MODELING AND OPTIMIZATION OF WATER QUALITY IN STREAMS

by 632

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CHAPTER 1

INTRODUCTION

The disposal of sewage and waste from industry into streams has caused serious pollution problems. In proposing a mathematical model for prediction, management and control of water quality in streams, it is necessary to make a good compromise between the ability to describe the complex process of waste assimilation as closely as possible, and the ability to solve the complex mathematical equations.

Among various aspects of water quality, the biological oxygen demand (BOD) and dissolved oxygen (DO) concentrations are of primary importance in many situations. Owing to excessive discharge of BOD from sewage and effluents from industry, the dissolved oxygen concentration may be reduced to very low values. A certain minimum concentration of dissolved oxygen is necessary for sustaining fish and plant life and to prevent anaerobic oxidation of the BOD. The modeling of biodegradable pollutants is more complex than that for most conservative pollutants. By suitable simplifications, the mathematical equations for BOD and DO balances can be used to represent the behavior of other non-biodegradable pollutants. For example, the equation for BOD can be used to describe the concentration of a salt by omitting the sedimentation and BOD decay terms.

In the present analysis, a steady state one dimensional dispersion model is developed to represent the BOD and DO balances in a stream having several outfalls of waste and intakes of water. The model also includes continuous additions and removals of BOD and DO along the stream. For the purpose of analysis the stream is divided into several segments, each segment beginning at the location of a waste outfall or water intake. In order to incorporate changing properties of the stream, the parameters of the second order ordinary differential equations representing BOD and DO balances, may be changed from segment to segment. An analytical solution is derived by applying continuity equations at the boundary of each segment, and equilibrium conditions at the ends of the stream which is assumed to be infinitely long.

The model developed is extended to incorporate waste treatment plant models at the location of waste outfalls, and water treatment plant models at the intakes. A cost-benefit function which includes the cost of waste and water treatment as well as the benefits derived from the stream is formulated. A search technique is used to find the optimum decision variables that would minimize the cost-benefit function for the system, within certain constraints on water quality.

The optimization studies made so far have neglected the effect of dispersion in streams. The effect of longitudinal

mixing due to the velocity distribution across the stream cross section can be considered if a dispersion model is employed. It is important in streams with low flow where pollution problems are more likely. The longitudinal mixing also becomes important when artificial aeration of the stream is used as means of controlling water quality. For streams in which the longitudinal mixing component is negligible, the continuous concentration profiles obtained by using a dispersion model are more realistic than the discontinuous profiles obtained by using plug flow especially in the regions near the waste outfalls.

Due to the steady state, isothermal assumptions made in the analysis, it has limited application when diurnal variations due to photosynthesis are large and when there is a wide variation in seasonal flows and temperature. By making quasi-steady state assumptions over small periods of time, the steady state approach may be employed for studying changes in optimum solutions under different water quality goals, and for examining alternate methods for control of water quality.

With suitable modification, a similar approach can be applied in the modeling and optimization of reactors with multiple side injections. The model can also be extended to include temperature effects on the BOD and DO balances by incorporating an energy balance and the appropriate temperature dependence into the rate constants.

CHAPTER 2

LITERATURE SURVEY

The literature survey made here deals mainly with two aspects: 1) development of deterministic models for describing water quality in streams, 2) water quality management models using a deterministic approach.

DETERMINISTIC MODELS FOR STREAM QUALITY

Most studies related to modeling of water quality in streams consider the biological oxygen demand (BOD) and dissolved oxygen (DO) concentrations as primary indicators of water quality. Two reasons have been given in support of this choice (2.27, 2.33). First, the biodegradation of organic pollutants is one of the major problems causing pollution in streams. Secondly, the modeling of BOD and DO is usually more complex than that for other conservative or nonconservative pollutants, and by suitable simplification in the model, one can reduce the mathematical equations to represent the behavior of other pollutants in the stream.

The factors that affect BOD and DO balances in streams are varied and complex, and the degree of importance of each individual phenomenon also differs greatly from stream to stream. Most studies have contributed to the understanding of one particular phenomenon, such as reaeration, BOD decay, photo-synthesis and the like. Therefore, it is preferred to review

individual aspects separately. First, flow models are considered, since the study of flow patterns is usually made independently (by tracer techniques, etc.), without involving other factors which affect the BOD and DO balance. Next, the kinetic expressions for BOD decay, natural reaeration studies, and studies relating to other factors such as photosynthesis, respiration, benthal demand, etc., are reviewed briefly.

Flow Models

Streeter and Phelps (2.54) were the first to propose mathematical equations for stream quality. They proposed a steady state, plug flow model, with a first order BOD decay process and a first order process for atmospheric reaeration proportional to the DO deficit (see Table 2.1). Modifications for the constant cross sectional area, uniform flow assumed by Streeter and Phelps, were made by O'Connor (2.34), who derived analytical expressions for the case of area and flow rate changing linearly or exponentially with distance along the stream. In a later article (2.15) DiToro and O'Connor presented an analytical solution for the unsteady state plug flow case resulting from a random component of velocity. Earlier Thomann (2.47) used systems analysis techniques to solve the unsteady state case by a transfer function approach. Recently DiToro (2.14) discussed the application of the method of characteristics in solving unsteady state plug flow equations

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Table 2.1 Summary of developments in modeling of flow of pollutants in streams

Names of Investigators	Nature of model and Assumptions made	Mathematical Model	Remarks	References
Streeter and Phelps (1925)	Plug flow model: Steady state, uniform flow, uniform cross-sectional area	$-U\frac{dC}{d\ell} + S(C,\ell)=0$	Analytical solutions for first order and sinusoidal source inputs	2.54, 2.45
O'Connor (1967)	Steady state with area and flow changing with distance along stream	$-\frac{Q(\ell)}{A(\ell)}\frac{\partial C}{\partial \ell} + S(C,\ell)=0$ $Q(\ell)=Q_o(1+q\ell)$ $A(\ell)=A_o(1+q\ell)$	Solutions were also derived for area and flow rate changing exponentially with distance.	2.45
O'Connor and DiToro (1968)	Unsteady state due to time varying velocity	$\frac{\partial C}{\partial t} + \frac{Q(t)}{A(\ell,t)}\frac{\partial C}{\partial \ell} - S(C,\ell,t)=0$	Analytical solution by Laplace transformation	2.15
Diachishin (1963)	Dispersion Model: One, two and three dimensional equations describing transport of conservative waste in streams	$\frac{\partial C}{\partial t} = D_x\frac{\partial^2 C}{\partial x^2} + D_y\frac{\partial^2 C}{\partial y^2} + D_z\frac{\partial^2 C}{\partial z^2} - U_x\frac{\partial C}{\partial x} - U_y\frac{\partial C}{\partial y} - U_z\frac{\partial C}{\partial z}$	Analytical solution derived for δ-input	2.13

Table 2.1 (continued)

Dobbins, Bella, Dresnack (1964)	One dimensional dispersion model incorporating sources and sinks of waste along stream flow	$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial \ell^2} - U\frac{\partial C}{\partial \ell} + S(\ell, t, C)$	Analytical solution for the steady-state case, numerical solution for unsteady state	2.3 2.5 2.16 2.17
Hays (1968)	Dead zone dispersion model: A dead zone bottom region which is completely mixed, flowing top zone represented by one dimensional longitudinal dispersion model	$\frac{\partial C_a}{\partial t} = D_a\frac{\partial^2 C}{\partial \ell^2} - U\frac{\partial C_a}{\partial \ell} - k_a(C_d - C_a)$ $\frac{\partial C_d}{dt} = k_d(C_a - C_d)$	Analytical solution derived for δ-input	2.30, 2.46
Fischer (1969)	Other factors affecting flow: Effect of area changes and	Please refer to Reference (2.20) for details of mathematical derivations.	Analytical relationships derived to predict theoretical effect	2.20
Wu	Effect of wind velocity on the velocity profile of a stream	Please refer to Reference (2.63) for details of mathematical derivations	Analytical relationships derived to predict theoretical effect	2.63

for streams.

The phenomenon of mixing represented by the longitudinal dispersion term was introduced in stream equations by Dobbins (2.16). He obtained an analytical solution for a steady state case, with a single input, assuming the BOD and DO concentrations at the point of input to be known. Armstrong and Gloyna (2.3) performed experiments with radioactive tracers to study the longitudinal dispersion in streams with bottom deposits. They used the dead zone concept proposed by Hays and Krenkel (2.3) to divide the stream into two zones, and they also considered the sorption of tracer by the stream bed and plants. The unsteady state equations which they used to describe the tracer concentration as a function of position and time were solved by finite difference techniques. They concluded that a one dimensional model is adequate to represent mixing phenomenon in most streams, but that a two dimensional representation is worthwhile in modeling the entrance region. Patterson (2.46) obtained an analytical solution for the dead zone dispersion model used by Armstrong and Gloyna for the homogeneous BOD equation using transformation of variables. However, analytical solutions for the nonhomogeneous partial differential equations especially for the case of DO are difficult to obtain. Diachischin (2.13) solved analytically the case of unsteady two and three dimensional

dispersion advection transport, but without any reaction term.

Numerical solutions for unsteady state flow models have been investigated by several workers (2.5, 2.17, 2.26, 2.28). Dresnack and Dobbins (2.17) proposed a two step implicit finite difference scheme to improve stability of the solution. Bella et. al. (2.5) applied an implicit finite difference scheme which incorporates changing area and stream properties and inputs changing with time and distance. Recently a numerical scheme using a finite element method has also been presented by Guyman (2.26) for solving stream equations. Work using the one dimensional dispersion model is reported and work for the two dimensional dispersion model is reported to be in progress.

Among other factors which affect the flow pattern in a stream, the effect of bends on the representation of the longitudinal dispersion was studied by Fischer (2.20). Theoretical relations were developed to predict the change in longitudinal dispersion due to bends along a stream. The change in velocity distribution of the stream brought about by the sheering effect of winds across the surface of the stream was studied by Wu (2.63). Theoretical relationships between the wind velocity and direction and its effect on the velocity distribution were developed.

In general, it can be seen that the one dimensional

approximation is often made for representing water quality in streams. The difficulty in formulating meaningful boundary conditions, when applying a three dimensional model to the BOD and DO equations as well as the difficulty in solving these equations, have been some of the major drawbacks limiting the use of a three dimensional approach.

Some of the flow models for streams are summarized in Table 2.1.

Kinetic Expression for BOD Decay

In the process of self purification of streams the oxidation of organic pollutants is brought about by micro-organisms present in the stream that use these organic nutrients to synthesize non-harmful products. Due to the presence of a wide variety of nutrients (pollutants) and also a wide variety of bacterial species in streams, the mathematical models that have been proposed for BOD decay are only useful in approximating the over all BOD decay process. As shown in Table 2.2, most of these models do not consider the concentrations of either individual pollutants or individual bacterial species.

Streeter and Phelps (2.54) assumed that the BOD decay rate was proportional to the unoxidized BOD. The first order model proposed by Streeter and Phelps is most commonly used in writing stream BOD balances (2.24, 2.55); however, several other models have also been investigated (2.18, 2.40, 2.47).

Table 2.2 Summary of models investigated for BOD reaction in streams

Name of Investigators	Description of Model	Mathematical Model	Remarks	References
Streeter and Phelps (1925)	First order rate of BOD decay proportional to the unreacted BOD	$\frac{dL}{dt} = -KL$	Note: All models denote batch growth	2.54
Robertson (1923)	Rate of BOD decay proportional to product of cell concentration and BOD concentration	$\frac{dL}{dt} = -K'LX$	Verified by Wezernack and Gannon (2.61) for actual stream data	2.47 2.61
Monod (1942)	Cell growth model for single substrate utilization	$\frac{dL}{dt} = -\frac{1\ \mu XL}{Y(K_s+L)}$	Verified by Sanders (2.48 and Stratton and McCarty (2.53) for actual stream data	2.40
Gaudy (1963)	Removal of heterogeneous substrate	$\frac{dL_c}{dt} = k_c(L_c^o - L_c)$ $\frac{dL_T}{dt} = k_n(L_N^o - L_N) + k_c(L_c^o - L_c)$ $t>ta$	The mathematical model is proposed by Biguira et al. (2.6)	2.23

One of the reasons favoring the use of a first order model is the fact that it is more suitable for obtaining analytical solutions. Besides, there has not been sufficient evidence in the literature to show the advanatges of other models over this one. This particular model has also been used in the present analysis.

The model proposed by Robertson (2.47) which takes into account the presence of bacterial species (see Table 2.2), was reported to give better representation of actual stream data by Wezernack, et. al. (2.61). Monod's model for cell growth (2.4) which also takes into account the concentration of bacterial mass, was investigated for representation of nitrification in streams by Stratton and McCarty (2.53), and Knowles, et. al. (2.34).

Gaudy (2.23) proposed a model for the removal of heterogeneous substrate consisting mainly of carbonaceous and nitrogenous organic matter in the activated sludge process in a sequential manner. Klein (2.32) recommended the above model for representation of stream BOD kinetics. The mathematical model for the sequential removal of carbonaceous and nitrogenous BOD was proposed by Biguira, et. al. (2.6). Courchaine (2.11) and other workers (2.2, 2.22) however, did not agree with the sequential removal representation in their study on specific streams.

In general there has been considerable disagreement in modeling of BOD kinetics in streams for reasons mentioned earlier. In order to overcome the discrepancy, Goodman and Dobbins (2.24) recommended the use of an empirical first order BOD constant over small segments of a stream, that would give the closest fit to actual data. The above representation has been used in the present analysis.

Reaeration Rate

Adeney and Becker (2.1) in 1919 showed that the absorption of oxygen into water occurs as a first order kinetic process in which the rate of absorption is proportional to the oxygen saturation deficit. In 1924, Lewis and Whitman (2.36) proposed a similar model for gas absorption based on their two film theory. The conditions prevailing during stream aeration, however, are very different from the assumptions on which the two film theory is based. Because of this, the coefficient of proportionality, k_2, in the case of stream aeration may be better related to other independent variables such as wind velocity and stream depth by using the surface renewal model for mass transfer (2.12).

Several workers have attempted to correlate the reaeration coefficient, k_2, with the hydraulic and physical characteristics of a stream. O'Connor and Dobbins (2.44) presented a theoretical development of the mechanism of aeration under isotropic and

nonisotropic conditions using the surface renewal mass transfer concept. They developed the relationships shown in Table 2.3 and experimentally verified their correlations for reaeration coefficient, k_2. Krenkel (2.35) and Churchill et. al. (2.10) also developed correlations of a similar nature using actual stream data. They also derived temperature correlations which can be used to predict the effect of temperature on the reaeration coefficient. Frotescque and Pearson (2.21) proposed an interesting surface renewal theory based on large scale eddy formulation. Kotaoka and Miyauchi (2.31) further developed the eddy formulation theory and verified their results experimentally. Thackson et. al. (2.56) related k_2 with the Froud number and shear velocity at the surface.

In addition to natural reaeration, studies have been made to predict the reaeration coefficient under artificial turbulence (2.19, 2.62).

The studies reported above have lead to a better prediction of reaeration in natural streams. The prediction of k_2 under artificial turbulence would be of great value in controlling water quality by the use of mechanical aerators.

Other Factors Affecting BOD and DO Balances in Streams

In several streams, ecological and other natural processes are more important in their influence on BOD and DO profiles than the effect of external impoundments. In 1964, Dobbins (2.16) extended the Streeter Phelps equation to incorporate some of

Table 2.3 Summary of theoretical developments of natural reaeration in streams

Names of Investigators	Description of study		Remarks	References
Adeney and Becker (1919)	Rate of reaeration is directly proportional to the DO deficit	$\frac{dD_{DO}}{dt} = k_2 D_{DO}$	This basic assumption is made by all subsequent workers	2.1
O'Connor and Dobbins (1956)	Reaeration coefficient is related to surface renewal rate and depth	$k_2 \alpha \frac{D_L^{1/2} S_e^{1/2}}{H^{5/4}}$ (non isotropic) $k_2 \alpha \frac{D^{1/2} U^{1/2}}{H^{3/2}}$ (isotropic)	Surface renewal rate defined for the first time in terms of hydraulic properties	2.4
Krenkel and Thackson (1962,1963)	k_2 is related to energy dissipated per unit mass. Also temperature correlations are derived.	$k_2 \alpha D^m H^n$		2.35 2.56
Churchill et al. (1962)	Effect of vertical diffusion on reaeration rate. Relationships based on dimensionless hydraulic parameters are developed.	$k_2 \alpha \phi (\frac{\epsilon}{r})$		2.10

Table 2.3 (continued)

Frotescque and Pearson	A surface renewal theory based on formation of large scale eddies is proposed.	$k_{RL} \alpha (\frac{D \sqrt{U^2}}{\lambda})^{1/2}$		2.21
Kataoka and Miyauchi (1 69)	Further development of surface renewal theory based on microscopic eddy formulation is made	$k_{RL} \alpha (\frac{E}{\upsilon})^{1/4} D^{1/2}$		2.31

these factors, namely,

1) Removal of BOD by sedimentation or absorption.

2) Addition of BOD along the stretch by scour of bottom deposits or by diffusion of partially decomposed organics from the benthal layer into the water above.

3) Addition of BOD along the stretch by local run-off.

4) Removal of oxygen due to the benthal activity.

5) Addition of oxygen due to photosynthesis and removal by respiration of fixed plants.

This model will be presented in detail in Chapter 3.

The photosynthetic production of oxygen which leads to the diurnal variations in DO, has been studied and modeled by several workers (2.2, 2.4, 2.41). O'Connor and DiToro (2.43) approximated this photosynthetic term by a Fourier series as given by

$$P(\ell,t) = P_m \{\frac{2p}{\pi} + 2 \sum_{n=1}^{\infty} a_n \cos[2\pi n(t-p/2)]\} \qquad (2.1)$$

where p is the period of sunlight and P_m is the maximum photosynthetic production.

The photosynthetic term, given by Equation (2.1) makes the DO model time varying. Analytical solutions were derived by O'Connor (2.45) for a plug flow model. However, in the case of a dispersion model, a numerical scheme is required to solve the second order partial differential equations (2.17).

The respiration term R is incorporated in the stream water quality equations as a sink of dissolved oxygen (2.4, 2.45). The ratio of production to respiration, P/R, is used to classify ecosystems (2.2). The changes in the ratio in the process of self purification in specific streams have been measured by several workers (2.2, 2.41).

The growth of attached stream bacteria which account for the benthic demand for oxygen was modeled by Sanders (2.48). O'Connell et. al. (2.42) and Bain (2.4) also studied the effect of benthic algae and incorporated this effect into their stream water quality equations.

Due to a lack of sufficient information, most models used for prediction and management of stream quality use a constant rate of addition or removal of BOD and DO over small segments, in order to account for unknown factors (2.24).

DETERMINISTIC MODELS FOR WATER QUALITY MANAGEMENT IN STREAMS

Before formal mathematical modeling was used for water quality management, an effluent standard policy based on prior experience was prescribed as a means of meeting stream standards. With the development of mathematical models to predict water quality in streams, several goals have been considered in order to allocate waste inputs in an optimum manner.

The uniform treatment policy was formulated on the basis of fair play (2.25). The recent trend is to seek optimum

solutions based on maximizing certain economic and social goals. Kneese (2.33) has discussed several measures in order to make such solutions feasible.

Various optimization techniques have been used in order to solve the constrained optimization problem. Sobel (2.52) discussed various linear and nonlinear programming techniques in order to obtain minimum cost solutions. Liebman and Lynn (2.37) presented a dynamic programming approach to obtain a minimum cost solution. Shih and DeFilippi (2.49)also used the dynamic programming approach. The mathematical equations for stream quality and the objective function in their study were more complex than those of previous investigators.

A solution based on a policy which compromises between equal treatment, and minimum cost of treatment, namely, a "uniform zonal treatment" was proposed by Liebman and Marks (2.38). Bramhall and Mills (2.7) considered a choice of alternative means of water quality control in their model. They found a solution for an optimum combination of waste treatment and flow augmentation by simulation. Tarrasov et. al. (2.55) applied the distributed maximum principle to obtain an optimal policy for artificial aeration as means of controlling the DO concentration for an unsteady plug flow model of a stream. Hamilton et. al. (2.27) in their recent work, discuss various systems analysis concepts for implementing water quality control programs on a regional basis.

and social factors, such as cost of waste and water treatment (2.51), recreational benefits (2.8, 2.39), effluent charging schemes (2.29, 2.60) and the like have been made. These play an important role in formulating more efficient water quality management programs.

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NOTATION

A = cross sectional area of stream at distance ℓ, sq. miles.

A_o = cross sectional area of stream at $\ell=0$, sq. miles.

a_n = constants in Fourier series expansion, n=1, ...,∞ (dimensionless).

C = concentration of pollutant, ppm.

C_a = concentration of pollutant in moving zone, ppm.

D = longitudinal dispersion coefficient, sq. miles/day.

D_a = longitudinal dispersion coefficient in the moving zone in Hays' model, sq. miles/day.

D_{DO} = DO deficit, ppm.

E = energy dissipated.

H = depth, miles.

K_s = saturation concentration, ppm.

K = first order BOD decay constant, 1/day.

K' = rate constant in Robertson's model, /ppm/day.

k_a = mass transfer coefficient for moving zone adjusted for the volume, 1/day.

k_c = rate constant for carbonaceous BOD decay, 1/day.

k_d = mass transfer coefficient for dead zone adjusted for the volume, 1/day.

k_N = rate constant for nitrogeneous BOD decay, 1/day.

k_{RL} = average mass transfer coefficient, 1/day/sq. miles.

k_2 = reaeration rate constant, 1/day.

L = BOD concentration, ppm.

L^o = ultimate BOD, ppm.

L_c = oxidized carbonaceous BOD or oxygen consumed in converting carbonaceous BOD, ppm.

L_c^o = ultimate carbonaceous BOD, ppm.

L_N = oxidized nitrogenous BOD or oxygen consumed in converting nitrogenous BOD, ppm.

L_N^o = ultimate nitrogenous BOD, ppm.

L_T = total BOD oxidized, ppm.

ℓ = distance along the stream, miles.

O = dissolved oxygen concentration, ppm.

P = photosynthetic production of oxygen, ppm/day.

P_m = maximum photosynthetic production of oxygen, ppm/day.

p = period of sunlight, days.

Q = volumetric flow rate of stream at distance ℓ, cu. miles/day.

Q_o = volumetric flow rate at ℓ=0, cu. miles/day.

q = parameter for increment of flow and area (dimensionless).

r = hydraulic radius, miles.

S = source or sink, ppm/day.

S_e = slope of stream (dimensionless).

t = time, days.

t_a = dead time for initialization of nitrogenous BOD reaction, days.

U = stream velocity, miles/day.

X = cell concentration, ppm.

x,y,z = coordinate directions, miles.

Y = yield factor (dimensionless).

Greek letters

ϵ = vertical diffusion coefficient, sq. miles/day.

λ = integral scale length, feet.

μ = rate constant in Monod's equation, 1/day.

υ = kinematic viscosity of liquid, day/sq. miles.

α = proportionality constant.

CHAPTER 3

A DISPERSION MODEL FOR A STREAM WITH SEVERAL WASTE INPUTS AND WATER INTAKES

The disposal of sewage and wastes from industry into streams has caused serious pollution problems. In proposing a useful mathematical model for prediction, management and control of water quality, one has to make a good compromise between the ability to describe the complex process of waste assimilation as closely as possible and the ability to solve the mathematical equations. In addition, quality of water includes innumerable aspects, and therefore it becomes necessary to choose the most severely affected water quality parameters under a given situation.

Some of the important factors to be considered in modeling this problem are 1) selecting a suitable flow model, 2) incorporating the most significant physical and biological processes affecting water quality, 3) formulating proper boundary conditions at the outfalls of wastes and intakes of water.

The model considered in this study is a one dimensional steady state dispersion model (3.2) describing the BOD and dissolved oxygen (DO) balance in a stream. It has been shown (3.5) that a one dimensional dispersion model is equivalent to considering the velocity profile effect in laminar flow. The

variation in velocity across the cross section may be important in streams under low flow conditions due to the retarding effect of the banks and bottom. These slow moving streams are more likely to face the possible hazards of pollution. The dissolved oxygen concentration is selected as the water quality parameter to be controlled, since it is one of the most severely affected parameters in many situations. A certain minimum dissolved oxygen concentration is necessary for sustaining aquatic life and to prevent harmful effects due to anaerobic biodegradation. Also, the modeling of dissolved oxygen is more complex than that for most other pollutants. By suitable simplifications it is possible to reduce the model to describe the behavior of other non-biodegradable pollutants.

In the analysis made it has been possible to obtain an analytical solution to predict BOD and DO oxygen profiles in streams.

MATHEMATICAL MODELING

The one dimensional dispersion model for a single pollutant is (3.4)

$$\frac{\partial}{\partial \ell}\left(D \frac{\partial C}{\partial \ell}\right) - \frac{\partial}{\partial \ell} (UC) + (S) = \frac{\partial C}{\partial t} \qquad (3.1)$$

where C is the concentration of the pollutant at time t, along the stream length ℓ, D is the axial dispersion coefficient, U is the mean velocity and S includes any sources and sinks of the pollutant in the system. An equation of the same form can

be written for dissolved oxygen. Under steady state conditions the derivative with respect to t vanishes and Equation (3.1) becomes an ordinary second order differential equation.

To develop steady state models for BOD and DO in a given length of a stream, the following assumptions are made:

(1) Steady state, uniform cross section and uniform flow.

(2) Parameters such as dispersion coefficient D, and other rate constants expressed by an average value for a given reach.

(3) Uniform temperature.

(4) First order rate processes for BOD decay due to bacterial action and sedimentation which depend only on the BOD concentration.

(5) A first order rate for atmospheric reaeration of oxygen, proportional to the DO deficit.

(6) A uniform BOD addition rate due to the local run off, scouring or resuspension of bottom deposits and the life, given by La.

(7) A uniform dissolved oxygen loss D_B, due to benthic activity. In case of oxygen production the appropriate sign can be incorporated.

(8) Negligible diurnal variations in DO due to photosynthesis.

Based on the above assumptions we can write the equations for

BOD and DO as follows

$$D\frac{d^2L}{d\ell^2} - U\frac{dL}{d\ell} - k_1L - k_3L + La = 0 \qquad (3.2)$$

$$D\frac{d^2O}{d\ell^2} - U\frac{dO}{d\ell} - k_1L + k_2(O_s - O) - D_B = 0 \qquad (3.3)$$

It can be seen that the above equations do not take into account material added and lost due to the waste outfalls and water intakes. This is taken care of by formulating appropriate boundary conditions where the waste enters the stream and the water intakes are located as shown later.

For the purpose of analysis the stream is divided into n+m+1 segments, where n is the number of waste inputs and m is the number of water intakes, as shown in Fig. 3.1. Every segment starts just below a new input or intake.

Equations (3.2) and (3.3) applied to the ith segment can be written as,

$$D^i\frac{d^2L^i}{d\ell^2} - U^i\frac{dL^i}{d\ell} - k_1^iL^i - k_3^iL^i + La^i = 0 \qquad (3.4)$$

$$D^i\frac{d^2O^i}{d\ell^2} - U^i\frac{dO^i}{d\ell} - k_1^iL^i + k_2^i(O_s^i - O^i) - D_B^i = 0 \qquad (3.5)$$

The superscripts i denote properties in the ith segment. The average parameters D, k_1, k_2, O_s, etc., may change from segment to segment due to change in flow pattern, bacterial activity, average temperature and so on. If necessary, a

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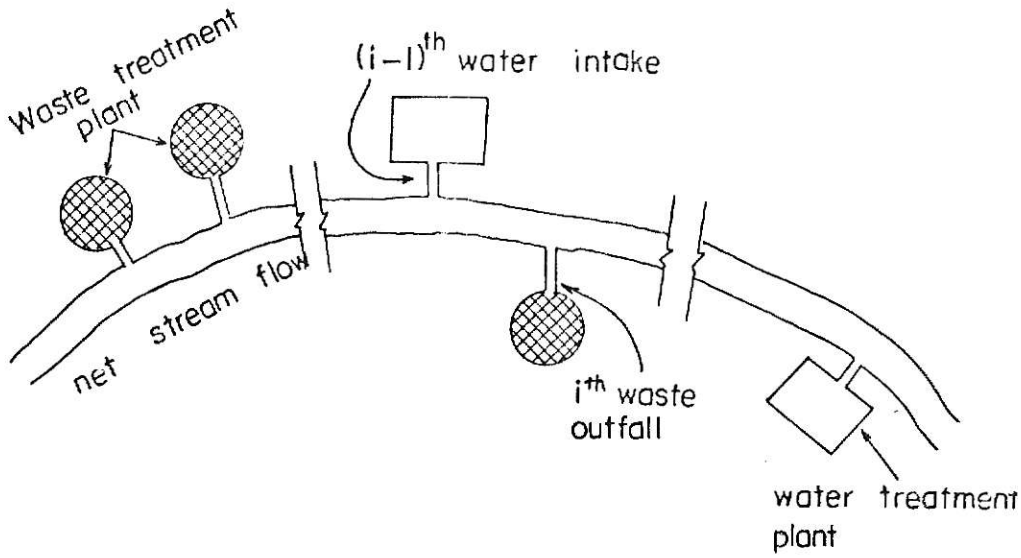
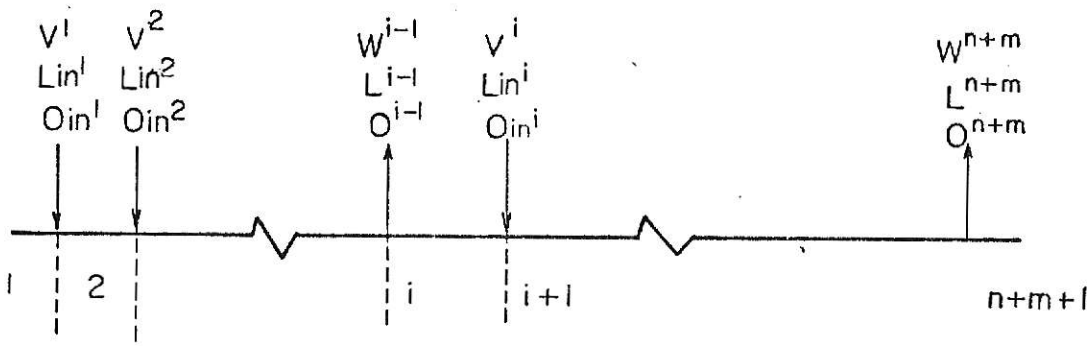


(a) Steam with several waste inputs and water intakes.



(b) One dimensional representation of the above system.

Fig.3·1. Representation of a stream with several waste inputs and water intakes.

segment may be further divided into smaller segments to take care of changing stream properties. It may be noted that the advantage gained by such a scheme is similar to that gained in difference modeling.

To model the regions near the waste outfalls, the input can be visualized as a side stream injected into an open flow system. The mixing at the point of entry is assumed to be instantaneous and complete in keeping with the other assumptions of the model. Boundary conditions similar to those for open entrance regions in reactors can be applied as shown below (3.3, 3.7).

For a waste input at $\ell = \ell_i$ between the ith and (i+1)th segments,

$$(L^i)_{\ell \to \ell_i^-} = (L^{i+1})_{\ell \to \ell_i^+} \tag{3.6a}$$

$$(0^i)_{\ell \to \ell_i^-} = (0^{i+1})_{\ell \to \ell_i^+} \tag{3.6b}$$

Equations (3.6a) and (3.6b) follow from the continuity of concentration profile. By the continuity of flux of material across $\ell = \ell_i$, we can write

$$(-D^i A^i \frac{dL^i}{d\ell})_{\ell \to \ell_i^-} + (U^i L^i A^i)_{\ell \to \ell_i^-} + (V^i Lin^i A^i)_{\ell = \ell_i} =$$

(Material flowing in by diffusion) (Material flowing in by convection) (Material input into the system = at $\ell = \ell_i$)

$$(-D^{i+1}A^{i+1}\frac{dL^{i+1}}{d\ell})_{\ell\to\ell_i+} + (U^{i+1}A^{i+1}L^{i+1})_{\ell\to\ell_i+} \qquad (3.7a)$$

(Material flowing out due to diffusion) (Material flowing out due to convection)

where V^i is the volumetric flow rate of the waste input per unit area of the stream and Lin^i and Oin^i are its BOD and DO concentrations, respectively. Similarly for DO,

$$(-D^iA^i\frac{dO^i}{d\ell})_{\ell\to\ell_i-} + (U^iA^iO^i)_{\ell\to\ell_i-} + (V^iA^iOin^i)_{\ell=\ell_i}$$

$$= (-D^{i+1}A^{i+1}\frac{dO^{i+1}}{d\ell})_{\ell\to\ell_i+} + (U^{i+1}A^{i+1}O^{i+1})_{\ell\to\ell_i+} \qquad (3.7b)$$

It may be noted that the reaction terms are absent since the material balance is over a section which has essentially zero volume.

Similarly for water removed from the stream at $\ell=\ell_j$ between the jth and j+1th segments,

$$(L^j)_{\ell\to\ell_j-} = (L^{j+1})_{\ell\to\ell_j+} \qquad (3.8a)$$

$$(O^j)_{\ell\to\ell_j-} = (O^{j+1})_{\ell\to\ell_j+} \qquad (3.8b)$$

$$(-D^jA^j\frac{dL^j}{d\ell})_{\ell\to\ell_j-} + (U^jA^jL^j)_{\ell\to\ell_j-} - (W^jA^jL^j)_{\ell=\ell_j}$$

$$= (-D^{j+1}A^{j+1}\frac{dL^{j+1}}{d\ell})_{\ell\to\ell_j+} + (U^{j+1}A^{j+1}L^{j+1})_{\ell\to\ell_j+} \qquad (3.9a)$$

$$(-D^{j}A^{j}\frac{dO^{j}}{d\ell})_{\ell\to\ell_j^-} + (U^{j}A^{j}O^{j})_{\ell\to\ell_j} - (W^{j}A^{j}O^{j})_{\ell=\ell_j}$$

$$= (-D^{j+1}A^{j+1}\frac{dO^{j+1}}{d\ell})_{\ell\to\ell_j^+} + (U^{j+1}A^{j+1}O^{j+1})_{\ell\to\ell_j^+} \qquad (3.9b)$$

where, W^{j} is the volumetric flow rate of water intake per unit area of the stream.

In addition, if the stream extends long enough above and below the region where the inputs of waste and intakes of water are located we can write the following end conditions.

$$(\frac{dL^{1}}{d\ell})_{\ell\to-\infty} = 0 \qquad (3.10a)$$

$$(\frac{dO^{1}}{d\ell})_{\ell\to-\infty} = 0 \qquad (3.10b)$$

$$(\frac{dL^{n+m+1}}{d\ell})_{\ell\to+\infty} = 0 \qquad (3.11a)$$

$$(\frac{dO^{n+m+1}}{d\ell})_{\ell\to+\infty} = 0 \qquad (3.11b)$$

Equations (3.10a), (3.10b), (3.11a) and (3.11b) state that in the absence of a disturbance the rate processes will reach equilibrium.

ANALYTICAL SOLUTION

To solve for the BOD and DO profiles for the (m+n+1) segments the differential equations given by Equations (3.4) and (3.5) for every segment must be solved.

Applying the method of homogeneous and particular integrals

we see that the solutions to Equations (3.4) and (3.5) are given by

$$L^i = L_p^i + L_h^i \tag{3.12a}$$

$$0^i = 0_p^i + 0_h^i \tag{3.12b}$$

where

$$L_p^i = \frac{La^i}{k_1^i + k_3^i} \tag{3.13a}$$

$$L_h^i = P_1^i \exp(m_1^i \ell) + Q_1^i \exp(m_2^i \ell) \tag{3.14a}$$

$$0_p^i = 0_s^i - \frac{D_B^i}{k_2^i} - \frac{k_1^i La^i}{k_2^i (k_1^i + k_3^i)} - \frac{k_1^i P_1^j \exp(m_1^i \ell)}{(k_2^i - k_1^i - k_3^i)} - \frac{k_1^i Q_1^i \exp(m_2^i \ell)}{(k_2^i - k_1^i - k_3^i)} \tag{3.13b}$$

$$0_h^i = P_2^i \exp(n_1^i \ell) + Q_2^i \exp(n_2^i \ell) \tag{3.14b}$$

and

$$m_1^i, m_2^i = \frac{U^i \mp \sqrt{(U^i)^2 + 4(k_1^i + k_3^i) D^i}}{2D^i} \tag{3.15a}$$

$$n_1^i, n_2^i = \frac{U^i \mp \sqrt{(U^i)^2 + 4k_2^i D^i}}{2D^i} \tag{3.15b}$$

P_1^i, Q_1^i, P_2^i, Q_2^i being arbitrary constants.

The $4(n+m+1)$ arbitrary constants P_1^i, Q_1^i, P_2^i, Q_2^i for the $(m+n+1)$ segments can be solved for by the $4(n+m) + 4$ linear algebraic equations obtained from the boundary conditions given by Equations (3.8a), (3.8b), (3.9a), and (3.9b) applied at the $(n+m)$ boundaries of the segments, and Equations (3.10a), (3.10b), (3.11a) and (3.11b) applied at either ends of the system. Substitution and solution of the equations is given in Appendix 3A.

In the special case, when the volumetric flow rate of the inflows is small as compared to the main flow in the stream so that the flow remains constant from segment to segment (see Fig. 3.2), the solution reduces to an interesting linear form. The concentrations of BOD and DO in the ith segment are obtained to be,

$$L^i = \frac{La^i}{k_1 + k_3} + \frac{U}{D} \sum_{j=1}^{i-1} \frac{qin^j}{(m_2 - m_1)} \exp\{m_1(\ell-\ell_j)\}$$

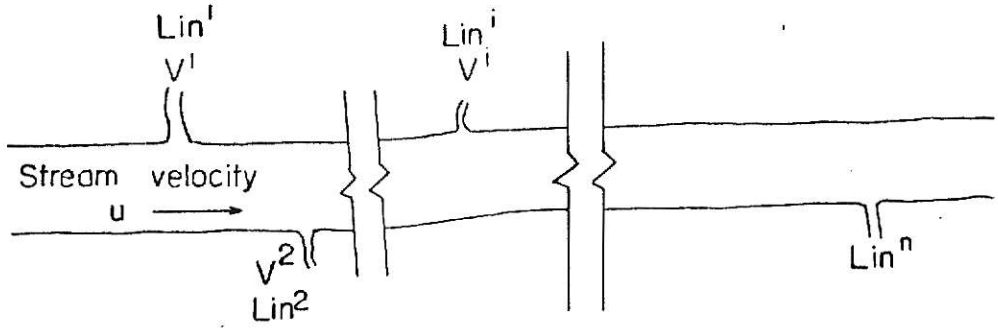
$$+ \frac{U}{D} \sum_{j=1}^{n} \frac{qin^j}{(m_2 - m_1)} \exp\{m_2(\ell-\ell_j)\} \qquad (3.16a)$$

$$O^i = O_s - \frac{D_B}{k_2} - \frac{k_1 La}{k_2(k_1+k_3)} + \frac{k_1}{D(k_2-k_1-k_3)} \sum_{j=1}^{i-1} \left(\frac{qin^j}{(n_2-n_1)} \exp\{n_1(\ell-\ell_j)\}\right.$$

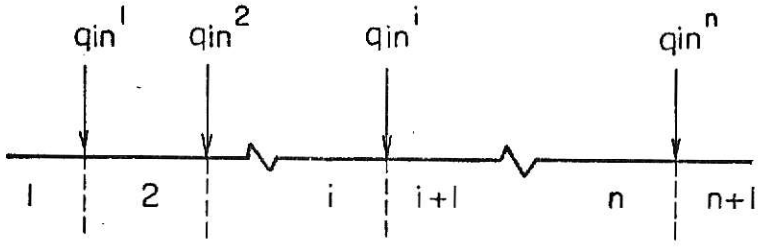
$$\left. - \frac{qin^j}{(m_2-m_1)} \exp\{m_1(\ell-\ell_j)\}\right) + \frac{k_1}{D(k_2-k_1-k_3)} \sum_{j=1}^{n}$$

$$\left(\frac{qin^j}{(n_2-n_1)} \exp\{n_2(\ell-\ell_j)\} - \frac{qin^j}{(m_2-m_1)} \exp\{m_2(\ell-\ell_j)\}\right)$$

(3.16b)



(a) Segment of a stream with several waste inputs having small volumetric flow rates as compared to the main stream flow.



(b) One dimensional model of the above system.

Fig.3·2. Representation of a stream with several waste inputs having small volumetric flow rates as compared to the main stream flow.

where the amount of BOD entering at the ith outfall is expressed in terms of qin*, and the amount of DO entering is neglected. The equations for every segment of the stream are given by Equations (3.2) and (3.3). Equations (3.16a) and (3.16b) denote that in this situation the effect of several inputs is obtained by superimposing the effect of each individual input. The proof of this result is given in Appendix 3B.

APPLICATION OF RESULTS IN STUDYING WATER POLLUTION CONTROL PROBLEMS

The model proposed above has eight parameters, D, U, k_1, k_2, k_3, D_B, La and O_s which are characteristic of individual streams. Some observations are made on the effect of variation in D, k_1 and k_2 on the BOD and DO profiles. These parameters vary widely from stream to stream and depend mainly on the flow pattern, biological activity and average temperature of the stream.

Figure 3.3 shows the effect of the dispersion coefficient D on the BOD and DO concentration profiles in a hypothetical stream. It is observed that an increase in dispersion coefficient is effective in reducing the peak concentration of BOD at the input. It also has a favorable effect in flattening the DO profile. The minimum DO concentration increases with increasing D. The location of the critical

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*qin denotes amount of BOD introduced per unit cross sectional area of the stream at the section of entry.

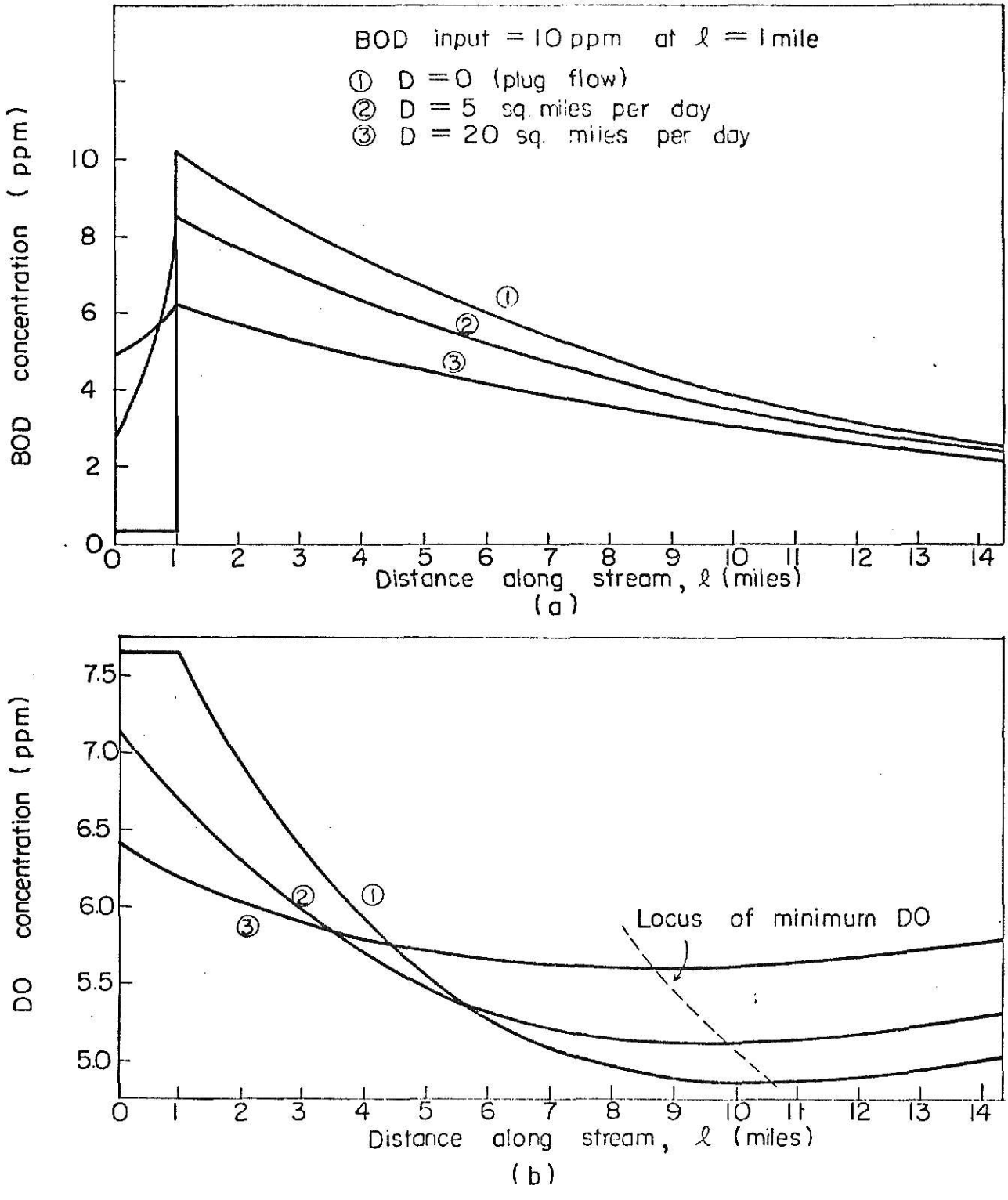


Fig. 3.3 Effect of variation in dispersion coefficient D, on BOD and DO concentration profiles.

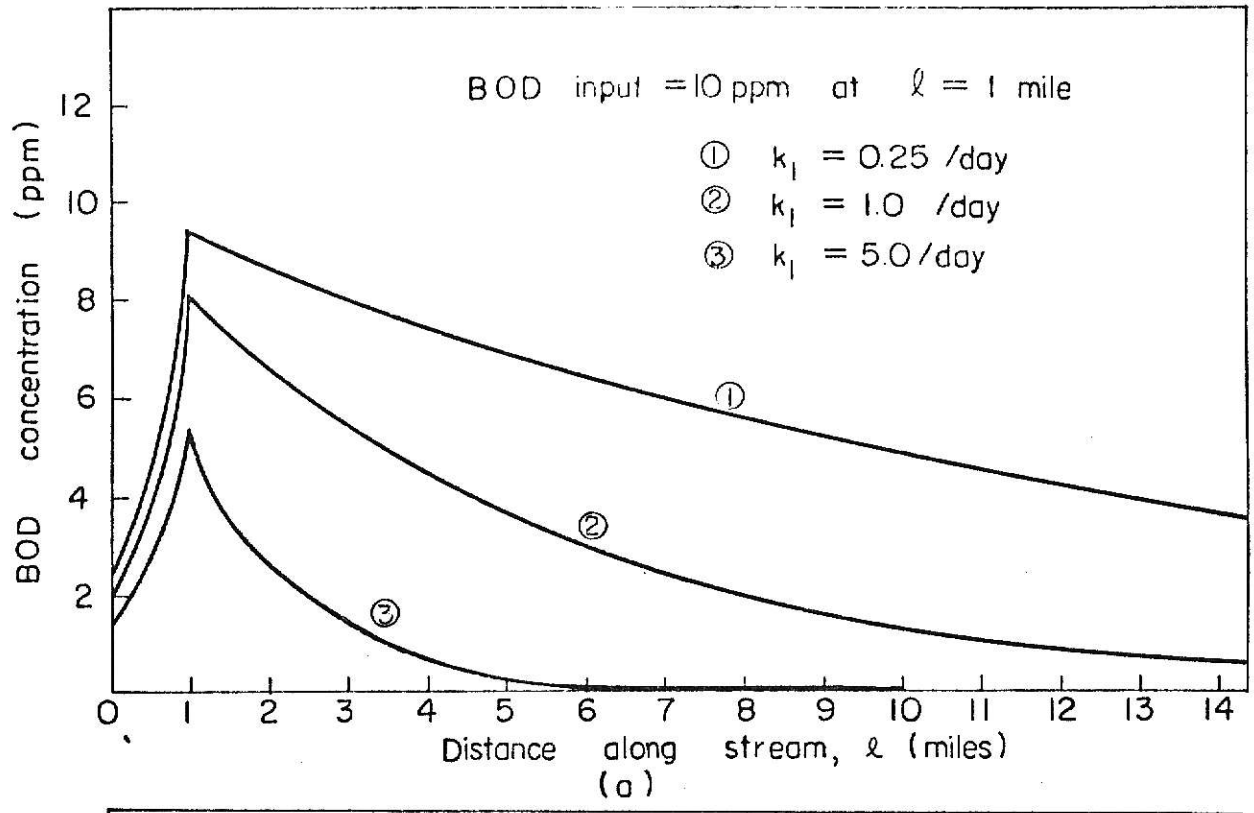
zone (minimum DO concentration) is also shifted towards the input.

The normal values of stream parameters used for illustration in Fig. 3.3 to 3.6 are, D=3 square miles/day, U=5 miles/day, k_1=0.4/day, k_2=0.5/day, k_3=0.15 day, La=0.1 ppm/day, D_B=0.1 ppm/day, O_s=8 ppm.

An increase in BOD decay rate k_1, is equivalent to increasing the rate of DO consumption for the BOD reaction (see Fig. 3.4). If the reaeration coefficient is not sufficiently large to cope with the DO depletion, it will result in critical DO concentrations. A good estimation of k_1 is important since the system is highly sensitive to k_1.

Figure 3.5 illustrates the sensitivity of the model to the reaeration coefficient, k_2. It can be seen that a considerable amount of interaction is involved between D, k_1 and k_2 which appear as product terms in exponential decay coefficients for the BOD and DO profiles as given in Equations (3.13), (3.14) and (3.15). For example, an increase in k_1 also results in decreasing the peak concentration of BOD at the outfall as seen in Fig. 3.4. A similar effect was observed on increasing D.

Figure 3.6 illustrates the effect of distributing the BOD load in reducing the maximum BOD concentration and in increasing the minimum DO concentration. Inefficient disposal of BOD input will lead to serious pollution problems as shown



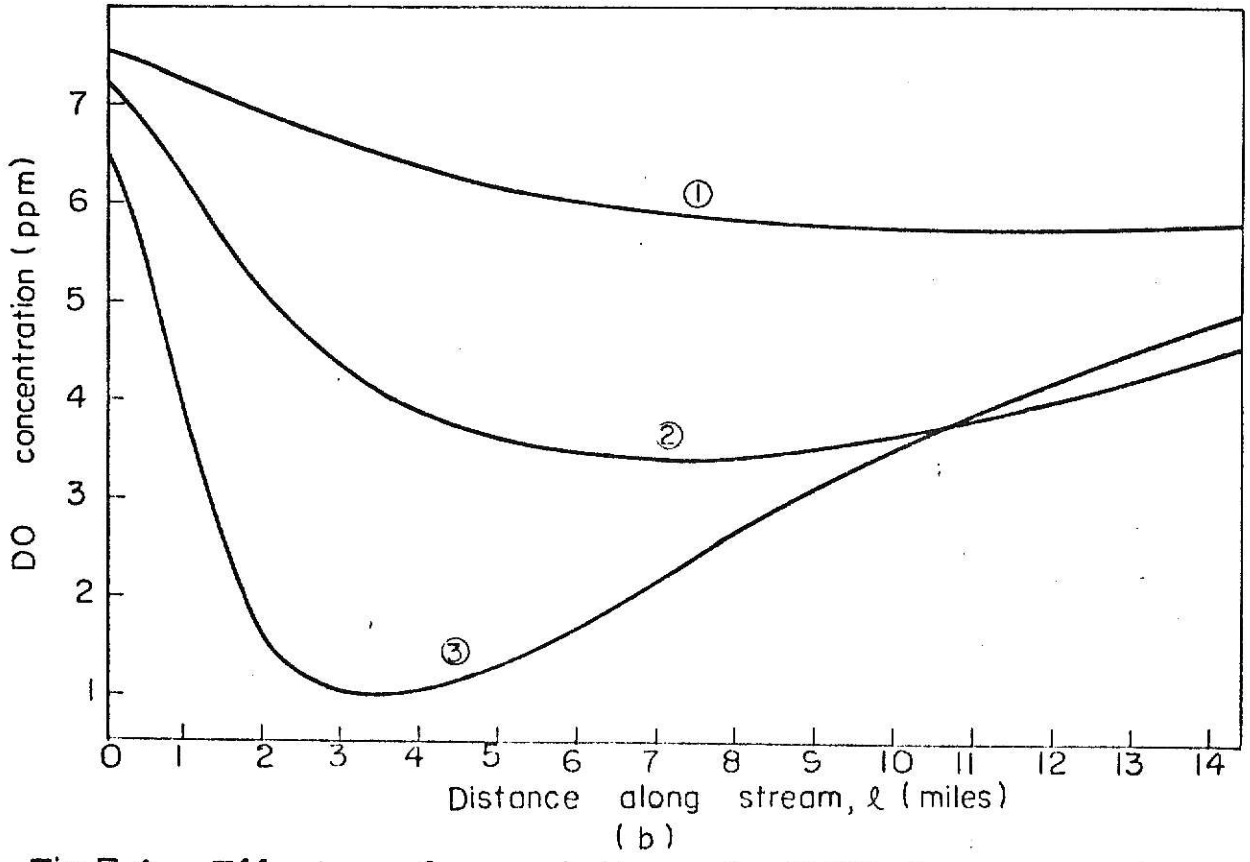


Fig.34. Effect of variation in BOD decay constant, k_1, on BOD and DO concentration profiles.

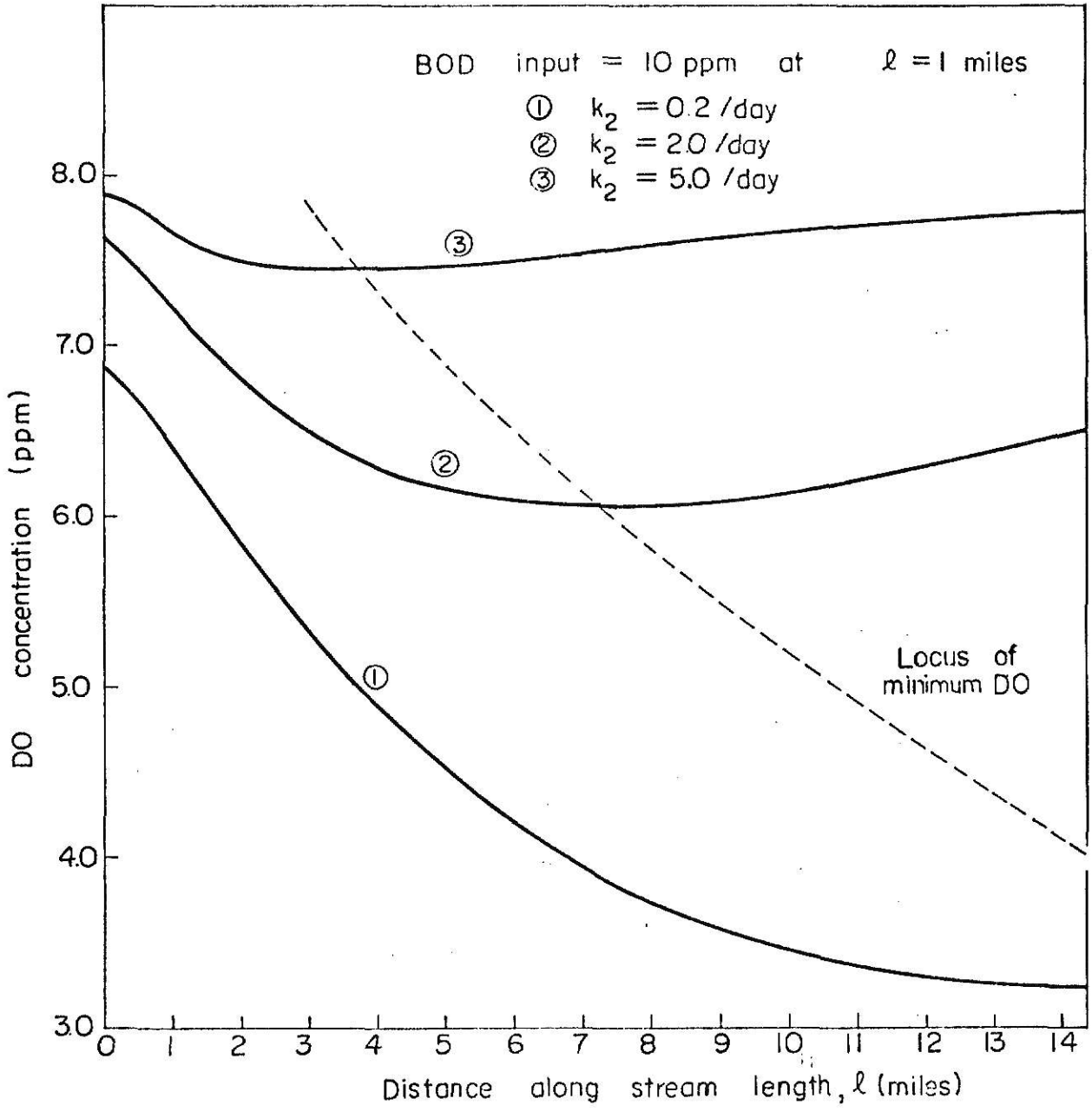
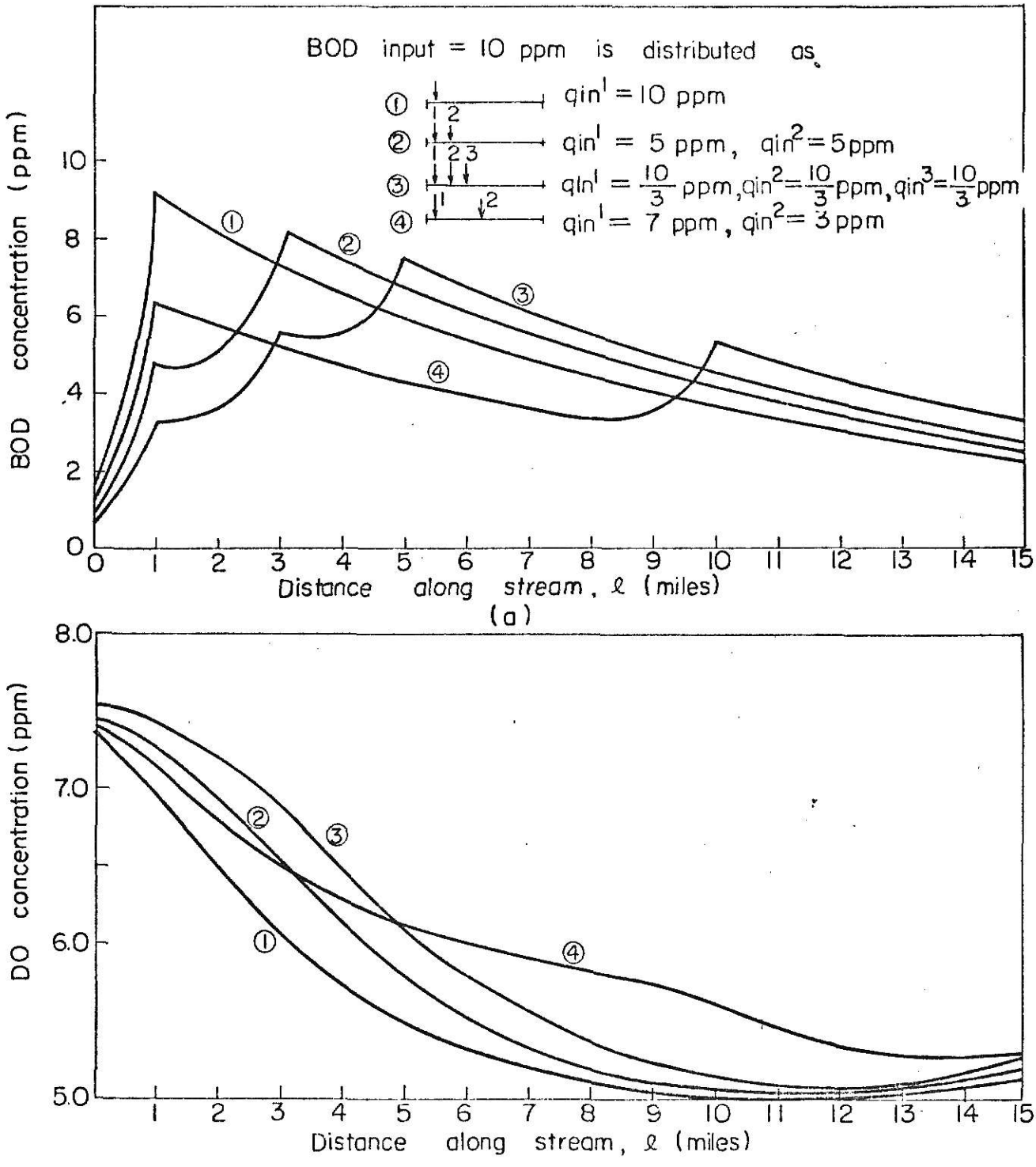


Fig 35. Effect of variation in reaeration coefficient, k_2, on the DO concentration profile.



Fig·3·6 Effect of distributing waste on BOD and DO concentration.

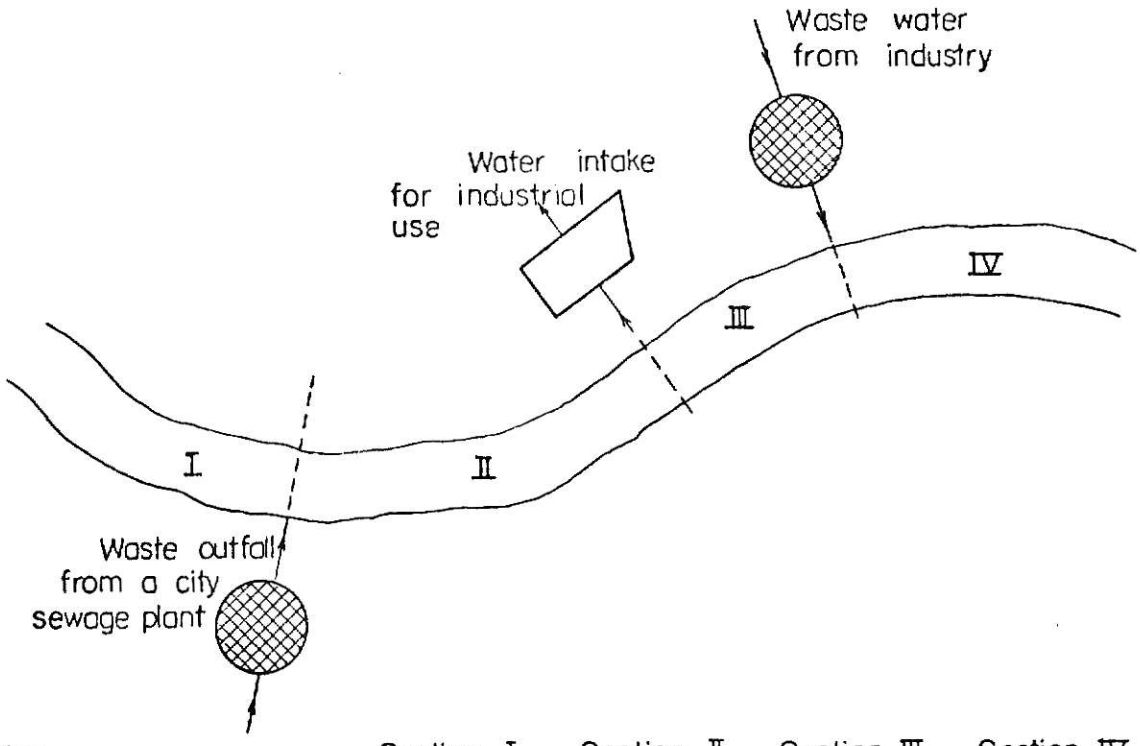
in Case 1 of Fig. 3.6. Diverting part of the waste as in Case 4 is more effective than Cases 2 or 3.

Figure 3.8 illustrates the solution for BOD and DO profiles for a case which may represent a simplified model of industrial and municipal waste inputs into a stream near an urban area (See Fig. 3.7). As shown in Fig. 3.7, waste enters the stream at 5 miles and 10 miles and there is a water intake at 8 miles. Cases 2 and 3 illustrate the effect of increasing the diffusion coefficient in different regions of the polluted stream. It is seen that increasing mixing in the earlier section of the reach leads to better distribution of BOD but the DO profile is not significantly changed. However, increasing D in the later sections leads to better DO concentrations. Such a study would be of value when water quality is to be controlled by introducing artificial mixing.

Artificial mixing will also have a favorable effect on the reaeration rate. This has been neglected in the above illustration. Similar studies can be made for studying the feasibility of flow augmentation and artificial reaeration, in controlling pollution.

CONCLUDING REMARKS

An analytical solution has been developed for BOD and DO profiles in streams with axial dispersion. The model may be



Parameter	Section I 0–5 miles	Section II 5–8 miles	Section III 8–10 miles	Section IV 10–20 miles
U (miles/day)	4	4.5	3.5	4
D (sq. miles/day)	3	4	3	3
k_1 (/day)	0.25	0.5	0.3	0.25
k_2(/day)	0.5	0.3	0.5	0.5
k_3(/ day)	0.15	0.1	0.15	0.15
La (ppm/day)	0.1	0.2	0.2	0.1
D_B (ppm/day)	0.1	0.2	0.2	0.2
O_S (ppm/day)	8.0	7.5	8.0	8.0
BOD concentration of waste discharged at the end of section. (ppm)	40	—	60	—
DO concentration of waste discharged at the end of section. (ppm)	5	—	6	—
Flow rate of outflow/unit area of stream (miles/day)	0.5	—	0.5	—
Flow rate of inflow/unit area of stream (miles/day)	—	1.0	—	—

Fig. 3·7. Hypothetical stream flowing near an urban area.

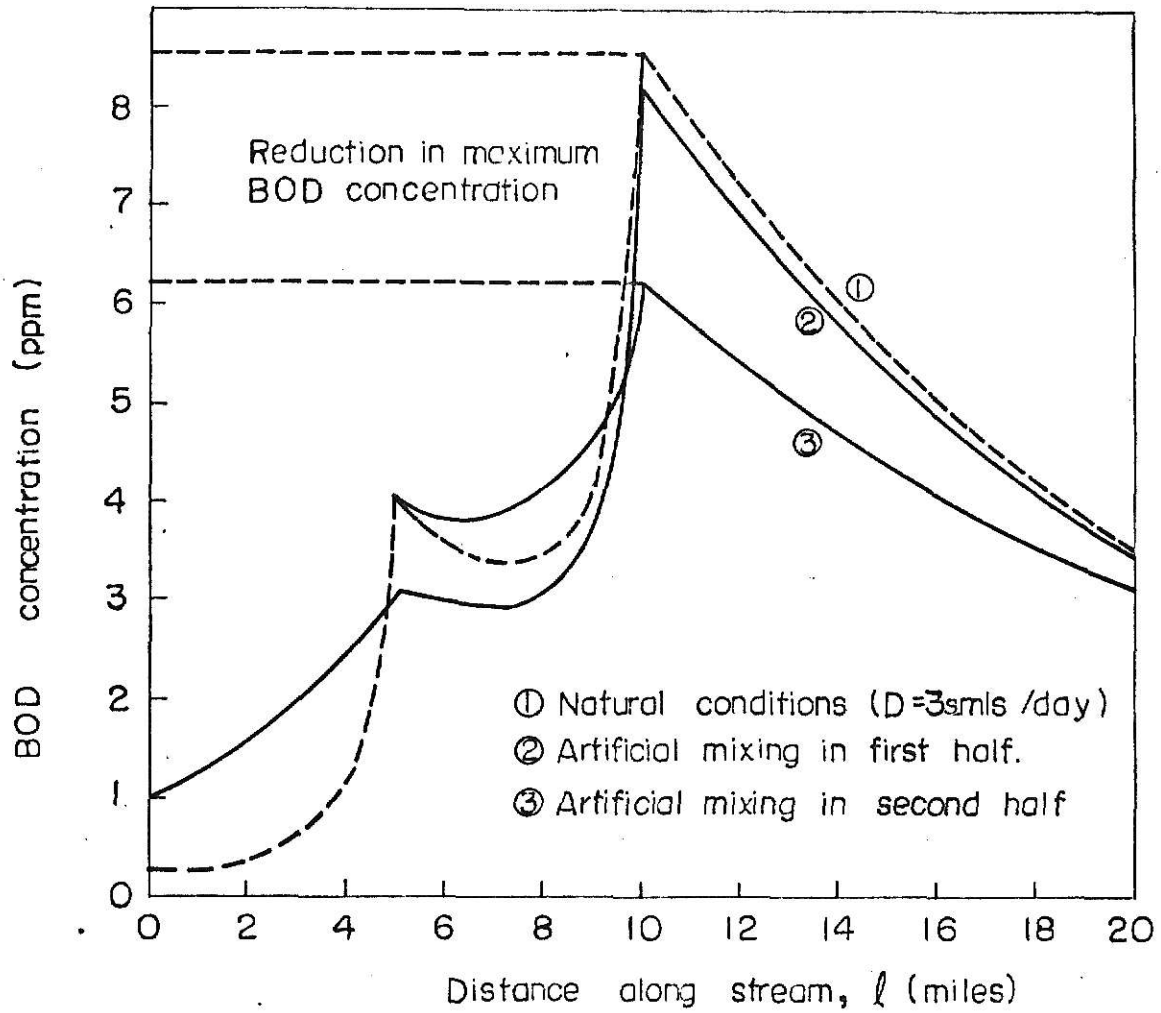


Fig.3·8a. Improvement in BOD level due to artificial mixing with D = 3 square miles/day under natural conditions and 20 square miles/day where artificial mixing is employed.

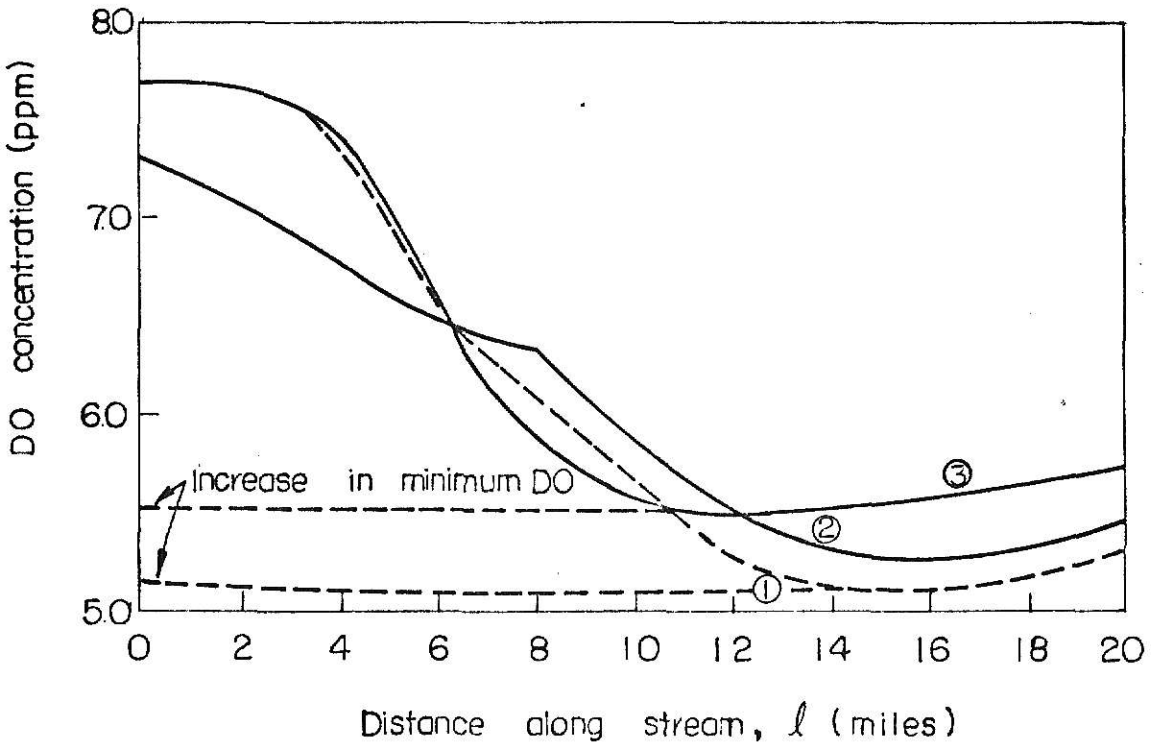


Fig.3·8b. Improvement in DO level due to artificial mixing with D = 3 square miles/day under natural conditions and 20 square miles/day where artificial mixing is employed.

used for optimization in water quality management and control. The modeling is more realistic than the plug flow model which is generally used for stream equations, because the concentration profiles are not discontinuous as in the latter case. The interaction between several waste discharges, the effect of upstream water quality depletion in the neighborhood of a downstream discharge, and the effect of changing stream properties may be simultaneously considered. Since the model includes continuous additions and removal of BOD and DO, it is suitable for representing situations where agricultural run-off is a major polluting factor.

Although the equations are derived under the assumption of first order reaction rates, the model may be easily extended to non-linear reaction models (3.1). Numerical approximations such as the collocation method (3.6) could be used to satisfy the differential equations as well as the boundary conditions.

The model could also be used to predict concentration profiles in tubular reactors with multiple inputs such as those to dilute heat effects or to increase the yield in multiple reactions.

It is realized that the deterministic approach used in the problem will have its limitations in modeling a stochastic environment. However, as indicated earlier, such analysis is useful for the purpose of design of water quality management models. It also gives insight into alternative methods for controlling water quality.

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NOTATION

C = concentration of a single reacting pollutant, ppm.

D = dispersion coefficient, sq. miles/day.

D_B = oxygen uptake due to benthic activity, ppm/day.

k_1 = rate constant for BOD decay, 1/day.

k_2 = rate constant for atmospheric reaeration, 1/day.

k_3 = rate constant for BOD sedimentation, 1/day.

L = concentration of BOD in stream at length, ℓ, ppm.

La = BOD addition due to local runoff and other factors, ppm/da

Lin = BOD concentration of incoming waste, ppm.

ℓ = length along stream, miles downstream.

ℓ_i = location of ith waste outfall/water intake, miles downstre

m = number of water intakes.

m_1, m_2 = roots of characteristic equation of Equation(3), 1/mile.

n = number of water outfalls.

n_1, n_2 = roots of characteristic equation of Equation(3), 1/mile.

O = dissolved oxygen concentration in stream at length, ℓ, ppm.

Oin = DO concentration of incoming waste, ppm.

O_s = DO saturation concentration, ppm.

qin = amount of BOD introduced per unit area of stream cross section, ppm.

S = sources or sinks of pollutant, ppm/day.

V = Volumetric flow rate of waste input per unit area of stream cross section, miles/day.

W = volumetric flow rate of water intake per unit area of stream cross section, miles/day.

Superscripts

APPENDIX 3A

The solution for the 4(m+n+1) arbitrary constants P_1^i, Q_1^i, P_2^i, Q_2^i, i=1, 2, ..., m+n+1 appearing in the expression for the BOD and DO concentrations is obtained from the 4(m+n+1) linear simultaneous equations resulting from substituting Equations (3.12) to (3.14) into the boundary conditions expressed by Equations (3.6) to (3.11). The substitution and simplification is presented below.

Since the solution for L^i is independent of O^i, the arbitrary constants P_1^i, Q_1^i, can be obtained by substituting Equations (3.12a), (3.13a), (3.14a) and (3.15a) in boundary conditions (3.6a) and (3.7a).

$$P_1^i e^{m_1^i \ell_i} + Q_1^i e^{m_2^i \ell_i} - P_1^{i+1} e^{m_1^{i+1} \ell_i} - Q_1^{i+1} e^{m_2^{i+1} \ell_i}$$

$$= - \frac{La^i}{k_1^i + k_3^i} + \frac{La^{i+1}}{k_1^{i+1} + k_3^{i+1}} \qquad (3A\text{-}1)$$

and

$$P_1^i(-m_1^i D^i A^i + U^i A^i) e^{m_1^i \ell_i} + Q_1^i(-m_2^i D^i A^i + U^i A^i) e^{m_2^i \ell_i} - P_1^{i+1}(-m_1^{i+1} D^{i+1}$$

$$A^{i+1} + U^{i+1} A^{i+1}) e^{m_1^{i+1} \ell_i} - Q_1^{i+1}(-m_2^{i+1} D^{i+1} A^{i+1} + U^{i+1} A^{i+1}) e^{m_2^{i+1} \ell_i}$$

$$= V^i A^i Lin^i - U^i L_p{}^i A^i + U^{i+1} L_p{}^{i+1} A^{i+1} \qquad (3A\text{-}2)$$

Similarly for a water intake at $\ell = \ell_j$, Equations (3.12a),

(3.13a), (3.14a), and (3.15a) are substituted in Equations (3.8a) and (3.9a) to obtain

$$P_1^j e^{m_1^j \ell_j} + Q_1^j e^{m_2^j \ell_j} - P_1^{j+1} e^{m_1^{j+1} \ell_j} - Q_1^{j+1} e^{m_2^{j+1} \ell_j}$$

$$= - \frac{La^j}{k_1^j + k_3^j} + \frac{La^{j+1}}{k_1^{j+1} + k_3^{j+1}} \qquad (3A-3)$$

and

$$-P_1^j m_1^j A^j e^{m_1^j \ell_j} - Q_1^j m_2^j A^j D^j e^{m_2^j \ell_j} + P_1^{j+1} m_1^{j+1} A^{j+1} D^{j+1} e^{m_1^{j+1} \ell_j}$$

$$+ Q_1^{j+1} m_2^{j+1} D^{j+1} A^{j+1} e^{m_2^{j+1} \ell_j} = 0 \qquad (3A-4)$$

The terms on the right hand side of Equation (3A-4) vanish, since by material balance, $V^j A^j L^j = A^j W^j L^{j+1} + A^{j+1} V^{j+1} L^{j+1}$.

The end conditions (3.10a) and (3.11a) imply that P_1^1 and Q_1^{n+m+1} must equal zero in order that L^1 and L^{m+n+1} may reach steady state values equal to L_p^1 and L_p^{n+m+1} respectively. Since m_1^1 is a negative root and m_2^{m+n+1} is a positive root, the terms $P_1^1 \exp(m_1^1 \ell)$ and $Q_1^{m+n+1} \exp(m_2^{m+n+1} \ell)$ in the expressions for L^1 and L^{m+n+1} will vanish as ℓ approaches $-\infty$ and $+\infty$ respectively.

In matrix notation, we can express the solution to the unknown column vector of coefficients $\{C_1\}$ given by,

$$\{C_1\} = [P_1^1 \; Q_1^1 \; P_1^2 \; Q_1^2 \ldots P_1^i \; Q_1^i \ldots P_1^{n+m+1} \; Q_1^{n+m+1}] \qquad (3A-5)$$

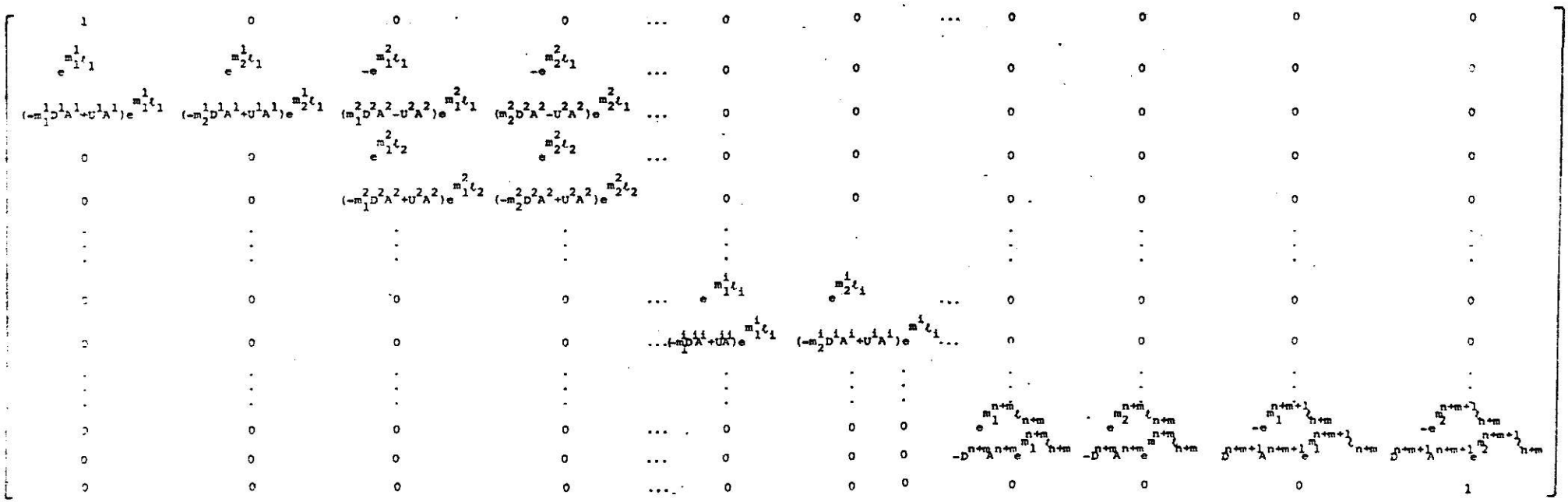
as

$$\{C_1\} = (X_1)^{-1} \{Y_1\} \qquad (3A\text{-}6)$$

where (X_1) is a square matrix of size $2(m+n+1) \times 2(m+n+1)$ formed by the coefficients of arbitrary constants and is given on page 56. In writing this matrix it is assumed that an intake of water is located at $\ell=\ell_{i-1}$ and an outfall at $\ell=\ell_i$ as shown in Fig. 3.1. Since the stream parameters are assumed to be known, the elements of (X_1) are defined. Also (X_1) is non-singular and hence can be inverted.

The vector $\{Y_1\}$ is given by

$$\{Y_1\} = \begin{Bmatrix} 0 \\ -L_p^i + L_p^2 \\ -V^1A^1Lin^1 - U^1A^1L_p^1 + U^2A^2L_p^2 \\ -L_p^2 + L_p^3 \\ -V^2A^2Lin^2 - U^2A^2L_p^2 + U^3A^3L_p^3 \\ \vdots \\ -L_p^i + L_p^{i+1} \\ -V^iA^iLin^i - U^iA^iL_p^i + U^{i+1}A^{i+1}L_p^{i+1} \\ \vdots \\ -L_p^{n+m} + L_p^{n+m+1} \\ 0 \\ 0 \end{Bmatrix}$$



Matrix (X_1) obtained from substitution of boundary conditions for computing the BOD profile in a stream. Note: The matrix for computing DO profile can be obtained by replacing m_1^i, m_2^i by n_1^i, n_2^i respectively, in the above matrix.

and $\{Y_2\}$ is given by,

$$
\left\{
\begin{array}{c}
0 \\
-O_p^1 + O_p^2 \\
-V^1A^1Oin^1 + D^1A^1\frac{dO_p^1}{d\ell} - D^2A^2\frac{dO_p^2}{d\ell} - U^1A^1O_p^1 + U^2A^2O_p^2 \\
-O_p^2 + O_p^3 \\
-V^2A^2Oin^2 + D^2A^2\frac{dO_p^2}{d\ell} - D^3A^3\frac{dO_p^3}{d\ell} - U^2A^2O_p^2 + U^3A^3O_p^3 \\
\vdots \\
-O_p^i + O_p^{i+1} \\
-V^iA^iOin^i + D^iA^i\frac{dO_p^i}{d\ell} - D^{i+1}A^{i+1}\frac{dO_p^{i+1}}{d\ell} - U^iA^iO_p^i + U^{i+1}A^{i+1}O_p^{i+1} \\
\vdots \\
-O_p^{n+m} + O_p^{n+m+1} \\
D^{n+m}A^{n+m}\frac{dO_p^{n+m}}{d\ell} - D^{n+m+1}A^{n+m+1}\frac{dO_p^{n+m+1}}{d\ell} \\
0
\end{array}
\right\}
$$

where O_p^is are determined using P_1^i, Q_1^i from Equation (3A-6).

APPENDIX 3B

We can apply the method of mathematical induction to prove that the BOD and DO concentration profiles are given by Equations (3.16a) and (3.16b) for the special case when several inputs of waste having negligible volume as compared to the main stream flow, are discharged at locations $\ell=\ell_i$, i=1, ...,n along the stream. In the derivation it is further assumed that the stream properties remain unchanged along the stream length.

The solution for the case of a single input at $\ell=\ell_1$ can be derived by a procedure similar to the one outlined in Appendix 3A as follows.

The general solution to the equations

$$D\frac{d^2L}{d\ell^2} - U\frac{dL}{d\ell} - k_1L + La = 0 \qquad (3B\text{-}1a)$$

and

$$D\frac{d^2O}{d\ell^2} - U\frac{dO}{d\ell} - k_1L + k_2(O_S-O)-D_B=0 \qquad (3B\text{-}1b)$$

with boundary conditions at $\ell=\ell_1$ given by

$$(L^1)_{\ell\to\ell_1^-} = (L^2)_{\ell\to\ell_1^+} \qquad (3B\text{-}2a)$$

$$(O^1)_{\ell\to\ell_1^-} = (O^2)_{\ell\to\ell_1^+} \qquad (3B\text{-}2b)$$

and

$$\left[\left(-D\frac{dL^1}{d\ell}\right)_{\ell\to\ell_1^-} + (UL^1)_{\ell\to\ell_1^-}\right] - \left[\left(-D\frac{dL^2}{d\ell}\right)_{\ell\to\ell_1^+} + (UL^2)_{\ell\to\ell_1^+}\right]$$

$$= -\,ULin^1 \qquad (3B\text{-}3a)$$

$$\left[-\left(D\frac{dO^1}{d\ell}\right)_{\ell\to\ell_1^-} + (Uo^1)_{\ell\to\ell_1^-}\right] - \left[-D\frac{dO^2}{d\ell}\right)_{\ell\to\ell_1^+} + (Uo^2)_{\ell\to\ell_1^+}\Big] = 0 \qquad (3B\text{-}3b)$$

can be written as

$$L^1 = \frac{La}{k_1+k_3} + P_1^1 e^{m_1\ell} + Q_1^1 e^{m_2\ell} \qquad (3B\text{-}4a)$$

$$O^1 = O_S - \frac{DB}{k_2} - \frac{k_1 La}{k_2(k_1+k_3)} - \frac{k_1 P_1^1 e^{m_1\ell}}{(k_2-k_1-k_3)} - \frac{k_1 Q_1^1 e^{m_2\ell}}{k_2-k_1-k_3}$$

$$+ P_2^1 e^{n_1\ell} + Q_2^1 e^{n_2\ell} \qquad (3B\text{-}4b)$$

for $-\infty < \ell < \ell_1$

and

$$L^2 = \frac{La}{k_1+k_3} + P_1^2 e^{m_1\ell} + Q_1^2 e^{m_2\ell} \qquad (3B\text{-}5a)$$

$$O^2 = O_S - \frac{DB}{k_2} - \frac{k_1 La}{k_2(k_1+k_3)} - \frac{k_1 P_1^2 e^{m_1\ell}}{(k_2-k_1-k_3)} - \frac{k_1 Q_1^2 e^{m_2\ell}}{k_2-k_1-k_3}$$

$$+ P_2^2 e^{n_1\ell} + Q_2^2 e^{n_2\ell} \qquad (3B\text{-}5b)$$

for $\ell_1 < \ell < +\infty$,

where P_1^1, Q_1^1, P_2^1, Q_2^1, P_1^2, Q_1^2, P_2^2 and Q_2^2 are arbitrary constants. We may note that on substituting the boundary conditions given by Equations (3B-2a), and (3B-2b) into Equations (3B-3a) and (3B-3b), the latter simplify to

$$-D\left(\frac{dL^1}{d\ell}\right)_{\ell \to \ell_1^-} + D\left(\frac{dL^2}{d\ell}\right)_{\ell \to \ell_1^+} = -U\ Lin^1 \qquad \text{(3B-6a)}$$

and

$$-D\left(\frac{dO^1}{d\ell}\right)_{\ell \to \ell_1^-} + D\left(\frac{dO^2}{d\ell}\right)_{\ell \to \ell_1^+} = 0 \qquad \text{(3B-6b)}$$

On applying the end conditions,

$$\frac{dL}{d\ell} = 0 \qquad \frac{dL}{d\ell} = 0$$

$$\text{at } \ell \to -\infty, \text{ and} \qquad \text{at } \ell \to +\infty$$

$$\frac{dO}{d\ell} = 0 \qquad \frac{dO}{d\ell}$$

we obtain,

$$P_1^1 = 0$$

$$P_2^1 = 0$$

$$Q_1^2 = 0, \quad Q_2^2 = 0.$$

To solve for the BOD profile, the expressions for L^1, L^2, given by Equations (3B-4a) and (3B-4b) are substituted into

the boundary conditions (3B-2) and (3B-6) to obtain

$$\frac{La}{k_1+k_3} + Q_1^1 e^{m_2 \ell_1} = \frac{La}{k_1+k_3} + P_1 e^{m_1 \ell_1}$$

and

$$-Dm_2 Q_1^1 e^{m_2 \ell_1} + Dm_1 P_1^2 e^{m_1 \ell_1} = U\ Lin^1$$

The two equations obtained above when solved for Q_1^1 and P_1^2 give,

$$Q_1^1 = \frac{U\ Lin^1 e^{-m_1 \ell_1}}{D(m_2-m_1)} \qquad (3B\text{-}7a)$$

$$P_1^2 = \frac{U\ Lin^1 e^{-m_2 \ell_1}}{D(m_2-m_1)} \qquad (3B\text{-}7b)$$

Hence, the BOD profiles in the two segments can be written as,

$$L^1 = \frac{La}{k_1+k_3} + \frac{U\ Lin^1}{D(m_2-m_1)} e^{m_2(\ell-\ell_1)}, \quad -\infty < \ell < \ell_1 \qquad (3B\text{-}8a)$$

$$L^2 = \frac{La}{k_1+k_3} + \frac{U\ Lin^1}{D(m_2-m_1)} e^{m_1(\ell-\ell_1)}, \quad \ell_1 < \ell < \infty \qquad (3B\text{-}8b)$$

Similarly on applying the boundary conditions for oxygen at $\ell=\ell_1$, we get

$$O_S - \frac{D_B}{k_2} - \frac{k_1 La}{k_2(k_1+k_3)} - \frac{k_1 U\ Lin^1}{D(k_2-k_1-k_3)(m_2-m_1)} + Q_2^1 e^{n_2 \ell_1}$$

$$= O_S - \frac{D_B}{k_2} - \frac{k_1 La}{k_2(k_1+k_3)} - \frac{k_1\ U\ Lin^1}{D(k_2-k_1-k_3)(m_2-m_1)} + P_2^2\ e^{n_1 \ell_1}$$

or

$$Q_2^1 e^{n_2 \ell_1} = P_2^2 \, e^{n_1 \ell_1}$$

and

$$-D\left[\frac{-k_1 U \, \mathrm{Lin}^1 m_2}{D(k_2-k_1-k_3)(m_2-m_1)} + n_2 Q_2^1 e^{n_2 \ell_1}\right] + D\left[\frac{-k_1 U \, \mathrm{Lin}^1}{D(k_2-k_1-k_3)(m_2-m_1)} + n_1 P_2^2 e^{n_1 \ell_1}\right] = 0$$

On solving for Q_2^1, P_2^1 we obtain

$$Q_2^1 = \frac{U \, \mathrm{Lin}^1 k_1 e^{-m_1 \ell_1}}{D(k_2-k_1-k_3)(n_2-n_1)} \qquad \text{(3B-9a)}$$

$$P_2^2 = \frac{U \, \mathrm{Lin}^1 k_1 e^{-m_2 \ell_1}}{D(k_2-k_1-k_3)(n_2-n_1)} \qquad \text{(3B-9b)}$$

On substituting the above expressions and Equations (3B-7a) and (3B-7b) into Equations (3B-4b) and (3B-5b) the expressions for oxygen concentration profiles are obtained as

$$O^1 = O_S - \frac{D_B}{k_2} - \frac{k_1 La}{k_2(k_1+k_3)} + \frac{U \, \mathrm{Lin}^1 k_1}{D(k_2-k_1-k_3)}\left(\frac{e^{n_2(\ell-\ell_1)}}{n_2-n_1} - \frac{e^{m_2(\ell-\ell_1)}}{m_2-m_1}\right)$$

$$-\infty < 0 < \ell_1 \qquad \text{(3B-10a)}$$

$$O^2 = O_S - \frac{D_B}{k_2} - \frac{k_1 La}{k_2(k_1+k_3)} + \frac{U \, \mathrm{Lin}^1 k_1}{D(k_2-k_1-k_3)}\left(\frac{e^{n_1(\ell-\ell_1)}}{n_2-n_1} - \frac{e^{m_1(\ell-\ell_1)}}{m_2-m_1}\right)$$

$$\ell_1 < \ell < \infty \qquad \text{(3B-10b)}$$

Thus Equation (16) is true for n=1.

Let us assume that it is valid for n=p. Then on introducing an additional input Lin^k between i and i+1th input two new segments denoted by (i, k) and k, i+1) are formed. To obtain an expression for the concentration profiles in these segments we proceed as follows. First, we notice that the solution for the original (i+1)th segment between i and i+1th input is given by,

$$L_p^{i+1} = \frac{La}{k_1+k_3} + \sum_{j=1}^{i} \frac{ULin^j e^{m_1(\ell-\ell_j)}}{D(m_2-m_1)} + \sum_{j=i+1}^{p} \frac{ULin^j e^{m_2(\ell-\ell_j)}}{D(m_2-m_1)} \qquad \text{(3B-11a)}$$

$$O_p^{i+1} = O_S - \frac{D_B}{k_2} - \frac{k_1 La}{k_2(k_1+k_3)} + \sum_{j=1}^{i} \frac{U\,Lin^j\,k_1}{D(k_2-k_1-k_3)} \left[\frac{e^{n_1(\ell-\ell_j)}}{n_2-n_1} - \frac{e^{m_1(\ell-\ell_j)}}{m_2-m_1}\right] + \sum_{j=i+1}^{p} \frac{U\,Lin^j k_1}{D(k_2-k_1-k_3)} \left[\frac{e^{n_2(\ell-\ell_j)}}{n_2-n_1} - \frac{e^{m_2(\ell-\ell_j)}}{m_2-m_1}\right] \qquad \text{(3B-11b)}$$

satisfies the differential equations, but not the boundary conditions for the new system. Hence it is a particular solution. Let the general solution to the segments (i, k) and (k, i+1) be represented by,

$$L^{i,k} = L_p^{i+1} + P_1^{i,k} e^{m_1(\ell-\ell_k)} + Q_1^{i,k} e^{m_2(\ell-\ell_k)} \qquad \text{(3B-12a)}$$

$$O^{i,k} = O_p^{i+1} + P_2^{i,k} e^{n_1(\ell-\ell_k)} + Q_2^{i,k} e^{n_2(\ell-\ell_k)} \qquad \text{(3B-12b)}$$

and

$$L^{k,i+1} = L_p^{i+1} + P_1^{k,i+1} e^{m_1(\ell-\ell_k)} + Q_1^{k,i+1} e^{m_2(\ell-\ell_k)} \qquad (3B\text{-}13a)$$

$$O^{k,i+1} = O_p^{i+1} + P_2^{k,i+1} e^{n_1(\ell-\ell_k)} + Q_2^{k,i+1} e^{n_2(\ell-\ell_k)} \qquad (3B\text{-}13b)$$

where $P_1^{i,k}$, $Q^{i,k}$, $P_2^{i,k}$, $Q_2^{i,k}$, $P_1^{k,i+1}$, $P_2^{k,i+1}$ are arbitrary constants. Equations (3B-8) and (3B-9) hold for any values of the inputs Lin^1 to Lin^p. In the special case when all the inputs are zero, except Lin^k, the solution must reduce to that for a single input obtained previously. By similar argument, therefore, the arbitrary constants,

$$P_1^{i,k},\ P_2^{i,k},\ Q_1^{k,i+1} \text{ and } Q_2^{k,i+1}$$

must equal zero.

In order that Equations (3B-12a) and (3B-13a) for BOD satisfy the boundary conditions at $\ell=\ell_k$, namely,

$$(L^{i,k})_{\ell\to\ell_k^-} = (L^{k,i+1})_{\ell\to\ell_k^+} \qquad (3B\text{-}14a)$$

and

$$\left(-D\frac{dL^{i,k}}{d\ell}\right)_{\ell\to\ell_k^-} - \left(-D\frac{dL^{k,i+1}}{d\ell}\right)_{\ell\to\ell_k^+} = U\ Lin^k \qquad (3B\text{-}14b)$$

$Q_1^{i,k}$ must equal $P_1^{k,i+1}$ (from (3B-14a)) and

$$Q_1^{i,k} = P_1^{k,i+1} = \frac{U\ Lin^k}{D(m_2-m_1)}$$

Similarly, for oxygen, on applying the boundary conditions

$$(O^{i,k})_{\ell \to \ell_k^-} = (O^{k,i+1})_{\ell \to \ell_k^+}$$

and

$$\left(-D \frac{dO^{i,k}}{d\ell}\right)_{\ell \to \ell_k^-} - \left(-D \frac{dO^{i,k}}{d\ell}\right)_{\ell \to \ell_k^+} = 0$$

we obtain

$$Q_2^{i,k} = P_2^{k,i+1} = \frac{U \, Lin^k}{D(n_2 - n_1)}$$

Hence the BOD and DO profiles for the $k+1^{th}$ segment after the k^{th} input in the new system is given by

$$L^{k+1} = \frac{La}{k_1+k_3} + \sum_{j=1}^{k} \frac{U \, Lin^j}{D(m_2-m_1)} e^{m_1(\ell-\ell_j)}$$

$$+ \sum_{j=k+1}^{p+1} \frac{U \, Lin^j}{D(m_2-m_1)} e^{m_2(\ell-\ell_j)}$$

$$O^{k+1} = O_S - \frac{D_B}{k_2} - \frac{k_1 La}{k_2(k_1+k_3)} + \sum_{j=1}^{k} \frac{k_1 U \, Lin^j}{D(k_2-k_1-k_3)} \left[\frac{e^{n_1(\ell-\ell_j)}}{n_2-n_1} - \frac{e^{m_1(\ell-\ell_j)}}{m_2-m_1}\right] + \sum_{j=k+1}^{p+1} \frac{k_1 U \, Lin^j}{D(k_2-k_1 k_3)} \left[\frac{e^{n_2(\ell-\ell_j)}}{n_2-n_1} - \frac{e^{m_2(\ell-\ell_j)}}{m_2-m_1}\right]$$

Therefore, if the solution is true for n=p, it is true for n=p+1. Since it is true for n=1, it is true for any positive integer n. Also, it can be verified that the solution satisfies the differential equation and the boundary conditions, and is, therefore, indeed the solution to the BOD and DO profiles for the system.

CHAPTER 4

MANAGEMENT OF OPTIMUM WATER QUALITY IN A STREAM

There exist a wide range of alternatives in the choice of water quality goals for regulating waste inputs to streams (4.1, 4.8, 4.10, 4.11, 4.14). A review of the current literature shows that the maximization of net benefits or minimization of the total cost benefit function for the system is accepted as a desirable objective in the managment of a regional water quality program (4.2, 4.10). The solution based on a policy of minimizing the total cost benefit function is difficult to implement in actual practice due to the well-known externalities in the system that lead to unfair treatment costs to individual waste dischargers. However, several methods for redistributing resources, such as a system of effluent charges (4.6, 4.8, 4.15) and the pooling of waste treatment at one place and discharging at several locations along the stream (4.12), are proposed in order to make such a solution feasible.

In the following analysis, the optimal waste discharge policy along a stream is investigated based on a program of minimizing the total cost benefit function. A steady state one dimensional dispersion model for determining the optimum biological oxygen demand (BOD) and dissolved oxygen (DO) profiles in the stream is employed. The mathematical form of the water quality benefit function is based on the general

response studied by Brown and Mar (4.2). The effect of longitudinal dispersion on the BOD and DO concentration profiles is considered. This factor would be significant especially when water quality is controlled by artificial mixing and aeration.

It may be noted that due to dispersion the waste introduced at a point not only degrades water quality downstream, but also affects water quality upstream. Hence the conventional dynamic programming approach is not applicable for finding the optimum solution. The feasibility of obtaining a solution by a search technique capable of handling the nonlinear constraints on water quality (4.4, 4.9) is studied.

THE MODEL

The system to be optimized consists of a stream along which are located several inputs consisting of treated effluents from sewage and industrial waste treatment plants, tributaries and run-offs. The quantity and quality of the incoming effluents to the treatment plants are assumed to be fixed and known. Water intakes are those needed for city or industrial usage. The stream water is treated by water treatment plants before being supplied for use in order to bring the water quality to a specified standard. Parts of the stream length are also used for recreation and navigation.

It is desired to maintain an optimum BOD and DO profile in the stream so as to minimize the total cost benefit function

comprising of the sum of costs of waste treatment, water treatment and other water quality control measures (such as flow augmentation and stream aeration), less the total benefits derived from maintaining a certain water quality in the stream. This objective has to be realized within the constraints imposed on the minimum allowable water quality in the stream.

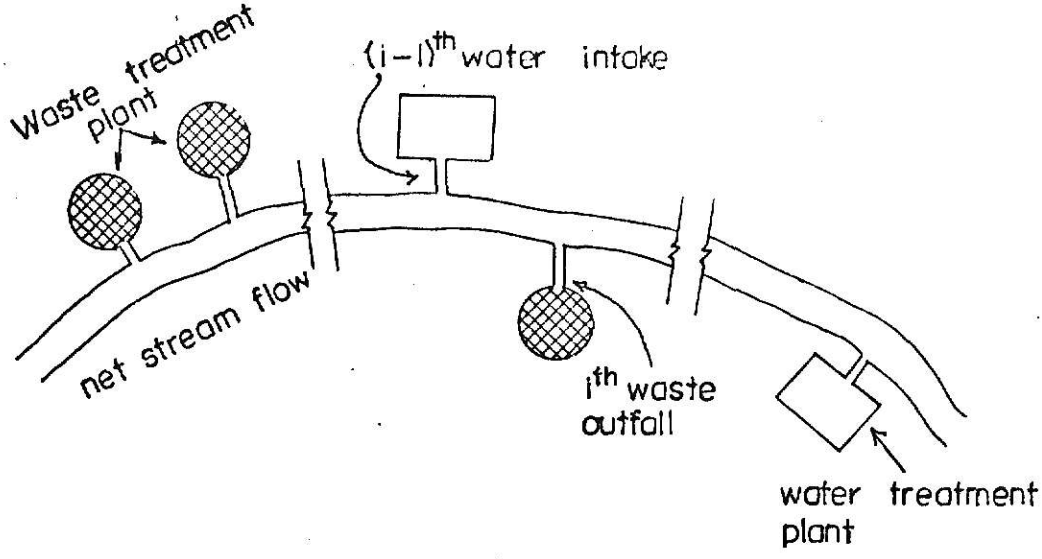
For the purpose of analysis, the stream is divided into several segments along its length (see Fig. 4.1). Each new segment begins just below the site of a water intake or waste outfall as described in Chapter 3. Additional segments may be made to incorporate changing properties in the stream. The cost benefit function for each segment is defined as,

$$S^i = \alpha^i C_T^i + \beta^i C_W^i - BF^i \qquad (4.1)$$

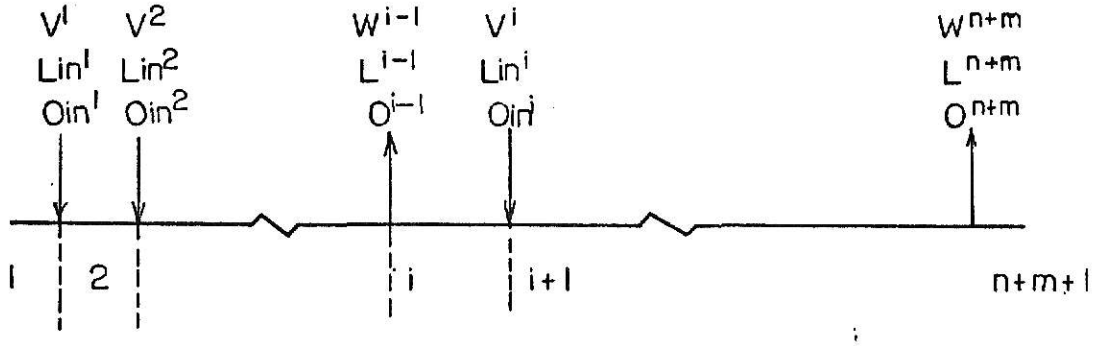
where, C_T^i is the cost of the waste treatment plant located at the end of the ith segment, C_W^i is the cost of water treatment, and BF^i is the benefit function for the ith segment. α^i and β^i denote the existence of a waste treatment plant or water treatment plant and take values 0 (nonexisting) or 1 (existing) BF^i is the total value estimated for the recreational and aesthetic benefits derived from the segment.

The total cost benefit functions for a stream consisting of n_T segments can be written as

$$S = \sum_{i=1}^{n_T} (\alpha^i C_T^i + \beta^i C_W^i - BF^i) \qquad (4.2)$$



(a) Steam with several waste inputs and water intakes.



(b) One dimensional representation of the above system.

Fig.4.1 Representation of a stream with several waste inputs and water intakes.

In addition, the constraints on water quality imply the existence of a discontinuous damage function, that is, at low levels of contamination the damages are small enough to be neglected, whereas at higher levels the damages are too large to be tolerated.

For the BOD and DO concentrations the required standards to be met are

$$BOD \leq L_A^i \tag{4.3}$$

$$DO \geq DO_A^i \tag{4.4}$$

where L_A^i and DO_A^i are the maximum allowable BOD concentration and minimum allowable DO concentration, respectively, and are specified for every segment.

Once the objective function and constraints have been defined, two main aspects have to be studied in order to solve the optimization problem. These are 1) the performance equations of the system, or the relationship between the state variables namely BOD and DO and the decision variables, usually degrees of treatment or amounts of waste input, and 2) the nature of the cost function and the benefit function as related to the variables of the system.

The mathematical model and its solution for BOD and DO concentration profiles in the stream is discussed in Chapter 3,

however, the particular form of the model to be used in this chapter is presented in the next section.

Mathematical Model for the BOD and DO Concentrations in the Stream

Based on the assumption of one dimensional, steady-state, uniform cross sectional area and uniform stream properties the equations representing the BOD and DO balances for a segment of a stream can be written as,

$$D^i \frac{d^2L^i}{d\ell^2} - U^i \frac{dL^i}{d\ell} - k_1^i L^i - k_3^i L^i + La^i = 0 \qquad (4.5)$$

and

$$D^i \frac{d^2O^i}{d\ell^2} - U^i \frac{dO^i}{d\ell} - k_1^i L^i + k_2^i (O_s^i - O^i) - D_B^i = 0 \qquad (4.6)$$

with the boundary conditions given as follows:

For an outfall of waste at $\ell = \ell_i$,

$$(L^i)_{\ell \to \ell_i -} = (L^{i+1})_{\ell \to \ell_i +} \qquad (4.7)$$

$$-D^i A^i \left(\frac{dL^i}{d\ell}\right)_{\ell \to \ell_i -} + U^i A^i (L^i)_{\ell \to \ell_i -} + V^i A^i L_{in}^i = -D^{i+1} A^{i+1} \left(\frac{dL^{i+1}}{d\ell}\right)_{\ell \to \ell_i} + U^{i+1} A^{i+1} (L^{i+1})_{\ell \to \ell_i +} \qquad (4.8)$$

$$(O^i)_{\ell \to \ell_i -} = (O^{i+1})_{\ell \to \ell_i +} \qquad (4.9)$$

$$-D^{i}A^{i}\left(\frac{dO^{i}}{d\ell}\right)_{\ell\to\ell_i-} + U^{i}A^{i}(O^{i})_{\ell\to\ell_i-} + V^{i}A^{i}O^{i}_{in} = -D^{i+1}A^{i}\left(\frac{dO^{i+1}}{d\ell}\right)_{\ell\to\ell_i+}$$

$$+ U^{i+1}A^{i+1}(O^{i+1})_{\ell\to\ell_i+} \tag{4.10}$$

For an intake of water at $\ell=\ell_j$,

$$(L^{j})_{\ell\to\ell_j-} = (L^{j+1})_{\ell\to\ell_j+} \tag{4.11}$$

$$(D^{j}A^{j}\left(\frac{dL}{d\ell}\right)_{\ell\to\ell_j-} - (U^{j}A^{j}L^{j})_{\ell\to\ell_j-} = W^{j}A^{j}L^{j} - D^{j+1}A^{j+1}\left(\frac{dL^{j+1}}{d\ell}\right)_{\ell\to\ell_j+}$$

$$+ U^{j+1}A^{j+1}(L^{j+1})_{\ell\to\ell_j+} \tag{4.12}$$

$$(O^{j})_{\ell\to\ell_j-} = (O^{j+1})_{\ell\to\ell_j+} \tag{4.13}$$

$$-D^{j}A^{j}\left(\frac{dO^{j}}{d\ell}\right)_{\ell\to\ell_j-} + U^{j}A^{j}(O^{j})_{\ell\to\ell_j-} + W^{j}A^{j}O^{j} - D^{j+1}A^{j+1}\left(\frac{dO^{j+1}}{d\ell}\right)_{\ell\to\ell_j+}$$

$$+ U^{j+1}A^{j+1}(O^{j+1})_{\ell\to\ell_j+} \tag{4.14}$$

The end conditions for the system are given by

$$\left(\frac{dL^{1}}{d\ell}\right)_{\ell\to-\infty} = 0 \tag{4.15}$$

$$\left(\frac{dO^1}{d\ell}\right)_{\ell \to -\infty} = 0 \tag{4.16}$$

and

$$\left(\frac{dL^{m+n+1}}{d\ell}\right)_{\ell \to +\infty} = 0 \tag{4.17}$$

$$\left(\frac{dO^{m+n+1}}{d\ell}\right)_{\ell \to +\infty} = 0 \tag{4.18}$$

where m is the number of water intakes, n is the number of waste outfalls and m+n+1 is the total number of segments in the stream. The last four equations denote that at a point far enough from a disturbance, the BOD and DO approach equilibrium values.

The solution to the above system is obtained in the form,

$$L^i = \frac{La^i}{k_1^i + k_3^i} + P_1^i \exp(m_1^i \ell) + Q_1^i \exp(m_2^i \ell) \tag{4.19}$$

$$O^i = O_s^i - \frac{D_B^i}{k_2^i} - \frac{k_1^i La^i}{k_2^i (k_1^i + k_3^i)} - \frac{k_1^i}{(k_2^i - k_1^i - k_3^i)} P_1^i \exp(m_1^i \ell)$$

$$- \frac{k_1^i}{(k_2^i - k_1^i - k_3^i)} Q_1^i \exp(m_2^i \ell) + P_2^i \exp(n_1^i \ell) + Q_2^i \exp(n_2^i \ell \tag{4.20}$$

where, the arbitrary constants P_1^i , Q_1^i are obtained by substituting Equation (4.13) in the boundary conditions given by Equations (4.7), (4.8), (4.11), (4.12) and end conditions

given by Equations (4.15) and (4.17). P_2^i , Q_2^i are obtained similarly, by substituting Equation (4.20) in the boundary conditions given by Equations (4.9), (4.10), (4.13), (4.14) and end conditions given by Equations (4.16) and (4.18). In matrix notation, the relation between the arbitrary constants and system parameters and inputs can be expressed by the following equations.

$$\{C_1\} = (X_1)^{-1} \{Y_1\} \tag{4.21}$$

$$\{C_2\} = (X_2)^{-1} \{Y_2\} \tag{4.22}$$

where $\{C_1\}$, $\{C_2\}$ are column vectors made up of the arbitrary constants P_1^i , Q_1^i and P_2^i , Q_2^i , respectively. (X_1), (X_2) are matrices comprising of the system parameters, which are generally assumed to be fixed. However, a parameter may also be treated as a decision variable by allowing a choice of location, reaeration rate, or diffusion rates, for example. The column vectors $\{Y_1\}$ and $\{Y_2\}$ are primarily made up of the inputs and outputs at the segment boundary.

Since the inputs to the stream depend upon the degrees of treatment, which are generally treated as decision variables, the arbitrary constants will be obtained as a linear combination of the sum of various input functions, as seen from Equations (4.21) and (4.22). The relationship between the state variables,

decision variables and parameters has now been completely defined.

Cost of Waste Treatment

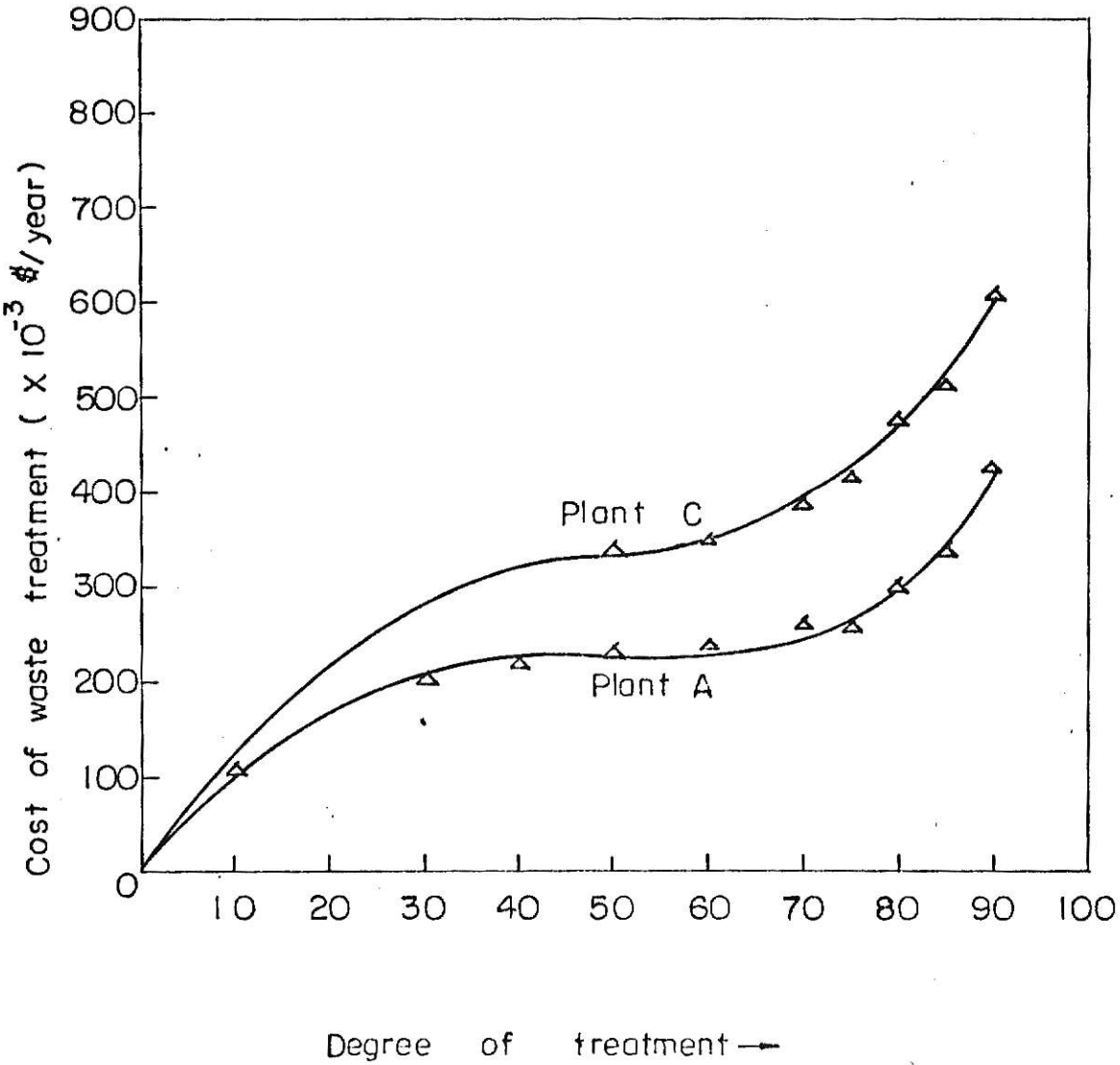
The cost of waste treatment depends upon several factors such as plant size, type of pollutant and degree of treatment as well as the specific process used for treating the waste. A relation between cost of treatment and process variables for a given plant size and design may be expressed as,

$$C_T^i = C_T^i (\nu^i, I^i, \phi^i, V^i) \tag{4.23}$$

where, ν^i is the waste characteristic index characterizing the composition and source of waste, I^i is the concentration of the key pollutant in the raw waste, ϕ^i is the degree of treatment, and V^i is the volume of waste handled. For a plant of fixed size and incoming effluent quality, a typical curve for BOD treatment which relates cost to the degree of treatment is shown in Fig. 4.2. A polynomial function relating cost and degree of treatment can be written as

$$C_T^i = a_T^i + b_T^i \phi^i + c_T^i (\phi^i)^2 + d_T^i (\phi^i)^3 + \ldots \tag{4.24}$$

Although a continuous polynomial approximation may not be strictly realistic, on fitting cost data used by Shih and DeFilippi (4.14) in their analysis, a 4th degree polynomial showed a good fit for their data (Fig. 4.2). Also, this



Fig·4·2. Cost curves for waste treatment plants

approximation is not necessary if it is not sufficiently accurate. As discussed later, since the approach to finding the solution consists of a discrete search, data in a tabular form can also be used.

Cost of Water Treatment

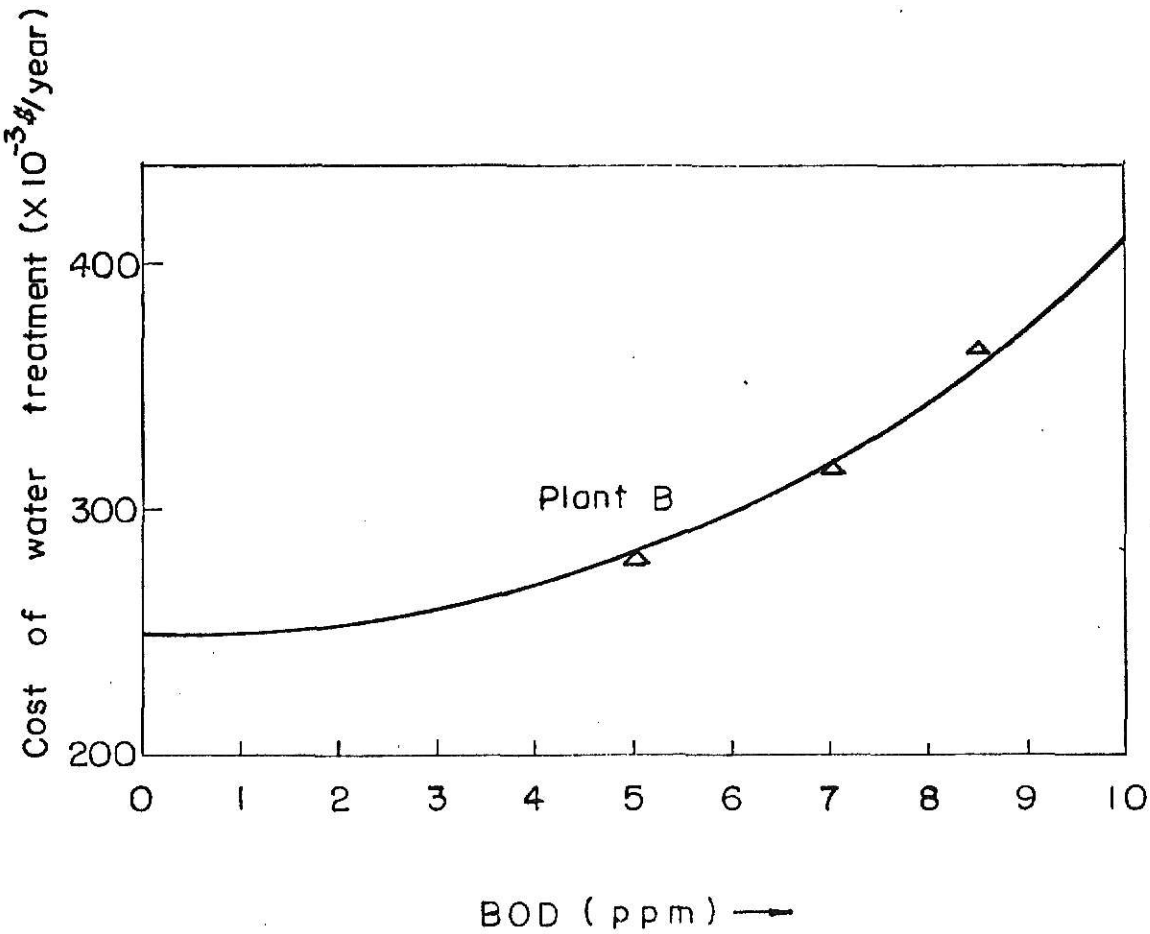
For a given process design, the cost of water treatment depends upon the nature of incoming water, purity of water required for consumption and size of the treatment plant. This can be expressed as

$$C_W^j = (\delta^j, L_q^j, W^j) \tag{4.25}$$

where δ^j is the maximum allowable concentration of the key pollutant in the purified water from the jth treatment plant, L_q^j is the pollutant concentration of the incoming water to the treatment plant, and W^j is the volume of water processed. Once again for a fixed size of plant and degree of purity, the cost can be related to the BOD concentration of the incoming water; that is,

$$C_W^j = a_W^j + b_W^j L_q^j + c_W^j (L_q^j)^2 + \ldots \tag{4.26}$$

On fitting the data used by Shih and DeFilippi it was found that a third degree polynomial gives a good representation of the cost function (Fig. 4.3).



Fig·4·3. Cost curve for water treatment plant.

Benefit Function

The recreational and aesthetic value of water has been described in terms of the price society is willing to pay to maintain a certain quality of water (4.10). Although an exact quantitative estimate of the benefit function is difficult to obtain as priorities change from person to person, Brown and Mar (4.2) have described the qualitative nature of the benefit function in terms of the major uses of stream water. In their analysis it is assumed that the total benefit function is comprised of the sum of three different responses 1) those that are independent of water quality, such as navigation 2) those that decrease linearly with water quality such as direct water use for consumption and 3) those that decrease rapidly at a given threshold of water quality such as aesthetic and recreational benefits. A typical response is shown in Fig. 4.4. A mathematical model chosen to represent the above response is,

$$BF^i = p^i + q^i \, L_A^i - \bar{L}^i + V^i \exp\left(-W^i \left(\frac{\bar{L}^i}{L_0^i}\right)\right) \qquad (4.27)$$

where, BF^i is the benefit per mile of stream within the reach i, p^i represents those benefits which are independent of water quality, q^i is a parameter for representing those benefits decreasing linearly with water quality and V^i and W^i are parameters for representing recreational and aesthetic benefits

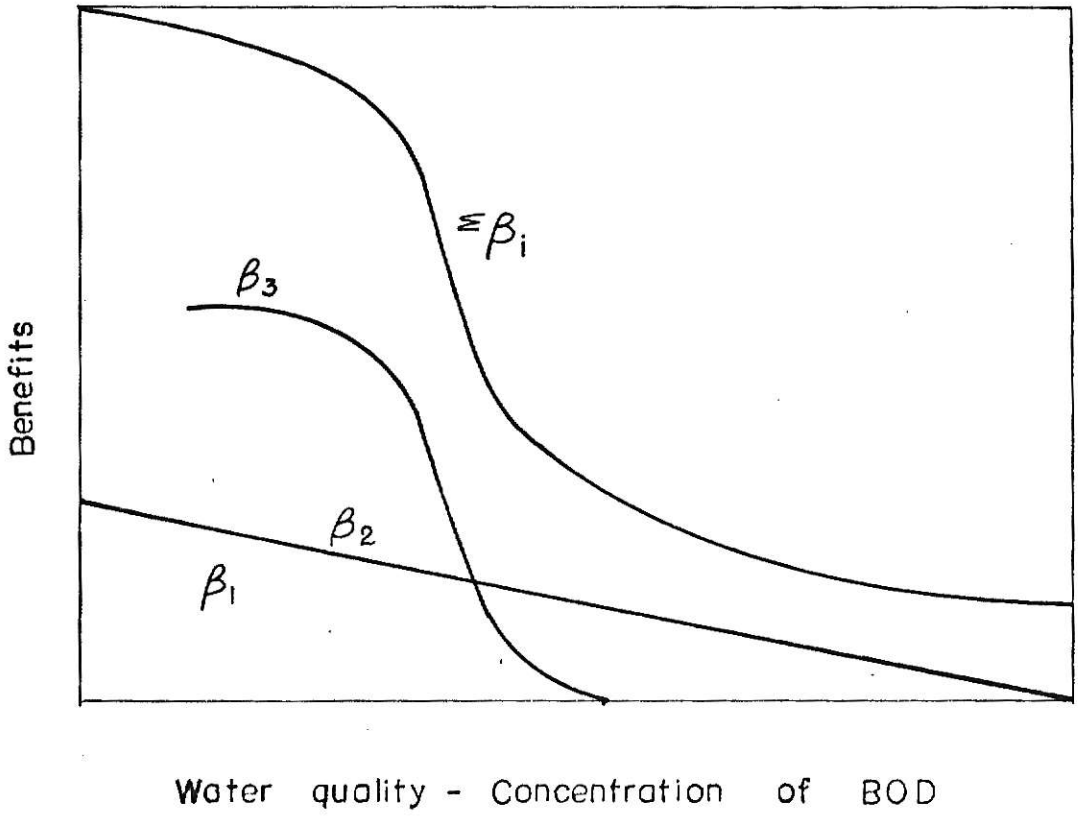


Fig 4.4 Variation of benefits from a water resource with water quality.

and L_o^i is the BOD concentration in the neighborhood of which the recreational and aesthetic benefits decrease sharply. The average water quality within a reach, $\bar{L}^i$ can be obtained from,

$$\bar{L}^i = \frac{1}{\ell_i - \ell_{i-1}} \int_{\ell_{i-1}}^{\ell_i} L^i d\ell \qquad (4.28)$$

Equation (4.28) can be integrated analytically by substituting for L^i from Equation (4.19).

It may be noted that in the present analysis, the effect of deterioration in water quality is also included in the water treatment cost equation. This effect may be deleted from the benefit function when all of the water intakes are considered individually; however, when only the large intakes are treated individually this term should be retained in the benefit function.

SOLUTION:

A search technique may be used in order to obtain the decision variables x_ℓ's, ℓ=1, 2, . . ., N, which would minimize the objective function given by Equation (4.2). The constraints on the state variables of the system (BOD and DO concentrations), are defined by Equations (4.3) and (4.4). The decision variables are the degrees of waste treatment at individual waste treatment plants and the location of the waste outfalls or water intakes.

It may be noted that the stagewise dynamic programming approach is not applicable for this system, since any change in decision variables, affects the BOD and DO concentration in the upstream as well as the downstream direction.

The sequential unconstrained minimization technique (SUMT) (4.4) is used to solve this constrained minimization problem. The original SUMT program has been modified by Lai (4.9) to perform a discrete search as well as to include a heuristic technique for pulling back the system variables into the feasible region.

The computational procedure for implementing the search based on the SUMT formulation is outlined below. Details of the modified SUMT program are given in Appendix 4B.

(1) An initial guess of decision variables is assumed.

(2) The BOD and DO concentration profiles for every segment are calculated using Equations (4.19), (4.20), (4.21), and (4.22).

(3) The constraints given by Equations (4.3) and (4.4) are checked for violation for every segment. The maximum concentration of BOD appears at the point of input, hence only these points need be checked. However, the minimum DO concentration for every segment is difficult to obtain analytically. A sequential search technique is used to find the minimum DO in

each reach, the details of which are given in Appendix 4A.

(4) If the constraints are violated the decision variables are readjusted till a feasible point is obtained. The computer flow chart for selecting a feasible starting point is shown in Appendix 4B.

(5) The objective function S, which is the sum of cost benefit for every segment S^i, i=1, . . .,n_T is next calculated. The cost of waste treatment C_T^i is directly related to the degree of treatment ϕ^i as shown in Equation (4.24). The BOD concentration at the intake of a water treatment plant L_q^i can be found from Equation (4.19) and then used in Equation (4.26) to obtain the cost of water treatment. The benefit function is calculated by use of Equations (4.27) and (4.28).

(6) The function

$$P = S + r_k^2 \sum \frac{1}{g_i} \tag{4.29}$$

is evaluated, where r_k is a monotonically decreasing sequence, g_i are the inequality constraints,

$$g_i \geq 0 \tag{4.30}$$

defined by taking appropriate signs in Equations (4.3) and (4.4).

(7) The P function is minimized by the Hooke and Jeeves pattern search. For every move into the infeasible region the system is brought back into the feasible region by use of the heuristic

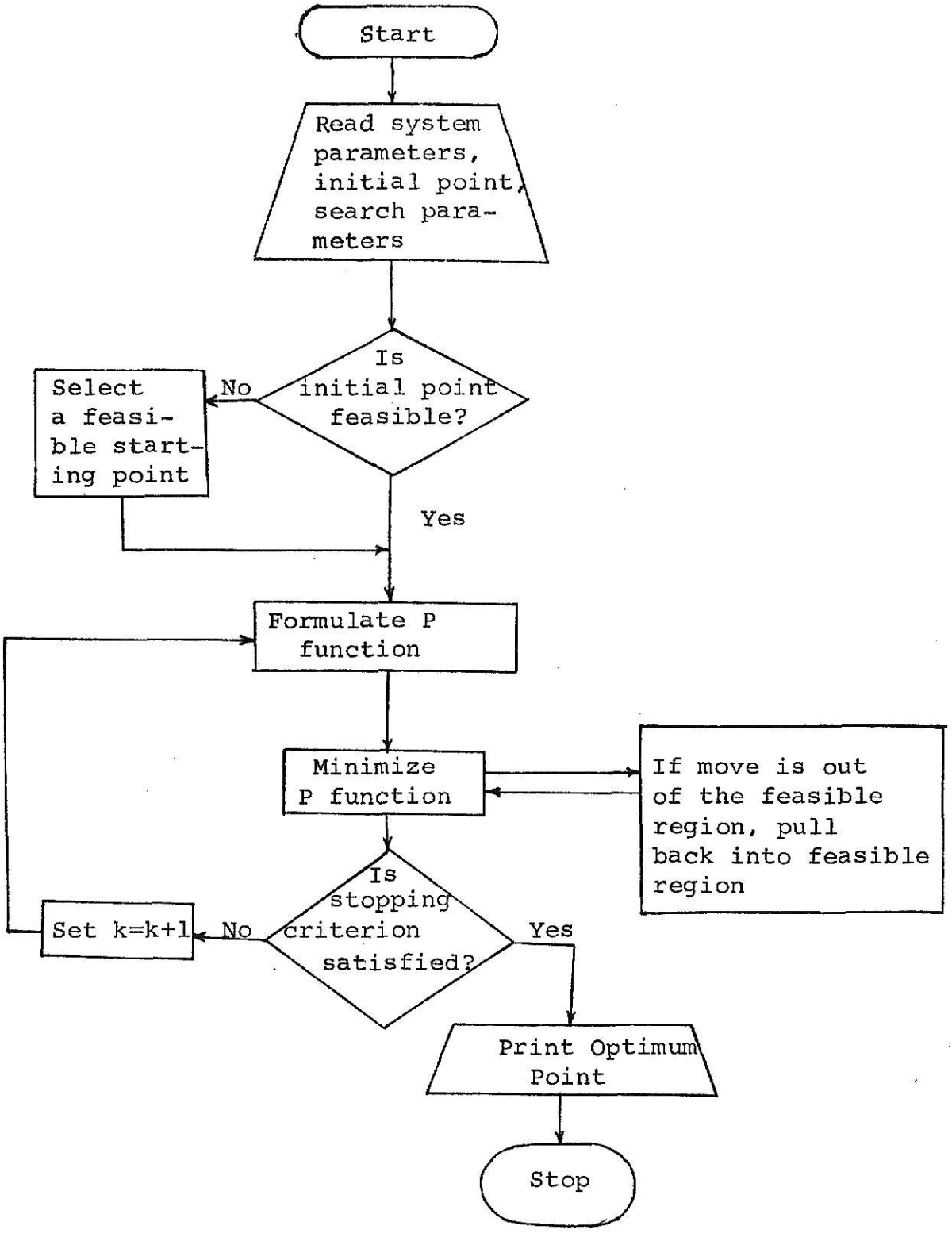


Fig. 4.5 General scheme for solving a constrained optimization problem by search technique (4.9).

technique described in Appendix 4B.

(8) The parameter r_k is reduced and a new P function formulated. The strategy is to reduce r_k to almost zero, so that the P function coincides with S.

(9) After every minimization of the P function, the stopping criterion, which is $\epsilon = | 1 - |\frac{S}{P}| | \leq \theta$ is checked.

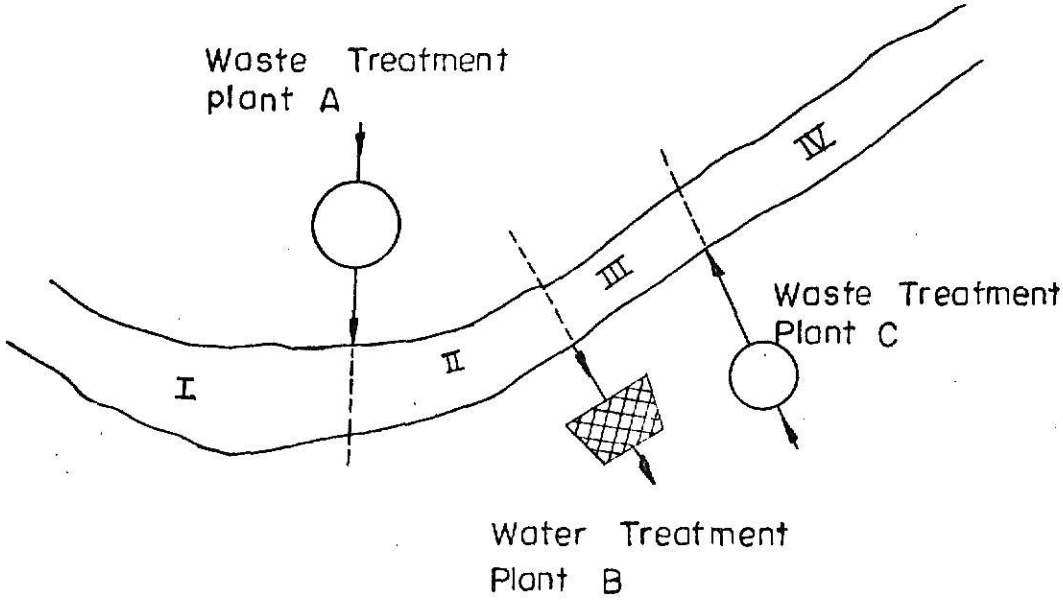
The basic condition necessary for the success of this search is that the function S must be convex and the constraints must form a convex set. A computer flow diagram describing the above procedure is shown in Fig. 4.5.

APPLICATION TO A HYPOTHETICAL PROBLEM

To illustrate the method outlined above, a section of a stream having two inputs of waste effluents from treatment plants and an intake of water which is treated at a water treatment plant before consumption is seen in Fig. 4.6. The quality and quantity of inputs to the treament plants, and the minimum acceptable quality from the water treatment plant are fixed. The nature of benefit function assumed is shown in Fig. 4.7. The cost curves for the waste treatment plants and water treatment plant (shown in Fig. 4.2 and 4.3) are obtained by fitting data used by Shih and DeFilippi (4.14) in their work. The polynominal coefficients for cost curves are given in Table 4.1. The inputs, outputs to the system, stream parameters and constraints are given in Table 4.2.

Table 4.1 Polynomial Coefficients for Cost of Waste and Water Treatment Plants

Waste treatment plant	a_T	b_T	c_T	d_T	e_T
Plant A (size 5 mgd)	1.54	1170.71	-1772.67	0.0	1205.8
Plant C (size 10 mgd)	0.0	1530.96	-2080.48	0.0	1389.33
Water treatment plant	a_W	b_W	c_W	d_W	e_W
Plant B (size 10 mgd)	248.34	0.0	1.22	0.04	0.0



Fig·4·6. Hypothetical stream with two waste treatment plants and one water treatment plant for optimization study.

Table 4.2 Stream parameters and inputs and outputs of treatment plants for hypothetical stream management problem described in Fig. 4.7

Parameter	Section-I 0-5 miles	Section-II 5-8 miles	Section III 8-10 miles	Section-IV 10-20 miles
U (miles/day)	4	4.5	3.5	4
D (sq miles/days)	3	4	3	3
k_1 (1/day)	0.25	0.5	0.3	0.25
k_2 (1/day)	0.5	0.3	0.5	0.5
k_3 (1/day)	0.15	0.1	0.15	0.15
La (ppm/day)	0.1	0.2	0.2	0.1
D_B (ppm/day)	0.1	0.2	0.2	0.2
O_S (ppm)	8.0	7.5	8.0	8.0
BOD conc. of waste coming to treatment plant (ppm)	-	120	-	200
DO conc. of waste coming to treatment plant (ppm)	5	-	6	-
Flow rate of waste to treatment plant/unit area of stream (miles/day)	0.5	-	0.5	-
Flow rate of water intake per unit area of stream (miles/day)	-	1.0	-	-

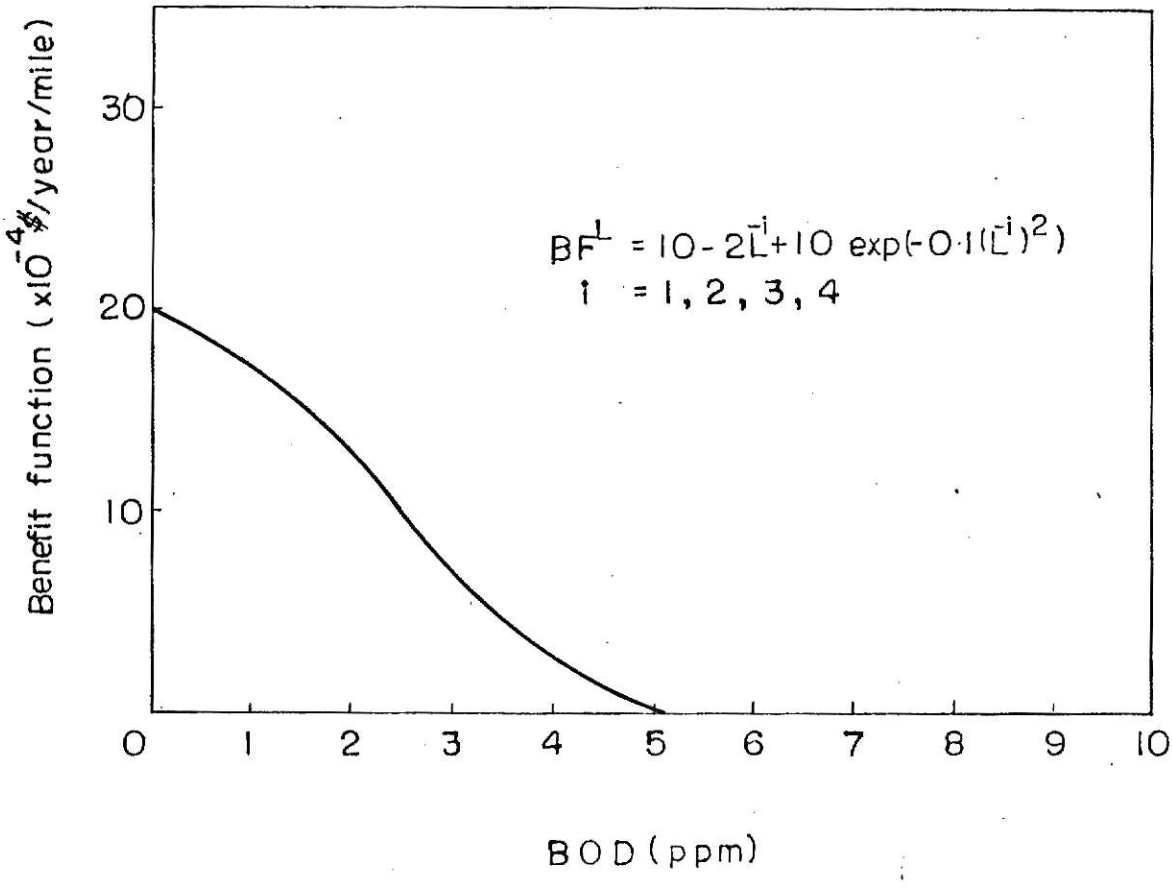
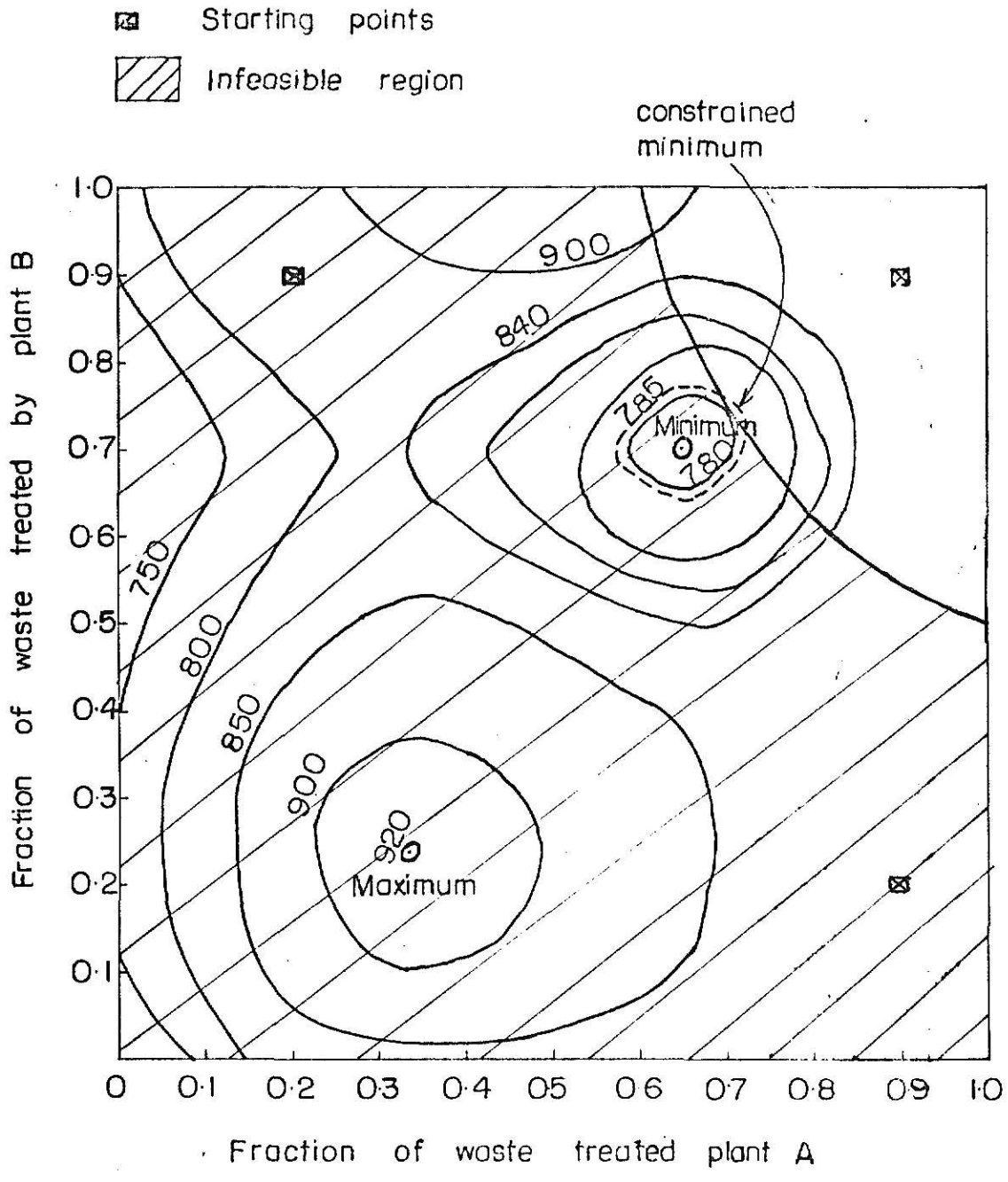


Fig. 4·7 Benefit function response for illustrative example.

First, to determine the nature of the response surface, the system is extensively simulated in order to get a response surface for the two decision variable case -- namely the degree of treatment at plants A and C (see Fig. 4.8). The response surface is highly undulated. The feasible region does not have any interior minima. By starting at several starting points within and outside the feasible region the search converged to a unique point, which is indeed the constrained minima. The computer time and number of iterations depend upon the starting point, stopping criterion and the initial step size. Since the search near the infeasible region is heuristic, it is not very efficient as seen in the results given in Table 4.3. Some experience in the handling of parameters r_k, and θ, is necessary to improve the search near the feasible region. The optimum BOD and DO profiles are plotted in Fig. 4.9.

The effect of degree of dispersion on the optimum BOD and DO profiles is investigated for the same system; however, in this problem the benefit function is not considered. Since the optimum solution in this problem lies on the DO constraint, and the minimum DO is increased by increasing the dispersion coefficient, one can expect lesser degrees of treatment required with high dispersion in order to maintain the minimum DO. The results are tabulated in Table 4.4. The optimum BOD and DO profile is plotted in Fig. 4.10. In each case several starting



Fig·4·8 Nature of the response surface for the problem discribed on fig. 4·7.

Table 4.3 Results of search to find optimum treatments for plants A and C.

Starting Point	No. of iterations for P function	Total no. of pattern moves	Final Point	Objective function value 1000$/yr	Stopping Criterion θ
0.6, 0.8 (infeasible) selected feasible point 0.65, 0.8	6	124	0.7337, 0.7212	784.8	0.0005
0.6, 0.8 (infeasible) selected feasible point 0.65, 0.8	8	165	0.7337, 0.72	784.6	0.0001
0.9, 0.2 (infeasible) selected feasible 0.1, 0.9	7	248	0.735, 0.72	785.6	0.0005
0.9, 0.9 (feasible)	7	148	0.735, 0.719	784.8	0.0005

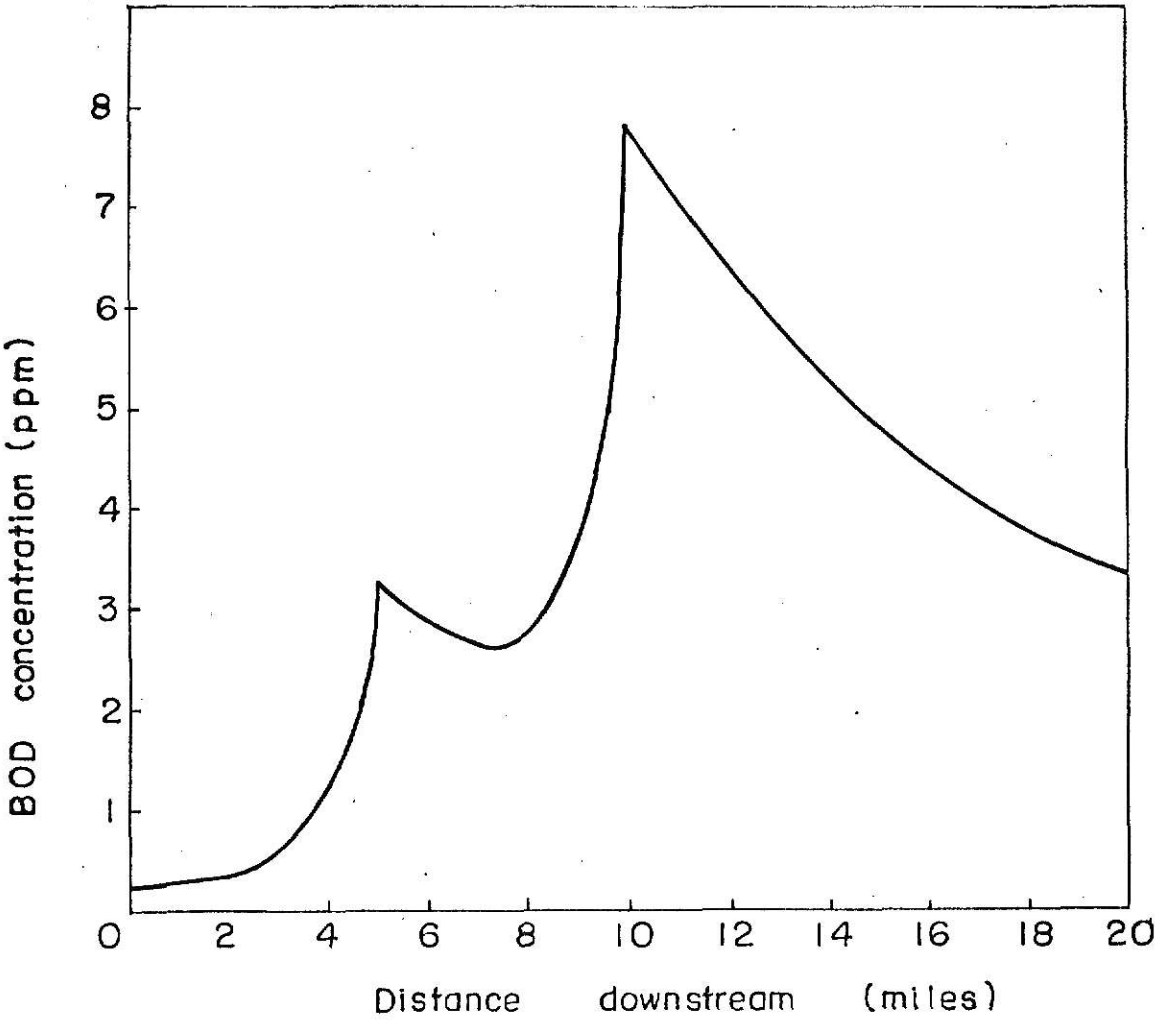
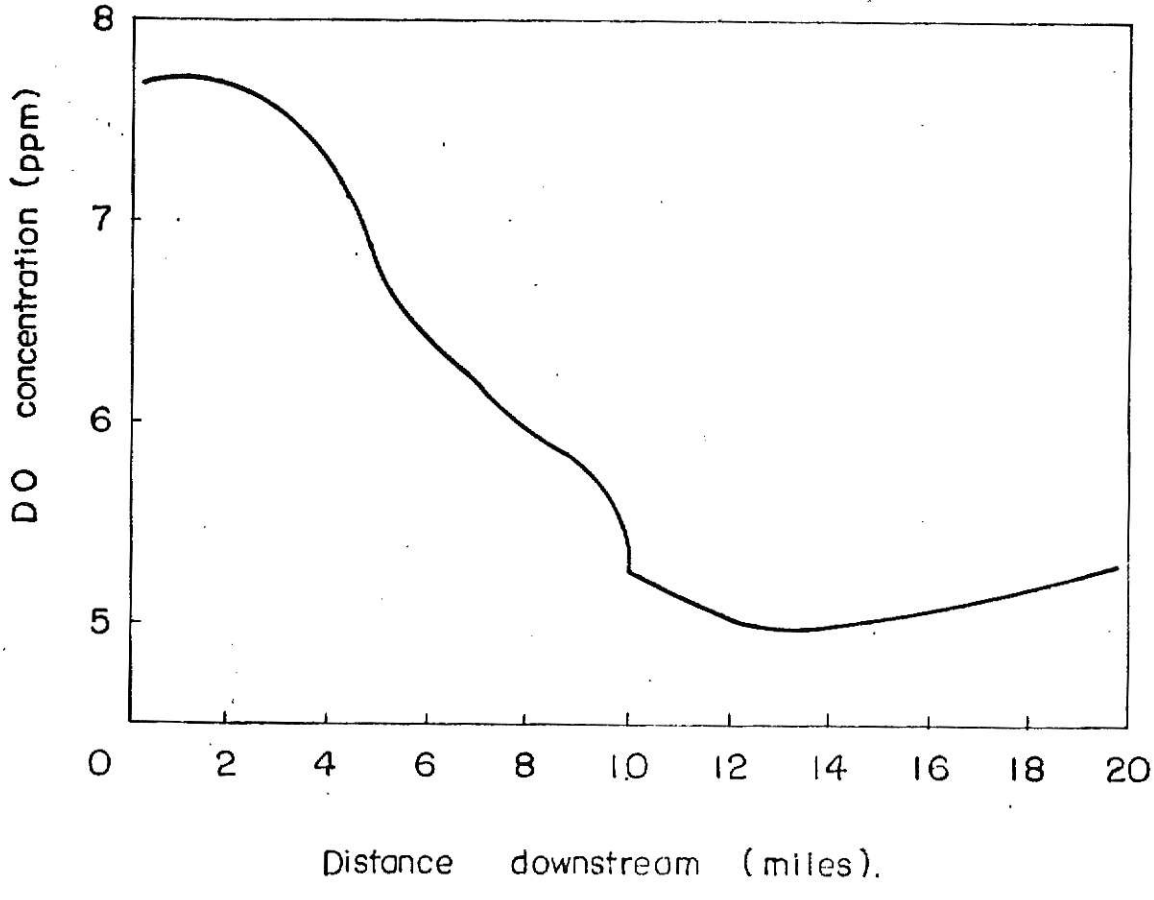


Fig.4·9a. Optimum BOD profile for problem described in fig.4·6.

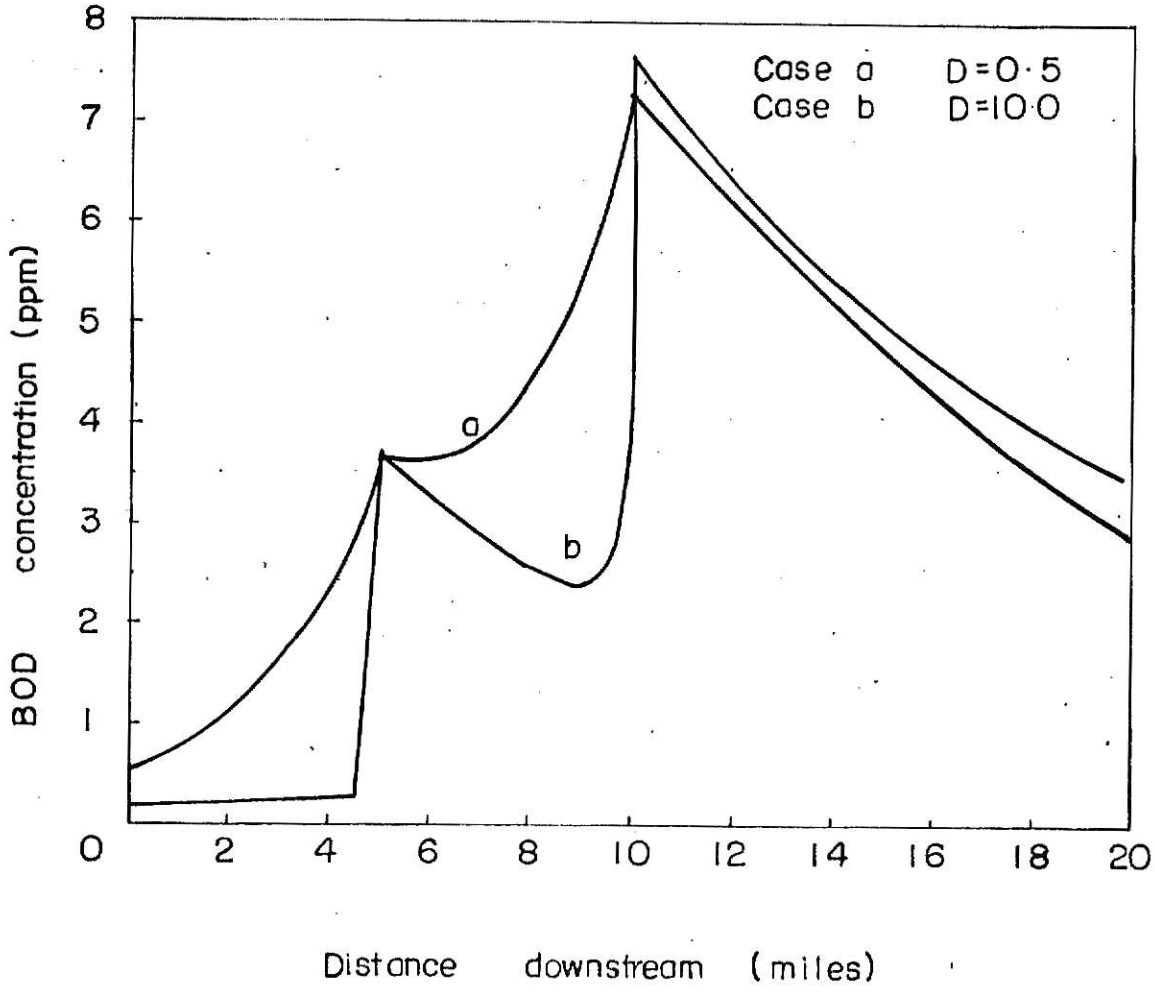


Fig·4·9b. Optimum DO profile for problem described in fig. 4·6.

Table 4.4 Optimum degrees of treatment for varying degrees of dispersion.

Dispersion Coefficient, D (sq. miles/day)	Optimum treatments		Objective function value (1000$/yr)
	Plant A	Plant C	
0.5	0.727	0.779	943.5
3.0	0.71	0.736	914.4
10.0	0.674	0.668	882.0

Note: The benefit function is assumed to be zero.



Fig·4·10a. Optimum BOD profile under different degrees of longitudinal mixing.

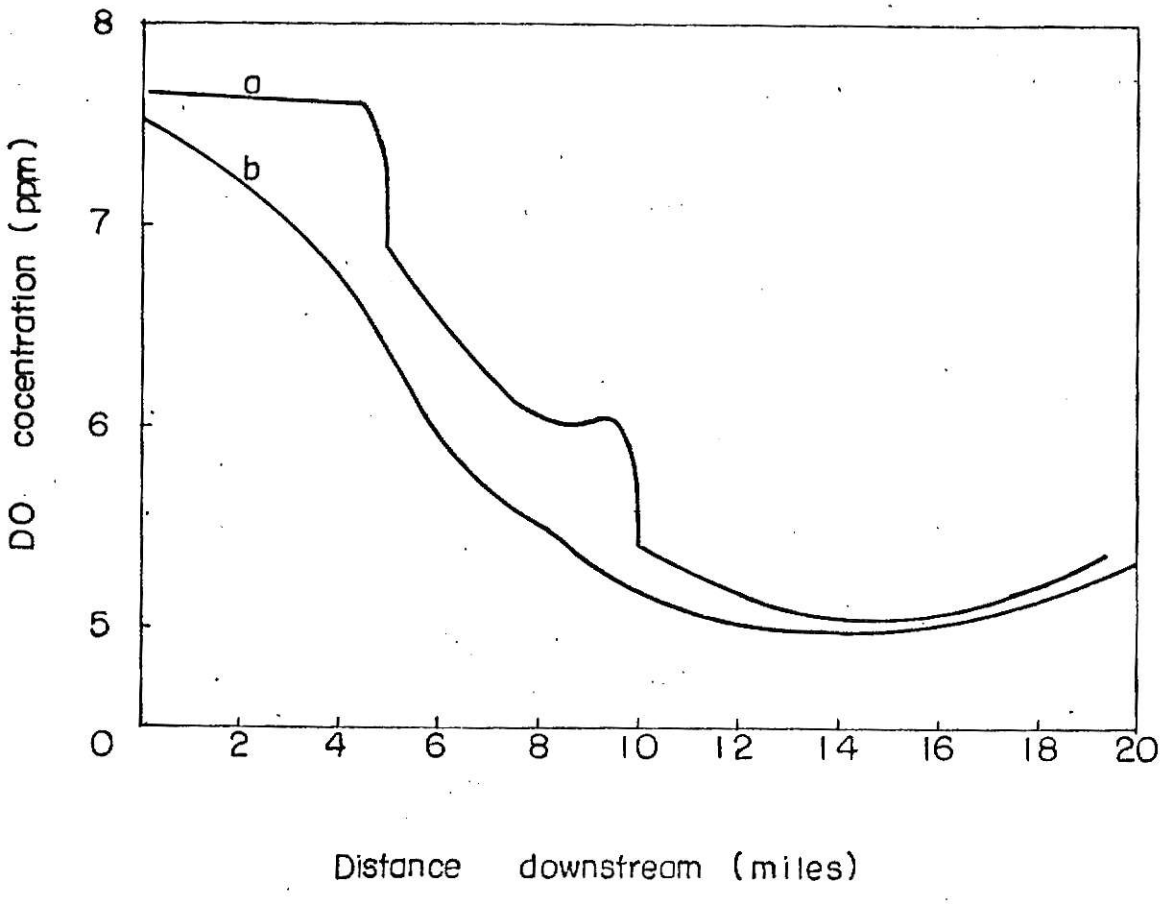


Fig.4·10b. Optimum DO profile under different degrees of longitudinal mixing.

points and stopping criterion were used to guarantee the optimum point.

The technique is also used to solve for the case of three equal inputs to three equally spaced treatment plants situated along a stream. The problem variables and parameters are described in Table 4.5. Fixing the treatments at points A and C at the optimum treatments found, the search is continued to locate plant B between A and C so as to require minimum treatment. By locating the plant B at 1.2 miles downstream of A, rather than 2.5 miles as located originally, the treatment required for plant B is reduced from 0.73 to 0.69. This case may represent the optimum location of a new waste discharge from the point of view of minimizing waste disposal cost.

Parameters such as dispersion coefficient D and reaeration constant k_2, could also be treated as decision variables in order to locate an optimum site for an artificial aerator, or the optimum amount of mixing in a preliminary analysis. Flow augmentation could also be considered as an additional measure for water quality control, if the cost data is available.

CONCLUDING REMARKS

An optimization model based on the policy of minimizing the cost-benefit function on a regional basis is formulated. The physical model for water quality in the stream includes the dispersion effect and hence is especially suitable for streams

Table 4.5 Stream parameters and inputs to waste treatment plants at the ends of Sections I, II and III for optimal location problem.

Parameter	Section I 0-5 miles	Section II 5-7 miles	Section III 7.5-10 miles	Section IV 10-20 miles
U (miles/day)	4.0	4.5	5.0	5.5
D (sq miles/day)	3	4	3	3
k_1 (1/day)	0.25	0.5	0.3	0.25
k_2 (1/day)	0.5	0.3	0.5	0.5
k_3 (1/day)	0.15	0.1	0.15	0.15
La (ppm/day)	0.1	0.2	0.2	0.1
D_B (ppm/day)	0.1	0.2	0.2	0.2
O_S (ppm)	8.0	7.5	8.0	8.0
BOD conc. of waste coming to treatment plant (ppm)	100	100	100	-
DO conc. of waste coming to treatment plant (ppm)	5	5	5	-
Flow rate of waste to treatment plant per unit area of stream cross section (miles/day)	0.5	0.5	0.5	-
Flow rate of water intake per unit area of stream cross section (miles/day)	-	-	-	-

Table 4.6 Optimum degrees of treatments for problem described in Table 4.5.

Optimal Treatments			Objective Function Value
Plant A	Plant B	Plant C	
0.684	0.731	0.65	720.23

with low flow, and streams for which artificial turbulence is introduced to induce aeration and mixing.

A search scheme is investigated for obtaining the optimum solution. The scheme is versatile as the number and type of decision variables can be increased without any additional effort in programming. The number of state variables does not pose any additional difficulty as in dynamic programming. The constraints on DO have been known to create considerable computing difficulty in the dynamic programming solution of the plug flow case which has been investigated before by several workers (4.10, 4.15).

The scheme has its limitations however, especially since the system is highly non-linear. As the number of decision variables increases, there may be more than one local minimum in the feasible region, and the search may converge to a sub-optimal point. Also the feasible region may not be a continuous space. In order to overcome this problem, a number of widely distributed starting points may be selected. This would involve more computer time. The heuristic technique is not very reliable in the near feasible region, and manipulation of the parameters to aid the search may be required in order to obtain an optimum solution lying on the boundary of the feasible region.

An iterative dynamic programming approach may be a possible

approach to the problem. Since the effect of a downstream discharge on upstream water quality is small, it may be neglected in the first iteration. The solution thus obtained can be made use of, in the next iteration and the previous decisions corrected. The process may be repeated till there is very slight improvement in the solution.

The general model proposed may require additional constraints in order to obtain a fair allocation of policies to individual waste discharges. It is well-known that the problem of stream pollution involves considerable externality (4.8). For example, the consequence of insufficient waste treatment by a discharger upstream, penalizes water users and waste dischargers downstream because of low quality of water available to water users, and higher treatments required to maintain stream standards for waste dischargers. On the other hand, the allocation of treatments based on the least cost objective, may lead to the benefit of the downstream waste dischargers. A compromise may be obtained, if a minimum amount of treatment is enforced on all waste treatment plants. The constrained optimization scheme proposed, can easily incorporate this modification in policy by imposing additional constraints on the decision variables, namely degrees of treatment.

As mentioned earlier, the mathematical form of the benefit function may not be rigorous. However, it would be of value in

examining changes in optimal solutions due to changes in the nature of the benefit function caused by priorities and aesthetic values a particular community may have, as well as due to recreational uses of the stream.

Introducing financial constraints (such as limiting total funds available for waste treatment) and changing the objective function (such as maximizing the benefit to cost ratio) may be some of the studies that can be made for a given system.

A sensitivity analysis with respect to the level of flows and temperature (since the model is isothermal) would be useful for cases in which seasonal variations in these two factors are large. This would be useful for examining how good the steady state assumption is. Even though the model may not be directly applicable, it would be useful to study the effect of alternative means of controlling water quality.

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NOTATION

a,b = end points for the interval for Golden Section method, miles.

$a_T^i, b_T^i, c_T^i, d_T^i, e_T^i$ = polynomial coefficients for cost function of waste treatment plant located at the end of the ith reach, 1000$/year.

$a_w^i, b_w^i, c_w^i, d_w^i, e_w^i$ = polynomial coefficients for cost function of water treatment plant located at the end of ith reach, 1000$/year.

B = tolerance limit for violation of inequality constraint (dimensionless).

BF^i = benefit function for the ith reach, 1000$/year/mile.

C_T^i = cost of the ith waste treatment, 1000$/year.

C_w^i = cost of the jth water treatment, 1000$/year.

D^i = dispersion coefficient in the ith reach, dq. miles/day.

D_B^i = oxygen uptake due to benthic activity in the ith reach, ppm/day.

DO_A^i = minimum allowable DO concentration in the ith reach, ppm.

k_1^i = rate constant for BOD decay, 1/day.

k_2^i = rate constant for reaeration, 1/day.

k_3^i = rate constant for BOD sedimentation, 1/day.

g_i = ith inequality constraint, arbitrary.

h_j = jth equality constraint, arbitrary.

L = BOD concentration, ppm.

L_A^i = maximum allowable BOD concentration in the ith reach, ppm.

La^i = BOD addition due to the local run off, ppm/day.

Lin^i = BOD concentration of waste effluent from treatment plant, ppm.

L_q^i = BOD concentration of water to water treatment plant, ppm.

ℓ = length along stream, miles.

ℓ_i = location of the ith waste outfall/water intake, miles.

m = number of water intakes.

m_1, m_2 = roots of characteristic equation of Equation (5), 1/mile.

n = number of waste outfalls.

n_1, n_2 = roots of characteristic equation of Equation (6), 1/mile.

n_T = total number of segments.

O = DO concentration, ppm.

O_m^i = DO concentration of effluent from waste treatment plant, ppm.

O_s^i = oxygen saturation value, ppm.

$P(x,r_k)$ = P function value at the kth iteration, 1000$/year.

q^i = gradient factor for benefits diminishing linearly with water quality, 1000$/year/ppm/mile.

r_k = penalty coefficient in P function formulation, at the kth iteration (dimensionless).

S = total cost-benefit function = $\sum_i S^i$, 1000$/year.

S^i = cost-benefit function for the ith reach, 1000$/year.

TGH = weight of violation (dimensionless).

v^i = parameter for recreational and aesthetic benefits, 1000$/year/mile.

w^i = parameter for recreational and aesthetic benefits, $1/(ppm)^2$.

x_ℓ = decision variable of the ℓth kind, arbitrary.

Greek Letters

α^i = existence flag for waste treatment plant at the beginning of the ith reach (dimensionless).

β^i = existence flag of water treatment plant at the beginning of the ith reach (dimensionless).

$\beta_1, \beta_2, \beta_3$ = components of benefit function, BF, 1000$/year/mile.

ν = waste characteristic index (in this case BOD concentration of effluent to treatment plant), ppm.

δ = maximum allowable concentration of pollutant in purified water from water treatment plant, ppm.

ϕ = degree of treatment (dimensionless).

θ = stopping criterion, $\left| 1 - \left|\frac{S}{P}\right| \right| \leq \theta$ (dimensionless).

Superscript Letters

i denotes parameters associated with the ith reach.

Subscript Letters

T denotes cost coefficients for waste treatment plants

W denotes cost coefficients for water treatment plant

i denotes type of inequality constraint

j denotes type of equality constraint

k denotes iteration number

ℓ denotes type of decision variable

Appendix 4A

The distance at which DO is minimum in a reach can be found by differentiating Equation (20) and equating to zero

$$\frac{dO^i}{d\ell} = \frac{-k_1^i}{(k_2^i - k_1^i - k_3^i)} P_1^i m_1^i e^{m_1^i \ell_1} \frac{-k_1^i}{(k_2^i - k_1^i - k_3^i)} Q_1^i m_2^i e^{m_2^i \ell_1}$$

$$+ P_2^i n_1^i e^{n_1^i \ell_1} + Q_2^i n_2^i e^{n_2^i \ell_1} = 0 \qquad (4A\text{-}1)$$

The minimum DO can then be found by substituting the value of ℓ obtained from the above equation in Equation (4.20).

Equation (4A-1) must be solved by a search, the computational efforts required for which are of the same magnitude as searching for the minimum DO from Equation(4.20) by a one dimensional search in ℓ. Hence the minimum DO is found directly by the use of a sequential search namely the Golden Section method.

The Golden Section method can be used for unimodal functions and is based on the principle of sequentially reducing the uncertainty interval in which the minimum is located, by eliminating a portion of the original interval.

In this search procedure, given a unit interval, the first trial (or guess) is placed at

$$\alpha_1 = \frac{2}{1 + \sqrt{5}} = 0.618033989$$

The second trial point is placed symmetrically with respect to the first trial point at

$$\alpha_2 = 0.38196011$$

Based on the unimodality property, and the values of the functions at these two points the interval of uncertainty can be reduced to

$$L_1 = 0.618033989$$

by eliminating a portion of the original interval. The process is repeated for the next trial point. At the end of n such symmetrical trial points placed symmetrically the original interval can be reduced to

$$L_n = (0.618033989)^{n-1}$$

Fig.4.11 shows the computer flow diagram for implementing the above sequential search.

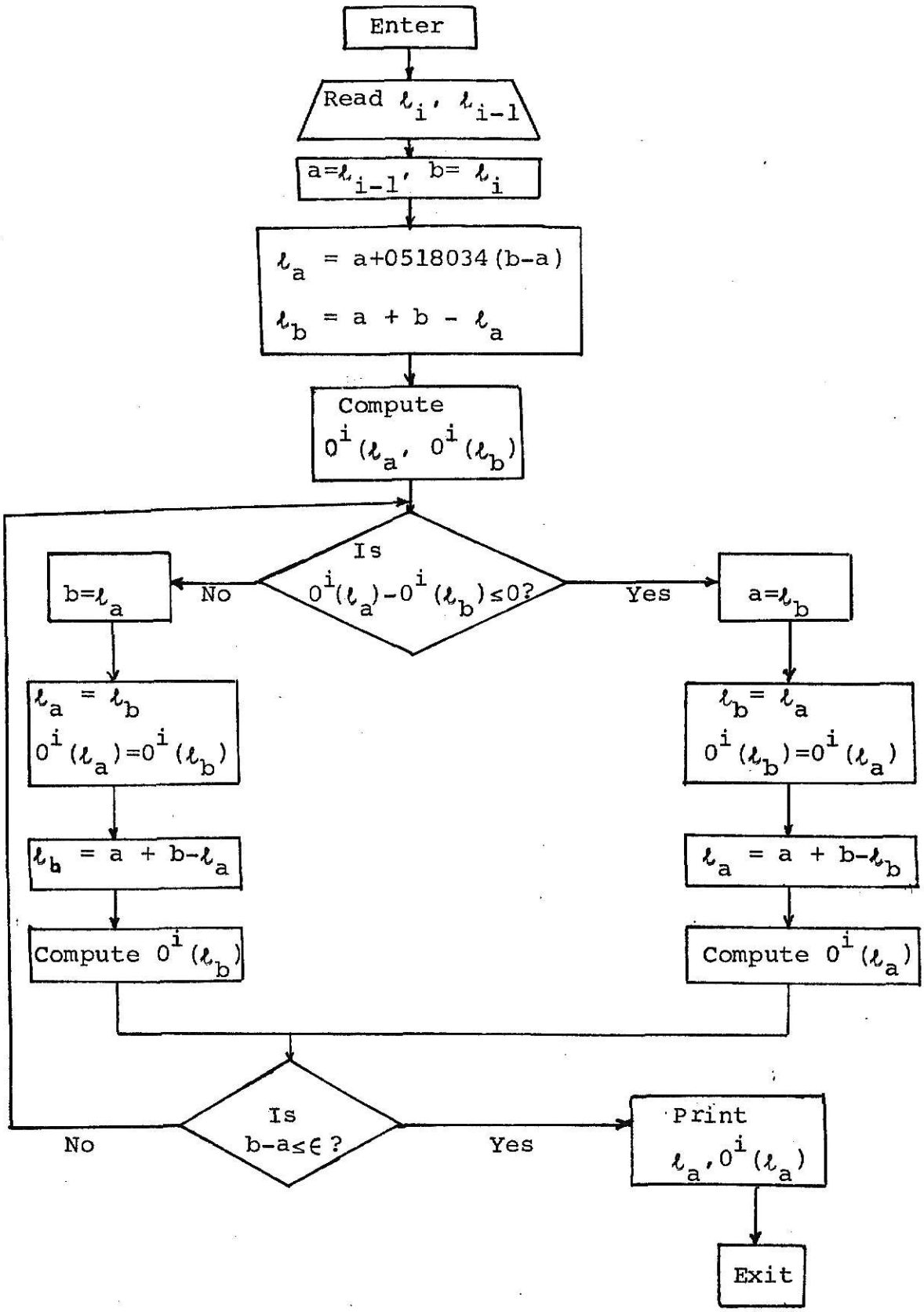

Fig. 4.11 Computer flow diagram for finding minimum DO by Golden Section method.

Appendix 4B

The sequential unconstrained minimization technique (SUMT) for solving the general nonlinear programming problems with linear or nonlinear equality and/or inequality constraints was originally proposed by Carroll (4.3) and further developed by Fiacco and McCormick (4.5). The basic scheme of this technique is that a constrained minimization problem is transformed into a sequence of unconstrained minimization problems which can be solved by any available technique for solving unconstrained minimization problems.

The technique used here is a modified version of the original SUMT program (4.5) which was modified by Lai (4.9) to incorporate a discrete unconstrained minimization searching technique (namely the Hooke and Jeeves pattern search (4.7) and to include a heuristic technique (4.13) to handle the inequality constraints of the problem. The general nonlinear programming problem with nonlinear (and/or linear) inequality and/or equality constraints is to choose x to minimize f(x) subject to

$$g_i(x) \geq 0, \quad i = 1, 2, \ldots, m \tag{4B-1}$$

and

$$h_j(x) = 0, \quad j = 1, 2, \ldots, \ell \tag{4B-2}$$

where x is an n dimensional vector (x_1, x_2, x_n). The SUMT technique for solving this problem is based on the minimization

of a function

$$P(x,r_k)=f(x)+r_k\sum_{i=1}^{m}\frac{1}{g_i(x)}+r_k^{-1/2}\sum_{j=1}^{\ell}h_j^2(x) \qquad (4B\text{-}3)$$

over a strictly monotonic decreasing sequence $\{r_k\}$. The basic condition necessary to guarantee that minimization of P functions over the sequence of $\{r_k\}$ leads to minimization of f(x) is that the P function must be convex.

The intuitive concept of P function is described below.

Since the sequence of $\{r_k\}$ is strictly monotonically decreasing, as $r_k \to 0$, the third term of the P function defined in Equation (4B-3), will approach infinity unless $h_j(x)=0$, $j=1, 2, \ldots, \ell$. Thus, while minimizing the P function, the formulation of P functions will force all equality constraints to be zero. For the second term of the P function in the feasible region bounded by inequality constraints, $r_k\sum_{i=1}^{m}\frac{1}{g_i(x)}$ will approach infinity as x approaches one of the boundaries of the inequality constraints. Hence the value of x will tend to remain inside the feasible region.

When $r_k \to 0$, the second term of the P function approaches zero, while the third term is forced to approach zero as described before. In other words, as $r_k \to 0$, $P(x, r_k) \to f(x)$, where x is the optimum point which yields minimum $P(x, r_k)$.

The mathematical proofs can be seen in Ref. (4.3), (4.4) and (4.5). The logic diagrams for the basic search for

minimizing the P function as well as for selecting a feasible starting point, and the heuristic search in the near-feasible region are shown on subsequent pages.

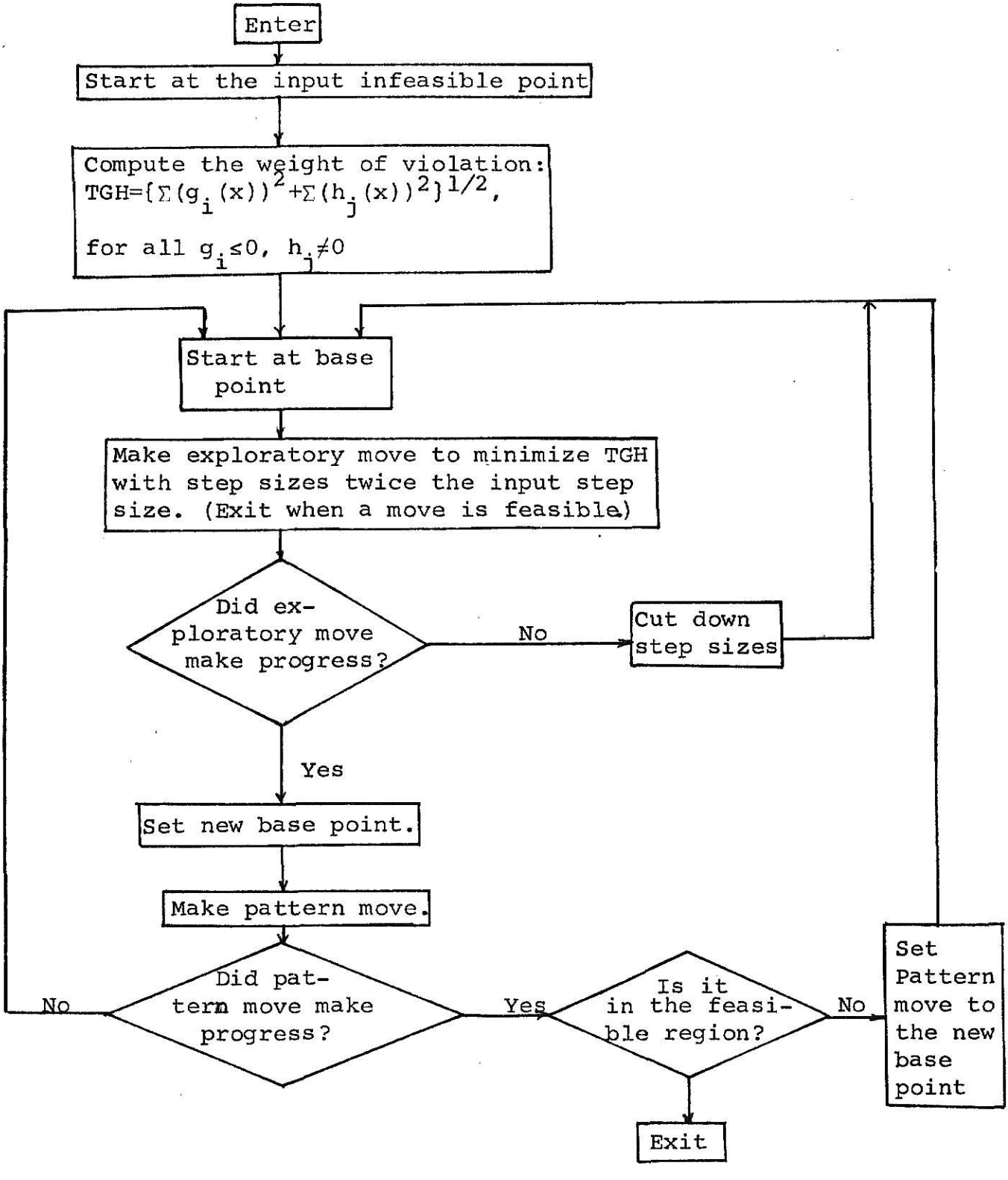

Fig. 4.12 Descriptive flow diagram for selecting a feasible starting point.

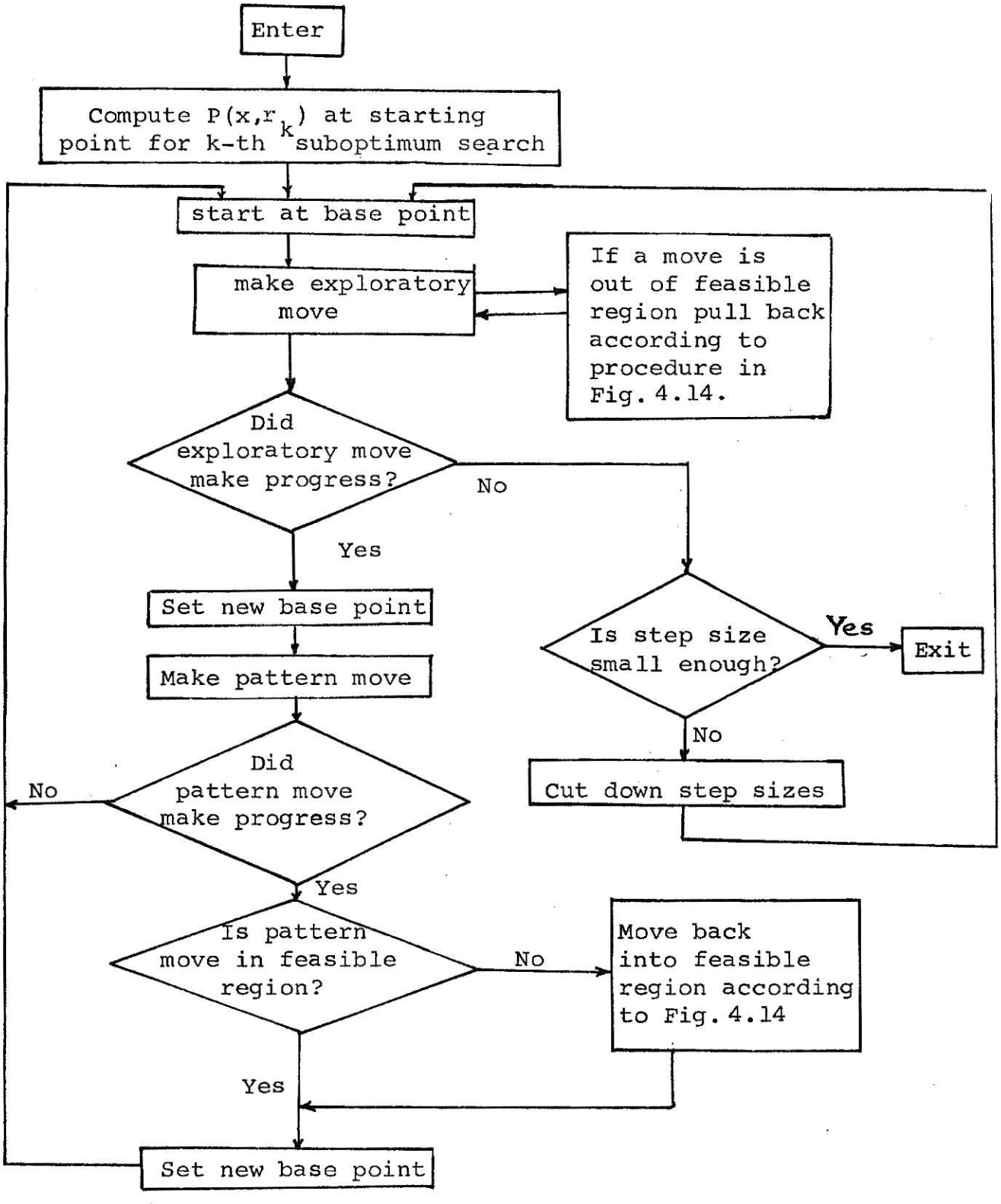

Fig. 4.13 Descriptive flow diagram for Hooke and Jeeves Pattern search for minimizing P function

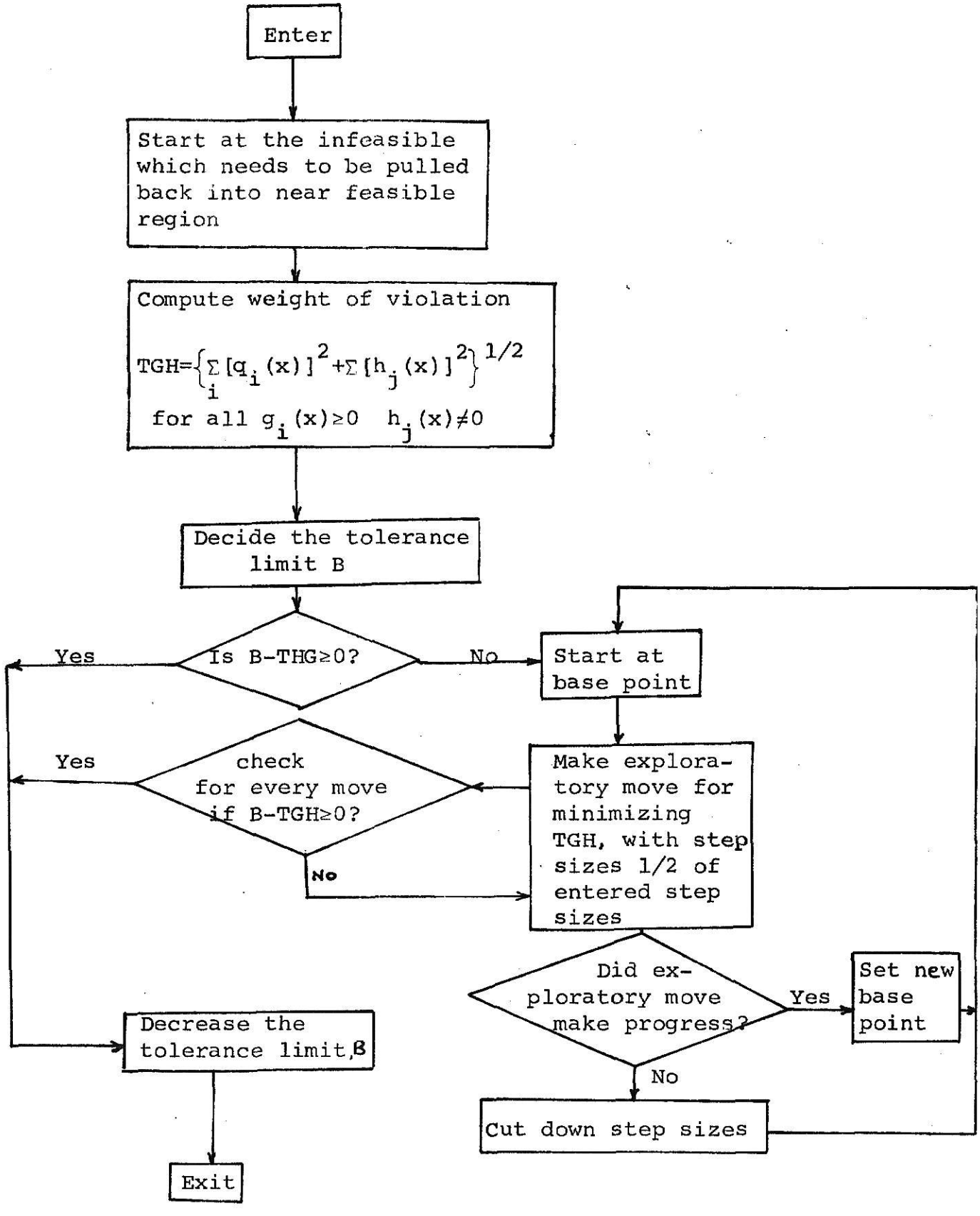

Fig. 4.14 Descriptive flow diagram for moving back an infeasible point into near feasible region.

Chapter 5

DISCUSSION AND RECOMMENDATION FOR FUTURE RESEARCH

DISCUSSION

A one-dimensional steady state dispersion model to predict BOD and DO concentration in a stream is developed in Chapter 3. The model includes point as well as continuous additions and removals of BOD and DO along the stream, and hence is suitable for describing pollution due to industrial and sewage plants as well that due to agricultural runoff, where input is more or less continuous. By adjusting the parameters of the stream equations, the model can be used to represent widely varying flow patterns, cross sectional areas and other factors affecting BOD and DO concentrations for a given stream. The model includes most factors that affect BOD and DO concentrations in streams, and hence is fairly general. However, it assumes steady state, isothermal conditions and negligible photosynthesis and hence is limited in application to cases where these factors are not significant.

In Chapter 4, an optimization model is formulated in order to minimize the cost-benefit function for a stream used for water supply, waste disposal and recreational and navigation purposes, with constraints imposed on water quality. The search scheme proposed for solving the constrained optimization problem is easy to implement and also versatile in application,

as the objective function and decision variables can be changed without any additional effort in programming. However, when the number of decision variables is large, the objective function may not be unimodal, and the search may converge to a sub-optimal point. Under such conditions, a large number of randomly selected starting points may be used.

Due to the steady state and isothermal assumptions that are made, the model may not have direct application when there is a large variation in seasonal flows and temperature. The model could, however, be used to examine alternate policies and alternate means of controlling water quality.

RECOMMENDATIONS FOR FUTURE RESEARCH

Application of the Model for a Specific Stream

Water quality data, geophysical data and waste input data available for polluted streams and rivers such as the Ohio, Delaware, Hudson and Sacramento rivers, may be examined with the help of the proposed water quality model.

First, the stream or river may be divided into segments having more or less uniform properties. Parameters, such as BOD decay rate, reaeration coefficient etc. which have been estimated may be used directly. Other parameters may be estimated from water quality data. Since analytic solution for the system can be derived, parameter estimation techniques, such as Marquardt's method (5.12) may be used to estimate unknown

parameters.

Optimization studies in Chapter 4 may be employed to obtain optimal waste handling, making use of the mathematical model developed for stream water quality equations. Optimal locations for new treatment plants may be selected, making use of the model.

Application of the Model to Examine Thermal Effects

There is serious concern over water quality changes due to thermal discharges from nuclear power plants. To examine the temperature effect, a heat balance equation can be written in a manner similar to that for BOD and DO balances. Since the heat of reaction for biological reactions is small the coupling is only one way; i.e., temperature affects BOD level which in turn affects DO level. By linearizing the mathematical equations over small segments, the analytical approach may be applied to get an approximate solution. Numerical techniques may be applied in order to obtain more accurate solutions.

Development of Unsteady State Models

Solution of the time varying BOD and DO model would be of great value in controlling water quality. The resulting partial differential equations require numerical techniques to obtain a stable solution (5.4). The solution would be of value in examining additional measures of water quality

control, such as time variation of flow of affluent discharged to take maximum advantage of diurnal temperature and photosynthesis cycle. Effluent charging (5.5) and introduction of political and economic constraints may be incorporated in the water quality control model to obtain realistic and meaningful solutions.

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MODELING AND OPTIMIZATION OF WATER QUALITY IN STREAMS

by

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AN ABSTRACT FOR A MASTER'S THESIS

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An analytical solution is developed to predict the BOD and DO profiles in a stream having several outfalls of waste and intakes of water. The model also incorporates continuous additions and removals of BOD and DO along the stream. A steady state one dimensional dispersion model is used to represent the longitudinal mixing in the stream. Changes in stream properties are considered by dividing the stream into several segments and changing the parameters of the BOD and DO equations as needed in each segment. By applying continuity equations at the boundaries of these segments and equilibrium conditions at either ends of the infinitely long stream, the arbitrary constants in the solutions to the differential equations for BOD and DO can be determined.

The model is extended to incorporate waste treatment plants at the location of waste outfalls and water treatment plants at the intakes. A cost-benefit function which consists of the sum of total cost of waste treatment and the cost of water treatment minus the benefits derived from the stream, is formulated. In order to optimize the system, by minimizing the total cost benefit function within certain constraints on water quality, a search scheme is proposed. The feasibility of obtaining a solution by a sequential minimization technique, employing a discrete search is studied for cases with two and three decision variables.

The analysis procedure can be used to examine existing water quality programs and to study the influence of alternate policies and constraints. Alternate means of water quality management programs may also be investigated using this approach.