AUTOMATIC GRAIN CLASSIFICATION CONSIDERATIONS

by

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TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

1.1 Introductory Remarks

All forms of grain sample analysis include a determination of the amount of material in the sample which is not the grain being analysed. The methods which are presently employed make use of various sieves of appropriate sizes, followed by a manual hand-picking of the remainder. The hand-picking process is carried out separately since it is both tedious and time consuming. To this end, it is desirable to automate the hand-picking part of the analysis by an appropriate scheme. Such a scheme should possess the capability of distinguishing between a variety of grains which includes wheat, barley, oats, rye, soybeans and milo.

The purpose of this report is two-fold. First it presents a review of the pertinent literature. Second, it extends an initial feasibility study which was reported recently by Vyas*. The approach entertained by Vyas was based on pattern recognition techniques. The types of grains considered were:

(1) corn, (2) wheat, (3) barley, (4) oats and (5) milo.

In this study, two additional types of grain namely rye and soybeans are also considered. Again, the study by Vyas was restricted to linear classifiers. In this report, quadratic classifiers are also considered.

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*A. H. Vyas, "A Pattern Recognition Approach to Grain Sample Analysis", Master of Science report, Electrical Engineering Department, Kansas State University, Manhattan, Kansas.

1.2 General Remarks Pertaining to Pattern Recognition Problems

The basic ideas associated with pattern recognition problems are best introduced by referring to Figure 1-1.

The signal acquisition stage acquires a set of signals from each of the classes which are to be classified. The signals acquired may be one-dimensional or multi-dimensional.

The output of the signal acquisition stage is denoted by C_i, $i=1,2,\ldots,K$, where C_i represents the i^{th} class, each of which consists of N_i signals. The j^{th} signal belonging to class i is denoted by x_{ij}. Thus for each i, j varies from 1 to N_i.

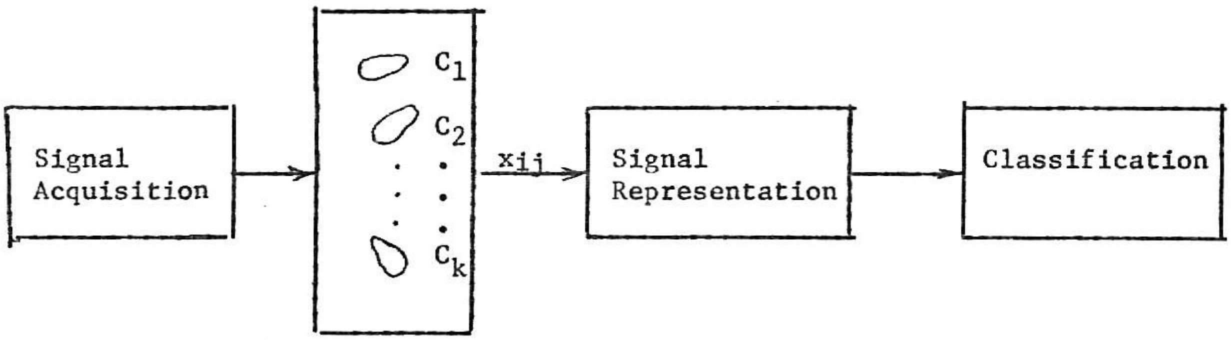

Figure 1.1 Block diagrams representation of a pattern recognition problem.

Consider a typical signal x_{ij} which is fed into a signal representation stage as shown in Figure 1-1. The role of the signal representation stage is to seek some "measurements", "features" or "attributes" which will help discriminate a signal x_{lj} from another signal x_{mj}, where $l \neq m$. The output of this stage corresponding to the input x_{ij} is denoted by P_{ij} which may be in the form of a finite n-dimensional vector or a finite multi-dimensional array. This P_{ij} is generally referred to as a pattern corresponding to the signal x_{ij}.

The classification stage in Figure 1-1 is a device which is trained to recognize a set of patterns $\{P_{ij}\}$. Consequently the set of patterns $\{P_{ij}\}$ whose classification is a known apriori is referred to as the training set. Once the classifier has been trained using the training set, it is conceivable that it will make errors while classifying patterns not belonging to the training set. There is a large number of training procedures available in the literature*. The procedure best suited for a specific application is generally dictated by the nature of the signal representation stage.

In conclusion, it is remarked that although the signal representation stage in Figure 1-1 plays a crucial role with respect to the overall system complexity and performance, very little theory is available which enables one to select the "best" measurements or features to represent the set of signals $\{x_{ij}\}$. Feature selection techniques vary from one pattern recognition problem to another. Some aspects of signal representation for the problem are briefly considered in the following section.

1.3 Signal Representation for Grain Classification

The signal representation technique used in this study is simple in that it concerns only the size and shape of a given kernel. Consider a black and white image of a grain kernel as shown in Figure 1-2.

The image frame in Figure 1-2 is devided into an (32x32) array. Again, each element of this array which contains the kernel is represented by a

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*For an excellent summary, see "A Survey of Pattern Recognition", by W. G. Wee, IEEE Proc. of the Seventh Symposium on Adaptive Processing, December 1968, pp. 2-E-13.

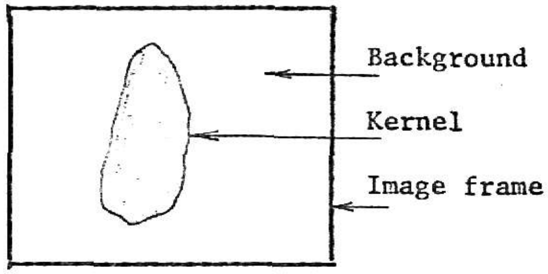

Figure 1-2 Black and White Image of a Kernel

"one" while the other elements are represented by "zeroes". An example of the coded image which results by this process is shown in Figure 1-3. Information pertaining to the size and shape of this image frame is extracted by means of a two-dimensional spectral analysis. The transform used for spectral analysis is analogous to the familiar Discrete Fourier Transform and is called the Walsh Hadamard or BIFORE (Binary Fourier Representation) transform. A brief introduction concerned with two-dimensional BIFORE transform is provided in Chapter III.

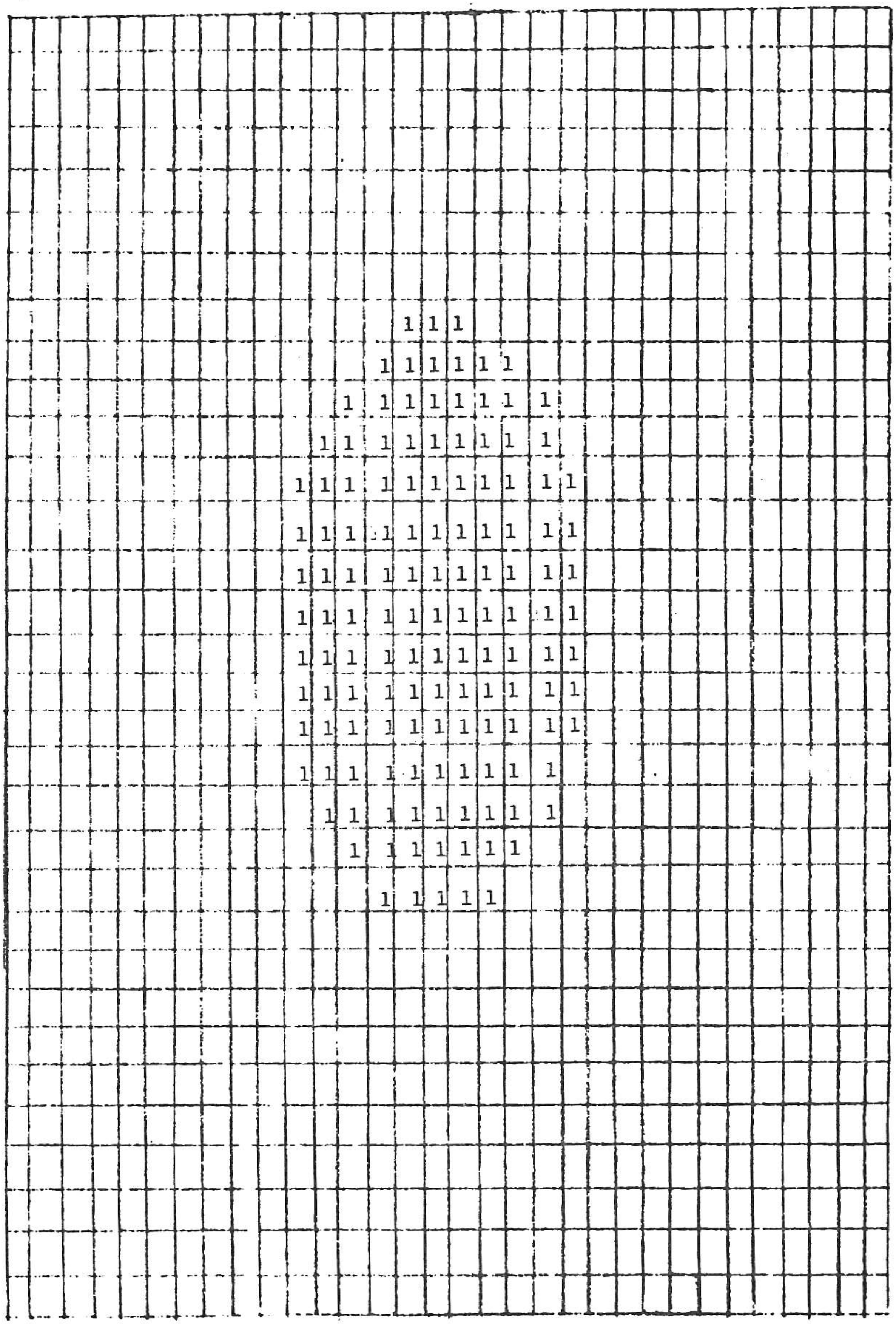

Figure 1-3. Typical output from microfilm reader.

(Blank elements of the array consist of zeros).

CHAPTER II

REVIEW OF THE LITERATURE

2.1 Introduction

A review of the literature published in the past decade shows that the only research effort besides that by Vyas [9] is one which was initiated in 1971 by Edison and Brogan [1]. This study is in progress at the present time. Two progress reports have been written, a summary of which is presented in what follows.

2.2 The Edison - Brogan Approach [1]

Data Acquisition

Basically, six categories or classes C_j, $j=1,2,....,6$ of grain were considered. These were the following:

(1) Corn

(2) Wheat

(3) Soybeans

(4) Oats

(5) Barley

(6) Rye

Physical measurements of length, width, height and projected area were made for 1000 kernels each of Wheat, Oats, Soybeans, Corn, Barley and Rye. Ten varieties of each kind were included except for Rye where only seven varieties were used as shown in Table 2-1.

A semi-automatic measuring system was assembled to obtain size data on grain samples. The system was utilized to measure the length, width, height

Table 2-1

List of Grain Varieties

Corn	Oats	Barley
Ia 4542 LR	Kata	Princes
Nebr 807	Jaycee	Dixon
NC + 50 LF	Pettis	Primas II
Nebr 501 D	Santee	Larker
Nebr 708	Neal	Otis
Nebr 611	Burnett	Trebi
NC + 53 MR	Garry	Liberty
NC + 11 SC MR	MO - 205	Kearney
NC + GO LF	Russell	Chase
Nebr 808	Garland	Custer

Soybeans	Wheat	Rye
Clark 63	Trapper	Elbon
Hawkeye	Trader	Cougar
SRF 300	Gage	Dakold
Kent	Scout 66	Pearl
Buson	Guide	Elbon
Cutler	Chanute	Frontier
Corsay	Lancer	Elk
Amsay	Scoutland	
Calland	Centurk	
Wayne	Satanta	

and projected area of a kernel and to record this data on punched paper tape for computer processing. This recording of the data was done using a analog-to-digital convertor by sampling a d.c. voltage which was proportional to the measurement. The sampled value was recorded on punched paper tape under control of a foot-actuated switch. This measuring process made it possible to obtain area and dimensional data on 100 kernels in less than 1 hour. The accuracy of the area measurement is ±0.003 square inches and for the linear measurements is about ±0.006 inches.

Several statistics were computed. Those that are pertinent to the present study are listed in Appendix 2-1.

Data Utilization

Three parameters, namely area, the ratio length / width and the ratio length / depth were used. This identification algorithm utilised a learning technique. The algorithm was based upon the assumption that all grains do not have equal probability of occurring in a particular sample. A "learning parameter" α was defined which changes the probability of occurrence of each grain as more information is gathered during a given run.

Let A, L, D, and W denote the area, length, depth and width of a kernel. Again, let the parameters A, L/D and L/W be denoted by a 3-dimensional measurement vector $\underline{m}$, where

$$\underline{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} \qquad (2\text{-}1)$$

where m_1=A, m_2=L/D and m_3=L/W. The mean values of $\underline{m}$ for each of the six categories C_j, j=1,2,...,6 are designated q_i, i=1,2,...,6.

Clearly, $\underline{m}$ in (2-1) can be looked upon as a random vector. It was assumed that the probability function $P(\underline{m}/C_j)$ associated with category C_j was Gaussian. That is

$$P(\underline{m}/C_j) = K\, e^{-\frac{1}{2}\, \underline{e}_j^T Q_j^{-1} \underline{e}_j}, \quad j=1,2,\ldots,6 \qquad (2\text{-}2)$$

where $K = \dfrac{1}{(2\pi)^{3/2}|Q_j|^{1/2}}$

$\underline{e}_j = \underline{m} - q_j$ denotes the transpose of $\underline{e}_j$

and

Q_j is the (3x3) covariance matrix of category C_j while $|Q_j|$ denotes its determinant.

The covariance matrix Q_j in (2-2) was estimated using the experimental data collected in the data acquisition stage.

When all classes are equally likely to be present in the sample population maximizing this conditional probability function is equivalent to maximizing the probability that random measurement $\underline{m}$ really came from category C_j. However, the occurrence of a given category is also a random phenomenon, with a discrete probability distribution. If each of the six categories is equally likely, then $P(C_j) = 1/6$. This may be an appropriate assumption initially, but as more and more samples are investigated, additional knowledge is accumulated. For example, it may become apparent, after observing 10 straight samples of Rye, that the probability of occurrence of Rye is much higher than 1/6 for that particular population. What one should do is assign each sample to the category which results in the maximum probability that the random variable $\underline{m}$ belongs to category C_j, given all accumulated information. That is one must maximize

$$P(\underline{m},\ C_j|\underline{m}) = P(C_j|\underline{m}) \tag{2-3}$$

where P(E) denotes the probability of the event E.

Using Baye's rule, $P(\underline{m},\ C_j)$ can be written as

$$P(C_j,\ \underline{m}) = P(C_j|\underline{m})\ P(\underline{m}) = P(\underline{m}|C_j)\ P(C_j) \tag{2-4}$$

Thus

$$P(C_j|\underline{m}) = \frac{P(\underline{m}|C_j)\ P(C_j)}{P(\underline{m})} \tag{2-5}$$

where

$$P(\underline{m}) = \sum_{i=1}^{6} P(C_i|\underline{m}) = \sum_{i=1}^{6} P(\underline{m}|C_i)\ P(C_i)$$

Finally

$$P(C_j|\underline{m}) = \frac{P(\underline{m}|C_j)\ P(C_j)}{\sum_{i=1}^{6} P(\underline{m}|C_i)\ P(C_i)} \tag{2-6}$$

where

$P(\underline{m}|C_j)$ is given by (2-2).

Thus using the measurement vector $\underline{m}$ corresponding to a kernel which was to be classified as belonging to one of the categories C_j, $j=1,2,\ldots,6$, the decision process was carried out sequentially as follows:

(1) Assign apriori probabilities to each category i.e $P(C_j) = 1/6$ for example.

(2) Obtain measurements of $\underline{m}$ for a given sample and compute $P(\underline{m}|C_j)$ for $j=1,2,\ldots,6$ using Eq. (2-2).

(3) Compute the total probability for this particular $\underline{m}$ by summing over all six categories,

$$\sum_{i=1}^{6} P(\underline{m}|C_i)\ P(C_i)$$

(4) Form the six ratios $P(C_j|\underline{m})$ according to Eq. (2-6).

(5) Assign $\underline{m}$ to that category j for which $P(C_j|\underline{m})$ is largest.

(6) Based on the current sample $\underline{m}$, the a posteriori probability that C_j is present in the population is, by definition, the quantity computed in Eq. (2-6). However, the new $P(C_j)$ to be used with the next sample, is taken as a weighted average of the old probability (based on apriori estimates and all past data) and the posteriori probability.

$$P(C_j)_{new} = \alpha\ P(C_j)_{old} + (1-\alpha)\ P(C_j|\underline{m}) \tag{2-7}$$

The parameter α is a "learning parameter". If α=0, all the weight is placed on the most recent estimate. If α=1, all the weight is placed on apriori estimate and no learning occurs. The value α=1 corresponds to the previously reported results and an accuracy of 80 to 85% was obtained.

The above scheme is being investigated. For example, by resetting all categories to be equally likely after each group of 50 samples, and using α=0.5, the results in Table 2.1 were obtained.

Table 2.2

Category	errors | no. of samples
1. corn	0 | 150
2. wheat	36 | 1000
3. soybeans	0 | 200
4. oats	2 | 1000
5. barley	93 | 1000
6. rye	15 | 850

With respect to the Table 2-2, it is reported that in the wheat category, 31 of 36 errors occurred on 100 samples of Scout 66 wheat and 50 samples of Trader wheat. Again, in the barley category, 72 of the 93 errors occurred on 100 samples of Trebi barley. These results imply that it is extremely difficult to separate certain types of wheat and barley from other types of grain.

The above summary was obtained from two progress reports [1] pertaining to a study which is being continued. However, no major changes in the identification scheme is expected. An attempt will be made to make the "best value" for the learning parameter α in (2-7). Some optical data having to do with reflectance properties of different types of grain will be incorporated into the identification process, if it proves to be useful in reducing errors.

APPENDIX 2-1

Summary For 1000 Kernels of Corn

	Length	Width	Depth	Area	L/W	L/D
Mean	.47296	.32082	.20403	.11474	1.4924	2.4288
Standard Deviation	.061624	.030223	.040493	.015850	.27507	.64476

Covariance Matrix

	Length	Width	Depth	Area
Length	$.37975\text{x}10^{-2}$	$-.68995\text{x}10^{-3}$	$-.13501\text{x}10^{-2}$	$.64562\text{x}10^{-3}$
Width	$-.68995\text{x}10^{-3}$	$.91349\text{x}10^{-3}$	$.11001\text{x}10^{-3}$	$.10482\text{x}10^{-3}$
Depth	$-.13501\text{x}10^{-2}$	$.11001\text{x}10^{-3}$	$.16396\text{x}10^{-2}$	$-.32088\text{x}10^{-3}$
Area	$.64562\text{x}10^{-3}$	$.10482\text{x}10^{-3}$	$-.32088\text{x}10^{-3}$	$.25122\text{x}10^{-3}$

Summary For 1000 Kernels of Wheat

	Length	Width	Depth	Area	L/W	L/D
Mean	.23654	.10991	.099534	.018779	2.1736	2.4012
Standard Deviation	.01784	.013054	.011926	.0026173	.24188	.27427

Covariance Matrix

	Length	Width	Depth	Area
Length	$.31834\text{x}10^{-3}$	$.94983\text{x}10^{-4}$	$.76844\text{x}10^{-4}$	$.31225\text{x}10^{-4}$
Width	$.94983\text{x}10^{-4}$	$.17041\text{x}10^{-3}$	$.91380\text{x}10^{-4}$	$.25208\text{x}10^{-4}$
Depth	$.76844\text{x}10^{-4}$	$.91380\text{x}10^{-4}$	$.14223\text{x}10^{-3}$	$.17609\text{x}10^{-4}$
Area	$.31225\text{x}10^{-4}$	$.25208\text{x}10^{-4}$	$.17609\text{x}10^{-4}$	$.68503\text{x}10^{-5}$

APPENDIX 2-1 (continued)

Summary For 1000 Kernels of Soybeans

	Length	Width	Depth	Area	L/W	L/D
Mean	.28732	.25276	.21151	.051916	1.1402	1.3650
Standard Deviation	.027126	.021707	.020077	.0077072	.10834	.14362

Covariance Matrix

	Length	Width	Depth	Area
Length	$.73587\times10^{-3}$	$.24517\times10^{-3}$	$.21014\times10^{-3}$	$.16373\times10^{-3}$
Width	$.24517\times10^{-3}$	$.47121\times10^{-3}$	$.28920\times10^{-3}$	$.12070\times10^{-3}$
Depth	$.21014\times10^{-3}$	$.28920\times10^{-3}$	$.40311\times10^{-3}$	$.96131\times10^{-4}$
Area	$.16373\times10^{-3}$	$.12070\times10^{-3}$	$.96131\times10^{-4}$	$.59401\times10^{-4}$

Summary for 1000 Kernels of Oats

	Length	Width	Depth	Area	L/W	L/D
Mean	.42686	.10456	.079946	.031001	4.1254	5.4582
Standard Deviation	.066858	.014559	.013060	.0069298	.69964	1.1862

Covariance Matrix

	Length	Width	Depth	Area
Length	$.44700\times10^{-2}$	$.37456\times10^{-3}$	$.18833\times10^{-3}$	$.35167\times10^{-3}$
Width	$.37456\times10^{-3}$	$.21197\times10^{-3}$	$.10991\times10^{-3}$	$.75719\times10^{-4}$
Depth	$.18833\times10^{-3}$	$.10991\times10^{-3}$	$.17058\times10^{-3}$	$.41528\times10^{-4}$
Area	$.35167\times10^{-3}$	$.75719\times10^{-4}$	$.41528\times10^{-4}$	$.48022\times10^{-4}$

APPENDIX 2-1 (continued)

Summary for 1000 Kernels of Barley

	Length	Width	Depth	Area	L/W	L/D
Mean	.34533	.12407	.098683	.028413	2.8255	8.5730
Standard Deviation	.048295	.015670	.015548	.0046847	.53775	.69878

Covariance Matrix

	Length	Width	Depth	Area
Length	$.23324 \times 10^{-2}$	$.10932 \times 10^{-3}$	$.87966 \times 10^{-4}$	$.16251 \times 10^{-3}$
Width	$.10932 \times 10^{-3}$	$.24556 \times 10^{-3}$	$.31626 \times 10^{-4}$	$.37646 \times 10^{-4}$
Depth	$.87966 \times 10^{-4}$	$.31626 \times 10^{-4}$	$.24174 \times 10^{-3}$	$.21240 \times 10^{-4}$
Area	$.16251 \times 10^{-3}$	$.37646 \times 10^{-4}$	$.21240 \times 10^{-4}$	$.21946 \times 10^{-4}$

Summary For 1000 Kernels of Rye

	Length	Width	Depth	Area	L/W	L/D
Mean	.26217	.086631	.082740	.017688	3.0533	3.7997
Standard Deviation	.028599	.010500	.010051	.0028382	.37982	.42519

Covariance Matrix

	Length	Width	Depth	Area
Length	$.81794 \times 10^{-3}$	$.11483 \times 10^{-3}$	$.93622 \times 10^{-4}$	$.62497 \times 10^{-4}$
Width	$.11483 \times 10^{-3}$	$.11026 \times 10^{-3}$	$.62282 \times 10^{-4}$	$.20322 \times 10^{-4}$
Depth	$.93622 \times 10^{-4}$	$.62282 \times 10^{-4}$	$.10103 \times 10^{-3}$	$.15757 \times 10^{-4}$
Area	$.62497 \times 10^{-4}$	$.20322 \times 10^{-4}$	$.15757 \times 10^{-4}$	$.80557 \times 10^{-5}$

CHAPTER III

THE TWO DIMENSIONAL BIFORE TRANSFORM

3.1 Definition

The shape and size of a kernel are used as criteria to distinguish it from a kernel belonging to a different type of grain. Each kernel is coded in the form of a (32x32) array of zeros and ones and can be represented by a matrix as follows:

$$\left[\underline{f}(x_1,x_2)\right] = \begin{bmatrix} f(0,0) & f(0,1) & \cdots\cdots\cdots & f(0,N_2-1) \\ f(1,0) & f(1,1) & \cdots\cdots\cdots & f(1,N_2-1) \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \\ f(N_1-1,0) & f(N_1-1,1) & \cdots\cdots\cdots & f(N_1-1,N_2-1) \end{bmatrix} \quad (3\text{-}1)$$

where [1] $N_1=N_2=32$ and each of the $f(i,j)$ is a zero or a one[2]. Then the two dimensional BIFORE transform (2-BT) of the data matrix $\underline{f}(x_1,x_2)$ in (3-1) is defined as

$$F(u_1,u_2) = \frac{1}{N_1N_2} \sum_{x_1=0}^{N_1-1} \sum_{x_2=0}^{N_2-1} f(x_1,x_2)(-1)^{<x,u>} \quad (3\text{-}2)$$

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1. Note that N_1 need not be equal to N_2

2. In general $f(i,j)$ can be any finite real number

where

$F(u_1,u_2)$ is a transform coefficient

$f(x_1,x_2)$ is an input data point

$$u_i = 0,1,\ldots,(N_i-1);i=1,2 \tag{3-3}$$

$$<x_i,u_i>= \sum_{m=0}^{n_i-1} u_i(m)x_i(m)$$

$$<x,u> = <x_1,u_1> + <x_2,u_2>$$

and

$$n_i = \log_2 N_i, i=1,2.$$

The terms $u_i(m)$ and $x_i(m)$ in (3-3) are the binary representations of u_i and x_i respectively. For example,

$$[u_i]_{\text{decimal}} = [u_i(k_i-1),\ u_i(k_i-2),\ldots,\ u_i\ (1),\ u_i(0)]_{\text{binary}} \tag{3-4}$$

where

$$u_i\ (\ ) \in \{0,1\}.$$

Alternately, (3-2) can be written in the form of a matrix to obtain

$$\left[\underline{F}(u_1,u_2)\right] = \frac{1}{N_1N_2} \left[H(n_1)\right] \left[\underline{f}(x_1,x_2)\right] \left[H(n_2)\right] \tag{3-5}$$

where

$\left[\underline{F}(u_1,u_2)\right]$ is a $(N_1 x N_2)$ transform matrix corresponding to the data matrix $\left[\underline{f}(x_1,x_2)\right]$

$\left[H(n_1)\right]$ and $\left[H(n_2)\right]$ are $(N_1 x N_1)$ and $(N_2 x N_2)$ Hadamard matrices with $n_i = \log_2 N_i$, $i=1, 2$.

The Hadamard matrices in (3-5) are defined by the recurrence relation

$$H(k+1) = \left[\begin{array}{c|c} H(k) & H(k) \\ \hline H(k) & -H(k) \end{array}\right] \qquad k=0,1,\ldots,n_i \tag{3-6}$$

$$H(0) = 1$$

From (3-6) it follows that the elements of a Hadamard matrix are either +1 or -1.

Using the fact that the 2-BT is an orthogonal transform, it can be shown that (3-7) the corresponding inverse transform (2-IBT) is defined as

$$f(x_1,x_2)= \sum_{u_1=0}^{N_1-1} \sum_{u_2=0}^{N_2-1} F(u_1,u_2)\,(-1)^{<x,u>} \qquad (3\text{-}7)$$

or alternately as

$$\left[\underline{f}(x_1,x_2)\right] = \left[H(n_1)\right]\left[\underline{F}(u_1,u_2)\right]\left[H(n_2)\right] \qquad (3\text{-}8)$$

3.2 The 2-BT Power Spectrum

A BIFORE power spectrum which is analogous to a two-dimensional Fourier power spectrum can be defined as

$$P(z_1,z_2) = \sum_{u_1=\left[2^{z_1-1}\right]}^{2^{z_1}-1} \sum_{u_2=\left[2^{z_2-1}\right]}^{2^{z_2}-1} F^2(u_1,u_2) \qquad (3\text{-}9)$$

where

$z_i=0,\ 1,\ \ldots.,\ k_i;\ n_i = \log_2 N_i,\quad i=1,2.$

and

$\left[2^{z_i-1}\right]$ is the integer part of 2^{z_i-1}.

From (3-9) it follows that the 2-BT power spectrum can be expressed in matrix form to obtain

$$\left[\underline{P}(k_1,k_2)\right] = \begin{bmatrix} P(0,0) & P(0,1) \;.\;.\;.\;. & P(0,n_2) \\ P(1,0) & P(1,1) \;.\;.\;.\;. & P(1,n_2) \\ & .\;.\;.\;. & \\ P(n_1,0) & P(n_1,1) \;.\;.\;.\;. & P(n_1,n_2) \end{bmatrix} \qquad (3\text{-}10)$$

Inspection of (3-10) reveals that the number of 2-BT power spectrum points is given by

$$\Sigma = (1+n_1)\ (1+n_2); \quad n_i = \log_2 N_i,\ i=1,2 \tag{3-11}$$

In this study, we attempt to characterize the <u>size</u> and <u>shape</u> of grain kernels by the $(1+\log_2 N_1)\ (1+\log_2 N_2)$ power spectrum points given by the matrix $\left[\underline{P}(k_1,k_2)\right]$ in (3-10). The motivation for this choice of the power spectrum will be considered in Section 3-4.

3.3 A Numerical Example

A numerical example will be helpful to clarify the discussion made above. Suppose the 2-BT power spectrum of the pattern shown in Figure 3-1 are desired.

0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Figure 3-1. An (8x8) pattern

From Figure 3-1 it follows that $N_1=N_2=8$. Thus (3-5) yields

$$\left[\underline{F}(u_1,u_2)\right] = \frac{1}{64}\left[H(3)\right]\left[\underline{f}(x_1,x_2)\right]\left[H(3)\right] \tag{3-12}$$

Using (3-6) to evaluate $\left[H(3)\right]$ and subsequently substituting in (3-12) there results the following matrix equation.

$$\left[\underline{F}(u_1,u_2)\right] = \frac{1}{64}\left[H(3)\right]\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\left[H(3)\right] \tag{3-13}$$

where

$$\left[H(3)\right] = \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ \hline 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{array}\right] \tag{3-14}$$

Evaluation of (3-13) yields the transform matrix

$$\left[\underline{F}(u_1,u_2)\right] = \frac{1}{64}\begin{bmatrix} 16 & 0 & 0 & 4 & 0 & -4 & -16 & 0 \\ 0 & 0 & 0 & -4 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & -4 & 0 \\ -4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 12 & 0 & 0 & 0 & 0 & 0 & -12 & 0 \\ -4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix} \quad (3\text{-}15)$$

Substitution of the values of $F(u_1,u_2)$ obtained from (3-15) into (3-9) results in the following 2-BT power spectrum in matrix form.

$$\left[P(k_1,k_2)\right] = \frac{1}{256}\begin{bmatrix} 16 & 0 & 1 & 17 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 12 & 0 & 0 & 12 \end{bmatrix} \quad (3\text{-}16)$$

3.4 Motivation for Using the 2-BT Power Spectrum.

We attempt to characterize the size and shape of grain kernels by the $(1+\log_2 N_1)\ (1+\log_2 N_2)$ power spectrum points given by the matrix $\left[\underline{P}(k_1,k_2)\right]$ in (3-10). Three reasons can be quoted for doing so.

(1). The power spectrum represents the distribution of power in a given two-dimensional pattern. This is best illustrated by a simple example with $N_1=2, N_2=4$ and

$$\left[\underline{f}(x_1,x_2)\right] = \begin{bmatrix} 3 & 0 & 3 & 4 \\ 3 & 8 & 7 & 8 \end{bmatrix} \quad (3\text{-}17)$$

Applying (3-5) and (3-9) to (3-12) results in the following 2-BT power spectrum

$$\begin{array}{lll} P(0,0) = 81/4, & P(0,1) = 1/4, & P(0,2) = 1 \\ P(1,0) = 4, & P(1,1) = 1, & P(1,2) = 1 \end{array} \quad (3\text{-}18)$$

Now, it can be shown that $\left[\underline{f}(x_1,x_2)\right]$ in (3-17) can be decomposed into the following mutually orthogonal sub-patterns:

$$\left[\underline{f}_{00}(x_1,x_2)\right] = \begin{bmatrix} 4.5 & 4.5 & 4.5 & 4.5 \\ 4.5 & 4.5 & 4.5 & 4.5 \end{bmatrix}$$

$$\left[\underline{f}_{01}(x_1,x_2)\right] = \begin{bmatrix} -0.5 & 0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix}$$

$$\left[\underline{f}_{02}(x_1,x_2)\right] = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$\left[\underline{f}_{10}(x_1,x_2)\right] = \begin{bmatrix} -2 & -2 & -2 & -2 \\ 2 & 2 & 2 & 2 \end{bmatrix} \quad (3\text{-}19)$$

$$\left[\underline{f}_{11}(x_1,x_2)\right] = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

and

$$\left[\underline{f}_{12}(x_1,x_2)\right] = \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{bmatrix}$$

It is straight forward to verify that

$$\underline{f}(x_1,x_2) = \sum_{i=0}^{2}\left[\underline{f}_{0i}(x_1,x_2)\right] + \sum_{i=0}^{2}\left[\underline{f}_{1i}(x_1,x_2)\right] \quad (3\text{-}20)$$

Inspection of the sub-patterns in (3-19) reveals that $\left[\underline{f}_{ij}(x_1,x_2)\right]$ is 2^i - periodic with respect to the first dimension (i.e. the columns) and

2^j - periodic in the second dimension (i.e. the rows). If $\sum_{i,j} \left[\underline{f}_{ij}(x_1,x_2)\right]^2$ denotes the sum of the squares of the elements in the sub-pattern $\left[\underline{f}_{i,j}(x_1,x_2)\right]$, divided by N_1N_2, then it can be verified that

$$\begin{aligned} &\sum_{i,j} \left[\underline{f}_{00}(x_1,x_2)\right]^2 = 81/4 && \sum_{i,j} \left[\underline{f}_{01}(x_1,x_2)\right]^2 = 1/4 \\ &\sum_{i,j} \left[\underline{f}_{02}(x_1,x_2)\right]^2 = 1 && \sum_{i,j} \left[\underline{f}_{10}(x_1,x_2)\right]^2 = 4 \qquad (3\text{-}21) \\ &\sum_{i,j} \left[\underline{f}_{11}(x_1,x_2)\right]^2 = 1 && \sum_{i,j} \left[\underline{f}_{12}(x_1,x_2)\right]^2 = 1 \end{aligned}$$

Comparing (3-18) and (3-21) it is clear that each of the 2-BT power spectrum points represents the average power in one of the mutually orthagonal sub-patterns $\left[\underline{f}_{ij}(x_1,x_2)\right]$. Thus the 2-BT power spectrum represents the distribution of power in a given two-dimensional pattern. In the problem at hand, such two-dimensional patterns are coded images (see Figure 1-3) which characterize the sizes and shapes of grain kernels.

(2). From the periodicities present in the sub-patterns $\left[\underline{f}_{ij}(x_1,x_2)\right]$, the analogy between the 2-BT power spectrum and the familiar discrete Fourier power spectrum is apparent. Again, the property that the Fourier power spectrum is invariant with respect to cyclic shifts without rotation of a pattern as illustrated in Figure 3-2 is also valid for the BIFORE spectrum. However, the 2-BT power spectrum yields considerable data compression since it consists of $(1+\log_2 N_1)(1+\log_2 N_2)$ spectrum points in contrast to $(N_1/2+1)(N_2/2+1)$ independent discrete Fourier spectrum points.

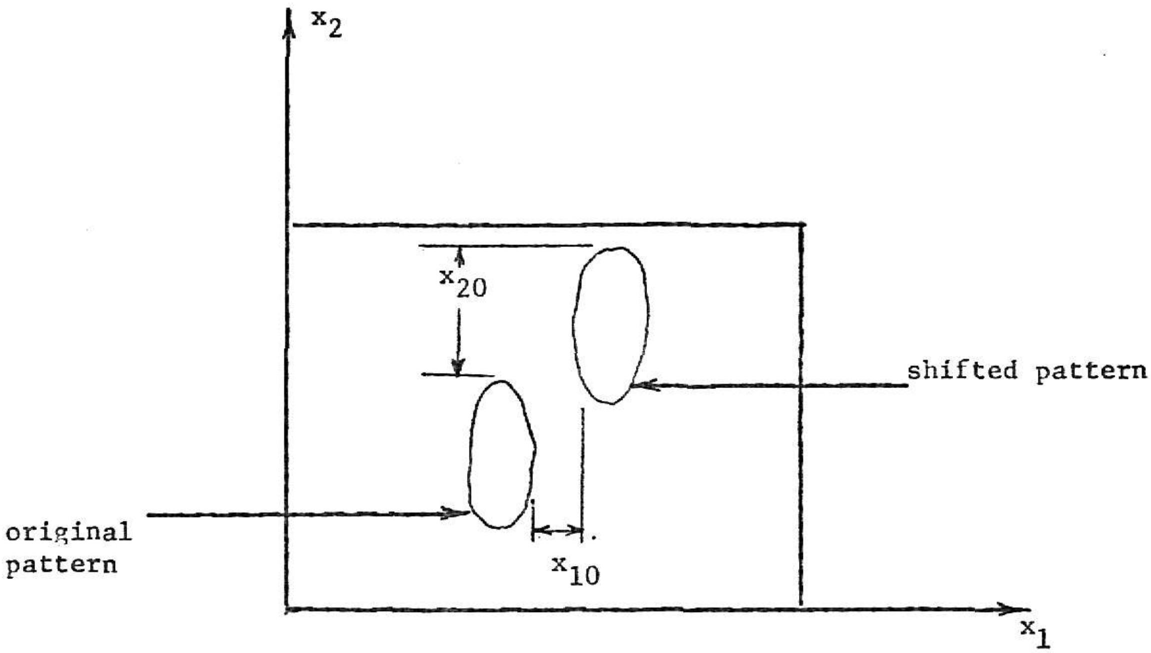


Figure 3-2 Illustration of the shift-invariance property.

(3). The 2-BT power spectrum can be computed rapidly using an algorithm called the fast BIFORE transform (FBT). The corresponding computer program is included in Appendix 4-1. Since only real arithmetic operations are involved, the corresponding implementation is simpler relative to that of the Fourier spectrum which requires complex arithmetic operations.

CHAPTER IV

EXPERIMENTAL RESULTS

4.1 Data Collection

A block diagram of the set up used to obtain the data is shown in Figure 4-1. Each kernel was placed at the base of a microfilm reader and projected on its screen. A (32x32) grid was attached to the screen for each kernel. Each square of the grid which was occupied by the kernel image was coded by a '1' and by a '0' otherwise. A typical output obtained from this stage of the data processing is shown in Figure 4-2. Before any such data was taken, the microfilm reader was calibrated each time using a standard circular pattern which is shown in Figure 4-3. The image coded pattern corresponding to this calibration pattern is shown in Figure 4-4.

The above output of the film reader in the form of a (32x32) array of ones and zeros was punched IBM cards and used as data for the computer program listed in Appendix 4-1. This program was used to compute the 2-BT (BIFORE TRANSFORM) power spectra of several grain kernels placed in random orientations. The corresponding 2-BT power spectra that resulted were in the form of (6x6) matrix i.e. 36 components. Twenty out of the 36 components whose magnitudes were relatively large were chosen. These are tabulated in Appendix 4-2. For convenience each spectrum point in this table has been multiplied by a scale factor of 10^3.

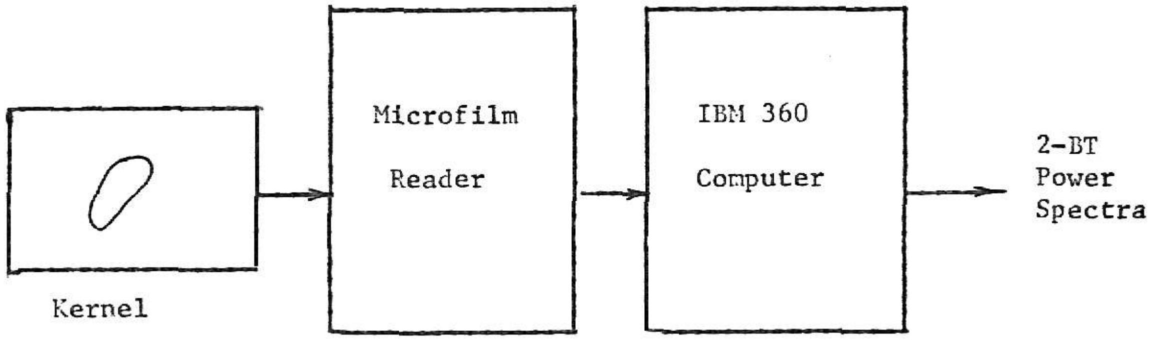


Figure 4-1. Block diagrm of set up to gather data

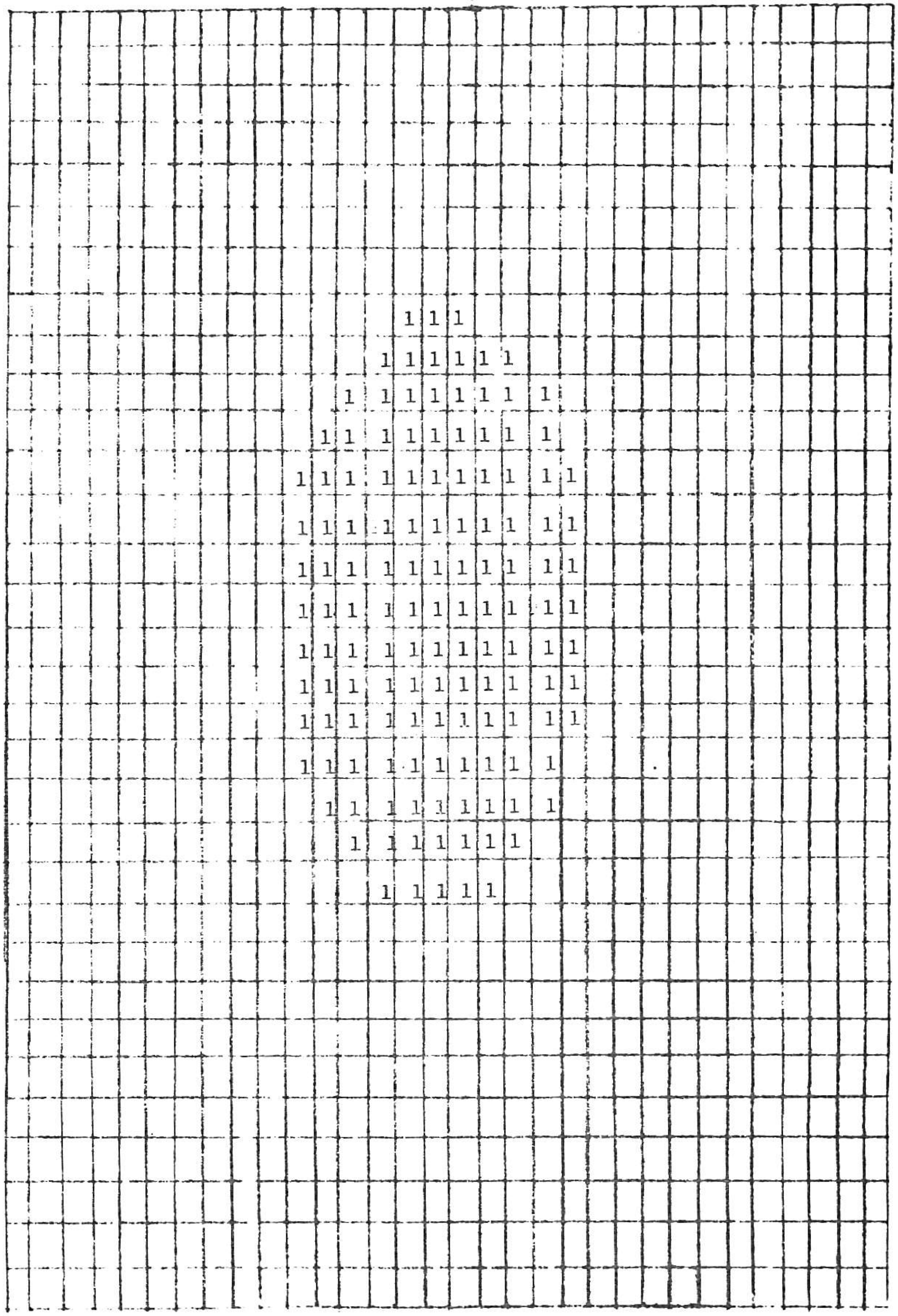


Figure 4-2. Typical output from microfilm reader.

(Blank elements of the array consist of zeros).

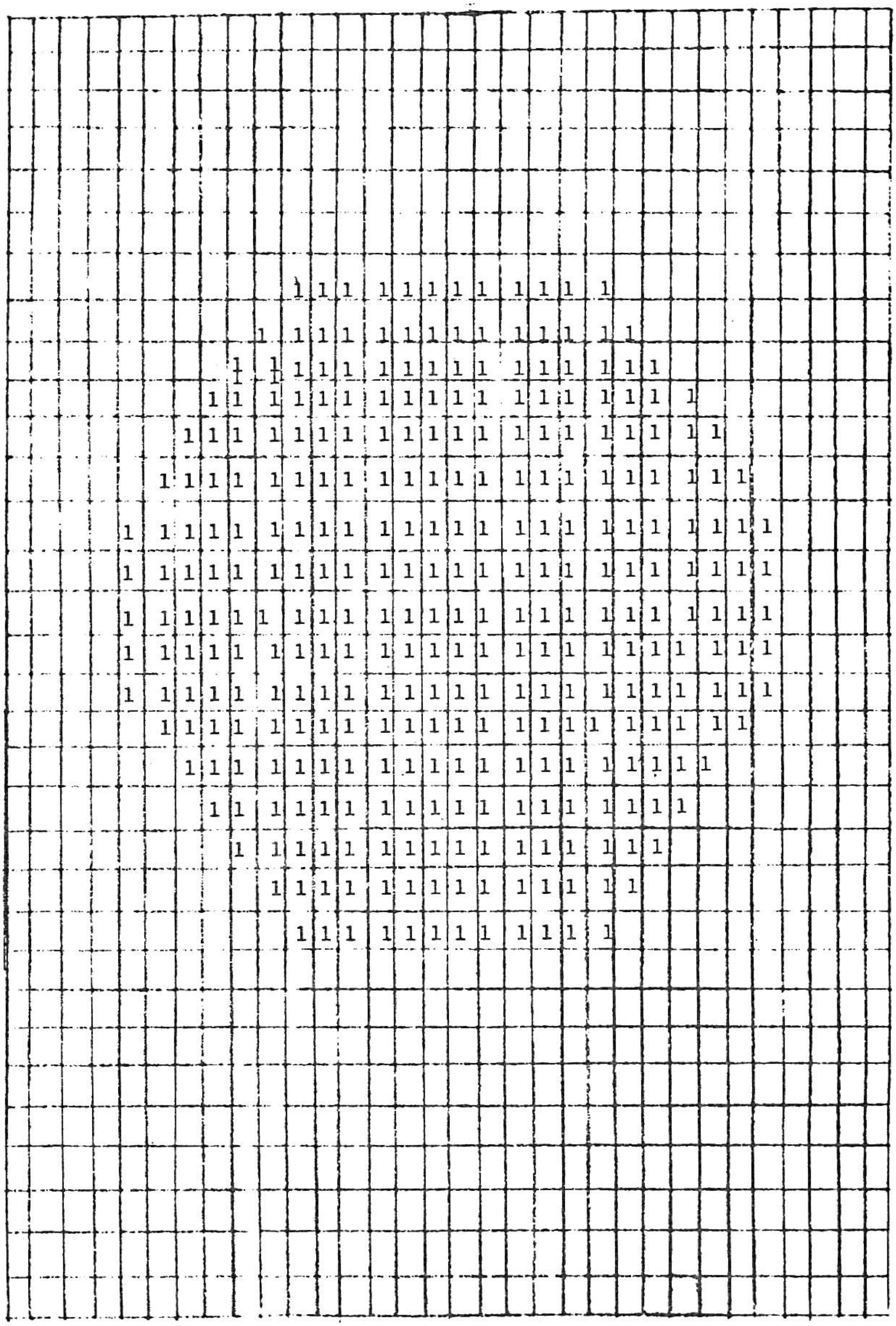


Figure 4-3. Image of calibration pattern.

(Blank elements of the array consist of zeros)

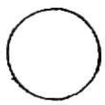


Figure 4-4. Calibration pattern for microfilm reader

APPENDIX 4-1

A listing of the computer program used to compute the 2-BT power spectra listed in Appendix 4-1 is presented in the appendix. For the purpose of illustration, the computation of the 2-BT power-spectrum for a Rye kernel is included in the listing which follows:

```
FORTRAN IV G LEVEL  18              MAIN              DATE = 72117          10/32/53            PAGE 0001

      C     EET SPECTRUM OF CCRN
                 DIMENSION A(32,32),B(6)
                 READ(5,10)M,N
              10 FORMAT(2I2)
                 JJ=16
              15 CONTINUE
                 DO 20 I=1,M
              20 READ (5,30) (A(I,J),J=1,N)
              30 FORMAT(32F2.1)
              35 FORMAT(20X,32F3.0)
                 DO 40 I=1,M
              40 WRITE (6,35) (A(I,J),J=1,N)
                 CALL ADJUST (M,N,A)
                 LL=0
                 MS=1
             120 K=0
                 L=0
                 NS=1
             140 PSP=0.0
                 DO 150 I=1,MS
                 IF(I.LE.LL) GO TO 150
                 DO 145 J=1,NS
                 IF (J.LE.L) GO TO 145
                 PSP=PSP+A(I,J)**2
             145 CONTINUE
             150 CONTINUE
                 K=K+1
                 B(K)=PSP
                 L=NS
                 NS=2*NS
                 IF(NS.LE.N) GO TO 140
                 WRITE(6,165)(B(I),I=1,K)
             165 FORMAT(///20X,6F10.5)
                 LL=MS
                 MS=2*MS
                 IF(MS.LE.M) GO TO 120
                 JJ=JJ-1
                 IF(JJ.GE.1)GO TO 15
                 STOP
                 END
```

```
FORTRAN IV G LEVEL  18              ADJUST              DATE = 72117          10/32/53          PAGE 0001

                SUBROUTINE ADJUST (M,N,A)
                DIMENSION A(M,N),X(32)
                DO 30 J=1,N
                DO 20 I=1,M
             20 X(I)=A(I,J)
                CALL FFT (M,X)
                DO 30 I=1,M
             30 A(I,J)=X(I)
                DO 60 I=1,M
                DO 50 J=1,N
             50 X(J)=A(I,J)
                CALL FFT (N,X)
                DO 60 J=1,N
             60 A(I,J)=X(J)
                RETURN
                END
```

```
      SUBROUTINE BFT (NUM,X)
      CIMENSICN IPOWER(10),X(32),Y(32)
   25 ITER=0
      IREM=NUM
   31 IREM=IREM/2
      IF (IREM.EQ.0) GO TO 32
      ITER=ITER+1
      GO TO 31
   32 CONTINUE
      CO 90 M=1,ITER
      IF (M.EQ.1) NUMP=1
      IF (M.NE.1) NUMP=NUMP*2
      MNUM=NUM/NUMP
      MNUM2=MNUM/2
      CO 60 MP=1,NUMP
      IB= (MP-1) * MNUM
      CO 60 MP2=1,MNUM2
      MNUM21 = MNUM2 + MP2 + IB
      IBA =IB + MP2
      Y(IBA)=X(IBA)+X(MNUM21)
      Y(MNUM21)=X(IBA)-X(MNUM21)
   6C CONTINUE
      CO 7C I=1,NUM
   7C X(I) =Y(I)
   9C CONTINUE
      CO 12C I=1,NUM
      Y(I)=Y(I)/NUM
  12C X(I)=Y(I)
      RETURN
      END
```

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

0.06546	0.00002	0.00005	0.00037	0.00117	0.03204
0.00003	0.0	0.00002	0.00002	0.00005	0.00024
0.00015	0.00001	0.00002	0.00006	0.00031	0.00104
0.00117	0.00002	0.00018	0.00064	0.00104	0.00330
0.00917	0.00008	0.00027	0.00085	0.00525	0.00488
0.07599	0.00012	0.00055	0.00155	0.00781	0.04150

APPENDIX 4-2

In this appendix, the 2-BT power spectra of various kernels of Wheat, Soybean, Barley, Oats, Milo and Rye are tabulated. The spectrum points are denoted by $P(i,j)$, i, $j = 0, 1, \ldots, 5$.

Wheat (hulled oats)

SET#	P(0,0)	P(4,0)	P(5,0)	P(5,4)	P(0,5)	P(4,5)	P(5,5)	P(3,3)	P(4,3)	P(5,3)	P(1,5)	P(3,5)	P(3,4)	P(4,4)	P(5,1)	P(3,0)	P(5,2)	P(0,4)	P(2,4)	P(0,5)
1	10.51	5.44	16.28	7.20	10.64	13.31	25.63	0.50	1.13	1.77	0.08	1.28	1.01	5.55	0.11	0.23	0.27	0.45	0.17	0.34
2	12.39	13.28	26.55	3.17	8.00	12.45	24.90	0.15	0.37	0.73	0.19	3.54	0.50	1.59	0.06	0.85	0.24	0.29	0.73	0.73
3	13.73	4.53	18.68	8.06	13.37	14.65	29.30	0.52	1.53	2.20	0.12	0.85	0.67	6.84	0.06	0.34	0.31	0.29	0.71	0.31
4	11.75	1.01	13.02	12.57	22.29	3.54	27.10	0.14	0.70	1.04	0.08	0.92	0.46	1.53	0.08	0.21	0.40	10.41	0.14	0.27
5	11.12	11.72	23.44	2.44	7.75	12.94	25.88	0.18	0.37	0.73	0.18	4.03	0.55	1.22	0.06	0.54	0.18	0.09	0.49	0.98
6	11.96	5.03	17.43	7.57	11.54	14.16	27.34	0.46	1.22	1.95	0.12	1.10	0.98	6.10	0.09	0.33	0.31	0.29	0.18	0.43
7	12.61	17.65	35.69	4.03	4.81	7.69	15.38	0.26	0.40	0.79	0.11	2.01	1.07	2.01	0.05	4.51	0.21	0.58	0.20	0.76
8	12.61	15.24	30.47	2.81	4.96	10.86	21.73	0.35	0.46	0.92	0.14	4.94	0.34	1.40	0.08	2.25	0.15	0.37	0.63	0.82
9	10.72	11.05	22.60	3.05	7.40	12.57	25.15	0.17	0.82	1.16	0.20	3.97	0.64	1.53	0.08	0.60	0.21	0.13	0.69	1.01
10	5.73	13.91	27.82	2.56	5.64	8.91	17.82	0.26	0.46	0.92	0.11	2.26	0.76	1.28	0.05	3.60	0.15	0.28	0.17	0.70
11	10.92	14.63	29.25	2.81	5.29	9.40	18.80	0.17	0.52	1.04	0.17	3.36	0.52	1.40	0.08	3.34	0.27	0.24	0.47	0.58
12	11.54	10.53	22.28	3.42	7.92	13.92	26.86	0.21	0.43	0.85	0.18	3.78	0.85	1.83	0.06	0.19	0.24	0.14	0.52	1.04
13	6.57	3.44	10.21	8.67	7.58	10.13	20.26	0.17	0.46	0.92	0.14	1.90	1.43	6.04	0.14	0.16	0.34	0.86	0.29	0.52
14	7.05	8.79	17.56	2.44	5.00	10.50	21.00	0.15	0.37	0.73	0.18	4.76	0.65	1.22	0.06	1.51	0.19	0.06	0.21	0.55
15	5.65	9.32	18.63	1.34	4.44	8.42	16.85	0.20	0.27	0.55	0.17	3.36	0.34	0.67	0.02	3.48	0.21	0.07	0.20	0.46
16	6.41	7.08	14.16	4.15	6.16	10.01	20.02	0.40	0.67	1.34	0.12	3.17	1.60	2.10	0.06	0.60	0.31	0.09	0.37	0.55
17	17.90	18.62	37.25	3.30	11.25	12.33	24.66	0.35	0.70	1.40	0.14	0.67	1.07	1.65	0.08	0.64	0.21	0.25	0.26	0.27
18	17.64	19.41	28.47	3.42	14.32	15.38	33.20	0.24	0.49	0.98	0.15	2.08	0.31	2.81	0.09	0.24	0.24	0.18	0.09	0.67
19	15.14	1.66	17.38	10.99	24.96	4.15	30.76	0.15	1.16	0.71	0.12	1.22	0.49	0.98	0.03	0.54	0.67	9.32	0.12	0.31
20	13.28	3.30	17.15	9.52	16.34	10.25	28.81	0.43	0.67	0.59	0.12	1.59	0.43	6.00	0.12	0.56	0.43	2.70	0.24	0.09
21	14.19	4.91	19.68	7.31	17.15	10.50	29.79	0.58	0.85	1.83	0.12	1.59	0.37	4.52	0.08	0.56	0.37	2.56	0.31	0.03
22	16.12	3.88	20.68	8.54	18.34	10.74	31.74	0.79	0.92	2.20	0.15	1.95	0.49	5.79	0.09	0.59	0.31	1.89	0.34	0.55
23	14.90	5.03	20.22	7.69	15.68	12.88	30.52	0.49	1.25	2.01	0.14	1.16	0.21	6.29	0.14	0.31	0.40	0.80	0.32	0.52
24	14.12	1.05	17.43	11.69	27.16	3.17	31.74	0.21	1.16	1.59	0.0	1.10	0.55	0.61	0.09	0.22	0.67	10.65	0.15	0.31
25	16.87	14.87	31.88	2.81	9.54	15.26	29.05	0.11	0.40	0.79	0.20	3.36	0.40	1.77	0.08	0.10	0.34	0.22	0.32	0.70
26	13.28	17.96	35.92	2.69	5.92	9.03	18.07	0.15	0.37	0.73	0.12	1.71	0.61	1.34	0.03	3.82	0.16	0.60	0.09	1.28
27	9.73	3.14	13.26	9.64	16.65	5.98	24.66	0.07	0.26	0.52	0.11	1.65	1.01	1.05	0.08	0.39	0.52	6.75	0.17	0.27
28	9.92	4.44	14.59	8.06	14.56	8.06	24.90	0.58	0.61	0.71	0.09	1.71	0.67	2.81	0.12	0.21	0.43	4.33	0.21	0.49
29	9.54	6.70	16.33	6.10	11.54	11.47	24.41	0.43	0.73	0.59	0.12	0.98	0.49	3.78	0.09	0.02	0.31	1.65	0.12	0.31
30	9.35	11.41	22.81	1.83	8.87	11.35	22.71	0.14	0.34	0.67	0.14	1.65	0.46	0.92	0.05	1.74	0.27	0.11	0.26	0.70

Wheat (triumph)

SET#	P([illegible],0)	P(4,0)	P(5,0)	P(5,4)	P(0,5)	P(6,5)	P(5,5)	P(3,3)	P(4,3)	P(5,3)	P(1,5)	P(3,5)	P(3,4)	P(4,4)	P(5,1)	P(3,0)	P(5,2)	P(0,4)	P(2,4)	P([illegible],5)
1	14.66	3.73	18.89	8.79	19.84	8.06	30.27	0.76	0.92	2.08	0.12	1.95	0.55	3.17	0.09	0.44	0.43	4.82	0.15	0.31
2	17.90	5.58	24.00	6.96	19.09	11.84	33.45	0.53	1.01	2.01	0.11	1.77	0.64	4.94	0.08	0.51	0.40	1.03	0.29	0.64
3	14.43	6.10	20.58	6.71	16.56	11.35	20.03	0.50	0.82	1.89	0.08	1.40	0.46	4.21	0.11	0.40	0.34	1.67	0.32	0.64
4	13.50	1.41	15.15	11.60	23.09	4.27	29.05	0.32	1.07	1.53	0.08	1.28	0.52	1.53	0.14	0.19	0.64	9.36	0.17	0.34
5	13.05	13.45	26.90	2.32	9.84	13.31	26.61	0.14	0.46	0.92	0.14	2.87	0.58	1.16	0.11	0.39	0.27	0.11	0.38	0.46
6	10.51	6.34	17.26	6.47	12.16	11.35	25.63	0.69	0.82	1.65	0.14	1.53	0.52	4.09	0.05	0.24	0.21	1.53	0.23	0.46
7	10.31	11.09	22.19	2.69	9.12	12.45	24.90	0.24	0.37	0.73	0.15	2.69	0.73	1.34	0.03	0.71	0.24	0.17	0.37	0.49
8	12.18	4.91	17.47	7.93	12.83	13.06	27.59	0.38	1.31	1.89	0.14	1.16	0.40	6.65	0.08	0.35	0.21	0.59	0.23	0.40
9	13.96	12.86	26.99	2.56	11.40	14.28	28.56	0.20	0.27	0.67	0.11	2.26	0.34	2.01	0.08	0.16	0.21	0.04	0.14	0.52
10	11.12	4.82	16.36	7.81	13.85	10.47	26.37	0.43	0.79	1.71	0.06	1.22	0.31	4.88	0.12	0.38	0.37	2.33	0.24	0.49
11	9.54	7.37	17.03	5.62	11.11	11.72	24.41	0.46	0.85	1.59	0.12	1.10	0.55	3.42	0.06	0.04	0.12	1.36	0.24	0.37
12	10.72	5.22	16.30	7.57	13.89	10.25	25.88	0.43	0.61	1.59	0.09	1.22	0.43	4.15	0.06	0.26	0.37	2.75	0.24	0.43
13	10.12	11.65	23.30	2.32	8.07	11.84	23.68	0.20	0.34	0.67	0.20	2.87	0.58	1.16	0.05	1.33	0.27	0.08	0.44	0.70
14	15.14	3.11	18.77	9.52	21.47	7.08	30.76	0.46	0.85	1.95	0.06	1.95	0.92	2.81	0.09	0.48	0.43	5.65	0.12	0.24
15	10.12	12.43	24.86	2.56	8.37	10.86	21.73	0.23	0.46	0.92	0.20	1.53	0.89	1.28	0.08	2.06	0.15	0.08	0.23	0.82
16	14.90	9.12	24.52	4.27	12.17	15.99	30.52	0.63	0.52	1.28	0.17	1.89	0.64	3.23	0.05	0.28	0.40	0.13	0.23	0.34
17	11.75	4.39	16.59	8.42	15.15	10.13	27.10	0.35	0.64	1.65	0.08	1.28	0.40	4.82	0.11	0.43	0.34	2.85	0.32	0.46
18	15.38	16.53	33.07	3.30	9.72	12.08	24.17	0.20	0.58	1.16	0.14	1.77	1.07	1.65	0.11	0.93	0.21	0.11	0.35	0.46
19	19.23	13.34	32.90	3.17	12.79	15.87	31.74	0.12	0.49	1.10	0.15	2.56	0.43	2.32	0.0	0.15	0.43	0.20	0.12	0.37
20	10.31	12.62	25.24	1.95	8.30	11.23	22.46	0.18	0.37	0.73	0.12	2.20	0.61	0.98	0.03	1.85	0.37	0.08	0.18	0.61
21	12.61	3.38	16.40	5.40	18.11	7.69	28.08	0.66	0.52	1.65	0.05	2.01	0.70	3.48	0.11	0.40	0.52	5.04	0.14	0.21
22	14.43	7.45	22.23	5.74	14.60	13.31	30.03	0.47	0.95	1.53	0.14	1.40	0.34	4.58	0.08	0.27	0.46	0.51	0.29	0.58
23	13.96	9.82	24.00	4.03	12.13	14.53	29.54	0.29	0.76	1.16	0.11	2.26	0.40	3.23	0.08	0.06	0.27	0.19	0.14	0.52
24	7.72	6.16	16.33	4.15	8.70	10.99	21.97	0.34	0.55	1.10	0.15	1.59	0.79	2.08	0.09	0.40	0.31	0.81	0.40	0.55
25	9.92	5.29	15.44	7.57	11.72	11.72	24.90	0.31	0.92	1.59	0.12	0.98	0.49	5.37	0.06	0.11	0.24	1.50	0.15	0.37

Soybeans

SET#	P(0,0)	P(4,0)	P(5,0)	P(5,4)	P(0,5)	P(4,5)	P(5,5)	P(3,3)	P(4,3)	P(5,3)	P(1,5)	P(3,5)	P(3,4)	P(4,4)	P(5,1)	P(3,0)	P(5,2)	P(0,4)	P(2,4)	P(2,5)
1	65.46	9.17	75.59	7.81	32.04	4.88	41.50	0.64	0.85	1.95	0.24	3.30	1.04	5.25	0.12	1.17	0.55	1.17	0.31	1.04
2	63.48	15.03	80.57	9.03	24.90	4.64	34.67	0.40	0.49	1.34	0.12	4.39	0.73	5.49	0.06	1.98	0.31	2.08	0.61	0.61
3	54.02	9.87	66.01	5.86	31.65	5.86	41.50	0.43	1.10	2.32	0.09	3.17	0.61	3.91	0.15	1.98	0.37	0.78	0.49	0.73
4	53.12	13.43	68.48	7.57	27.44	4.39	36.62	0.21	1.16	1.95	0.21	3.91	0.49	5.62	0.06	1.85	0.55	0.92	0.46	0.67
5	87.56	2.42	86.65	10.13	39.87	5.49	47.61	0.23	1.19	1.89	0.14	2.62	0.64	4.46	0.11	0.08	0.58	2.34	0.20	0.46
6	85.63	0.75	75.32	6.10	54.41	4.15	46.88	0.37	1.46	2.69	0.15	2.08	1.71	6.59	0.24	0.08	0.61	0.73	0.40	0.73
7	79.10	9.23	90.15	10.01	30.07	4.88	38.57	0.34	0.73	1.34	0.18	2.44	1.10	5.86	0.12	1.63	0.43	2.64	0.34	1.04
8	78.01	13.69	94.48	9.52	24.96	4.64	33.69	0.67	0.43	1.46	0.18	3.17	0.79	4.88	0.12	2.55	0.37	3.25	0.55	0.73
9	26.92	10.36	38.33	3.66	25.67	10.01	38.57	0.43	0.31	0.98	0.09	2.44	1.34	1.59	0.12	1.01	0.37	0.50	0.18	0.37
10	27.56	10.13	38.54	3.17	26.40	10.01	39.55	0.37	0.31	0.85	0.15	2.69	1.22	1.34	0.06	1.17	0.43	0.47	0.09	0.31
11	48.28	9.91	60.04	5.98	31.97	5.00	41.26	0.26	1.31	2.26	0.11	3.36	0.34	4.82	0.05	1.74	0.27	0.40	0.35	0.82
12	47.85	12.28	62.07	5.86	28.93	5.13	39.06	0.18	1.34	1.95	0.18	4.27	0.49	4.39	0.12	1.79	0.31	0.61	0.31	0.55
13	44.08	10.43	56.56	4.76	30.99	5.74	41.26	0.23	1.31	1.89	0.17	3.72	0.46	3.48	0.11	1.95	0.40	0.37	0.35	0.64
14	50.89	9.45	62.18	5.74	33.13	4.52	42.24	0.17	1.50	2.26	0.23	3.48	0.27	4.58	0.05	1.70	0.34	0.43	0.41	0.89
15	32.64	14.47	48.26	4.27	22.45	9.40	35.40	0.47	0.95	1.89	0.17	2.75	1.25	2.38	0.11	0.94	0.40	0.28	0.26	0.64
16	35.14	13.75	50.11	5.13	24.44	8.30	36.62	0.34	0.85	1.59	0.15	3.30	1.26	3.30	0.06	1.14	0.24	0.23	0.21	0.43

Barley

SET#	P(5,0)	P(4,0)	P(6,0)	P(5,4)	P(0,5)	P(6,5)	P(5,5)	P(3,3)	P(4,3)	P(5,3)	P(1,5)	P(0,5)	P(3,4)	P(4,4)	P(5,1)	P(3,0)	P(6,2)	P(0,4)	P(2,4)	P(2,5)
1	20.61	9.94	31.17	3.54	11.34	21.12	35.40	0.38	0.46	1.28	0.29	2.14	0.64	2.26	0.05	0.52	0.34	0.40	0.20	0.52
2	22.62	24.22	48.23	4.88	7.20	10.25	20.51	0.37	0.55	1.10	0.12	1.95	1.16	2.44	0.09	1.53	0.18	0.95	0.18	0.98
3	21.74	13.05	35.05	3.78	10.03	20.14	33.45	0.17	0.58	1.04	0.08	2.75	0.58	2.38	0.08	0.19	0.34	0.55	0.20	0.46
4	20.89	16.00	22.37	10.74	24.75	8.54	30.13	0.98	0.85	2.44	0.15	2.32	1.10	5.86	0.09	0.08	0.49	2.99	0.21	0.37
5	30.50	30.17	61.86	6.35	5.49	8.06	18.07	0.52	0.49	1.22	0.31	3.05	2.93	3.05	0.09	0.74	0.31	0.08	0.18	1.16
6	29.88	8.95	40.15	3.54	10.06	27.71	41.26	0.17	0.64	1.04	0.17	2.26	0.70	1.77	0.11	1.30	0.34	0.72	0.26	1.07
7	31.59	30.84	62.90	6.10	5.31	9.52	19.04	0.21	0.18	0.61	0.31	2.81	2.38	2.81	0.03	0.40	0.18	0.64	0.21	1.10
8	30.90	1.50	31.74	7.57	19.47	19.29	45.41	0.73	1.05	2.32	0.24	2.56	1.40	7.08	0.18	0.23	0.67	0.23	0.18	0.92
9	21.17	1.29	22.75	11.11	24.83	8.42	36.38	0.50	0.55	2.01	0.17	2.50	1.62	6.04	0.11	0.15	0.40	3.14	0.26	0.46
10	20.33	8.29	29.39	4.15	12.79	20.26	35.64	0.55	0.73	1.46	0.15	1.71	0.85	2.69	0.15	0.68	0.49	0.20	0.37	0.73
11	22.91	6.23	31.68	4.52	12.19	22.58	37.35	0.29	0.82	1.40	0.11	1.77	0.76	2.99	0.14	0.64	0.40	0.40	0.26	0.70
12	22.32	24.28	48.57	5.00	8.29	10.13	20.26	0.11	0.27	0.55	0.23	1.16	1.62	2.50	0.05	1.77	0.27	0.51	0.29	0.45
13	26.60	28.51	57.02	4.76	6.15	9.16	18.31	0.26	0.52	1.04	0.11	2.14	1.68	2.38	0.08	1.79	0.34	0.42	0.20	0.76
14	22.32	10.11	33.37	3.05	8.80	24.29	36.87	0.04	0.08	0.89	0.14	2.99	0.82	1.40	0.11	0.77	0.27	0.56	0.20	0.64
15	28.21	0.21	21.97	11.47	40.63	4.15	37.11	0.18	1.53	3.17	0.06	1.46	0.98	1.95	0.12	0.16	0.37	11.76	0.15	0.37
16	28.21	0.61	26.46	12.45	30.91	7.57	41.59	0.61	1.46	2.44	0.15	2.81	2.01	5.13	0.15	0.01	0.49	2.14	0.21	0.55
17	30.90	24.00	55.70	4.60	12.50	14.50	25.90	0.10	0.60	1.20	0.21	1.22	0.37	2.32	0.09	0.42	0.37	1.79	0.12	0.48
18	37.40	2.90	19.70	5.30	43.80	4.20	28.70	0.20	0.70	1.80	0.23	0.28	0.38	3.20	0.62	0.50	0.93	28.73	0.18	0.37
19	34.70	2.50	11.90	8.70	62.60	5.70	23.70	0.10	0.20	1.90	0.11	0.22	0.46	2.87	0.38	0.32	0.52	26.63	0.14	0.27
20	36.40	0.30	25.70	10.00	21.60	14.20	56.20	1.50	0.30	3.30	0.09	2.81	1.53	12.70	0.19	0.30	0.43	0.31	0.31	0.73
21	43.70	16.40	61.60	5.80	2.80	29.30	35.20	0.20	0.90	1.30	0.05	2.40	0.83	3.91	0.12	1.67	0.43	0.28	0.34	0.79
22	45.90	2.20	14.20	7.00	67.20	6.00	26.60	0.20	0.40	3.90	0.05	0.02	0.58	3.23	0.08	0.10	1.50	24.93	0.17	0.34
23	54.90	21.90	78.50	7.30	17.30	5.40	28.80	0.70	0.60	1.60	0.09	3.87	0.67	4.03	0.09	1.68	0.49	2.20	0.31	0.67
24	37.80	1.60	14.10	11.10	61.80	5.30	22.60	0.40	0.50	2.50	0.03	1.16	0.46	2.62	0.09	0.10	0.70	22.48	0.17	0.34
25	42.90	2.10	20.30	7.60	64.00	5.90	31.30	0.10	0.30	2.60	0.08	1.93	0.88	3.17	0.18	0.74	0.61	17.87	0.15	0.31
26	29.20	0.90	19.60	13.10	19.80	11.40	50.10	1.20	0.80	1.80	0.20	3.36	0.07	14.87	0.08	0.46	0.95	0.10	0.20	0.07

Oats

SET#	P(0,0)	P(4,0)	P(5,0)	P(5,4)	P(0,5)	P(4,5)	P(5,5)	P(3,3)	P(4,3)	P(5,3)	P(1,5)	P(3,5)	P(3,4)	P(4,4)	P(5,1)	P(3,0)	P(5,2)	P(0,4)	P(2,4)	P(2,5)
1	43.27	1.88	14.02	25.76	33.19	5.98	60.79	1.11	1.31	2.50	0.11	3.23	0.89	10.07	0.17	0.03	0.76	0.11	0.47	0.70
2	37.77	1.17	7.77	6.71	72.01	5.74	17.82	0.29	1.01	2.38	0.11	1.16	0.56	2.75	0.20	0.16	0.76	33.94	0.17	0.34
3	38.53	1.88	11.22	29.17	31.45	7.93	54.93	1.11	0.76	2.14	0.14	2.99	1.62	9.09	0.17	0.03	0.52	0.18	0.17	0.70
4	42.46	0.69	22.48	7.93	10.67	21.61	68.12	1.66	1.13	3.97	0.11	1.28	1.98	17.03	0.14	0.33	0.40	1.35	0.23	1.25
5	25.65	0.27	22.31	14.65	27.40	9.52	40.04	1.19	1.10	2.32	0.12	2.56	1.10	7.81	0.15	0.07	0.61	1.57	0.18	0.43
6	26.28	0.26	16.30	11.47	44.22	4.15	30.76	0.34	1.34	1.95	0.09	1.59	0.92	2.08	0.18	0.30	0.85	17.53	0.18	0.24
7	29.88	11.44	42.28	4.27	6.94	28.20	37.84	0.38	0.89	1.40	0.17	1.89	0.46	2.26	0.11	0.94	0.52	1.20	0.29	0.64
8	29.88	0.24	25.56	11.35	24.70	12.33	45.65	1.33	1.25	2.99	0.11	3.23	1.31	9.95	0.14	0.19	0.34	0.21	0.32	0.40
9	31.24	10.84	43.44	3.78	2.15	30.15	39.31	0.47	0.40	1.28	0.26	5.43	0.40	2.50	0.05	1.26	0.52	0.18	0.63	1.31
10	33.71	0.93	31.46	9.77	7.39	34.67	47.36	0.67	1.89	2.56	0.24	1.71	0.92	5.98	0.09	0.11	0.55	1.50	0.34	0.43
11	28.54	0.76	11.55	20.63	43.56	3.78	35.40	0.17	0.95	2.38	0.11	1.28	0.64	1.77	0.20	0.04	0.64	14.03	0.23	0.34
12	31.24	0.53	16.31	19.90	27.24	8.67	49.07	1.48	0.95	2.14	0.20	2.99	1.43	11.66	0.20	0.24	0.76	0.19	0.23	0.21
13	34.07	0.36	29.77	8.67	12.68	25.76	50.54	1.42	1.07	2.75	0.11	2.38	0.70	9.95	0.17	0.10	0.40	1.09	0.60	0.82
14	32.29	1.40	13.28	19.04	47.54	5.13	34.67	0.49	1.10	1.34	0.09	1.71	0.79	2.20	0.27	0.24	0.73	14.94	0.09	0.31
15	35.16	0.37	31.10	8.79	10.41	30.27	50.29	0.98	1.53	2.93	0.21	1.95	0.43	8.54	0.15	0.04	0.49	1.68	0.46	0.61
16	34.79	0.36	31.11	7.93	12.04	27.95	50.54	1.02	1.62	2.59	0.14	2.01	0.46	9.22	0.11	0.06	0.58	1.52	0.50	0.58
17	30.60	1.10	21.70	8.70	44.40	5.70	34.40	0.40	1.70	3.20	0.05	1.90	0.70	2.80	0.05	0.60	0.80	13.70	0.20	0.40
18	35.20	2.70	29.20	7.30	27.30	8.70	27.20	0.30	2.20	4.70	0.05	2.70	0.90	1.60	0.05	0.10	0.50	10.20	0.37	0.40
19	29.30	12.30	39.20	5.30	20.20	8.70	35.20	0.40	0.40	2.10	0.15	3.10	2.20	3.30	0.09	0.60	0.90	0.60	0.37	0.20
20	30.60	0.90	8.30	8.80	58.30	3.30	80.30	0.20	2.10	3.10	0.82	0.90	0.60	2.30	0.04	0.30	0.90	27.30	0.20	0.20
21	28.20	31.40	62.70	4.40	5.20	7.80	75.60	0.10	0.50	1.00	0.34	2.10	1.04	2.20	0.03	2.60	0.20	0.70	0.40	0.20
22	20.30	23.10	46.10	4.20	6.10	9.80	19.50	0.40	0.60	1.10	0.18	2.40	0.90	2.10	0.06	2.50	0.30	2.30	0.20	1.00
23	29.40	29.70	58.30	7.20	9.30	5.30	25.40	0.30	0.90	2.10	0.20	3.40	0.90	2.10	0.05	3.10	0.60	0.60	0.30	1.20
24	37.30	2.40	28.20	9.20	7.40	8.30	23.60	0.40	0.50	2.10	0.20	1.30	0.80	3.10	0.09	3.20	0.50	0.60	0.20	1.00
25	26.80	0.90	35.30	10.20	34.70	9.70	39.80	0.60	2.30	10.80	0.02	2.30	0.60	2.20	0.09	0.07	0.20	13.20	0.30	0.50
26	27.90	0.50	24.70	9.60	40.00	4.30	37.40	0.50	1.70	2.60	0.05	1.30	0.50	1.70	0.11	0.09	0.30	11.30	0.20	0.50

Rye

SET#	P(2,0)	P(4,0)	P(5,0)	P(5,4)	P(0,5)	P(4,5)	P(5,5)	P(3,3)	P(4,3)	P(5,3)	P(1,5)	P(3,5)	P(3,4)	P(4,4)	P(5,1)	P(3,0)	P(5,2)	P(0,4)	P(2,4)	P(2,5)
1	15.63	10.62	26.58	3.17	6.68	18.60	31.25	0.16	0.73	1.10	0.27	4.80	1.40	0.65	0.09	0.22	0.31	0.26	0.58	0.73
2	16.12	23.16	46.33	3.91	3.67	6.10	12.21	0.21	0.37	0.73	0.09	1.10	1.22	1.96	0.06	6.39	0.24	0.47	0.15	1.28
3	14.90	0.37	13.81	14.53	21.71	6.47	30.52	0.78	0.64	1.28	0.11	1.90	0.95	4.94	0.14	0.02	0.76	6.05	0.08	0.34
4	14.66	0.15	12.21	12.21	29.24	3.42	26.37	0.34	1.53	1.22	0.06	1.10	0.50	1.34	0.24	0.11	0.49	14.07	0.18	0.37
5	8.43	1.10	9.80	11.47	13.89	7.32	22.95	0.24	0.79	1.22	0.03	1.46	0.85	5.25	0.09	0.24	0.37	5.23	0.12	0.24
6	9.92	3.17	13.64	9.28	10.01	12.94	24.90	0.21	1.04	1.34	0.12	1.34	1.04	7.45	0.09	0.51	0.55	0.41	0.31	0.49
7	8.25	0.83	9.23	11.35	17.78	3.78	22.71	0.20	0.64	1.53	0.08	0.79	0.40	1.90	0.14	0.13	0.46	8.90	0.14	0.27
8	8.61	12.44	24.89	2.56	4.20	8.91	17.82	0.17	0.46	0.92	0.29	3.97	0.46	1.28	0.05	3.55	0.15	0.33	0.38	0.46
9	14.90	0.87	1.39	6.10	9.52	18.07	25.79	0.79	0.67	1.83	0.24	1.34	1.71	3.91	0.09	0.25	0.37	0.17	0.24	0.61
10	12.83	15.09	30.16	2.21	5.86	1.47	22.95	0.31	0.55	1.10	0.12	4.80	0.37	1.10	0.03	1.05	0.18	0.26	0.40	0.79
11	14.43	1.06	15.64	12.33	20.16	7.69	30.03	0.56	0.21	1.40	0.08	1.77	1.01	5.92	0.08	0.10	0.58	5.12	0.23	0.40
12	13.50	0.40	13.09	12.82	26.50	2.81	27.59	0.17	1.13	1.04	0.08	0.92	0.46	0.92	0.20	0.05	0.46	12.76	0.11	0.21
13	6.57	6.97	13.93	4.76	5.78	10.13	20.26	0.32	0.64	1.28	0.11	3.60	1.86	2.38	0.08	0.36	0.21	0.14	0.32	0.64
14	7.22	3.03	10.42	9.16	9.02	10.62	21.24	0.17	0.52	1.16	0.11	1.16	0.70	6.77	0.11	0.16	0.40	1.46	0.14	0.34
15	8.25	11.47	22.93	2.08	5.36	9.64	15.29	0.29	0.46	0.92	0.11	3.60	0.40	1.04	0.05	2.85	0.15	0.11	0.38	0.58
16	6.57	4.83	11.49	7.20	7.06	10.13	20.26	0.23	0.58	1.16	0.11	2.26	1.92	4.46	0.08	0.04	0.34	0.40	0.35	0.70
17	8.97	9.79	19.58	3.05	5.20	11.84	23.68	0.11	0.40	0.79	0.20	5.55	0.95	1.53	0.05	0.76	0.21	0.19	0.38	0.89
18	9.16	3.98	10.35	11.96	19.41	3.42	23.93	0.15	0.55	1.22	0.06	0.85	0.43	1.71	0.09	0.14	0.31	9.70	0.09	0.18
19	12.18	4.74	17.44	7.69	9.81	15.99	27.59	0.63	1.01	2.01	0.17	0.92	1.86	5.68	0.11	0.47	0.34	0.04	0.08	0.70
20	7.39	8.96	17.91	2.44	5.19	10.74	21.48	0.09	0.43	0.85	0.18	4.64	0.92	1.22	0.03	1.34	0.24	0.06	0.21	0.73
21	8.07	13.57	27.13	2.69	3.63	7.08	14.16	0.24	0.37	0.73	0.15	2.69	0.67	1.34	0.03	5.39	0.18	0.23	0.37	0.61
22	12.16	13.99	23.27	3.95	7.12	15.01	27.59	0.29	0.46	0.92	0.11	4.33	1.19	0.92	0.08	0.09	0.27	0.18	0.63	1.01
23	10.92	4.54	15.88	8.18	10.79	13.55	26.12	0.32	1.13	1.65	0.17	1.28	1.07	6.65	0.14	0.35	0.27	0.36	0.08	0.34
24	11.33	3.63	12.01	12.82	17.68	6.71	26.61	0.47	0.58	1.28	0.11	1.77	1.19	5.31	0.11	0.03	0.40	6.02	0.23	0.34
25	8.61	6.10	14.79	6.47	5.62	13.55	23.19	0.35	0.76	1.65	0.20	3.23	3.27	2.62	0.08	0.03	0.21	0.11	0.35	0.56

Milo

SCTN	P(2,0)	P(4,0)	P(5,0)	P(5,4)	P(0,5)	P(4,5)	P(5,5)	P(3,3)	P(4,3)	P(5,3)	P(1,5)	P(3,5)	P(3,4)	P(4,4)	P(5,1)	P(3,0)	P(5,2)	P(0,4)	P(2,4)	P(2,5)
1	8.61	7.79	16.49	4.76	11.12	10.38	23.19	0.38	0.64	1.54	0.14	1.16	0.58	1.77	0.08	0.09	0.34	2.20	0.17	0.40
2	7.90	7.09	15.03	5.25	10.68	10.13	22.22	0.38	0.82	1.77	0.14	0.79	0.34	2.14	0.02	0.03	0.15	2.62	0.11	0.27
3	5.08	5.39	10.79	5.74	7.77	8.91	17.82	0.29	0.52	1.04	0.08	0.79	0.21	2.87	0.05	0.26	0.21	2.48	0.14	0.27
4	5.95	5.71	11.80	5.74	9.02	9.16	19.29	0.41	0.52	1.28	0.05	0.79	0.21	2.62	0.08	0.10	0.40	2.83	0.05	0.27
5	6.10	6.44	12.88	5.13	8.51	9.77	19.53	0.34	0.67	1.34	0.03	0.98	0.31	2.56	0.06	0.28	0.12	2.15	0.09	0.24
6	7.90	6.52	14.54	5.98	11.22	9.40	22.22	0.26	0.64	1.40	0.17	1.04	0.64	2.14	0.08	0.06	0.21	2.94	0.20	0.40
7	9.35	7.99	17.47	5.00	11.92	10.38	24.17	0.20	0.58	1.28	0.20	1.28	0.89	1.65	0.08	0.07	0.34	2.19	0.23	0.40
8	14.19	7.75	22.28	5.37	17.43	10.01	29.79	0.18	0.98	1.71	0.15	1.71	1.22	1.10	0.06	0.21	0.37	2.75	0.27	0.49
9	7.22	6.39	13.69	5.74	10.24	9.40	21.24	0.47	0.64	1.53	0.11	1.16	0.52	2.26	0.08	0.07	0.21	2.80	0.11	0.34
10	9.92	8.56	18.59	4.64	12.24	10.99	24.90	0.15	0.67	1.34	0.15	1.10	0.79	1.59	0.03	0.07	0.31	1.94	0.21	0.43
11	6.26	6.72	13.44	4.76	8.41	9.89	19.78	0.32	0.70	1.40	0.11	1.04	0.27	2.38	0.02	0.40	0.15	1.91	0.14	0.34
12	9.54	6.88	16.51	5.86	12.68	9.77	24.41	0.24	0.92	1.59	0.06	1.22	0.85	1.71	0.15	0.02	0.21	3.04	0.21	0.49
13	6.57	5.62	12.21	6.23	9.96	9.16	20.26	0.26	0.64	1.28	0.08	0.79	0.40	2.62	0.11	0.07	0.34	3.06	0.11	0.27
14	8.97	7.76	16.80	5.00	1.70	10.38	23.68	0.26	0.70	1.53	0.17	1.04	0.64	1.77	0.08	0.04	0.27	2.33	0.22	0.40
15	7.90	6.22	14.18	5.98	11.31	9.40	22.22	0.50	0.89	1.77	0.08	1.04	0.46	2.14	0.08	0.03	0.21	3.20	0.17	0.40
16	7.72	6.94	14.60	5.37	10.44	9.77	21.97	0.43	0.61	1.46	0.12	1.22	0.61	2.08	0.09	0.11	0.24	2.50	0.12	0.43
17	7.70	6.70	14.50	5.60	10.70	9.80	22.00	0.40	0.70	1.60	0.12	0.98	0.49	2.20	0.06	0.25	0.18	2.70	0.18	0.43
18	9.40	7.90	17.40	5.00	11.60	10.80	24.20	0.14	0.64	1.30	0.14	1.16	0.76	1.89	0.05	0.10	0.40	2.11	0.16	0.40

CHAPTER V

GRAIN CLASSIFICATION RESULTS

5.1 Elements of Pattern Classification

Some basic concepts of pattern classification are best introduced by referring to Figure 5.1. Let X_{ij} represent the j^{th} pattern belonging to class i, i=1, 2, ..., k, where k is the total no. of classes. Generally X_{ij} is in the form of a vector but it can also be a multi-dimensional array. The set of patterns X_{ij}, j=1, 2, ..., N_i is denoted by $\{X_{ij}\}$ and consists of a total of N samples, N_i of which belong to class i, i=1, 2, ..., k. This set of patterns $\{X_{ij}\}$ is called the training set since it is used to "train" or "teach" the classifier to classify a pattern X (whose classification is unknown) as belonging to a particular class. Once a classifier has been trained, it is capable of classifying incoming patterns on its own. The overall performance of the classifier is generally measured in terms of the number of errors it makes. Many different types of training algorithms are available [8]. The choice of a particular algorithm is generally dictated by the nature of the problem which has to be solved.

5.2 The Training Algorithm

The specific algorithm used in the present grain classification study uses the "least-squares mapping" approach. The basic idea is to derive a linear transformation A which maps (in the least-square sense) the training samples belonging to class i into the unit vector V_i, i=1, 2, ..., k, where k is the total number of classes. V_i is a vector, whose elements are all zero, except for the i^{th} element which is unity. Once the classifier is trained, the transformation A is obtained in the form a matrix. Then, to classify a pattern 'X' whose classification is not known, the following steps are used;

(1) Compute Z=AX. Then Z is the mapping of X into the unit vector space.

(2) Find the distance of Z from the unit vectors V_i, i=1, 2, ..., k. If Z falls closest to 'V_{i_o}', then it is decided that 'X' belongs to calss i_o.

For a detailed discussion of the above training algorithm, the reader may consult references [4] and [5]. A listing of the computer program associated with this algorithm is included in Appendix 5-1.

5.3 Classification of Wheat, Soybeans, Barley, Oats, Milo and Rye

The 2-BT power spectrum points tabulated in Appendix 4-2 are used to obtain the training set. The following 20 of the 36 power spectrum points are used.

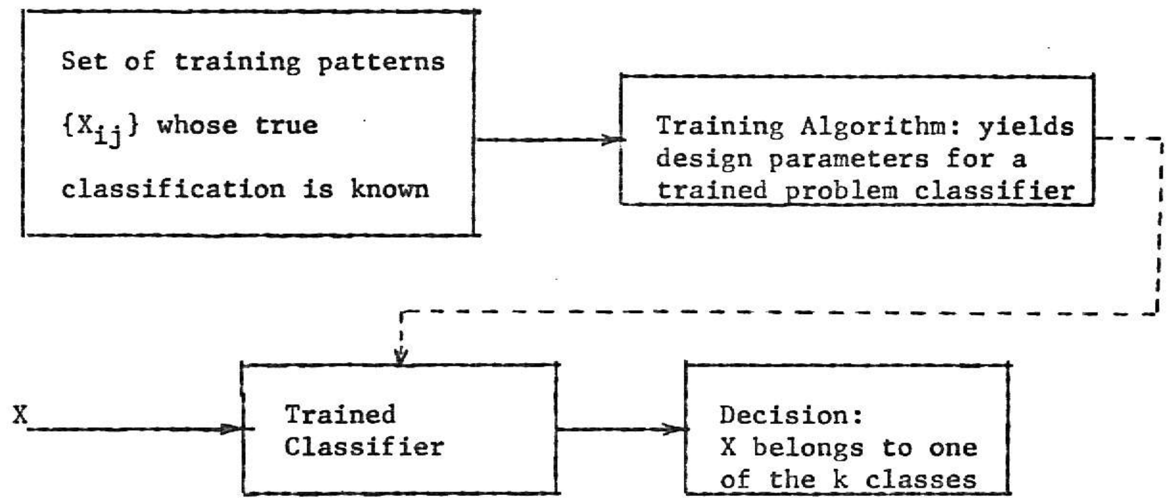
P(0,0), P(4,0), P(5,0), P(5,4), P(0,5)

P(4,5), P(5,5), P(3,3), P(4,3), P(5,3)

P(1,5), P(3,5), P(3,4), P(4,4), P(5,1)

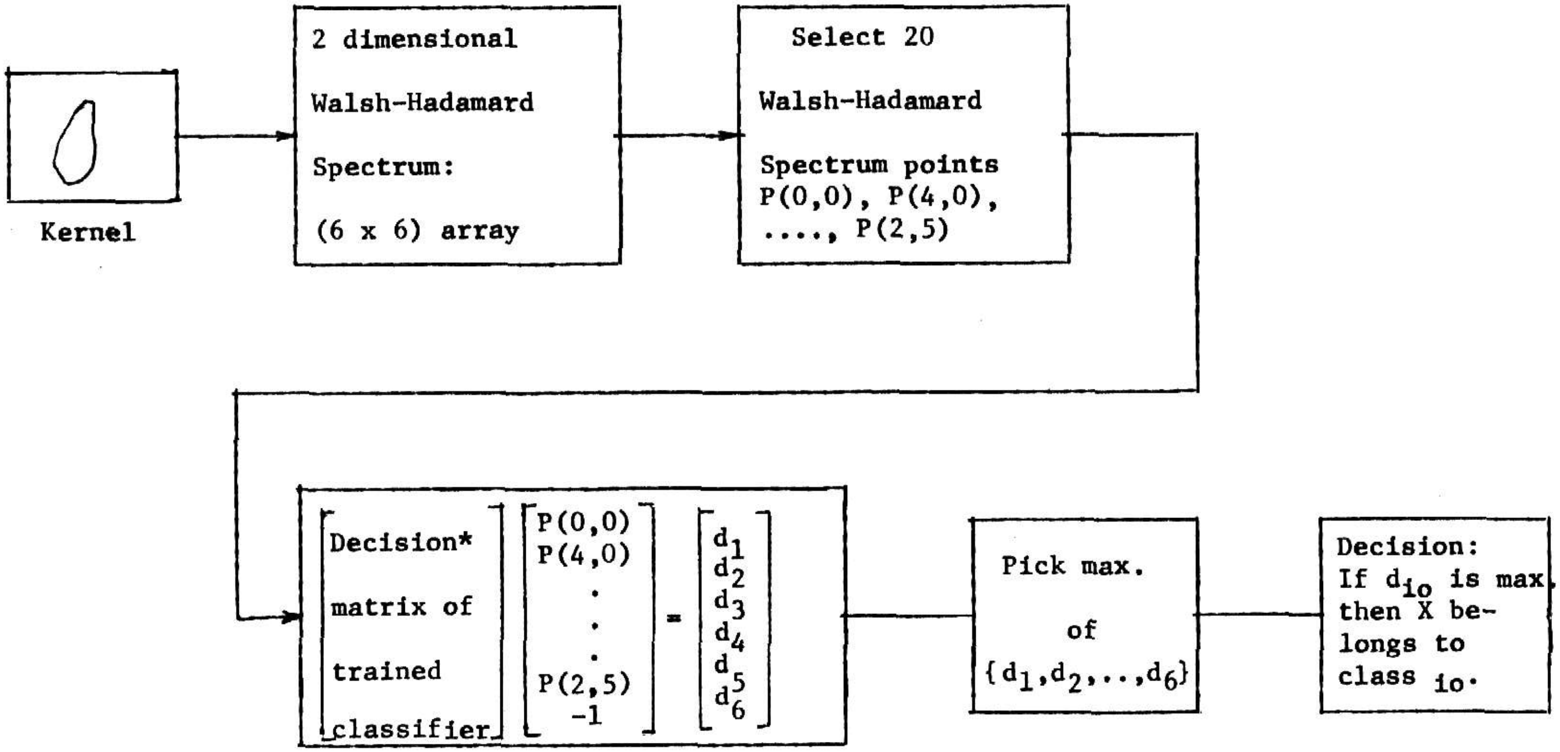
P(3,0), P(5,2), P(0,4), P(2,4), P(2,5)

Thus each X_{ij} in Figure 5-1 is a (20 x 1) vector in this application. The "decision matrix" referred to in Figure 5-2 is directly related to the transformation A, referred to in Section 5.2. It is obtained using the training algorithm as evident from Appendix 5-1 (see page 51). Let class k, k=1, 2, ...,6 denote wheat, soybean, barley, oat, rye and milo respectively.



X: A pattern with unknown classification

Figure 5-1: A pattern classification scheme



*See Appendix 5-1, Page 51

Figure 5-2. Summary of the classification of Wheat, Soybeans, Barley, Oats, Milo and Rye.

5.4 Classification Results

After the decision matrix was obtained, the training samples were classified using the trained classifier as shown in Figure 5-2.

Two varieties of wheat were considered: (1) Hulled Oats and (2) Triumph. The overall results pertaining to both these varieties may be summarized by two confusion matrices F_1 and F_2 listed in Table 5-1.

Table 5-1

$$F_1 = \begin{array}{cl} \begin{bmatrix} 10 & 0 & 2 & 1 & 9 & 8 \\ 0 & 16 & 0 & 0 & 0 & 0 \\ 1 & 1 & 18 & 4 & 2 & 0 \\ 0 & 0 & 3 & 23 & 0 & 0 \\ 3 & 0 & 0 & 0 & 21 & 1 \\ 0 & 0 & 0 & 0 & 0 & 18 \end{bmatrix} & \begin{array}{l} \text{Wheat (hulled oats)} \\ \text{Soybeans} \\ \text{Barley} \\ \text{Oats} \\ \text{Rye} \\ \text{Milo} \end{array} \end{array} \quad (5\text{-}1)$$

$$F_2 = \begin{array}{cl} \begin{bmatrix} 16 & 0 & 2 & 0 & 3 & 4 \\ 0 & 16 & 0 & 0 & 0 & 0 \\ 1 & 1 & 18 & 4 & 2 & 0 \\ 0 & 0 & 3 & 22 & 0 & 1 \\ 0 & 0 & 0 & 0 & 24 & 1 \\ 0 & 0 & 0 & 0 & 0 & 18 \end{bmatrix} & \begin{array}{l} \text{Wheat (triumph)} \\ \text{Soybeans} \\ \text{Barley} \\ \text{Oats} \\ \text{Rye} \\ \text{Milo} \end{array} \end{array} \quad (5\text{-}2)$$

Each element f_{ij}, $i \neq j$ of F_1 and F_2 denotes that f_{ij} kernels of the i-th class are misclassified as belonging to class j.

We make the following observations with respect to the confusion matrices F_1 and F_2:

(1) There is substantial confusion of the hulled oats variety of wheat with milo and rye (see F_1).

(2) There is some confusion between barley and oats (see F_1 and F_2).

(3) Some confusion between the triumph variety of wheat and rye is present (see F_2).

5.5 *Classification of Wheat, Soybean, Barley, Oats and Rye*

Here, milo is ommitted in the interest of comparing[1] the classification results with those of A. R. Edison and W.L. Brogan [1] discussed in Chapter II. The resulting confusion matrix F_3 is as follows:

$$F_3 = \begin{array}{cl} \begin{bmatrix} 20 & 0 & 0 & 0 & 5 \\ 0 & 16 & 0 & 0 & 0 \\ 2 & 1 & 19 & 4 & 0 \\ 0 & 1 & 6 & 19 & 0 \\ 3 & 0 & 1 & 0 & 21 \end{bmatrix} & \begin{array}{l} \text{Wheat (Triumph} \\ \text{Soybeans} \\ \text{Barley} \\ \text{Oats} \\ \text{Rye} \end{array} \end{array} \qquad (5\text{-}3)$$

Following observations can be made with respect to confusion matrix F_3:

(1) There is still some confusion between wheat and rye although it is less than that in the previous case.

(2) Again, confusion between barley and oats is present in this case also.

5.6 Quadratic Classifier Considerations

The specific algorithm used for the grain classification in Section 5.3 uses the "least-squares mapping" approach (see Section 5.2). The approach is equivalent to implementing linear boundaries called hyperplanes in the pattern space. This training procedure can easily be extended to implement quadratic boundaries. The key to designing a quadratic classifier is the notion of a quadratic discriminant function which is defined as

---

[1]This will be discussed in Chapter VI.

$$g_i(X) = \underbrace{\sum_{j=1}^{d} \omega_{jj}x_j^2 + \sum_{j=1}^{d-1}\sum_{k=j+1}^{d-1} \omega_{jk}x_jx_k}_{\text{quadratic terms}} + \underbrace{\sum_{j=1}^{d} \omega_j x_j}_{\text{linear terms}} - \theta, \; i=1,2,\ldots,k \qquad (5\text{-}4)$$

From (5-4) it follows that a quadratic discriminant $g_i(X)$ function has $[(d+1)(d+2)]/2$ parameters or weights consisting of

d parameters as coefficients of X_j^2 terms ω_{jj}

d parameters as coefficients of x_j terms ω_j

$d(d-1)/2$ parameters as coefficients of x_j x_k terms, $j \neq k$ ω_{jk}

and a threshold θ.

The parameters and the threshold listed above are obtained from the training process.

To explain the implementation associated with (5-1), we first define the M-dimensional vector G, whose components $f_1, f_2, \ldots, f_M$ are functions of the x_i, $i=1, 2, \ldots, d$. The first d components of G are $x_1^2, x_2^2, \ldots, x_d^2$; the next $d(d-1)/2$ components are all pairs $x_1x_2, x_1x_3, \ldots, x_{d-1}x_d$; the last d components are $x_1, x_2, \ldots, x_d$. The total number of these components are $M=[d(d+3)]/2$.

Thus for every pattern vector $X' = (x_1, x_2, \ldots, x_d)$ in a d-dimensional space, there is a unique vector $G' = (f_i, f_2, \ldots, f_M)$ in an M dimensional space. In other words a quadratic discriminant function in d-dimensional space is equivalent to a linear discriminant function in an M dimensional space.

A 5-class computer program was used with the following 5 of the 36 power spectrum points instead of 20 used in the case of the linear classifier (see Section 5.3).

P(0,0), P(5,0), P(5,4), P(0,5), P(5,5)

The corresponding confusion matrices F_4 and F_5 are listed in Table 5-3.

Table 5-3

$$F_4 = \begin{bmatrix} 21 & 0 & 0 & 0 & 9 \\ 0 & 16 & 0 & 0 & 0 \\ 0 & 1 & 21 & 4 & 0 \\ 0 & 1 & 6 & 19 & 0 \\ 8 & 0 & 1 & 0 & 16 \end{bmatrix} \begin{array}{l} \text{wheat (hulled oats)} \\ \text{soybean} \\ \text{barley} \\ \text{oats} \\ \text{rye} \end{array} \qquad (5\text{-}5)$$

$$F_5 = \begin{bmatrix} 20 & 0 & 0 & 0 & 5 \\ 0 & 16 & 0 & 0 & 0 \\ 2 & 1 & 19 & 4 & 0 \\ 0 & 1 & 6 & 19 & 0 \\ 3 & 0 & 0 & 0 & 22 \end{bmatrix} \begin{array}{l} \text{wheat (triumph)} \\ \text{soybean} \\ \text{barley} \\ \text{oats} \\ \text{rye} \end{array} \qquad (5\text{-}6)$$

We make the following observations with respect to confusion matrices F_4 and F_5:

(1) There is considerable amount of confusion between both varieties of wheat and rye.

(2) There is still some confusion between barley and oats.

APPENDIX 5-1

This appendix provides a listing of a computer program for a training algorithm which uses the least-squares mapping principle. Details pertaining to the algorithm are available in reference (4) and (5).

The program results in the computation of confusion matrix F_1 with respect to classes C_i, $i = 1, 2, \ldots, 6$ where C_i, $i= 1,2, \ldots, 6$ denotes wheat (hulled oats) soybeans, barley, oats, rye and milo respectively.

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```
FORTRAN IV G LEVEL  18                MAIN              DATE = 72112         15/20/10              PAGE 0001

      C
      C     # OF CLASSES: MAX.  10
      C     #OF SAMPLES IN EACH CLASS: MAX.  100
      C     DIMENSION OF SAMPLES & UNIT VECTORS: MAX.  20
      C     FIRST CARD IN THE MAIN PROGRAM SHOULD BE CHANGED DEPENDING ON THE
      C     COMPILER USED
      C     FIRST DATA CARD: # OF CLASSES,DIMENSIONS OF SAMPLES & UNIT VECTORS IN 3I5
      C     SECOND CARD : # OF SAMPLES IN EACH CLASS IN FORMATS OF I5
      C     THIRD CARD : PROBABILITY OF EACH CLASS IN FORMATS OF F8.4
      C     READ IN UNIT VECTORS , ONE PER CARD, EACH COMPONENT IN F8.4
      C     SAMPLES , ONE PER CARD,WITH COMPONENTS IN F8.4
      C
            DATA NR,NW/5,6/
            DIMENSION NSAMP(10),PROB(10),V(10,10),X(21,100),P(2100,10)
            DIMENSION Y(21,21),XX(21,21),A(10,21),E(10,21),VX(10,21),D(10,100)
            DIMENSION ICF(10,10)
            READ(NR,1)NCLAS,NCOMP,LV
          1 FORMAT(10I5)
            WRITE(NW,2)NCLAS,NCOMP
          2 FORMAT(10X,'THIS PROBLEM HAS',I5,'  CLASSES & EACH CLASS IS',I5,
           1' DIMENSIONAL'///)
            N=NCLAS
            READ(NR,1)(NSAMP(I),I=1,N)
            READ(NR,20)(PROB(I),I=1,N)
          3 FORMAT(///)
            DO 4 I=1,N
          4 WRITE(NW,5)I,NSAMP(I),PROB(I)
          5 FORMAT(10X,'CLASS',I5,'  HAS',I5,'  SAMPLES &  ITS PROBABILITY
           1IS',F10.4)
            L=NCOMP
            DO 10 J=1,N
         10 READ (NR,20)(V(I,J),I=1,LV)
         20 FORMAT(10F8.4)
            WRITE(NW,3)
            WRITE(NW,25)
         25 FORMAT(10X,'  V  MATRIX'///)
            DO 30 I=1,LV
         30 WRITE(NW,35)(V(I,J),J=1,N)
         35 FORMAT(10X,11F10.3)
            L1=L+1
            DO 80 K=1,N
            NI=NSAMP(K)
            FI=FLOAT(NI)
            PI=PROB(K)
            DO 40 J=1,NI
            READ(NR,20)(X(I,J),I=1,L)
         40 X(L1,J)=-1.
            M=1
            DO 41 J=1,NI
            DO 41 I=1,L1
            P(M,K)=X(I,J)
         41 M=M+1
              WRITE(NW,3)
            WRITE(NW,45)K
         45 FORMAT(10X,'AUGMENTED PATTERNS OF CLASS',I5///)
            DO 50 I=1,L1
         50 WRITE(NW,35)(X(I,J),J=1,NI)
            DO 60 I1=1,L
```

```
      DO 60 I2=1,L1
      Y(I1,I2)=0.0
      DO 55 J=1,NI
   55 Y(I1,I2)=Y(I1,I2)+X(I1,J)*X(I2,J)
      Y(I1,I2)=PI*Y(I1,I2)/FI
      IF(K.EQ.1)XX(I1,I2)=0.0
   60 XX(I1,I2)=XX(I1,I2)+Y(I1,I2)
      DO 65 I=1,L1
      DO 65 J=2,NI
   65 X(I,1)=X(I,1)+X(I,J)
      DO 70 I=1,LV
      DO 70 J=1,L1
      A(I,J)=PI*V(I,K)*X(J,1)/FI
      IF(K.EQ.1)VX(I,J)=0.0
   70 VX(I,J)=VX(I,J)+A(I,J)
   80 CONTINUE
      CALL INVERS(XY,L1)
      DO 120 I=1,LV
      DO 120 J=1,L1
      A(I,J)=0.0
      DO 120 K=1,L1
  120 A(I,J)=A(I,J)+VX(I,K)*XX(K,J)
      WRITE(NW,3)
      WRITE(NW,125)
  125 FORMAT(10X,'TRANSFORMATION MATRIX A'//)
      DO 130 I=1,LV
  130 WRITE(NW, 35)(A(I,J),J=1,L1)
      DO  135 I=1,N
      DO  135 J=1,L1
      F(I,J)=0.0
      DO 135 K=1,LV
  135 F(I,J)=F(I,J)+V(K,I)*A(K,J)
        WRITE(NW,3)
      WRITE(NW,140)
  140 FORMAT(10X,'DECISION MATRIX'///)
      DO 142 I=1,N
  142 WRITE(NW,35)(F(I,J),J=1,L1)
      DO 145 I=1,N
      DO 145 J=1,N
  145 ICF(I,J)=0
      DO 185 K=1,N
      NI=NSAMP(K)
      M=1
      DO 150 J=1,NI
      DO 150 I=1,L1
      X(I,J)=P(M,K)
  150 M=M+1
      DO 160 I=1,N
      DO 160 J=1,NI
      D(I,J)=0.0
      DO 160 M=1,L1
  160 D(I,J)=D(I,J)+F(I,M)*X(M,J)
      WRITE(NW,3)
      DO 161 ID=1,N
  161 WRITE(NW,35)(D(ID,JD),JD=1,NI)
      DO 180 J=1,NI
      LS=1
      C=D(1,J)
```

```
FORTRAN IV G LEVEL  18                MAIN                DATE = 72112          15/20/10                PAGE 0003

            DO 170 I=1,N
            IF(C.LT.D(I,J))GO TO 165
            GO TO 170
      165   LS=I
            C=D(I,J)
      170   CONTINUE
      180   ICF(K,LS)=ICF(K,LS)+1
      185   CONTINUE
              WRITE(NW,3)
            WRITE(NW,190)
      190   FORMAT(15X,'CONFUSION MATRIX'///)
            DO 195 I=1,N
      195   WRITE(NW,200)(ICF(I,J),J=1,N)
      200   FORMAT(10X,10I5)
            STOP
            END
```

```
FORTRAN IV G LEVEL  18              INVERS              DATE = 72112          15/20/10              PAGE 0001

             SUBROUTINE INVERS(Z,N)
             DIMENSION U(21,21),V(21,21),Y(21,21),Z(21,21)
             DO 5 I=1,N
             DO 5 J=1,N
             U(I,J)=0.0
        5    IF (I.EQ.J)U(I,J)=1.0
       1     L=1
      10     K=L
        11   C=ABS(Z(K,L))
             IF(Q-0.000001)20,20,30
      20     Z(K,L)=0.0
             K=K+1
             IF(K-N)11,11,90
      30     DO 31 M=1,N
             Y(K,M)=Z(K,M)
             Z(K,M)=Z(L,M)
             Z(L,M)=Y(K,M)
             V(K,M)=U(K,M)
             U(K,M)=U(L,M)
      31     U(L,M)=V(K,M)
      40     Y(L,L)=Z(L,L)
             DO 41 M=1,N
             Z(L,M)=Z(L,M)/Y(L,L)
      41     U(L,M)=U(L,M)/Y(L,L)
             K=1
      50     IF(K-L)52,51,52
      51     K=K+1
             IF(K-N)50,50,70
        52   C=ABS(Z(K,L))
             IF(Q-0.000001)53,53,54
       53    Z(K,L)=0.0
             K=K+1
             IF(K-N)50,50,60
      54     Y(K,L)=Z(K,L)
             DO 55 M=1,N
             Z(K,M)=Z(K,M)-Z(L,M)*Y(K,L)
      55     U(K,M)=U(K,M)-U(L,M)*Y(K,L)
             K=K+1
             IF(K-N)50,50,60
      60     L=L+1
             IF(L-N)10,10,70
       70    DO 80 I=1,N
             DO 80 J=1,N
       80    Z(I,J)=U(I,J)
       90    RETURN
             END
```

THIS PROBLEM HAS 6 CLASSES & EACH CLASS IS 20 DIMENSIONAL

CLASS	1	HAS	30	SAMPLES &	ITS PROBABILITY	IS	0.1667
CLASS	2	HAS	16	SAMPLES &	ITS PROBABILITY	IS	0.1667
CLASS	3	HAS	26	SAMPLES &	ITS PROBABILITY	IS	0.1667
CLASS	4	HAS	26	SAMPLES &	ITS PROBABILITY	IS	0.1667
CLASS	5	HAS	25	SAMPLES &	ITS PROBABILITY	IS	0.1667
CLASS	6	HAS	18	SAMPLES &	ITS PROBABILITY	IS	0.1667

V MATRIX

1.000	0.0	0.0	0.0	0.0	0.0
0.0	1.000	0.0	0.0	0.0	0.0
0.0	0.0	1.000	0.0	0.0	0.0
0.0	0.0	0.0	1.000	0.0	0.0
0.0	0.0	0.0	0.0	1.000	0.0
0.0	0.0	0.0	0.0	0.0	1.000

AUGMENTED PATTERNS OF CLASS 1

10.510	12.390	13.730	11.750	11.120	11.960	12.610	12.610	10.920	9.730	10.920
11.540	6.570	7.050	5.650	6.410	17.900	17.640	15.140	13.280	14.190	16.120
14.900	16.120	16.670	13.280	9.730	9.920	9.540	9.350			
5.440	13.280	4.580	1.010	11.720	5.080	17.650	15.240	11.050	13.910	14.630
10.530	3.440	8.790	9.320	7.080	18.620	10.410	1.660	3.300	4.910	3.880
5.930	1.050	14.670	17.960	3.140	4.440	6.700	11.410			
16.280	26.550	15.660	13.320	23.440	17.430	35.690	30.470	22.600	27.820	29.250
27.280	10.210	17.580	18.630	14.160	37.250	28.470	17.360	17.150	19.680	20.680
25.220	17.430	31.880	35.920	13.260	14.590	16.330	22.810			
7.200	3.170	6.560	12.570	2.440	7.570	4.030	2.810	3.050	2.560	2.610
3.420	8.670	2.440	1.340	4.150	3.300	3.420	10.990	9.520	7.610	6.540
7.690	11.690	7.210	2.690	9.640	8.060	6.100	1.330			
10.640	8.500	13.370	22.290	7.730	11.540	4.810	4.960	7.400	5.840	5.290
7.930	7.580	5.060	4.440	6.160	11.250	14.320	24.960	16.360	17.150	18.340
15.860	27.160	9.540	5.920	16.650	14.560	11.540	8.870			
13.310	12.450	14.650	3.540	12.940	14.160	7.690	10.860	12.570	6.910	9.400
13.920	10.130	10.500	8.420	10.010	12.330	15.380	4.150	10.250	10.500	10.740
12.860	3.170	15.260	9.030	5.980	8.060	11.470	11.350			
25.610	24.930	29.300	27.100	25.880	27.340	15.380	21.730	25.150	17.820	18.610
26.360	20.260	21.000	16.850	20.020	24.660	33.200	30.760	28.810	29.790	31.740
30.520	31.740	29.950	18.570	24.660	24.900	24.410	22.710			
0.500	0.150	0.520	0.140	0.180	0.460	0.260	0.350	0.170	0.260	0.170
0.210	0.170	0.150	0.200	0.400	0.350	0.240	0.150	0.430	0.580	0.790
0.490	0.210	0.110	0.150	0.070	0.580	0.430	0.140			
1.130	0.370	1.530	0.700	0.370	1.220	0.400	0.460	0.820	0.460	0.520
0.430	0.460	0.370	0.270	0.670	0.700	0.490	1.160	0.670	0.850	0.920
1.250	1.160	0.400	0.370	0.260	0.610	0.730	0.340			

1.770	0.730	2.200	1.040	0.730	1.950	0.790	0.920	1.160	0.920	1.040
0.850	0.520	0.730	0.550	1.340	1.400	0.980	0.710	0.590	1.630	2.200
2.010	1.570	0.790	0.730	0.520	0.710	0.590	0.670			
[illegible]	0.190	0.120	0.080	0.180	0.120	0.110	0.140	0.200	0.110	0.170
0.180	0.140	0.180	0.170	0.120	0.140	0.150	0.120	0.120	0.120	0.150
0.140	0.0	0.200	0.120	0.110	0.090	0.120	0.140			
1.260	3.540	0.850	0.920	4.030	1.100	2.010	4.940	3.970	2.260	3.350
3.760	1.900	4.760	3.360	3.170	0.670	2.080	1.220	1.590	1.590	1.950
1.160	1.190	2.360	1.710	1.650	1.710	0.980	1.650			
1.010	0.500	0.670	0.460	0.550	0.980	1.070	0.340	0.640	0.760	0.520
0.830	1.430	0.650	0.340	1.600	1.070	0.310	0.490	0.430	0.370	0.490
0.210	0.550	0.400	0.610	1.010	0.670	0.490	0.460			
5.550	1.590	6.840	1.530	1.220	6.100	2.010	1.400	1.530	1.280	1.400
1.830	6.040	1.220	0.670	2.160	1.650	2.810	0.960	6.030	4.520	5.790
6.290	0.610	1.770	1.340	1.050	2.810	3.780	0.920			
0.110	0.060	0.040	0.080	0.060	0.090	0.050	0.080	0.080	0.050	0.060
0.060	0.140	0.260	0.020	0.060	0.080	0.090	0.030	0.120	0.080	0.040
0.140	0.090	0.060	0.030	0.080	0.120	0.090	0.050			
0.230	0.850	0.340	0.210	0.540	0.330	4.510	2.250	0.600	3.600	3.340
0.190	0.160	1.510	3.480	0.600	0.640	0.240	0.540	0.560	0.560	0.590
0.310	0.220	0.100	3.820	0.390	0.210	0.020	1.740			
0.270	0.240	0.310	0.400	0.160	0.310	0.210	0.150	0.210	0.150	0.270
0.240	0.340	0.150	0.210	0.310	0.210	0.240	0.670	0.430	0.370	0.310
0.470	0.670	0.340	0.160	0.520	0.430	0.320	0.270			
0.450	0.290	0.290	10.410	0.090	0.290	0.680	0.370	0.130	0.280	0.340
0.140	0.860	0.060	0.070	0.090	0.250	0.180	9.320	2.700	2.560	1.890
0.800	10.650	0.220	0.600	6.750	4.330	1.650	0.110			
0.170	0.730	0.210	0.140	0.490	0.180	0.200	0.630	0.690	0.170	0.470
0.320	0.290	0.210	0.200	0.370	0.260	0.090	0.120	0.240	0.310	0.340
0.320	0.150	0.320	0.090	0.170	0.210	0.120	0.260			
0.340	0.730	0.510	0.270	0.980	0.430	0.760	0.820	1.010	0.700	0.560
1.040	0.520	0.550	0.460	0.550	0.270	0.670	0.310	0.090	0.030	0.550
0.520	0.310	0.700	1.280	0.270	0.490	0.310	0.700			
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000			

AUGMENTED PATTERNS OF CLASS 2

65.460	63.480	54.020	53.120	87.560	85.830	79.100	78.010	26.920	27.560	48.260
47.650	44.080	50.890	32.640	35.160						
9.170	15.030	9.870	13.430	2.420	0.750	9.230	13.690	10.360	10.130	9.910
12.260	10.430	9.450	14.470	13.750						
75.990	80.570	67.010	68.480	86.650	75.320	90.150	94.480	38.330	38.940	60.040
62.070	56.560	67.150	48.260	50.110						
7.810	9.030	6.660	7.570	10.130	6.100	10.910	9.520	3.660	3.170	5.960
5.860	4.760	5.740	4.270	5.130						
32.140	24.900	31.650	27.440	39.870	54.410	30.030	24.960	25.670	26.400	31.970
28.950	30.990	37.130	22.450	24.440						
4.560	4.640	6.660	4.390	5.490	4.150	4.680	4.640	10.010	10.010	5.000
3.150	5.740	4.520	9.400	8.300						
41.500	34.670	41.500	36.620	47.610	46.880	38.570	33.690	38.570	39.550	41.260
39.360	41.260	42.240	35.400	36.620						
0.640	0.400	0.430	0.210	0.230	0.370	0.340	0.670	0.430	0.370	0.260
0.180	0.230	0.170	0.470	0.340						

0.850	0.490	1.100	1.160	1.190	1.460	0.730	0.430	0.310	0.310	1.310
1.340	1.310	1.500	0.950	0.850						
1.950	1.340	2.320	1.950	1.390	2.690	1.340	1.460	0.980	0.850	2.260
1.950	1.890	2.260	1.690	1.590						
0.240	0.120	0.090	0.210	0.140	0.150	0.180	0.180	0.090	0.150	0.110
0.230	0.170	0.230	0.170	0.150						
3.370	4.390	2.170	3.910	2.520	2.080	2.440	3.170	2.440	2.690	3.360
4.270	3.720	2.480	2.750	3.300						
1.040	0.730	0.610	0.490	0.640	1.710	1.100	0.790	1.340	1.220	0.340
0.490	0.460	0.270	1.250	1.280						
5.250	5.400	2.910	5.620	4.460	6.590	5.860	4.880	1.590	1.340	4.820
4.390	3.480	4.580	2.330	3.300						
0.120	0.060	0.150	0.060	0.110	0.240	0.120	0.120	0.120	0.060	0.050
0.120	0.110	0.050	0.110	0.060						
1.170	1.930	1.980	1.850	0.080	0.080	1.630	2.550	1.010	1.170	1.740
1.790	1.950	1.700	0.940	1.140						
0.550	0.310	0.370	0.550	0.580	0.610	0.430	0.370	0.370	0.430	0.270
0.310	0.400	0.340	0.400	0.240						
1.170	2.080	0.780	0.920	2.340	0.730	2.640	3.250	0.500	0.470	0.400
0.610	0.370	0.430	0.280	0.230						
0.310	0.610	0.490	0.460	0.250	0.400	0.340	0.550	0.180	0.090	0.350
0.310	0.350	0.410	0.260	0.210						
1.040	0.610	0.730	0.670	0.460	0.730	1.040	0.730	0.370	0.310	0.820
0.550	0.640	0.690	0.640	0.430						
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	-1.000						

AUGMENTED PATTERNS OF CLASS 3

20.610	22.620	21.740	20.890	30.900	29.880	31.590	30.900	21.170	20.330	22.910
27.320	26.600	27.320	28.210	28.210	30.900	37.400	34.700	36.600	43.700	40.900
54.900	37.800	47.900	29.200							
9.940	24.220	17.050	16.000	30.170	8.950	30.840	1.560	1.290	8.290	8.280
24.280	28.510	10.110	0.210	0.610	24.000	2.900	2.500	0.300	16.400	2.200
21.970	1.600	7.100	0.900							
31.170	48.530	35.050	22.370	61.860	40.150	62.900	31.740	22.750	29.390	31.880
48.570	57.020	37.370	21.970	26.460	55.700	19.700	11.900	25.700	61.600	14.200
76.900	14.100	20.300	19.600							
3.340	4.680	2.780	10.740	6.350	3.540	6.100	7.570	11.110	4.150	4.520
5.000	4.760	3.050	11.470	12.450	4.600	5.300	8.700	10.000	5.800	7.000
7.300	11.100	7.600	13.100							
11.340	7.200	10.030	24.750	5.490	10.060	5.310	19.470	24.830	12.790	12.190
8.290	6.150	6.800	40.830	30.910	12.500	43.800	62.600	21.800	2.800	67.200
17.300	61.800	64.000	19.800							
21.120	10.250	26.140	8.540	8.060	27.710	9.520	19.290	8.420	20.260	22.560
10.130	9.160	24.290	4.150	7.570	14.500	4.200	5.700	14.200	29.300	6.000
5.400	5.300	5.900	11.400							
35.400	20.510	37.450	30.130	18.070	41.260	19.040	45.410	36.380	35.640	37.350
20.260	18.310	34.870	37.110	41.990	25.900	28.700	23.700	56.200	35.200	26.600
28.810	28.600	31.300	50.100							
0.380	0.370	0.170	0.980	0.520	0.170	0.210	0.730	0.500	0.550	0.290
0.110	0.260	0.040	0.180	0.610	0.100	-0.250	0.100	1.500	0.200	0.200
0.700	0.400	0.100	1.200							
0.460	0.550	0.580	0.650	0.490	0.640	0.180	1.650	0.950	0.730	0.820
0.270	0.520	0.080	1.530	1.460	0.600	0.700	0.200	0.300	0.900	0.400

0.600	0.500	0.300	0.800							
1.260	1.100	1.040	2.440	1.220	1.040	0.610	2.320	2.010	1.460	1.400
0.530	1.040	0.890	3.170	2.440	1.200	1.800	1.900	3.300	1.300	3.900
1.600	2.500	2.600	1.600							
0.290	0.120	0.080	0.150	0.310	0.170	0.310	0.240	0.170	0.150	0.110
0.230	0.110	0.140	0.060	0.150	0.210	0.230	0.110	0.090	0.050	0.050
0.090	0.030	0.080	0.200							
2.140	1.930	2.750	2.320	3.050	2.260	2.810	2.560	2.530	1.710	1.770
1.160	2.140	2.990	1.460	2.810	1.220	0.280	0.220	2.810	2.400	0.020
3.870	1.160	1.930	3.360							
0.640	1.160	0.580	1.100	2.930	0.700	2.360	1.400	1.620	0.850	0.760
1.620	1.680	0.820	0.980	2.010	0.370	0.380	0.460	1.530	0.830	0.560
0.670	0.460	0.680	0.070							
2.260	2.440	2.380	5.860	3.050	1.770	2.810	7.080	6.040	2.690	2.990
2.570	2.380	1.400	1.950	5.130	2.320	3.230	2.870	12.700	3.910	3.230
4.030	2.620	2.170	14.870							
0.050	0.090	0.080	0.090	0.090	0.110	0.030	0.180	0.110	0.150	0.140
0.050	0.080	0.110	0.120	0.150	0.090	0.620	0.380	0.190	0.120	0.080
0.090	0.090	0.180	0.080							
0.520	1.530	0.190	0.060	0.740	1.300	0.400	0.230	0.150	0.680	0.640
1.770	1.770	0.770	0.160	0.010	0.420	0.500	0.320	0.300	1.670	0.130
1.660	0.190	0.740	0.460							
0.340	0.180	0.340	0.490	0.310	0.340	0.180	0.670	0.400	0.490	0.430
0.270	0.340	0.270	0.370	0.490	0.370	0.930	0.520	0.430	0.430	1.500
0.490	0.700	0.610	0.950							
0.400	0.950	0.550	2.990	0.080	0.720	0.640	0.230	3.140	0.200	0.400
0.510	0.420	0.560	11.760	2.140	1.790	28.730	26.630	0.310	0.260	24.930
2.200	22.480	17.870	0.100							
0.290	0.190	0.200	0.210	0.180	0.260	0.210	0.180	0.260	0.370	0.260
0.290	0.230	0.200	0.150	0.210	0.120	0.180	0.140	0.310	0.340	0.170
0.310	0.170	0.150	0.200							
0.520	0.980	0.460	0.370	1.160	1.070	1.100	0.920	0.460	0.730	0.730
0.460	0.760	0.640	0.370	0.550	0.480	0.370	0.270	0.730	0.790	0.340
0.670	0.340	0.310	0.070							
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000							

AUGMENTED PATTERNS OF CLASS 4

43.270	37.770	38.530	42.460	25.650	26.280	29.680	29.880	31.240	33.710	28.540
31.240	34.070	37.290	35.160	34.790	30.600	35.200	29.300	30.600	26.200	20.300
29.400	37.300	26.600	27.900							
1.860	1.170	1.880	0.690	0.270	0.260	11.440	0.240	10.840	0.930	0.760
0.550	0.360	1.400	0.370	0.360	1.100	2.700	12.300	0.900	31.400	23.100
29.750	2.400	0.900	0.500							
14.020	7.770	11.220	22.480	22.310	16.300	42.280	25.960	43.440	31.460	11.550
16.310	29.770	13.280	31.100	31.110	21.700	29.200	39.200	8.300	62.700	46.100
58.300	28.200	35.300	24.700							
25.760	6.710	29.170	7.930	14.650	11.470	4.270	11.350	3.780	9.770	20.630
19.930	8.670	19.040	8.790	7.930	8.700	7.300	5.300	8.600	4.400	4.200
7.200	9.200	10.200	9.600							
33.190	72.010	31.450	10.670	27.400	44.220	6.940	24.700	2.150	7.390	43.560
27.240	12.680	47.940	10.410	12.040	44.900	27.300	20.200	58.300	5.200	6.100
9.300	7.400	34.700	40.000							

5.980	5.740	7.930	21.610	9.520	4.150	28.200	12.330	30.150	34.670	3.780
6.670	25.760	8.130	30.270	27.950	5.700	8.700	8.700	3.300	7.800	9.800
5.330	8.390	9.700	4.300							
60.790	17.820	54.930	68.120	40.040	30.760	37.840	45.650	39.310	47.360	35.470
49.370	50.540	34.670	50.290	50.540	34.400	27.200	35.200	80.390	75.600	19.590
25.400	23.690	39.600	37.400							
1.120	0.290	1.110	1.660	1.190	0.340	0.280	1.330	0.470	0.670	0.170
1.480	1.420	0.490	0.980	1.020	0.400	0.300	0.400	0.200	0.100	0.400
0.300	0.400	0.600	0.500							
1.310	1.010	0.760	1.130	1.100	1.340	0.890	1.250	0.460	1.890	0.950
0.950	1.070	1.100	1.530	1.620	1.700	2.200	0.400	2.190	0.500	0.600
0.900	0.590	2.300	1.700							
2.500	2.380	2.140	3.970	2.320	1.950	1.400	2.990	1.280	2.560	2.360
2.140	2.750	1.340	2.930	2.990	3.200	4.700	2.100	3.100	1.300	1.100
2.100	2.190	10.800	2.600							
0.120	0.110	0.140	0.110	0.120	0.090	0.170	0.110	0.260	0.240	0.110
0.200	0.110	0.090	0.210	0.140	0.050	0.050	0.150	0.820	0.340	0.160
0.200	0.200	0.020	0.050							
3.230	1.160	7.990	1.280	2.560	1.590	1.890	3.230	5.430	1.710	1.260
2.990	2.380	1.710	1.950	2.010	1.900	2.700	3.100	0.900	2.100	2.400
3.400	1.200	2.300	1.300							
0.390	0.580	1.620	1.980	1.100	0.920	0.460	1.310	0.400	0.920	0.640
1.430	0.790	0.790	0.430	0.400	0.700	0.900	2.200	0.600	1.040	0.900
0.350	0.810	0.600	0.500							
10.070	2.750	9.690	17.030	7.810	2.080	2.260	9.950	2.500	5.960	1.770
11.660	9.950	7.200	8.540	9.220	2.800	1.600	3.300	2.300	2.200	2.100
2.100	3.170	2.200	1.700							
0.170	0.290	0.170	0.140	0.150	0.180	0.110	0.140	0.050	0.090	0.200
0.200	0.170	0.270	0.150	0.110	0.050	0.050	0.090	0.040	0.030	0.060
0.050	0.010	0.090	0.110							
0.030	0.160	0.030	0.330	0.070	0.300	0.940	0.190	1.260	0.110	0.040
0.240	0.100	0.240	0.040	0.060	0.600	0.100	0.600	0.300	2.600	2.500
3.100	3.270	0.970	0.090							
0.700	0.750	0.520	0.400	0.610	0.850	0.520	0.340	0.520	0.550	0.640
0.700	0.400	0.730	0.490	0.580	0.830	0.500	0.900	0.900	0.200	0.300
0.600	0.500	0.200	0.300							
0.110	33.940	0.180	1.550	1.570	17.530	1.200	0.210	0.180	1.500	14.030
0.190	1.090	14.940	1.680	1.520	13.700	10.200	0.600	27.300	0.700	2.300
0.630	0.600	17.200	11.300							
0.470	0.170	0.170	0.230	0.160	0.180	0.290	0.320	0.630	0.340	0.230
0.230	0.670	0.090	0.460	0.500	0.200	0.370	0.370	0.200	0.400	0.200
0.370	0.250	0.300	0.200							
0.710	0.340	0.700	1.250	0.430	0.240	0.640	0.400	1.310	0.430	0.340
0.210	0.820	0.310	0.610	0.580	0.400	0.400	0.200	0.200	0.200	1.000
1.220	1.000	0.500	0.500							
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000							

AUGMENTED PATTERNS OF CLASS 5

15.630	14.120	14.900	14.660	8.430	9.920	8.250	8.610	14.930	12.830	14.430
3.530	6.570	7.220	8.250	6.570	8.970	9.160	12.180	7.390	8.070	12.180
19.920	11.330	8.610								
10.620	23.160	0.370	0.150	1.130	3.170	0.830	12.440	0.870	15.090	1.060

0.470	6.970	2.030	11.470	4.830	9.790	0.980	4.740	8.960	13.570	10.990
4.540	0.630	6.100								
26.580	46.330	17.810	12.210	9.800	13.640	9.230	24.590	1.390	30.180	15.640
13.760	13.930	10.420	22.930	11.490	19.580	10.350	17.440	17.910	27.130	23.270
15.880	12.010	14.790								
3.170	3.910	14.530	12.210	11.470	9.280	11.350	2.560	6.100	2.207	12.330
12.320	4.760	9.160	2.080	7.200	3.050	11.960	7.690	2.440	2.690	3.050
8.180	12.820	6.470								
6.680	3.630	21.710	29.240	13.890	10.010	17.780	4.200	9.520	5.800	20.130
26.570	5.730	9.020	5.360	7.060	5.200	19.410	9.810	5.190	3.630	7.130
1[illegible].790	17.680	8.630								
18.830	6.100	6.470	3.420	7.320	12.940	3.780	8.910	18.070	1.470	7.690
2.810	10.130	10.620	9.640	10.130	11.840	3.420	15.990	10.740	7.080	15.010
13.550	6.710	17.550								
31.250	12.210	30.520	26.370	22.950	24.900	22.710	17.820	29.790	22.950	30.030
27.590	20.260	21.240	19.290	20.260	23.680	23.930	27.590	21.480	14.160	27.590
26.120	26.610	27.190								
0.157	0.210	0.780	0.340	0.240	0.210	0.200	0.170	0.790	0.310	0.560
0.170	0.320	0.170	0.290	0.230	0.110	0.150	0.630	0.090	0.240	0.290
0.320	0.470	0.350								
0.730	0.370	0.640	1.530	0.790	1.040	0.640	0.460	0.670	0.550	0.210
1.130	0.640	0.520	0.460	0.560	0.400	0.550	1.010	0.430	0.370	0.460
1.190	0.580	0.760								
1.100	0.730	1.280	1.220	1.220	1.340	1.530	0.920	1.830	1.100	1.400
1.340	1.280	1.160	0.920	1.160	0.790	1.220	2.010	0.850	0.730	0.920
1.650	1.280	1.650								
0.270	0.090	0.110	0.060	0.050	0.120	0.060	0.290	0.240	0.120	0.080
0.080	0.110	0.110	0.110	0.110	0.200	0.060	0.170	0.160	0.150	0.110
0.170	0.110	0.200								
4.870	1.130	1.900	1.100	1.460	1.340	0.790	3.970	1.340	4.800	1.770
0.920	3.600	1.160	3.600	2.260	5.550	0.850	0.920	4.640	2.690	4.330
1.280	1.770	2.230								
1.400	1.220	0.950	0.500	0.850	1.040	0.400	0.460	1.710	0.370	1.010
0.460	1.860	0.700	0.400	1.920	0.950	0.430	1.060	0.920	0.670	1.190
1.070	1.190	2.270								
0.850	1.960	4.940	1.340	5.250	7.450	1.900	1.260	3.910	1.100	5.920
0.920	2.380	6.770	1.040	4.460	1.530	1.710	5.660	1.220	1.340	0.920
6.650	5.310	2.620								
0.090	0.060	0.140	0.240	0.090	0.090	0.140	0.050	0.090	0.030	0.050
0.200	0.080	0.110	0.050	0.060	0.050	0.090	0.110	0.030	0.030	0.060
0.140	0.110	0.080								
0.220	6.390	0.020	0.110	0.240	0.510	0.130	3.550	0.250	1.650	0.100
0.150	0.360	0.160	2.850	0.040	0.760	0.140	0.470	1.340	5.390	0.090
0.350	0.030	0.030								
0.310	0.240	0.760	0.490	0.370	0.550	0.460	0.150	0.370	0.180	0.550
0.460	0.210	0.490	0.150	0.340	0.210	0.310	0.340	0.240	0.180	0.270
0.270	0.400	0.210								
0.260	0.470	6.050	14.070	5.230	0.410	8.900	0.330	0.170	0.260	5.120
12.760	0.140	1.460	0.110	0.400	0.190	9.700	0.040	0.060	0.230	0.150
4.300	6.020	0.110								
0.550	0.150	0.080	0.180	0.120	0.310	0.140	0.380	0.240	0.400	0.230
0.120	0.320	0.140	0.380	0.350	0.380	0.090	0.080	0.210	0.370	0.630
0.300	0.230	0.350								
0.730	1.280	0.340	0.370	0.240	0.490	0.270	0.460	0.610	0.790	0.400
0.210	0.640	0.340	0.580	0.700	0.890	0.180	0.700	0.730	0.610	1.010
0.340	0.340	0.580								
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000								

AUGMENTED PATTERNS OF CLASS 6

8.610	7.900	5.080	5.950	6.100	7.900	9.350	14.190	7.220	9.920	6.260
9.540	6.570	8.770	7.900	7.720	7.700	9.400				
7.790	7.090	5.390	5.710	6.440	6.520	7.990	7.750	6.390	8.560	6.720
6.440	5.620	7.760	6.220	6.940	6.700	7.900				
16.470	15.030	10.790	13.600	12.880	14.540	17.470	22.260	13.690	18.590	13.440
16.520	12.310	17.800	14.160	14.840	14.500	17.400				
4.760	5.250	5.740	5.740	5.130	5.980	5.000	5.370	5.740	4.640	4.760
5.860	6.230	5.000	5.980	5.370	5.600	5.000				
11.120	10.880	7.770	9.020	8.510	11.220	11.920	17.430	10.240	12.240	8.410
12.480	9.960	7.700	11.310	10.440	10.700	11.800				
10.360	10.130	8.910	9.160	9.770	9.400	10.360	10.010	9.400	10.990	9.890
9.770	9.160	10.360	9.400	9.770	9.800	10.800				
23.170	22.220	17.820	19.290	19.530	22.220	24.170	29.790	21.240	24.900	19.760
24.410	20.260	27.680	22.220	21.970	22.000	24.200				
0.360	0.380	0.290	0.410	0.340	0.260	0.200	0.180	0.470	0.150	0.320
0.240	0.260	0.260	0.500	0.430	0.400	0.140				
0.640	0.820	0.520	0.320	0.670	0.640	0.580	0.980	0.640	0.670	0.700
0.920	0.640	0.790	0.890	0.610	0.700	0.640				
1.540	1.770	1.040	1.280	1.340	1.400	1.280	1.710	1.530	1.340	1.430
1.590	1.280	1.530	1.770	1.460	1.600	1.300				
0.140	0.140	0.060	0.050	0.030	0.170	0.200	0.150	0.110	0.150	0.110
0.160	0.080	0.170	0.080	0.120	0.120	0.140				
1.160	0.790	0.790	0.790	0.980	1.040	1.280	1.710	1.160	1.100	1.040
1.220	0.790	1.040	1.040	1.220	0.980	1.160				
0.580	0.340	0.210	0.210	0.310	0.640	0.690	1.220	0.520	0.790	0.270
0.450	0.400	0.640	0.460	0.610	0.490	0.760				
1.770	2.140	2.870	2.620	2.560	2.140	1.650	1.100	2.260	1.590	2.380
1.720	2.420	1.770	2.140	2.060	2.200	1.890				
0.060	0.020	0.050	0.080	0.060	0.080	0.060	0.060	0.080	0.030	0.020
0.150	0.110	0.080	0.080	0.090	0.060	0.050				
0.090	0.030	0.260	0.100	0.280	0.060	0.070	0.210	0.070	0.070	0.400
0.020	0.070	0.040	0.030	0.110	0.250	0.100				
0.340	0.150	0.210	0.400	0.120	0.210	0.340	0.370	0.210	0.310	0.150
0.220	0.340	0.270	0.210	0.240	0.180	0.400				
2.310	2.620	2.480	2.530	2.150	2.940	2.190	2.750	2.800	1.940	1.910
3.040	3.060	2.330	3.200	2.500	2.700	2.110				
0.170	0.110	0.140	0.050	0.090	0.200	0.230	0.270	0.110	0.210	0.140
0.210	0.110	0.220	0.170	0.120	0.180	0.160				
0.450	0.270	0.270	0.270	0.240	0.400	0.400	0.490	0.340	0.430	0.340
0.490	0.270	0.400	0.400	0.430	0.430	0.400				
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000				

TRANSFORMATION MATRIX A

-0.018	-0.008	0.007	-0.000	0.011	-0.001	-0.002	-0.049	0.027	-0.080	-0.078
-0.038	-0.085	0.021	-0.188	0.017	0.029	-0.008	0.739	0.161	-0.270	
-0.004	-0.021	0.015	-0.019	0.016	-0.011	0.002	-0.003	-0.013	-0.042	0.121
0.013	-0.015	-0.002	0.119	0.005	-0.206	-0.027	-0.069	-0.093	-0.145	

-0.007	0.027	0.005	0.006	0.012	0.030	-0.003	-0.209	-0.233	0.057	-0.065
0.044	0.033	0.042	0.992	-0.047	0.167	0.010	-0.778	0.048	0.588	
0.019	0.025	-0.018	0.026	-0.014	0.011	0.009	0.384	0.265	0.052	-0.250
0.022	0.023	-0.052	-0.526	0.079	0.242	0.022	0.136	-0.072	0.728	
-0.013	-0.034	0.009	0.012	-0.012	-0.007	-0.003	-0.384	-0.004	-0.035	-0.070
0.092	0.240	0.038	0.082	0.077	0.006	0.027	0.035	0.042	-0.259	
0.023	0.011	-0.017	-0.025	-0.012	-0.023	-0.004	0.261	-0.042	0.048	0.362
-0.134	-0.196	-0.047	-0.418	-0.139	-0.239	-0.023	-0.063	-0.086	-1.641	

DECISION MATRIX

-0.018	-0.008	0.007	-0.000	0.011	-0.001	-0.002	-0.049	0.027	-0.000	-0.078
-0.138	-0.085	0.021	-0.188	0.027	0.029	-0.008	0.739	0.161	-0.270	
-0.004	-0.021	0.015	-0.019	0.016	-0.011	0.002	-0.003	-0.013	-0.042	0.121
0.013	-0.015	-0.002	0.119	0.003	-0.206	-0.027	-0.069	-0.093	-0.145	
-0.007	0.027	0.005	0.006	0.012	0.030	-0.003	-0.209	-0.233	0.057	-0.065
0.044	0.033	0.042	0.992	-0.047	0.167	0.010	-0.778	0.048	0.588	
0.019	0.025	-0.018	0.026	-0.014	0.011	0.009	0.384	0.265	0.052	-0.250
0.022	0.023	-0.052	-0.586	0.079	0.242	0.022	0.136	-0.072	0.728	
-0.013	-0.034	0.009	0.012	-0.012	-0.007	-0.003	-0.384	-0.004	-0.035	-0.070
0.092	0.240	0.038	0.082	0.077	0.006	0.027	0.035	0.042	-0.259	
0.023	0.011	-0.017	-0.025	-0.012	-0.023	-0.004	0.261	-0.042	0.048	0.362
-0.134	-0.196	-0.047	-0.418	-0.139	-0.239	-0.023	-0.063	-0.086	-1.641	

0.226	0.622	0.299	0.279	0.454	0.256	0.357	0.513	0.584	0.329	0.459
0.452	0.370	0.189	0.354	0.269	0.171	0.193	0.303	0.392	0.308	0.370
0.432	0.232	0.262	0.384	0.248	0.324	0.296	0.425			
-0.012	0.057	0.018	-0.027	0.070	-0.013	0.056	0.070	0.013	0.100	0.072
0.015	-0.091	0.101	0.137	-0.035	0.130	0.724	0.069	0.120	0.126	0.130
0.072	0.027	0.105	0.065	-0.017	0.001	0.047	0.106			
0.144	-0.089	0.044	-0.086	0.070	0.150	0.110	-0.073	-0.159	0.031	-0.069
0.081	0.177	0.098	-0.158	-0.100	0.292	0.451	-0.194	0.094	-0.028	-0.019
0.086	-0.127	0.426	0.231	0.050	-0.003	0.117	0.052			
0.076	-0.021	0.196	0.089	-0.078	0.090	0.112	0.167	0.071	0.049	0.119
-0.045	-0.299	-0.075	-0.024	0.090	0.067	-0.167	0.276	-0.083	0.150	0.154
0.055	0.353	-0.082	-0.036	-0.048	0.061	-0.090	-0.130			
0.250	0.215	0.119	0.458	0.251	0.246	0.487	0.344	0.314	0.377	0.418
0.311	0.757	0.611	0.499	0.550	-0.256	-0.099	0.377	0.321	0.063	0.091
0.323	0.334	0.303	0.272	0.577	0.254	0.071	0.118			
0.316	0.217	0.324	0.287	0.233	0.271	-0.122	-0.021	0.177	0.115	-0.029
0.178	0.086	0.074	0.192	0.226	0.596	0.398	0.170	0.156	0.382	0.274
0.332	0.181	0.285	0.085	0.190	0.363	0.559	0.429			

-0.041	0.136	0.192	0.197	-0.358	-0.149	-0.076	-0.040	0.111	0.057	0.231
0.088	0.210	0.267	0.071	0.030						
0.933	0.813	0.874	0.741	1.240	1.378	1.003	0.926	0.558	0.604	0.845
0.830	0.825	0.896	0.470	0.602						
0.165	0.069	-0.024	0.090	0.036	0.084	0.138	-0.006	0.332	0.334	-0.008
0.149	0.028	-0.106	0.286	0.291						

0.017	0.059	0.104	0.072	0.037	-0.188	-0.091	0.170	-0.087	-0.082	0.006
0.023	0.052	0.005	0.141	0.022						
-0.036	0.159	-0.087	0.094	-0.205	-0.243	0.122	-0.002	-0.012	-0.015	-0.050
0.175	-0.006	-0.051	-0.099	0.054						
-0.029	-0.237	-0.058	-0.194	0.250	0.117	-0.097	-0.048	0.099	0.101	-0.015
-0.165	-0.109	-0.010	0.130	0.301						
0.112	0.137	0.092	0.029	-0.171	0.155	-0.073	0.059	0.199	0.238	0.194
0.236	0.040	0.059	0.032	0.049	0.022	-0.116	0.065	-0.551	-0.017	-0.037
-0.158	0.048	-0.025	0.166							
0.133	0.036	0.100	-0.096	0.029	0.114	0.059	0.207	0.246	0.081	0.092
0.179	0.050	0.116	0.245	0.363	0.167	-0.181	0.035	0.195	0.021	-0.141
0.526	0.039	0.371	0.098							
0.409	0.549	0.560	0.492	0.810	0.519	0.844	0.338	0.133	0.275	0.437
0.476	0.663	0.672	0.048	0.088	0.789	0.656	0.974	0.424	0.706	0.878
0.470	0.549	0.768	0.331							
0.121	0.091	0.107	0.559	0.140	0.266	-0.070	0.392	0.042	0.243	0.192
-0.067	0.137	0.004	0.369	0.303	-0.010	0.193	-0.062	0.368	0.537	0.477
0.356	0.431	0.035	0.386							
-0.092	-0.031	-0.021	-0.327	0.234	0.059	0.158	0.240	0.534	-0.057	0.019
0.141	0.093	0.177	0.229	0.432	-0.427	0.274	0.026	0.118	0.052	-0.266
-0.165	-0.139	-0.017	0.144							
0.326	0.219	0.162	0.332	-0.042	-0.114	0.082	-0.236	-0.157	0.171	0.067
0.176	0.057	-0.028	0.076	-0.235	0.458	-0.027	-0.038	-0.054	-0.299	0.058
0.040	0.071	-0.131	-0.126							
0.061	0.037	-0.175	-0.226	0.118	0.118	0.079	0.082	0.263	-0.051	0.079
-0.005	0.244	-0.002	0.091	0.164	0.107	-0.150	-0.024	-0.061	0.053	0.197
0.134	-0.037	-0.408	0.182							
-0.083	-0.141	-0.150	-0.216	0.210	0.025	-0.016	0.305	-0.070	-0.191	-0.112
0.065	-0.011	-0.014	-0.090	-0.053	0.153	0.002	0.250	-0.001	0.299	-0.015
-0.026	-0.001	-0.045	0.337							
-0.370	0.769	0.296	0.401	0.172	0.199	0.511	0.093	0.399	0.259	0.236
0.235	0.125	0.393	0.236	0.135	0.064	-0.200	0.381	-0.094	0.246	0.414
0.469	-0.287	0.279	-0.172							
1.241	0.458	1.076	0.701	0.421	0.501	0.516	0.448	0.538	1.031	0.620
0.726	0.711	0.686	0.823	0.824	0.628	0.862	0.224	1.204	0.620	0.228
0.577	0.485	1.035	0.397							
-0.006	0.073	0.133	0.113	0.129	0.334	-0.154	0.155	0.211	0.065	0.190
0.171	-0.066	0.104	-0.076	-0.066	0.089	0.169	0.272	-0.057	-0.226	0.090
-0.029	0.358	-0.040	-0.022							
-0.064	-0.196	-0.183	0.029	-0.050	-0.177	0.065	-0.084	-0.391	-0.112	-0.012
-0.195	-0.003	-0.166	0.016	-0.004	-0.040	0.296	-0.103	0.009	0.307	0.066
-0.132	0.483	0.177	0.278							
0.260	0.448	0.155	0.275	0.263	0.457	0.250	0.388	0.020	0.336	0.303
0.219	0.201	0.329	0.422	0.342	0.283	0.221	0.108	0.210	0.030	0.439
0.165	0.280	0.037								
0.028	0.031	-0.043	-0.049	-0.092	-0.108	-0.111	0.122	-0.223	0.193	0.006
-0.109	-0.036	-0.092	0.104	-0.112	0.037	-0.053	-0.079	0.061	0.092	-0.016
0.008	-0.047	-0.107								

0.243	0.195	0.190	-0.091	0.075	0.119	-0.031	-0.139	-0.032	-0.227	0.205
-0.040	0.015	0.266	-0.145	0.051	0.129	-0.041	0.296	0.109	-0.270	0.039
0.293	0.063	0.092								
0.232	0.146	0.261	0.348	-0.029	-0.047	0.033	0.062	0.473	0.061	-0.033
0.193	0.014	-0.293	0.084	-0.131	-0.046	0.001	0.142	-0.060	0.148	0.114
-0.077	0.012	0.160								
0.451	0.516	0.403	0.405	0.643	0.589	0.431	0.539	0.176	0.332	0.493
0.436	0.493	0.527	0.385	0.754	0.644	0.460	0.386	0.615	0.624	0.359
0.408	0.640	0.950								
-0.104	-0.337	0.035	0.113	0.140	-0.011	0.429	0.029	0.586	0.306	0.026
0.205	0.113	0.263	0.150	0.097	-0.048	0.411	0.146	0.065	-0.124	0.065
0.183	0.051	-0.132								

0.270	0.191	0.312	0.210	0.221	0.266	0.252	0.225	0.182	0.266	0.273
0.236	0.238	0.154	0.230	0.194	0.261	0.236				
-0.007	-0.001	-0.050	-0.078	-0.012	-0.008	0.024	0.116	-0.023	0.028	-0.013
0.008	-0.058	-0.144	-0.039	-0.014	-0.023	0.000				
0.003	-0.024	-0.046	0.103	0.014	0.017	0.093	0.020	0.044	0.071	-0.064
0.073	0.094	-0.072	-0.051	0.085	-0.025	0.139				
0.038	0.030	-0.146	-0.052	-0.069	-0.086	-0.078	0.118	-0.001	-0.045	-0.046
0.003	-0.094	0.101	0.107	-0.024	0.010	-0.058				
0.019	-0.041	0.159	0.061	0.095	0.162	0.158	0.195	0.076	0.093	0.078
0.184	0.162	0.180	0.029	0.057	0.067	0.147				
0.676	0.244	0.771	0.756	0.751	0.649	0.551	0.327	0.723	0.586	0.775
0.495	0.659	0.783	0.724	0.672	0.709	0.536				

CONFUSION MATRIX

10	0	2	1	9	8
1	14	0	0	0	0
2	1	18	4	2	0
0	0	3	23	0	0
3	0	0	0	21	1
0	0	0	0	0	18

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

6.1 Conclusions

While the pattern recognition approach considered in this report works satisfactorily with respect to most types of grain, it has difficulty distinguishing between the following:

(1) milo and rye with respect to the two types of wheat considered, and

(2) barley and oats.

This conclusion is in agreement with the results of Edison and Brogan (see Table 2-2) in that wheat and barley are the most difficult to separate relative to the other types of grain.

6.2 Recommendations For Future Work

On the basis of the results obtained from the present study, the following recommendations are made for future work:

(1) Obtain % correct classification scores using the same varieties of grain kernels considered by Edison and Brogan, (see Table 2-1). These can be obtained from the U. S. Marketing Research Center, Manhattan, Kansas. The number of kernels in each type of grain should be approximately the same as those used by Edison and Brogan (see page 6).

(2) It is recalled that the notion of a "learning parameter" α was introduced by Edison and Brogan [see Eq. 2-7]. An analogous effect can be obtained in the case of the pattern classifier used in this study. Basically it involves perbubation of boundaries

implemented in the pattern space. A method to accomplish this is suggested by Powell [4].

(3) In conclusion, it is remarked that the main advantage of the method proposed in this report over that considered by Edison and Brogan is that it is simpler to implement. This is because it avoids the need to measure the length, width and projected area of a given kernel prior to its classification.

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AUTOMATIC GRAIN CLASSIFICATION CONSIDERATIONS

by

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AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1972

ABSTRACT

All forms of grain sample analysis include a determination of the amount of material in the sample which is not the grain being analysed. The methods which are presently employed make use of various sieves of appropriate sizes, followed by a manual hand-picking of the remainder. The hand-picking process is carried out seperately since it is both tedious and time consuming. To this end, it is desirable to automate the hand-picking part of the analysis by an appropriate scheme. Such a scheme should possess the capability of distinguishing between a variety of grains which includes wheat, barley, oats, rye, and soybeans.

The purpose of this report is two-fold. First, it presents a review of the pertinent literature. Second, it extends an initial feasibility study which was reported recently by Vyas. The approach entertained by Vyas was based on pattern recognition techniques. The types of grains considered were:

(1) corn, (2) wheat, (3) barley, (4) oats and (5) milo.

In this study, two additional types of grain namely rye and soybeans are also considered. Again, the study by Vyas was restricted to linear classifiers. In this report, quadratic classifiers are also considered.

The results of this study show that certain varieties of wheat, barley, and oats are the most difficult to separate. This is in agreement with the results of a study conducted by Edison and Brogan at the University of Nebraska.

The classification results presented in this report were obtained from the analysis of a small number of grain samples. Thus it is recommended that

the method used in this study be applied to classify a large number of grain samples consisting of the various varieties of wheat, oats, barley, rye and soybeans considered by Edison and Brogan. The corresponding results can be compared with those obtained by Edison and Brogan.