INVESTIGATION OF LOCALIZED BENDING MOMENT IN AN UPRIGHT CYLINDRICAL CONTAINER SUBJECTED TO HYDROSTATIC LOADING

by

KARL J. SVATY, JR.

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Approved by:

Frank McCormick

Major Professor

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NOMENCLATURE

a - radius of the container or tank

b - inner radius of plate in contact with base

β - parameter of shell

d - depth of container or tank

D - flexural rigidity of plate

D_s - flexural rigidity of shell

E - Young's modulus

γ - specific weight of liquid in the container

h_p - thickness of plate

h_s - thickness of shell

M_r - radial moment in the plate

M_θ - tangential moment in the plate

M_x - vertical moment in the shell

M_ϕ - horizontal moment in shell

N_ϕ - resultant of hoop stresses in the shell

N_x - vertical resultant in the shell

q - hydrostatic pressure on the plate

Q_r - shear in the plate

Q_x - shear in the shell

ν - Poisson's ratio (taken as 0.3)

R - r/a

r - independent variable in the plate

w - vertical deflection of the plate

w_s - horizontal deflection of the shell

x - independent variable in the shell

INTRODUCTION

This investigation was concerned with determining the localized bending moment at the connection between the base and wall of an upright cylindrical tank. The tank was subjected to a hydrostatic loading. To determine the moment at the connection, the theory of thin plates and shells with small deflection was used. The equations of the plate and shell were then solved using assumed boundary conditions and a computer-aided graphic solution. The deflection curve of the base plate was then verified experimentally.

The general procedure of this report was to consider the small rigid rotation of the connection of the base and wall. Using boundary conditions derived from the consideration of the small rotation, the moment at the connection was determined.

The results were then compared with approximate solutions prepared by Hofmann [5] and Den Hartog [2].

DERIVATION OF EQUATIONS

1. Thin Plate Theory

In the analysis of the bottom of the container, thin plate theory with small deflections was used. The assumptions used in derivation were:

1) Planes perpendicular to the middle surface remain perpendicular after bending.
2) The stresses in the transverse (z) direction are small and were neglected.
3) The middle surface of the plate is a neutral surface.
4) The vertical displacement of any point in the plate is the same as the vertical displacement of a point directly above or below the middle surface; this results from the second assumption, considering the thickness of the plate to be small in comparison to the radius.

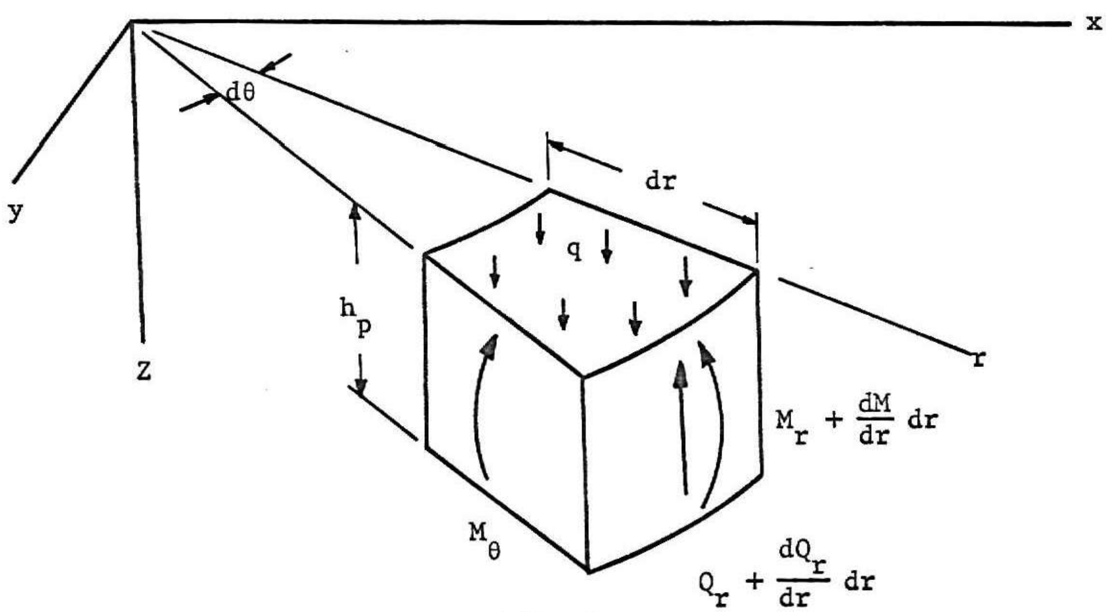

Fig. 1

Element of the Plate

Two stress resultants are not shown because they vanish due to symmetry. These resultants are the twisting moment and the shear on the θ face.

Summing moments on an element of the plate, an expression was obtained relating the moments and shear. The equations relating the moment and deflection were substituted into the equation determined above from the element (Fig. 1). This results in the governing differential equation for an axially symmetric plate.

$$\frac{1}{r}\frac{d}{dr}\ r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{q}{D} \tag{1}$$

This differential equation was then integrated and cast into dimensionless form

$$\frac{D}{qa^4}w = C_1 + C_2\ \mathrm{logeR} + C_3R^2 + C_4R^2\ \mathrm{logeR} + \frac{1}{64}R^4 \tag{2}$$

where D is the flexural rigidity of the plate

$$D = \frac{Eh_p^3}{12(1 - \nu^2)} \tag{a}$$

where E is Young's modulus, h_p is the thickness of the plate and ν is Poisson's ratio. Also, in the above expression for deflection, q represents the hydrostatic pressure on the plate, a is the radius of the tank or plate and R represents the dimensionless ratio of r/a. The quantity r is the independent parameter of the plate. The constants of integration are shown as C_1 through C_4 in Eq. (2).

The equations for slope and moment in dimensionless form, as they were used later, are

$$\frac{D}{qa^3}\frac{dw}{dr} = C_2\frac{1}{2R} + C_3 2R + C_4R(2\mathrm{logeR} + 1) + R^3/16 \tag{3}$$

$$\frac{M_r}{qa^2} = C_2\frac{(\nu-1)}{R^2} + C_3 2(1+\nu) + C_4\left[2(1+\nu)\ \log_e R + 3 + \nu\right] + (3+\nu)\ R^2/16 \tag{4}$$

These expressions for the plate were used in conjunction with the shell equations, which are presented next.

For a more complete derivation of the equations of a plate see Timoshenko [7].

2. Thin Shell Theory

The same assumptions used for the thin plate were used in the derivation of the equations of the shell. However, there was no longer a neutral surface because of hoop stresses in the shell which, by the theory of thin shell were superimposed on the bending stresses [8].

A free-body diagram of an element of the shell is shown in Fig. 2. Since the shell is axially symmetric, there are two stress resultants that vanish, the twisting moments and the shear on a vertical section.

The equilibrium equations of the shell has four unknowns, so an additional relationship was needed. This was a stress-strain relationship for the hoop stresses.

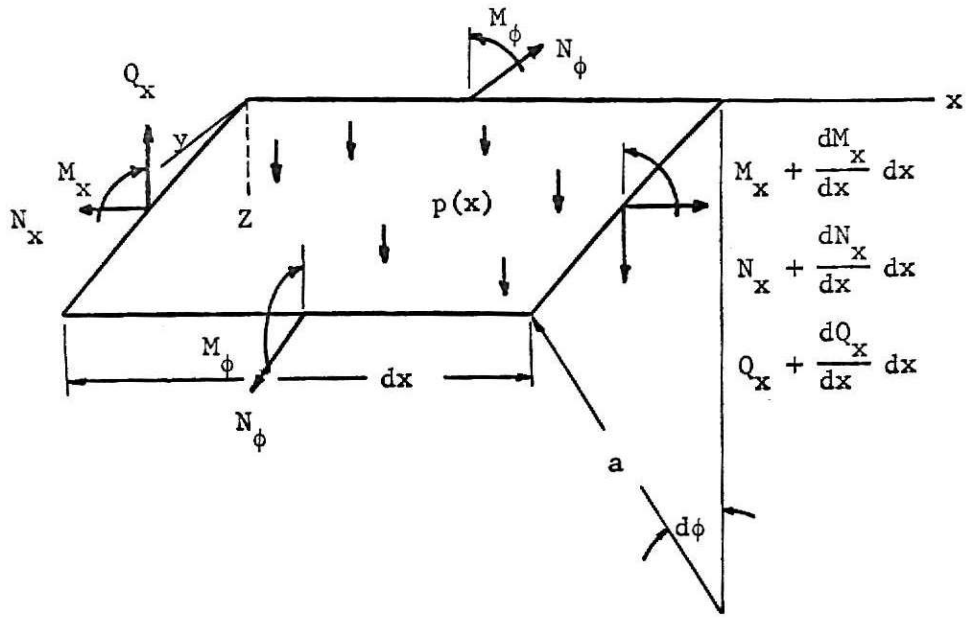

Fig. 2

Element of a Thin Shell

The expression for hoop stress, along with the equations for the moment in terms of deflection w_s for the shell, were substituted into the equilibrium equations. The resulting expression was the governing differential equation for the thin shell. The solution was easily obtained and contained four arbitrary constants. It was noted that the solution was in powers of e and, as x approaches large values, the deflection w_s approaches infinite values. This was not possible. Therefore, the two constants which are coefficients of the positive powers of e must vanish. The resulting expression for w_s in dimensionless form was

$$\frac{4D_s\beta^5}{\gamma} w_s = e^{-\beta x} \left[A_1 \cos\beta x + A_2 \sin\beta x\right] + \beta(d-x) \tag{5}$$

and the moment

$$\frac{2\beta^3}{\gamma} M_x = e^{-\beta x} \left[A_1 \sin\beta x - A_2 \cos\beta x\right] \tag{6}$$

where

$$\beta^4 = \frac{3(1-\nu^2)}{a^2 h_s^2} \tag{7}$$

Again, D_s is the flexural rigidity of the shell and γ is the specific weight of the fluid in the container. The moment M_x in the moment on a horizontal section of the shell. A_1 and A_2 represent the constants of integration and h_s is the thickness of the wall.

See Timoshenko [7] for a complete derivation.

SOLUTION OF THE PROBLEM

This investigation was concerned with determining the localized moment at the connection of the plate and wall of a cylindrical container. Using the expressions developed previously and the boundary conditions following, a solution was developed. A diagram of the tank and its deflected shape is given to assist in describing the assumed boundary conditions.

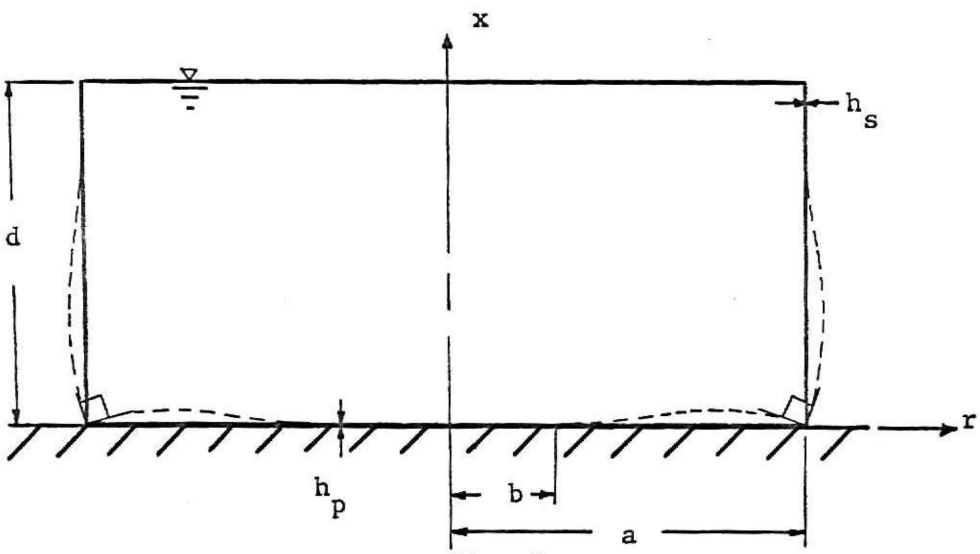

Fig. 3

Diametrical Section of Cylindrical Container

The first and second conditions are rather apparent from the figure. It was assumed that the connection of the plate and shell was rigid; that is, the slope of the wall and plate and also, the moment of the wall and plate, were equal at the connection

$$\left.\frac{dw}{dx}\right|_{x = 0} = \left.\frac{dw}{dr}\right|_{r = a} \tag{8}$$

$$\left.M_x\right|_{x = 0} = \left.M_r\right|_{r = a} \tag{9}$$

A third assumption is that the horizontal deflection at the base was zero.

$$w_s\Big|_{x = 0} = 0 \tag{10}$$

The third assumption may give rise to small errors due to stretching of the plate by the shear force in the shell; however, in examining the foundation upon which the tank rests, it was observed that at the junction of the wall and base there was an unknown frictional force resisting this shear in the wall in conjunction with membrane stresses in the plate. Therefore, the assumption that the horizontal deflection was zero was used in the solution since this produced only small errors.

A fourth assumed boundary condition at the joint was that

$$w\Big|_{r = a} = 0 \tag{11}$$

the vertical deflection was zero. This assumption arises from the fact that a rigid foundation was assumed for the base of the container.

Examining the plate within the radius b, it was noted that the plate was in contact with the base over this central region. This indicates that the deflection, slope, and curvature are zero at $r = b$. Therefore,

$$w\Big|_{r = b} = 0 \tag{12}$$

$$\frac{dw}{dr}\Big|_{r = b} = 0 \tag{13}$$

$$M_r\Big|_{r = b} = 0 \tag{14}$$

From the equations derived previously, there were six unknown constants and seven boundary conditions; the seventh unknown was the inner radius b. This was the point at which the plate was in contact with the base (see Fig. 3). This complicated the problem considerably because it became non-linear. The method used to find a solution was to examine the shell and plate individually and then combine the results.

For the plate, an inner radius b was assumed and the moment and slope at the edge ($r = a$) was calculated, using the four assumed boundary conditions for the plate. This process was continued until a curve of slope versus moment could be drawn for the plate. A computer was used here to find the solution (see Appendix I for program). The expressions for the plate with boundary conditions applied were set up in matrix form; the matrix was then inverted and the unknown constants of integration were found. Knowing these constants the moment and slope at the connection could easily be found. The coordinates for the slope versus moment curve are found in Table No. 1.

A similar approach was used for the shell; however, this problem was linear, which simplifies matters. In this case the solution was obtained using Eq. (10) and expressed in the same dimensionless parameters as used for the plate. Again a curve of slope versus moment could be drawn for the shell. This curve was expressed in terms of three dimensionless constants which were related to the dimensions of the container being considered. The expression used to calculate the shell curve was

$$2C_1^{\frac{1}{2}}\left[3(1-\nu^2)\right]^{\frac{1}{4}}\left(\frac{D}{qa^3}\ \frac{dw}{dr}\right)+\frac{1}{2C_1\left[3(1-\nu^2)\right]^{\frac{1}{2}}} - \frac{1}{2C_2C_1^{\frac{1}{2}}\left[3(1-\nu^2)\right]^{3/4}} = \frac{M_x}{qa^2} \tag{15}$$

where the dimensionless constants in Eq. (15) were

$$C_1 = \frac{a}{h_s} \tag{a}$$

$$C_2 = \frac{d}{h_s} \tag{b}$$

$$C_3 = \frac{h_p}{h_s} \qquad \text{(c)}$$

The computer solution (see Appendix II) was used to determine the intercepts for the curve of slope versus moment for the shell. Only the intercepts were found since Eq. (15) was linear.

Now having two curves of slope versus moment, the boundary conditions of equal slope and moment at the rigid connection were used to solve for the moment.

These curves have been plotted for varying parameters C_1, C_2, and C_3. It is interesting to note that as C_1 approaches infinity, the slope and moment approace zero for all finite values of C_2 and C_3 (see Fig. 4). This is equivalent to the statement that, as the radius increases, the moment at the joint is reduced. Examining Fig. 5 it is observed that the moment is bounded as C_2 approaches infinity. An important item to note here is that the depth is not really a controlling factor of the localized moment at the edge. Figure 6 demonstrates the effect of the thickness of the wall and plate on the localized moment. It is again noticed that as C_3 approaches infinity the moment at the joint is bounded. Allowing C_3 to approach infinity is equivalent to increasing the thickness of the plate while keeping the wall thickness constant. It is noted here that, as the thickness of the plate is reduced, the moment is also reduced. This results in not only a saving of material but also a reduction in the stresses. Knowing the dimensions of a particular tank, the proper curve may be selected and the moment and slope can easily be calculated and hence the stresses determined.

The parametric curves of slope versus moment for the wall were given only for a few values of the parameters C_1, C_2, and C_3. If other ratios are desired, they may be calculated using the computer program in Appendix II or from Eq. (10).

Table No. 1 Slope vs. Moment of the Plate for $\nu = 0.3$

ASSUMED RADIUS	SLOPE	MOMENT
0.01000	0.26922769D-01	-0.15133717D 00
0.02000	0.25915308D-01	-0.14790715D 00
0.03000	0.25093083D-01	-0.14522955D 00
0.04000	0.24356054D-01	-0.14273317D 00
0.05000	0.23669236D-01	-0.14038171D 00
0.06000	0.23016092D-01	-0.13811988D 00
0.07000	0.22387516D-01	-0.13591705D 00
0.08000	0.21778014D-01	-0.13375459D 00
0.09000	0.21184061D-01	-0.13162044D 00
0.10000	0.20603299D-01	-0.12950649D 00
0.20000	0.15319162D-01	-0.10877703D 00
0.30000	0.10841557D-01	-0.88370273D-01
0.40000	0.71791395D-02	-0.68642238D-01
0.50000	0.43492757D-02	-0.50217204D-01
0.60000	0.23221046D-02	-0.33748219D-01
0.70000	0.10181311D-02	-0.19876684D-01
0.80000	0.31261881D-03	-0.92266452D-02
0.90000	0.40394876D-04	-0.24037614D-02
0.91000	0.29541994D-04	-0.19548705D-02
0.92000	0.20814159D-04	-0.15507650D-02
0.93000	0.13987831D-04	-0.11920286D-02
0.94000	0.88362536D-05	-0.87924364D-03
0.95000	0.51294805D-05	-0.61299154D-03
0.96000	0.26344078D-05	-0.39385262D-03
0.97000	0.11147996D-05	-0.22240619D-03
0.98000	0.33131570D-06	-0.99230550D-04
0.99000	0.41538599D-07	-0.24903026D-04

$$\text{Radius} = r/a \tag{16}$$

$$\text{Slope} = D/qa^3 \, (dw/dr) \tag{17}$$

$$\text{Moment} = M_r/qa^2 \tag{18}$$

Slope vs. Moment

$C_2 = 200 \qquad C_3 = 1.00$

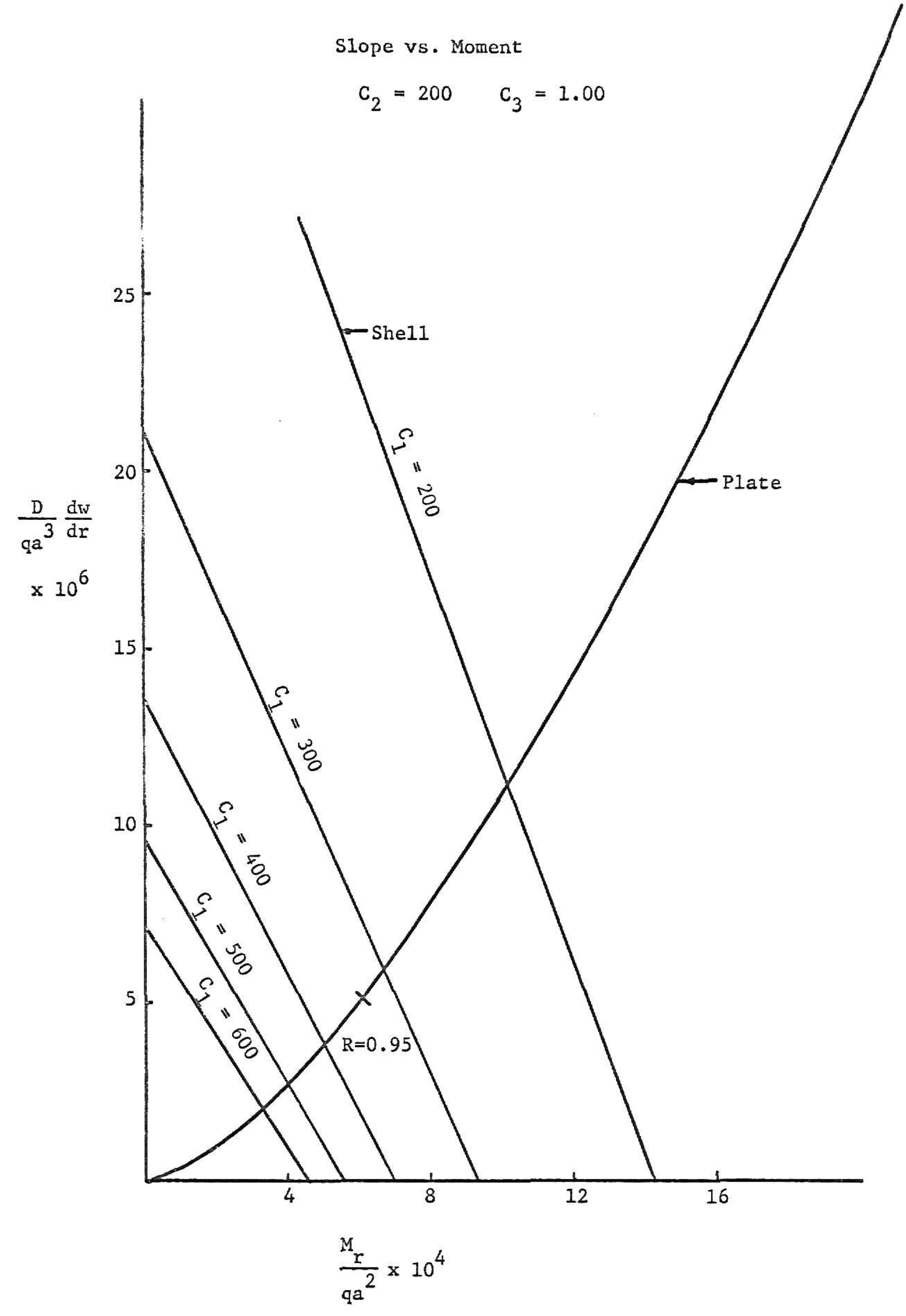

Fig. 4

Slope vs. Moment

$C_1 = 200 \qquad C_3 = 1.0$

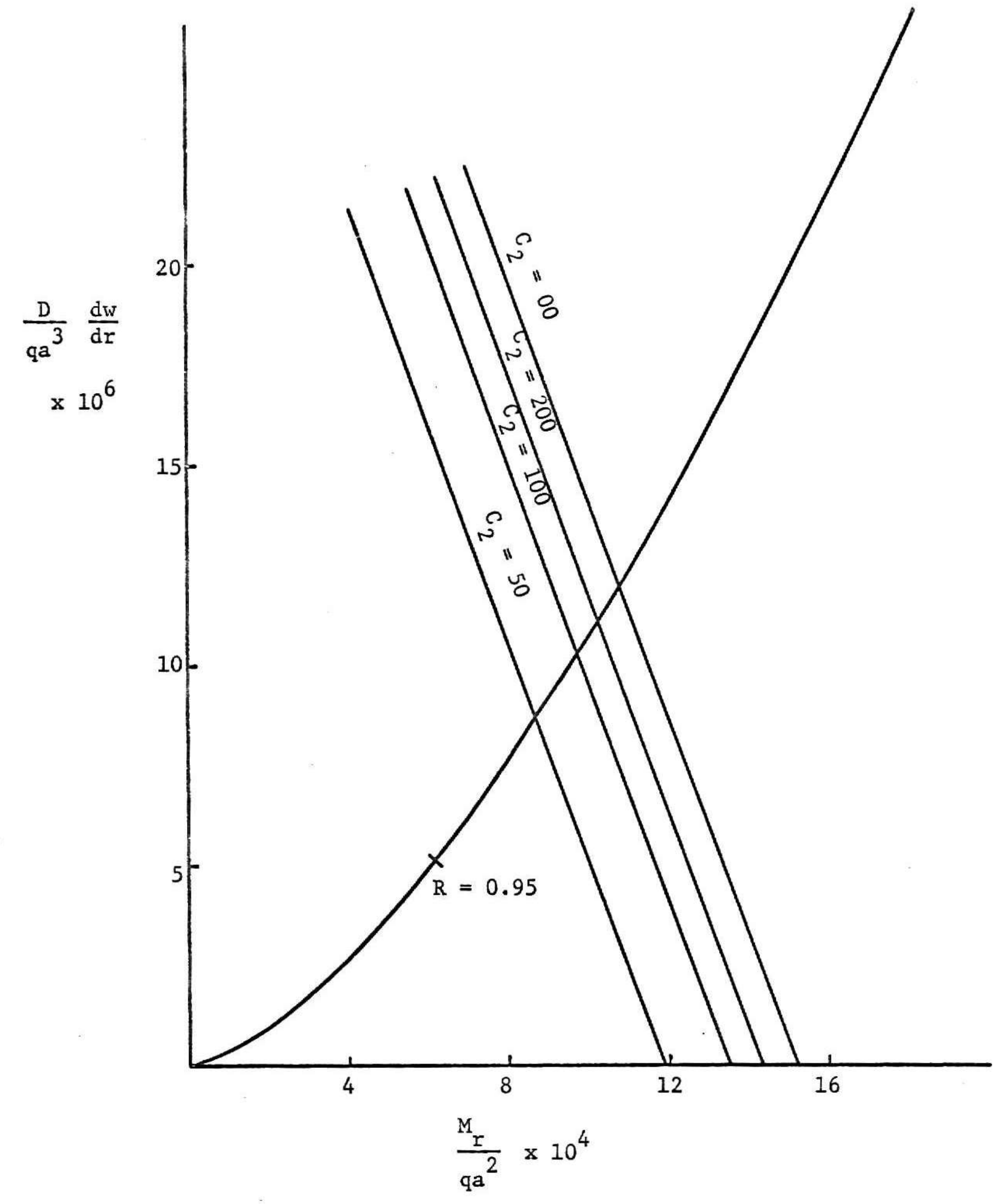

Fig. 5

Slope vs. Moment

$C_1 = 200 \qquad C_2 = 200$

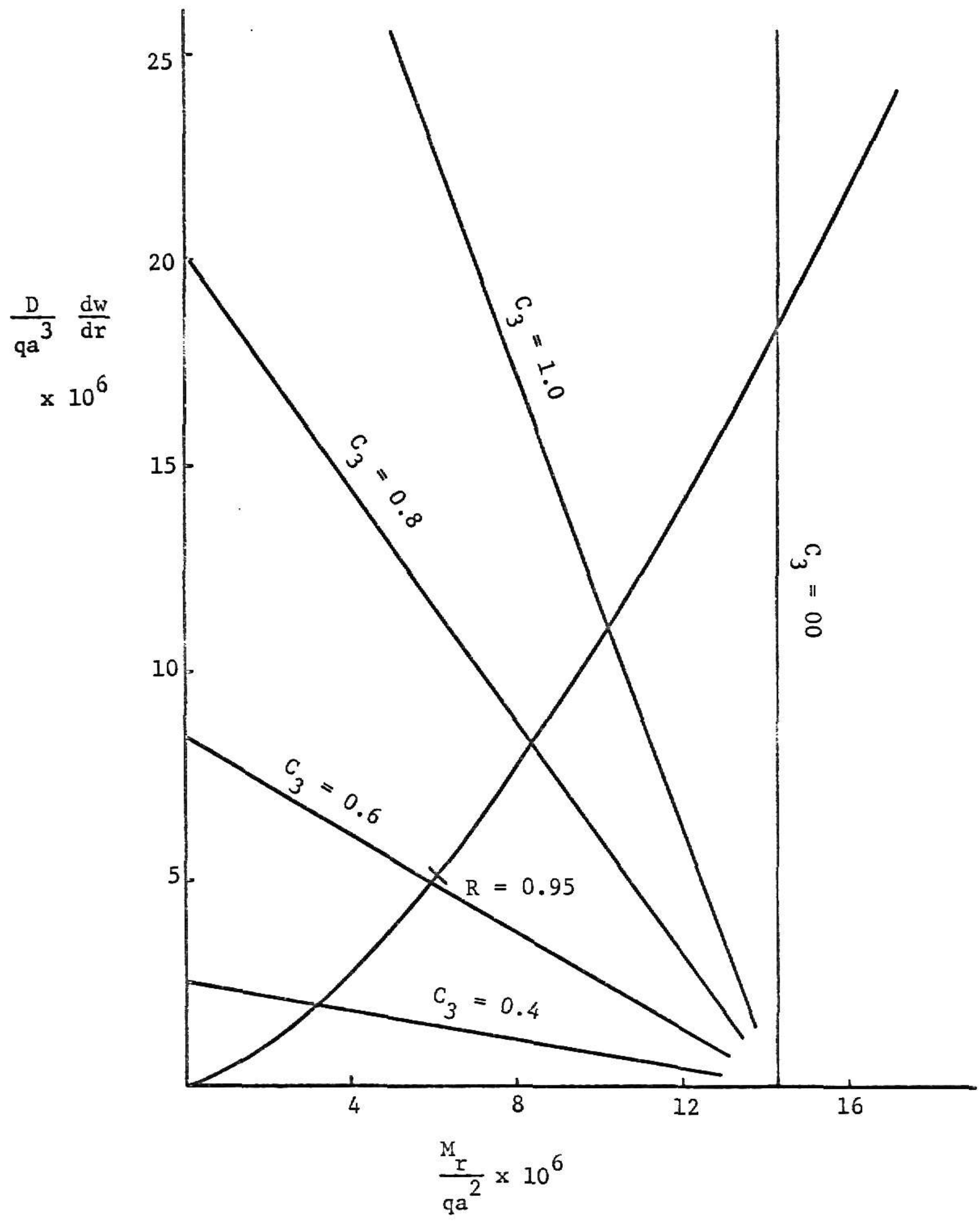

Fig. 6

EXPERIMENTAL VERIFICATION

Having an analytic solution to the problem, it was interesting to see how well this fits the physical situation. In this investigation it was decided to examine the deflection curve of the base plate of the container experimentally. The deflection curve was selected because of the intuitive feeling for it.

The apparatus for the experimental investigation (see Fig. 7) consisted of a 0.75 inch steel plate to provide a rigid foundation, a supporting stand, four 0.0001" dial gages, a cylindrical container for the load, and a plexiglass plate for the model.

The rigid base plate was turned on a lathe to obtain a flat surface and the edge at this time was also turned to put a square edge on the base plate. Holes were then drilled along radial lines as indicated on the plan view to allow the stems of the dial gages to rest on the plexiglass plate. The base plate was then supported on a steel stand to which the dial gages were attached.

The dial gages used were Starrett gages with 2 inch dials and 100 divisions per revolution. These were mounted on supports attached to the supporting stand. Lengthened stems were used on the dial gages to extend through the holes drilled in the supporting base plate and measure the deflection of the plexiglass plate. Gages used in this fashion unwind when the load is applied.

The plexiglass was cut from 0.060 ± 0.001 inch sheet 2 inches larger in radius than the base plate. The overhang was used to apply a uniform cantilever ring load to the plexiglass plate (see Fig. 7). By drawing a free-body diagram of this arrangement, it can be seen that such a loading applied a uniform bending moment to that portion of the plate directly above

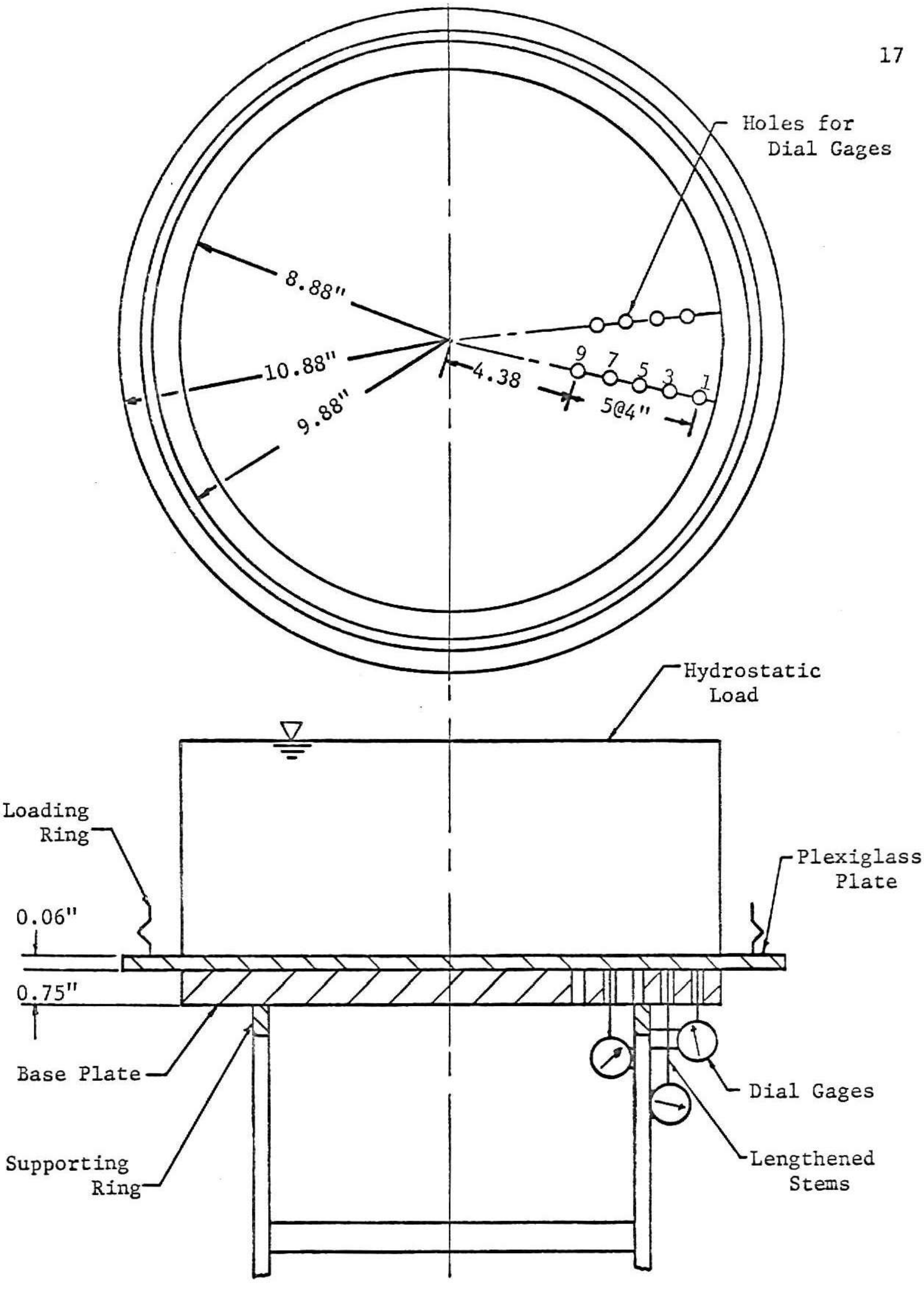

Fig. 7

Diagram of Appartus

the rim of the steel base plate. This is equivalent to the bending moment applied to the bottom of a tank by the wall; however, there is no stretching of the plexiglass plate by membrane forces as in the bottom plate of the tank.

The uniform pressure over the plexiglass was obtained by filling a plastic bag with water. The bag was contained with a sheet metal ring, whose diameter was equal to that of the base plate. The sheet metal ring was not attached, but was allowed to rest on the plexiglass plate directly above the edge of the base plate. In this way, the weight of the ring was transferred directly to the base plate.

The uniform moment, as described previously, was produced by the cantilevered portion of the plexiglass plate. This was loaded by use of a stiff ring on which weights were placed. The moment at the edge of the base plate was calculated by simply assuming that a segment of the outer ring of the plexiglass plate was a cantilever beam. This does not yield the correct moment, since the actual moment is reduced by the tangential moments. Therefore, the calculated moment was reduced by 10%. The value of the 10% was arrived at, because this is approximately ν^2 (see Eq. 2a), which Den Hartog [2] states is the approximate increase in stiffness of a plate over a beam.

The method used to record deflections was first to deflect the plate and begin the readings five minutes after loading. This time was decided upon by examinations made by Myers [6], who plotted a curve of stress relaxation versus time. It is evident, from his curve that, after 5 minutes, the stress relaxation was negligible. Measured deflections are recorded in Table No. 2. It is noted from this table that corresponding readings seem to somewhat lower on each successive loading. During any one loading, the creep in the plate was not noticed but, as shown in the table, it became apparent after several loadings. It is important to note that the deflection of the steel base plate, when checked under the load, could not be read on the dial gages used in the investigation. The maximum stress in the plexiglass plate was calculated, using the reduced moment and found to be 1500 psi.

To compute the theoretical deflections, the properties of the material had to be known. To determine E and ν, a simple tension speciment was used with electrical resistance strain gages mounted in both the longitudinal and transverse directions.

The gages used were Budd EC5-644B-350. In first attempts, heating of the material directly adjacent to the gage made the gage unstable, and proper readings could not be made. This problem was solved by applying a low voltage (0.917 VDC) to a full bridge which was temperature compensated. A B & F Model 1-7005G input conditioner was used for the voltage supply. The signal from the bridge was then amplified 1000 times using a B & F Model 700-10D amplifier. The signal after amplification was displayed on a Digitec Model 252 digital voltmeter. The value of 0.917 VDC was necessary to have the change in milli-volts of the amplified signal equal to the strain in micro-strain. The equipment used for power supply and amplifier was of very good quality and no drift was found to be present.

Young's modulus and Poisson's ratio were determined using a tension specimen that was loaded to the maximum stress expected, held for five minutes, after which the readings were taken as the specimen was unloaded. The total time required to take the readings during the unloading process was approximately five minutes. It was noted that after the gages were completely unloaded the strain did not return to zero (see Fig. 10). Since there was no drift in the instruments this value represents creep in the specimen during the test. The value of 532,000 psi was found for E and Poisson's ratio was determined to be 0.40.

The results of the experimental investigation of the circular plate are given in Fig. 9, along with a theoretical deflection curve that was plotted using the computer program in Appendix III.

EXPLANATION OF PLATE 1

Arrangement of the Experiment

PLATE 1

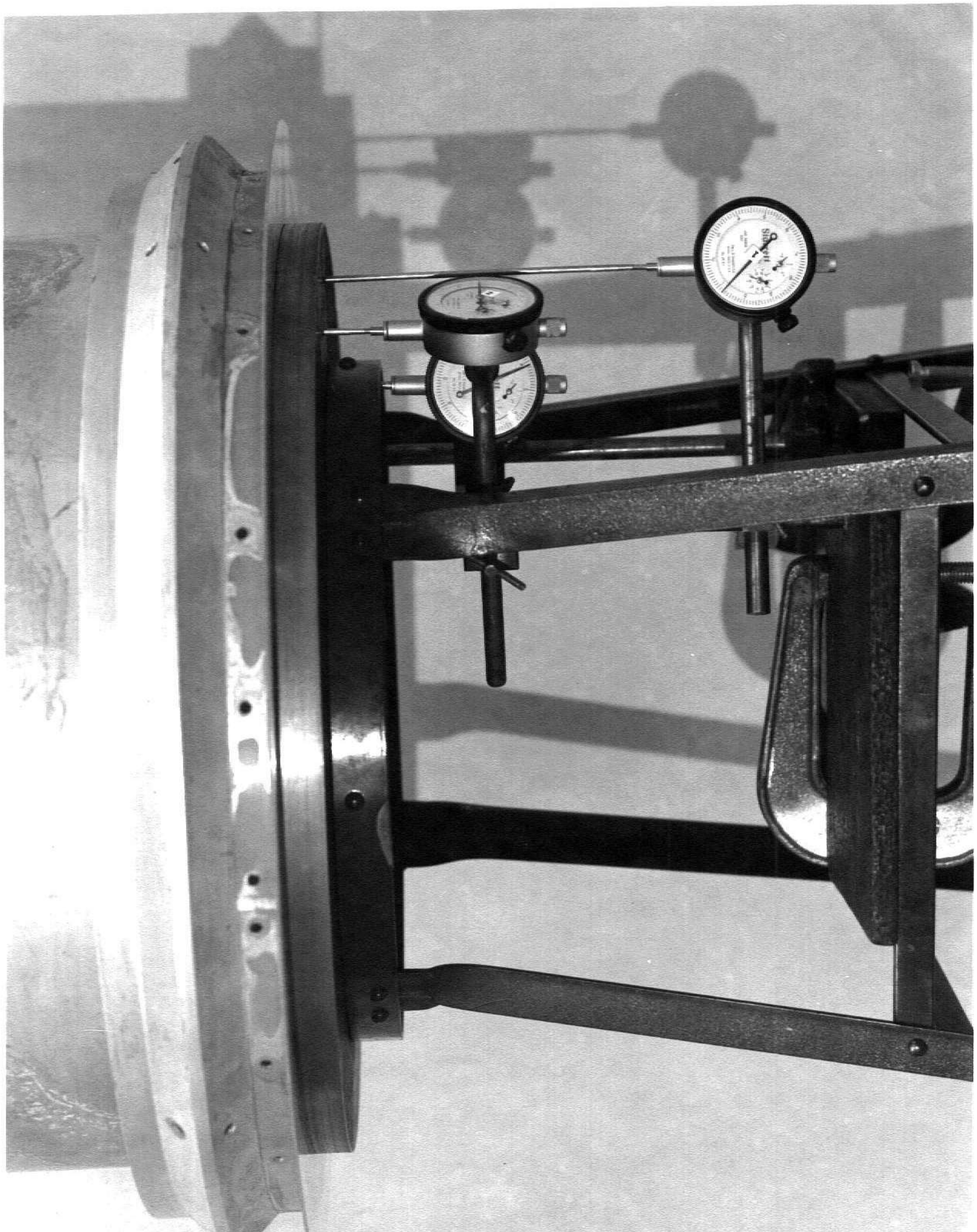

Table No. 2 Experimental Data for Deflection Curve of Circular Plate

		1	2	3	4	5	6	7
	Pts.	1	2	3	4	5	6	7
	Gage Position from edge of base plate (inches)	1/2	1	1 1/2	2	2 1/2	3	3 1/2
Trial No. 1	Initial Reading	.0400	.0700	.0400	.0400	.0300	.0800	.0300
	Load	.0288	.0571	.0274	.0330	.0266	.0791	.0300
	Final Reading	.0400	.0700	.0400	.0400	.0300	.0800	.0300
	Deflection	.0112	.0129	.0126	.0070	.0034	.0009	.0000
Trial No. 2	Initial Reading	.0400	.0700	.0400	.0300	.0300	.0800	.0300
	Load	.0287	.0582	.0290	.0238	.0272	.0794	.0300
	Final Reading	.0401	.0700	.0400	.0302	.0300	.0800	.0300
	Deflection	.0113	.0118	.0110	.0062	.0028	.0006	.0000

Deflection vs. Radius

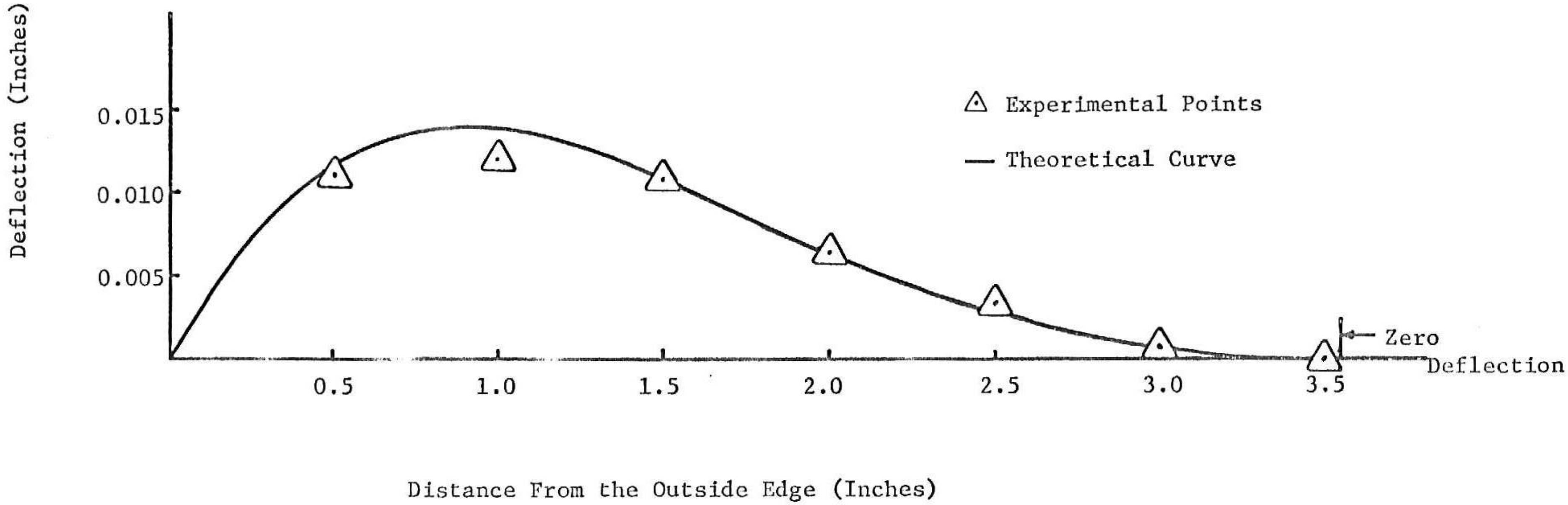

Fig. 9

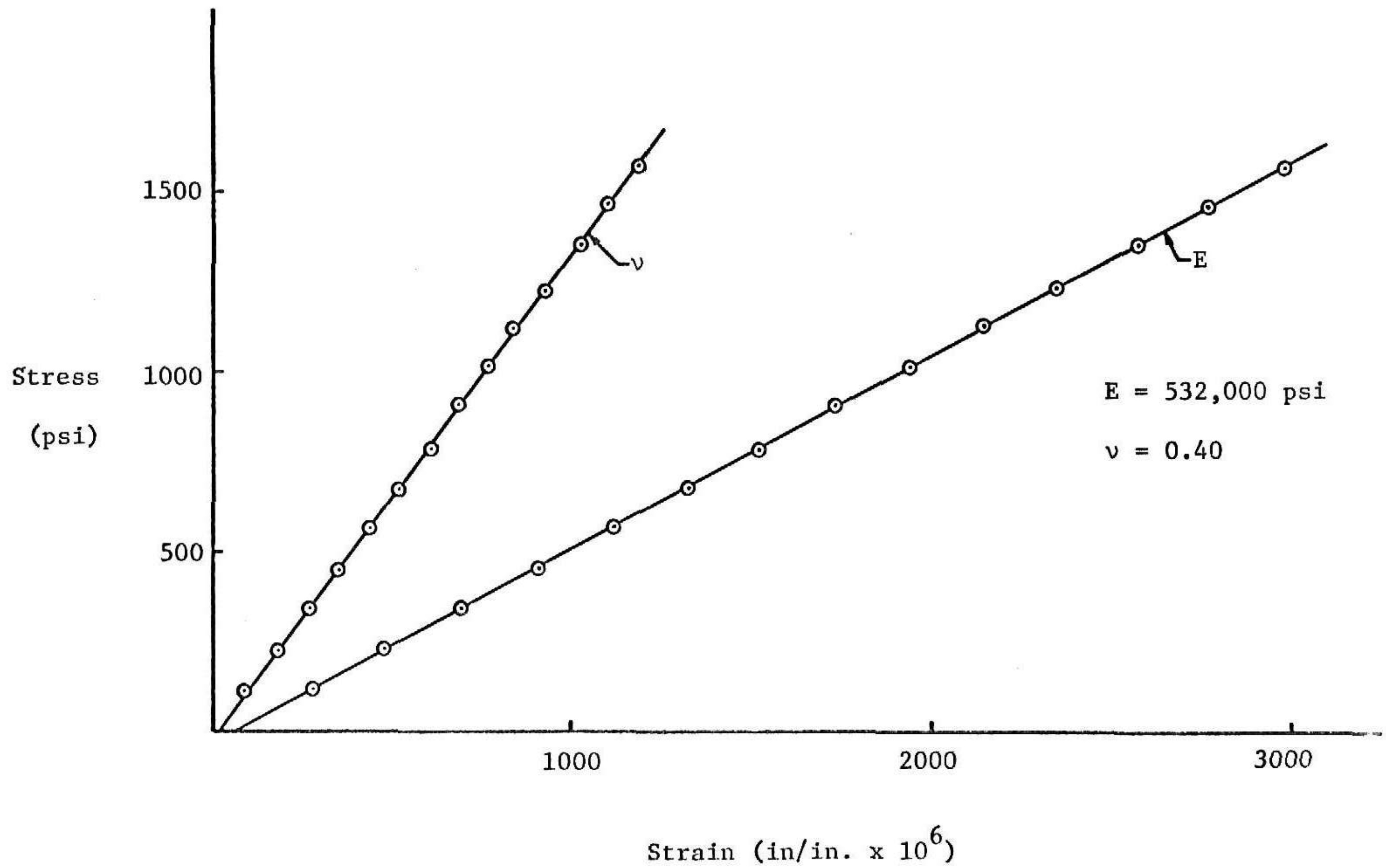

Fig. 10

Material Properties Determination

CONCLUSION

Hofmann [5] has investigated this problem previously and the solution obtained here is very similar to his; however, Hofmann, to simplify calculation, allowed Poisson's ratio to be zero. Noting the equations for the shell after setting $\nu = 0$, the moment equation is reduced to that of a beam; the other expressions are unchanged.

Plotting the curve of slope versus moment at the edge of the plate, the difference of the moment for values of $\nu = 0$ and $\nu = 0.3$ may be observed (see Fig. 11).

The linear part of the curve in Fig. 11 is given by

$$\frac{M_r}{qa^2} = \frac{D}{qa^3}(1 + \nu)\frac{dw}{dr} + \frac{1}{8} \qquad (19)$$

Equation 19 was derived for a uniformly loaded circular plate supported at its boundary with a uniformly distributed moment applied at the edge.

It may be seen from the curves of slope versus moment for the plate that, for the same slope, with an increasing value of ν there is a corresponding increase in the value of the moment at the edge. This is as it should be, because as ν increases, the flexural rigidity of the plate increases, resulting in a higher moment for the same slope. Also, note that Figures 4, 5, and 6 represent only a small part of this curve. The previous plots of slope versus moment for the plate have been shown for R greater that 0.9; note this point on Fig. 11. The curves of slope versus moment for $\nu = 0.0$ and $\nu = 0.3$ for R greater that 0.9 are very close; thus, using the value of $\nu = 0.0$ for the plate and shell has little effect on the solution.

In conclusion, using a value of 0.3 for ν seems more realistic than

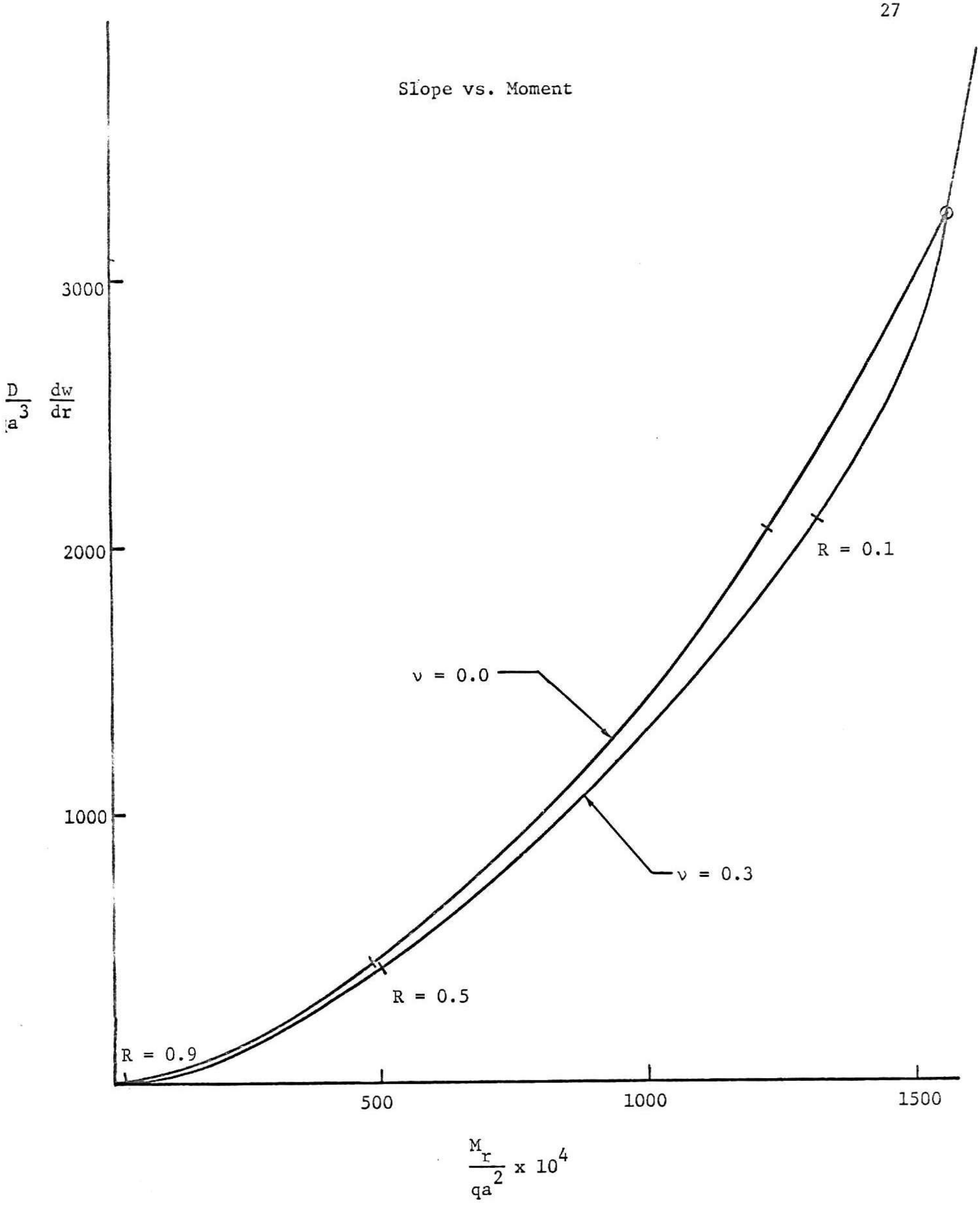

Fig. 11

Comparison with Hofmann

zero, but it produces little change in the solution.

Den Hartog [2] has also suggested an approximate solution to this problem by using the theory of beams on elastic foundations. Using this theory, he assumes no rotation of the connection between the plate and shell. He also assumes the pressure in the bottom three feet of the shell to be uniform. He indicates that these assumptions are not actually true; but, that they produce an approximate solution much more easily than any other method which may be used.

Den Hartog examines a tank 30 feet in height, 50 feet in radius and with a wall and base thickness equal to 0.75 inches. He finds the moment at the connection

$$M\Big|_{r = a} = 1600 \text{ in-lb/in.} \tag{20}$$

where $E = 30 \times 10^6$ psi.

Using the method of this investigation, with $\nu = 0.3$, the moment is (see Fig. 12)

$$M_r\Big|_{r = 50'} = 2.6\ qa^2 = 1200 \text{ in-lb/in.} \tag{21}$$

for constants

$$C_1 = \frac{600}{.75} \qquad C_2 = \frac{360}{.75} \qquad C_3 = \frac{.75}{.75} \tag{a}$$

This is a substantial reduction in the calculated moment. It was then apparent that allowing the joint to rotate and calculating the moment was much more reasonable than assuming a fixed wall at the base. Therefore, the procedure given for calculating the moment is of special interest to individuals interested in designing cylindrical containers.

In summary, the procedure presented for determining the moment at the connection of the wall and plate was to plot the curve of slope versus moment for the plate at the connection. Using this curve, and determining the

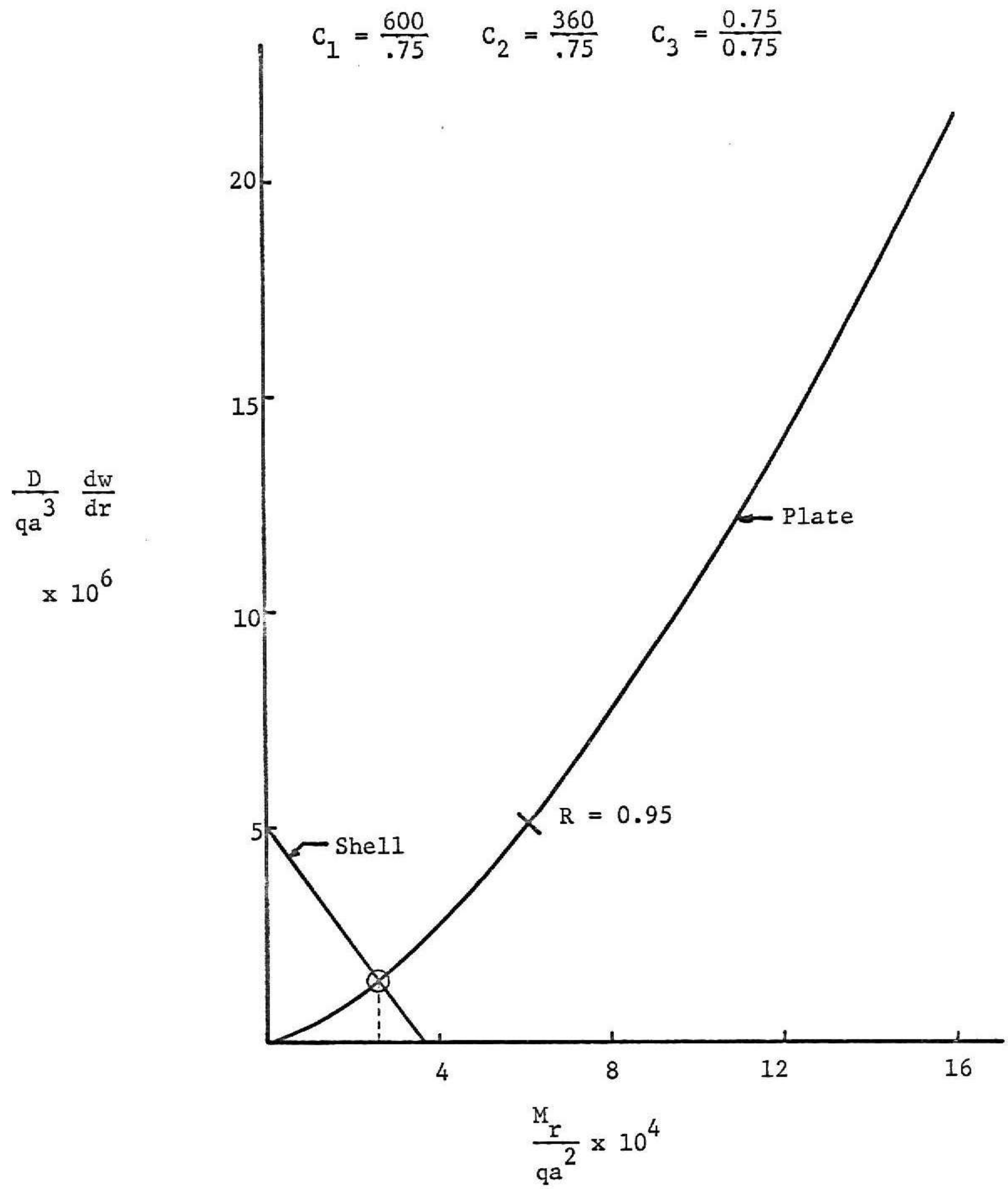

Fig. 12

Solution to Example Given by Den Hartog

intercepts of the shell curve of slope versus moment at the base, a solution was found. This solution was then compared to two approximate solutions. It was found that the procedure presented in this report yields a more reasonable solution than those of the approximations; however, the approximation of Hofmann's was very close to the solution given in this investigation.

RECOMMENDATIONS FOR FURTHER RESEARCH

In this investigation the horizontal deflection of the shell was assumed to be zero at the base; however, the horizontal deflection is not zero and, if it were taken into account, it might change the moment at the joint significantly.

Another variation of the problem would be to consider the tank on an elastic foundation rather than on a rigid foundation. This would increase the complexity considerably but, at the same time, the results would be of much more interest to the designer, since the majority of the large storage containers are constructed on a compacted soil foundation or on an asphaltic concrete base.

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APPENDIX I

COMPUTER PROGRAM OF PLATE CURVE

Appendix I contains the computer program in Fortran language used to calculate Table No. 1 in the text. This calculation provides the slope and moment at the edge of the tank for a given inner radius b. The equations are set up in matrix form, the constants of integration were then found. Knowing the constants, the slope and moment may be found for any radius assumed.

```
$JOB              KJS,RUN=CHECK
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
  101 FORMAT (//10X,'ASSUMED RADIUS',13X,'SLOPE',22X,'MOMENT'//)
  102 FORMAT (10X,F10.5,15X,D16.8,10X,D16.8)
  405 FORMAT (1H1)
C     THIS PROGRAM DETERMINES THE SLOPE VS.MOMENT CURVE AT THE EDGE O
C     THE PLATE (DIMENSIONLESS)
C     THE SLOPE IS EQUAL TO (D/PA**3) (DW/DR)
C     THE MOMENT IS EQUAL TO M/PA**2
C      A RADIUS IS ASSUMED AND THE SLOPE AND MOMENT AT THE EDGE ARE
C     CALCULATED
      DIMENSION A(3,3),C(3),AIV(3,3),ACI(3)
      AN=0.3000
      B=0.0000
      WRITE (3,101)
  675 IF (0.09500-B) 710,710,712
  712 B=B+0.01000
      GO TO 720
  710 IF (0.8800-B) 713,715,715
  715 B=B+0.1000
      GO TO 720
  713 B=B+0.01000
  720 A(1,1)=DLOG(B)
      A(1,2)=B*B-1.000
      A(1,3)=B*B*DLOG(B)
      A(2,1)=1.000/B
      A(2,2)=2.000*B
      A(2,3)=B*2.000*DLOG(B)+B
      A(3,1)=(AN-1.000)/(B*B)
      A(3,2)=2.0*(1.000+AN)
      A(3,3)=2.000*(1.00+AN)*DLOG(B)+3.00+AN
      C(1)=(1.0-B**4)/64.000
      C(2)=-B**3/16.000
      C(3)=-B**2*(3.00+AN)/16.000
      CALL MATINV (A,AIV,3,6)
      CALL MATMUL (AIV,C,ACI,3,3,1)
      C2=ACI(1)
      C3=ACI(2)
      C4=ACI(3)
      C1=-C3-1.00/64.000
      AMOA=C2*(1.0-AN)-C3*2.0*(1.0+AN)-(C4+1.0/16.0)*(3.0+AN)
      SLOP=C2+2.0*C3+C4+1.0/16.0
      WRITE (3,102) B,SLOP,AMOA
      IF (0.98500-B) 601,675,675
  601 CONTINUE
      WRITE (3,405)
      STOP
      END
```

```
      SUBROUTINE MATMUL (A,B,C,M,N,L)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      DIMENSION A(M,N),B(N),C(N)
      DO 100 I=1,M
      SUM=0.0
      DO 102 K=1,N
  102 SUM=SUM+A(I,K)*B(K)
  101 C(I)=SUM
  100 CONTINUE
      RETURN
      END
C     SUBROUTINE FOR MATRIX INVERSION
```

```
      SUBROUTINE MATINV (B,AINV,N,N2)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      DIMENSION B(N,N),AINV(N,N),A(3,6)
      N1=N+1
      DO 300 I=1,N
      DO 302 J=1,N
  302 A(I,J)=B(I,J)
      DO 301 J=N1,N2
  301 A(I,J)=0.0
  300 A(I,I+N)=1.0
      DO 200 J=1,N
      DIV=A(J,J)
      S=1.0/DIV
      DO 201 K=J,N2
  201 A(J,K)=A(J,K)*S
      DO 202 I=1,N
      IF(I-J) 203,202,203
  203 AIJ=-A(I,J)
      DO 204 K=J,N2
  204 A(I,K)=A(I,K)+AIJ*A(J,K)
  202 CONTINUE
  200 CONTINUE
      DO 400 I=1,N
      DO 400 J=1,N
  400 AINV(I,J)=A(I,J+N)
      RETURN
      END
```

APPENDIX II

COMPUTER PROGRAM FOR SHELL CURVE

Appendix II presents the computer solution of slope versus moment. The slope versus moment curve was found in terms of the dimensionless parameters for the plate. In calculating the curve of slope versus moment for the shell, three constants were needed which were related to the dimensions of the container being considered. These constants were

$$C_1 = \frac{a}{h_s} \qquad C_2 = \frac{d}{h_s} \qquad C_3 = \frac{h_p}{h_s}$$

The coordinates of the curve of slope versus moment for the shell were calculated by the program on the following page. Note, from Eq. (15) in the text, that the shell curve was linear. Therefore, only the intercepts were calculated. The curve of slope versus moment for the shell was then plotted with the curve for the plate.

ILLEGIBLE

THE FOLLOWING DOCUMENT (S) IS ILLEGIBLE DUE TO THE PRINTING ON THE ORIGINAL BEING CUT OFF

ILLEGIBLE

```
$JOB            KJS,RUN=CHECK
C       THESE ARE THE ABSCISSA AND ORDINATE FOR THE GRAPH OF THE CONTAI
C       THESE VALUES ARE PLOTTED ON THE GRAPH OF SLOPE VS. MOMENT FOR T
C       PLATE.  THE SOLUTION IS THEN FOUND FOR SEVERAL DIFFERENT CASES
C       C1=RADIUS OF TANK/THICKNESS OF SHELL
C       C2=DEPTH OF TANK/THICKNESS OF SHELL
C       C3=THICKNESS OF PLATE/THICKNESS OF SHELL
    302 FORMAT (//10X,'C2 VARIES FROM 50 TO 1200 WHILE C1= ',F10.5)
    305 FORMAT (10X,F10.5,10X,E16.8,10X,E16.8)
    307 FORMAT (//10X,'CONSTANT C2',15X,'ABS',24X,'ORD'/)
    320 FORMAT (/10X,'C1=RADIUS OF TANK/THICKNESS OF SHELL'/)
    321 FORMAT (/10X,'C2=DEPTH OF TANK/THICKNESS OF SHELL'/)
    322 FORMAT (/10X,'C3=THICKNESS OF PLATE/THICKNESS OF SHELL'/)
    325 FORMAT (10X,E16.8,10X,E16.8)
    326 FORMAT (//5X,'FOR LIMITING CASE OF C2 APPROACHING INFINITY')
    327 FORMAT (13X,'ABSL',20X,'ORDL')
    399 FORMAT (20X,'C3= ',F10.5)
    900 FORMAT (1H1)
        AN=0.300
        C3=0.2000
    850 CONTINUE
        WRITE (3,320)
        WRITE (3,321)
        WRITE (3,322)
        C1=50.00
    826 CONTINUE
        WRITE (3,302) C1
        WRITE (3,399) C3
    806 C2=50.00
        ABSL=1.0/(2.0*C1*(3.0*(1.0-AN**2))**0.5)
        ORDL=+C3**3/(4.0*C1**1.50*(3.0*(1.0-AN**2))**0.750)
        WRITE (3,326)
        WRITE (3,327)
        WRITE (3,325) ABSL,ORDL
        WRITE (3,307)
    805 CONTINUE
        ABS=1.0/(2.0*(3.0*(1.0-AN**2)*C1**2)**0.5)-1.0/(2.0*(3.0*(1.0-A
      12)*C2**(4.0/3.0)*C1**(2.0/3.0))**0.7500)
        ORD=((C3**3)/(2.0*(3.0*(1.0-AN**2)*C1**2)**0.250))*ABS
        WRITE (3,305) C2,ABS,ORD
        C2=C2+50.00
        IF (1225.00-C2) 807,807,805
    807 CONTINUE
        WRITE (3,900)
        C1=C1+50.00
        IF (1225.00-C1) 825,825,826
    825 CONTINUE
        C3=C3+0.2000
        IF (2.050-C3) 851,850,850
    851 CONTINUE
        STOP
        END
```

APPENDIX III

COMPUTER PROGRAM FOR PLATE DEFLECTION

Appendix III presents a Fortran program similar to that of Appendix I, since it uses the same matrix equations to compute the constants of integration and then computes a deflection curve and moment curve for the assumed inner radius b. This program was used to calculate the theoretical deflection curve which was compared to the experimental deflection curve in Fig. 10.

```
$JOB              KJS,RUN=CHECK
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
  205 FORMAT (12X,'DEFLECTION CURVE FOR EDGE MOMENT OF M/QA**2= ',D16
  206 FORMAT (1H1)
  102 FORMAT (10X,F10.5,15X,D16.8,10X,D16.8)
  405 FORMAT (1H1)
C     THIS PROGRAM DETERMINES THE SLOPE VS.MOMENT CURVE AT THE EDGE O
C     THE PLATE (DIMENSIONLESS)
C     THE SLOPE IS EQUAL TO (D/PA**3) (DW/DR)
C     THE MOMENT IS EQUAL TO M/PA**2
C      A RADIUS IS ASSUMED AND THE SLOPE AND MOMENT AT THE EDGE ARE
C     CALCULATED
      DIMENSION A(3,3),C(3),AIV(3,3),ACI(3)
      AN=0.3000
      B=0.0000
  675 CONTINUE
  712 B=B+0.01000
  720 A(1,1)=DLOG(B)
      A(1,2)=B*B-1.000
      A(1,3)=B*B*DLOG(B)
      A(2,1)=1.000/B
      A(2,2)=2.000*B
      A(2,3)=B*2.000*DLOG(B)+B
      A(3,1)=(AN-1.000)/(B*B)
      A(3,2)=2.0*(1.000+AN)
      A(3,3)=2.000*(1.00+AN)*DLOG(B)+3.00+AN
      C(1)=(1.0-B**4)/64.000
      C(2)=-B**3/16.000
      C(3)=-B**2*(3.00+AN)/16.000
      CALL MATINV (A,AIV,3,6)
      CALL MATMUL (AIV,C,ACI,3,3,1)
      C2=ACI(1)
      C3=ACI(2)
      C4=ACI(3)
      C1=-C3-1.00/64.000
      AMQA=C2*(1.0-AN)-C3*2.0*(1.0+AN)-(C4+1.0/16.0)*(3.0+AN)
      WRITE (3,205) AMQA
  418 FORMAT (/12X,'RADIUS',12X,'DEFLECTION',20X,'MOMENT  M/PA**2')
      WRITE (3,418)
      R=B
  202 CONTINUE
      W=C1+C2*DLOG(R)+C3*R*R+C4*R*R*DLOG(R)+R**4/64.00
      AMP=C2*(AN-1.0)/R**2+C3*2*(1.0+AN)+C4*(2.0*(1.0+AN)*DLOG(R)+3.0
     1N)+R*R*(3.0+AN)/16.00
      WRITE (3,102) R,W,AMP
      R=R+0.0100
      IF(1.00-R) 201,202,202
  201 CONTINUE
      WRITE (3,206)
      IF (0.98500-B) 601,675,675
  601 CONTINUE
      WRITE (3,405)
      STOP
      END
```

```
      SUBROUTINE MATMUL (A,B,C,M,N,L)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      DIMENSION A(M,N),B(N),C(N)
      DO 100 I=1,M
      SUM=0.0
      DO 102 K=1,N
  102 SUM=SUM+A(I,K)*B(K)
  101 C(I)=SUM
  100 CONTINUE
      RETURN
      END
C     SUBROUTINE FOR MATRIX INVERSION
```

```
      SUBROUTINE MATINV (B,AINV,N,N2)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      DIMENSION B(N,N),AINV(N,N),A(3,6)
      N1=N+1
      DO 300 I=1,N
      DO 302 J=1,N
  302 A(I,J)=B(I,J)
      DO 301 J=N1,N2
  301 A(I,J)=0.0
  300 A(I,I+N)=1.0
      DO 200 J=1,N
      DIV=A(J,J)
      S=1.0/DIV
      DO 201 K=J,N2
  201 A(J,K)=A(J,K)*S
      DO 202 I=1,N
      IF(I-J) 203,202,203
  203 AIJ=-A(I,J)
      DO 204 K=J,N2
  204 A(I,K)=A(I,K)+AIJ*A(J,K)
  202 CONTINUE
  200 CONTINUE
      DO 400 I=1,N
      DO 400 J=1,N
  400 AINV(I,J)=A(I,J+N)
      RETURN
      END
```

INVESTIGATION OF LOCALIZED BENDING MOMENT IN AN UPRIGHT CYLINDRICAL CONTAINER SUBJECTED TO HYDROSTATIC LOADING

by

KARL J. SVATY, JR.

B.S., Kansas State University, 1968

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Applied Mechanics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1970

ABSTRACT

This investigation describes the procedure for determining the localized moment at the connection between the wall and the base of an upright cylindrical container. The moment was found by using a computer-aided graphic solution.

The purpose of this study was to determine accurately the value of the moment at the connection. The deflection curve of the base of the container was found analytically and also determined experimentally to verify the theory used in the investigation.

The theory of thin plates and shells was used in this report. The comparison of the theoretical deflection curve and experimental curve was very good. This indicates the theory was satisfactory.

The moment at the base, as computed in this report, produces a more reasonable value, compared to that of other approximate methods. This would make the procedure presented in this investigation of interest to designers.