GEOMETRY IN THE ELEMENTARY
MATHEMATICS CURRICULUM

by

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TABLE OF CONTENTS

Page

LIST OF TABLES

INTRODUCTION

Whenever new ideas or concepts in mathematics have been introduced there has been a subsequent rearrangement of the curricular content of mathematics. Much discussion and questioning began when geometry was included in the elementary mathematics curriculum.

Brief contact with geometry, in the sense of working with circles, rectangles, and squares and contact with some of the basic concepts in art work, had been experienced by elementary school children prior to the inclusion of geometry in the mathematics curriculum. With the inclusion of geometry in the mathematics curriculum the children were able to see it as a mathematical area, with properties, definitions, and conditions to be dealt with in reference to all mathematical concepts.

According to Egsgard (5) a large number of our elementary teachers looked back to geometry as a study of deductive proofs about properties of plane figures using the method established by Euclid more than 1500 years ago. They never really understood the principle of deductive proof and, therefore, were afraid to even think of teaching geometry to their students.

Their fears have been alleviated, to some degree, for the geometry of the elementary school deals primarily with observable planar and spatial relationships. The world is full of these relationships. Doesn't symmetry exist in many plant leaves? Weren't geometric shapes used in the construction of the classroom? Children are able to see and discover such concepts but basic geometric groundwork is a necessary part of their discovery process.

"Introducing them [the children] to the idea of geometry as being concerned with shape and size in the material world will help them to realize and appreciate that mathematics is something that plays an important role in the world in which we live" (Egsgard, 5:478).

The study of geometry should also prepare them for better adjustment to the abstractions which they must make when they deal with geometry in high school.

In no way can we say that the task of the elementary teacher, who is introducing the child to the world of mathematics, is an easy one but it can be a very rewarding one.

Geometry is with us, it is important, and it can be fun for both the student and the teacher. As Robinson said: "Geometry can extend and enrich the study of arithmetic" (9:3).

Andre Revuz has stated some principles which he feels can serve as landmarks to guide the teaching of geometry from situations to theories. Several of these hold particular interest for the elementary teacher. Revuz has first stated that geometry is not to be taught separately but in connection with all parts of mathematics. He also stated that the teaching of geometry must begin in the kindergarten for it is sure that children very easily and spontaneously build models of spatial relationships at an early age. He also feels that it is important to allow the children to undertake deduction when they are ready for it, not at an age we define for them.

Elementary teachers have a responsibility to take note of aspects of geometry which are being taught in the elementary schools. Once the aspects of geometry being taught are determined, the elementary teachers must decide if these aspects are too general or not extensive enough to help children get a complete picture of mathematics. The total elementary mathematics program helping prepare the children for situations they will meet throughout later years.

The available research should help teachers reach decisions and conclusions about what is being tried at the various elementary levels and provide them with methods for determining how students are progressing in developing a geometric awareness. The teacher is then provided with a place from which to guide further development.

PURPOSE

The purpose of this study is threefold. 1. There will be identification of the geometric concepts of concern to those who have conducted the research reviewed. 2. The conclusions the researchers have reached and the methods and procedures they have used will be compared. 3. Based on the study of these reports general recommendations for inclusion of certain geometric concepts at the elementary level will be made.

KINDERGARTEN AND FIRST GRADE STUDIES

Two studies concerned with kindergarten and first grade children were available for review.

Starlin, dealing with first grade children was attempting to identify "gifted" children, meaning those having high inherent ability, who could then serve as peer tutors. His method of identification was the ability to identify seven geometric figures, not commonly presented at this level.

Rea and Reys working with kindergarten children were interested in giving teachers a base on which to build appropriate learning activities. The children's geometric capabilities regarding vocabulary, shape matching, spatial relationships, and geometric reproductions as identified on the Comprehensive Mathematics Inventory proved most interesting.

Starlin's Study

As previously mentioned Starlin (11) chose identification of the pentagon, hexagon, octagon, diamond, trapezoid, parallelogram, and ellipse because most first graders don't know the names, and the identification of these shapes is not usually a part of a first grade curriculum.

First grade subjects. Starlin's subjects were ten boys and ten girls in the self contained class of Mrs. Starlin, during the 1967-1968 school year. It should be noted that because of the relationship existing between the researcher and the teacher some bias may be present.

Procedure and findings. The study was undertaken using flash cards of the pentagon, hexagon, octagon, diamond, trapezoid, ellipse, and parallelogram. A Standard Behavior chart was used for charting the frequency of the students' responses.

For five days a one minute sample of each child's responses was recorded. Results at the end of five days showed Dru, one of the students, two to four correct per minute; four others, one to two correct per minute; and the other fifteen, zero to two correct per minute.

On the basis of this Dru was identified as the first peer tutor. Before the tutoring began Starlin tried to increase Dru's mean number of correct responses per minute from three to thirty. This was to demonstrate the feasibility of increasing the academic performance of a "gifted" student.

For seventeen days Dru's regular teacher spent five minutes a day teaching her the figures she did not know. This consisted of equating numbers with the prefixes, when possible, and drill. Occasional praise was an integral part of these sessions and within the first week her middle frequency reached thirty per minute.

Dru then began tutoring and she used the same techniques which had been used with her. She spent fifteen minutes a day with her group of four pupils and after five days all knew the seven figures.

As an added incentive before these four new tutors began work with their groups they were told that if everyone learned the seven figures there would be an ice cream party. Eighteen days from Dru's beginning work with her group every student knew the names of the figures. They also improved in their frequency of correct responses per minute.

It was determined that the ice cream party did serve as a favorable stimulus for all.

<u>Conclusions</u>. Starlin concluded that on the basis of his results it is feasible to teach geometric figures, at least the seven used, in the first grade. More importantly, the results suggest that the feasibility of introducing first graders to some of the curricular areas traditionally presented at higher levels be explored.

<u>Rea and Reys Study</u>

Rea and Reys felt that because of the influence of the world today on people of all ages particularly young children, findings concerning the abilities of entering kindergarten children from studies conducted over a year before their study were questionable (7).

They also noted that few instruments which will provide a baseline of data for developing programs and materials for these children have been produced.

After reviewing contemporary mathematics programs for kindergarten children, Rea and Reys decided to provide teachers with information which they might use in planning appropriate learning activities for these children.

Kindergarten subjects. Seven hundred and twenty-seven entering kindergarten children, in metropolitan St. Louis, were used in the sample. They were selected from thirty kindergartens broadly representative of subpopulations found in modern urban areas.

Procedure and findings. Data were gathered through the use of the Comprehensive Mathematics Inventory, experimental form, which was administered during the first full week of school. The instrument is divided into two parts and the total testing time is forty minutes. The following were the outcomes of the geometry subtest.

The questions on the subtest were classified as shape matching, vocabulary, and spatial relationships, or geometric reproductions. For reference the subtest has been included in Appendix A.

Matching shapes was found to be the easiest part of the test for the children, with 97 percent answering the most difficult item correctly.

Two kinds of questions were asked on the vocabulary section, to label a geometric shape and to identify parts of a geometric shape.

Although over 50 percent could correctly label a circle and a square, only 23 percent could correctly label a triangle, rectangle, and diamond. In identification of parts of the shapes 80 percent were able to correctly identify lines, sides, and corners. The researchers felt it noteworthy that there was a higher percentage on the second question.

Spatial relations questions were correctly answered by over half of the subjects. The researchers felt that problems encountered in determining the number of sides or corners were inherent in counting abilities, although it was noted that the side omitted by the child was the one closest to him.

In the final section at least 80 percent of the children were able to draw parallel and perpendicular lines after viewing pictures of them. Their dexterity in drawing simple geometric figures was noted to be adequate.

Conclusions. From these findings the researchers concluded that children entering school definitely possess many intuitive notions of geometry that are essential for further learning and that this would be an appropriate area to begin planning learning activities for them.

Comparison of Starlin and Rea and Reys Studies

While Starlin's study was limited to rote work it did indicate that first grade children are capable of making associations of numerical concepts and properties of geometric figures.

Rea and Reys' study gave definite evidence that the study of shapes and spatial relationships, vocabulary, and geometric reproduction is possible at the kindergarten level. These findings might provide a base on which to investigate Starlin's suggestion that a study of other topics for use with first graders is necessary.

Transfer of the findings to all kindergarten and first grade children may prove difficult. Rea and Reys did identify and attempt to some degree to randomize their subjects but they also qualified their findings by saying ". . . these findings should not be indiscriminatingly applied to a particular school district, school, or classroom" (7:390). Starlin, however, made no mention of student background or assessment.

Of added interest is the fact that Rea and Reys used no type of instruction before the test was given. This would imply that children bring with them a wealth of information which if properly directed will assist them in their fulfillment of a quest for knowledge.

It was also noted that Starlin used extraneous rewards while Rea and Reys employed none.

The methods used in both studies were clearly identified and provide us with a basis for knowing how their conclusions were made. One shortcoming in the procedure of Rea and Reys was in not stating if the test was administered on a one to one basis or in groups. There was also no mention made of the validity and reliability of the instrument.

The recording and interpretation of data was a weakness in both studies. It was perhaps the intent of the researchers, however, to present the data, so that it would be fully understood by the people who would be primarily interested in their findings, these being kindergarten and first grade teachers, who may not have a firm background in the interpretation of statistical information.

If attention was given to possible existing extraneous variables in either study no mention was made.

FOURTH, FIFTH, AND SIXTH GRADE STUDIES

While there were no studies available for review dealing with second and third grade exclusively there were three available which dealt with fourth, fifth, and sixth grade students.

Weaver was concerned with identifying levels of geometric understanding by using an instrument which he had developed. His premise was that the instrument would be sensitive to the identification of the different levels of understanding centering about the ability to classify plane geometric figures in terms of non-disjoint categories on the basis of common characteristics or properties (13:322). The categories he used were: polygon, quadrilateral, rectangle, simple closed curve, square, and triangle. Each of the twelve figures used was identified according to the categories listed by a yes, no, or not sure answer. The latter was included as an indication of relative unfamiliarity with a particular classification of geometric figures. An example from the inventory Weaver used and all of the figures used can be found in Appendix B.

D'Augustine sought to investigate some factors which might serve as predictors of predetermined levels of success with respect to selected topics in geometry and point set topology when taught through a programmed text which he had developed.

The third study, that of Denmark and Karlin, dealt exclusively with the feasibility of teaching geometric construction at this level.

Their study was limited to only one fifth grade class but their findings do suggest that more extensive work is indeed feasible.

Weaver Preliminary Study

To explore the feasibility of developing his inventory, Weaver worked with samples of fourth, fifth, and sixth grade children; elementary education majors in preservice training programs; and elementary school teachers in in-service education programs. In a description of his pilot study Weaver chose to report the results from three groups of in-service teachers.

Group A was composed almost exclusively of elementary teachers. All were registered in a university course in informal geometry. The inventory was administered about midway through their course, all items on the inventory having been introduced and discussed.

Group B was predominantly elementary teachers from the same suburban school system participating in a year's in-service program led by a mathematics educator from a nearby university. The inventory was administered approximately four months after the group had had an orientation session on geometry and they had made progress in updating their mathematics program.

Group C was also primarily elementary teachers participating in a year's in-service program. The inventory was administered near the end of the program before a limited amount of work with informal geometry was introduced. They were also involved in the early stages of updating their mathematics program.

The hypothesis stated was: Opportunity to learn was a highly crucial factor in determining level of group performance on the inventory.

Weaver (13) surmised on the basis of the groups that A definitely had the greatest opportunity to learn with the most systematic experience, while B had a greater opportunity than C in terms of planned study and exposure to geometric ideas through textbooks used in elementary school classrooms.

Using the formula $\bar{D} = R_x - R_y/12$, the mean difference between corresponding item ranks was computed. It was found that for B and C, $\bar{D} = 1.42$; for A and C, $\bar{D} = 3.17$; and for A and B, $\bar{D} = 3.25$. Thus B and C agree more closely in rank order of mean rights. For the mean number of right responses on each classification, calculated using $\bar{D} = R_x - R_y/6$, it was found that for B and C, $\bar{D} = 0.67$; for A and C, $\bar{D} = 1.67$; and for A and B, $\bar{D} = 1.33$. Again finding that B and C agree more closely. The percentage norms for total scores of seventy, seventy-one, or seventy-two, this being the highest possible, were 57 percent for A, 32 percent for B, and 5 percent for C.

Thus it was concluded that difference in opportunity to learn was directly related to difference in performance on the inventory and the hypothesis was retained.

The principle significance of the inventory is not in measuring achievement, per se, rather its principle significance is felt to be more diagnostic in nature in connection with the identification of degrees or levels of understanding for groups or classes and for individuals.

Weaver's Study

Eight months later Weaver (14) conducted a study involving fourth, fifth, and sixth grade students. He sought to discover if his inventory was sensitive to differences in the understanding of pupils who have had relatively much instruction in non-metric geometry and those who have had little instruction. He also hoped to discover if pupil performance on the inventory was in any way related to grade level.

Weaver subjects. For his study he selected two districts and from each drew three classes of a conventional and three classes of a contemporary program. The districts and the classes were selected to provide diversity rather than homogeneity within a total sample. The diversity was further reflected in that the three conventionals in District A had specialist teachers. The conventional classes all had little instruction in non-metric geometry while the contemporary classes had much.

Procedure and findings. The inventory was administered by each teacher following a standardized set of instructions, during the seven to ten weeks prior to the close of the school year.

Tables illustrating the response mean rights follow (14:687-688).

One should note that for the contemporary program in District A the fifth grade mean falls well below the fourth while the sixth grade mean reaches very close to the ceiling of the test.

The conventionals in District B show no corresponding trend to the same group in A, while the contemporaries show a steady decrease in mean rights, again not correlating to the same group in A.

For the contemporary programs in both districts there is a markedly higher mean rights at each grade level than for the conventional programs.

TABLE I

RESPONSE MEAN RIGHTS BY GRADE, DISTRICT, AND PROGRAM FOR TOTAL INVENTORY

District	Program	Grade	N	Mean Right
A	Conventional	4	31	35.2
A	Conventional	5	27	36.6
A	Conventional	6	19	37.5
A	Contemporary	4	27	56.3
A	Contemporary	5	33	44.9
A	Contemporary	6	29	66.1
B	Conventional	4	33	36.6
B	Conventional	5	29	42.6
B	Conventional	6	30	33.9
B	Contemporary	4	31	63.4
B	Contemporary	5	33	58.9
B	Contemporary	6	17	55.2

<u>Conclusions</u>. The inventory indicates that subjects exposed to a contemporary program do score higher than those exposed to a conventional program. This supports the hypothesis from the preliminary study, that opportunity to learn does affect performance on the inventory. No clear trend between grade level and level of geometric understanding was evidenced.

Weaver has also concluded that this investigation has emphasized the importance of questions or hypotheses in connection with research in mathematics education.

This led him to a conjecture regarding the single factor most critical in accounting for observed difference in mean rights. His follow up statement about this conjecture was most unique and will be given further attention in the comparison of the three studies in this section.

<u>D'Augustine's Studies</u>

As stated earlier D'Augustine used a programmed text which he had developed. The text itself developed the concepts of points and sets of points, and by a logical sequential structure the properties of lines, line segments, collinearity, and broken lines evolved. From these it was hoped the properties of simple closed curves, convexity and non-convexity, boundedness, and interior and exterior would be developed. By extension of these properties the properties of various polygons could be established.

In his pilot study D'Augustine (2) used a sample of thirteen boys and thirteen girls randomly assigned to a sixth grade class.

They spent one hour for five consecutive days using the text developed by the researcher. At the end of this period a test was administered and on the basis of scores students were classified as to predetermined levels of success. Nine showed excellent success, eight moderate success, and nine limited success.

An analysis of the test items showed these topics to be highly teachable: interior, exterior, and boundary points; congruency; simple closed curves; properties and definition of a triangle; collinearity; countable and uncountable sets of points; properties of lines and line segments; and properties of broken lines.

The researcher felt that the programmed unit proved to be a highly efficient experimental tool but it did have a limitation in that each student did not receive a wide variety of instructional techniques to assist in his mastery of the concepts explored (2:412).

Feeling the very definite need for further study of this topic, D'Augustine undertook an extensive research project in this area two years later. He was attempting to discover factors which relate to a student's achievement with geometric topics and at what grade levels certain topics are learned with a high degree of efficiency in terms of time and expended effort.

D'Augustine subjects. The study consisted of all fifth and sixth grade students from Leonard Wesson Elementary School and all seventh grade students from Richards Junior High, both located in Tallahassee, Florida. In this report only the findings for the fifth and sixth grade students are being considered.

Three cells or groups were formed at each grade level, with thirty students placed in each cell using a table of random numbers.

Procedure and findings. The students had pursued a standard curriculum for their grade and pretests gave no evidence of their familiarity with the topics to be studied.

The group in one of the three cells, designated control, received a pretest and posttest but no instruction in the topics of geometry and topology. Another group, randomly designated group T, received thirty minute daily sessions in the programmed text, while the group, randomly designated F, received fifty minute daily sessions.

The programmed text was basically the one developed in the preliminary study previously described. The researcher did note that there were sequences of development beginning with the concept of path, followed by investigation of properties of geometrical figures and culminating in the study of properties of squares, rectangles, and parallelograms. The emphasis being to cause the students to discover what properties are necessary to receive a specific class name.

This implies that during the two year lapse between studies attention was given to a refinement of the text.

A linear hypothesis model program was used to test these hypotheses: There are no significant differences in the mean levels of achievement on the criterion posttest attributable to differences in working time; sex; or grade levels after mean criterion posttests have been adjusted to Mental Age, Chronological Age, Reading Ability, and Arithmetic Ability.

Several combinations of the covariates is possible but as far as the population studied was concerned only Reading Ability and Arithmetic Ability had significant effects upon various experimental mean posttest scores. Mental Age and Chronological Age had no significant effect.

On the basis of the data obtained none of the hypotheses relating to treatments could be rejected.

An analysis of the mean completion time represented the first non-linear set of means found in the study. The sixth grade thirty minute group was most efficient in terms of completion time while the fifty minute fifth grade cell proved to be least efficient.

The hypothesis tested was: There are no significant differences in the mean levels of completion time for any experimental cell.

The results of the Analysis of Variance caused rejection at almost every level of comparison. The only difference not significant at the .05 level for the groups being considered here was for the thirty minute fifth grade group and the fifty minute sixth grade group.

Conclusions. The researcher felt that the shorter working periods proved most effective at each grade level and if covariate adjustment is allowed the programmed text proved to be most efficient at the sixth grade level (3:197).

On the basis of this study it was shown that Reading Ability and Arithmetic Ability do serve as factors relating to a student's achievement with certain geometrical and topological topics and in terms of time and expended effort it appears that there is a higher degree of efficiency for short periods of instruction.

Denmark and Karlin's Study

In seeking an answer to their query about teaching geometric construction to intermediate grade children Denmark and Karlin (4) were investigating the children's intellectual maturity for learning the geometric concepts and the presence of the necessary muscular skills required to perform certain constructions.

Denmark and Karlin subjects. Their population consisted of one fifth grade class from the University School of Florida State University. There were fourteen boys and fourteen girls randomly assigned with no attempt made to group.

They followed a course standard in most Florida schools, but it is important to note that on the basis of the California Test Bureau Test administered at the beginning of the year the mean I.Q. was found to be 126, with the mean Reading Ability at the 7.4 level, Language Ability at the 6.5 level, and Arithmetic Ability at the 6.6 level.

Procedure and findings. The class used two worktexts developed by Hawley and Suppes. These contain instruction in some of the basic Euclidean constructions with compass and straightedge.

Each student was allowed to work at his own rate in class meetings of approximately fifty minutes. Anecdotal records were kept by twelve undergraduate students who were assigned two to three students grouped by the teacher on a homogeneous basis according to ability and Mental Age to date.

On the average the students required several weeks to complete both books. As a book was completed a specifically written criterion test was administered.

The first test contained questions requiring the students to construct an equivalent triangle with given base and fixed vertex; construct circles with radii less than, greater than, or equal to a given distance; bisect angles and line segments; compare size of angles and line segments by using the compass; connect pairs of points with a line segment and then determine the number of line segments, points of intersection and the number of triangles formed; to double a line segment; and construct certain geometric figures. There was also a twelve item vocabulary section.

The results of the test were set up in a frequency distribution. With a possible score of forty-nine, the scores ranged from a low of twenty-eight to a high of forty-seven. The mean score for the class was 38.5, or 78.6 percent of the total possible.

In the subjective opinion of the researchers a score of twenty-eight was satisfactory while forty-seven was excellent, the class average indicating more than reasonable accomplishment.

To test this subjective opinion the thirty-seven item part of the test was given to a senior high geometry class at the University School. The results show that the twenty-eight fifth graders had a range from thirty-six to twenty with the mean being thirty. The senior high class of twenty students exhibited a range from thirty-five to twenty-two with the mean being thirty. The researchers determined that these results supported their judgment of the fifth grade class achievement.

Test Two covered bisecting angles; constructing perpendicular lines; comparing the size of angles with a compass; reproducing triangles and quadrilaterals; constructing similar figures; and responding to thirteen true-false items testing comprehension of generalizations.

The results were recorded in an interval distribution with the highest possible score being eighty-eight. The scores ranged from a low of ten to a high of seventy-nine with the class mean being 50.0 or 56.8 percent of the total possible.

In the subjective judgment of the researchers a score less than forty was poor and the class average of fifty was not satisfactory enough to justify the instructional time used.

Item analysis of this test showed that the students could: bisect an angle; construct the perpendicular bisector of a line segment; copy a triangle and a quadrilateral; and construct a perpendicular to a line through a point on the line.

Particular difficulty was noted with comparing size of angles, attributed by Denmark and Karlin to lack of precision using a compass; dropping a perpendicular to the line from a point not on the line, attributed to questionable constructions; constructing a square or rectangle, due to confusion with perpendicular lines; and constructing similar triangles, where the students failed to get equivalent angle measure.

Conclusions. Denmark and Karlin (4) concluded that their study supports the opinion that a considerable amount of geometry can be taught in the elementary schools and to a greater extent than is now done.

The difficulty in handling the compass may suggest that certain necessary muscular skills are not yet developed.

Comparison of Fourth, Fifth, and Sixth Grade Studies

All three of the above studies concerning geometry with fourth, fifth, and sixth grade students present valuable information for those attempting to implement such activities in their classrooms.

The findings of all of the studies support the presentation of geometric concepts which cause the child to acquire specific knowledge of the properties of geometric figures and then using these properties to classify the figures in broader, general categories. Finally applying this knowledge they are able to compare properties and perform constructions which permit them to extend their understanding through a discovery of other characteristics.

In their major investigations, Weaver (14) and D'Augustine (3) selected their populations from broad areas, providing for some randomness. Denmark and Karlin (4) not only were limited to one class but to a University school. This could cause one to question the representativeness of the population studied.

All studies used appropriate means for testing their hypotheses. It is interesting though that Weaver (14) neglected to indicate whether the higher mean rights for the contemporary programs was statistically significant.

D'Augustine's use of in-service teachers in his preliminary study is quite unique and indicates that some elementary school teachers do lack a knowledge of basic geometric concepts.

Weaver's study (14) seems to support this fact for in District A where specialized teachers were used there was a higher degree of student performance on the inventory than for the corresponding group in District B even though both were receiving conventional treatment.

D'Augustine (3) found the Reading Ability and Arithmetic Ability of the students does have a significant effect on geometric understanding, so while teacher knowledge is not wholly responsible, it does appear to influence student achievement.

By using programmed materials perhaps D'Augustine (3) and Denmark and Karlin (4) were trying to eliminate the teacher as a variable.

A research study to determine teacher knowledge influence on student achievement would be interesting.

As cited earlier Weaver (14) leaves his findings in a most unusual way. Where he stated that he had a conjecture regarding the single factor that is most critical in accounting for observed differences in mean rights, he did not state what his conjecture was. Rather he asks: "What is your conjecture regarding that factor? How might you test the validity of your conjecture in a more extended investigation?" (14:690)

Denmark and Karlin do a similar thing by introducing the question: "What is the value of teaching geometry in the elementary school?" (4:73), but never answering it. D'Augustine leaves one pondering: "Which geometric and topological topics are appropriate at various grade levels to clarify and simplify other types of mathematical concepts?" (3:192)

All are using tactics which cause the consumer of their material to look again and to really study their conjectures and outcomes. It causes one to weigh the results of their findings before immediately applying similar tactics to one's own class situation. One is also made aware that these researchers do not conclude that their research is definitive. Rather they suggest extension of their work and other investigations which might provide useful information.

CHILDREN AGES SEVEN TO ELEVEN
A COMPREHENSIVE STUDY

There was one study available for review which covered the expanse from ages seven to eleven. The material covered, being quite extensive, will be treated separately. Following this there will be a brief comparison of all six studies.

Shah's Study

The purpose of Shah's study (10) was to determine to what extent certain content in geometry was satisfactory for children within the range seven to eleven.

The content included concepts of plane figures with three to twelve sides, nets of figures, symmetry of figures about a line, reflection of figures in a plane mirror, rotation of figures about a point, translation, bending and stretching of figures with no cutting, and networks.

Shah subjects. For the study three hundred and seventy-four pupils, within the age range of seven to twelve, were selected from eleven schools. They represented several religious denominations, urban and rural areas, different ethnic groups and different socio-economic levels. The sample was sufficiently large and dispersed to be considered representative.

Procedure and findings. The student teachers had all read compulsory mathematics books for a course developed along modern trends. They were given the freedom to use whatever concrete materials they considered desirable.

The classes were held for a period of two weeks. It was felt that the children may have had previous experiences which might have found expression in the exercises presented.

During the course of the presentation the student teachers prepared and administered tests. These were used as a basis for preparing the final objective test.

A Kuder-Richardson Formula 21 reliability coefficient of 0.93 was obtained for the final test and therefore it was felt to be valid.

The test consisted of nine parts and was administered three parts at a time. The children worked simultaneously with no time limit set.

It was found that the content regarding matching polygons to numbers was understood by almost 100 percent of each age group. Material concerning nets of solids was found to be reasonably understood by all age groups, while symmetry about a line was easily understood. Reflection in a mirror caused little difficulty for any age group. Rotation about a point was found to be poorly understood by the seven to eight age group with very little more understanding shown by the eight to nine age group, while the other age group percentages remained high. Similarly, when working with translation the two younger groups did not perform well while the two older groups did. All age groups performed well with bending and stretching of two dimensional figures but bending and stretching of three dimensional figures caused difficulty, especially for the younger children. With networks the seven to eight age group performed poorly while the others performed reasonably well.

With a possible score of 108 on the test it was found that 50 percent of the seven to eight age group scored in the middle range (50-59), while 85.1 percent of the eight to nine age group, 96.9 percent of the nine to ten age group, and 100 percent of the ten to eleven age group scored in this range.

The score and percentage means are illustrated in the following table (10:122).

TABLE II

SCORE AND PERCENTAGE MEANS BY AGE GROUPS ON FINAL OBJECTIVE TEST

Age Groups	Score Means (108)	Percentage Means (100)
7- 8	48.2	44.6
8- 9	68.6	63.5
9-10	78.7	72.8
10-11	81.0	75.0

Conclusions. The researcher noted that performance became better as age increased and the results of various aspects of the test give good indications of the kind of geometry which can be successfully studied by children of different ages.

Description of the subtests has been included in Appendix C so that an idea of the type of test used can be formulated.

Comparison of All Six Studies

The sample drawn for the Shah study (10) was, statistically, the most representative of all of the studies, although transfer might be questioned because the study took place in Trinidad.

The fact that a reliability coefficient for the test used in the Shah study (10) was indicated causes acceptance of the findings more readily than those obtained in the studies of Rea and Reys (7), Starlin (11), Weaver (14), Denmark and Karlin (4), or D'Augustine (3).

Shah's study (10) supports the supposition made by Starlin, that is, it is possible to present more advanced topics to first grade children than has been done previously.

The fact that no programmed text was used in the Shah study (10) also indicated that these topics can be presented by well-informed teachers. This fact supports the findings of Weaver's preliminary study (13), that opportunity to learn does influence achievement, suggesting that our elementary teachers need the opportunity for growth in this area so that they may properly prepare the children they teach for further geometric experiences.

The fact that the findings of all of the studies point to the inclusion of geometric topics at the elementary levels prompts one to accept their relative significance to the topic under consideration. That is, all of the studies point to the fact and give some statistical backing for the presentation of selected geometric topics to elementary school children.

CONCLUSION

Overall, it can be seen that the presentation of geometric shapes to elementary children does not have to be limited to the square, circle, triangle, and rectangle. Even primary children have exhibited the capability of learning other shapes such as the pentagon, hexagon, and octagon and they can associate numerical identification with the prefixes for these figures.

Extended investigation of shapes has shown that the children can classify the shapes in broader categories on the basis of general properties. This was substantiated by both Weaver and D'Augustine. Elementary children can follow a logical development from basic general properties to an application of these to specific forms.

There is also evidence that children in kindergarten through grade six have the capabilities to handle the vocabulary connected with basic geometric concepts. This extends to their understanding of point, line, line segment, base, side, corner, inside, and outside of figures. A question of muscular development could cause a problem for some children in using the compass and protractor for constructions but there is evidence that most can successfully use them.

It is apparent that concepts, such as symmetry, nets of solids, reflections, and bending and stretching of two dimensional shapes can be presented from first through sixth grade, while rotation about a point, translation and networks can be successfully presented from fourth through sixth grades.

Topics presented at the primary levels can be extended at the intermediate level with the overall coverage helping to prepare the children for geometric encounters at higher levels.

To say that we now have all of the information necessary to establish a fail-safe geometric program or an approach to an established one is erroneous. Much further research is necessary as an investigation of the presentation of geometry at the elementary level continues.

The findings cited above can lead us to surmise that geometry is important and it is unrealistic to postpone the study of geometry until it can be approached in a systematic and rigorous way (Robinson, 9:3).

As Brune has stated in our shunning of geometry at the elementary level we have forgotten that geometry could not be separated from life and it could not be effectively dealt with in the traditional tenth grade crash program (1:446).

Brune's statement suggests that geometry has a place in the elementary mathematics curriculum. Other mathematical concepts are presented in what we might call a spiral. The research cited indicates that geometry has a place in that spiral. It is not necessary to wait until a child is fourteen or fifteen years old to present him with geometry. He is capable of handling certain concepts long before that age.

Concrete objects are used to introduce almost all mathematical concepts at the primary level. So it should be with geometry. Gibney has indicated that geometry should progress from concrete objects to the non-metric phase (6:472). Many high school students have difficulty making transfers to geometrical abstractions because they have not experienced this very important stage in geometric development.

The fact that Reading Ability has been found to influence geometric achievement places stronger emphasis on the importance of concrete representations at the primary level where reading achievement is not fully developed.

Using the research surveyed as a basis the following topics are suggested for implementation at the elementary level emphasizing ideas of interest and presenting material of value for the study of mathematics.

The topics suggested are: Vocabulary treatment of geometric concepts; planar and spatial relationships; symmetry; geometric constructions and reproductions; property relationships and comparisons; rotations about lines and points; bending and stretching of two dimensional figures; and figure identification and classification.

With all of the above topics opportunity to learn should be carefully noted by the teacher thus providing all children with opportunity for experience in geometry so that they may acquire background and knowledge.

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APPENDIX A

CMI Geometry Subtest

Shape Matching

107 Match wooden □ on card.
108 Match wooden ○ on card.
109 Match wooden △ on card.
110 Match wooden ◇ on card.
111 Match wooden on card.
112 Match wooden ▭ on card.
113 Match wooden △ on board with cutout.
114 Match wooden ▭ on board with cutout.
115 Match wooden ◇ on board with cutout.
116 Match wooden ○ on board with cutout.
117 Match wooden □ on board with cutout.
118 Match wooden on board with cutout.

Vocabulary

119 What is this? (Show □)
120 What is this? (Show ○)
121 What is this? (Show △)
122 What is this? (Show ▭)
123 What is this? (Show ◇)
124 Point to side of □
126 Point to corner of □
128 Point to side of △
132 Which is line? (Picture with rope and line)

Spatial Relationships

125 How many sides does □ have?
127 How many corners does □ have?
129 How many sides does △ have?
130 Put finger inside circle.
131 Put finger outside circle.
134 Which line is longest? (Show card with four lines of unequal length).
135 Which line is shortest? (Same as card above).
137 Which lines are closest together? (Show card with three parallel lines, different distances apart).
138 Which lines are farthest apart? (Same card as above).

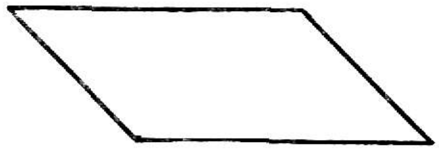
Geometric Reproduction

133 Draw a line.
136 Draw a pair of lines. (Show card with pair of parallel lines).
139 Draw a line like this. (Show card with broken or dotted line).
140 Draw lines like this. (Show card with perpendicular lines).

APPENDIX B

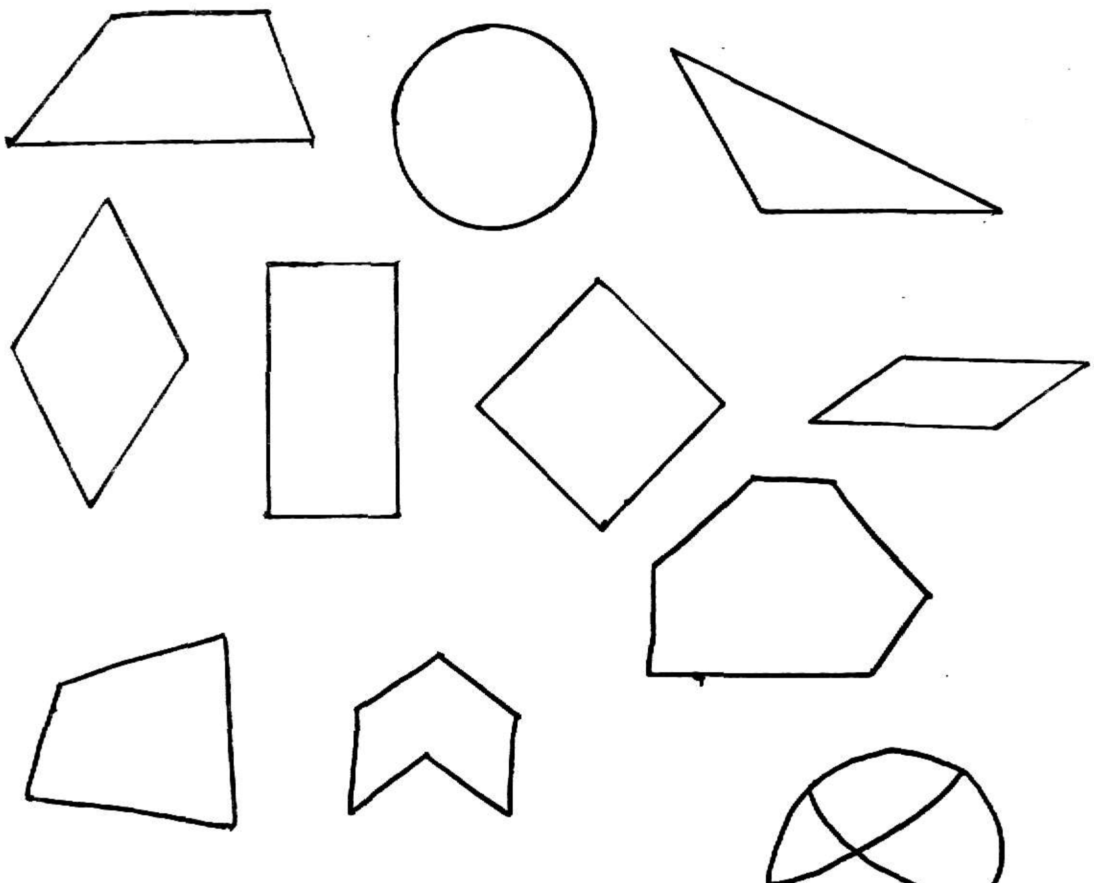
Inventory: Form G_1 (13:324-326)

1 This is a drawing of a . . .



polygon	Yes	No	Not Sure
quadrilateral	Yes	No	Not Sure
rectangle	Yes	No	Not Sure
simple closed curve	Yes	No	Not Sure
square	Yes	No	Not Sure
triangle	Yes	No	Not Sure

The other figures used are:



APPENDIX C

Description of Shah Subtests

G1 Matching polygons to numbers

Polygons having 3, 4, 5, 6, 7, 8, 9, and 12 sides were given, and the numbers were arranged randomly. Candidates were asked to match each figure with a number by writing the number inside the figure.

G2 Nets of solids

The solids shown were a rectangular box; a ball; a short, closed cylindrical box; a long, open tube; a rectangular pyramid; and a triangular pyramid. The nets of these figures were also drawn. Both solids and nets were labeled. Candidates were given the following directions: "One of the figures shows the faces of one of the solids. Match each solid with its 'faces' by writing the letter given for the solid on the faces."

G3 Symmetry about a line

Candidates were asked to draw a line in each of the figures so that, when the figure is folded about that line, one part will fit onto the other part.

The figures given were all two-dimensional and were in the shape of a heart, a triangle, a man's face, a rhombus, a deck of a boat, a side of a house with triangular cross section, a side of a boat, and letters A, E, and C.

For another set of figures, candidates were asked to draw as many lines as possible along which you can fold the figure so that one part will fit onto the other part. These figures were a rectangle, a square, a rhombus, and two circles touching each other.

A third set of figures were given, where only half of each figure was drawn, and candidates were asked to complete the figures. These were a leaf, a man's face, a triangle, an archdoor, the number "8", a butterfly, the letter "x", and an insect body.

G4 Reflection in a mirror

Candidates were given the following information for exercises: "With the following, the broken line (----) is a mirror. A figure is given on one side of the mirror. Draw on the other side of the mirror what you will see when you look in the mirror." The figures were a profile of a man's face, an upside-down "2", the letter S, a bowl, a fish, a champagne glass, the number "3", and the letter P.

G5 Rotation about a point

1. The following were given: a triangle with one side extended to a pole, a circle with a pole (as with a speed-limit sign), a triangle, and a circle. Candidates had this information: "the following figures are turned in the direction of the broken line (----). Show the figures by drawing in the new direction.

2. For another question an equilateral triangle with the numbers 1, 2, 3 written at the vertices and an arrow pointing in a clockwise direction were given. The triangle was turned about its center. Candidates' instructions were: "The figure below is turned around about 0 in the direction of the arrow. Show the position of the corners of the figure in (a), (b), (c), (d), as the figure is turning, by writing in the numbers 1, 2, and 3." The letters (a), (b), (c), (d) were labels of equilateral triangles.

3. Here the triangle was given as in (2), but the direction of the arrow was changed to counterclockwise. Instructions were: "Show the positions of the corners in (e), (f), (g), (h)." (These letters were labels for equilateral triangles.)

4. This question was similar to (3) except that here the numerals 1, 2, 3, and 4 were written on the circumference of a circle at equal intervals.

5. This question was similar to (4) except that the direction of the arrow was counterclockwise.

G6 Translation

Figures were given with one point translated from A to A. Candidates were asked to show where other points on the figures were to move. The figures given were a line segment, an arc of a circle, the letter S, an airplane, a boat, a car, and a rocket.

G7 Bending and stretching of two-dimensional figures

1. Instructions for this question were: "Some of the figures below can be obtained by bending and stretching a circle, but no cutting is done. Check those obtained by bending and stretching the circle." Here three "closed" and three "open" figures were given.

2. For this question, instructions were: "Some of the figures below can be obtained by bending and stretching a rubber string." Three "closed" and three "open" figures were given.

3&4. For these questions instructions were: "In the following, one figure is obtained from another by bending and stretching, but no cutting or joining is done. Match figures by putting the letter from one onto another." For (3) the figures were a "Y", a cross (+), a box with compartments, a box cross section with three compartments. For (4) all the figures were open and comprised of segments. They had two, three, and four segments.

G8 Bending and stretching of three-dimensional figures

Solids were given and candidates were asked to match figures. The figures were a ball, a doughnut, a sphere with two handles, a sphere with three handles, and a solid with four holes.

G9 Networks

1. Four figures were given, which could be traced by a single path, and four figures were given which could not be traced by a single

path. The instructions were given: "Show which figure you can trace by going over any part only once. You can start anywhere you like. Do this by drawing arrows in the direction you are going. Put a check on those you can trace by going over each part only once, and put a cross (x) on those you cannot trace."

2. Eight figures were given, for which joinings were required to obtain single paths.

GEOMETRY IN THE ELEMENTARY
MATHEMATICS CURRICULUM

by

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AN ABSTRACT OF A MASTER'S REPORT

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The scientific and technological advancements being made in the world today have caused educators to look closely at many curricular areas. Mathematics is one of these areas. More advanced concepts are being presented at earlier ages.

One of the areas receiving the attention of researchers who are attempting to determine the feasibility of presenting advanced topics at the elementary level is geometry. This study reviews six available research projects. The purpose of this study was to identify the geometric concepts with which the researchers were concerned, to compare the findings of the research reviewed, and on the basis of these findings to suggest topics suitable for inclusion in the elementary mathematics curriculum.

Two of the studies dealt only with kindergarten or first grade children. The topics investigated in these studies were: vocabulary understanding, shape matching, spatial relationships, geometric reproductions, and the identification of certain plane figures.

It was found that a child of this age is capable of handling the topics cited. The findings suggested a base on which beginning learning activities could be built.

Three studies concerning children in the fourth, fifth, and sixth grade were available for review. The problems of concern here were: levels of geometric understanding in relation to general categories of classification; the feasibility of teaching geometric constructions; and the investigation of factors which might serve as predictors of predetermined levels of geometric and topological success.

The findings of all three studies supported the inclusion of more advanced geometric topics at this age. It was found that children in these grades can successfully build from general geometric properties to specific figures and apply their knowledge to classification and construction work. It was also found that reading and arithmetic ability do influence student achievement with some geometric topics.

The last study available for review was one in which the subjects ranged from ages seven to eleven and the scope of material ranged from determination of the number of sides of given polygons to work with networks. From this study it was concluded that the topics covered very definitely have a place at some level in the elementary school.

The total overview of all six studies led to the suggestion of the following topics for inclusion in the mathematics curriculum of the elementary school: vocabulary treatment of geometric concepts; planar and spatial relationships; symmetry; geometric constructions and reproductions; property relationships and comparisons; rotations about lines and points; bending and stretching of two dimensional figures; and figure identification and classification.