + .6S_t/S_p)^2}\right\}\frac{dS_p}{S_p}$	$E_s = (3 - 4.07\frac{w}{D_t})\,\frac{dS_p}{S_p}$
C_p	$E_c = -0.079\frac{w}{m}\,\frac{dC_p}{C_p}$	$E_c = (4.07\frac{w}{D_t} - 1)\,\frac{dC_p}{C_p}$

Suppose that none of the packed bed variables is to be allowed to contribute more than 5 percent to the error in predicting pressure drop. Table 9 shows that porosity, or V should be known within 2.5 percent of its true value when flow is turbulent; when flow is laminar, porosity may be known within 2.5 percent for small values of w/m, or within 1.57 percent when w/m reaches the upper limit of 15. When flow is turbulent, total packing surface, S_p, should be known within 1.67 percent when w/D_t is small and within 2.81 percent when w/D_t reaches the upper limit of 0.3; laminar flow requires that packing surface be known within 2.5 percent when both w/m and S_t/S_p are small, within 2.92 percent when S_t/S_p reaches its upper limit of 0.35, within 1.14 percent when w/m reaches its upper limit of 15, and within 1.23 percent when w/m and S_t/S_p both reach their limiting values. For turbulent flow, the total packing perimeter, Cp, should be known within 5 percent for small values of w/D_t and within 7.6 percent when w/D_t reaches its upper limit of 0.3; laminar flow requires no knowledge of C_p when w/m is small and requires accuracy of 4.22 percent when w/m reaches its upper limit of 15. Transition flow requires intermediate degrees of accuracy for these variables.

For turbulent flow, the sum of possible errors for all three packed bed variables may be as large as 9.17 percent for small values of w/D_t and 12.91 percent for the limiting w/D_t of 0.3 if it is desired to maintain the predicted pressure drop within 15 percent of the true value. Laminar flow requires that this sum of errors be within 5.0 percent plus any large error in C_p when w/m is near zero and within 7.12 percent when S_t/S_p and w/m reach their respective limits of 0.35 and 15.

S_p must be known with the greatest degree of accuracy, V requires less accuracy, and C_p may be less well defined than either of the others when each of the variables is expected to contribute the same degree of accuracy to the predicted pressure drop. In general, an error in pressure drop of less than ± 8.5 percent will result if the value of each of these variables is known within ± 1 percent.

Determination of Packing Surface From Pressure Drop

Knowledge of the surface area of irregular objects is often desired. Certain relations between surface area and particle size, catalytic activity, mass transfer rates, or ionic activity exist. Often, a quantitative measure of surface area can be used to determine when a pulverizing operation is satisfactorily completed, or when fibers are of a desired fineness or texture.

Most tests for surface area have been conducted by allowing flow through the material in question to be laminar. Allowing the flow to be transitional would require very cumbersome calculations, and truly turbulent flow requires tremendous pressure drops that are not required when laminar flow is maintained. Thus, it is valid to assume that any surface area determination will require the use of equation (20) and not the complete equation for pressure drop.

$$\Delta P/L_t = \frac{50\mu U_o S_p^2}{9gV^2}(1 + 0.6S_t/S_p)^2(10)^{0.0343w/m}$$

$$\text{or} \quad S_p = 0.6V\left(\frac{*\Delta Pg}{2L_t\mu U_o}\right)^{1/2}(10)^{-0.01715w/m} - 0.6S_t$$

for highly laminar flow (20)

V, L_t, μ, U_o, ΔP, g, and S_t are always known when surface area is sought. A typical determination of V is from the density of the material and the total volume of the sample. S_t is usually negligible but can be included when the necessity arises.

w/m is the only term which must be approximated when equation (20) is used. Fortunately, w/m can be predicted quite accurately from porosity when only the general nature of the particles of the material in question is known. Table 10 shows how w/m can be approximated for many common shapes that are approximated by fibers, dusts, crystals, or sands.

Table 10. w/m from porosity for some common geometrical shapes.

Object	w/m as a function of porosity
Sphere	$6\ (1 - V/V_t)/(V/V_t)$
Cube	"
Tetrahedron	$6.788(1 - V/V_t)/(V/V_t)$
Octahedron	$6.364(1 - V/V_t)/(V/V_t)$
Circular fiber	$2\pi\ (1 - V/V_t)/(V/V_t)$
Square fiber	$8(1 - V/V_t)/(V/V_t)$
Triangular fiber	$5.196(1 - V/V_t)/(V/V_t)$
Circular disc	(Diam./thickness) $(1 - V/V_t)/(V/V_t)$
Square plate	"
Equilateral triangular plate	$\sqrt{4/3}$(edge/thickness)$(1 - V/V_t)/(V/V_t)$
Ribbon	2(width/thickness) $(1 - V/V_t)/(V/V_t)$

Uniformity of particle size or degree of conformity to the geometrical shapes listed effect very little the estimated value for w/m. Ordinarily, a mixture of sizes will possess a slightly lower value for w/m than will the parent particle of uniform size. Only 0.0395 of the error in estimating w/m manifests itself as relative error in calculating the final surface area, that is, a discrepancy of ± 1.0 in evaluating w/m results in ± 3.95 percent uncertainty in estimating surface area. Ordinarily, errors in evaluating w/m should range between zero, for materials of known shape to ±0.5 for materials of irregular shape.

Hoffing and Lockhart (15) presented information by which the method of determining surface area might be tested. The surface area of diatomacious earth was determined by both Nitrogen adsorption and permeability. They used Carman's conclusions to determine surface area by permeability. These results are as follows:

Information reported	Air permeability	Water permeability
Vol. of cake, V_t	0.756 cm^3	5.90 cm^3
Area of cylinder, A_t	0.378 cm^3	1.77 cm^2
Inverse flow rate	8.87 sec/cm^3	27.3 sec/cm^3
Pressure drop, ΔP	704 gm/cm^2	704 gm/cm^2
Porosity, V/V_t	0.714	0.849
Viscosity, μ	0.000185 poise	0.00947 poise
Surface area, S_p/V_p (by methods of Carman)	74600 cm^2/cm^3	74300 cm^2/cm^3
Surface area, S_p/V_p by nitrogen adsorption	78800 cm^2/cm^3	78800 cm^2/cm^3
Derived information		
w/m, assuming circular fibers	2.51	1.12
Surface area, S_p/V_p by equation (20)	75800 cm^2/cm^3	74200 cm^2/cm^3

This example does not show the full advantage of equation (20) over that proposed by Carman, although slightly better agreement was obtained by equation (20). Equation (20) can be expected to apply to plate-like or ribbon-like materials and Carman's equation is known to be inapplicable for such materials.

One Way in Which Active Surface Might be Estimated

Studies concerned with mass transfer and catalysis in packed beds have shown that all of the surface area of solid particles is not exposed to the transient fluid mass.

The proportion of unavailable surface might be comparable to an apparent unavailable free volume. The following equation, similar to that of Carman, represents how pressure drop would be expressed for hypothetical packed beds where the fluid path is not obstructed.

$$\frac{\Delta P}{L_t} = \frac{50}{9} \frac{\mu U_o V_t S_p^2}{V^3} (1 + 0.6\, S_t/S_p)^2$$

Actually, the fluid path is obstructed and the proportionality between the above equation and equation (20) may be a measure of the surface about which appreciable quantities of material flow. This residual term is proposed to have the following interpretation:

$$\frac{\text{active surface}}{\text{total surface}} = \frac{\text{available volume}}{\text{free volume}} = (V_t/V)\,(10)^{-0.0343w/m}$$

Turbulent flow is not understood well enough so that the effect of increasing flow rate upon available surface can be predicted. Present knowledge does not preclude the possibility that the same fractional free volume is available in turbulent flow.

Comparison of transfer rates in packed beds requires that the Reynold's number be the same for each bed. Reynold's number has been determined as

$$Re = \frac{B}{A} \frac{w \rho U_o}{\mu (1 + 0.6\ S_t/S_p)^2}$$

$$= \rho\, 9wW/200A_t\, \mu\, (1 + 0.6\ S_t/S_p)^2 (10)^{1.766w/D_t\ +\ 0.0343w/m}$$

Taecker and Hougen (16) determined heat transfer coefficients for several packed beds. The material from which the packing was made and the operating conditions were maintained very nearly constant so that a direct comparison of heat transfer coefficients to available surface can be made. The following information illustrates heat transfer factors, j_h, for their beds at Re = 3.0. Values of w, m, and S_p that were used to ascertain the Reynold's number are also included. The coefficients are not point values but have been obtained from plots of j_h vs. Re for the range of interest.

Packing	S_p/V_t	m	w	j_h	$(V_t/V)\ (10)^{-0.0343w/m}$
Raschig ring	111	0.0057	0.0350	0.103	0.976
" "	111	0.0057	0.0350	0.094	0.976
" "	58	0.0123	0.0698	0.095	0.896
" "	29	0.0248	0.1396	0.082	0.895
Partition ring	36	0.0186	0.1440	0.062	0.813
Berl saddle	155	0.0038*	0.0358	0.11	0.806

*Required free volume based on manufacturers information for similar packing as presented by Perry (12).

Figure 16 illustrates the manner in which j_h decreases as the proposed measure for available surface decreases. A decrease in $(V_t/V)\ (10)^{-0.0343w/m}$ represents a proposed decrease in available surface. This comparison suggests that the surface available for transfer may be proportional to $(V_t/V)\ (10)^{-0.0343w/m}$.

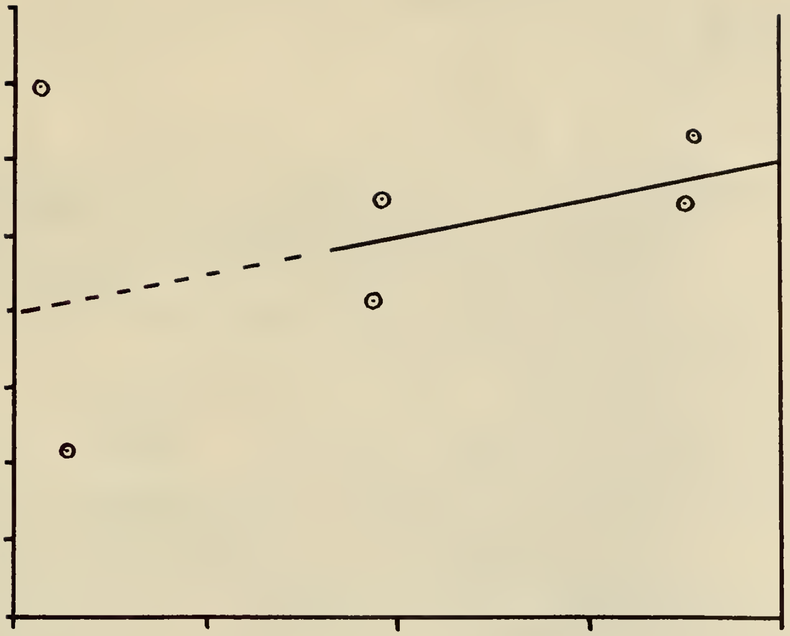


Fig. 16. Heat transfer factor versus a measure of available surface area, Re constant at 3.0.

Efficient Packing

The packed bed is used where large surfaces are required for mass and heat transfer operations. Increased pressure drop through a bed increases the cost required in accomplishing a given amount of transfer. Thus, packing efficiency may be defined as follows:

$$\text{packing eff.} = \frac{\text{rate of trans.}}{\text{power cons.}} \quad \frac{\text{income per unit trans.}}{\text{cost per unit power}}$$

Heat transfer rates parallel mass transfer rates so that efficiency may be evaluated from the standpoint of heat transfer.

$$\text{packing eff.} = \frac{h\ Sp\ \Delta T}{U_o A t\ \Delta P}\ \text{(income ratio)}$$

This efficiency is a dimensionless quantity if consistent energy units are used throughout. Accurate evaluation of efficiency requires specific knowledge of heat transfer coefficients and operating conditions for each operation. However, efficiency may be indexed by reference to some well known relationships.

Most transfer processes are accomplished where flow is turbulent. Certain general relationships may be established for this case. They are as follows:

The heat transfer coefficient, h, may be estimated by the results of the investigation by Taecker and Hougen (16). The Reynold's number that they used has been adjusted to include "w" rather than their approximation for particle diameter. This equation represents their tests with Raschig rings and partition rings.

$$h = 0.723 C \rho U_o \left(\frac{C\mu}{k}\right)^{-0.667} \left(\frac{\rho U_o w}{\mu}\right)^{-0.41}$$

Pressure drop in turbulent flow may be estimated by the results of this investigation.

$$\Delta P = L_t \rho U_o^2 (w/m)/4gm$$

for highly turbulent flow and large diameter columns.

Substituting these approximations into the equation for packing efficiency produces the following relationship.

$$\text{Packing eff.} = (\text{inc. ratio}) \frac{2.89 g k^{0.67} C^{0.33} \Delta T}{\mu^{0.26} \rho^{0.41} U_o^{\ 2.41}} \cdot \frac{(Sp/Vt)m^2}{w^{1.41}}$$

The group of fluid and empty column variables, income ratio, k, C, Δ T, μ, ρ, U_o, and constant factors, are controlled independently of the type of packed column so that $(S_p/V_t)m^2 \div w^{1.41}$ becomes a measure of the expected amount of transfer per unit of pumping power input. Including "w" to the 1.0 power rather than the 1.41 power will not alter the accuracy of this approximation by a greater degree than the order of accuracy involved in substituting "w" for the particle diameter used by Taecker and Hougen. Thus, this equation may be considered an index to the usefulness of a packing material for transfer operations.

$$\text{Packing index} = \frac{m(Sp/Vt)}{w/m} = \frac{(V/Vt)}{(w/m)}$$

when flow is turbulent

Similar analysis of heat transfer during laminar flow produces this index to the packing material:

$$\text{Packing index} = (V/V_t)\ (10)^{-0.0343\ w/m}$$

when flow is laminar

These developments have not considered the availability of surface area which has been previously discussed. In a more detailed analysis, the indexes derived above should be multiplied by the fraction of available surface. Inclusion of the proposed measure for available surface would produce these indexes:

$$\text{Packing ind.} = \frac{(10)^{-0.0343\ w/m}}{(w/m)} \text{ for turbulent flow}$$

$$= (10)^{-0.0686\ w/m} \text{ for laminar flow}$$

The measure of available surface does not change the overall picture as to the effect of w/m on packing efficiency.

Generally speaking, it is desirable to use packing materials that produce low values for w/m. The range of w/m noted for several packing materials that were studied are listed below. The more desirable units are placed at the top of the list.

Packing material	Range of w/m
Wire ring	1.21- 1.39
Berl saddle	3.95- 6.33
Raschig ring	4.28- 8.23
Cylinder	6.37- 9.65
Sphere	6.80-10.53

Mixing wire rings with other packing materials produces favorable values of w/m.

CONCLUSIONS

Pressure drop in packed beds has been found to depend on three properties of the packing. These properties are the void space within the bed, the total surface of the packing, and the perimeter of the packing. The perimeter represents boundries which must be circumvented by the fluid in passing through the bed and has been defined as the locus of tangent points to the packing that would be intersected by a line that moved throughout the bed remaining oriented parallel to the column wall.

Other factors incluencing pressure drop are the size of the column, fluid density, fluid viscosity, and the superficial velocity of the fluid. Normal degrees of roughness of the packing material do not noticeably influence pressure drop. For laminar flow, three fifths of the column wall tends to reduce pressure drop in turbulent flow, this effect is determined by the ratio of packing width to column diameter. Packing width is defined as total packing surface divided by total packing perimeter.

Pressure drop through beds of widely varying properties and for flow ranging from completely laminar to highly turbulent can be expressed mathematically or graphically by reference to the variables which are mentioned above. Graphical correlation of pressure drop requires consideration of the following factors*.

1. $F = \phi(Re)$, primary representation.
2. $Re = w\rho U_oB/\mu A(1 + 0.6\ S_t/S_p)^2$
3. $F = f/B(w/m)$
4. $f = \Delta Pgm/L_t \rho U_o^2$

---

*Table of nomenclature is on page 78.

5. B $= \phi(w/D_t)$, auxiliary representation.

6. A $= \phi(w/m)$, auxiliary representation.

Mathematical correlation requires use of the following equations*:

$$\Delta P/L_t = A \mu U_o (1 + 0.6 S_t/S_p)^2/gm^2 + B^* \rho U_o^2/gm$$

$$A = \frac{50}{9}(10)^{0.03430\ w/m} \qquad \pm 15\%$$

$$B^* = 0.25(w/m)(10)^{-1.766\ w/D_t} \qquad \pm 25\%$$

These conclusions have been derived from information representing eight different packing materials and a mixture comprised of four of them. They have also been proven applicable to materials such as diatomacious earth. The range of variables that was studied is as follows*:

$$0.344 \leqslant V/V_t \leqslant 93.5$$

$$0.020 \leqslant S_t/S_p \leqslant 0.305$$

$$1.21 \leqslant w/m \leqslant 11.11$$

$$0.017 \leqslant w/D_t \leqslant 0.282$$

$$0.00008 \leqslant Re \leqslant 97.0$$

A Reynold's number equal to unity indicates the center of the transition region as flow varies from laminar to turbulent.

The term, w/m, has been identified as an index to the power required by different packing materials when a given rate of mass or heat transfer is desired. Small values of w/m indicate the minimum in pumping costs.

*Table of nomenclature is on page 78.

TABLE OF NOMENCLATURE

a	=	ratio between the effective flow area and A_t.
A	=	laminar flow constant = $\Delta Pgm^2/L_t \mu U_o(1 + 0.6\ S_t/S_p) - B'N$.
Am.l.	=	the predicted value for A.
A_t	=	cross-sectional area of the column.
A'	=	$A(1 + 0.6\ S_t/S_p)^2$.
B	=	$B^*/(w/m)$.
Bm.l.	=	predicted value for B.
B^*	=	turbulent flow constant = $\Delta Pgm/L_t \rho U_o^2 - A'/N$.
B'	=	$B^*/(1 + 0.6\ S_t/S_p)^2$.
B_o^*	=	constant for an orifice = $\Delta Pg/\rho U^2$.
C	=	heat capacity, or perimeter contributed by a specific packing unit.
C_p	=	total perimeter of the packing.
^oC	=	temperature, degrees Centigrade.
cm	=	centimeter.
d	=	differential element.
D	=	equivalent hydraulic diameter, or nominal particle diameter for a specific packing unit.
D_p	=	nominal diameter of a packing unit.
D_s	=	diameter of a sphere having the same volume as the packing unit.
D_t	=	diameter of the column.
D_v	=	effective nominal diameter of a packing unit corresponding to a given void fraction.
e	=	height of an element of surface roughness, or, 2.71828.
E	=	relative error.
E_c	=	error in predicted pressure drop due to an error in perimeter determination.

E_s = error in predicted pressure drop due to an error in surface determination.

E_v = error in predicted pressure drop due to an error in porosity determination.

f = $\Delta P g m / L_t \rho U_o^2$.

f_d = friction factor for dense packing arrangement.

f_l = friction factor for loose packing arrangement.

ft = foot.

F = friction factor = f/B^*.

^oF = temperature, degrees Fahrenheit.

g = acceleration due to gravity, taken as 32.15 ft. per sq. sec. in the Manhattan area.

gm = gram.

h = heat transfer coefficient.

ΔH = vertical displacement.

j_h = heat transfer factor = $\frac{hA_t}{CW}\left(\frac{C\mu}{k}\right)^{2/3}$.

k = arbitrary constant, or thermal conductivity.

K = arbitrary constant.

lb = pound.

L = true length of flow path, or length of a specific packing unit.

L_t = length of the packed zone for which ΔP is measured.

m = estimated hydraulic radius = V/S_p.

mm = pressure, millimeters of Mercury.

n = number of items.

N = $m \rho U_o / \mu$.

ΔP = pressure loss due to frictional resistance along L_t.

r = ratio between true length of flow path and L_t.

Re	= Reynold's number = $m \rho U_o B^* / \mu A(1 + 0.6 S_t/S_p)^2$.
$^\circ R$	= temperature, degrees Rankine.
s_p	= surface of the packing unit.
sq.	= square.
s_s	= surface of a sphere having the same volume as the packing unit.
sec.	= second.
S_p	= total surface of the packing.
S_t	= surface of the column wall.
t	= thickness of a specific packing unit.
U	= true fluid velocity.
U_o	= velocity based on the empty column.
v	= volume of a specific packing unit.
v_p	= volume of the packing unit.
V	= free or void volume within the packed zone.
V_p	= total volume of the packing.
V_t	= $V + V_p$ = volume of the empty column.
w	= S_p/C_p = width of the barrier to be circumvented by the fluid.
W	= mass rate of flow.
z	= ratio between effective hydraulic radius and m.
γ	= arbitrary function.
δ	= constant arising because of the geometrical nature of a duct.
δ'	= special case for δ.
Δ	= incremental element.
θ	= angle of orientation of an element of surface.
μ	= absolute viscosity.

π = 3.14159

ρ = mass density of the fluid.

ϕ = function of.

$<$ = less than.

$\leqslant$ = less than or equal to.

$\cong$ = approximated by.

$\propto$ = proportional to.

$\sqrt{\ }$ = square root.

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BIBLIOGRAPHY

(1) Blake, C. F.
The resistance of packing to fluid flow. American Institute of Chem. Engineers, Transactions. 14: 415-421, 1921-22.

(2) Brownell, Lloyd E., and Donald L. Katz.
Flow of fluids through porous media — Part I Single homogenous fluids. Chem. Engg. Progress. 43: 537-545, October, 1947.

(3) Brownell, Lloyd E., H. S. Dombrowski, and C. A. Dickey.
Pressure drop through porous media — Part IV New data and revised correlation. Chem. Engg. Progress. 46: 415-422, August, 1950.

(4) Burke, S. P., and W. B. Plummer.
Gas flow through packed columns. Industrial and Engg. Chem. 20: 1196-1200, November, 1928.

(5) Carman, P. C.
Fluid flow through granular beds. Transactions of the Institution of Chem. Engineers. 15: 150-166, 1937.

(6) Fowler, J. L., and K. L. Hertel.
Flow of a gas through porous media. Journal of Applied Physics. 11:496-502, July, 1940.

(7) Hodgman, Charles D.
Handbook of chemistry and physics, 28th ed. Cleveland: Chemical Rubber Publishing Company, 1944.

(8) Leva, Max, and Milton Grummer.
Pressure drop through packed tubes. A series of three articles concluded in: Chem. Engg. Progress. 43: 713-718, December, 1947.

(9) Meyer, Walter G., and Lincoln T. Work.
Flow of fluids through packed solids. American Institute of Chem. Engg., Transactions. 32: 13-18, 1937.

(10) Morcom, A. R.
Fluid flow through granular materials. Transactions of the Institution of Chem. Engineers. 24: 30-43, 1946.

(11) Oman, A. O., and K. M. Watson.
Pressure drop in granular beds. Process Engg. Data, a series of seven articles, University of Wisconsin. Reprinted from: Tech. section of the national petroleum news (now petroleum processing). May through November, 1944.

(12) Perry, John H.
Chemical engineers handbook, 2nd ed. New York: McGraw-Hill, 1941.

(13) Rouse, Hunter.
Fluid mechanics for hydraulic engineers, 1st ed. New York: McGraw-Hill, 1937.

(14) Sullivan, R. R., and K. L. Hertel.
The flow of air through porous media. Journal of Applied Physics. 11: 761-765, December, 1940.

(15) Hoffing, E. H., and Frank J. Lockhart.
Resistance to filtration. Chem. Engg. Progress. 47: 3-10, January, 1951.

(16) Taecker, R. G., and O. A. Hougen.
Heat and mass transfer of the gas film in the flow of gases through commercial tower packings. Chem. Engg. Progress. 45: 188-193, March, 1949.

(17) Badger, Walter L., and Warren L. McCabe.
Elements of chemical engineering, 2nd ed. New York: McGraw-Hill, 1936.

APPENDIX

EXPLANATION OF PLATE X

Cross-sections of packed beds.

Packing materials shown:

0.792 inch glass ball.

1.028 inch Berl saddle.

1.032 inch Raschig ring.

0.512 inch Berl saddle.

0.522 inch Raschig ring.

PLATE X

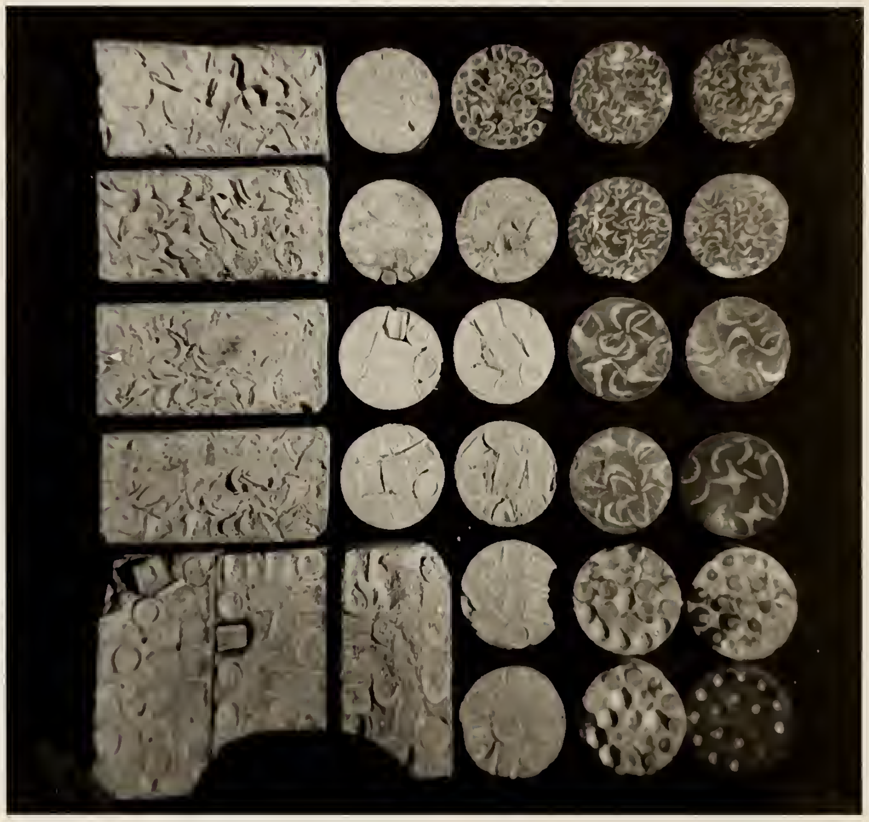


Table 11. Author's original data.

Bed #1, 1.028 inch Berl saddle in the 3.10 inch column.

volume of unit = 1.445×10^{-4} cu.ft.
surface of unit = 0.0343 sq.ft.
% void = 77.0
S_t/S_p = 0.285
w/m = 4.54

Run #	Temp. °C	Press., mm	$\rho \frac{lb}{ft^3}$	$\mu \frac{lb}{ft-sec}$	$\frac{W}{A_t} \frac{lb}{ft^2-sec}$	$\frac{\Delta P}{L_t} \frac{lb}{ft^3}$
D	34.8	---	55.8	0.323	2.42	28.5
E	39.4	---	55.6	0.234	0.947	8.1
4	36.0	---	55.7	0.295	4.61	42.6
5	38.3	---	55.6	0.252	2.66	25.1
6	41.0	---	55.5	0.210	0.926	6.9
51	28.8	---	62.2	5.52×10^{-4}	5.73	0.9
50	28.6	---	"	5.54 "	7.72	1.5
49	28.5	---	"	5.56 "	9.93	2.4
48	28.4	---	"	5.58 "	13.3	4.1
47	28.2	---	"	5.60 "	16.9	7.1
46	28.1	---	"	5.60 "	20.9	12.2
45	28.0	---	"	5.62 "	24.8	17.0
44	27.9	---	"	5.63 "	27.2	21.8
43	27.8	---	"	5.64 "	32.4	26.5
25	23.0	737	0.0719	1.22×10^{-5}	0.314	1.3
24	23.0	741	0.0724	" "	0.422	2.3
23	23.2	747	0.0729	" "	0.527	3.6
22	23.4	760	0.0742	" "	0.735	6.6
21	23.2	780	0.0761	" "	0.970	11.1
20	22.8	803	0.0780	" "	1.18	15.6
19	23.1	824	0.0803	" "	1.28	18.0

Table 11 (cont.)

Bed #2, 0.645 inch wire ring in the 6.08 inch column

diameter of wire = 0.0677 inch
length of wire = 1.92 inch
% void = 81.6
S_t/S_p = 0.059
w/m = 1.39

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
A	33.2	---	55.8	0.363	0.444	15.6
B	37.5	---	55.7	0.266	1.31	33.3
C	35.8	---	55.7	0.300	0.338	10.4
1	37.2	---	55.7	0.272	1.10	28.7
2	38.6	---	55.6	0.247	0.537	12.8
3	39.2	---	55.6	0.237	0.368	6.3
42	28.1	---	62.2	5.60×10^{-4}	4.18	0.9
41	28.0	---	62.2	5.62 "	5.15	1.1
40	28.0	---	62.2	5.62 "	5.79	1.3
39	28.0	---	62.2	5.26 "	7.12	2.0
38	27.9	---	62.2	5.64 "	7.85	2.4
37	27.7	---	62.2	5.66 "	8.31	2.6
28	23.0	765	0.0746	1.22×10^{-5}	0.242	1.0
27	22.6	778	0.0760	1.22 "	0.276	1.3
26	22.0	793	0.0777	1.22 "	0.309	1.6

Table 11 (cont.)

Bed #3, 0.792 inch glass ball in the 4.06 inch column

% void = 42.3
St/Sp = 0.225
w/m = 8.20

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{Lt}$ $\frac{lb}{ft^3}$
F	33.1	---	55.8	0.365	1.84	83.3
G	36.5	---	55.7	0.286	1.16	40.6
H	39.1	---	55.6	0.238	0.766	22.8
I	41.2	---	55.5	0.207	0.378	10.3
7	35.0	---	55.8	0.318	1.92	70.6
8	36.3	---	55.7	0.289	1.07	34.4
9	37.5	---	"	0.266	0.371	10.6
93	37.0	---	"	0.276	2.78	91.9
94	38.9	---	55.6	0.242	1.28	36.7
95	38.2	---	"	0.254	0.660	20.2
86	27.2	---	62.2	5.71×10^{-4}	4.17	1.6
85	27.1	---	"	5.72 "	4.42	1.7
84	27.0	---	"	5.74 "	5.57	2.6
83	26.8	---	"	5.77 "	7.34	4.9
82	26.7	---	"	5.78 "	9.23	9.6
81	26.6	---	"	5.79 "	13.0	16.6
80	26.4	---	"	5.82 "	16.6	23.8
79	26.3	---	"	5.83 "	18.3	27.6
18	22.7	734	0.0717	1.22×10^{-5}	0.171	1.5
17	22.8	735	0.0718	" "	0.202	2.0
16	"	737	0.0720	" "	0.258	3.0
15	"	742	0.0725	" "	0.346	5.2
14	"	753	0.0735	" "	0.455	8.5
13	"	763	0.0747	" "	0.549	12.0
12	22.5	780	0.0763	" "	0.686	17.9
10	22.7	782	0.0764	" "	0.701	18.6
11	22.1	791	0.0768	" "	0.755	21.1

Table 11 (cont.)

Bed #4, 1.028 inch Berl saddle in the 6.08 inch column

volume of unit = 1.445×10^{-4} cu. ft.
surface of unit = 0.0343 sq. ft.
% void = 71.8
S_t/S_p = 0.118
w/m = 5.94

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
90	35.9	---	55.7	0.297	1.51	20.4
91	37.6	---	55.6	0.264	1.06	12.9
92	39.1	---	"	0.238	0.630	6.9
70	23.1	---	62.3	6.28×10^{-4}	5.49	1.1
69	23.0	---	"	6.30 "	5.85	1.3
68	22.7	---	"	6.34 "	6.99	1.8
67	22.2	---	"	6.40 "	7.82	2.1
66	21.8	---	"	6.46 "	8.20	2.3
56	24.0	767	0.0746	1.23×10^{-5}	0.183	1.0
55	23.9	776	0.0755	" "	0.210	1.3
54	23.2	791	0.0772	1.22 "	0.248	1.7
53	21.0	814	0.0801	" "	0.301	2.2
52	18.9	832	0.0824	1.21 "	0.336	2.6

Table 11 (cont.)

Bed #5, 0.645 inch wire ring in the 3.10 inch column

diameter of wire = 0.0677 inch
length of wire = 1.92 inch
% void = 83.6
S_t/S_p = 0.131
w/m = 1.21

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
87	37.6	---	55.6	0.265	2.71	58.4
88	38.5	---	"	0.249	1.38	29.1
89	39.1	---	"	0.238	0.780	15.9
78	26.2	---	62.2	5.84×10^{-4}	5.30	1.2
77	26.0	---	"	5.87 "	6.89	1.6
76	25.7	---	"	5.90 "	8.55	2.4
75	25.4	---	"	5.94 "	12.1	3.8
74	25.2	---	"	5.98 "	16.5	6.5
73	25.0	---	"	6.00 "	23.0	12.0
72	24.8	---	"	6.03 "	28.4	17.6
71	24.4	---	"	6.09 "	31.3	22.7
62	21.3	744	0.0730	1.22×10^{-5}	0.239	1.2
61	24.0	746	0.0726	" "	0.303	1.6
60	25.0	750	0.0728	1.23 "	0.389	2.8
59	25.6	779	0.0755	" "	0.784	8.8
58	26.1	816	0.0789	" "	1.07	16.3
57	24.7	851	0.0826	" "	1.30	22.3

Table 11 (cont.)

Bed #6, 1.032 inch Raschig ring in the 6.08 inch column

length of unit = 1.048 inch
thickness of unit = 0.162 inch
% void = 68.9
S_t/S_p = 0.157
w/m = 5.63

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
125	38.1	---	55.6	0.255	1.28	14.9
126	38.8	---	"	0.243	0.864	10.0
127	39.3	---	"	0.235	0.605	6.7
122	29.5	---	62.2	5.43×10^{-4}	8.26	3.8
123	29.7	---	"	5.41 "	5.72	1.6
124	30.0	---	"	5.39 "	4.27	1.0
96	22.7	608	0.0791	1.22×10^{-5}	0.331	3.6
97	23.8	790	0.0772	1.23 "	0.278	2.8
98	23.6	764	0.0746	1.22 "	0.190	1.4
99	23.4	753	0.0735	1.23 "	0.146	0.8

Table 11 (cont.)

Bed #7, 0.645 inch wire ring in the 4.06 inch column

diameter of wire = 0.0677 inch
length of wire = 1.92 inch
% void = 83.2
S_t/S_p = 0.098
w/m = 1.25

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
131	37.2	---	55.7	0.272	2.82	60.3
132	37.7	---	55.6	0.263	1.36	28.2
133	38.2	---	"	0.254	0.660	14.0
115	29.1	---	62.2	5.48×10^{-4}	6.10	1.0
114	29.0	---	"	5.50 "	8.28	1.7
113	28.9	---	"	5.51 "	11.1	3.0
112	28.7	---	"	5.54 "	14.8	4.9
111	28.4	---	"	5.57 "	16.6	7.6
110	24.8	745	0.0724	1.23×10^{-5}	0.250	1.2
109	25.0	750	0.0728	" "	0.358	2.1
108	24.9	771	0.0748	" "	0.558	4.7
107	24.1	795	0.0774	" "	0.730	7.2

Table 11 (cont.)

Bed #8, 0.792 inch glass ball in the 3.10 inch column

% void = 44.0
S_t/S_p = 0.305
w/m = 7.64

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
128	38.5	---	55.6	0.249	3.72	93.9
129	39.1	---	"	0.238	1.87	46.1
130	39.8	---	"	0.227	1.06	25.1
121	30.8	---	62.2	5.28×10^{-4}	5.34	2.3
120	30.4	---	"	5.33 "	7.28	4.1
119	30.1	---	"	5.37 "	10.9	11.2
118	29.9	---	"	5.39 "	15.7	18.0
117	29.8	---	"	5.40 "	22.9	32.9
116	29.6	---	"	5.42 "	32.0	61.0
106	23.1	741	0.0724	1.22×10^{-5}	0.178	1.9
105	"	743	0.0727	" "	0.253	3.6
104	23.0	750	0.0733	" "	0.381	7.6
103	22.8	758	0.0742	" "	0.538	14.3
102	22.4	784	0.0767	" "	0.834	32.2
101	21.8	815	0.0801	" "	1.11	56.1
100	20.8	859	0.0847	" "	1.37	61.0

Table 11 (cont.)

Bed #9, 0.792 inch glass ball in the 6.08 inch column

% void = 38.8
St/Sp = 0.142
w/m = 9.47

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
159	23.3	---	62.3	6.25×10^{-4}	9.24	7.3
160	23.6	---	"	6.20 "	6.62	3.5
161	23.7	---	"	6.19 "	4.04	1.6
134	26.7	825	0.0796	1.23×10^{-5}	0.326	5.8
135	26.8	778	0.0748	" "	0.222	3.1
136	26.3	752	0.0726	" "	0.140	1.3

Bed #10, 1.028 inch Berl saddle in the 4.06 inch column

volume of unit = 1.445×10^{-4} cu. ft.
surface of unit = 0.0343 sq. ft.
% void = 72.7
St/Sp = 0.182
w/m = 5.70

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
148	19.6	---	62.3	6.83×10^{-4}	20.9	8.2
149	19.7	---	"	6.78 "	16.5	5.4
150	19.9	---	"	6.76 "	12.8	3.2
151	20.1	---	"	6.73 "	9.55	1.8
152	20.3	---	"	6.70 "	7.14	1.1
143	21.6	831	0.0815	1.22×10^{-5}	0.763	7.6
144	22.4	800	0.0783	" "	0.626	5.4
145	22.8	777	0.0758	" "	0.512	3.8
146	22.9	751	0.0733	" "	0.382	2.2
147	22.6	741	0.0724	" "	0.257	1.1

Table 11 (cont.)

Bed #11, 1.032 inch Raschig ring in the 3.10 inch column

length of unit = 1.048 inch
thickness of unit = 0.162 inch
% void = 74.5
S_t/S_p = 0.354
w/m = 4.28

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
153	21.7	---	62.3	6.48×10^{-4}	36.2	19.5
154	21.9	---	"	6.44 "	27.7	11.1
155	22.1	---	"	6.41 "	20.6	6.1
156	22.4	---	"	6.38 "	16.4	3.4
157	22.6	---	"	6.35 "	12.3	1.8
158	22.8	---	"	6.32 "	8.68	0.9
137	23.1	825	0.0807	1.22×10^{-5}	1.29	13.9
138	23.1	792	0.0774	" "	1.02	9.1
139	22.8	762	0.0744	" "	0.817	4.8
140	22.4	752	0.0736	" "	0.600	3.4
141	22.0	744	0.0729	" "	0.488	2.2
142	21.8	738	0.0723	" "	0.342	1.1

Table 11 (cont.)

Bed #12, 1.0115 inch metal Raschig ring in the 6.08 inch column

length of unit = 1.0034 inch
thickness of unit = 1/32 inch
distance between extremities of the packed zone = 31.7 inches
% void = 92
S_t/S_p = 0.126
w/m = 4.43

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
184	35.3	---	55.7	0.313	1.205	11.8
185	36.6	---	"	0.284	1.346	11.3
186	36.5	---	"	0.287	1.317	11.4
177	27.0	768	0.0740	1.235×10^{-5}	0.275	2.1
174	26.6	772	0.0744	1.23 "	0.295	2.4
176	26.2	787	0.0761	" "	0.342	3.1
175	25.8	787	000761	" "	0.347	3.2

Bed #12a, same specifications as bed #12

distance between extremities of the packed zone = 31.6 inches

Run #	Temp. °C	Press., mm	$\frac{lb}{ft^3}$	$\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{P}{L_t}$ $\frac{lb}{ft^3}$
196	36.2	---	55.7	0.294	1.142	9.5
197	37.2	---	"	0.273	1.171	9.1
198	37.9	---	55.6	0.260	1.139	8.9
193	26.1	776	0.0749	1.23×10^{-5}	0.273	2.0
195	"	780	0.0753	" "	0.278	2.1
194	25.8	825	0.0797	" "	0.353	3.2
192	25.3	826	0.0800	" "	0.362	3.3

Table 11 (cont.)

Bed #13, 0.522 inch Raschig ring in the 4.06 inch column

length of unit = 0.533 inch
thickness of unit = 0.0888 inch
% void = 59.5
S_t/S_p = 0.103
w/m = 7.20

Run #	Temp. °C	Press., mm	ρ $\frac{lb}{ft^3}$	μ $\frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}$ $\frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}$ $\frac{lb}{ft^3}$
189	35.2	---	55.7	0.315	1.462	107.0
190	35.4	---	"	0.310	1.478	105.4
191	37.1	---	"	0.275	0.2955	22.8
162	27.0	734	0.0707	1.235×10^{-5}	0.132	2.4
173	27.4	737	0.0709	" "	0.136	2.4
172	27.5	739	0.0711	" "	0.207	5.3
163	27.0	737	0.0709	" "	0.222	5.8
171	27.8	742	0.0712	" "	0.308	11.1
164	26.9	741	0.0713	" "	0.320	11.7
165	27.0	750	0.0722	" "	0.435	20.2
170	27.7	752	0.0724	" "	0.449	21.7
166	27.1	770	0.0741	" "	0.637	41.8
169	27.6	772	0.0743	" "	0.639	41.9
168	27.4	789	0.0758	" "	0.791	61.4
167	27.4	791	0.0761	" "	0.807	64.2

Table 11 (concl.)

Bed #14, mixture of 5-0.645 inch wire rings, 1-0.792 inch glass ball, 1-0.512 inch Berl saddle, and 1-0.522 inch Raschig ring in the 3.10 inch column.
volume of simplest composite = 2.37×10^{-4} cu. ft.
surface of simplest composite = 0.0486 sq. ft.
perimeter of simplest composite = 2.48 ft.
% void = 53.0
S_t/S_p = 0.157
w/m = 3.57

Run #	Temp. °C	Press., mm	$\rho\ \frac{lb}{ft^3}$	$\mu\ \frac{lb}{ft\text{-}sec}$	$\frac{W}{A_t}\ \frac{lb}{ft^2\text{-}sec}$	$\frac{\Delta P}{L_t}\ \frac{lb}{ft^3}$
187	37.2	---	55.7	0.273	0.990	51.4
188	37.0	---	"	0.270	1.013	51.0
183	27.6	737	0.0709	1.235×10^{-5}	0.177	2.4
182	27.9	740	0.0710	" "	0.288	5.8
181	28.0	747	0.0718	" "	0.395	10.6
180	"	758	0.0728	" "	0.560	20.4
179	27.6	794	0.0764	" "	0.878	45.1
178	27.0	867	0.0836	" "	1.315	91.2

Table 12. Individual values of the laminar flow constant.

Ref.	Packing	Bed #	D_t,in.	D_p,in.	%void	S_t/S_p	w/m	F'	Run #	N	A	Run #	N	A
Aut.	Wire ring	2	6.08	0.645	81.6	0.059	1.39	0.3	A	0.008	6.16	B	0.030	6.02
									C	0.007	6.51	1	0.025	6.05
									2	0.013	6.10	3	0.009	6.02
"	" "	7	4.06	"	83.2	0.098	1.25	0.2	131	0.071	5.80	132	0.036	5.91
									133	0.018	6.08			
"	" "	5	3.10	"	83.6	0.131	1.21	0.3	87	0.072	6.08	88	0.040	6.31
									89	0.024	6.42			
"	Glass ball	3	4.06	0.792	42.3	0.225	8.20	0.8	F	0.040	11.25	G	0.033	10.98
									H	0.026	11.27	I	0.014	11.80
									7	0.049	10.40	8	0.030	10.00
									9	0.011	9.72	93	0.123	10.93
									94	0.043	10.16	95	0.021	10.84
"	" "	8	3.10	"	44.0	0.305	7.64	0.8	128	0.129	9.59	129	0.068	9.88
									130	0.040	9.91			
"	Berl saddle	1	3.10	1.028	77.0	0.285	4.54	0.6	D	0.106	9.41	E	0.057	9.44
									4	0.220	8.01	5	0.149	9.64
									6	0.063	9.19			
"	" "	4	6.08	"	71.8	0.118	5.94	0.6	90	0.054	8.43	91	0.043	8.53
									92	0.032	8.59			
"	Raschig ring	6	6.08	1.032	68.9	0.157	5.63	1.0	125	0.065	11.58	126	0.046	12.00
									127	0.033	11.90			
"	" "	12	"	1.0115	92	0.126	4.43	0.5	184	0.056	10.43	185	0.070	9.79
									186	0.067	10.01			
"	" "	12a	"	"	"	"	"	"	196	0.057	9.34	197	0.063	9.49
									198	0.064	9.93			

Table 12 (cont.)

Ref.	Packing	Bed#	D_t,in.	D_p,in.	%void	S_t/S_p	w/m	ϵ'	Run#	N	A	Run#	N	A
Aut.	Raschig ring	13	4.06	0.522	59.5	0.103	7.20	1.0	189	0.024	9.90	190	0.025	9.79
									191	0.004	9.52			
"	Mixture	14	3.10	varied	53.0	0.157	3.57	0.6	187	0.020	8.59	188	0.020	8.19
(14)	Glass ball	--	1.523	0.0273	39.08	0.0198	9.353	---	--	0.01	11.58	--	-----	-----
"	" "	--	"	"	39.04	"	9.368	---	--	0.01	11.63	--	-----	-----
"	" "	--	0.3234	"	39.11	0.0935	9.341	---	--	0.01	11.61	--	-----	-----
"	" "	--	"	"	39.24	"	9.290	---	--	0.01	11.64	--	-----	-----
(3)	" "	1	2.07	0.2083	41.2	0.114	8.56	1.2	148	0.003	10.82	149	0.003	11.33
									150	0.003	14.14	151	0.004	10.82
									152	0.005	10.08	153	0.007	9.43
									154	0.007	9.90	155	0.009	10.18
									156	0.010	10.36	157	0.011	10.62
									158	0.012	9.94	159	0.013	10.80
									160	0.015	10.86	161	0.017	10.75
									162	0.017	11.22	163	0.019	11.00
									164	0.022	11.08	165	0.025	10.90
									167	0.031	10.90	168	0.034	10.78
									169	0.038	10.65	170	0.042	10.45
									171	0.048	10.43	172	0.055	9.85
									173	0.058	10.07	175	0.063	10.00
									176	0.068	9.93	177	0.077	9.52
									178	0.079	9.88	179	0.085	9.78
									180	0.094	9.76	181	0.096	9.82
									182	0.105	9.70	183	0.114	9.67
									184	0.122	9.78	185	0.128	9.95
									186	0.138	10.11	187	0.143	10.12
									188	0.152	10.20	189	0.159	10.30
									190	0.169	10.32	191	0.174	10.40
									192	0.183	10.47	193	0.192	10.55

Table 12 (cont.)

Ref.	Packing	Bed	D_t, in.	D_p, in.	% void	S_t/S_p	w/m	I'	Run	N	A	Run	N	A
(3)	Glass ball	1	2.07	0.2083	41.2	0.114	8.56	1.2	27	0.194	8.53	28	0.224	8.60
									29	0.260	9.02	30	0.311	8.44
									31	0.359	9.65			
"	Smooth saddle	2	"	0.1316	93.1	0.041	5.35	0.9	135	0.001	13.82	136	0.001	13.61
									137	0.001	11.58	138	0.003	9.47
									139	0.003	8.96	140	0.004	11.10
									141	0.005	9.20	142	0.006	10.04
									143	0.007	10.60	144	0.008	10.75
									145	0.009	9.25	146	0.011	11.35
									147	0.012	11.78	148	0.014	11.80
									149	0.016	12.18	150	0.018	12.50
									151	0.020	12.45	152	0.022	12.67
									153	0.025	12.11	154	0.029	11.86
									155	0.034	11.66	156	0.038	11.52
									157	0.044	11.42	158	0.048	11.34
									159	0.057	10.97	160	0.041	19.82
									161	0.095	10.60	162	0.115	10.62
									163	0.132	10.54	164	0.147	10.64
									165	0.164	10.55	171	0.166	9.85
									172	0.196	10.12	173	0.228	10.58
									174	0.260	11.40	175	0.284	10.34
									176	0.325	10.40	177	0.386	9.95
									178	0.409	10.32	179	0.462	9.97
									180	0.507	10.44	181	0.559	10.70
"	Rough saddle	3	"	0.130	93.5	0.044	5.02	0.8	86	0.000	10.08	87	0.001	7.71
									88	0.002	6.78	89	0.002	7.65
									90	0.002	7.75	91	0.002	7.31
									92	0.003	7.53	93	0.003	7.12
									94	0.004	7.48	99	0.007	8.03
									95	0.004	7.64	96	0.005	7.64
									97	0.005	7.57	98	0.006	7.70
									100	0.007	8.36	101	0.008	8.38
									102	0.009	8.22	103	0.009	8.08

Table 12 (cont.)

Ref.	Packing	Bed#	D_t,in.	D_p,in.	%void	S_t/S_p	w/m	B'	Run#	N	A	Run#	N	A
(3)	Rough saddle	3	2.07	0.130	93.5	0.044	5.02	0.8	104	0.010	8.07	105	0.012	7.83
									106	0.013	8.03	107	0.015	8.00
									108	0.016	7.97	109	0.018	8.00
									110	0.019	7.92	111	0.020	7.81
									112	0.023	7.70	113	0.026	7.68
									114	0.030	7.68	295	0.029	7.87
									296	0.042	7.95	297	0.056	8.02
									298	0.066	8.00	299	0.083	8.08
									300	0.101	8.10	301	0.121	8.06
									302	0.145	8.03	303	0.165	7.91
									304	0.202	7.98	305	0.227	8.01
									306	0.262	8.08	307	0.309	8.01
									308	0.348	7.99	273	0.417	6.91
									274	0.482	7.15			
"	Berl saddle	4	6*	1.00	72.5	0.113	6.33	0.6	10	0.008	7.47	11	0.010	5.90
									17	0.010	7.05	21	0.012	7.45
									53	0.013	7.54	52	0.017	6.53
									51	0.021	6.76	12	0.022	6.92
									22	0.024	5.97	9	0.025	7.73
									20	0.025	6.06	50	0.026	6.66
									49	0.029	6.69	19	0.032	6.12
									13	0.033	7.45	48	0.033	6.66
									23	0.034	6.91	8	0.036	7.21
									24	0.037	6.39	47	0.037	6.62
									18	0.040	6.07	46	0.042	6.61
									54	0.043	7.01	25	0.045	6.33
									45	0.047	6.53	14	0.048	7.37
									44	0.051	6.51	43	0.056	6.49
									7	0.059	7.05	42	0.061	6.39
									15	0.064	7.22	26	0.064	6.19
									41	0.068	6.42	27	0.075	6.14
									16	0.082	7.01	28	0.083	6.16
									29	0.088	6.18	67	0.114	6.17

Table 12 (cont.)

Ref.	Packing	Bed#	Dt,in.	Dp,in.	%void	St/Sp	w/m	B'	Run#	N	A	Run#	N	A
(3)	Berl saddle	4	6*	1.00	72.5	0.113	6.33	0.6	34	0.117	6.80	30	0.139	6.76
									55	0.137	6.23	31	0.152	6.75
									32	0.182	6.28	35	0.180	6.86
									37	0.197	6.73	56	0.200	6.22
									40	0.210	6.70	36	0.223	6.82
									5	0.250	6.89	38	0.270	6.48
									57	0.273	6.22	39	0.308	6.74
									68	0.308	6.19	4	0.348	6.81
									58	0.362	5.22	59	0.432	5.88
									3	0.453	6.82	69	0.518	5.90
									60	0.550	5.81			
"	Raschig ring**	5	"	"	70.7	0.121	6.36	0.9	148	0.001	7.57	149	0.003	7.60
									150	0.005	8.07	151	0.008	7.97
									152	0.012	7.77	167	0.015	6.32
									153	0.017	7.39	154	0.022	7.68
									155	0.027	7.80	156	0.033	7.58
									157	0.040	7.48	158	0.052	7.45
									168	0.053	8.68	136	0.067	9.45
									169	0.079	6.14	159	0.080	8.52
									170	0.110	8.31	171	0.127	7.48
									160	0.130	8.31	172	0.157	7.54
									103	0.193	8.58	135	0.228	6.58
									161	0.254	8.20	162	0.312	8.27
									134	0.374	7.47	104	0.382	7.22
									163	0.404	8.06	137	0.410	6.73
(5)	Hexaganol prism	---		0.185	37.7	0.066	9.39	---	--	-----	12.32	--	-----	-----
"	" "	---		"	42.6	0.065	7.66	---	--	-----	9.30	--	-----	-----
"	Cube	---		0.220	34.4	0.075	11.11	---	--	-----	14.10	--	-----	-----
"	"	---		"	39.7	0.078	9.10	---	--	-----	10.80	--	-----	-----

Table 12 (concl.)

Ref.	Packing	Bed #	D_t, in.	D_p, in.	% void	S_t/S_p	w/m	B'	Run #	N	A	Run #	N	A
(5)	Cube	--	---	0.220	44.8	0.074	7.40	---	--	-----	8.62	--	-----	-----

* This value assumed because of lack of evidence of the true value.

** The surface of this unit was calculated from the volume of the unit and its overall demensions. The calculated value of 0.0430 sq ft was used for these estimations instead of the reported value of 0.0390 sq ft because it was in agreement with information published by the manufacturer.

Table 13. Individual values of the turbulent flow constant.

Ref.	Packing	Bed#	D_t,in.	D_p,in.	%void	S_t/S_p	w/m	A'	Run#	N	B^*	Run#	N	B^*
Aut.	Wire ring	2	6.08	0.645	81.6	0.059	1.39	6.7	42	45.9	0.484	41	56.1	0.409
									40	63.4	0.372	39	78.0	0.418
									38	85.6	0.396	37	90.7	0.389
									28	122	0.209	27	139	0.212
									26	156	0.222			
"	" "	5	3.10	"	83.6	0.131	1.21	7.1	78	64.0	0.485	77	82.9	0.394
									76	102	0.385	75	144	0.307
									74	195	0.293	73	270	0.289
									72	339	0.282	71	363	0.300
									62	139	0.285	61	175	0.234
									60	223	0.271	59	449	0.225
									58	610	0.238	57	744	0.234
"	" "	7	4.06	"	83.2	0.098	1.25	6.9	115	77.3	0.270	114	103	0.280
									113	138	0.287	112	184	0.268
									111	228	0.272	110	140	0.253
									109	199	0.221	108	310	0.229
									107	406	0.214			
"	Glass ball	3	"	0.792	42.3	0.225	8.20	13.7	86	59.1	1.30	85	62.2	1.21
									84	78.3	1.17	83	102	1.34
									82	129	1.69	81	181	1.50
									80	230	1.30	79	254	1.27
									18	113	0.813	17	134	0.791
									16	170	0.746	15	236	0.733
									14	300	0.734	13	362	0.735
									12	453	0.719	10	463	0.717
									11	500	0.705			
"	" "	8	3.10	"	44.0	0.305	7.64	14.3	121	87.9	1.25	120	118	1.21
									119	176	1.54	118	251	1.20
									117	367	1.04	116	510	1.00
									106	126	1.07	105	179	1.07

Table 13 (cont.)

Ref.	Packing	Bed#	D_t,in.	D_p,in.	%void	S_t/S_p	w/m	A'	Run#	N	B*	Run#	N	B*
Aut.	Glass ball	8	3.10	0.792	44.0	0.305	7.64	14.3	104	270	1.01	103	381	0.98
									102	591	0.746	101	787	0.99
									100	976	1.00			
"	"	9	6.08	"	38.8	0.142	9.47	13.7	161	45.5	1.07	160	74.5	0.93
									159	121	1.08	136	79.4	1.01
									135	126	1.01	134	185	0.901
"	Berl saddle	1	3.10	1.028	77.0	0.285	4.54	10.9	51	147	0.918	50	196	0.844
									49	253	0.837	48	336	0.806
									47	425	0.857	46	525	0.974
									45	624	0.962	44	683	1.02
									43	810	0.883	25	364	0.509
									24	486	0.524	23	609	0.515
									22	848	0.502	21	1120	0.495
									20	1360	0.493	19	1480	0.498
"	"	4	6.08	"	71.8	0.118	5.94	10.2	70	93.9	0.688	69	100	0.728
									68	119	0.696	67	131	0.682
									66	136	0.675	56	159	0.718
									55	183	0.698	54	218	0.683
									53	264	0.651	52	298	0.615
"	"	10	4.06	"	72.7	0.182	5.70	10.7	152	119	0.393	151	159	0.375
									150	212	0.388	149	273	0.405
									148	343	0.390	147	236	0.389
									146	350	0.368	145	470	0.373
									144	575	0.369	143	700	0.368
"	Raschig ring	6	6.08	1.032	68.9	0.157	5.63	10.4	124	104	1.30	123	138	1.23
									122	198	1.36	99	155	1.14
									98	203	1.17	97	295	1.13
									96	354	1.07			

Table 13 (cont.)

Ref.	Packing	Bed#	D_t, in.	D_p, in.	%void	S_t/S_p	w/m	A'	Run#	N	B*	Run#	N	B*
Aut.	Raschig ring	11	3.10	1.032	74.5	0.354	4.28	11.5	158	234	0.359	157	330	0.365
									156	439	0.406	155	549	0.470
									154	734	0.479	153	954	0.497
									142	478	0.350	141	681	0.363
									140	838	0.368	139	1140	0.284
									138	1420	0.401	137	1800	0.365
"	"	12	6.08	1.0115	92	0.126	4.43	9.0	177	334	0.961	174	358	0.937
									176	416	0.936	175	422	0.918
"	"	12a	"	"	"	"	"	"	193	325	0.915	195	331	0.944
									194	421	0.953	192	422	0.965
"	"	13	4.06	0.522	59.5	0.103	7.20	11.1	162	55.5	1.40	173	57.0	1.35
									172	87.0	1.32	163	93.2	1.28
									171	129	1.30	164	134	1.27
									165	183	1.22	170	188	1.27
									166	267	1.23	169	266	1.23
									168	332	1.21	167	339	1.22
"	Mixture	14	3.10	varied	53.0	0.157	3.57	8.8	183	78.8	0.846	182	126	0.815
									181	176	0.814	180	249	0.802
									179	391	0.768	178	585	0.765
(1)	Glass ring	--	4[x]	0.269[xx]	72	0.050	6.32	9.7	--	15.2	2.48	--	25.0	2.24
									--	35.0	2.10	--	47.5	2.03
									--	65.0	1.95	--	102	1.85
"	"	--	"	0.233	67	0.042	6.53	9.8	--	16.8	2.27	--	25.4	2.08
									--	33.8	2.07	--	41.9	1.98
									--	58.5	1.93	--	76.2	1.89
									--	102	1.85			
"	"	--	"	0.394	80	0.073	5.24	9.2	--	20.4	1.54	--	28.0	1.25
									--	40.0	1.16	--	52.0	1.13

Table 13 (cont.)

Ref.	Packing	Bed#	D_t,in.	D_p,in.	%void	S_t/S_p	w/m	A'	Run#	N	B*	Run#	N	L*
(1)	Glass ring	--	4^x	0.394	60	0.073	5.24	9.2	--	72.0	1.10	--	92.0	1.10
									--	122	1.07	--	152	1.08
"	" "	--	"	0.484	84.5	0.092	4.62	8.9	--	45.3	1.49	--	64.0	1.45
									--	65.5	1.39	--	113	1.32
									--	139	1.31	--	208	1.25
"	Raschig ring	--	6^x	1.00	72	0.147	5.35	10.1	--	91.4	1.05	--	162	0.786
									--	242	0.698	--	396	0.673
									--	527	0.678	--	635	0.674
(4)	Lead shot	--	0.705	0.0583	37.4	0.088	10.03	13.6	--	9.1	1.37	--	10.7	1.34
									--	12.9	1.34	--	20.2	1.44
"	" "	--	1.47	"	38.0	0.043	9.79	12.7	--	12.5	1.82	--	17.3	1.66
"	" "	--	2.07	"	37.5	0.030	10.00	"	--	11.5	1.89	--	25.0	1.91
"	" "	--	"	"	36.3	"	10.53	13.3	--	5.2	1.64			
"	" "	--	1.34	0.121	38.3	0.098	9.67	13.4	--	19.1	1.89	--	24.0	1.96
									--	27.4	2.03	--	29.5	2.04
									--	38.3	1.87	--	42.5	1.90
									--	46.7	1.86	--	49.8	1.81
									--	53.2	1.90			
"	" "	--	2.07	"	39.0	0.064	9.39	12.6	--	5.2	1.59			
"	" "	--	"	"	37.0	0.062	10.20	13.4	--	4.4	1.67			
"	" "	--	1.34	0.250	42.1	0.214	8.26	13.6	--	16.2	1.47	--	20.0	1.46
									--	26.3	1.37	--	30.0	1.30
									--	43.0	1.68	--	86.0	1.28

Table 13 (cont.)

Ref.	Packing	Rod#	D_t, in.	D_p, in.	%void	S_t/S_p	w/m	A'	Run#	N	f*	Run#	N	f*
(4)	Lead shot	--	3.00	0.250	39.7	0.090	9.10	12.7	--	5.30	1.03	--	8.1	1.03
									--	8.0	1.51	--	18.0	1.83
									--	23.4	1.74	--	30.5	1.70
"	"	--	"	"	37.5	0.087	10.00	13.6	--	8.9	1.75	--	14.4	1.84
									--	20.3	1.66	--	53.5	1.78
(11)	Celite Cylinder	--	4.026	0.267	36.1	0.075	9.65	13.0	--	112	1.89	--	121	1.88
									--	134	1.86	--	138	1.86
"	"	--	"	"	36.5	"	9.48	12.9	--	89.9	1.93	--	97.1	1.88
									--	103	1.85			
"	"	--	"	"	37.2	0.076	9.19	12.6	--	88.2	1.81	--	89.0	1.80
									--	93.4	1.80	--	98.6	1.79
"	"	--	"	"	45.5	0.088	6.55	10.3	--	193	1.08	--	221	1.07
									--	260	1.05	--	255	1.06
"	"	--	"	"	45.7	"	6.47	10.2	--	163	1.13	--	179	1.06
									--	197	1.06	--	200	1.07
"	"	--	"	"	46.1	0.089	6.37	"	--	159	1.08	--	170	1.08
									--	182	1.07			
"	Raschig ring	--	"	0.385	56.3	0.078	7.95	11.4	--	254	1.51	--	288	1.48
									--	316	1.48	--	360	1.47
"	"	--	"	"	55.8	0.077	8.12	11.5	--	216	1.50	--	240	1.50
									--	262	1.52	--	288	1.49
"	"	--	"	"	55.55	"	8.21	11.6	--	192	1.59	--	209	1.57
									--	224	1.57	--	241	1.56

Table 13 (cont.)

Ref.	Packing	Bed	Dt, in.	D_p, in.	% void	S_t/S_p	w/m	A'	Run#	N	B*	Run#	N	B*
(11)	Raschig ring	--	4.026	0.305	55.5	0.077	6.22	11.6	--	177	1.60	--	192	1.60
									--	208	1.58			
"	" "	--	"	"	55.45	"	6.23	"	--	112	1.64	--	122	1.68
									--	134	1.69	--	147	1.62
									--	156	1.63	--	162	1.62
									--	175	1.59	--	192	1.58
"	" "	--	"	"	61.35	0.069	6.44	10.2	--	328	0.937	--	414	0.938
									--	466	0.922			
"	" "	--	"	"	62.07	0.090	6.26	10.1	--	317	1.00	--	383	0.99
									--	431	0.98			
"	" "	--	"	"	62.13	"	6.25	"	--	278	0.920	--	321	0.920
									--	338	0.916	--	362	0.917
"	" "	--	"	"	62.3	0.091	6.19	"	--	186	0.97	--	214	0.95
									--	240	0.951	--	266	0.940
									--	299	0.926	--	346	0.926
"	Celite sphere	--	"	0.2166	37.85	0.058	9.85	13.0	--	109	1.94	--	127	1.82
									--	145	1.81			
"	" "	--	"	"	37.90	"	9.84	"	--	92.5	1.90	--	102	1.89
									--	108	1.88			
"	" "	--	"	"	37.75	"	9.89	"	--	79.7	1.98	--	85.0	1.97
									--	99.7	1.93			
"	" "	--	"	"	37.85	"	9.85	"	--	50.4	2.06	--	57.5	2.07
									--	64.0	2.04	--	67.7	2.03
									--	73.8	2.02	--	85.1	1.99

Table 13 (cont.)

Ref.	Packing	Bed#	D_t, in.	D_p, in.	%void	S_t/S_p	w/m	A'	Run#	N	B*	Run#	N	b*
(11)	Celite sphere	--	4.026	0.2166	46.90	0.068	6.60	10.3	--	170	1.47	--	204	1.46
"	" "	--	"	"	46.80	0.067	6.83	"	--	151	1.42	--	166	1.42
									--	193	1.40			
"	" "	--	"	"	46.90	0.068	6.60	"	--	140	1.41	--	155	1.40
									--	174	1.38			
"	" "	--	"	"	46.40	0.067	6.94	10.4	--	86.0	1.38	--	103	1.35
									--	113	1.34	--	123	1.34
									--	136	1.32	--	143	1.32
									--	160	1.31			
"	Berl saddle	--	"	0.5xxxx	72.05	0.113	4.93	9.4	--	517	0.672	--	585	0.666
									--	662	0.671	--	728	0.667
"	" "	--	"	"	71.33	0.110	5.10	9.5	--	453	0.918	--	547	0.907
									--	620	0.905			
"	" "	--	"	"	71.05	0.109	5.18	"	--	423	0.906	--	502	0.897
									--	525	0.897			
"	" "	--	"	"	71.25	0.110	5.12	"	--	264	0.932	--	362	0.918
									--	435	0.912	--	481	0.911
"	" "	--	"	"	76.30	0.133	3.95	8.9	--	625	0.666	--	725	0.658
									--	775	0.659	--	695	0.659
"	" "	--	"	"	76.35	"	"	"	--	610	0.664	--	725	0.645
									--	820	0.649			
"	" "	--	"	"	75.90	0.131	4.03	"	--	540	0.675	--	670	0.671
									--	764	0.666			

Table 13 (cont.)

Ref.	Packing	Bed	D_t, in.	D_p, in.	%void	S_t/S_p	w/m	A'	Run	N	P*	Run#	N	P*
(11)	Berl saddle	--	4.026	0.5xxx	76.15	0.132	[illegible]	8.9	--	401	0.665	--	461	0.661
									--	559	0.655	--	615	0.657
									--	675	0.653	--	738	0.654
(3)	Glass ball	--	2.067	0.2083	41.2	0.114	6.56	12.4	128	8.6	1.45	129	9.0	1.42
									130	9.4	1.41	66	9.8	1.67
									67	10.9	1.66	68	13.2	1.69
									69	15.6	1.61	70	17.6	1.49
									71	19.5	1.70	72	22.0	1.64
									73	25.4	1.64	74	29.7	1.59
									75	33.2	1.53	76	38.6	1.43
									77	44.0	1.36	78	49.6	1.39
									79	55.0	1.30	80	61.0	1.17
									81	69.4	1.26	82	74.4	1.25
									83	80.7	1.25	84	91.6	1.25
									85	103	1.53	86	115	1.17
									87	133	1.19	88	162	1.23
									89	165	1.25	90	206	1.28
									91	230	1.25	92	249	1.31
"	Smooth saddle	--	"	0.1316	93.1	0.041	5.35	8.9	76	10.5	1.27	77	12.3	1.31
									78	13.9	1.37	79	15.3	1.36
									80	17.2	1.25	81	20.8	1.26
									82	24.1	1.23	83	26.6	1.25
									84	31.0	1.24	85	35.1	1.16
									86	36.3	1.29	87	43.4	1.25
									88	49.4	1.33	89	52.5	1.26
									90	56.0	1.25	91	62.4	1.10
									92	72.8	0.97	93	77.4	1.08
									94	84.5	1.11	95	94.4	1.06
									96	105	1.02	97	130	0.97
									43	167	0.909	44	174	0.865
									45	189	0.891	46	234	0.959
									47	260	0.809	48	274	0.805

Table 13 (cont.)

Ref.	Packing	Bed"	D_t, in.	D_p, in.	%void	S_t/S_p	w/m	A'	Run"	N	B*	Run'	N	B*
(3)	Smooth saddle	--	2.067	0.1316	93.1	0.041	5.35	8.9	49	310	0.798	50	353	0.781
									51	389	0.769	52	428	0.801
									53	475	0.785	54	526	0.773
									55	599	0.750	56	672	0.728
									66	691	0.663	57	737	0.708
									62	737	0.769	58	779	0.722
									59	800	0.725			
"	Rough saddle	--	"	0.130	93.5	0.044	5.02	8.7	256	10.8	1.00	198	10.9	1.00
									257	12.4	1.06	258	14.0	1.04
									199	15.0	0.99	259	15.6	1.09
									260	17.2	1.13	200	18.5	1.01
									201	23.1	1.01	202	28.9	0.98
									203	34.9	0.92	204	40.7	0.99
									205	47.1	0.95	206	58.0	0.88
									207	67.7	0.89	208	78.9	0.840
									209	90.3	0.828	210	102	0.814
									211	112	0.812	212	126	0.802
									213	134	0.802	214	140	0.792
									215	154	0.781	216	174	0.744
									217	193	0.720	218	215	0.719
									219	240	0.694	228	242	0.434
									229	295	0.433	230	344	0.427
									231	394	0.643	232	460	0.615
									233	525	0.619	235	675	0.594
									236	736	0.598			
"	Perl saddle	--	6^x	1.00	72.5	0.113	6.33	10.4	119	12.3	0.56	118	13.8	0.59
									117	15.0	0.62	116	17.0	0.61
									83	19.9	0.58	115	20.5	0.65
									114	24.0	0.65	113	26.1	0.66
									112	27.6	0.67	84	30.4	0.627
									111	30.7	0.68	110	32.0	0.68
									109	33.9	0.686	85	35.6	0.613

Table 13 (cont.)

Ref.	Packing	Bed Dt, in.	Dp, in.	% void	St/Sa	a/m	Z'	Run	N	H*	Run	N	H*
(3)	Berl saddle	-- 6x	1.00	72.5	0.113	6.33	10.4	108	38.5	0.695	91	34.4	0.2[illegible]9
								86	39.4	0.622	93	41.9	0.4[illegible]7
								87	45.3	0.603	88	48.3	0.620
								94	51.0	0.445	89	53.6	0.632
								95	57.0	0.611	96	62.4	0.626
								97	66.3	0.614	154	70.0	0.660
								98	73.1	0.652	99	79.3	0.644
								100	86.3	0.655	101	94.3	0.659
								103	103	0.663	104	110	[illegible]
								153	110	0.675	105	115	0.6[illegible]3
								106	121	0.658	152	122	0.683
								107	126	0.655	151	135	0.691
								150	149	0.643	149	162	0.671
								148	177	0.5[illegible]	147	192	0.656
								146	212	0.661	145	232	0.6[illegible]
								144	244	0.662	143	260	0.6[illegible]1
								126	266	0.649	127	292	0.648
								128	313	0.642	129	325	0.634
								130	337	0.649	131	360	0.697
								132	416	0.654	133	456	0.632
								134	482	0.634	135	509	0.634
								136	560	0.639	137	586	0.647
								138	625	0.620	139	665	0.622
								140	710	0.611	141	784	0.614
								142	858	0.612			
"	Raschig ring	-- 6x	1.00xxx	70.7	0.121	6.36	10.5	99	8.5	0.89	53	9.0	0.97
								90	9.2	0.89	91	10.2	0.9[illegible]
								52	10.4	1.01	100	10.5	0.94
								79	10.6	0.99	92	11.1	0.95
								51	11.9	1.03	101	11.9	0.96
								57	12.2	0.92	93	12.5	0.97
								50	13.3	1.05	102	13.6	1.00
								49	14.5	1.06	94	14.6	1.02
								95	15.6	1.04	48	16.0	1.09

Table 13 (cont.)

Ref.	Packing	Bed#	D_t,in.	D_p,in.	%void	S_t/S_p	w/m	A'	Run#	N	B*	Run#	N	B*
(3)	Raschig ring	--	6^x	1.00^{xxx}	70.7	0.121	6.36	10.5	96	16.5	1.05	97	17.3	1.06
									47	17.5	1.08	98	18.4	1.07
									58	18.6	0.99	46	19.0	1.06
									59	19.6	1.02	78	20.4	1.03
									45	20.8	1.07	60	20.9	1.04
									44	21.9	1.08	20	22.1	1.05
									61	22.6	1.02	43	23.4	1.07
									42	24.2	1.08	77	24.5	1.07
									41	25.2	1.08	62	25.3	1.05
									40	26.2	1.08	63	26.4	0.82
									39	27.2	1.07	64	27.6	1.05
									38	28.1	1.09	65	28.6	1.07
									37	29.1	1.08	76	29.4	1.04
									36	29.7	1.08	66	30.4	1.07
									35	30.5	1.08	67	31.4	1.07
									21	35.1	1.09	75	35.5	1.09
									74	43.1	1.08	22	43.9	1.09
									73	49.3	1.07	23	52.0	1.06
									72	54.8	1.10	24	57.1	1.06
									71	59.4	1.06	70	64.3	1.08
									25	67.0	1.02	69	70.3	1.09
									26	70.9	1.02	27	76.6	1.03
									28	84.5	0.96	68	90.4	1.11
									29	88.8	1.00	6	93.0	0.98
									30	97.1	0.93	31	104	0.98
									32	111	0.99	33	120	0.97
									34	125	0.98	14	136	0.95
									7	143	0.96	15	156	0.97
									8	162	0.99	16	175	0.96
									9	182	0.98	17	195	1.03
									10	199	0.98	18	212	0.98
									11	220	0.97	19	231	0.99
									12	236	0.897	13	248	0.99
									1	249	1.00	2	280	0.96

Table 13 (concl.)

Ref.	Packing	Bed#	D_t, in.	D_p, in.	%void	S_t/S_p	w/m	A'	Run#	N	B^*	Run#	N	B^*
(3)	Raschig ring	--	6^x	1.00^{xxx}	70.7	0.121	6.36	10.5	3	313	0.99	4	345	0.965
									5	408	0.960			

x These values were assumed because evidence of the true value was not available.

xx The reported value for surface area of this bed of 288 sq. ft. per cu. ft. was found to disagree with the reported packing dimensions and voids or number of units packed. A value of 238 was estimated from packing and packed column measurements and was used for these calculations.

xxx The surface area of this unit was determined as indicated in Table 12.

xxxx Reported information indicates that the surface area of this unit was 310 sq. ft. per cu. ft. of packing. Manufacturers information, which amounts to 378 sq. ft. per cu. ft. of packing, was used for these estimations.

Table 14. Supplementary descriptions of packing materials.

Ref.	Packing	Description
(1)	Glass ring	diameter = 0.269 inch; length = 0.281 inch; thickness = 0.032 inch.
"	" "	diameter = 0.233 inch; length = 0.219 inch; thickness = 0.032 inch.
"	" "	diameter = 0.394 inch; length = 0.375 inch; thickness = 0.032 inch.
"	" "	diameter = 0.484 inch; length = 0.449 inch; thickness = 0.030 inch.
"	Clay raschig ring	diameter = 1.00 inch; length = 1.03 inch; thickness = 0.14 inch.
(3)	Smooth nickel saddle	diameter = 0.1316 inch; surface = 6.35×10^{-4} sq ft; volume = 7.84×10^{-8} cu ft.
"	Rough nickel packing	diameter = 0.0108 foot; surface = 6.35×10^{-4} sq ft; volume = 7.84×10^{-8} cu ft.
"	Clay Berl saddle	diameter = 1.00 inch; surface = 0.0343 sq ft; volume = 1.35×10^{-4} cu ft.
"	Clay Raschig ring	diameter = 1.00 inch; length = 1.00 inch; volume = 1.93×10^{-4} cu ft.
(5)	Hexagonal prism	diameter = 0.47 cm; length = 0.48 cm.
(11)	Celite cylinder	diameter = 0.267 inch; length = 0.344 inch.
"	Raschig ring	diameter = 0.385 inch; length = 0.397 inch; thickness = 0.0836 in.
"	Clay Berl saddle	diameter = 0.5 inch; surface = 378 sq ft per cu ft of packing.

Table 15. Calculated values for friction factors and Reynold's numbers.

Ref.	Packing	Run	F	Re	Run#	F	Re	Run#	F	Re
Aut.	Wire ring	A	2640	0.00039	B	665	0.00147	C	3070	0.00034
		1	801	0.00122	2	1560	0.00064	33	2220	0.00044
		42	1.94	2.24	41	1.63	2.74	40	1.47	3.08
		39	1.52	3.[illegible]	38	1.47	4.18	37	1.43	4.41
		28	0.82	5.93	27	0.80	6.79	26	0.82	7.61
		87	367	0.00256	88	721	0.00143	89	1230	0.00086
		78	2.34	2.36	77	1.80	3.69	76	1.75	3.81
		75	1.40	5.38	74	1.29	7.29	73	1.24	10.1
		72	1.19	12.6	71	1.25	13.5	62	1.32	5.20
		61	1.06	[illegible]	60	1.19	8.33	59	0.95	16.8
		58	0.98	22.8	57	0.96	27.8	131	319	0.00228
		132	531	0.00147	133	1300	0.00074	115	1.28	3.13
		114	1.23	4.19	113	1.20	5.61	112	1.09	7.50
		111	1.07	9.24	110	1.08	5.70	109	0.91	8.15
		108	0.89	12.6	107	0.82	10.5			
"	Sphere	F	395	0.00269	G	466	0.00222	H	665	0.00175
		I	1180	0.00094	7	293	0.00330	8	466	0.00202
		9	1230	0.00074	93	126	0.00830	94	331	0.00290
		95	724	0.00135	86	1.65	4.00	85	1.55	4.20
		84	1.46	5.28	83	1.59	6.93	82	1.94	6.75
		81	1.71	12.3	80	1.47	15.5	79	1.43	17.2
		18	1.01	7.65	17	0.96	9.10	16	0.89	11.5
		15	0.85	16.0	14	0.84	20.3	13	0.84	24.5
		12	0.81	30.6	10	0.81	31.3	11	0.79	33.8
		128	157	0.00610	129	306	0.00321	130	520	0.00189
		121	2.10	4.16	120	1.98	5.55	119	2.42	8.36
		118	1.88	11.9	117	1.61	17.3	116	1.54	24.1
		106	1.75	5.96	105	1.71	8.52	104	1.58	12.7
		103	1.72	1[illegible].0	102	1.47	28.0	101	1.50	37.2
		100	1.50	45.9	161	0.98	3.90	160	0.80	6.36
		159	0.86	10.4	13[illegible]	0.85	6.79	135	0.81	10.8
		134	0.70	13.9						
"	Berl saddle	D	296	0.00403	E	542	0.00216	4	122	0.00835
		5	216	0.00565	6	465	0.00240	51	2.40	5.70
		50	2.18	7.45	49	2.13	9.58	48	2.03	12.7
		47	2.14	16.1	46	2.41	19.9	45	2.37	23.7
		44	2.52	25.8	43	2.17	30.3	25	1.31	13.8
		24	1.32	18.4	23	1.29	23.0	22	1.25	32.2
		21	1.25	42.4	20	1.21	51.5	19	1.22	56.0
		90	199	0.00479	91	252	0.00381	92	342	0.00292
		70	0.88	6.35	69	0.91	8.90	68	0.86	10.6
		67	0.84	11.7	66	0.83	12.1	56	0.85	14.2
		55	0.83	15.3	54	0.80	19.4	53	0.76	23.5

Table 15 (cont.)

Ref.	Packing	Run	F	Re	Run	F	Re	Run	F	Re
Aut.	Berl saddle	52	0.71	26.6	152	0.73	7.60	151	0.67	10.1
		150	0.66	13.5	149	0.67	17.4	148	0.64	21.8
		147	0.66	15.0	146	0.60	22.2	145	0.60	30.0
		144	0.59	36.6	143	0.58	44.5			
"	Raschig ring	125	260	0.00460	126	409	0.00339	127	567	0.00244
		124	1.84	7.60	123	1.72	10.1	122	1.86	14.5
		99	1.59	11.3	98	1.61	14.8	97	1.54	21.6
		96	1.45	25.8	158	1.21	8.93	157	1.18	9.77
		156	1.28	13.0	155	1.45	16.2	154	1.47	21.7
		153	1.51	26.2	142	1.11	14.1	141	1.10	20.1
		140	1.13	24.8	139	0.87	33.8	138	1.21	42.0
		137	1.10	53.3	184	330	0.00402	185	246	0.00503
		186	265	0.00461	177	1.50	24.0	174	1.47	25.7
		176	1.46	29.9	175	1.43	30.3	196	290	0.00410
		197	267	0.00453	198	274	0.00460	193	1.44	23.4
		195	1.48	23.8	194	1.48	30.3	192	1.50	30.4
		189	404	0.00250	190	384	0.00260	191	2340	0.00042
		162	1.39	5.77	173	1.34	5.93	172	1.26	9.04
		163	1.22	9.69	171	1.21	13.4	164	1.17	13.9
		165	1.11	19.0	170	1.16	19.5	166	1.10	27.9
		169	1.10	27.8	168	1.08	34.5	167	1.09	35.2
"	Mixture	187	783	0.00149	188	745	0.00149	183	1.46	5.86
		182	1.35	10.5	181	1.32	13.1	180	1.28	18.5
		179	1.21	29.1	178	1.19	43.5			
(1)	Raschig ring	--	2.50	1.96	--	2.10	3.22	--	1.90	4.52
		--	1.78	6.13	--	1.68	8.40	--	1.56	13.2
		--	2.12	2.30	--	1.84	3.48	--	1.76	4.63
		--	1.64	5.74	--	1.56	8.01	--	1.50	10.4
		--	1.45	14.0	--	2.05	2.14	--	1.64	2.94
		--	1.44	4.20	--	1.36	5.45	--	1.28	7.55
		--	1.24	9.63	--	1.19	12.8	--	1.18	15.9
		--	2.12	4.08	--	1.99	5.76	--	1.86	7.70
		--	1.75	10.2	--	1.71	12.5	--	1.61	16.8
		--	1.54	6.82	--	1.12	12.1	--	0.95	18.0
		--	0.94	29.5	--	0.92	39.3	--	0.92	47.4
(4)	Sphere	--	31.5	0.0245	--	20.5	0.0415	--	9.35	0.103
		--	7.55	0.125	--	5.95	0.163	--	5.15	0.20
		--	4.4	0.23	--	3.85	0.275	--	3.15	0.355
		--	2.7	0.48	--	2.5	0.535	--	2.35	0.595
		--	2.1	0.69	--	2.0	0.775	--	1.95	0.855
		--	1.59	1.21	--	1.45	3.3	--	1.33	2.3
		--	1.17	2.68	--	19.5	0.046	--	13.6	0.073
		--	5.05	0.225	--	4.05	0.285	--	3.05	0.39

Table 15 (cont.)

Ref.	Packing	Run#	F	Re	Run#	F	Re	Run#	F	Re
(4)	Sphere	--	2.3	0.675	--	1.46	1.70	--	1.33	2.10
		--	1.12	2.91	--	11.0	0.087	--	7.8	0.120
		--	4.0	0.265	--	3.5	0.31	--	3.0	0.405
		--	1.74	1.06	--	1.34	1.94	--	1.08	4.2
		--	22.5	0.040	--	10.1	0.102	--	6.0	0.168
		--	3.2	0.375	--	2.0	0.81	--	1.78	0.92
		--	24	0.045	--	11.7	0.095	--	6.35	0.190
		--	2.7	0.685	--	2.25	0.86	--	1.55	2.4
		--	1.51	3.0	--	1.51	3.4	--	1.49	3.65
		--	1.33	4.75	--	1.33	5.3	--	1.29	5.8
		--	1.25	6.2	--	1.29	6.6	--	33.5	0.028
		--	11.5	0.091	--	6.0	0.197	--	3.5	0.35
		--	3.3	0.38	--	2.3	0.69	--	2.15	0.765
		--	12.5	0.081	--	8.55	0.131	--	6.35	0.178
		--	2.35	0.65	---	2.35	1.16	--	2.2	1.43
		--	1.94	1.88	--	1.80	2.15	--	2.05	3.05
		--	1.48	6.15	--	4.25	0.32	--	2.1	0.685
		--	1.58	1.05	--	1.90	1.03	--	1.55	2.35
		--	1.39	3.0	--	1.30	3.95	--	18.2	0.0625
		--	5.9	0.133	--	4.3	0.235	--	2.0	0.64
		--	1.82	0.89	--	1.47	1.46	--	1.25	2.35
		--	1.13	3.3	--	0.91	8.8			
(3)	"	148	3390	0.00029	153	1130	0.00077	158	708	0.00141
		163	461	0.00220	164	400	0.00255	175	128	0.00724
		180	84.4	0.0108	184	65.2	0.0140	189	52.5	0.0184
		27	35.2	0.0224	33	12.4	0.0735	38	8.00	0.122
		101	5.27	0.191	106	4.27	0.271	111	3.25	0.401
		116	2.72	0.555	121	2.36	0.714	126	2.11	0.915
		66	2.07	1.13	71	1.65	2.25	76	1.23	4.45
		81	1.03	8.05	86	0.90	13.3	91	0.92	26.6
		92	0.96	28.8						
"	Smooth saddle	136	10100	0.00014	141	1560	0.00063	146	965	0.00126
		151	540	0.00246	156	265	0.00465	161	98.9	0.0116
		171	52.7	0.0202	176	28.6	0.0397	181	17.4	0.0685
		186	10.3	0.120	191	7.80	0.169	196	5.25	0.264
		201	3.70	0.417	205	3.00	0.664	73	2.64	0.839
		212	2.50	1.00	76	1.95	1.28	81	1.56	2.53
		86	1.41	4.43	91	1.14	7.60	96	1.01	12.8
		46	0.92	28.5	51	0.75	47.4	56	0.68	82.0
		59	0.68	97.0						
"	Rough saddle	88	4690	0.00016	93	2300	0.00034	98	1380	0.00062
		103	1030	0.00094	108	520	0.00166	113	302	0.00276
		298	124	0.00692	303	50.6	0.0171	308	24.4	0.0362
		277	10.9	0.0779	282	6.10	0.151	288	3.37	0.296
		249	2.51	0.431	197	1.88	0.869	199	1.54	1.56
		202	1.26	3.01	208	0.94	8.21	213	0.85	14.0

Table 15 (cont.)

Ref.	Packing	Run#	F	Re	Run#	F	Re	Run#	F	Re
(3)	Rough saddle	21[illegible]	0.75	21.4	231	0.66	41.0	236	0.60	76.7
"	Berl saddle	21	755	0.00109	22	312	0.00212	19	238	0.00286
		24	216	0.00328	25	174	0.00403	7	149	0.00520
		27	104	0.00672	34	72.5	0.0104	35	47.6	0.0161
		5	34.8	0.0218	4	25.0	0.0310	60	1[illegible].8	0.0487
		70	9.50	0.0747	71	6.85	0.109	76	4.55	0.182
		124	2.42	0.472	82	1.90	0.706	117	1.41	1.34
		113	1.14	2.32	109	1.07	3.02	93	0.72	4.00
		95	0.86	5.06	99	0.83	7.05	153	0.83	9.80
		151	0.83	12.0	146	0.76	16.9	127	0.74	26.0
		132	0.73	37.0	137	0.72	52.3	142	0.67	76.4
"	Raschig ring	167	514	0.00147	157	224	0.00429	159	125	0.00746
		103	54.4	0.0180	104	23.8	0.0356	165	18.2	0.0491
		105	12.1	0.0690	140	9.25	0.0985	141	7.24	0.132
		142	6.03	0.166	122	5.68	0.182	82	4.57	0.232
		114	4.15	0.286	117	3.65	0.344	120	3.43	0.392
		55	2.41	0.655	90	2.10	0.845	92	1.97	1.01
		50	1.91	1.22	46	1.81	1.46	58	1.61	1.70
		60	1.60	1.91	42	1.56	2.21	63	1.26	2.40
		37	1.49	2.65	67	1.45	2.86	73	1.33	4.50
		70	1.28	5.87	28	1.12	7.70	31	1.12	9.47
		7	1.07	13.1	17	1.12	17.8	12	0.98	21.6
		4	1.03	31.5						
(11)	Cylinder	1	1.09	16.0	2	1.08	17.2	3	1.06	19.1
		4	1.05	19.7	5	1.14	12.7	6	1.10	13.7
		7	1.09	14.6	8	1.11	12.3	9	1.09	12.4
		10	1.10	13.0	11	1.09	13.6	12	0.90	23.6
		13	0.89	27.0	14	0.87	31.9	15	0.87	31.2
		16	0.96	19.8	17	0.90	21.8	18	0.90	24.0
		19	0.90	24.3	20	0.94	19.1	21	0.94	20.4
		22	0.93	21.6						
"	Raschig ring	1	1.12	31.0	2	1.09	35.2	3	1.09	38.5
		4	1.06	44.0	5	1.09	28.7	6	1.09	29.6
		7	1.10	32.4	8	1.08	35.6	9	1.14	23.8
		10	1.13	21.0	11	1.12	27.8	12	1.12	30.0
		13	1.16	22.0	14	1.15	23.8	15	1.14	25.6
		16	1.21	13.9	17	1.24	15.2	18	1.22	16.6
		19	1.19	18.3	20	1.16	19.4	21	1.17	20.1
		22	1.15	21.0	23	1.14	23.0	24	0.86	36.3
		25	0.85	45.9	26	0.84	51.6	27	0.94	34.6
		28	0.93	41.8	29	0.91	47.0	30	0.88	30.0
		31	0.87	34.6	32	0.87	36.5	33	0.86	41.2
		34	0.95	19.9	35	0.93	22.9	36	0.92	25.7
		37	0.91	28.5	38	0.89	32.0	39	0.88	37.3

Table 1[illegible] (concl.)

Ref.	Packing	Run	F	Re	Run	F	Re	Run	F	Re
(11)	Berl saddle	1	1.09	[illegible].0	2	1.06	51.0	3	1.08	57.6
		4	1.07	63.2	5	1.11	40.5	6	1.09	46.9
		7	1.09	55.4	8	1.06	36.4	9	1.06	45.[illegible]
		10	1.06	47.7	11	1.12	25.4	12	1.11	32.3
		13	1.10	36.8	14	1.09	43.0	15	1.04	46.0
		16	1.02	[illegible]3.3	17	1.02	57.0	18	1.02	65.9
		19	1.04	4[illegible].9	20	1.00	53.4	21	1.01	60.3
		22	1.03	43.6	23	1.02	50.4	24	1.01	57.4
		25	1.04	29.7	26	1.03	35.6	27	1.02	41.5
		28	1.02	45.5	29	1.01	50.0	30	1.01	54.7
"	Sphere	1	1.04	16.4	2	0.97	19.4	3	0.96	22.1
		4	1.03	14.1	5	1.02	15.6	6	1.01	16.5
		7	1.06	12.2	8	1.07	13.0	9	1.04	15.3
		10	1.17	7.69	11	1.16	8.77	12	1.13	9.75
		13	1.12	10.3	14	1.11	11.2	15	1.06	13.0
		16	1.12	22.6	17	1.10	27.1	18	1.09	20.1
		19	1.08	22.1	20	1.06	25.7	21	1.06	16.6
		22	1.07	20.6	23	1.05	25.2	24	1.07	11.6
		25	1.04	13.9	26	1.02	15.2	27	1.01	16.5
		28	1.00	16.3	29	0.99	19.2	30	0.99	21.5

The information in this table was calculated by using the predicted values of A and B. Thus the accuracy for predicting pressure drop can be ascertained by comparing F to (1 + 1/Re).

-- Information lacking run numbers is treated in the same order as it appears in Tables 12 or 13.

Table 16. Pressure drop through nine regularly spaced one-fourth inch orifices in a three-fourth inch pipe.

Run #	Spacing, orif.diams.	ΔH, inches	Flow, gm/sec	E_o^*
1	4.94	20.50	36.9	0.417
2	"	20.12	36.1	0.425
3	"	26.94	42.5	0.412
4	"	26.62	42.1	0.414
5	0.875	26.75	72.1	0.1422
6	"	26.25	69.8	0.1487
7	"	21.44	63.9	0.1446
8	"	20.94	62.8	0.1468

The flow system is described in Plate XI.

* $E_o^* = g\Delta H/9U^2$, all terms in consistent units and U based on the area of the orifice.

EXPLANATION OF PLATE XI

A. General layout, one eighth inch = one inch.

a. Water level in the reservoir.
b. Thirty gallon tank open to the atmosphere.
c. 1" outlet from tank.
d. Internal adaption to 1 1/4" pipe.
e. 1 1/4" gate valve, wide open for all runs.
f. 1 1/4" close nipple.
g. 1 1/4" elbow.
h. 1 1/4" close nipple.
i. 1 1/4" x 1" reducing coupling.
j. 1" pipe 28 1/2" in length exiting to the atmosphere.
ΔH. Elevation representing energy lost in the orifices.

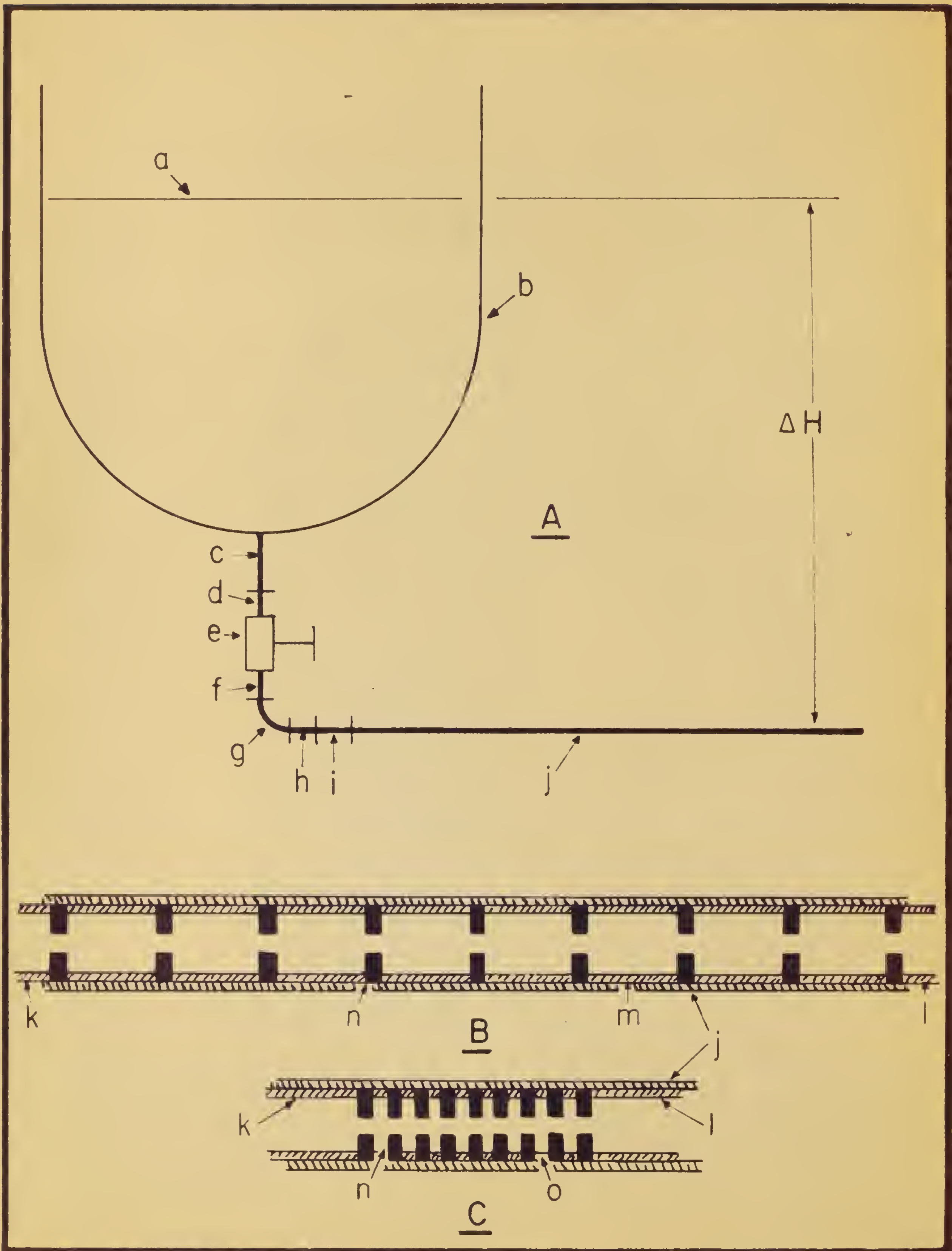
B. The dispersed arrangement for the orifices, one-half inch = one inch.

k. Upstream insert of 3/4" pipe, 13 11/16" long.
l. Downstream insert of 3/4" pipe, 10 1/2" long.
m. Long spacer made of 3/4" pipe, 1 15/64" long.
n. Orifice made of washer 1" o.d. x 1/4" i.d. x 3/16" thick.

C. The close arrangement for the orifices, one-half inch = one inch.

o. Short spacer made of 3/4" pipe, 7/32" long.

PLATE XI



PRESSURE DROP FOR SINGLE PHASE
FLOW THROUGH PACKED BEDS

by

ROBERT HAMBLETT CROWTHER

B. Ch. E., Fenn College, 1950

AN ABSTRACT OF A THESIS

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Pressure drop in laminar or turbulent flow is correlated by considering the surface of the packing material, the porosity of the packed bed, the fluid variables, the size of the confining column, and the degree to which different packing materials obstruct flow. The effect of each of these factors is determined according to a simple mathematical or graphical solution.

Experimental results of seven investigators, including the author, are used to support the conclusions. These results represent seventy-five beds packed with eight different types of packing materials and a mixture of four of them. The correlation is of such a nature that it may be extended to novel packing units.

The observed accuracy of predicting pressure drop is 15 percent for laminar flow and 25 percent for turbulent flow. Accuracy is intermediate to these figures for transition flow.

Determination of surface area of porous media by permeability is discussed. Calculations for surface area are outlined. The maximum error to be encountered in determining surface area is estimated to lie within zero to 2 percent in cases where accurate measurements of porosity, pressure loss, and flow rate may be made, and where the approximate shape of the granules is known.

Auxiliary investigations were concerned with the appearance of the cross-section of a packed bed and the effect of spacing upon pressure loss through a series of orifices.