A GENERALIZATION OF DELTA MODULATION

by

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INTRODUCTION

Delta modulation (DM) is a method of encoding analog signals into a digital form for transmission through a data channel. The delta modulated signal is then received and decoded by the demodulator back into its original form.

This report briefly summarizes some of the early probabilistic work in delta modulation. A review and extension of a deterministic approach in a previous report is presented, and a generalization of delta modulation to a system having a five-level output quantization is made. The new system is investigated and compared to the basic delta modulation system.

Basic Delta Modulation

The basic scheme for a delta modulator-demodulator system is shown in Fig. 1.

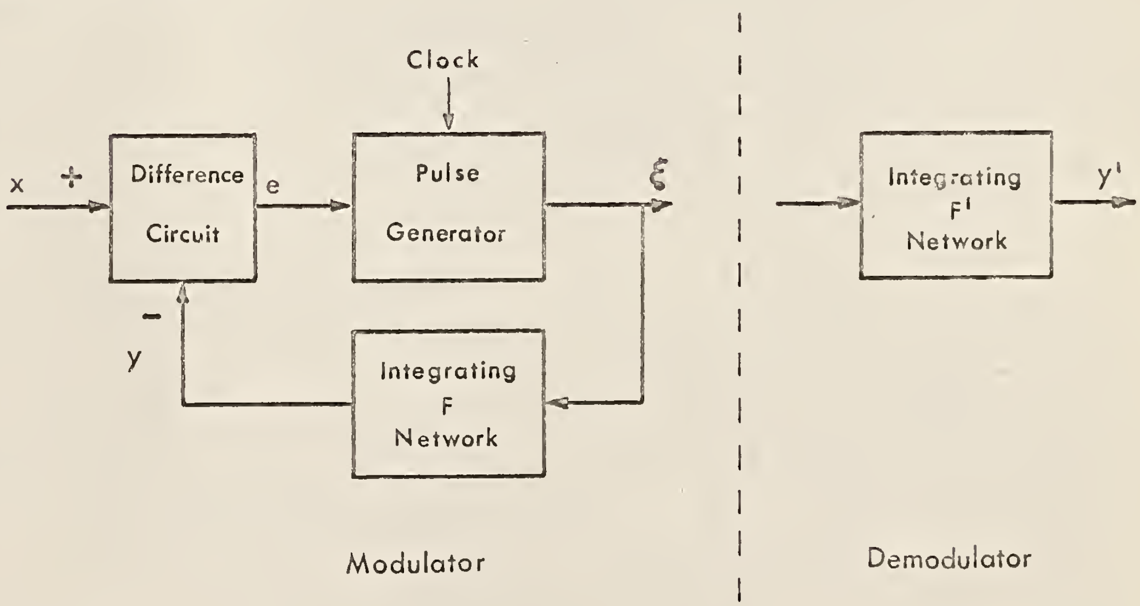

Fig. 1. Basic delta modulator and demodulator.

The output of the modulator, ξ, is a train of single-weighted pulses whose frequency is controlled by the clock and whose sign is determined by the output of the difference circuit, e. This train of pulses, after being sent across the data channel, is integrated by the demodulator at F' to produce a stepped waveform, y', that is a quantized representation of the input signal, x. An identical integrator in the modulator, F, produces a similar signal, y, that is compared to the input to produce the error signal, e, that controls the pulse generator.

Figure 2 shows an analog waveform and its quantized representation after passing through a perfect delta modulation system. Also shown is the output pulse train, ξ.

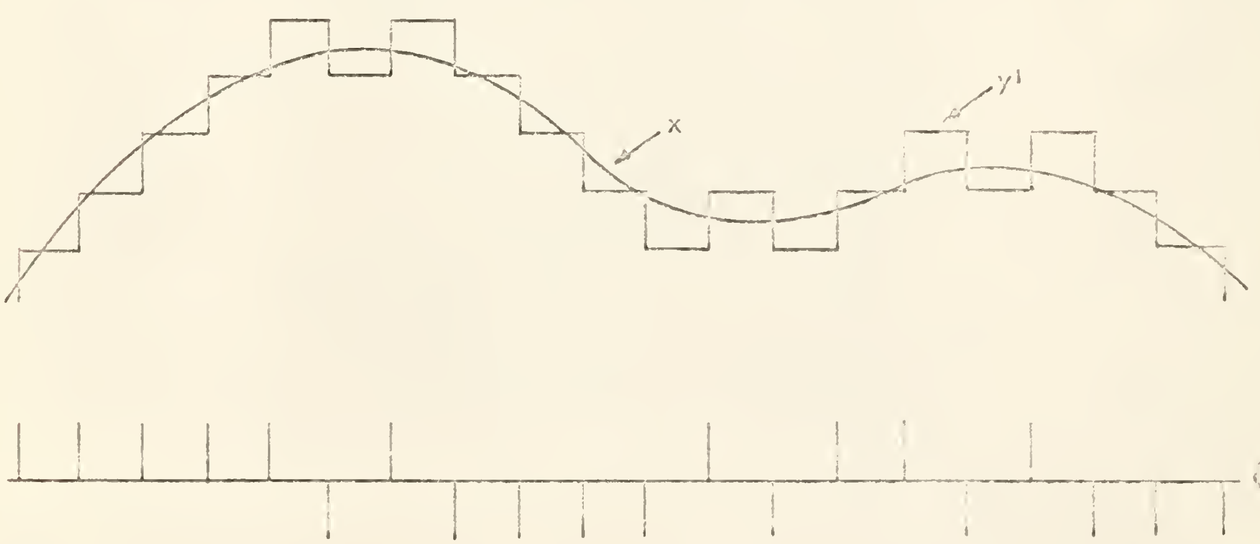

Fig. 2. Modulator input and modulator and demodulator output signals.

Pulse Code Modulation

The current standard, and perhaps most widely used form of analog-to-digital coding, is pulse code modulation (PCM).

Compared to delta modulation, pulse code modulation is a much more complicated and sophisticated scheme (Fig. 3).

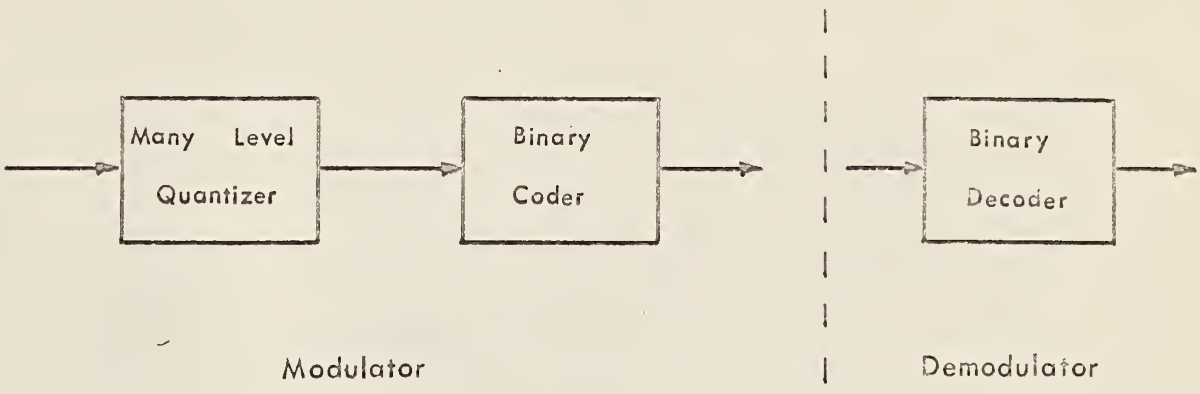

Fig. 3. Pulse code modulator and demodulator.

The input signal is constantly sampled and the sampled values quantized and assigned to one of a finite number of levels. The level number is then coded into binary form and the corresponding set of pulses is sent out. Typically, a PCM system might have 256 levels which would require an eight-digit code. Level 43, for example, would be represented by the pulse set 00101011.

The PCM demodulator consists of a binary decoder. The set of pulses corresponding to one sample point is sorted out, given binary weighting, and added to reconstruct the sample point. These are usually smoothed with a low-pass filter.

PREVIOUS WORK

F. de Jager's Paper

Delta modulation was first described by Deloraine, Van Merlo, and DerJavitch in a 1946 French patent (2). The simplicity of single-weighted pulse systems aroused interest in delta modulation. In 1952 F. de Jager (3) presented a qualitative description which formed a basis for much of the early work in delta modulation. He found that a single integrating network for F gave satisfactory results, and tried cascading a second integrating network with the first in order to improve the signal-to-noise ratio. Although this resulted in noise reduction, it also gave rise to stability problems in the higher frequency range. He compromised by using a single integrator followed by a low-pass filter with cutoff at 3800 cps. Actually, the integrator was a shallow-skirted low-pass filter with cutoff at 200 cps. de Jager found that the signal-to-noise ratio varies as the 3/2 power of the pulse frequency for single integration and as the 5/2 power for "double integration." He stated that the bandwidth for DM is about fifty per cent greater than that for PCM, and the quality of DM is comparable to an eight-digit PCM.

L. H. Zetterberg's Paper

In 1954 L. H. Zetterberg (8) published a paper in which he compared delta modulation to pulse code modulation. In a comparison of channel capacities, Zetterberg found that

$$C = \frac{1}{T} \log 2$$

for PCM, and

$$C = \frac{1}{T} \log (2 \cos \pi/n+1)$$

for DM. C is the channel capacity, T the pulse interval, and n the number of quantizing levels. For the same pulse frequency, the difference between the channel capacity for DM and PCM is less than five per cent when the number of quantizing levels is greater than ten. In a practical application, n will be greater than thirty, and the channel capacities will be essentially equivalent.

Zetterberg also performed an error analysis in which he classified errors as "sampling errors" and "quantization errors." Sampling inherently results in errors that can be reduced mainly by increasing the sampling rate. Quantization errors were subdived into (1) those resulting from overloading and (2) those caused by granulation.

For DM, overloading occurs when the input level is greater than the saturation level, or when the slope of the input signal is greater than the height of one quantizing level. For PCM, overloading occurs when the amplitude of the input signal is greater than the maximum of the quantizing code.

Zetterberg calculated signal-to-noise ratios and found that for certain cases the effect of granulation in DM is very much less than in PCM. Also, in higher frequencies, overloading in DM will be less troublesome than in PCM.

Zetterberg found that bandwidth required for DM is approximately twice that for PCM when the frequency range of the input signal is about 3.5 kc.

B. L. Barber's Report

In 1961 B. L. Barber (1) demonstrated that delta modulation is derived from pulse duration modulation (PDM) and pulse amplitude modulation (PAM). Previously, the only comparisons were to pulse code modulation for matters of convenience. For pulse duration, the input signal is added to a triangular wave (see Fig. 4). The sum is fed to a signum device, the output of which is the PDM signal. Demodulation is accomplished by averaging the modulated signal over the period of the triangular wave. This can be realized by a holding filter with holding time equal to the period of the triangular wave.

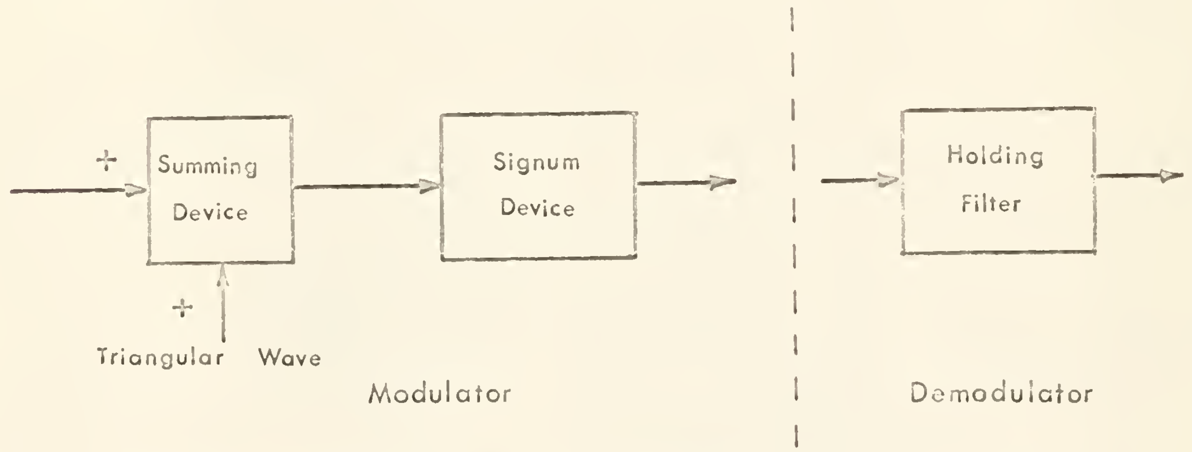

Fig. 4. Pulse duration modulator and demodulator.

Pulse amplitude modulation (Fig. 5) is accomplished by periodically sampling the input signal. The demodulator is ideally a steeply-skirted low-pass filter whose cutoff frequency is 1/T. T is the period of the sampling function.

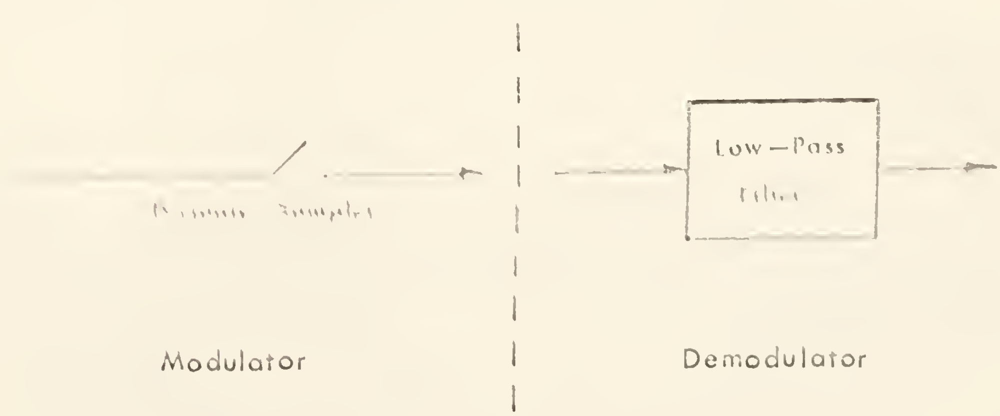

Fig. 5. Pulse amplitude modulator and demodulator.

Barber proposed to let the PDM and its demodulator be separated by a PAM and its demodulator as shown in Fig. 6. This he called pulse duration-amplitude modulation (PDAM).

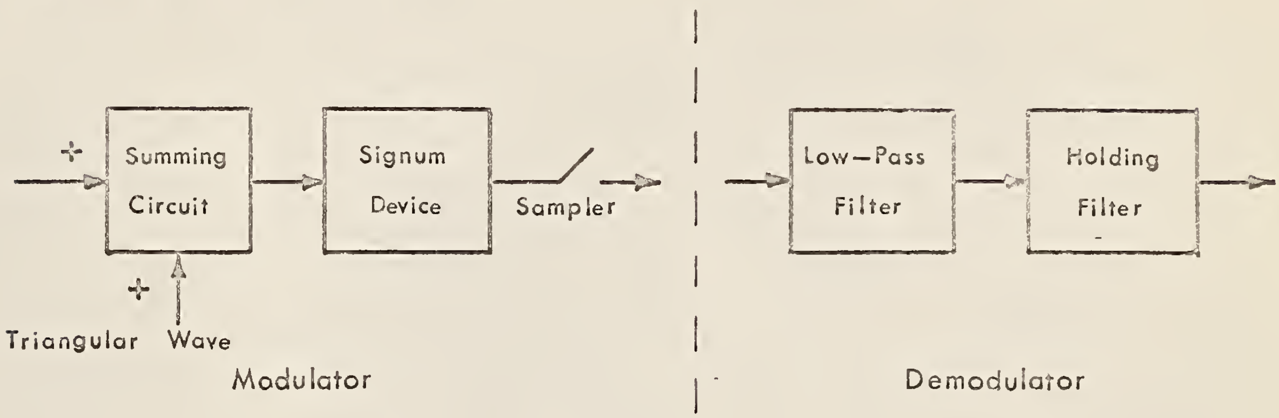

Fig. 6. Pulse duration-amplitude modulator and demodulator.

If the triangular wave is removed from the PDAM modulator, the weakened system can be improved by providing negative feedback so that the output of the holding filter replaces the triangular wave. This arrangement, shown in Fig. 7, is indeed delta modulation.

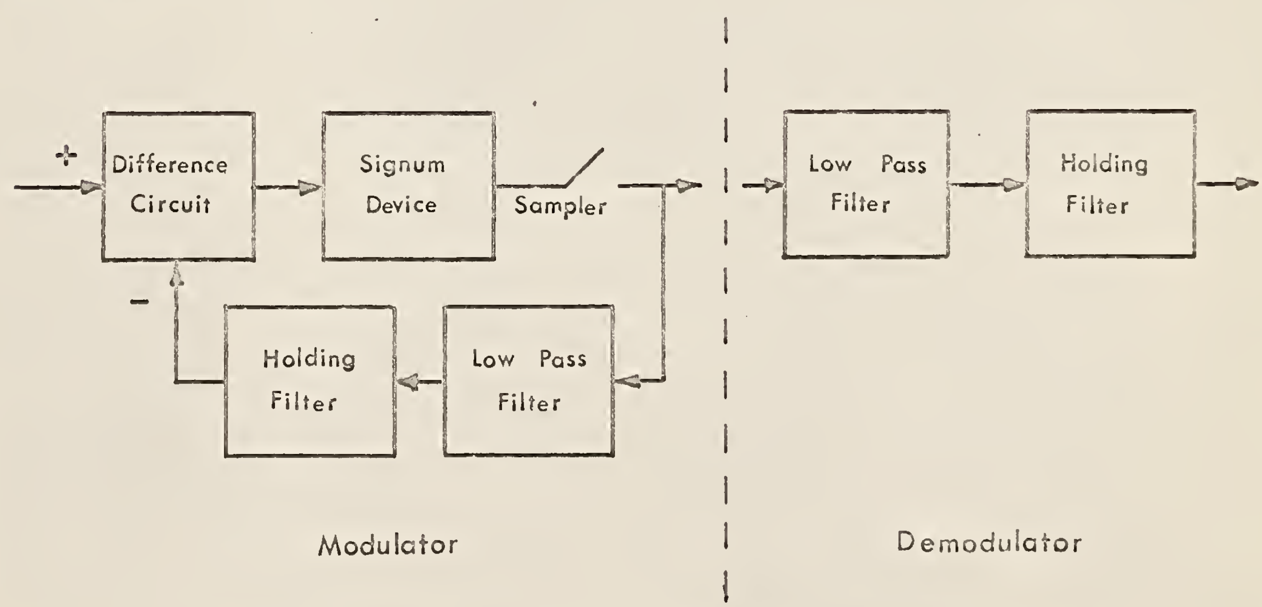

Fig. 7. Delta modulation as derived from PDAM.

J. S. Tripp's Report

Z-Transform Description of Delta Modulator-Demodulator Model

J. S. Tripp (7) used Barber's description of DM as a basis for devising a mathematical model with which he performed a deterministic analysis. The nature of the system lends itself particularly well to Z-transforms. The PAM modulator becomes the sampling operator Z. The demodulator was degraded, because of reasons of stability, to a shallow-skirted low-pass filter represented in Z-transform notation by (1-z)/s. The holding filter is represented by $(1-z^m)/mTs$ where T is the period of the sampling operator and mT (m an integer) is the period of the triangular wave now removed from the system. Note that $z = e^{-sT}$. The complete Z-transform model is shown in Fig. 8.

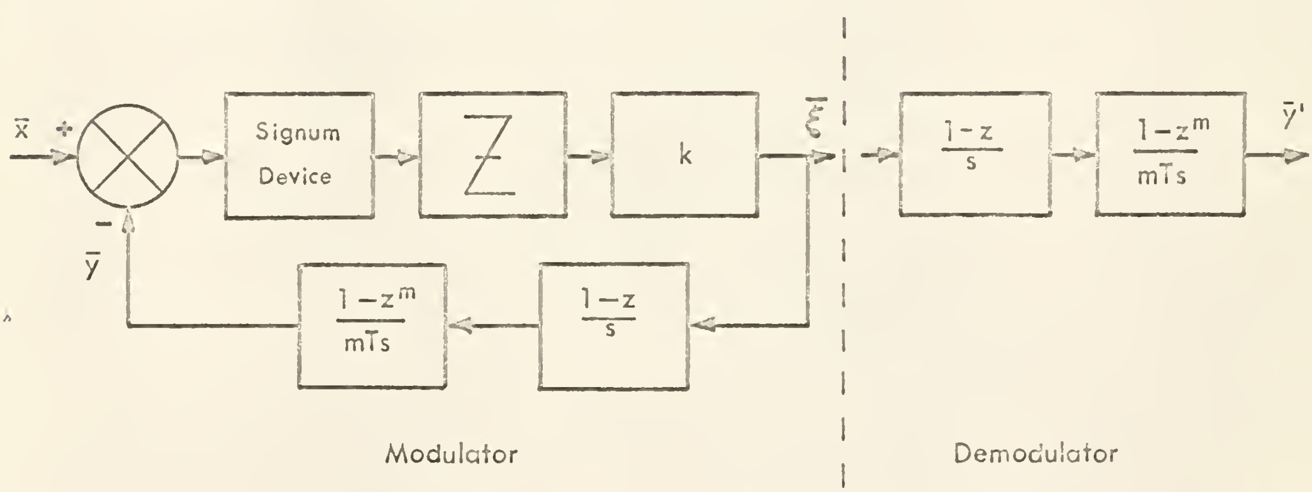

Fig. 8. Z-transform model of delta modulation system.

The k was added to represent the gain of the modulator, and the signum function will be defined later. The derivation of a recursion formula for the output of the demodulator proceeds directly as follows.

Recursion Relation for Model's Output

Let the Laplace-transformed values of x and y be $\bar{x}$ and $\bar{y}$. The input to the signum device is $(\bar{x} - \bar{y})$, and

$$\bar{y} = \left[\frac{1-z^m}{mTs}\right]\left[\frac{1-z}{s}\right] kZ \left[\operatorname{sgn}(\bar{x} - \bar{y})\right].$$

Note that Z may be taken inside the signum function since the order of sampling and quantization is immaterial. The Z-transformed equation becomes

$$Z\bar{y} = \frac{k}{mT} Z\left[\frac{1}{s^2} (1-z^m)(1-z) \operatorname{sgn} (Z\bar{x} - Z\bar{y})\right].$$

By use of the relation

$$Z\left\{F(s)\cdot Z\left[G(s)\right]\right\} = Z\left[F(s)\right]\cdot Z\left[G(s)\right],$$

it follows that

$$Z\bar{y} = \frac{k}{mT}\left[Z\left(\frac{1}{s^2}\right)\right]\left[(1-z^m)(1-z) \operatorname{sgn} (Z\bar{x} - Z\bar{y})\right].$$

Now $\quad Z\left(\frac{1}{s^2}\right) = \frac{Tz}{(1-z)^2}$.

Then

$$Z\bar{y} = \frac{k}{m}\left(\frac{1-z^m}{1-z}\right) z \operatorname{sgn} (Z\bar{x} - Z\bar{y}),$$

or

$$Z\bar{y} = zZ\bar{y} + \frac{k}{m}\left[z \operatorname{sgn} (Z\bar{x} - Z\bar{y}) - z^{m+1} \operatorname{sgn} (Z\bar{x} - Z\bar{y})\right].$$

The delay operator, z, may be taken inside the signum function.

$$Z\bar{y} - zZ\bar{y} = \frac{k}{m}\left[\operatorname{sgn} (zZ\bar{x} - zZ\bar{y}) - \operatorname{sgn} (z^{m+1} Z\bar{x} - z^{m+1} Z\bar{y})\right].$$

By writing this equation in terms of the sampled and delayed values of x and y, one obtains the following recursion relation,

$$y_n = y_{n-1} + \frac{k}{m}\left[\operatorname{sgn}(x_{n-1} - y_{n-1}) - \operatorname{sgn}(x_{n-m-1} - y_{n-m-1})\right].$$

It should be noted that this recursion relation yields solution values at only the sampled instants. A complete solution would be difficult to obtain directly by any method, but it may be reasoned that the output of the demodulator will be continuous at all points. Straight line interpolation between the sampled instants will not be erroneous for this model.

Tripp also made a Z-transform calculation for the modulator output, ξ. This was unnecessary since the modulator output is known to be the sampled output of the signum device. Calculation of y_n will also yield $e_n = x_n - y_n$ and one has only to take $\operatorname{sgn}(e_n)$ to find the modulator output.

It should also be noted that it was not necessary to specify anything about the behavior of the input function, x, for the development of the recursion relation. The final solution requires knowledge of x at the sampling instants only.

Definition of Signum Function

Tripp chose to define the signum function as follows:

$$\operatorname{sgn}(e) = \begin{cases} 1, & e > 0 \\ 0, & e = 0 \\ -1, & e < 0. \end{cases}$$

This does not agree completely with the conventional delta modulators which are binary in nature. A DC level is usually represented by a series of pulses of alternating sign. It is not necessary, however, to supply both pulse polarities; the effect of one pulse polarity can be achieved by intentionally making the integrator memory short by use of an RC passive integrator with a discharge resistor. With this scheme, a specific number of pulses is required to hold the integrator voltage at a given DC level. This simplifies the pulse generating circuitry at the expense of increased bandwidth, and is usually referred to as modified delta modulation.

AN EXTENSION OF TRIPP'S WORK

Results for Step Inputs

Tripp reasoned that an improvement in performance could be expected in the three-level device over the two-level device, and proceeded to calculate response and signal-to-noise values for step inputs. Tripp confined his investigation to inputs that were powers of two; m and k were also powers of two. He found the output to be nearly that of a servo for input values between k/m and k. Below x = k/m there was no response, and the system saturated for x>k. The system response also demonstrated predictable fluctuations for the cases he studied. Generalizing Tripp's results, one finds that after building up linearly to the input value, the output remains constant until the $(m+1)^{st}$ sample. There it drops to (x - k/m) for a specific number of intervals before returning to x, where it remains until the $2(m+1)^{st}$ sample. Here again it drops to (x - k/m) as before and the procedure seems to be repeated indefinitely. The period is m+1 and the number of drops are xm/k.

For powers of two inputs, the average output signal is m/(m+1) times the input. This could be made to be exactly the value of the input by means of a suitable gain, (m+1)/m, in the demodulator. This system behavior permitted Tripp to formulate an empirical formula for the signal-to-noise ratio for his special set of input functions.

The noise signal was defined as the deviation of the output signal from its average. The rms value of the noise signal, N, is given by

$$N = \frac{x}{(m+1)} \sqrt{(m+1) \cdot \frac{k}{xm} - 1}.$$

The signal-to-noise ratio is then

$$S/N = \frac{m}{\sqrt{(m+1) \cdot \frac{k}{xm} - 1}}.$$

A closer investigation revealed that the model behaves in this manner for all input values that are integer multiples of k/m. For inputs that are not integer multiples of k/m, however, the model behaves quite differently. The output builds up linearly to the input value and then oscillates about that value with modulator output pulses alternating negative and positive. The system is always periodic in m+1 samples for step inputs. The number of negative pulses, n, (for positive inputs) is the smallest integer equal to or greater than

$$\frac{m}{2} \cdot \frac{(k-x)}{k}.$$

The undershoot is x - (m+1 - 2n) · (k/m), and the overshoot is k/m - undershoot.

From these relationships, calculations could be made for signal-to-noise ratios as before, and an empirical formula derived. Because of the complexity of the resulting formula, however, these calculations were done by a digital computer.

Results show that the system operates in two modes. Mode 1 occurs when the input value is an integer multiple of k/m, and the signal-to-noise ratios behave as Tripp predicted for his specialized case. The system lapses into mode 2 when the input is not an integer multipe of k/m. The signal-to-noise values for each mode may be plotted as a smooth curve as shown in Fig. 9. It is easily seen that the oscillations of the output about the input value in mode 2 cause a marked degeneration in signal-to-noise quality.

Tables 1 and 2 show typical behavior of the three-level model for type 1 and type 2 inputs. Table 3 gives a complete listing of steady state signal-to-noise values. A value of m = 64 was chosen for uniformity in all examples. It is understood, however, that the signal-to-noise ratios also vary as some power of m. For small inputs, S/N will vary as $m^{1/2}$; for large inputs as $m^{3/2}$.

It should be noted that the signal value was taken to be the average value of the output signal over a period of m+1 samples. This cannot be completely justified for the second mode of operation, but was done to facilitate comparison of the two modes.

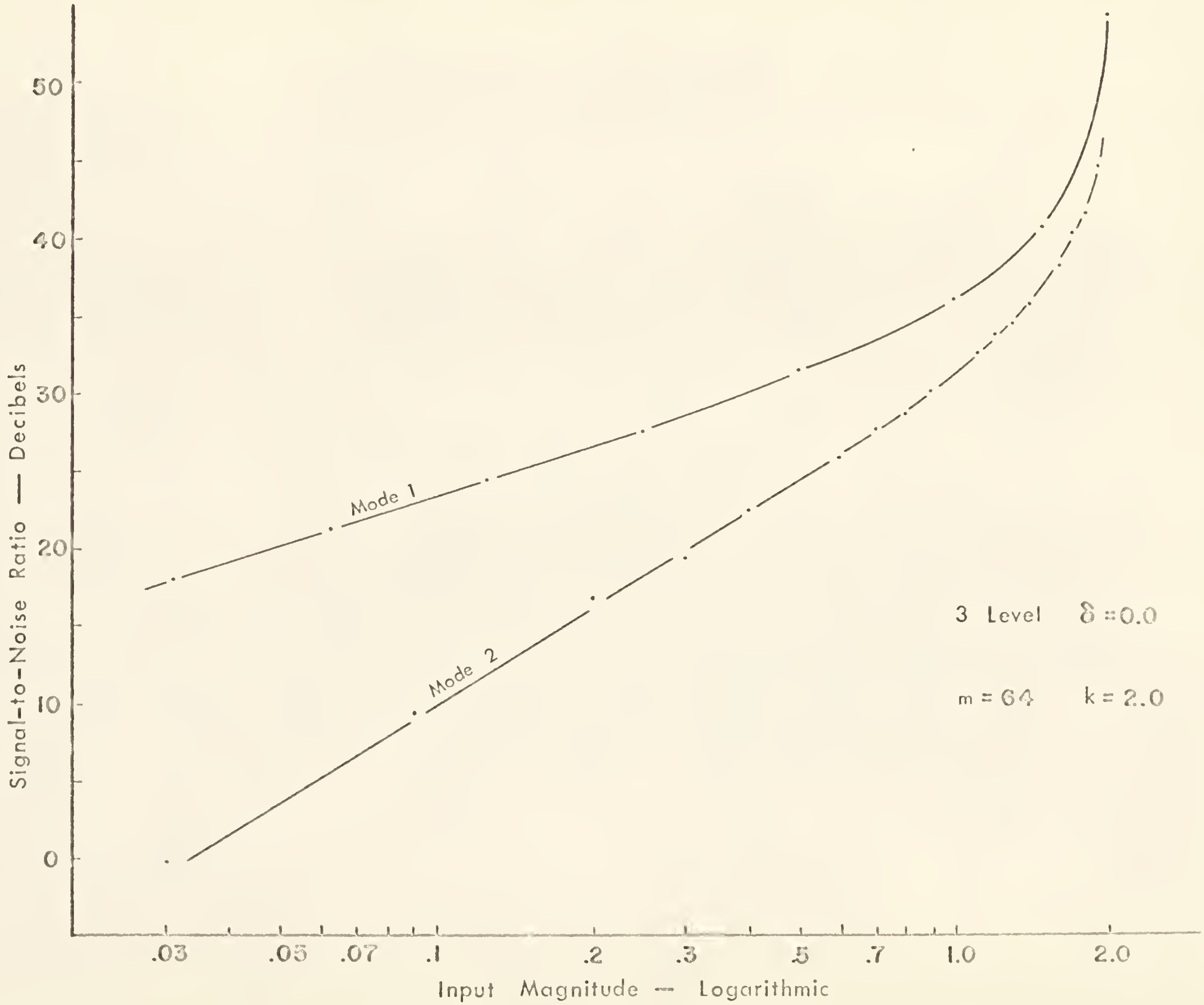

Fig. 9. Signal-to-noise ratio vs. step input magnitude.

TABLE 1

3 LEVEL MOD-DEMOD MODEL .12500000 STEP INP

M = 64 GAIN = 2.00000000 DEAD SPACE = .000000

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(
0	.0000000	.1250000	1.0	65	.0937500	.0312500	1
1	.0312500	.0937500	1.0	66	.0937500	.0312500	1
2	.0625000	.0625000	1.0	67	.0937500	.0312500	1
3	.0937500	.0312500	1.0	68	.0937500	.0312500	1
4	.1250000	.0000000	.0	69	.1250000	.0000000	
5	.1250000	.0000000	.0	70	.1250000	.0000000	
6	.1250000	.0000000	.0	71	.1250000	.0000000	
7	.1250000	.0000000	.0	72	.1250000	.0000000	
8	.1250000	.0000000	.0	73	.1250000	.0000000	
9	.1250000	.0000000	.0	74	.1250000	.0000000	
10	.1250000	.0000000	.0	75	.1250000	.0000000	
11	.1250000	.0000000	.0	76	.1250000	.0000000	
12	.1250000	.0000000	.0	77	.1250000	.0000000	
13	.1250000	.0000000	.0	78	.1250000	.0000000	
14	.1250000	.0000000	.0	79	.1250000	.0000000	
15	.1250000	.0000000	.0	80	.1250000	.0000000	
16	.1250000	.0000000	.0	81	.1250000	.0000000	
17	.1250000	.0000000	.0	82	.1250000	.0000000	
18	.1250000	.0000000	.0	83	.1250000	.0000000	
19	.1250000	.0000000	.0	84	.1250000	.0000000	
20	.1250000	.0000000	.0	85	.1250000	.0000000	
21	.1250000	.0000000	.0	86	.1250000	.0000000	
22	.1250000	.0000000	.0	87	.1250000	.0000000	
23	.1250000	.0000000	.0	88	.1250000	.0000000	
24	.1250000	.0000000	.0	89	.1250000	.0000000	
25	.1250000	.0000000	.0	90	.1250000	.0000000	
26	.1250000	.0000000	.0	91	.1250000	.0000000	
27	.1250000	.0000000	.0	92	.1250000	.0000000	
28	.1250000	.0000000	.0	93	.1250000	.0000000	
29	.1250000	.0000000	.0	94	.1250000	.0000000	
30	.1250000	.0000000	.0	95	.1250000	.0000000	
31	.1250000	.0000000	.0	96	.1250000	.0000000	
32	.1250000	.0000000	.0	97	.1250000	.0000000	
33	.1250000	.0000000	.0	98	.1250000	.0000000	
34	.1250000	.0000000	.0	99	.1250000	.0000000	
35	.1250000	.0000000	.0	100	.1250000	.0000000	
36	.1250000	.0000000	.0	101	.1250000	.0000000	
37	.1250000	.0000000	.0	102	.1250000	.0000000	
38	.1250000	.0000000	.0	103	.1250000	.0000000	
39	.1250000	.0000000	.0	104	.1250000	.0000000	
40	.1250000	.0000000	.0	105	.1250000	.0000000	
41	.1250000	.0000000	.0	106	.1250000	.0000000	
42	.1250000	.0000000	.0	107	.1250000	.0000000	
43	.1250000	.0000000	.0	108	.1250000	.0000000	
44	.1250000	.0000000	.0	109	.1250000	.0000000	
45	.1250000	.0000000	.0	110	.1250000	.0000000	
46	.1250000	.0000000	.0	111	.1250000	.0000000	
47	.1250000	.0000000	.0	112	.1250000	.0000000	
48	.1250000	.0000000	.0	113	.1250000	.0000000	
49	.1250000	.0000000	.0	114	.1250000	.0000000	
50	.1250000	.0000000	.0	115	.1250000	.0000000	
51	.1250000	.0000000	.0	116	.1250000	.0000000	
52	.1250000	.0000000	.0	117	.1250000	.0000000	
53	.1250000	.0000000	.0	118	.1250000	.0000000	
54	.1250000	.0000000	.0	119	.1250000	.0000000	
55	.1250000	.0000000	.0	120	.1250000	.0000000	
56	.1250000	.0000000	.0	121	.1250000	.0000000	
57	.1250000	.0000000	.0	122	.1250000	.0000000	
58	.1250000	.0000000	.0	123	.1250000	.0000000	
59	.1250000	.0000000	.0	124	.1250000	.0000000	
60	.1250000	.0000000	.0	125	.1250000	.0000000	
61	.1250000	.0000000	.0	126	.1250000	.0000000	
62	.1250000	.0000000	.0	127	.1250000	.0000000	
63	.1250000	.0000000	.0	128	.1250000	.0000000	
64	.1250000	.0000000	.0	129	.1250000	.0000000	

TABLE 1 (continued)

3 LEVEL MOD-DEMOD MODEL .12500000 STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .00000000

N	Y(N)	E(N)	S(N)
30	.0937500	.0312500	1.0
31	.0937500	.0312500	1.0
32	.0937500	.0312500	1.0
33	.0937500	.0312500	1.0
34	.1250000	.0000000	.0
35	.1250000	.0000000	.0
36	.1250000	.0000000	.0
37	.1250000	.0000000	.0
38	.1250000	.0000000	.0
39	.1250000	.0000000	.0
40	.1250000	.0000000	.0
41	.1250000	.0000000	.0
42	.1250000	.0000000	.0
43	.1250000	.0000000	.0
44	.1250000	.0000000	.0
45	.1250000	.0000000	.0
46	.1250000	.0000000	.0
47	.1250000	.0000000	.0
48	.1250000	.0000000	.0
49	.1250000	.0000000	.0
50	.1250000	.0000000	.0
51	.1250000	.0000000	.0
52	.1250000	.0000000	.0
53	.1250000	.0000000	.0
54	.1250000	.0000000	.0
55	.1250000	.0000000	.0
56	.1250000	.0000000	.0
57	.1250000	.0000000	.0
58	.1250000	.0000000	.0
59	.1250000	.0000000	.0
60	.1250000	.0000000	.0
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78	.1250000	.0000000	.0
79	.1250000	.0000000	.0
80	.1250000	.0000000	.0
81	.1250000	.0000000	.0
82	.1250000	.0000000	.0
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84	.1250000	.0000000	.0
85	.1250000	.0000000	.0
86	.1250000	.0000000	.0
87	.1250000	.0000000	.0
88	.1250000	.0000000	.0
89	.1250000	.0000000	.0
90	.1250000	.0000000	.0
91	.1250000	.0000000	.0
92	.1250000	.0000000	.0
93	.1250000	.0000000	.0
94	.1250000	.0000000	.0

N	Y(N)	E(N)	S(N)
195	.0937500	.0312500	1.0
196	.0937500	.0312500	1.0
197	.0937500	.0312500	1.0
198	.0937500	.0312500	1.0
199	.1250000	.0000000	.0
200	.1250000	.0000000	.0
201	.1250000	.0000000	.0
202	.1250000	.0000000	.0
203	.1250000	.0000000	.0
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251	.1250000	.0000000	.0
252	.1250000	.0000000	.0
253	.1250000	.0000000	.0
254	.1250000	.0000000	.0
255	.1250000	.0000000	.0
256	.1250000	.0000000	.0
257	.1250000	.0000000	.0
258	.1250000	.0000000	.0
259	.1250000	.0000000	.0

TABLE 2

3 LEVEL MOD-DEMOD MODEL .10000000 STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .00000000

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
0	.0000000	.1000000	1.0	65	.0625000	.0375000	1.0
1	.0312500	.0687500	1.0	66	.0625000	.0375000	1.0
2	.0625000	.0375000	1.0	67	.0625000	.0375000	1.0
3	.0937500	.0062500	1.0	68	.0625000	.0375000	1.0
4	.1250000	-.0250000	-1.0	69	.1250000	-.0250000	-1.0
5	.0937500	.0062500	1.0	70	.0625000	.0375000	1.0
6	.1250000	-.0250000	-1.0	71	.1250000	-.0250000	-1.0
7	.0937500	.0062500	1.0	72	.0625000	.0375000	1.0
8	.1250000	-.0250000	-1.0	73	.1250000	-.0250000	-1.0
9	.0937500	.0062500	1.0	74	.0625000	.0375000	1.0
10	.1250000	-.0250000	-1.0	75	.1250000	-.0250000	-1.0
11	.0937500	.0062500	1.0	76	.0625000	.0375000	1.0
12	.1250000	-.0250000	-1.0	77	.1250000	-.0250000	-1.0
13	.0937500	.0062500	1.0	78	.0625000	.0375000	1.0
14	.1250000	-.0250000	-1.0	79	.1250000	-.0250000	-1.0
15	.0937500	.0062500	1.0	80	.0625000	.0375000	1.0
16	.1250000	-.0250000	-1.0	81	.1250000	-.0250000	-1.0
17	.0937500	.0062500	1.0	82	.0625000	.0375000	1.0
18	.1250000	-.0250000	-1.0	83	.1250000	-.0250000	-1.0
19	.0937500	.0062500	1.0	84	.0625000	.0375000	1.0
20	.1250000	-.0250000	-1.0	85	.1250000	-.0250000	-1.0
21	.0937500	.0062500	1.0	86	.0625000	.0375000	1.0
22	.1250000	-.0250000	-1.0	87	.1250000	-.0250000	-1.0
23	.0937500	.0062500	1.0	88	.0625000	.0375000	1.0
24	.1250000	-.0250000	-1.0	89	.1250000	-.0250000	-1.0
25	.0937500	.0062500	1.0	90	.0625000	.0375000	1.0
26	.1250000	-.0250000	-1.0	91	.1250000	-.0250000	-1.0
27	.0937500	.0062500	1.0	92	.0625000	.0375000	1.0
28	.1250000	-.0250000	-1.0	93	.1250000	-.0250000	-1.0
29	.0937500	.0062500	1.0	94	.0625000	.0375000	1.0
30	.1250000	-.0250000	-1.0	95	.1250000	-.0250000	-1.0
31	.0937500	.0062500	1.0	96	.0625000	.0375000	1.0
32	.1250000	-.0250000	-1.0	97	.1250000	-.0250000	-1.0
33	.0937500	.0062500	1.0	98	.0625000	.0375000	1.0
34	.1250000	-.0250000	-1.0	99	.1250000	-.0250000	-1.0
35	.0937500	.0062500	1.0	100	.0625000	.0375000	1.0
36	.1250000	-.0250000	-1.0	101	.1250000	-.0250000	-1.0
37	.0937500	.0062500	1.0	102	.0625000	.0375000	1.0
38	.1250000	-.0250000	-1.0	103	.1250000	-.0250000	-1.0
39	.0937500	.0062500	1.0	104	.0625000	.0375000	1.0
40	.1250000	-.0250000	-1.0	105	.1250000	-.0250000	-1.0
41	.0937500	.0062500	1.0	106	.0625000	.0375000	1.0
42	.1250000	-.0250000	-1.0	107	.1250000	-.0250000	-1.0
43	.0937500	.0062500	1.0	108	.0625000	.0375000	1.0
44	.1250000	-.0250000	-1.0	109	.1250000	-.0250000	-1.0
45	.0937500	.0062500	1.0	110	.0625000	.0375000	1.0
46	.1250000	-.0250000	-1.0	111	.1250000	-.0250000	-1.0
47	.0937500	.0062500	1.0	112	.0625000	.0375000	1.0
48	.1250000	-.0250000	-1.0	113	.1250000	-.0250000	-1.0
49	.0937500	.0062500	1.0	114	.0625000	.0375000	1.0
50	.1250000	-.0250000	-1.0	115	.1250000	-.0250000	-1.0
51	.0937500	.0062500	1.0	116	.0625000	.0375000	1.0
52	.1250000	-.0250000	-1.0	117	.1250000	-.0250000	-1.0
53	.0937500	.0062500	1.0	118	.0625000	.0375000	1.0
54	.1250000	-.0250000	-1.0	119	.1250000	-.0250000	-1.0
55	.0937500	.0062500	1.0	120	.0625000	.0375000	1.0
56	.1250000	-.0250000	-1.0	121	.1250000	-.0250000	-1.0
57	.0937500	.0062500	1.0	122	.0625000	.0375000	1.0
58	.1250000	-.0250000	-1.0	123	.1250000	-.0250000	-1.0
59	.0937500	.0062500	1.0	124	.0625000	.0375000	1.0
60	.1250000	-.0250000	-1.0	125	.1250000	-.0250000	-1.0
61	.0937500	.0062500	1.0	126	.0625000	.0375000	1.0
62	.1250000	-.0250000	-1.0	127	.1250000	-.0250000	-1.0
63	.0937500	.0062500	1.0	128	.0625000	.0375000	1.0
64	.1250000	-.0250000	-1.0	129	.1250000	-.0250000	-1.0

TABLE 2 (continued)

3 LEVEL MOD-DEMOD MODEL .10000000 STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .00000000

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
130	.0625000	.0375000	1.0	195	.0625000	.0375000	1.0
131	.0625000	.0375000	1.0	196	.0625000	.0375000	1.0
132	.0625000	.0375000	1.0	197	.0625000	.0375000	1.0
133	.0625000	.0375000	1.0	198	.0625000	.0375000	1.0
134	.1250000	-.0250000	-1.0	199	.1250000	-.0250000	-1.0
135	.0625000	.0375000	1.0	200	.0625000	.0375000	1.0
136	.1250000	-.0250000	-1.0	201	.1250000	-.0250000	-1.0
137	.0625000	.0375000	1.0	202	.0625000	.0375000	1.0
138	.1250000	-.0250000	-1.0	203	.1250000	-.0250000	-1.0
139	.0625000	.0375000	1.0	204	.0625000	.0375000	1.0
140	.1250000	-.0250000	-1.0	205	.1250000	-.0250000	-1.0
141	.0625000	.0375000	1.0	206	.0625000	.0375000	1.0
142	.1250000	-.0250000	-1.0	207	.1250000	-.0250000	-1.0
143	.0625000	.0375000	1.0	208	.0625000	.0375000	1.0
144	.1250000	-.0250000	-1.0	209	.1250000	-.0250000	-1.0
145	.0625000	.0375000	1.0	210	.0625000	.0375000	1.0
146	.1250000	-.0250000	-1.0	211	.1250000	-.0250000	-1.0
147	.0625000	.0375000	1.0	212	.0625000	.0375000	1.0
148	.1250000	-.0250000	-1.0	213	.1250000	-.0250000	-1.0
149	.0625000	.0375000	1.0	214	.0625000	.0375000	1.0
150	.1250000	-.0250000	-1.0	215	.1250000	-.0250000	-1.0
151	.0625000	.0375000	1.0	216	.0625000	.0375000	1.0
152	.1250000	-.0250000	-1.0	217	.1250000	-.0250000	-1.0
153	.0625000	.0375000	1.0	218	.0625000	.0375000	1.0
154	.1250000	-.0250000	-1.0	219	.1250000	-.0250000	-1.0
155	.0625000	.0375000	1.0	220	.0625000	.0375000	1.0
156	.1250000	-.0250000	-1.0	221	.1250000	-.0250000	-1.0
157	.0625000	.0375000	1.0	222	.0625000	.0375000	1.0
158	.1250000	-.0250000	-1.0	223	.1250000	-.0250000	-1.0
159	.0625000	.0375000	1.0	224	.0625000	.0375000	1.0
160	.1250000	-.0250000	-1.0	225	.1250000	-.0250000	-1.0
161	.0625000	.0375000	1.0	226	.0625000	.0375000	1.0
162	.1250000	-.0250000	-1.0	227	.1250000	-.0250000	-1.0
163	.0625000	.0375000	1.0	228	.0625000	.0375000	1.0
164	.1250000	-.0250000	-1.0	229	.1250000	-.0250000	-1.0
165	.0625000	.0375000	1.0	230	.0625000	.0375000	1.0
166	.1250000	-.0250000	-1.0	231	.1250000	-.0250000	-1.0
167	.0625000	.0375000	1.0	232	.0625000	.0375000	1.0
168	.1250000	-.0250000	-1.0	233	.1250000	-.0250000	-1.0
169	.0625000	.0375000	1.0	234	.0625000	.0375000	1.0
170	.1250000	-.0250000	-1.0	235	.1250000	-.0250000	-1.0
171	.0625000	.0375000	1.0	236	.0625000	.0375000	1.0
172	.1250000	-.0250000	-1.0	237	.1250000	-.0250000	-1.0
173	.0625000	.0375000	1.0	238	.0625000	.0375000	1.0
174	.1250000	-.0250000	-1.0	239	.1250000	-.0250000	-1.0
175	.0625000	.0375000	1.0	240	.0625000	.0375000	1.0
176	.1250000	-.0250000	-1.0	241	.1250000	-.0250000	-1.0
177	.0625000	.0375000	1.0	242	.0625000	.0375000	1.0
178	.1250000	-.0250000	-1.0	243	.1250000	-.0250000	-1.0
179	.0625000	.0375000	1.0	244	.0625000	.0375000	1.0
180	.1250000	-.0250000	-1.0	245	.1250000	-.0250000	-1.0
181	.0625000	.0375000	1.0	246	.0625000	.0375000	1.0
182	.1250000	-.0250000	-1.0	247	.1250000	-.0250000	-1.0
183	.0625000	.0375000	1.0	248	.0625000	.0375000	1.0
184	.1250000	-.0250000	-1.0	249	.1250000	-.0250000	-1.0
185	.0625000	.0375000	1.0	250	.0625000	.0375000	1.0
186	.1250000	-.0250000	-1.0	251	.1250000	-.0250000	-1.0
187	.0625000	.0375000	1.0	252	.0625000	.0375000	1.0
188	.1250000	-.0250000	-1.0	253	.1250000	-.0250000	-1.0
189	.0625000	.0375000	1.0	254	.0625000	.0375000	1.0
190	.1250000	-.0250000	-1.0	255	.1250000	-.0250000	-1.0
191	.0625000	.0375000	1.0	256	.0625000	.0375000	1.0
192	.1250000	-.0250000	-1.0	257	.1250000	-.0250000	-1.0
193	.0625000	.0375000	1.0	258	.0625000	.0375000	1.0
194	.1250000	-.0250000	-1.0	259	.1250000	-.0250000	-1.0

TABLE 3

3 LEVEL MOD-DEMOD MODEL ERROR DATA STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = 0.00000000

INPUT	AVERAGE	MAX ERROR	RMS ERROR	S/N RATIO	IN DB
.030000	.030769	-.032500	.031246	.985	-.133
.031250	.030769	.031250	.003846	8.000	18.061
.040000	.030769	.040000	.031246	.985	-.133
.050000	.030769	.050000	.031246	.985	-.133
.060000	.030769	.060000	.031246	.985	-.133
.062500	.061539	.031250	.005397	11.403	21.141
.070000	.092308	-.055000	.031217	2.957	9.417
.080000	.092308	-.045000	.031217	2.957	9.417
.090000	.092308	-.035000	.031217	2.957	9.417
.100000	.092308	.037500	.031217	2.957	9.417
.125000	.123077	.031250	.007509	16.389	24.291
.200000	.215384	-.050000	.031068	6.932	16.818
.250000	.246154	.031250	.010267	23.977	27.596
.300000	.276923	.050000	.030949	8.947	19.034
.400000	.400000	-.037500	.030619	13.064	22.321
.500000	.492308	.031250	.013461	36.571	31.263
.600000	.584615	.037500	.029885	19.562	25.828
.700000	.707692	-.050000	.029228	24.212	27.681
.800000	.769231	.050000	.028846	26.667	28.519
.900000	.892308	-.037500	.027967	31.905	30.077
1.000000	.984615	.031250	.015623	63.023	35.990
1.100000	1.076923	.037500	.026333	40.896	32.234
1.200000	1.261539	-.050000	.025000	48.000	33.624
1.300000	1.261539	.050000	.024249	52.024	34.324
1.400000	1.384615	-.037500	.022550	61.402	35.764
1.500000	1.476923	.031250	.013734	107.542	40.632
1.600000	1.569231	.037500	.019374	80.995	38.169
1.700000	1.692308	-.050000	.016654	101.614	40.139
1.800000	1.753846	.050000	.015020	116.770	41.347
1.900000	1.876923	-.037500	.010793	173.898	44.806
2.000000	1.969231	.031250	.003846	512.000	54.185

Signum Function with Dead Space

Since the oscillatory behavior in mode 2 results from the input not being an integer multiple of the output, one would suspect that a small dead space in the signum function would improve the S/N characteristics of the system. A new signum function with dead space could be defined as follows. The width of the dead space is 2δ.

$$\mathrm{sgn}\ (e) = \begin{cases} 1, & e > \delta \\ 0, & |e| \leq \delta \\ -1, & e \leq -\delta \end{cases}$$

Tripp investigated this possibility, again using inputs of powers of two as test cases. He concluded that the addition of a dead space offered no advantage since it had no effect for $\delta<k/m$ and reduced the output values for $\delta \geq k/m$. For inputs that are not multiples of k/m, however, a considerable improvement can be seen. An optimum value of $\delta = k/2m$ was chosen. This is a logical value since the total deviation of the output about the input value, x, is k/m; the chosen value of δ will place the output within x at least once in its oscillation about x. This causes a signum value of 0 to be taken, and minimizes the fluctuations of the output with an increased signal-to-noise ratio. Figure 10 shows the S/N curve for dead space added. It will be noted that the entire curve falls on the line for mode 1 without dead space.

Table 4 shows the behavior of the output for a case that operated in mode 2 before the dead space was added. Mode 1 cases are unaffected by the dead space. Table 5 gives a list of steady state signal-to-noise values with dead space added.

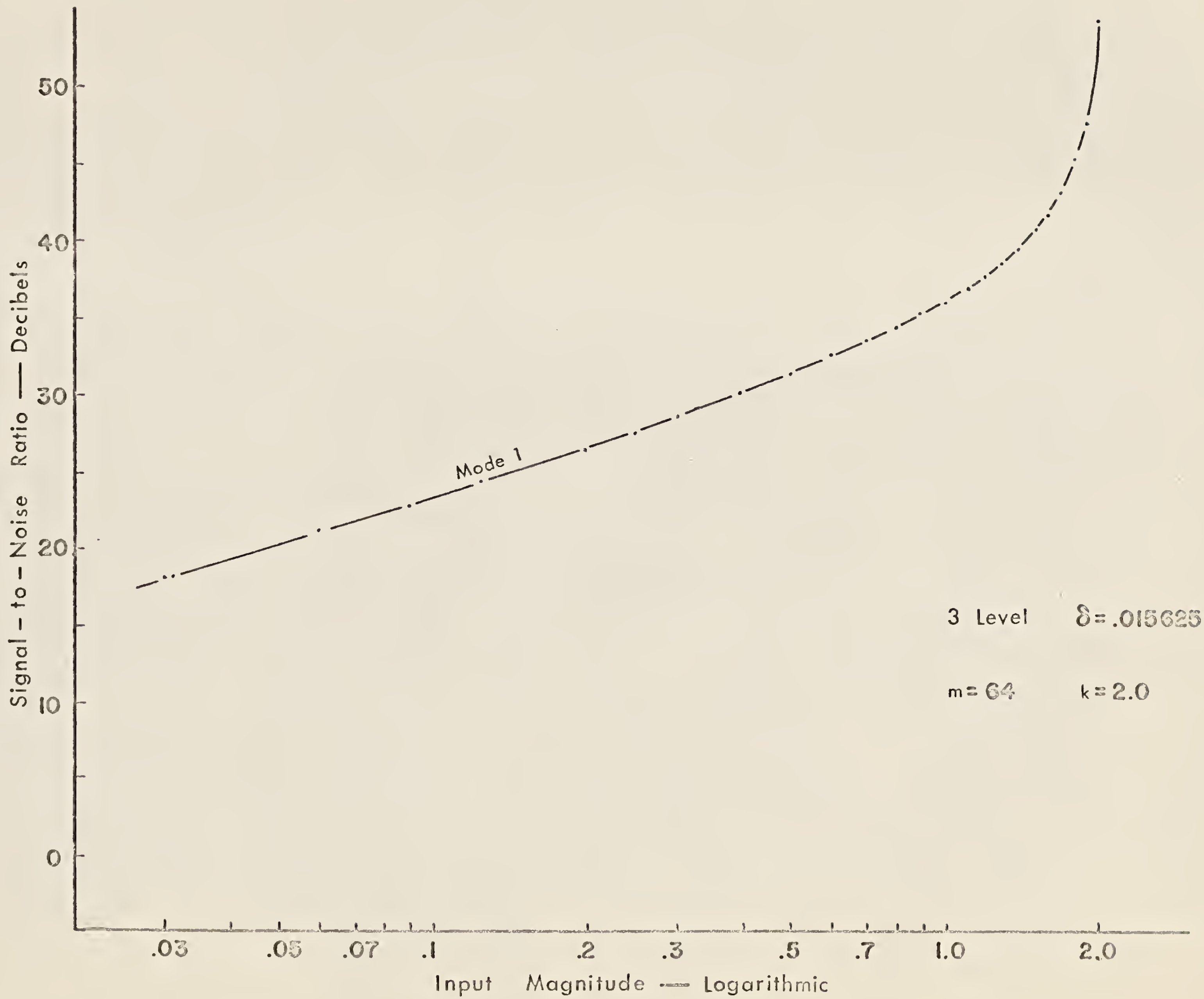

Fig. 10. Signal-to-noise ratio vs. step input magnitude.

Results for Ramp Inputs

One is now concerned with the behavior of the delta modulation system for higher order input functions. A small change was required in the basic computer program to give ramp output values. The slope of the ramp was defined to be the change of input to the model in (m+1)T seconds. The sign of the slope was initially positive and changed each time a saturation value was reached.

Results showed that, for the basic three-level device, the output oscillated regularly about the input value with pattern changes every m+1 intervals. Generally, the fluctuations became more violent each period of m+1 samples.

Figures 11-13 show output behavior for the first three periods for a typical input value. Table 6 is a complete tabulation of the output values. In general, the average error remains constant within each section of m+1 samples. To further aid in the analysis, the average error and standard deviation of error were calculated for each period of m+1 samples. The average error over one period was defined to be

$$\langle e_i \rangle = \frac{1}{m+1} \sum_{n=(i-1)(m+1)}^{i(m+1)} (x_n - y_n) = \frac{1}{m+1} \sum_{n=(i-1)(m+1)}^{i(m+1)} e_n \quad , \; i = 1,2,3\ldots$$

The standard deviation is then

$$\sigma_i = \sqrt{\frac{1}{m+1} \sum_{n=(i-1)(m+1)}^{i(m+1)} \cdot e_n^2 - \langle e_i \rangle^2}, \; i = 1,2,3\ldots$$

Standard deviation of error, which is a measure of noise, is plotted in Fig. 14 for the first three periods over a wide

TABLE 4

3 LEVEL MOD-DEMOD MODEL .10000000 STEP INPU

M = 64 GAIN = 2.00000000 DEAD SPACE = .0156250

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N
0	.0000000	.1000000	1.0	65	.0625000	.0375000	1.
1	.0312500	.0687500	1.0	66	.0625000	.0375000	1.
2	.0625000	.0375000	1.0	67	.0625000	.0375000	1.
3	.0937500	.0062500	.0	68	.0937500	.0062500	.
4	.0937500	.0062500	.0	69	.0937500	.0062500	.
5	.0937500	.0062500	.0	70	.0937500	.0062500	.
6	.0937500	.0062500	.0	71	.0937500	.0062500	.
7	.0937500	.0062500	.0	72	.0937500	.0062500	.
8	.0937500	.0062500	.0	73	.0937500	.0062500	.
9	.0937500	.0062500	.0	74	.0937500	.0062500	.
10	.0937500	.0062500	.0	75	.0937500	.0062500	.
11	.0937500	.0062500	.0	76	.0937500	.0062500	.
12	.0937500	.0062500	.0	77	.0937500	.0062500	.
13	.0937500	.0062500	.0	78	.0937500	.0062500	.
14	.0937500	.0062500	.0	79	.0937500	.0062500	.
15	.0937500	.0062500	.0	80	.0937500	.0062500	.
16	.0937500	.0062500	.0	81	.0937500	.0062500	.
17	.0937500	.0062500	.0	82	.0937500	.0062500	.
18	.0937500	.0062500	.0	83	.0937500	.0062500	.
19	.0937500	.0062500	.0	84	.0937500	.0062500	.
20	.0937500	.0062500	.0	85	.0937500	.0062500	.
21	.0937500	.0062500	.0	86	.0937500	.0062500	.
22	.0937500	.0062500	.0	87	.0937500	.0062500	.
23	.0937500	.0062500	.0	88	.0937500	.0062500	.
24	.0937500	.0062500	.0	89	.0937500	.0062500	.
25	.0937500	.0062500	.0	90	.0937500	.0062500	.
26	.0937500	.0062500	.0	91	.0937500	.0062500	.
27	.0937500	.0062500	.0	92	.0937500	.0062500	.
28	.0937500	.0062500	.0	93	.0937500	.0062500	.
29	.0937500	.0062500	.0	94	.0937500	.0062500	.
30	.0937500	.0062500	.0	95	.0937500	.0062500	.
31	.0937500	.0062500	.0	96	.0937500	.0062500	.
32	.0937500	.0062500	.0	97	.0937500	.0062500	.
33	.0937500	.0062500	.0	98	.0937500	.0062500	.
34	.0937500	.0062500	.0	99	.0937500	.0062500	.
35	.0937500	.0062500	.0	100	.0937500	.0062500	.
36	.0937500	.0062500	.0	101	.0937500	.0062500	.
37	.0937500	.0062500	.0	102	.0937500	.0062500	.
38	.0937500	.0062500	.0	103	.0937500	.0062500	.
39	.0937500	.0062500	.0	104	.0937500	.0062500	.
40	.0937500	.0062500	.0	105	.0937500	.0062500	.
41	.0937500	.0062500	.0	106	.0937500	.0062500	.
42	.0937500	.0062500	.0	107	.0937500	.0062500	.
43	.0937500	.0062500	.0	108	.0937500	.0062500	.
44	.0937500	.0062500	.0	109	.0937500	.0062500	.
45	.0937500	.0062500	.0	110	.0937500	.0062500	.
46	.0937500	.0062500	.0	111	.0937500	.0062500	.
47	.0937500	.0062500	.0	112	.0937500	.0062500	.
48	.0937500	.0062500	.0	113	.0937500	.0062500	.
49	.0937500	.0062500	.0	114	.0937500	.0062500	.
50	.0937500	.0062500	.0	115	.0937500	.0062500	.
51	.0937500	.0062500	.0	116	.0937500	.0062500	.
52	.0937500	.0062500	.0	117	.0937500	.0062500	.
53	.0937500	.0062500	.0	118	.0937500	.0062500	.
54	.0937500	.0062500	.0	119	.0937500	.0062500	.
55	.0937500	.0062500	.0	120	.0937500	.0062500	.
56	.0937500	.0062500	.0	121	.0937500	.0062500	.
57	.0937500	.0062500	.0	122	.0937500	.0062500	.
58	.0937500	.0062500	.0	123	.0937500	.0062500	.
59	.0937500	.0062500	.0	124	.0937500	.0062500	.
60	.0937500	.0062500	.0	125	.0937500	.0062500	.
61	.0937500	.0062500	.0	126	.0937500	.0062500	.
62	.0937500	.0062500	.0	127	.0937500	.0062500	.
63	.0937500	.0062500	.0	128	.0937500	.0062500	.
64	.0937500	.0062500	.0	129	.0937500	.0062500	.

TABLE 4 (continued)

3 LEVEL MOD-DEMOD MODEL .10000000 STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .01562500

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
130	.0625000	.0375000	1.0	195	.0625000	.0375000	1.0
131	.0625000	.0375000	1.0	196	.0625000	.0375000	1.0
132	.0625000	.0375000	1.0	197	.0625000	.0375000	1.0
133	.0937500	.0062500	.0	198	.0937500	.0062500	.0
134	.0937500	.0062500	.0	199	.0937500	.0062500	.0
135	.0937500	.0062500	.0	200	.0937500	.0062500	.0
136	.0937500	.0062500	.0	201	.0937500	.0062500	.0
137	.0937500	.0062500	.0	202	.0937500	.0062500	.0
138	.0937500	.0062500	.0	203	.0937500	.0062500	.0
139	.0937500	.0062500	.0	204	.0937500	.0062500	.0
140	.0937500	.0062500	.0	205	.0937500	.0062500	.0
141	.0937500	.0062500	.0	206	.0937500	.0062500	.0
142	.0937500	.0062500	.0	207	.0937500	.0062500	.0
143	.0937500	.0062500	.0	208	.0937500	.0062500	.0
144	.0937500	.0062500	.0	209	.0937500	.0062500	.0
145	.0937500	.0062500	.0	210	.0937500	.0062500	.0
146	.0937500	.0062500	.0	211	.0937500	.0062500	.0
147	.0937500	.0062500	.0	212	.0937500	.0062500	.0
148	.0937500	.0062500	.0	213	.0937500	.0062500	.0
149	.0937500	.0062500	.0	214	.0937500	.0062500	.0
150	.0937500	.0062500	.0	215	.0937500	.0062500	.0
151	.0937500	.0062500	.0	216	.0937500	.0062500	.0
152	.0937500	.0062500	.0	217	.0937500	.0062500	.0
153	.0937500	.0062500	.0	218	.0937500	.0062500	.0
154	.0937500	.0062500	.0	219	.0937500	.0062500	.0
155	.0937500	.0062500	.0	220	.0937500	.0062500	.0
156	.0937500	.0062500	.0	221	.0937500	.0062500	.0
157	.0937500	.0062500	.0	222	.0937500	.0062500	.0
158	.0937500	.0062500	.0	223	.0937500	.0062500	.0
159	.0937500	.0062500	.0	224	.0937500	.0062500	.0
160	.0937500	.0062500	.0	225	.0937500	.0062500	.0
161	.0937500	.0062500	.0	226	.0937500	.0062500	.0
162	.0937500	.0062500	.0	227	.0937500	.0062500	.0
163	.0937500	.0062500	.0	228	.0937500	.0062500	.0
164	.0937500	.0062500	.0	229	.0937500	.0062500	.0
165	.0937500	.0062500	.0	230	.0937500	.0062500	.0
166	.0937500	.0062500	.0	231	.0937500	.0062500	.0
167	.0937500	.0062500	.0	232	.0937500	.0062500	.0
168	.0937500	.0062500	.0	233	.0937500	.0062500	.0
169	.0937500	.0062500	.0	234	.0937500	.0062500	.0
170	.0937500	.0062500	.0	235	.0937500	.0062500	.0
171	.0937500	.0062500	.0	236	.0937500	.0062500	.0
172	.0937500	.0062500	.0	237	.0937500	.0062500	.0
173	.0937500	.0062500	.0	238	.0937500	.0062500	.0
174	.0937500	.0062500	.0	239	.0937500	.0062500	.0
175	.0937500	.0062500	.0	240	.0937500	.0062500	.0
176	.0937500	.0062500	.0	241	.0937500	.0062500	.0
177	.0937500	.0062500	.0	242	.0937500	.0062500	.0
178	.0937500	.0062500	.0	243	.0937500	.0062500	.0
179	.0937500	.0062500	.0	244	.0937500	.0062500	.0
180	.0937500	.0062500	.0	245	.0937500	.0062500	.0
181	.0937500	.0062500	.0	246	.0937500	.0062500	.0
182	.0937500	.0062500	.0	247	.0937500	.0062500	.0
183	.0937500	.0062500	.0	248	.0937500	.0062500	.0
184	.0937500	.0062500	.0	249	.0937500	.0062500	.0
185	.0937500	.0062500	.0	250	.0937500	.0062500	.0
186	.0937500	.0062500	.0	251	.0937500	.0062500	.0
187	.0937500	.0062500	.0	252	.0937500	.0062500	.0
188	.0937500	.0062500	.0	253	.0937500	.0062500	.0
189	.0937500	.0062500	.0	254	.0937500	.0062500	.0
190	.0937500	.0062500	.0	255	.0937500	.0062500	.0
191	.0937500	.0062500	.0	256	.0937500	.0062500	.0
192	.0937500	.0062500	.0	257	.0937500	.0062500	.0
193	.0937500	.0062500	.0	258	.0937500	.0062500	.0
194	.0937500	.0062500	.0	259	.0937500	.0062500	.0

TABLE 5

3 LEVEL MOD-DEMOD MODEL ERROR DATA STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = 0.01562500

INPUT	AVERAGE	MAX ERROR	RMS ERROR	S/N RATIO	IN DB
.030000	.030770	.030000	.003846	8.000	18.062
.031250	.030770	.031250	.003846	8.000	18.062
.040000	.030770	.040000	.003846	8.000	18.062
.050000	.061539	.018750	.005397	11.403	21.141
.060000	.061539	.028750	.005397	11.403	21.141
.062500	.061539	.031250	.005397	11.403	21.141
.070000	.061539	.038750	.005397	11.403	21.141
.080000	.092308	.017500	.006557	14.078	22.971
.090000	.092308	.027500	.006557	14.078	22.971
.100000	.092308	.037500	.006557	14.078	22.971
.125000	.123077	.031250	.007510	16.389	24.291
.200000	.184615	.043750	.009045	20.409	26.197
.250000	.246153	.031250	.010267	23.977	27.596
.300000	.307692	.018750	.011275	27.290	28.720
.400000	.400000	.025000	.012500	32.000	30.103
.500000	.492308	.031250	.013462	36.571	31.263
.600000	.584615	.037500	.014213	41.131	32.283
.700000	.676923	.043750	.014787	45.778	33.213
.800000	.800000	.018750	.015309	52.255	34.363
.900000	.892308	.025000	.015534	57.442	35.185
1.000000	.984615	.031250	.015623	63.023	35.990
1.100000	1.076923	.037500	.015578	69.128	36.793
1.200000	1.169231	.043750	.015399	75.926	37.601
1.300000	1.292308	.018750	.014943	86.485	38.739
1.400000	1.384615	.025000	.014423	96.000	39.645
1.500000	1.476923	.031250	.013733	107.542	40.632
1.600000	1.569231	.037500	.012846	122.152	41.738
1.700000	1.661539	.043750	.011717	141.801	43.034
1.800000	1.784615	.018750	.009687	184.223	45.307
1.900000	1.876923	.025000	.007509	249.928	47.956
2.000000	1.969231	.031250	.003846	512.000	54.185

EXPLANATION OF PLATE I

Three-level delta modulation, $\delta = 0.0$, $m = 64$, $k = 2.0$

Output for ramp input slope 0.1

Fig. 11. Period 1 (samples 0-64)

Fig. 12. Period 2 (samples 65-129)

Fig. 13. Period 3 (samples 130-194)

PLATE I

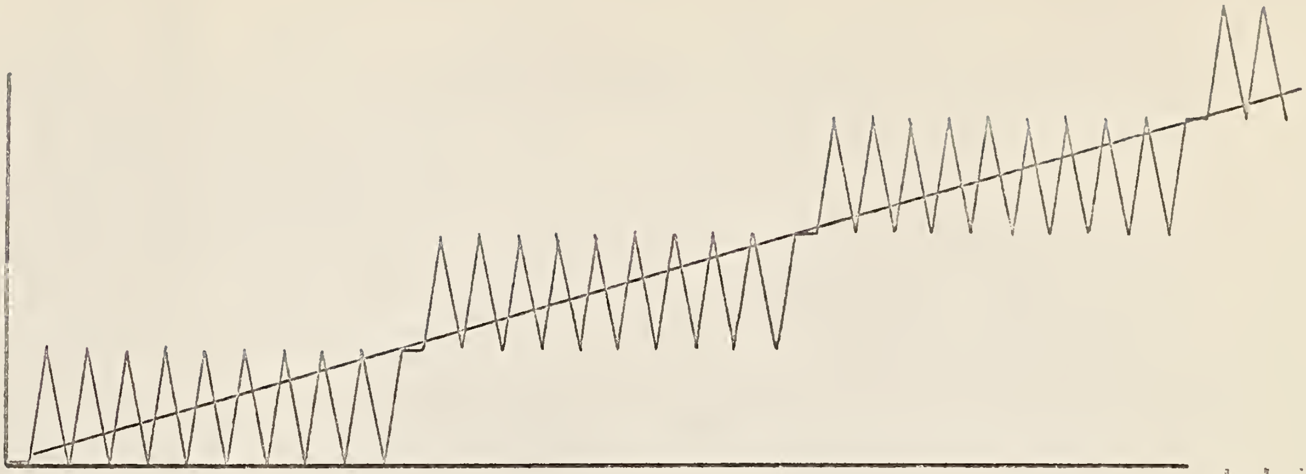

Fig. 11

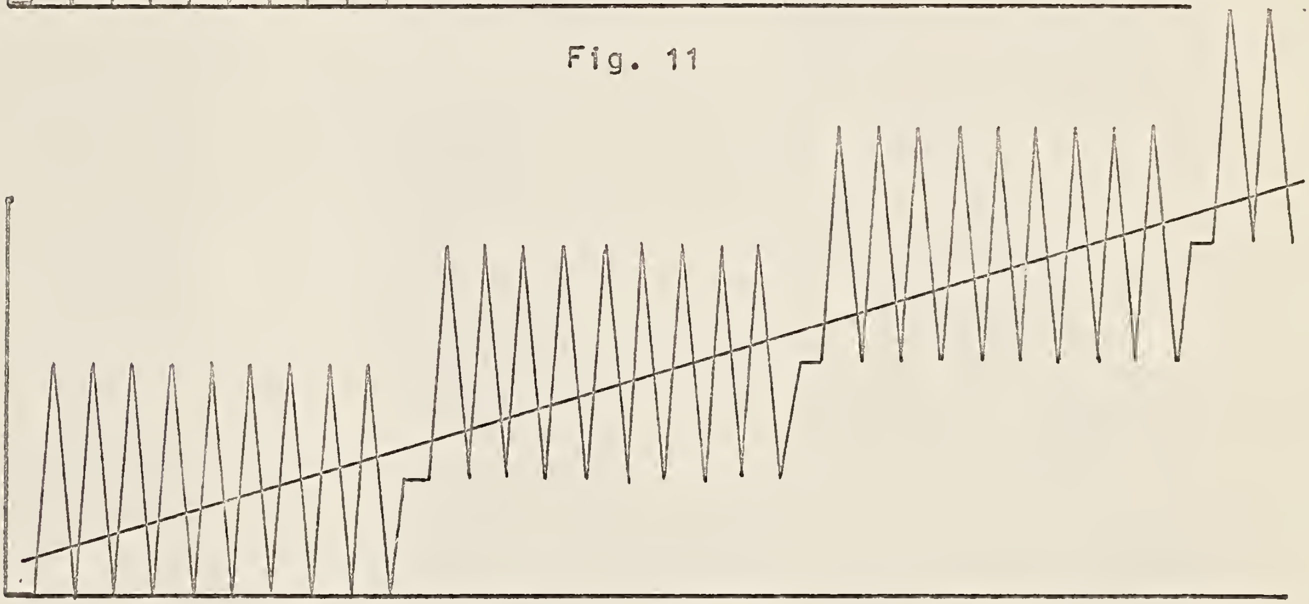

Fig. 12

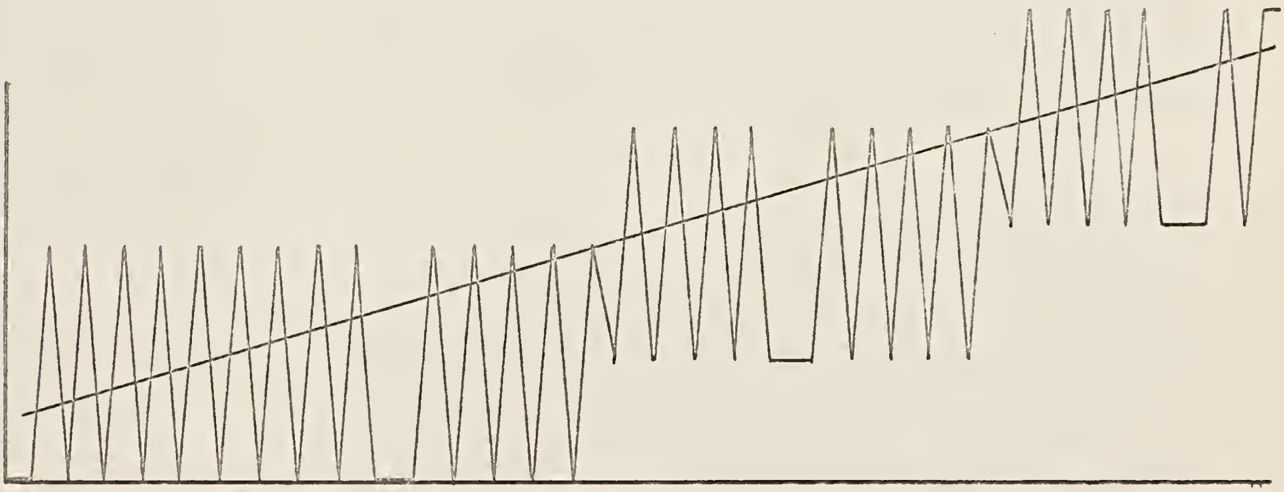

Fig. 13

TABLE 6

3 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .00000000

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
0	.0000000	.0000000	.0	65	.0937500	.0078125	1.0
1	.0000000	.0015625	1.0	66	.0937500	.0093750	1.0
2	.0312500	-.0281250	-1.0	67	.1562500	-.0515625	-1.0
3	.0000000	.0046875	1.0	68	.0937500	.0125000	1.0
4	.0312500	-.0250000	-1.0	69	.1562500	-.0484375	-1.0
5	.0000000	.0078125	1.0	70	.0937500	.0156250	1.0
6	.0312500	-.0218750	-1.0	71	.1562500	-.0453125	-1.0
7	.0000000	.0109375	1.0	72	.0937500	.0187500	1.0
8	.0312500	-.0187500	-1.0	73	.1562500	-.0421875	-1.0
9	.0000000	.0140625	1.0	74	.0937500	.0218750	1.0
10	.0312500	-.0156250	-1.0	75	.1562500	-.0390625	-1.0
11	.0000000	.0171875	1.0	76	.0937500	.0250000	1.0
12	.0312500	-.0125000	-1.0	77	.1562500	-.0359375	-1.0
13	.0000000	.0203125	1.0	78	.0937500	.0281250	1.0
14	.0312500	-.0093750	-1.0	79	.1562500	-.0328125	-1.0
15	.0000000	.0234375	1.0	80	.0937500	.0312500	1.0
16	.0312500	-.0062500	-1.0	81	.1562500	-.0296875	-1.0
17	.0000000	.0265625	1.0	82	.0937500	.0343750	1.0
18	.0312500	-.0031250	-1.0	83	.1562500	-.0265625	-1.0
19	.0000000	.0296875	1.0	84	.0937500	.0375000	1.0
20	.0312500	.0000000	.0	85	.1250000	.0078125	1.0
21	.0312500	.0015625	1.0	86	.1250000	.0093750	1.0
22	.0625000	-.0281250	-1.0	87	.1875000	-.0515625	-1.0
23	.0312500	.0046875	1.0	88	.1250000	.0125000	1.0
24	.0625000	-.0250000	-1.0	89	.1875000	-.0484375	-1.0
25	.0312500	.0078125	1.0	90	.1250000	.0156250	1.0
26	.0625000	-.0218750	-1.0	91	.1875000	-.0453125	-1.0
27	.0312500	.0109375	1.0	92	.1250000	.0187500	1.0
28	.0625000	-.0187500	-1.0	93	.1875000	-.0421875	-1.0
29	.0312500	.0140625	1.0	94	.1250000	.0218750	1.0
30	.0625000	-.0156250	-1.0	95	.1875000	-.0390625	-1.0
31	.0312500	.0171875	1.0	96	.1250000	.0250000	1.0
32	.0625000	-.0125000	-1.0	97	.1875000	-.0359375	-1.0
33	.0312500	.0203125	1.0	98	.1250000	.0281250	1.0
34	.0625000	-.0093750	-1.0	99	.1875000	-.0328125	-1.0
35	.0312500	.0234375	1.0	100	.1250000	.0312500	1.0
36	.0625000	-.0062500	-1.0	101	.1875000	-.0296875	-1.0
37	.0312500	.0265625	1.0	102	.1250000	.0343750	1.0
38	.0625000	-.0031250	-1.0	103	.1875000	-.0265625	-1.0
39	.0312500	.0296875	1.0	104	.1250000	.0375000	1.0
40	.0625000	.0000000	.0	105	.1562500	.0078125	1.0
41	.0625000	.0015625	1.0	106	.1562500	.0093750	1.0
42	.0937500	-.0281250	-1.0	107	.2187500	-.0515625	-1.0
43	.0625000	.0046875	1.0	108	.1562500	.0125000	1.0
44	.0937500	-.0250000	-1.0	109	.2187500	-.0484375	-1.0
45	.0625000	.0078125	1.0	110	.1562500	.0156250	1.0
46	.0937500	-.0218750	-1.0	111	.2187500	-.0453125	-1.0
47	.0625000	.0109375	1.0	112	.1562500	.0187500	1.0
48	.0937500	-.0187500	-1.0	113	.2187500	-.0421875	-1.0
49	.0625000	.0140625	1.0	114	.1562500	.0218750	1.0
50	.0937500	-.0156250	-1.0	115	.2187500	-.0390625	-1.0
51	.0625000	.0171875	1.0	116	.1562500	.0250000	1.0
52	.0937500	-.0125000	-1.0	117	.2187500	-.0359375	-1.0
53	.0625000	.0203125	1.0	118	.1562500	.0281250	1.0
54	.0937500	-.0093750	-1.0	119	.2187500	-.0328125	-1.0
55	.0625000	.0234375	1.0	120	.1562500	.0312500	1.0
56	.0937500	-.0062500	-1.0	121	.2187500	-.0296875	-1.0
57	.0625000	.0265625	1.0	122	.1562500	.0343750	1.0
58	.0937500	-.0031250	-1.0	123	.2187500	-.0265625	-1.0
59	.0625000	.0296875	1.0	124	.1562500	.0375000	1.0
60	.0937500	.0000000	.0	125	.1875000	.0078125	1.0
61	.0937500	.0015625	1.0	126	.1875000	.0093750	1.0
62	.1250000	-.0281250	-1.0	127	.2500000	-.0515625	-1.0
63	.0937500	.0046875	1.0	128	.1875000	.0125000	1.0
64	.1250000	-.0250000	-1.0	129	.2500000	-.0484375	-1.0

TABLE 6 (continued)

3 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .00000000

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
130	.1875000	.0156250	1.0	195	.2500000	.0546875	1.0
131	.1875000	.0171875	1.0	196	.2500000	.0562500	1.0
132	.2500000	-.0437500	-1.0	197	.3125000	-.0046875	-1.0
133	.1875000	.0203125	1.0	198	.2500000	.0593750	1.0
134	.2500000	-.0406250	-1.0	199	.3125000	-.0015625	-1.0
135	.1875000	.0234375	1.0	200	.2500000	.0625000	1.0
136	.2500000	-.0375000	-1.0	201	.3125000	.0015625	1.0
137	.1875000	.0265625	1.0	202	.3125000	.0031250	1.0
138	.2500000	-.0343750	-1.0	203	.3750000	-.0578125	-1.0
139	.1875000	.0296875	1.0	204	.3125000	.0062500	1.0
140	.2500000	-.0312500	-1.0	205	.3750000	-.0546875	-1.0
141	.1875000	.0328125	1.0	206	.3125000	.0093750	1.0
142	.2500000	-.0281250	-1.0	207	.3750000	-.0515625	-1.0
143	.1875000	.0359375	1.0	208	.3125000	.0125000	1.0
144	.2500000	-.0250000	-1.0	209	.3750000	-.0484375	-1.0
145	.1875000	.0390625	1.0	210	.3125000	.0156250	1.0
146	.2500000	-.0218750	-1.0	211	.3750000	-.0453125	-1.0
147	.1875000	.0421875	1.0	212	.3125000	.0187500	1.0
148	.2500000	-.0187500	-1.0	213	.3750000	-.0421875	-1.0
149	.1875000	.0453125	1.0	214	.3125000	.0218750	1.0
150	.1875000	.0468750	1.0	215	.3125000	.0234375	1.0
151	.1875000	.0484375	1.0	216	.3125000	.0250000	1.0
152	.2500000	-.0125000	-1.0	217	.3750000	-.0359375	-1.0
153	.1875000	.0515625	1.0	218	.3125000	.0281250	1.0
154	.2500000	-.0093750	-1.0	219	.3750000	-.0328125	-1.0
155	.1875000	.0546875	1.0	220	.3125000	.0312500	1.0
156	.2500000	-.0062500	-1.0	221	.3750000	-.0296875	-1.0
157	.1875000	.0578125	1.0	222	.3125000	.0343750	1.0
158	.2500000	-.0031250	-1.0	223	.3750000	-.0265625	-1.0
159	.1875000	.0609375	1.0	224	.3125000	.0375000	1.0
160	.2500000	.0000000	.0	225	.3437500	.0078125	1.0
161	.2187500	.0328125	1.0	226	.3437500	.0093750	1.0
162	.2812500	-.0281250	-1.0	227	.4062500	-.0515625	-1.0
163	.2187500	.0359375	1.0	228	.3437500	.0125000	1.0
164	.2812500	-.0250000	-1.0	229	.4062500	-.0484375	-1.0
165	.2187500	.0390625	1.0	230	.3437500	.0156250	1.0
166	.2812500	-.0218750	-1.0	231	.4062500	-.0453125	-1.0
167	.2187500	.0421875	1.0	232	.3437500	.0187500	1.0
168	.2812500	-.0187500	-1.0	233	.4062500	-.0421875	-1.0
169	.2187500	.0453125	1.0	234	.3437500	.0218750	1.0
170	.2187500	.0468750	1.0	235	.3437500	.0234375	1.0
171	.2187500	.0484375	1.0	236	.3437500	.0250000	1.0
172	.2812500	-.0125000	-1.0	237	.4062500	-.0359375	-1.0
173	.2187500	.0515625	1.0	238	.3437500	.0281250	1.0
174	.2812500	-.0093750	-1.0	239	.4062500	-.0328125	-1.0
175	.2187500	.0546875	1.0	240	.3437500	.0312500	1.0
176	.2812500	-.0062500	-1.0	241	.4062500	-.0296875	-1.0
177	.2187500	.0578125	1.0	242	.3437500	.0343750	1.0
178	.2812500	-.0031250	-1.0	243	.4062500	-.0265625	-1.0
179	.2187500	.0609375	1.0	244	.3437500	.0375000	1.0
180	.2812500	.0000000	.0	245	.3750000	.0078125	1.0
181	.2500000	.0328125	1.0	246	.3750000	.0093750	1.0
182	.3125000	-.0281250	-1.0	247	.4375000	-.0515625	-1.0
183	.2500000	.0359375	1.0	248	.3750000	.0125000	1.0
184	.3125000	-.0250000	-1.0	249	.4375000	-.0484375	-1.0
185	.2500000	.0390625	1.0	250	.3750000	.0156250	1.0
186	.3125000	-.0218750	-1.0	251	.4375000	-.0453125	-1.0
187	.2500000	.0421875	1.0	252	.3750000	.0187500	1.0
188	.3125000	-.0187500	-1.0	253	.4375000	-.0421875	-1.0
189	.2500000	.0453125	1.0	254	.3750000	.0218750	1.0
190	.2500000	.0468750	1.0	255	.3750000	.0234375	1.0
191	.2500000	.0484375	1.0	256	.3750000	.0250000	1.0
192	.3125000	-.0125000	-1.0	257	.4375000	-.0359375	-1.0
193	.2500000	.0515625	1.0	258	.3750000	.0281250	1.0
194	.3125000	-.0093750	-1.0	259	.4375000	-.0328125	-1.0

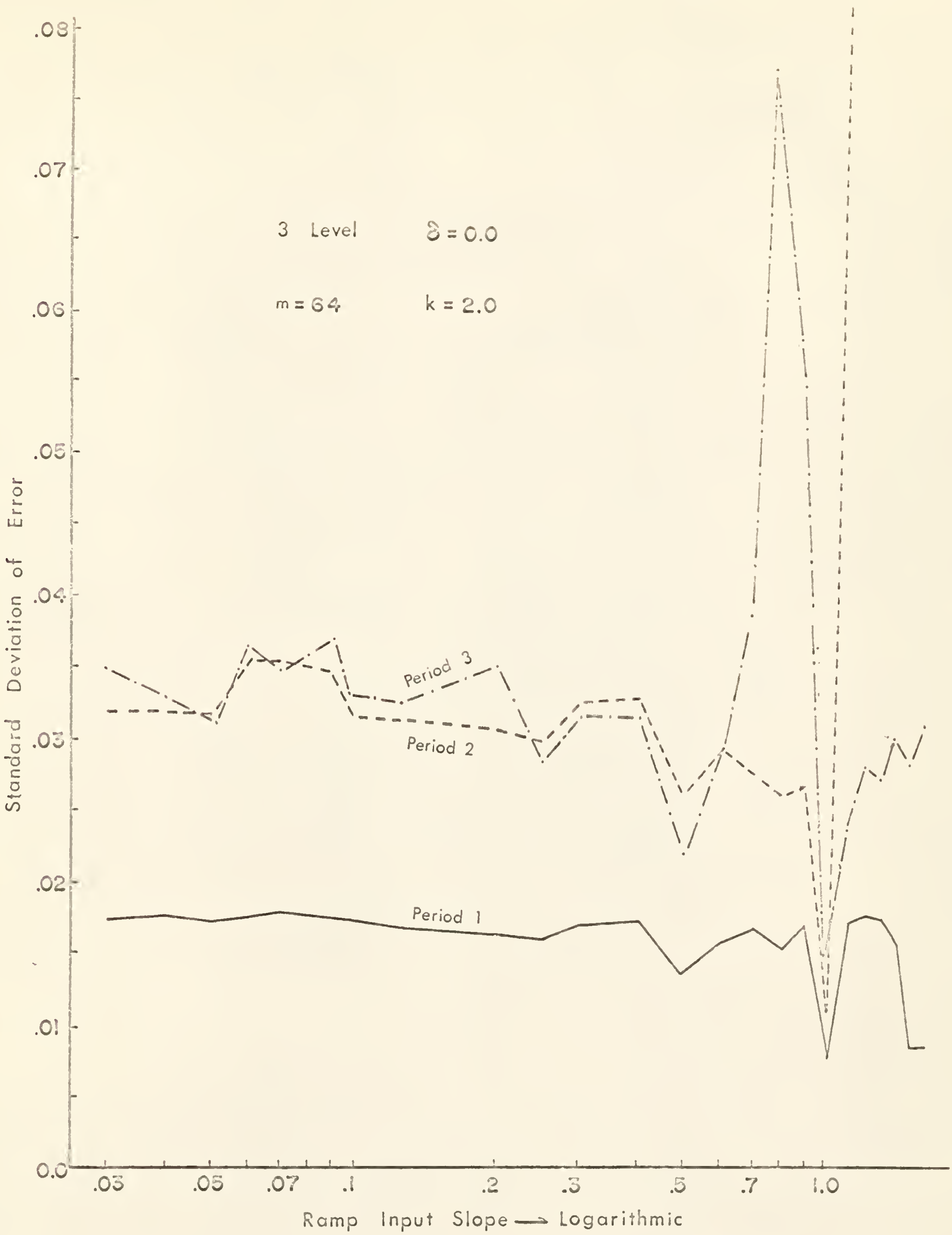

Fig. 14. Standard deviation of error vs. ramp input slope.

range of ramp inputs for the basic three-level device. Large errors are caused when the slope of the input changes sign. Errors generally increase from one period to the next as the output begins to lag the input.

System Modifications to Improve Ramp Performance

Tripp investigated placing a delay in the feedback loop (Fig. 15) in order to minimize the output lag. The results of

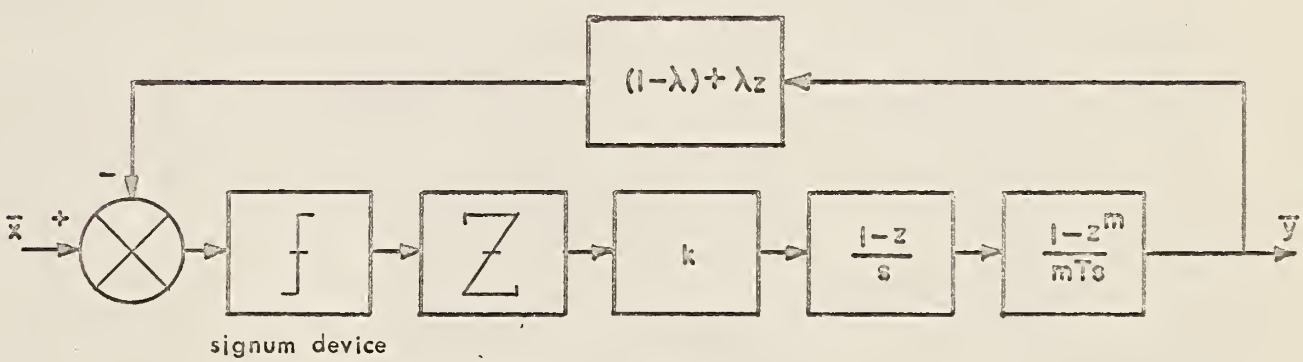

Fig. 15. Delta modulator with delay in feedback loop.

this study showed that the effect of delay was to increase the average error and standard deviation of error without improving the output lag. Tripp also tried placing the identity operator, $1/(m+\tau s)$, in the feedback loop. (See Fig. 16.) He concluded that, with the proper choice of τ, the average output lag could be made zero at the expense of greatly increased noise.

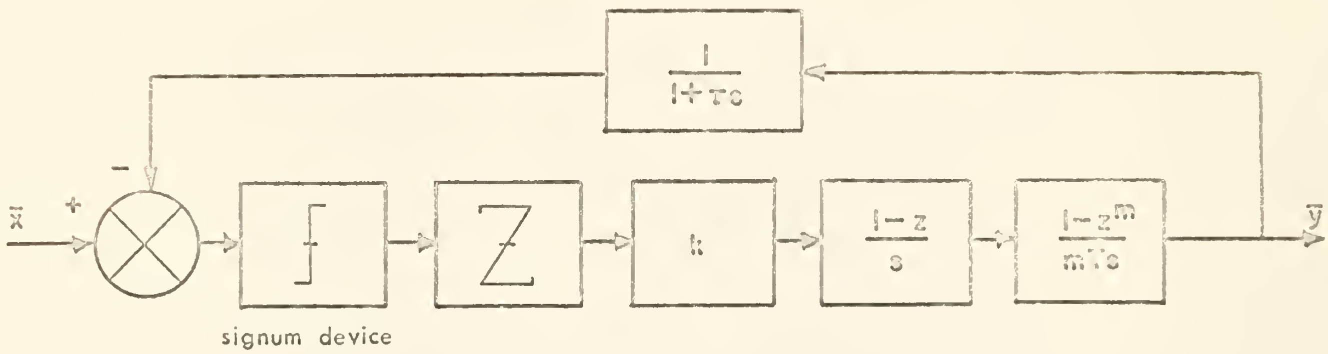

Fig. 16. Delta modulator with $\frac{1}{1+\tau s}$ in feedback loop.

Derivative compensation (Fig. 17) resulted in increased output lag.

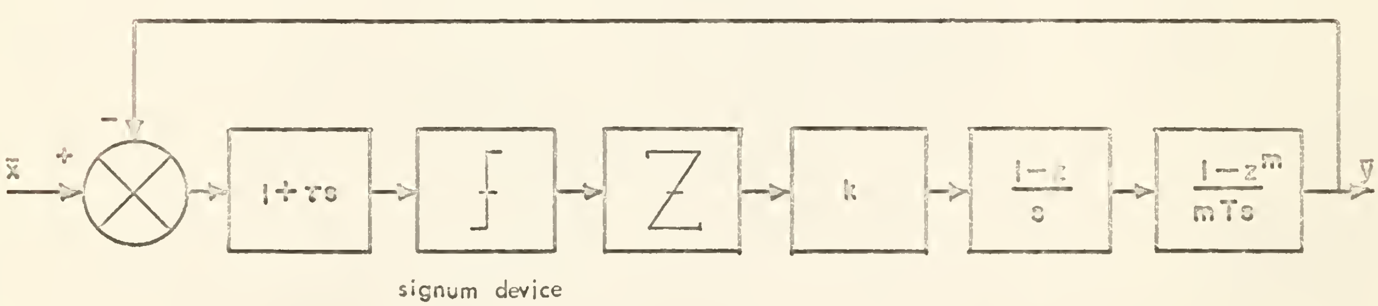

Fig. 17. Delta modulator with derivative compensation in forward loop.

The addition of dead space in the signum function resulted in a greatly decreased standard deviation of error at the expense of an increased average error and output lag. Figures 18 and 19 are plots of the standard deviation of error for the first three periods over a wide range of inputs with a dead space of k/2m and k/m in the signum device. A dead space of $\delta = k/2m$ was chosen as

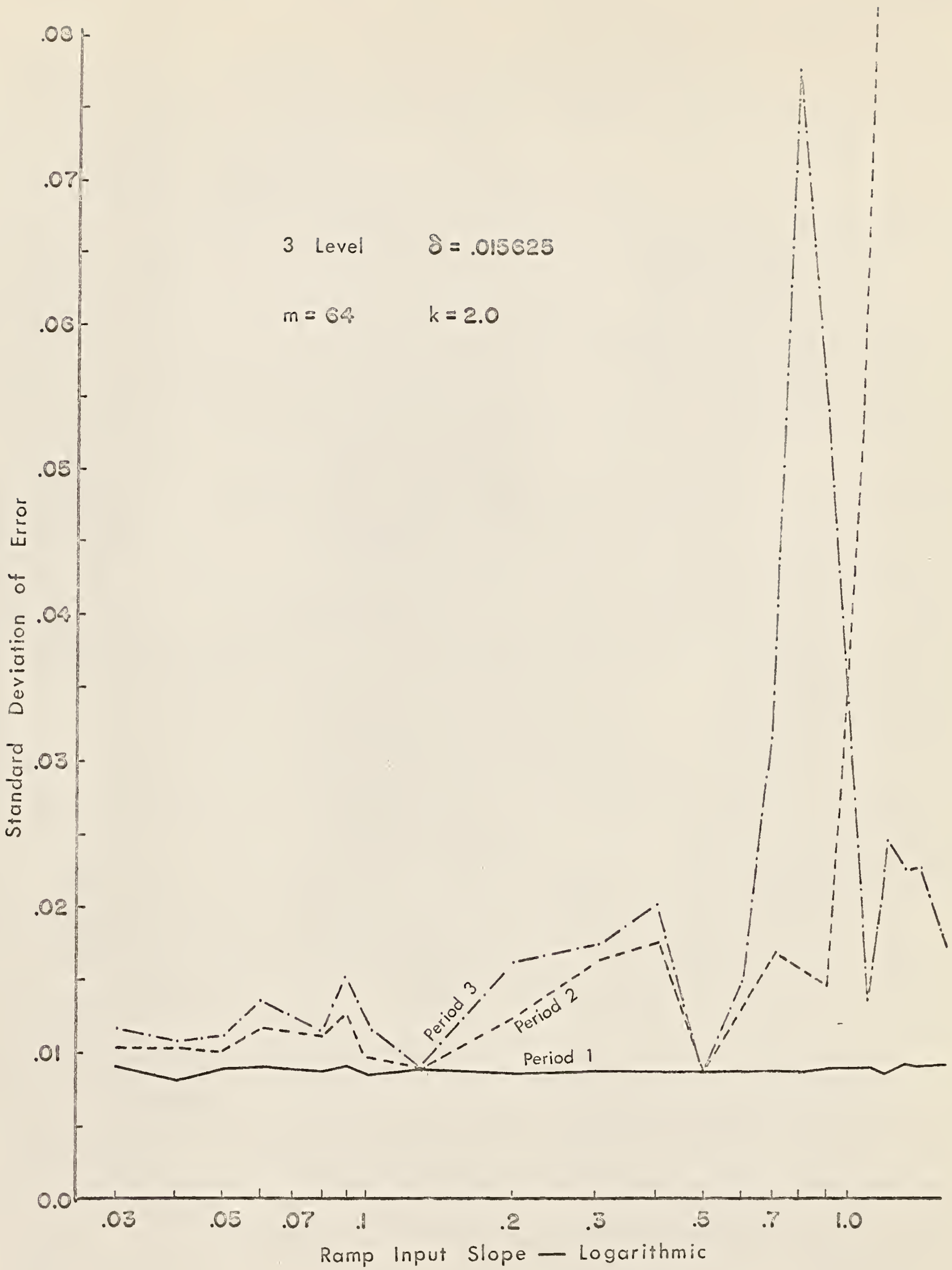

Fig. 18. Standard deviation of error vs. ramp input slope.

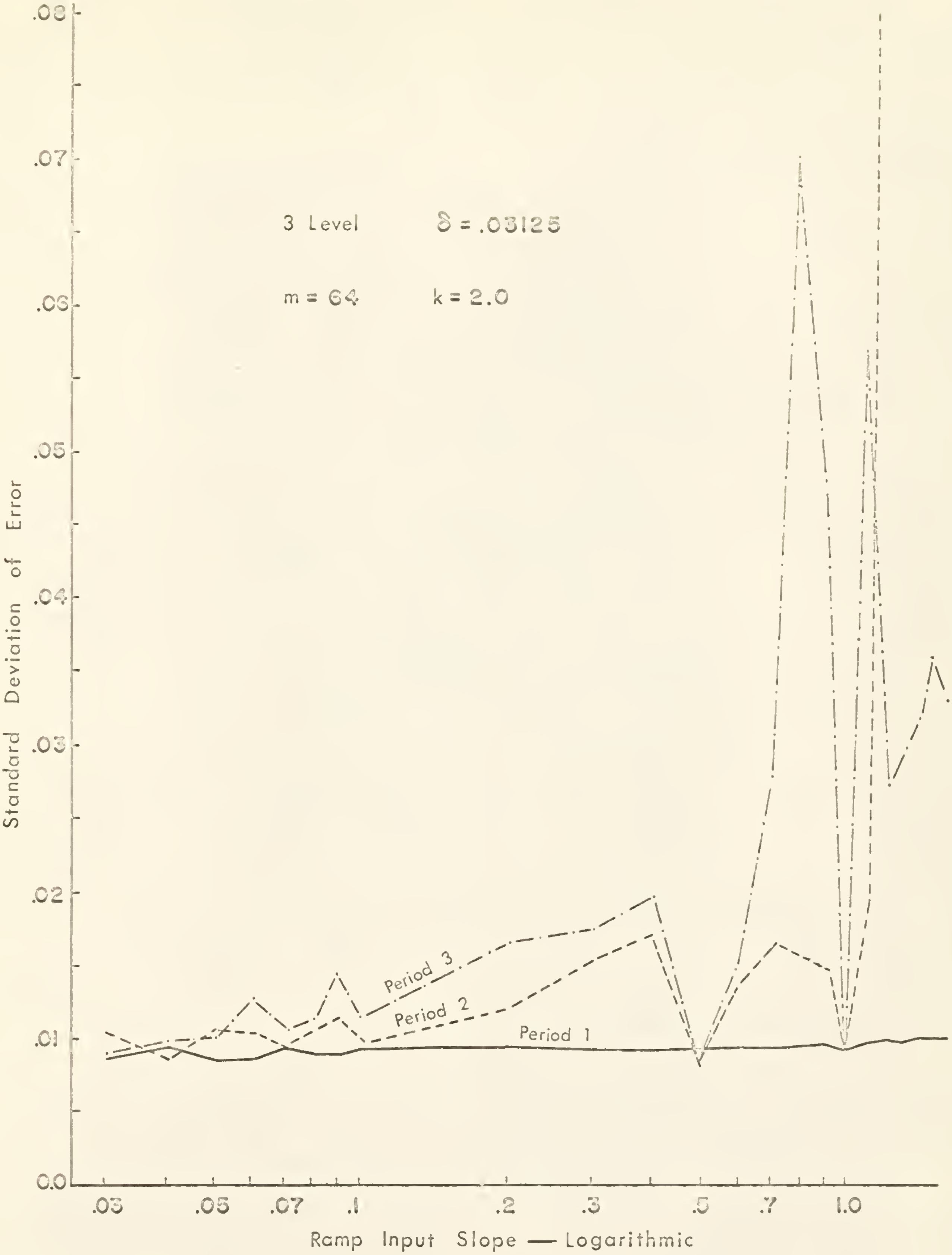

Fig. 19. Standard deviation of error vs. ramp input slope.

optimum since varying δ between 2k/m and k/(4m) did not have much effect on the output noise, although increasing δ did increase the delay.

Output values are listed in Tables 7 and 8 for a typical input function with dead spaces of k/2m and k/m in the signum function. The first three periods of output for $\delta = k/2m$ and $\delta = k/m$ are shown in Plates II and III.

GENERALIZATION OF DELTA MODULATION

Definition of Five-Level Signum Function

After reviewing the behavior of the delta modulator model, Tripp suggested a five-level device be substituted for the three-level signum function. The five levels may be defined by a generalized Rademacher function as follows:

$$\mathrm{sgn}_{\delta}(e) = \begin{cases} 1, & e > \delta \\ 1/3, & 0 < e \leq \delta \\ 0, & e = 0 \\ -1/3, & 0 > e \geq \delta \\ -1, & e < -\delta. \end{cases}$$

A Coder and Decoder for Five-Level Output

To insure accurate reception and demodulation after passing through an imperfect data channel, it is necessary that the five-level pulses be regenerated into unit height. This can be done by representing the higher level pulses by two pulses of a single level, the second pulse occuring half way between the original pulse and its following pulse.

An encoder for this purpose is shown in Fig. 26. The five-level pulses are fed to a signum device which produces an output

TABLE 7

3 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .01562500

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
0	.0000000	.0000000	.0	65	.0937500	.0078125	.0
1	.0000000	.0015625	.0	66	.0937500	.0093750	.0
2	.0000000	.0031250	.0	67	.0937500	.0109375	.0
3	.0000000	.0046875	.0	68	.0937500	.0125000	.0
4	.0000000	.0062500	.0	69	.0937500	.0140625	.0
5	.0000000	.0078125	.0	70	.0937500	.0156250	.0
6	.0000000	.0093750	.0	71	.0937500	.0171875	1.0
7	.0000000	.0109375	.0	72	.1250000	-.0125000	.0
8	.0000000	.0125000	.0	73	.1250000	-.0109375	.0
9	.0000000	.0140625	.0	74	.1250000	-.0093750	.0
10	.0000000	.0156250	.0	75	.1250000	-.0078125	.0
11	.0000000	.0171875	1.0	76	.0937500	.0250000	1.0
12	.0312500	-.0125000	.0	77	.1250000	-.0046875	.0
13	.0312500	-.0109375	.0	78	.1250000	-.0031250	.0
14	.0312500	-.0093750	.0	79	.1250000	-.0015625	.0
15	.0312500	-.0078125	.0	80	.1250000	.0000000	.0
16	.0312500	-.0062500	.0	81	.1250000	.0015625	.0
17	.0312500	-.0046875	.0	82	.1250000	.0031250	.0
18	.0312500	-.0031250	.0	83	.1250000	.0046875	.0
19	.0312500	-.0015625	.0	84	.1250000	.0062500	.0
20	.0312500	.0000000	.0	85	.1250000	.0078125	.0
21	.0312500	.0015625	.0	86	.1250000	.0093750	.0
22	.0312500	.0031250	.0	87	.1250000	.0109375	.0
23	.0312500	.0046875	.0	88	.1250000	.0125000	.0
24	.0312500	.0062500	.0	89	.1250000	.0140625	.0
25	.0312500	.0078125	.0	90	.1250000	.0156250	.0
26	.0312500	.0093750	.0	91	.1250000	.0171875	1.0
27	.0312500	.0109375	.0	92	.1562500	-.0125000	.0
28	.0312500	.0125000	.0	93	.1562500	-.0109375	.0
29	.0312500	.0140625	.0	94	.1562500	-.0093750	.0
30	.0312500	.0156250	.0	95	.1562500	-.0078125	.0
31	.0312500	.0171875	1.0	96	.1250000	.0250000	1.0
32	.0625000	-.0125000	.0	97	.1562500	-.0046875	.0
33	.0625000	-.0109375	.0	98	.1562500	-.0031250	.0
34	.0625000	-.0093750	.0	99	.1562500	-.0015625	.0
35	.0625000	-.0078125	.0	100	.1562500	.0000000	.0
36	.0625000	-.0062500	.0	101	.1562500	.0015625	.0
37	.0625000	-.0046875	.0	102	.1562500	.0031250	.0
38	.0625000	-.0031250	.0	103	.1562500	.0046875	.0
39	.0625000	-.0015625	.0	104	.1562500	.0062500	.0
40	.0625000	.0000000	.0	105	.1562500	.0078125	.0
41	.0625000	.0015625	.0	106	.1562500	.0093750	.0
42	.0625000	.0031250	.0	107	.1562500	.0109375	.0
43	.0625000	.0046875	.0	108	.1562500	.0125000	.0
44	.0625000	.0062500	.0	109	.1562500	.0140625	.0
45	.0625000	.0078125	.0	110	.1562500	.0156250	.0
46	.0625000	.0093750	.0	111	.1562500	.0171875	1.0
47	.0625000	.0109375	.0	112	.1875000	-.0125000	.0
48	.0625000	.0125000	.0	113	.1875000	-.0109375	.0
49	.0625000	.0140625	.0	114	.1875000	-.0093750	.0
50	.0625000	.0156250	.0	115	.1875000	-.0078125	.0
51	.0625000	.0171875	1.0	116	.1562500	.0250000	1.0
52	.0937500	-.0125000	.0	117	.1875000	-.0046875	.0
53	.0937500	-.0109375	.0	118	.1875000	-.0031250	.0
54	.0937500	-.0093750	.0	119	.1875000	-.0015625	.0
55	.0937500	-.0078125	.0	120	.1875000	.0000000	.0
56	.0937500	-.0062500	.0	121	.1875000	.0015625	.0
57	.0937500	-.0046875	.0	122	.1875000	.0031250	.0
58	.0937500	-.0031250	.0	123	.1875000	.0046875	.0
59	.0937500	-.0015625	.0	124	.1875000	.0062500	.0
60	.0937500	.0000000	.0	125	.1875000	.0078125	.0
61	.0937500	.0015625	.0	126	.1875000	.0093750	.0
62	.0937500	.0031250	.0	127	.1875000	.0109375	.0
63	.0937500	.0046875	.0	128	.1875000	.0125000	.0
64	.0937500	.0062500	.0	129	.1875000	.0140625	.0

TABLE 7 (continued)

3 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .01562500

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
130	.1875000	.0156250	.0	195	.3125000	-.0078125	.0
131	.1875000	.0171875	1.0	196	.2812500	.0250000	1.0
132	.2187500	-.0125000	.0	197	.3125000	-.0046875	.0
133	.2187500	-.0109375	.0	198	.3125000	-.0031250	.0
134	.2187500	-.0093750	.0	199	.3125000	-.0015625	.0
135	.2187500	-.0078125	.0	200	.3125000	.0000000	.0
136	.1875000	.0250000	1.0	201	.2812500	.0328125	1.0
137	.2187500	-.0046875	.0	202	.3125000	.0031250	.0
138	.2187500	-.0031250	.0	203	.3125000	.0046875	.0
139	.2187500	-.0015625	.0	204	.3125000	.0062500	.0
140	.2187500	.0000000	.0	205	.3125000	.0078125	.0
141	.1875000	.0328125	1.0	206	.2812500	.0406250	1.0
142	.2187500	.0031250	.0	207	.3125000	.0109375	.0
143	.2187500	.0046875	.0	208	.3125000	.0125000	.0
144	.2187500	.0062500	.0	209	.3125000	.0140625	.0
145	.2187500	.0078125	.0	210	.3125000	.0156250	.0
146	.2187500	.0093750	.0	211	.3125000	.0171875	1.0
147	.2187500	.0109375	.0	212	.3437500	-.0125000	.0
148	.2187500	.0125000	.0	213	.3437500	-.0109375	.0
149	.2187500	.0140625	.0	214	.3437500	-.0093750	.0
150	.2187500	.0156250	.0	215	.3437500	-.0078125	.0
151	.2187500	.0171875	1.0	216	.3125000	.0250000	1.0
152	.2500000	-.0125000	.0	217	.3437500	-.0046875	.0
153	.2500000	-.0109375	.0	218	.3437500	-.0031250	.0
154	.2500000	-.0093750	.0	219	.3437500	-.0015625	.0
155	.2500000	-.0078125	.0	220	.3437500	.0000000	.0
156	.2187500	.0250000	1.0	221	.3125000	.0328125	1.0
157	.2500000	-.0046875	.0	222	.3437500	.0031250	.0
158	.2500000	-.0031250	.0	223	.3437500	.0046875	.0
159	.2500000	-.0015625	.0	224	.3437500	.0062500	.0
160	.2500000	.0000000	.0	225	.3437500	.0078125	.0
161	.2187500	.0328125	1.0	226	.3125000	.0406250	1.0
162	.2500000	.0031250	.0	227	.3437500	.0109375	.0
163	.2500000	.0046875	.0	228	.3437500	.0125000	.0
164	.2500000	.0062500	.0	229	.3437500	.0140625	.0
165	.2500000	.0078125	.0	230	.3437500	.0156250	.0
166	.2500000	.0093750	.0	231	.3437500	.0171875	1.0
167	.2500000	.0109375	.0	232	.3750000	-.0125000	.0
168	.2500000	.0125000	.0	233	.3750000	-.0109375	.0
169	.2500000	.0140625	.0	234	.3750000	-.0093750	.0
170	.2500000	.0156250	.0	235	.3750000	-.0078125	.0
171	.2500000	.0171875	1.0	236	.3437500	.0250000	1.0
172	.2812500	-.0125000	.0	237	.3750000	-.0046875	.0
173	.2812500	-.0109375	.0	238	.3750000	-.0031250	.0
174	.2812500	-.0093750	.0	239	.3750000	-.0015625	.0
175	.2812500	-.0078125	.0	240	.3750000	.0000000	.0
176	.2500000	.0250000	1.0	241	.3437500	.0328125	1.0
177	.2812500	-.0046875	.0	242	.3750000	.0031250	.0
178	.2812500	-.0031250	.0	243	.3750000	.0046875	.0
179	.2812500	-.0015625	.0	244	.3750000	.0062500	.0
180	.2812500	.0000000	.0	245	.3750000	.0078125	.0
181	.2500000	.0328125	1.0	246	.3437500	.0406250	1.0
182	.2812500	.0031250	.0	247	.3750000	.0109375	.0
183	.2812500	.0046875	.0	248	.3750000	.0125000	.0
184	.2812500	.0062500	.0	249	.3750000	.0140625	.0
185	.2812500	.0078125	.0	250	.3750000	.0156250	.0
186	.2812500	.0093750	.0	251	.3750000	.0171875	1.0
187	.2812500	.0109375	.0	252	.4062500	-.0125000	.0
188	.2812500	.0125000	.0	253	.4062500	-.0109375	.0
189	.2812500	.0140625	.0	254	.4062500	-.0093750	.0
190	.2812500	.0156250	.0	255	.4062500	-.0078125	.0
191	.2812500	.0171875	1.0	256	.3750000	.0250000	1.0
192	.3125000	-.0125000	.0	257	.4062500	-.0046875	.0
193	.3125000	-.0109375	.0	258	.4062500	-.0031250	.0
194	.3125000	-.0093750	.0	259	.4062500	-.0015625	.0

TABLE 8

3 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .03125000

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
0	.0000000	.0000000	.0	65	.0937500	.0078125	.0
1	.0000000	.0015625	.0	66	.0937500	.0093750	.0
2	.0000000	.0031250	.0	67	.0937500	.0109375	.0
3	.0000000	.0046875	.0	68	.0937500	.0125000	.0
4	.0000000	.0062500	.0	69	.0937500	.0140625	.0
5	.0000000	.0078125	.0	70	.0937500	.0156250	.0
6	.0000000	.0093750	.0	71	.0937500	.0171875	.0
7	.0000000	.0109375	.0	72	.0937500	.0187500	.0
8	.0000000	.0125000	.0	73	.0937500	.0203125	.0
9	.0000000	.0140625	.0	74	.0937500	.0218750	.0
10	.0000000	.0156250	.0	75	.0937500	.0234375	.0
11	.0000000	.0171875	.0	76	.0937500	.0250000	.0
12	.0000000	.0187500	.0	77	.0937500	.0265625	.0
13	.0000000	.0203125	.0	78	.0937500	.0281250	.0
14	.0000000	.0218750	.0	79	.0937500	.0296875	.0
15	.0000000	.0234375	.0	80	.0937500	.0312500	.0
16	.0000000	.0250000	.0	81	.0937500	.0328125	1.0
17	.0000000	.0265625	.0	82	.1250000	.0031250	.0
18	.0000000	.0281250	.0	83	.1250000	.0046875	.0
19	.0000000	.0296875	.0	84	.1250000	.0062500	.0
20	.0000000	.0312500	.0	85	.1250000	.0078125	.0
21	.0000000	.0328125	1.0	86	.0937500	.0406250	1.0
22	.0312500	.0031250	.0	87	.1250000	.0109375	.0
23	.0312500	.0046875	.0	88	.1250000	.0125000	.0
24	.0312500	.0062500	.0	89	.1250000	.0140625	.0
25	.0312500	.0078125	.0	90	.1250000	.0156250	.0
26	.0312500	.0093750	.0	91	.1250000	.0171875	.0
27	.0312500	.0109375	.0	92	.1250000	.0187500	.0
28	.0312500	.0125000	.0	93	.1250000	.0203125	.0
29	.0312500	.0140625	.0	94	.1250000	.0218750	.0
30	.0312500	.0156250	.0	95	.1250000	.0234375	.0
31	.0312500	.0171875	.0	96	.1250000	.0250000	.0
32	.0312500	.0187500	.0	97	.1250000	.0265625	.0
33	.0312500	.0203125	.0	98	.1250000	.0281250	.0
34	.0312500	.0218750	.0	99	.1250000	.0296875	.0
35	.0312500	.0234375	.0	100	.1250000	.0312500	.0
36	.0312500	.0250000	.0	101	.1250000	.0328125	1.0
37	.0312500	.0265625	.0	102	.1562500	.0031250	.0
38	.0312500	.0281250	.0	103	.1562500	.0046875	.0
39	.0312500	.0296875	.0	104	.1562500	.0062500	.0
40	.0312500	.0312500	.0	105	.1562500	.0078125	.0
41	.0312500	.0328125	1.0	106	.1250000	.0406250	1.0
42	.0625000	.0031250	.0	107	.1562500	.0109375	.0
43	.0625000	.0046875	.0	108	.1562500	.0125000	.0
44	.0625000	.0062500	.0	109	.1562500	.0140625	.0
45	.0625000	.0078125	.0	110	.1562500	.0156250	.0
46	.0625000	.0093750	.0	111	.1562500	.0171875	.0
47	.0625000	.0109375	.0	112	.1562500	.0187500	.0
48	.0625000	.0125000	.0	113	.1562500	.0203125	.0
49	.0625000	.0140625	.0	114	.1562500	.0218750	.0
50	.0625000	.0156250	.0	115	.1562500	.0234375	.0
51	.0625000	.0171875	.0	116	.1562500	.0250000	.0
52	.0625000	.0187500	.0	117	.1562500	.0265625	.0
53	.0625000	.0203125	.0	118	.1562500	.0281250	.0
54	.0625000	.0218750	.0	119	.1562500	.0296875	.0
55	.0625000	.0234375	.0	120	.1562500	.0312500	.0
56	.0625000	.0250000	.0	121	.1562500	.0328125	1.0
57	.0625000	.0265625	.0	122	.1875000	.0031250	.0
58	.0625000	.0281250	.0	123	.1875000	.0046875	.0
59	.0625000	.0296875	.0	124	.1875000	.0062500	.0
60	.0625000	.0312500	.0	125	.1875000	.0078125	.0
61	.0625000	.0328125	1.0	126	.1562500	.0406250	1.0
62	.0937500	.0031250	.0	127	.1875000	.0109375	.0
63	.0937500	.0046875	.0	128	.1875000	.0125000	.0
64	.0937500	.0062500	.0	129	.1875000	.0140625	.0

TABLE 8 (continued)

3 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .03125000

N	Y(N)	E(N)	S(N)
130	.1875000	.0156250	.0
131	.1875000	.0171875	.0
132	.1875000	.0187500	.0
133	.1875000	.0203125	.0
134	.1875000	.0218750	.0
135	.1875000	.0234375	.0
136	.1875000	.0250000	.0
137	.1875000	.0265625	.0
138	.1875000	.0281250	.0
139	.1875000	.0296875	.0
140	.1875000	.0312500	.0
141	.1875000	.0328125	1.0
142	.2187500	.0031250	.0
143	.2187500	.0046875	.0
144	.2187500	.0062500	.0
145	.2187500	.0078125	.0
146	.1875000	.0406250	1.0
147	.2187500	.0109375	.0
148	.2187500	.0125000	.0
149	.2187500	.0140625	.0
150	.2187500	.0156250	.0
151	.1875000	.0484375	1.0
152	.2187500	.0187500	.0
153	.2187500	.0203125	.0
154	.2187500	.0218750	.0
155	.2187500	.0234375	.0
156	.2187500	.0250000	.0
157	.2187500	.0265625	.0
158	.2187500	.0281250	.0
159	.2187500	.0296875	.0
160	.2187500	.0312500	.0
161	.2187500	.0328125	1.0
162	.2500000	.0031250	.0
163	.2500000	.0046875	.0
164	.2500000	.0062500	.0
165	.2500000	.0078125	.0
166	.2187500	.0406250	1.0
167	.2500000	.0109375	.0
168	.2500000	.0125000	.0
169	.2500000	.0140625	.0
170	.2500000	.0156250	.0
171	.2187500	.0484375	1.0
172	.2500000	.0187500	.0
173	.2500000	.0203125	.0
174	.2500000	.0218750	.0
175	.2500000	.0234375	.0
176	.2500000	.0250000	.0
177	.2500000	.0265625	.0
178	.2500000	.0281250	.0
179	.2500000	.0296875	.0
180	.2500000	.0312500	.0
181	.2500000	.0328125	1.0
182	.2812500	.0031250	.0
183	.2812500	.0046875	.0
184	.2812500	.0062500	.0
185	.2812500	.0078125	.0
186	.2500000	.0406250	1.0
187	.2812500	.0109375	.0
188	.2812500	.0125000	.0
189	.2812500	.0140625	.0
190	.2812500	.0156250	.0
191	.2500000	.0484375	1.0
192	.2812500	.0187500	.0
193	.2812500	.0203125	.0
194	.2812500	.0218750	.0
195	.2812500	.0234375	.0
196	.2812500	.0250000	.0
197	.2812500	.0265625	.0
198	.2812500	.0281250	.0
199	.2812500	.0296875	.0
200	.2812500	.0312500	.0
201	.2812500	.0328125	1.0
202	.3125000	.0031250	.0
203	.3125000	.0046875	.0
204	.3125000	.0062500	.0
205	.3125000	.0078125	.0
206	.2812500	.0406250	1.0
207	.3125000	.0109375	.0
208	.3125000	.0125000	.0
209	.3125000	.0140625	.0
210	.3125000	.0156250	.0
211	.2812500	.0484375	1.0
212	.3125000	.0187500	.0
213	.3125000	.0203125	.0
214	.3125000	.0218750	.0
215	.3125000	.0234375	.0
216	.2812500	.0562500	1.0
217	.3125000	.0265625	.0
218	.3125000	.0281250	.0
219	.3125000	.0296875	.0
220	.3125000	.0312500	.0
221	.3125000	.0328125	1.0
222	.3437500	.0031250	.0
223	.3437500	.0046875	.0
224	.3437500	.0062500	.0
225	.3437500	.0078125	.0
226	.3125000	.0406250	1.0
227	.3437500	.0109375	.0
228	.3437500	.0125000	.0
229	.3437500	.0140625	.0
230	.3437500	.0156250	.0
231	.3125000	.0484375	1.0
232	.3437500	.0187500	.0
233	.3437500	.0203125	.0
234	.3437500	.0218750	.0
235	.3437500	.0234375	.0
236	.3125000	.0562500	1.0
237	.3437500	.0265625	.0
238	.3437500	.0281250	.0
239	.3437500	.0296875	.0
240	.3437500	.0312500	.0
241	.3437500	.0328125	1.0
242	.3750000	.0031250	.0
243	.3750000	.0046875	.0
244	.3750000	.0062500	.0
245	.3750000	.0078125	.0
246	.3437500	.0406250	1.0
247	.3750000	.0109375	.0
248	.3750000	.0125000	.0
249	.3750000	.0140625	.0
250	.3750000	.0156250	.0
251	.3437500	.0484375	1.0
252	.3750000	.0187500	.0
253	.3750000	.0203125	.0
254	.3750000	.0218750	.0
255	.3750000	.0234375	.0
256	.3437500	.0562500	1.0
257	.3750000	.0265625	.0
258	.3750000	.0281250	.0
259	.3750000	.0296875	.0

EXPLANATION OF PLATE II

Three-level delta modulation, $\delta = .015625$, $m = 64$, $k = 2.0$

Output for ramp input slope 0.1

Fig. 20. Period 1 (samples 0-64)

Fig. 21. Period 2 (samples 65-129)

Fig. 22. Period 3 (samples 130-194)

PLATE II

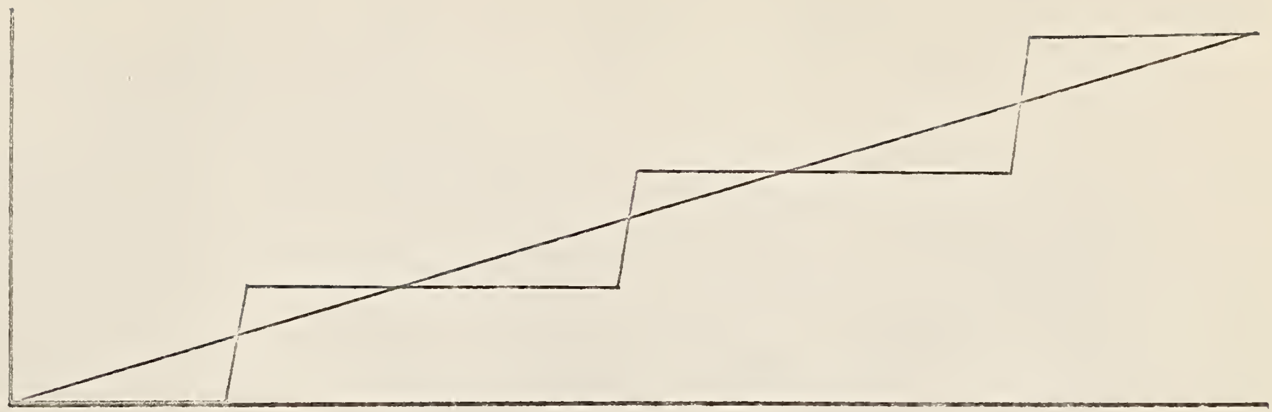

Fig. 20

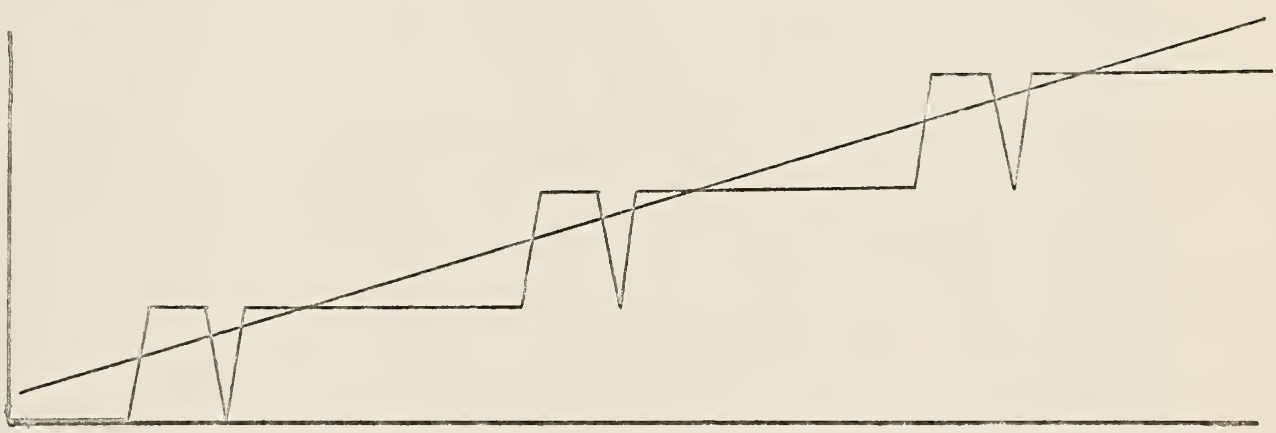

Fig. 21

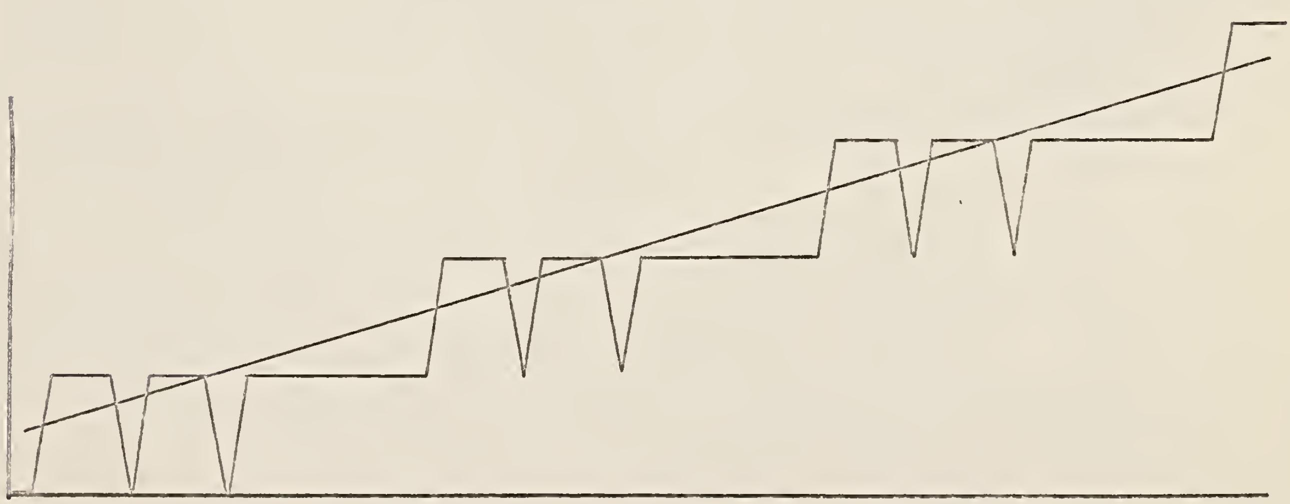

Fig. 22

EXPLANATION OF PLATE III

Three-level delta modulation, $\delta = .03125$, $m = 64$, $k = 2.0$

Output for ramp input slope 0.1

Fig. 23. Period 1 (samples 0-64)

Fig. 24. Period 2 (samples 65-129)

Fig. 25. Period 3 (samples 130-194)

PLATE III

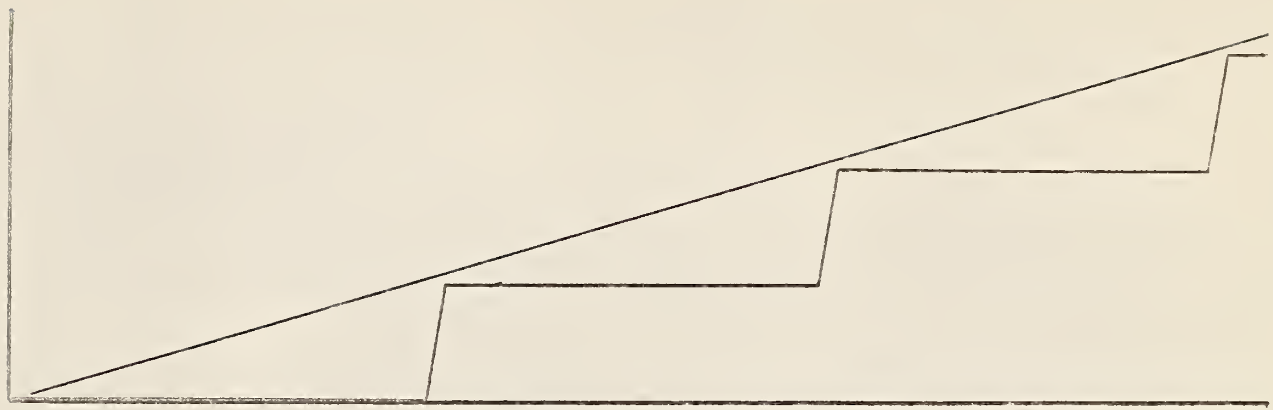

Fig. 23

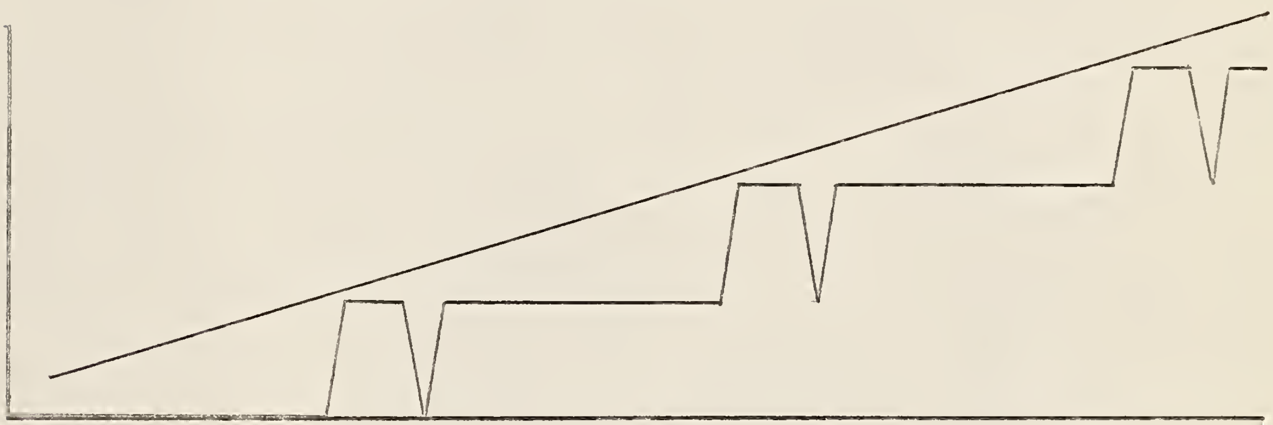

Fig. 24

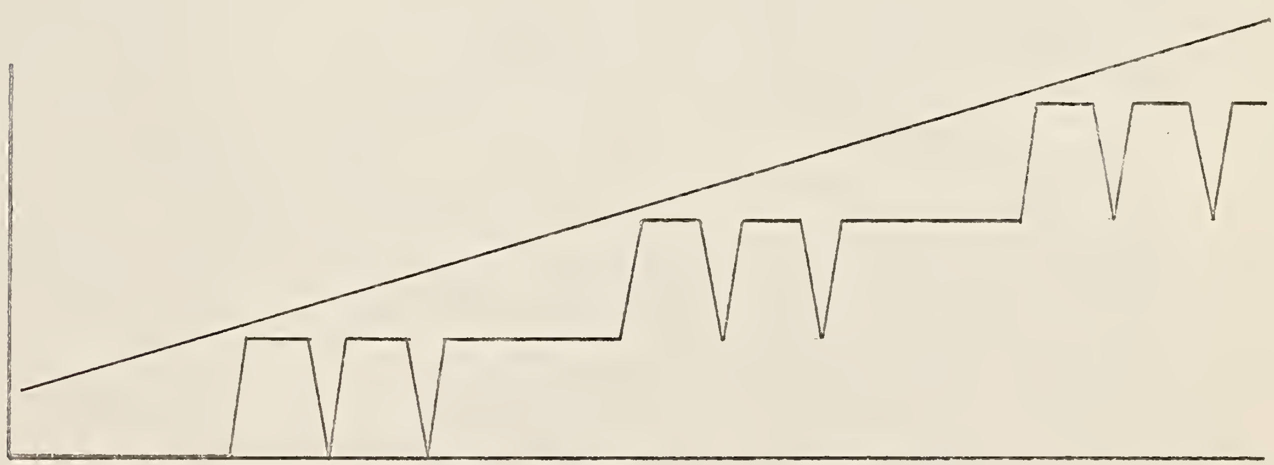

Fig. 25

only for the higher level pulses. The output of the signum device is delayed by time t = T/2 and added to the original sequence. The new pulse train is then fed to a limiting amplifier which reduces the pulses to unit height before they are transmitted.

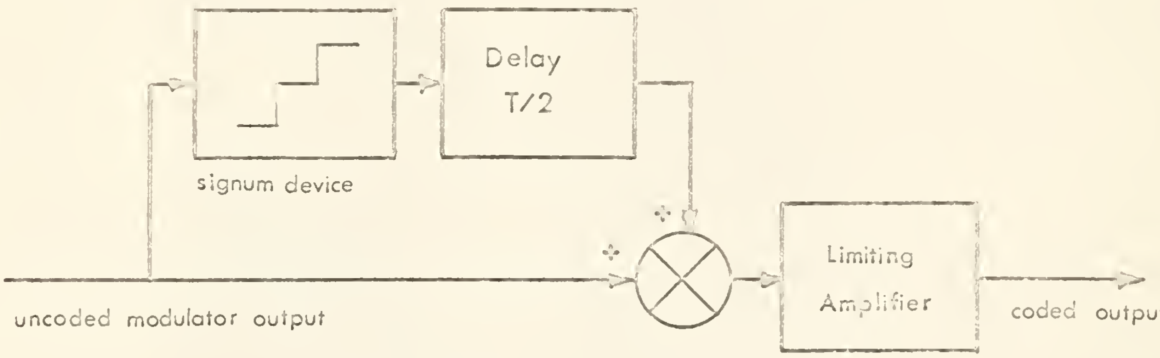

Fig. 26. Three-level encoder for five-level delta modulator.

Decoding is accomplished by multiplying the received sequence by two and adding the product to the same sequence delayed by t = T/2. The sum is then sampled at nT + T/2 instants to reconstruct the original signal. (See Fig. 27.)

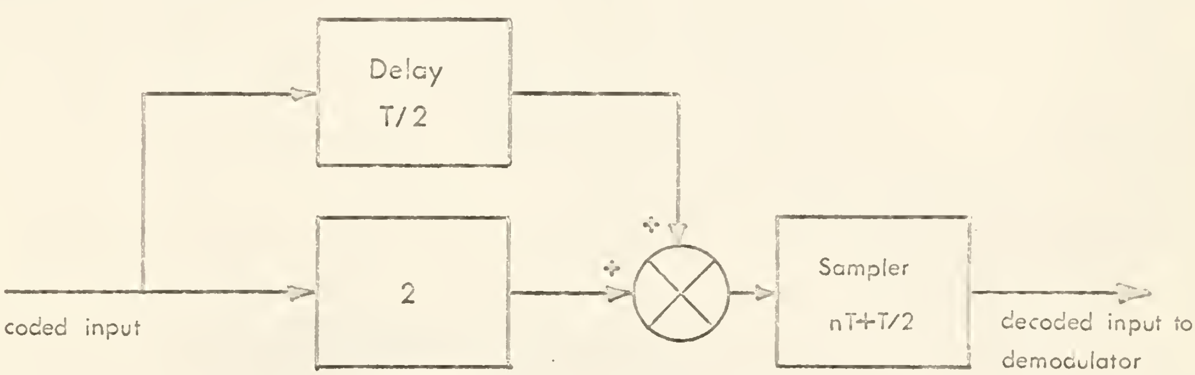

Fig. 27. Decoder for encoded five-level delta modulator.

This is accomplished at the expense of more complex circuitry (comparable to that for PCM) and increased bandwidth. There is also an inherent delay of T/2 seconds between the transmitted and reconstructed signals.

Results for Step Inputs

To analyze the new system on the computer, a perfect channel was again assumed and the original program modified for a five-level signum function. The delay in the demodulator was not taken into account. Results for step inputs show an improvement over the original three-level case. Steady state signal-to-noise ratios are shown in Fig. 28 for a wide range of input with $\delta = k/2m$.

The system now operates in three modes whose S/N ratios follow smooth curves as shown in the figure. The addition of the middle level in the signum function caused some of the mode 2 points of the original three-level system to be raised to a third mode which lies between modes 1 and 2. Several values were tried for δ, and $\delta = k/2m$ was chosen as optimum for step inputs. Tables 9-11 show typical behavior for the three modes of operation. There is no noise improvement for the five-level case over the three-level case with dead space, but there is a slight decrease in average error. Table 12 is a tabulation of error data for $\delta = k/2m$. Figure 29 shows the average output error plotted against input for the original three-level device, the three-level device with dead space, and the five-level device.

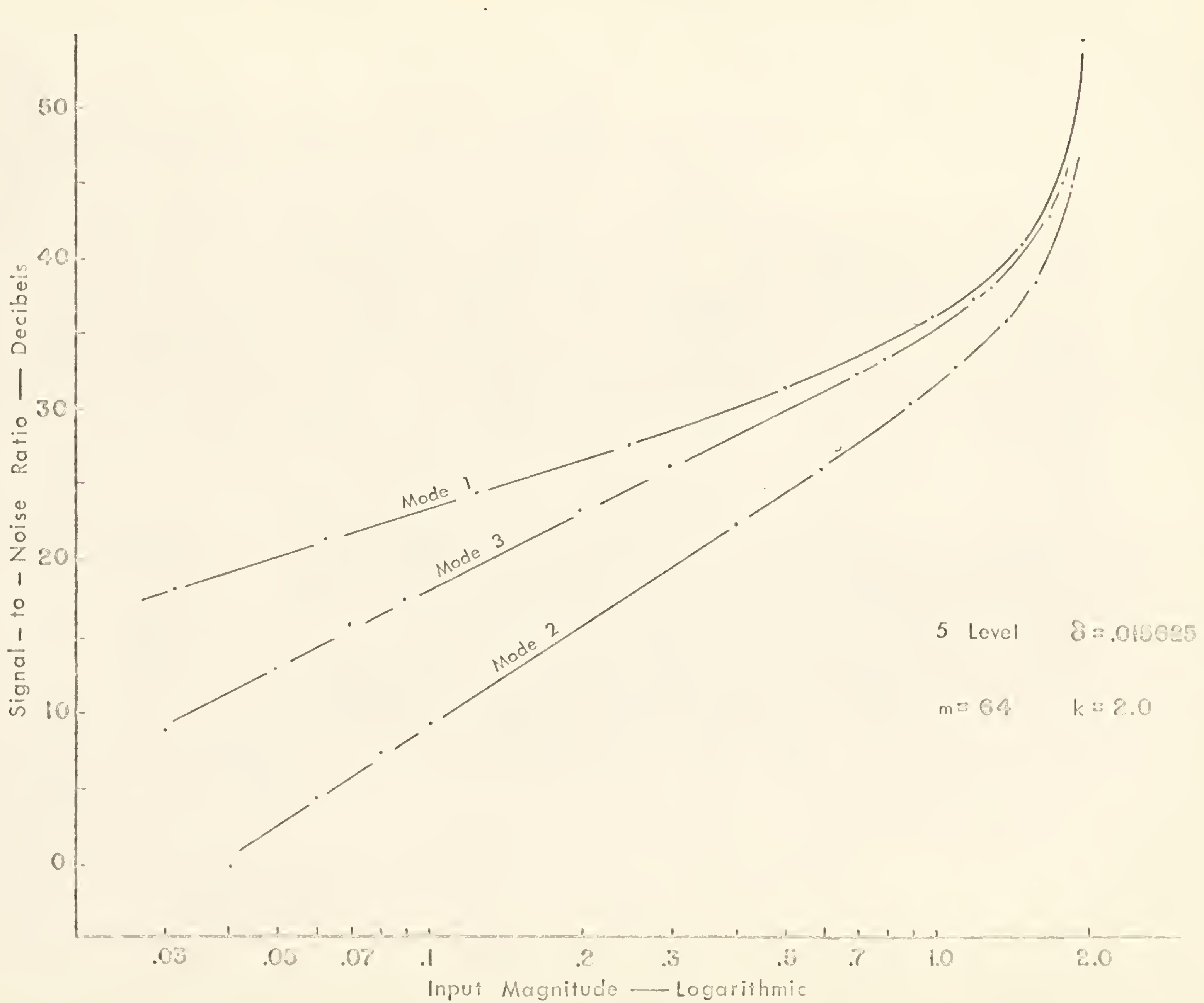

Fig. 28. Signal-to-noise ratio vs. step input magnitude.

TABLE 9

5 LEVEL MOD-DEMOD MODEL .12500000 STEP INPU

M = 64 GAIN = 2.00000000 DEAD SPACE = .0156250

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N
0	.0000000	.1250000	1.0	65	.0937500	.0312500	1.
1	.0312500	.0937500	1.0	66	.0937500	.0312500	1.
2	.0625000	.0625000	1.0	67	.0937500	.0312500	1.
3	.0937500	.0312500	1.0	68	.0937500	.0312500	1.
4	.1250000	.0000000	.0	69	.1250000	.0000000	.
5	.1250000	.0000000	.0	70	.1250000	.0000000	.
6	.1250000	.0000000	.0	71	.1250000	.0000000	.
7	.1250000	.0000000	.0	72	.1250000	.0000000	.
8	.1250000	.0000000	.0	73	.1250000	.0000000	.
9	.1250000	.0000000	.0	74	.1250000	.0000000	.
10	.1250000	.0000000	.0	75	.1250000	.0000000	.
11	.1250000	.0000000	.0	76	.1250000	.0000000	.
12	.1250000	.0000000	.0	77	.1250000	.0000000	.
13	.1250000	.0000000	.0	78	.1250000	.0000000	.
14	.1250000	.0000000	.0	79	.1250000	.0000000	.
15	.1250000	.0000000	.0	80	.1250000	.0000000	.
16	.1250000	.0000000	.0	81	.1250000	.0000000	.
17	.1250000	.0000000	.0	82	.1250000	.0000000	.
18	.1250000	.0000000	.0	83	.1250000	.0000000	.
19	.1250000	.0000000	.0	84	.1250000	.0000000	.
20	.1250000	.0000000	.0	85	.1250000	.0000000	.
21	.1250000	.0000000	.0	86	.1250000	.0000000	.
22	.1250000	.0000000	.0	87	.1250000	.0000000	.
23	.1250000	.0000000	.0	88	.1250000	.0000000	.
24	.1250000	.0000000	.0	89	.1250000	.0000000	.
25	.1250000	.0000000	.0	90	.1250000	.0000000	.
26	.1250000	.0000000	.0	91	.1250000	.0000000	.
27	.1250000	.0000000	.0	92	.1250000	.0000000	.
28	.1250000	.0000000	.0	93	.1250000	.0000000	.
29	.1250000	.0000000	.0	94	.1250000	.0000000	.
30	.1250000	.0000000	.0	95	.1250000	.0000000	.
31	.1250000	.0000000	.0	96	.1250000	.0000000	.
32	.1250000	.0000000	.0	97	.1250000	.0000000	.
33	.1250000	.0000000	.0	98	.1250000	.0000000	.
34	.1250000	.0000000	.0	99	.1250000	.0000000	.
35	.1250000	.0000000	.0	100	.1250000	.0000000	.
36	.1250000	.0000000	.0	101	.1250000	.0000000	.
37	.1250000	.0000000	.0	102	.1250000	.0000000	.
38	.1250000	.0000000	.0	103	.1250000	.0000000	.
39	.1250000	.0000000	.0	104	.1250000	.0000000	.
40	.1250000	.0000000	.0	105	.1250000	.0000000	.
41	.1250000	.0000000	.0	106	.1250000	.0000000	.
42	.1250000	.0000000	.0	107	.1250000	.0000000	.
43	.1250000	.0000000	.0	108	.1250000	.0000000	.
44	.1250000	.0000000	.0	109	.1250000	.0000000	.
45	.1250000	.0000000	.0	110	.1250000	.0000000	.
46	.1250000	.0000000	.0	111	.1250000	.0000000	.
47	.1250000	.0000000	.0	112	.1250000	.0000000	.
48	.1250000	.0000000	.0	113	.1250000	.0000000	.
49	.1250000	.0000000	.0	114	.1250000	.0000000	.
50	.1250000	.0000000	.0	115	.1250000	.0000000	.
51	.1250000	.0000000	.0	116	.1250000	.0000000	.
52	.1250000	.0000000	.0	117	.1250000	.0000000	.
53	.1250000	.0000000	.0	118	.1250000	.0000000	.
54	.1250000	.0000000	.0	119	.1250000	.0000000	.
55	.1250000	.0000000	.0	120	.1250000	.0000000	.
56	.1250000	.0000000	.0	121	.1250000	.0000000	.
57	.1250000	.0000000	.0	122	.1250000	.0000000	.
58	.1250000	.0000000	.0	123	.1250000	.0000000	.
59	.1250000	.0000000	.0	124	.1250000	.0000000	.
60	.1250000	.0000000	.0	125	.1250000	.0000000	.
61	.1250000	.0000000	.0	126	.1250000	.0000000	.
62	.1250000	.0000000	.0	127	.1250000	.0000000	.
63	.1250000	.0000000	.0	128	.1250000	.0000000	.
64	.1250000	.0000000	.0	129	.1250000	.0000000	.

TABLE 9 (continued)

5 LEVEL MOD-DEMOD MODEL .12500000 STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .01562500

N	Y(N)	E(N)	S(N)
130	.0937500	.0312500	1.0
131	.0937500	.0312500	1.0
132	.0937500	.0312500	1.0
133	.0937500	.0312500	1.0
134	.1250000	.0000000	.0
135	.1250000	.0000000	.0
136	.1250000	.0000000	.0
137	.1250000	.0000000	.0
138	.1250000	.0000000	.0
139	.1250000	.0000000	.0
140	.1250000	.0000000	.0
141	.1250000	.0000000	.0
142	.1250000	.0000000	.0
143	.1250000	.0000000	.0
144	.1250000	.0000000	.0
145	.1250000	.0000000	.0
146	.1250000	.0000000	.0
147	.1250000	.0000000	.0
148	.1250000	.0000000	.0
149	.1250000	.0000000	.0
150	.1250000	.0000000	.0
151	.1250000	.0000000	.0
152	.1250000	.0000000	.0
153	.1250000	.0000000	.0
154	.1250000	.0000000	.0
155	.1250000	.0000000	.0
156	.1250000	.0000000	.0
157	.1250000	.0000000	.0
158	.1250000	.0000000	.0
159	.1250000	.0000000	.0
160	.1250000	.0000000	.0
161	.1250000	.0000000	.0
162	.1250000	.0000000	.0
163	.1250000	.0000000	.0
164	.1250000	.0000000	.0
165	.1250000	.0000000	.0
166	.1250000	.0000000	.0
167	.1250000	.0000000	.0
168	.1250000	.0000000	.0
169	.1250000	.0000000	.0
170	.1250000	.0000000	.0
171	.1250000	.0000000	.0
172	.1250000	.0000000	.0
173	.1250000	.0000000	.0
174	.1250000	.0000000	.0
175	.1250000	.0000000	.0
176	.1250000	.0000000	.0
177	.1250000	.0000000	.0
178	.1250000	.0000000	.0
179	.1250000	.0000000	.0
180	.1250000	.0000000	.0
181	.1250000	.0000000	.0
182	.1250000	.0000000	.0
183	.1250000	.0000000	.0
184	.1250000	.0000000	.0
185	.1250000	.0000000	.0
186	.1250000	.0000000	.0
187	.1250000	.0000000	.0
188	.1250000	.0000000	.0
189	.1250000	.0000000	.0
190	.1250000	.0000000	.0
191	.1250000	.0000000	.0
192	.1250000	.0000000	.0
193	.1250000	.0000000	.0
194	.1250000	.0000000	.0
195	.0937500	.0312500	1.0
196	.0937500	.0312500	1.0
197	.0937500	.0312500	1.0
198	.0937500	.0312500	1.0
199	.1250000	.0000000	.0
200	.1250000	.0000000	.0
201	.1250000	.0000000	.0
202	.1250000	.0000000	.0
203	.1250000	.0000000	.0
204	.1250000	.0000000	.0
205	.1250000	.0000000	.0
206	.1250000	.0000000	.0
207	.1250000	.0000000	.0
208	.1250000	.0000000	.0
209	.1250000	.0000000	.0
210	.1250000	.0000000	.0
211	.1250000	.0000000	.0
212	.1250000	.0000000	.0
213	.1250000	.0000000	.0
214	.1250000	.0000000	.0
215	.1250000	.0000000	.0
216	.1250000	.0000000	.0
217	.1250000	.0000000	.0
218	.1250000	.0000000	.0
219	.1250000	.0000000	.0
220	.1250000	.0000000	.0
221	.1250000	.0000000	.0
222	.1250000	.0000000	.0
223	.1250000	.0000000	.0
224	.1250000	.0000000	.0
225	.1250000	.0000000	.0
226	.1250000	.0000000	.0
227	.1250000	.0000000	.0
228	.1250000	.0000000	.0
229	.1250000	.0000000	.0
230	.1250000	.0000000	.0
231	.1250000	.0000000	.0
232	.1250000	.0000000	.0
233	.1250000	.0000000	.0
234	.1250000	.0000000	.0
235	.1250000	.0000000	.0
236	.1250000	.0000000	.0
237	.1250000	.0000000	.0
238	.1250000	.0000000	.0
239	.1250000	.0000000	.0
240	.1250000	.0000000	.0
241	.1250000	.0000000	.0
242	.1250000	.0000000	.0
243	.1250000	.0000000	.0
244	.1250000	.0000000	.0
245	.1250000	.0000000	.0
246	.1250000	.0000000	.0
247	.1250000	.0000000	.0
248	.1250000	.0000000	.0
249	.1250000	.0000000	.0
250	.1250000	.0000000	.0
251	.1250000	.0000000	.0
252	.1250000	.0000000	.0
253	.1250000	.0000000	.0
254	.1250000	.0000000	.0
255	.1250000	.0000000	.0
256	.1250000	.0000000	.0
257	.1250000	.0000000	.0
258	.1250000	.0000000	.0
259	.1250000	.0000000	.0

TABLE 10

5 LEVEL MOD-DEMOD MODEL .10000000 STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .01562500

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
0	.0000000	.1000000	1.0	65	.0625000	.0375000	1.0
1	.0312500	.0687500	1.0	66	.0625000	.0375000	1.0
2	.0625000	.0375000	1.0	67	.0625000	.0375000	1.0
3	.0937500	.0062500	.3	68	.0833333	.0166666	1.0
4	.1041667	-.0041666	-.3	69	.1250000	-.0249999	-1.0
5	.0937500	.0062500	.3	70	.0833333	.0166666	1.0
6	.1041667	-.0041666	-.3	71	.1250000	-.0249999	-1.0
7	.0937500	.0062500	.3	72	.0833333	.0166666	1.0
8	.1041667	-.0041666	-.3	73	.1250000	-.0249999	-1.0
9	.0937500	.0062500	.3	74	.0833333	.0166666	1.0
10	.1041667	-.0041666	-.3	75	.1250000	-.0249999	-1.0
11	.0937500	.0062500	.3	76	.0833333	.0166666	1.0
12	.1041667	-.0041666	-.3	77	.1250000	-.0249999	-1.0
13	.0937500	.0062500	.3	78	.0833333	.0166666	1.0
14	.1041667	-.0041666	-.3	79	.1250000	-.0249999	-1.0
15	.0937500	.0062500	.3	80	.0833333	.0166666	1.0
16	.1041667	-.0041666	-.3	81	.1250000	-.0249999	-1.0
17	.0937500	.0062500	.3	82	.0833333	.0166666	1.0
18	.1041667	-.0041666	-.3	83	.1250000	-.0249999	-1.0
19	.0937500	.0062500	.3	84	.0833333	.0166666	1.0
20	.1041667	-.0041666	-.3	85	.1250000	-.0249999	-1.0
21	.0937500	.0062500	.3	86	.0833333	.0166666	1.0
22	.1041667	-.0041666	-.3	87	.1250000	-.0249999	-1.0
23	.0937500	.0062500	.3	88	.0833333	.0166666	1.0
24	.1041667	-.0041666	-.3	89	.1250000	-.0249999	-1.0
25	.0937500	.0062500	.3	90	.0833333	.0166666	1.0
26	.1041667	-.0041666	-.3	91	.1250000	-.0249999	-1.0
27	.0937500	.0062500	.3	92	.0833333	.0166666	1.0
28	.1041667	-.0041666	-.3	93	.1250000	-.0249999	-1.0
29	.0937500	.0062500	.3	94	.0833333	.0166666	1.0
30	.1041667	-.0041666	-.3	95	.1250000	-.0249999	-1.0
31	.0937500	.0062500	.3	96	.0833333	.0166666	1.0
32	.1041667	-.0041666	-.3	97	.1250000	-.0249999	-1.0
33	.0937500	.0062500	.3	98	.0833333	.0166666	1.0
34	.1041667	-.0041666	-.3	99	.1250000	-.0249999	-1.0
35	.0937500	.0062500	.3	100	.0833333	.0166666	1.0
36	.1041667	-.0041666	-.3	101	.1250000	-.0249999	-1.0
37	.0937500	.0062500	.3	102	.0833333	.0166666	1.0
38	.1041667	-.0041666	-.3	103	.1250000	-.0249999	-1.0
39	.0937500	.0062500	.3	104	.0833333	.0166666	1.0
40	.1041667	-.0041666	-.3	105	.1250000	-.0249999	-1.0
41	.0937500	.0062500	.3	106	.0833333	.0166666	1.0
42	.1041667	-.0041666	-.3	107	.1250000	-.0249999	-1.0
43	.0937500	.0062500	.3	108	.0833333	.0166666	1.0
44	.1041667	-.0041666	-.3	109	.1250000	-.0249999	-1.0
45	.0937500	.0062500	.3	110	.0833333	.0166666	1.0
46	.1041667	-.0041666	-.3	111	.1250000	-.0249999	-1.0
47	.0937500	.0062500	.3	112	.0833333	.0166666	1.0
48	.1041667	-.0041666	-.3	113	.1250000	-.0249999	-1.0
49	.0937500	.0062500	.3	114	.0833333	.0166666	1.0
50	.1041667	-.0041666	-.3	115	.1250000	-.0249999	-1.0
51	.0937500	.0062500	.3	116	.0833333	.0166666	1.0
52	.1041667	-.0041666	-.3	117	.1250000	-.0249999	-1.0
53	.0937500	.0062500	.3	118	.0833333	.0166666	1.0
54	.1041667	-.0041666	-.3	119	.1250000	-.0249999	-1.0
55	.0937500	.0062500	.3	120	.0833333	.0166666	1.0
56	.1041667	-.0041666	-.3	121	.1250000	-.0249999	-1.0
57	.0937500	.0062500	.3	122	.0833333	.0166666	1.0
58	.1041667	-.0041666	-.3	123	.1250000	-.0249999	-1.0
59	.0937500	.0062500	.3	124	.0833333	.0166666	1.0
60	.1041667	-.0041666	-.3	125	.1250000	-.0249999	-1.0
61	.0937500	.0062500	.3	126	.0833333	.0166666	1.0
62	.1041667	-.0041666	-.3	127	.1250000	-.0249999	-1.0
63	.0937500	.0062500	.3	128	.0833333	.0166666	1.0
64	.1041667	-.0041666	-.3	129	.1250000	-.0249999	-1.0

TABLE 10 (continued)

5 LEVEL MOD-DEMOD MODEL .10000000 STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .01562500

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
30	.0625000	.0375000	1.0	195	.0625000	.0375000	1.0
31	.0625000	.0375000	1.0	196	.0625000	.0375000	1.0
32	.0625000	.0375000	1.0	197	.0625000	.0375000	1.0
33	.0625000	.0375000	1.0	198	.0625000	.0375000	1.0
34	.1250000	-.0249999	-1.0	199	.1250000	-.0249999	-1.0
35	.0625000	.0375000	1.0	200	.0625000	.0375000	1.0
36	.1250000	-.0249999	-1.0	201	.1250000	-.0249999	-1.0
37	.0625000	.0375000	1.0	202	.0625000	.0375000	1.0
38	.1250000	-.0249999	-1.0	203	.1250000	-.0249999	-1.0
39	.0625000	.0375000	1.0	204	.0625000	.0375000	1.0
40	.1250000	-.0249999	-1.0	205	.1250000	-.0249999	-1.0
41	.0625000	.0375000	1.0	206	.0625000	.0375000	1.0
42	.1250000	-.0249999	-1.0	207	.1250000	-.0249999	-1.0
43	.0625000	.0375000	1.0	208	.0625000	.0375000	1.0
44	.1250000	-.0249999	-1.0	209	.1250000	-.0249999	-1.0
45	.0625000	.0375000	1.0	210	.0625000	.0375000	1.0
46	.1250000	-.0249999	-1.0	211	.1250000	-.0249999	-1.0
47	.0625000	.0375000	1.0	212	.0625000	.0375000	1.0
48	.1250000	-.0249999	-1.0	213	.1250000	-.0249999	-1.0
49	.0625000	.0375000	1.0	214	.0625000	.0375000	1.0
50	.1250000	-.0249999	-1.0	215	.1250000	-.0249999	-1.0
51	.0625000	.0375000	1.0	216	.0625000	.0375000	1.0
52	.1250000	-.0249999	-1.0	217	.1250000	-.0249999	-1.0
53	.0625000	.0375000	1.0	218	.0625000	.0375000	1.0
54	.1250000	-.0249999	-1.0	219	.1250000	-.0249999	-1.0
55	.0625000	.0375000	1.0	220	.0625000	.0375000	1.0
56	.1250000	-.0249999	-1.0	221	.1250000	-.0249999	-1.0
57	.0625000	.0375000	1.0	222	.0625000	.0375000	1.0
58	.1250000	-.0249999	-1.0	223	.1250000	-.0249999	-1.0
59	.0625000	.0375000	1.0	224	.0625000	.0375000	1.0
60	.1250000	-.0249999	-1.0	225	.1250000	-.0249999	-1.0
61	.0625000	.0375000	1.0	226	.0625000	.0375000	1.0
62	.1250000	-.0249999	-1.0	227	.1250000	-.0249999	-1.0
63	.0625000	.0375000	1.0	228	.0625000	.0375000	1.0
64	.1250000	-.0249999	-1.0	229	.1250000	-.0249999	-1.0
65	.0625000	.0375000	1.0	230	.0625000	.0375000	1.0
66	.1250000	-.0249999	-1.0	231	.1250000	-.0249999	-1.0
67	.0625000	.0375000	1.0	232	.0625000	.0375000	1.0
68	.1250000	-.0249999	-1.0	233	.1250000	-.0249999	-1.0
69	.0625000	.0375000	1.0	234	.0625000	.0375000	1.0
70	.1250000	-.0249999	-1.0	235	.1250000	-.0249999	-1.0
71	.0625000	.0375000	1.0	236	.0625000	.0375000	1.0
72	.1250000	-.0249999	-1.0	237	.1250000	-.0249999	-1.0
73	.0625000	.0375000	1.0	238	.0625000	.0375000	1.0
74	.1250000	-.0249999	-1.0	239	.1250000	-.0249999	-1.0
75	.0625000	.0375000	1.0	240	.0625000	.0375000	1.0
76	.1250000	-.0249999	-1.0	241	.1250000	-.0249999	-1.0
77	.0625000	.0375000	1.0	242	.0625000	.0375000	1.0
78	.1250000	-.0249999	-1.0	243	.1250000	-.0249999	-1.0
79	.0625000	.0375000	1.0	244	.0625000	.0375000	1.0
80	.1250000	-.0249999	-1.0	245	.1250000	-.0249999	-1.0
81	.0625000	.0375000	1.0	246	.0625000	.0375000	1.0
82	.1250000	-.0249999	-1.0	247	.1250000	-.0249999	-1.0
83	.0625000	.0375000	1.0	248	.0625000	.0375000	1.0
84	.1250000	-.0249999	-1.0	249	.1250000	-.0249999	-1.0
85	.0625000	.0375000	1.0	250	.0625000	.0375000	1.0
86	.1250000	-.0249999	-1.0	251	.1250000	-.0249999	-1.0
87	.0625000	.0375000	1.0	252	.0625000	.0375000	1.0
88	.1250000	-.0249999	-1.0	253	.1250000	-.0249999	-1.0
89	.0625000	.0375000	1.0	254	.0625000	.0375000	1.0
90	.1250000	-.0249999	-1.0	255	.1250000	-.0249999	-1.0
91	.0625000	.0375000	1.0	256	.0625000	.0375000	1.0
92	.1250000	-.0249999	-1.0	257	.1250000	-.0249999	-1.0
93	.0625000	.0375000	1.0	258	.0625000	.0375000	1.0
94	.1250000	-.0249999	-1.0	259	.1250000	-.0249999	-1.0

TABLE 11

5 LEVEL MOD-DEMOD MODEL .20000000 STEP INPU

M = 64 GAIN = 2.00000000 DEAD SPACE = .0156250

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N
0	.0000000	.2000000	1.0	65	.1666667	.0333333	1.
1	.0312500	.1687500	1.0	66	.1666667	.0333333	1.
2	.0625000	.1375000	1.0	67	.1666667	.0333333	1.
3	.0937500	.1062500	1.0	68	.1666667	.0333333	1.
4	.1250000	.0750000	1.0	69	.1666667	.0333333	1.
5	.1562500	.0437500	1.0	70	.1666667	.0333333	1.
6	.1875000	.0125000	.3	71	.1875000	.0125000	.
7	.1979167	.0020833	.3	72	.1875000	.0125000	.
8	.2083333	-.0083333	-.3	73	.2083333	-.0083333	-.
9	.1979167	.0020833	.3	74	.1875000	.0125000	.
10	.2083333	-.0083333	-.3	75	.2083333	-.0083333	-.
11	.1979167	.0020833	.3	76	.1875000	.0125000	.
12	.2083333	-.0083333	-.3	77	.2083333	-.0083333	-.
13	.1979167	.0020833	.3	78	.1875000	.0125000	.
14	.2083333	-.0083333	-.3	79	.2083333	-.0083333	-.
15	.1979167	.0020833	.3	80	.1875000	.0125000	.
16	.2083333	-.0083333	-.3	81	.2083333	-.0083333	-.
17	.1979167	.0020833	.3	82	.1875000	.0125000	.
18	.2083333	-.0083333	-.3	83	.2083333	-.0083333	-.
19	.1979167	.0020833	.3	84	.1875000	.0125000	.
20	.2083333	-.0083333	-.3	85	.2083333	-.0083333	-.
21	.1979167	.0020833	.3	86	.1875000	.0125000	.
22	.2083333	-.0083333	-.3	87	.2083333	-.0083333	-.
23	.1979167	.0020833	.3	88	.1875000	.0125000	.
24	.2083333	-.0083333	-.3	89	.2083333	-.0083333	-.
25	.1979167	.0020833	.3	90	.1875000	.0125000	.
26	.2083333	-.0083333	-.3	91	.2083333	-.0083333	-.
27	.1979167	.0020833	.3	92	.1875000	.0125000	.
28	.2083333	-.0083333	-.3	93	.2083333	-.0083333	-.
29	.1979167	.0020833	.3	94	.1875000	.0125000	.
30	.2083333	-.0083333	-.3	95	.2083333	-.0083333	-.
31	.1979167	.0020833	.3	96	.1875000	.0125000	.
32	.2083333	-.0083333	-.3	97	.2083333	-.0083333	-.
33	.1979167	.0020833	.3	98	.1875000	.0125000	.
34	.2083333	-.0083333	-.3	99	.2083333	-.0083333	-.
35	.1979167	.0020833	.3	100	.1875000	.0125000	.
36	.2083333	-.0083333	-.3	101	.2083333	-.0083333	-.
37	.1979167	.0020833	.3	102	.1875000	.0125000	.
38	.2083333	-.0083333	-.3	103	.2083333	-.0083333	-.
39	.1979167	.0020833	.3	104	.1875000	.0125000	.
40	.2083333	-.0083333	-.3	105	.2083333	-.0083333	-.
41	.1979167	.0020833	.3	106	.1875000	.0125000	.
42	.2083333	-.0083333	-.3	107	.2083333	-.0083333	-.
43	.1979167	.0020833	.3	108	.1875000	.0125000	.
44	.2083333	-.0083333	-.3	109	.2083333	-.0083333	-.
45	.1979167	.0020833	.3	110	.1875000	.0125000	.
46	.2083333	-.0083333	-.3	111	.2083333	-.0083333	-.
47	.1979167	.0020833	.3	112	.1875000	.0125000	.
48	.2083333	-.0083333	-.3	113	.2083333	-.0083333	-.
49	.1979167	.0020833	.3	114	.1875000	.0125000	.
50	.2083333	-.0083333	-.3	115	.2083333	-.0083333	-.
51	.1979167	.0020833	.3	116	.1875000	.0125000	.
52	.2083333	-.0083333	-.3	117	.2083333	-.0083333	-.
53	.1979167	.0020833	.3	118	.1875000	.0125000	.
54	.2083333	-.0083333	-.3	119	.2083333	-.0083333	-.
55	.1979167	.0020833	.3	120	.1875000	.0125000	.
56	.2083333	-.0083333	-.3	121	.2083333	-.0083333	-.
57	.1979167	.0020833	.3	122	.1875000	.0125000	.
58	.2083333	-.0083333	-.3	123	.2083333	-.0083333	-.
59	.1979167	.0020833	.3	124	.1875000	.0125000	.
60	.2083333	-.0083333	-.3	125	.2083333	-.0083333	-.
61	.1979167	.0020833	.3	126	.1875000	.0125000	.
62	.2083333	-.0083333	-.3	127	.2083333	-.0083333	-.
63	.1979167	.0020833	.3	128	.1875000	.0125000	.
64	.2083333	-.0083333	-.3	129	.2083333	-.0083333	-.

TABLE 11 (continued)

5 LEVEL MOD-DEMOD MODEL .20000000 STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .01562500

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
30	.1666667	.0333333	1.0	195	.1666667	.0333333	1.0
31	.1666667	.0333333	1.0	196	.1666667	.0333333	1.0
32	.1666667	.0333333	1.0	197	.1666667	.0333333	1.0
33	.1666667	.0333333	1.0	198	.1666667	.0333333	1.0
34	.1666667	.0333333	1.0	199	.1666667	.0333333	1.0
35	.1666667	.0333333	1.0	200	.1666667	.0333333	1.0
36	.1875000	.0125000	.3	201	.1875000	.0125000	.3
37	.1875000	.0125000	.3	202	.1875000	.0125000	.3
38	.2083333	-.0083333	-.3	203	.2083333	-.0083333	-.3
39	.1875000	.0125000	.3	204	.1875000	.0125000	.3
40	.2083333	-.0083333	-.3	205	.2083333	-.0083333	-.3
41	.1875000	.0125000	.3	206	.1875000	.0125000	.3
42	.2083333	-.0083333	-.3	207	.2083333	-.0083333	-.3
43	.1875000	.0125000	.3	208	.1875000	.0125000	.3
44	.2083333	-.0083333	-.3	209	.2083333	-.0083333	-.3
45	.1875000	.0125000	.3	210	.1875000	.0125000	.3
46	.2083333	-.0083333	-.3	211	.2083333	-.0083333	-.3
47	.1875000	.0125000	.3	212	.1875000	.0125000	.3
48	.2083333	-.0083333	-.3	213	.2083333	-.0083333	-.3
49	.1875000	.0125000	.3	214	.1875000	.0125000	.3
50	.2083333	-.0083333	-.3	215	.2083333	-.0083333	-.3
51	.1875000	.0125000	.3	216	.1875000	.0125000	.3
52	.2083333	-.0083333	-.3	217	.2083333	-.0083333	-.3
53	.1875000	.0125000	.3	218	.1875000	.0125000	.3
54	.2083333	-.0083333	-.3	219	.2083333	-.0083333	-.3
55	.1875000	.0125000	.3	220	.1875000	.0125000	.3
56	.2083333	-.0083333	-.3	221	.2083333	-.0083333	-.3
57	.1875000	.0125000	.3	222	.1875000	.0125000	.3
58	.2083333	-.0083333	-.3	223	.2083333	-.0083333	-.3
59	.1875000	.0125000	.3	224	.1875000	.0125000	.3
60	.2083333	-.0083333	-.3	225	.2083333	-.0083333	-.3
61	.1875000	.0125000	.3	226	.1875000	.0125000	.3
62	.2083333	-.0083333	-.3	227	.2083333	-.0083333	-.3
63	.1875000	.0125000	.3	228	.1875000	.0125000	.3
64	.2083333	-.0083333	-.3	229	.2083333	-.0083333	-.3
65	.1875000	.0125000	.3	230	.1875000	.0125000	.3
66	.2083333	-.0083333	-.3	231	.2083333	-.0083333	-.3
67	.1875000	.0125000	.3	232	.1875000	.0125000	.3
68	.2083333	-.0083333	-.3	233	.2083333	-.0083333	-.3
69	.1875000	.0125000	.3	234	.1875000	.0125000	.3
70	.2083333	-.0083333	-.3	235	.2083333	-.0083333	-.3
71	.1875000	.0125000	.3	236	.1875000	.0125000	.3
72	.2083333	-.0083333	-.3	237	.2083333	-.0083333	-.3
73	.1875000	.0125000	.3	238	.1875000	.0125000	.3
74	.2083333	-.0083333	-.3	239	.2083333	-.0083333	-.3
75	.1875000	.0125000	.3	240	.1875000	.0125000	.3
76	.2083333	-.0083333	-.3	241	.2083333	-.0083333	-.3
77	.1875000	.0125000	.3	242	.1875000	.0125000	.3
78	.2083333	-.0083333	-.3	243	.2083333	-.0083333	-.3
79	.1875000	.0125000	.3	244	.1875000	.0125000	.3
80	.2083333	-.0083333	-.3	245	.2083333	-.0083333	-.3
81	.1875000	.0125000	.3	246	.1875000	.0125000	.3
82	.2083333	-.0083333	-.3	247	.2083333	-.0083333	-.3
83	.1875000	.0125000	.3	248	.1875000	.0125000	.3
84	.2083333	-.0083333	-.3	249	.2083333	-.0083333	-.3
85	.1875000	.0125000	.3	250	.1875000	.0125000	.3
86	.2083333	-.0083333	-.3	251	.2083333	-.0083333	-.3
87	.1875000	.0125000	.3	252	.1875000	.0125000	.3
88	.2083333	-.0083333	-.3	253	.2083333	-.0083333	-.3
89	.1875000	.0125000	.3	254	.1875000	.0125000	.3
90	.2083333	-.0083333	-.3	255	.2083333	-.0083333	-.3
91	.1875000	.0125000	.3	256	.1875000	.0125000	.3
92	.2083333	-.0083333	-.3	257	.2083333	-.0083333	-.3
93	.1875000	.0125000	.3	258	.1875000	.0125000	.3
94	.2083333	-.0083333	-.3	259	.2083333	-.0083333	-.3

TABLE 12

5 LEVEL MOD-DEMOD MODEL ERROR DATA STEP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = 0.01562500

INPUT	AVERAGE	MAX ERROR	RMS ERROR	S/N RATIO	IN DB
.030000	.030769	.030000	.011029	2.790	8.912
.031250	.030769	.031250	.003846	8.000	18.062
.040000	.030769	.040000	.031246	.985	-.133
.050000	.051282	.029167	.011601	4.421	12.910
.060000	.051282	.039167	.031025	1.652	4.365
.062500	.061539	.031250	.005397	11.403	21.141
.070000	.071795	.028333	.011574	6.203	15.852
.080000	.071795	.038333	.030799	2.331	7.351
.090000	.092308	.027500	.012103	7.627	17.647
.100000	.092308	.037500	.031217	2.957	9.417
.125000	.123077	.031250	.007510	16.389	24.291
.200000	.194872	.033333	.013393	14.551	23.257
.250000	.246153	.031250	.010267	23.977	27.596
.300000	.297436	.029167	.014848	20.032	26.035
.400000	.400000	-.037500	.030619	13.064	22.321
.500000	.492308	.031250	.013462	36.571	31.263
.600000	.584615	.037500	.029885	19.562	25.828
.700000	.687179	.033333	.016942	40.561	32.162
.800000	.789744	.029167	.017420	45.335	33.129
.900000	.892308	-.037500	.027967	31.905	30.077
1.000000	.984615	.031250	.015623	63.023	35.990
1.100000	1.076923	.037500	.026333	40.897	32.233
1.200000	1.179487	.033333	.016623	70.953	37.019
1.300000	1.282051	.029167	.016374	78.296	37.875
1.400000	1.384615	-.037500	.022550	61.402	35.764
1.500000	1.476923	.031250	.013734	107.542	40.632
1.600000	1.569231	.037500	.019374	80.995	38.169
1.700000	1.671795	.033333	.012137	137.741	42.781
1.800000	1.774359	.029167	.010698	165.865	44.395
1.900000	1.876923	-.037500	.010793	173.898	44.806
2.000000	1.969231	.031250	.003846	512.000	54.185

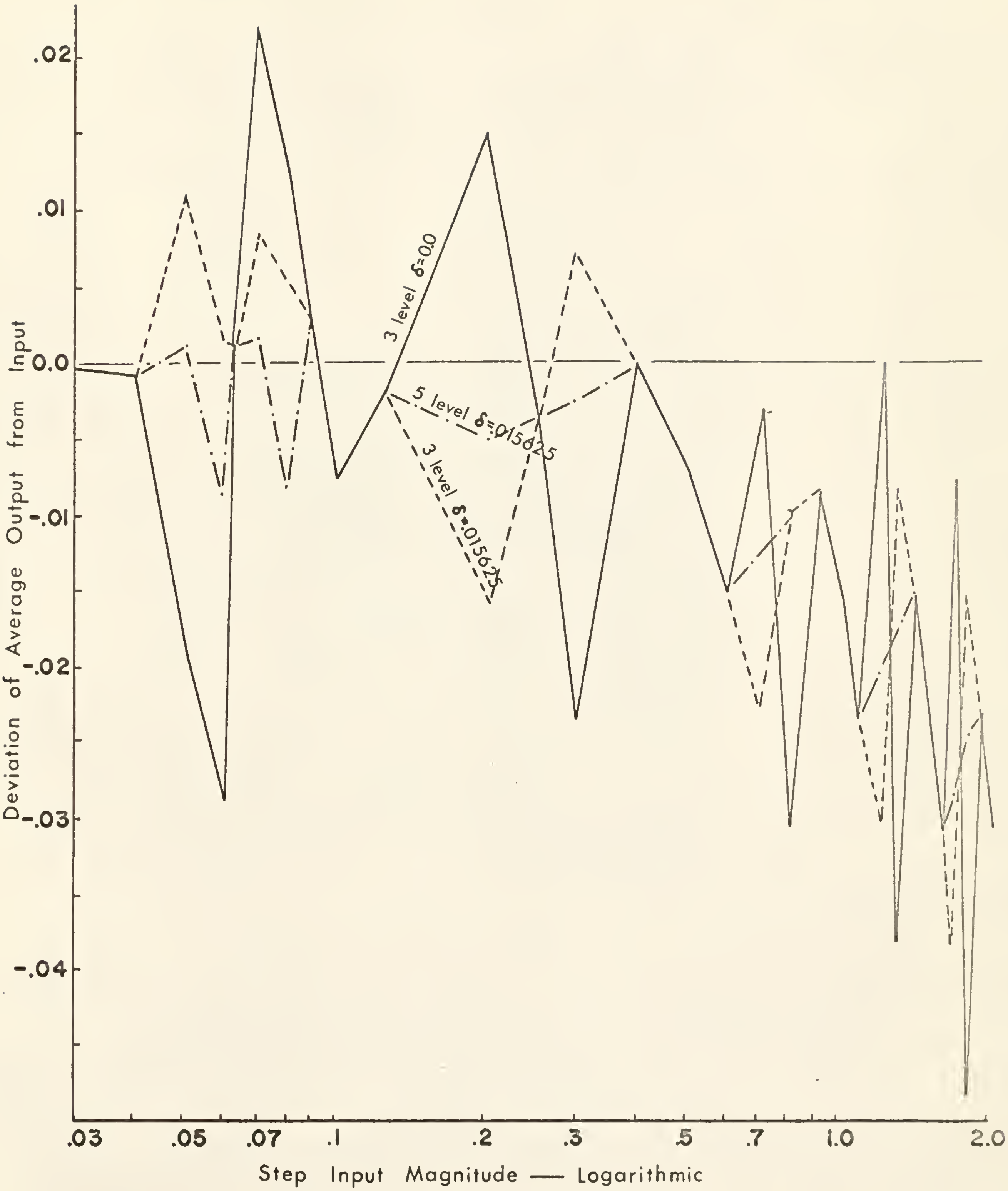

Fig. 29. Average error vs. input magnitude.

Results for Ramp Inputs

For ramp inputs, the five-level device again shows a noise improvement over the original three-level device (see Figs. 30 and 31), but at the expense of increased output delay. Plates IV and V show typical ramp response characteristics for the five-level case with $\delta = k/2m$ and $\delta = k/m$. The output values are tabulated in Tables 13 and 14. Only the case of $\delta = k/m$ showed an improvement over the three-level case with dead space; however, its delay was nearly twice that of the optimum three-level case with dead space for large ramp inputs.

Conclusions for Five-Level Device

The generalization of delta modulation to a five-level device shows an improvement over the basic delta modulation system. The cost of improvement is great, however, in the complexity of equipment needed for the five-level device. There is also a trade-off between high signal-to-noise and small delay. One can be had only at the expense of the other. A noise improvement nearly equal to that of the five-level device is obtained by adding a dead space to the basic system. Once again, however, there is a trade-off between low noise and increased delay.

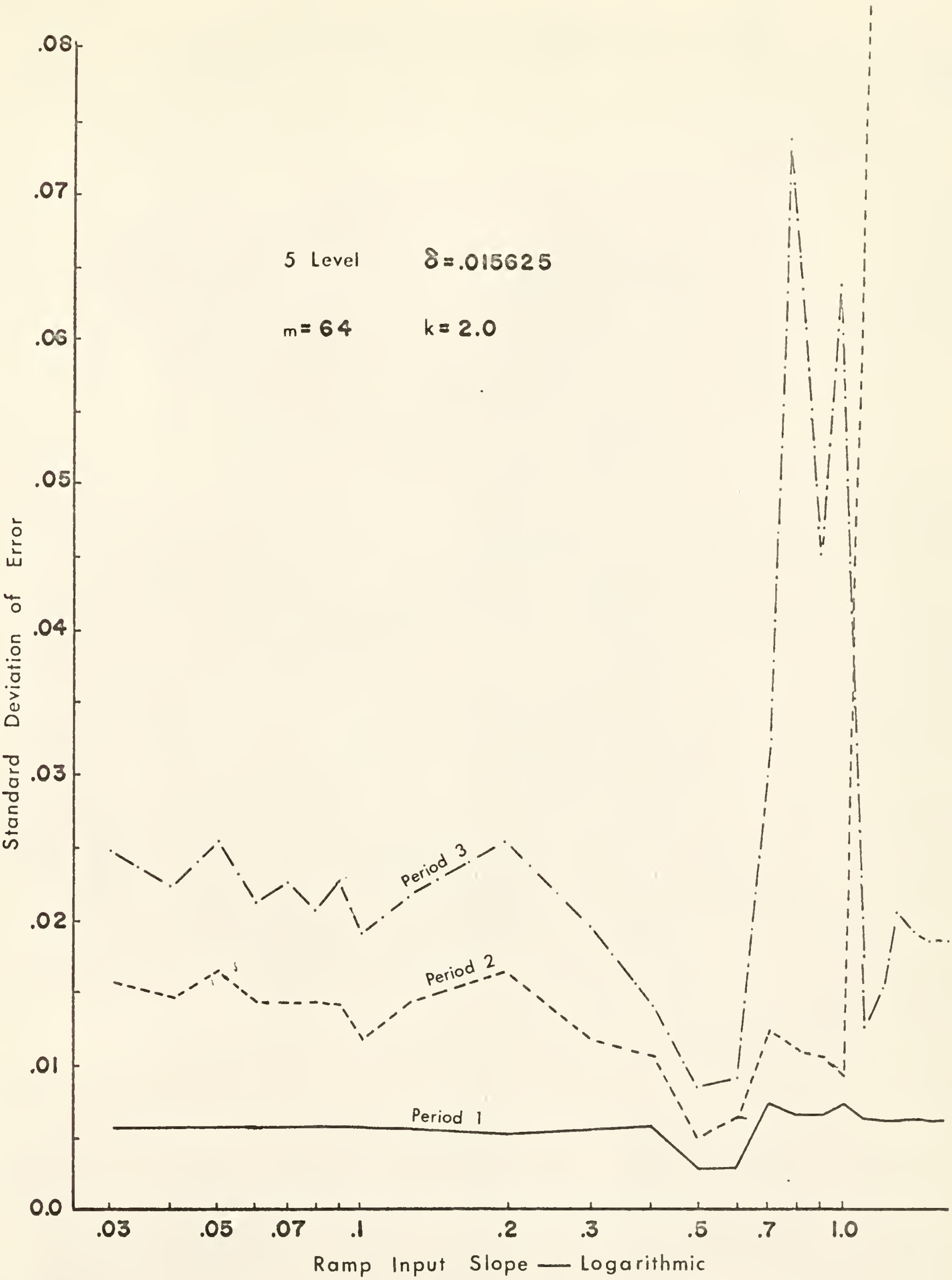

Fig. 30. Standard deviation of error vs. ramp input slope.

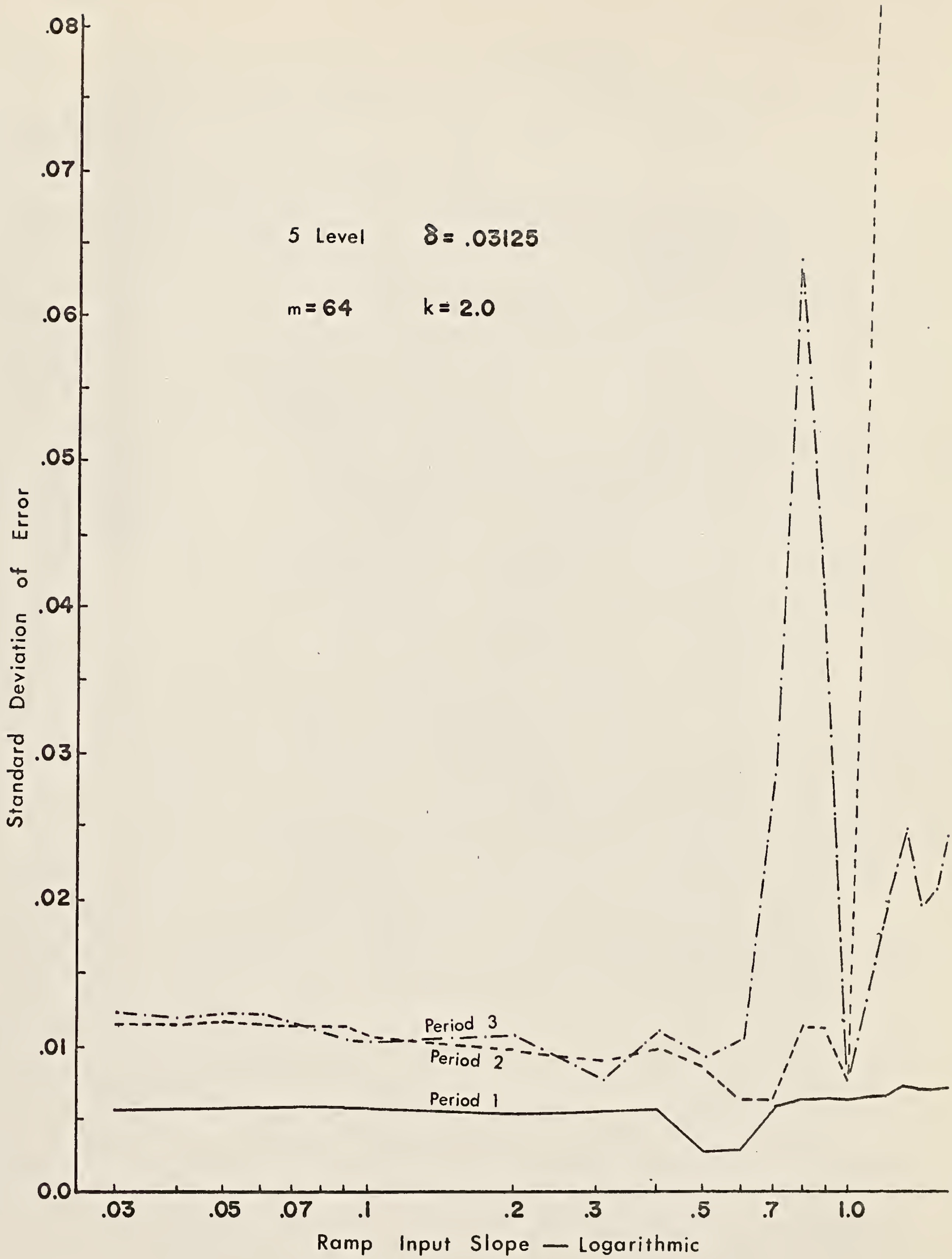

Fig. 31. Standard deviation of error vs. ramp input slope.

EXPLANATION OF PLATE IV

Five-level delta modulation, $\delta = .015625$, m = 64, k = 2.0

Output for ramp input slope 0.1

Fig. 32. Period 1 (samples 0-64)

Fig. 33. Period 2 (samples 65-129)

Fig. 34. Period 3 (samples 130-194)

PLATE IV

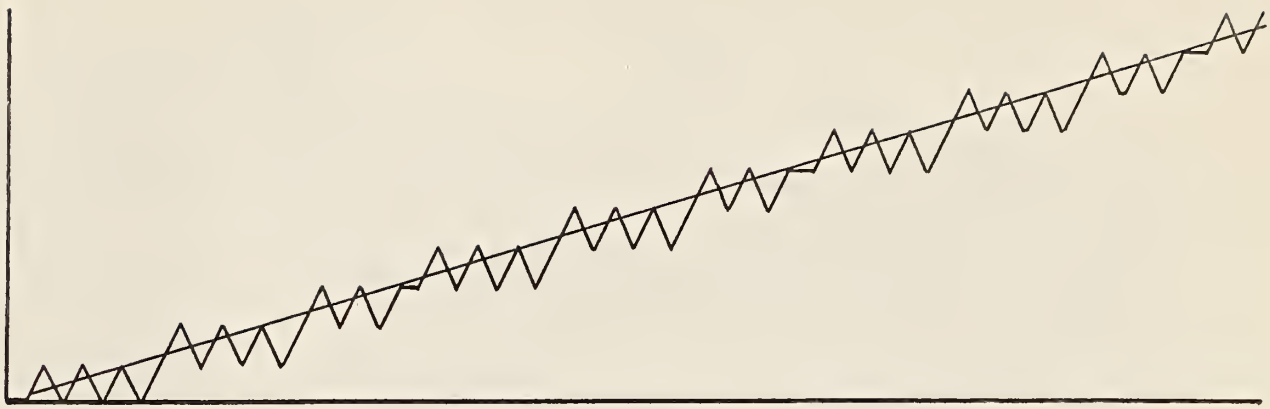

Fig. 32

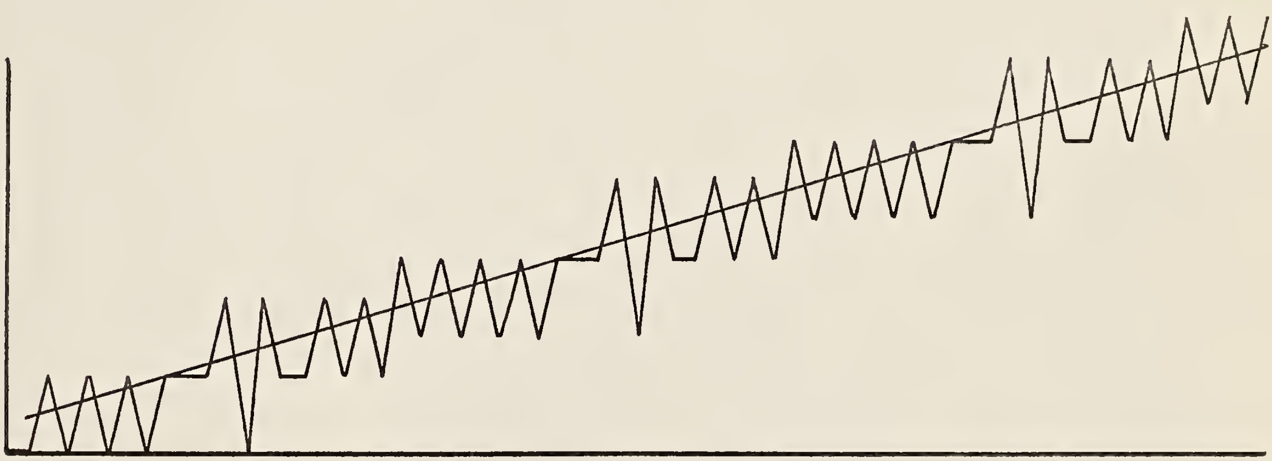

Fig. 33

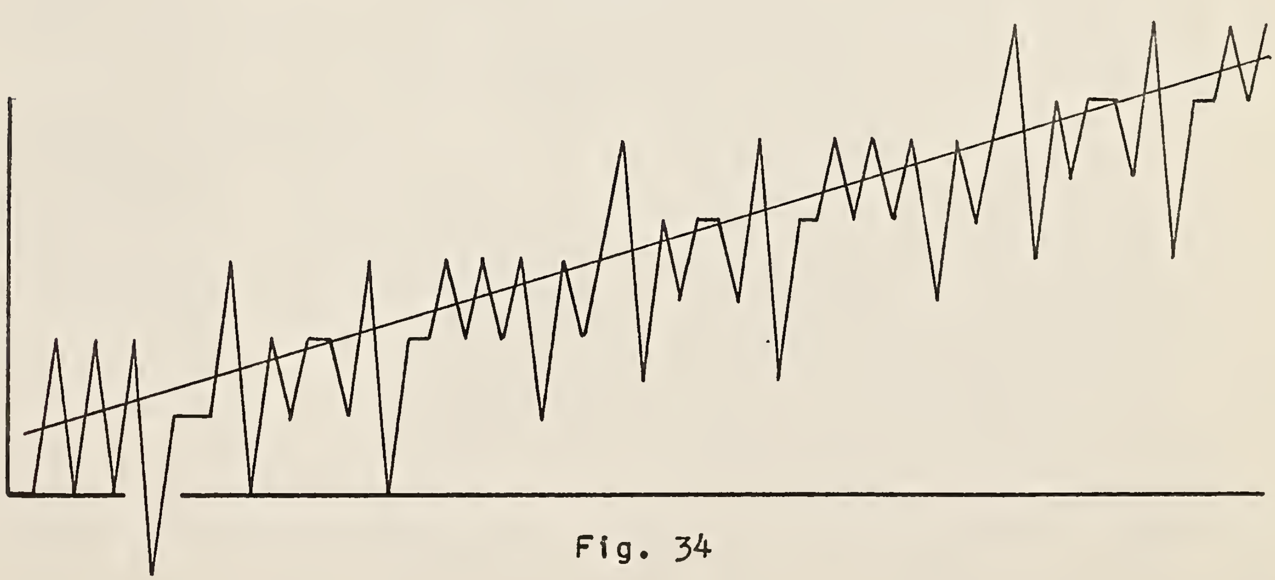

Fig. 34

EXPLANATION OF PLATE V

Five-level delta modulation, $\delta = .03125$, $m = 64$, $k = 2.0$

Output for ramp input slope 0.1

Fig. 35. Period 1 (samples 0-64)

Fig. 36. Period 2 (samples 65-129)

Fig. 37. Period 3 (samples 130-194)

PLATE V

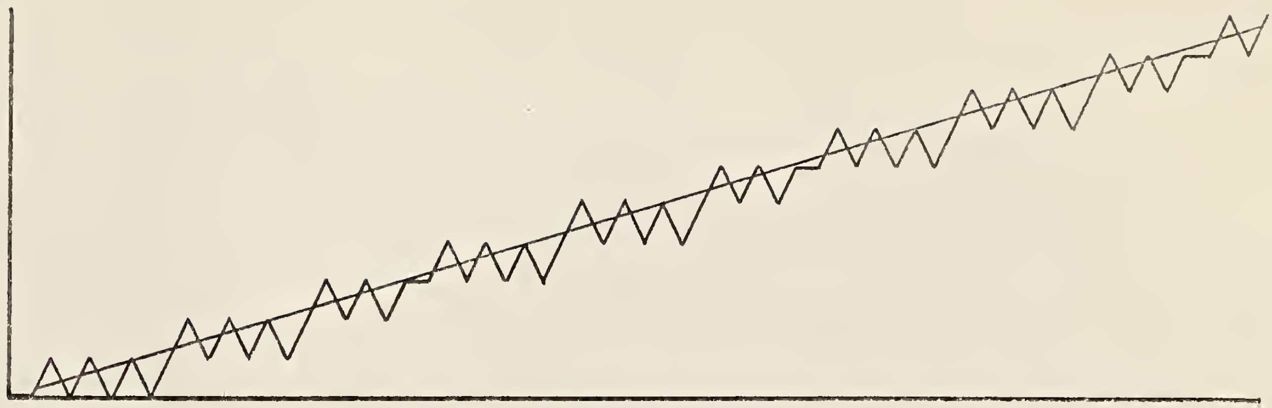

Fig. 35

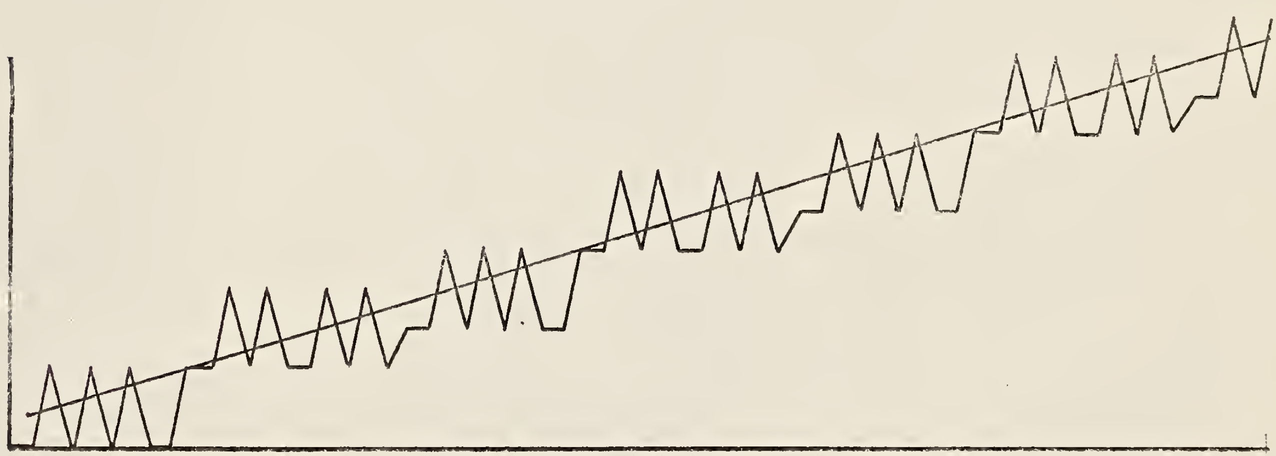

Fig. 36

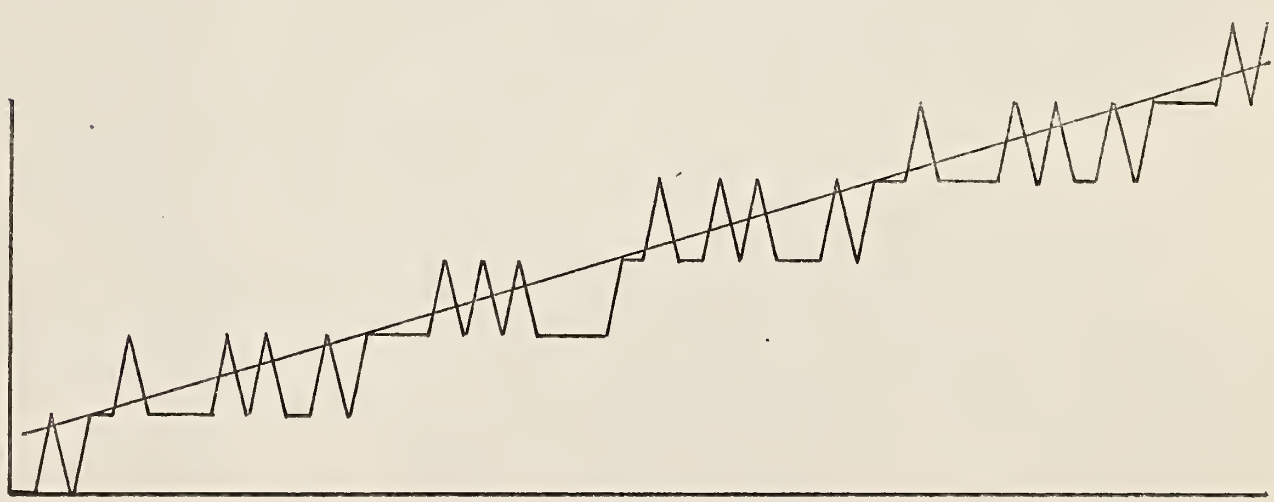

Fig. 37

TABLE 13

5 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .01562500

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N)
0	.0000000	.0000000	.0	65	.0937500	.0078125	.3
1	.0000000	.0015625	.3	66	.0937500	.0093750	.3
2	.0104167	-.0072916	-.3	67	.1145833	-.0098958	-.3
3	.0000000	.0046875	.3	68	.0937500	.0125000	.3
4	.0104167	-.0041666	-.3	69	.1145833	-.0067708	-.3
5	.0000000	.0078125	.3	70	.0937500	.0156250	.3
6	.0104167	-.0010416	-.3	71	.1145833	-.0036458	-.3
7	.0000000	.0109375	.3	72	.0937500	.0187500	1.0
8	.0104167	.0020833	.3	73	.1145833	-.0005208	-.3
9	.0208333	-.0067708	-.3	74	.1145833	.0010416	.3
10	.0104167	.0052083	.3	75	.1145833	.0026041	.3
11	.0208333	-.0036458	-.3	76	.1354167	-.0166666	-1.0
12	.0104167	.0083333	.3	77	.0937500	.0265625	1.0
13	.0208333	-.0005208	-.3	78	.1354167	-.0135416	-.3
14	.0104167	.0114583	.3	79	.1145833	.0088541	.3
15	.0208333	.0026041	.3	80	.1145833	.0104166	.3
16	.0312500	-.0062499	-.3	81	.1354167	-.0088541	-.3
17	.0208333	.0057291	.3	82	.1145833	.0135416	.3
18	.0312500	-.0031249	-.3	83	.1354167	-.0057291	-.3
19	.0208333	.0088541	.3	84	.1145833	.0166666	1.0
20	.0312500	.0000000	.0	85	.1458333	-.0130208	-.3
21	.0312500	.0015625	.3	86	.1250000	.0093750	.3
22	.0416667	-.0072916	-.3	87	.1458333	-.0098958	-.3
23	.0312500	.0046875	.3	88	.1250000	.0125000	.3
24	.0416667	-.0041666	-.3	89	.1458333	-.0067708	-.3
25	.0312500	.0078125	.3	90	.1250000	.0156250	.3
26	.0416667	-.0010416	-.3	91	.1458333	-.0036458	-.3
27	.0312500	.0109375	.3	92	.1250000	.0187500	1.0
28	.0416667	.0020833	.3	93	.1458333	-.0005208	-.3
29	.0520833	-.0067708	-.3	94	.1458333	.0010416	.3
30	.0416667	.0052083	.3	95	.1458333	.0026041	.3
31	.0520833	-.0036458	-.3	96	.1666667	-.0166666	-1.0
32	.0416667	.0083333	.3	97	.1250000	.0265625	1.0
33	.0520833	-.0005208	-.3	98	.1666667	-.0135416	-.3
34	.0416667	.0114583	.3	99	.1458333	.0088541	.3
35	.0520833	.0026041	.3	100	.1458333	.0104166	.3
36	.0625000	-.0062499	-.3	101	.1666667	-.0088541	-.3
37	.0520833	.0057291	.3	102	.1458333	.0135416	.3
38	.0625000	-.0031249	-.3	103	.1666667	-.0057291	-.3
39	.0520833	.0088541	.3	104	.1458333	.0166666	1.0
40	.0625000	.0000000	.0	105	.1770833	-.0130208	-.3
41	.0625000	.0015625	.3	106	.1562500	.0093750	.3
42	.0729167	-.0072916	-.3	107	.1770833	-.0098958	-.3
43	.0625000	.0046875	.3	108	.1562500	.0125000	.3
44	.0729167	-.0041666	-.3	109	.1770833	-.0067708	-.3
45	.0625000	.0078125	.3	110	.1562500	.0156250	.3
46	.0729167	-.0010416	-.3	111	.1770833	-.0036458	-.3
47	.0625000	.0109375	.3	112	.1562500	.0187500	1.0
48	.0729167	.0020833	.3	113	.1770833	-.0005208	-.3
49	.0833333	-.0067708	-.3	114	.1770833	.0010416	.3
50	.0729167	.0052083	.3	115	.1770833	.0026041	.3
51	.0833333	-.0036458	-.3	116	.1979167	-.0166666	-1.0
52	.0729167	.0083333	.3	117	.1562500	.0265625	1.0
53	.0833333	-.0005208	-.3	118	.1979167	-.0135416	-.3
54	.0729167	.0114583	.3	119	.1770833	.0088541	.3
55	.0833333	.0026041	.3	120	.1770833	.0104166	.3
56	.0937500	-.0062499	-.3	121	.1979167	-.0088541	-.3
57	.0833333	.0057291	.3	122	.1770833	.0135416	.3
58	.0937500	-.0031249	-.3	123	.1979167	-.0057291	-.3
59	.0833333	.0088541	.3	124	.1770833	.0166666	1.0
60	.0937500	.0000000	.0	125	.2083333	-.0130208	-.3
61	.0937500	.0015625	.3	126	.1875000	.0093750	.3
62	.1041667	-.0072916	-.3	127	.2083333	-.0098958	-.3
63	.0937500	.0046875	.3	128	.1875000	.0125000	.3
64	.1041667	-.0041666	-.3	129	.2083333	-.0067708	-.3

TABLE 13 (continued)

5 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPU

M = 64 GAIN = 2.00000000 DEAD SPACE = .0156250

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N
130	.1875000	.0156250	.3	195	.2916667	.0130708	.
131	.1875000	.0171875	1.0	196	.2708333	.0354166	1.
132	.2291667	-.0229166	-1.0	197	.3333333	-.0255708	-1.
133	.1875000	.0203125	1.0	198	.2708333	.0385416	1.
134	.2291667	-.0197916	-1.0	199	.3333333	-.0223958	-1.
135	.1875000	.0234375	1.0	200	.2708333	.0416666	1.
136	.2291667	-.0166666	-1.0	201	.3333333	-.0192708	-1.
137	.1666667	.0473958	1.0	202	.2708333	.0447916	1.
138	.2083333	.0072916	.3	203	.2916667	.0255208	1.
139	.2083333	.0088541	.3	204	.3125000	.0062500	.
140	.2083333	.0104166	.3	205	.3125000	.0078125	.
141	.2500000	-.0296874	-1.0	206	.3541667	-.0322916	-1.
142	.1875000	.0343750	1.0	207	.2916667	.0317708	1.
143	.2291667	-.0057291	-.3	208	.3333333	-.0083333	-.
144	.2083333	.0166666	1.0	209	.2916667	.0348958	1.
145	.2291667	-.0026041	-.3	210	.3333333	-.0052083	-.
146	.2291667	-.0010416	-.3	211	.3333333	-.0036458	-.
147	.2083333	.0213541	1.0	212	.2916667	.0395833	1.
148	.2500000	-.0187499	-1.0	213	.3541667	-.0213541	-1.
149	.1875000	.0453125	1.0	214	.2916667	.0427083	1.
150	.2291667	.0052083	.3	215	.3125000	.0234375	1.
151	.2291667	.0067708	.3	216	.3333333	.0041666	.
152	.2500000	-.0124999	-.3	217	.3541667	-.0151041	-.
153	.2291667	.0098958	.3	218	.3333333	.0072916	.
154	.2500000	-.0093749	-.3	219	.3541667	-.0119791	-.
155	.2291667	.0130208	.3	220	.3333333	.0104166	.
156	.2500000	-.0062499	-.3	221	.3541667	-.0088541	-.
157	.2083333	.0369791	1.0	222	.3125000	.0343750	1.
158	.2500000	-.0031249	-.3	223	.3541667	-.0057291	-.
159	.2291667	.0192708	1.0	224	.3125000	.0375000	1.
160	.2500000	.0000000	.0	225	.3437500	.0078125	.
161	.2812500	-.0296875	-1.0	226	.3854167	-.0322916	-1.
162	.2187500	.0343750	1.0	227	.3229167	.0317708	1.
163	.2604167	-.0057291	-.3	228	.3645833	-.0083333	-.
164	.2395833	.0166666	1.0	229	.3229167	.0348958	1.
165	.2604167	-.0026041	-.3	230	.3645833	-.0052083	-.
166	.2604167	-.0010416	-.3	231	.3645833	-.0036458	-.
167	.2395833	.0213541	1.0	232	.3229167	.0395833	1.
168	.2812500	-.0187499	-1.0	233	.3854167	-.0213541	-1.
169	.2187500	.0453125	1.0	234	.3229167	.0427083	1.
170	.2604167	.0052083	.3	235	.3437500	.0234375	1.
171	.2604167	.0067708	.3	236	.3645833	.0041666	.
172	.2812500	-.0124999	-.3	237	.3854167	-.0151041	-.
173	.2604167	.0098958	.3	238	.3645833	.0072916	.
174	.2812500	-.0093749	-.3	239	.3854167	-.0119791	-.
175	.2604167	.0130208	.3	240	.3645833	.0104166	.
176	.2812500	-.0062499	-.3	241	.3854167	-.0088541	-.
177	.2395833	.0369791	1.0	242	.3437500	.0343750	1.
178	.2812500	-.0031249	-.3	243	.3854167	-.0057291	-.
179	.2604167	.0192708	1.0	244	.3437500	.0375000	1.
180	.2812500	.0000000	.0	245	.3750000	.0078125	.
181	.3125000	-.0296875	-1.0	246	.4166667	-.0322916	-1.
182	.2500000	.0343750	1.0	247	.3541667	.0317708	1.
183	.2916667	-.0057291	-.3	248	.3958333	-.0083333	-.
184	.2708333	.0166666	1.0	249	.3541667	.0348958	1.
185	.2916667	-.0026041	-.3	250	.3958333	-.0052083	-.
186	.2916667	-.0010416	-.3	251	.3958333	-.0036458	-.
187	.2708333	.0213541	1.0	252	.3541667	.0395833	1.
188	.3125000	-.0187499	-1.0	253	.4166667	-.0213541	-1.
189	.2500000	.0453125	1.0	254	.3541667	.0427083	1.
190	.2916667	.0052083	.3	255	.3750000	.0234375	1.
191	.2916667	.0067708	.3	256	.3958333	.0041666	.
192	.3125000	-.0124999	-.3	257	.4166667	-.0151041	-.
193	.2916667	.0098958	.3	258	.3958333	.0072916	.
194	.3125000	-.0093749	-.3	259	.4166667	-.0119791	-.

TABLE 14

5 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPUT

M = 64 GAIN = 2.00000000 DEAD SPACE = .03125000

N	Y(N)	E(N)	S(N)
0	.0000000	.0000000	.0
1	.0000000	.0015625	.3
2	.0104167	-.0072916	-.3
3	.0000000	.0046875	.3
4	.0104167	-.0041666	-.3
5	.0000000	.0078125	.3
6	.0104167	-.0010416	-.3
7	.0000000	.0109375	.3
8	.0104167	.0020833	.3
9	.0208333	-.0067708	-.3
10	.0104167	.0052083	.3
11	.0208333	-.0036458	-.3
12	.0104167	.0083333	.3
13	.0208333	-.0005208	-.3
14	.0104167	.0114583	.3
15	.0208333	.0026041	.3
16	.0312500	-.0062499	-.3
17	.0208333	.0057291	.3
18	.0312500	-.0031249	-.3
19	.0208333	.0088541	.3
20	.0312500	.0000000	.0
21	.0312500	.0015625	.3
22	.0416667	-.0072916	-.3
23	.0312500	.0046875	.3
24	.0416667	-.0041666	-.3
25	.0312500	.0078125	.3
26	.0416667	-.0010416	-.3
27	.0312500	.0109375	.3
28	.0416667	.0020833	.3
29	.0520833	-.0067708	-.3
30	.0416667	.0052083	.3
31	.0520833	-.0036458	-.3
32	.0416667	.0083333	.3
33	.0520833	-.0005208	-.3
34	.0416667	.0114583	.3
35	.0520833	.0026041	.3
36	.0625000	-.0062499	-.3
37	.0520833	.0057291	.3
38	.0625000	-.0031249	-.3
39	.0520833	.0088541	.3
40	.0625000	.0000000	.0
41	.0625000	.0015625	.3
42	.0729167	-.0072916	-.3
43	.0625000	.0046875	.3
44	.0729167	-.0041666	-.3
45	.0625000	.0078125	.3
46	.0729167	-.0010416	-.3
47	.0625000	.0109375	.3
48	.0729167	.0020833	.3
49	.0833333	-.0067708	-.3
50	.0729167	.0052083	.3
51	.0833333	-.0036458	-.3
52	.0729167	.0083333	.3
53	.0833333	-.0005208	-.3
54	.0729167	.0114583	.3
55	.0833333	.0026041	.3
56	.0937500	-.0062499	-.3
57	.0833333	.0057291	.3
58	.0937500	-.0031249	-.3
59	.0833333	.0088541	.3
60	.0937500	.0000000	.0
61	.0937500	.0015625	.3
62	.1041667	-.0072916	-.3
63	.0937500	.0046875	.3
64	.1041667	-.0041666	-.3
65	.0937500	.0078125	.3
66	.0937500	.0093750	.3
67	.1145833	-.0098958	-.3
68	.0937500	.0125000	.3
69	.1145833	-.0067708	-.3
70	.0937500	.0156250	.3
71	.1145833	-.0036458	-.3
72	.0937500	.0187500	.3
73	.0937500	.0203125	.3
74	.1145833	.0010416	.3
75	.1145833	.0026041	.3
76	.1354167	-.0166666	-.3
77	.1145833	.0057291	.3
78	.1354167	-.0135416	-.3
79	.1145833	.0088541	.3
80	.1145833	.0104166	.3
81	.1354167	-.0088541	-.3
82	.1145833	.0135416	.3
83	.1354167	-.0057291	-.3
84	.1145833	.0166666	.3
85	.1250000	.0078125	.3
86	.1250000	.0093750	.3
87	.1458333	-.0098958	-.3
88	.1250000	.0125000	.3
89	.1458333	-.0067708	-.3
90	.1250000	.0156250	.3
91	.1458333	-.0036458	-.3
92	.1250000	.0187500	.3
93	.1250000	.0203125	.3
94	.1458333	.0010416	.3
95	.1458333	.0026041	.3
96	.1666667	-.0166666	-.3
97	.1458333	.0057291	.3
98	.1666667	-.0135416	-.3
99	.1458333	.0088541	.3
100	.1458333	.0104166	.3
101	.1666667	-.0088541	-.3
102	.1458333	.0135416	.3
103	.1666667	-.0057291	-.3
104	.1458333	.0166666	.3
105	.1562500	.0078125	.3
106	.1562500	.0093750	.3
107	.1770833	-.0098958	-.3
108	.1562500	.0125000	.3
109	.1770833	-.0067708	-.3
110	.1562500	.0156250	.3
111	.1770833	-.0036458	-.3
112	.1562500	.0187500	.3
113	.1562500	.0203125	.3
114	.1770833	.0010416	.3
115	.1770833	.0026041	.3
116	.1979167	-.0166666	-.3
117	.1770833	.0057291	.3
118	.1979167	-.0135416	-.3
119	.1770833	.0088541	.3
120	.1770833	.0104166	.3
121	.1979167	-.0088541	-.3
122	.1770833	.0135416	.3
123	.1979167	-.0057291	-.3
124	.1770833	.0166666	.3
125	.1875000	.0078125	.3
126	.1875000	.0093750	.3
127	.2083333	-.0098958	-.3
128	.1875000	.0125000	.3
129	.2083333	-.0067708	-.3

TABLE 14 (continued)

5 LEVEL MOD-DEMOD MODEL .10000000 RAMP INPU

M = 64 GAIN = 2.00000000 DEAD SPACE = .0312500

N	Y(N)	E(N)	S(N)	N	Y(N)	E(N)	S(N
130	.1875000	.0156250	.3	195	.2916667	.0130208	.
131	.1875000	.0171875	.3	196	.2916667	.0145833	.
132	.2083333	-.0020833	-.3	197	.3125000	-.0046874	-.
133	.1875000	.0203125	.3	198	.2916667	.0177083	.
134	.2083333	.0010416	.3	199	.2916667	.0192708	.
135	.2083333	.0026041	.3	200	.2916667	.0208333	.
136	.2291667	-.0166666	-.3	201	.3125000	.0015625	.
137	.2083333	.0057291	.3	202	.3125000	.0031250	.
138	.2083333	.0072916	.3	203	.3125000	.0046875	.
139	.2083333	.0088541	.3	204	.3125000	.0062500	.
140	.2083333	.0104166	.3	205	.3125000	.0078125	.
141	.2291667	-.0088541	-.3	206	.3333333	-.0114583	-.
142	.2083333	.0135416	.3	207	.3125000	.0109375	.
143	.2291667	-.0057291	-.3	208	.3333333	-.0083333	-.
144	.2083333	.0166666	.3	209	.3125000	.0140625	.
145	.2083333	.0182291	.3	210	.3125000	.0156250	.
146	.2291667	-.0010416	-.3	211	.3333333	-.0036458	-.
147	.2083333	.0213541	.3	212	.3125000	.0187500	.
148	.2291667	.0020833	.3	213	.3125000	.0203125	.
149	.2291667	.0036458	.3	214	.3125000	.0218750	.
150	.2291667	.0052083	.3	215	.3125000	.0234375	.
151	.2291667	.0067708	.3	216	.3125000	.0250000	.
152	.2500000	-.0124999	-.3	217	.3333333	.0057291	.
153	.2291667	.0098958	.3	218	.3333333	.0072916	.
154	.2500000	-.0093749	-.3	219	.3541667	-.0119791	-.
155	.2291667	.0130208	.3	220	.3333333	.0104166	.
156	.2500000	-.0062499	-.3	221	.3541667	-.0088541	-.
157	.2291667	.0161458	.3	222	.3333333	.0135416	.
158	.2291667	.0177083	.3	223	.3333333	.0151041	.
159	.2291667	.0192708	.3	224	.3333333	.0166666	.
160	.2291667	.0208333	.3	225	.3333333	.0182291	.
161	.2500000	.0015625	.3	226	.3333333	.0197916	.
162	.2500000	.0031250	.3	227	.3333333	.0213541	.
163	.2708333	-.0161458	-.3	228	.3541667	.0020833	.
164	.2500000	.0062500	.3	229	.3541667	.0036458	.
165	.2500000	.0078125	.3	230	.3541667	.0052083	.
166	.2708333	-.0114583	-.3	231	.3750000	-.0140624	-.
167	.2500000	.0109375	.3	232	.3541667	.0083333	.
168	.2708333	-.0083333	-.3	233	.3750000	-.0109374	-.
169	.2500000	.0140625	.3	234	.3541667	.0114583	.
170	.2500000	.0156250	.3	235	.3541667	.0130208	.
171	.2500000	.0171875	.3	236	.3541667	.0145833	.
172	.2708333	-.0020833	-.3	237	.3750000	-.0046874	-.
173	.2500000	.0203125	.3	238	.3541667	.0177083	.
174	.2708333	.0010416	.3	239	.3541667	.0192708	.
175	.2708333	.0026041	.3	240	.3541667	.0208333	.
176	.2916667	-.0166666	-.3	241	.3750000	.0015625	.
177	.2708333	.0057291	.3	242	.3750000	.0031250	.
178	.2708333	.0072916	.3	243	.3750000	.0046875	.
179	.2708333	.0088541	.3	244	.3750000	.0062500	.
180	.2708333	.0104166	.3	245	.3750000	.0078125	.
181	.2916667	-.0088541	-.3	246	.3958333	-.0114583	-.
182	.2708333	.0135416	.3	247	.3750000	.0109375	.
183	.2916667	-.0057291	-.3	248	.3958333	-.0083333	-.
184	.2708333	.0166666	.3	249	.3750000	.0140625	.
185	.2708333	.0182291	.3	250	.3750000	.0156250	.
186	.2916667	-.0010416	-.3	251	.3958333	-.0036458	-.
187	.2708333	.0213541	.3	252	.3750000	.0187500	.
188	.2916667	.0020833	.3	253	.3750000	.0203125	.
189	.2916667	.0036458	.3	254	.3750000	.0218750	.
190	.2916667	.0052083	.3	255	.3750000	.0234375	.
191	.2916667	.0067708	.3	256	.3750000	.0250000	.
192	.3125000	-.0124999	-.3	257	.3958333	.0057291	.
193	.2916667	.0098958	.3	258	.3958333	.0072916	.
194	.3125000	-.0093749	-.3	259	.4166667	-.0119791	-.

SUMMARY

A deterministic approach has been taken in analyzing a basic delta modulation system and extending it to a more general five-level device. The main virtue of delta modulation is its simplicity; one of the difficulties is its inability to transmit a constant amplitude. Modifications have been suggested to compensate for this difficulty. Such a modification is the addition of a third, or zero, level. The system still exhibits an oscillatory behavior for certain input values. This can be overcome by adding a dead space to the device, but at the expense of adding a steady state error to step inputs and a delay for higher order input functions. A further sophistication would be the modification of the three-level device to a five (or higher) level device. Results show, however, that the advantages to be gained over a three-level device with dead space are few. The basic simplicity of delta modulation has been lost, and the device becomes as complicated, in principle, as pulse code modulation.

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APPENDIX A

```
C      THREE LEVEL DELTA MODULATOR-DEMODULATOR MODEL              STEP INPUT
       DIMENSION SGN(257),N(288),Y(288),E(288),S(288)
1      FORMAT(I5,3F10.8,2I5)
2      FORMAT(1H1,26H   3 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H STEP INPUT,
      14X,26H   3 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H STEP INPUT/1HK,6H
      2 M =,I4,7X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8,4X,6H   M =,I4,
      37X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8/1HL,30H   N  Y(N)
      4 E(N)      S(N),3(4X,30H   N  Y(N)        E(N)      S(N))/)
3      FORMAT(1H ,I4,F11.7,F10.7,F5.1,3(4X,I4,F11.7,F10.7,F5.1))
4      FORMAT(1H )
5      FORMAT(1H ,34X,I4,F11.7,F10.7,F5.1,38X,I4,F11.7,F10.7,F5.1)
6      FORMAT(1H ,102X,I4,F11.7,F10.7,F5.1)
7      FORMAT(1H1,26H   3 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H STEP INPUT/
      11HK,6H   M =,I4,7X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8/1HL,64H
      2   PERIOD  AVERAGE    MAX ERROR   RMS ERROR   S/N RATIO    IN DB/)
8      FORMAT(1H ,I7,3F12.7,F12.4,F9.4)
10     READ(1,1)M,GAIN,D,STEP,NMIN,NOLIST
C      M IS NUMBER OF SAMPLING INTERVALS  (RANGE IS 8-256 IN POWERS OF 2)
C      GAIN IS THE GAIN FACTOR              D IS WIDTH OF DEAD SPACE
C      STEP IS HEIGHT OF STEP INPUT
C      NMIN IS MINIMUM NUMBER OF PERIODS  (PERIOD IS M+1 INTERVALS)
C      NOLIST .NE. 0 CAUSES SUPPRESSION OF LISTING OF DATA POINTS
       IF(M.EQ.0) STOP
       REWIND 5
C      DETERMINE NUMBER OF PERIODS PER PAGE (NPPP) AND TOTAL NUMBER
C      OF PERIODS (NP).
       NPPP=256/M
       NP=0
20     NP=NP+NPPP
       IF(NP.LT.NMIN) GO TO 20
       MP1=M+1
C      DETERMINE NUMBER OF POINTS PER PAGE (NSPP) AND EXTRA POINTS
C      (NEXTRA) FOR LISTING PURPOSES.
       NSPP=MP1*NPPP
       N1=NSPP/4
       NEXTRA=NSPP-N1*4
C      INITIALIZE.
       CON=GAIN/FLOAT(M)
       SIGNUM=0.
       YNM1=0.
       DO 40 I=1,MP1
40     SGN(I)=0.
       DO 170 II=1,NP,NPPP
       IJ=(II-1)/NPPP*NSPP-1
       DO 90 IM=1,NPPP
C      RESET FOR ERROR CALCULATIONS THIS PERIOD.
       ERMAX=0.
       AVE=0.
       IN=(IM-1)*MP1
```

```
      DO 80 IL=1,MP1
      IK=IL+IN
      I=IK+IJ
C     I IS THE SAMPLING INSTANT NUMBER.
      J=MOD(I,MP1)+1
C     J POINTS TO N-M-1 SIGNUM VALUE.
C     CALCULATE CURRENT VALUE OF Y.
      YN=YNM1+CON*(SIGNUM-SGN(J))
C     DETERMINE ERROR.
      ERROR=STEP-YN
C     DETERMINE SIGNUM VALUE.
      IF(ABS(ERROR).GT.D) GO TO 60
      SIGNUM=0.
      GO TO 70
60    SIGNUM=SIGN(1.0,ERROR)
C     STORE VALUES.
70    Y(IK)=YN+SIGN(.00000005,YN)
      E(IK)=ERROR
      S(IK)=SIGNUM
      N(IK)=I
      SGN(J)=SIGNUM
      YNM1=YN
C     CALCULATE FOR ERROR DATA.
      IF(ABS(ERROR).GT.ABS(ERMAX)) ERMAX=ERROR
80    AVE=AVE+Y(IK)
C     CALCULATE ERROR DATA.
      AVE=AVE/FLOAT(MP1)
      J1=IN+1
      J2=IN+MP1
      RMSER=0.
      DO 85 J=J1,J2
85    RMSER=RMSER+(AVE-Y(J))**2
      RMSER=SQRT(RMSER/FLOAT(MP1))
      STON=AVE/RMSER
      DB=20.*ALOG(STON)/ALOG(10.)
      WRITE(5)AVE,ERMAX,RMSER,STON,DB
90    CONTINUE
C     WRITE OUTPUT.
      IF(NOLIST.NE.0) GO TO 170
      WRITE(3,2)STEP,STEP,M,GAIN,D,M,GAIN,D
      IF(NEXTRA.EQ.2) GO TO 120
      N2=N1
      N3=N2+N1
      N4=N3+N1
      GO TO 130
120   N2=N1
      N3=N2+N1+1
      N4=N3+N1
130   DO 140 K1=1,N1
      N2=N2+1
      N3=N3+1
      N4=N4+1
```

```
      WRITE(3,3) N(K1),Y(K1),E(K1),S(K1),N(N2),Y(N2),E(N2),S(N2),N(N3),Y
     1(N3),E(N3),S(N3),N(N4),Y(N4),E(N4),S(N4)
      IF(MOD(K1,MP1).EQ.0) WRITE(3,4)
140   CONTINUE
      IF(NEXTRA-1) 170,160,150
150   N2=N2+1
      N4=N4+1
      WRITE(3,5)N(N2),Y(N2),E(N2),S(N2),N(N4),Y(N4),E(N4),S(N4)
      GO TO 170
160   N4=N4+1
      WRITE(3,6) N(N4),Y(N4),E(N4),S(N4)
170   CONTINUE
C     WRITE ERROR DATA.
      REWIND 5
      WRITE(3,7)STEP,M,GAIN,D
      DO 180 I=1,NP
      READ(5)AVE,ERMAX,RMSER,STON,DB
180   WRITE(3,8)I,AVE,ERMAX,RMSER,STON,DB
      GO TO 10
      E N D
```

APPENDIX B

```
C     THREE LEVEL DELTA MODULATOR-DEMODULATOR MODEL             RAMP INPUT
      REAL MAX,INPUT
      DIMENSION SGN(257),N(288),Y(288),E(288),S(288)
1     FORMAT(I5,3F10.8,2I5)
2     FORMAT(1H1,26H   3 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H RAMP INPUT,
     14X,26H   3 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H RAMP INPUT/1HK,6H
     2 M =,I4,7X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8,4X,6H   M =,I4,
     37X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8/1HL,30H   N  Y(N)
     4 E(N)      S(N),3(4X,30H   N  Y(N)        E(N)      S(N))/)
3     FORMAT(1H ,I4,F11.7,F10.7,F5.1,3(4X,I4,F11.7,F10.7,F5.1))
4     FORMAT(1H )
5     FORMAT(1H ,34X,I4,F11.7,F10.7,F5.1,38X,I4,F11.7,F10.7,F5.1)
6     FORMAT(1H ,102X,I4,F11.7,F10.7,F5.1)
7     FORMAT(1H1,26H   3 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H RAMP INPUT/
     11HK,6H   M =,I4,7X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8/1HK,18H
     2   AVERAGE ERROR =,F10.7,11X,15HMAXIMUM ERROR =,F10.7/1HL,64H   PE
     3RIOD      MAXIMUM ERROR       AVERAGE ERROR       SD OF ERROR/)
8     FORMAT(1H ,I7,3F19.7)
10    READ(1,1)M,GAIN,D,RAMP,NMIN,NOLIST
C     M IS NUMBER OF SAMPLING INTERVALS  (RANGE IS 8-256 IN POWERS OF 2)
C     GAIN IS THE GAIN FACTOR              D IS WIDTH OF DEAD SPACE
C     RAMP IS SLOPE OF RAMP INPUT FOR M INTERVALS
C     RAMP WILL CHANGE SLOPE WHEN IT REACHES THE SATURATION VALUE
C     NMIN IS MINIMUM NUMBER OF PERIODS  (PERIOD IS M+1 INTERVALS)
C     NOLIST .NE. 0 CAUSES SUPPRESSION OF LISTING OF DATA POINTS
      IF(M.EQ.0) STOP
      REWIND 5
C     DETERMINE NUMBER OF PERIODS PER PAGE (NPPP) AND TOTAL NUMBER
C     OF PERIODS (NP).
      NPPP=256/M
      NP=0
20    NP=NP+NPPP
      IF(NP.LT.NMIN) GO TO 20
      MP1=M+1
C     DETERMINE NUMBER OF POINTS PER PAGE (NSPP) AND EXTRA POINTS
C     (NEXTRA) FOR LISTING PURPOSES.
      NSPP=MP1*NPPP
      N1=NSPP/4
      NEXTRA=NSPP-N1*4
      DELTA=RAMP/FLOAT(M)
C     DELTA IS RAMP INCREMENT PER SAMPLING INTERVAL.
C     INITIALIZE.
      CON=GAIN/FLOAT(M)
      MAX=1.0*GAIN
      INPUT=0.
      SIGNUM=0.
      YNM1=0.
      ERMXMX=0.
      AVER=0.
```

```
      DO 40 I=1,MP1
40    SGN(I)=0.
      DO 170 II=1,NP,NPPP
      IJ=(II-1)/NPPP*NSPP-1
      DO 90 IM=1,NPPP
C     RESET FOR ERROR CALCULATION THIS PERIOD.
      ER=0.
      ERMAX=0.
      ERSQ=0.
      IN=(IM-1)*MP1
      DO 80 IL=1,MP1
      IK=IL+IN
      I=IK+IJ
C     I IS THE SAMPLING INSTANT NUMBER.
      J=MOD(I,MP1)+1
C     J POINTS TO N-M-1 SIGNUM VALUE.
C     CALCULATE CURRENT VALUE OF Y.
      YN=YNM1+CON*(SIGNUM-SGN(J))
C     DETERMINE ERROR.
      ERROR=INPUT-YN
C     DETERMINE SIGNUM VALUE.
      IF(ABS(ERROR).GT.D) GO TO 60
      SIGNUM=0.
      GO TO 70
60    SIGNUM=SIGN(1.0,ERROR)
C     STORE VALUES.
70    Y(IK)=YN+SIGN(.00000005,YN)
      E(IK)=ERROR
      S(IK)=SIGNUM
      N(IK)=I
      SGN(J)=SIGNUM
C     CALCULATE NEW INPUT VALUE.
      IF(ABS(INPUT).GE.MAX) DELTA=-DELTA
      INPUT=INPUT+DELTA
C     CALCULATE FOR ERROR DATA.
      ER=ER+ERROR
      ERSQ=ERSQ+ERROR**2
      IF(ABS(ERROR).GT.ABS(ERMAX)) ERMAX=ERROR
80    YNM1=YN
C     CALCULATE ERROR DATA.
      AVEER=ER/FLOAT(MP1)
      AVER=AVER+AVEER
      SDEV=SQRT(ERSQ/FLOAT(MP1)-AVEER**2)
      WRITE(5)ERMAX,AVEER,SDEV
      IF(ABS(ERMAX).GT.ABS(ERMXMX)) ERMXMX=ERMAX
90    CONTINUE
C     WRITE OUTPUT.
      IF(NOLIST.NE.0) GO TO 170
      WRITE(3,2)RAMP,RAMP,M,GAIN,D,M,GAIN,D
      IF(NEXTRA.EQ.2) GO TO 120
      N2=N1
      N3=N2+N1
```

```
      N4=N3+N1
      GO TO 130
120   N2=N1
      N3=N2+N1+1
      N4=N3+N1
130   DO 140 K1=1,N1
      N2=N2+1
      N3=N3+1
      N4=N4+1
      WRITE(3,3) N(K1),Y(K1),E(K1),S(K1),N(N2),Y(N2),E(N2),S(N2),N(N3),Y
     1(N3),E(N3),S(N3),N(N4),Y(N4),E(N4),S(N4)
      IF(MOD(K1,MP1).EQ.0) WRITE(3,4)
140   CONTINUE
      IF(NEXTRA-1) 170,160,150
150   N2=N2+1
      N4=N4+1
      WRITE(3,5)N(N2),Y(N2),E(N2),S(N2),N(N4),Y(N4),E(N4),S(N4)
      GO TO 170
160   N4=N4+1
      WRITE(3,6) N(N4),Y(N4),E(N4),S(N4)
170   CONTINUE
C     WRITE ERROR DATA.
      REWIND 5
      AVER=AVER/FLOAT(NP)
      WRITE(3,7)RAMP,M,GAIN,D,AVER,ERMXMX
      DO 180 I=1,NP
      READ(5)ERMAX,AVEER,SDEV
180   WRITE(3,8) I,ERMAX,AVEER,SDEV
      GO TO 10
      E N D
```

APPENDIX C

```
C     FIVE LEVEL DELTA MODULATOR-DEMODULATOR MODEL                  STEP INPUT
      DIMENSION SGN(257),N(288),Y(288),E(288),S(288)
1     FORMAT(I5,3F10.8,2I5)
2     FORMAT(1H1,26H    5 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H STEP INPUT,
     14X,26H    5 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H STEP INPUT/1HK,6H
     2 M =,I4,7X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8,4X,6H    M =,I4,
     37X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8/1HL,30H    N  Y(N)
     4 E(N)      S(N),3(4X,30H    N  Y(N)         E(N)      S(N))/)
3     FORMAT(1H ,I4,F11.7,F10.7,F5.1,3(4X,I4,F11.7,F10.7,F5.1))
4     FORMAT(1H )
5     FORMAT(1H ,34X,I4,F11.7,F10.7,F5.1,38X,I4,F11.7,F10.7,F5.1)
6     FORMAT(1H ,102X,I4,F11.7,F10.7,F5.1)
7     FORMAT(1H1,26H    5 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H STEP INPUT/
     11HK,6H    M =,I4,7X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8/1HL,64H
     2   PERIOD  AVERAGE    MAX ERROR   RMS ERROR   S/N RATIO    IN DB/)
8     FORMAT(1H ,I7,3F12.7,F12.4,F9.4)
10    READ(1,1)M,GAIN,D,STEP,NMIN,NOLIST
C     M IS NUMBER OF SAMPLING INTERVALS  (RANGE IS 8-256 IN POWERS OF 2)
C     GAIN IS THE GAIN FACTOR                D IS WIDTH OF DEAD SPACE
C     STEP IS HEIGHT OF STEP INPUT
C     NMIN IS MINIMUM NUMBER OF PERIODS  (PERIOD IS M+1 INTERVALS)
C     NOLIST .NE. 0 CAUSES SUPPRESSION OF LISTING OF DATA POINTS
      IF(M.EQ.0) STOP
      REWIND 5
C     DETERMINE NUMBER OF PERIODS PER PAGE (NPPP) AND TOTAL NUMBER
C     OF PERIODS (NP).
      NPPP=256/M
      NP=0
20    NP=NP+NPPP
      IF(NP.LT.NMIN) GO TO 20
      MP1=M+1
C     DETERMINE NUMBER OF POINTS PER PATE (NSPP) AND NUMBER OF
C     EXTRA POINTS (NEXTRA) FOR LISTING.
      NSPP=MP1*NPPP
      N1=NSPP/4
      NEXTRA=NSPP-N1*4
C     INITIALIZE.
      TOL=.00000009
      CON=GAIN/FLOAT(M)
      SIGNUM=0.
      RND=0.
      YNM1=0.
      DO 40 I=1,MP1
40    SGN(I)=0.
      DO 170 II=1,NP,NPPP
      IJ=(II-1)/NPPP*NSPP-1
      DO 90 IM=1,NPPP
C     RESET FOR ERROR CALCULATIONS THIS PERIOD.
      ERMAX=0.
```

```
      AVE=0.
      IN=(IM-1)*MP1
      DO 80 IL=1,MP1
      IK=IL+IN
      I=IK+IJ
C     I IS THE SAMPLING INSTANT NUMBER.
      J=MOD(I,MP1)+1
C     J POINTS TO N-M-1 SIGNUM VALUE.
C     CALCULATE CURRENT VALUE OF Y.
      YN=YNM1+CON*(SIGNUM-SGN(J))
C     DETERMINE ERROR.
      ERROR=STEP-YN
C     DETERMINE SIGNUM VALUE.
      IF(ABS(ERROR).LE.TOL) GO TO 50
      IF(ABS(ERROR).GT.D) GO TO 60
      SIGNUM=SIGN(.333333333,ERROR)
C     ADJUST FOR ROUND-OFF.
      RND=RND+SIGN(1.,ERROR)
      IF(ABS(RND).NE.3.) GO TO 70
      SIGNUM=SIGNUM+SIGN(.000000001,RND)
      RND=0.
      GO TO 70
50    SIGNUM=0.
      GO TO 70
60    SIGNUM=SIGN(1.0,ERROR)
C     STORE VALUES.
70    Y(IK)=YN+SIGN(.00000005,YN)
      E(IK)=ERROR
      S(IK)=SIGNUM
      N(IK)=I
      SGN(J)=SIGNUM
C     CALCULATE FOR ERROR DATA.
      AVE=AVE+Y(IK)
      IF(ABS(ERROR).GT.ABS(ERMAX)) ERMAX=ERROR
80    YNM1=YN
C     CALCULATE ERROR DATA.
      AVE=AVE/FLOAT(MP1)
      J1=IN+1
      J2=IN+MP1
      RMSER=0.
      DO 85 J=J1,J2
85    RMSER=RMSER+(AVE-Y(J))**2
      RMSER=SQRT(RMSER/FLOAT(MP1))
      STON=AVE/RMSER
      DB=20.*ALOG(STON)/ALOG(10.)
      WRITE(5)AVE,ERMAX,RMSER,STON,DB
90    CONTINUE
C     WRITE OUTPUT.
      IF(NOLIST.NE.0) GO TO 170
      WRITE(3,2)STEP,STEP,M,GAIN,D,M,GAIN,D
      IF(NEXTRA.EQ.2) GO TO 120
      N2=N1
```

```
      N3=N2+N1
      N4=N3+N1
      GC TO 130
120   N2=N1
      N3=N2+N1+1
      N4=N3+N1
130   DC 140 K1=1,N1
      N2=N2+1
      N3=N3+1
      N4=N4+1
      WRITE(3,3) N(K1),Y(K1),E(K1),S(K1),N(N2),Y(N2),E(N2),S(N2),N(N3),Y
     1(N3),E(N3),S(N3),N(N4),Y(N4),E(N4),S(N4)
      IF(MOD(K1,MP1).EQ.0) WRITE(3,4)
140   CONTINUE
      IF(NEXTRA-1) 170,160,150
150   N2=N2+1
      N4=N4+1
      WRITE(3,5)N(N2),Y(N2),E(N2),S(N2),N(N4),Y(N4),E(N4),S(N4)
      GC TO 170
160   N4=N4+1
      WRITE(3,6) N(N4),Y(N4),E(N4),S(N4)
170   CONTINUE
C     WRITE ERRCR DATA.
      REWIND 5
      WRITE(3,7)STEP,M,GAIN,D
      DO 180 I=1,NP
      READ(5)AVE,ERMAX,RMSER,STON,DB
180   WRITE(3,8)I,AVE,ERMAX,RMSER,STON,DB
      GO TO 10
      E N D
```

APPENDIX D

```
C     FIVE LEVEL DELTA MODULATOR-DEMODULATOR MODEL              RAMP INPUT
      REAL MAX,INPUT
      DIMENSION SGN(257),N(288),Y(288),E(288),S(288)
1     FORMAT(I5,3F10.8,2I5)
2     FORMAT(1H1,26H    5 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H RAMP INPUT,
     14X,26H    5 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H RAMP INPUT/1HK,6H
     2 M =,I4,7X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8,4X,6H    M =,I4,
     37X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8/1HL,30H    N   Y(N)
     4 E(N)       S(N),3(4X,30H    N   Y(N)        E(N)       S(N))/)
3     FORMAT(1H ,I4,F11.7,F10.7,F5.1,3(4X,I4,F11.7,F10.7,F5.1))
4     FORMAT(1H )
5     FORMAT(1H ,34X,I4,F11.7,F10.7,F5.1,38X,I4,F11.7,F10.7,F5.1)
6     FORMAT(1H ,102X,I4,F11.7,F10.7,F5.1)
7     FORMAT(1H1,26H    5 LEVEL MOD-DEMOD MODEL,17X,F10.8,11H RAMP INPUT/
     11HK,6H    M =,I4,7X,6HGAIN =,F11.8,7X,12HDEAD SPACE =,F11.8/1HK,18H
     2    AVERAGE ERROR =,F10.7,11X,15HMAXIMUM ERROR =,F10.7/1HL,64H    PE
     3RIOD       MAXIMUM ERROR       AVERAGE ERROR       SD OF ERROR/)
8     FORMAT(1H ,I7,3F19.7)
10    READ(1,1)M,GAIN,D,RAMP,NMIN,NOLIST
C     M IS NUMBER OF SAMPLING INTERVALS  (RANGE IS 8-256 IN POWERS OF 2)
C     GAIN IS THE GAIN FACTOR               D IS WIDTH OF DEAD SPACE
C     RAMP IS SLOPE OF RAMP INPUT FOR M INTERVALS
C     RAMP WILL CHANGE SLOPE WHEN IT REACHES THE SATURATION VALUE
C     NMIN IS MINIMUM NUMBER OF PERIODS  (PERIOD IS M+1 INTERVALS)
C     NOLIST .NE. 0 CAUSES SUPPRESSION OF LISTING OF DATA POINTS
      IF(M.EQ.0) STOP
      REWIND 5
C     DETERMINE NUMBER OF PERIODS PER PAGE (NPPP) AND TOTAL NUMBER
C     OF PERIODS (NP).
      NPPP=256/M
      NP=0
20    NP=NP+NPPP
      IF(NP.LT.NMIN) GO TO 20
      MP1=M+1
C     DETERMINE NUMBER OF POINTS PER PAGE (NSPP) AND NUMBER OF
C     EXTRA POINTS FOR LISTING PURPOSES.
      NSPP=MP1*NPPP
      N1=NSPP/4
      NEXTRA=NSPP-N1*4
      DELTA=RAMP/FLOAT(M)
C     DELTA IS RAMP INCREMENT PER SAMPLING INTERVAL.
C     INITIALIZE.
      CON=GAIN/FLOAT(M)
      TOL=.00000009
      MAX=1.0*GAIN
      ERMXMX=0.
      AVER=0.
      RND=0.
      INPUT=0.
```

```
      SIGNUM=0.
      YNM1=0.
      DO 40 I=1,MP1
40    SGN(I)=0.
      DO 170 II=1,NP,NPPP
      IJ=(II-1)/NPPP*NSPP-1
      DO 90 IM=1,NPPP
C     RESET FOR ERROR CALCULATIONS THIS PERIOD.
      ER=0.
      ERMAX=0.
      ERSQ=0.
      IN=(IM-1)*MP1
      DO 80 IL=1,MP1
      IK=IL+IN
      I=IK+IJ
C     I IS THE SAMPLING INSTANT NUMBER.
      J=MOD(I,MP1)+1
C     J POINTS TO N-M-1 SIGNUM VALUE.
C     CALCULATE CURRENT VALUE OF Y.
      YN=YNM1+CON*(SIGNUM-SGN(J))
C     DETERMINE ERROR.
      ERROR=INPUT-YN
C     DETERMINE SIGNUM VALUE.
      IF(ABS(ERROR).LE.TOL) GO TO 50
      IF(ABS(ERROR).GT.D) GO TO 60
      SIGNUM=SIGN(.333333333,ERROR)
C     ADJUST FOR ROUND-OFF.
      RND=RND+SIGN(1.,ERROR)
      IF(ABS(RND).NE.3.) GO TO 70
      SIGNUM=SIGNUM+SIGN(.000000001,RND)
      RND=0.
      GO TO 70
50    SIGNUM=0.
      YN=INPUT
      GO TO 70
60    SIGNUM=SIGN(1.0,ERROR)
C     STORE VALUES.
70    Y(IK)=YN+SIGN(.00000005,YN)
      E(IK)=ERROR
      S(IK)=SIGNUM
      N(IK)=I
      SGN(J)=SIGNUM
C     CALCULATE FOR ERROR DATA.
      ER=ER+ERROR
      ERSQ=ERSQ+ERROR**2
      IF(ABS(ERROR).GT.ABS(ERMAX)) ERMAX=ERROR
C     CALCULATE NEW INPUT VALUE.
      IF(ABS(INPUT).GE.MAX) DELTA=-DELTA
      INPUT=INPUT+DELTA
80    YNM1=YN
C     CALCULATE ERROR DATA.
      AVEER=ER/FLOAT(MP1)
```

```
      AVER=AVER+AVEER
      SCEV=SQRT(ERSQ/FLCAT(MP1)-AVEER**2)
      WRITE(5)ERMAX,AVEER,SDEV
      IF(ABS(ERMAX).GT.ABS(ERMXMX)) ERMXMX=ERMAX
90    CCNTINUE
C     WRITE OUTPUT.
      IF(NOLIST.NE.0) GC TO 170
      WRITE(3,2)RAMP,RAMP,M,GAIN,D,M,GAIN,D
      IF(NEXTRA.EQ.2) GO TO 120
      N2=N1
      N3=N2+N1
      N4=N3+N1
      GC TO 130
120   N2=N1
      N3=N2+N1+1
      N4=N3+N1
130   DC 140 K1=1,N1
      N2=N2+1
      N3=N3+1
      N4=N4+1
      WRITE(3,3) N(K1),Y(K1),E(K1),S(K1),N(N2),Y(N2),E(N2),S(N2),N(N3),Y
     1(N3),E(N3),S(N3),N(N4),Y(N4),E(N4),S(N4)
      IF(MOD(K1,MP1).EQ.0) WRITE(3,4)
140   CCNTINUE
      IF(NEXTRA-1) 170,160,150
150   N2=N2+1
      N4=N4+1
      WRITE(3,5)N(N2),Y(N2),E(N2),S(N2),N(N4),Y(N4),E(N4),S(N4)
      GC TO 170
160   N4=N4+1
      WRITE(3,6) N(N4),Y(N4),E(N4),S(N4)
170   CCNTINUE
C     WRITE ERROR DATA.
      REWIND 5
      AVER=AVER/FLOAT(NP)
      WRITE(3,7)RAMP,M,GAIN,D,AVER,ERMXMX
      DC 180 I=1,NP
      READ(5)ERMAX,AVEER,SDEV
180   WRITE(3,8) I,ERMAX,AVEER,SDEV
      GO TO 10
      E N D
```

A GENERALIZATION OF DELTA MODULATION

by

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Delta modulation is a fairly recent development in the field of digital coding. Early investigations into the nature of delta modulation were probabilistic analyses that revealed the capacity of the system to handle information.

Barber showed that delta modulation may be regarded as a hybrid of pulse duration modulation and pulse amplitude modulation. Tripp used this new approach to devise an idealized mathematical model which forms the basis for a deterministic analysis. The deterministic approach reveals the behavior of a system and adapts itself to system improvement.

The main virtue of delta modulation is its simplicity; one of the difficulties is its inability to transmit a constant amplitude. Modifications have been suggested to compensate for this difficulty. Such a modification is the addition of a third quantization level to the basic binary system.

This report uses the techniques developed by Tripp to analyze the response of the three-level system. Investigation reveals that the three-level delta modulation system demonstrates servo response for step inputs and will follow ramps with a certain amount of delay and error. The addition of a dead space will minimize the noise at the expense of increased error.

A further generalization of the system to one having five-level quantization is made. Results show that even though a slight improvement over the three-level device with dead space is made, the advantages to be gained by the five-level device are few. The basic simplicity of delta modulation is lost and the improvement will not justify the cost of increased bandwidth and equipment complexity.