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PROBABILITY-BASED LOAD FACTORS
AND LOAD COMBINATION FACTORS
IN STRUCTURAL DESIGN

by

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Abstract

Probability-based load factors and load combinations are presented which are compatible with the loads specified in ANSI A58 Standards. The load effects considered are due to dead, occupancy live, snow, wind and earthquake loads. The load criteria which are presented were developed by numerous investigators on the basis of probability of failure calculated by simplified theory, using statistical data on structural loads and resistances. The report presents the rationale and methodology used by the investigators for selecting the load factors which are intended to apply to all types of structural materials used in building construction.

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I. Introduction

Structural safety standards are developed to satisfy the ultimate limit states (for human safety) followed by checks of the serviceability limit states. Generally, any design specification must list the limit states to be considered and must present load and resistance factors for use in checking these limit states. The existence of safety factors is due to uncertainties associated with the calculations of resistance and loads. Examples of uncertainties associated with the resistance of materials are; 1) variability in material properties such as strength of steel; 2) variability in member geometry such as cross-sectional area; and 3) variability in the ratio of strength used in calculations to the actual member strength represented by available test data. Examples of uncertainties related to the loads are variations of live loads with time and variations caused by idealization of loads and their locations. In addition load and resistance factors account for uncertainties in structural analysis and modeling such as the assumption that connections are rigid, and the use of a rectangular stress block in concrete design. These uncertainties in resistances and loads indicate that they should be considered as random variables in statistical studies. As a result a single-value factor of safety is unacceptable because it implies that both loads and resistances can be obtained with the same degree of uncertainty

in different loading conditions. Therefore a statistical approach to the safety criteria is considered in this report because of the necessity of dealing with random variables.

Some of the advantages of probabilistic limit states design are;

1) more consistent reliability is attained for different design situations because the different variabilities of the strengths and loads are considered independently; 2) the reliability level can be chosen to reflect the consequences of failure; 3) it gives a better understanding of the fundamental structural requirements; 4) it simplifies the understanding of different design processes by encouraging the same philosophy and procedure to all materials of construction.

2. Structural Reliability

2.1 Normal Distribution

To determine the reliability of a structure, it is assumed that the mean, variance, and the type of distribution function of random variables (loads and resistance) are known. A mathematical model is first derived which relates the resistance and load variables for the limit state of interest,

$g(X_1, X_2, \dots, X_n) = 0$. The terms X_i are the resistance or load variables, and failure occurs when $g < 0$ for any state of interest. Safety is assured by assigning a small probability P_f to the event that the limit state will be reached, $g < 0$ [Ref. 3],

$$P_f = \int_{x_1} \int_{x_2} \dots \int_{x_n} f_x(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \quad (1)$$

which in this $f_x(x_1, x_2, \dots, x_n)$ is the joint probability density function for $X_1, X_2, \dots, X_n$ and the integration is performed over the region where $g < 0$. To perform the reliability analysis, the mathematical model must be linearized at some point. This point is called the checking point and the linearized form of the model is

$$Z = g(X_1^*, X_2^*, \dots, X_n^*) + \sum (X_i - X_i^*) (\partial g / \partial X_i)_{X_i^*}$$

where $(X_1^*, X_2^*, \dots, X_n^*)$ is the checking point. In this equation the partial derivatives are evaluated at the checking point $(X_1^*, X_2^*, \dots, X_n^*)$. By assuming the mean values of variables as the checking point, the reliability index, β ,

can be approximated as shown below,

$$\bar{Z} = g(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) \quad (2)$$

$$\sigma_Z^2 = (\partial g / \partial x_1)^2 \sigma_{X_1}^2 + (\partial g / \partial x_2)^2 \sigma_{X_2}^2 + \dots \quad (3)$$

$$\sigma_Z = [\sum (\partial g / \partial x_i)^2 \sigma_{X_i}^2]^{1/2} \quad (4)$$

$$\beta = \bar{Z} / \sigma_Z \quad (5)$$

in which $\bar{Z}$ and σ_Z are the mean and standard deviation of the linearized form, respectively, and $\sigma(X_i)$ is the standard deviation of X_i . It should be noted here that by linearizing the mathematical model, $g(\cdot)$, big errors can be obtained at greater distances from X_i^* . Figure 1 shows the density functions of Z for a two-variable problem $g(R, Q) = 0$. The mathematical model for this example is $g = R - Q$, in which R is the resistance and Q is the load effect and failure occurs when $g < 0$. Here the probability of failure is computed as,

$$P_f = P(R < Q) = \int_0^{\infty} F_R(x) \cdot f_Q(x) dx \quad (6)$$

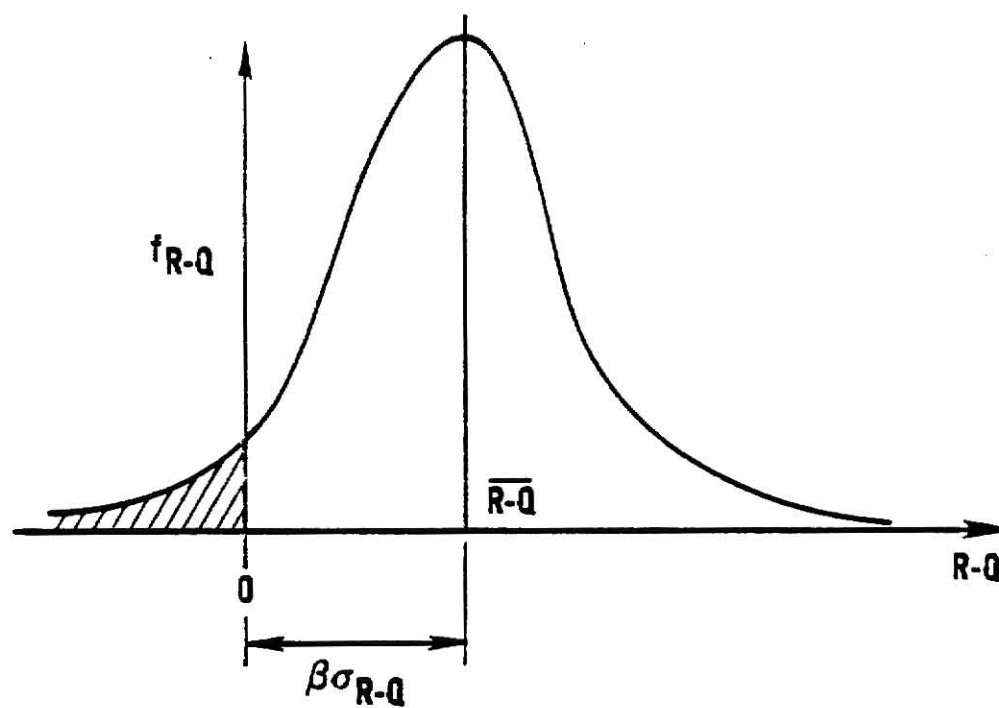
where F_R is the cumulative probability distribution function (c.d.f.) of R and f_Q is the probability density function of Q . If it is assumed that both R and Q are normally distributed and statistically independent, then $R - Q$ is normal with mean $\bar{R} - \bar{Q}$ and variance $\sigma_R^2 + \sigma_Q^2$ [Refs. 1, 6]. Therefore the probability of failure for a normal distribution is

$$P_f = P(R - Q < 0) = [1 / (2\pi)^{1/2} \cdot \sigma_{R-Q}] \int_{-\infty}^0 \exp\{(-1/2)[(x - (\bar{R} - \bar{Q})) / \sigma_{R-Q}]^2\} dx \quad (7)$$

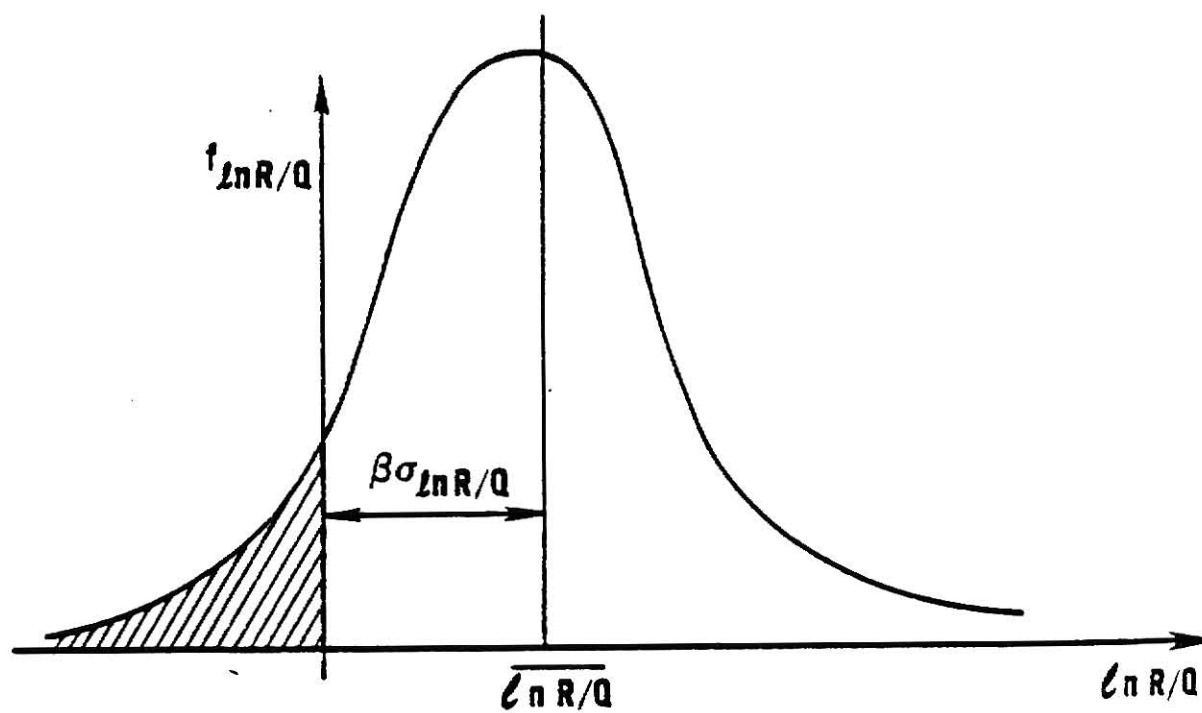
So the relationship between P_f and β is as shown below [Ref. 3],

$$\beta = \text{mean / standard deviation}$$

$$\beta = (\bar{R} - \bar{Q}) / [\sigma_R^2 + \sigma_Q^2]^{1/2} \quad (8)$$



(a) $g = R-Q$



(b) $g = \ln(R/Q)$

Figure 1 - Illustration of the Reliability Index Concept
(Ref. 4)

$$P_F = \Phi \left[-(\bar{R} - \bar{Q}) / (\sigma_R^2 + \sigma_Q^2)^{1/2} \right] \quad (9)$$

$$P_F = \Phi(-\beta) \quad (10)$$

$$\beta = \Phi^{-1}(1 - P_F) \quad (11)$$

in which $\Phi(\)$ is the cumulative distribution function for the standard normal variate and $\Phi^{-1}(\)$ is a statistical symbol. Its function is opposite to that of the cumulative distribution function; in other words for a given probability of occurrence it calculates the value of X . Some of the numerical values of P_F and β for a normal distribution are shown below [Ref.2].

P_F	β
0.01	2.33
0.001	3.09
0.0001	3.72
0.00001	4.26

There are some shortcomings in linearizing the $g(\)$ function at the mean values of X -variables. First, by linearizing $g(\)$ at the mean values significant errors may be introduced by neglecting higher order terms of the Taylor series expansion used to linearize the model, at increasing distances from the linearized point. Second, β depends on how the limit state is formulated; and by linearizing at the mean value points, β becomes variant to the different mechanically equivalent formulations of the same problem. The function $g(\)$ and its partial derivatives are independent of how the problem is formulated only on the surface where $g(\)=0$. So to avoid this problem, $g(\)$ must be linearized

at some point on the failure surface. This procedure is explained as follows. The variables, first, are transformed to reduced variables with zero mean and unit standard deviation by

$$x_i = (X_i - \bar{X}_i) / \sigma(X_i) \quad (12)$$

and the limit state is $g_1(x_1, x_2, \dots, x_n) = 0$ with failure occurring when $g_1 < 0$. Then the reliability index is defined as the shortest distance between the surface $g_1(\) = 0$ and the origin. See Figure 2. The checking point is computed as shown below [Refs.3,4],

$$\alpha_i = (\partial g_1 / \partial x_i) / [\sum (\partial g_1 / \partial x_i)^2]^{1/2} \quad (13)$$

in which α_i is the direction cosine between the gradient vector associated with the surface and the i th coordinate axis. See Appendix A for the derivation of direction cosines for the general case. Then,

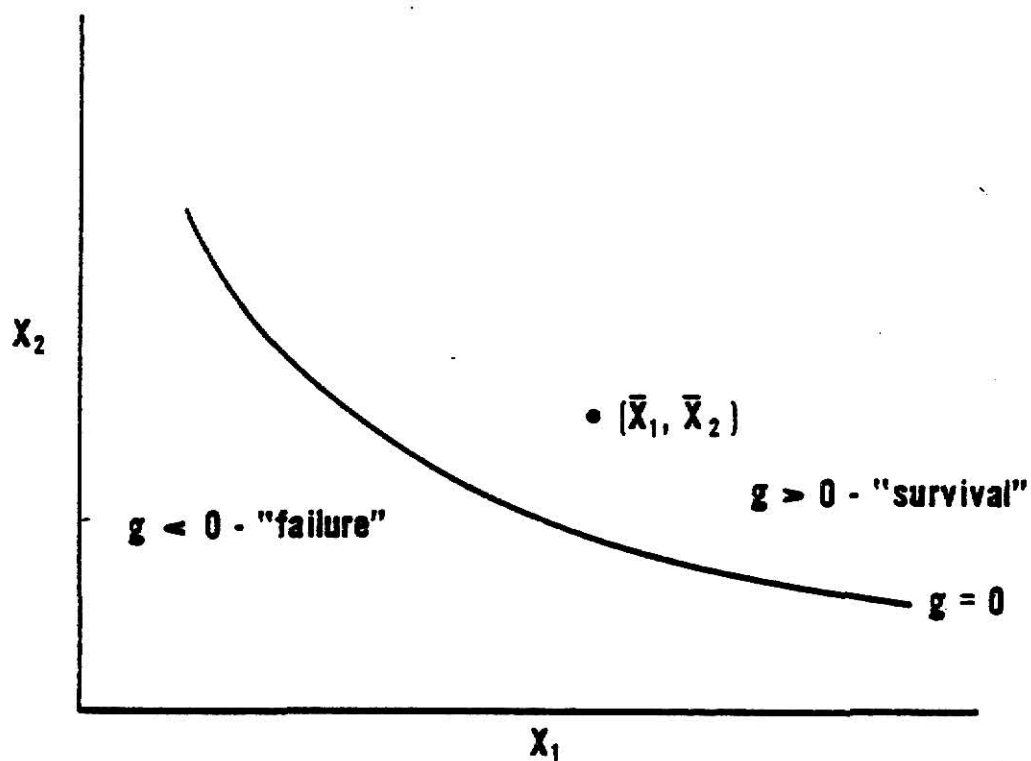
$$x_i^* = -\alpha_i \cdot \beta \quad (14)$$

$$g_1(x_1^*, x_2^*, \dots, x_n^*) = 0$$

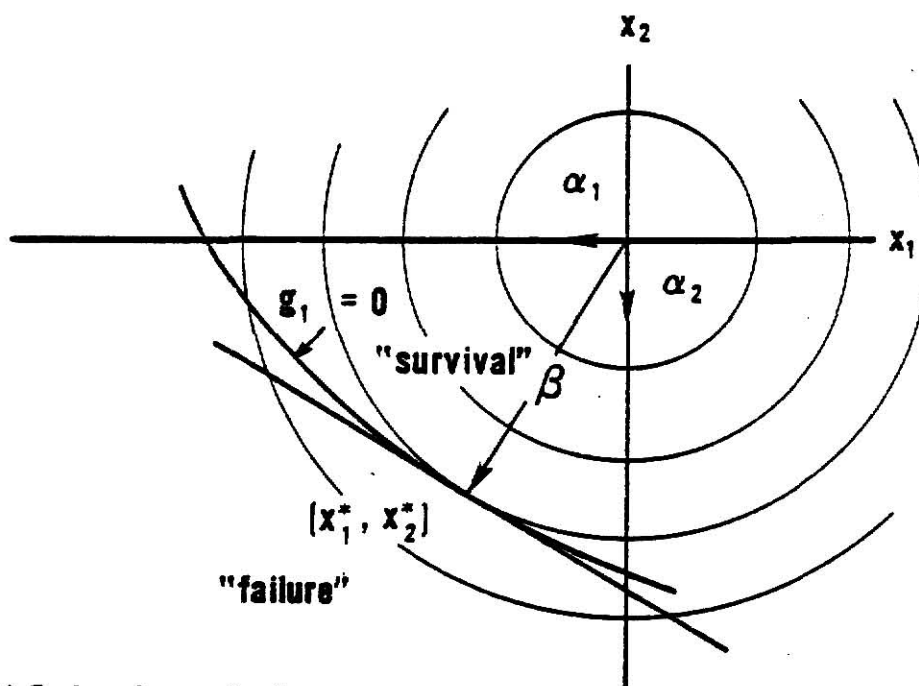
β , the reliability index, is obtained by searching for the direction cosines α_i to minimize the distance from the origin to surface.

2.2 Approximate Method For Non-Normal Distributions

The procedure described prior to this gives values of β , the reliability index, which are related to the probability of failure if the distribution functions are normal and the $g(\)$



(a) Original coordinates



(b) Reduced coordinates

Figure 2 - Formulation of Safety Analysis in Original and Reduced Variable Coordinates
(Ref. 4); x_1^* is the checking point corresponding to the shortest distance from the origin, on $g_1(\) = 0$.

function is linearized. In other cases it gives an approximate relationship as explained before. For non-normal variables, the idea is to transfer them to equivalent normal variables. This transformation may be done by approximating the actual distribution by a normal distribution at a point on the failure surface. The justification for this is that if normalization takes place at the point of failure (minimum β), the probability of failure obtained by the approximate method should match the actual probability of failure very closely. The mean and standard deviation of the equivalent normal variable are calculated [Refs. 3,14] such that at the checking point (X_i), the cumulative probability, and the probability density functions of the actual and the equivalent normal are equal. For example, if the actual distribution curve of a variable is log-normal, at the checking point where $g_1(\cdot) = 0$ the probability of x_i being less than x_i^* is the same as for the equivalent normal distribution. See Figure 3. In this figure the shaded areas must be equal. These guidelines suggest that the equivalent mean and standard deviation must be as shown below [Refs. 14].

$$\sigma_i^N = \phi(\Phi^{-1}[F_i(X_i^*)]) / f_i(x^*) \quad (15)$$

$$\bar{X}_i^N = X_i^* - \Phi^{-1}[F_i(X_i^*)] \cdot \sigma_i^N \quad (16)$$

where F_i and f_i are non-normal cumulative distribution and density functions of X_i , and ϕ represents the standard normal distribution function.

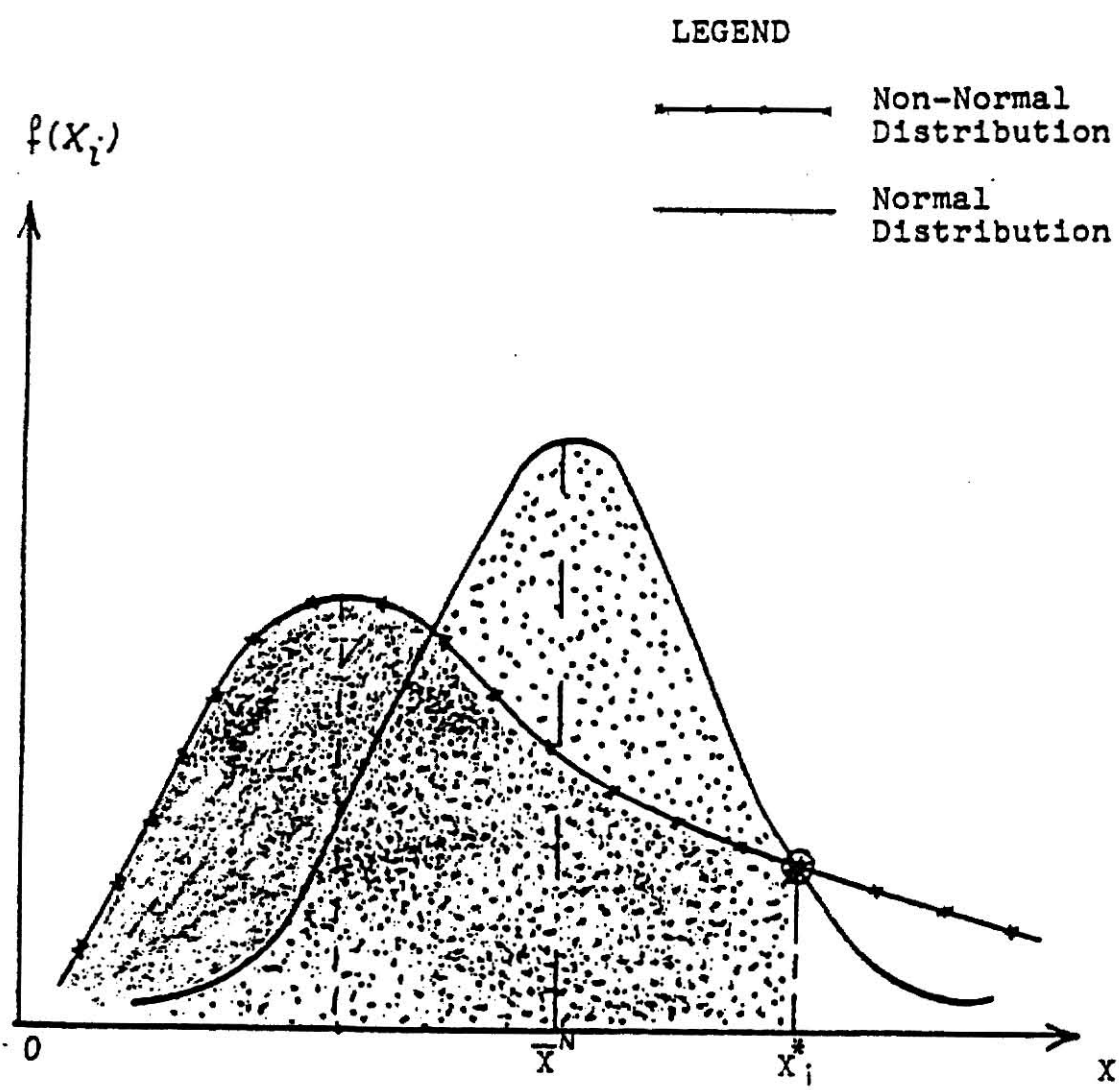


Figure 3 - Estimate of Equivalent Normal Distribution by Non-Normal Distribution

The following summarizes the procedure that must be taken to calculate β .

1. Define the limit state function.
2. Make the initial guess at β .
3. Set the initial checking point values $X_i^* = \bar{X}_i$, for all i .
4. Compute the mean and standard deviation of the normal for those variables that are non-normal.
5. Compute $\partial g / \partial X_i$, evaluated at X_i^* .
6. Compute α_i as $\alpha_i = [(\partial g / \partial X_i) \cdot \sigma_i^N] / \{\sum [(\partial g / \partial X_i) \cdot \sigma_i^N]^2\}^{1/2}$
7. Compute new values of X_i^* from $X_i^* = \bar{X}_i - \alpha_i \cdot \beta \cdot \sigma_i^N$
8. Repeat step 4 to 7 until α_i stabilizes.
9. Compute the values of β necessary for $g(X_1^*, X_2^*, \dots, X_n^*) = 0$.
10. Repeat step 4 to 9 until the values of β differ by some small amount.

See Appendix B for a numerical example.

3. Statistical Properties of Structural Loads

In evaluating the load statistics, the sources of information were the load subcommittees within American National Standard Institute Committee A58.

In this study, the load effect Q_i is considered to be related to structural loads as $Q_i = C_i \cdot B_i \cdot A_i$, in which C_i is the influence coefficient; B_i is the modeling parameter; and A_i is the structural load.

Dead Load The dead load, D , is assumed to remain constant throughout the life of the structure. It appears that dead load has a normal distribution curve with the ratio of mean load to the calculated nominal load close to unity. The coefficient of variation, V_d , is 0.06-0.15, with a typical value of 0.10. In this study it is assumed that $\bar{D}/D_n = 1.05$ and $V_d = 0.10$ for all construction materials.

Live Load The total live load on a floor can be thought of as having a component relatively constant, referred to as "arbitrary point-in-time live load" and an extraordinary component which arises from infrequent clustering of people or activities such as remodeling. The arbitrary point-in-time live load appears to be fitted best by a Gamma probability distribution function. However, it appears that the maximum live load, L , fits best by an Extreme Value Type I distribution curve [Refs. 3,7,10,11]. See Appendix C for a review on Extreme Value Type I distribution.

Wind Load Statistical analysis of data has shown that the mean and coefficient of variation are dependent on geographical locations. However, the analysis has shown that the probability distribution of the annual extreme for extra-tropical winds is Extreme Value Type I [Refs. 3,7,10,11].

Snow Load The snow load distribution is derived using climatological data and field studies. The U.S. Army Cold Regions Research and Engineering Laboratory analysis of current data indicates that the cumulative distribution function for annual extreme ground snow load is log-normal with parameters varying from site to site [Refs. 3,7,10,11].

Load Combinations Most structural loads vary with time and more than one time-varying load will be acting on a structure at any given time. When more than one time-varying load acts, it is very unlikely that each load will reach its peak lifetime value at the same time. In this study Turkstra's rule has been adopted in dealing with load combinations [Ref. 4]. Turkstra's rule says that the maximum of a combination of load effects will occur when one of the loads is at its lifetime maximum value while others assume their instantaneous values. The minimum value of β , calculated from the load combinations would provide a lower bound on the reliability of the element. Table 1 represents the load parameters used to develop the proposed load factors and load combination factors [Ref. 3]. The data given in Table 1 is the best expected values of the variables presented. The available

Table 1 - Load Distributions and Parameters (Ref. 3)

Load	$\bar{X}/X_n$	** v_X	*** cdf
D	1.05	0.10	Normal
L	Eqs. 17 or 18	0.25	Type I
L_{apt}	Eq. 19	Varies	Gamma
W	0.78	0.37	Type I
W_{ann}	0.33	0.59	Type I
W_{apt}	* $U = -0.021$	* $\alpha = 18.7$	Type I
S	0.82	0.26	Type II
S_{ann}	0.20	0.73	Lognormal
E	(Site dependent)	* $k = 2.3$	Type II

* Parameters used in the probability distribution functions of Type I or Type II

** Coefficient of variation

*** Cumulative distribution function

data to estimate the most expected values is provided by different agencies such as NASA, The U.S. Army Cold Regions Research and Engineering Laboratory Report, and The American Concrete Institute.

The nominal live load, Eq.17, is the value in ANSI A58.1-1972.

$$L_n = [1 - \min \{0.0008A_T, 0.6, 0.23(1 + D_n/L_o)\}] L_o \quad (17)$$

in which D_n = nominal dead load, L_o = the basic unreduced live load given in ANSI A58.1-1972, and A_T = the tributary area in square feet. This equation is used to determine the values of β corresponding to existing practice. Eq.18 presents the 1980 version of the A58 Standard.

$$L_n = [0.25 + 15/(A_i)] L_o \quad (18)$$

in which A_i is the influence area in square feet. Similarly, for the arbitrary point-in-time live load,

$$L/L_n = 0.24/\text{Eq.17 or 18} \quad (19)$$

The variability of the arbitrary point-in-time live load is dependent on the influence area (A_i). The larger the influence area, the smaller the variability gets.

4. Statistical Properties of Reinforced and Prestressed Concrete Members

Three major assumptions were made in determining the strength to be used in calculations [Ref.3].

1. The variabilities of the material properties and dimensions correspond to average quality construction.
2. The strength of concrete in structures was based on the assumption of a slow rate of loading, corresponding to failure in a test lasting one hour. Because of rapid rates of loading, in the case of wind or earthquake loads the concrete and reinforcement strengths were assumed to increase by 5%.
3. Long time strength changes of the concrete and possible corrosion of the reinforcement were ignored.

Table 2 gives the tensile and compressive strengths, which are assumed to have a normal distribution curve [Refs. 3,13]. Table 3 shows the strength distribution of various reinforced and prestressed concrete members [Refs. 3,13]. For the purpose of this study, beams are assumed to have effective depths in excess of 10 inches and no moment re-distribution or strain hardening is considered.

The information given in Table 2 and Table 3 is based on the studies of North American construction practice and the data are provided by various agencies such as ACI and ASCE. Mean values given in Table 2 such as the dimension properties are the mean deviation from the nominal values tested.

Table 2 - Basic Variables (Refs. 3,13)

Property	Mean	V [*]	σ ^{**}
<u>Concrete Normal Control</u>			
Compressive strength in structure loaded to failure in one hour.			
f'_c = 3000 psi	2760 psi	0.18	-
= 4000 psi	3390 psi	0.18	-
= 5000 psi	4028 psi	0.15	-
Tensile strength in structure, loaded to failure in one hour.			
f'_c = 3000 psi	306 psi	0.18	-
= 4000 psi	339 psi	0.18	-
= 5000 psi	366 psi	0.18	-
<u>Reinforcement</u>			
Grade 40, Static Yield	45.3 ksi	0.116	5.3 ksi
Grade 60, Static Yield	67.5 ksi	0.098	6.6 ksi
Grade 270 Prestressing Strand, Tensile Strength in Static Test	281 ksi	0.025	7.0 ksi
<u>Dimensions***</u>			
Overall depth - Nominal			
Slab (1696 Swedish Slabs)	+0.03 in	-	0.47 in
(99 Slabs)	+0.21 in	-	0.26 in
Beam (108 beams)	-0.12 in	-	0.25 in
(24 beams)	+0.81 in	-	0.55 in
Effective depth - Nominal			
One-way Slab; Top Bars			
(1696 Swedish Slabs)	-0.75 in	-	0.63 in
(99 Slabs)	-0.04 in	-	0.37 in
Values Used	-0.40 in	-	0.50 in
One-way Slab; Bottom Bars			
(2805 Swedish Slabs)	-0.13 in	-	0.34 in
(96 Slabs)	-0.16 in	-	0.35 in
Values Used	-0.13 in	-	0.35 in
Beam, Top Bars	-0.22 in	-	0.53 in
Beam Stem Width - Nominal Width	+0.10 in	-	0.15 in
Column width, breadth - Nominal	+0.06 in	-	0.25 in
Cover, bottom steel in beams	+0.06 in	-	0.45 in
	-0.35 in	-	0.28 in

1 psi = 6895 Pa ; 1 in = 25.4 mm

* Coefficient of variation

** Standard deviation

*** Mean deviation from actual

Table 3 - Resistance Statistics (Refs. 3,13)

Action	Type of Member	Details	$\bar{R}/R_n$	V_R
Flexure Reinforced Concrete	Continuous one-way slabs	5 in. thick, Grade 40	1.22	0.16
		5 in. thick, Grade 60	1.21	0.15
	Two-way slabs	5 in. thick, Grade 60	1.16	0.15
		7 in. thick, Grade 60	1.12	0.14
	One-way pan joists	13 in. overall depth, Grade 60	1.13	0.135
	Beams, Grade 40, $f'_c = 5$ ksi	$\rho = 0.005 = 0.09 \rho_b$	1.18	0.14
		$\rho = 0.019 = 0.35 \rho_b$	1.14	0.14
	Beams, Grade 60, $f'_c = 5$ ksi	$\rho = 0.006 = 0.14 \rho_b$	1.04	0.08
		$\rho = 0.015 = 0.31 \rho_b$	1.09	0.11
		$\rho = 0.027 = 0.57 \rho_b$	1.05	0.11
	$\rho = 0.034 = 0.73 \rho_b$	1.01	0.12	
Flexure, Reinforced Concrete - Overall Values			1.05	0.11
Flexure Prestressed Concrete	Plant Precast Pretensioned	$e_u = 0.054$	1.06	0.057
		$e_u = 0.122$	1.05	0.061
		$e_u = 0.228$	1.06	0.083
		$e_u = 0.295$	1.04	0.097
	Cast-in-Place Post-tensioned	$e_u = 0.054$	1.02	0.061
		$e_u = 0.122$	1.05	0.083
		$e_u = 0.228$	1.03	0.111
		$e_u = 0.295$	1.05	0.144
Flexure, Plant Precast Pretensioned, Overall Value			1.06	0.08
Cast-in-Place Post-tensioned, Overall Value			1.04	0.095
Axial Load and Flexure	Short Columns, Compression Failures	$f'_c = 3$ ksi	1.05	0.16
		$f'_c = 5$ ksi	0.95	0.14
	Short Columns, Tension Failures	$f'_c = 3$ and 5 ksi	1.05	0.12
	Slender Columns, $kl/h = 20$, Compression Failures	$f'_c = 5$ ksi	1.10	0.17
	Slender Columns, $kl/h = 20$, Tension Failures	$f'_c = 5$ ksi	0.95	0.12
Shear	Beams with $a/d \geq 2.5$, $\rho_v = 0.008$	No stirrups	0.93	0.21
		Min stirrups	1.00	0.19
		$\rho_v f_y = 150$ psi	1.09	0.17

1 ksi = 6.9 N/mm²; 1 in = 25.4 mm

ρ = steel ratio, ρ_b = balanced steel ratio, $\rho_s = A_{ps} \cdot f_{ps} / b d f'_c$ partial reinforcing index, f'_c = concrete strength, $\rho_w = A_w / b d$ web steel ratio. $\bar{R}/R_n$ is the ratio of mean strength to nominal strength.

5. Selection of Target Reliability or Existing Reliability

For the new approach in determining the load and combination load factors, the reliability indices can be estimated by reviewing the reliabilities in the existing standards which have resulted in satisfactory performance.

Considering the gravity loads, dead plus live load on floors or dead plus snow load on roofs, the factors are formulated in current design standards as shown below.

In the allowable stress design specification,

$$R_n / F.S. = D_n + L_n \quad (20)$$

In the plastic design of steel structures,

$$R_n = 1.7(D_n + L_n) \quad (21)$$

In reinforced concrete structures,

$$\phi R_n = 1.4D_n + 1.7L_n \quad (22)$$

The reliability indices associated with reinforced concrete and steel beams are given in Fig.4 for L_o/D_n and S_n/D_n [Ref.5].

L_o is the basic live load tabulated in building codes. For given live and dead loads, in this figure, nominal resistance is calculated according to the design equations given above and the reliability index is determined as shown in the example in the appendix and Fig.2. In Fig.4, in each case β decreases as L_o/D_n or S_n/D_n increases. When viewing the similarity, it should be noted that reinforced concrete beams have practical ranges of

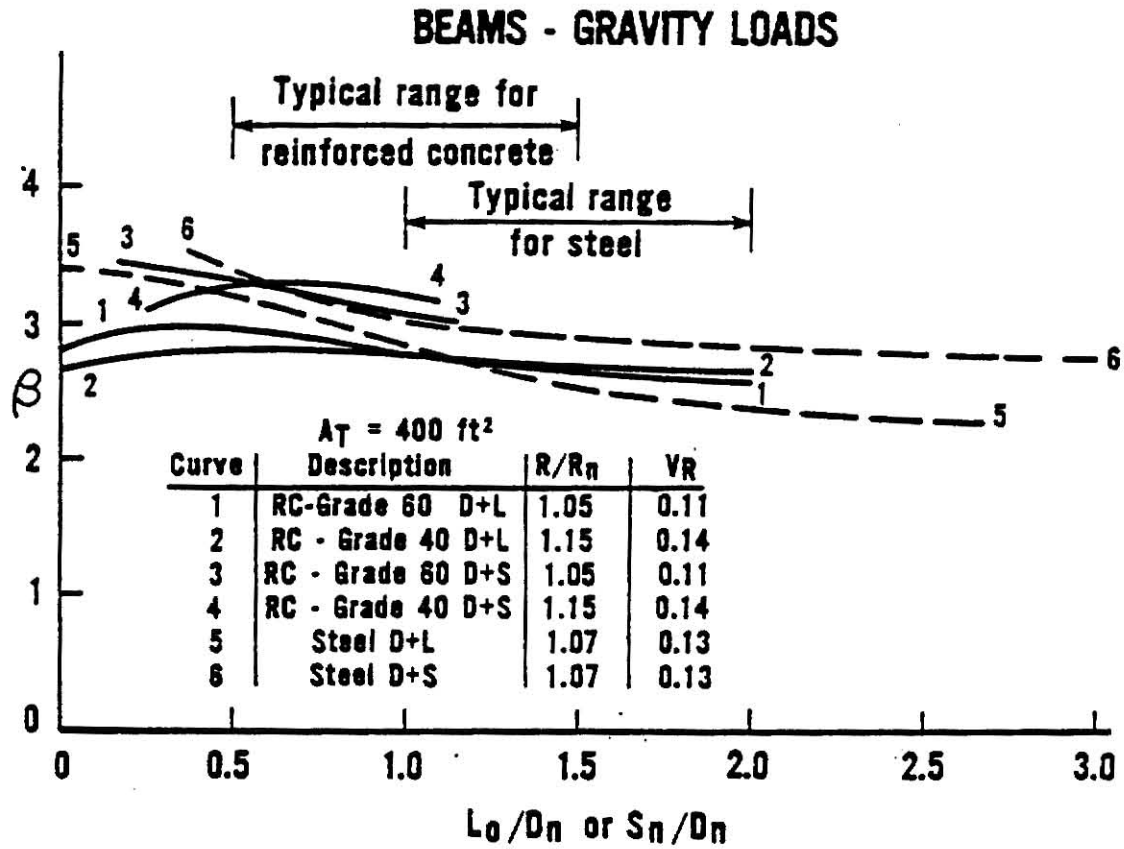


Figure 4-Reliability Index for Steel and Reinforced Concrete Beams
 Conforming to Current Criteria - Gravity Loads ($100 \text{ ft}^2 = 9.3 \text{ m}^2$)
 (Ref. 5)

L_o/D_n or S_n/D_n of 0.5 to 1.5, while for steel beams this range is from 1 to 2. So the analysis of current standards gives values of β , the reliability index, of 2.8 and 3.1 for concrete beams for the D+L and D+S combination.

Looking at gravity and environmental loads, the major load combinations are D+L+W and D+L+E. The variation of β with various L_o/D_n and W_n/D_n ratios is shown in Fig.5 for reinforced concrete beams [Ref. 5]. In each design situation, R_n is determined as below.

In the allowable stress design,

$$R_n = F.S. (D_n + L_n + W_n) (3/4) \quad (23)$$

In the plastic design of steel,

$$R_n = 1.3 (D_n + L_n + W_n) \quad (24)$$

In reinforced concrete design,

$$\phi R_n = 0.75 (1.4 D_n + 1.7 L_n + 1.7 W_n) \quad (25)$$

The effect of the rate of loading has been considered by multiplying R/R_n by 1.05 for reinforced concrete members. In Fig.5, β decreases as W_n/D_n increases, and β increases as L_o/D_n increases. It can also be seen that β for wind approaches a value of 2 where wind is the major load component. With greater live and dead loads, β increases to that of the D+L case. It is possible that the reliability under wind is less because of factors such as load-sharing and load re-distribution among members.

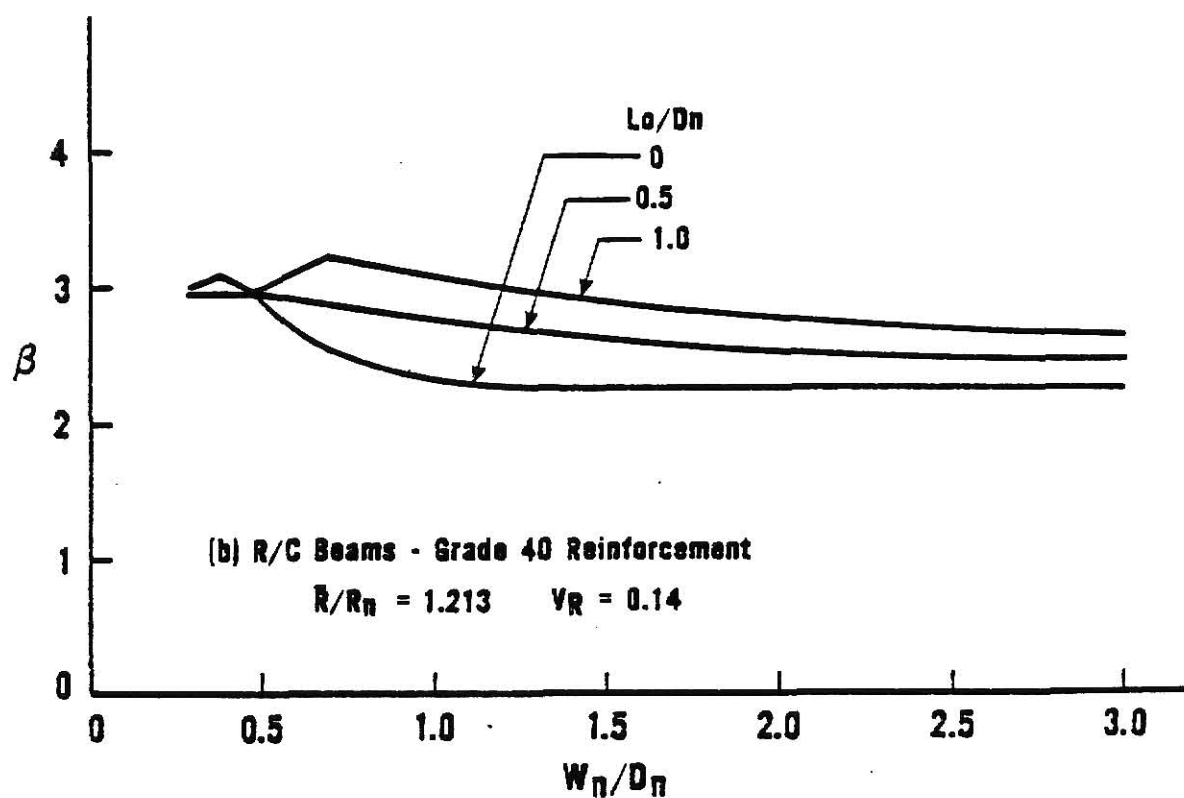
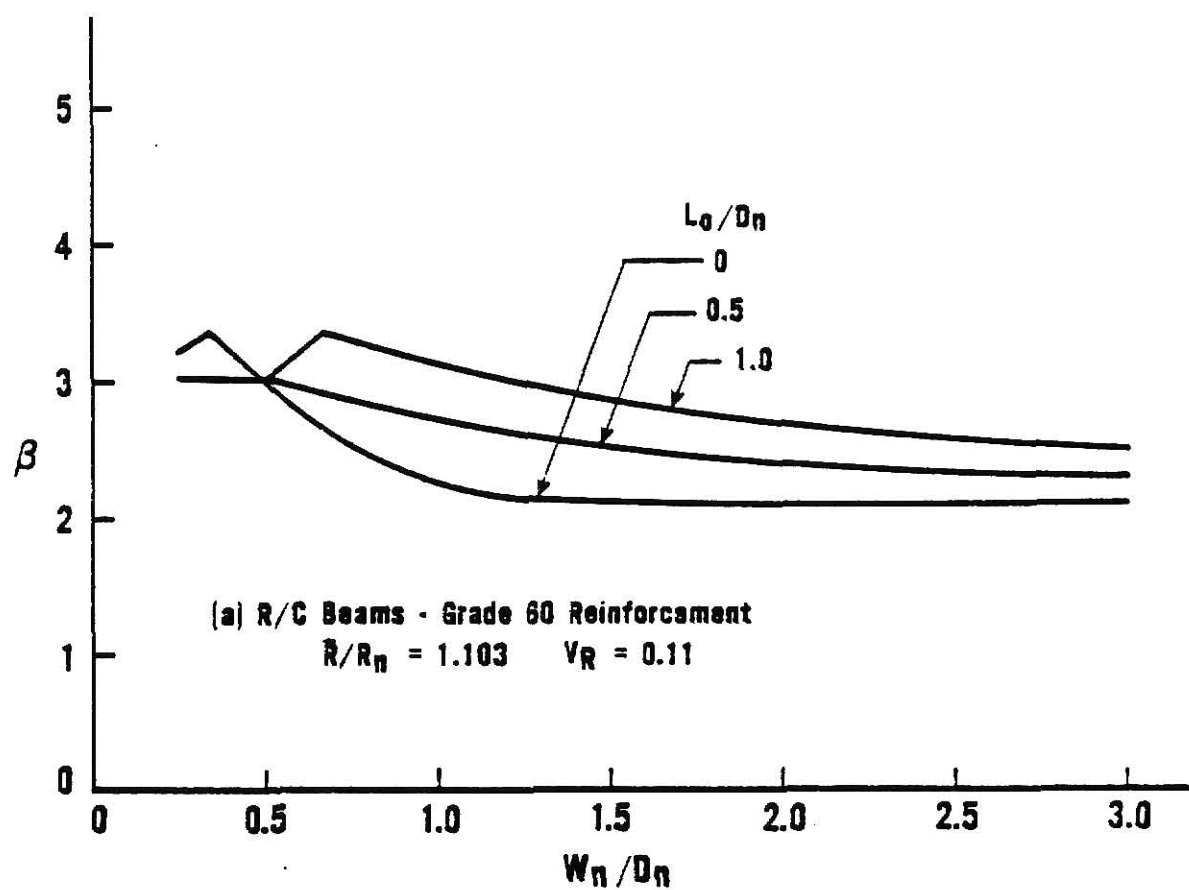


Figure 5 - Reliability Index for Reinforced Concrete Beams Conforming to Current Criteria - Gravity Plus Wind Load - $A_T = 400 \text{ ft}^2$ (37 m^2) (Ref. 5)

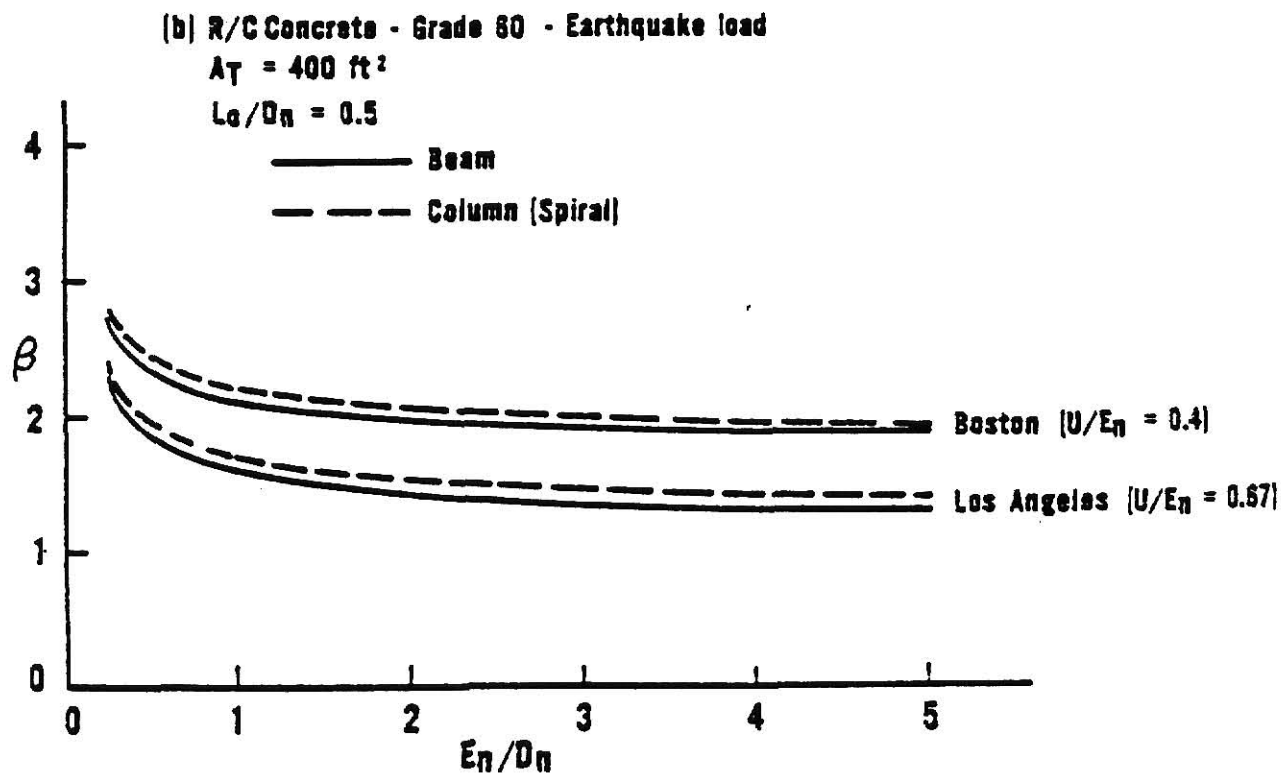
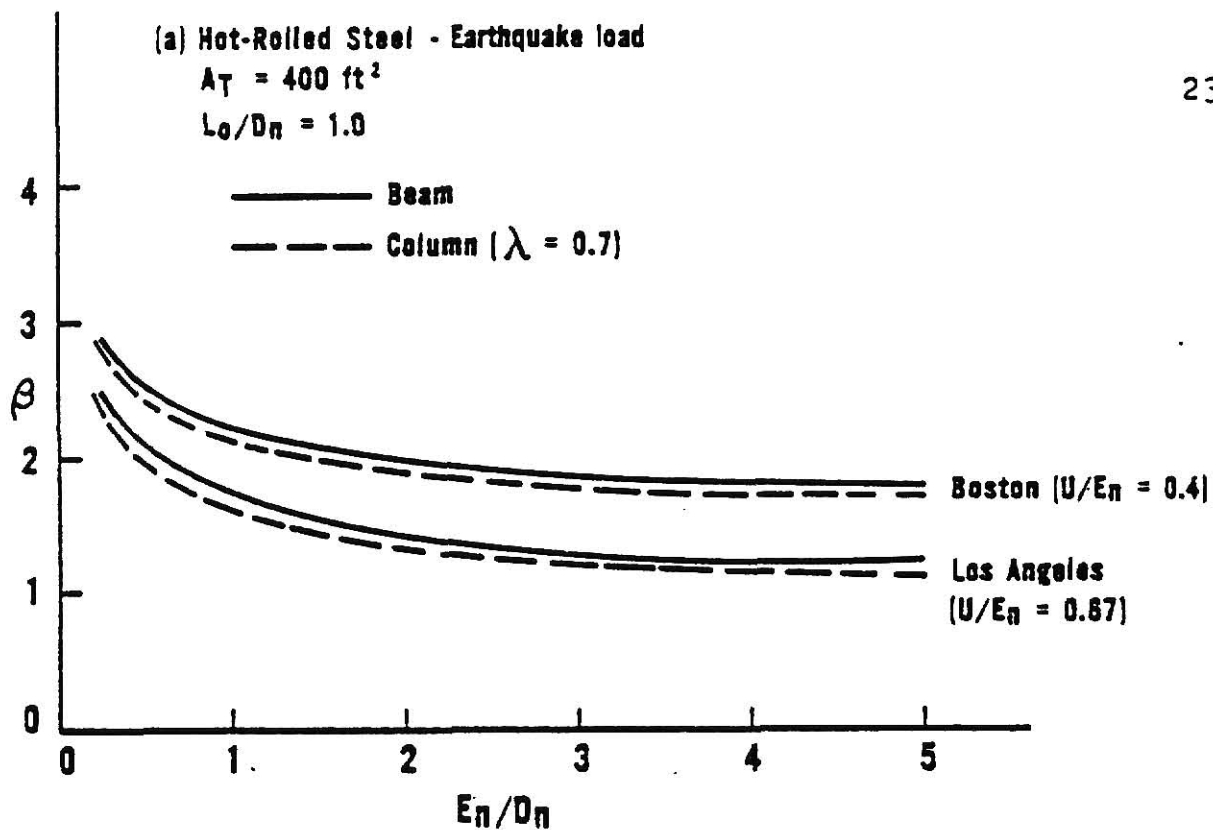


Figure 6 - Reliability Index for Steel and Reinforced Concrete Beams - Gravity Plus Earthquake Load ($100 \text{ ft}^2 = 9.3 \text{ m}^2$) (Refs. 5,3)

Values of β for the earthquake loading case $D+L+E$, are given in Fig.6 for Boston and Los Angeles [Refs. 5,3]. Due to the high variability of the earthquake loads as compared to dead loads, the β Vs. E_n/D_n curves flatten out rapidly to values which reflect essentially only the contribution of the earthquake load effect. When the effects of wind or earthquake counteract the effect of gravity loads, the reliability tends to be lower than when the loads are additive, as indicated in Fig.7 [Ref. 5]. The descending branch of the curves in Fig.7 is a result of the minimum strength that the member has even if it is not designed to resist counteracting load effects.

It has been shown that the β values for many flexural and compression members fall within the range 2.5 to 3.0 for the $D+L$, $D+S$, and $D+L+W$ load combinations. Therefore, the target β the reliability index, for $D+L$ and $D+S$ is 3.0; for $D+L+W$, β is 2.5; and for $D+L+E$, β is 1.75.

These values are slightly more conservative than indicated by the current practice when the time-dependent load (L_n, W_n, S_n, E_n) is large compared with the permanent load and less conservative when the permanent load is a major component.

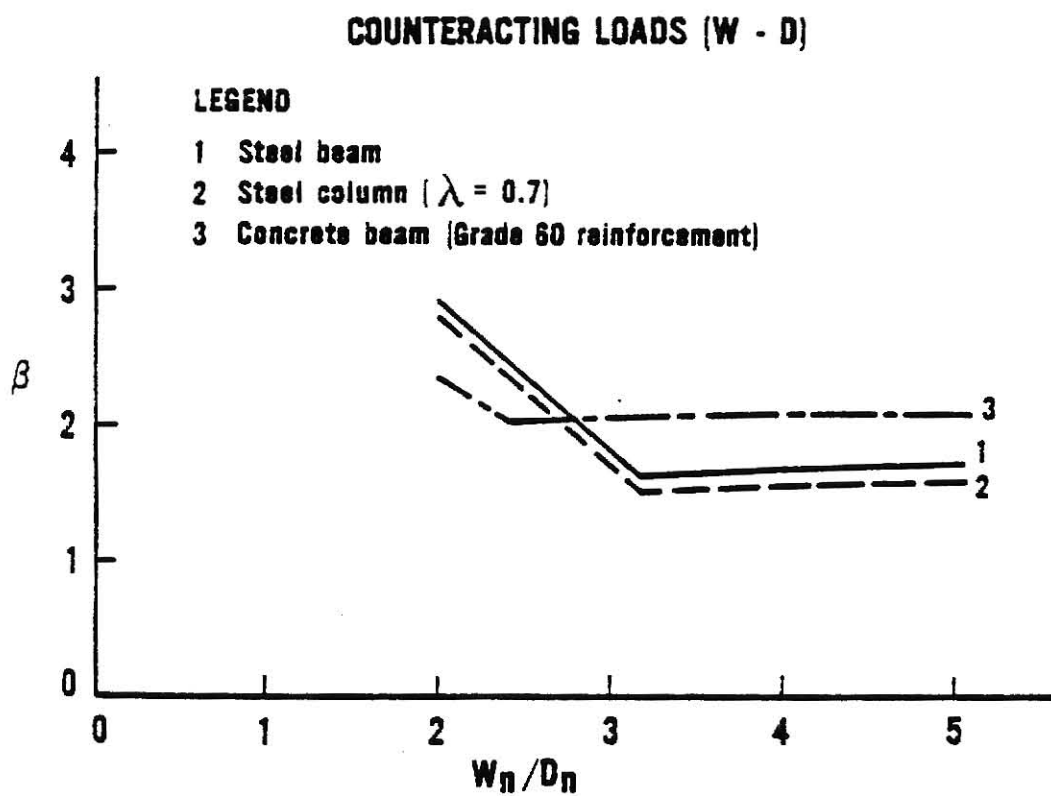


Figure 7 - Current Reliability Index for Steel and Reinforced Concrete Members - Counteracting Loads
(Ref. 5)

6. Derivation of Load And Resistance Factors

6.1 Basic Theory

This section presents the bases for the derivation of load and resistance factors for use in design. As explained before, the probability of failure can be expressed as:

$$P_f = P[R/Q < 1.0] \quad (26)$$

$$\text{or,} \quad P_f = P[\ln(R/Q) < 0] \quad (27)$$

Both of these are true regardless of the actual frequency distributions of R and Q. If we assume $\ln(R/Q)$ is normally distributed, (R/Q) will be log-normally distributed. A log-normal distribution of (R/Q) has been assumed because Q tends to be skewed; and because theoretically a log-normal distribution better represents the products of random variables (R/Q) .

Assuming $Y = \ln(R/Q)$, the mean and standard deviation of Y are:

$$\bar{Y} = \overline{\ln(R/Q)} = \ln(\bar{R}/\bar{Q}) \quad (28)$$

$$\sigma_Y^2 = \sigma^2(\ln(R/Q)) = \sigma^2(\ln R) + \sigma^2(\ln Q) \quad (29)$$

The function Y is the same as function g in Fig.1. As was explained before, the safety criteria is as shown below.

$$\overline{\ln(R/Q)} \geq \beta \cdot \sigma(\ln R/Q) \quad (30)$$

$$\text{or,} \quad \overline{\ln(R/Q)} \geq \beta [\sigma^2(\ln R) + \sigma^2(\ln Q)]^{1/2} \quad (31)$$

For a log-normal distribution the coefficient of variation is [Ref. 11],

$$V_R = [(\exp(\sigma^2(\ln R)) - 1)]^{1/2} \quad (32)$$

For $V_R \leq 0.6$, V_R can be approximated as [Ref. 12];

$$V_R^2 \approx \sigma^2(\ln R) \quad (33)$$

The error in this approximation is less than 2% for $V_R=0.3$ rising to about 10% for $V_R=0.6$. So we can approximate,

$$\overline{\ln(R/Q)} \geq \beta[V_R^2 + V_Q^2]^{1/2} \quad (34)$$

The right side of this equation can be further simplified by the use of the separation function as shown below.

$$[V_R^2 + V_Q^2]^{1/2} \approx \alpha V_R + \alpha V_Q \quad (35)$$

in which α is a "separation function" having values between 0.707 and 1.0. For V_R/V_Q between 1/3 and 3, $\alpha=0.75 \pm 0.06$. So from above,

$$\overline{\ln(R/Q)} \geq \beta \cdot \alpha \cdot V_R + \beta \cdot \alpha \cdot V_Q \quad (36)$$

$$\text{or,} \quad \overline{R/Q} \geq \exp[\beta \cdot \alpha \cdot V_R + \beta \cdot \alpha \cdot V_Q] \quad (37)$$

$$\text{or,} \quad \overline{R} \langle \exp[-\beta \cdot \alpha \cdot V_R] \rangle \geq \overline{Q} \langle \exp[\beta \cdot \alpha \cdot V_Q] \rangle \quad (38)$$

This equation is the same as ACI (1971) format. If θ_r and θ_q are taken as;

$$\overline{R} = \theta_r \cdot R \quad (39)$$

$$\overline{Q} = \theta_q \cdot Q \quad (40)$$

then the safety criteria $\phi R \geq \lambda Q$ can be written as;

$$\theta_r \cdot R \langle \exp[-\beta \cdot \alpha \cdot V_R] \rangle \geq \theta_q \cdot Q \langle \exp[\beta \cdot \alpha \cdot V_Q] \rangle \quad (41)$$

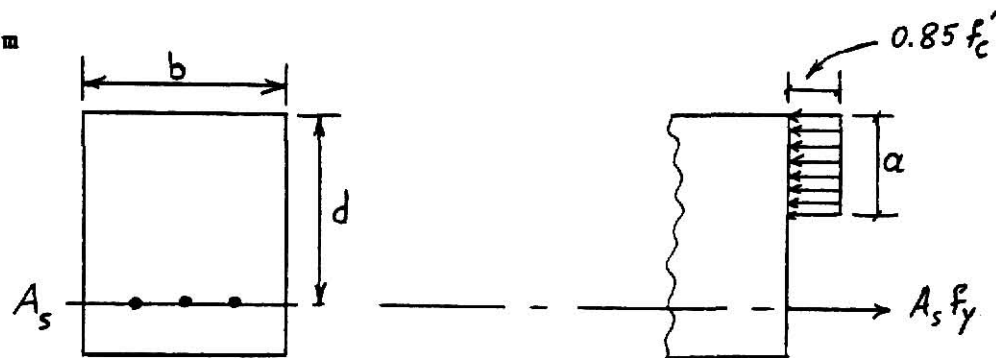
in which ϕ is a resistance factor and λ is a load factor. From the above equations, it can be seen that load and resistance factors are defined as;

$$\phi = \theta_r \cdot \langle \exp[-\beta \cdot \alpha \cdot V_R] \rangle \quad (42)$$

$$\lambda = \theta_q \cdot \langle \exp[\beta \cdot \alpha \cdot V_Q] \rangle \quad (43)$$

6.2 Example Calculation For Flexure of a Reinforced Concrete

Beam



The properties of the beam in the calculations are assumed according to ACI 1971, and are listed in Table 4 [Ref.12]. The beam strength is calculated as shown below.

$$R=M_n=A_s \cdot f_y \cdot (d - a/2)$$

$$a/2=(A_s \cdot f_y)/(2)(0.85)(f'_c \cdot b)$$

in which A_s is the area and f_y is the yield strength of steel; d is the depth of reinforcing bars and a is the width of the stress block.

$$R=M_n = (3.24)(60)\{18-[(3.24)(60)/(2)(0.85)(4)(12)]\}/12$$

$$R=M_n = 253 \text{ ft-K}$$

Now, using the mean strength and dimensions, we can calculate the mean strength.

$$\bar{M}_n=\bar{A}_s \cdot \bar{F}_y \cdot (\bar{d}-\bar{a}/2)$$

$$\bar{a}/2 = (3.24)(62)/(2)(0.85)(3.8)(12.05) = 2.58 \text{ in.}$$

$$\bar{M}_n = (3.24)(62)(17.85-2.58)/12 = 256 \text{ ft-K}$$

Since the average of measured to calculated strength, P , is given in Table 4 as 1.06,

$$\bar{R} = \bar{M}_n \cdot P = (256)(1.06) = 271 \text{ ft-K}$$

$$U_r=\bar{R}/R = 1.071$$

Here, it is necessary to indicate the effect of the accuracy of the design equation by calculating V_R , the coefficient of variation of strength. The coefficient of variation will be computed in a number of stages:

$V(a/2)$: Since $(a/2)$ is the product of a number of variables,

$$V(a/2) = [V(A_s)^2 + V(f_y)^2 + V(f_c)^2 + V(b)^2]^{1/2}$$

$$V(a/2) = [0.06^2 + 0.07^2 + 0.18^2 + 0.025^2]^{1/2} = 0.204$$

$V(d-a/2)$: Since $(d-a/2)$ is the sum of two variables the below equation holds,

$$\sigma(d-a/2) = [\sigma^2(d) + \sigma^2(a/2)]^{1/2}.$$

And since,

coefficient of variation = standard deviation/mean

$$\sigma(a/2) = V(a/2) \cdot \bar{a}/2 = (2.58)(0.204) = 0.526 \text{ in.}$$

$$\sigma(d-a/2) = [0.45^2 + 0.526^2]^{1/2} = 0.69 \text{ in.}$$

$$V(d-a/2) = \sigma(d-a/2) / (\bar{d} - \bar{a}/2) = 0.69 / 15.27 = 0.045$$

$V(M_n)$: Since M_n is calculated as the product of A_s , f_y , and $(d-a/2)$,

$$V(M_n) = [V(A_s)^2 + V(f_y)^2 + V(d-a/2)^2]^{1/2}$$

$$V(M_n) = [0.06^2 + 0.07^2 + 0.045^2]^{1/2} = 0.103$$

V_R : Finally, R is the product of M_n and P ,

$$V_R = [0.103^2 + 0.04^2]^{1/2} = 0.11$$

So in conclusion, $R=253 \text{ ft-K}$, $\bar{R}=271 \text{ ft-K}$, $V_R=1.071$, and

$$V_R=0.11.$$

Now, the resistance factor ϕ can be calculated as described in the earlier part of this chapter.

Table 4 - Statistical distributions assumed in calculations (Ref. 12)

	Specified	Mean <i>in situ</i>	$\mu = \frac{\text{Mean}}{\text{Specified}}$	σ	V
<i>Material Strengths, M</i>					
Concrete strength	4000 psi	3800 psi	—	—	0.18
Concrete tensile strength	—	—	—	—	0.18
Yield strength	60 ksi	62 ksi	—	—	0.07
<i>Dimensions, F</i>					
<i>b</i> -beam, column (in.)	12	12.05	1.004	0.3	0.025
<i>d</i> -beam (in.)	18	17.85	0.992	0.45	0.025
<i>h</i> -column (in.)	12	12.05	1.004	0.3	0.025
<i>s</i> -stirrups (in.)	9	9.0	1.00	0.25	0.03
<i>A_s</i> -beam (in. ²)	3.24	—	1.00	—	0.06
<i>A_s</i> -column (in. ²)	2.16	—	1.00	—	0.06
<i>A_s</i> -stirrups (in. ²)	0.22	0.22	1.00	—	0.06
<i>Accuracy of code equations, P</i>					
<i>M_u</i> -under-reinforced beams	—	—	1.06	—	0.04
<i>P_u</i> -axially loaded columns	—	—	0.98	—	0.05
<i>V_c</i> -shear carried by concrete	—	—	1.10	—	0.15
<i>V_s</i> -shear carried by stirrups	—	—	1.20	—	0.15
<i>Loadings, S</i>					
Dead load	—	—	1.0	—	0.07
Maximum floor load in 30 year life	—	—	0.7	—	0.30
<i>Structural analysis, E</i>					
Dead load effects	—	—	1.0	—	0.08
Live load effects	—	—	1.0	—	0.20

1 Ksi = 6.9 N/mm²

If it is assumed that $\beta=3.5$ and $\alpha=0.75$ for an under-reinforced or ductile concrete beam,

$$\phi = \phi_r \cdot \{\exp[-\beta \cdot \alpha \cdot V_R]\}$$

$$\phi = 1.071 \{\exp[(-3.5)(0.75)(0.11)]\}$$

$$\phi = 0.802$$

The corresponding load factors are calculated for live plus dead loads.

$$\lambda = \phi_r \cdot \{\exp[\beta \cdot \alpha \cdot V_Q]\}$$

But, $Q=D+L$ and since the coefficient of variation of D is much smaller than that of L , it is desired to separate these variables. Therefore

$$\lambda_Q = \bar{Q} \{\exp[\beta \cdot \alpha \cdot V_Q]\}$$

using the first two terms of the expansion for $\exp(x)$, and also taking $\bar{V}_Q = V_Q$ gives,

$$\bar{Q} \{\exp[\beta \cdot \alpha \cdot V_Q]\} = \bar{Q} [1 + \beta \cdot \alpha \cdot \bar{V}_Q]$$

Since $\bar{V}_Q = \sigma_Q / \bar{Q}$

$$\bar{Q} \{\exp[\beta \cdot \alpha \cdot V_Q]\} = (\bar{D} + \bar{L}) \{1 + [\beta \cdot \alpha \cdot \sigma_Q / (\bar{D} + \bar{L})]\}$$

$$\text{or, } \bar{Q} \{\exp[\beta \cdot \alpha \cdot V_Q]\} = (\bar{D} + \bar{L}) \{1 + [\beta \cdot \alpha \cdot (\bar{D}^2 \cdot V_D^2 + \bar{L}^2 \cdot V_L^2)^{1/2} / (\bar{D} + \bar{L})]\}$$

Using the separation function,

$$\bar{Q} \{\exp[\beta \cdot \alpha \cdot V_Q]\} \cong (\bar{D} + \bar{L}) \{1 + [(\beta \cdot \alpha^2 \cdot \bar{D} \cdot V_D + \beta \cdot \alpha^2 \cdot \bar{L} \cdot V_L) / (\bar{D} + \bar{L})]\}$$

$$\text{or, } \bar{Q} \{\exp[\beta \cdot \alpha \cdot V_Q]\} \cong \bar{D} \{1 + [\beta \cdot \alpha^2 \cdot V_D]\} + \bar{L} \{1 + [\beta \cdot \alpha^2 \cdot V_L]\}$$

From this equation the load factors for D and L are;

$$\lambda(D) = \phi_d \cdot \exp[\beta \cdot \alpha^2 \cdot V_D] \quad (44)$$

$$\lambda(L) = \phi_l \cdot \exp[\beta \cdot \alpha^2 \cdot V_L] \quad (45)$$

Now, taking $\beta = 3.5$ for a ductile member; V_D is affected by

variations in the load, V_{sd} , and variations due to the structural analysis, V_{ed} , (see Table 4).

$$V_D^2 = V_{sd}^2 + V_{ed}^2$$

$$V_D^2 = 0.07^2 + 0.08^2$$

$$V_D = 0.106$$

Thus, from Eq.44; $\lambda(D) = 1.23$

Also V_L is affected by the same variables as V_D . See Table 4.

$$V_L^2 = V_{sl}^2 + V_{el}^2$$

$$V_L^2 = 0.3^2 + 0.2^2 = 0.36^2$$

$$V_L = 0.361$$

Thus, from Eq.45; $\lambda(L) = 1.42$

In summary;

For a ductile member, with $\beta = 3.5$;

$$\phi R \geq 1.23D + 1.42L$$

For a brittle member, with $\beta = 4.0$, similar calculations show

$$\phi R \geq 1.27D + 1.58L$$

7. Probability-Based Design

It can be seen from Figs. 8,9, and 10 that the resistance factor (ϕ) is insensitive to the time-varying load in the combination, when that load is very small. These figures were obtained for load and resistance factors corresponding to β of 3 for the gravity loads, and β of 2.5 for D+L+W combination [Refs. 4,5]. For a given limit state and β , the factored loads and factored resistances can be obtained according to Chapter 2 of this report, describing the reliability index. The load and resistance factors are then calculated for a given set of nominal loads and resistances as shown below.

$$\phi_i = x_i / x_{ni}$$

in which x_i = the checking point value of load or resistance corresponding to the given β , and x_{ni} = nominal value of that load or resistance specified in the appropriate specification. From these figures, the load factor for dead load is seen to be very low due to its small variability compared to other load variabilities. A comparison of these figures shows that ϕ is in the same range for D+L and D+L+W combinations. From these figures it appears that ϕ_D is independent of the magnitude of the time-varying loads in the equation. So, taking ϕ_D as constant in the load combination will not significantly change the target reliability. However, the load factors on the time-varying load in the combination increases as that load increases because its

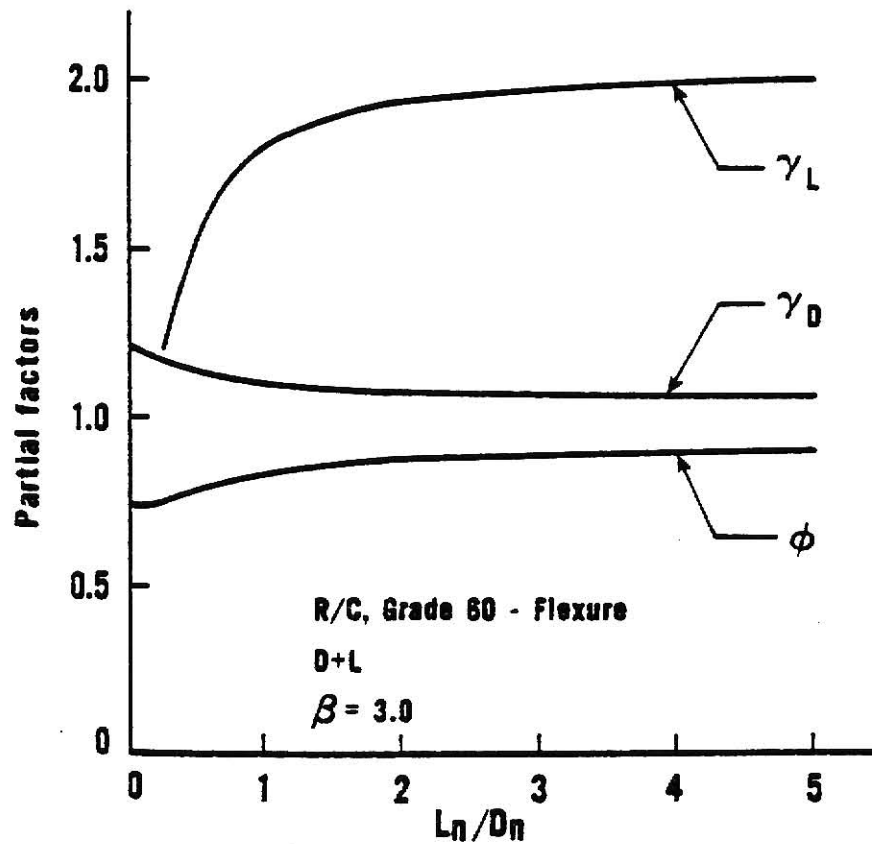
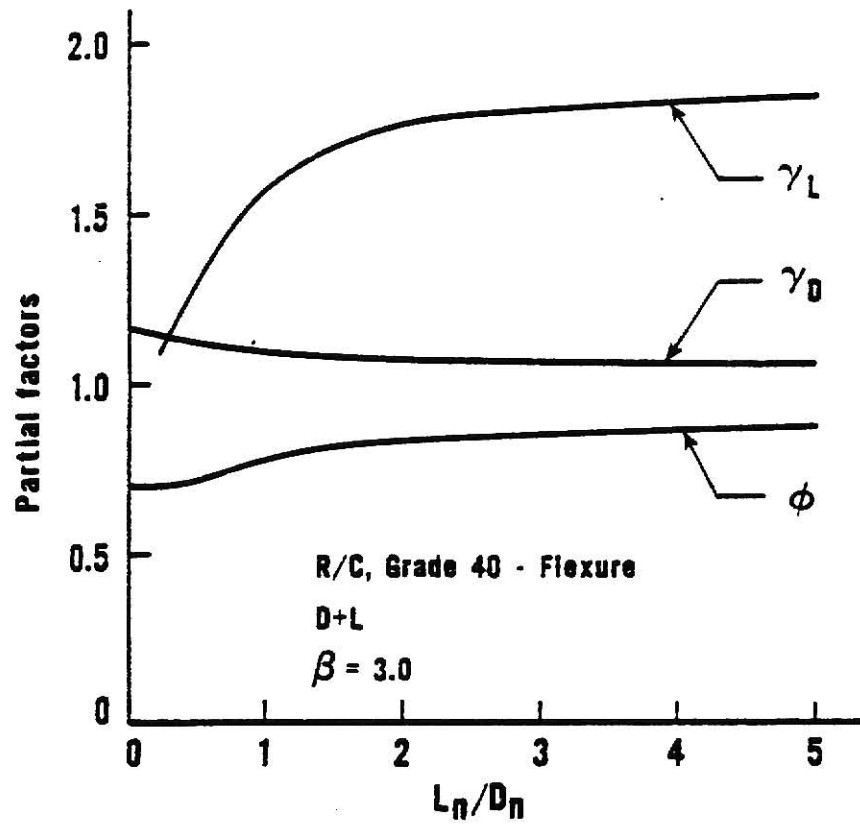


Figure 8 - Load and Resistance Factors for Flexure in Reinforced Concrete Beams (Live Loads)
(Refs. 4,5)

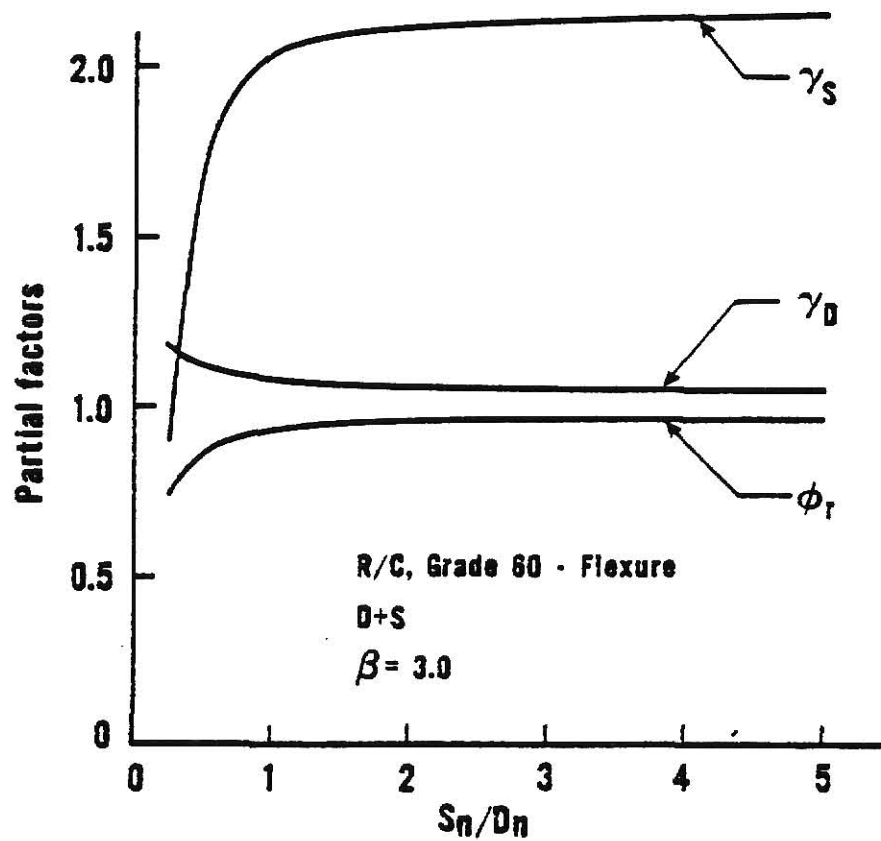
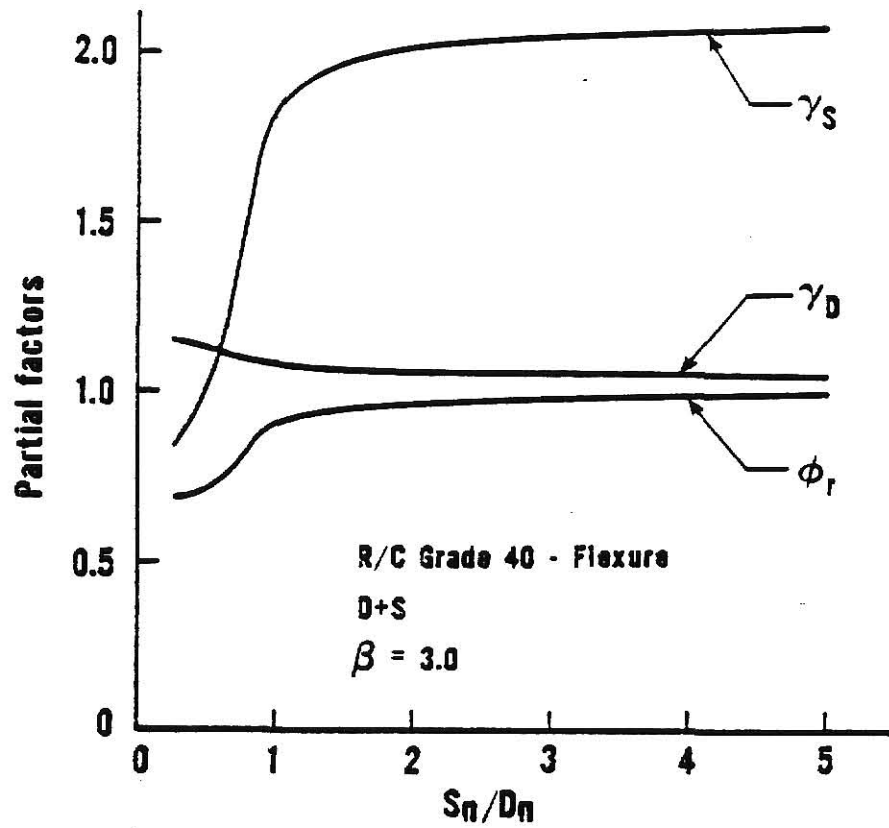


Figure 9 - Load and Resistance Factors for Flexure in Reinforced Concrete Beams (Snow Loads)
(Refs. 4,5)

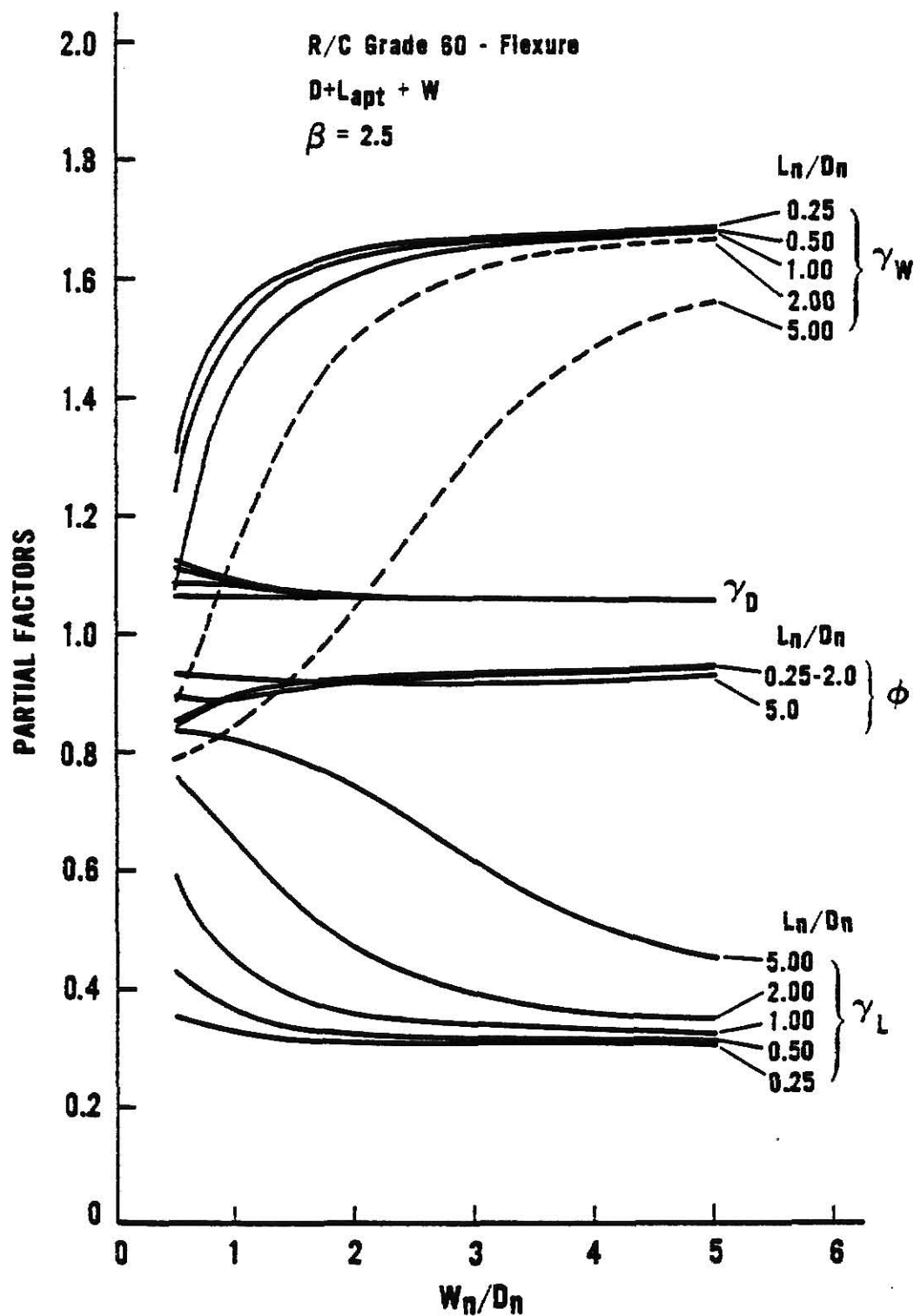


Figure 10 - Load and Resistance Factors for Flexure in Reinforced Concrete Beams (Gravity plus Wind Load)
(Refs. 4,5)

higher variability becomes increasingly more important in determining the total load effect. So it can be concluded that if load factors for the time-varying loads were taken constant as in the current design procedure, there would be some deviation from the ideal constant reliability for certain load situations. It should be noted here that while the resistance factor will depend on the material and limit state of interest, the load factors will be independent of these conditions. To determine the optimal set of load factors to be applied to all situations, and a set of resistance factors that only depend on the material and limit state the following procedure must be met. (1) Define some function which measures the difference between the target reliability and the reliability associated with constant load and resistance set. (2) Select the load factors so as to minimize this function. To do this, first the resistance R_{ni}^{II} was calculated with a given set of nominal loads and associated target reliability. Then, the nominal resistance R_{ni}^{I} was calculated with a design equation which prescribed a set of load factors that were constant for all load ratios. To simplify the procedure, the difference in nominal resistance is being minimized by selecting a set of ψ_i and ϕ factors as shown below [Refs. 3,5],

$$I(\phi, \psi_i) = \sum [R_{ni}^{\text{II}} - R_{ni}^{\text{I}}]^2 \cdot P_i \quad (46)$$

over a predefined set of combinations of dead, live, snow, wind, and earthquake loads; where P_i is the relative weight assigned

Table 5a - Weights for D+S - Values of $P_{\perp}$ (Refs. 2,3)

	S_n/D_n						
Material	0.25	0.50	1.0	1.5	2.0	3.0	5.0
Steel	0	10	20	25	35	7	3
R/C	30	40	20	5	5	0	0
Light Gage & Aluminum	0	0	6	17	22	33	22
Glulam	0	2	16	32	32	18	0
Masonry	36	36	20	6	2	0	0

Table 5b - Weights for D+L - Values of $P_{\perp}$ (Refs. 2,3)

	L_n/D_n						
Material	0.25	0.5	1.0	1.5	2.0	3.0	5.0
Steel	0	10	20	25	35	7	3
R/C	10	45	30	10	5	0	0
Light Gage & Aluminum	0	0	6	17	22	33	22
Glulam	0	5	26	26	26	12	5
Masonry	36	36	20	6	2	0	0

to the i th load combination. Table 5 gives the values of P_i for D+L and D+S load combinations [Refs.2,3]. Values of P_i are the best estimates for the relative likelihood of these load situations. The justification for Eq.46 is that for a constant optimal set of load factors over all likely combination of loads, the resistance factors will depend only on the material and the limit state of interest. So this is the resistance factor that measures the limit state or the reliability index. Also in Eq.46, the reason for minimizing the square of the difference between R_n^{II} and R_n^{I} is to penalize equally deviations from target reliability which are conservative and those which are unconservative. Studies of the sensitivity of the optimal safety factors to various assumptions showed that they were considerably more sensitive to the range of L_n/D_n , etc., than to the distribution of P_i within that range [Refs. 2,3].

8. Combination Load Factors

The selection of load factors can not be made independently of the resistance side of the design equation. In this report, it was assumed that the resistance factor for ordinary steel and reinforced concrete lies in the range of 0.80-0.90. The justification for this range is that when ϕ is more than 0.90 for ordinary member, there is little margin for further adjustment to reflect improvements in fabrication or quality control that tend to reduce variability. Here first as a simple measure of comparison only one load at a time is considered. Setting $\phi=0.80$ or 0.85, Table 6 presents the required γ_Q for the prescribed β [Ref. 5]. This analysis suggests that the load factors values are approximately for $\gamma_D=1.2-1.3$, $\gamma_L=1.6-1.7$, $\gamma_W=1.3-1.4$, and $\gamma_E=1.4-1.5$.

8.1 Gravity Load Combinations

For different construction materials, the safety factors for D+L and D+S combinations can be determined by calculating R_n for β of 3.0 and minimizing Eq.46 to get the optimal ϕ , γ_L , γ_S . The term γ_D is taken as a constant equal to 1.2 from Table 6 and Figures 8, 9, and 10; because γ_D is very insensitive to the magnitude of other load factors in the limit state equation. As shown in Table 7, the next step is to select

Table 6 - Required Load Factors- One Load (Ref. 5)

Member (1)	ϕ (2)	LOAD						
		γ_D		γ_L		γ_S	γ_W	γ_E
		$\beta = 2.5$ (3)	$\beta = 3.0$ (4)	$\beta = 2.5$ (5)	$\beta = 3.0$ (6)	$\beta = 3.0$ (7)	$\beta = 2.5$ (8)	$\beta = 2.0$ (9)
Hot-rolled steel beam	0.80	1.18	1.28	1.52	1.78	1.78	1.30	1.44
	0.85	1.25	1.35	1.61	1.89	1.89	1.38	1.52
Concrete beam—	0.80	1.18	1.33	1.44	1.70	1.66	1.28	1.39
Grade 40 rein- forced	0.85	1.26	1.41	1.53	1.81	1.77	1.36	1.48
Concrete beam—	0.80	1.18	1.29	1.52	1.77	1.79	1.36	1.51
Grade 60 rein- forced	0.85	1.26	1.37	1.61	1.88	1.90	1.45	1.61

Table 7 - Optimal Load and Resistance Factors for Gravity Loads (Ref. 3)

Material	Combination	Optimum Values		Optimum ϕ for $\gamma_D = 1.2, \gamma_L = 1.6$
		ϕ	γ_L, γ_S	
Steel Beam ($\beta_o = 3$)	D + L	0.96	2.10	0.78
	D + S	1.05	2.32	0.79
R/C Beam, Gr. 60 ($\beta_o = 3$)	D + L	0.87	1.83	0.81
	D + S	0.93	1.93	0.84
R/C Beam, Gr. 40 ($\beta_o = 3$)	D + L	0.82	1.61	0.81
	D + S	0.85	1.56	0.86
Glulam Beam [*] ($\beta_o = 2.5$)	D + L	0.59	1.38	0.66
	D + S	0.59	1.08	0.77
Brick Masonry Wall [*] ($\beta_o = 7.5$)	D + L	0.38	4.10	0.22
Brick Masonry Wall [*] ($\beta_o = 5.0$)	D + L	0.52	2.45	0.41
Concrete Masonry Wall [*] ($\beta_o = 6.5$)	D + L	0.41	3.28	0.27
Concrete Masonry Wall [*] ($\beta_o = 5.0$)	D + L	0.49	2.38	0.40

^{*}
 $\bar{R}/R_n$ assumed to equal to 1.0 for illustration.

one factor to be used with both live and snow loads so to constrain further the final load criteria. This factor should be very close to the γ_L and γ_S factors calculated earlier (Table 7). At the same time, ϕ should be close to the desirable range of 0.80-0.85 for flexure in reinforced concrete beams. The range of ϕ was selected as such to allow different quality control procedures. It was found that $\gamma_L = \gamma_S = 1.6$ will fulfill the necessary requirement for ϕ regarding the values in Table 6. Again Eq.46 can be used in calculating the optimal ϕ corresponding to $\gamma_D = 1.2$ and $\gamma_L = \gamma_S = 1.6$, as shown in Table 7. An additional condition is to place some restriction on the factored load in case the time-varying load approaches zero, according to Table 6. The gravity load combinations are shown below.

$$U = 1.2D_n + (1.6L_n \text{ or } 1.6S_n) \quad (47)$$

$$U \geq 1.4D_n \quad (48)$$

8.2 Gravity Plus Wind Load Combinations

In this case the maximum of the checking equations given below govern the design.

$$U = (\gamma_D)D_n + (\gamma_{L_1})L_n + (\gamma_W)W_n \quad (49)$$

$$U = (\gamma_D)D_n + (\gamma_L)L_n + (\gamma_{W_1})W_n \quad (50)$$

in which $(\gamma_{L_1})L_n$ and $(\gamma_{W_1})W_n$ are the factored arbitrary point-in-time live and wind loads, respectively. Again, optimal

load and resistance factors were determined by first calculating R_n corresponding to β of 2.5 for Eq.49 and β of 3.0 for Eq.50 and minimizing Eq.46. ψ_D is taken as 1.2. The result for a reinforced concrete beam with grade 60 reinforcement and an influence area of 1000 sq. ft. is shown in Table 8. Keeping the limit state the same, the ϕ -factor should stay the same as D+L and D+S combinations. But as Table 8 shows some reduction in ϕ , ψ_L , and ψ_W is necessary. Moreover, ψ_L should approach 1.6 as the influence of W_n approaches zero, corresponding to the D+L case. If by taking $\psi_L=1.6$, $\psi_{L_1}=0.5$, $\psi_W=1.3$, and $\psi_{W_1}=0.10$, also regarding Table 6; the ϕ -factor falls in the desired range (0.80-0.85), (see Table 8). The final load factors in the combination are shown below.

$$U = 1.2D_n + 0.5L_n + 1.3W_n \quad (51a)$$

$$U = 1.2D_n + 1.6L_n + 0.10W_n \quad (51b)$$

8.3 Gravity and Earthquake Combinations

Similarly, an optimal set of load factors was determined by calculating R_n corresponding to β of 1.75 for reinforced concrete beams, for Boston and Los Angeles. The related influence area to find the arbitrary point-in-time live load is taken as 1000 sq. ft.. To make ψ_E site independent, the results of the R_n calculation were combined to find optimal ϕ , ψ_L , and ψ_E for reinforced concrete beams. The term ψ_D is again taken equal to

Table 8-Load and Resistance Factors for Gravity Plus Wind Loads
(Ref. 3)

Material	Eq.	Optimum Values			Optimum ϕ when $\gamma_W = 1.3$		
		ϕ	γ_L	γ_W	$\gamma_{L_1} = 0.3$	$\gamma_{L_1} = 0.4$	$\gamma_{L_1} = 0.5$
Steel Beam	51 a	1.11	0.61	1.71	0.85	0.87	0.89
	51 b	0.93	1.97	0.08	-	0.81	-
Concrete Beam	51 a	1.06	0.49	1.76	0.82	0.83	0.84
	51 b	0.96	1.63	0.14	-	0.81	-

Table 9-Load Factors for Earthquake Loads (Ref. 3)

Material	Combinations	ϕ	γ_L	γ_E	Optimal ϕ when $\gamma_E = 1.5$		
					$\gamma_{L_1} = 0.0$	$\gamma_{L_1} = 0.2$	$\gamma_{L_1} = 0.5$
Steel Beam	D + L + E	1.25	0.39	2.31	0.82	0.85	0.90
R/C Beam	D + L + E	1.21	0.38	2.37	0.80	0.82	0.85

1.2. Table 9 shows the calculated load factors for earthquake loads. Since the ϕ -factor was large, the ψ_E factor was adjusted to bring down the ϕ -factor. It was found that by making $\psi_{L_1}=0.2$, and $\psi_E=1.5$, according to Table 6; the corresponding ϕ will fall in the optimal range. However, $0.2L_n$ in many cases would be less than the arbitrary point-in-time live load. So ψ_{L_1} was increased to 0.5 in order to be compatible with the arbitrary point-in-time live load. The load factor for earthquake load is higher than the factor for wind load ($\psi_W=1.3$) because of the big difference in the coefficient of variation of wind load (0.30-0.40) compared to the earthquake load (greater than 1.00). The analysis of D+S+E combination determined that the snow load factor was close to zero. However for conservatism in areas with heavy snow and earthquake loads ψ_{S_1} is set equal to 0.2. The final load factors in the combination are shown below.

$$U = 1.2D_n + 1.5E_n + (0.5L_n \text{ or } 0.2S_n) \quad (52)$$

8.4 Counteracting Loads

Common cases when loads counteract one another are the wind or earthquake loads opposing the gravity loads.

$$U = E - D \quad (53)$$

$$U = W - D \quad (54)$$

To simplify the minimizing problem, there should be some constraints placed on the safety factors. First, since $\psi_D=1.2$ or

1.1 for the additive case, it is reasonable to take $\psi_D=0.9$ for the counteracting case. Further, the ϕ -factor must stay the same (0.80-0.85) for both additive and counteracting cases. So, fixing $\psi_D=0.90$ and $\phi=0.85$, ψ_E (or ψ_W) can be calculated by finding R_n and then minimizing Eq.46.

The optimal value of ψ_W (or ψ_E) depends on ϕ . In the case where $\phi=0.85$ and $\psi_D=0.9$, ψ_W was found to be 1.22. To be consistent with the additive combinations, the load factors for counteractive case were chosen as below.

$$U = 0.9D_n - 1.3W_n \quad (55)$$

$$U = 0.9D_n - 1.5E_n \quad (56)$$

Other counteracting load combinations may be treated similarly. Some of the cases which may be important to consider are;

$$D + L_{apt} + S \quad (57)$$

$$D + W_{ann} + S \quad (58)$$

in which L_{apt} is the arbitrary point-in-time live load and W_{ann} is the maximum annual wind load. In these cases, the load factors for L_n and W_n to give the desired $\phi(0.80-0.85)$ are $\psi_{L_1}=0.5$ and $\psi_{W_1}=0.80$.

In sum, the load combinations and load factors recommended for use by the individual material specification writers in their design specifications are:

$$1.4D_n$$

$$1.2D_n + 1.6L_n$$

$$1.2D_n + 1.6S_n + (0.5L_n \text{ or } 0.8W_n)$$

$$1.2D_n + 1.3W_n + 0.5L_n$$

$$1.2D_n + 1.5E_n + (0.5L_n \text{ or } 0.2S_n)$$

$$0.9D_n - (1.3W_n \text{ or } 1.5E_n)$$

For other loading combinations, the same methodology can be used to derive the related safety factors.

9. Determination of Resistance Factors

The first step in calculating the ϕ -factor for concrete members is selecting a target β . As was shown previously, the reliability indices associated with current practice are as below [Ref. 3].

- a. Reinforced concrete beams in flexure, 2.6 to 3.2
- b. Plant produced pretensioned beams in flexure, 3.2 to 4.0
- c. Tied columns, compression failure, 3.0 to 3.5
- d. Spiral columns, compression failure, 2.6 to 3.3
- e. Shear, beams with stirrups, 1.9 to 2.4

In many practical instances, the design is governed by the D+L combination. Accordingly, Figure 11 (11a through 11e) presents curves relating the reliability β , $\bar{R}/R_n$, V_R , and ϕ for the design criterion,

$$R_n \geq 1.2D_n + 1.6L_n$$

These curves are computed for an influence area of 1000 sq. ft.; and are presented in terms of the most common live load $L=50$ psf in offices. Table 10 presents the ϕ values based on β of 3.0 for ductile failure, and β of 3.5 for brittle failure. Finally, in Fig. 12 a comparison of existing design and the future design is presented.

Table 10 - Values of Resistance Factors for Reinforced Concrete Members (Ref. 3)

Action	Type of Member	$\bar{R}/R_n$	V_R	Range of ϕ for $L_o/D_n = 0.25 - 2.0$
<u>Flexure, Reinforced Concrete, $\beta = 3.0$</u>				
	Beam, Grade 40, $\rho = 0.35\rho_b$	1.14	0.14	0.82 - 0.84
	Beam, Grade 60, $\rho = 0.57\rho_b$	1.05	0.11	0.80 - 0.85
	Beam, Grade 60, $\rho = 0.73\rho_b$	1.01	0.12	0.76 - 0.80
	Two way slabs, Grade 60	1.16	0.15	0.83 - 0.86
	Continuous, one-way slabs	1.22	0.16	0.85 - 0.88
<u>Flexure, Plant Produced Pretensioned Concrete, $\beta = 3.0$</u>				
	Double T $\omega_p = 0.054$	1.06	0.057	0.86 - 0.95
	Beam $\omega_p = 0.228$	1.06	0.083	0.83 - 0.90
	Beam $\omega_p = 0.295$	1.04	0.10	0.80 - 0.86
<u>Flexure, Cast-in-Situ Post-Tensioned Concrete $\beta = 3.0$</u>				
	$\omega_p = 0.228$	1.03	0.11	0.78 - 0.83
	$\omega_p = 0.295$	1.05	0.14	0.76 - 0.79
<u>Tied Columns, Compression Failures, $\beta = 3.5$</u>				
	3000 psi Concrete, short	1.05	0.16	0.65 - 0.69
	5000 psi Concrete, short	0.95	0.14	0.61 - 0.66
	5000 psi Concrete, $l/h = 20$	1.10	0.17	0.66 - 0.70
<u>Spiral Columns, Compression Failures, $\beta = 3.0$</u>				
	3000 psi Concrete, short	1.05	0.16	0.74 - 0.76
	5000 psi Concrete, short	0.95	0.14	0.69 - 0.72
<u>Shear, $\beta = 3.5$</u>				
	Beams without stirrups	0.93	0.21	0.50 - 0.52
	Beams with minimum stirrups	1.00	0.19	0.60 - 0.64
	Beams with $\rho_v f_y = 150$	1.09	0.17	0.66 - 0.70

Note: 1 psi = 6895 N/m²

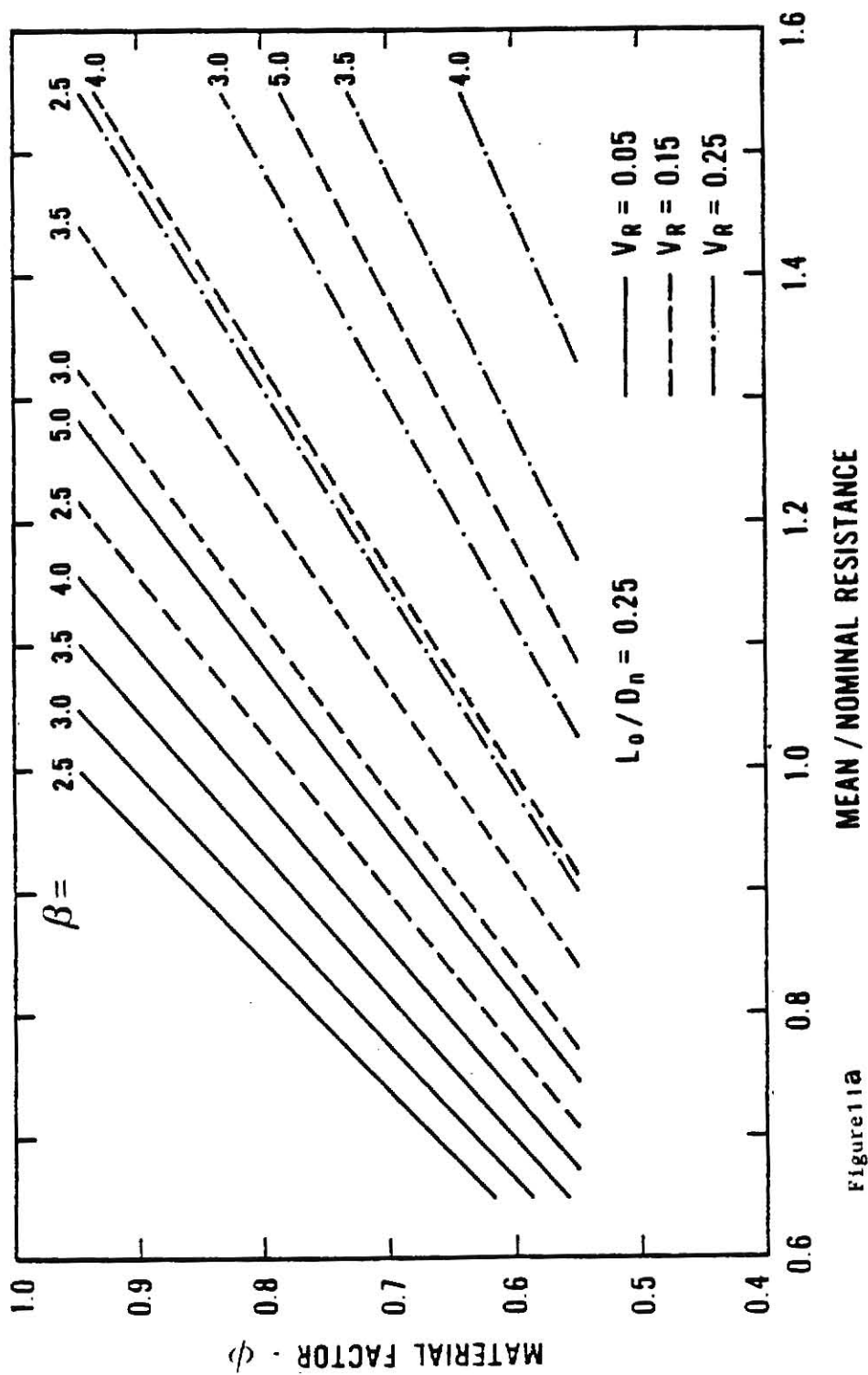


Figure 11 - Aids to the Selection of Resistance Factors
(Ref. 5)

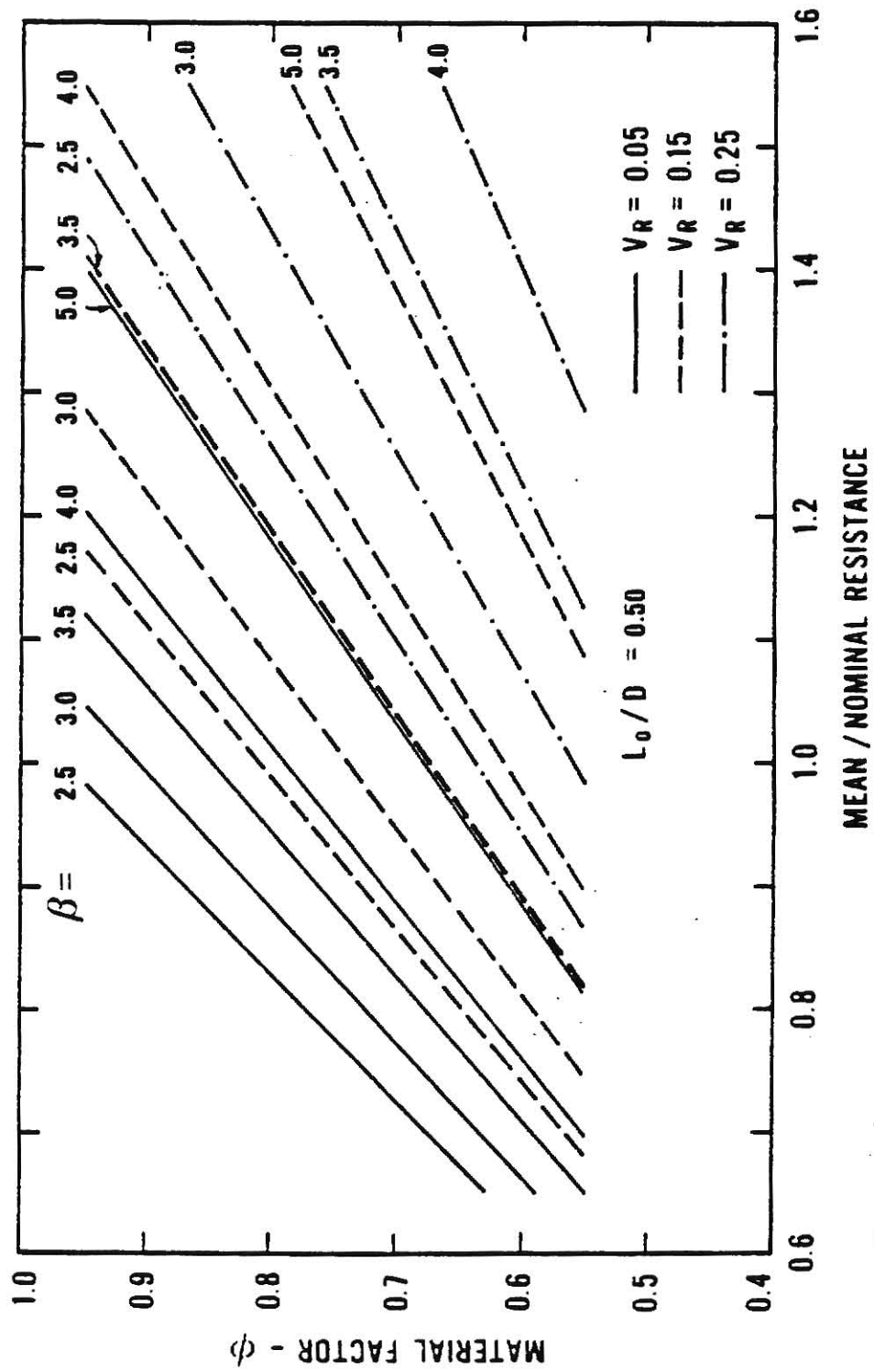


Figure 11b

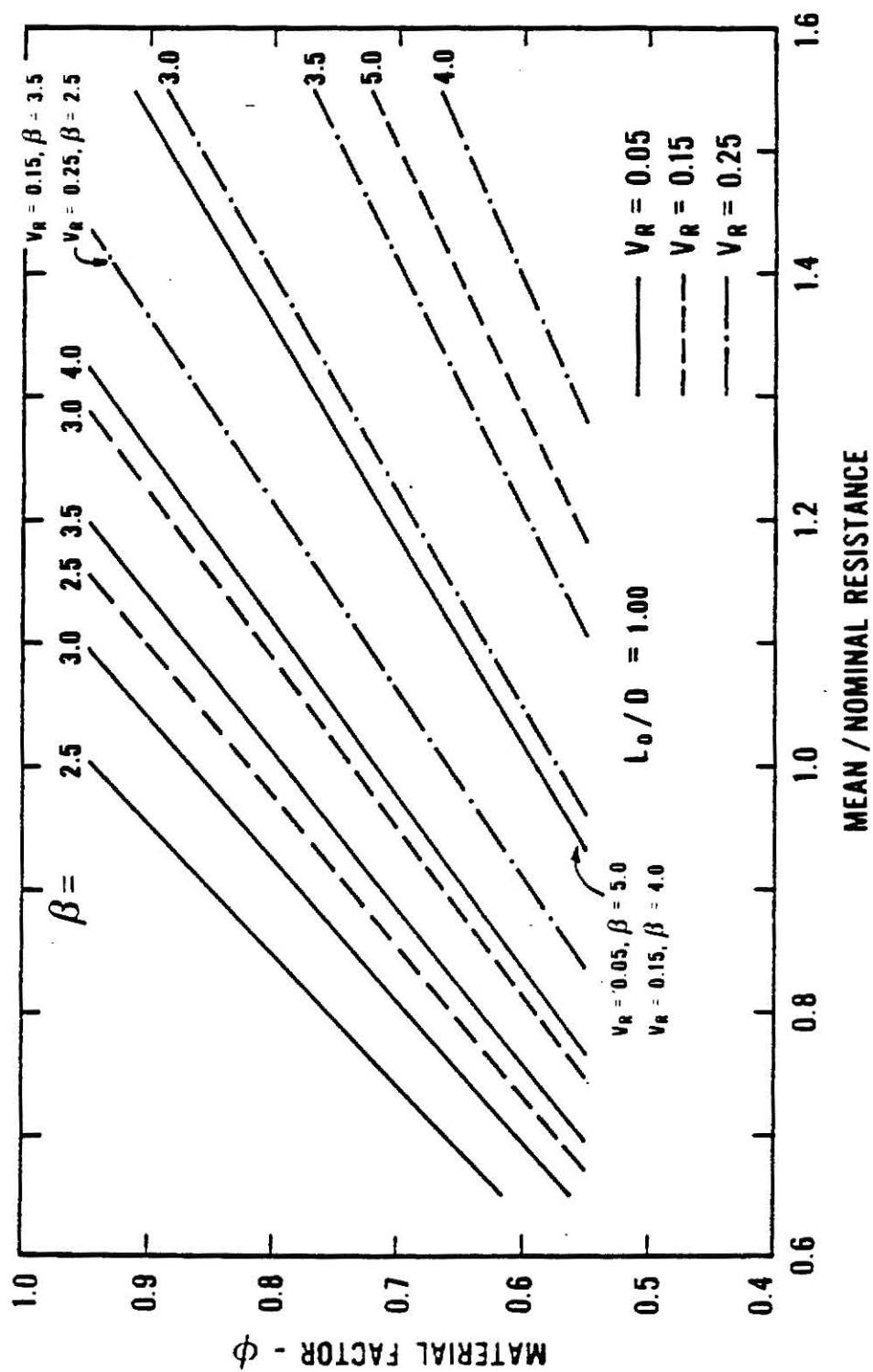


Figure 11C

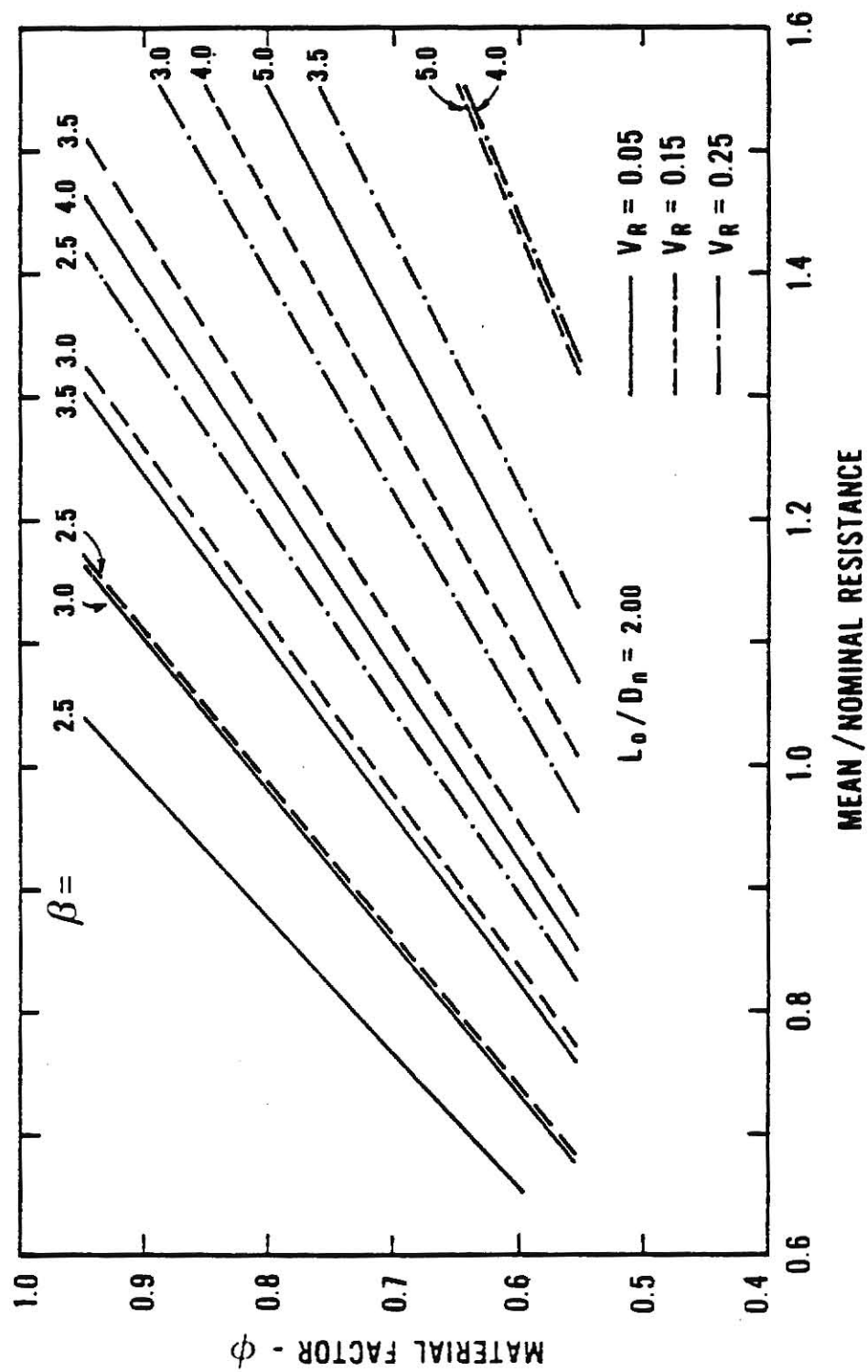


Figure 11d

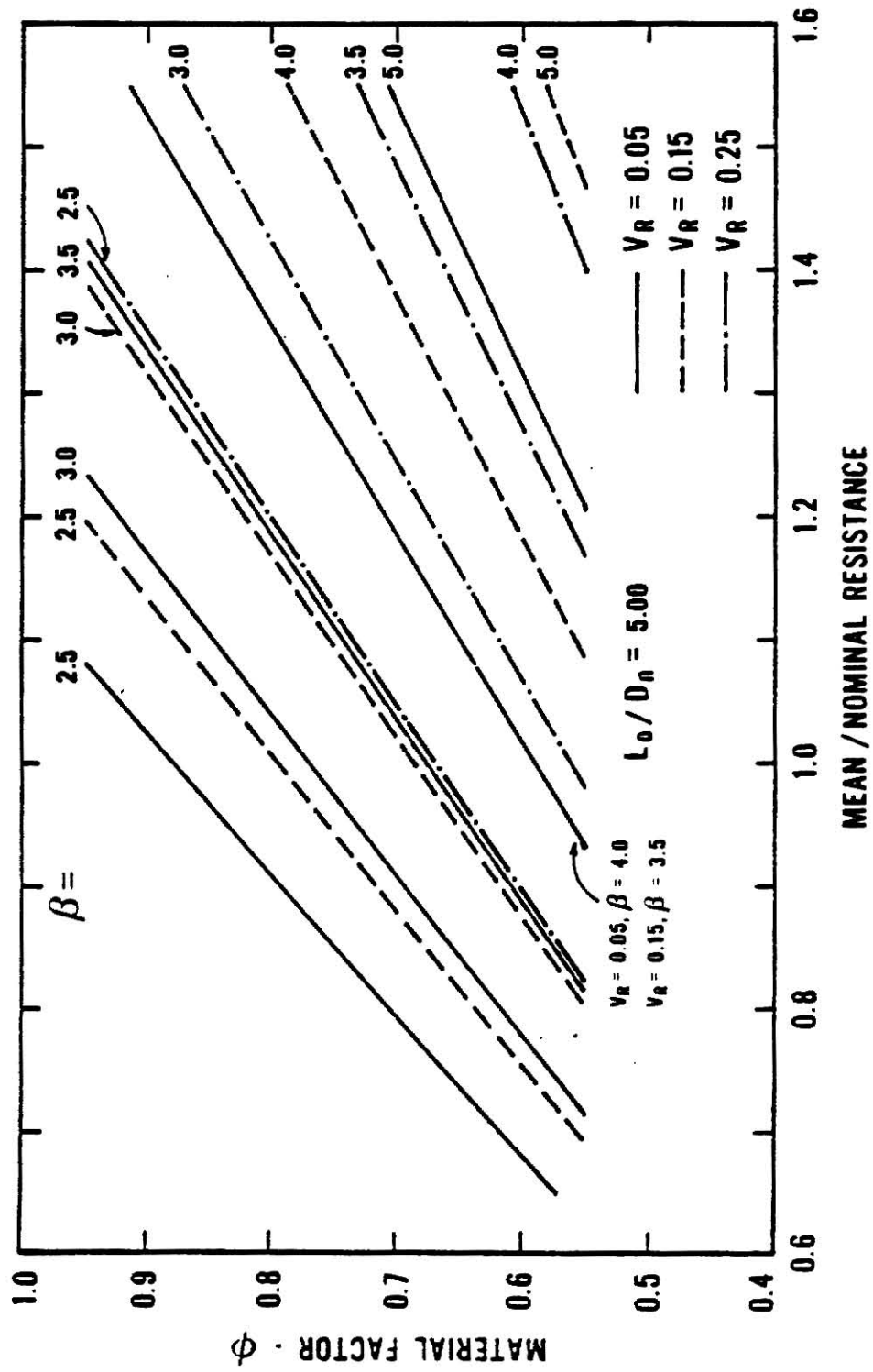
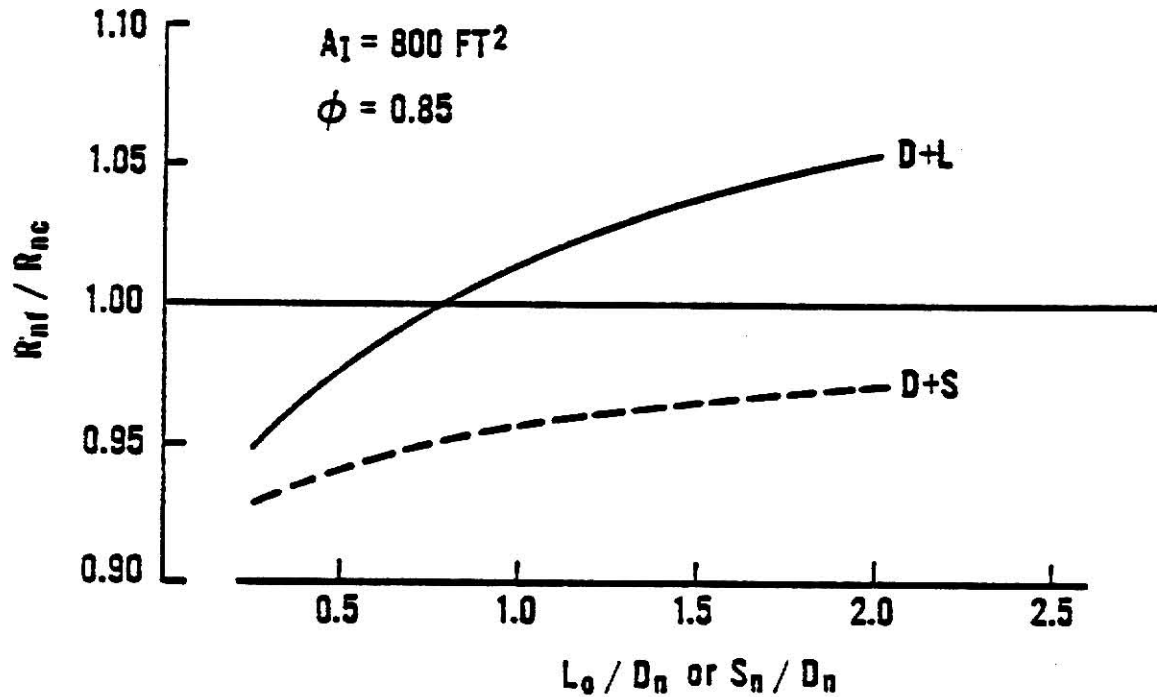
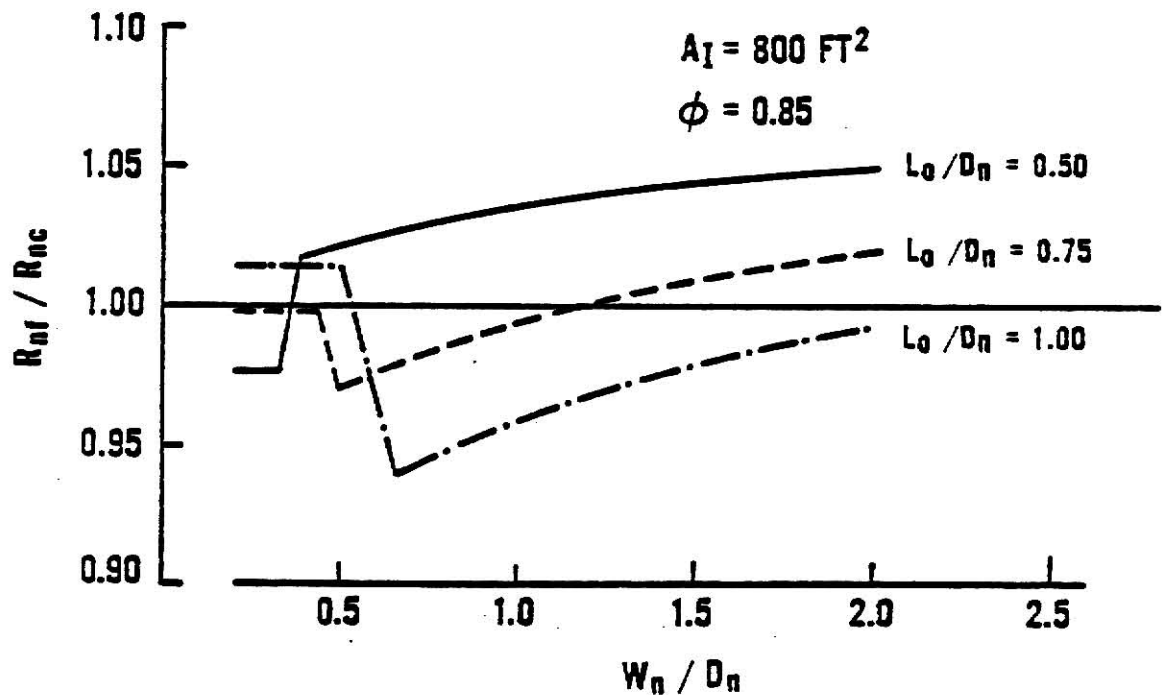


Figure 11e



(a) R/C BEAMS - GRAVITY LOADS



(b) R/C BEAMS - GRAVITY PLUS WIND LOADS

Figure 12 - Comparison of Designs Using Existing and Proposed Criteria for Reinforced Concrete Beams ($100 \text{ ft}^2 = 9.3 \text{ m}^2$) (Ref. 3), R_{nf} = future nominal resistance, R_{nc} = current nominal resistance.

10. Summary and Conclusions

This report has described the development of a set of load and load combination factors for use with loads in the American National Standard A58. The scope of these load criteria is the same as of the A58 Standard, which covers dead, live, wind, snow, and earthquake loads. The given criterion does not apply to vehicle loads on bridges, nuclear reactor constructions, and other loads that are considered outside the A58 Standard.

The load factors were developed by first evaluating the reliabilities associated with the current practice. The procedure for this evaluation is described in Chapter 2. Then, the load factors were developed consistent with these reliabilities, taking the statistical and probabilistic information of various types of structural loads and structural capacities (resistances) in to the consideration. The data for loads and resistances were obtained from the load subcommittees of American National Standard Institute Committee A58.

The proposed load and load combination factors for A58 Standard are as follows:

$$1.4D_n$$

$$1.2D_n + 1.6L_n$$

$$1.2D_n + 1.6S_n + (0.5L_n \text{ or } 0.8W_n)$$

$$1.2D_n + 1.3W_n + 0.5L_n$$

$$1.2D_n + 1.5E_n + (0.5L_n \text{ or } 0.2S_n)$$

$$0.9D_n - (1.3W_n \text{ or } 1.5E_n)$$

In conclusion, it should be mentioned that by setting the load and resistance factors constant over all range of nominal loads and resistance the reliability index does not stay constant. However by setting the load factors constant, the resistance factors can be changed to fulfill a relatively constant reliability index.

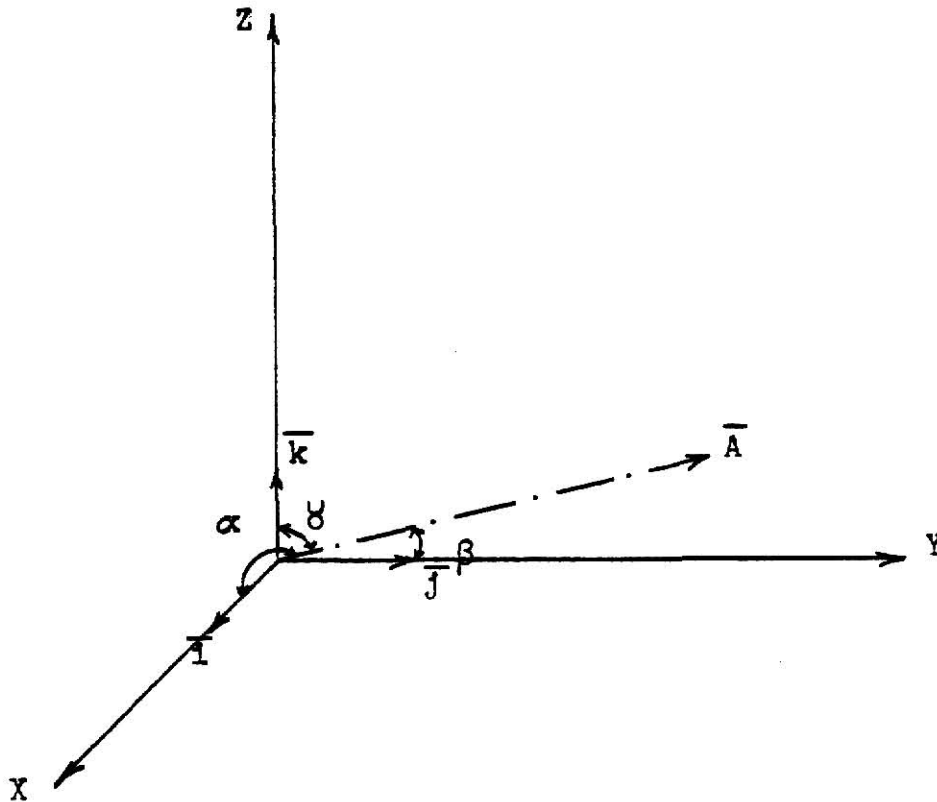
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Direction Angles - α , β , γ , angles between $\vec{A}$ and the standard $\vec{i}$, $\vec{j}$, and $\vec{k}$ basis vectors.

So, the cosines of the three direction angles are,

$$\cos \alpha = (\vec{A} \cdot \vec{i}) / |\vec{A}|$$

$$\cos \beta = (\vec{A} \cdot \vec{j}) / |\vec{A}|$$

$$\cos \gamma = (\vec{A} \cdot \vec{k}) / |\vec{A}|$$

$$\text{since; } \vec{A} = a\vec{i} + b\vec{j} + c\vec{k}$$

$$a = \vec{A} \cdot \vec{i}$$

$$b = \vec{A} \cdot \vec{j}$$

$$c = \vec{A} \cdot \vec{k}$$

$$|\overline{A}| = [a^2 + b^2 + c^2]^{1/2}$$

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Then;

$$\cos \alpha = a / |\overline{A}|$$

$$\cos \beta = b / |\overline{A}|$$

$$\cos \gamma = c / |\overline{A}|$$

Appendix B - Example Reliability Calculations

In the following example the 10-step procedure described in Chapter 2.2 will be followed to determine the optimum β .

Suppose a beam, 16WF31 section with yield stress $F_y = 36$ Ksi and plastic section modulus $Z = 54 \text{ in}^3$ supports a moment of 1140 in-Kip. Statistics of F_y and Z are:

F_y : Log-Normal- $\bar{F}_y = 38$ Ksi, $V(F_y) = 0.10$

Z : Normal- $\bar{Z} = 54$ in. $V(Z) = 0.05$

and the limit state, $g(\) = F_y \cdot Z - 1140 = 0$

Step 1. Limit State, $g(\) = F_y \cdot Z - 1140 = 0$

Step 2. Initial Guess, $\beta = 3.0$

Step 3. $X_i^* = \bar{X}_i$, $F_y^* = 38$, $Z^* = 54$

Step 4. the mean and standard deviation of the equivalent normal distribution for the Log-Normal distribution are as below.

$$\sigma^N = \phi(\Phi^{-1}[F_i(X_i^*)]) / f_i(x^*)$$

$$\bar{X}_i = X_i^* - \Phi^{-1}[F_i(X_i^*)] \cdot \sigma^N$$

F_y : Log-Normal Distribution, $\bar{F}_y = 38$, $V(F_y) = 0.10$

Because of the special properties of log-normal and normal distribution, we can take their coefficients of variation equal.

So, $V(F_y) = 0.10$ for the equivalent normal distribution. In a normal distribution the equation below holds.

$$V = \sigma / \mu$$

in which V = coefficient of variation, σ = standard deviation, and μ =

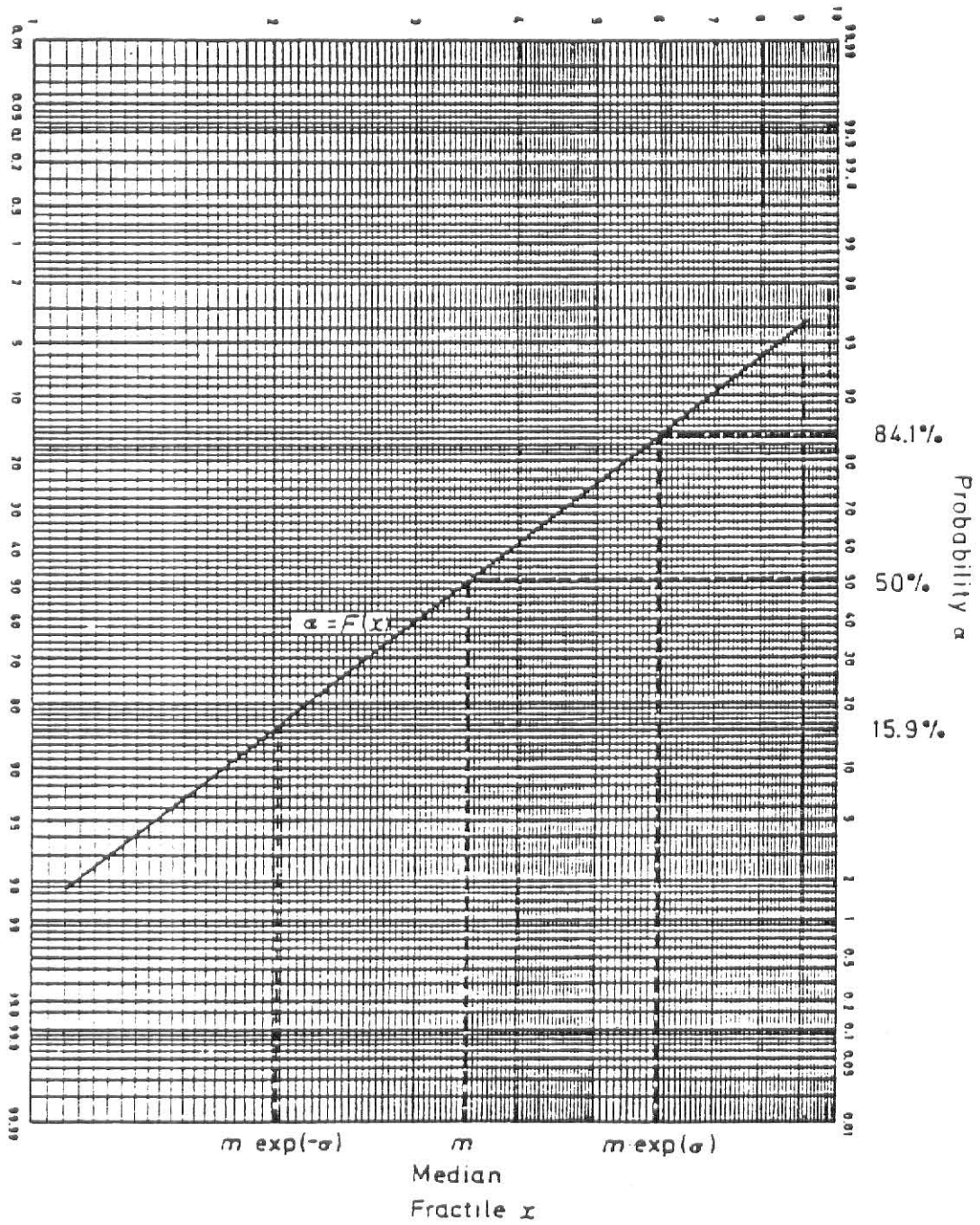


Figure 13 Lognormal distribution, probability paper
(Ref. 8)

mean. Therefore, the equivalent standard deviation is,

$$\sigma = \mu \cdot V$$

$$\sigma = (38)(0.10) = 3.8$$

To find the equivalent mean, we need to find the cumulative distribution value of the log-normal distribution at X_i^* , $F(X_i^*)$.

As shown in Fig.13 the median value, m , corresponds to 50% probability of occurrence in a log-normal distribution [Ref. 8]. Further, $m \cdot \exp(\sigma)$, corresponds to 84.1% and $m \cdot \exp(-\sigma)$, corresponds to 15.9% probability of occurrence. The calculation below evaluates these values.

In a log-normal distribution, the coefficient of variation is;

$$V = [\exp(\sigma^2) - 1]^{1/2}$$

$$\text{so, } V = 0.10 = [\exp(\sigma^2) - 1]^{1/2}$$

$$\text{therefore, } \sigma = 0.0997$$

$$\text{also, } \text{mean} = m \cdot \exp(\sigma^2/2)$$

$$\text{therefore, } 38 = m \cdot \exp(0.0997^2/2)$$

$$m = 37.811$$

$$m \cdot \exp(\sigma) = (37.811) \cdot \exp(0.0997) = 41.775$$

$$m \cdot \exp(-\sigma) = (37.811) \cdot \exp(-0.0997) = 34.223$$

From Fig.13, and the above calculations, the probability of the variate being equal to or less than $X_i^* = 38$ can be obtained as shown below.

$$\text{Log}(X_i^*) = \text{Log}(38) = 1.57978$$

$$\text{Log}(m) = \text{Log}(37.811) = 1.57761$$

$$\text{Log}(m \cdot \exp(\sigma)) = \text{Log}(41.775) = 1.62091$$

$$\text{Log}(m.\exp(-U)) = \text{Log}(34.223) = 1.53431$$

So the probability $F(38)$ computes according to the straightline in Figure 13.

$$F(38) = 50 + [(1.57978 - 1.57761) / (1.62091 - 1.57761)](84.1 - 50)$$

$$F(38) = 51.709\%$$

Now assuming a standard normal distribution with this probability of occurrence, we can proceed by using the Standard Normal Distribution Table given in Ref. 6.

$$\Phi^{-1}(0.51709)$$

$$\text{or } \Phi^{-1}(0.51709 - 0.5) = 0.04279$$

and therefore from Chapter 2,

$$\bar{X}_i = 38 - (0.04279)(3.3)$$

$$\bar{X}_i = 37.83$$

Step 5.

$$\partial g / \partial F_y \Big|_{\bar{Z}} = Z = 54$$

$$\partial g / \partial Z \Big|_{\bar{F}_y} = F_y = 38$$

Step 6.

$$\sigma_Z = (\text{mean})(\text{coeff. of variation}) = (54)(0.05) = 2.7$$

$$\alpha_{F_y} = [(54)(3.8)] / [(54)^2(3.8)^2 + (38)^2(2.7)^2]^{1/2} = 0.894$$

$$\alpha_Z = [(38)(2.7)] / [(54)^2(3.8)^2 + (38)^2(2.7)^2]^{1/2} = 0.447$$

Step 7. New values of X_i^* are

$$X_Z^* = 54 - (0.447)(3.0)(2.7) = 50.38$$

$$X_{F_y}^* = 37.83 - (0.894)(3.0)(3.8) = 27.64$$

Here, we repeat steps 4 through 7 until the estimates of α_i stabilize. The results of these calculations and other steps taken in this problem are tabulated in Table 11 [Ref. 3].

$X_i^* = \bar{X}_i - \alpha_i \cdot \beta \cdot \sigma_i^N$ is used to calculate β in the limit state equation.

Table 11-Illustration of Reliability Calculations-
Iterative Solution Steps (Ref. 3)

Step 2	β	3.0			5.001		5.136	
Steps 3,7	F_y^*	38	27.64	29.02	23.95	24.46	24.19	24.21
	Z^*	54	50.38	50.17	47.60	47.34	47.14	47.11
Step 4	$\overline{F}_y^N$	37.83	36.31	36.71	34.90	35.12	35.00	35.01
	σ_y^N	3.80	2.76	2.90	2.40	2.45	2.42	2.42
	$\overline{Z}^N$	54	54	54	54	54	54	54
	σ_Z^N	2.7	2.7	2.7	2.7	2.7	2.7	2.7
Step 5	$\frac{\partial g}{\partial F_y}$	54	50.38	50.17	47.6	47.34	47.14	47.11
	$\frac{\partial g}{\partial Z}$	38	27.64	29.02	23.95	24.46	24.19	24.21
Step 6	α_{F_y}	0.894	0.881	0.880	0.870	0.869	0.868	0.868
	α_{F_Z}	0.447	0.473	0.474	0.493	0.495	0.497	0.497
Step 8	β	5.001			5.136		5.144	

Appendix C - Extreme Value Type I Distribution

Type I distributions are sometimes called the log-Weibull distributions or they are sometimes called "double exponential" distributions. The function below is the general form of the Extreme Value Type I distribution.

$$P[X \leq x] = \exp\{-\exp[-(x-\xi)/\theta]\}$$

where ξ , and θ are parameters.

The name 'extreme value' is attached to these distributions because they can be obtained as limiting distributions (as $n \rightarrow \infty$) of the greatest value among n independent random variables each having the same continuous distribution. By replacing X by $-X$, limiting distributions of least values are obtained.

PROBABILITY-BASED LOAD FACTORS
AND LOAD COMBINATION FACTORS
IN STRUCTURAL DESIGN

by

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B.S., Kansas State University, 1980

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

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KANSAS STATE UNIVERSITY
Manhattan, Kansas

1983

Abstract

Probability-based load factors and load combinations are presented which are compatible with the loads specified in ANSI A58 Standards. The load effects considered are due to dead, occupancy live, snow, wind and earthquake loads. The load criteria which are presented were developed by numerous investigators on the basis of probability of failure calculated by simplified theory, using statistical data on structural loads and resistances. The report presents the rationale and methodology used by the investigators for selecting the load factors which are intended to apply to all types of structural materials used in building construction.