ESTIMATION AND HYPOTHESES TESTING OF THE FITNESS COEFFICIENT

by

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Introduction

Estimation and hypothesis testing regarding fitness in population of two or more types are of importance in evolution and natural selection, in experimental studies of competition in laboratory populations and in mixed cultures of microbial populations used in biotechnology.

A common experimental method in fitness studies is to follow changes in the frequencies of different types within a population over time. One important technique employed by evolutionary biologist is to model these changes as a Markov process.

In this study, we use a Markovian process to study a mixture of two types where one type has a fitness advantage over the other. We present a maximum likelihood estimate of the fitness coefficient and construct a test of hypothesis for this coefficient for a fixed population of size N.

Model

Consider a population with two types A and a and relative fitness coefficients w and 1 respectively. Let

$$P_{ij} = P[x(k) = j \mid x(k) + y(k) = N,\ x(k - 1) = i,\ y(k - 1) = N - i] = Pr[j, N - j \mid i, N - i] \quad (1)$$

$$k = 1,2,\ldots m$$

$$i = 1,2,\ldots N$$

where m represents the number of generations and N = the total population size. Thus

$$P_{ij} = \binom{N}{j}\left[\frac{wi}{wi + N - i}\right]^{j}\left[\frac{N - i}{wi + N - i}\right]^{N-j}$$

$$i,j, = 0,1,\ldots N \tag{2}$$

where w is the fittness coefficient. Using the fact that the process is Markovian

$$P = \text{Prob}[(i_m, N - i_m), (i_{m-1}, N - i_{m-1})\cdots$$
$$(i_1, N - i_1)(i_0, n - i_0)] \tag{3}$$

can be written as

$$P = \prod_{k=1}^{m}\left\{\binom{N}{i_k}\left[\frac{wi_{k-1}}{wi_{k-1} + N - i_{k-1}}\right]^{i_k}\left[\frac{N - i_{k-1}}{wi_{k-1} + N - i_{k-1}}\right]^{N - i_k}\right\}. \tag{4}$$

Estimation

To obtain a maximum likelihood estimate of the coefficient w we may solve the equation

$$\frac{\partial \ln P}{\partial \hat{w}} = 0 \; . \tag{5}$$

From (4) and (5)(with $w = \hat{w}$) one obtains

$$\frac{\partial \ln P}{\partial \hat{w}} = \sum_{k=1}^{m} \left\{ \frac{i_k}{\hat{w}} - \frac{N i_{k-1}}{\hat{w} i_{k-1} + N - i_{k-1}} \right\} = 0 \ . \qquad (6)$$

To obtain a solution for $\hat{w}$ we may use the bisection method to solve the following equation

$$\sum_{k=1}^{m} \frac{i_k}{\hat{w}} = \sum_{k=1}^{m} \frac{N i_{k-1}}{\hat{w} i_{k-1} + N - i_{k-1}} \ . \qquad (7)$$

Hypothesis testing.

A locally best test of size α may be given for $w_{\bullet}$ as

$$H_0: w \le 1$$
$$H_a: w > 1 \ . \qquad (8)$$

We will let ψ be the test function and let $\Delta = \frac{\partial \ln P}{\partial w}\Big|\ w = 1$. Then the locally best test is given by (Ferguson, 1967),

$$\psi = \begin{cases} 1 & \text{if} \quad \Delta > c \\ \gamma & \text{if} \quad \Delta = c \\ 0 & \text{if} \quad \Delta < c \end{cases} \qquad (9)$$

where $E_{H_0}(\psi) = \alpha$, $w = 1$, $\Delta = i_m - i_0$ and c = constant. (10)

If we substitute (10) into (9) we obtain

$$\psi = \begin{cases} 1 & \text{if } i_m - i_0 > c \\ \gamma & \text{if } i_m - i_0 = c \\ 0 & \text{if } i_m - i_0 < c \end{cases} . \qquad (11)$$

Because i_0 is a positive constant and N is a positive constant such that $N > i_0$, (11) can be written as

$$\psi = \begin{cases} 1 & \text{if } \frac{i_m}{N} > \frac{c + i_0}{N} \\ \gamma & \text{if } \frac{i_m}{N} = \frac{c + i_0}{N} \\ 0 & \text{if } \frac{i_m}{N} < \frac{c + i_0}{N} \end{cases} . \qquad (12)$$

The test is now written in terms of the frequency $\frac{i_m}{N}$ of one type. The reason one performs this transformation is that it will be easy to find a density function for $\frac{i_m}{N}$. The quanity $\frac{i_m}{N}$ is the frequency of type A, while $1 - \frac{i_m}{N}$ is the frequency of type a. These frequencies will be denoted by x and 1 - x, respectively, where $x \epsilon [0, 1]$.

<u>Determination of the Density Function of Gene Frequency When Fitness Equals 1</u>

Let $\phi(p,x;t)$ be the conditional probability density that the frequency is x at time (t) given the initial frequency was p at time 0. Kimura (1962) has shown that the distribution of $\frac{i_m}{N}$ can be calculated by using the Fokker Plank equation

$$\frac{\partial\phi}{\partial t} = \frac{1}{2N}\frac{\partial^2}{\partial x^2}[x(1-x)\phi] - s\frac{\partial}{\partial x}[x(x-1)\phi] \tag{13}$$

$$(0 < x < 1)$$

where s is the selection coefficient. In terms of our model,

$$w = 1 + s. \tag{14}$$

Under the null hypotheses $s = 0$ (which implies $w = 1$) equation (13) becomes

$$\frac{\partial\phi}{\partial t} = \frac{1}{2N}\frac{\partial^2}{\partial x^2}[x(1-x)\phi] \tag{15}$$

with initial condition $\phi(p,x,0) = \delta(x - p)$ (16)

where $\delta(x - p)$ is the Dirac Delta Function. To solve (15) we let

$$\phi(x,t) = X(x)T(t). \tag{17}$$

Now substitute (17) into Equation (15) and use the separation of variables technique to obtain

$$\frac{1}{T(t)}\frac{dT(t)}{dt} = -\lambda \tag{18}$$

and

$$x(1 - x) \frac{d^2X(x)}{dt^2} + 2(1 - 2x) \frac{dX(x)}{dt} - 2(1 - N\lambda) X(x) = 0. \qquad (19)$$

Equation (19) is a hypergeometric equation which has the general form

$$x(1 - x) X''(x) + [\gamma - (\alpha + \beta + 1)x]X'(x) - \alpha\beta X(x) = 0. \qquad (20)$$

This form has a solution which is a hypergeometric function denoted by $F(\alpha, \beta, \gamma, x)$. Equation (19) may be solved if we let $\alpha = 1 - i$, $\beta = i + 2$, $\gamma = 2$. $F(\alpha, \beta, \gamma, x)$ has a series representation which is given by

$$F(1 - i, i + 2, 2, x) = 1 + \frac{(1 - i)(i + 2)}{1 \times 2} x + \frac{(1 - i)(2 - i)(i + 2)(i + 3)}{1 \times 2 \times 2 \times 3} x^2 + \frac{(1 - i)(2 - i)(3 - i)(i + 2)(i + 3)(i + 4)}{1 \times 2 \times 2 \times 3 \times 3 \times 4} x^3 + \ldots. \qquad (21)$$

It is tedious to express solutions to (15) in terms of the hypergeometric function. Instead we write the solution in terms of gegenbauer polynomials. Thus,

$$\phi(p,x;t) = \sum_{i=1}^{\infty} \frac{(2i + 1)(1 - r^2)}{i(i + 1)} T_{i - 1}(r) T_{i - 1}(z) \exp\left(\frac{-i(i + 1)t}{2N}\right) \qquad (22)$$

where $$r = 1 - 2p \qquad (23)$$

$$z = 1 - 2x \ . \tag{24}$$

The gegenbauer polynomials are given by

$$T_{i-1}(z) = \frac{i(i+1)}{2} F\left(i + 2, 1 - i, 2; \frac{1-z}{2}\right) \tag{25}$$

$$T_{i-1}(r) = \frac{i(i+1)}{2} F\left(i + 2, 1 - i, 2; \frac{1-r}{2}\right) \ . \tag{26}$$

Thus we can now calculate a test for the hypotheses H_0: $w = 1$ from knowledge of the distribution of x or $\frac{i_w}{N}$.

Using eq. (22) and the program in appendix A we generated $\phi(P,x;t)$ for different values of $\frac{N}{t}$ and an initial frequency $P = \frac{1}{2}$. Results are presented in Tables 1.1 - 1.19. The solution for $\phi(P,x;t)$ excludes the frequency classes 0 and 1. The frequency of the 0 class for $p = \frac{1}{2}$ is

$$\frac{1}{2}[1 - \int_0^1 \phi(p,x;t)dx].$$

Hence, this frequency was included in the calculations of cumulative probabilities in Tables 1.1 - 1.19. From these tables, the critical values for rejection ([from (12)]) were generated for different α levels and N/t values. Results are presented in Table 2.1.

Calculation of Power.

In the previous section a method was presented to estimate the distribution of $\frac{i_m}{N}$. In this section we shall consider how the power of such a test can be calculated. Two methods shall be discussed, the first method will be a method due to Stratton (1941). The second method will be by simulation.

Consider the Fokker Plank equation with $s \neq 0$

$$\frac{\partial \phi}{\partial t} = \frac{1}{2N} \frac{\partial^2}{\partial x^2} x(x - 1)\phi - s\frac{\partial}{\partial x} x(1 - x)\phi. \tag{27}$$

$$0 < x < 1$$

The power of a particular test can be calculated by solving (27) for ϕ given some value of s. In 1941, Stratton gave a solution to Equation (27) in the form

$$\phi(P,x;t) = \sum_{k=0}^{\infty} C_k \exp^{(-\lambda_{kt} + 2cx)} V_{i,k}^{(1)}(z) \tag{28}$$

where
$$V_{1,k}^{(1)}(z) = \sum_{n=1}^{\infty} f_n^{\,k} T_n^{\,1}(z) , \tag{29}$$

and the $f_n^{\,k}$'s are constants described by Stratton. The primed summation means the sum over even values when k is even or the sum over odd values when k is odd

$$C_k = \frac{(1 - r^2)\, e^{-c(1 - r)}\, V_{ik}^{(1)}(r)}{\sum_{r=0,1}' \frac{(n + 1)(n + 2)}{(2n + 3)} \left(f_n^{\,k}\right)^2} \tag{30}$$

where $r = 1 - 2p \quad c = Ns$.

The Stratton solution involved the method of separation of variable in which the x component was represented as an oblate spheroid equation. This solution must be represented as a series solution. Stratton produced a limited set of constants. Hence, his solution to (28) was not adequate for computing the power.

The problem for generating coefficients for the Fokker Plank equation was a time consuming process. Hence, a simulation of the stochastic process was performed using the program SIMULATE (Appendix B). The results of SIMULATE are given in table 3.1 - 3.16. From these tables we generated (for $N = 40$, $N/t = 8$ and different Ns values) the power of the test statistic for each of the α levels in Tables 2.1. These results are presented in Table 4.1.

Conclusion

In this study we present a maximum likelihood extimator for the relative fitness coefficient in a population of two types and a locally best test for testing the null hypothesis of no differential selection between types. The distribution of the test statistic is investigated and critical values for rejection as different α levels given. An example of how the test can be applied is given below.

Let us assume we are interested in testing the null hypothesis of no selection (H_0: $w = 1$) against the alternative hypothesis there is selection, when the population size is $n = 40$ over 5 generations. Formally stated

$$H_0: w \leq 1$$

$$H_a: w > 1 \ .$$

The test statistic is given in equation (12) to be $\frac{i_m}{N}$.

We now look in Table 2.1 for α = .05, N/t = 8 and find the critical value to be .778286. Thus, if $\frac{i_m}{N} > .778286$ we reject H_0. If we wish to test the power, we can refer to Table 4.1. We note from this table that for α = .05 and Ns = 8 the β value is .764. Hence, the power is 1 - .764 = .236. For a given fitness or selection coefficient s, the power can be increased by increasing N. An Ns value in the neighberhood of 15 for initial frequency of $\frac{1}{2}$ gives a very high power as seen in Table 4.1.

TABLES 1.1 - 1.19. Results give the values for phi ($\phi(P,x;t)$) and the cumulative probabilities for different frequencies (x) and $P = \frac{1}{2}$.

Table 1.1 N/T = 2.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.6662787	0.0066628	0.510	0.9757209	0.4446618
0.020	0.6782525	0.0134453	0.520	0.9753168	0.4544149
0.030	0.6900228	0.0203455	0.530	0.9746435	0.4641614
0.040	0.7015867	0.0273614	0.540	0.9737010	0.4738984
0.050	0.7129417	0.0344908	0.550	0.9724897	0.4836233
0.060	0.7240853	0.0417317	0.560	0.9710096	0.4933334
0.070	0.1350148	0.0490818	0.570	0.9692613	0.5030260
0.080	0.7457279	0.0565391	0.580	0.9672449	0.5126984
0.090	0.7562222	0.0641013	0.590	0.9649611	0.5223480
0.100	0.7664952	0.0717663	0.600	0.9624101	0.5319722
0.110	0.7765446	0.0795317	0.610	0.9595927	0.5415681
0.120	0.7863684	0.0873954	0.620	0.9565094	0.5511332
0.130	0.7959642	0.0953551	0.630	0.9531609	0.5606648
0.140	0.8053299	0.1034083	0.640	0.9495479	0.5701603
0.150	0.8144634	0.1115530	0.650	0.9456712	0.4796170
0.160	0.8233627	0.1197866	0.660	0.9415316	0.5890323
0.170	0.8320259	0.1281069	0.670	0.9371300	0.5984036
0.180	0.8404510	0.1365114	0.680	0.9324674	0.6077283
0.190	0.8486362	0.1449977	0.690	0.9275447	0.6170037
0.200	0.8565797	0.1535635	0.700	0.9223631	0.6262273
0.210	0.8642797	0.1622063	0.710	0.9169237	0.6353966
0.220	0.8717345	0.1709237	0.720	0.9112276	0.6445089
0.230	0.8789426	0.1797131	0.730	0.9052760	0.6535616
0.240	0.8859022	0.1885721	0.740	0.8990704	0.6625523
0.250	0.8926120	0.1974982	0.750	0.8926120	0.6714784
0.260	0.8990704	0.2064890	0.760	0.8859022	0.6803375
0.270	0.9052760	0.2155417	0.770	0.8789426	0.6891269
0.280	0.9112276	0.2246540	0.480	0.8717345	0.6978442
0.290	0.9169237	0.2338232	0.790	0.8642797	0.7064840
0.300	0.9223631	0.2430469	0.800	0.8565797	0.7150528
0.310	0.9275447	0.2523223	0.810	0.8486362	0.7235392
0.320	0.9324674	0.2616470	0.820	0.8404510	0.7319437
0.330	0.9371300	0.2710183	0.830	0.8320259	0.7402640
0.340	0.9415316	0.2804336	0.840	0.8233627	0.7484976
0.350	0.9456712	0.2898903	0.850	0.8144634	0.7566422
0.360	0.9495479	0.2993858	0.860	0.8053299	0.7646955
0.370	0.9531609	0.3089174	0.870	0.7959642	0.7726552
0.380	0.9565094	0.3184825	0.880	0.7863684	0.7808188
0.390	0.9595927	0.3280784	0.890	0.7765446	0.7882843
0.400	0.9624101	0.3377025	0.900	0.7664952	0.7959492
0.410	0.9649611	0.3473521	0.910	0.7562222	0.8035115
0.420	0.9692613	0.3667172	0.920	0.7457279	0.8109687
0.430	0.9692613	0.3667172	0.930	0.7350148	0.8183189
0.440	0.9710096	0.3764273	0.940	0.7240853	0.8255597
0.450	0.9724897	0.3861522	0.950	0.7129417	0.8326892
0.460	0.9737010	0.3958892	0.960	0.7015867	0.8397050
0.470	0.9746435	0.4056356	0.970	0.6900228	0.8466053
0.480	0.9753168	0.4153888	0.980	0.6782525	0.8533878
0.490	0.9757209	0.4251460	0.990	0.6662787	0.8600506
0.500	0.9758555	0.4349046			

Table 1.2 N/T = 3.00

x	Phi	Cum Prob.
0.010	0.4585114	0.0045851
0.020	0.4846700	0.0094318
0.030	0.5107763	0.0145396
0.040	0.5368030	0.0199076
0.050	0.5627228	0.0255348
0.060	0.5885090	0.0314199
0.070	0.6141348	0.0375613
0.080	0.6395742	0.0439570
0.090	0.6648011	0.0506050
0.100	0.6897899	0.0575029
0.110	0.7145157	0.0646481
0.120	0.7389534	0.0720376
0.130	0.7630788	0.0796684
0.140	0.7868679	0.0875371
0.150	0.8102972	0.0956401
0.160	0.8333436	0.1039735
0.170	0.8559846	0.1125333
0.180	0.8781981	0.1213153
0.190	0.8999684	0.1303149
0.200	0.9212565	0.1395275
0.210	0.9420598	0.1489481
0.220	0.9623523	0.1585716
0.230	0.9821147	0.1683928
0.240	1.0013279	0.1784061
0.250	1.0199736	0.1886058
0.260	1.0380343	0.1989861
0.270	1.0554927	0.2095411
0.280	1.0723323	0.2202644
0.290	1.0885373	0.2311498
0.300	1.1040925	0.2421907
0.310	1.1189832	0.2533805
0.320	1.1331955	0.2647125
0.330	1.1467161	0.2761796
0.340	1.1595325	0.2877750
0.350	1.1716326	0.2994913
0.360	1.1830053	0.3113213
0.170	1.1936399	0.3232577
0.380	1.2035267	0.3352930
0.390	1.2126564	0.3474196
0.400	1.2210207	0.3596298
0.410	1.2286117	0.3719159
0.420	1.2354226	0.3842701
0.430	1.2414471	0.3966846
0.440	1.2466795	0.4091514
0.450	1.2511151	0.4216625
0.460	1.2547498	0.4342100
0.470	1.2575803	0.4467858
0.480	1.2596039	0.4593819
0.490	1.2608188	0.4719901
0.500	1.2612239	0.4846023
0.510	1.2608188	0.4972105
0.520	1.2596039	0.5098065
0.530	1.2575803	0.5223823
0.540	1.2547498	0.5349298
0.550	1.2511151	0.5474410
0.560	1.2466795	0.5599078
0.570	1.2414471	0.5723222
0.580	1.2354226	0.5846765
0.590	1.2286117	0.5969626
0.600	1.2210107	0.6091728
0.610	1.2126564	0.6212994
0.620	1.2035267	0.6333346
0.630	1.1936399	0.6452710
0.640	1.1830053	0.6571011
0.650	1.1716326	0.6688174
0.660	1.1595325	0.6804127
0.670	1.1467161	0.6918799
0.680	1.1331955	0.7032118
0.690	1.1189832	0.7144017
0.700	1.1040925	0.7254426
0.710	1.0885373	0.7363280
0.720	1.0723323	0.7470513
0.730	1.0554927	0.7576062
0.740	1.0380343	0.7679866
0.750	1.0799736	0.7781863
0.760	1.0013279	0.7881996
0.770	0.9821147	0.7980207
0.780	0.9623523	0.8076442
0.790	0.9420598	0.8170648
0.800	0.9212565	0.8262774
0.810	0.8999624	0.8352770
0.820	0.8781981	0.8440590
0.830	0.8559846	0.8526189
0.840	0.8333436	0.8609523
0.850	0.8102972	0.8690553
0.860	0.7868679	0.8769239
0.870	0.7630788	0.8845547
0.880	0.7389534	0.8919443
0.890	0.7146157	0.8990894
0.900	0.6897899	0.9059873
0.910	0.6648011	0.9126353
0.920	0.6395742	0.9190311
0.930	0.6141348	0.9251724
0.940	0.5885090	0.9310575
0.950	0.5627228	0.9366847
0.960	0.5368030	0.9420528
0.970	0.5107763	0.9471605
0.980	0.4846700	0.9520072
0.990	0.4585114	0.9565924

Table 1.3 N/T = 4.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.2536058	0.0093814	0.510	1.4917926	0.5223807
0.020	0.2834278	0.0122156	0.520	1.4894892	0.5372755
0.030	0.3140439	0.0153561	0.530	1.4856550	0.5521321
0.040	0.3453945	0.0188100	0.560	1.4802974	0.5669351
0.050	0.3774185	0.0225842	0.550	1.4734264	0.5816693
0.060	0.4100535	0.0266847	0.560	1.4650550	0.5963199
0.070	0.4432357	0.0311171	0.570	1.4551991	0.6108719
0.080	0.4769004	0.0358861	0.580	1.4438774	0.6253107
0.090	0.5109819	0.0409959	0.590	1.4311113	0.6396218
0.100	0.5454138	0.0464501	0.600	1.4169250	0.6537910
0.110	0.5801290	0.0522513	0.610	1.4013453	0.6678045
0.120	0.6150597	0.0584019	0.620	1.3844019	0.6816485
0.130	0.6501381	0.0649033	0.630	1.3661269	0.6953098
0.140	0.6852958	0.0717563	0.640	1.3465549	0.7087753
0.150	0.7204646	0.0789609	0.650	1.3257233	0.7220325
0.160	0.7555762	0.0865167	0.660	1.3036715	0.7350693
0.170	0.7905626	0.0944223	0.670	0.2804416	0.7478737
0.180	0.8253558	0.1026759	0.680	1.2560778	0.7604344
0.190	0.8598887	0.1112748	0.690	1.2306265	0.7727407
0.200	0.8940944	0.1202157	0.700	1.2041362	0.7847821
0.210	0.9279069	0.1294948	0.710	1.1766575	0.7965486
0.220	0.9612610	0.1391074	0.720	1.1482429	0.8080311
0.230	0.9940925	0.1490483	0.730	1.1189468	0.8192205
0.240	1.0263381	0.1593117	0.740	1.0888253	0.8301088
0.250	1.0579360	0.1698911	0.750	1.0579360	0.8406882
0.260	1.0888253	0.1807793	0.760	1.0263381	0.8509515
0.270	1.1189468	0.1919688	0.770	0.9940925	0.8608925
0.280	1.1482429	0.2034512	0.780	0.9612610	0.8705051
0.290	1.1766575	0.2152178	0.790	0.9279069	0.8797841
0.300	1.2041362	0.2272591	0.800	0.8940944	0.8887251
0.310	1.2306265	0.2395654	0.810	0.8598887	0.8973240
0.320	1.2560778	0.2521262	0.820	0.8253558	0.9055775
0.330	1.2804416	0.2649306	0.830	0.7905626	0.9134832
0.340	1.3036715	0.2779673	0.840	0.7555762	0.9210389
0.350	1.3257233	0.2912245	0.850	0.7204646	0.9282436
0.360	1.3465549	0.3046901	0.860	0.6852958	0.9350965
0.370	1.3661269	0.3183514	0.870	0.6501381	0.9415979
0.380	1.3844019	0.3321954	0.880	0.6150597	0.9477485
0.390	1.4013453	0.3462088	0.890	0.5801290	0.9535498
0.400	1.4169250	0.3603781	0.900	0.5454138	0.9590039
0.410	1.4311113	0.3746892	0.910	0.5109819	0.9641138
0.420	1.4438774	0.3891280	0.920	0.4769004	0.9688828
0.430	1.4551991	0.4036800	0.930	0.4432357	0.9733151
0.440	1.4650550	0.4183305	0.940	0.4100535	039774156
0.450	1.4734264	0.4330648	0.950	0.3774185	0.9811898
0.460	1.4802974	0.4478678	0.960	0.3453945	0.9846438
0.470	1.4856550	0.4627243	0.970	0.3140439	0.9877842
0.480	1.4894892	0.4776192	0.980	0.2834278	0.9906185
0.490	1.4917926	0.4925371	0.990	0.2536058	0.9931546
0.500	1.4925609	0.5074627			

Table 1.4   N/T = 5.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.1256323	0.0034488	0.510	1.6912883	0.5253753
0.020	0.1506004	0.0049548	0.520	1.6876805	0.5422521
0.030	0.1772852	0.0067277	0.530	1.6816796	0.5590689
0.040	0.2056447	0.0087841	0.540	1.6733032	0.5758020
0.050	0.2356297	0.0111404	0.550	1.6625763	0.5924277
0.060	0.2671838	0.0138123	0.560	1.6495307	0.6089230
0.070	0.3002440	0.0168147	0.570	1.6342051	0.6252651
0.080	0.3347407	0.0201621	0.580	1.6166450	0.6414315
0.090	0.3705982	0.0238681	0.590	1.5969023	0.6574006
0.100	0.4077347	0.0279454	0.600	1.5750356	0.6731509
0.110	0.4460631	0.0324061	0.610	1.5511096	0.6886620
0.120	0.4854908	0.0372610	0.620	1.5251949	0.7039140
0.130	0.5259205	0.0425202	0.630	1.4973680	0.7188876
0.140	0.5672503	0.0481927	0.640	1.4677111	0.7335647
0.150	0.6093741	0.0542864	0.650	1.4363115	0.7479279
0.160	0.6521823	0.0608082	0.660	1.4032614	0.7619605
0.170	0.6955618	0.0677639	0.670	1.3686578	0.7756470
0.180	0.7393965	0.0751578	0.680	1.3326022	0.7889731
0.190	0.7835680	0.0829935	0.690	1.2951998	0.8019251
0.200	0.8279556	0.0912731	0.700	1.2565598	0.8144907
0.210	0.8724371	0.0999974	0.710	1.2167945	0.8266586
0.220	0.9168891	0.1091663	0.720	1.1760194	0.8384188
0.230	0.9611871	0.1187782	0.730	1.1343521	0.8497623
0.240	1.0052068	0.1288303	0.740	1.0919128	0.8606815
0.250	1.0488233	0.1393185	0.750	1.0488233	0.8711697
0.260	1.0919128	0.1502376	0.760	1.0052068	0.8812218
0.270	1.1343524	0.1615812	0.770	0.9611871	0.8908336
0.280	1.1760194	0.1733413	0.780	0.9168891	0.9000025
0.290	1.2167945	0.1855093	0.790	0.8724341	0.9087269
0.300	1.2565598	0.1980749	0.800	0.8279556	0.9170064
0.310	1.2951998	0.2110269	0.810	0.7835680	0.9248421
0.320	1.3326022	0.2243529	0.820	0.7393965	0.9322361
0.330	1.3686578	0.2380395	0.830	0.6955618	0.9391917
0.340	1.4032614	0.2520721	0.840	0.6521823	0.9457135
0.350	1.4363115	0.2664352	0.850	0.6093741	0.9518073
0.360	1.4677111	0.2811123	03860	0.5672503	0.9574798
0.370	1.4973680	0.2960860	0.870	0.5259205	0.9627390
0.380	1.5251949	0.3113380	0.880	0.4854908	0.9675939
0.390	1.5511096	0.3268491	0.890	0.4460631	0.9720545
0.400	1.5750356	0.3425994	0.900	0.4077347	0.9761319
0.410	1.5969023	0.3585684	0.910	0.3705982	0.9798378
0.420	1.6166450	0.3747349	0.920	0.3347407	0.9831853
0.430	1.6342051	0.3910769	0.930	0.3002440	0.9861877
0.440	1.6495307	0.4075722	0.940	0.2671838	0.9888595
0.450	1.6625763	0.4241980	0.950	0.2356297	0.9912158
0.460	1.6733032	0.4409310	0.960	0.2056447	0.9932723
0.470	1.6816796	0.4577478	0.970	0.1772852	0.9950451
0.480	1.6876805	0.4746246	0.980	0.1506004	0.9965511
0.490	1.6912883	0.4915375	0.990	0.1256323	0.9978075
0.500	1.6924920	0.5084624			

Table 1.5 N/T = 6.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0583953	0.0012953	0.510	1.8694579	0.5280503
0.020	0.0759132	0.0020544	0.520	1.8643624	0.5466939
0.030	0.0956026	0.0030104	0.530	1.8558930	0.5652529
0.040	0.1174995	0.0041854	0.540	1.8440844	0.5836937
0.050	0.1416265	0.0056017	0.550	1.8289850	0.6019836
0.060	0.1679933	0.0072816	0.560	1.8106564	0.6200901
0.070	0.1965962	0.0092476	0.570	1.7891733	0.6379819
0.080	0.2274177	0.0115217	0.580	1.7646233	0.6556281
0.090	0.2604271	0.0141260	0.590	1.7371062	0.6729991
0.100	0.2955802	0.0170818	0.600	1.7067334	0.6900665
0.110	0.3328195	0.0204100	0.610	1.6736280	0.7068028
0.120	0.3720746	0.0241308	0.620	1.6379232	0.7231820
0.130	0.4132622	0.0282634	0.630	1.5997626	0.7391796
0.140	0.4562867	0.0328262	0.640	1.5592988	0.7547726
0.150	0.5010406	0.0378367	0.650	1.5166933	0.7699395
0.160	0.5474049	0.0433107	0.660	1.4721148	0.7846607
0.170	0.5952500	0.0492632	0.670	1.4257393	0.7989181
0.180	0.6444357	0.0557076	0.680	1.3777487	0.8126956
0.190	0.6948126	0.0626557	0.690	1.3283300	0.8259789
0.200	0.7462224	0.0701179	0.700	1.2776744	0.8387556
0.210	0.7984986	0.0781029	0.710	1.2259763	0.8510154
0.220	0.8514678	0.0866176	0.720	1.1734325	0.8627497
0.230	0.9049501	0.0956671	0.730	1.1202411	0.8739521
0.240	0.9587601	0.1052547	0.740	1.0666003	0.8846181
0.250	1.0127080	0.1153818	0.750	1.0127080	0.8947452
0.260	1.0666003	0.1260478	0.760	0.9587601	0.9043328
0.270	1.1202411	0.1372502	0.770	0.9049501	0.9133823
0.280	1.1734325	0.1489845	0.780	0.8514678	0.9218970
0.290	1.2259763	0.1612443	0.790	0.7984986	0.9298820
0.300	1.2776744	0.1740210	0.800	0.7462224	0.9373442
0.310	1.3283300	0.1873043	0.810	0.6948126	0.9442923
0.320	1.3777487	0.2010818	0.820	0.6444357	0.9507367
0.330	1.4257393	0.2153392	0.830	0.5952500	0.9566892
0.340	1.4721148	0.2300603	0.840	0.5474049	0.9621632
0.350	1.5166933	0.2452273	0.850	0.5010406	0.9671736
0.360	1.5592988	0.2608202	0.860	0.4562867	0.9717365
0.370	1.5997626	0.2768179	0.870	0.4132622	0.9758691
0.380	1.6379232	0.2931971	0.880	0.3720746	0.9795899
0.390	1.6736280	0.3099334	0.890	0.3328195	0.9829181
0.400	1.7067334	0.3270007	0.900	0.2955802	0.9858739
0.410	1.7371062	0.3443718	0.910	0.2604271	0.9884781
0.420	1.7646233	0.3620180	0.920	0.2274177	0.9907523
0.430	1.7891733	0.3799097	0.930	0.1965962	0.9927183
0.440	1.8106564	0.3980163	0.940	0.1679933	0.9946982
0.450	1.8289850	0.4163062	0.950	0.1416265	0.9958145
0.460	1.8440844	0.4347470	0.960	0.1174995	0.9969895
0.470	1.8558930	0.4533059	0.970	0.0956026	0.9979455
0.480	1.8643624	0.4719496	0.980	0.0759132	0.9987046
0.490	1.8695479	0.4906441	0.990	0.0583953	0.9992886
0.500	1.8711587	0.5093557			

Table 1.6 N/T = 7.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0260798	0.0004940	0.510	2.0319314	0.5304902
0.020	0.0370663	0.0008647	0.520	2.0251856	0.5507421
0.030	0.0501608	0.0013663	0.530	2.0139819	0.5708819
0.040	0.0654950	0.0020212	0.540	1.9983789	0.5908657
0.050	0.0831881	0.0028531	0.550	1.9784584	0.6106503
0.060	0.1033448	0.0038865	0.560	1.9543243	0.6301935
0.070	0.1260548	0.0051471	0.570	1.9261022	0.6494546
0.080	0.1513911	0.0066610	0.580	1.8939386	0.6683939
0.090	0.1794089	0.0084551	0.590	1.8579999	0.6869739
0.100	0.2101450	0.0105565	0.600	1.8184712	0.7051587
0.110	0.2436166	0.0129927	0.610	1.7755552	0.7229142
0.120	0.2798211	0.0157909	0.620	1.7294706	0.7402089
0.130	0.3187357	0.0189783	0.630	1.6804509	0.7570134
0.140	0.3603167	0.0225814	0.640	1.6287428	0.7733008
0.150	0.4044995	0.0266264	0.650	1.5746044	0.7890469
0.160	0.4511992	0.0311384	0.660	1.5183036	0.8042299
0.170	0.5003100	0.0361415	0.670	1.4601164	0.8188311
0.180	0.5517063	0.0416586	0.680	1.4003251	0.8328343
0.190	0.6052427	0.0477110	0.690	1.3392162	0.8462265
0.200	0.6607552	0.0543186	0.700	1.2770786	0.8589973
0.210	0.7180615	0.0614992	0.710	1.2142020	0.8711393
0.220	0.7769626	0.0692688	0.720	1.1508746	0.8826481
0.230	0.8372433	0.0776413	0.730	1.0873816	0.8935219
0.240	0.8986742	0.0855280	0.740	1.0240031	0.9037619
0.250	0.9610123	0.0962381	0.750	0.9610123	0.9133720
0.260	1.0240031	0.1064781	0.760	0.8986742	0.9223588
0.270	1.0873816	0.1173520	0.770	0.8372433	0.9307312
0.280	1.1508746	0.1288607	0.780	0.7769626	0.9385008
0.290	1.2142020	0.1410027	0.790	0.7180615	0.9456814
0.300	1.2770785	0.1537735	0.800	0.6607552	0.9522890
0.310	1.3392162	0.1671657	0.810	0.6052427	0.9583414
0.320	1.4003251	0.1957701	0.820	0.5517063	039638585
0.330	1.4601164	0.1957701	0.830	0.5003100	0.9688616
0.340	1.5183036	0.2109531	0.840	0.4511992	0.9733736
0.350	1.5746044	0.2266992	0.850	0.4044995	0.9774186
0.360	1.6287428	0.2429866	0.860	0.3603167	0.9810217
0.370	1.6804509	0.2597911	0.870	0.3187357	0.9842091
0.380	1.7294706	0.2770858	0.880	0.2798211	0.9870073
0.390	1.7755552	0.2948414	0.890	0.2436166	0.9894435
0.400	1.8184712	0.3130261	0.900	0.2101450	0.9915449
0.410	1.8579999	0.3316061	0.910	0.1794089	0.9933390
0.420	1.8939386	0.3505455	0.920	0.1513911	0.9948529
0.430	1.9261022	0.3698065	0.930	0.1260548	0.9961135
0.440	1.9543243	0.3893497	0.940	0.1033448	0.9971469
0.450	1.9784584	0.4091343	0.950	0.0831881	0.9979788
0.460	1.9983789	0.4291181	0.960	0.0654950	0.9986337
0.470	2.0139819	0.4492579	0.970	0.0501608	0.9991354
0.480	2.0251856	0.4695098	0.980	0.0370663	0.9995060
0.490	2.0319314	0.4898291	0.990	0.0260798	0.9997668
0.500	2.0341839	0.5101709			

Table 1.7 N/T = 8.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0113424	0.0001134	0.510	2.1822216	0.5326705
0.020	0.0177278	0.0002907	0.520	2.1736783	0.5544073
0.030	0.0258517	0.0005492	0.530	2.1595003	0.5760023
0.040	0.0359156	0.0009084	0.540	2.1397782	0.5974001
0.050	0.0481165	0.0013895	0.550	2.1146379	0.6185465
0.060	0.0626439	0.0020160	0.560	2.0842394	0.6393889
0.070	0.0796774	0.0028128	0.570	2.0487752	0.6598766
0.080	0.0993838	0.0038066	0.580	2.0084691	0.6799613
0.090	0.1219147	0.0050257	0.590	1.9635743	0.6995971
0.100	0.1474039	0.0064998	0.600	1.9143707	0.7187408
0.110	0.1759653	0.0082594	0.610	1.8611630	0.7373524
0.120	0.2076906	0.0103363	0.620	1.8042781	0.7553952
0.130	0.2426475	0.0127628	0.630	1.7440622	0.7728358
0.140	0.2808780	0.0155716	0.640	1.6808781	0.7896446
0.150	0.3223968	0.0187956	0.650	1.6151019	0.8057956
0.160	0.3671905	0.0224675	0.660	1.5471201	0.8212668
0.170	0.4152164	0.0266196	0.670	1.4773264	0.8360401
0.180	0.4664018	0.0312836	0.680	1.4061183	0.8501013
0.190	0.5206441	0.0364901	0.690	1.3338942	0.8631102
0.200	0.5778105	0.0422682	0.700	1.2610497	0.8760507
0.210	0.6377384	0.0486456	0.710	1.1879751	0.8879305
0.220	0.7002360	0.0556479	0.720	1.1150518	0.8990810
0.230	0.7650831	0.0632988	0.730	1.0426494	0.9095075
0.240	0.8320325	0.0716191	0.740	0.9711233	0.9192187
0.250	0.9008115	0.0806272	0.750	0.9008115	0.9282268
0.260	0.9711233	0.0903384	0.760	0.8320325	0.9365471
0.270	1.0426494	0.1007649	0.770	0.7650831	0.9441980
0.280	1.1150518	0.1119155	0.780	0.7002360	0.9512003
0.290	1.1879751	0.1237952	0.790	0.6377384	0.9575777
0.300	1.2610497	0.1364057	0.800	0.5778105	0.9633558
0.310	1.3338942	0.1497446	0.810	0.5206441	0.9685623
0.320	1.4061183	0.1638058	0.820	0.4664018	0.9732263
0.330	1.477.264	0.1785791	0.830	0.4152164	0.9773784
0.340	1.5471201	0.1940503	0.840	0.3671905	0.9810503
0.350	1.6151019	0.2102013	0.850	0.3223968	0.9842743
0.360	1.6808781	0.2270101	0.860	0.2808780	0.9870831
0.370	1.7440622	0.2444507	0.870	0.2426475	0.9895096
0.380	1.8042781	0.2624935	0.880	0.2076906	0.9915865
0.390	1.8611630	0.2811051	0.890	0.1759653	0.9933461
0.400	1.9143707	0.3002488	0.900	0.1474039	0.9948202
0.410	1.9635743	0.3198846	0.910	0.1219147	0.9960393
0.420	2.0084691	0.3399693	0.920	0.0993838	0.9970332
0.430	2.0487752	0.3604570	0.930	0.0796774	0.9978299
0.440	2.0842394	0.3812994	0.940	0.0626439	0.9984564
0.450	2.1146379	0.4024458	0.950	0.0481165	0.9989375
0.460	2.1397782	0.4238436	0.960	0.0359156	0.9992967
0.470	2.1595003	0.4454386	0.970	0.0258517	0.9995552
0.480	2.1736783	0.4671754	0.980	0.0177278	0.9997325
0.490	2.1822216	0.4889976	0.990	0.0113424	0.998459
0.500	2.1850755	0.5108483			

Table 1.8 N/T = 9.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0048428	0.0000484	0.510	2.3226967	0.5648324
0.020	0.0083584	0.0001320	0.520	2.3122204	0.5579546
0.030	0.0131576	0.0002636	0.530	2.2948480	0.5809030
0.040	0.0194682	0.0004583	0.540	2.2707109	0.6036101
0.050	0.0273256	0.0007335	0.550	2.2399910	0.6260101
0.060	0.0375700	0.0011092	0.560	2.2029187	0.6480392
0.070	0.0498423	0.0016076	0.570	2.1597711	0.6696370
0.080	0.0645811	0.0022535	0.580	2.1108684	0.6907456
0.090	0.0820184	0.0030736	0.590	2.0565712	0.7113113
0.100	0.1023759	0.0040974	0.600	1.9972763	0.7312841
0.110	0.1258613	0.0053560	0.610	1.9334132	0.7506182
0.120	0.1526641	0.0068827	0.620	1.8654389	0.7692726
0.130	0.1829523	0.0087122	0.630	1.7938344	0.7892110
0.140	0.2168686	0.0108809	0.640	1.7190986	0.8044020
0.150	0.2545274	0.0134261	0.650	1.6417446	0.8208194
0.160	0.2960111	0.0163862	0.660	1.5622934	0.8364426
0.170	0.3413681	0.0197999	0.670	1.4812698	0.8512550
0.180	0.3906101	0.0237060	0.680	1.3991969	0.8652470
0.190	0.4437102	0.0281431	0.690	1.3165909	0.8784129
0.200	0.5006012	0.0331491	0.700	1.2339568	0.8907525
0.210	0.5611750	0.0387609	0.710	1.1517834	0.9022703
0.220	0.6252814	0.0450137	0.720	1.0705390	0.9129757
0.230	0.6927288	0.0519410	0.730	0.9906674	0.9228824
0.240	0.7632839	0.0595738	0.740	0.9125842	0.9320082
0.250	0.8366732	0.0679406	0.750	0.8366732	0.9403750
0.260	0.9125842	0.0770664	0.760	0.7632839	0.9480078
0.270	0.9906674	0.0869731	0.770	0.6927288	0.9549351
0.280	1.0705390	0.0976785	0.780	0.6252814	0.9611879
0.290	1.1517834	0.1090963	0.790	0.5611750	0.9667997
0.300	1.2339568	0.1215359	0.800	0.5006012	0.9718057
0.310	1.3165909	0.1347018	0.810	0.4437102	0.9762428
0.320	1.3991969	0.1486938	0.820	0.3906101	0.9801489
0.330	1.4812698	0.1635065	0.830	0.3413681	0.9835625
0.340	1.5622934	0.1791294	0.840	0.2960111	0.9865227
0.350	1.6417446	0.1955468	0.850	0.2545274	0.9890679
0.360	1.7190986	0.2127378	0.860	0.2168686	0.9912366
0.370	1.1938344	0.2306762	0.870	0.1829523	0.9930661
0.380	1.8654389	0.2493306	0.880	0.1526641	0.9945928
0.390	1.9334132	0.2686647	0.890	0.1258613	0.9958514
0.400	1.9972763	0.2886374	0.900	0.1023759	0.9968752
0.410	2.0565712	0.3092032	0.910	0.0820184	0.9976953
0.420	2.1108684	0.3303118	0.920	0.0645811	0.9983411
0.430	2.1597711	0.3519096	0.930	0.0498423	0.9988396
0.440	2.2029187	0.3739387	0.940	0.0375700	0.9992153
0.450	2.2399910	0.3963386	0.950	0.0275256	0.9994905
0.460	2.2707109	0.4190458	0.960	0.0194682	0.9996852
0.470	2.2948480	0.4419942	0.970	0.0131576	0.9998168
0.480	2.3122204	0.4651164	0.980	0.0083584	0.9999004
0.490	2.3226967	0.4883434	0.990	0.0048428	0.9999488
0.500	2.3261976	0.5116054			

Table 1.9 N/T = 10.00

x	Phi	Cum Prob.
0.010	0.0020404	0.0000204
0.020	0.0039002	0.0000594
0.030	0.0066352	0.0001258
0.040	0.0104618	0.0002304
0.050	0.0156161	0.0003865
0.060	0.0223509	0.0006100
0.070	0.0309334	0.0009194
0.080	0.0416406	0.0013358
0.090	0.0547561	0.0018833
0.100	0.0705649	0.0025890
0.110	0.0893490	0.0034825
0.120	0.1113820	0.0045963
0.130	0.1369242	0.0059655
0.140	0.1662169	0.0076277
0.150	0.1994777	0.0096225
0.160	0.2368948	0.0119914
0.170	0.2786223	0.0147777
0.180	0.3247754	0.0180254
0.190	0.3754264	0.0217797
0.200	0.4306012	0.0260857
0.210	0.4902749	0.0309884
0.220	0.5543709	0.0365322
0.230	0.6227576	0.0427597
0.240	0.6952481	0.0497122
0.250	0.7715995	0.0574282
0.260	0.8515132	0.0659433
0.270	0.9346362	0.0752897
0.280	1.0205633	0.0854953
0.290	1.1088396	0.0965837
0.300	1.1989644	0.1085734
0.310	1.2803951	0.1214773
0.320	1.3825527	0.1353029
0.330	1.4748268	0.1500511
0.340	1.5665825	0.1657169
0.350	1.6571662	0.1822886
0.360	1.7469134	0.1997477
0.370	1.8321555	0.2180693
0.380	1.9152272	0.2372216
0.390	1.9944746	0.2571663
0.400	2.0692624	0.2778589
0.410	2.1389811	0.2992487
0.420	2.2030547	0.3212793
0.430	2.2609473	0.3438888
0.440	2.3121694	0.3670105
0.450	2.3562840	0.3905733
0.460	2.3929121	0.4145024
0.470	2.4217367	0.4387198
0.490	2.4550428	0.4876953
0.500	2.4592335	0.5122876
0.510	2.4550428	0.5368381
0.520	2.4425073	0.5612631
0.530	2.4217367	0.5854805
0.540	2.3929121	0.6094096
0.550	2.3562840	0.6329725
0.560	2.3121694	0.6560941
0.570	2.2609473	0.6787036
0.580	2.2030547	0.7007342
0.590	2.1389811	0.7221240
0.600	2.0692624	0.7428166
0.610	1.9941746	0.7627613
0.620	1.9152272	0.7819136
0.630	1.8321555	0.8002352
0.640	1.7459134	0.8176943
0.650	1.6571662	0.8342660
0.660	1.5665825	0.8499318
0.670	1.4748268	0.8646801
0.680	1.3825527	0.8785056
0.690	1.2903951	0.8914095
0.700	1.1989644	0.9033992
0.710	1.1088396	0.9144876
0.720	1.0205633	0.9246932
0.730	0.9346362	0.9340396
0.740	0.8515132	0.9425547
0.750	0.7715995	0.9502707
0.760	0.6952481	0.9572232
0.770	0.6227576	0.9634508
0.780	0.5543709	0.9689945
0.790	0.4902749	0.9738972
0.800	0.4306012	0.9782032
0.810	0.3754265	0.9819575
0.820	0.3247754	0.9852053
0.830	0.2786223	0.9879915
0.840	0.2368948	0.9903604
0.850	0.1994777	0.9923552
0.860	0.1662169	0.9940174
0.870	0.1369242	0.9953866
0.880	0.1113820	0.9965004
0.890	0.0893490	0.9973939
0.900	0.0705649	0.9980996
0.910	0.0547561	0.9986471
0.920	0.0416406	0.9990635
0.930	0.0309334	0.9993729
0.940	0.0223509	0.9995964
0.950	0.0156161	0.9997525
0.960	0.0104618	0.9998572
0.970	0.0066352	0.9999235
0.990	0.0020404	0.9999829

Table 1.10 N/T = 11.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0008511	0.0000085	0.510	2.5805116	0.5387294
0.020	0.0018056	0.0000266	0.520	2.5657984	0.5643874
0.030	0.0033224	0.0000598	0.530	2.5414387	0.5898018
0.040	0.0055843	0.0001156	0.540	2.5076735	0.6148785
0.050	0.0088021	0.0002037	0.550	2.4648352	0.6395269
0.060	0.0132129	0.0003358	0.560	2.4133432	0.6636603
0.070	0.0190789	0.0005266	0.570	2.3536979	0.6871973
0.080	0.0266848	0.0007934	0.580	2.2864734	0.7100620
0.090	0.0363346	0.0011568	0.590	2.2123098	0.7321851
0.100	0.0483471	0.0016402	0.600	2.1319042	0.7535042
0.110	0.0630520	0.0022708	0.610	2.0460012	0.7719642
0.120	0.0807835	0.0030786	0.620	1.9553827	0.7935180
0.130	0.1018748	0.0040973	0.630	1.8608571	0.8121266
0.140	0.1266519	0.0053639	0.640	1.7632493	0.8297591
0.150	0.1554262	0.0069181	0.650	1.6633893	0.8463930
0.160	0.1884878	0.0088030	0.660	1.5621018	0.8620140
0.170	0.2260988	0.0110640	0.670	1.4601960	0.8766157
0.180	0.2684653	0.0137488	03680	1.3584889	0.8902005
0.190	0.3058318	0.0169072	0.690	1.2576306	0.9027768
0.200	0.3682737	0.0205899	0.700	1.1584264	0.9143611
0.210	0.4258921	0.0248488	0.710	1.0614989	0.9249761
0.220	0.4887080	0.0297359	0.720	0.9674468	0.9434186
0.230	0.5566782	0.0353027	0.730	0.8768061	0.9434184
0.240	0.6296913	0.0414996	0.740	0.7900460	0.9513190
0.250	0.7075653	0.0486752	0.750	0.7075653	0.9586947
0.260	0.7900460	0.0565757	0.760	0.6293913	0.9646916
0.270	0.8768061	0.0653438	0.770	0.5566782	0.9702584
0.280	0.9674468	0.0750182	0.780	0.4887080	0.9751455
0.290	1.0614989	0.0856332	0.790	0.4258921	0.9794044
0.300	1.1584264	0.0972175	0.800	0.3682737	0.9830871
0.310	1.2576306	0.1097938	0.810	0.3158318	0.9862454
0.320	1.3584559	0.1233787	0.820	0.2684853	0.9889303
0.330	1.4601960	0.1379809	0.830	0.3360988	0.9911913
0.340	1.5621018	0.1536013	0.840	0.1884878	0.9930762
0.350	1.6633893	0.1702352	0.850	0.1554262	0.9946304
0.360	1.7632493	0.1878677	0.860	0.1266519	0.9969157
0.370	1.8608571	0.2064763	0.870	0.1018748	0.9969157
0.380	1.9553827	0.2260301	0.880	0.0807838	0.9977235
0.390	2.0460012	0.2464901	0.890	0.0630520	0.9983540
0.400	2.1319042	0.2678092	0.900	0.0483471	0.9988375
0.410	2.2123098	0.2899323	0.910	0.0363346	0.9992009
0.420	2.2864734	0.3127970	0.920	0.0266848	0.9994677
0.430	2.3536979	0.3363340	0.930	0.0190789	0.9996585
0.440	2.4133432	0.3604674	0.940	0.0132129	0.9997906
0.450	2.4648352	0.3851158	0.950	0.0088021	0.9998787
0.460	2.5076735	0.4101925	0.960	0.0055843	0.9999345
0.470	2.5414387	0.4656069	0.970	0.0033224	0.9999677
0.480	2.5657984	0.4612649	0.980	0.0018056	0.9999858
0.490	2.5805116	0.4870700	0.990	0.0008511	0.9999843
0.500	2.5854322	0.5129243			

Table 1.11 N/T = 12.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0003523	0.0000035	0.510	2.7000622	0.5405284
0.020	0.0008308	0.0000118	0.520	2.6830594	0.5673590
0.030	0.0016542	0.0000284	0.530	2.6549312	0.5939083
0.040	0.0029648	0.0000580	0.540	2.6159887	0.6200682
0.050	0.0049355	0.0001074	0.550	2.5666604	0.6457348
0.060	0.0077711	0.0001851	0.560	2.5074851	0.6708097
0.070	0.0117083	0.0003022	0.570	2.4391039	0.6952007
0.080	0.0170159	0.0004723	0.580	2.3622491	0.7188232
0.090	0.0239923	0.0007123	0.590	2.2777336	0.7416005
0.100	0.0329637	0.0010419	0.600	2.1864373	0.7634649
0.110	0.0442798	0.0014847	0.610	2.0892942	0.7843578
0.120	0.0583097	0.0020678	0.620	1.9872777	0.8042606
0.130	0.0754355	0.0028221	0.630	1.8818364	0.8230445
0.140	0.0960461	0.0037826	0.640	1.7726288	0.8407708
0.150	0.1205297	0.0049879	0.650	1.6620094	0.8573909
0.160	0.1492653	0.0064806	0.660	1.5505141	0.8728960
0.170	0.1826145	0.0083067	0.670	1.4390971	0.8872870
0.180	0.2209118	0.0105158	0.680	1.3286681	0.9005737
0.190	0.2644558	0.0131604	0.690	1.2200812	0.9127745
0.200	0.3135000	0.0162954	0.700	1.1141249	0.9239157
0.210	0.3682437	0.0199778	0.710	1.0115131	0.9340309
0.220	0.4288234	0.0242660	0.720	0.9128787	0.9431596
0.230	0.4953056	0.0292191	0.730	0.8187680	0.9513473
0.240	0.5676791	0.0348959	0.740	0.7296371	0.9586437
0.250	0.6458501	0.0413544	0.750	0.6458501	0.9641022
0.260	0.7296371	0.0486508	0.760	0.5676791	0.9707790
0.270	0.8187680	0.0568384	0.770	0.4953056	0.9757320
0.280	0.9128787	0.0659672	0.780	0.4288234	0.9800203
0.290	1.0115131	0.0760824	0.790	0.3682437	0.9837027
0.300	1.1141249	0.0872236	0.800	0.3135000	0.9868377
0.310	1.2200812	0.0994244	0.810	0.2644558	0.9894823
0.320	1.3286681	0.1127111	0.820	0.2209118	0.9916914
0.330	1.4390971	0.1271021	0.830	0.1826145	0.9935175
0.340	1.5505141	0.1426072	0.840	0.1492653	0.9950102
0.350	1.6620095	0.1592273	0.850	0.1205297	0.9962155
0.360	1.7726288	0.1769536	0.860	0.0960461	0.9971759
0.370	1.8813864	0.1957675	0.870	0.0754355	0.9979303
0.380	1.9872777	0.2156402	0.880	0.0583097	0.9985134
0.390	2.0892942	0.2365332	0.890	0.0442798	0.9989562
0.400	2.1864373	0.2583976	0.900	0.9329637	0.9992858
0.410	2.2777336	0.2811749	0.910	0.0129923	0.9995258
0.420	2.3622491	0.3047974	0.920	0.0170159	0.9996959
0.430	2.4391039	0.3291884	0.930	0.0117083	0.9998130
0.440	2.5074841	0.3542633	0.940	0.0077711	0.9998907
0.450	2.5666604	0.3799299	0.950	0.0049355	0.9999401
0.460	2.6159887	0.4060898	0.960	0.0029648	0.9999697
0.470	2.6549312	0.4326391	0.970	0.0016452	0.9999863
0.480	2.6830594	0.4594697	0.980	0.0008308	039999946
0.490	2.7000622	0.4864703	0.990	0.0003523	0.9999981
0.500	2.7057508	0.5135278			

Table 1.12 N/T = 13.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0001450	0.0000014	0.510	2.8144491	0.5422489
0.020	0.0003804	0.0000053	0.520	2.4950503	0.5701994
0.030	0.0008199	0.0000135	0.530	2.7629838	0.5978292
0.040	0.0015672	0.0000291	0.540	2.7186417	0.6250156
0.050	0.0027556	0.0000567	0.550	2.6625629	0.6516413
0.060	0.0045513	0.0001022	0.560	2.5954237	0.6775955
0.070	0.0071554	0.0001737	0.570	2.5180254	0.7027758
0.080	0.0108059	0.0002818	0.580	2.4312803	0.7270886
0.090	0.0157782	0.0004396	0.590	2.3361954	0.7504505
0.100	0.0223845	0.0006634	0.600	2.2338551	0.7717891
0.110	0.0309722	0.0009732	0.610	2.1254029	0.7940431
0.120	0.0419206	0.0013924	0.620	2.0120214	0.8141633
0.130	0.0556367	0.0019487	0.630	1.8949134	0.8331124
0.140	0.0725491	0.0026742	0.640	1.7752822	0.8508653
0.150	0.0931011	0.0036052	0.650	1.6543127	0.8674084
0.160	0.1177419	0.0047826	0.660	1.5331533	0.8827399
0.170	0.1469176	0.0062518	0.670	1.4128996	0.8968689
0.180	0.1810603	0.0080624	0.680	1.2845788	0.9098147
0.190	0.2205772	0.0102682	0.690	1.1791363	0.9216061
0.200	0.2658389	0.0129266	0.700	1.0674251	0.9322803
0.210	0.3171674	0.0160983	0.710	0.9601962	0.9418823
0.220	0.3748247	0.0198465	0.720	0.8580919	0.9504632
0.230	0.4390012	0.0242365	0.730	0.7316422	0.9580796
0.240	0.5098052	0.0293346	0.740	0.6712623	0.9647923
0.250	0.5872534	0.0352071	0.750	0.5872534	0.9706648
0.260	0.6712623	0.0419197	0.760	0.5098052	0.9757628
0.270	0.7616422	0.0495362	0.770	0.4390012	0.9801528
0.280	0.8580919	0.0581171	0.780	0.3748247	0.9839011
0.290	0.9601962	0.0677190	0.790	0.3171674	0.9870728
0.300	1.0674251	0.0783933	0.800	0.2658358	0.9897312
0.310	1.1791363	0.0901846	0.810	0.2205772	0.9919369
0.320	1.2945788	0.1031304	0.820	0.1810603	0.9937475
0.330	1.4128996	0.1172594	0.830	0.1469176	0.9952167
0.340	1.5331533	0.1325910	0.840	0.1177419	0.9963941
0.350	1.6543127	0.1491341	0.850	0.0931011	0.9973251
0.360	1.7752822	0.1668869	0.860	0.0725491	0.9980506
0.370	1.8949134	0.1858360	0.860	0.0725491	0.9980506
0.380	2.0120214	0.2059563	0.880	0.0419206	0.9990262
0.390	2.1256029	0.2272103	0.890	0.0309722	0.9993359
0.400	2.2338551	0.2495488	0.900	0.0223845	0.9995598
0.410	2.3361954	0.2729108	0.910	0.0157782	0.9997176
0.420	2.4312803	0.2972236	0.920	0.0108059	0.9998256
0.430	2.5180254	0.3224039	0.930	0.0071554	0.9998972
0.440	2.5954237	0.3483581	0.940	0.0045513	0.9999427
0.450	2.6625629	0.3749837	0.950	0.0027556	0.9999702
0.460	2.7186417	0.4021701	0.960	0.0015672	0.9999859
0.470	2.7629838	0.4398000	0.970	0.0008199	0.9999941
0.480	2.7950503	0.4577505	0.980	0.0003804	0.9999979
0.490	2.8144491	0.4858950	0.990	0.0001450	0.9999994
0.500	2.8209419	0.5141044			

Table 1.13 N/T = 14.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0000594	0.0000006	0.510	2.9242784	0.5439007
0.020	0.0001735	0.0000023	0.520	2.9023822	0.5729245
0.030	0.0004048	0.0000064	0.530	2.8662160	0.6015867
0.040	0.0008253	0.0000146	0.540	2.8162644	0.6297494
0.050	0.0015330	0.0000300	0.550	2.7531919	0.6572813
0.060	0.0026562	0.0000565	0.560	2.6778303	0.6840596
0.070	0.0043577	0.0001001	0.570	2.5911616	0.7099712
0.080	0.0068387	0.0001685	0.580	2.4942994	0.7349142
0.090	0.0103409	0.0002719	0.590	2.3884667	0.7587989
0.100	0.0151491	0.0004234	0.600	2.2749735	0.7815486
0.110	0.0215910	0.0006393	0.610	2.1551911	0.8031005
0.120	0.0300370	0.0009397	0.620	2.0305276	0.8234058
0.130	0.0408973	0.0013486	0.630	1.9024023	0.8424298
0.140	0.0546183	0.0019848	0.640	1.7722212	0.8601520
0.150	0.0716760	0.0026116	0.650	1.6413525	0.8765655
0.160	0.0925691	0.0035373	0.660	1.5111056	0.8916766
0.170	0.1178092	0.0047154	0.670	1.3827094	0.9055034
0.180	0.1479101	0.0061945	0.680	1.2572970	0.9180767
0.190	0.1833757	0.0080282	0.690	1.1358884	0.9294355
0.200	0.2246859	0.0102751	0.700	1.0193802	0.9396293
0.210	0.2722833	0.0129979	0.710	0.9085365	0.9487147
0.220	0.3265574	0.0162635	0.720	0.8039835	0.9567645
0.230	0.3878306	0.0201418	0.730	0.7062074	0.9638166
0.240	0.4563432	0.0247052	0.740	0.6155554	0.9699722
0.250	0.5322394	0.0300276	0.750	0.5322394	0.9752946
0.260	0.6155554	0.0361832	0.760	0.4563432	0.9798580
0.270	0.7062074	0.0432452	0.770	0.3878306	0.9837363
0.280	0.8039835	0.0512851	0.780	0.3265574	0.9870019
0.290	0.9085365	0.0603704	0.790	0.2722833	0.9897247
0.300	1.0193802	0.0705642	0.800	0.2246859	0.9919716
0.310	1.1358884	0.0819231	0.810	0.1833757	0.9938053
0.320	1.2572970	0.0944961	0.820	0.1479101	0.9952844
0.330	1.3827094	0.1083232	0.830	0.1178092	0.9964625
0.340	1.5111054	0.1234343	0.840	0.0925691	0.9973882
0.350	1.6413525	0.1398478	0.850	0.0716760	0.9981050
0.360	1.7722212	0.1575700	0.860	0.0546183	0.9986411
0.370	1.9024023	0.1765940	0.870	0.0408973	0.9990607
0.380	2.0305276	0.1968993	0.880	0.0300370	0.9993605
0.390	2.1551911	0.2184512	0.890	0.0215910	0.9994564
0.400	2.2749735	0.2412009	0.900	0.0151491	0.9997279
0.410	2.3884667	0.2650856	0.910	0.0103406	0.9998313
0.420	2.4942994	0.2900286	0.920	0.0068267	0.9998997
0.440	2.6778303	0.3427185	0.940	0.0026562	0.9999698
0.450	2.7531919	0.3702504	0.950	0.0015330	0.9999852
0.460	2.8162644	0.3984131	0.960	0.0008253	0.9999934
0.470	2.8662160	0.4270752	0.970	0.0004048	039999975
0.480	2.9023822	0.4560991	0.980	0.0001735	0.9999992
0.490	2.9242784	0.4853418	0.990	0.0000594	0.9999998
0.500	2.9316101	0.5146579			

Table 1.14 N/T = 15.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0000242	0.0000002	0.510	3.0300460	0.5454917
0.020	0.0000788	0.0000010	0.520	3.0055550	0.5755472
0.030	0.0001992	0.0000030	0.530	2.9651353	0.6051986
0.040	0.0004333	0.0000074	0.540	2.9093755	0.6342923
0.050	0.0008502	0.0000159	0.550	2.8390814	0.6626831
0.060	0.0015455	0.0000313	0.560	2.7552586	0.6902357
0.070	0.0026461	0.0000578	0.570	2.6590909	0.7168266
0.080	0.0043153	0.0001009	0.580	2.5519145	0.7423458
0.090	0.0067577	0.0001685	0.590	2.4351904	0.7666977
0.100	0.0102227	0.0002707	0.600	2.3104734	0.7898024
0.110	0.0150080	0.0004208	0.610	2.1793811	0.8115962
0.120	0.0214605	0.0006354	0.620	2.0435611	0.8320318
0.130	0.0299770	0.0009352	0.630	1.9046593	0.8510784
0.140	0.0410023	0.0013452	0.640	1.7642894	0.8687213
0.150	0.0550251	0.0018955	0.650	1.6240033	0.8849614
0.160	0.0725725	0.0026212	0.660	1.4852654	0.8998140
0.170	0.0942018	0.0035632	0.670	1.3494287	0.9133083
0.180	0.1204897	0.0047681	0.680	1.2177156	0.9254855
0.190	0.1520207	0.0062883	0.690	1.0912018	0.9363975
0.200	0.1893719	0.0081820	0.700	0.9708046	0.9461055
0.210	0.2330975	0.0105130	0.710	0.8572760	0.9546783
0.220	0.2837115	0.0133501	0.720	0.7511994	0.9621903
0.230	0.3416696	0.0167668	0.730	0.6529909	0.9687202
0.240	0.4073506	0.0208403	0.740	0.5629040	0.9743492
0.250	0.4810382	0.0256507	0.750	0.4810382	0.9791596
0.260	0.5629040	0.0312797	0.760	0.4073506	0.9832331
0.270	0.6529909	0.0378096	0.770	0.3416696	0.9866498
0.280	0.7511994	0.0453216	0.780	0.2837115	0.9894869
0.290	0.8572760	0.0538944	0.790	0.2330975	0.9918179
0.300	0.9708046	0.0636024	0.800	0.1893719	0.9937116
0.310	1.0912018	0.0745145	0.810	0.1520207	0.9952318
0.320	1.2177156	0.0866916	0.820	0.1204897	0.9964367
0.330	1.3494287	0.1001859	0.830	0.0942018	0.9973787
0.340	1.4852654	0.1150386	0.840	0.0725725	0.9981045
0.350	1.6240033	0.1312786	0.850	0.0550251	0.9986547
0.360	1.7642894	0.1489215	0.860	0.0410023	0.9990647
0.370	1.9046593	0.1679681	0.870	0.0299770	0.9993645
0.380	2.0435611	0.1884037	0.880	0.0214605	0.9995791
0.390	2.1793811	0.2101975	0.890	0.0150080	0.9997292
0.400	2.3104734	0.2333022	0.900	0.0102227	0.9998314
0.410	2.4351904	0.2576541	0.910	0.0067577	0.9998990
0.420	2.5519145	0.2831733	0.920	0.0043153	0.9999422
0.430	2.6590909	0.3097642	0.930	0.0026461	0.9999686
0.440	2.7552586	0.3373168	0.940	0.0015455	0.9999841
0.450	2.8390814	0.3657076	0.950	0.0008503	0.9999926
0.460	2.9093755	0.3948014	0.960	0.0004333	0.9999969
0.470	2.9641353	0.4244527	0.970	0.0001992	0.9999989
0.480	3.0055550	0.4545083	0.980	0.0000789	0.9999997
0.490	3.0300460	0.4848087	0.990	0.0000242	0.9999999
0.500	3.0382498	0.5151912			

Table 1.15 N/T = 16.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0000098	0.0000001	0.510	3.1321633	0.5470280
0.020	0.0000357	0.0000005	0.520	3.1049843	0.6086795
0.030	0.0000978	0.0000014	0.530	3.0601639	0.6086795
0.040	0.0002269	0.0000037	0.540	2.9984070	0.6386635
0.050	0.0004703	0.0000084	0.550	2.9206769	0.6678703
0.060	0.0008970	0.0000174	0.560	2.8281722	0.6961520
0.070	0.0016026	0.0000334	0.570	2.7222990	0.7233750
0.080	0.0027161	0.0000606	0.580	2.6046382	0.7494214
0.090	0.0044049	0.0001046	0.590	2.4769094	0.7741905
0.100	0.0068811	0.0001734	0.600	2.3409320	0.7975998
0.110	0.0104062	0.0002775	0.610	2.1985860	0.8195857
0.120	0.0152949	0.0004304	0.620	2.0517708	0.8401034
0.130	0.0219185	0.0006496	0.630	1.9023672	0.8591270
0.140	0.0304050	0.0009567	0.640	1.7521988	0.8766490
0.150	0.0421389	0.0013781	0.650	1.6029985	0.8926790
0.160	0.0567566	0.0019456	0.660	1.4563776	0.9072428
0.170	0.0751414	0.0026970	0.670	1.3137993	0.9203808
0.180	0.0979139	0.0036762	0.680	1.1765580	0.9321464
0.190	0.1257211	0.0049334	0.690	1.0457629	0.9426040
0.200	0.1592213	0.0065256	0.700	0.9223281	0.9518273
0.210	0.1990683	0.0085163	0.710	0.8069676	0.9598970
0.220	0.2458917	0.0109752	0.720	0.7001961	0.9668989
0.230	0.3002765	0.0139780	0.730	0.6023340	0.9729223
0.240	0.3627414	0.0176054	0.740	0.5135183	0.9780574
0.250	0.4337160	0.0219425	0.750	0.4331760	0.9823946
0.260	0.5135183	0.0270777	0.760	0.3627414	0.9860220
0.270	0.6023340	0.0331011	0.770	0.3002765	0.9890248
0.280	0.7001961	0.0401030	0.780	0.2458917	0.9914837
0.290	0.8069676	0.0481727	0.790	0.1990683	0.9934744
0.300	0.9223281	0.0573960	0.800	0.1592213	0.9950666
0.310	1.0457629	0.0678536	0.810	0.1257210	0.9963238
0.320	1.1765580	0.0796192	0.820	0.0979139	0.9973029
0.330	1.3137993	0.0927572	0.830	0.0751414	0.9980544
0.340	1.4563776	0.1073210	0.840	0.0567566	0.9986219
0.350	1.6029985	0.1233509	0.850	0.0421389	0.9990433
0.360	1.7521988	0.1408729	0.860	0.0307051	0.9993504
0.370	1.9023672	0.1598966	0.870	0.0219185	0.9995695
0.380	2.0517708	0.1804143	0.880	0.0152949	0.9997225
0.390	2.1985860	0.2024002	0.890	0.0104061	0.9998266
0.400	2.3409320	0.2258095	0.900	0.0068812	0.9998954
0.410	2.4769094	0.2505786	0.910	0.0044050	0.9999394
0.420	2.6046382	0.2766250	0.920	0.0027160	0.9999666
0.430	2.7222990	0.3038480	0.930	0.0016027	0.9999826
0.440	2.8281722	0.3321297	0.940	0.0008970	0.9999916
0.450	2.9206769	0.3613364	0.950	0.0004704	0.9999963
0.460	2.9984070	0.3913205	0.960	0.0002269	0.9999985
0.470	3.0601639	0.4219222	0.970	0.0000978	0.9999995
0.480	3.1049843	0.4529720	0.980	0.0000358	0.9999999
0.490	3.1321933	0.4842936	0.990	0.0000098	1.0000000
0.500	3.1412712	0.5157063			

Table 1.16 N/T = 17.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0000040	0.0000000	0.510	3.2309767	0.5485149
0.020	0.0000161	0.0000002	0.520	3.2010197	0.5805251
0.030	0.0000479	0.0000007	0.530	3.1516574	0.6120416
0.040	0.0001185	0.0000019	0.540	3.0837237	0.6428789
0.050	0.0002596	0.0000045	0.550	2.9983556	0.6728624
0.060	0.0005194	0.0000097	0.560	2.8969642	0.7018324
0.070	0.0009685	0.0000193	0.570	2.7811994	0.7296441
0.080	0.0017056	0.0000364	0.580	2.6529079	0.7561731
0.090	0.0028650	0.0000650	0.590	2.5140886	0.7813140
0.100	0.0046218	0.0001113	0.600	2.3668441	0.8049825
0.110	0.0071997	0.0001833	0.610	2.2133315	0.8271158
0.120	0.0108772	0.0002920	0.620	2.0557132	0.8476729
0.130	0.0159918	0.0004520	0.630	1.8961098	0.8666340
0.140	0.0229554	0.0006814	0.640	1.7365555	0.8839996
0.150	0.0322014	0.0010034	0.650	1.5789584	0.8997891
0.160	0.0442927	0.0014463	0.660	1.4250655	0.9140398
0.170	0.0598099	0.0020444	0.670	1.2764345	0.9268041
0.180	0.0793990	0.0028384	0.680	1.1344117	0.9381483
0.190	0.1037507	0.0038759	0.690	1.0001168	0.9481494
0.200	0.1335876	0.0052118	0.700	0.8744356	0.9568938
0.210	0.1696474	0.0069083	0.710	0.7580183	0.9644740
0.220	0.2126633	0.0090349	0.720	0.6412857	0.9709868
0.230	0.2633416	0.0116683	0.730	0.5544397	0.9765312
0.240	0.3223369	0.0148917	0.740	0.4674804	0.9812060
0.250	0.3902260	0.0187940	0.750	0.3902260	0.9851083
0.260	0.4674804	0.0234688	0.760	0.3223369	0.9883317
0.270	0.5544397	0.0290132	0.770	0.2633416	0.9909651
0.280	0.6512857	0.0355260	0.780	0.2126633	0.9930917
0.290	0.7580183	0.0431062	0.790	0.1696474	0.9947882
0.300	0.8744356	0.0518506	0.800	0.1335876	0.9961241
0.310	1.0001168	0.0618517	0.820	0.1037507	0.9971616
0.310	1.0001168	0.0618517	0.810	0.1037507	0.9971616
0.320	1.1344117	0.0731958	0.820	0.0793989	0.9979556
0.330	1.2764345	0.0859602	0.830	0.0598099	0.9989966
0.340	1.4250655	0.1002108	0.840	0.0442927	0.9989966
0.350	1.5789584	0.1160004	0.850	0.0322014	0.9993186
0.360	1.7365555	0.1333660	0.860	0.0229445	0.9995480
0.370	1.8961098	0.1523271	0.870	0.0159918	0.9997080
0.380	2.0557132	0.1728842	0.870	0.0159918	0.9997080
0.390	2.2133315	0.1950175	0.890	0.0071996	0.9998887
0.400	2.3668441	0.2186860	0.900	0.0046219	0.9999349
0.410	2.5140886	0.2438269	0.910	0.0028651	0.9999636
0.420	2.6529079	0.2703559	0.920	0.0017055	0.9999806
0.430	2.7811994	0.2981679	0.930	0.0009687	0.9999903
0.440	2.8969642	0.3271376	0.940	0.0005195	0.9999955
0.450	2.9983556	0.3571211	0.950	0.0002597	0.9999981
0.460	3.0837237	0.3879584	0.960	0.0001185	0.9999993
0.470	3.1516574	0.4194749	0.970	0.0000480	0.9999998
0.480	3.2010197	0.4514851	0.980	0.0000164	1.0000000
0.490	3.2309767	0.4837949	0.990	0.0000039	1.0000000
0.500	3.2410164	0.5162051			

Table 1.17 N/T = 18.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0000016	0.0000000	0.510	3.3267805	0.5499567
0.020	0.0000073	0.0000001	0.520	3.2939586	0.5828963
0.030	0.0000234	0.0000003	0.530	3.2399189	0.6152955
0.040	0.0000618	0.0000009	0.540	3.0656368	0.6469519
0.050	0.0001430	0.0000024	0.550	3.0724401	0.6776763
0.060	0.0003002	0.0000054	0.560	2.9319722	0.7072960
0.070	0.0005841	0.0000112	0.570	2.8361477	0.7356575
0.080	0.0010692	0.0000219	0.580	2.6971009	0.7626285
0.090	0.0018599	0.0000405	0.590	2.5471300	0.7880998
0.100	0.0030983	0.0000715	0.600	2.3886382	0.8119862
0.110	0.0049718	0.0001212	0.610	2.2240739	0.8342269
0.120	0.0077207	0.0001984	0.620	2.0558699	0.8547856
0.130	0.0116455	0.0003149	0.630	1.8863912	0.8736495
0.140	0.0171130	0.0004860	0.640	1.7178800	0.8908283
0.150	0.0245611	0.0007316	0.650	1.5524112	0.9063524
0.160	0.0345010	0.0010766	0.660	1.3918545	0.9202710
0.170	0.0475173	0.0015518	0.670	1.2378438	0.9326494
0.180	0.0642645	0.0021944	0.680	1.0917556	0.9435670
0.190	0.0854596	0.0030490	0.960	0.9546961	0.9531139
0.210	0.1443051	0.0056108	0.710	0.1707210	0.9684961
0.220	0.1835827	0.0074466	0.720	0.6046703	0.9745428
0.230	0.2305202	0.0097518	0.730	0.5094082	0.9796369
0.240	0.2859007	0.0126108	0.740	0.4247807	0.9838847
0.250	0.3504452	0.0161153	0.750	0.3504452	0.9873892
0.260	0.4247807	0.0203631	0.760	0.2859007	0.9902482
0.270	0.5094082	0.0254572	0.770	0.2305202	0.9925534
0.280	0.6046704	0.0315039	0.780	0.1835827	0.9943892
0.290	0.7107210	0.0386111	0.790	0.1443051	0.9958323
0.300	0.8274974	0.0468861	0.800	0.1118715	0.9969510
0.310	0.9546961	0.0564330	0.810	0.0854596	0.9978056
0.320	1.0917556	0.0673506	0.820	0.0642644	0.9984482
0.330	1.2378438	0.0797290	0.830	0.0475172	0.9989234
0.340	1.3918545	0.0936476	0.840	0.0345010	0.9992684
0.350	1.5524112	0.1091717	0.850	0.0245610	0.9995140
0.360	1.7178800	0.1263505	0.860	0.0171131	0.9996851
0.370	1.8863912	0.1452144	0.870	0.0116455	0.9998016
0.380	2.0558699	0.1647731	0.880	0.0077209	0.9998788
0.390	2.2240735	0.1880138	0.890	0.0049712	0.9999285
0.400	2.3886382	0.2119002	0.900	0.0030968	0.9999595
0.410	2.5471300	0.2373715	0.910	0.0018602	0.9999781
0.420	2.6971009	0.2643425	0.920	0.0010685	0.9999888
0.430	2.8361477	0.2927040	0.930	0.0005845	0.9999946
0.450	3.0724401	0.3530481	0.950	0.0001431	0.9999991
0.460	3.1656368	0.3847045	0.960	0.0000616	0.9999997
0.470	3.2399189	0.4171037	0.970	0.0000239	0.9999999
0.480	3.2939586	0.4500433	0.980	0.0000078	1.0000000
0.490	3.3267805	0.4833111	0.990	0.0000016	1.0000000
0.500	3.3377880	0.5166889			

Table 1.18 N/T = 19.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0000007	0.0000000	0.510	3.4198278	0.5513574
0.020	0.0000033	0.0000000	0.520	3.3840571	0.5851980
0.030	0.0000114	0.0000002	0.530	3.3252092	0.6184501
0.040	0.0000322	0.0000005	0.540	3.2444146	0.6508942
0.050	0.0000786	0.0000013	0.550	3.1432091	0.6823263
0.060	0.0001732	0.0000030	0.560	3.0234883	0.7125612
0.070	0.0003517	0.0000065	0.570	2.8874530	0.7414357
0.080	0.0006690	0.0000132	0.580	2.7375461	0.7688112
0.090	0.0012053	0.0000253	0.590	2.5763849	0.7945780
0.100	0.0020735	0.0000460	0.600	2.4066893	0.8186419
0.110	0.0034275	0.0000803	0.610	2.2312105	0.8409540
0.120	0.0054710	0.0001350	0.620	2.0526610	0.8614807
0.130	0.0084662	0.0002196	0.630	1.8736495	0.8802172
0.140	0.0127422	0.0003471	0.640	1.6966217	0.8971834
0.150	0.0187022	0.0005341	0.650	1.5238093	0.9124215
0.160	0.0268290	0.0008024	0.660	1.3571889	0.9259933
0.170	0.0376883	0.0011793	0.670	1.1984510	0.9379779
0.180	0.0519283	0.0016985	0.680	1.0489793	0.9484677
0.190	0.0702764	0.0024013	0.690	0.9098420	0.9575661
0.200	0.0935302	0.0033366	0.700	0.7817923	0.9653840
0.210	0.1225453	0.0045321	0.710	0.6652795	0.9720368
0.220	0.1582166	0.0061442	0.720	0.5604680	0.9776415
0.230	0.2014561	0.0081588	0.730	0.4672637	0.9823141
0.240	0.2531647	0.0106904	0.740	0.3853454	0.9861676
0.250	0.3142009	0.0138324	0.750	0.3142009	0.9893096
0.260	0.3853454	0.0176859	0.760	0.2531647	0.9918412
0.270	0.4672637	0.0223585	0.770	0.2014561	0.9938558
0.280	0.5604681	0.0279632	0.780	0.1582165	0.9954379
0.290	0.6652795	0.0346160	0.790	0.1225454	0.9966634
0.300	0.7817923	0.0424339	0.800	0.0935303	0.9975987
0.310	0.9098420	0.0515323	0.810	0.0702763	0.9983015
0.320	1.0498793	0.0620221	0.820	0.0519280	0.9988207
0.330	1.1984510	0.0740067	0.830	0.0376881	0.9991976
0.340	1.3571889	0.0875785	0.840	0.0268292	0.9994659
0.350	1.5238093	0.1028166	0.850	0.0187022	0.9996529
0.360	1.6966217	0.1197828	0.860	0.0127424	0.9997804
0.370	1.8736495	0.1385193	0.870	0.0084660	0.9998650
0.380	2.0526610	0.1590460	0.880	0.0054713	0.9999197
0.390	2.2312105	0.1813581	0.890	0.0034257	0.9999540
0.400	2.4066893	0.2054250	0.900	0.0020741	0.9999747
0.410	2.5763849	0.2311888	0.910	0.0012063	0.9999868
0.420	2.7375461	0.2585643	0.930	0.0006671	0.9999935
0.430	2.8874530	0.2874388	0.930	0.0003525	0.9999970
0.440	3.0234883	0.3176737	0.940	0.0001738	0.9999987
0.450	3.1432091	0.3491058	0.950	0.0000788	0.9999995
0.460	3.2444146	0.3815499	0.960	0.0000313	0.9999998
0.470	3.3252092	0.4148020	0.970	0.0000131	1.0000000
0.480	3.3840571	0.4486426	0.980	0.0000042	1.0000000
0.490	3.4198278	0.4828409	0.990	0.0000012	1.0000000
0.500	3.4318290	0.4171591			

Table 1.19 N/T = 20.00

x	Phi	Cum Prob.	x	Phi	Cum Prob.
0.010	0.0000003	0.0000000	0.510	3.5103378	0.5527202
0.020	0.0000015	0.0000000	0.520	3.4715369	0.5874356
0.030	0.0000056	0.0000001	0.530	3.4077548	0.6215131
0.040	0.0000167	0.0000002	0.540	3.3202906	0.6547160
0.050	0.0000432	0.0000007	0.550	3.2109056	0.6868251
0.060	0.0000997	0.0000017	0.560	3.0817680	0.7176427
0.070	0.0002114	0.0000038	0.570	2.9353862	0.7469966
0.080	0.0004180	0.0000080	0.580	2.7745326	0.7747419
0.090	0.0007799	0.0000158	0.590	2.6021624	0.8007636
0.100	0.0013855	0.0000296	0.600	2.4213276	0.8249768
0.110	0.0023593	0.0000532	0.610	2.2350934	0.8473278
0.120	0.0038710	0.0000919	0.620	2.0464559	0.8677923
0.130	0.0061457	0.0001534	0.630	1.8582678	0.8863750
0.140	0.0094736	0.0002481	0.640	1.6731710	0.9031067
0.150	0.0142196	0.0003903	0.650	1.4935417	0.9180421
0.160	0.0208319	0.0005986	0.660	1.3214460	0.9312566
0.170	0.0298479	0.0008971	0.670	1.1586095	0.9428427
0.180	0.0418979	0.0013161	0.680	1.0063998	0.9529067
0.190	0.0577049	0.0018931	0.690	0.8658219	0.9615649
0.200	0.0780801	0.0026739	0.700	0.7375263	0.9689402
0.210	0.1039125	0.0037131	0.710	0.6218277	0.9751584
0.220	0.1361538	0.0050746	0.720	0.5187327	0.9803458
0.230	0.1757963	0.0068326	0.730	0.4279749	0.9846255
0.240	0.2238462	0.0090710	0.740	0.3490562	0.9881161
0.250	0.2812897	0.0118839	0.750	0.2812897	0.9909290
0.260	0.3490560	0.0153745	0.760	0.2238462	0.9931674
0.270	0.4279749	0.0196542	0.770	0.1757963	0.9949254
0.280	0.5187327	0.0248416	0.780	0.1361536	0.9962869
0.290	0.6218279	0.0310598	0.790	0.1039128	0.9973261
0.300	0.7375264	0.0384351	0.800	0.0780804	0.9981069
0.310	0.8658219	0.0470933	0.810	0.0577047	0.9986839
0.320	1.0063998	0.0571573	0.820	0.0418970	0.9991029
0.330	1.1586095	0.0687434	0.830	0.0298475	0.9994014
0.340	1.3214460	0.0819579	0.840	0.0208326	0.9996097
0.350	1.4935417	0.0968933	0.850	0.0142195	0.9997519
0.360	1.6731710	0.1136250	0.860	0.0094742	0.9998466
0.370	1.8582678	0.1322077	0.870	0.0061779	0.9999081
0.380	2.0464559	0.1526722	0.880	0.0038717	0.9999468
0.390	2.2350931	0.1750232	0.890	0.0023544	0.9999703
0.400	2.4213276	0.1992364	0.900	0.0013870	0.9999842
0.410	2.6021624	0.2252581	0.910	0.0007828	0.9999920
0.420	2.7745326	0.2530034	0.920	0.0004131	0.9999962
0.430	2.9353862	0.2823573	0.930	0.0002129	0.9999983
0.440	3.0817680	0.3131749	0.940	0.0001006	0.9999993
0.450	3.2109056	0.3452840	0.950	0.0000432	0.9999997
0.460	3.3202906	0.3784869	0.960	0.0000140	0.9999999
0.470	3.4077548	0.4125644	0.970	0.0000101	1.0000000
0.480	3.4715369	0.4472798	0.980	0.0000025	1.0000000
0.490	3.5103378	0.4823832	0.990	0.0000028	1.0000000
0.500	3.5233608	0.5176168			

Table 2.1. Critical values for rejection at different α and N/t levels. N = population size, t = generation time and α = size of the rejection region.

N/T	α .001	.005	.01	.025	.05	.075	.1
4	--------	--------	.977817	.934109	.883881	.845487	.81324
5	--------	.969745	.944840	.8972239	.845776	.810213	.774497
6	.985058	.944249	.916692	.867896	.818856	.783886	.755480
7	.9673011	.9211669	.892648	.844020	.796535	.763173	.73632
8	.951739	.901475	.872361	.824272	.778286	.746418	.720881
9	.93427	.883235	.854298	.807197	.762876	.732320	.708029
10	.918474	.867176	.838479	.792561	.749649	.720328	.697165
11	.904472	.852918	.824731	.779702	.738330	.710025	.687792
12	.891329	.839932	.812344	.768522	.728354	.701072	.679568
13	.879375	.828525	.801219	.758504	.719460	.693180	.672419
14	.868530	.818077	.791225	.749447	.711599	.686095	.666020
15	.858421	.808475	.782201	.741353	.704543	.679601	.660138
16	.848972	.799582	.773966	.739046	.698019	.673926	.655021
17	.840106	.791856	.766335	.727238	.692116	.668587	.650148
18	.832221	.784233	.759132	.720897	.686738	.663820	.645908
19	.824757	.777232	.752227	.715287	.681684	.659268	.641848
20	.817545	.776548	.746697	.709745	.677112	.655265	.638143

Tables 3.2 - 3.16. Simulation results for N = 40, t = 5 [N/t = 8] and different w levels. Results include frequencies (or counts) and cumulative frequencies for all possible outcomes, i(i = 0,1,2...40) based on 10,000 replications.

Table 3.1

time = 5 N = 40 w = 1.050

index= 0	count= 2	cum= 0.000
index= 1	count= 5	cum= 0.001
index= 2	count= 6	cum= 0.001
index= 3	count= 13	cum= 0.003
index= 4	count= 17	cum= 0.004
index= 5	count= 32	cum= 0.007
index= 6	count= 44	cum= 0.012
index= 7	count= 59	cum= 0.018
index= 8	count= 93	cum= 0.027
index= 9	count= 97	cum= 0.037
index= 10	count= 140	cum= 0.051
index= 11	count= 164	cum= 0.067
index= 12	count= 227	cum= 0.090
index= 13	count= 270	cum= 0.117
index= 14	count= 317	cum= 0.149
index= 15	count= 356	cum= 0.184
index= 16	count= 385	cum= 0.223
index= 17	count= 423	cum= 0.265
index= 18	count= 474	cum= 0.312
index= 19	count= 531	cum= 0.365
index= 20	count= 547	cum= 0.420
index= 21	count= 558	cum= 0.476
index= 22	count= 515	cum= 0.527
index= 23	count= 558	cum= 0.583
index= 24	count= 517	cum= 0.635
index= 25	count= 562	cum= 0.691
index= 26	count= 466	cum= 0.738
index= 27	count= 448	cum= 0.783
index= 28	count= 425	cum= 0.825
index= 29	count= 337	cum= 0.859
index= 30	count= 337	cum= 0.892
index= 31	count= 266	cum= 0.919
index= 32	count= 218	cum= 0.941
index= 33	count= 170	cum= 0.958
index= 34	count= 151	cum= 0.973
index= 35	count= 97	cum= 0.983
index= 36	count= 65	cum= 0.989
index= 37	count= 58	cum= 0.995
index= 38	count= 25	cum= 0.998
index= 39	count= 16	cum= 0.999
index= 40	count= 9	cum= 1.000

Table 3.2

time = 5 N = 40 w = 1.125

index= 0	count= 0	cum= 0.000
index= 1	count= 0	cum= 0.000
index= 2	count= 3	cum= 0.000
index= 3	count= 7	cum= 0.001
index= 4	count= 10	cum= 0.002
index= 5	count= 9	cum= 0.003
index= 6	count= 20	cum= 0.005
index= 7	count= 22	cum= 0.007
index= 8	count= 42	cum= 0.011
index= 9	count= 73	cum= 0.019
index= 10	count= 83	cum= 0.027
index= 11	count= 94	cum= 0.036
index= 12	count= 108	cum= 0.047
index= 13	count= 140	cum= 0.061
index= 14	count= 208	cum= 0.082
index= 15	count= 229	cum= 0.105
index= 16	count= 246	cum= 0.129
index= 17	count= 332	cum= 0.163
index= 18	count= 332	cum= 0.196
index= 19	count= 402	cum= 0.236
index= 20	count= 470	cum= 0.283
index= 21	count= 475	cum= 0.331
index= 22	count= 515	cum= 0.382
index= 23	count= 514	cum= 0.433
index= 24	count= 575	cum= 0.491
index= 25	count= 597	cum= 0.551
index= 26	count= 536	cum= 0.604
index= 27	count= 545	cum= 0.659
index= 28	count= 526	cum= 0.711
index= 29	count= 489	cum= 0.760
index= 30	count= 498	cum= 0.810
index= 31	count= 429	cum= 0.853
index= 32	count= 355	cum= 0.888
index= 33	count= 312	cum= 0.920
index= 34	count= 274	cum= 0.947
index= 35	count= 193	cum= 0.966
index= 36	count= 138	cum= 0.980
index= 37	count= 90	cum= 0.989
index= 38	count= 59	cum= 0.995
index= 39	count= 37	cum= 0.999
index= 40	count= 13	cum= 1.000

Table 3.3

time = 5 N = 40 w = 1.150

index= 0	count= 3	cum= 0.000
index= 1	count= 0	cum= 0.000
index= 2	count= 1	cum= 0.000
index= 3	count= 5	cum= 0.001
index= 4	count= 6	cum= 0.002
index= 5	count= 8	cum= 0.002
index= 6	count= 11	cum= 0.003
index= 7	count= 25	cum= 0.006
index= 8	count= 28	cum= 0.009
index= 9	count= 44	cum= 0.013
index= 10	count= 59	cum= 0.019
index= 11	count= 59	cum= 0.025
index= 12	count= 82	cum= 0.033
index= 13	count= 127	cum= 0.046
index= 14	count= 153	cum= 0.061
index= 15	count= 213	cum= 0.082
index= 16	count= 207	cum= 0.103
index= 17	count= 280	cum= 0.131
index= 18	count= 290	cum= 0.160
index= 19	count= 367	cum= 0.197
index= 20	count= 404	cum= 0.237
index= 21	count= 424	cum= 0.280
index= 22	count= 511	cum= 0.331
index= 23	count= 563	cum= 0.387
index= 24	count= 532	cum= 0.440
index= 25	count= 580	cum= 0.498
index= 26	count= 574	cum= 0.556
index= 27	count= 550	cum= 0.611
index= 28	count= 581	cum= 0.669
index= 29	count= 556	cum= 0.724
index= 30	count= 519	cum= 0.776
index= 31	count= 484	cum= 0.825
index= 32	count= 405	cum= 0.865
index= 33	count= 340	cum= 0.899
index= 34	count= 326	cum= 0.932
index= 35	count= 222	cum= 0.954
index= 36	count= 175	cum= 0.971
index= 37	count= 136	cum= 0.985
index= 38	count= 76	cum= 0.993
index= 39	count= 45	cum= 0.997
index= 40	count= 29	cum= 1.000

Table 3.4

time = 5 N = 40 w = 1.200

index= 0	count= 0	cum= 0.000
index= 1	count= 0	cum= 0.000
index= 2	count= 1	cum= 0.000
index= 3	count= 2	cum= 0.000
index= 4	count= 0	cum= 0.000
index= 5	count= 3	cum= 0.001
index= 6	count= 10	cum= 0.002
index= 7	count= 7	cum= 0.002
index= 8	count= 16	cum= 0.004
index= 9	count= 23	cum= 0.006
index= 10	count= 35	cum= 0.010
index= 11	count= 44	cum= 0.014
index= 12	count= 65	cum= 0.021
index= 13	count= 92	cum= 0.030
index= 14	count= 112	cum= 0.041
index= 15	count= 120	cum= 0.053
index= 16	count= 137	cum= 0.067
index= 17	count= 177	cum= 0.084
index= 18	count= 216	cum= 0.106
index= 19	count= 271	cum= 0.133
index= 20	count= 320	cum= 0.165
index= 21	count= 374	cum= 0.203
index= 22	count= 445	cum= 0.247
index= 23	count= 504	cum= 0.297
index= 24	count= 527	cum= 0.350
index= 25	count= 522	cum= 0.402
index= 26	count= 616	cum= 0.464
index= 27	count= 621	cum= 0.526
index= 28	count= 555	cum= 0.582
index= 29	count= 614	cum= 0.643
index= 30	count= 597	cum= 0.703
index= 31	count= 545	cum= 0.757
index= 32	count= 520	cum= 0.809
index= 33	count= 502	cum= 0.859
index= 34	count= 396	cum= 0.899
index= 35	count= 345	cum= 0.933
index= 36	count= 237	cum= 0.957
index= 37	count= 194	cum= 0.977
index= 38	count= 125	cum= 0.989
index= 39	count= 80	cum= 0.997
index= 40	count= 30	cum= 1.000

Table 3.5

time = 5 N = 40 w = 1.175

index= 0	count= 0	cum= 0.000
index= 1	count= 1	cum= 0.000
index= 2	count= 1	cum= 0.000
index= 3	count= 4	cum= 0.001
index= 4	count= 2	cum= 0.001
index= 5	count= 10	cum= 0.002
index= 6	count= 11	cum= 0.003
index= 7	count= 26	cum= 0.005
index= 8	count= 26	cum= 0.008
index= 9	count= 36	cum= 0.012
index= 10	count= 47	cum= 0.016
index= 11	count= 54	cum= 0.022
index= 12	count= 71	cum= 0.029
index= 13	count= 85	cum= 0.037
index= 14	count= 132	cum= 0.051
index= 15	count= 163	cum= 0.067
index= 16	count= 191	cum= 0.086
index= 17	count= 221	cum= 0.108
index= 18	count= 270	cum= 0.135
index= 19	count= 304	cum= 0.166
index= 20	count= 334	cum= 0.199
index= 21	count= 424	cum= 0.241
index= 22	count= 471	cum= 0.288
index= 23	count= 538	cum= 0.342
index= 24	count= 533	cum= 0.396
index= 25	count= 570	cum= 0.453
index= 26	count= 585	cum= 0.511
index= 27	count= 601	cum= 0.571
index= 28	count= 603	cum= 0.631
index= 29	count= 596	cum= 0.691
index= 30	count= 559	cum= 0.747
index= 31	count= 519	cum= 0.799
index= 32	count= 447	cum= 0.844
index= 33	count= 403	cum= 0.884
index= 34	count= 335	cum= 0.917
index= 35	count= 283	cum= 0.946
index= 36	count= 223	cum= 0.968
index= 37	count= 148	cum= 0.983
index= 38	count= 99	cum= 0.993
index= 39	count= 48	cum= 0.997
index= 40	count= 26	cum= 1.000

Table 3.6

time = 5 N = 40 w = 1.1875

index= 0	count= 1	cum= 0.000
index= 1	count= 0	cum= 0.000
index= 2	count= 3	cum= 0.000
index= 3	count= 1	cum= 0.001
index= 4	count= 7	cum= 0.001
index= 5	count= 7	cum= 0.002
index= 6	count= 5	cum= 0.002
index= 7	count= 18	cum= 0.004
index= 8	count= 13	cum= 0.005
index= 9	count= 42	cum= 0.010
index= 10	count= 37	cum= 0.013
index= 11	count= 46	cum= 0.018
index= 12	count= 74	cum= 0.025
index= 13	count= 90	cum= 0.034
index= 14	count= 114	cum= 0.046
index= 15	count= 141	cum= 0.060
index= 16	count= 172	cum= 0.077
index= 17	count= 213	cum= 0.098
index= 18	count= 253	cum= 0.124
index= 19	count= 267	cum= 0.150
index= 20	count= 370	cum= 0.187
index= 21	count= 413	cum= 0.229
index= 22	count= 485	cum= 0.277
index= 23	count= 507	cum= 0.328
index= 24	count= 544	cum= 0.382
index= 25	count= 540	cum= 0.436
index= 26	count= 616	cum= 0.498
index= 27	count= 534	cum= 0.551
index= 28	count= 627	cum= 0.614
index= 29	count= 611	cum= 0.675
index= 30	count= 555	cum= 0.731
index= 31	count= 567	cum= 0.787
index= 32	count= 448	cum= 0.832
index= 33	count= 476	cum= 0.880
index= 34	count= 339	cum= 0.914
index= 35	count= 288	cum= 0.942
index= 36	count= 226	cum= 0.965
index= 37	count= 168	cum= 0.982
index= 38	count= 101	cum= 0.992
index= 39	count= 56	cum= 0.998
index= 40	count= 25	cum= 1.000

Table 3.7

time = 5 N = 40 w = 1.025

index= 0	count= 0	cum= 0.000
index= 1	count= 3	cum= 0.000
index= 2	count= 11	cum= 0.001
index= 3	count= 15	cum= 0.003
index= 4	count= 38	cum= 0.007
index= 5	count= 41	cum= 0.011
index= 6	count= 63	cum= 0.017
index= 7	count= 88	cum= 0.026
index= 8	count= 116	cum= 0.037
index= 9	count= 158	cum= 0.053
index= 10	count= 195	cum= 0.073
index= 11	count= 220	cum= 0.095
index= 12	count= 250	cum= 0.120
index= 13	count= 288	cum= 0.149
index= 14	count= 361	cum= 0.185
index= 15	count= 411	cum= 0.226
index= 16	count= 426	cum= 0.268
index= 17	count= 438	cum= 0.312
index= 18	count= 545	cum= 0.367
index= 19	count= 500	cum= 0.417
index= 20	count= 501	cum= 0.467
index= 21	count= 533	cum= 0.520
index= 22	count= 552	cum= 0.575
index= 23	count= 543	cum= 0.630
index= 24	count= 476	cum= 0.677
index= 25	count= 486	cum= 0.726
index= 26	count= 444	cum= 0.770
index= 27	count= 398	cum= 0.810
index= 28	count= 389	cum= 0.849
index= 29	count= 311	cum= 0.880
index= 30	count= 289	cum= 0.909
index= 31	count= 239	cum= 0.933
index= 32	count= 177	cum= 0.951
index= 33	count= 181	cum= 0.969
index= 34	count= 124	cum= 0.981
index= 35	count= 82	cum= 0.989
index= 36	count= 55	cum= 0.995
index= 37	count= 26	cum= 0.997
index= 38	count= 15	cum= 0.999
index= 39	count= 9	cum= 1.000
index= 40	count= 3	cum= 1.000

Table 3.8

time = 5 N = 40 w = 1.0125

index= 0	count= 2	cum= 0.000
index= 1	count= 4	cum= 0.001
index= 2	count= 8	cum= 0.001
index= 3	count= 19	cum= 0.003
index= 4	count= 35	cum= 0.007
index= 5	count= 51	cum= 0.012
index= 6	count= 64	cum= 0.018
index= 7	count= 94	cum= 0.028
index= 8	count= 122	cum= 0.040
index= 9	count= 151	cum= 0.055
index= 10	count= 198	cum= 0.075
index= 11	count= 246	cum= 0.099
index= 12	count= 291	cum= 0.129
index= 13	count= 302	cum= 0.159
index= 14	count= 351	cum= 0.194
index= 15	count= 419	cum= 0.236
index= 16	count= 442	cum= 0.280
index= 17	count= 466	cum= 0.327
index= 18	count= 527	cum= 0.379
index= 19	count= 514	cum= 0.431
index= 20	count= 558	cum= 0.486
index= 21	count= 508	cum= 0.537
index= 22	count= 574	cum= 0.595
index= 23	count= 544	cum= 0.649
index= 24	count= 529	cum= 0.702
index= 25	count= 452	cum= 0.747
index= 26	count= 444	cum= 0.791
index= 27	count= 382	cum= 0.830
index= 28	count= 360	cum= 0.866
index= 29	count= 300	cum= 0.896
index= 30	count= 263	cum= 0.922
index= 31	count= 204	cum= 0.942
index= 32	count= 182	cum= 0.961
index= 33	count= 125	cum= 0.973
index= 34	count= 90	cum= 0.982
index= 35	count= 75	cum= 0.990
index= 36	count= 52	cum= 0.995
index= 37	count= 26	cum= 0.996
index= 38	count= 17	cum= 0.999
index= 39	count= 5	cum= 1.000
index= 40	count= 4	cum= 1.000

Table 3.9

time = 5 N = 20 w = 1.025

index= 0	count= 120	cum= 0.012
index= 1	count= 150	cum= 0.027
index= 2	count= 219	cum= 0.049
index= 3	count= 354	cum= 0.084
index= 4	count= 407	cum= 0.125
index= 5	count= 454	cum= 0.170
index= 6	count= 569	cum= 0.227
index= 7	count= 653	cum= 0.293
index= 8	count= 662	cum= 0.259
index= 9	count= 682	cum= 0.427
index= 10	count= 725	cum= 0.499
index= 11	count= 712	cum= 0.571
index= 12	count= 755	cum= 0.646
index= 13	count= 768	cum= 0.723
index= 14	count= 648	cum= 0.788
index= 15	count= 553	cum= 0.843
index= 16	count= 502	cum= 0.2893
index= 17	count= 372	cum= 0.930
index= 18	count= 291	cum= 0.960
index= 19	count= 219	cum= 0.982
index= 20	count= 185	cum= 1.000

Table 3.10

time = 5 N = 20 w = 1.050

index= 0	count= 112	cum= 0.011
index= 1	count= 126	cum= 0.024
index= 2	count= 174	cum= 0.041
index= 3	count= 242	cum= 0.065
index= 4	count= 353	cum= 0.101
index= 5	count= 437	cum= 0.144
index= 6	count= 531	cum= 0.198
index= 7	count= 615	cum= 0.259
index= 8	count= 590	cum= 0.318
index= 9	count= 689	cum= 0.387
index= 10	count= 790	cum= 0.466
index= 11	count= 739	cum= 0.540
index= 12	count= 770	cum= 0.617
index= 13	count= 680	cum= 0.685
index= 14	count= 694	cum= 0.754
index= 15	count= 602	cum= 0.814
index= 16	count= 554	cum= 0.870
index= 17	count= 472	cum= 0.917
index= 18	count= 352	cum= 0.952
index= 19	count= 242	cum= 0.976
index= 20	count= 236	cum= 1.000

Table 3.11

time = 5 N = 20 w = 1.075

index= 0	count= 79	cum= 0.008
index= 1	count= 113	cum= 0.019
index= 2	count= 161	cum= 0.035
index= 3	count= 228	cum= 0.058
index= 4	count= 315	cum= 0.090
index= 5	count= 397	cum= 0.129
index= 6	count= 420	cum= 0.171
index= 7	count= 516	cum= 0.223
index= 8	count= 590	cum= 0.282
index= 9	count= 668	cum= 0.349
index= 10	count= 746	cum= 0.423
index= 11	count= 742	cum= 0.497
index= 12	count= 731	cum= 0.571
index= 13	count= 763	cum= 0.647
index= 14	count= 740	cum= 0.721
index= 15	count= 669	cum= 0.788
index= 16	count= 579	cum= 0.2846
index= 17	count= 516	cum= 0.897
index= 18	count= 426	cum= 0.940
index= 19	count= 305	cum= 0.970
index= 20	count= 296	cum= 1.000

Table 3.12

time = 5 N = 20 w = 1.100

index= 0	count= 69	cum= 0.007
index= 1	count= 91	cum= 0.016
index= 2	count= 152	cum= 0.031
index= 3	count= 195	cum= 0.051
index= 4	count= 251	cum= 0.076
index= 5	count= 354	cum= 0.111
index= 6	count= 410	cum= 0.152
index= 7	count= 501	cum= 0.202
index= 8	count= 568	cum= 0.259
index= 9	count= 623	cum= 0.321
index= 10	count= 645	cum= 0.386
index= 11	count= 741	cum= 0.460
index= 12	count= 772	cum= 0.537
index= 13	count= 768	cum= 0.614
index= 14	count= 764	cum= 0.690
index= 15	count= 743	cum= 0.765
index= 16	count= 655	cum= 0.830
index= 17	count= 570	cum= 0.887
index= 18	count= 495	cum= 0.937
index= 19	count= 320	cum= 0.969
index= 20	count= 313	cum= 1.000

Table 3.13

time = 5 N = 20 w = 1.150

index= 0	count= 58	cum= 0.006
index= 1	count= 74	cum= 0.013
index= 2	count= 96	cum= 0.023
index= 3	count= 154	cum= 0.038
index= 4	count= 205	cum= 0.059
index= 5	count= 256	cum= 0.084
index= 6	count= 348	cum= 0.119
index= 7	count= 390	cum= 0.158
index= 8	count= 464	cum= 0.204
index= 9	count= 592	cum= 0.264
index= 10	count= 626	cum= 0.326
index= 11	count= 709	cum= 0.397
index= 12	count= 790	cum= 0.476
index= 13	count= 840	cum= 0.560
index= 14	count= 843	cum= 0.644
index= 15	count= 793	cum= 0.724
index= 16	count= 748	cum= 0.799
index= 17	count= 619	cum= 0.861
index= 18	count= 568	cum= 0.917
index= 19	count= 432	cum= 0.961
index= 20	count= 395	cum= 1.000

Table 3.14

time = 5 N = 20 w = 1.125

index= 0	count= 54	cum= 0.005
index= 1	count= 75	cum= 0.013
index= 2	count= 127	cum= 0.026
index= 3	count= 174	cum= 0.043
index= 4	count= 256	cum= 0.069
index= 5	count= 297	cum= 0.098
index= 6	count= 375	cum= 0.136
index= 7	count= 434	cum= 0.179
index= 8	count= 544	cum= 0.234
index= 9	count= 586	cum= 0.292
index= 10	count= 672	cum= 0.359
index= 11	count= 754	cum= 0.435
index= 12	count= 733	cum= 0.508
index= 13	count= 807	cum= 0.589
index= 14	count= 779	cum= 0.667
index= 15	count= 724	cum= 0.739
index= 16	count= 687	cum= 0.808
index= 17	count= 616	cum= 0.869
index= 18	count= 543	cum= 0.924
index= 19	count= 403	cum= 0.964
index= 20	count= 360	cum= 1.000

Table 3.15

time= 5 N=20 w= 1.200

index= 0	count= 46	cum= 0.005
index= 1	count= 45	cum= 0.009
index= 2	count= 70	cum= 0.016
index= 3	count= 113	cum= 0.027
index= 4	count= 173	cum= 0.045
index= 5	count= 187	cum= 0.063
index= 6	count= 283	cum= 0.092
index= 7	count= 330	cum= 0.125
index= 8	count= 428	cum= 0.168
index= 9	count= 475	cum= 0.215
index= 10	count= 579	cum= 0.273
index= 11	count= 654	cum= 0.338
index= 12	count= 736	cum= 0.412
index= 13	count= 778	cum= 0.490
index= 14	count= 858	cum= 0.576
index= 15	count= 860	cum= 0.661
index= 16	count= 834	cum= 0.745
index= 17	count= 756	cum= 0.821
index= 18	count= 706	cum= 0.891
index= 19	count= 554	cum= 0.947
index= 20	count= 535	cum= 1.000

Table 3.16

time = 5 N = 20 w = 1.100

index= 0	count= 383	cum= 0.038
index= 1	count= 127	cum= 0.051
index= 2	count= 157	cum= 0.067
index= 3	count= 212	cum= 0.088
index= 4	count= 211	cum= 0.109
index= 5	count= 240	cum= 0.133
index= 6	count= 249	cum= 0.158
index= 7	count= 309	cum= 0.189
index= 8	count= 357	cum= 0.225
index= 9	count= 400	cum= 0.265
index= 10	count= 389	cum= 0.303
index= 11	count= 465	cum= 0.350
index= 12	count= 478	cum= 0.398
index= 13	count= 544	cum= 0.452
index= 14	count= 518	cum= 0.504
index= 15	count= 580	cum= 0.562
index= 16	count= 627	cum= 0.625
index= 17	count= 637	cum= 0.688
index= 18	count= 631	cum= 0.751
index= 19	count= 603	cum= 0.812
index= 20	count=1883	cum= 1.000

Table 4.1. Type II error (β) or the power ($1 - \beta$) of the test statistics $\frac{i_w}{N}$ for N/t = 8 at different α and Ns levels.

Ns | α	.001	.01	.025	.05	.075	.1
2	.998	.974	.942	.922	.887	.853
5	.995	.964	.919	.858	.803	.752
6	.993	.952	.898	.830	.769	.715
7	.993	.943	.883	.805	.739	.681
7.5	.992	.939	.879	.797	.723	.665
8	.988	.929	.857	.764	.694	.633
15	.809	.395	.263	.210	.137	.104

APPENDIX A

```
{The program gegen computes the p.d.f. φ(P,x,t) and the c.d.f for
different values of x(x = i_w /N)
const nterm=18;
 time=10.0;
      ngen=20.0;
      phat=0.5;
      step=100;
var i,ix,n:integer;
   ffx,ee,ffp,xx,nt,ff,p,phi,pp,ii,x,fx,ncoff:real;

   procedure gegan(var x:real);{gegan computes the gegenbauer
polynomial}
          begin
        fx:=1.0;
        ncoff:=1.0;
          for n := 1 to nterm do
           begin
            ncoff:=ncoff*((n-i)*(i+n+1))/(n*n+n)*x;
            fx:=fx+ncoff;
           {writeln(n,ncoff,fx)}{debug statement}
           end;{for}
     end;{proc}
begin
phi:=0.0;ff:=0.0;p:=phat;
writeln('Number of Gen     ',ngen:5:2,'    Number of terms in poly
=',nterm:2);
nt:=ngen/time;
writeln('Time=',time:5:2,'    P=',PHAT:5:3,"    NGEN/TIME=',nt:5:2,'
step=',step);
writeln('       x        Phi        Cum Prob.');
   for ix:=1 to step-1 do
   begin
     x:=ix/step;
      for i:=1 to nterm do
          begin
            gegan(x);
            ffx:=fx;             gegan(p);
            pp:=p*(1.0-p)*(2.0*i+1);
            ii:=i*(i+1.0);
            ee:=exp(-0.5*time*ii/ngen);
            phi:=phi+(ffx*fx*ee*ii*pp);
   end(for);
            ff:=ff+phi*1/step;
            if ix mod 5=0 then
            writeln(x:10:3,phi:12:7,ff:12:7);
            if ix=99 then
            writeln(x:10:3,phi:12:7,ff:12:7);
            phi:=0.0
       end{for}
end.
```

APPENDIX B

```
PROGRAM Simulate;
   CONST
       min = 0;
       max =20;
       ngen=10;{number of generations}
       w=1.1;{parameter for selectivity}
       n=20; {number in generation}
       nrep=10000:
   TYPE
       RealArray =Array[min..max] of integer;
   VAR
       printer:test;
       count:RealArray;
      isum,index,ii, i,j,k:Integer;
      dummy,phat,sum,cell:real;
   Begin{main program}
   assign(printer,'lst:');
   rewrite(printer);
   writeln(printer,' time= ',ngen,'N= ',n,'w=',w);
   sum:=0.0;
    for ii:=min to max
     do {intial count vector}
      count[ii]:=0 ;
       for i:=1 to nrep do
        begin{i loop}
         { writeln(printer,'nrep= ',i);}
          phat:=0.5;
           for j:=1 to ngen do
             begin{j loop}

               for k:=1 to n do
               begin {k loop}
                       cell:=random;
                          if cell<phat then sum:=sum+1;
                             { writeln(phat,sum,cell);}
             end;{k loop}
             phat:=(sum*w)/(sum*w-sum+n); {compute new phat}
             index:=trunc(sum);
  {      writeln(printer,' ngen= ',ngen,' phat= ',phat:8:f,' index=
",index);}
             sum:=0.0;    {intialize sum}
          end;{j loop}
         {writeln(printer,phat:8:,' index=',index);}
         count[index]:=count[index]+1;
   {     writeln(printer,' index+ ',index, " count= ",count[index]);}
      end;{i loop}
      sum:=0.0;
      for ii:=min to max do
      begin
      sum:=sum+count[ii]*1.0;
      dummy:=sum/nrep;
```

```
      writeln (printer,'index= ',ii, '  count=',count[ii],'
cum=',dummy:7:3);
      ent{ii};
   close(printer)
END.{program}
```

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ESTIMATION AND HYPOTHESES
TESTING OF THE FITNESS COEFFICIENT

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AN ABSTRACT OF A MASTER'S REPORT

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The fitness coefficient of a type in a population is a measure of its reproductive value. In this study we model the dynamics of a population with two types of individuals as a Markov process. The maximum likelihood technique is used to develop a method of estimating the fitness coefficient of one type relative to the other. A locally best test is then used to construct a test of hypothesis concerning selection for one type over the other. The distribution of the test statistic and its power are determined from solutions to the Fokker-Plank equation.