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AN ANALYSIS OF MINIMIZATION OF ENERGY REQUIREMENTS WHEN THERMAL COMFORT IS MAINTAINED BY AN ENVIRONMENTAL CONTROL SYSTEM

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LIONEL ROBERT WHITMER

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Robert L. Gorton

Major Professor

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INTRODUCTION

Since there is an increasing emphasis on energy conservation due to the so-called "energy crisis", it is the purpose of this thesis to determine those combinations of environmental variables within an enclosed environment which minimize the energy consumption. This is accomplished by placing the restriction of thermal comfort for human occupants on the allowable range of the variables and by investigating the effects of the environmental variables on energy consumption. The main variables, air temperature, humidity (water vapor pressure), and air velocity, are selected for an environmental control system such that the thermal comfort constraint is satisfied and the energy requirements are minimized. To place emphasis on the importance of conservation of energy in environmental control systems, it is appropriate to quote Dr. P. E. McNall, Jr. at the Conference on Energy Conservation at Henniker, NH., [57]*, it was stated that "Approximately 20% of U.S. energy is used in the heating and cooling of spaces occupied by people (homes, factories, offices, schools, etc.). This is primarily gas and oil with some electricity." This is further supported by a report of the U.S. Dept. of Commerce which states that 33.6% of total energy used in the United States is by residential and commercial buildings, as shown in Table 1 [95]. Space heating, which occurs mostly during winter months, accounts for 53% and air conditioning makes up 8% of the residential and commercial energy use. While the 8% is less than 3% of the total national annual usage, it is 42% of the summer total for residential and commercial buildings and represents an annual national energy expenditure of more than 1.5×10^{15} Btu (in 1968 and increasing at 10 percent per year).

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*Numbers in brackets refer to references listed on page 131.

Basic Pattern of Energy Use

Transportation	25.2%
Industrial	41.2%
Residential & Commercial	33.6%
	100.0%

of the 33.6% Residential & Commercial

by type of use ...

Space Heating	53%
Water Heating	12%
Air Conditioning	8%
Refrigeration	7%
Lighting	5%
Other Electrical	5%
Cooking	4%
Clothes Drying	1%
Misc	5%
	100%

Table 1. United States Energy Use

It is obvious that in order to design an energy conservation system the optimum combination of environmental variables must be determined. Further, as pointed out by Dr. McNall, "Quality of life is largely dependent on our environment. If we remove the stress caused by people *fighting* their environment, we release more energy for man's higher pursuits. It is part of the well-documented energy use per capita vs living standard correlation." This exemplifies the necessary condition of restricting the environmental variables to values which provide thermal comfort while minimizing energy consumption.

From the above, it is seen that the provision for thermal comfort (thermal neutrality) be the primary concern. Even though quantitation of human comfort is difficult, the conditions for thermal comfort have been shown to be predictable. In this study the "Comfort Equation" of Dr. P.O. Fanger based upon KSU-ASHRAE* studies is used as the definition of thermal comfort conditions [18].

The study presents the techniques that can be applied to a model of the energy consuming elements of an environmental control system. It yields the values for the environmental variables that are restricted to values that maintain thermal comfort, and that further require the minimization of energy consumption for the specified system parameters.

*KSU-ASHRAE refers to sponsored research conducted at Kansas State University for the American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.

LITERATURE SURVEY

Historically Leonardo da Vinci is referred to as, the father of environmental control. This was based upon his work for the Duke of Milan in 1482. During this time he devised a forced-air central heating system and a water-pumping mechanism for the castle at Milan. To mark other environmental control events, in 1607 Galieo invented the thermometer, and, in 1660, Wren designed a gravity exhaust ventilating system for the House of Parliament. Early recognition of the value of environmental control was given in an exhaustive review of the "History and Art of Warming and Ventilating Rooms and Buildings" published in 1845 by Walter Bernan. The introductory essay in this study discussed the value and benefits of proper environmental control. Mr. Bernan documented the need for artificial heat for "personal comfort", and it was predicted that "the formation and regulation of artificial climate will assume the character of an art for developing and expanding the mind and body for preserving health and prolonging life: and the skillful practice of the art, as a means of saving fuel, will become essential not to the well-being only, but to the existence of communities" [6].

The sensation of comfort or lack of awareness of discomfort is a complex, subjective reaction which results from a combination of physical, physiological and psychological factors. The factors are separated into three groups: those factors associated with the physical environment; those factors associated with the person, or organismic factors; and those factors associated with his behavior, or reciprocative factors [80,81,83,85,86,84,65,66]. Some of these factors such as air temperature, exert a significant influence on comfort, while others affect comfort only slightly. Some of the factors predicted to affect comfort sensations are given in Table 2.

Further, the problem of defining comfort criteria is complicated by the variation of an individual's reaction from day to day and by variations among individuals. It has been found however that dry bulb temperature, air motion, relative humidity and mean radient temperature (temperature of the surrounding surfaces), have the greatest influence on comfort. These factors define the thermal environment and affect the heat exchange of people. To achieve thermal comfort, the thermal environment should be such that the body is in thermal equilibrium, a necessary but not sufficient condition for comfort. ASHRAE defines thermal comfort as "that condition of mind which expresses satisfaction with the thermal environment" (ASHRAE Standard 55-66, Section 2.2) [3].

The newly-proposed standard (ASHRAE standard 55-66R) defines in section 2.1 [4], "Acceptable Thermal Environment - an environment in which at least 80% of normally clothed men and women living in the United States and Canada, while engaged in indoor sedentary or near sedentary activities, would express thermal comfort." The new standard is represented by a quadrilateral plotted as water vapor pressure versus adjusted dry bulb temperature (defined as the average of mean radiant temperature plus dry bulb temperature) graph. A summary of the proposed standard is shown in Figure 1.

Organized efforts to establish criteria for thermal comfort were initiated during the period from 1913 to 1923. John Sheppard, at Teacher's Normal College in Chicago, is reported to have introduced the

Table 2. Environmental Factors Affecting Comfort

Physical Factors:	
Air Temperature	Odors-Inspired Gases
Air Motion	Air Pressure
Relative Humidity	Area-Volume
Mean Radiant Temperature	Force Field
Noise	Ion Count
Lighting	Color
Organismic Factors:	
Age	Rhythmicity
Sex	Psyche
Genetics	Sensory Processes
Body Type	Drive
Reciprocative Factors:	
Clothing	Social
Physical Activity	Diet
Mental Activity	Incentive
Exposure	Health

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

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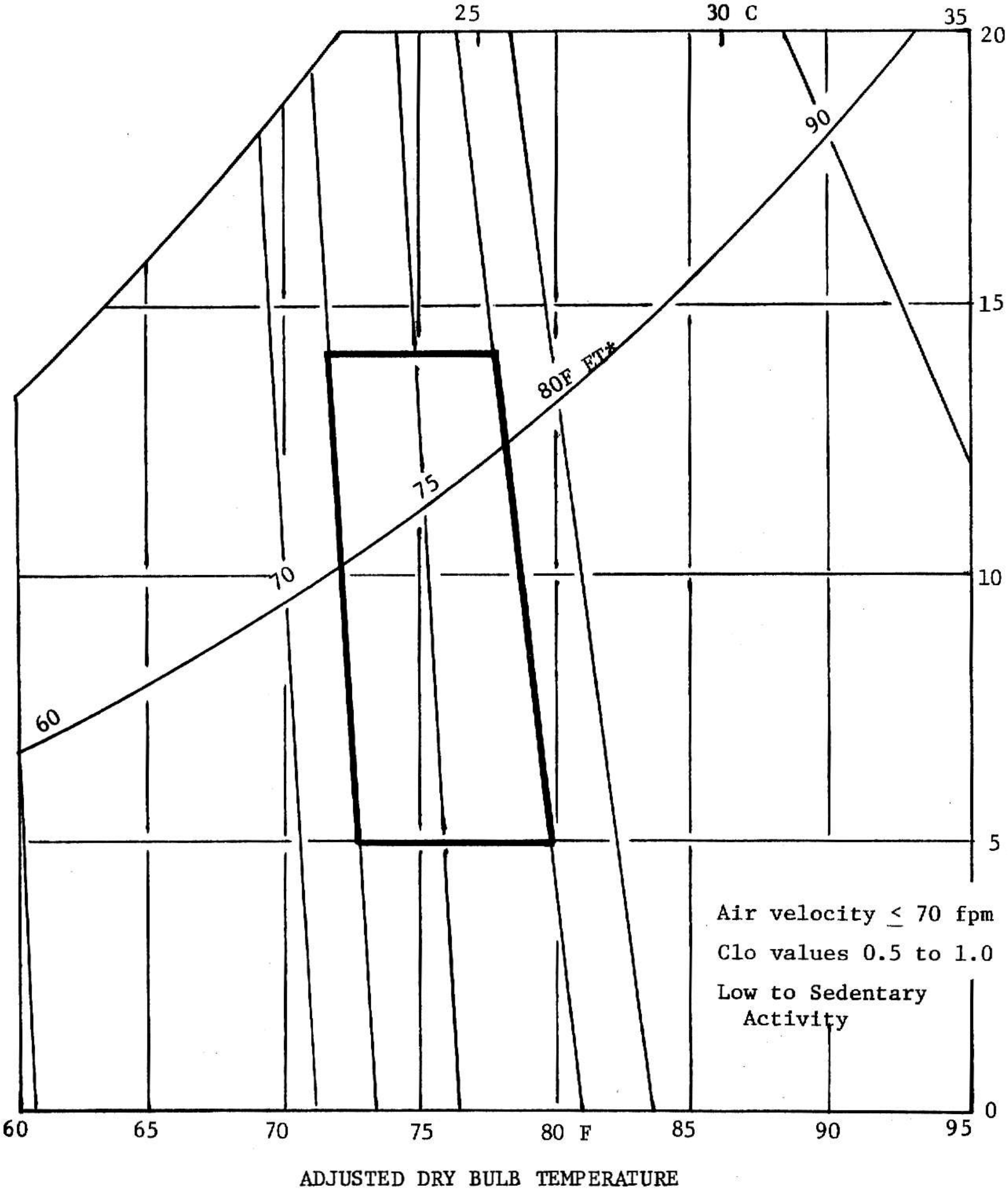

Figure 1. Comfort Envelope —— Proposed ASHRAE Standards.

term "comfort zone" [37]. E. Vernon Hill prepared his "synthetic air chart" [48]. The New York Commission on Ventilation was appointed and began its numerous experiments [78]. ASHRAE published in 1923, the original research of Houghten and Yaglou [41], who mapped out the combinations of ambient temperature and humidity which produced "feelings of equal warmth" which they equated to "feelings of equal comfort" (or discomfort). These studies were made for the guidance of heating, ventilating and air-conditioning engineers, and established the so-called "comfort zone" in terms of the "effective temperature" scale which combined the effects of air temperature, velocity and relative humidity. The papers of Houghten and Yaglou in 1923 were the first real attempt to relate comfort to temperature and humidity [42]. Obviously, since subjects were stripped to the waist, the information did not give data the engineer needed, so the next investigation studied the effects of clothing on comfort [102]. In 1929, Houghten, Teague, Miller and Yant [40] published results that show the heat and moisture losses from the human body as a function of the effective temperature and air motion. Further modification of the comfort chart resulted from work, published in 1929, by Yaglou and Drinker [101]. Their experiments were carried out at the Harvard School of Public Health to determine effects of summer climate on the comfort zone. The French Engineer Missenard, in 1931, was the first to propose an explanation of effective temperature in terms of the applicable heat transfer coefficients involved. Hardy and Dubois in 1938 [33] measured quantitatively the total heat loss and the proportions due to radiation and convection from men exposed to various atmospheric conditions in the temperature range of 22° C to 35° C. Radiation accounted for about 70% of the total loss at 22° C and at 26° C, but this percentage fell rapidly to zero as skin and air temperatures approached each other. Vaporization dissipated 18% to 30% of the heat at the lower air temperature but accounted for about 100% at 35° C. Convection remained fairly uniform at about 15% until the air temperature rose above 32° C. Convection was significantly increased by slight movements of the body or the air.

In 1938, research by Houghten [39] showed a relation between air velocity and the difference between the temperature of moving air and the room air.

The effect of air movement upon heat losses from the clothed human body was investigated by Winslow, Gagge and Herrington [96] in 1938. Also in 1938, Gagge et al, investigated the effect of clothing on the physiological reactions of the human body to varying environmental temperatures. He found that the gross physiological responses of the clothed body when compared with the nude body were broadly identical, and that the skin temperature was the prime factor controlling sensations of thermal pleasantness.

In 1939 Herrington [35] calculated an equation for the heat exchange of the clothed human body. Also in 1939, DuBois [11] determined heat loss from the human body and Winslow, et al. [97] carried out a study of the nude body's physiological reactions and sensations of pleasantness under varying atmospheric conditions. It was an attempt by Winslow to analyze the influence of widely varying conditions of air temperature, wall temperature, air movement, and humidity upon physiological reactions and human comfort.

Published in 1940 was a recommendation of a new environmental index, the operative temperature [20]. This index combined the effects of dry bulb temperature, air movement and mean radiant temperature and is derived by considering the thermal exchange of the body with the surroundings. Effects of humidity were not included since the index is used within the comfort zone.

The cooling effect produced by three cold walls of equal temperature was reported by Houghten and McDermott in 1933. Thermal radiation effects were considered in a report published in 1941 [38]. Two test rooms were used, one heated with a forced warm air system and the other with hot water radiators. Eight subjects reported comfort sensations in each room and a relationship between effective temperature and mean radiant temperature was formulated.

Evidence from various sources [87,73] indicated that in the zone of thermal neutrality (Zone of vaso-motor regulation: heat production equal to the net heat loss by convection, radiation and evaporation with no change in stored energy and without sweating or shivering.), the effective index over-emphasizes the effect of relative humidity on comfort. In 1947, Yaglou proposed that the over-emphasis resulted from the use of instantaneous thermal impressions and the resulting adsorption and desorption phenomena, that is, the heat of adsorption giving a sense of warmth as moisture was adsorbed on skin and clothing [100]. Likewise, a cooling effect when the moisture evaporated. To correct the effective temperature index (ET) it was proposed that lines of constant mean skin temperature replace the ET lines.

Leopold [54] developed an argument for accurate control of environmental conditions, showing that even though persons may individually have a wide zone of comfort, it is necessary to maintain close control of the environment for maximum group comfort. He proposed the use of a discomfort index rather than a comfort index. This procedure was also used by Chrenks [10] in England.

Research by Glickman, et. al. [31] using 15 subjects exposed to two levels of relative humidity (30% and 80%) found that the effective temperature index appeared to be adequate for subjects in the dynamic state but for equilibrium conditions the ET placed too much emphasis on relative humidity. Measurements in these tests included physiological as well as subjective reactions. As pointed out by Bedford [5], the establishment of the proper ET does not ensure that the environment will be pleasant. Factors such as air movement, thermal gradients, and radiant effects should be considered.

In 1953, Inouye [46] studied the effects of environments with widely different relative humidities on the heat loss of uniformly and lightly clad men and on their subjective sensations of thermal comfort. The environments were selected to slowly cool the skin without inducing visible sweat or apparent shivering during a period of three hours. He concluded that non-fasting men, lightly clad in a uniform suit, showed greater heat loss by evaporation in environments maintained at $80^{o}F$, $76^{o}F$, or $72^{o}F$ with a 30% relative humidity (RH) than with an 80% RH. The effects attributed to RH appeared more readily at the higher temperatures.

With the advent of radiant heating, new and increased interest in the effects of radiation on comfort was developed. Several projects were undertaken and reported in the literature [36,10,68]. These projects dealt with specific problems of panel location, effect of floor surface temperatures, etc., and the results were such that a comfort zone with mean radiant temperature (MRT) as a variable could not be defined.

In 1956, Fahnestock and Werden [14] reported that in the dry bulb temperature range of 73 to 77^{o}F, variations in relative humidity from 25 to 60% did not affect comfort sensations for sedentary or slightly active healthy men and women, normally clothed in uniform environments. Jennings and Givoni [47] published in 1959 studies dealing with human reactions to environments in the 80 to 105^{o}F zone, or outside the comfort zone.

Another study, by Koch, Jennings and Humphreys [50], published in 1960, initiated a comprehensive program of re-evaluation of the comfort chart. The tests were conducted with dry bulb temperatures of from 68^{o}F to 94^{o}F and with relative humidities of from 20% to 90%. Twenty subjects, under still air conditions and wearing summer clothing, were exposed for periods of three hours while seated at rest. They were asked to report their impressions on a number of scales which included thermal sensation, sensation of humidity, pleasantness, air motion, perspiration and sensation of warmth or coolness from surrounding surfaces. The data show that, over the range of variables studied, the effect of relative humidity is small, and that the optimum comfort line for these subjects was 77.6^{o}F at 30% RH and 76.5^{o}F at 85% RH.

In 1961, studies for the ASHRAE environmental research program were proposed by Nevins and Humphreys [69] to provide a "Complete" comfort chart. The objective of this program was to provide the air conditioning engineer with a base-line comfort chart in terms of dry bulb temperature and relative humidity, and to determine factors for this chart for variations in mean radiant temperature, activity, clothing and the like.

Fahnestock, et. al. [13,12], reported comfort and physiological responses to work in an environment of 75°F and 45% relative humidity.

In 1965, Chatonnet and Cabanac [8] immersed healthy, young men in a warm bath at 38°C to show that internal body temperature, independent of skin temperature, can cause thermal discomfort. The subjects registered their thermal comfort over an hour period and reported comfortably warm for the first half hour but with the rising rectal temperature they reported progressive discomfort even though the skin temperature was held at 38°C.

In experiments conducted by Nevins et al. [70,71,91] 61 subjects engaged in sedentary and walking activity levels and wearing light shoes showed no serious discomfort caused by floor temperature during 3-hour exposures for floor surface temperature as high as 29°C. For 48 subjects in test conditions with cold floors and wearing light shoes it was indicated that floor surface temperatures of 17-18°C yielded the lower comfort limit [67].

Nevins et al. published in 1966 [72] thermal comfort conditions, namely for dry bulb temperature and relative humidity, for seated persons from 72 tests using 360 male and 360 female subjects. In these experiments the subjects wore standard clothing and were exposed to the test environment for three hours. The results showed a strong linear effect of temperature and a smaller, but substantial, linear effect of relative humidity with an interaction effect between temperature and humidity that was statistically significant at the 5% probability level. The sensation votes were correlated directly with measures of the physical environment and comfort was judged by the thermal environment in which man enjoys a neutral temperature sensation and is neither warm nor cold.

In 1966, ASHRAE issued its first standard (55-66) on thermal comfort conditions [3] in which thermal comfort is defined as "that condition of mind which expresses satisfaction with the thermal environment". This definition implies that factors other than a sense of heat and cold are necessary in the judgement of comfort. For example those factors with a physiological basis such as local skin temperature, hypothalamic temperature, heart rate, circulatory effects and exercise affect the comfort of a person.

In experiments by Gagge et al. [26] attention has been called to the fact that men who have normally slightly higher internal body temperature generally preferred a cooler environment than those with slightly lower internal temperature. These studies point to the importantance of internal body temperature as a factor in thermal comfort.

McNall et al. [59] determined the thermally neutral conditions for three levels of activity in 1967. As the activity level is increased the air temperature must be reduced to maintain thermal balance. For metabolic rates of approximately 600, 800, and 1000 Btu per hour, the thermally neutral temperatures were 72, 66 and 60^{o}F respectively. This compares with a temperature of 78^{o}F for college-age students seated at rest. College-age students, dressed in the standard clothing, were used in the activity studies and exposed for three-hour periods. The males and females preferred similar temperatures for thermal neutrality; however, the comfort zone for men at each metabolic rate spanned a wider range of temperatures than for the females. Relative humidity did affect the thermal comfort region for women at the 1000 Btu/hour metabolic rate, while it had little effect upon men and women at the 600 and 800 Btu/hour activity levels.

The radiant environment and the effect of high temperature radiation on comfort was studied at the Pierce Foundation Laboratory, Yale University [23,22,25,24,21,77]. Gagge et al. [24] introduced the concept "effective radiant field" (ERF) to express the heat exchange by radiation. The effective radiant field is defined as the heat exchange by radiation (per unit body surface area) between the environment and a man-shaped object with a hypothetical black-body radiating surface temperature equal to the ambient air temperature.

Schlegel and McNall [88] exposed 90 sedentary subjects clothed in standard uniforms (0.6 clo) to an asymmetric radiant field. In comparison with similar experiments in uniform radiant fields, no discomfort as a result of the asymmetry was found. Further, 234 subjects were exposed to more extreme asymmetries by McNall and Biddison [58]. In two sets of experiments the subjects were exposed to a ceiling with a temperature of 30°C higher and of 15°C lower than the remaining chamber surface temperatures. In a third set, subjects were exposed to a wall 11°C lower than ambient temperature. In another test the wall was 30°C warmer than ambient temperature. The results showed discomfort due to asymmetry only in the last test where the wall was 30°C warmer.

In an attempt to formulate conditions for comfort, Fanger [18] introduced the "Comfort Equation" based on the KSU-ASHRAE Studies. He recognized that when a person is comfortable (thermally neutral) there were two highly probable relationships: (1) one between his average skin temperature and metabolism (a direct measure of activity) and (2) one between his regulatory skin sweating and metabolism. By using these two physiological criteria for comfort in the heat balance equation for man in equilibrium with his thermal environment, he has been able to predict comfort over a wide range of the following variables: (1) air temperature, (2) humidity (water vapor partial pressure), (3) mean radiant temperature, (4) relative air velocity, (5) activity level and (6) insulation value of the clothing. The solutions of the "Comfort Equation" were compared with experimental data and found to be in good agreement. Using this equation it is possible to calculate, for a type of activity and a clothing ensemble, all those combinations of the above variables which create thermal comfort. It is noted that the range on the variables is restricted to the values of the experimental tests upon which the "Comfort Equation" was based.

Other theoretical models have been set up by Morse and Kowalczewski [63], 1967, by Nishi et al. [75], 1969, by Ibamato et. al [45], 1969, and by Gagge et al. [27], 1969. Nishi and Ibamato have expressed the influence of clothing and environmental variables by a hypothetical skin temperature necessary to keep heat balance for the body. Gagge et al. have studied the relationship between skin wettedness and discomfort. Wettedness is defined as the ratio of the actual heat loss by skin sweating to the maximum possible evaporative heat loss to the environment, if the skin surface were theoretically 100% wet and if all evaporation occurs on the skin surface. Problems occur when predicting evaporative cooling when a person is clothed.

Lee, Fan, Hwang and Shaikh [53] have used Fanger's "Comfort Equation" as the basis for systems analysis and optimization of life support systems for confined spaces in aircraft and space vehicles. Maes [55] used the basic experimental data in the "Comfort Equation" to write a computer program for calculating thermal comfort in passenger cabins in large airplanes. Choa [9] used Lee's (et al.) results and applied a search technique in the optimization of an environmental system.

Wyon [99] studied operating personnel and their thermal comfort by assessment of their physical environment. Thermal transients and their effect where investigated by Gagge et al [26,29], Hardy [34,32] and Stolwizk [66]. In one experiment they exposed nude subjects alternately to cold and neutral and to hot and neutral environments. When proceeding from neutral to cold or warm environments, the changing thermal sensation was found to be correlated with the actual skin temperature and sweat rate in the same way as under steady state conditions. But when these transients were reversed, i.e. proceeding from a cold or hot to a neutral environment, they felt almost immediately comfortable, even though their skin temperature had not yet reached the steady state level considered comfortable. Gagge explains this by the rate of change of skin temperature which might cause a sensation that compensates for and predominates over the sensation of discomfort caused by the skin temperature itself.

In a study involving different comfort conditions for summer or winter occupants, McNall et al. [60] exposed college-age subjects to the same conditions as Nevins et al. [72] had previously. For these subjects, seated at rest for three-hour periods and dressed in the Kansas State University standard clothing (0.6 clo), no significant difference in preferred conditions of temperature and humidity was detected.

Further studies by McNall et al. [62,61] looked at the relative effects of convection and radiation and at metabolic rates of subjects at four different activity levels. The influence of the mean radiant temperature and velocity can be seen and shows reasonable agreement with the "Comfort Equation".

In 1970, Nishi and Gagge [74] investigated the effect of moisture permeation of clothing. Sprague et al. [90] studied the influence of periodic fluctuations in air temperature, mean radiant temperature and relative humidity. They found that no serious occupancy complaints occur due to temperature fluctuations if

$$\Delta t^2 (\text{cph}) < 4.6 \qquad (^\circ\text{C}^2/\text{hr})$$

where Δt is the peak to peak amplitude of the air temperature ($^\circ$C)

cph is the cycling frequency (hr^{-1}).

Also in 1970, Fanger [19] published his book, Thermal Comfort, which developed along with his comfort equation, equations to calculate the Predicted Mean Vote, PMV, of average persons exposed to thermal environments.

He established the approximate exponential relationship between the change in vote per unit change in thermal load and activity level based on his data and on the evidence presented by Nevins et al. [72] and McNall et al, [59].

In 1971, Gagge et al. [28] defined a new effective temperature as the dry bulb temperature at the point of intersection of the loci of constant percentage body wettedness due to regulatory sweating and the 50% relative humidity line on the psychrometic chart. This recent comfort index has been calculated by a computer program developed by Woods and Rohles [98] in 1972. It calculates the value of Gagge's effective temperature index when given dry bulb temperature and relative humidity at 1 met activity, 0.6 clo, still air, and mean radiant temperature equal to dry bulb temperature for 1,2, and 3 hour exposures.

Rohles [83] proposed that instead of talking about "thermal" comfort, one should consider "environmental" comfort. The selection of non-thermal environmental factors should be applied to an environmental control system. This of course has not been attained and requires systematic, objective and interdisciplinary research efforts for focus on environmental comfort. Rohles [82] states "However, to specify the non-thermal parameters in environmental comfort requires as a starting point, the thermal conditions where most of the people are comfortable".

It remains, based on documented data (detailed in the original reference) [19], that Fanger's thermal comfort equation can be used as a mathematical tool to provide the constraint of "comfort" for optimization of an environmental control system.

Techniques for optimization of systems have been investigated by numerous people. The theory used in this study is based upon works by Kuhn and Tucker [52], Tucker [94], Fan et al. [16,17,15], Hwang et al. [43,44] and Beveridge and Schechter [7].

With the background of the above mentioned works the method of application of an optimization technique to an environmental control system with the constraint of the comfort equation shall provide the combination of variables that require minimum consumption of energy and the starting point or basis for environmental comfort and total economic optimization.

BODY OF STUDY

Part 1

Solution of Fanger's Thermal Comfort Equation

Because it is used as a constraint on allowable values of the environmental variables for determination of energy consumption in an environmental control system, the equation proposed by Fanger [18] was thoroughly investigated.

A similar procedure as Lee et al. [53] reported, was used in the solution of the comfort equation. The results needed in this study are different from those of Lee due to some terms which were neglected by the former investigation.

The work by Fanger established a general comfort equation based upon the following variables: (1) air temperature, (2) humidity (water vapor partial pressure), (3) mean radiant temperature, (4) relative air velocity, (5) activity level (internal heat production) and (6) insulation value of the clothing (clo-value).

Thermal comfort sensation was assumed to be a function of the mean skin temperature and rate of sweat secretion. Experiments have shown that for different levels of activity there are certain values of the mean skin temperature and sweat secretion rate that provide comfort sensation for each subject. Fanger [18,19] recognized that, under thermal comfort conditions, the mean skin temperature and rate of sweat secretion could be related to the activity levels by the following equations.

$$\bar{t}_{sk} = 35.7 - 0.032 \frac{M}{A_{Du}} (1-\eta) \tag{1}$$

$$\bar{E}_{rsw} = A_{Du}(0.42)[\frac{M}{A_{Du}} (1-\eta) - 50] \tag{2}$$

where

$\bar{t}_{sk}$ = average mean skin temperature during a state of comfort, ^oC

$\bar{E}_{rsw}$ = average rate of regulatory sweating during a state of comfort, kcal/hr

A_{Du} = DuBois body surface area, m^2

M = metabolic rate, kcal/hr

η = external mechanical efficiency, dimensionless.

The metabolic rate indicates the activity level of a person. For college-age individuals (males), the metabolic rates per unit DuBois body surface area for four different activity levels are as follows [18,53]:

(a) sedentary: M/A_{Du} = 52 kcal/m^2hr

(b) low activity level: M/A_{Du} = 83 kcal/m^2hr

(c) medium activity level: M/A_{Du} = 111 kcal/m^2hr

(d) high activity level: M/A_{Du} = 132 kcal/m^2hr.

When no external work is performed by the subjects, the metabolic rate is equal to the internal heat production rate. However, if external work is performed, a part of the metabolic energy is converted into work with a conversion efficiency η. According to Fanger, Equations (1) and (2) represent the basic conditions for thermal comfort. Given an activity level, the comfort values for $\bar{t}_{sk}$ and $\bar{E}_{rsw}$ can be obtained by solving simultaneously these two equations. However, to maintain steady state conditions, the heat production rate inside the body must be equal to the heat dissipation rate which is a function of the environmental conditions. The comfort equation was obtained by using basic heat balance and the two equations above. Under steady state conditions, the double heat balance relationship for the body is (no heat storage in thermally neutral state):

$$H - \bar{E}_{rsw} - E_d - L - D = K = R + C \qquad (3)$$

where

H = internal heat production in the human body

$\bar{E}_{rsw}$ = heat loss by sweat evaporation from the skin

E_d = heat loss by water vapor diffusion through the skin

L = heat loss by latent respiration

D = heat loss by dry respiration

K = heat loss by conduction through the clothing

R = heat loss by radiation at the outer surface of the clothing

C = heat loss by convection from the outer surface of the clothed body.

Fanger [18] performed the heat balance calculations and obtained the following equation which must be satisfied for thermal comfort.

$$\frac{A}{M_{Du}} (1-\eta) - 0.35 [43 - 0.061 \frac{M}{A_{Du}} (1-\eta) - P_a]$$

$$- 0.42 [\frac{M}{A_{Du}} (1-\eta) - 50] - 0.0023 \frac{M}{A_{Du}} (44 - P_a)$$

$$- 0.0014 \frac{M}{A_{Du}} (34 - t_a) = [35.7 - 0.032 \frac{M}{A_{Du}} (1-\eta) - t_{cl}]/0.18 I_{cl} =$$

$$4.8 \times 10^{-8} f_{cl} f_{eff} [(t_{cl} + 273)^4 - (t_{mrt} + 273)^4]$$

$$+ f_{cl} h_c (t_{cl} - t_a), \quad \text{kcal/m}^2\text{hr} \qquad (4)$$

where

P_a = partial pressure of water vapor in ambient air, mm Hg

t_a = air temperature, ^oC

t_{cl} = outer temperature of clothed body, ^oC

I_{cl} = dimensionless overall heat transfer resistance from skin to the outer surface of the clothed body

f_{cl} = ratio of the surface area of the clothed body to the nude body

f_{eff} = ratio of the effective radiation area of the clothed body to the surface area of the clothed body

t_{mrt} = mean radiant temperature, ^oC

h_c = convective heat transfer coefficient, kcal/m^{2}hr ^oC.

The products, $A_{Du}\ f_{cl}\ f_{eff}$ and $A_{Du}\ f_{cl}$, represent the effective heat transfer area of the clothed body for radiation and convection respectively.

For free and forced convections, the values of h_c had been taken to be [18]

$$h_c = 2.05\ (t_{cl} - t_a)^{0.25}\ \text{kcal/m}^2\text{/hr/}^\circ\text{C}\quad \text{(free convection)} \tag{5}$$

and

$$h_c = 10.4\ v^{0.5}\ \text{kcal/m}^2\text{/hr/}^\circ\text{C}\quad \text{(forced convection)} \tag{6}$$

respectively, where

v = relative air velocity < 2.6 m/sec.

It should be noted that Equations (5) and (6) are in agreement with the commonly used formulas for free and forced convections respectively [56].

For a motionless person, the relative air velocity is equal to the actual air velocity. The mean radiant temperature, in relation to a given person placed at a given point with a given body position and a given clothing, is defined as that uniform temperature of a black enclosure, which gives the same heat loss by radiation from the person as in the actual enclosure under study.

Since Equation (4) consists of two separate equations, it first can be solved for the left part for the outer surface temperature of the clothed body, t_{cl}. This gives rise to the following expression.

$$\begin{aligned} t_{cl} = {} & 35.7 + \frac{M}{A_{Du}}\,[-0.081459 I_{cl} - 0.032 + \eta(0.108243 I_{cl} + 0.032)] \\ & - (0.063 I_{cl} + 0.000414 I_{cl}\frac{M}{A_{Du}})P_a - 0.000252 I_{cl}\frac{M}{A_{Du}}\,t_a \\ & - 1.071 I_{cl},\ ^\circ\text{C} \end{aligned} \tag{7}$$

Let, for simplicity

$$\alpha = -0.081459I_{cl} - 0.032 + \eta(0.108243I_{cl} + 0.032) \qquad (8)$$

Next the left hand side of the first equality (left part) in Equation (4) is equated to the right hand side of the second equality sign of the same equation and using Equations (7) and (8), to obtain the following equation.

$$\frac{M}{A_{Du}}(1-\eta) - 15.05 + 0.02135\frac{M}{A_{Du}}(1-\eta) + 0.35\,P_a - 0.42\frac{M}{A_{Du}}(1-\eta)$$

$$+ 21 - 0.1012\frac{M}{A_{Du}} + 0.0023\frac{M}{A_{Du}}P_a - 0.0476\frac{M}{A_{Du}} + 0.0014\frac{M}{A_{Du}}t_a$$

$$= 4.8 \times 10^{-8} f_{cl}\, f_{eff}\, [(\alpha\frac{M}{A_{Du}} - (0.063\, I_{cl} + 0.000414\, I_{cl}\frac{M}{A_{Du}})\, P_a$$

$$- 0.000252\frac{M}{A_{Du}} I_{cl}\, t_a - 1.071\, I_{cl} + 308.7)^4 - (t_{mrt} + 273)^4]$$

$$+ f_{cl}\, h_c\, [(35.7 + \alpha\frac{M}{A_{Du}} - (0.063\, I_{cl} + 0.000414\, I_{cl}\frac{M}{A_{Du}})\, P_a$$

$$- (1 + 0.000252\, I_{cl}\frac{M}{A_{Du}})\, t_a - 1.071\, I_{cl})] \qquad (9)$$

Taking the assumptions made by Fanger [18] in arriving at the above final form of the heat balance equation as follows:

(1) The mean skin temperature, $\bar{t}_{sk}$, and the internal body temperature are important parameters for thermal comfort. The mean skin temperature

for comfort lies between $27^{\circ}C$ and $37^{\circ}C$ for the metabolic range used by Fanger.

(2) The heat of vaporization of water at $35^{\circ}C$ is assumed to be equal to 575 kcal/kg.

In addition, Fanger [18] has used the following conditions in his calculations:

(1) For each activity level the temperature of air, t_a, is maintained equal (or approximately equal) to the mean radiant temperature, or

$$t_a = t_{mrt} \; .$$

(2) The relative humidity is maintained at 45%.

(3) The external mechanical efficiency of the body, η, is zero.

In the present study it was assumed that the convective heat loss from the body is by forced convection only. The value of convective heat transfer coefficient, h_c, is given by Equation (6).

Since there is a transition zone between free and forced convection, the lower limit of the applicability of Equation (6) is not clearly defined. However, it has been found in practical calculations that the lower limit of Equation (6) is approximately 0.1 m/sec [18]. This lower limit of validity is adopted as noted in this work. Also since velocities above 2.6 m/sec produce drafts it was adopted as the upper limit. Thus the value of v is restricted to the range

$$0.1 < v < 2.6. \tag{10}$$

Taking the above experimental conditions and the assumptions into consideration, the comfort equation, or Equation (9), becomes as follows

$$0.45255 \frac{M}{A_{Du}} + (0.35 + 0.0023 \frac{M}{A_{Du}}) P_a + 0.0014 \frac{M}{A_{Du}} t_a$$

$$+ 5.95 = 4.8 \times 10^{-8} f_{cl} f_{eff} [(- (0.063 I_{cl}$$

$$+ 0.0004141 I_{cl} \frac{M}{A_{Du}}) P_a - 0.000252 I_{cl} \frac{M}{A_{Du}} t_a + 308.7$$

$$- (0.081459 I_{cl} + 0.032) \frac{M}{A_{Du}} - 1.071 I_{cl})^4$$

$$- (t_a + 273)^4] + 10.4 f_{cl} v^{0.5} [- (0.063 I_{cl}$$

$$+ 0.000414 I_{cl} \frac{M}{A_{Du}}) P_a - (1 + 0.000252 I_{cl} \frac{M}{A_{Du}}) t_a$$

$$+ 35.7 = (0.081459 I_{cl} + 0.032) \frac{M}{A_{Du}} - 1.071 I_{cl}], \text{ kcal/m}^2\text{h} . \tag{11}$$

This comfort equation, Equation (11), contains the following variables:

$$I_{cl}, f_{cl}, \frac{M}{A_{Du}}, f_{eff}, v, t_a, P_a .$$

However, in most practical situations in life support or environmental control systems, only the thermal environmental variables P_a, t_a and v can be controlled. The other variables, namely, M/A_{Du}, I_{cl}, f_{cl} and f_{eff} cannot be changed easily and will be considered as parameters in the present study.

In Table 3 are listed the values used for the parameters in the solution of the comfort equation. Four types of activities for both males and females and four arbitrary levels of activity are considered. There are total of twelve sets of parameter values for $\frac{M}{A_{Du}}$, I_{cl}, f_{cl}, f_{eff}.

With the assumption that $\frac{M}{A_{Du}}$, I_{cl}, f_{cl} and f_{eff} are constants for each type of activity, the comfort equation can be written in the following form

$$A + CP_a + Dt_a + 5.95 = E[(-FP_a - Gt_a + W)^4$$

$$- (t_a + 273)^4] + X\sqrt{v}\,(- FP_a - Zt_a + U), \qquad (12)$$

where

$A = 0.45255\ M/A_{Du}$

$C = 0.35 + 0.0023\ M/A_{Du}$

$D = 0.0014\ M/A_{Du}$

$E = 4.8 * 10^{-8}\ f_{cl}\ f_{eff}$

$F = 0.063\ I_{cl} + 0.000414\ I_{cl}\ M/A_{Du}$

$G = 0.000252\ I_{cl}\ M/A_{Du}$

$B = -\ (0.081459\ I_{cl} + 0.032)\ M/A_{Du} - 1.071\ I_{cl}$

$W = 308.7 + B$

$X = 10.4\ f_{cl}$

$Z = 1 + G$

$U = 35.7 + B$.

From examination of Equation (12) it is revealed that Equation (12) depends on the fourth powers of t_a and P_a, but only on the square root of V. Therefore, the best way to solve this equation is to assume values for t_a and P_a, and then calculate v from Equation (12). The solution of the comfort equation for v is

$$v = \left\{ \frac{A + CP_a + Dt_a + 5.95 - E[(-FP_a - Gt_a + W)^4 - (t_a + 273)^4]}{X(-FP_a - Zt_a + U)} \right\}^2$$

$$= \left\{ \frac{\text{DATA 1} - \text{DATA 2}}{\text{DATA 3}} \right\}^2 . \qquad (13)$$

The solution of Equation (13) has been performed on an IBM 360/50 computer. The computer flow diagram is shown in Appendix A and the computer program is listed in Appendix B. The computational results are shown in Figures 2 through 13, where the different activity levels are listed on each figure.

The peculiar behavior of the results shown is attributed to the characteristic upper and lower limits of Equation (13). The lower limit is zero, which occurs when the numerator equals to zero, and the upper limit is infinite, which occurs when the denominator equals to zero. These two limits can be seen clearly in the figures. Obviously, the comfort equation is usable or feasible only in the restricted range of Equation (10). This feasibility range is an important point and requires some investigation.

Table 3. Values of Parameters used in Solution of Comfort Equation (For Figures 2 through 13)

	type of activity	M/A_{Du} kcal/m^{2}hr	I_{cl} clo	f_{cl}	f_{eff}
male	sedentary	52	0.6	1.1	0.65
	low	83	0.6	1.1	0.75
	medium	111	0.6	1.1	0.75
	high	132	0.6	1.1	0.75
female	sedentary	40	0.6	1.1	0.65
	low	66	0.6	1.1	0.75
	medium	87	0.6	1.1	0.75
	high	110	0.6	1.1	0.75
arbitrary	sedentary	50	0.0	1.0	0.65
	low	80	0.0	1.0	0.75
	medium	100	0.0	1.0	0.75
	high	150	0.0	1.0	0.75

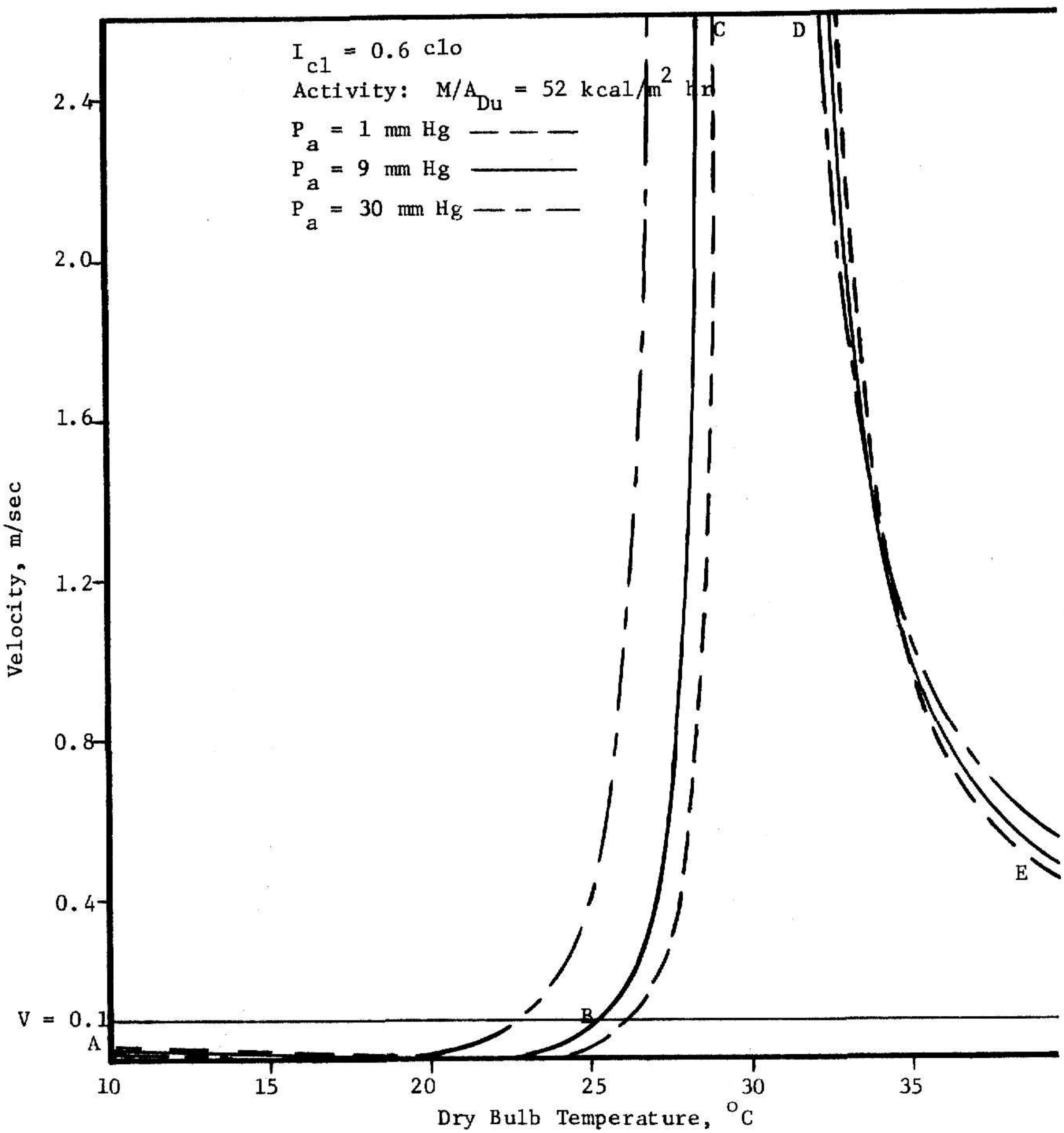

Figure 2. Solution of Comfort Equation for Male (Sedentary).

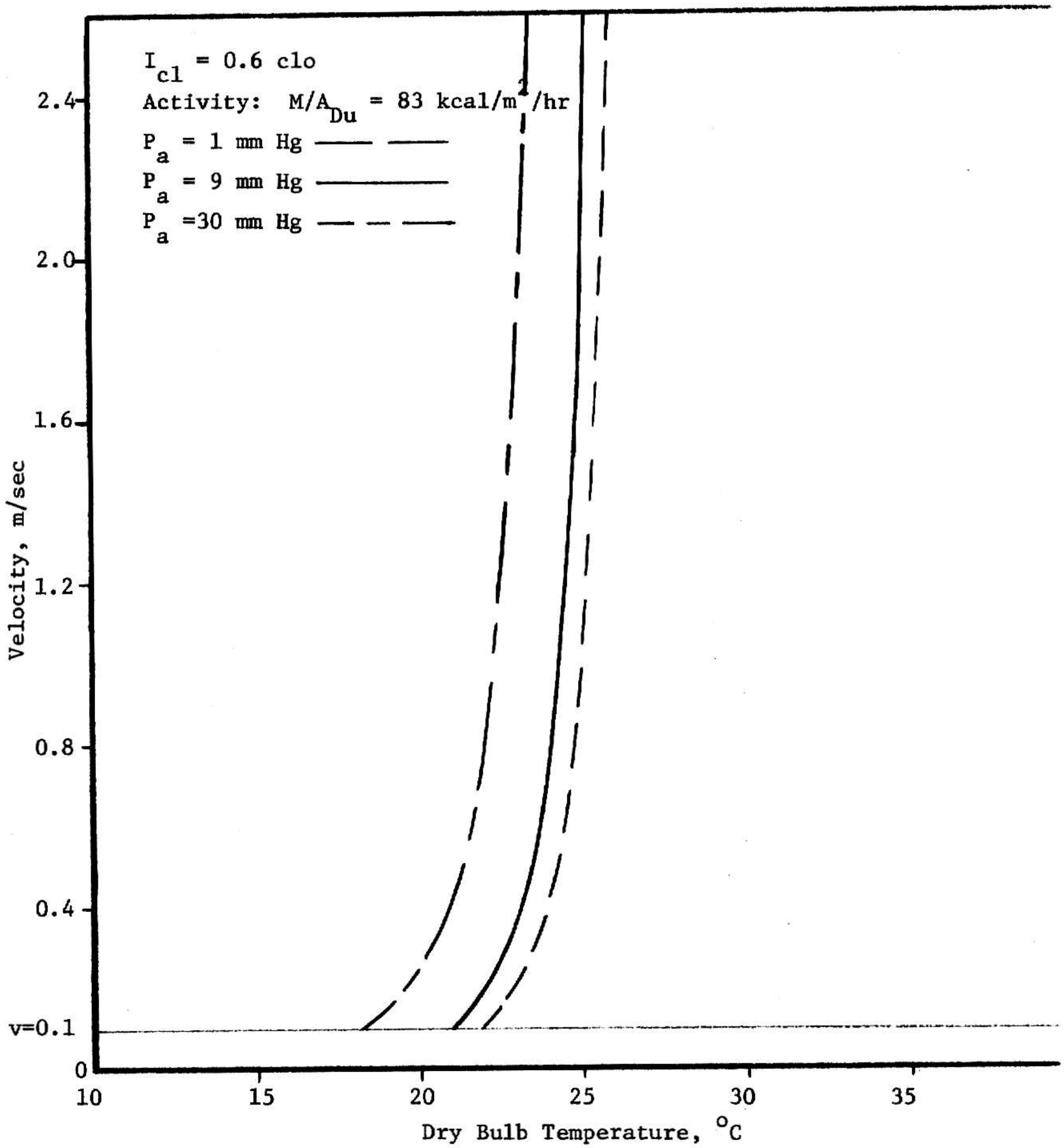

Figure 3. Solution of comfort equation.

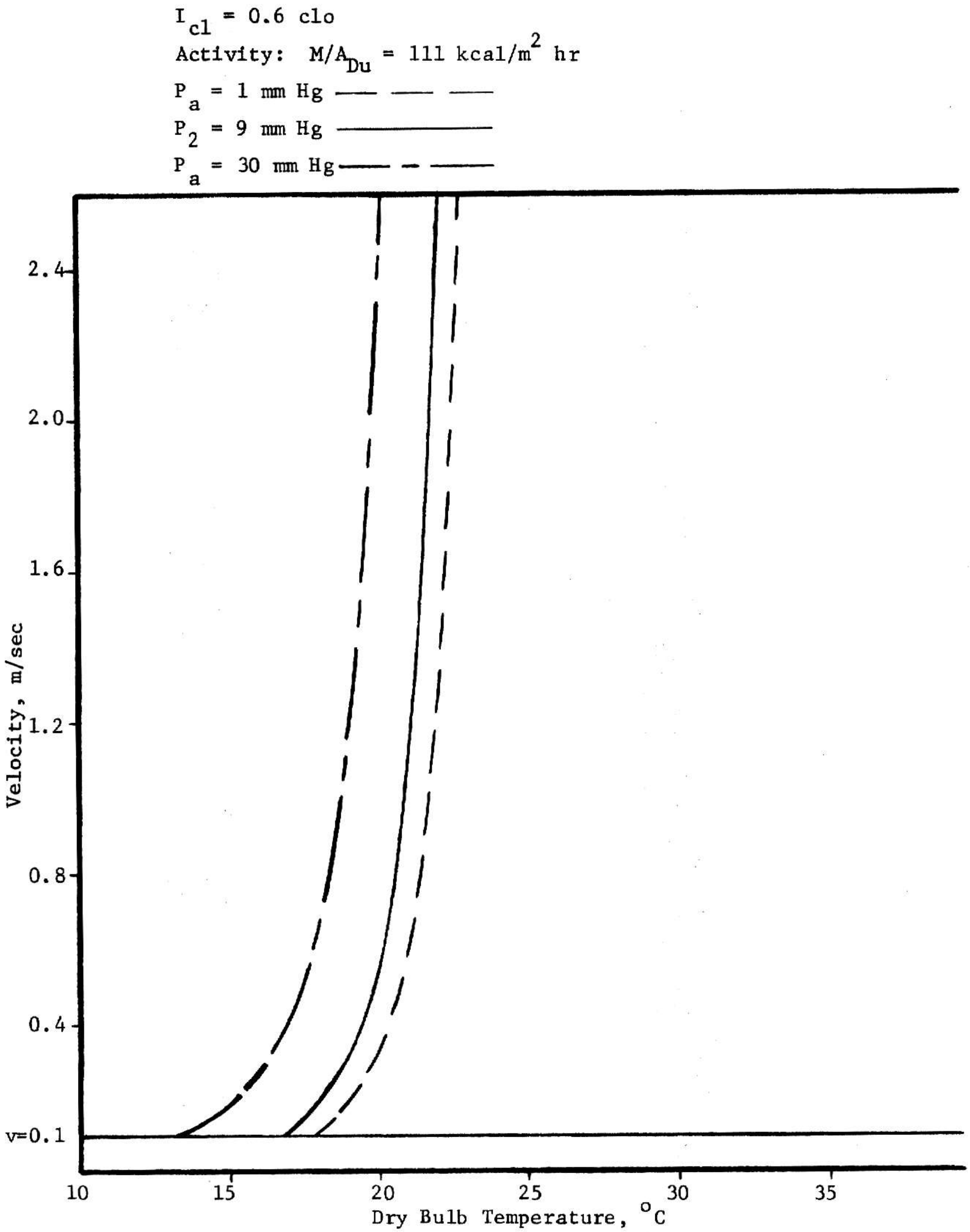

Figure 4. Solution of Comfort Equation for Male (Medium Activity).

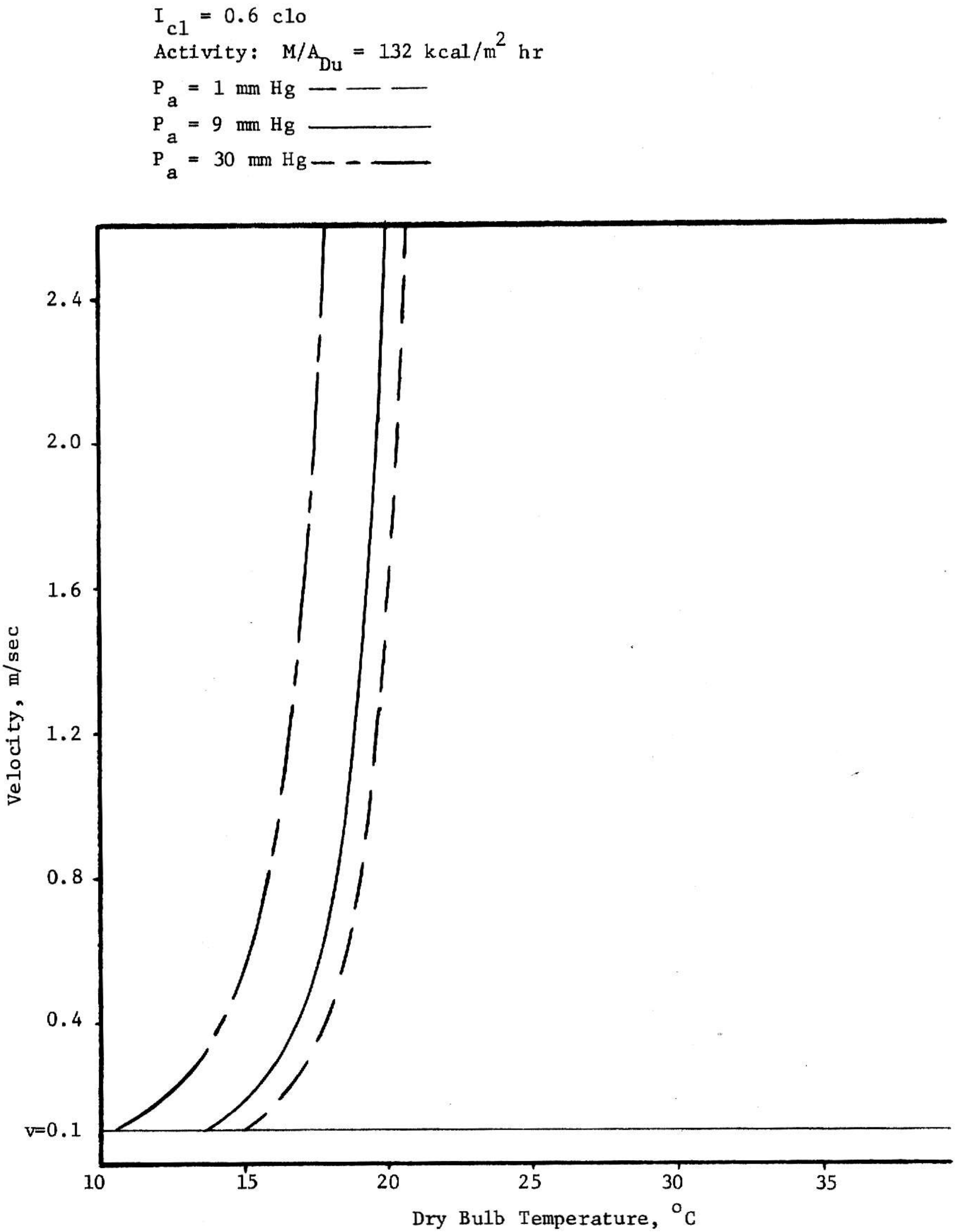

Figure 5. Solution of Comfort Equation for Male (High Activity).

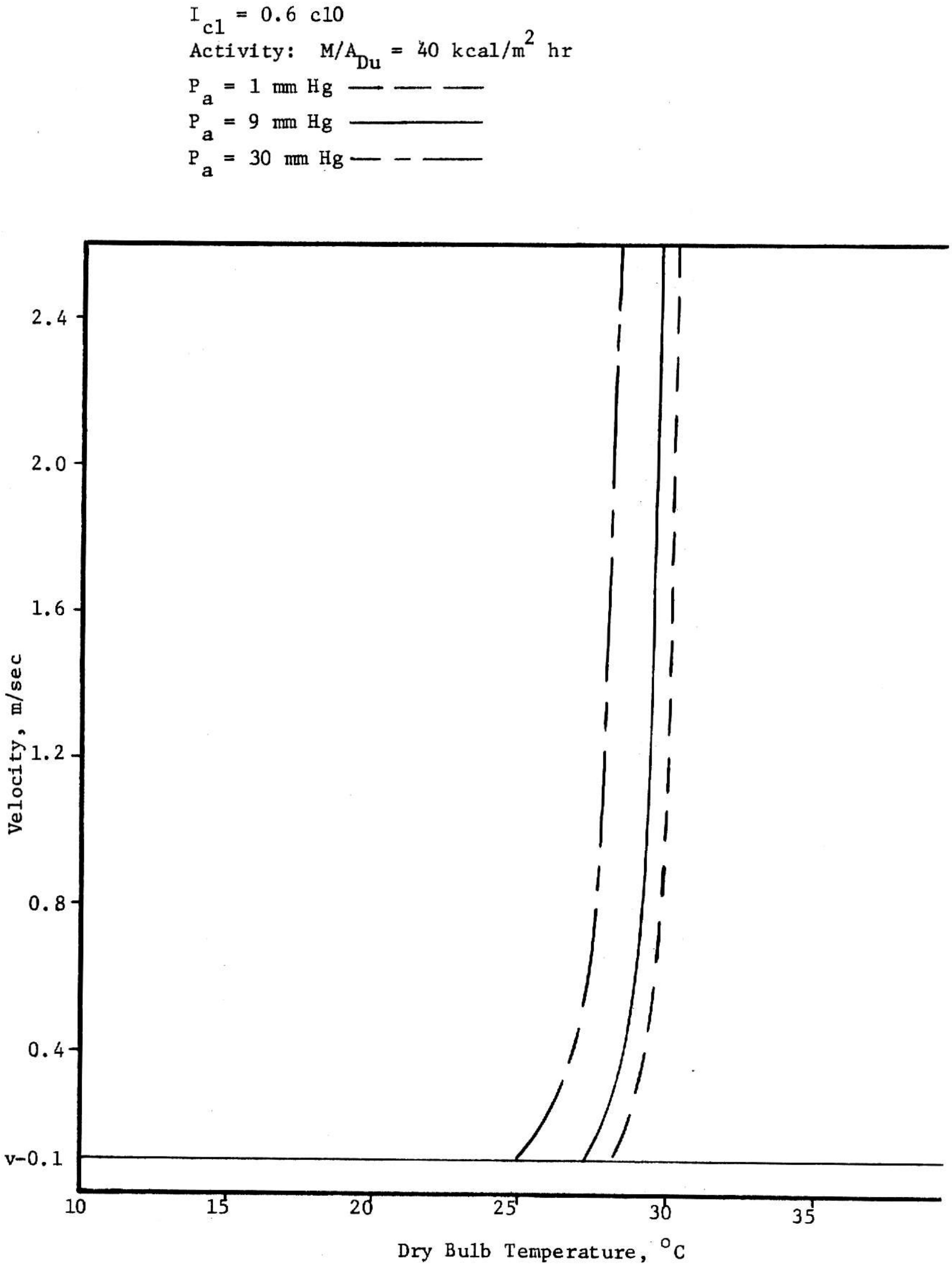

Figure 6. Solution of Comfort Equation for Female (Sedentary).

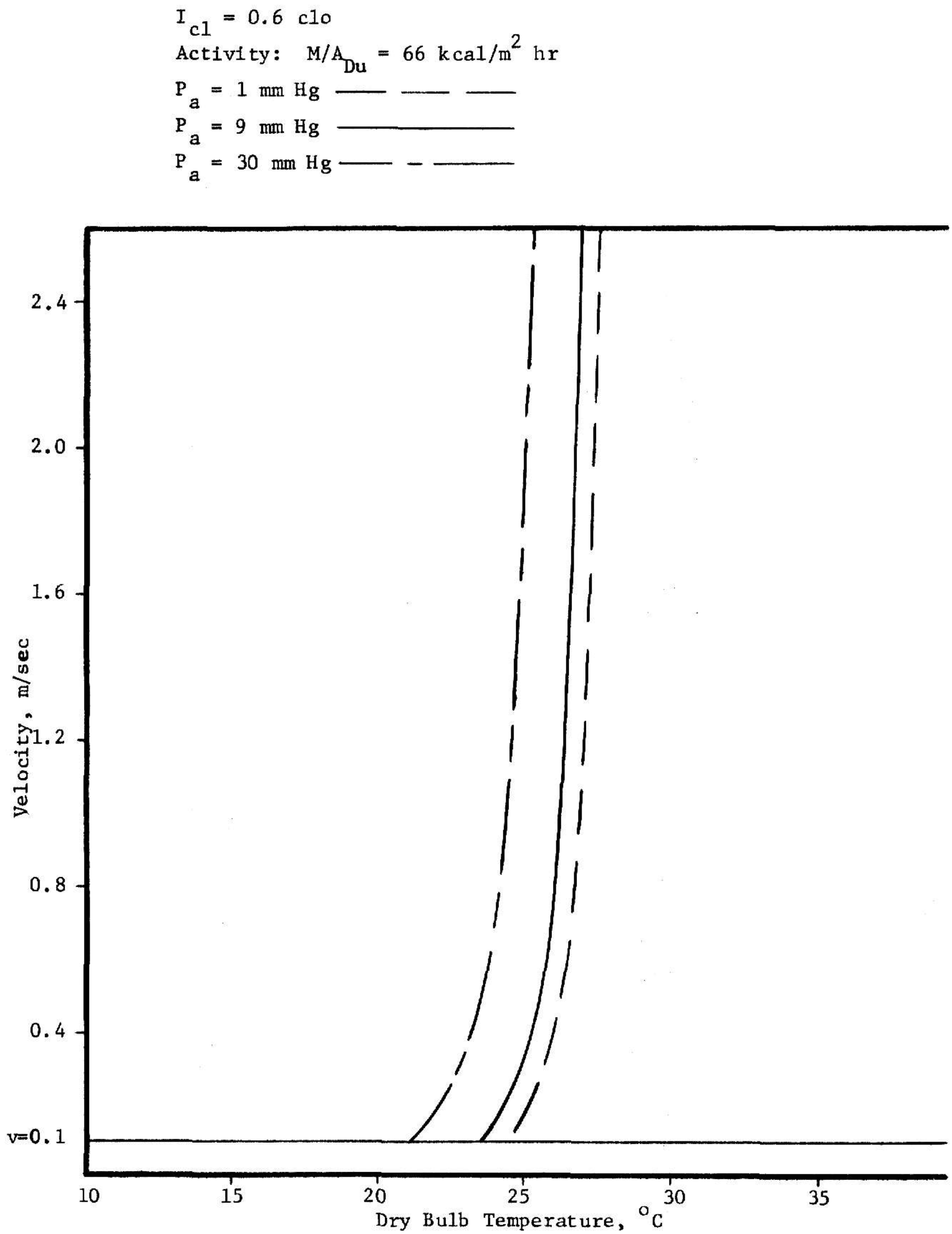

Figure 7. Solution of Comfort Equation for Female (Low Activity).

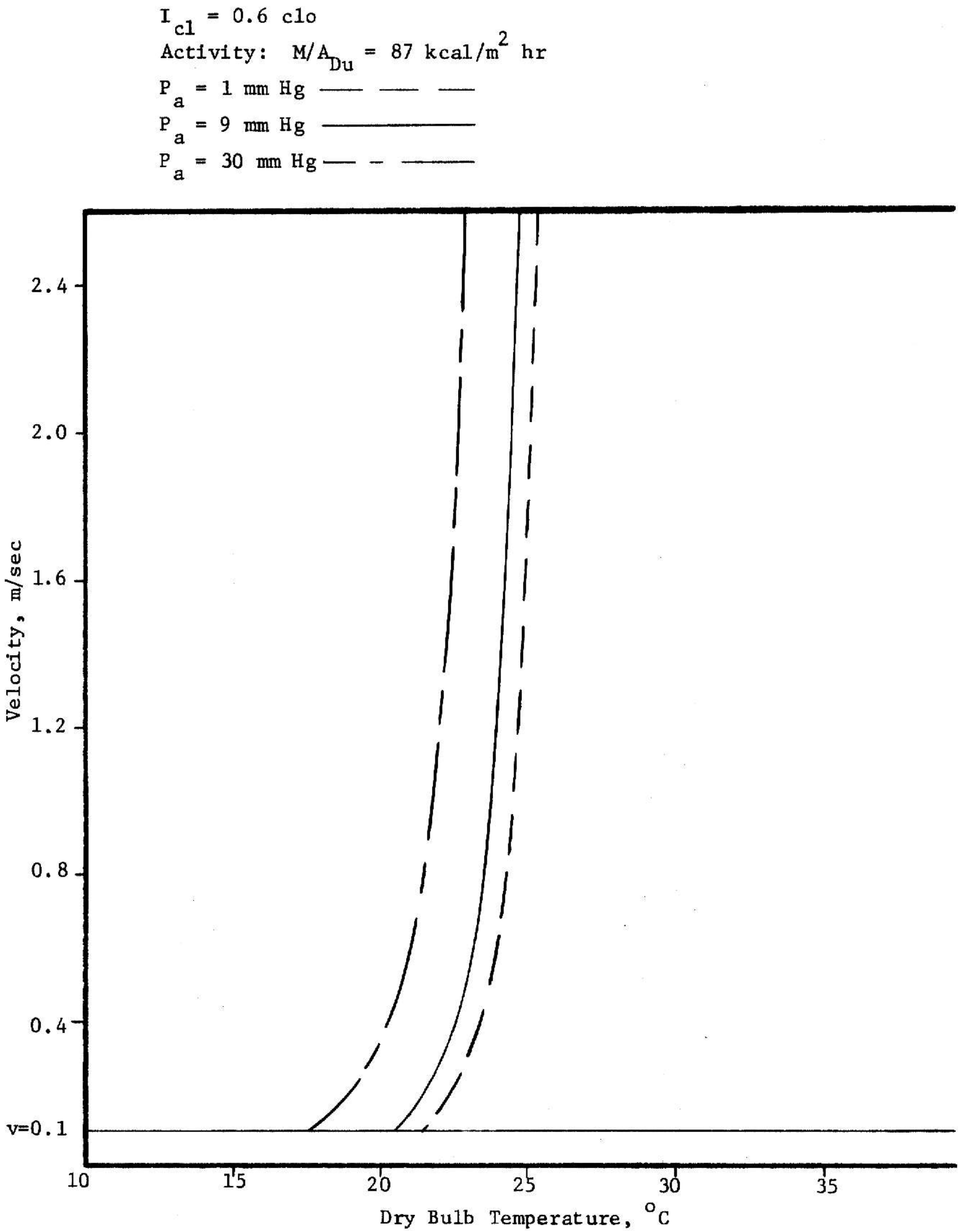

Figure 8. Solution of Comfort Equation for Female (Medium Activity).

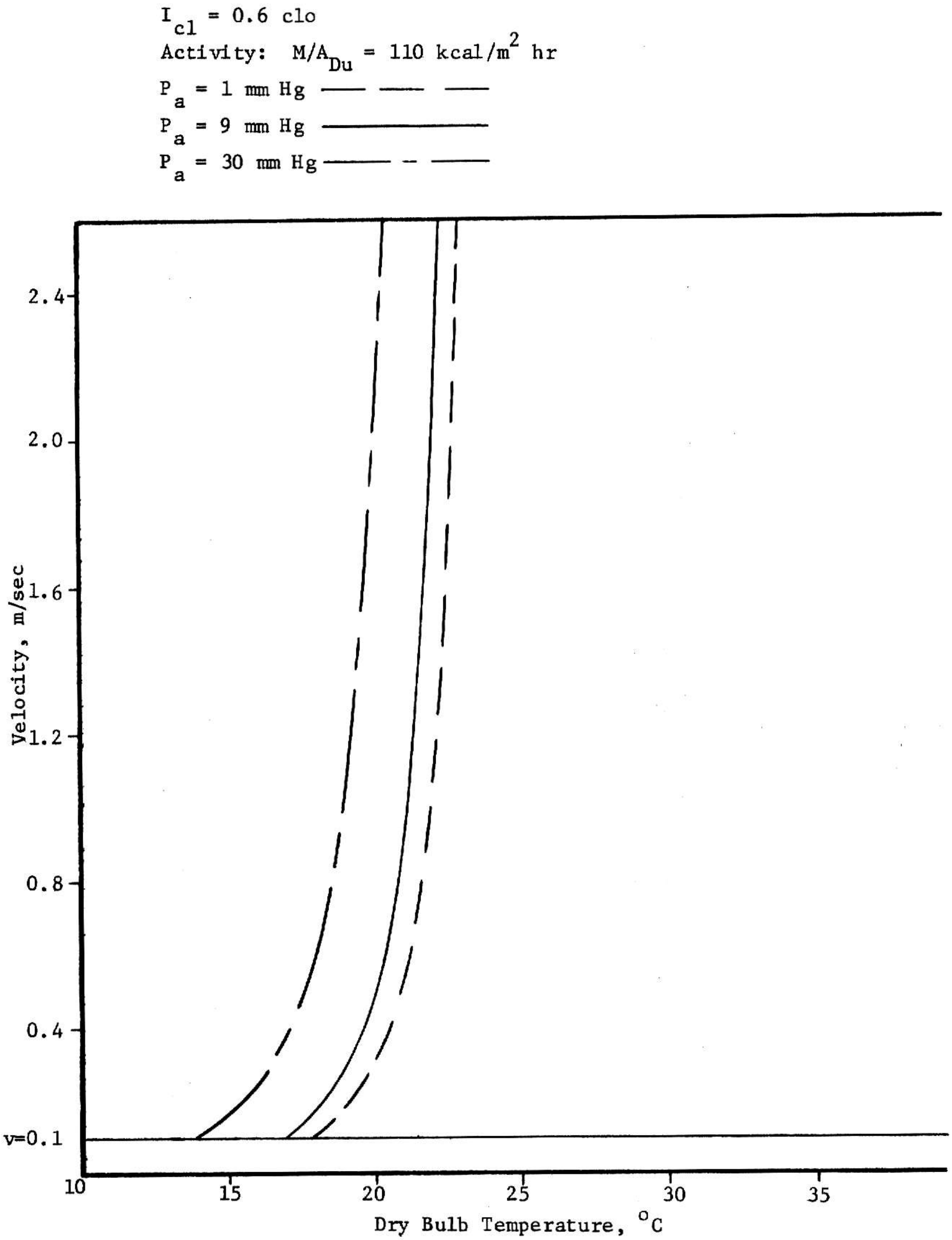

Figure 9. Solution of Comfort Equation for Female (High Activity).

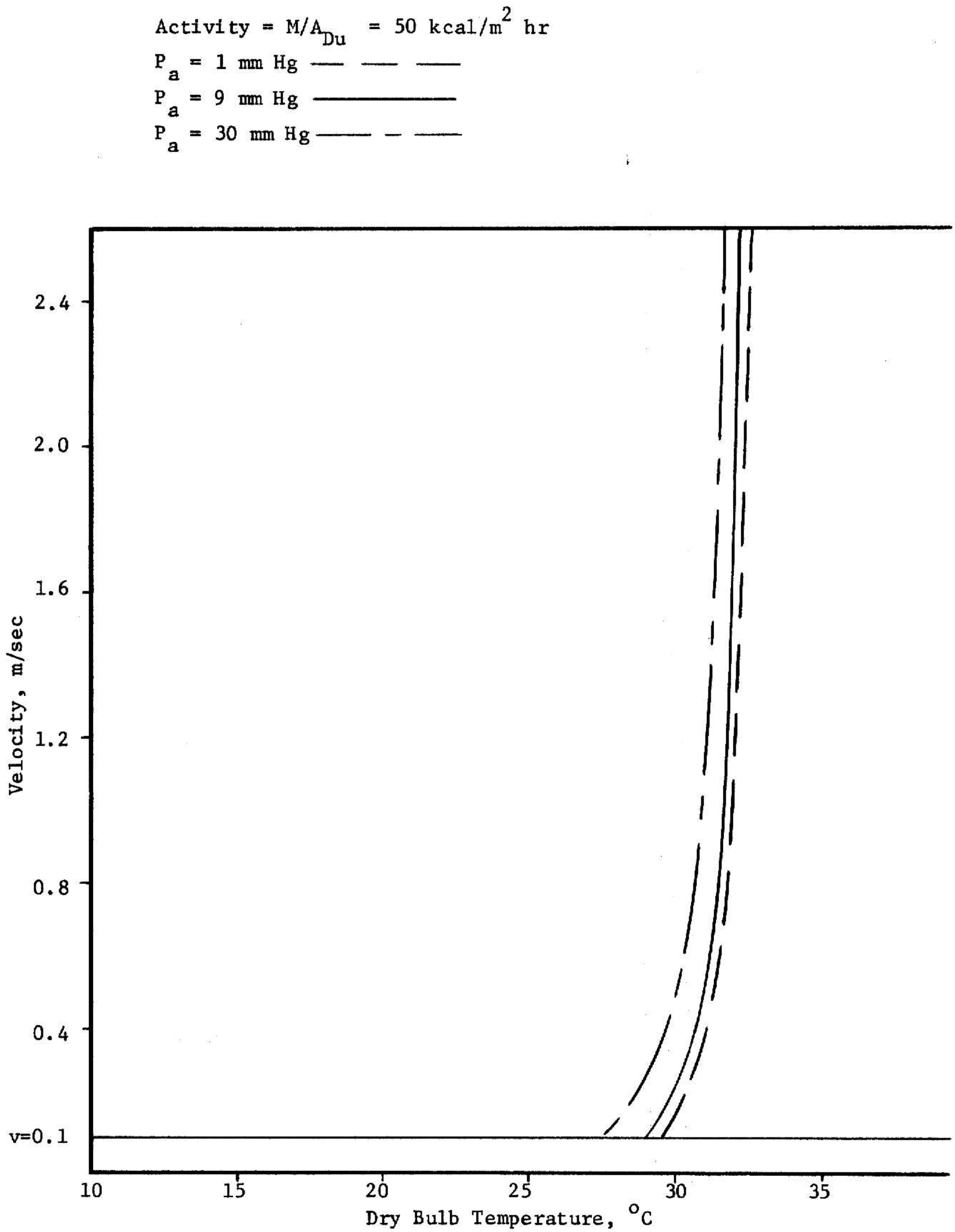

Figure 10. Solution of Comfort Equation at 0.0 clo (Sedentary).

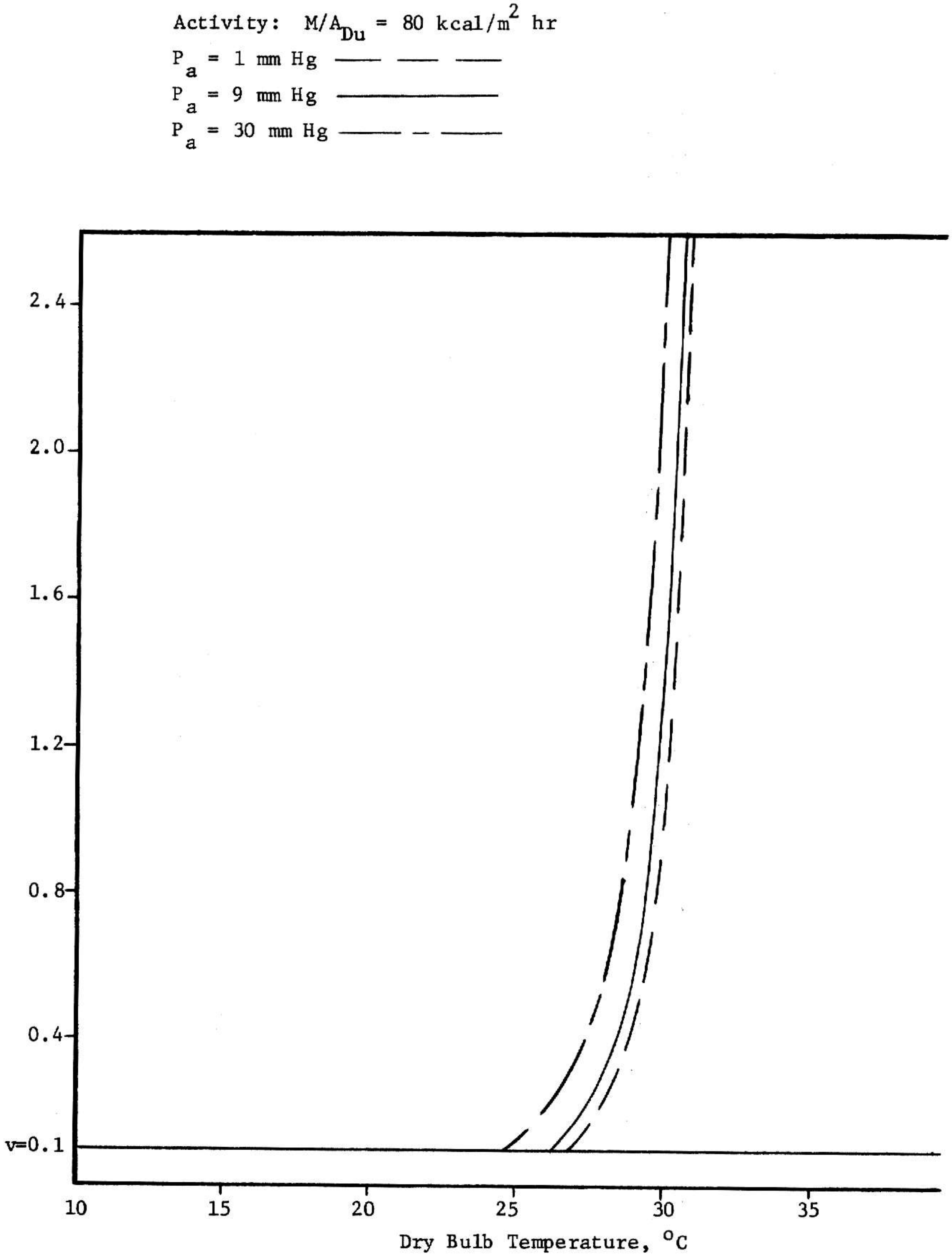

Figure 11. Solution of Comfort Equation at 0.0 clo (Low Activity).

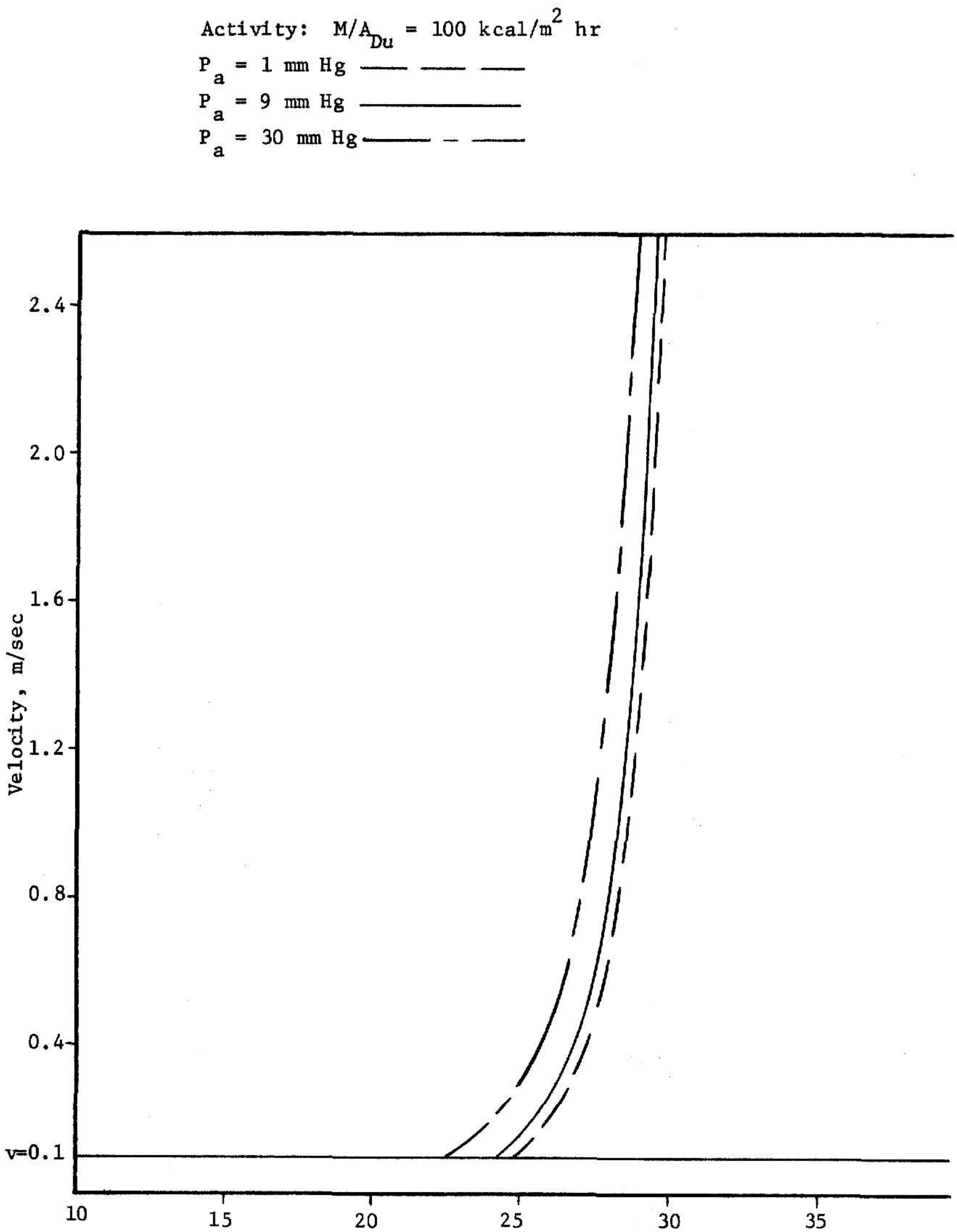

Figure 12. Solution of Comfort Equation at 0.0 clo (Medium Activity).

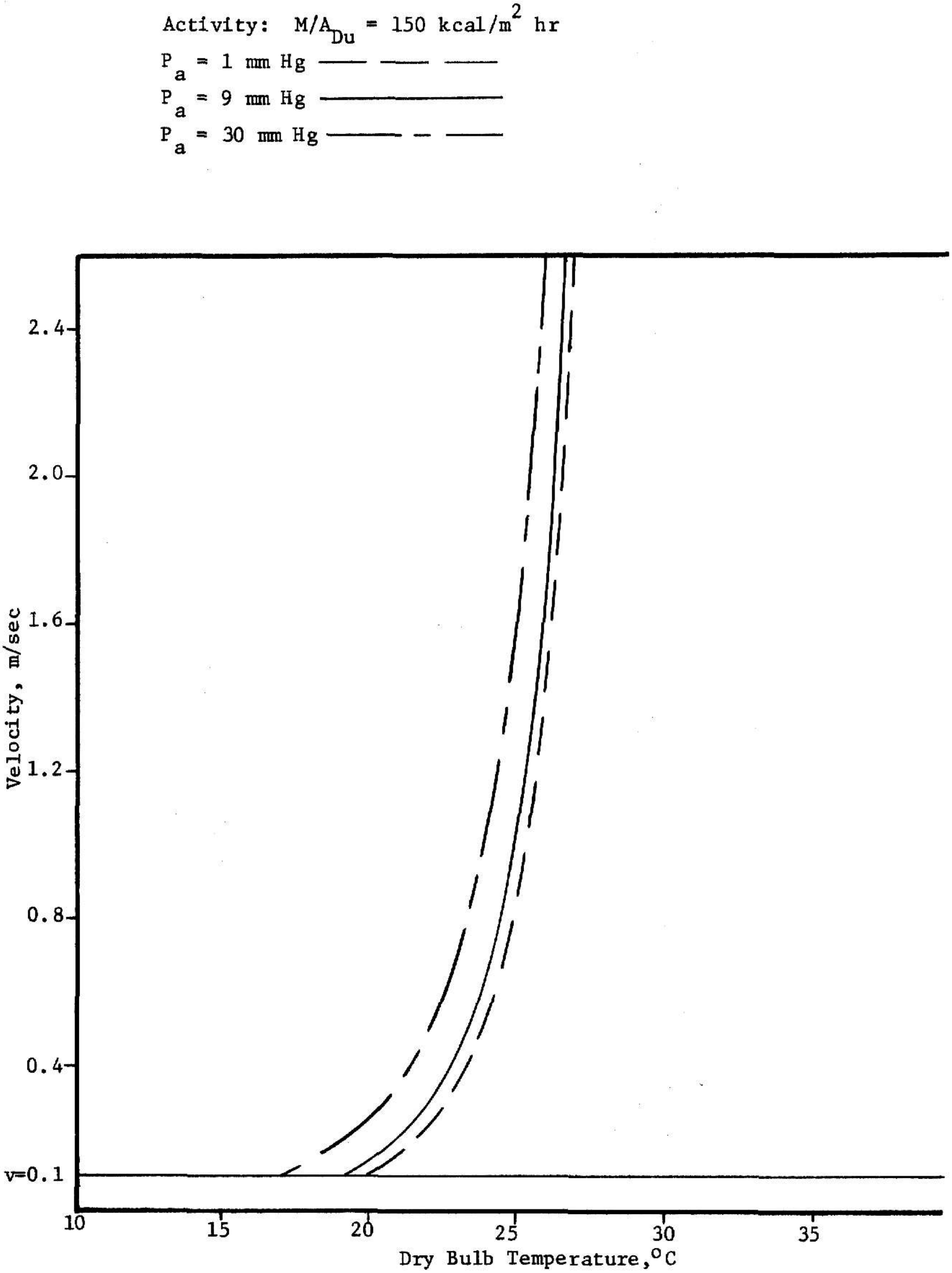

Figure 13. Solution of Comfort Equation for 0.0 clo (High Activity).

Feasibility Study of the Comfort Equation

The only feasible solutions of the comfort equation occur in the zone represented by the segment BC (see Figure 2). The segment AB is infeasible because of the assumption of forced convection and therefore the relative velocity of air must be larger than 0.1 m/sec. It should be noted that the comfort equation [18] is also usable for free convection, provided that Equation (5) is used. However, for free convection it is seen that the relative velocity of air, v, does not appear in the comfort equation.

The feasible regions for $P_a = 1$ mm Hg and $P_a = 15$ mm Hg are given in Table 4 and Table 5 respectively. The feasible regions are listed for the different parameters that were in Table 3.

It is of practical interest to obtain the upper and lower mathematical limits of Equation (13). At the upper limit, where v approaches infinity, Equation (13) yields

$$X(-FP_a - Zt_a + U) = 0. \qquad (14)$$

This equation can be solved easily since it is a linear relationship. At the lower limit, where v equals zero, Equation (13) yields

$$A + CP_a + Dt_a + 5.95 - E[(-FP_a - Gt_a + W)^4 - (t_a + 273)^4] = 0. \qquad (15)$$

Equation (15) involves a fourth power relationship in t_a and P_a and cannot be solved easily. Therefore, the Newton-Raphson method [93,16] was used to solve Equation (15). The method is described in Appendix C. The computer flow diagram is shown in Appendix D and the procedure used in the computer program is given in Appendix E.

For this study the physical limits imposed by Equation (10) are important. To investigate the feasible range of these physical limits of $v = 0.1$ and $v = 2.6$, Equation (13) is solved for P_a as a function of t_a by the Newton-Raphson method. An example is given by the computer program in Appendix F.

The results for the calculations above for the mathematical and physical limits are shown in Figures 14 through 25. To illustrate the feasible region more clearly the physical bound of 100% relative humidity line and the 50% relative humidity line are drawn on each figure.

To calculate the values for the above limits an arbitrary range of P_a was chosen as

$$0 \leq P_a \leq 40 \text{ mm Hg.}$$

As can be seen from Equation (15), four values of P_a are obtained for each value of t_a. It is worth examining all four roots for feasibility. Equation (15) can be rewritten as

$$f(P_a, t_a) = A + CP_a + Dt_a + 5.95 - E[(-FP_a - Gt_a + W)^4 - (t_a + 273)^4]. \tag{16}$$

With $t_a = 20^oC$, values of the function f have been calculated as a function of P_a. However, it has been found [53] that all roots except one are not within the selected range of P_a. In fact, no root has been found to exist even within the range of $-40 \leq P_a \leq 0$, which is obviously physically impossible. Therefore, Equation (15) only has one physically feasible root as shown in Figures 14 through 25. The upper bound selected for calculation, $P_a = 40$ mm Hg, can be seen to be well above the 100% relative humidity line and hence is definitely not physically feasible.

Table 4. Feasible Range of Dry Bulb Temperature at P_a = 1.0 mm Hg.

Activity M/A_{Du} $kcal/m^2hr$	DBT, °C
(0.6 clo - normal)	
52	$26.3 \leq t_a \leq 29.2$
83	$22.0 \leq t_a \leq 25.9$
111	$18.0 \leq t_a \leq 23.0$
132	$15.0 \leq t_a \leq 20.8$
40	$28.3 \leq t_a \leq 30.4$
66	$24.6 \leq t_a \leq 27.7$
87	$21.5 \leq t_a \leq 25.5$
110	$18.1 \leq t_a \leq 23.1$
(CASE I: Nude condition)	
50	$29.6 \leq t_a \leq 32.7$
80	$26.9 \leq t_a \leq 30.9$
100	$24.9 \leq t_a \leq 29.8$
150	$19.9 \leq t_a \leq 27.0$

Table 5. Feasible Range of Dry Bulb Temperature at P_a = 15.0 mm Hg.

Activity M/A_{Du} $kcal/m^2hr$		DBT, ^oC
	(0.6 clo)	
52		$24.7 \leq t_a \leq 28.2$
83		$20.3 \leq t_a \leq 24.8$
111		$16.0 \leq t_a \leq 21.7$
132		$12.8 \leq t_a \leq 19.4$
40		$26.7 \leq t_a \leq 29.4$
66		$22.9 \leq t_a \leq 26.7$
87		$19.6 \leq t_a \leq 24.3$
110		$16.1 \leq t_a \leq 21.8$
	(CASE I: Nude condition)	
50		$28.6 \leq t_a \leq 32.3$
80		$25.9 \leq t_a \leq 30.6$
100		$23.8 \leq t_a \leq 29.4$
150		$18.4 \leq t_a \leq 26.5$

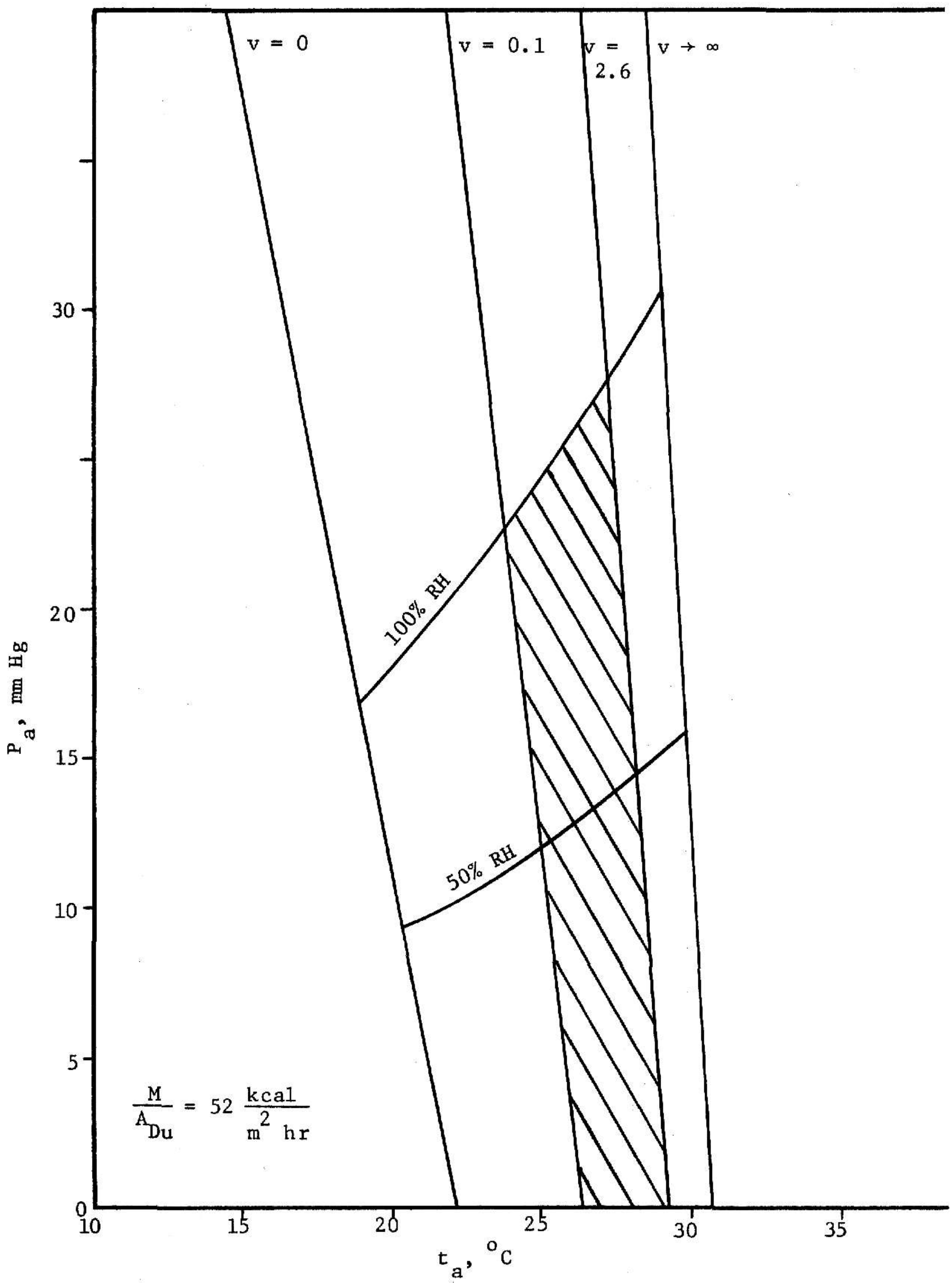

Figure 14. Feasible Region of the Comfort Equation.

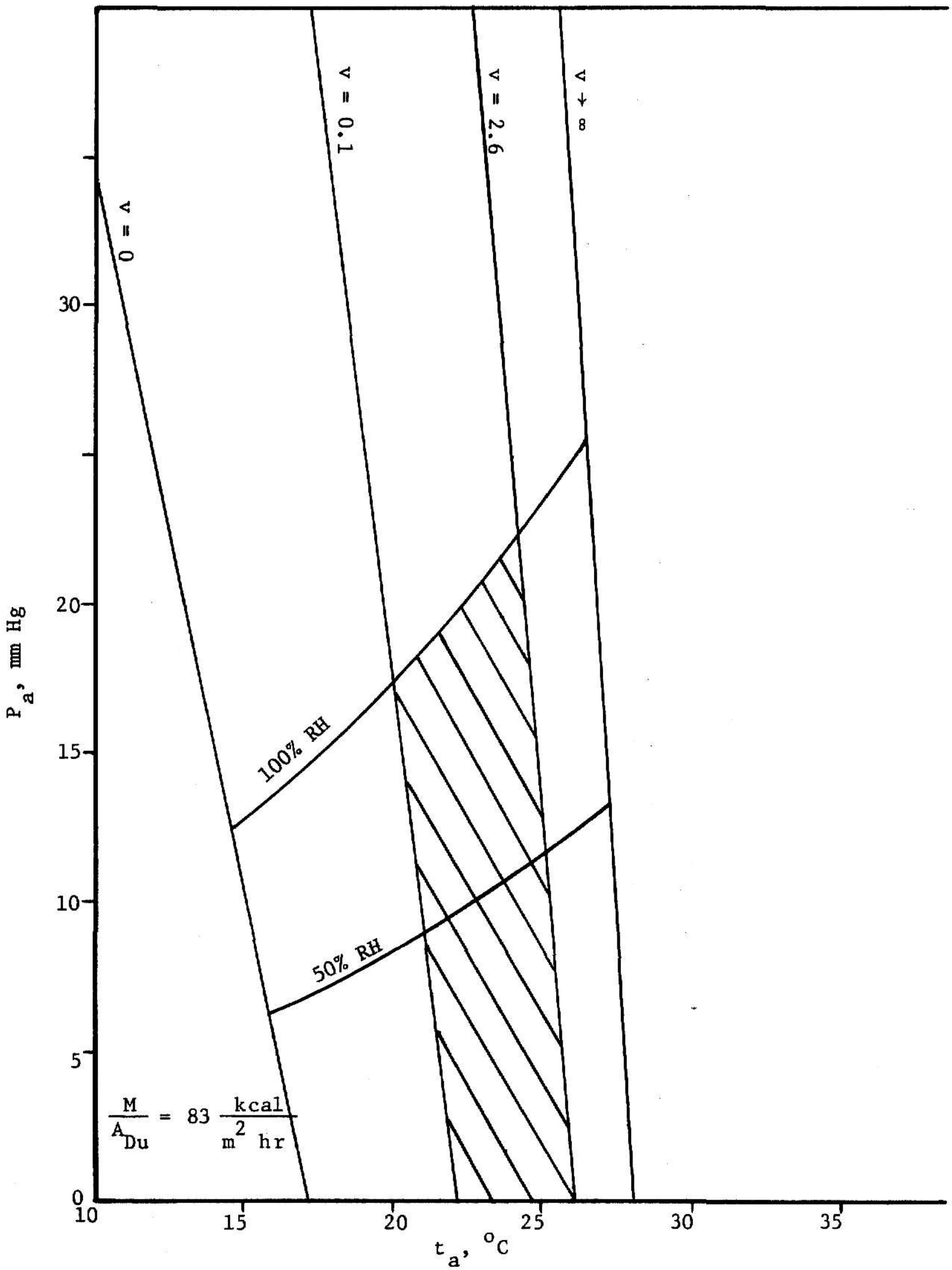

Figure 15. Feasible Region of the Comfort Equation.

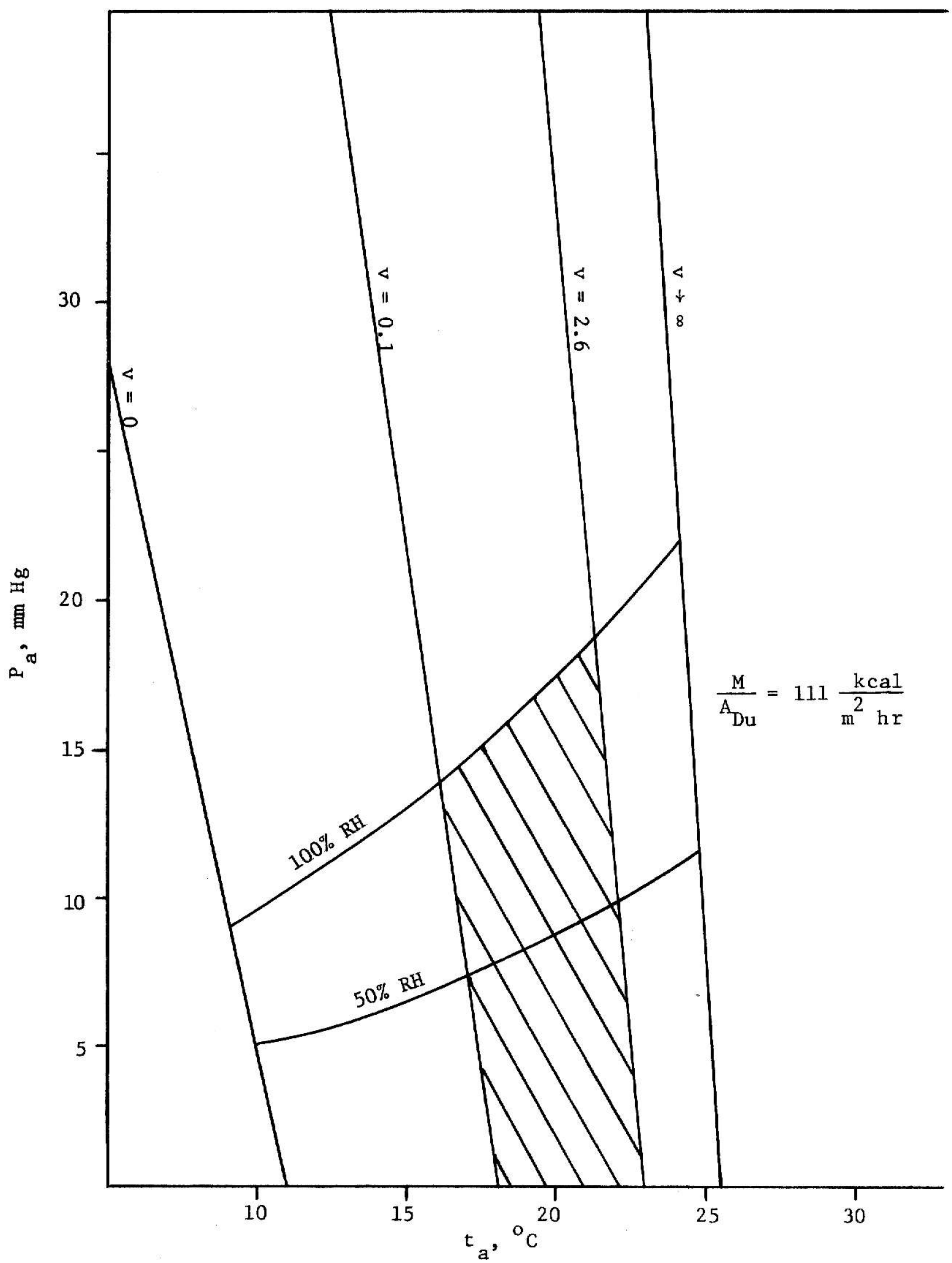

Figure 16. Feasible Region of the Comfort Equation.

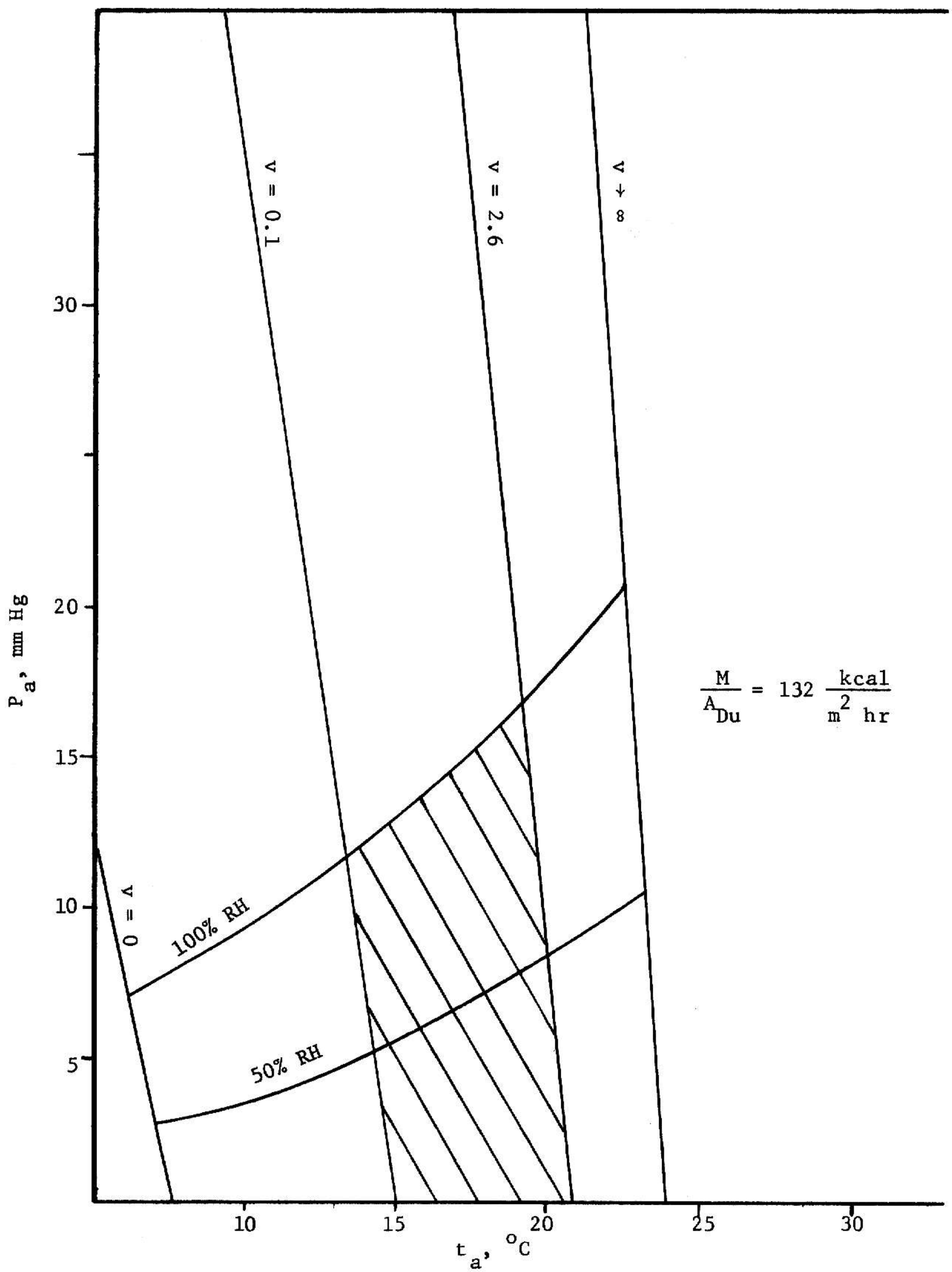

Figure 17. Feasible Region of the Comfort Equation.

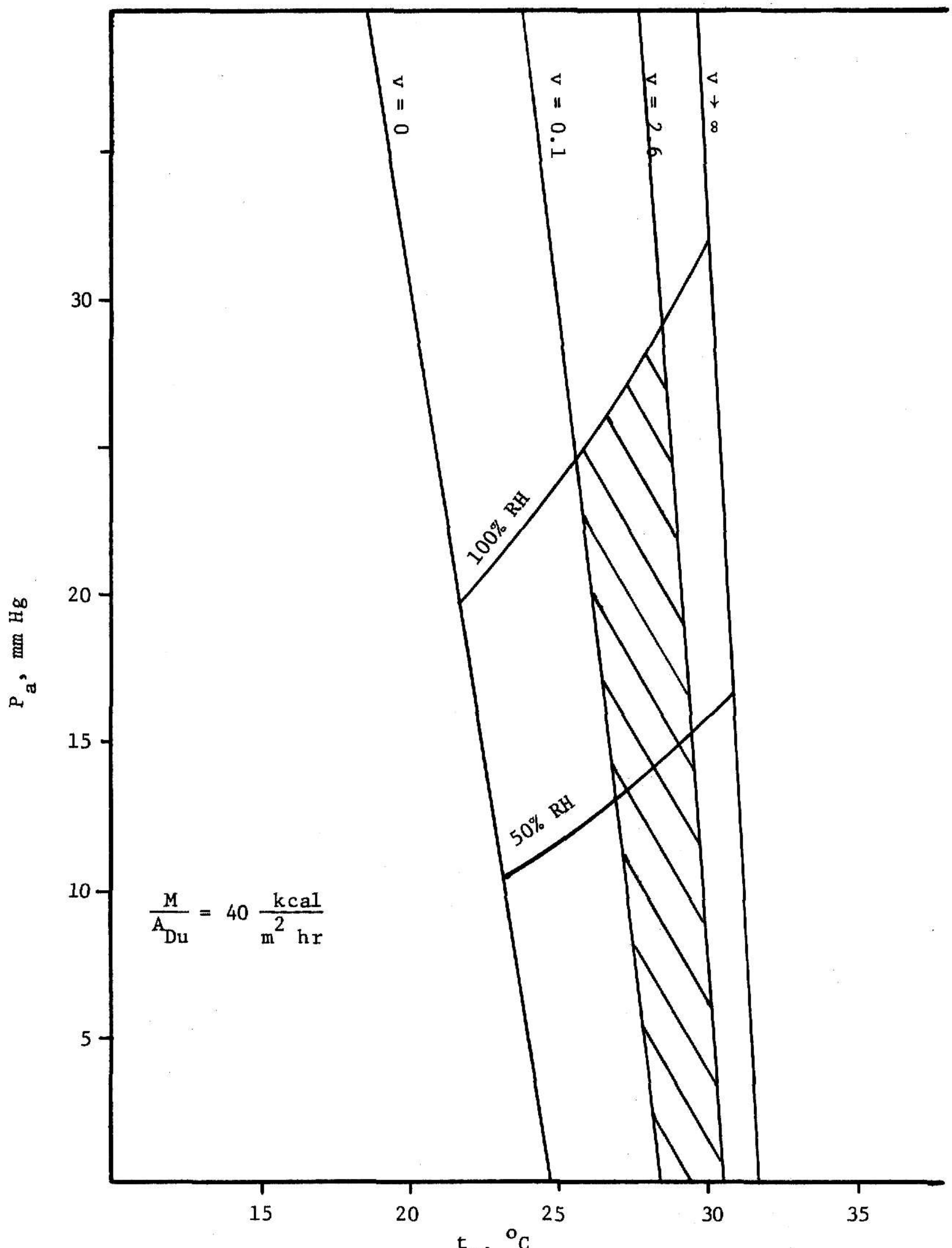

Figure 18. Feasible Region of the Comfort Equation.

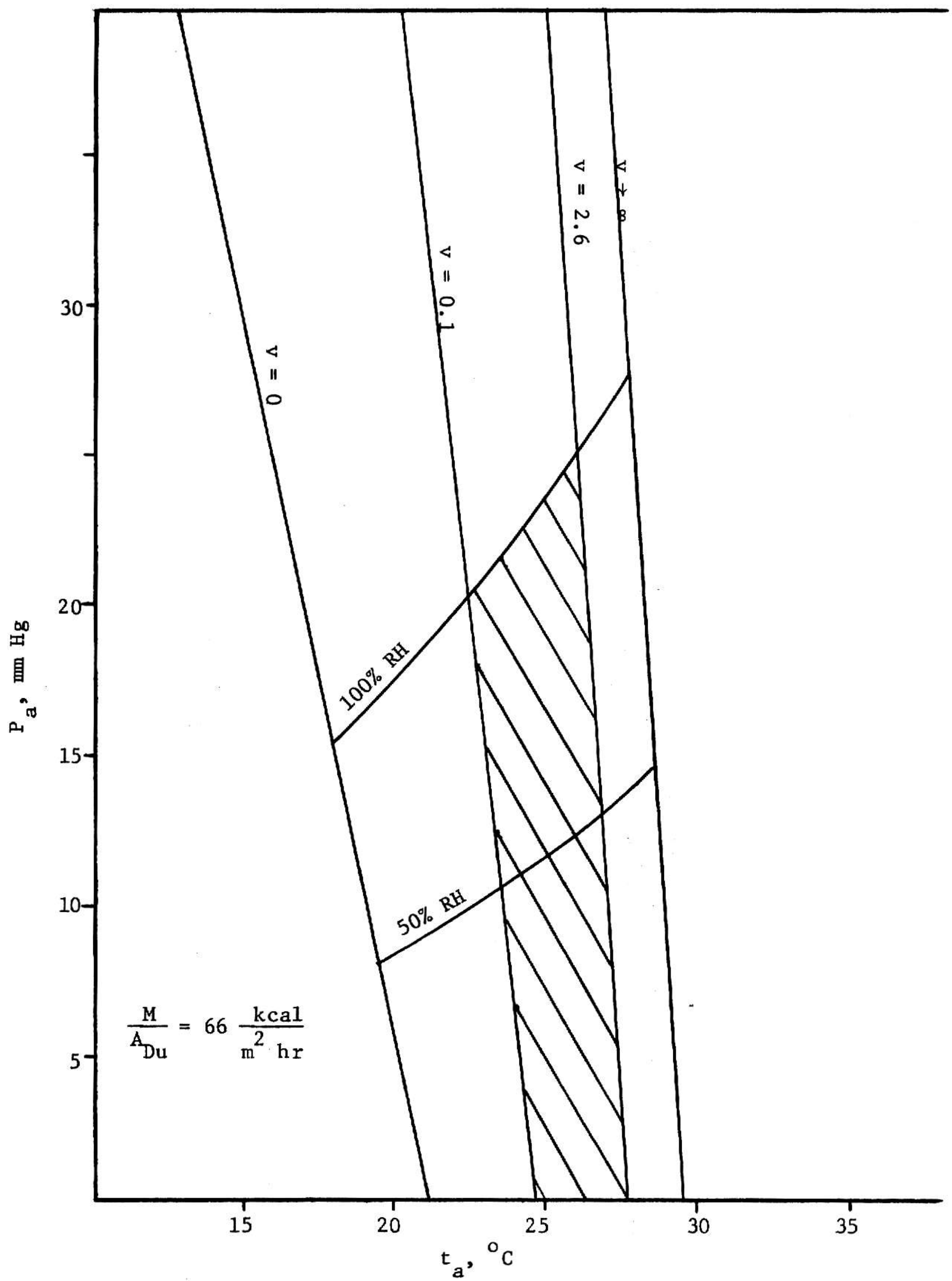

Figure 19. Feasible Region of the Comfort Equation.

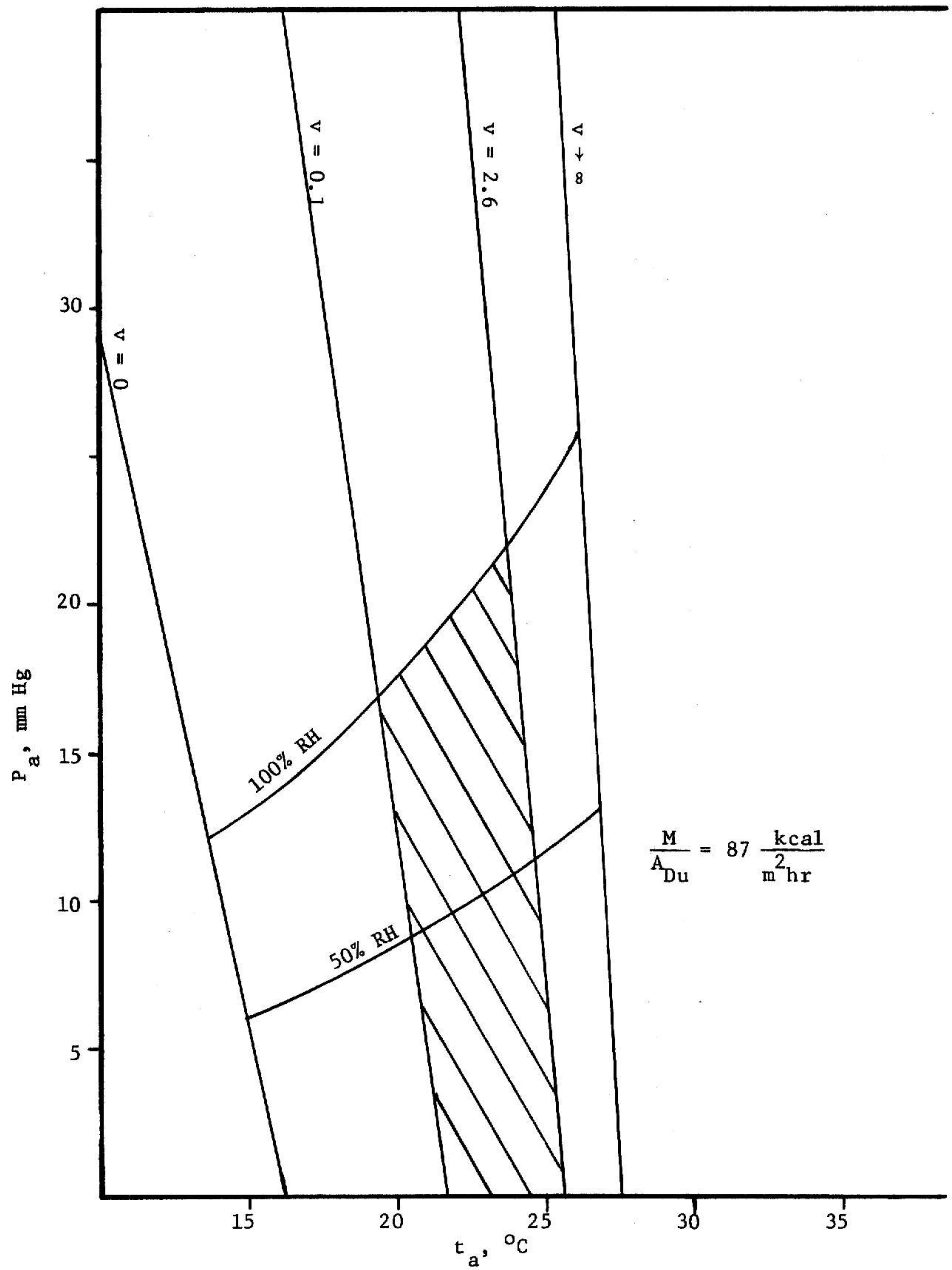

Figure 20. Feasible Region of the Comfort Equation

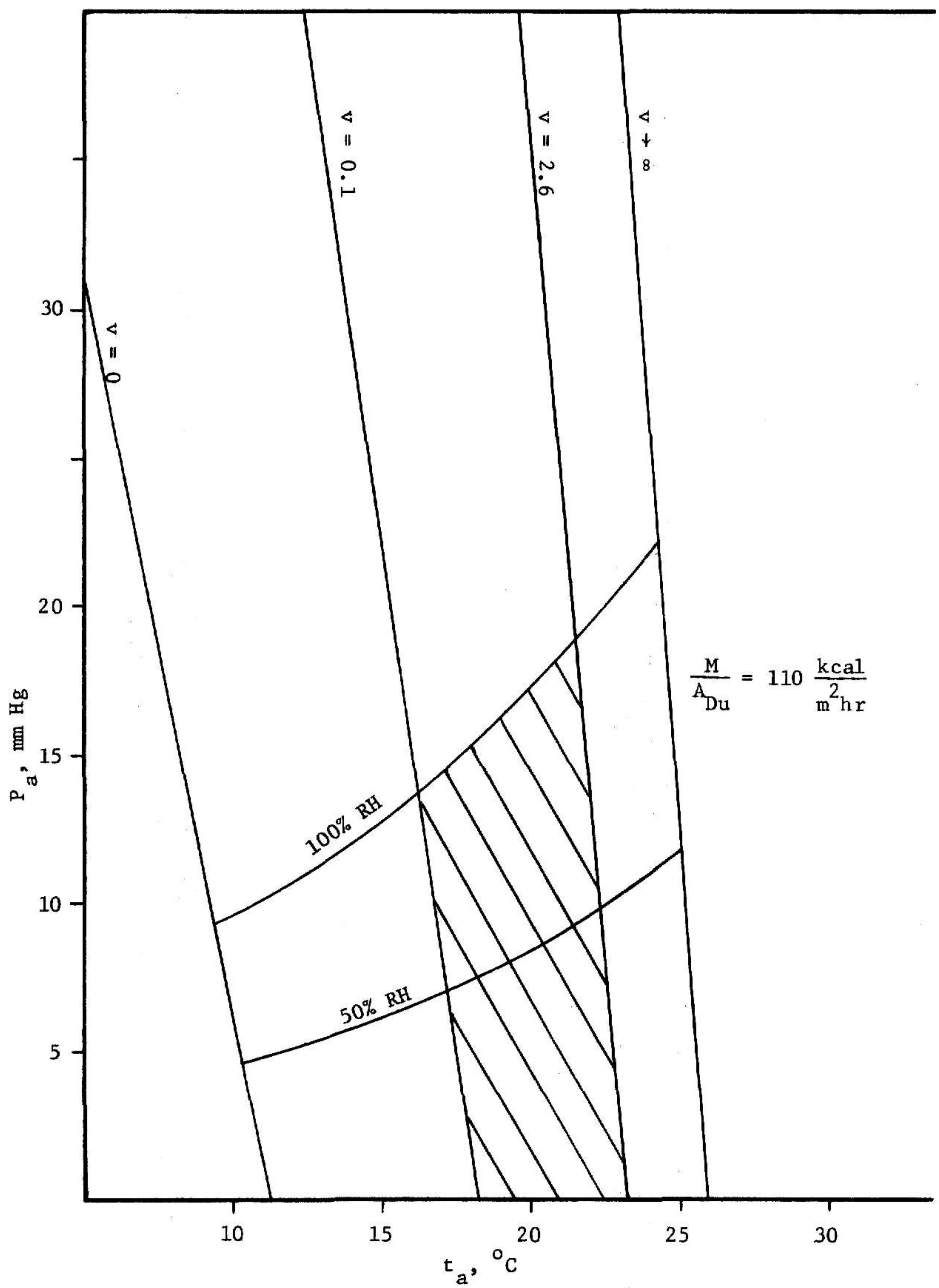

Figure 21. Feasible Region of the Comfort Equation

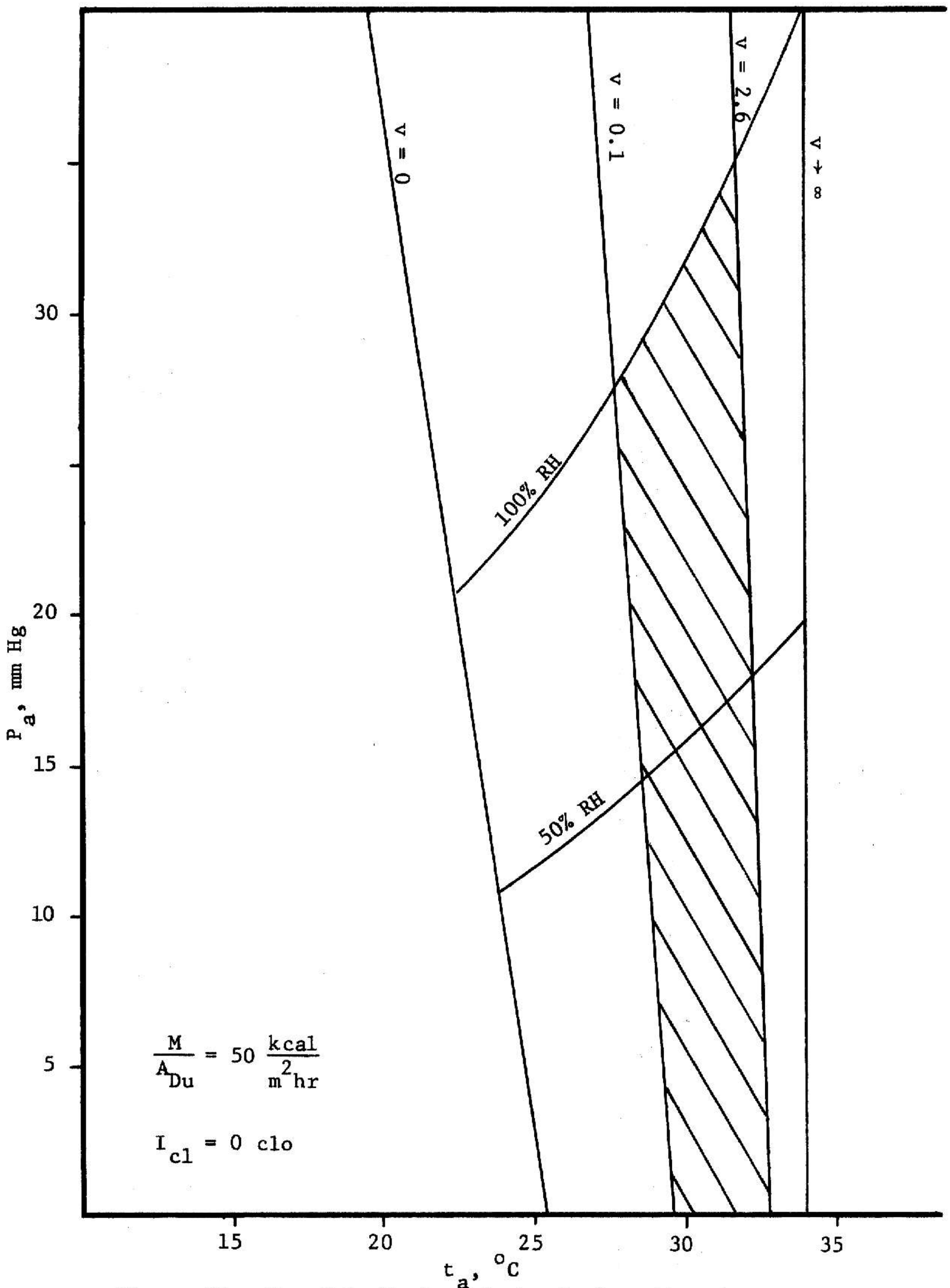

Figure 22. Feasible Region of the Comfort Equation

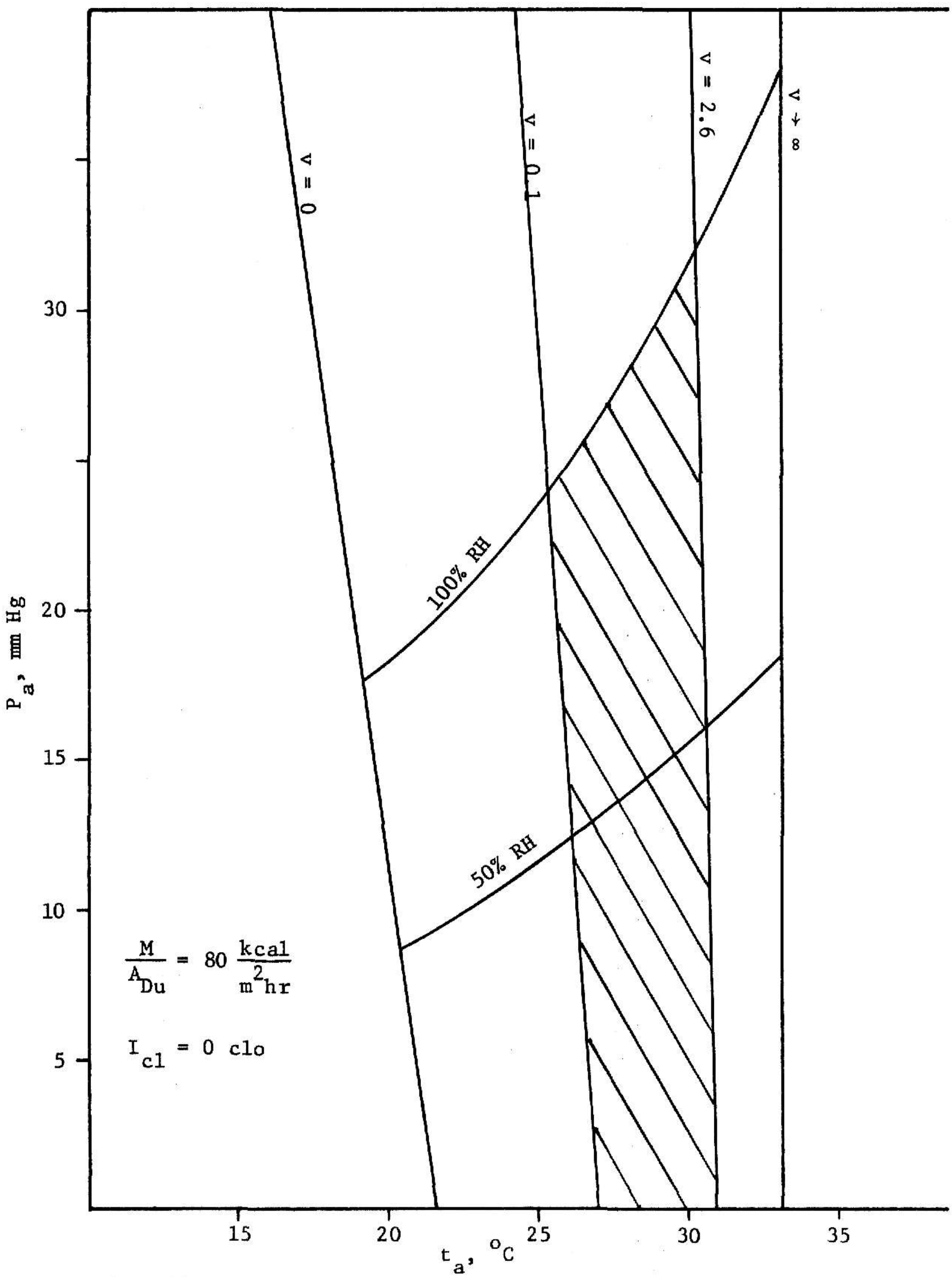

Figure 23. Feasible Region of the Comfort Equation

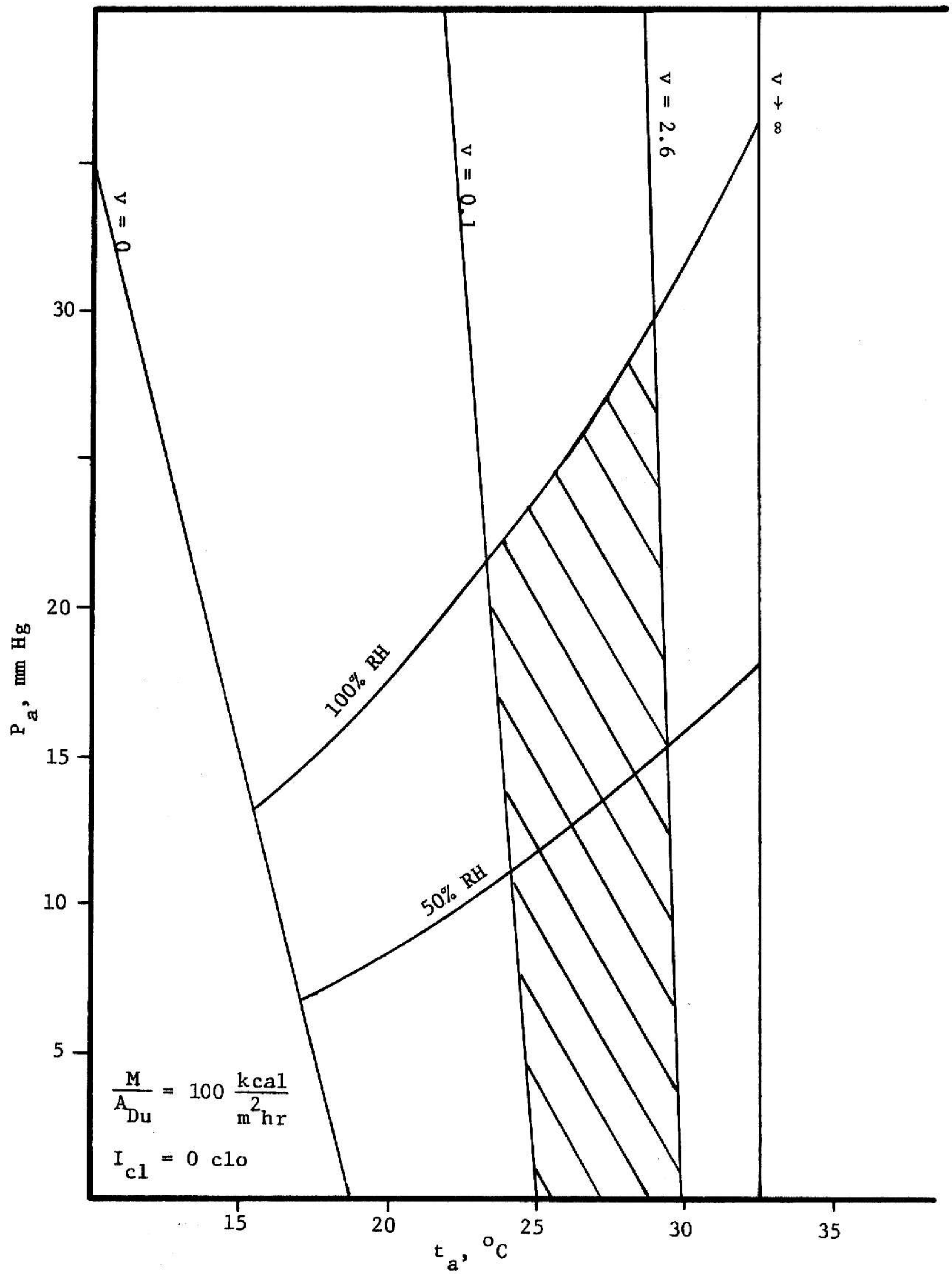

Figure 24. Feasible Region of the Comfort Equation

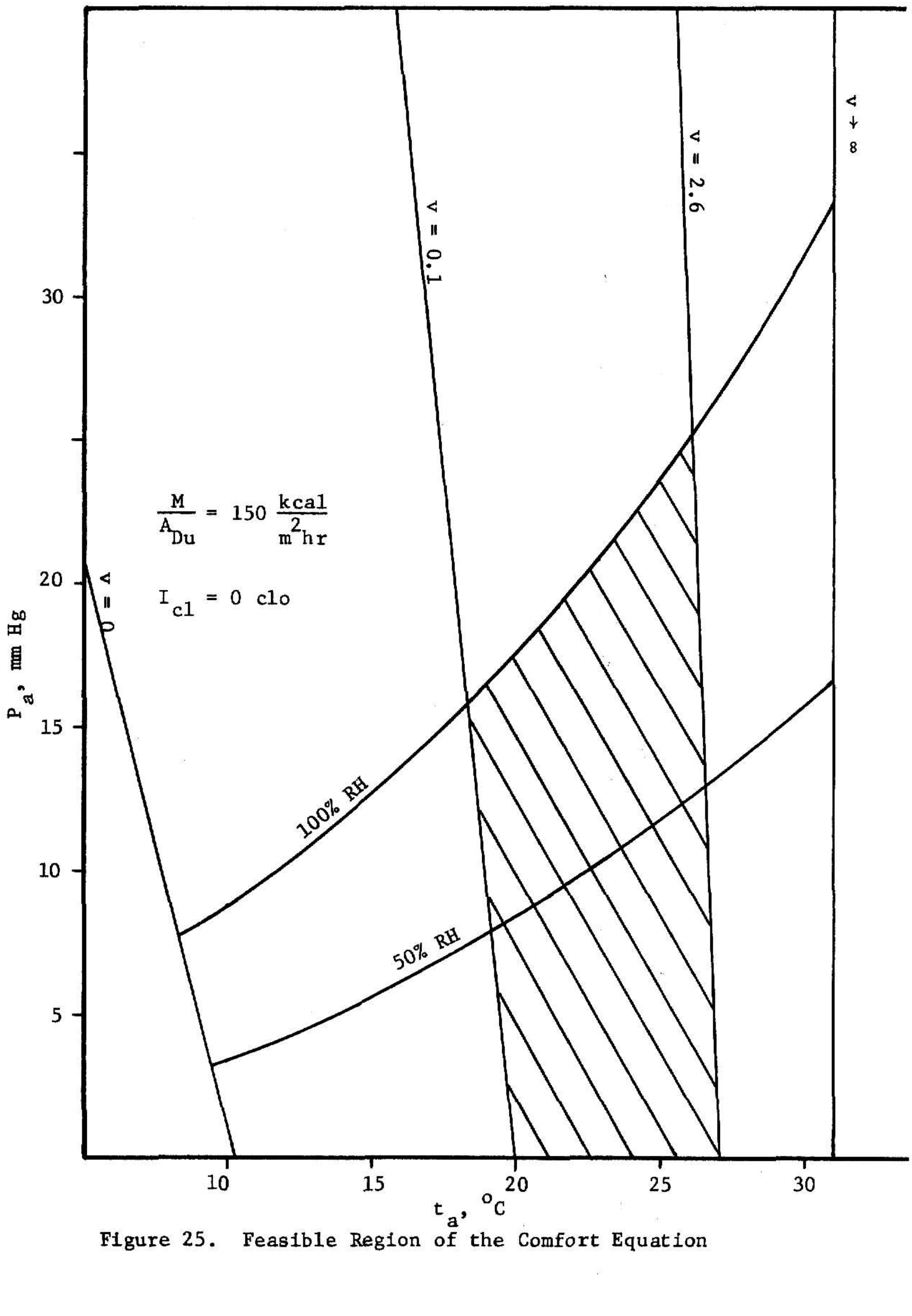

Figure 25. Feasible Region of the Comfort Equation

Part 2. Simulation and Optimization of an Environmental Control System

DEFINITION OF A CONTROL SYSTEM MODEL

For purposes of demonstrating the various mathematical techniques, a system was modeled. A very simple model, referred to as System I, was established as illustrated in Figure 26.

The model considers only the basic interactions between existing conditions outside and inside the control system. To keep it simple internal contributions and external influences upon the space besides the air control volume are not simulated. The air enclosure was assumed to be a well stirred system with uniform, incompressible air flow. To maintain equilibrium and provide thermal comfort conditions inside the space, the incoming air could disturb the system in four different ways. These conditions of disturbance are:

(1) When the outside dry bulb temperature and vapor pressure (humidity) are higher than those inside, the external air will present heat to the system.

(2) When the outside dry bulb temperature and vapor pressure (humidity) are lower than those inside, the external air will absorb heat from the system.

(3) When the outside dry bulb temperature is higher than that inside but the vapor pressure (humidity) is lower than that inside, then the external air will either yield or absorb heat in the system.

(4) When the outside dry bulb temperature is lower than that of the inside but the vapor pressure is higher than that inside, then the external air will either yield or absorb heat in the system.

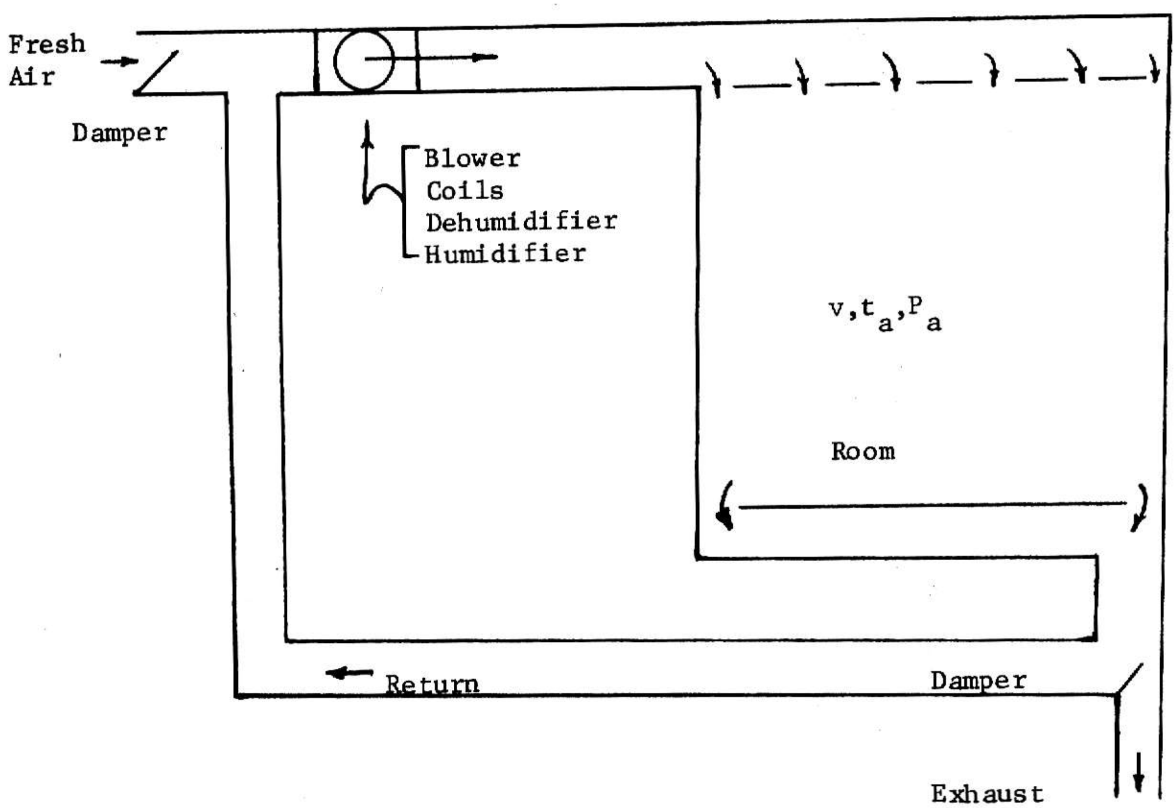

Figure 26. Sketch of System I.

For calculational purposes the enclosure was considered to be 3 x 4 x 5 meters [89]. The length of the duct was assumed to be 30.48 meters (100 ft.). The duct design was allowed to vary in diameter in order to maintain a constant pressure gradient of 0.2 inches of water per 100 feet of length. For the above duct length the total pressure drop would be 0.2 inches of water (49.76 pascals). The cross-sectional area of the room perpendicular to the direction of the air flow was 3 x 4 meters. It was assumed that a ventilation rate of 283 liters per minute (10 cubic feet per minute) of fresh air was required [1].

In this study the direction of heat flow will be considered positive when a situation as is (1) above occurs. That is, it was assumed in the model simulation that the thermal potential of the external conditions were greater than the inside conditions and that heat gain was evidenced, and negatively experienced values would indicate the opposite or heat loss from the space to the external air. With this in mind the external air brings in thermal energy by three different avenues:
(1) Sensible heat, (2) Latent heat, and (3) Frictional plus kinetic (due to conversion of velocity pressure or kinetic energy into thermal energy) heat. The total of which gives the energy required to maintain equilibrium within the environmental control system to meet the objective of providing thermal comfort conditions within the working space. The sensible heat is due to the difference between the outside and inside dry bulb temperatures. This relationship is shown below.

$$S_1 = f_1(t_2 - t_a) \tag{17}$$

S_1 = sensible heat, kcal/hour

t_2 = outside temperature, ^oC

t_a = inside temperature, ^oC

The latent heat is due to the difference between the outside and the inside partial pressure of water vapor. This relationship is shown below.

$$S_2 = f_2\ (P_{a2} - P_a)$$

S_2 = latent heat, Kcal/hour (18)

P_{a2} = outside partial pressure of water vapor, mm Hg

P_a = inside partial pressure of water vapor, mm hg

The frictional heat plus kinetic which is brought about by the total pressure required to overcome friction and provide velocity is (shown in a following section) to be as follows.

$$S_3 = f_3(v, v^3) \tag{19}$$

where

S_3 = frictional plus kinetic heat, Kcal/hour

v = air velocity in the room, meters/sec.

Therefore, the total thermal energy local from the external air is represented by the following relationship.

$$S = S_1 + S_2 + S_3 \tag{20}$$

where

S = total heat rate brought in by the external air which caused the disturbance of the comfort condition inside the space.

This relationship can also be expressed as:

$$S = C_1(t_2 - t_a) + C_2\ (P_{a2} - P_a) + C_3\ v + C_4\ v^3 \tag{21}$$

or

$$S = C_1 t_a + C_2 P_a + C_3 v + C_4 v^3 + C_k \tag{22}$$

where

C_k = constant = function of outside conditions (t_2, P_2).

So that for existing outside conditions the total energy load subjected to the thermal equilibrium is given as a function of three variables, namely, t_a, P_a, and v.

LAGRANGE MULTIPLIER TECHNIQUE

To optimize (minimize the energy load of) the above environmental control system, with the equality constraint of the comfort equation from Part 1) used as the equilibrium condition, the problem was approached by the Lagrange Multiplier Method. This technique treats each variable independently and is composed of finding in generals terms a set of x variables which minimize (or maximize) the objective function

$$S = f(x_1, x_2, \ldots, x_n) \tag{23}$$

subject to m equality contraints

$$g_j (x_1, x_2, \ldots, x_n) = 0, \qquad j = 1, \ldots, m \tag{24}$$

Then select or define the Lagrange multipliers, λ_j, in such a way that if each equality constraint, g_j, is multiplied by a Lagrange multiplier [7,44] and substracted from the objective function, one obtains the Lagrangian function

$$L(x,\lambda) = f(x_1, \ldots, x_n) - \sum_{j=1}^{m} \lambda_j g_j (x_1, \ldots, x_n) \tag{25}$$

This new "objective function" will yield the optimal solution (necessary condition) by setting the partial derivatives with respect to $x_1, \ldots, x_n$ and $\lambda_1, \ldots, \lambda_m$ equal to zero as follows.

$$\frac{\partial L}{\partial x_i} = \frac{\partial f}{\partial x_i} - \sum_{j=1}^{m} \lambda_j \frac{\partial g_i}{\partial x_i} = 0, \qquad i = 1, \ldots, n \tag{26}$$

$$-\frac{\partial L}{\partial \lambda_i} = g_j(x_1, \ldots, x_n) = 0, \qquad j = 1, \ldots, m \tag{27}$$

The points obtained by solving the above equations will be the stationary points (optimal points).

The stationary point obtained may be either a local maximum, a local minimum or a saddle point. It is important to note and heed that the modified objective function cannot be tested in the usual way to determine if the stationary point is a maximum or a minimum. This, incidently, is a point that is often missed in some texts and technical papers, where it is falsely asserted that sufficiency conditions can be used to test the Lagrangian function [7]. Since the algebra is tedious the development for only two variables with one equality constraint is presented in Appendix G to obtain the sufficient condition of optimality.

PRELIMINARY CALCULATIONS OF SYSTEM I TERMS

Before an optimization technique can be applied to the simple system given in Figure 26, one must first determine the values for the parameters that are involved in the objective function, Equation (22). The sensible heat exchange due to the incoming air is a function of the incoming air's

In order to compute the specific volume of the incoming air the outside temperature and vapor pressure must be given or measured. If one was given the outside temperature and considered the relative humidity at 100%, the saturated vapor pressure could be calculated. This relationship was determined and shown in Appendix H. In this manner, when given the outside temperature at 100% RH, the mass flow rate can be computed as follows.

$$M = \frac{Q}{V_{sp}} \tag{30}$$

where

M = mass flow rate of air, gm/hr

Q = ventilation flow rate required, liters/hr

V_{sp} = can be obtained from Equation (29).

For ordinary air temperatures the specific heats of dry air and water vapor can be taken as 0.238 and 0.46 cal/gm ^{o}C respectively. Therefore, the specific heat of air is as follows

$$CS_2 = 0.238 + 0.46\ W_2,\ \text{cal/gm}\ ^{o}\text{C} \tag{31}$$

where

W_2 = outside absolute humidity, gm water/gm air

Since the humidity ratio, W_2, is related to the partial pressure of water vapor as [1] follows,

$$W_2 = 0.62198\ \frac{P_{a2}}{P - P_{a2}} \tag{32}$$

where

P_{a2} = outside water vapor partial pressure, mm Hg

P = total pressure = 760 mm Hg

the specific heat of air becomes

$$CS_2 = 0.238 + 0.46\ [0.62198\ P_{a2}/(760 - P_{a2})] \tag{33}$$

Therefore the sensible heat component becomes a function of outside temperature (w 100% RH), t_2, and room temperature, t_a. When the outside temperature is given the load is simply a function of the room temperature variable as

$$S_1 = M_2 * CS_2 * (t_2 - t_a) * 10^{-3}, \quad \text{Kcal/hr.} \tag{34}$$

or

$$S_1 = C_1\ t_a + C_{01} \tag{35}$$

where

$C_1 = -\ M_2 * CS_2 * 10^{-3}$

$C_{01} = -\ C_1\ t_2$

The latent heat load is computed from

$$S_2 = M_2 * H_{vap} * (W_2 - W_a), \text{ Kcal/hr} \tag{36}$$

where

H_{vap} = heat of vaporization of water = 0.575 Kcal/gm

M_2 = calculated from Equation (30) and

CS_2 = calculated from Equation (33)

or

$$S_2 \;= M_2 * H_{vap} * K_{22}\,(P_{a2} - P_a)$$

where from Equation (32) and since 760 >> P_{a2}, P_a we get

$$K_{22} \cong 0.00083 \text{ gm water/gm air/mm Hg}$$

or

$$S_2 \;= C_2\, P_a + C_{02} \qquad (37)$$

where

$$C_2 \;= -\, M_2 * H_{vap} * K_{22}$$

$$C_{02} = -\, C_2\, P_{a2}$$

The frictional plus velocity pressure load component is calculated for the system based upon the following assumptions:

(1) The velocity of air in the duct was uniform.

(2) The air was incompressible and in the enclosed area was completely mixed.

(3) The air in the duct was under constant pressure and temperature

(4) The density of air was taken as 0.0721 lb./cu.ft. (1.155 kg/m^3) at one atmosphere for between 0^oC and 50^oC [76].

(5) The effects of the roughness of the duct and of heat transfer on the friction factor was considered negligible.

(6) The friction loss of air flow in the room was considered negligible.

(7) The frictional losses due to fittings, expansion, or contraction were not considered.

Since the velocity in the room was assumed to be uniform, the total volumetric flow rate is as follows

$$Q_T = A_r v \tag{38}$$

where

Q_r = volumetric flow rate of air, m^3/sec

A_r = cross-sectional area of the room, perpendicular to the direction of air flow, m^2

v = air velocity in the room, m/sec.

The velocity pressure head of the forced air is given as the squared room air velocity times the kinetic energy factor, α_k, divided by two times the acceleration of gravity, g, [1]. This may be expressed in energy terms by multiplying by the density of air, ρ, and the total volumetric flow rate, Q_T, as below.

$$S_{32} = \frac{\alpha_k}{2g} \rho Q_T \, v^2 \tag{39}$$

where

S_{32} = kinetic energy term, Kg-m/sec

Substitution of Equation (38) into Equation (39) yields the following

$$S_{32} = \frac{\alpha_k}{2g} \rho A_r \, C_f \, v^3, \qquad \text{Kcal/hr} \tag{40}$$

where

C_f = 8.4258 (conversion from Kg-m/sec to Kcal/hr).

The frictional load is calculated from the total pressure gradient, P_d, times the total volumetric flow rate as follows.

$$S_{31} = P_d Q_T C_f \qquad (41)$$

where

S_{31} = frictional load, Kcal/hr

Using the above, the frictional plus kinetic thermal load is determined as

$$S_3 = S_{31} + S_{32}, \quad \text{Kcal/hr} \qquad (42)$$

or

$$S_3 = P_d A_r C_f \; v + \frac{\alpha k}{2g} \rho A_r C_f v^3 \qquad (43)$$

which can be expressed as

$$S_3 = C_3 v + C_4 v^3 \qquad (44)$$

where C_3, C_4 = coefficients of constant terms from Equation (43).

From above the objective function is again expressed as

$$S = S_1 + S_2 + S_3, \text{ Kcal/hr.} \qquad (20)$$

or

$$S = C_1 t_a + C_2 P_a + C_3 v + C_k + C_4 v^3 \qquad (22)$$

where

$$C_k = C_{01} + C_{02}$$

and the total energy load is a function of the three variables, t_a, P_a and v.

SIMULATION OF A LINEAR RADIATION COEFFICIENT

Since the algebra in the mathematical manipulation of the fourth power relationship of the radiation component in the comfort equation constraint was quite involved, it was decided to facilitate the procedure by linearizing this component. A linear radiation coefficient was used by Gagge et al. [24] which was defined as simply as follows:

"h_r, the linear radiation coefficient describing how heat is exchanged by the skin or outside surface of the body with a 4Π black body enclosure at uniform temperature. This linear coefficient, as used in the present paper, is evaluated by the term $4\sigma\,(T_s)^3\,A_r/A$, where σ is the Stefan-Boltzmann constant; T_s is the absolute value of the skin temperature, 92.5F; and A_r/A is the fraction of the total body surface radiating to the 4Π enclosure. For a resting-sitting subject $A_r/A = 0.77$ and h_r becomes 0.92 Btu/sq ft/hr/F".

They described further in the paper that h_r would vary $\pm$ 10% as the MRT deviates 30F above or below a skin temperature of 92.5F.

To find a more representative value for h_r it was decided to try an average value for T^3 that would give a more representative h_r from the temperatures found in the feasible zones in Part 1. So that for the given parameters of the comfort equation a specified average temperature was used in computing an average h_r value.

Setting up the simplified comfort equation constraint, Equation (4) now is represented as the Equation (24) constraint or

$$g = M(1-\eta) - 0.35\,A_{Du}\,[43 - 0.061\,\frac{M}{A_{Du}}\,(1-\eta) - P_a]$$

$$- 0.42\,A_{Du}\,[\frac{M}{A_{Du}}\,(1-\eta) - 50] - 0.0023\,M\,(44 - P_a)$$

$$- 0.0014M\,(34 - t_a) - h_r\,A_{eff}\,(t_{cl} - t_{mrt}) - h_c\,A_{Du}$$

$$(t_{cl} - t_a) = 0, \quad \text{Kcal/hr} \tag{24a}$$

where

$$A_{eff} = f_{eff} * f_{cl} * A_{Du},\ m^2$$

$$h_r \quad = \text{linear radiation coefficient, } Kcal/m^2/hr/^{o}C$$

and where the other terms are defined previously in Part 1.

Solving for t_{cl} from the left hand side of Equation (4), as in Part 1, and using the assumptions made previously, substitute for t_{cl} into Equation (24a) to get

$$g = AP_a + Bt_a - DV^{0.5} + EP_a v^{0.5} + Ft_a v^{0.5} + C = 0 \qquad (24b)$$

for variables t_a, P_a and v, where

$$A = 035\ A_{Du} + 0.0023\ M + h_r\ A_{eff}\ (0.063\ I_{cl} + 0.000414\ I_{cl} \frac{M}{A_{Du}})$$

$$B = 0.0014\ M + h_r\ A_{eff}\ (1 + 0.000252\ I_{cl} \frac{M}{AD_u})$$

$$E = 10.4\ f_{cl}\ A_{Du}\ (0.063\ I_{cl} + 0.000414\ I_{cl} \frac{M}{AD_u})$$

$$F = 10.4\ f_{cl}\ A_{Du}\ (1 + 0.000252\ I_{cl} \frac{M}{A_{Du}})$$

$$D = 371.28\ f_{cl}\ A_{Du} + 10.4\ f_{cl}\ M\ (-\ 0.081459\ I_{cl} - 0.032)$$

$$- 10.4\ f_{cl}\ A_{Du}\ I_{cl}\ (1.071)$$

Therefore the comfort equation was solved for a range of values covering the feasible zones and compared Gagge's h_r with an averaged h_r with an h_r computed from the fourth power relationship. Also the radiation loads calculated by each method and the velocity terms were compared. The results are shown in Table 6 through 9. The computer program is given in Appendix I.

Table 6. Comparison of Linear Radiation Coefficients

Temperature of Air t_a, °C	Vapor Pressure P_a, mm Hg	Activity Level MET, $\frac{M/A_{Du}}{50}$	Clothing Insulation I_{cl}, clo	Linear Radiation Coefficient h_r, kcal/m^2/hr/C	Average Radiation Coefficient Ave h_r, kcal/m^2/hr/C	Fourth Power Radiation Coefficient h_{RF}, kcal/m^2/hr/C	Percentage Deviation of Ave h_r Ave Pct,%	Percentag Deviation of h_r Lin Pct,%
	(Note: Average Temperature = 27.78, 23.33, 21.11 °C for Insulation Values of 0.0, 0.6, 1.0 clo Respectively)							
30.00	5.0	1.0	0.0	5.75	5.40	5.45	0.95	5.43
30.00	10.0	1.0	0.0	5.75	5.40	5.45	0.95	5.43
30.00	14.0	1.0	0.0	5.75	5.40	5.45	0.95	5.43
26.67	5.0	1.0	0.6	5.75	5.16	5.27	2.01	9.07
27.78	5.0	1.0	0.6	5.75	5.16	5.30	2.54	8.48
26.67	10.0	1.0	0.6	5.75	5.16	5.26	1.88	9.21
27.78	10.0	1.0	0.6	5.75	5.16	5.29	2.42	8.61
a5.56	14.0	1.0	0.6	5.75	5.16	5.23	1.25	9.91
26.67	14.0	1.0	0.6	5.75	5.16	5.26	1.79	9.31
27.78	14.0	1.0	0.6	5.75	5.16	5.29	2.32	8.72
24.44	5.0	1.0	1.0	5.75	5.05	5.15	1.99	11.58
25.56	5.0	1.0	1.0	5.75	5.05	5.18	2.53	10.96
24.44	10.0	1.0	1.0	5.75	5.05	5.14	1.78	11.82
25.56	10.0	1.0	1.0	5.75	5.05	5.17	2.33	11.19
23.33	14.0	1.0	1.0	5.75	5.05	5.10	1.08	12.62
24.44	14.0	1.0	1.0	5.75	5.05	5.13	1.61	12.01
26.67	5.0	1.6	0.0	5.69	5.40	5.34	1.16	6.67
27.78	5.0	1.6	0.0	5.69	5.40	5.37	0.61	6.09
28.89	5.0	1.6	0.0	5.69	5.40	5.40	0.06	5.51
30.00	5.0	1.6	0.0	5.69	5.40	5.42	0.48	4.94
26.67	10.0	1.6	0.0	5.69	5.40	5.34	1.16	6.67
27.78	10.0	1.6	0.0	5.69	5.40	5.37	0.61	6.09
28.89	10.0	1.6	0.0	5.69	5.40	5.40	0.06	5.51
26.67	14.0	1.6	0.0	5.69	5.40	5.34	1.16	6.67
27.78	14.0	1.6	0.0	5.69	5.40	5.37	0.61	6.09
28.89	14.0	1.6	0.0	5.69	5.40	5.40	0.06	5.51
22.22	5.0	1.6	0.6	5.69	5.16	5.09	1.47	11.89
23.33	5.0	1.6	0.6	5.69	5.16	5.12	0.91	11.27
24.44	5.0	1.6	0.6	5.69	5.16	5.14	0.37	10.67
22.22	10.0	1.6	0.6	5.69	5.16	5.08	1.62	12.05
23.33	10.0	1.6	0.6	5.69	5.16	5.11	1.06	11.43

Table 7. Comparison of Radiation Loads

Temperature of Air t_a, °C	Vapor Pressure P_a, mm Hg	Activity Level MET, $\frac{M/A_{Du}}{50}$	Clothing Insulation I_{cl}, clo	Linear Radiation Load RL, kcal/hr	Average Radiation Load Ave Rad, kcal/hr	Fourth Power Radiation Load Rad, kcal/hr	Radiation Load Calculated from Equation (13) R13, kcal/hr
		(Note: Average Temperature = 27.78, 23.33, 21.11 °C for Insulation Values of 0.0, 0.6, 1.0 clo Respectively)					
30.00	5.0	1.0	0.0	30.11	28.29	28.49	28.56
30.00	10.0	1.0	0.0	30.11	28.29	28.49	28.56
30.00	14.0	1.0	0.0	30.11	28.29	28.49	28.56
28.89	20.0	1.0	0.0	38.26	35.95	36.01	36.09
26.67	5.0	1.0	0.6	31.43	28.24	28.82	28.82
27.78	5.0	1.0	0.6	22.40	20.12	20.66	20.65
26.67	10.0	1.0	0.6	29.40	26.42	26.93	26.92
27.78	10.0	1.0	0.6	20.37	18.30	18.71	18.75
25.56	14.0	1.0	0.6	36.81	33.07	33.49	33.49
26.67	14.0	1.0	0.6	27.78	24.96	25.42	25.41
27.78	14.0	1.0	0.6	18.75	16.84	17.20	17.24
24.44	5.0	1.0	1.0	32.00	28.11	28.61	28.68
25.56	5.0	1.0	1.0	22.43	19.70	19.95	20.21
24.44	10.0	1.0	1.0	28.47	25.01	25.40	25.46
25.56	10.0	1.0	1.0	18.89	16.59	16.95	16.99
23.33	14.0	1.0	1.0	35.13	30.86	31.12	31.20
24.44	14.0	1.0	1.0	25.64	22.52	22.84	22.89
26.67	5.0	1.6	0.0	47.07	44.64	44.02	44.13
27.78	5.0	1.6	0.0	38.99	36.98	36.67	36.76
28.89	5.0	1.6	0.0	30.92	29.32	29.23	29.30
30.00	5.0	1.6	0.0	22.84	21.66	21.72	21.77
26.67	10.0	1.6	0.0	47.07	44.64	44.02	44.13
27.78	10.0	1.6	0.0	38.99	36.98	36.67	36.76
28.89	10.0	1.6	0.0	30.92	29.32	29.23	29.30
26.67	14.0	1.6	0.0	47.07	44.64	44.02	44.13
27.78	14.0	1.6	0.0	38.99	36.98	36.67	36.76
28.89	14.0	1.6	0.0	30.92	29.32	29.23	29.30

Table 8. Comparison of Velocity Terms

Temperature of Air t_a, °C	Vapor Pressure P_a, mm Hg	Activity Level MET, $\frac{M/A_{Du}}{50}$	Clothing Insulation I_{cl}, clo	Linear Velocity $v\ell$, m/sec	Average Velocity Ave v, m/sec	Velocity from Fourth Power Relation vf, m/sec	Velocity from Equation (13) v13, m/sec
30.00	5.0	1.0	0.0	0.15	0.16	0.16	0.16
30.00	10.0	1.0	0.0	0.19	0.21	0.21	0.21
30.00	14.0	1.0	0.0	0.23	0.25	0.25	0.25
26.67	5.0	1.0	0.6	0.12	0.15	0.14	0.14
27.78	5.0	1.0	0.6	0.41	0.47	0.46	0.45
26.67	10.0	1.0	0.6	0.20	0.24	0.23	0.23
27.78	10.0	1.0	0.6	0.68	0.75	0.73	0.74
25.56	14.0	1.0	0.6	0.10	0.13	0.12	0.12
26.67	14.0	1.0	0.6	0.30	0.34	0.34	0.34
27.78	14.0	1.0	0.6	1.01	1.09	1.03	1.07
24.44	10.0	1.0	1.0	0.22	0.27	0.24	0.27
25.56	10.0	1.0	1.0	0.84	0.93	0.91	0.91
23.33	14.0	1.0	1.0	0.12	0.15	0.14	0.15
24.44	14.0	1.0	1.0	0.39	0.45	0.41	0.44
26.67	5.0	1.6	0.0	0.10	0.12	0.12	0.12
27.78	5.0	1.6	0.0	0.22	0.24	0.24	0.24
28.89	5.0	1.6	0.0	0.49	0.52	0.52	0.52
30.00	5.0	1.6	0.0	1.18	1.23	1.22	1.22
26.67	10.0	1.6	0.0	0.13	0.15	0.15	0.15
27.78	10.0	1.6	0.0	0.27	0.29	0.29	0.29
28.89	10.0	1.6	0.0	0.58	0.61	0.61	0.61
26.67	14.0	1.6	0.0	0.15	0.17	0.17	0.17
27.78	14.0	1.6	0.0	0.31	0.33	0.34	0.34
28.89	14.0	1.6	0.0	0.65	0.68	0.68	0.69
22.22	5.0	1.6	0.6	0.10	0.13	0.13	0.13
23.33	5.0	1.6	0.6	0.25	0.28	0.27	0.29

(Note: Average Temperature = 27.78, 23.33, 21.11 °C for Insulation Values of 0.0, 0.6, 1.0 clo Respectively)

Table 9. Comparison of Heat Exchange Loads

Temperature of Air t_a, °C	Vapor Pressure P_a, mm Hg	Activity Level MET, $\frac{M/A_{Du}}{50}$	Clothing Insulation I_{cl}, clo	Sweat Rate SWR, kcal/m^2/hr	Diffusion DIFF, kcal/m^2/hr	Latent Respiratory LAT, kcal/m^2/hr	Dry Respiratory DRY, kcal/m^2/hr	Radiation Fourth Power Relationship RAD, kcal/m^2/hr	Convection CONV, kcal/m^2/hr	Velocity v, m/sec	Predicted Mean Vote PMV*
30.00	5.0	1.0	0.0	0.00	12.30	4.48	0.28	15.83	16.28	0.16	0.00
30.00	10.0	1.0	0.0	0.00	10.55	3.91	0.28	15.83	18.60	0.21	0.00
30.00	14.0	1.0	0.0	0.00	9.15	3.45	0.28	15.83	20.46	0.25	0.00
26.67	5.0	1.0	0.6	0.00	12.30	4.48	0.51	15.97	15.31	0.14	0.00
27.78	5.0	1.0	0.6	0.00	12.30	4.48	0.44	11.44	20.40	0.46	-0.02
26.67	10.0	1.0	0.6	0.00	10.55	3.91	0.51	14.92	18.76	0.23	0.00
27.78	10.0	1.0	0.6	0.00	10.55	3.91	0.44	10.39	23.86	0.73	0.00
25.56	14.0	1.0	0.6	0.00	9.15	3.45	0.59	18.56	16.42	0.12	0.01
26.67	14.0	1.0	0.6	0.00	9.15	3.45	0.51	14.08	21.52	0.34	-0.01
27.78	14.0	1.0	0.6	0.00	9.15	3.45	0.44	9.56	26.62	1.03	-0.03
24.44	5.0	1.0	0.6	0.00	12.30	4.48	0.67	15.90	14.83	0.12	-0.01
24.56	5.0	1.0	0.6	0.00	12.30	4.48	0.59	11.20	20.23	0.44	0.06
24.44	10.0	1.0	1.0	0.00	10.55	3.91	0.67	14.11	19.12	0.24	0.06
25.56	10.0	1.0	1.0	0.00	10.55	3.91	0.59	9.42	24.52	0.91	0.08
23.33	14.0	1.0	1.0	0.00	9.15	3.45	0.75	17.29	17.20	0.14	-0.02
24.44	14.0	1.0	1.0	0.00	9.15	3.45	0.67	12.69	22.55	0.41	0.07
26.67	5.0	1.6	0.0	12.60	11.66	7.18	0.82	24.46	21.66	0.12	0.00
27.78	5.0	1.6	0.0	12.60	11.66	7.18	0.70	20.37	26.27	0.24	0.00
28.89	5.0	1.6	0.0	12.60	11.66	7.18	0.57	16.24	30.88	0.52	0.00
30.00	5.0	1.6	0.0	12.60	11.66	7.18	0.45	12.06	35.49	1.22	0.00
26.67	10.0	1.6	0.0	12.60	9.91	6.26	0.82	24.46	24.33	0.15	0.00
27.78	10.0	1.6	0.0	12.60	9.91	6.26	0.70	20.37	28.94	0.29	0.00
28.89	10.0	1.6	0.0	12.60	9.91	6.26	0.57	16.24	33.55	0.61	0.00
26.67	14.0	1.6	0.0	12.60	8.51	5.52	0.82	24.46	26.47	0.17	0.00
27.78	14.0	1.6	0.0	12.60	8.51	5.52	0.70	20.37	31.08	0.34	0.00
28.89	14.0	1.6	0.0	12.60	8.51	5.52	0.57	16.24	35.69	0.68	0.00
22.22	5.0	1.6	0.6	12.60	11.66	7.18	1.32	23.03	21.48	0.13	0.03

(* See Appendix T for this Relationship)

It is noted that the radiation coefficient as computed by the averaged temperature method was within 4% error of the actual value from the fourth power relationship. Greater accuracy can be accomplished if for each set of parameters an appropriate averaged temperature is used in the computation of the linear radiation coefficient.

From this exercise we now have formulated a workable comfort equation that can be used as the constraint in analytically involved optimization techniques.

OPTIMIZATION ATTEMPT BY LAGRANGE MULTIPLIER TECHNIQUE

According to the procedure presented previously on the Lagrange Multiplier Method, it was attempted to minimize the energy equation subject to the modified comfort equation used as the equality constraint.

Therefore, minimize

$$S = C_1 t_a + C_2 P_a + C_3 v + C_4 v^3 + C_k = f(t_a, P_a, v) \quad (45)$$

subject to

$$g(t_a, P_a, v) = AP_a + Bt_a - Dv^{0.5} + EP_a v^{0.5} + Ft_a v^{0.5} + C$$
$$= 0 \quad (46)$$

Lagrangian Function,

$$L = S - \lambda g \quad (47)$$

necessary conditions for optimality (stationary point)

$$\frac{\partial L}{\partial t_a} = \frac{\partial f}{\partial t_a} - \lambda \frac{\partial g}{\partial v} = 0 \quad (48)$$

$$\frac{\partial L}{\partial P_a} = \frac{\partial f}{\partial P_a} - \lambda \frac{\partial g}{\partial P_a} = 0 \quad (49)$$

$$\frac{\partial L}{\partial v} = \frac{\partial f}{\partial v} - \lambda \frac{\partial g}{\partial v} = 0 \quad (50)$$

$$-\frac{\partial L}{\partial \lambda} = g(t_a, P_a, v) = 0 \quad (51)$$

carrying out the differentiation to get,

$$\frac{\partial L}{\partial t_a} = C_1 - \lambda(B + Fv^{.5}) = 0 \quad (48)$$

$$\frac{\partial L}{\partial P_a} = C_2 - \lambda(A + Ev^{.5}) = 0 \quad (49)$$

$$\frac{\partial L}{\partial v} = C_3 + 3C_4\, v^2 - \lambda\left(-\frac{D}{2v^{.5}} + \frac{EP_a}{2v^{.5}} + \frac{Ft_a}{2v^{0.5}}\right) = 0 \tag{50}$$

$$-\frac{\partial L}{\partial \lambda} = AP_a + Bt_a - Dv^{.5} + EP_a v^{.5} + Ft_a\, v^{.5} + C = 0 \tag{51}$$

From Equation (49)

$$\lambda = \frac{C_2}{A+Ev^{.5}}$$

and from Equation (48)

$$\lambda = \frac{C_1}{B + Fv^{.5}}$$

equating (49) and (48) to get

$$(B+Fv^{.5})C_2 - (A+Ev^{.5})C_1 = 0$$

or

$$v^{.5} = \frac{AC_1 - BC_2}{FC_2 - EC_1} \tag{52}$$

squaring to give rise to

$$v = \left(\frac{AC_1 - BC_2}{FC_2 - EC_1}\right)^2$$

From which substitution into Equations (50) and (51) gives values for t_a and P_a. However, upon solving the above equations it is found that the minimum occurs for values of the variables which are not physically possible to obtain. It points out that (though the technique is correct) this is because the constraint equation is the result of a heat balance and the range of variables that meets this constraint are not restricted in the above technique. Therefore the variables must be restricted within physical bounds to allow a feasible solution and an optimal solution to occur for combinations of the environmental variables as in the feasible zones in Part 1. Therefore to minimize the energy consumption within an environmental control system the variables have to be restricted by feasible bounds before applying optimization techniques.

Part 3. Optimization by Use of Search Techniques

Since many systems of practical interest cannot be optimized by analytical methods, numerical techniques must be applied. These methods are characterized by one essential idea, namely, to run an organized and exhaustive search to find a new feasible point which is better than the existing one. These techniques are applied iteratively until no further improvement is possible or until the optimum has been located to within the desired accuracy.

There are many different searching methods that can be applied in optimization of a system model. These techniques have been developed for unconstrained objective functions. They are easily adapted for use on digital computers and by modification of available computer programs can be forced to search within specified boundaries.

HOOKE AND JEEVES SEARCH TECHNIQUE

One of the simplest and most efficient methods for solving the unconstrained nonlinear minimization problem is the Hooke and Jeeves pattern search technique [16]. It consists of searching the local nature of the objective function in the space and then moving in a favorable direction for reducing the functional value. This technique is described in detail in Appendix J.

Initially values of room temperature and room vapor pressures are chosen arbitrarily. For the given parameters of the outside environment and the system model the physical constraint on relative humidity between 0% and 100% was first checked. If satisfied the comfort equation constraint as given in Equation (13) was used to compute the room velocity and the velocity constraint, Equation (10), was checked. If satisfied the search technique was applied to the objective function where the initial values of t_a and P_a became the starting coordinates. Every new point obtained in the search technique was checked with the relative humidity and velocity constraints. This procedure was repeated until the minimum value was obtained. This optimization process is illustrated by the flow diagram in Figure 27. The computer flow diagram for the Hooke and Jeeves pattern search is given in Appendix K, and the computer program used in optimization is presented in Appendix L.

The results from application of the optimization procedure by the Hooke and Jeeves search technique of the system model (Figure 26) are summarized in Table 10. Thus for a given outside temperature and outside vapor pressure and the parameters of activity and clothing insulation the combination of optimum values for room temperature, t_a, room vapor pressure, P_a, and room velocity, v, are found which give the minimum total energy load requirement.

SIMPLEX PATTERN SEARCH TECHNIQUE

Nelder and Mead [64] developed the Sequential Simplex pattern search technique for finding the optimum value of a function of several variables. This method is recommended in Beveridge and Schechter [7] since it is conceptually simple and easy to program and requires only one additional experiment for each move. Himsworth (1962) has stated that the Simplex method is always more efficient than other techniques, the more so if the number of variables is large and the experimental error large [7]. This technique is described in detail in Appendix M.

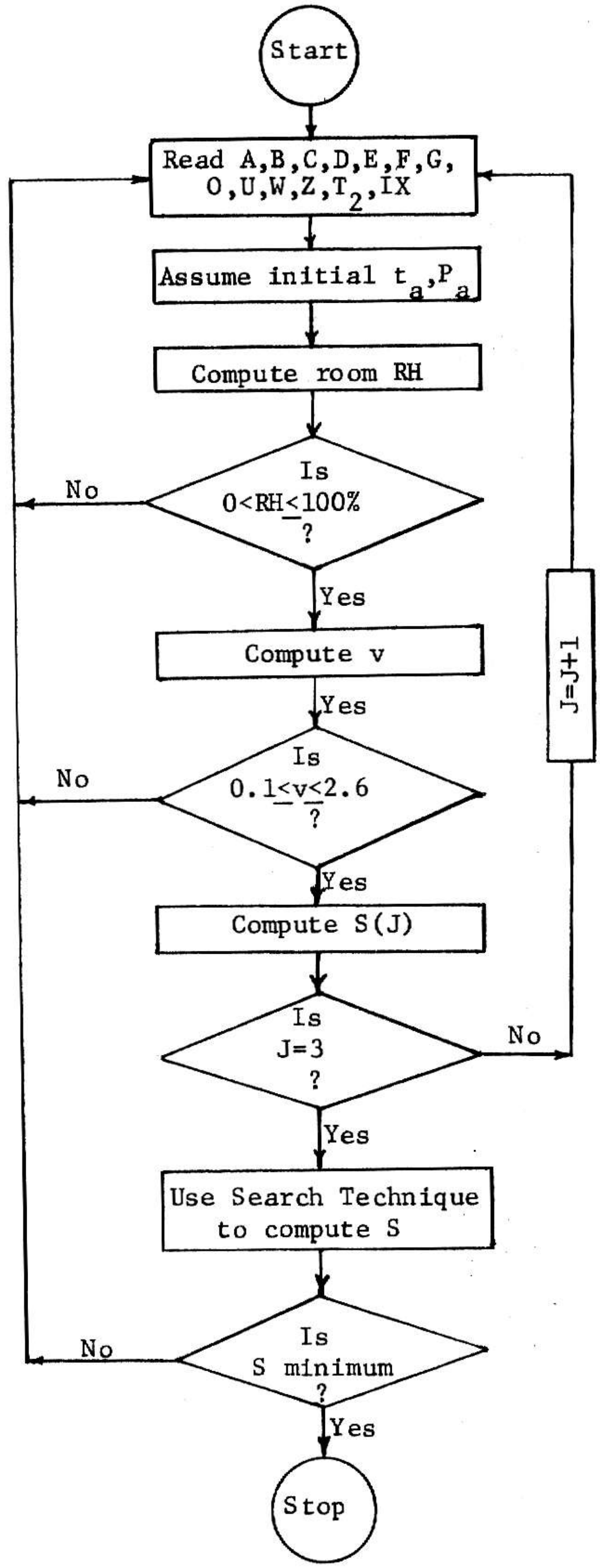

Figure 27. Diagram of Computer Search Optimization

Table 10. Optimum Results from Hooke and Jeeves Technique

Outside Temperature T_2, °C	Activity $ACT, M/A_{Du}$	Room Temperature t_a, °C	Room Velocity v, m/sec	Room Vapor Pressure P_a, mm Hg	Room Relative Humidity RH, %	Sensible Heat Load S_1, kcal/hr	Latent Heat Load S_2, kcal/hr	Frictional + Kinetic Heat Load S_3, kcal/hr	Minimum Total Energy Required S, kcal/hr
(Note: Room Vapor Pressure Constraint $0 < RH \leq 100$)									
50	52	23.86	0.10	22.26	99.99	118.44	651.19	50.80	820.43
	83	20.08	0.11	17.67	99.98	135.57	688.20	55.71	879.48
	111	16.13	0.10	13.80	99.94	153.48	719.15	50.61	923.24
	132	13.27	0.10	11.49	100.00	166.40	737.35	50.40	954.15
40	52	23.86	0.10	22.26	99.99	72.15	317.38	50.78	440.31
	83	20.08	0.10	17.67	99.98	91.32	351.12	51.42	493.86
	111	16.13	0.10	13.80	99.94	109.90	384.35	50.61	544.86
	132	13.27	0.10	11.49	100.00	123.75	407.22	50.57	581.54
(Note: Increased Ventilation Ten-fold, one case only)									
50	52	24.50	0.10	23.13	100.00	1155.36	6440.81	50.38	7646.55
(Note: Room Vapor Pressure Constraint $20 < RH \leq 70$)									
50	52	24.56	0.10	16.25	69.95	115.25	699.64	50.80	865.69
	83	20.55	0.10	12.73	70.00	133.45	727.53	51.30	912.28
	111	16.67	0.10	9.99	69.98	151.04	749.16	50.61	950.81
	132	13.76	0.10	8.30	69.99	164.18	762.40	50.49	977.07
(Note: Room Vapor Pressure Constraint $20 < RH \leq 70$ and $1 < P_a \leq 14$)									
50	50	25.13	0.10	14.00	58.30	112.70	717.50	51.24	881.44
	80	21.00	0.10	13.06	69.83	131.39	724.94	50.81	907.14
	100	18.25	0.10	11.00	69.45	143.85	741.23	50.63	935.71
40	50	25.13	0.10	14.00	58.30	68.85	385.50	51.24	505.59
	80	21.00	0.10	13.06	69.83	87.95	393.62	50.80	532.37
	100	18.25	0.10	11.00	69.73	100.67	411.41	50.62	562.70

Using the same procedure as shown in Figure 27, the optimization of System I is performed by the Simplex method. The computer flow diagram for this method is given in Appendix N, and an example of the computer program using this optimization procedure is presented in Appendix O.

The results from application of the sequential simplex pattern search technique in optimization of System I are given in Table 11. Again the optimum combination of t_a, P_a and v are determined which yield the minimum energy load requirement for System I.

Further the sequential simplex pattern method was used in optimization of a system by Chao [9]. The computer program for this application is given in Appendix P. The results from using this computer program are summarized in Table 12 and are similar to the results obtained in Tables 10 and 11.

Table 11. Sequential Simplex Pattern Search Optimization Results

Outside Temperature T_2, °C	Activity $ACT, M/A_{Du}$	Room Temperature t_a, °C	Room Velocity v,m/sec	Room Vapor Pressure P_a,mm Hg	Room Relative Humidity RH, %	Sensible Heat Load S_1,kcal/hr	Latent Heat Load S_2,kcal/hr	Frictional + Kinetic Heat Load S_3,kcal/hr	Minimum Total Energy Required S,kcal/hr
(Note: Room Vapor Pressure Constraint $0 < RH \leq 100$)									
50	52	23.84	0.10	22.23	100.00	118.52	651.39	50.34	820.25
	83	19.93	0.10	17.51	99.95	136.22	689.51	50.32	876.05
	111	16.12	0.10	13.79	100.00	153.52	719.15	50.32	922.99
	132	13.27	0.10	11.49	99.97	166.41	737.38	50.32	904.11
40	52	23.84	0.10	22.23	100.00	72.23	317.58	50.34	440.15
	83	19.93	0.10	17.52	99.99	92.34	352.13	50.37	494.84
	111	16.12	0.10	13.79	100.00	109.90	384.31	50.32	544.53
	132	13.27	0.10	11.49	100.00	123.75	407.22	50.32	581.29
(Note: Room Vapor Pressure Constraint $20 < RH \leq 70$)									
50	52	24.53	0.10	16.22	70.00	115.39	699.79	50.32	865.50
	83	20.55	0.10	12.73	69.98	133.45	727.57	50.32	911.34
	111	16.66	0.10	9.99	69.99	151.04	749.15	50.33	950.52
	132	13.76	0.10	8.30	69.97	164.18	762.42	50.32	976.92

Table 12. Optimal Results from Another Simplex Pattern Search

utside emperature T_{23} °C	Activity ACT, M/AD_u	Room Temperature t_a, °C	Room Velocity v, m/sec	Room Vapor Pressure P_a, mm Hg	Room Relative Humidity RH, %	Sensible Heat Load S_1, kcal/hr	Latent Heat Load S_2, kcal/hr	Frictional + Kinetic Heat Load S_3, kcal/hr	Minimum Total Energy Required S, kcal/hr
			(Note: Room Vapor Pressure Constraint $0 < RH \leq 100$)						
50	52	24.00	0.11	22.45	99.99	117.11	644.66	52.17	813.94
	83	19.93	0.10	17.51	99.95	136.22	689.51	50.32	876.05
	111	16.12	0.10	13.79	100.00	153.52	719.15	50.61	923.28
	132	13.27	0.10	11.48	99.89	166.42	737.47	50.32	904.21
40	52	23.99	0.10	22.44	99.98	71.94	317.24	50.32	440.18
	83	19.93	0.10	17.52	99.99	92.34	352.14	50.32	494.80
	111	16.12	0.10	13.79	100.00	109.90	384.31	50.36	544.57
	132	13.27	0.10	11.48	99.89	123.01	410.15	50.37	583.53

Part 4. Analysis and Optimization by the Method of Lagrange Multipliers and the Kuhn-Tucker Conditions

The analytical approach by Lagrange Multipliers in Part 2 of this study failed to give feasible results since the physical inequality constraints were not included in the analysis procedure. This classical method is of use mainly in theoretical analyses and is well suited for certain types of problems. In contrast to the previous attempt of ignoring the inequality restrictions, optimization can be carried out by converting each inequality relationship into an equality restriction, with subsequent optimization of a function subject to a set of equality constraints.

Modification of the inequality restrictions may be carried out by the introduction of "slack" variables or functions, one slack variable being introduced for each inequality constraint in the system. This allows one to find the stationary points by the method of constrained variation. However, once the constraints are in this form, the method of Lagrangian multipliers can also be applied. This approach is preferred, since the value of the multiplier can be of some assistance in determining the character of the stationary point.

In general the problem is to determine the optimum value of some $f(x_i)$, $i = 1$ to n, subject to m constraints, where the equality constraints are,

$$\phi_k = g_k + S_k(X_{n+k}) = 0 \qquad \text{for } k = 1,2, \ldots, m \tag{53}$$

g_k being the original inequality

$$g_k \leq 0 \tag{54}$$

and S_k the nonnegative slack function, a function of the slack variable X_{n+k}; then optimizing the function

$$L = f - \sum_{k=1}^{m} \lambda_k \phi_k \tag{55}$$

where λ_k multipliers, L is thus a function of n + m variables.

The restricted stationary points, those satisfying Equation (53), can then be found by solving the n + 2m equations,

$$\frac{\partial L}{\partial x_j} = 0 \quad \text{for } j = 1,2, \ldots, (n+m) \tag{56}$$

and

$$\frac{\partial L}{\partial \lambda_k} = 0$$

which is equivalent to

$$\phi_k = 0 \quad \text{for } k = 1,2, \ldots, m \tag{57}$$

From this it is seen that a slack variable can appear only in one equation of the set of necessary conditions, and that equation must have the structure

$$x_{n+k} \lambda_k = 0 \tag{58}$$

This equation arises when differentiating L with respect to the slack variable, and has this form due to the fact that the slack variable appears only once in L and is always a quadratic multiplied by a Lagrangian multiplier. Because of this structure, it is seen that a nonzero value of the Lagrangian multiplier means that the solution is necessarily found on the boundary. Of course, the solution may still lie on the boundary if the Lagrangian multiplier vanishes. One can say, however, that the stationary point can be found only in the interior if the Lagrangian multiplier vanishes. A nonzero Lagrangian multiplier implies that the solution is on the boundary. Remembering these two conclusions, the slack variable need not be introduced. This formulation is the basis for the method of Kuhn and Tucker [7,52].

The method of Lagrange multipliers can now be generalized to handle the problem involving inequality constraints and non-negative variables. The necessary conditions for solving this problem are the Kuhn-Tucker conditions.

A point $(x_1, x_2, \ldots, x_n)$ which optimizes (minimizes) the objective function

$$S = f(x_1, \ldots, x_n) \tag{59}$$

subject to the inequality constraints

$$g_k(x_1, \ldots, x_n) \leq 0, \qquad k = 1, \ldots, m \tag{60}$$

exists if there is a set of Lagrangian multipliers, $\lambda_1, \ldots, \lambda_m$ that satisfies the following set of conditions.

$$\frac{\partial L}{\partial x_j} = \frac{\partial f}{\partial x_j} - \sum_{k=1}^{m} \lambda_k \frac{\partial g_k}{\partial x_j} \geq 0, \qquad j = 1, \ldots, n \quad \text{(for minimization)} \tag{61}$$

$$\sum_{j=1}^{n} x_j \frac{\partial L}{\partial x_j} = 0, \qquad j = 1, \ldots, n \tag{62}$$

$$\lambda_k g_k = 0, \qquad k = 1, \ldots, m \tag{63}$$

$$g_k \leq 0, \qquad k = 1, \ldots, m \tag{64}$$

$$x_j \geq 0, \qquad j = 1, \ldots, n \tag{65}$$

$$\lambda_k \leq 0, \qquad k = 1, \ldots, m \text{ (for minimization)} \tag{66}$$

These conditions are also sufficient for a global minimum if f(x) and g_k, $k = 1, \ldots, m$, are all convex and differentiable functions [44].

Equation (66) is based on the fact that if $\lambda > 0$, the stationary point can not be a minimum [52]. Note that the size of λ will be affected by several factors such as the type of optimization problem (whether maximization or minimization), the type of inequality constraints (whether $g_k(x) \leq 0$ or $g_k(x) \geq 0$), and the type of Lagrangian function (whether $L(x,\lambda) = f(x) - \sum_k \lambda_k g_k(x)$ or $L(x,\lambda) = f(x) + \sum_k \lambda_k g_k(x)$).

The Kuhn-Tucker conditions (the necessary conditions given by equations (61) through (65) for $x \geq 0$) provide the candidates for local minimum points. A local optimal point which satisfies the Kuhn-Tucker conditions will be the global optimal point if the objective function is convex (for minimization) or concave (for maximization), and if the constraints form a closed convex set in which the optimal point lies. The convexity or concavity of a function can be identified by the Hessian matrix (for example the matrix of the second partial derivatives of f(x) with respect to x) and applying Sylvester's theorem. A set of points satisfying the constraints, $g_k(x) \leq 0$, $k = 1,2, \ldots, m$, will be a closed convex set if $g_k(x)$, $k = 1, \ldots, m$ are all convex functions.

MINIMIZATION OF ENERGY FOR SYSTEM I

For the system given in Figure 26, the method of Lagrangian multipliers and the Kuhn-Tucker conditions was applied to find the optimal values of t_a, P_a and v, which give the minimum energy load requirement. Using the equations from Part 2 for the objective function and comfort equation constraint and the physical constraints on vapor pressure and velocity form Part 1 one gets the following.

Minimize

$$S = C_1 t_a + C_2 P_a + C_3 v + C_4 v^3 + C_k = f(t_a, P_a, v, v^3) \tag{45}$$

subject to the constraints

$$g_1(t_a, P_a, v) = AP_a + Bt_a - Dv^{0.5} + EP_a v^{0.5} + Ft_a v^{0.5} + C = 0 \tag{46}$$

$$g_2(v) = 0.1 - v \leq 0 \tag{67}$$

$$g_3(v) = v - 2.6 \leq 0 \tag{68}$$

$$g_4(P_a) = 1 - P_a \leq 0 \tag{69}$$

$$g_5(P_a) = P_a - 30 \leq 0 \tag{70}$$

Applying Kuhn-Tucker conditions

$$\frac{\partial L}{\partial t_a} = \frac{\partial f}{\partial t_a} - \left(\lambda_1 \frac{\partial g_1}{\partial t_a} + \lambda_2 \frac{\partial g_2}{\partial t_a} + \lambda_3 \frac{\partial g_3}{\partial t_a} + \lambda_4 \frac{\partial g_4}{\partial t_a} + \lambda_5 \frac{\partial g_5}{\partial t_a}\right) = 0 \tag{71}$$

$$\frac{\partial L}{\partial P_a} = \frac{\partial f}{\partial P_a} - \left(\lambda_1 \frac{\partial g_1}{\partial P_a} + \lambda_2 \frac{\partial g_2}{\partial p_a} + \lambda_3 \frac{\partial g_3}{\partial P_a} + \lambda_4 \frac{\partial g_4}{\partial P_a} + \lambda_5 \frac{\partial g_5}{\partial P_a}\right) = 0 \qquad (72)$$

$$\frac{\partial L}{\partial v} = \frac{\partial f}{\partial v} - \left(\lambda_1 \frac{\partial g_1}{\partial v} + \lambda_2 \frac{\partial g_2}{\partial v} + \lambda_3 \frac{\partial g_3}{\partial v} + \lambda_4 \frac{\partial g_4}{\partial v} + \lambda_5 \frac{\partial g_5}{\partial v}\right) = 0 \qquad (73)$$

$$\lambda_1 g_1 = 0 \qquad (74)$$

$$\lambda_2 g_2 = 0 \qquad (75)$$

$$\lambda_3 g_3 = 0 \qquad (76)$$

$$\lambda_4 g_4 = 0 \qquad (77)$$

$$\lambda_5 g_5 = 0 \qquad (78)$$

$$g_1 = 0 \qquad (79)$$

$$g_2 \leq 0 \qquad (80)$$

$$g_3 \leq 0 \qquad (81)$$

$$g_4 \leq 0 \qquad (82)$$

$$g_5 \leq 0 \qquad (83)$$

$$\lambda_1 \leq 0 \quad \text{(minimum)} \qquad (84)$$

$$\lambda_2 \leq 0 \quad \text{(minimum)} \qquad (85)$$

$$\lambda_3 \leq 0 \quad \text{(minimum)} \qquad (86)$$

$$\lambda_4 \leq 0 \quad \text{(minimum)} \qquad (87)$$

$$\lambda_5 \leq 0 \quad \text{(minimum)} \qquad (88)$$

These conditions are also sufficient for a global minimum if f and g_k (k = 1 to 5), are all convex and differentiable functions. Carrying out the above procedure yields the following.

$$\frac{\partial L}{\partial t_a} = C_1 - [\lambda_1(B + Fv^{0.5})] = 0 \tag{71}$$

$$\frac{\partial L}{\partial P_a} = C_2 - [\lambda_1(A + Ev^{0.5}) + \lambda_4(-1) + \lambda_5] = 0 \tag{72}$$

$$\frac{\partial L}{\partial v} = C_3 + 3C_4v^2 - [\lambda_1\left(-\frac{D}{2v^{0.5}} + \frac{EP_a}{2v^{0.5}} + \frac{Ft_a}{2v^{0.5}}\right) - \lambda_2 + \lambda_3] = 0 \tag{73}$$

$$\lambda_1(AP_a + Bt_a - Dv^{0.5} + EP_aV^{0.5} + Ft_aV^{0.5} + C) = 0 \tag{74}$$

$$\lambda_2(.1 - v) = 0 \tag{75}$$

$$\lambda_3(v - 2.6) = 0 \tag{76}$$

$$\lambda_4(1 - P_a) = 0 \tag{77}$$

$$\lambda_5(P_a - 30) = 0 \tag{78}$$

$$AP_a + Bt_a - Dv^{0.5} + EP_aV^{0.5} + Ft_av^{0.5} + C = 0 \tag{79}$$

$$.1 - v \leq 0 \tag{80}$$

$$v - 2.6 \leq 0 \tag{81}$$

$$1 - P_a \leq 0 \tag{82}$$

$$P_a - 30 \leq 0 \tag{83}$$

$$\lambda_1 \leq 0 \tag{84}$$

$$\lambda_2 \leq 0 \tag{85}$$

$$\lambda_3 \leq 0 \tag{86}$$

$$\lambda_4 \leq 0 \tag{87}$$

$$\lambda_5 \leq 0 \tag{88}$$

From Equation (84) and Equation (74) and Equation (79)

$$\lambda_1 < 0 \tag{89}$$

and from Equation (71)

$$\lambda_1 = \frac{C_1}{B+Fv^{0.5}} \tag{90}$$

then substitution of Equation (90) into Equation (72) to get

$$C_1 \frac{(A+Ev^{0.5})}{(B+Fv^{0.5})} - \lambda_4 + \lambda_5 = C_2 \tag{91}$$

From Equation (77) either $\lambda_4 = 0$ or $P_a = 1$, likewise

From Equation (78) either $\lambda_5 = 0$ or $P_a = 30$, likewise

From Equation (75) either $\lambda_2 = 0$ or $v = 0.1$, likewise

From Equation (76) either $\lambda_3 = 0$ or $v = 2.6$.

From the above propose $\lambda_4 = 0$ and $\lambda_5 = 0$, therefore Equation (91) gives rise to

$$v^{0.5} = \frac{AC_1 - BC_2}{FC_2 - EC_1} \tag{92}$$

which gives a negative value when the constants are substituted into Equation (92), and thus this result is not feasible and violates the physical constraint on velocity.

Next propose $\lambda_4 < 0$, i.e. $P_a = 1$, and hence $\lambda_5 = 0$, therefore from Equation (72) and Equation (90)

$$\frac{C_1(A+Ev^{0.5})}{B+Fv^{0.5}} - \lambda_4 = C_2 \tag{93}$$

and from Equation (79)

$$A + Bt_a - Dv^{0.5} + Ev^{0.5} + Ft_a v^{0.5} + C = 0 \tag{94}$$

such that λ_4, t_a and v are unknown and there exist only two equations.

Therefore propose $\lambda_2 = 0$ and $\lambda_3 = 0$, which yields from Equation (73)

$$C_3 + 3C_4v^2 - \left\{\frac{C_1}{B+Fv^{0.5}} \left(- \frac{D}{2v^{0.5}} + \frac{E}{2v^{0.5}} + \frac{Ft_a}{2v^{0.5}}\right)\right\} = 0 \tag{95}$$

Solving from Equations (94) and (95) for t_a and $v^{0.5}$ a value for $v >> 2.6$ was found which violates the boundary condition in Equation (81).

Select $\lambda_2 < 0$, i.e. $v = 0.1$ and hence $\lambda_3 = 0$, to get from Equations (93) and (94)

$$\lambda_4 = \frac{C_1(A+0.317E)}{B+0.317F} - C_2 \tag{96}$$

and

$$t_a = \frac{0.317D - A - C - 0.317E}{B + 0.317F} \tag{97}$$

which gives a positive value for λ_4 which violates Equation (87).

Therefore select $\lambda_3 < 0$, ie. $v = 2.6$ and hence $\lambda_2 = 0$, and it is found that λ_4 is positive which violates Equation (87). Since no feasible value of velocity exists which meets the above constraints then the proposal of $\lambda_4 < 0$ is invalid.

Propose $\lambda_5 < 0$, ie. $P_a = 30$ and hence $\lambda_4 = 0$, then from Equation (72)

$$C_2 - \lambda_1 (A+Ev^{0.5}) - \lambda_5 = 0 \tag{98}$$

Substitute Equation (90) into equation (98) to get

$$\lambda_5 = C_2 - \frac{C_1}{B+Fv^{0.5}} (A+Ev^{0.5}) \tag{99}$$

and from Equation (79)

$$30A + Bt_a - Dv^{0.5} + 30Ev^{0.5} + Ft_a v^{0.5} + C = 0 \tag{100}$$

which gives rise to two equations and three unknowns.

Therefore, select as before $\lambda_2 = 0$, $\lambda_3 = 0$, which yields from Equation (73)

$$C_3 + 3C_4 v^2 - \left\{\frac{C_1}{B+Fv^{0.5}} \left(\frac{-D}{2v^{0.5}} + \frac{30E}{2v^{0.5}} + \frac{Ft_a}{2v^{0.5}}\right)\right\} = 0 \tag{101}$$

Solving from Equation (100) and (101) for t_a and v it is found that $v \gg 2.6$ which violates Equation (81).

Propose $\lambda_2 < 0$, i.e. $v = 0.1$ and hence $\lambda_3 = 0$, which gives from Equations (99) and (100)

$$\lambda_5 = C_2 - \frac{C_1}{B+0.317F}[A + 0.317E] \tag{102}$$

and

$$t_a = \frac{0.317D - 30A - C - 30E(0.317)}{B+0.317F} \tag{103}$$

this results in a negative value for λ_5 which meets the above constraints.

Likewise, propose $\lambda_3 < 0$, $v = 2.6$ and hence $\lambda_2 = 0$, which yields again a negative value for λ_5 and satisfies the constraints. So this results in the following possible solutions,

$$\lambda_1 < 0 \tag{89}$$

$$\lambda_4 = 0 \tag{104}$$

$$\lambda_5 < 0 \tag{105}$$

$$P_a = 30 \text{ mm Hg (upper boundary)} \tag{106}$$

and either

$$\lambda_2 < 0, \text{ i.e. } v = 0.1 \text{ m/sec and hence } \lambda_3 = 0 \tag{107}$$

or

$$\lambda_3 < 0, \text{ i.e. } v = 2.6 \text{ m/sec and hence } \lambda_2 = 0 \tag{108}$$

The computer program for System I is given in Appendix Q. The results from this program show that, for all values of reasonable constants chosen, the minimum required energy load occurred at the lower boundary condition of velocity. Therefore the optimal solution would be Equation (107).

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Table 13. Lagrange Multiplier and Kuhn-Tucker Conditions Optimization Results for System I.

Outside Temperature T_2, °C	Insulation Icl/clo	Activity ACT,M/AD_u	Room Temperature t_a, °C	Room Velocity v, m/sec	Room Vapor Pressure P_a, mm Hg	Sensible Heat S_1,Kcal/hr	Latent Heat S_2,Kcal/hr	Friction + Kinetic Heat Load S_3,Kcal/hr	Minimum Tota Energy Required S,Kcal/hr
50	0	50	28.24	0.10	20	98.01	664.29	50.32	812.62
			32.12	2.60	20	80.52	664.29	1411.05	3155.86
		80	25.55	0.10	20	110.12	664.29	50.32	824.73
			30.44	2.60	20	88.10	664.29	1411.05	2163.44
		100	23.49	0.10	20	119.38	664.29	50.32	833.99
			29.29	2.60	20	93.29	664.29	1411.05	2168.63
		150	18.42	0.10	20	142.24	664.29	50.32	856.85
			26.42	2.60	20	106.20	664.29	1411.05	2181.54
	0.6	50	24.47	0.10	20	114.98	664.29	50.32	829.59
			28.00	2.60	20	99.06	664.29	1411.05	2174.40
		80	20.28	0.10	20	133.86	664.29	50.32	848.47
			24.70	2.60	20	113.96	664.29	1411.05	2189.30
		100	17.28	0.10	20	147.38	664.29	50.32	861.99
			22.48	2.60	20	123.92	664.29	1411.05	2199.26
		150	9.94	0.10	20	180.44	664.29	50.32	895.05
			17.03	2.60	20	148.47	664.29	1411.05	2223.81
	1.0	50	21.85	0.10	20	126.78	664.29	50.32	841.39
			25.25	2.60	20	111.46	664.29	1411.05	2186.80
		80	16.65	0.10	20	150.19	664.29	50.32	864.80
			20.88	2.60	20	131.13	664.29	1411.05	2206.47

Table 13 -- Continued

T_2	I_{cl}	ACT	t_a	v	P_a	S_1	S_2	S_3	S
40	0.6	50	23.36	0.10	30	76.59	241.85	50.32	368.76
			27.30	2.60	30	58.44	241.85	1411.05	1711.34
		80	19.06	0.10	30	96.34	241.85	50.32	388.51
			23.90	2.60	30	74.09	241.85	1411.05	1726.99
		100	15.97	0.10	30	110.60	241.85	50.32	402.77
			21.62	2.60	30	84.57	241.85	1411.05	1737.47
		150	8.39	0.10	30	145.46	241.85	50.32	437.63
			16.01	2.60	30	110.39	241.85	1411.05	1763.29
40	0	50	28.64	0.10	14	52.26	382.52	50.32	485.10
			32.26	2.60	14	35.62	382.52	1411.05	1829.19
		80	25.98	0.10	14	64.52	382.52	50.32	497.36
			30.59	2.60	14	43.29	382.52	1411.05	1836.86
		100	23.96	0.10	14	73.82	382.52	50.32	506.86
			29.45	2.60	14	48.54	382.52	1411.05	1842.11
		150	18.97	0.10	14	96.78	382.52	50.32	529.62
			26.62	2.60	14	61.58	382.52	1411.05	1855.15
	0.6	50	25.14	0.10	14	68.40	382.52	50.32	501.24
			28.43	2.60	14	53.25	382.52	1411.05	1846.82
		80	21.01	0.10	14	87.40	382.52	50.32	520.24
			25.18	2.60	14	68.22	382.52	1411.05	1861.79
		100	18.06	0.10	14	100.95	382.52	50.32	533.79
			23.00	2.60	14	78.22	382.52	1411.05	1871.79
		150	10.86	0.10	14	134.08	382.52	50.32	566.92
			17.65	2.60	14	102.86	382.52	1411.05	1896.43

The procedure used in this analysis is shown in the computer flow diagram in Appendix R. For the parameters selected the optimal results from the above analytical method are given in Table 13.

DEFINITION OF SYSTEM II MODEL

Now that optimization techniques have been applied to a relatively simple environmental control system model, System I, it follows that a more realistic model is in order. Therefore, using the same comfort and physical constraints, a more complete model was devised. The basic schematic of this system, System II, is shown in Figure 28.

With this system, as in System I, concern was primarily with the control volume of air and the energy components that affect the principal variables of the system. The total energy of System II is comprised of external and internal components acting on the space. Looking at the loads of this system we find the following which perturbs the equilibrium condition of the control volume.

$$\text{Energy Load} = \text{supply air} + \text{fan} + \text{wall transmission} + \text{exterior radiation} + \text{lights} + \text{people} + \text{miscellaneous equipment} \tag{109}$$

The thermal loads which result in cooling or heating requirements are accomplished by the coils which are furnished by a circulating pump or pumps. The humidity balance is maintained by a dehumidifier and a humidifier. Since the controls and equipment are not of direct concern they are not detailed in this study. Therefore the effectiveness coefficients and/or efficiences were not included in the analysis. As stated before interest lay only in the energy load requirements that must be demanded

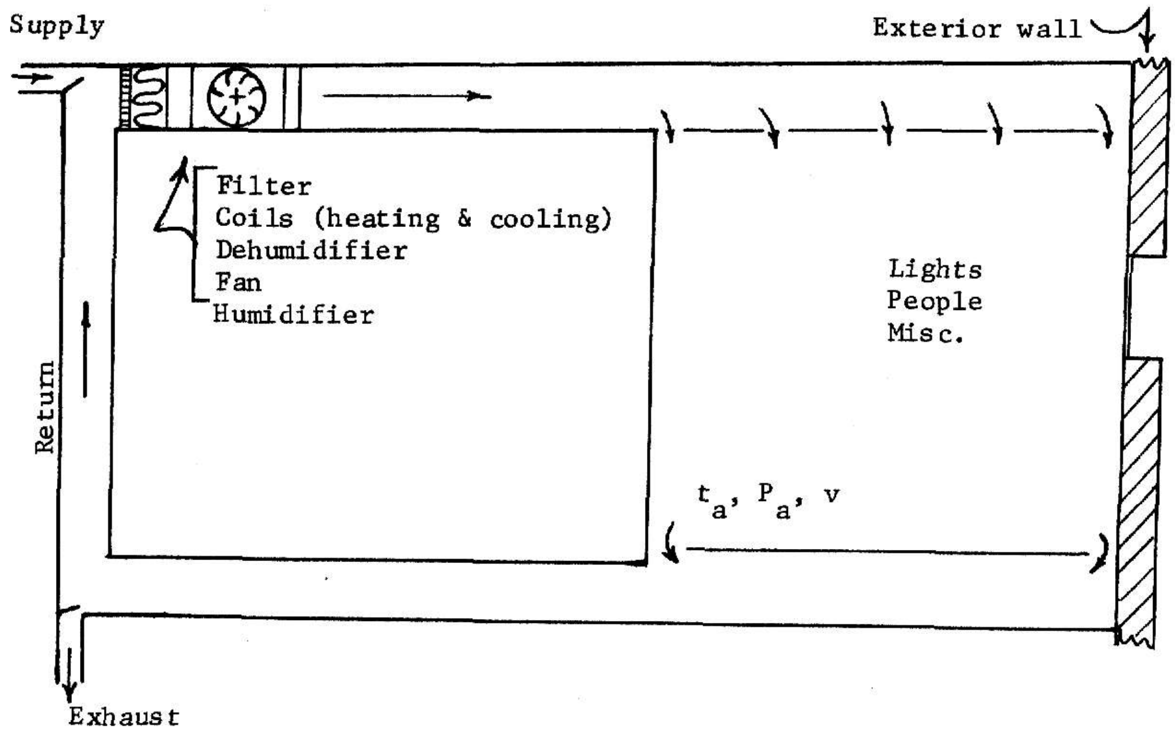

Figure 28. Sketch of System II.

from the equipment that is involved. The indirect (to air system) energy analysis is reserved until the consumption demand from keeping the space in equilibrium (comfort) condition is evaluated. Therefore, the energy consumption required for the pump(s), refrigeration, heating, and motor(s) were not included in the total energy load.

The physical set-up is similar to that presented for System I, which of course can be stated in general parameter terms. Using the assumptions of Part 2, System I, for System II in this model, we can look at the components that act on the space variables.

The supply air consists of the outside ventilation requirement and the infiltration into the space. The infiltration is considered to be negligible and thus the sensible and insensible loads of the outside air are given, as before, by

$$S_1 = C_1 t_a + C_{01} \tag{35}$$

and

$$S_2 = C_2 P_a + C_{02} \tag{37}$$

The fan acts upon the system to provide a total pressure to overcome friction and provide velocity. In this way the thermal load, neglecting internal fan losses, that acts upon the air consists of static pressure and velocity pressure (as it reduces along the duct run, energy conversion causes a temperature rise) [2]. The resistance imposed by the duct system is overcome by expenditure of mechanical energy of the fan. This energy is a function of the flow rate and static pressure rise. The total fan load, static plus velocity, provided to the air as thermal load must be removed at the coils. Therefore the total required load values are the sum of the fan load and the thermal load. This may be expressed from Equation (44) as follows.

$$S_3 = 2(C_3 v + C_q v^3) \qquad (110)$$

The heat transmission through the exterior wall from the outside environment (assuming the wall is homogeneous, contains no moisture and is nonporous) is given by

$$q_{trans} = h_i\ (t_{wi} - t_a) = \frac{k_w}{x_w}\ (t_{wo} - t_{wi}) = h_o(t_2 - t_{wo}) \qquad (111)$$

where

q_{trans} = rate of heat transfer, kcal/hr m^2

h_i = inside air heat transfer coefficient, kcal/hr m^2 ^oC

h_o = outside air heat transfer coefficient, kcal/hr m^2 ^oC

t_{wi} = inside wall temperature, ^oC

k_w = thermal wall conductivity, kcal/hr $m^2/^oC$ per cm

x_w = wall thickness, cm.

t_{wo} = outside wall temperature, ^oC

t_a = room air temperature, ^oC

t_2 = outside temperature, ^oC

which by defining an overall heat transfer coefficient, U, may be given by

$$S_4 = Q_{trans} = UA_w\ (t_2 - t_a) \qquad (112)$$

where

$$Q_{trans} = q_{trans}\ A_w,\ \text{kcal/hr}$$

$$A_w \quad = \text{surface area of exterior wall, m}^2$$

$$U^{-1} = \frac{1}{h_i} + \frac{x_w}{k_w} + \frac{1}{h_o}\ ,\quad U \text{ in kcal/hr m}^2\ {}^oC$$

This U value may also be represented as

$$U = \left(\frac{1}{h_i} + \frac{1}{U_{ex}}\right)^{-1} \tag{113}$$

where

$$U_{ex} = \text{external wall heat transfer coefficient, kcal/hr m}^2\ {}^oC$$

or

$$U_{ex} = \left(\frac{1}{h_o} + \frac{x_w}{k_w}\right)^{-1} = \frac{h_i\ U}{h_i - U} \tag{114}$$

From this Equations (114), (112) and (111) are used to find

$$t_{wi} = \frac{U}{h_i}\ t_o + \frac{(h_i - U)}{h_i}\ t_a \tag{115}$$

The heat transmission load, Equation (112), may be expressed as follows

$$S_4 = C_4 t_a + C_{04} \tag{116}$$

where

$$C_4 = -UA_w$$

$$C_{04} = UA_w t_2$$

By defining the mean radiant temperature, t_{mrt}, as the surface area weighted average of the surface temperatures of the walls of the space [49], where the inside walls have a surface temperature equal to approximately air temperature, we find

$$t_{mrt} = \frac{A_w t_{wi} + A_{iw} t_a}{A}, \ {}^oC \tag{117}$$

where

A_{iw} = surface area of inside walls, m^2

A = $A_w + A_{iw}$ = total surface area, m^2

This permits the expression of mean radiant temperature, t_{mrt}, in terms of the dry bulb temperature, t_a, by substitution of Equation (115) into Equation (117) as

$$t_{mrt} = \frac{UA_w}{h_i A} t_2 + \left(\frac{(h_i - U)A_w}{h_i A} + \frac{A_{iw}}{A}\right) t_a \tag{118}$$

or

$$t_{mrt} = F_1 t_2 + F_2 t_a \tag{119}$$

Therefore for poor insulation the effect of outside temperature on mean radiant temperature results in significant deviation from air temperature. However, when the effect of insulation is maximized and the effect of external temperature conditions minimized the value of mean radiant temperature will follow the value of air temperature more closely. For general interior spaces with no exterior walls the mean radiant temperature can be considered approximately equal to the air temperature.

The exterior radiation component consists of direct solar radiation, diffuse sky radiation and reflected solar radiation. The effect upon the heat transmission load depends upon several factors. Some of these are wall surface type, atmospheric clearness, cloud cover type, orientation angle, time of day and year, and location. It is impossible to represent as a steady-state heat transmission because of continuous flucuations. If windows are present the type of glass, spacing, shading, angles of incidence, and other factors affect the calculation of energy transmission [79]. Therefore in this study the exterior radiation load is represented as an averaged load upon the space as

$$SR = SF * A_e * ERF \tag{120}$$

where

SR = average external radiation load, kcal/hr

SF = solar factor

A_e = effective surface area

ERF = effective radiant load

The interior load caused by lighting upon the system is usually a constant component which depends upon the type of lights, use factor, and allowance factor. Therefore, for a given floor area, the lighting load is represented as

$$LT = W * A_f * U_f * S_a * C_n \tag{121}$$

where

LT = load due to lights, kcal/hr

W = wattage factor for type of lights, watts/m^2

A_f = floor area, m^2

U_f = use factor

S_a = separate allowance factor

C_n = conversion factor = 0.859 kcal/watt hr

The human occupancy load is a function of the activity of the people in the space and the number of people that are present in the room. The sensible and insensible components are given as (from Equation (3))

$$S_5 = D + R + C, \qquad \text{kcal/hr} \tag{122}$$

and

$$S_6 = \bar{E}_{rsw} + E_d + L, \quad \text{kcal/hr} \tag{123}$$

where

D = heat loss by dry respiration

R = heat loss by radiation

C = heat loss by convection

$\bar{E}_{rsw}$ = heat loss by sweat evaporation

E_d = heat loss by diffusion of water vapor

L = heat loss by latent respiration

This results in the following equations

$$S_5 = 0.0014M(34-t_a) + h_r A_{eff}(t_{cl}-t_{mrt}) + h_c A_{Du}(t_{cr}-t_a) \tag{124}$$

and

$$S_6 = 0.42\, A_{Du} \left[\frac{M}{A_{Du}} - 50\right] + 0.35\, A_{Du} \left[43 - 0.061 \frac{M}{A_{Du}} - P_a\right]$$

$$+ 0.0023\, M\, (44 - P_a) \qquad (125)$$

where the terms are defined in Part 1 and Part 2, and the mechanical efficiency of the body is taken as zero.

From Equation (7), the clothing temperature is given as

$$t_{cl} = 35.7 + \frac{M}{A_{Du}} [-0.081459\, I_{cl} - 0.032] - (0.063\, I_{cl} +$$

$$0.000414\, I_{cl} \frac{M}{A_{Du}})\, P_a - 0.000252\, I_{cl} \frac{M}{A_{Du}}\, t_a - 1.071\, I_{cl} \qquad (126)$$

By substitution of Equations (6) and (126) into Equation (124), the sensible human load is

$$S_5 = A_5 P_a + B_5 t_a + D_5 v^{0.5} + E_5 v^{0.5} P_a + F_5 v^{0.5} t_a$$

$$+ G_5 t_{mrt} + C_5 \qquad (127)$$

where

$$A_5 = -\, h_r A_{eff}\, (0.063 I_{cl} + 0.000414\, I_{cl} \frac{M}{A_{Du}})$$

$$B_5 = -\, 0.0014\, M - h_r\, A_{eff}\, (0.000252\, I_{cl} \frac{M}{A_{Du}})$$

$$C_5 = 0.0476\, M + h_r\, A_{eff}\, C_{15}$$

$$C_{15} = 35.7 + \frac{M}{A_{Du}} (-0.081459\, I_{cl} - 0.032) - 1.071\, I_{cl}$$

$$D_5 = 10.4\, f_{cl}\, A_{Du}\, C_{15}$$

$$E_5 = -\ 10.4\ f_{cl}\ A_{Du}\ (0.063\ I_{cl} + 0.00414\ I_{cl}\ \frac{M}{A_{Du}})$$

$$F_5 = -\ 10.4\ f_{cl}\ A_{Du}\ (1 + 0.000252\ I_{cl}\ \frac{M}{A_{Du}})$$

$$G_5 = -\ h_r\ A_{eff}$$

and by reducing Equation (125) it is found that the insensible human load is

$$S_6 = A_6\ P_a + C_6 \tag{128}$$

where

$$A_6 = -\ 0.35\ A_{Du} - 0.0023\ M$$

$$C_6 = 0.49985\ M - 5.95\ A_{Du}\ .$$

The final load components are the internal contribution of any sensible heat and moisture gains by miscellaneous equipment in the space. For this study all miscellaneous loads are taken as zero.

The total energy load for the system under consideration is represented by Equation (109). This Equation is represented in general terms as

$$S = S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + SR + LT \tag{129}$$

or as

$$S = C_1 t_a + C_2 P_a + 2C_3 v + 2C_q v^3 + C_4 t_a + A_5 P_a + B_5 t_a + D_5 v^{0.5}$$

$$+ E_5 v^{0.5} P_a + F_5 v^{0.5}\ t_a + G_5 t_{mrt} + A_6 P_a + K \tag{130}$$

where

$$K = C_{01} + C_{02} + C_{04} + C_5 + C_6 + SR + LT$$

Collecting terms on Equation (130) it is found that

$$S = K_1 t_a + K_2 P_a + K_3 v + K_4 v^3 + D_5 v^{0.5} + E_5 v^{0.5} P_a$$

$$+ F_5 v^{0.5} t_a + G_5 t_{mrt} + K \qquad (131)$$

where

$$K_1 = C_1 + C_4 + B_5$$

$$K_2 = C_2 + A_5 + A_6$$

$$K_3 = 2C_3$$

$$K_4 = 2C_q$$

This Equation gives the energy demand of the system, System II, in terms of the principal variables of the system, namely, t_a, P_a, v and t_{mrt}. This is, of course, similar to the mathematical model of System I with the inclusion of the mean radiant temperature, t_{mrt}, variable. If it is assumed that the inside wall heat transfer coefficient is close to unity Equation (115) becomes approximated by

$$t_{wi} \cong t_a + U(t_2 - t_a) \qquad (132)$$

which shows that as the value of U decreases (insulation increases to an optimal value) to a minimum or ideally approaches zero, the t_{mrt} (see Equation (117)) or t_{wi} approaches the t_a value.

From Equation (119), when the outside temperature, t_2, is taken as a parameter of the outside environmental conditions, one acknowledges that the mean radiant temperature, t_{mrt}, is actually a function of dry bulb temperature, t_a. Therefore, since t_{mrt} is not an independent variable, but dependent upon t_a, substitution of Equation (119) into Equation (131) yields

$$S = K_G t_a + K_2 P_a + K_3 v + K_4 v^3 + D_5 v^{0.5} + E_s v^{0.5} P_a$$
$$+ F_5 v^{0.5} t_a + K_F \tag{133}$$

where

$$K_G = K_1 + G_5 F_2$$

$$K_F = K + G_5 F_1 t_2$$

Equation (133) is the energy requirement for the system as a function of t_a, P_a and v.

ANALYSIS OF ENVIRONMENTAL CONTROL SYSTEM II

Following the minimization procedure used in optimization of System I, the method of Lagrangian multipliers and the Kuhn-Tucker conditions is applied to find the optimal values of t_a, P_a and v, which give the minimum energy load requirement. The comfort equation constraint is the modified version of Equation (46) where $t_{mrt} \neq t_a$.

Therefore the comfort constraint equations is given by

$$g = AP_a + B_1 t_a - Dv^{0.5} + EP_a v^{0.5} + Ft_a v^{0.5} +$$
$$Gt_{mrt} + C = 0 \tag{134}$$

where A,D,E,F and C were defined previously and

$$B_1 = -B_5$$

$$G = -G_5$$

$$C = M - C_5 - C_6$$

Substitution of Equation (119) into Equation (134) gives rise to

$$g = AP_a + B_2 t_a - Dv^{0.5} + EP_a v^{0.5} + Ft_a v^{0.5} + C_g = 0 \qquad (135)$$

where

$$B_2 = B_1 + GF_2$$

$$C_g = C + GF_1$$

Minimize

$$S = K_G t_a + K_2 P_a + K_3 v + K_4 v^3 + D_5 v^{0.5} + E_5 v^{0.5} P_a$$

$$+ F_5 v^{0.5} t_a + K_F = f(t_a, P_a, v) \qquad (133)$$

subject to the constraints

$$g_1 = AP_a + B_2 t_a - Dv^{0.5} + EP_a v^{0.5} + Ft_a v^{0.5} + C_g = 0 \qquad (135)$$

$$g_2 = 0.1 - v \leq 0 \qquad (136)$$

$$g_3 = v - 2.6 \leq 0 \qquad (137)$$

$$g_4 = 1 - P_a \leq 0 \qquad (138)$$

$$g_5 = P_a - 20 \leq 0 \qquad (139)$$

Applying the Kuhn-Tucker conditions

$$\frac{\partial L}{\partial t_a} = K_G + F_5 v^{0.5} - [\lambda_1 (B_2 + F v^{0.5})] = 0 \tag{140}$$

$$\frac{\partial L}{\partial P_a} = K_2 + E_5 v^{0.5} - [\lambda_1 (A + E v^{0.5}) - \lambda_4 + \lambda_5] = 0 \tag{141}$$

$$\frac{\partial L}{\partial v} = K_3 + 3K_4 v^2 + \frac{D_5}{2v^{0.5}} + \frac{E_5 P_a}{2v^{0.5}} + \frac{F_5 t_a}{2v^{0.5}}$$

$$- [\lambda_1 (- \frac{D}{2v^{0.5}} + \frac{EP_a}{2v^{0.5}} + \frac{Ft_a}{2v^{0.5}}) - \lambda_2 + \lambda_3] = 0 \tag{142}$$

$$\lambda_1 (AP_a + B_2 t_a - Dv^{0.5} + EP_a v^{0.5} + Ft_a v^{0.5} + C_g) = 0 \tag{143}$$

$$\lambda_2 (0.1 - v) = 0 \tag{144}$$

$$\lambda_3 (v - 2.6) = 0 \tag{145}$$

$$\lambda_4 (1 - P_a) = 0 \tag{146}$$

$$\lambda_5 (P_a - 20) = 0 \tag{147}$$

$$AP_a + B_2 t_a - Dv^{0.5} + EP_a v^{0.5} + Ft_a v^{0.5} + C_g = 0 \tag{148}$$

$$0.1 - v \leq 0 \tag{149}$$

$$v - 2.6 \leq 0 \tag{150}$$

$$1 - P_a \le 0 \tag{151}$$

$$P_a - 20 \le 0 \tag{152}$$

$$\lambda_1 \le 0 \tag{153}$$

$$\lambda_2 \le 0 \tag{154}$$

$$\lambda_3 \le 0 \tag{155}$$

$$\lambda_4 \le 0 \tag{156}$$

$$\lambda_5 \le 0 \tag{157}$$

From Equations (153), (143) and (148) it is found that

$$\lambda_1 < 0 \tag{158}$$

and from Equation (140)

$$\lambda_1 = \frac{K_G + F_5 v^{0.5}}{B_2 + F v^{0.5}} \tag{159}$$

If Equation (159) is substituted into Equation (141) it gives rise to

$$K_2 + E_5 v^{0.5} - \left(\frac{K_G + F_5 v^{0.5}}{B_2 + F v^{0.5}} (A + E v^{0.5}) - \lambda_4 + \lambda_5\right) = 0 \tag{160}$$

Looking at Equations (146) and (147) it is seen that either $\lambda_4 = 0$ or $P_a = 1$ and either $\lambda_5 = 0$ or $P_a = 20$. Combining these facts with Equations (156) and (157) Proposal # 1 is made, that $\lambda_4 = 0$ and $\lambda_5 = 0$. Hence Equation (160) becomes

$$K_2 + E_5 v^{0.5} - \left(\frac{K_G + F_5 v^{0.5}}{B_2 + Fv^{0.5}} (A + Ev^{0.5})\right) = 0 \tag{161}$$

Solving Equation (161) for $v^{0.5}$ for various parameter values negative answers were found which violate the physical boundaries or velocity.

Next, Proposal # 2, $\lambda_4 < 0$, hence $P_a = 1$ and $\lambda_5 = 0$, results in

$$K_2 + E_5 v^{0.5} - \left(\frac{K_G + F_5 v^{0.5}}{B_2 + Fv^{0.5}} (A + Ev^{0.5}) - \lambda_4\right) = 0 \tag{162}$$

and from Equation (148) it is found that

$$A + B_2 t_a - Dv^{0.5} + Ev^{0.5} + Ft_a v^{0.5} + C_g = 0 \tag{163}$$

which involves two equations and three unknowns. Therefore, looking at Equations (144) and (145) it is found that either $\lambda_2 = 0$ or $v = 0.1$ and either $\lambda_3 = 0$ or $v = 2.6$. Combining these facts with Equation (154) and (155) Proposal # 3 is made , such that $\lambda_2 = 0$ and $\lambda_3 = 0$. Henceforth, from Equation (142) it is found that

$$K_3 + 3K_4 v^2 + \frac{D_5}{2v^{0.5}} + \frac{E_5}{2v^{0.5}} + \frac{F_5 t_a}{2v^{0.5}} - \left(\lambda_1 \left(- \frac{D}{2v^{0.5}} + \frac{E}{2v^{0.5}} + \frac{Ft_a}{2v^{0.5}}\right)\right) = 0 \tag{164}$$

Substitution of Equation (159) for λ_1 yields two equations, namely, Equations (163) and (164) and two unknowns. Solving for v a value is found that is much larger than the upper physical boundary condition and thus this value violates Equation (150).

Within Proposal # 2, if one selects $\lambda_2 < 0$, hence $v = 0.1$ and $\lambda_3 = 0$, as Proposal # 4 to get from Equation (162)

$$\lambda_4 = \frac{K_G + F_5(0.1)^{0.5}}{B_2 + F(0.1)^{0.5}} - (K_2 + E_5(0.1)^{0.5}) \tag{165}$$

which upon substitution of the constant values yields a positive solution for λ_4 which violates Equation (156).

Again within Proposal # 2, if one selects $\lambda_3 < 0$, hence $v = 2.6$ and $\lambda_2 = 0$, as Proposal # 5 to get from Equation (162)

$$\lambda_4 = \frac{K_G + F_5(2.6)^{0.5}}{B_2 + F(2.6)^{0.5}} - (K_2 + E_5(2.6)^{0.5}) \tag{166}$$

which gives results that yield a positive solution for λ_4 that violates Equation (156).

From the foregoing Proposal # 2 is then not possible since it does not have a feasible solution that satisfies the velocity constraints. Therefore, Proposal # 6 is made which selects $\lambda_5 > 0$, hence $P_a = 20$ and $\lambda_4 = 0$, from which Equation (160) yields

$$K_2 + E_5v^{0.5} - \frac{K_G + F_5v^{0.5}}{B_2 + Fv^{0.5}}(A + Ev^{0.5}) - \lambda_5 = 0 \tag{167}$$

and from Equation (148) it is found that

$$20A + B_2t_a - Dv^{0.5} + 20Ev^{0.5} + Ft_av^{0.5} + C_g = 0 \tag{168}$$

which again involves two equations and three unknowns. So as before try Proposal # 3, that $\lambda_2 = 0$ and $\lambda_3 = 0$. This yields from Equation (142) as

$$K_3 + 3K_4v^2 + \frac{D_5}{2v^{0.5}} + \frac{20E_5}{2v^{0.5}} + \frac{F_5t_a}{2v^{0.5}} - \left\{\lambda_1\left(-\frac{D}{2v^{0.5}} + \frac{20E}{2v^{0.5}} + \frac{Ft_a}{2v^{0.5}}\right)\right\} = 0$$

Substitution of Equation (159) for λ_1 yields two equations, namely, Equations (168) and (169) and two unknowns. Solving for v a value is found that is much larger than the upper physical boundary condition which violates Equation (150).

Within Proposal # 6, try Proposal # 4, where $\lambda_2 < 0$, hence $v = 0.1$ and $\lambda_3 = 0$, to get from Equation (167)

$$\lambda_5 = K_2 + E_5(0.1)^{0.5} - \frac{K_G + F_5(0.1)^{0.5}}{B_2 + F(0.1)^{0.5}}(A + E(0.1)^{0.5}) \tag{170}$$

which gives a negative value for λ_5 when the values for the parameters are inserted. This negative value satisfies the constraint of Equation (157).

Likewise Proposal # 5 is made, within Proposal # 6, where $\lambda_3 > 0$, hence $v = 2.6$ and $\lambda_2 = 0$, which gives negative values for λ_5. Therefore both # 4 and # 5 Proposals must be checked to determine the minimum energy from Equation (133).

The dry bulb temperature, t_a, is found from Equation (148) as

$$t_a = \frac{Dv^{0.5} - EP_a v^{0.5} - AP_a - C_g}{B_2 + Fv^{0.5}} \tag{171}$$

where the values for P_a are from Proposal # 6 (the upper physical boundary placed on water vapor pressure, in this case $P_a = 20$) and the values for v are from either Proposal # 4 or Proposal # 5.

The computer flow diagram used in this procedure is the same as in System I optimization which is given in Appendix R. An example of the computer program used for System II is presented in Appendix S. The optimal results for selected parameters are given in Table 14. The optimal values which gave the minimum energy load requirement, occurred at the lower physical boundary placed on the velocity variable. Hence when Proposal # 4 is made within Proposal # 6 the optimum solution is found.

Table 14. Lagrange Multiplier and Kuhn-Tucker Conditions Optimization Results for System II.

(Note: Solar Radiation Load = 73.53 Kcal/hr. and Lighting Load - 391.67 Kcal/hr.)

Outside Temperature T_2, °C	Clothing Insulation I_{cl}, clo	Activity ACT, M/AD_u	Room Temperature t_a, °C	Room Velocity v, m/sec	Room Vapor Pressure P_a, mm Hg	S_1	S_2	S_3	S_4	S_5	S_6	Minimum Total Energy Required S, Kcal/hr.
40	0	50	28.70	0.10	14	52.02	382.52	100.64	58.18	66.52	22.56	1147.64
			32.28	2.60	14	35.53	382.52	2822.10	39.74	66.52	22.56	3833.90
		80	26.03	0.10	14	64.28	382.52	100.64	71.90	95.13	47.81	1227.48
			30.61	2.60	14	43.20	382.52	2822.10	48.32	95.13	47.81	3904.10
		100	24.00	0.10	14	73.61	382.52	100.64	82.33	114.29	64.65	1283.23
			29.47	2.60	14	48.44	382.52	2822.10	54.18	114.29	64.65	3951.20
		150	19.01	0.10	14	96.61	382.52	100.64	108.06	162.21	106.74	1421.97
			26.64	2.60	14	61.50	382.52	2822.10	68.78	162.21	106.74	4068.90
	0.6	50	25.18	0.10	14	68.19	382.52	100.64	76.27	66.45	22.56	1181.83
			28.44	2.60	14	53.17	382.52	2822.10	59.48	66.45	22.56	3871.30
		80	21.05	0.10	14	87.22	382.52	100.64	97.55	95.05	47.81	1275.99
			25.19	2.60	14	68.14	382.52	2822.10	76.22	95.05	47.81	3956.80
		100	18.10	0.10	14	100.79	382.52	100.64	112.74	114.21	64.65	1340.75
			23.02	2.60	14	78.15	382.52	2822.10	87.41	114.21	64.65	4014.10
		150	10.88	0.10	14	133.99	382.52	100.64	149.87	162.13	106.74	1501.08
			17.66	2.60	14	102.80	382.52	2822.10	114.99	162.13	106.74	4156.30
	1.0	50	22.74	0.10	14	79.43	382.52	100.64	88.85	66.43	22.56	1205.63
			25.88	2.60	14	64.98	382.52	2822.10	72.68	66.43	22.56	3896.20

Table 14 -- Continued

T_2	I_{cl}	ACT	t_a	v	P_a	S_1	S_2	S_3	S_4	S_5	S_6	S
40	1.0	80	17.62	0.10	14	102.99	382.52	100.64	115.20	95.02	47.81	1309.38
			21.59	2.60	14	84.70	382.52	2822.10	94.74	95.02	47.81	3991.90
		100	14.05	0.10	14	119.41	382.52	100.64	133.56	114.19	64.65	1380.16
			18.75	2.60	14	97.78	382.52	2822.10	109.37	114.19	64.65	4055.60
		150	5.38	0.10	14	159.31	382.52	100.64	178.19	162.10	106.74	1554.69
			11.80	2.60	14	129.78	382.52	2822.10	145.17	162.10	106.74	4213.40
50	0	50	28.70	0.10	14	95.95	711.95	100.64	109.65	66.29	22.56	1572.23
			32.28	2.60	14	79.81	711.95	2822.10	91.21	66.29	22.56	4258.90
		80	26.03	0.10	14	107.95	711.95	100.64	123.37	94.86	47.81	1651.78
			30.61	2.60	14	87.31	711.95	2822.10	99.79	94.86	47.81	4328.80
		100	24.00	0.10	14	117.08	711.95	100.64	133.80	114.02	64.65	1707.33
			29.47	2.60	14	92.45	711.95	2822.10	105.65	114.02	64.65	4375.90
		150	19.01	0.10	14	139.59	711.95	100.64	159.53	161.94	106.74	1845.58
			26.64	2.60	14	105.23	711.95	2822.10	120.25	161.94	106.74	4493.20
	0.6	50	25.18	0.10	14	111.78	711.95	100.64	127.74	66.20	22.56	1606.06
			28.44	2.60	14	97.08	711.95	2822.10	110.94	66.20	22.56	4295.90
		80	21.05	0.10	14	130.40	711.95	100.64	149.02	94.76	47.81	1699.78
			25.19	2.60	14	111.73	711.95	2822.10	127.68	94.76	47.81	4381.00
		100	18.10	0.10	14	143.69	711.95	100.64	164.21	113.92	64.65	1764.25
			23.02	2.60	14	121.52	711.95	2822.10	138.88	113.92	64.65	4438.10
		150	10.88	0.10	14	176.18	711.95	100.64	201.34	161.83	106.74	1923.87
			17.66	2.60	14	145.65	711.95	2822.10	166.46	161.83	106.74	4579.80

DISCUSSION OF RESULTS

The feasibility range of the comfort equation was shown in Part 1, in Figures 14 through 25. It is highlighted by cross-hatching to illustrate the narrow region of acceptable values that are physically allowable for the comfort equation solution. However, to achieve thermal comfort as defined by ASHRAE Standard 55-66R [4] further restrictions on the variables, shown in Figure 1, must be made that yield an even smaller region. When looking, as in this study, at only forced convection coupled with the above restrictions from Standard 55-66R the range of feasible values are reduced even more dramatically.

In Part 2, it was found that a linear radiation coefficient, for the radiation component in the comfort equation, could be calculated by correlation of an average temperature for each value of clothing insulation. The absolute average temperature value to the third power enabled evaluation of the linear radiation coefficient to differ less than 3% from the fourth power relationship for the range of values of the above Standard 55-66R and the forced convection restriction. The average temperature values of 27.78, 23.33 and 21.11 $^{\circ}C$ were taken in correlation to clothing insulation values of 0.0, 0.6 and 1.0 clo respectively. The results are shown in Tables 6 through 9, with the corresponding linear radiation coefficients, based on the average temperatures, given as 5.40, 5.16 and 5.05 $Kcal/m^2/hr/^{\circ}C$ respectively.

The application of search techniques yielded practically the same optimum values for all three attempts in the minimization of the total energy load for System I. In Part 3, tables 10, 11 and 12 present the optimum results for each technique. In each case the clothing insulation value was taken as 0.6 clo. In addition to the physical constraint on velocity, Equation (10), the relative humidity was constrained between 0 and 100% for all three techniques.

The Hooke and Jeeves Search yielded in the first case at an outside condition of 50^{o}C and 100% RH and at sedentary activity (52 Kcal/m^{2} hr) the optimum values of t_a = 23.86^{o}C, P_a = 22.26 mm Hg (RH = 99.99%) and v = 0.10 m/sec. Similarly the Simplex Pattern Search Technique and Another Programmed Simplex Pattern Search [9] using the same Technique gave for the first case the optimum values of t_a = 23.84^{o}C, P_a = 22.23 mm Hg (RH = 100.00%), v = 0.10 m/sec and t_a = 23.84 ^{o}C, P_a = 22.24 mm Hg (RH = 99.99%), v = 0.10 m/sec respectively.

The final solution in each case and the total number of evaluations required to converge to the minimum point depended upon the selection of the initial starting point values and the accuracy desired in final total energy value. For the limited number of cases run on the computer, there was no clear distinction in the superiority of technique or program involved. Each search, for the given outside conditions, resulted in a combination of the three variables such that the optimum velocity was at 0.1 m/sec (lower bound) and the optimum water vapor partial pressure was at the upper limit of the constraint on relative humidity. For all the cases investigated the optimal value of temperature was a function of the activity levels (at 0.6 clo) for the given outside conditions.

From the feasible regions in Part 1 it can be easily seen that for the given conditions the optimum point occurs, and hence the optimum value of dry bulb temperature specified, at the intersection of the 0.1 m/sec (lower bound) velocity line and the upper boundary constraint line on relative humidity (100%) for the particular activity level (@ 0.6 clo). Hence, if the lower forced convection restriction on the velocity were lowered, the optimum t_a and P_a values would decrease in the same fashion.

Other cases were run which included lowering the upper boundary on relative humidity (70%) and also the stipulation of a maximum value on partial pressure (P_a = 14 mm Hg), to correspond to ASHRAE Standard 55-66R, in combination with the lowered relative humidity upper constraint (70% RH). The latter cases were of particular interest in the specification of the upper vapor pressure limit at P_a = 14 mm Hg in combination with the 70% RH boundary. The results using the Hooke and Jeeves Search Technique at a sedentary activity level (50 Kcal/m^2/hr) were t_a = 25.13 ^oC, v = 0.10 m/sec and P_a = 14.00 mm Hg (RH = 58.30%). The optimal values at a low activity level (80 Kcal/m^2/hr) were t_a = 21.00 ^oC, v = 0.10 m/sec and P_a = 13.06 mm Hg (RH = 69.83%) and at a medium activity level (100 Kcal/m^2/hr) were t_a = 18.25 ^oC, v = 0.10 m/sec and P_a = 11.00 mm Hg (RH = 69.45%).

As before, the results show that, for the given outside conditions, to achieve the minimum total energy load the optimum combination of variables occur at the lower bound of velocity, the upper bound on vapor pressure (or relative humidity) and a dry bulb temperature at the intersection of these velocity and vapor pressure (or relative humidity) lines.

The analytical procedure using Lagrange Multipliers and Kuhn-Tucker Conditions for optimization of System I, Part 4, produced the same general results as those indicated in optimization by search techniques. Of course only one mathematical run through of the calculations are necessary when using the analytical analysis presented in Part 4.

Comparison of the optimization results obtained from the Hooke and Jeeves Pattern Search Technique to that of the Method of Lagrange Multipliers and Kuhn-Tucker Conditions for System I at outside conditions of 40°C and 100% RH, at sedentary activity level (50 kcal/m^2/hr) and for 0.6 clo insulation value yielded the following:

	t_a	v	P_a	S_1	S_2	S_3	S
(Search)	25.13	0.10	14.00	68.85	385.50	51.24	505.59
(Analytical)	25.14	0.10	14	68.40	382.52	50.32	501.24

This of course shows almost identical results, with optimization by the analytical procedure being exact. The advantage in addition to being an exact calculation is a much more concise computer program and the cheaper cost involved in running an analysis. Also the analytic procedure does not run the risk that a search entails, that is, the possibility of converging to an erroneous value when and if the accuracy deviation between succeeding improvement trials is met. Therefore the important aspect of reliability is available with the Lagrange Multiplier and Kuhn-Tucker Conditions type of analysis.

Table 13 gives the optimal results for application of the Lagrange Multiplier and Kuhn-Tucker Conditions to the model of System I. As proposed in the Part 4 analysis both boundary values on velocity were calculated, which quite evidently yielded the minimum energy value at the lower boundary condition for each case. Several cases were run in which the optimal results were presented at different upper boundary constraints on vapor pressure (20, 30 and 14 mm Hg).

The further application of this technique to the System II model (Figure 28) gives rise to the optimal combination of environmental variables, namely t_a, P_a and v, that are in extremely close agreement with the results obtained from the System I model. This is the desired solution for the two system models since this optimal combination of environmental variables must produce thermal neutrality in accordance with the thermal comfort constraint.

Table 14 presents the optimum values for the environmental variables studied, the energy components, and the minimum total energy load required for several cases of outside conditions, clothing insulation and activity levels. Again the minimum energy value occurred every time at the lower physical limit placed on velocity (v = 0.10 m/sec). The upper limit for vapor pressure at 14 mm Hg was particularly investigated due to the range proposed in ASHRAE Standard 55-66R [4].

CONCLUSIONS AND RECOMMENDATIONS

The investigation by this thesis produces an analytical technique which can be used for given parameters of any environmental control system model that will give the optimum combination of environmental variables that will minimize the energy consumption required for control. This optimum combination is restricted by Fanger's thermal comfort equation.

The approach in this study was on a load basis which affected the disturbance of the desired values of the environmental variables from their equilibrium position that the control system maintains for thermal comfort. The energy savings was reflected in the minimization of this total energy load required to maintain thermal comfort. Efficiencies for heating or cooling loads, for humidity maintenance, and for equipment components in the control system were not included. Minimization of the costs involved for the physical system and system components that were required to attain the comfort conditions, the optimization of the physical parameters of the building or the maximization of the stability of the system was not undertaken. All these investigations can not be successfully attempted without an understanding of the energy requirements of the system as was provided in this brief study.

The design of System I and System II in this work was not detailed since the main concern was the application of optimization techniques in the analysis of a system model. By these techniques the energy load requirements are minimized and the constraint of thermal comfort is maintained.

Since the thermal comfort equation of Fanger [18] was used to dictate the feasible range of values for the variables it was studied for several cases of parameters. Of particular value are Figures 14 through 25 which illustrate the narrow region available for each particular case. As evidenced in the study the intersection of the lower limit on velocity and the upper limit on vapor pressure (or relative humidity) yields the desired dry bulb temperature for the assumed outside conditions.

The relatively small velocity margin (0.22 miles per hour to 5.18 miles per hour) that was feasible for forced convection arises from the fact that above the upper limit too much draft creates uncomfortable conditions and below the lower limit free convection is said to occur. Fanger's equation can be used for velocities less than 0.1 meter per second where natural (free) convection, Equation (5), must be assumed, but then the velocity term does not appear in the comfort equation. The inclusion of the free convection part in the velocity feasibility range is recommended for further study.

In looking at Fanger's predicted mean vote equation [19], Appendix T, the feasible region could be enlarged by allowing an acceptable deviation (in terms of say $\pm$ 0.5 vote) from the comfort condition, PMV = 0, to occur so that less demands on the control equipment would be more feasible (and economical). It is therefore recommended that further investigation is needed to give feasible ranges for the independent environmental variables based upon PMV values. In this way control equipment can be designed that requires less demand of energy for this larger allowable set of conditions.

Since the comfort equation, Equation (9), is a heat balance equation which is based upon heat transfer data and Equations (1) and (2) it can be imporved in several ways. The heat transfer data can be investigated and improved and the ranges of the parameters used in Equation (9) can be improved.

The optimization results from search techniques are dependent upon initial value selection and accuracy of deviation of the optimal objective function value, as well as the skill of programming manipulation. The optimal value is reached after several evaluations of the objective function and expense is involved to yield a desirable solution. This is eliminated by the use of analytical techniques which give rise to an exact solution and hence a straight-forward optimum value is obtained that is reliable.

In summary a theoretical technique, Lagrange Multiplier and Kuhn-Tucker Conditions, was found that for a given set of parameters, such as clothing insulation, activity level, outside conditions, wall insulation, and others, an optimum combination of room air temperature, t_a, room vapor pressure, P_a, and room air velocity, v, are determined that satisfy the comfort constraint and minimize the consumption of energy. This procedure can be applied to different cases within a specified system and to many system models.

It is recommended that dynamic modeling be used in system analysis and in representation of the comfort constraint for further studies. Also special consideration would have to be incorporated in the above study for significant deviation of mean radiant temperature from air temperature and for radiant heating. Also further optimization studies can be investigated from the above basis in relation to physical building parameters, adjustment of the tolerable working environment, automatic controls and equipment modeling, and avenues of energy and their cost.

Lastly, further consideration should include non-thermal factors in addition to the thermal conditions in any environmental control system model.

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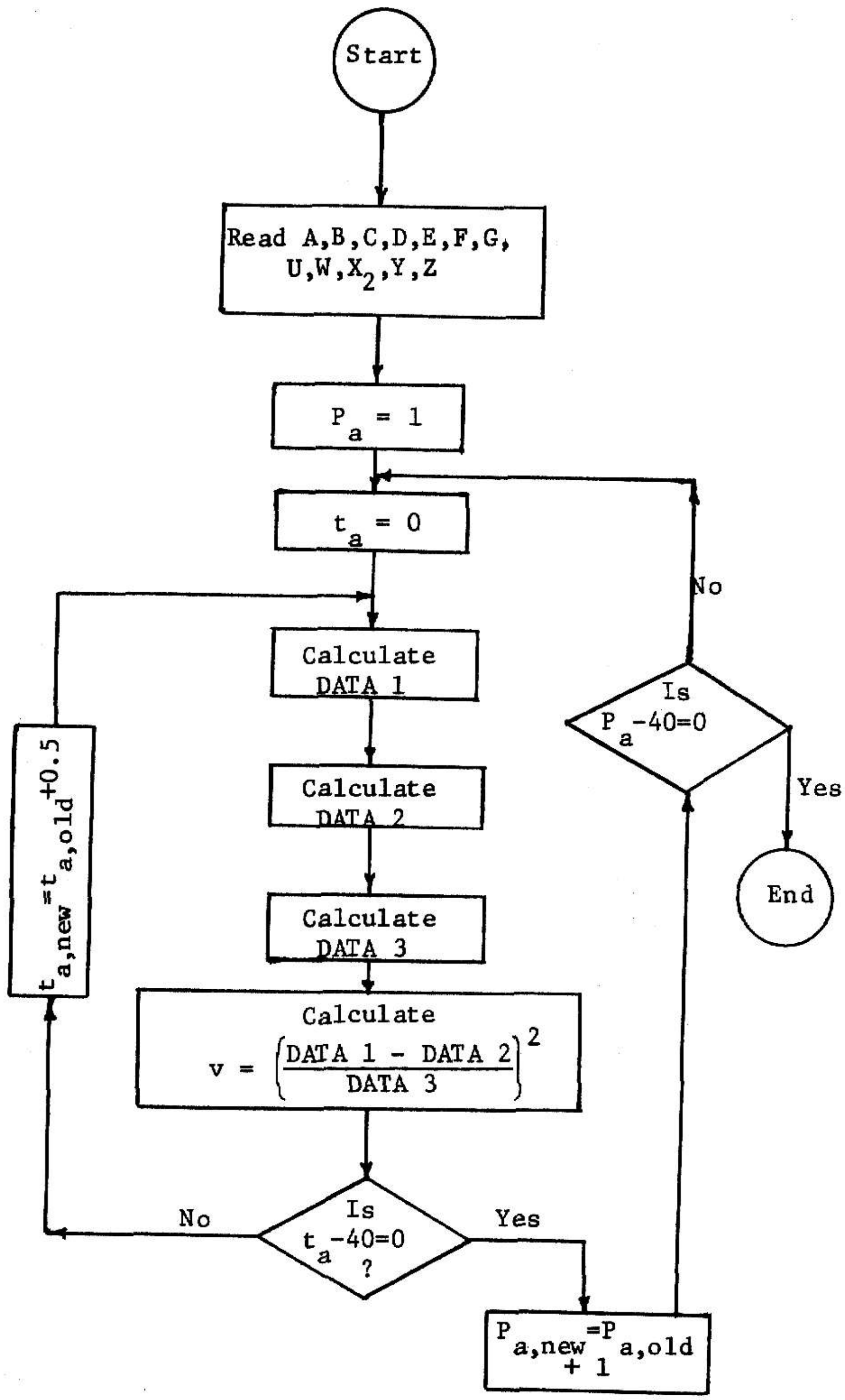
Appendix A

Computer Flow Diagram for Solution of Comfort Equation.



Appendix B

Computer Program Used in Study of Comfort Equation

```
$JOB           LRW,RUN=CHECK,TIME=25,PAGES=150,LINES=60,KP=29
C     SIMULATION OF EQUATION(13).
C
C  APPENDIX B. COMPUTER PROGRAM USED IN SOLUTION OF COMFORT EQUATION.
C
      DIMENSION ACT(8)
      DATA ACT/52.,83.,111.,132.,40.,66.,87.,110./
  100 FORMAT('1')
      WRITE(6,100)
      DO 93 K=1,8
      FEFF=0.75
      IF(ACT(K).LE.52.) FEFF=0.65
      CLO=0.6
      FCL=1.1
      IF(ACT(K).EQ.52.) PRINT 5
    5 FORMAT('0',20X'MALE')
      IF(ACT(K).EQ.40.) GO TO 6
      GO TO 8
    6 PRINT 7
    7 FORMAT('0',20X'FEMALE')
    8 CONTINUE
      A=0.0
      B=0.0
      C=0.0
      F=0.0
      G=0.0
      W=0.0
      Y=0.0
      Z=0.0
      A=.45255*ACT(K)
      B=-(.031459*CLO+.032)*ACT(K)-1.071*CLO
      C=.35+.0023*ACT(K)
      D=.0014*ACT(K)
      E=4.8E-8*FCL*FEFF
      F=.063*CLO+.000414*CLO*ACT(K)
      G=0.000252*CLO*ACT(K)
      W=303.7+B
      X=10.4*FCL
      Y=F
      Z=1.0+0.000252*CLO*ACT(K)
      U=35.7+B
      P=1.
   90 T=10.0
   20 DATA1=A+C*P+D*T+5.95
      DATA2=E*((-F*P-G*T+W)**4.-(T+273.)**4.)
      DATA3=X*(-Y*P-Z*T+U)
      V=(DATA1-DATA2)*(DATA1-DATA2)/(DATA3*DATA3)
   60 FORMAT(3F10.4)
      WRITE(6,60)P,T,V
      IF(T-40.) 10,50,50
   10 T=T+.5
      GO TO 20
   50 IF(P-13.)80,81,81
   80 P=P+4.
      GO TO 90
   81 IF(P-15.)62,63,63
   62 P=P+2.
      GO TO 90
   63 IF(P-30.)64,65,65
   64 P=P+15.
```

```
      GO TO 90
   65 IF(P-40.)66,70,70
   66 P=P+10.
      GO TO 90
   70 CONTINUE
   33 CONTINUE
      STOP
      END

  $ENTRY
```

Appendix C

Newton-Raphson Method

The Newton-Raphson method was used to solve the fourth order comfort equation in the feasibility part of the study. It is an extention of the Newton method for cases of finding solutions to sets of nonlinear simultaneous equations [17].

To use this method start by the assumption of an assumed solution, say, p_0, and attempt to determine an improved approximation based on a knowledge of the gradient of $f(\rho)$. The essence of the method is best understood by referring to the sketch in Figure 29 below [7].

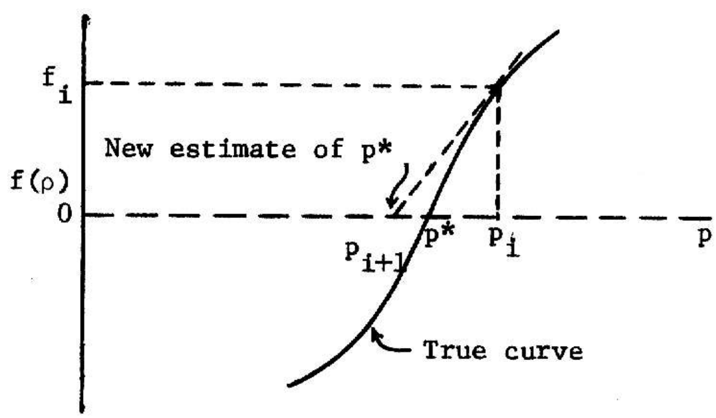


Figure 29. Newton-Raphson Method

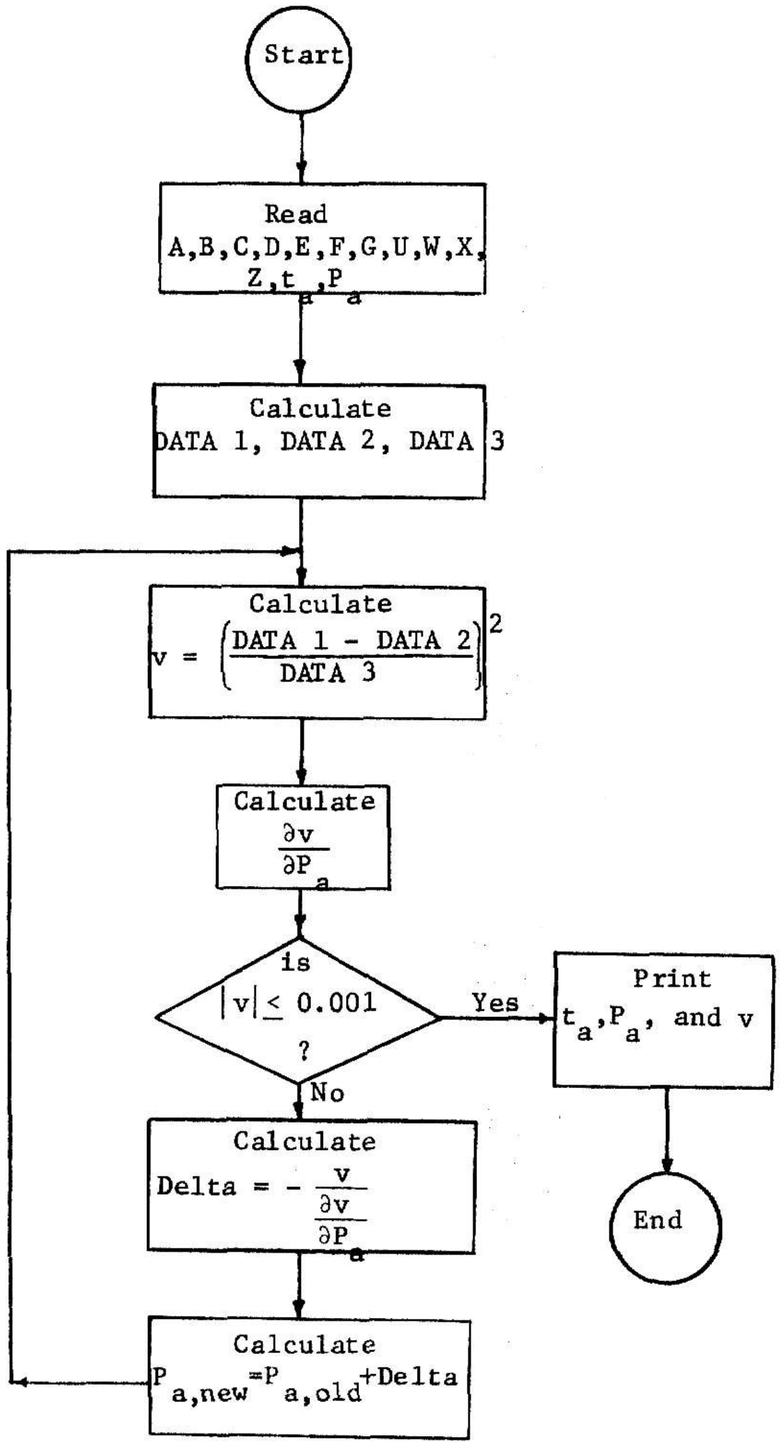
The most recent estimate of a root is p_i, at which point the function and its derivative have values of f_i and f_i', respectively. The problem is to determine a better estimate of the root based on knowledge of f_i and f_i'. Since f_i' is the tangent of the angle that the dashed line constructed tangent to the curve at the point p_i makes with the axis, we see that, if the new estimate is to be the intersection of the tangent line with the axis, then, by construction, $f_i' = f_i/(p_i - p_{i+1})$ or solving for the new approximation, $p_{i+1} = p_i - (f_i/f_i')$. This equation is to be used repeatedly until a sufficiently accurate estimate of the root is obtained.

Using this procedure solutions of s algebraic equations in s unknowns can be accomplished. If a system of s equations, as $g_i(X_1, \ldots, X_s) = 0$, for $i = 1, \ldots, s$, is the set to be solved for $X_1, \ldots, X_s$, then by Taylor series expansion of these equations about the point $(X_1, X_2, \ldots, X_s)_0$ and neglecting the second and higher order therms in the Taylor series expansion, we obtain linear algebraic equations that can be solved for $X_1, X_2, \ldots, X_s$.

Although this method does not always converge to a solution of the set of system equations, it is widely used and is one of the better methods that is available. Since multiple solutions to the system of equations may exit, there is also no certainty that the solution of interest will be found [17]. Therefore, knowledge of the system's feasible bounds and common logic are sometimes necessary to find the desired solution when several solutions are found.

Appendix D

Computer Flow Diagram for the Newton-Raphson Method.



Appendix E

Computer Program of Newton-Raphson Method to Solve Equation (15)

```
$JOB            LRW,RUN=CHECK,TIME=25,PAGES=150,LINES=60,KP=29
C
C  APPENDIX E. COMPUTER PROGRAM OF NEWTON-RAPHSON METHOD TO SOLVE EQUATION (15).
C
C      NEWTON-RAPHSON METHOD.
C      LOWER LIMIT FOR VELOCITY EQUAL TO ZERO.
  100 FORMAT('1')
      WRITE(6,100)
    1 FORMAT(I3,3F15.4)
   40 FORMAT(2F5.1)
      ACT=52.
      CLO=0.6
      FCL=1.1
      FEFF=.65
      A=.45255*ACT
      C=.35+.0023*ACT
      D=.0014*ACT
      E=4.8E-8*FCL*FEFF
      F=0.063*CLO+.000414*CLO*ACT
      G=0.000252*CLO*ACT
      B=-(.081459*CLO+.032)*ACT-1.071*CLO
      W=309.7+B
      Z=1.0+0.000252*CLO*ACT
      U=35.7+B
      Y=F
      X=10.4*FCL
      K=0
   70 READ(5,40) T,P
      K=K+1
   10 DATA1=A+C*P+D*T+5.95
      DATA2=E*((-F*P-G*T+W)**4-(T+273.)**4)
      DATA3=X*(-Y*P-Z*T+U)
      Q=DATA3**2
      R=DATA1-DATA2
      Q1=R**2
      BB=(-F*P-G*T+W)**3
      Q2=2.*R*((C+4.*E*F*BB))
      DELTA=-(Q1/Q2)
      S1=ABS(Q1)
      IF(S1-0.01) 20,20,30
   30 P=P+DELTA
      GO TO 10
   20 WRITE(6,1) K,P,T,Q1
      IF(K-3) 70,50,50
   50 STOP
      END

$ENTRY
```

Appendix F

Computer Program of Newton-Raphson Method for Feasible Range Limits

```
$JOB            LRW,RUN=CHECK,TIME=25,PAGES=150,LINES=60,KP=29
C
C  APPENDIX F. COMPUTER PROGRAM OF NEWTON-RAPHSON METHOD FOR FEASIBLE RANGE
C              LIMITS.
C
  100 FORMAT('1')
      WRITE(6,100)
C     NEWTON-RAPHSON METHOD.
C     UPPER BOUND FOR PHYSICAL CONSTRAINT ON VELOCITY.
    1 FORMAT(I3,3F15.4)
   40 FORMAT(2F5.1)
      ACT=52.
      FCL=1.1
      FEFF=.65
      CLO=0.6
      A=.45255*ACT
      C=.35+.0023*ACT
      D=.0014*ACT
      E=4.8E-8*FCL*FEFF
      B=-(.081459*CLO+.032)*ACT-1.071*CLO
      F=0.063*CLO+.000414*CLO*ACT
      G=0.000252*CLO*ACT
      W=308.7+B
      U=35.7+B
      Z=1.0+0.000252*CLO*ACT
      Y=F
      X=10.4*FCL
      K=0
   70 READ(5,40) T,P
      K=K+1
   10 DATA1=A+C*P+D*T+5.95
      DATA2=E*((-F*P-G*T+W)**4-(T+273.)**4)
      DATA3=X*(-Y*P-Z*T+U)
      Q=DATA3**2
      DQ=2.6*Q
      R=DATA1-DATA2
      Q1=R*R-DQ
      BB=(-F*P-G*T+W)**3
      Q2=2.*R*(C+4.*E*F*BB)+5.2*X*Y*DATA3
      DELTA=-(Q1/Q2)
      S1=ABS(Q1)
      IF(S1-0.01) 20,20,30
   30 P=P+DELTA
      GO TO 10
   20 WRITE(6,1) K,P,T,Q1
      IF(K-3) 70,50,50
   50 STOP
      END

$ENTRY
```

Appendix G

The Sufficient Condition of Optimality

In the part where optimization by Lagrange Multipliers is attempted, the necessary condition of optimality is that at an extreme point where the derivative of the function is zero. The sufficient condition for the function, say, $f(x_1, x_2)$, for two variables x_1 and x_2 and for one equality constraint for explanation purposes, is given below [45].

Expansion of Taylor's series for the function gives use to the following

$$f(x_1 + \Delta x_1, x_2 + \Delta x_2) = f(x_1, x_2) + df + \frac{1}{2!} d^2 f + \frac{1}{3!} d^3 f + \ldots \quad (G\text{-}1)$$

Since x_1 and x_2 must satisfy the relation

$$g(x_1, x_2) = 0$$

the differential dg must be zero, that is,

$$\frac{\partial g}{\partial x_1} dx_1 + \frac{\partial g}{\partial x_2} dx_2 = 0 \quad (G\text{-}2)$$

or

$$dx_1 = - \left(\frac{\partial g}{\partial x_2} / \frac{\partial g}{\partial x_1} \right) dx_2 \quad (G\text{-}3)$$

if $\frac{\partial g}{\partial x_1} \neq 0$.

Substituting the value of dx_1 given by Equation (G-3) into the differential gives rise to

$$df = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 = 0 \tag{G-4}$$

or

$$df = \left\{ \left(- \frac{\partial f}{\partial x_1} \frac{\partial g}{\partial x_2} \Big/ \frac{\partial g}{\partial x_1} \right) + \frac{\partial f}{\partial x_2} \right\} dx_2 = 0 \tag{G-5}$$

Since dx_2 can not be zero, a necessary for df to be zero in equation (G-5) is

$$\left(- \frac{\partial f}{\partial x_1} \frac{\partial g}{\partial x_2} \Big/ \frac{\partial g}{\partial x_1} \right) + \frac{\partial f}{\partial x_2} = 0$$

or

$$\frac{\partial f}{\partial x_1} \Big/ \frac{\partial g}{\partial x_1} = \frac{\partial f}{\partial x_2} \Big/ \frac{\partial g}{\partial x_2} \tag{G-6}$$

Let this common ratio be denoted by λ,

$$\frac{\partial f}{\partial x_1} \Big/ \frac{\partial g}{\partial x_1} = \frac{\partial f}{\partial x_2} \Big/ \frac{\partial g}{\partial x_2} = \lambda \tag{G-7}$$

we have

$$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial g}{\partial x_1} = 0 \tag{G-8}$$

and

$$\frac{\partial f}{\partial x_2} - \lambda \frac{\partial g}{\partial x_2} = 0 \tag{G-9}$$

The sufficient conditions for a stationary point to be a maximum, the term d^2f in Taylor's series must be negative. Assuming that x_1 and x_2 are not independent, the second differential of the function, f, is

$$d^2 f = d(f_{x_1} dx_1 + f_{x_2} dx_2) \tag{G-10}$$

where

$$f_{x_1} = \frac{\partial f}{\partial x_1} \tag{G-11}$$

and

$$f_{x_2} = \frac{\partial f}{\partial x_2} \tag{G-12}$$

therefore, we have

$$d^2 f = d(f_{x_1}) dx_1 + f_{x_1} d(dx_1) + d(f_{x_2}) dx_2 + f_{x_2} d(dx_2) \tag{G-13}$$

$$= (f_{x_1 x_1} dx_1 + f_{y_1 y_2} dx_2) dx_1 + f_{x_1} d^2 x_1 +$$

$$+ (f_{x_1 x_2} dx_1 + f_{x_2 x_2} dx_2) dx_2 + f_{x_2} d^2 x_2 \tag{G-14}$$

$$= f_{x_1 x_1} (dx_1)^2 + 2 f_{x_1 x_2} dx_1 dx_2 +$$

$$f_{x_2 x_2} (dx_2)^2 + f_{x_1} d^2 x_1 + f_{x_2} d^2 x_2 \tag{G-15}$$

where

$$f_{x_i x_j} = \frac{\partial^2 f}{\partial x_i \partial x_j} , \qquad i, j = 1, 2$$

Similiarly from Equation (G-2), we obtain

$$d^2g = g_{x_1x_1}(dx_1)^2 + 2g_{x_1x_2}dx_1dx_2 + g_{x_2x_2}(dx_2)^2 +$$

$$g_{x_1}d^2x_1 + g_{x_2}d^2x_2 = 0 \tag{G-16}$$

Now solving Equation (G-16) for d^2x_1, substituting the value of d^2x_1 so obtained in Equation (G-15), and collecting the terms gives

$$d^2f = \left(f_{x_1x_1} - \frac{f_{x_1}}{g_{x_1}} g_{x_1x_1}\right)(dx_1)^2 + 2\left(f_{x_1x_2} - \frac{f_{x_1}}{g_{x_1}} g_{x_1x_2}\right)dx_1dx_2 +$$

$$\left(f_{x_2x_2} - \frac{f_{x_1}}{g_{x_1}} g_{x_2x_2}\right)(dx_2)^2 + \left(f_{x_2} - \frac{f_{x_1}}{g_{x_1}} g_{x_2}\right)d^2x_2 \tag{G-17}$$

In the above equation, the last term is zero because the factor in the brackets is df (see Equation (G-5)). Substituting the value of dx_1 from Equation (G-3) and using λ as defined previously, we have

$$\lambda = \frac{f_{x_1}}{g_{x_1}} = \frac{f_{x_2}}{g_{x_2}} \tag{G-18}$$

and

$$d^2f = -\frac{dx_2^2}{dx_1^2}\Delta_3 \tag{G-19}$$

where

$$\Delta_3 = -\left\{g_{x_2}^2 \; [f_{x_1x_1} - \lambda g_{x_1x_1}] - 2g_{x_1}g_{x_2} \; [f_{x_1x_2} - \lambda g_{x_1x_2}]\right.$$

$$\left. + \; g_{x_1}^2 \; [f_{x_2x_2} - \lambda g_{x_2x_2}]\right\} \tag{G-20}$$

Thus the function $f(x_1,x_2)$ will have a maximum (or minimum) subject to the constraint $g(x_1,x_2) = 0$
if Δ_3 is positive (or negative).

If at a point the first and second derivatives vanish, then we must examine higher order terms) in order to develop sufficient conditions for optimality. In other words if Δ_3 is zero, then higher differentials of the function $f(x_1,x_2)$ should be examined to develop sufficient conditions.

Appendix H

Relationship of Saturated Vapor Pressure to Temperature

To formulate an empirical equation which could be used to obtain the saturated vapor pressure at any given temperature was desirable in study of System I and System II. Thus when a temperature is specified this equation is used to calculate the saturated vapor pressure.

A few values of saturated vapor pressure for the corresponding temperatures are tabulated below [78].

Temperature t, °C	Saturated Vapor pressure P_s, mm Hg	$\text{Log}_{10}\ P_s$	1/(273 + t) RTK
15	12.788	1.1060	0.003470
20	17.535	1.2440	0.003415
25	23.756	1.3755	0.003355
30	31.824	1.5030	0.003330
35	42.175	1.6250	0.003250

Since $\log_{10} P_s$ is linearly related to the temperature, their relationship can be expressed as follows:

$$\log P_s = a + b\ (\text{RTK}) \qquad \text{(H-1)}$$

Equation (H-1) describes the linear relationship between RTK and $\log_{10} P_s$. To solve for a and b, any two points on the straight line could be used. In this instance the two points used were: RTK = 0.00347, $\log_{10} P_s = 1.1060$ ($t = 15°C$) and RTK = 0.00333, $\log_{10} P_s = 31.824$ ($t = 30°C$).

The above values were substituted into Equation (H-1) to obtain the following two equations:

$$\log_{10} 12.788 = a + b\ (0.00347) \tag{H-2}$$

and

$$\log_{10} 31.824 = a + b\ (0.00333) \tag{H-3}$$

Substracting Equation (H-3) from Equation (H-2) gives

$$\log_{10} 12.788 - \log_{10} 31.828 = b\ (0.00347 - 0.00333)$$

or

$$\log_{10} \frac{12.788}{31.828} = b\ (0.00014) \tag{H-4}$$

Converting $\log_{10}$ to $\log_e$, Equation (H-4) becomes

$$\ln \frac{12.788}{31.828}\ b(0.0014)\ \ln 10$$

or

$$b = -\ 2303.81 \tag{H-5}$$

Substitution of b into Equation (H-2) yields

$$a = \log_{10} 12.788 - (-\ 2303.81)(0.00347)$$

or

$$a = 9.11 \tag{H-6}$$

Substituting the values of a and b into Equation (H-1), $\log_{10} P_s$ was obtained as

$$\log_{10} P_s = 9.11 - 2303.81\ (RTK) \tag{H-7}$$

Converting Equation (H-7) to $\log_e$ we get

$$\ln P_s = 20.98 = 5307.02\ (RTK) \tag{H-8}$$

which gives saturated vapor pressure as a function of temperature. It was found that in the temperature range of $8^{o}C < t < 36^{o}C$, Equation (H-8) gives the saturated vapor pressure at any temperature.

To find the relative humidity inside the room, in order to check the humidity constraint in Part 3 of this study, Equation (H-8) was used to find P_s in the room. Hence by the definition of relative humidity (RH) as the partial pressure of water vapor (P_a) at a specific temperature divided by the saturated vapor pressure (P_s) at that same temperature yields

$$RH = \frac{P_a}{P_s} \tag{H-9}$$

This allows the calculation of RH inside the room when the vapor pressure and temperature are known.

Appendix I

Computer Program for Simulation of Linear Radiation Coefficient

```
$JOB            LRW,RUN=CHECK,TIME=25,PAGES=150,LINES=60,KP=29
C
C  APPENDIX I. COMPUTER PROGRAM FOR SIMULATION OF LINEAR RADIATION COEFFICIENT.
C
      DIMENSION TA(13),PA(6),V(1),ACT(4),CLO(3),VA(1)
      DATA TA/15.56,18.89,20.,21.11,22.22,23.33,24.44,25.56,26.67,27.78
     1,28.89,30.,32.22/,PA/1.0,5.0,10.0,14.0,20.0,30.0/,ACT/1.0,1.6,2.0,
     1    3.0/,CLO/0.0,0.6,1.0/
      TAVE=25.56
      ADU=1.8
      AM=50.0
      HR=0.0
      AA=0.0
      HA=0.0
      C1A=0.0
      C3A=0.0
      CA=0.0
      VRA=0.0
      VX=0.0
      RADAV=0.0
      RXD=0.0
      SRTVA=0.0
      XM=0.0
      HCF=0.0
      VR=0.0
      TCLN=0.0
      RL=0.0
      TCL=0.0
      ERROR=0.0
      TCLI=0.0
      I=0
      J=1
      PMV=0.0
      CONV=0.0
      SWR=0.0
      Y=0.0
      DIFF=0.0
      XLAT=0.0
      DRY=0.0
      RAD=0.0
      BB=0.0
      Z=0.0
      ZP=0.0
      TSK=0.0
      TC=0.0
      A=0.0
      B=0.0
      D=0.0
      E=0.0
      F=0.0
      C=0.0
      C1=0.0
      C2=0.0
      C3=0.0
      XDM=0.0
      BC=0.0
      FC=0.0
      GC=0.0
      WC=0.0
      ZC=0.0
```

```
      UC=0.0
      DATA1=0.0
      DATA2=0.0
      DATA3=0.
      V13=0.
      R13=0.
      APCT=0.
      PCT=0.
  100 FORMAT('1')
      WRITE(6,100)
  101 FORMAT(3X'TA'8X'PA'8X'V'8X'VF'8X'ACT'7X'CLO'8X'RL'7X'RAD'8X'HR'7X'
     1PAV'7X'TCL'2X'TC'6X'PAGE'I3)
      WRITE(6,101)J
  102 FORMAT(17X'V13'6X'VAVE'6X'VRA'5X'LIN. VR'5X'SRTVF'3X'RAD AVE'6X'R1
     13'5X'AVE HR'6X'HRF'5X'AV PCT'4X'LIN PCT')
      WRITE(6,102)
      DO 24 K1=1,4
      DO 25 K2=1,3
      L=1
      DO 21 M=1,6
      DO 20 K=1,13
      Q=1.0
      Q1=1.1
      Q2=1.15
      XDM=AM*ACT(K1)
      XM=ACT(K1)*ADU*AM
      IF(CLO(K2).EQ.0.0) FCL=Q
      IF(CLO(K2).EQ.0.6) FCL=Q1
      IF(CLO(K2).EQ.1.0) FCL=Q2
      IF(CLO(K2).EQ.0.0) TAVE=27.78
      IF(CLO(K2).EQ.0.6) TAVE=23.33
      IF(CLO(K2).EQ.1.0) TAVE=21.11
      FEFF=0.71
      AEFF=ADU*FEFF*FCL
      TSK=35.7-0.032*XDM
      SIG=4.96E-8
      HR=4.*SIG*(TSK+273.0)**3
      HRA=4.*SIG*(TAVE+273.0)**3
      HA=HRA
      TC=35.7+XDM*(-0.081459*CLO(K2)-0.032)-(0.063*CLO(K2)+0.000414*CLO(
     1K2)*XDM)*PA(M)-0.000252*CLO(K2)*TA(K)*XDM-1.071*CLO(K2)
      A=0.35*ADU+0.0023*XM     +HR*AEFF*(0.063*CLO(K2)+0.000414*CLO(K2)*X
     1DM)
      B=0.0014*XM     +HR*AEFF*(1.0+0.000252*CLO(K2)*XDM)
      D=371.23*FCL*ADU+10.4*FCL*XM     *(-0.081459*CLO(K2)-0.032)-10.4*1.
     1071*FCL*ADU*CLO(K2)
      E=10.4*FCL*ADU*(0.063*CLO(K2)+0.000414*CLO(K2)*XDM)
      F=10.4*FCL*ADU*(1.0+0.000252*CLO(K2)*XDM)
      C1=0.45255+HR*FCL*FEFF*(0.081459*CLO(K2))+HR*FCL*FEFF*0.032
      C2=    5.95-35.7*HR*FCL*FEFF
      C3=1.071*CLO(K2)*HR*AEFF
      C=C1*XM     +C2*ADU+C3
      VR=-1.0*(C+A*PA(M)+B*TA(K))/(E*PA(M)+F*TA(K)-D)
      V(L)=VR*VR
      RL=HR*AEFF*(TC-TA(K))
      AA=0.35*ADU+0.0023*XM+HA*AEFF*(0.063*CLO(K2)+0.000414*CLO(K2)*XDM)
      BA=0.0014*XM+HA*AEFF*(1.0+0.000252*CLO(K2)*XDM)
      C1A=0.45255+HA*FCL*FEFF*(0.081459*CLO(K2))+HA*FCL*FEFF*0.032
      C2A=    5.95-35.7*HA*FCL*FEFF
      C3A=1.071*CLO(K2)*HA*AEFF
```

```
      CA=C1A*XM+C2A*ADU+C3A
      VRA=-1.0*(CA+AA*PA(M)+BA*TA(K))/(E*PA(M)+F*TA(K)-D)
      VX=VRA*VRA
      RADAV=HRA*AEFF*(TC-TA(K))
      VA(L)=V(L)
      IF(VA(L).GT.1.4) GO TO 60
      II=0
      MM=0
   41 HCF=10.4*SQRT(VA(L))
      IF(MM.GE.1) GO TO 200
      TCLI=TC
  200 TCLN=35.7-0.032*XDM-0.18*CLO(K2)*(3.4E-8*FCL*((TCLI+273.0)**4-(TA(
     1K)+273.0)**4)+FCL*HCF*(TCLI-TA(K)))
      IF(CLO(K2).EQ.0.0) GO TO 50
      ERROR=TCLN-TCLI
      IF(ABS(ERROR).LT.0.1) GO TO 300
      II=II+1
      IF(II.EQ.20) GO TO 90
      TCLI=TCLI+0.5*ERROR
      GO TO 200
   90 PRINT 91
   91 FORMAT('0'20X'ITERATION CARRIED 20 TIMES AND TERMINATED')
  300 CONTINUE
   50 CONTINUE
      TCL=TCLN
      SWR=0.42*(XDM-50.0)
      Y=43.2-0.061*XDM-PA(M)
      DIFF=0.35*Y
      XLAT=0.0023*XDM*(44.0-PA(M))
      DRY=0.0014*XDM*(34.0-TA(K))
      RAD=3.4E-8*FCL*((TCL+273.0)**4-(TA(K)+273.0)**4)
      MM=MM+1
      IF(MM.EQ.3) GO TO 40
   45 FORMAT('0'20X'**NATURAL CONVECTION**')
   46 FORMAT('0'20X'**ZERO OR NEGATIVE VELOCITY REQUIRED**')
   64 FORMAT('0'20X'** VELOCITY EXCEEDS 200 FPM**')
   68 FORMAT('0'20X'**EXCEEDS VELOCITY LIMITS**')
      SRTVA=((XDM-DIFF-XLAT-SWR-DRY-RAD)/(FCL*10.4*(TCL-TA(K))))
      VA(L)=(SRTVA)**2
      IF(SRTVA.LE.0.00) GO TO 47
      IF(VA(L).GT.2.6) GO TO 66
      IF(VA(L).GT.1.3) GO TO 66
      GO TO 41
   61 FORMAT ('0'20X'**VELOCITY EXCEEDS 270 FPM***')
   60 PRINT 61
      GO TO 99
   66 PRINT 68
      GO TO 99
   40 CONTINUE
      IF(SRTVA.LE.0.00) GO TO 47
      IF(VA(L).LT.0.1) GO TO 43
      IF(VA(L).GT.1.02) GO TO 65
      GO TO 70
   47 PRINT 46
      GO TO 99
   43 PRINT 45
      GO TO 44
   65 PRINT 64
      GO TO 44
   70 CONTINUE
```

```
   44 CONTINUE
      CONV=FCL*HCF*(TCL-TA(K))
      Z=EXP(-0.042*XDM)
      ZP=0.352*Z+0.032
      BB=XDM-DIFF-XLAT-SWR-CONV-DRY-RAD
      PMV=ZP*BB
      RXD=RAD*ADU
      AC=.45255*XDM
      BC=-(.081459*CLO(K2)+.032)*XDM-1.071*CLO(K2)
      CC=.35+.0023*XDM
      DC=.0014*XDM
      EC=4.8E-8*FCL*FEFF
      FC=.053*CLO(K2)+.000414*CLO(K2)*XDM
      GC=.000252*CLO(K2)*XDM
      WC=308.7+3C
      XC=10.4*FCL
      ZC=1.+.000252*CLO(K2)*XDM
      UC=35.7+BC
      DATA1=AC+CC*PA(M)+DC*TA(K)+5.95
      DATA2=FC*((-FC*PA(M)-GC*TA(K)+WC)**4-(TA(K)+273.)**4)
      DATA3=XC*(-FC*PA(M)-ZC*TA(K)+UC)
      V13=((DATA1-DATA2)/DATA3)**2
      R13=EC*((TC+273.)**4-(TA(K)+273.)**4)*ADU
      APCT=((RADAV-R13)/R13)*100.
      PCT=((RL-R13)/R13)*100.
      HRF=(4.8E-8*((TCL+273.)**4-(TA(K)+273.)**4))/(TCL-TA(K))
   11 FORMAT('0',12F10.2)
      WRITE(6,11) TA(K),PA(M),V(L),VA(L),ACT(K1),CLO(K2),RL,RXD,HR,PMV,T
     1CL,TC
   12 FORMAT(11X,11F10.2)
      WRITE(6,12) V13,VX,VRA,VR,SRTVA,RADAV,R13,HRA,HRF,APCT,PCT
      TEST=SWR+DIFF+XLAT+DRY+RAD+CONV
      WRITE(6,12) SWR,DIFF,XLAT,DRY,RAD,CONV,TEST,XM
   99 CONTINUE
      I=I+1
      IF(I.EQ.13) PRINT 100
      IF(I.EQ.13) J=J+1
      IF(I.EQ.13) WRITE(6,101)J
      IF(I.EQ.13) WRITE(6,102)
      IF(I.EQ.13) I=0
   20 CONTINUE
   21 CONTINUE
   25 CONTINUE
   24 CONTINUE
   14 FORMAT(' ',20X'THE END')
      PRINT 14
      STOP
      END

$ENTRY
```

Appendix J

Hooke and Jeeves Pattern Search

A direct search technique that is among the simplest and most efficient methods for solving the unconstrained nonlinear minimization problems. The technique consists of searching the local nature of the objective function in the space and then moving in a favorable direction for reducing the functional value [17].

The direct search method of Hooke and Jeeves is a sequential search routine for minimizing a function $f(\underline{x})$ of more than one variable, $\underline{x} = (x_1, x_2, \ldots, x_r)$. The argument $\underline{x}$ is varied until the minimum of $f(\underline{x})$ is obtained. The search routine determines the sequence of values for $\underline{x}$. The successive values of $\underline{x}$ can be intepreted as points in an r-dimensional space. The procedure consists of two types of moves: Exploratory and Pattern.

A move is defined as the procedure of going from a given point to a following point. A move is a success if the value of $f(\underline{x})$ decreases (for minimization); otherwise, it is a failure. The first type of move is an exploratory move which is designed to explore the local behavior of the objective function, $f(\underline{x})$. The success or failure of the exploratory moves is utilized by combining it into a pattern which indicates a probable direction for a successful move.

The exploratory move is performed as follows:

1. Introduce a starting point $\underline{x}$ with a prescribed step length δ_i in each of the independent variables x_i, $i = 1, 2, \ldots, r$.

2. Compute the objective function $f(\underline{x})$ where

   $\underline{x} = (x_1, x_2, \ldots, x_r)$. Set $i = 1$.

3. Compute $f_i(\underline{x})$ at the trial point

   $\underline{x} = (x_1, x_2, \ldots, x_i + \delta_i, x_{i+1}, \ldots, x_r)$.

4. Compare $f_i(\underline{x})$ with $f(\underline{x})$:

(i) If $f_i(\underline{x}) < f(\underline{x})$, set $f(\underline{x}) = f_i(\underline{x})$, $\underline{x} = (x_1, x_2, \ldots, x_i, \ldots, x_r)$ $= (x_1, x_2, \ldots, x_i + \delta_i, \ldots, x_r)$, and $i = i + 1$.

Consider this trial point as a starting point, and repeat from step 3.

(ii) If $f_i(\underline{x}) \geq f(\underline{x})$, set $\underline{x} = (x_1, x_2, \ldots, x_i - 2\delta_i, \ldots, x_r)$.

Compute $f_i(\underline{x})$, and see if $f_i(\underline{x}) < f(\underline{x})$. If this move is a success the new trial point is retained. Set $f(\underline{x}) = f_i(\underline{x})$, $\underline{x} = (x_1, x_2, \ldots, x_i, \ldots, x_r) = (x_1, x_2, \ldots, x_i - 2\delta_i, \ldots, x_r)$, and $i = i + 1$, and repeat from step 3. If again $f_i(\underline{x}) \geq f(\underline{x})$, then the move is a failure and $\underline{x}_i$ remains unchanged, that is, $\underline{x} = (x_1, x_2, \ldots, x_i, \ldots, x_r)$.

Set $i = i + 1$ and repeat from step 3.

The point $\underline{x}_B$ obtained at the end of the exploratory moves, which is reached by repeating step 3 until $i = 4$, is defined as a <u>base point</u>. The starting point introduced in step 1 of the exploratory move is a starting base point or point obtained by the pattern move.

The pattern move is designed to utilize the information acquired by the exploratory move, and executes the actual minimization of the function by moving in the direction of the established pattern. The pattern move is a simple step from the current base to the point

$$\underline{x} = \underline{x}_B + (\underline{x}_B - \underline{x}_B^*) \tag{J-1}$$

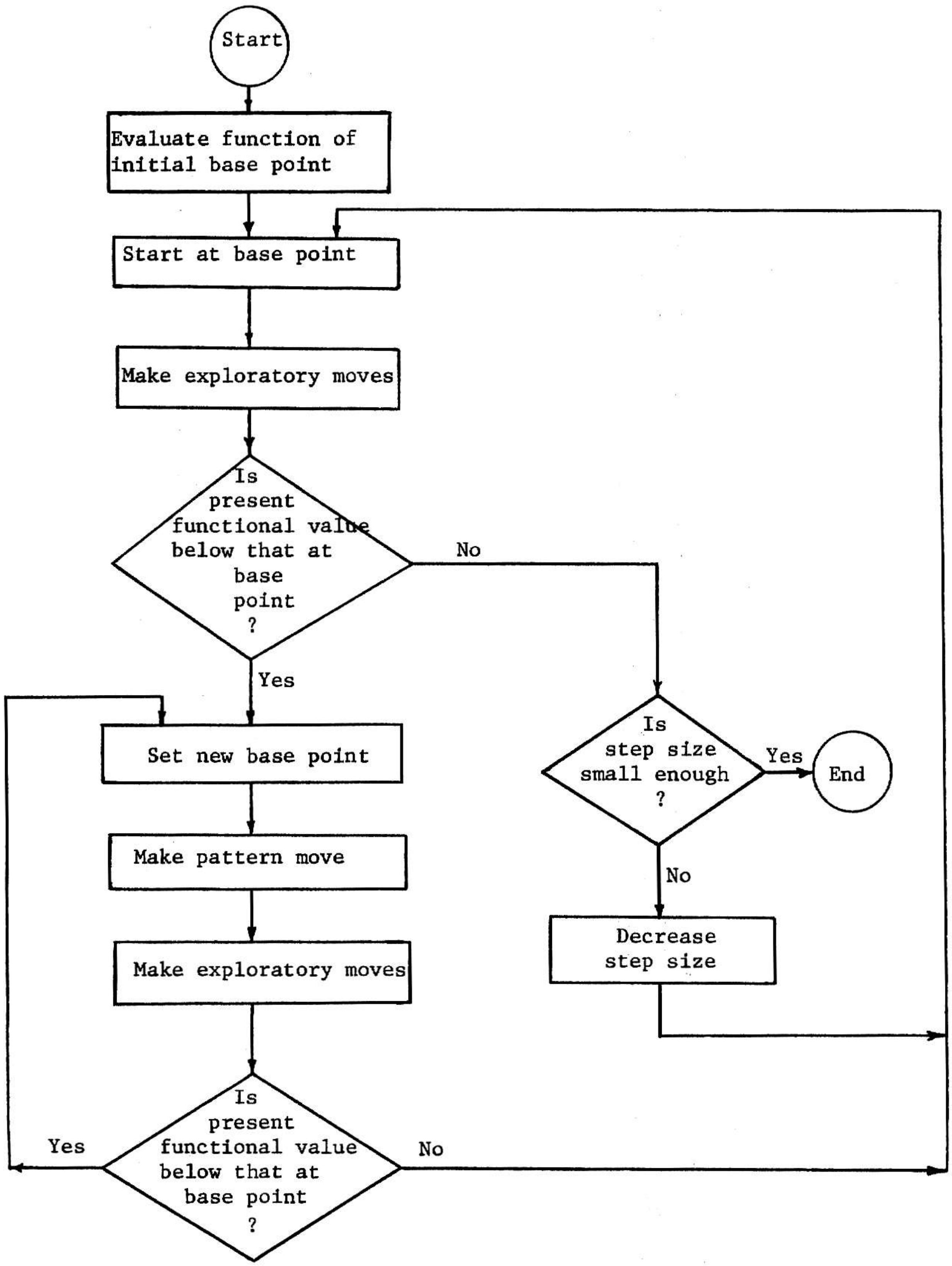
where $\underline{x}_B^*$ is either the starting base point or the preceding base point.

Following the pattern move a series of exploratory moves is conducted to further improve the pattern. If the pattern move followed by the exploratory moves brings no improvement, the pattern move is a failure. Then we return to the last base point which becomes a starting base and the process is repeated.

If the exploratory moves from any starting base do not yield a point which is better than this base, the lengths of all the steps are reduced and the moves repeated. Convergence is assumed when the step lengths, δ_i, have been reduced below predetermined limits.

Appendix K

Computer Flow Diagram for Hooke and Jeeves Pattern Search



Appendix L

Computer Program for Optimization by Hooke and Jeeves Pattern Search

```
$JOB           LRW,RUN=CHECK,TIME=25,PAGES=150,LINES=60,KP=29
C
C  APPENDIX I. COMPUTER PROGRAM FOR OPTIMIZATION BY HOOKE AND JEEVES PATTERN
C              SEARCH.
C
C  ...................................................................
C
C       FORTRAN PROGRAM FOR HOOKE AND JEEVES PATTERN SEARCH
C
C  PURPOSE
C       TO FIND A LOCAL MINIMUM OF A FUNCTION OF SEVERAL VARIABLES
C       BY THE METHOD OF HOOKE AND JEEVES
C
C       IT IS A GENERAL PROGRAM TO FIND THE DECISION VARIABLE VALUES
C       IN MULTIDIMENSIONAL PROBLEMS WITHOUT ANY CONSTRAINT .
C
C  DESCRIPTION OF THE PARAMETERS
C  SUBPROGRAM REQUIRED
C            FUNCTION SUBPROGRAM OBJECT(PHI)
C       THIS SUBPROGRAM  CALCULATES THE OBJECTIVE FUNCTION VALUE AT
C       THE POINT PHI, WHICH IS A N DIMENSIONAL VECTOR.
C       IN THE SUBPROGRAM OBJECT, THE EVALUATION COUNTER IS ADVANCED BY
C       ONE, EVERY TIME THE  SUBPROGRAM IS USED.
C       FOR THIS PURPOSE THE COUNTER,N, MUST BE PLACED IN THE COMMON BLOCK
C
C       THE DATA SUPPLIED
C            NUMBER AND TITLE OF THE PROBLEM
C            CONTROL FOR PRINTING
C            DIMENSIONS OF THE PROBLEM
C            STARTING STEP SIZE
C            INITIAL SEARCH ORIGIN
C            STOPPING STEP SIZE
C
C
C  DATA INPUT INFORMATION
C
C    FIRST CARD
C       NAME , TITLE
C       FORMAT  I5,6041
C       THE PROGRAM  READS THE FIRST CARD AS THE NUMBER OF THE PROBLEM
C       IF IT IS ZERO , IT STOPS . TITLE IS REPRODUCED BEFORE THE OUTPUT.
C
C    SECOND CARD
C       IPRINT          CONTROL FOR PRINTING
C       FORMAT (16I5)
C            IPRINT=2  PRINT EVERY DETAIL
C            IPRINT=1  PRINT ALL THE STEPS
C            IPRINT=0  SKIP THE PRINTING
C
C    THIRD CARD
C       ND  NS
C       FORMAT (16I5)
C       DIMENSION AND STAGES IN THE PROBLEM
C
C  ND*3/7 CARDS
C     BASEN,DELTA,DEL
C       FORMAT (7F10.4)
C       INITIAL SEARCH ORIGIN, INITIAL STEP SIZE, STOPPING STEP SIZE
C
C
```

```
C    METHOD
C        THE METHOD IS DESCRIBED IN THE FOLLOWING REPORT
C        HOOKE AND JEEVES PATTERN SEARCH SOLUTION TO  OPTIMAL PRODUCTION
C        PLANNING PROBLEMS
C        INSTITUTE FOR SYSTEMS DESIGN AND OPTIMIZATION REPORT NO. 18
C        KANSAS STATE UNIVERSITY,MANHATTAN,KANSAS
C
C
C    .................................................................
      DIMENSION X(50), BASEO(50),BASEN(50),DEL(50),TITLE(60)
      COMMON NS,ND,DELTA(50) ,N,IPRINT  ,NE,NB
      COMMON IX,T2
C     SET OUTSIDE TEMPERATURE CONDITION.
      T2=50.
      IX=1
      IY=1
  103 FORMAT (I5,60A1)
  110 FORMAT (10I5)
 1000 FORMAT ('-BEFORE EXPLORATORY MOVES   ',9X,I3,1X5E18.5/(40X5E18.5))
 1001 FORMAT ('-NO. OF STAGES  ',I3,'    NO. OF DIMENSIONS   ',I3)
 1002 FORMAT ('1     PROBLEM NO. ',I3,10X,60A1)
 1010 FORMAT (' AFTER EXPLORATORY  MOVES   ',9X,I3,1X5E18.5/(40X5E18.5))
 1011 FORMAT (' BASE POINT NUMBER  ',I5)
 1020 FORMAT ('-AFTER PATTERN MOVE        ',9X,I3,1X5E18.5/(40X5E18.5))
 1050 FORMAT (' TOTAL EVALUATIONS OF THE FUNCTION  =',I4)
 1120 FORMAT ('-STEP SIZE      ', 5E15.4 /(16X,5E15.4))
 1121 FORMAT (' STARTING POINT', 5E15.4 /(16X,5E15.4))
 1130 FORMAT ('-OBJECTIVE FUNCTION  ',E18.6/' OPTIMAL POINT ',5E18.5
     1/(15X,5E18.5))
 1200 FORMAT ('- FAILED PATTERN MOVE , RETURN TO LAST BASE POINT')
 1201 FORMAT (' FAILED EXPLORATORY MOVES ,CHECK THE STEP SIZE')
 1220 FORMAT ('- STEP SIZE REDUCED TO '  ,5E14.4/(23X,5E14.4))
 1221 FORMAT ('-  FINAL STEP SIZES    ',5E14.4/(23X,5E14.4))
  300 CONTINUE
 1250 N=0
C        SET EVALUATION COUNTER TO ZERO
      NN=1
      NE=0
      NBASE=0
C        READ PROBLEM NUMBER,IF IT IS ZERO PROGRAM GOES TO STOP
      READ(5,103,END=101) NAME,TITLE
      IF(NAME.EQ.0)  GO TO  101
      READ 110,IPRINT
      PRINT 1002,NAME,TITLE
C        NS = NUMBER OF STAGES
C        ND = NUMBER OF DIMENSIONS
      READ 110,NS ,ND
C        READ THE STARTING POINT,INITIAL STEP SIZE AND TERMINATING STEP SIZE
      PRINT 1001,NS,ND
   51 FORMAT(7E10.4)
      READ(5,51) (BASEN(I),I=1,ND),(DELTA(I),I=1,ND),(DEL(I),I=1,ND)
      PRINT 1120,(DELTA(I),I=1,ND)
      PRINT 1121,                 (BASEN(I),I=1,ND)
C        X = CURRENT VARIABLE VECTOR
C        BASEN = CURRENT BASE POINT
C        BASEO = LAST BASE POINT IN THE SEARCH
C        FX = FUNCTION VALUE AT X
C        FXBN = FUNCTIONAL VALUE AT CURRENT BASE POINT
C        FXBO = FUNCTIONAL VALUE AT OLD BASE POINT
      FXBN = OBJECT(BASEN)
```

```
    1 DO 10 I=1,ND
   10 X(I) = BASEN(I)
      FX = FXBN
      IF(IPRINT.GE.1)    PRINT 1000,NN,(X  (I),I=1,ND),FX
C        EXPLORATORY MOVES
      NBASE=NBASE+1
      CALL EXPLMV(FX,X)
      IF(IPRINT.GE.1)   PRINT 1010,NE, (X(I),I=1,ND),FX
      IF(IPRINT.EQ.2)              PRINT 1011,NBASE
      IF(FX.GE.FXBN)   GO TO 3
C        SET NEW BASE POINT
    2 DO 20 I=1,ND
      BASEO(I) = BASEN(I)
      BASEN(I) = X(I)
   20 CONTINUE
      FXBN = FX
      NNB=NE
C        PATTERN MOVE
      DO 21 I=1,ND
      X(I) = BASEN(I)*2.  -  BASEO(I)
   21 CONTINUE
      FX = OBJECT(X)
      IF(IPRINT.GE.1)   PRINT 1020,N,(X  (I),I=1,ND),FX
      IF(IPRINT.GE.1)   PRINT 1000,N,(X  (I),I=1,ND),FX
C        EXPLORATORY MOVES
      NBASE=NBASE+1
      CALL EXPLMV(FX,X)
      IF(IPRINT.GE.1)     PRINT 1010,NE, (X(I),I=1,ND) ,FX
      IF(IPRINT.EQ.2)              PRINT 1011,NBASE
      IF(FX.LT.FXBN)  GO TO 2
      IF(IPRINT.GE.1)         PRINT 1200
C        PATTERN MOVE HAS FAILED
      NN=NNB
      NBASE=NBASE-1
      GO TO 1
    3 CONTINUE
      IF(IPRINT.GE.1)         PRINT 1201
C        CHECKING OF THE CURRENT STEP SIZE
C        IF IT IS SMALL ENOUGH  STOP
C        IF IT IS LARGE REDUCE IT TO HALF AND GO BACK
      DO 30 I=1,ND
      IF(DELTA(I).GE.DEL(I))  GO TO 31
   30 CONTINUE
      GO TO 100
   31 DO 35 I=1,ND
      DELTA(I) = DELTA(I)*0.5
   35 CONTINUE
      IF(IPRINT.GE.1)         PRINT 1220,(DELTA(IS),IS=1,ND)
      GO TO 1
  100 PRINT 1130,FXBN,(BASEN(I),I=1,ND)
      PRINT 1221,(DELTA(IS),IS=1,ND)
      PRINT 1050,N
      IF(IX-4) 201,205,205
  201 IX=IX+1
      GO TO 300
  205 IF(IY-3) 206,207,207
  206 IY=IY+1
      T2=T2-10.
      IX=1
      GO TO 300
```

```
  207 STOP
  101 STOP
      END

      SUBROUTINE EXPLNV (FX,X)
      COMMON NS,ND,DELTA(50) ,N,IPRINT  ,NE,NB
      COMMON IX,T2
      DIMENSION X(50)
  100 FORMAT(' EXPLORATORY MOVE IN X(',I3,') DIRECTION',I3,5E18.4/
     1(40X,5E18.4))
  101 FORMAT(1H )
      IF(IPRINT.EQ.2)    PRINT 101
      DO 201 I=1,ND
      X(I) = X(I) + DELTA(I)
      FXI = OBJECT(X)
      NE = N
      IF(IPRINT.EQ.2)    PRINT 100,I,N,(X(J),J=1,ND),FXI
      IF(FXI-FX)  200,180,180
  180 X(I) = X(I) - 2.*DELTA(I)
      FXI = OBJECT(X)
      NE = N
      IF(IPRINT.EQ.2)    PRINT 100,I,N,(X(J),J=1,ND),FXI
      IF(FXI-FX) 200,181,181
  181 X(I) = X(I) + DELTA(I)
      NE=N-2
      IF(IPRINT.EQ.2)    PRINT 100,I,NE,(X(J),J=1,ND),FX
      GO TO 202
  200 FX = FXI
  202 CONTINUE
  201 CONTINUE
      RETURN
      END

      FUNCTION OBJECT (X)
      DIMENSION X(50)
      COMMON NS,ND,DELTA(50) ,N,IPRINT  ,NE,NB
      COMMON IX,T2
  104 FORMAT(6E13.6)
  999 FORMAT(9F16.2)
      N=N+1
      CLO=0.6
      FCL=1.1
      GO TO (20,21,22,23),IX
   20 ACT=50.
      FEFF=.65
      GO TO 40
   21 ACT=80.
      FEFF=0.75
      GO TO 40
   22 ACT=100.
      GO TO 40
   23 ACT=150.
   40 TK2=273.+T2
      R=62.361
      SP1=EXP(20.98-(5307.02/(273.+X(1))))
C     RELATIVE HUMIDITY INSIDE SPACE.
      RH=100.*X(2)/SP1
      IF(RH.LE.20.) GO TO 16
      IF(RH.GT.70.) GO TO 16
      IF(X(2).LE.1.) GO TO 16
```

```
      IF(X(2).GT.14.) GO TO 16
      A=.45255*ACT
      C=.35+.0023*ACT
      D=.0014*ACT
      E=4.8E-8*FCL*FEFF
      F=0.063*CLO+.000414*CLO*ACT
      G=0.000252*CLO*ACT
      B=-(.031459*CLO+.032)*ACT-1.071*CLO
      W=308.7+B
      Z=1.0+0.000252*CLO*ACT
      U=35.7+3
      Y=F
      O=10.4*FCL
      DATA1=A+C*X(2)+D*X(1)+5.95
      DATA2=E*((-F*X(2)-G*X(1)+W)**4-(X(1)+273.)**4)
      DATA3=O*(-Y*X(2)-Z*X(1)+U)
      V=((DATA1-DATA2)/DATA3)**2
      WRITE(6,104) V
      IF(V.LE..1) GO TO 16
      IF(V.GT.2.5) GO TO 16
      SP2=EXP(20.93-(5307.02/(273.+T2)))
C     OUTSIDE PARTIAL PRESSURE -- FUNCTION OF OUTSIDE TEMP.
      PA2=SP2
C     INSIDE HUMIDITY RATIO.
      W1=(0.62198*X(2))/(760.-X(2))
C     OUTSIDE HUMIDITY RATIO.
      W2=(0.62198*PA2)/(760.-PA2)
C     OUTSIDE SPECIFIC VOLUME.
      VSP2=R*TK2/(28.9645*(760.-PA2))
C     OUTSIDE VENTILATION REQUIRED.
      VENT=600.
C     OUTSIDE MASS FLOW RATE FOR SPECIFIED VENTILATION.
      XM2=VENT*28.32/VSP2
      CS1=.238+.46*W1
      CS2=.238+.46*W2
      RHO=1.155
      ARO=4.*3.
      QT=AFO*V
      XMAT=RHO*QT*3600.*1000.
      RATIO=(XM2/XMAT)*100.
C     SENSIBLE HEAT RATE -- FN. OF DBT, IN KCAL/HR.
      SEN=XM2*CS2*(T2-X(1))/1000.
C     LATENT HEAT RATE -- FN. OF PA, IN KCAL/HR.
      XLAT=XM2*(W2-W1)*575./1000.
C     PRESSURE GRADIENT IS 0.2 INCHES WATER.
      PG=0.2
C     CONVERT TO PASCALS.
      PO=PG*248.8
C     CONVERSION FACTOR.
      CF=8.425E
C     FRICTIONAL LOAD RATE -- FN. OF V, KCAL/HR.
      C3=PO*ARO*CF
      S31=C3*V
C     KINETIC ENERGY FACTOR.
      AK=1.003
C     LOCAL ACCELERATION OF GRAVITY.
      AG=1.0
C     KINETIC THERMAL LOAD RATE -- FN. OF V**3,KCAL/HR.
      C4=AK*RHO*AFO*CF/(2.*AG)
      S32=C4*V**3
```

```
C     FRICTIONAL + KINETIC LOAD.
      S3=S31+S32
C     TOTAL LOAD.
      T=SEN+XLAT+S3
      WRITE(6,999) T2,ACT,RH,SEN,XLAT,S3,T
      WRITE(6,999) RATIO,S31,S32
      OBJECT=T
      GO TO 17
   15 Y=10000000.
      OBJECT=Y
   17 RETURN
      END

$ENTRY
```

Appendix M

Sequential Simplex Pattern Search Technique

There are a number of direct pattern search techniques that are referred to as a sequential simplex pattern search. The technique used in this work was the method proposed by Nelder and Mead [66]. This method, as used for minimization of n variables, depends upon a simplex of (n+1) vertices or trial points in the n-dimensional space, followed by the replacement of the vertex with the highest value (objective function evaluated at this point) by another point with a lower value of the objective function. This procedure is repeated until the point corresponding to the minimum value of the objective function is obtained.

For a two dimensional problem, to illustrate this technique, minimize the objective function, $S = f(x_1, x_2)$. This requires a simplex with $(n+1) = 3$ points. To clarify the following discussion, the definitions of terms are presented below.

y_n = the value of the objective function at point P_n

P_n = nth point in n-dimensional space defining the current "simplex"

P_n = the vertex or point with the lowest value of the objective function (y_1) in the simplex or set of trial points

P_3 = the vertex or point with the highest value of the objective function (y_3) in the simplex or set of trial points; this point corresponds to P_{n+1} for n variables,

P_2 = the vertex or point at which the corresponding value of the objective function (y_2) lies between the value of the objective function (y_1) for point P_1 and that (y_3) for point P_3

P_4 = the centroid of the vertices or points, P_1 and P_2, with the value of the objective function (y_4). In general the centroid of a set of n points in a simplex is $P_c = \sum_{i=1}^{n} P_i/n$.

The operations through which a new point with a lower value of the objective function is found are reflection, expansion and contraction. Where the coefficients of reflection, expansion and contraction are defined as α , β and γ respectively. An illustration of these operations is shown in Figure 30. The values of the coefficients, α, β and γ, are 1, 1/2, and 2 respectively. These were considered best by Nelder and Mead [66] for faster convergence. However, the best values may be different for different problems and should be determined from experience.

The steps of the procedure for using the method are described as follows:

1. Vertices, P_1, P_2, and P_3 of the initial simplex are located according to the values of the objective function at each point having the relation $y_1 < y_3 < y_3$.

2. P_4, the centroid of P_1 and P_2, is determined.

3. First, P_3 is reflected to P_5 with respect to P_4, and if $y_1 < y_5 \leq y_2$, then P_3 is replaced by P_5 and we start the procedure again with a new simplex, i.e., we return to Step 1.

4. If $y_5 < y_1$, i.e., if the reflection has produced a new minimum, we expand P_5 to P_6. If $y_6 < y_1$, we replace P_3 by P_6 and restart the process by returning to step 1. But if $y_6 > y_1$, we have failed in expansion and must replace P_3 by P_5 before restarting.

Reflection: $P_5 = P_4 + \alpha(P_4 - P_3)$

Expansion: $P_6 = P_4 + \gamma(P_5 - P_4)$

Contraction: $P_7 = P_4 + \beta(P_3 - P_4)$

$\alpha=1, \quad \beta=1/2, \quad \gamma=2$

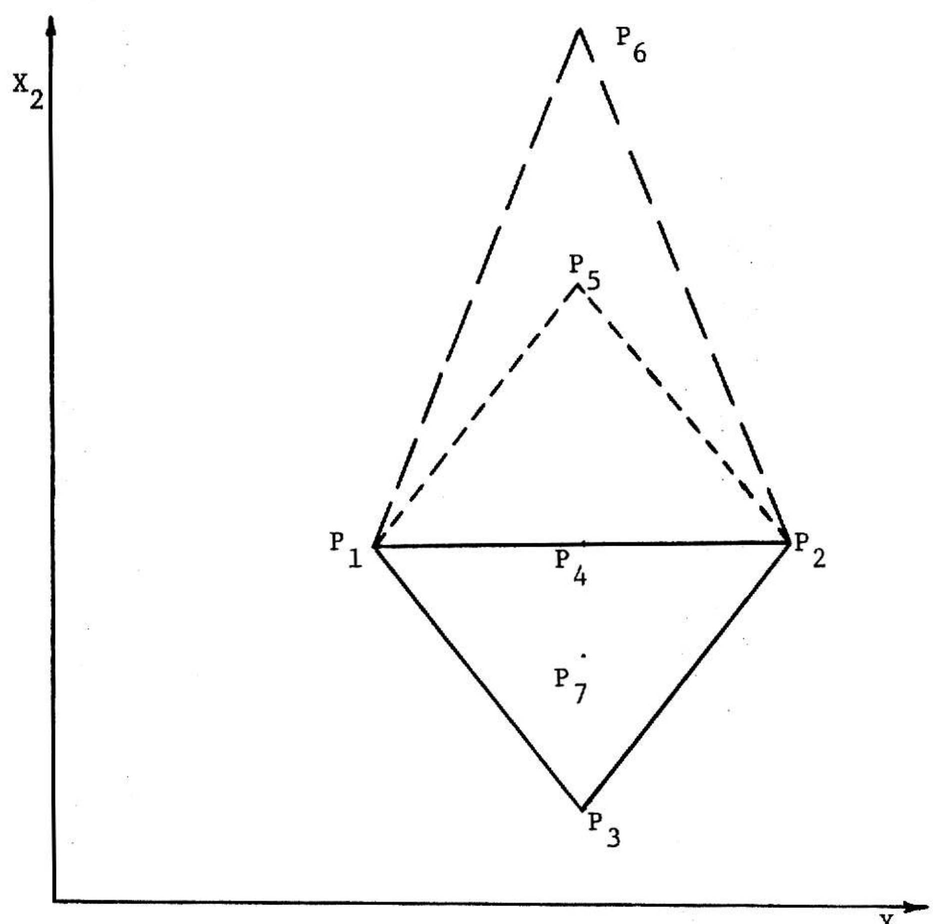


Figure 30. Simplex triangle

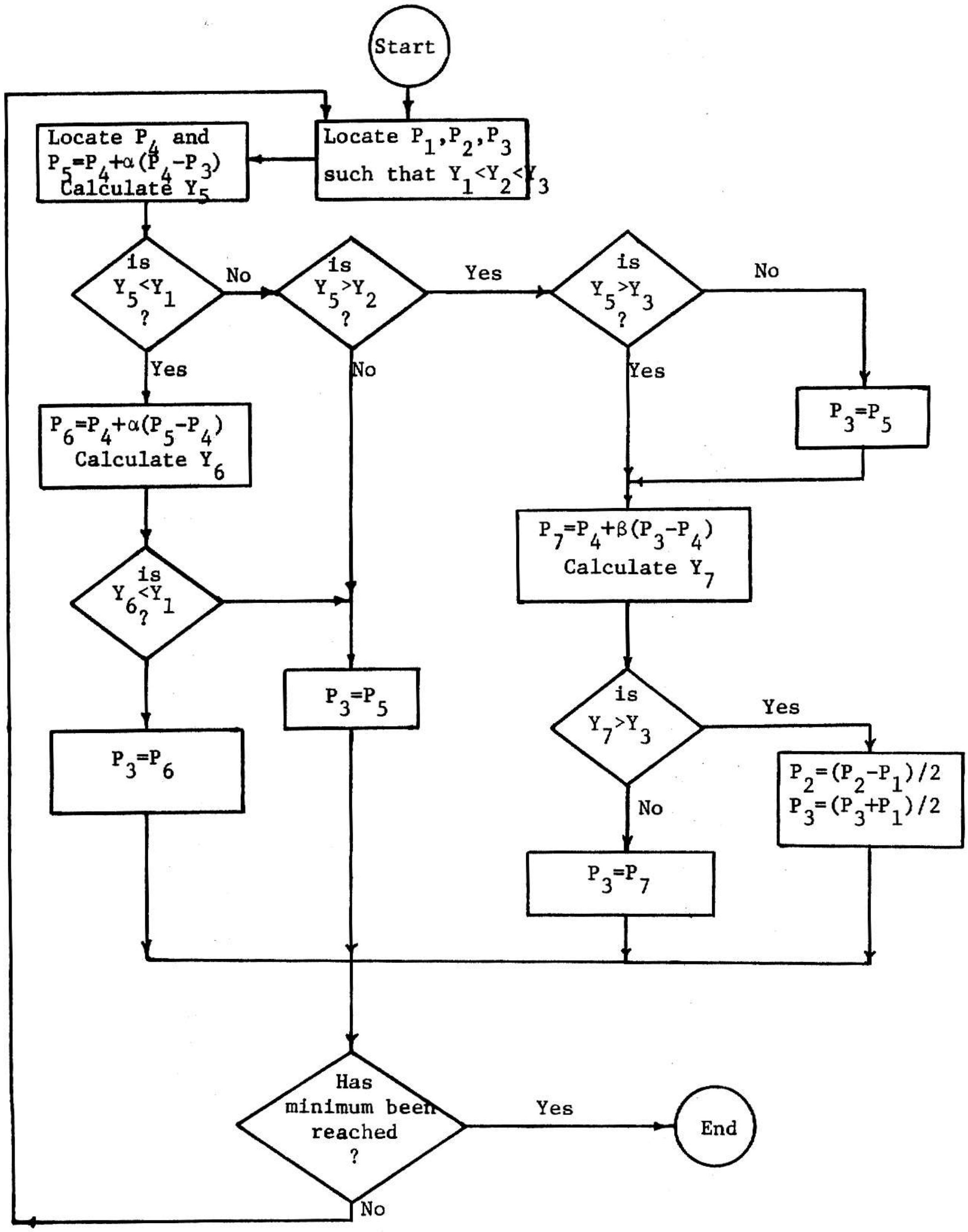
5. If, after reflection, we find that $y_5 > y_1$ and $y_5 > y_2$, we define a new P_3 to be either the old P_3 or P_5, depending on whichever has a lower y_n value, and then contract P_3 to P_7. We replace P_3 by P_7 and restart the procedure by returning to step 1, unless $y_7 > y_3$, that is, unless the contracted point has a higher value than P_3. For such a failed contraction, we replace P_2 and P_3 by $(P_2 + P_1)/2$ and $(P_3 + P_1)/2$ respectively and restart the process by returning to step 1.

The procedures used here for the two dimensional search can be extended to the n-dimensional problem [66]. The worst point of a simplex with (n+1) vertices is reflected, expanded or contracted in the same manner with respect to the centroid of the remaining n vertices until the stopping criterion is satisfied.

If there are constraints on the variables then any variable violating the constraint is given a large objective function value and contraction is then followed to bring the variable inside the constraints.

Appendix N

Computer Flow Diagram for Simplex Pattern Search



Appendix O

Computer Program for Optimization by Simplex Pattern Search

```
$JOB           LRW,RUN=CHECK,TIME=25,PAGES=150,LINES=60,KP=29
C
C  APPENDIX O. COMPUTER PROGRAM FOR OPTIMIZATION BY SIMPLEX PATTERN SEARCH.
C
      DIMENSION FX(50),D(40)
      COMMON IX,T2
C     SET OUTSIDE TEMPERATURE CONDITION.
      T2=50.
      IX=1
      IY=1
   10 FORMAT(4F10.3,3I3,F10.5)
   11 FORMAT('1')
      READ(5,10) FX(1),FX(2),D(1),D(2),N,ITUUT,ITMAX,EPSI
      WRITE(6,11)
  300 CALL SIMPX(FX,FY,N,D,ITOUT,ITMAX,EPSI,1.0,0.5,2.0)
      IF(IX-4) 201,205,205
  201 IX=IX+1
      GO TO 300
  205 IF(IY-3) 206,207,207
  206 IY=IY+1
      T2=T2-10.
      IX=1
      GO TO 300
  207 STOP
      END
C         SUBROUTINE SIMPX
C            TO FIND THE UNCONSTRAINED MINIMUM OF A FUNCTION OF MANY
C            VARIABLES BY SIMPLEX PATTERN SEARCH METHOD STARTING FROM
C            AN ARBITRARY POINT ENTERED.
C
C
C     ..............................................................
C
C                                        K. C. LAI , IE. KSU , 10/25/1969
C         PURPOSE
C         USAGE
C            CALL SIMPX(FX,FY,N,D,ITOUT,ITMAX,EPSI,ALPHA,BETA,GAMMA)
C
C         DESCRIPTION OF PARAMETERS
C            N       - NUMBER OF VARIABLES OF THE PROBLEM.
C            FX      - WITH (N) DIMENSIONS, THE ENTERRING POINT WHEN
C                      CALLING AND THE MINIMUM POINT WHEN RETURNING.
C            FY      - FUNCTION VALUE AT RETURNING POINT IN RETURN.
C            D       - WITH (N) DIMENSIONS, THE STEP-SIZES FOR EACH
C                      DIMENSION FOR INITIAL SIMPLEX SET-UP.
C            ITOUT   - FREQUENCY OF WITHIN-SEARCH INTERMEDIATE PRINT-OUT
C                      DESIRED. WHEN PUT ITOUT=THE NUMBER IN  ITMAX,
C                      NO INTERMEDIATE PRINT-OUT WILL BE OUTPUT.
C            ITMAX   - MAXIMUM NUMBER SEARCH ITERATION ASSIGNED. WHEN
C                      EXCEEDED, THE SEARCH WILL BE TERMINATED AND RETURN
C                      THE LAST MINIMUM DATA SEARCHED.
C            EPSI    - STOPPING CRITERION. WHEN EPSI .GE. SY, STANDARD
C                      DEVIATION OF FUNCTION VALUES EVALUATED AT CURRENT
C                      SIMPLEX VERTICES,RETURN THE MINIMUM DATA .
C            ALPHA   - REFLECTION COEFFICIENT, SUGGESTED VALUE IS 1.0 .
C            BETA    - CONTRACTION COEFFICIENT, SUGGESTED VALUE IS 0.5 .
C            GAMMA   - EXPANSION COEFFICIENT, SUGGESTED VALUE IS 2.0 .
C
C         REMARKS
C             1) IF ANY ADDITIONAL DATA NEEDED TO BE ENTERED TO SIMPX
```

```
C              AND/OR ITS NEEDED SUBROUTINE OBJN, USE COMMON STATEMENT
C              TO ENTER .
C          II) SIMPX WILL PRINT-OUT THE ENTERED DATA, THE INITIAL
C              SIMPLEX VERTICES, INTERMEDIATE OUTPUT WITH A FREQUENCY
C              OF ITOUT AND THE RETURNING DATA .
C
C      OUTPUT PARAMETERS
C          OY      - MINIMUM FUNCTION VALUE SEARCHED SO FAR.
C          OX      - WITH (N) DIMENSIONS, THE POINT WHICH PRODUCE OY.
C          ITER    - NUMBER OF SUCCESSFUL MOVES MADE .
C          NOPT    - NUMBER OF POINTS CALCULATED NOT INCLUDE THE ENTER
C                    POINT AND INITIAL SIMPLEX VERTICES.
C          NOCVN   - NUMBER OF CONTINUOUS CONVERGENT MOVES, OUTPUT
C                    FOR REFERENCE ONLY .
C          NORFT   - NUMBER OF REFLECTION MOVES MADE .
C          NOEXP   - NUMBER OF EXPANSION MOVES MADE .
C          NOCNT   - NUMBER OF CONTRACTION MOVES MADE .
C          NOCUT   - NUMBER OF CUT STEP-SIZE OPERATIONS MADE .
C          YMEAN   - THE MEAN OF FUNCTION VALUES AT CURRENT SIMPLEX
C                    VERTICES.
C          SY      - THE STANDARD DEVIATION OF FUNCTION VALUES AT
C                    CURRENT SIMPLEX VERTICES .
C          X       - WITH (N) DIMENSIONS, THE NEW SEARCHED POINT.
C          Y       - THE FUNCTION VALUE AT X.
C
C      SUBROUTINE NEEDED
C          SUBROUTINE OBJN(X,Y,N)     - FOR COMPUTE FUNCTION VALUE Y
C                                       AT X(I),A POINT, I=1,2,....N .
C
C      METHOD
C          THE METHOD IS DESCRIBED IN THE FALLOWING ARTICLES
C          L. T. FAN, C. L. HWANG AND F. A. TILLMAN , A SEQUENTIAL
C          SIMPLEX PATTERN SEARCH SOLUTION TO PRODUCTION PLANNING
C          PROBLEMS , AIIE TRANSACTIONS, SEPT. 1969 .
C
C          B. H. CARPENTER AND H. C. SWEENY , PROCESS IMPROVEMENT
C          WITH SIMPLEX SELF-DIRECTING EVOLUTIONARY OPERATION ,
C          CHEMICAL ENGINEERING, JULY 5, 1965 .
C
C     ..............................................................
C
      SUBROUTINE SIMPX(FX,FY,N,D,ITOUT,ITMAX,EPSI,ALPHA,BETA,GAMMA)
      DIMENSION X(45,40),FX(50),Y(45),D(40)
 1003 FORMAT(3X65(1H*))
 1004 FORMAT(5X18H**INITIAL POINT...)
 1005 FORMAT(12X2HX(I3,4H) = E11.5)
 1006 FORMAT(5X5HFY = E11.5,15H, EPSI USED IS E11.5,2H .)
 1007 FORMAT(5X20H**INITIAL SIMPLEX ..)
 1008 FORMAT(8X5HPOINTI3,3H ..)
 1009 FORMAT(10X3HY= E11.5,1H.)
 1010 FORMAT(20X)
 1011 FORMAT(5X5HOY = E11.5,9H  ITER = I4,10H  NOPT. = I4,10H  NOCVN =
     1I4)
 1012 FORMAT(7X9HNORFT = I4,4X8HNOEXP = I4,10H  NOCNT = I4,10H  NOCUT =
     1I4)
 1013 FORMAT(7X24HCURRENT SEARCHED DATA ../10X3HY= E11.5,1H.)
 1014 FORMAT(10X2HX(I3,4H) = E11.5,1H,5X3HOX(I3,4H) = E11.5,1H,)
 1015 FORMAT(7X8HYMEAN = E15.8,9H ,  SY = E15.8,2H .)
 1016 FORMAT(5X24H**CUT STEP-SIZES TIMES  I3,2H .)
```

```
 1018 FORMAT(5X//5X17HOPTIMAL RESULTS..//)
 1019 FORMAT(7X7HNOPT = I4,10H   ITER = I4,10H  NOCVN = I4/7X8HNOREFT = ,
     II+,10H  NOEXP = I4,10H  NOCNT = I4,7X8HNOCUT = I4)
 1020 FORMAT(7X14H* OPTIMAL Y = E11.5,8H,  YM = E11.5,7H, SY = E11.5,2H
     1,)
 1022 FORMAT(5X63(1H*))
 1023 FORMAT(5X26H** ITERATION NO. EXCEEDED I5,2H .)
 1025 FORMAT(5X20H...SIMPLEX SEARCH...)
      WRITE(6,1025)
      MULT=1
      NOPT=0
      NOCUT=0
      NOCVN=0
      ITER=0
      NOREFT=0
      NOEXP=0
      NOCNT=0
      FN=N
      NN=N+1
      CALL OBJN(FX,YF,N)
      LOCAT=1
      IWAY=1
      GO TO 510
C  . SET UP INITIAL SIMPLEX .
    2 DO 6 J=1,N
      DO 3 I=1,J
    3 X(I,J)=  FX(J)-D(J)
      FJ=J
      X(J+1,J)=  FX(J)+FJ*D(J)
      IF(J-N)4,6,6
    4 JM=J+2
      DO 5 I=JM,NN
    5 X(I,J)=  FX(J)
    6 CONTINUE
      WRITE(6,1010)
      DO 8 I=1,NN
      DO 7 J=1,N
    7 FX(J)=X(I,J)
      CALL OBJN(FX,YF,N)
    8 Y(I)=YF
      INI=1
C     REARRANGE ORDER (OVERALL) .
    9 I=1
      NS=N+1
   10 IF(Y(I)-Y(NS))13,11,11
   11 YTEM=Y(NS)
      Y(NS)=Y(I)
      Y(I)=YTEM
      DO 12 J=1,N
      FX(J)=X(NS,J)
      X(NS,J)=X(I,J)
   12 X(I,J)=  FX(J)
   13 IF(NS-I-1)15,15,14
   14 NS=NS-1
      GO TO 10
   15 I=I+1
      IF(I-N-1)16,17,17
   16 NS=N+1
      GO TO 10
   17 IF(IK(1) 65,65,501
```

```
  501 LOCAT=2
      IWAY=2
      GO TO 120
C     COMPUTE THE CENTROID .
   18 DO 20 J=1,N
      PXT=X(1,J)
      DO 19 I=2,N
   19 PXT=PXT+X(I,J)
   20 X(N+2,J)=PXT/FN
C     **MAKE REFLECTION MOVE .
      DO 21 J=1,N
      X(N+3,J)=X(N+2,J)+ALPHA*(X(N+2,J)-X(N+1,J))
   21 FX(J)=X(N+3,J)
      CALL OBJN(FX,YF,N)
      Y(N+3)=YF
      NOPT=NOPT+1
      LOCAT=3
      IWAY=3
      GO TO 500
   22 IF(Y(N+3)-Y(1))29,23,23
   23 IF(Y(N+3)-Y(N))24,26,26
  244 IWAY=7
   24 DO 25 I=1,N
   25 X(N+1,I)=X(N+3,I)
      Y(N+1)=Y(N+3)
      ITER=ITER+1
      NOREFT=NOREFT+1
      GO TO 100
   26 IF(Y(N+3)-Y(N+1))27,49,49
   27 DO 28 I=1,N
   28 X(N+1,I)=X(N+3,I)
      Y(N+1)=Y(N+3)
      ITER=ITER+1
      NOREFT=NOREFT+1
      GO TO 49
C     **MAKE EXPANSION MOVE .
   29 DO 30 J=1,N
      X(N+4,J)=X(N+2,J)+GAMMA*(X(N+3,J)-X(N+2,J))
   30 FX(J)=X(N+4,J)
      CALL OBJN(FX,YF,N)
      Y(N+4)=YF
      NOPT=NOPT+1
      LOCAT=4
      IWAY=4
      GO TO 500
   31 IF(Y(N+4)-Y(1))32,244,244
   32 DO 33 I=1,N
   33 X(N+1,I)=X(N+4,I)
      Y(N+1)=Y(N+4)
      ITER=ITER+1
      NOEXP=NOEXP+1
      GO TO 100
C     **MAKE CONTRACTION MOVE .
   49 DO 50 J=1,N
      X(N+5,J)=X(N+2,J)+BETA*(X(N+1,J)-X(N+2,J))
   50 FX(J)=X(N+5,J)
      CALL OBJN(FX,YF,N)
      Y(N+5)=YF
      NOPT=NOPT+1
      LOCAT=5
```

```
      IWAY=5
      GO TO 500
   51 IF(Y(N+5)-Y(N+1))52,60,60
   52 DO 53 I=1,N
   53 X(N+1,I)=X(N+5,I)
      Y(N+1)=Y(N+5)
      ITER=ITER+1
      NOCNT=NOCNT+1
      NOCVN=NOCVN+1
      GO TO 110
C   **CUT DOWN STEP-SIZES .
   60 DO 62 I=2,NM
      DO 61 J=1,N
      X(I,J)=(X(1,J)+X(I,J))/2.0
   61 FX(J)=X(I,J)
      CALL OBJN(FX,YF,N)
   62 Y(I)=YF
C   **REARRANGE ORDER (OVERALL ) .
      INI=0
      GO TO 9
   65 NOCUT=NOCUT+1
      NOPT=NOPT+N
      NOCVN=NOCVN+1
      LOCAT=6
      IWAY=6
      GO TO 120
  100 NOCVN=0
C   **REARRANGE ORDER ( SHOT-DOWN ).
  110 IOR=N
  111 IF(Y(IOR+1)-Y(IOR))112,120,120
  112 YTEM=Y(IOR+1)
      Y(IOR+1)=Y(IOR)
      Y(IOR)=YTEM
      DO 113 J=1,N
      FX(J)=X(IOR+1,J)
      X(IOR+1,J)=X(IOR,J)
  113 X(IOR,J)=FX(J)
      IF(IOR-1)120,120,114
  114 IOR=IOR-1
      GO TO 111
C   **TEST FOR OPTIMALITY .
  120 FNM=NM
      YM=Y(1)
      DO 121 I=2,NM
  121 YM=YM+Y(I)
      YM=YM/FNM
      SY=(Y(1)-YM)**2
      DO 122 I=2,NM
  122 SY=SY+(Y(I)-YM)**2
      SY=(SY/FN)**0.5
      IF(LOCAT-6) 123,500,123
  123 IF(LOCAT-2) 500,500,124
  124 IF(SY-EPSI) 125,125,18
  125 LOCAT=5
  500 IF(NOPT-ITMAX) 505,505,560
  505 GO TO (510,520,530,530,530,540,560,560),LOCAT
  510 WRITE(6,1004)
      DO 511 I=1,N
  511 WRITE(6,1005)I,FX(I)
      WRITE(6,1006)YF,EPSI
```

```
      WRITE(6,1003)
      GO TO 2
  520 WRITE(6,1007)
      DO 521 I=1,NM
      WRITE(6,1008)I
      DO 522 J=1,N
  522 WRITE(6,1005)J,X(I,J)
      WRITE(6,1009)Y(I)
  521 WRITE(6,1022)
      WRITE(6,1010)
      GO TO 18
  530 IF(NOPT-ITOUT*MULT) 533,531,531
  531 MULT=MULT+1
      IF(NOPT-ITOUT*MULT) 532,531,531
  532 WRITE(6,1011)Y(1),ITER,NOPT,NOCVN
      WRITE(6,1012)NORFT,NOEXP,NOCNT,NOCUT
      WRITE(6,1015)YM,SY
      WRITE(6,1013)YF
      DO 534 IN=1,N
  534 WRITE(6,1014)IN,FX(IN),IN,X(1,IN)
      WRITE(6,1003)
  533 IWAY=IWAY-2
      GO TO (22,31,51,123,18),IWAY
  540 WRITE(6,1016)NOCUT
      GO TO 123
  560 IF(LOCAT-8) 561,562,562
  561 WRITE(6,1023)ITMAX
      GO TO 563
  562 WRITE(6,1018)
  563 WRITE(6,1019)NOPT,ITER,NOCVN,NORFT,NOEXP,NOCNT,NOCUT
      WRITE(6,1020)Y(1),YM,SY
      DO 565 I=1,N
  565 WRITE(6,1005)I,X(1,I)
      WRITE(6,1003)
      DO 564 I=1,N
  564 FX(I)=X(1,I)
      FY=Y(1)
      RETURN
      END

      SUBROUTINE OBJN(X,Y,N)
      DIMENSION X(40)
      COMMON IX,T2
  104 FORMAT(6E13.6)
  999 FORMAT(9F16.2)
      CLO=0.6
      FCL=1.1
      GO TO (20,21,22,23),IX
   20 ACT=52.
      FEFF=.65
      GO TO 40
   21 ACT=83.
      FEFF=0.75
      GO TO 40
   22 ACT=111.
      GO TO 40
   23 ACT=132.
   40 TK2=273.+T2
      R=62.361
      SP1=EXP(20.98-(5307.02/(273.+X(1))))
```

```
C     RELATIVE HUMIDITY INSIDE SPACE.
      RH=100.*X(2)/SP1
      IF(RH.LE.0.) GO TO 16
      IF(RH.GT.100.) GO TO 16
      A=.45255*ACT
      C=.35+.0023*ACT
      D=.0014*ACT
      E=4.8E-1*FCL*FEFF
      F=0.304*CLO+.000414*CLO*ACT
      G=0.000252*CLO*ACT
      H=-(.031453*CLO+.032)*ACT-1.071*CLO
      W=308.7+3
      Z=1.0+0.000252*CLO*ACT
      U=35.7+H
      Y=F
      P=10.4*FCL
      DATA1=A+C*X(2)+D*X(1)+5.95
      DATA2=E*((-F*X(2)-G*X(1)+W)**4-(X(1)+273.)**4)
      DATA3=P*(-Y*X(2)-Z*X(1)+U)
      V=((DATA1-DATA2)/DATA3)**2
      WRITE(6,104) V
      IF(V.LT..1) GO TO 16
      IF(V.GT.2.0) GO TO 16
      SP2=EXP(20.99-(5307.02/(273.+T2)))
C     OUTSIDE PARTIAL PRESSURE -- FUNCTION OF OUTSIDE TEMP.
      PA2=SP2
C     INSIDE HUMIDITY RATIO.
      W1=(0.62193*X(2))/(760.-X(2))
C     OUTSIDE HUMIDITY RATIO.
      W2=(0.62198*PA2)/(760.-PA2)
C     OUTSIDE SPECIFIC VOLUME.
      VSP2=R*TK2/(28.9645*(760.-PA2))
C     OUTSIDE VENTILATION REQUIRED.
C     OUTSIDE MASS FLOW RATE FOR SPECIFIED VENTILATION.
      VENT=600.
      XM2=VENT*28.32/VSP2
      CS1=.238+.46*W1
      CS2=.238+.46*W2
      RHO=1.155
      ARO=4.*3.
      QT=ARO*V
C     ROOM MASS TRANSFER RATE IN GM./HR.
      XMT=RHO*QT*3600.*1000.
      RATIO=(XM2/XMAT)*100.
C     SENSIBLE HEAT RATE -- FN. OF DBT, IN KCAL/HR.
      SEN=XM2*CS2*(T2-X(1))/1000.
C     LATENT HEAT RATE -- FN. OF PA, IN KCAL/HR.
      XLAT=XM2*(W2-W1)*575./1000.
      PS=0.2
C     PRESSURE GRADIENT IS 0.2 INCHES WATER.
C     CONVERT TO PASCALS.
      PD=PS*248.8
C     CONVERSION FACTOR.
      CF=3.4253
C     FRICTIONAL LOAD RATE -- FN. OF V, KCAL/HR.
      C3=PD*ARO*CF
      AK=1.003
C     KINETIC ENERGY FACTOR.
      AK=1.003
C     LOCAL ACCELERATION OF GRAVITY.
```

```
      AG=1.0
C     KINETIC THERMAL LOAD RATE -- FN. OF V**3,KCAL/HR.
      C4=AK*RHO*ARD*CF/(2.*AG)
      S32=C4*V**3
C     FRICTIONAL + KINETIC LOAD.
      S3=S31+S32
C     TOTAL LOAD.
      T=SEN+XLAT+S3
      WRITE(6,998) T2,ACT,RH,SEN,XLAT,S3,T
      WRITE(6,999) RATIO,S31,S32
      Y=T
      GO TO 17
   15 Y=9000000.
   17 RETURN
      END

$ENTRY
```

Appendix P

Computer Program for Optimization by Another Simplex Pattern Search

```
$JOB            LRW,RUN=CHECK,TIME=25,PAGES=150,LINES=60,KP=29
C
C  APPENDIX P. COMPUTER PROGRAM FOR OPTIMIZATION BY ANOTHER  SIMPLEX
C                PATTERN SEARCH.
C
C     COMPUTER PROGRAM FOR OPTIMIZATION.
      DIMENSION DLTVX(27,28),S(30),DCVX(27,30)
      COMMON IX,T2
  101 FORMAT(10I5)
  102 FORMAT(7F10.4)
  103 FORMAT(/16H EVALUATION NO =I5/)
  104 FORMAT(6E13.6)
      T2=50.
      IX=1
      IY=1
      READ(5,101)NDIM,NOPT,NDIMP1,MAXNO,METHOD
      READ(5,102)ERROR,SUPLIM
      READ(5,102)((DLTVX(I,J),I=1,NDIM),J=1,NDIMP1)
  200 READ(5,102)(DCVX(I,1),I=1,NDIM)
      WRITE(6,101)NDIM,NOPT,NDIMP1,MAXNO,METHOD
      WRITE(6,104)ERROR,SUPLIM
      WRITE(6,104)(DCVX(I,1),I=1,NDIM)
      WRITE(6,104)((DLTVX(I,J),I=1,NDIM),J=1,NDIMP1)
  300 CALL GKCHEN(NDIM,METHOD,MAXNO,ERROR,SUPLIM,DLTVX,DCVX,S,KK)
      WRITE(6,104)S(NDIM+2),(DCVX(I,NDIM+2),I=1,NDIM)
      WRITE(6,104)((DCVX(I,J),I=1,NDIM),J=1,NOPT)
      WRITE(6,104)(S(I),I=1,NOPT)
      WRITE(6,103)KK
      IF(IX-4) 201,205,205
  201 IX=IX+1
      GO TO 300
  205 IF(IY-3) 206,207,207
  206 IY=IY+1
      T2=T2-10.
      IX=1
      GO TO 200
  207 STOP
      END

      SUBROUTINE GKCHEN(NDIM,METHOD, MAXNO,ERROR,SUPLIM,DLTVX,DCVX,S,KK)
      DIMENSION DLTVX(27,28),C(28),DCVX(27,30),S(30),CNTROX(27)
      COMMON IX,T2
  110 FORMAT(/19H THIS IS NEW METHOD/)
  111 FORMAT(/16H THIS IS SIMPLEX/)
  112 FORMAT(/12H THIS IS BOX/)
  113 FORMAT(/16H ****WARNING****/)
  114 FORMAT( /50H INADEQUATE GIVEN MAX. NO FOR FUNCTION EVALUATION,)
  115 FORMAT(47H INCREASING THE MAXNO OR CHANGING THE STEP SIZE/)
      GO TO (116,117,118),METHOD
  116 JMCHEN=1
      KCHEN=1
      ALPHO=1.0
      BETA=0.5
      COEFF=1.2
      GAMMA=2.0
      WRITE(6,110)
      GO TO 1
  117 JMCHEN=1
      KCHEN=2
      ALPHO=1.0
```

```
      BETA=0.5
      GAMMA=2.0
      WRITE(6,111)
      GO TO 1
  118 JMCHEN=NDIM
      ALPHO=1.3
      BETA=0.5
      WRITE(6,112)
    1 J=1
      KK=1
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      K=NDIM+JMCHEN
      KLT1=K-1
      DO 3 J=2,K
      DO 2 I=1,NDIM
    2 DCVX(I,J)=DCVX(I,1)+DLTVX(I,J-1)
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
    3 CONTINUE
    4 M=K
      ALPHA=ALPHO
      CALL CRDER(M,NDIM,S,DCVX)
      DO 5 I=1,KLT1
    5 C(I)=1.
      CALL CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
    6 DO 7 I=1,NDIM
    7 DCVX(I,K+1)=CNTROX(I)+ALPHA*(CNTROX(I)-DCVX(I,K))
      J=K+1
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      IF(KK-MAXNO)8,8,36
    8 GO TO (9,9,23),METHOD
    9 IF(S(K+1)-S(1))10,10,23
   10 DO 11 I=1,NDIM
   11 DCVX(I,K+2)=CNTROX(I)+GAMMA*(DCVX(I,K+1)-CNTROX(I))
      J=K+2
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      IF(KK-MAXNO) 12,12,36
   12 GO TO (16,13),KCHEN
   13 IF(S(K+2)-S(1)) 14,14,21
   14 S(K)=S(K+2)
      DO 15 L=1,NDIM
   15 DCVX(L,K)=DCVX(L,K+2)
      GO TO 35
   16 IF(S(K+2)-S(K+1)) 17,17,21
   17 S(K)=S(K+2)
      DO 18 L=1,NDIM
   18 DCVX(L,K)=DCVX(L,K+2)
      M=K
      CALL ORDER(M,NDIM,S,DCVX)
      CALL SCHECK(K,SUM,NDIM,S)
      IF(SUM-ERROR) 37,37,19
   19 CVALUE=2*NDIM-1
      DO 20 I=1,KLT1
      C(I)=CVALUE
   20 CVALUE=2*NDIM-2
      CALL CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
      ALPHA=ALPHO*COEFF
      GO TO 6
   21 S(K)=S(K+1)
      DO 22 L=1,NDIM
   22 DCVX(L,K)=DCVX(L,K+1)
```

```
      GO TO 35
   23 IF(S(K+1)-S(K-1)) 21,21,24
   24 IF(S(K+1)-S(K)) 25,25,27
   25 S(K)=S(K+1)
      DO 26 I=1,NDIM
   26 DCVX(I,K)=DCVX(I,K+1)
C     CONTRACTION MOVE.
   27 DO 28 I=1,NDIM
   28 DCVX(I,K+1)=CNTROX(I)+BETA*(DCVX(I,K)-CNTROX(I))
      J=K+1
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      IF(KK-MAXND) 29,29,36
   29 IF(S(K+1)-S(K)) 30,30,32
   30 S(K)=S(K+1)
      DO 31 I=1,NDIM
   31 DCVX(I,K)=DCVX(I,K+1)
      GO TO 35
   32 DO 34 J=2,K
      DO 33 I=1,NDIM
   33 DCVX(I,J)=(DCVX(I,1)+DCVX(I,J))/2.
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
   34 CONTINUE
      IF(KK-MAXND) 35,35,36
   35 CALL SCHECK(K,SUM,NDIM,S)
      IF(SUM-ERROR) 37,37,4
   36 WRITE(6,113)
      WRITE(6,114)
      WRITE(6,115)
      GO TO 40
   37 DO 38 I=1,KLT1
   38 C(I)=1.
      CALL CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
      DO 39 I=1,NDIM
   39 DCVX(I,K+1)=CNTROX(I)
      J=K+1
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
   40 RETURN
      END

      SUBROUTINE SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      DIMENSION S(30),DCVX(27,30),X(27),XOPT(27)
      COMMON IX,T2
    1 FORMAT(31H THE OPTIMUM FUNCTION VALUE IS E13.6)
    2 FORMAT(6E13.6)
    3 FORMAT(10I4)
  104 FORMAT(6E13.6)
  999 FORMAT(3F15.2)
      IF(J-1) 4,4,5
    4 KCONT=10
      ERR=10.
      GO TO 6
    5 KK=KK+1
    6 DO 7 I=1,NDIM
      X(I)=DCVX(I,J)
    7 CONTINUE
      CLO=0.6
      FCL=1.1
      GO TO (20,21,22,23),IX
   20 ACT=52.
      FEFF=.65
```

```
      GO TO 40
   21 ACT=83.
      FEFF=0.75
      GO TO 40
   22 ACT=111.
      GO TO 40
   23 ACT=132.
   40 TK2=273.+T2
      R=62.361
      SP1=EXP(20.98-(5307.02/(273.+X(1))))
      RH=100.*X(2)/SP1
      IF(RH.LE.0.) GO TO 16
      IF(RH.GT.100.) GO TO 16
      A=.45255*ACT
      C=.35+.0023*ACT
      D=.0014*ACT
      E=4.8E-8*FCL*FEFF
      F=0.063*CLO+.000414*CLO*ACT
      G=0.000252*CLO*ACT
      H=-(.001459*CLO+.032)*ACT-1.071*CLO
      W=308.7+H
      Z=1.0+0.000252*CLO*ACT
      U=35.7+H
      Y=F
      Q=10.4*FCL
      DATA1=A+C*X(2)+D*X(1)+5.95
      DATA2=E*((-F*X(2)-G*X(1)+W)**4-(X(1)+273.)**4)
      DATA3=Q*(-Y*X(2)-Z*X(1)+U)
      V=((DATA1-DATA2)/DATA3)**2
      WRITE(5,104) V
      IF(V.LE..1) GO TO 16
      IF(V.GT.2.5) GO TO 16
      SP2=EXP(20.98-(5307.02/(273.+T2)))
      PA2=SP2
      W1=(18.*X(2))/(29.*(760.-X(2)))
      W2=(18.*PA2)/(29.*(760.-PA2))
      VSP2=R*TK2/(29.3*(760.-PA2))
      VENT=500.
      XM2=VENT*28.32/VSP2
      CS1=.233+.46*W1
      CS2=.233+.46*W2
      RHO=1.155
      ARO=4.*3.
      QT=ARO*V
      XMAT=RHO*QT*3600.*1000.
      RATIO=(XM2/XMAT)*100.
      SFR=XM2*CS2*(T2-X(1))/1000.
      XLAT=XM2*(W2-W1)*575./1000.
C     PRESSURE GRADIENT IS 0.2 INCHES WATER.
      PS=0.2
C     CONVERT TO PASCALS.
      PD=PS*248.8
C     CONVERSION FACTOR.
      CF=3.4235
C     FRICTIONAL LOAD RATE -- FN. OF V, KCAL/HR.
      C3=PD*ARO*CF
      S31=C3*V
C     KINETIC ENERGY FACTOR.
      AK=1.003
C     LOCAL ACCELERATION OF GRAVITY.
```

```
      AG=1.0
C     KINETIC THERMAL LOAD RATE -- FN. OF V**3,KCAL/HR.
      C4=AK*RHC*ARO*CF/(2.*AG)
      S32=C4*V**3
C     FRICTIONAL + KINETIC LOAD.
      S3=S31+S32
C     TOTAL LOAD.
      T=SEN+XLAT+S3
      WRITE(6,999) T2,ACT,RH,SEN,XLAT,S3,T
      WRITE(6,999) RATIO,S31,S32
      S(J)=T
      IF(J-1) 9,9,11
    9 DO 10 I=1,NDIM
      XOPT(I)=X(I)
   10 CONTINUE
      SOPT=T
      IF(J-1) 17,17,12
   11 IF(S(1)-S(J)) 12,9,9
   12 IF(KK-KCONT) 14,13,13
   13 WRITE(6,1) SOPT
      WRITE(6,2) (XOPT(I),I=1,NDIM)
      WRITE(6,3) KK
      KCONT=KCONT+10
   14 IF(S(J)-ERR) 15,15,17
   15 WRITE(6,1) SOPT
      WRITE(6,2)(XOPT(I),I=1,NDIM)
      WRITE(6,3) KK
      ERR=ERR*0.1
      GO TO 17
   16 S(J)=SUPLIM
   17 RETURN
      END

      SUBROUTINE ORDER(M,NDIM,S,DCVX)
      DIMENSION S(30),DCVX(27,30)
      COMMON IX,T2
      K=4
      KLT1=K-1
      DO 5 I=1,KLT1
      M=M-1
      DO 4 J=1,M
      IF(S(M+1)-S(J)) 2,2,4
    2 A=S(M+1)
      S(M+1)=S(J)
      S(J)=A
      DO 3 L=1,NDIM
      B=DCVX(L,M+1)
      DCVX(L,M+1)=DCVX(L,J)
      DCVX(L,J)=B
    3 CONTINUE
    4 CONTINUE
    5 CONTINUE
      RETURN
      END

      SUBROUTINE CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
      DIMENSION C(28),CNTROX(27),DCVX(27,30)
      COMMON IX,T2
      CSUM=0.
      DO 1 I=1,KLT1
```

```
      1 CSUM=CSUM+C(I)
        DO 3 I=1,NDIM
        AXIS=0.
        DO 2 J=1,KLTI
        CNTROX(I)=AXIS+C(J)*DCVX(I,J)
        AXIS=CNTROX(I)
      2 CONTINUE
        CNTROX(I)=CNTROX(I)/CSUM
      3 CONTINUE
        RETURN
        END

        SUBROUTINE SCHECK(K,SUM,NDIM,S)
        DIMENSION S(30)
        COMMON IX,T2
        SAVG=0.
        DO 1 L=1,K
      1 SAVG=S(L)+SAVG
        AK=K
        SAVG=SAVG/AK
        SUM=0.
        DO 2 L=1,K
      2 SUM=SUM+(S(L)-SAVG)**2
        ANDIM=NDIM
        SUM=SUM**0.5/ANDIM
        RETURN
        END

         $ENTRY
  2     3     2  200     2
0.100000E-01 0.100000E 03
0.250000E 02 0.100000E 02
0.100000E 01-0.100000E 01 0.200000E 01 0.000000E 00

THIS IS SIMPLEX
```

Appendix Q

Computer Program for Optimization by Lagrange Multiplier and Kuhn-Tucker Conditions Optimization Technique

```
$JOB           ERW,RUN=CHECK,TIME=25,PAGES=150,LINES=60,KP=29
C
C  APPENDIX D. COMPUTER PROGRAM FOR OPTIMIZATION BY LAGRANGE MULTIPLER AND
C                KUHN-TUCKER CONDITIONS OPTIMIZATION TECHNIQUE.
C
C     LAGRANGE OPTIMIZATION TECH. WITH KUHN-TUCKER CONDITIONS.
C     THIS PROGRAM GIVES VALUES OF TEMP., PARTIAL PRESS., AND VELOCITY THAT
C     YIELDS A MINIMUM SUBJECT TO THE COMFORT EQUATION CONSTRAINT, PHYSICAL
C     LIMITATIONS, AND GIVEN PHYSICAL PARAMETERS OF A CONTROL SYSTEM.
      DIMENSION V(2)
      DATA V/0.1,2.6/
  100 FORMAT('1')
      WRITE(6,100)
  999 FORMAT(9F16.2)
  998 FORMAT(9F16.4)
    1 FORMAT(' ')
    2 FORMAT(11X'T2'15X'CLO'12X'ACT'13X'TAVE'12X'VENT'13X'RATIO')
    3 FORMAT(12X'V'16X'PA'13X'TA'14X'S'14X'SENS'13X'LAT'13X'S3')
    4 FORMAT(11X'H1'16X'H2'12X'CS1'14X'CS2'14X'S31'13X'S32')
      WRITE(6,1)
      WRITE(6,2)
      WRITE(6,3)
      WRITE(6,4)
C     SET OUTSIDE TEMP.
      T2=40.
      GO TO 25
   17 T2=35.
      GO TO 25
   18 T2=20.
      GO TO 25
   19 T2=10.
      GO TO 25
   16 T2=50.
   25 CONTINUE
C     SET CLO. VALUE.
      GO TO 11
   13 CLO=1.0
      GO TO 15
   12 CLO=0.6
      GO TO 15
   11 CLO=0.0
   15 CONTINUE
C     SET ACT. LEVEL.
C     SEDENTARY IX=1
C     LOW ACT. IX=2
C     MEDIUM ACT. IX=3
C     HIGH ACT. IX=4
      IX=0
   10 CONTINUE
      IX=IX+1
C     SET VENTILATION REQUIREMENT (CUBIC FEET PER HR.)
      VENT=600.
C     SET UPPER PHYSICAL LIMIT ON DESIRED PARTIAL PRESS. OF WATER VAPOR(MM HG.)
      PA=14.
C     SET INITIAL VALUES EQUAL TO ZERO.
      QT=0.0
      S31=0.0
      S32=0.0
      S3=0.0
      TA=0.0
```

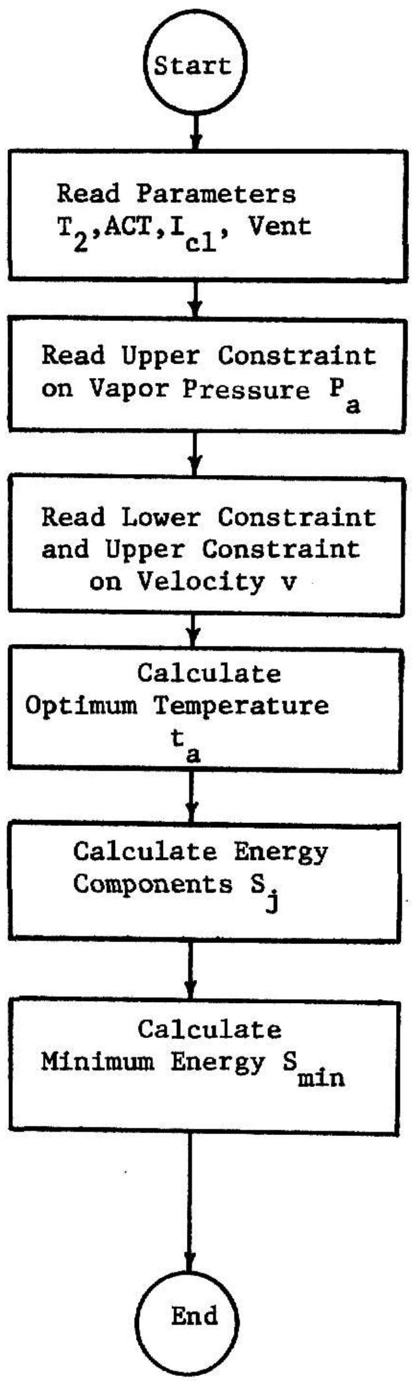
```
      IF(CLO.EQ.1.0) TAVE=23.33
      IF(CLO.EQ.0.0) TAVE=27.78
      IF(CLO.EQ.0.6) TAVE=25.56
      ADU=1.8
      FCL=1.1
      Q=1.0
      Q2=1.15
      IF(CLO     .EQ.0.0) FCL=Q
      IF(CLO     .EQ.1.0) FCL=Q2
      GO TO (20,21,22,23),IX
   20 ACT=50.
      FEFF=.65
      GO TO 40
   21 ACT=80.
      FEFF=0.75
      GO TO 40
   22 ACT=100.
      GO TO 40
   23 ACT=150.
   40 TK2=273.+T2
      R=32.361
      AEFF=ADU*FEFF*FCL
      XM=ACT*A(X)
      XOM=ACT
      SIG=4.96E-8
      HRA=4.*SIG*(TAVE+273.0)**3
      HA=HRA
      D=371.2*FCL*ADU+10.4*FCL*XM     *(-0.081459*CLO     -0.032)-10.4*1.
     1071*FCL*ADU*CLO
      E=10.4*FCL*ADU*(0.063*CLO     +0.000414*CLO     *XOM)
      F=10.4*FCL*ADU*(1.0+0.000252*CLO     *XOM)
      A=0.35*ADU+.0023*XM +HA*AEFF*(0.063*CLO     +0.000414*CLO     *XOM)
      B=.0014*XM   +HA*AEFF*(1.0+0.000252*CLO     *XOM)
      C1A=0.45255+HA*FCL*FEFF*(0.081459*CLO     )+HA*FCL*FEFF*0.032
      C2A=     5.95-35.7*HA*FCL*FEFF
      C3A=1.071*CLO     *HA*AEFF
      CA=C1A*XM +C2A*ADU+C3A
      C=CA
      SP2=EXP(20.93-(5307.02/(273.+T2)))
      PA2=SP2
      W2=(18.*PA2)/(29.*(760.-PA2))
      VSP2=R*TK2/(28.8*(760.-PA2))
      XM2=VENT*28.32/VSP2
      CS2=.238+.46*W2
      DO 200 I=1,2
      TA=(D*(V(I)**.5)-PA*A-C-PA*E*(V(I)**.5))/(B+F*(V(I)**.5))
      SP1=EXP(20.93-(5307.02/(273.+TA)))
      RH=100.*PA/SP1
      W1=(18.*PA)/(29.*(760.-PA))
      CS1=.238+.46*W1
      RHO=1.155
      ARC=4.*3.
      QT=ARC*V(I)
      XMAT=RHO*QT*3600.*1000.
      RATIO=(XM2/XMAT)*100.
      SEN=XM2*CS2*(T2-TA)/1000.
      XLAT=XM2*(W2-W1)*575./1000..
C     PRESSURE GRADIENT IS 0.2 INCHES WATER.
      PG=0.2
C     CONVERT TO PASCALS.
```

```
      PD=PG*243.8
C     CONVERSION FACTOR.
      CF=3.4255
C     FRICTIONAL LOAD RATE -- FN. OF V, KCAL/HR.
      C3=PD*ARO*CF
      S31=C3*V(I)
C     KINETIC ENERGY FACTOR.
      AK=1.003
C     LOCAL ACCELERATION OF GRAVITY.
      AG=1.0
C     KINETIC THERMAL LOAD RATE -- FN. OF V**3,KCAL/HR.
      C4=AK*RHO*ARO*CF/(2.*AG)
      S32=C4*V(I)**3
C     FRICTIONAL + KINETIC LOAD.
      S3=S31+S32
C     TOTAL LOAD.
      T=SEN+XLAT+S3
      S1=SEN
      S2=XLAT
      STOT=T
      WRITE(6,999) T2,CLO,ACT,TAVE,VENT,RATIO
      WRITE(6,939) V(I),PA,TA,STOT,S1,S2,S3
      WRITE(6,933) W1,W2,CS1,CS2,S31,S32
  200 CONTINUE
      WRITE(3,1)
      IF(IX.EQ.4) GO TO 50
      GO TO 10
   50 CONTINUE
      IF(CLO.EQ.0.0) GO TO 12
      IF(CLO.EQ.0.6) GO TO 13
      IF(T2.EQ.40.) GO TO 16
      IF(T2.EQ.50.) GO TO 17
      IF(T2.EQ.35.) GO TO 18
      IF(T2.EQ.20.) GO TO 19
   14 FORMAT(' ',20X'THE END')
      PRINT 14
      STOP
      END

$ENTRY
```

Appendix R

Computer Flow Chart for Lagrange Multiplier and Kuhn-Tucker Conditions Optimization Technique



Appendix S

Computer Program for System II

```
$JOB           LSU,RUN=CHECK,TIME=25,PAGES=150,LINES=60,KP=29
C
C  APPENDIX S. COMPUTER PROGRAM FOR SYSTEM II.
C
C      LAGRANGE OPTIMIZATION TECH. WITH KUHN-TUCKER CONDITIONS.
C      THIS PROGRAM GIVES VALUES OF TEMP., PARTIAL PRESS., AND VELOCITY THAT
C      YIELDS A MINIMUM SUBJECT TO THE COMFORT EQUATION CONSTRAINT, PHYSICAL
C      LIMITATIONS, AND GIVEN PHYSICAL PARAMETERS OF A CONTROL SYSTEM.
       DIMENSION V(2)
       DATA V/0.1,2.6/
   700 FORMAT('1')
       WRITE(6,700)
   999 FORMAT(9F16.2)
   911 FORMAT(9F16.4)
     1 FORMAT(' ')
     2 FORMAT(11X'T2'15X'CLO'12X'ACT'13X'TAVE'12X'VENT'13X'RATIO')
     3 FORMAT(12X'V'16X'PA'13X'TA'14X'S'14X'SENS'13X'LAT'13X'S3'13X'S4')
     4 FORMAT(10X'S31'15X'S32'12X'S5'15X'S6 '14X'SR'14X'SLT')
       WRITE(6,1)
       WRITE(6,2)
       WRITE(6,3)
       WRITE(6,4)
C      SET OUTSIDE TEMP.
       T2=40.
       GO TO 25
    17 T2=35.
       GO TO 25
    18 T2=20.
       GO TO 25
    19 T2=10.
       GO TO 25
    16 T2=50.
    25 CONTINUE
C      SET CLO. VALUE.
       GO TO 11
    13 CLO=1.0
       GO TO 15
    12 CLO=0.6
       GO TO 15
    11 CLO=0.3
    15 CONTINUE
C      SET ACT. LEVEL.
C      SEDENTARY IX=1
C      LOW ACT. IX=2
C      MEDIUM ACT. IX=3
C      HIGH ACT. IX=4
       IX=0
    10 CONTINUE
       IX=IX+1
C      SET VENTILATION REQUIREMENT (CUBIC FEET PER HR.)
       VENT=600.
C      SET UPPER PHYSICAL LIMIT ON DESIRED PARTIAL PRESS. OF WATER VAPOR(MM HG.)
       PA=16.
C      SET INITIAL VALUES EQUAL TO ZERO.
       JT=0.0
       S31=0.0
       S32=0.0
       S3=0.0
       S5=0.0
       TA=0.0
```

```
      IF(CLC.EQ.1.0) TAVE=23.33
      IF(CLC.EQ.0.0) TAVE=27.78
      IF(CLO.EQ.0.5) TAVE=25.56
      ADU=1.8
      FCL=1.1
      Q=1.0
      Q2=1.15
      IF(CLO    .EQ.0.0) FCL=Q
      IF(CLO    .EQ.1.0) FCL=Q2
      GO TO (20,21,22,23),IX
   20 ACT=50.
      FEFF=.55
      GO TO 40
   21 ACT=80.
      FEFF=0.75
      GO TO 40
   22 ACT=100.
      GO TO 40
   23 ACT=150.
   40 TK2=273.+T2
      R=62.361
      AEFF=ADU*FEFF*FCL
      XM=ACT*ADU
      XDM=ACT
      SIG=4.96E-8
      HRA=4.*SIG*(TAVE+273.0)**3
      HA=HRA
      D=371.28*FCL*ADU+10.4*FCL*XM    *(-0.081459*CLO    -0.032)-10.4*1.
     1071*FCL*ADU*CLO
      E=10.4*FCL*ADU*(0.063*CLO     +0.000414*CLO     *XDM)
      F=10.4*FCL*ADU*(1.0+0.000252*CLO     *XDM)
       A=0.35*ADU+.0023*XM  +HA*AEFF*(0.063*CLO     +0.000414*CLO     *XDM)
      B=.0014*XM   +HA*AEFF*(1.0+0.000252*CLO     *XDM)
      C1A=0.45255+HA*FCL*FEFF*(0.021459*CLO     )+HA*FCL*FEFF*0.032
      C2A=    5.95-35.7*HA*FCL*FEFF
      C3A=1.071*CLO     *HA*AEFF
      CA=C1A*XM  +C2A*ADU+C3A
      C=CA
      SP2=EXP(20.93-(5307.02/(273.+T2)))
      PA2=SP2
      W2=(18.*PA2)/(29.*(760.-PA2))
      VSP2=R*TK2/(28.8*(760.-PA2))
      XM2=VENT*28.32/VSP2
      CS2=.238+.46*W2
      AW=3.*5.
      HCI=3.*.253*10.76391*1.8
      AIW=79.
      ATOT=AW+AIW
      U=0.37*0.253*10.76391*1.8
      F1=(U*AW)/(HCI*ATOT)
      F2=(((HCI-U)*AW)/(HCI*ATOT))+(AIW/ATOT)
      WF=3.53*10.76391
      AF=12.
      CN=0.859
      SLT=WF*AF*CN
      TT=0.6
      SF=1.0
      AL=15.*TT
      ERF=2.25*0.253*10.76391
      SL=SF*AL*ERF
```

```
      C4=-U*AW
      C04=U*AW*T2
      A6=-0.35*ADU-0.0023*X4
      D5=D
      E5=-E
      C6=0.4*XF5*XM-5.95*ADU
      A5=-HA*AEFF*(0.063*CLO+0.000414*CLO*XDM)
      G5=-HA*AEFF
      B5=-E-G5
      C15=35.7+XDM*(-0.01459*CLO-0.032)-1.071*CLO
      C5=0.0475*XM+HA*AEFF*C15
      F5=-F
      G=-G5
      CG=C+G*F1
      B1=-B5
      B2=B1+G*F2
      DO 200 I=1,2
      TA=(D*(V(I)**.5)-A*PA-CG-PA*E*(V(I)**.5))/(B2+F*(V(I)**.5))
      SP1=EXP(20.98-(5307.02/(273.+TA)))
      RH=100.*PA/SP1
      W1=(18.*PA)/(29.*(760.-PA))
      CS1=.238+.46*W1
      RHO=1.155
      ARO=4.*3.
      QT=ARO*V(I)
      XMAT=RHO*QT*3600.*1000.
      RATIO=(XM2/XMAT)*100.
      SEN=XM2*CS2*(T2-TA)/1000.
      XLAT=XM2*(W2-W1)*575./1000.
      S1=SEN
      S2=XLAT
C     PRESSURE GRADIENT IS 0.2 INCHES WATER.
C     CONVERT TO PASCALS.
      PD=0.2*248.8
C     CONVERSION FACTOR.
      CF=8.4258
C     FRICTIONAL LOAD RATE -- FN. OF V, KCAL/HR.
      C3=PD*ARO*CF
      S31=C3*V(I)
C     KINETIC ENERGY FACTOR.
      AK=1.003
C     LOCAL ACCELERATION OF GRAVITY.
      AG=1.0
C     KINETIC THERMAL LOAD RATE -- FN. OF V**3,KCAL/HR.
      V4=AK*RHO*ARO*CF/(2.*AG)
      S32=V4*V(I)**3
C     FRICTIONAL + KINETIC LOAD.
      S3=S31+S32
C     FAN LOAD + COIL REMOVAL OF THE THERMAL LOAD.
      S3=2.*S3
      S4=C4*TA+C04
      S5=A5*PA+B5*TA+D5*V(I)**0.5+E5*PA*V(I)**.5+F5*TA*V(I)**.5+G5*(F1*T
     12+F2*TA)+C5
      S6=A6*PA+C6
C     TOTAL LOAD.
      STOT=S1+S2+S3+S4+S5+S6+S8+SLT
      WRITE(6,940) T2,CLO,ACT,TAVE,VENT,RATIO
      WRITE(6,929) V(I),PA,TA,STOT,S1,S2,S3,S4
      WRITE(6,930) S31,S32,S5,S6,S8,SLT
  200 CONTINUE
```

```
      WRITE(5,1)
      IF(IX.EQ.4) GO TO 50
      GO TO 10
   50 CONTINUE
      IF(CLD.EQ.0.0) GO TO 12
      IF(CLD.EQ.0.6) GO TO 13
      IF(T2.EQ.40.) GO TO 16
      IF(T2.EQ.50.) GO TO 17
      IF(T2.EQ.25.) GO TO 13
      IF(T2.EQ.23.) GO TO 19
   14 FORMAT(' ',20X'THE END')
      PRINT 14
      STOP
      END

$ENTRY
```

Appendix T

Fanger's Predicted Mean Vote Equation

Using the seven point psycho-physical ASHRAE scale as a measure for the thermal sensation, Fanger [20] established the approximate exponential relationship between the change in vote per unit change in thermal load and activity level based on his data and on the evidence presented by Nevins et. al [74] and McNall and his co-workers [60]. He derived the Predicted Mean Vote Equation, PMV, of average persons exposed to thermal environments to evaluate comfort from the knowledge of dry bulb temperature, relative air velocity, vapor pressure, and mean radiant temperature along with physical and physiological variables of clothing insulation, clothing temperature and activity level. The vote scale, clothing temperature equation and predicted mean vote equation are presented below. It is noted that at a PMV = 0, the neutral condition of thermal sensation, the predicted mean vote equation reduces to the comfort equation the constraint upon which this work was based.

ASHRAE vote scale:

- 3 cold

- 2 cool

- 1 slightly cool

0 neutral

+ 1 slightly warm

+ 2 warm

+ 3 hot

Clothing Temperature Equation:

$$t_{cl} = 35.7 - 0.032 \frac{M}{A_{Du}} (1-\eta) - 0.18\, I_{cl} \, [3.4 * 10^{-8} f_{cl}$$

$$[(t_{cl} + 273)^4 - (t_{mrt} + 273)^4] + f_{cl}\, h_c\, (t_{cl} - t_a)]$$

where $h_c = 10.4 \sqrt{v}$ in forced convection

$= 2.05\, (t_{cl} - t_a)^{0.25}$ in natural convection

Predicted Mean Vote Equation:

$$PMV = [0.352\, e^{-0.042\, (M/A_{Du})} + 0.032[[M/A_{Du}(1-\eta) - 0.35$$

$$(43 - 0.061 \frac{M}{A_{Du}} (1-\eta) - P_a) - 0.42\, (\frac{M}{AD_u} (1-\eta) - 50)$$

$$- 0.0023 \frac{M}{AD_u} (44 - P_a) - 0.0014 \frac{M}{A_{Du}} (34 - t_a)$$

$$- 3.4 * 10^{-8} f_{cl}\, [(t_{cl} + 273)^4 - (t_{mrt} + 273)^4]$$

$$- f_{cl}\, h_c\, (t_{cl} - t_a)]$$

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VITA

Lionel Robert Whitmer

Candidate for the Degree of

Master of Science

Thesis: AN ANALYSIS OF MINIMIZATION OF ENERGY REQUIREMENTS WHEN THERMAL COMFORT IS MAINTAINED BY AN ENVIRONMENTAL CONTROL SYSTEM

Major Field: Mechanical Engineering with emphasis on Bio-Environmental Engineering

Biographical:

Personal Data: Born October 12, 1945 at Dorrance, Kansas, the son of Robert L. and the late Cora R. Whitmer. Married, wife-Claudia, two youngsters - William and Jennifer.

Education: Graduated from Dorrance Rural High School in 1963; received a B.S.M.E. from Kansas State University in January, 1968; completed requirements for the M.S. degree in Mechanical Engineering at Kansas State University in August, 1974.

Professional: Worked as Summer Technical Student in summer, 1967 for Phillips Petroleum Company's Design Division of the Engineering Department in Bartlesville, Oklahoma; worked as Engineering Trainee and then Production Engineer for Phillips Petroleum Company in the Exploration and Production Department in Oklahoma City, Oklahoma, Great Bend, Kansas, and Casper, Wyoming, from February, 1968 until June, 1971. Student member of ASHRAE, member of Pi Tau Sigma, and member of First United Methodist Church.

AN ANALYSIS OF MINIMIZATION OF ENERGY REQUIREMENTS WHEN THERMAL COMFORT IS MAINTAINED BY AN ENVIRONMENTAL CONTROL SYSTEM

by

LIONEL ROBERT WHITMER

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ABSTRACT

This study presents optimization techniques that are used to determine the optimum combination of environmental variables within an enclosed environmental control system, such that the total energy load required is minimized subject to the constraint of thermal comfort. The analysis makes a steady state investigation of the energy components where the comfort criteria is specified by Fanger's thermal comfort equation.

The constraint, namely, the comfort equation is solved for various cases and the feasible regions are determined for the variables of dry bulb temperature, relative air velocity and partial pressure of water vapor. Besides the physical boundaries that are feasible, limitation of relative air velocity to forced convection values was specified.

Numerical and analytical procedures were used to optimize an environmental control system. The direct search techniques of Hooke and Jeeves and the Simplex Pattern Search were used to find the variable values within a space that would yield the minimum energy load required subject to the comfort constraint and the specified physical restrictions. The method of Lagrange Multipliers and the Kuhn-Tucker Conditions was used in the last part to analytically find the optimal values for two environmental control system models.

The results show that the feasible regions, earmarked by the comfort criteria, specify that only certain combinations of the variables studied will meet the narrow range of values available for each particular case. This is valuable for a quick determination of the variables. As evidenced in the study the intersection of the lower limit on velocity and the upper

limit on vapor pressure, for the given outside conditions, specified the desired dry bulb temperature for comfort and for optimization.

The optimization by the analytical procedure was straight-forward, more reliable and more economical to use than the search techniques. For all the cases investigated, for the given outside conditions, it was found that the optimum combination of the variables studied were the lower constraint placed on air velocity, the upper boundary placed on vapor pressure (or relative humidity), and the dry bulb temperature defined by the comfort equation and the values given above. These optimal values would then minimize the energy load consumption that would be required for the environmental control system. This procedure could be used for many environmental control system models and constitutes a step toward "environmental" comfort optimization as well as total system optimization.