APPLICATION OF LINEAR PROGRAMMING TO MILLING PROBLEMS WHICH INVOLVE BLENDING OF WHEAT

by

JOSEPH ELDON UNGER

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INTRODUCTION

The basic problem of economic life is programming. Whether a person is a farmer, a manufacturer, or a distributor, he is faced with the problem of programming when he is searching for the most profitable way to use the resources at his disposal. A buyer also uses programming when he searches for the least expensive combination of goods and services that will meet his needs.

One problem in economics is resource allocation. The problem of an individual business firm is how will it use the resources at its disposal to maximize its profits or to minimize its costs. "The concepts of maximization and minimization are best explained by the tools of mathematics."[1]

Within recent years remarkable progress has been made in the development of the technique known as "linear programming." Some, but not all, programming problems can be stated in linear terms. The analysis of such problems was called "linear programming" by Dantzig[2] who devised the "Simplex method" of solving them. Variations of the simplex method have been outlined in detail by Dantzig,[3] by Dorfman,[4] by Charnes, Cooper, and

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[1] Earl O. Heady, Economics of Agriculture Production and Resource Use, p. 5.

[2] G. B. Dantzig, "Maximization of a Linear Form Where Variables Are Subject to a System of Linear Inequalities," Headquarters, USAAF, November 1, 1919.

[3] G. B. Dantzig, Activity Analysis of Production and Allocation, T. C. Koopmans, ed. New York, 1951, Chaps. XXI and XXIII.

[4] Robert Dorfman, Application of Linear Programming to the Theory of the Firm, 1951.

Henderson,[1] and by others.

"Linear programming" is defined by Charnes, Cooper, and Henderson as being "concerned with the problem of planning a complex of interdependent activities in the best possible (optimal) fashion."[2]

The "simplex method" of computation is mechanical and simple. Division, multiplication, subtraction, and addition are the only operations required. "Programming calculations (although perhaps more tedious) are simpler than those required for correlation calculations."[3] For problems having only a few unknowns, this technique is adapted to hand or machine calculations and hence need not restrict the user to expensive high-speed computing machinery. However, if problems contain a large number of variables in a number of equations, electronic computers must be used for practicality.

A powerful and flexible set of tools for theoretical and empirical research is provided through the methods of programming. The scope of economics can be extended by the use of these tools which may be adapted to the solution of a wide variety of practical problems.[4] The use of linear programming will result in programs of maximum profit or minimum cost.

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[1] A. Charnes, W. W. Cooper, and A. Henderson, An Introduction to Linear Programming, 1953.

[2] Ibid., p. 1.

[3] Ibid., p. 2.

[4] Ibid., p. 1.

FIELD OF STUDY

This particular study, although many of the problems and principles involved are applicable to other types of mills, was confined to the industry engaged in the production of wheat flour.

There are many types of wheat flour milled by many different flour mills, but the principles involved in using linear programming in devising a satisfactory mill-mix can be applied to milling any certain type of wheat flour.

Only one process--blending wheat--was studied. This restriction of the area of study added the further convenience of avoiding such problems of buying, storing, etc., which would be encountered in the actual operation of the milling firm.

Blending, rather than other phases of milling operations, was selected for study because (a) it represents an important distinct phase of milling operations and hence of programming, (b) it has essential characteristics that make it typical of other blending problems that may be found in the grain marketing field, and (c) it was felt that the technique of linear programming was a better tool in blending wheat than the methods now used.

PURPOSE AND OBJECTIVES

The primary purpose and objective of this study was to investigate applications of linear programming in the blending of wheat for the milling industry. The technique of programming was used in this study in the blending of wheat for a mill-mix and in the blending of wheat to offer for sale in the cash market. Particular emphasis was placed on blending wheat for mill-mixes from stocks on hand from previous purchases by the milling firm. Wheat which has been purchased for milling purposes but is found to have undesirable milling characteristics must be sold in the cash market. Linear programming was used to determine if it is more profitable to sell each of the undesirable wheats separately or mix them in some proportion.

A second objective of this study was to determine whether linear programming is a better tool than present methods used in the blending of wheat for a mill-mix at minimum cost.

Blending is critically important to other operations in the milling industry. Intelligent programming of purchasing, manufacturing, or marketing generally requires a solution of blending problems as an initial or integral part of the whole process for it is in blending that the final outputs are determined to a large degree.

METHOD OF PROCEDURE

The lack of any published material on the subject of linear programming as applied to the blending of wheat for any specific purpose left but one alternative in accumulating sufficient information--that of formulating milling problems involving blending of wheat and solving them by linear programming.

This study was conducted in three stages. First, a realistic mill-mix problem was formulated from data obtained from millers and grain merchandisers. Since textbooks on milling did not list any mill-mix problems which included all the factors that must be considered, a method of obtaining these factors was required. This was achieved by constructing a questionnaire which was sent to various millers and grain merchandisers. The information obtained from these firms was used in the formulation of the realistic problem. This mill-mix problem was not solved since high speed electronic computing equipment was not available for this work. By omitting some of the variables this realistic problem would reduce to the mill-mix problem stated in Edgar S. Miller's Studies in Practical Milling.[1] It was therefore believed unnecessary to work the realistic problem leaving out some of the variables. With appropriate computational equipment it is possible to extend the calculations for programming to include all the variables.

Second, Edgar S. Miller's mill-mix problem was solved by linear programming. This problem was used for the following reasons: (1) it was stated as a real problem, (2) the solution was not obtained by programming, (3) a solution could be obtained by the use of linear programming, and (4) the solutions obtained by the two methods could be compared in order to determine if linear programming was the better method of solving such problems. Miller's problem, however, did not consider all the factors that are taken into account when blending a mill-mix; therefore, the task of formulating a problem actually confronting millers was

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[1] Edgar S. Miller, Studies in Practical Milling, Chapter XIII.

necessary. This was achieved by the formulation of the realistic problem.

Third, a problem was set up to show that linear programming could be used by millers in solving problems that dealt with the disposing of wheats purchased that were undesirable for milling or for wheat for which the miller had no use. This example was to demonstrate another possible use of linear programming in the milling industry.

ELEMENTS OF THE PROBLEM

Limitations

Only chemical tests, baking tests, varieties, quantities, and the price of wheat were considered in the formulation of the programming problems. Capital, labor, and other elements of cost were omitted. If the mill and blending equipment are given, such costs are variable only within limits. Labor costs are fixed largely by the milling operations as a whole, for blending activity has only an insignificant effect on these costs. The same is true of overhead and other related costs.

The Aim in Milling

The primary aim in the entire milling process is the profitable conversion of wheat to flour of specified uniform quality. It is the demands of the housewife and the baker who, from the flour performance viewpoint, dictate most of the specifications which govern the various mill-mixes. For example, most of the bread-eating public attach great importance to appearance and palatability and thus demand a loaf of good volume, soft texture, and with thin cell walls which do not crumble readily. Such a loaf requires flour from which bran has been eliminated. A small minority, however, prefer bread containing some bran because they like the taste.

Different mill-mixes must be formulated to meet these various demands of consumers. Regularity of flour quality must be maintained if consumer preferences are to be satisfied. It is up to the miller to convert wheat to flour and maintain regularity in flour quality from day to day and from month to month. It is believed that linear programming may be a tool that will enable the miller to maintain regularity while at the same time reduce his mill-mix cost.

Planning An Economic Mill-Mix

What is meant by an economic mill-mix? Smith describes it as "A wheat mixture, which, with the milling plant available, will produce a flour to satisfy all customers, at a competitive price, and at the same time, show a reasonable profit after being milled."[1] It has often been stated by millers that comparatively little skill is required to formulate an excellent wheat mixture, provided there is an unlimited quantity of good wheat available, and that comparative prices are of no importance. Since this situation does not prevail, the matter is much more difficult.

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[1] Leslie Smith, Flour Milling Technology, p. 313.

It is a question whether the present methods used by millers are as capable as is the technique of linear programming in planning economic mill-mixes.

Binning of Wheat

Grain as it is received at the mill elevator is usually binned by class, grade, variety, protein content, or other characteristics inasmuch as it is desirable to store each quality of milling stock separate from other qualities within the limits of available bin space. It is generally the practice of most mill operators to lot wheat by class (Hard Red Spring, Soft Red Winter, White Wheat, etc.), by subclass (Dark Northern Spring, Hard Winter, White Club, etc.), by numerical grade according to official federal grading standards, and also by the percentage of protein varying in ranges of one-half per cent. Any lot of wheat which is graded with a notation of "tough," "smutty," "weevily," etc., is binned separately so that it can be given special treatment as required before blending with other lots. For example, if a carload of wheat grades No. 1 Dark Hard Winter with a "tough" notation by reason of the fact that its moisture content is 14.7 per cent, it should be stored in a separate bin until it can be aerated and conditioned before blending.

Preparation of Mill-Mixes

In the production of uniform flours, the many important factors which must be considered may be separated into four major phases. These phases are the formulation and preparation of the mill-mix, the cleaning and conditioning of the mix, the milling process, and the flour treatment. All four phases are of great importance and are interdependent. Attention to these four main points insures a flour having a high degree of uniformity and quality, whereas a functional failure in any one of the major steps can and often does mean failure of the finished product to meet required standards of uniform flour quality and analysis.

In actual operation these four steps constitute a complex continuous process. An error at any point may not be discovered until the mill run is completed. This emphasizes the need of perfect control of each phase and coordination between phases.

The relative importance of the formulation of a mill-mix, as a part of the whole milling process, is recognized by millers. This paper is limited to the study of the first part of the first phase--that of formulating a mill-mix.

The principal factors to be taken into account in making up a blend are as follows:

1. Quantities and kinds of wheat available.
2. Relative suitability of wheats to yield flour meeting specified requirements.
3. Relative wheat costs.
4. Possible need to limit percentage of one or more wheats because of some defect.
5. Relative milling values.

The factors listed above will be considered in more detail.

Wheats Available. To construct and maintain a particular blend there must be available a sufficient supply of suitable wheats. A miller will usually use so much strong wheat, so much medium, and so much weak; but, he will aim to maintain regularity of flour quality. If at any time the strong wheats are of exceptionally high quality, the proportion of such (usually high priced) wheats may be reduced. If strong wheats are relatively lower in price, then more may be used and the quantity of medium reduced and weak increased. Conversely, if medium quality wheat is plentiful and cheap it can be incorporated more freely with correspondingly reduced amounts of both strong and weak wheats.

Wheat Suitability. This factor, together with cost, is of primary importance in blending a mill-mix. The miller must use all of his accumulated experience of previously used wheats and judgment of the now available wheats together with data furnished by the mill and laboratories regarding this wheat.

Costs. The cost must be considered simultaneously with suitability when formulating the mill-mix, and must of course be kept as low as reasonably possible. This normally devolves to the problem of reducing the amount of higher priced strong wheats and increasing the amounts of various filler and weak wheats which are cheaper but still maintain a good quality flour at minimum cost.

Defective Wheats. Sometimes a carload of wheat may be bought cheaply because it is in some way not up to standard. Likewise, something might have gone wrong during subsequent storage of a bin of wheat. The wheat may be tainted by garlic, bunt, or weevil; it may be insect-invested; or it may be sprouted. This would yield flour of abnormally high diastatic value. Because of these or other defects it may be necessary to limit the use of the wheat to 5 or 10 per cent in the blend or to eliminate it entirely.

Milling Value. In determining the percentage of each wheat to be used, the miller will bear in mind its probable behavior on the mill. For example, a miller would tend to avoid using wheat of a soft woolly nature as the bran is difficult to separate from the endosperm. The desirability of obtaining a satisfactory flour extraction and high moisture gain will likewise affect his choice.

Preliminary Testing of Wheat

In order to control the buying of wheats and their blending and preparation for milling, the miller must estimate such factors as bushel weight, protein content, moisture content, and percentage of impurities. In order to mill flour that will best meet the demands of his particular market he must also ascertain, by preliminary laboratory tests, water absorption, strength of mill-mix, mixing tolerance of flour, and other properties that control flour quality.

Test Milling of Wheats

Every mill-mix does not yield flour with the desired characteristics. To test the quality of the flours produced from various wheats (or wheats blended for a possible mill-mix), samples of the wheats are milled into flour and compared with commercially-made flours of the desired standards.

This test milling can be done in several ways. The first, which has been adopted by some large firms, is to build a separate complete mill of small output. Such a plant gives results comparable to those of a large commercial mill and enables large-scale tests to be made on every parcel of wheat, giving a full and reliable analysis of flour quality and flour yield.

If firms cannot afford the above method of testing wheats and if they want equally complete test results, the alternative is to mill the test samples separately on the commercial mill itself.

A third alternative is to use a miniature laboratory mill. This mill, if carefully handled, can give almost as high a flour extraction as a commercial mill.

Choice of Wheats for a Mill-Mix

The state of a miller's balance sheet depends largely upon the proper selection and purchase of the wheat for all his mixtures. Each individual miller decides for himself how he blends the different wheats to form his mill-mix. But once he has decided on this mix, his subsequent mixes must maintain regularity in flour quality if consumer preferences are to be satisfied.

It is comparatively easy to compile a good mixture and maintain a regular grade of flour when buying the best wheat, but it is an entirely different matter to maintain regularity when the wheats which must supply the backbone to the mixture are too high in price to be used at the usual percentages. In this case lower-priced substitutes have to be included in the mill-mix.

In the purchase and blending of wheat the miller's main objective is to make a profit. Secondary, he wants to produce and maintain regularity in the flour quality demanded by his particular market.[1]

Flour quality depends chiefly on strength, color, mixing tolerance, water absorption, loaf volume, and loaf texture. The difficult art of making profits depends (as far as choice of wheats is concerned) largely on buying wheat in sound condition that will best combine high flour yield with reasonably low prices and in blending the wheats purchased in the most efficient manner. Flour yield is governed by such factors as test weight, percentage of impurities, moisture content, skin thickness, etc.

Although the best known graded wheats give a fairly constant flour yield, the flour quality is not absolutely invariable; therefore, every consignment of wheat should be carefully tested. If any one variety or type of wheat is unobtainable, unduly expensive or otherwise unsatisfactory, it can usually be replaced by some other variety, type, or combination of wheats that will give the same final results.

Choice of wheats for a mill-mix, therefore, means striking a balance between quality, flour yield, availability, and price. It demands full use of laboratory testing equipment, careful study of the wheat market, much judgment in blending wheats, and experience in flour milling.[2]

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[1] J. F. Lockwood, Flour Milling, p. 107.

[2] Loc. cit.

Having decided upon his standard mill-mix, the miller is then faced with the problem of maintaining a standard of quality, even though circumstances will not permit his keeping the same wheat mix. This is where substitutes and the opportunity for using linear programming become important.

Methods of Blending Wheat

The composition of a mill-mix may call for the blending of many wheats of extremely diverse characteristics and monetary values. Accurate blending is, therefore, very important.

Different wheats need different conditioning treatment, and it is, therefore, usual to blend them after they have been cleaned, washed, and conditioned. It is not always, however, either practicable or necessary for every single bin of wheat to be washed and conditioned separately. Wheats of similar character may be blended before conditioning. Provision for blending must, therefore, be made both before and after conditioning; the final blending being the more important.

The modern tendency is to use synchronized automatic weighers, especially for the final blending of widely dissimilar wheats.[1] By this method absolute accuracy by weight can be relied on. This type of equipment is advantageous for mixes solved by programming since exact amounts of the various wheats are required to meet the specific requirements.

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[1] Smith, op. cit., p. 101.

Blends of Spring and Winter Wheats

It is a common and accepted practice in the milling industry to blend hard red winter wheats with hard red spring in varying ratios in order to produce flours which are especially well adapted to certain kinds of baking. Such blends may vary from a small amount of spring in a winter wheat blend to a small amount of winter in a spring wheat blend. The percentage composition of such blends is dictated by the market in which the flour is to be sold, freight rates, and the individual policy of each particular milling company.

Summary

The various points that were discussed regarding the blending of wheat must be kept in mind when formulating mill-mixes. Some of these points also have a bearing on problems that involve blending of wheat for grain merchandising. These factors that have been mentioned relevant to blending wheat for mill-mixes or for grain merchandising are pertinent if linear programming is used in solving such problems.

PROBLEM OF LINEAR PROGRAMMING

The problem of linear programming may be stated as follows: A set of m equations in n unknowns is specified. This may be written in the form $\sum_{i=1}^{n} \lambda_i P_i = P_o$, where the P_i and P_o represent column vectors, each with m components. The problem then is to find a set of $\lambda_i \geq 0$ which satisfies these equations and which maximizes a linear functional $f(\lambda_i) = \sum_{i=1}^{n} \lambda_i c_i$. The c_i may correspond to any set of relevant criteria but, by the nature of the problem in this study, they are restricted to prices of inputs.[1]

The problem revolves around variation in the values of P_i. The results of these variations may be studied in terms either of consequent variation in physical program variation (via the new activity levels so obtained) or of consequent variation in the value of the functional, or both. Because of the explicit importance of profits to business activity, attention in this study is centered on the changes in the profit position resulting from such structural variation.

As previously stated, "linear programming is concerned with optimal planning of interdependent activities subject to a complex of restrictions."[2] The activities and restrictions are stated in the form $\sum_{i=1}^{n} \lambda_i P_i = P_o$ with $\lambda_i \geq o$; $i = 1, 2, . . ., n$. The criteria by which optimality is to be judged are summarized in the values, associated with different λ_i, which can be attained by the functional $f(\lambda) = \sum_{i=1}^{n} c_i \lambda_i$ where $\lambda = (\lambda_1, \lambda_2, . . ., \lambda_n)$.

REALISTIC MILL-MIX PROBLEM

Determination of Variables

The complete list of variables entering into what is believed to be a realistic mill-mix problem are given in Tables 1 and 2.

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[1] Charnes, Cooper, Henderson, op. cit., p. 7-8.

[2] Ibid., p. 8.

Table 1. Chemical and physical properties, analysis of baking tests, and measurements of other factors considered when blending a mill-mix.

Bin	Protein content %	Moisture content %	Test wt. lbs.	Dough development time min.	Mixing tolerance index	Water absorption %	Ash content straight flour %	Loaf volume cc	Loaf texture %	Color %	Yield straight flour %	Infestation per 100g	Variety	Price (replacement) bu.
(A) 1	15.0	12.0	62.0	4.5	25	61.5	0.43	740	90.0	92.0	73.5	1	Pawnee	$2.22
(B) 2	13.5	11.5	60.5	4.0	27	61.0	0.45	700	89.0	90.5	72.0	0	Pawnee	2.17
(C) 3	12.5	12.5	60.2	3.5	30	63.0	0.47	680	88.5	91.5	71.5	3	Pawnee	2.15
(D) 4	11.5	13.0	58.5	4.0	35	59.0	0.46	670	87.5	91.0	69.0	4	Pawnee	2.02
(E) 5	14.2	12.5	60.5	3.0	25	60.4	0.49	720	89.0	91.5	72.6	1	Comanche	2.20
(F) 6	13.6	13.0	61.0	4.0	28	61.2	0.54	700	89.5	92.0	71.7	1	Comanche	2.18
(G) 7	12.5	11.0	59.7	4.5	40	58.9	0.42	690	90.0	90.0	72.0	0	Comanche	2.15
(H) 8	11.5	12.5	60.4	5.0	32	62.5	0.47	680	87.5	89.0	68.5	5	Comanche	2.00
(I) 9	13.5	11.0	59.0	4.0	27	61.3	0.44	730	89.5	91.0	69.0	1	Cheyenne	2.09
(J)10	14.5	12.8	61.2	4.5	30	60.5	0.52	740	89.0	90.5	72.5	0	Cheyenne	2.20
(K)11	11.5	13.6	60.5	4.0	29	58.4	0.48	660	87.0	89.0	71.5	2	Tenmarq	2.14
(L)12	14.0	12.4	60.8	5.0	30	62.5	0.41	740	89.0	91.0	71.0	2	Tenmarq	2.21
(M)13	12.5	13.7	58.2	4.0	32	61.7	0.47	690	88.0	90.5	72.0	0	Nebred	2.10
(N)14	11.5	12.7	61.0	4.5	35	60.2	0.44	680	88.5	91.0	71.5	1	Nebred	2.16
(O)15	15.0	11.8	60.5	3.5	29	62.0	0.43	740	91.0	92.0	71.0	1	Mida	2.30
(P)16	14.5	12.3	59.0	5.0	28	59.0	0.47	730	90.0	91.0	70.0	0	Mida	2.26
(Q)17	14.0	12.9	59.2	4.0	29	61.3	0.45	720	90.0	90.0	72.0	0	Mida	2.21
(R)18	12.5	13.2	60.5	4.5	33	58.7	0.44	690	88.0	89.5	69.0	0	Mida	2.15
(S)19	13.5	11.8	58.5	4.0	35	60.3	0.49	700	87.5	91.0	70.5	1	Pilot	2.20
(T)20	13.0	12.9	60.0	4.5	30	62.0	0.43	700	88.0	90.0	70.0	0	Pilot	2.18
(U)21	12.5	13.1	59.2	4.5	32	61.0	0.45	680	87.5	91.5	71.0	2	Cadet	2.16
(V)22	14.0	12.2	58.5	5.0	30	58.0	0.42	730	90.0	90.0	69.5	0	Cadet	2.23
(W)23	13.5	12.8	60.2	4.0	31	59.7	0.47	710	88.5	89.5	71.5	0	Ceres	2.20
(X)24	14.0	13.0	58.9	4.5	33	60.5	0.44	720	89.5	91.0	71.0	2	Ceres	2.24
(Y)25	13.5	11.9	60.0	4.5	31	61.0	0.43	710	89.0	91.0	72.0	0	Supreme	2.19

Table 2. Requirements to be met by flour milled for "Harina" baking purpose from wheats listed in Table 1.

Factor requirements	Measurement	Limitations
Protein content	Per cent	≥ 13.6
Moisture content	Per cent	≤ 13.5
Test weight	Pounds	≥ 60.0
Dough development time	Minutes	≥ 4.0
Mixing tolerance	Index	≤ 30
Water absorption	Per cent	≥ 61.0
Ash content (flour)	Per cent	≤ 0.46
Loaf volume	Cubic centimeters	≥ 710
Loaf texture	Per cent	≥ 89.0
Color	Per cent	≥ 91.0
Flour yield	Per cent	≥ 71.0
Infestation	Per 100 grams	≤ 2
Pawnee wheat	Per cent	= 30
Comanche wheat	Per cent	= 20
Hard red winter	Per cent	= 70
Mida wheat	Per cent	= 15
Hard red spring	Per cent	= 30
Price	Dollars	To be a minimum

Each miller would probably consider some factors that others do not; therefore, every miller may not believe this problem to be realistic. The 14 factors included in this problem are ones that it is believed the milling industry, as a whole, would consider when blending a mix. The variables included were ones which were most generally agreed upon in the questionnaires sent to millers and grain merchandisers and interviews with Professor Eugene Farrell and Professor Max Milner of the Milling Department at Kansas State College.

Questionnaires[1] were sent to five leading flour millers and grain merchandisers. Of these five only three filled out

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[1] Example of questionnaire that was sent to millers and grain merchandisers is shown in Appendix A.

the data requested. Most millers are rather particular about the type of information they give out since they are in a highly competitive business. Each milling concern has its own blending policy which aims to maintain quality and regularity in the flour produced. The requirements and restrictions set in this problem, therefore, will probably differ from those set by any one milling firm. Given a particular mill, the type of flour to be produced, the requirements of that flour, the restrictions on percentages of wheats to be used, and the wheats available to be used in the mix, a problem similar to the one that is to follow could be formulated to fit that miller's needs.

Table 1 displays a tabulation of the chemical and physical properties, analysis of baking tests made from flour of the various wheats, and measurements of other factors considered when blending a mill-mix. Table 2 states the requirements or restrictions that must be met by the mill-mix which was to be blended from the 25 bins of wheat listed in Table 1. It was supposed that these requirements were set with regard to the milling policy for flour milled for "Harina"[1] baking purposes.

Mathematical Statement of Problem

With the material that has been covered as a background, it is now possible to proceed with a mathematical statement of the problem. As stated before, the linear programming problem

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[1] Refers to "any type of flour" milled rather than a specific kind such as cake or bread flour.

requires specification of a set of "m" equations in "n" unknowns which may be written in the vector form of $\sum_{i=1}^{n} \lambda_i P_i = P_o$ where the P_i and P_o represent column vectors, each with "m" components. The requisite is to find a set of $\lambda_i \geq 0$ that satisfies these equations and simultaneously maximizes a linear functional $f(\lambda_i) = \sum_{i=1}^{n} \lambda_i c_i$.

The following definitions adapted by Charnes, Cooper, and Mellon[1] from M. K. Woods' "Elements in the Design of Mathematical Models for Programming," will be useful in the succeeding discussion: (1) "Model--a set of simultaneous equations or inequations which describe the interrelationships within an operation." (2) "Activity--a subdivision of an operation identified by principal output and a particular combination of input factors." (3) "Artificial Activity--an activity having no exact counterpart in reality, which is introduced into the formulation as a deliberate construct for conceptual or computational convenience." (4) "Program--a combined schedule of the levels of all activities and of the inputs and outputs of all items involved in an operation."

Tables 1 and 2 supply the necessary information for a mathematical formulation of the problem. Each of the variables listed in Table 2 have stipulated minimal or maximal specifications that must be met by the mill-mix. Each of the possible inputs, bins A, B, C, . . .,Y in Table 1 has certain ratings

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[1] A. Charnes, W. W. Cooper, B. Mellon, "Blending Aviation Gasolines--A Study in Programming Interdependent Activities in an Integrated Oil Company," Econometrics, April 1952, 20:147.

for the variables that must be considered when blending a mill-mix. This information may be utilized to form the following set of inequations:

Protein Content

$$15.0\ A + 13.5\ B + 12.5\ C + \ldots + 13.5\ Y \geq 13.6\ M'$$

Moisture Content

$$12.0\ A + 11.5\ B + 12.5\ C + \ldots + 11.9\ Y \leq 13.5\ M'$$

Test Weight

$$62.0\ A + 60.5\ B + 60.2\ C + \ldots + 60.0\ Y \geq 60.0\ M'$$

Dough Development Time

$$4.5\ A + 4.0\ B + 3.5\ C + \ldots + 4.5\ Y \geq 4.0\ M'$$

Mixing Tolerance

$$25\ A + 27\ B + 30\ C + \ldots + 31\ Y \leq 30\ M'$$

Water Absorption

$$61.5\ A + 61.0\ B + 63.0\ C + \ldots + 61.0\ Y \geq 61.0\ M'$$

Ash Content of Straight Flour

$$0.43\ A + 0.45\ B + 0.47\ C + \ldots + 0.43\ Y \leq 0.46\ M'$$

Loaf Volume

$$740\ A + 700\ B + 680\ C + \ldots + 710\ Y \geq 710\ M'$$

Loaf Texture

$$90.0\ A + 89.0\ B + 88.5\ C + \ldots + 89.0\ Y \geq 89.0\ M'$$

Color

$$92.0\ A + 90.5\ B + 91.5\ C + \ldots + 91.0\ Y \geq 91.0\ M'$$

Yield Straight Flour

$$73.5\ A + 72.0\ B + 71.5\ C + \ldots + 72.0\ Y \geq 71.0\ M'$$

Infestation

$$1\ A + 0\ B + 3\ C + \ldots + 0\ Y \leq 2\ M'$$

where, by definition

$$M' = A + B + C + D + E + F + G + H + \ldots + X + Y$$

Some of the inputs, A, B, C, D, . . . ,Y, which go into the makeup of M', may be zero. The inequations specify the requirements that must be met by the final mix (M'). Referring to protein content, for example, the inequation states that M' shall have a protein content of at least 13.6 per cent. For moisture content, M' shall not contain over 13.5 per cent moisture. Although the terms on the right-hand side of the inequations can be viewed as components of the requirements of vector P_0, it is better first to reduce the set of inequations to equations by introducing additional nonnegative quantities which will be appropriately defined. This will be done at a later stage.

For convenience of notation, the symbols[1] $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{25}$ will be used in place of A, B, C, D, etc., which referred to the various bins. Reference to this system of notation is as follows: λ_1 represents the amount (to be determined) of wheat from bin A going into the mix (M'); λ_2 represents the amount of wheat from bin B which will be used in the mix (M'); and so on.

Employing this new notation, the following set of relations is obtained:

$$\begin{aligned}
15.0\lambda_1 + 13.5\lambda_2 + 12.5\lambda_3 + \ldots + 13.5\lambda_{25} &\geq 13.6\ (\lambda_1+\lambda_2+\ldots+\lambda_{25}) \\
12.0\lambda_1 + 11.5\lambda_2 + 12.5\lambda_3 + \ldots + 11.9\lambda_{25} &\leq 13.5\ (\lambda_1+\lambda_2+\ldots+\lambda_{25}) \\
62.0\lambda_1 + 60.5\lambda_2 + 60.2\lambda_3 + \ldots + 60.0\lambda_{25} &\geq 60.0\ (\lambda_1+\lambda_2+\ldots+\lambda_{25}) \\
4.5\lambda_1 + 4.0\lambda_2 + 3.5\lambda_3 + \ldots + 4.5\lambda_{25} &\geq 4.0\ (\lambda_1+\lambda_2+\ldots+\lambda_{25}) \\
25\lambda_1 + 27\lambda_2 + 30\lambda_3 + \ldots + 31\lambda_{25} &\leq 30\ (\lambda_1+\lambda_2+\ldots+\lambda_{25}) \\
61.5\lambda_1 + 61.0\lambda_2 + 63.0\lambda_3 + \ldots + 61.0\lambda_{25} &\geq 61.0\ (\lambda_1+\lambda_2+\ldots+\lambda_{25}) \\
0.43\lambda_1 + 0.45\lambda_2 + 0.47\lambda_3 + \ldots + 0.43\lambda_{25} &\leq 0.46\ (\lambda_1+\lambda_2+\ldots+\lambda_{25}) \\
740\lambda_1 + 700\lambda_2 + 680\lambda_3 + \ldots + 710\lambda_{25} &\geq 710\ (\lambda_1+\lambda_2+\ldots+\lambda_{25}) \\
90.0\lambda_1 + 89.0\lambda_2 + 88.5\lambda_3 + \ldots + 89.0\lambda_{25} &\geq 89.0\ (\lambda_1+\lambda_2+\ldots+\lambda_{25})
\end{aligned} \tag{1}$$

---

[1] Symbols used in thesis are identified or defined in Appendix B.

$$92.0\lambda_1 + 90.5\lambda_2 + 91.5\lambda_3 + . . . + 91.0\lambda_{25} \geq 91.0 \ (\lambda_1+\lambda_2+...+\lambda_{25})$$

$$73.5\lambda_1 + 72.0\lambda_2 + 71.5\lambda_3 + . . . + 72.0\lambda_{25} \geq 71.0 \ (\lambda_1+\lambda_2+\cdots+\lambda_{25})$$

$$1\lambda_1 + 0\lambda_2 + 3\lambda_3 + . . . + 0\lambda_{25} \leq 2 \ (\lambda_1 + \lambda_2 + ... + \lambda_{25})$$

Associated with each of the variables λ_i in the above set of inequations is a vector with components representing elements in the given structure. Reference to Table 3 may serve to make the meaning of this statement clear. Under the column headed P_1 are structural components representing the properties of bin A going into the makeup of the mill-mix. For example, protein contributes 15.0 units, or 15.0 - 13.6 = 1.4 units in excess of minimal requirements of the blend mix; and moisture contributes 12.0 units, or 13.5 - 12.0 = 1.5 units below the maximum permitted. The remaining terms in this vector and the terms in the other vectors can be determined by the same procedure. (Later in the discussion all signs will be changed for computational convenience.)

It is desirable to simplify the set of inequations that have been formulated in order to construct the program matrix. Before doing so, however, the policy restrictions will be stated in a mathematical form. As a result of company policies and from previous blending experience, certain varieties of wheat are desired in the mill-mix. In Table 1 are listed the varieties that are available, and in Table 2 is found a statement on the percentages of each variety that must make up the mill-mix. Stated in mathematical form, this policy restriction becomes:

$$\begin{aligned}
&\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 30 \\
&\lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 = 20 \\
&\lambda_{15} + \lambda_{16} + \lambda_{17} + \lambda_{18} = 15 \\
&\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \ldots + \lambda_{13} + \lambda_{14} = 70 \\
&\lambda_{15} + \lambda_{16} + \lambda_{17} + \ldots + \lambda_{24} + \lambda_{25} = 30 \\
&\lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_{24} + \lambda_{25} = 100
\end{aligned} \tag{2}$$

where λ_i refer to the percentage of wheat from bins A, B, C, . . ., X, Y making up the mix. For example, 30 per cent of the mill-mix must come from bins A, B, C, and D since the mix is to contain 30 per cent of Pawnee wheat. The total mix, or 100 per cent, must be made from the 25 available bins of wheat. Of course, some of these bins may not be used in the final blend.

By transposing the relations into the right-hand side of the inequations (set 1), and multiplying by minus one, where necessary, to point all inequality signs in the same direction, the set of inequalities may be simplified to:

$$\begin{aligned}
&-1.4\lambda_1 + 0.1\lambda_2 + 1.1\lambda_3 \ldots + 0.1\lambda_{25} \leq 0 \\
&-1.5\lambda_1 - 2.0\lambda_2 - 1.0\lambda_3 \ldots - 1.6\lambda_{25} \leq 0 \\
&-2.0\lambda_1 - 0.5\lambda_2 - 0.2\lambda_3 \ldots + 0\lambda_{25} \leq 0 \\
&-0.5\lambda_1 + 0\lambda_2 + 0.5\lambda_3 \ldots -0.5\lambda_{25} \leq 0 \\
&-5\lambda_1 - 3\lambda_2 + 0\lambda_3 \ldots + 1\lambda_{25} \leq 0 \\
&-0.5\lambda_1 + 0\lambda_2 - 2.0\lambda_3 \ldots + 0\lambda_{25} \leq 0 \\
&-0.03\lambda_1 - 0.01\lambda_2 + 0.01\lambda_3 \ldots - 0.03\lambda_{25} \leq 0
\end{aligned} \tag{3}$$

$$-30\lambda_1 + 10\lambda_2 + 30\lambda_3 \ . \ . \ . + 0\lambda_{25} \leq 0$$

$$-1.0\lambda_1 + 0\lambda_2 + 0.5\lambda_3 \ . \ . \ . + 0\lambda_{25} \leq 0$$

$$-1.0\lambda_1 + 0.5\lambda_2 - 0.5\lambda_3 \ . \ . \ . + 0\lambda_{25} \leq 0$$

$$-2.5\lambda_1 - 1.0\lambda_2 - 0.5\lambda_3 \ . \ . \ . -1.0\lambda_{25} \leq 0$$

$$-1\lambda_1 - 2\lambda_2 + 1\lambda_3 \ . \ . \ . -2\lambda_{25} \leq 0$$

A statement of these conditions in the form of inequalities permits over-fulfillment of requirements if profits can be increased in that manner. That is, the moisture content of the mix for example, may be only 12.5 per cent, which would be 1.0 per cent less than the maximum permitted. It may prove more profitable to over-fulfill than to meet each requirement exactly. Imposing the condition that requirements must be met exactly would necessitate restatement of (1) in the form of a set of equations, replacing the inequality signs by equality signs. Such a restatement may introduce contradictory conditions. The program would then be infeasible.

The inequations in (3) may be reduced to a set of equivalent equations by introducing an appropriate set of new nonnegative unknowns, i.e., $\lambda_{26}, \lambda_{27}, \lambda_{28}, \ldots, \lambda_{37}$ on the left-hand side (one unknown to each inequation). The λ's so introduced are referred to as pseudo (slack) variables.

Corresponding to each of these pseudo variables is a slack vector, P_{26}, P_{27}, P_{28}, . . ., P_{37} with components as listed under the headings in Table 3. Meaning need not always be attached to these slack vectors, for they may be introduced only for the purpose of facilitating computations. This set has a definite physical meaning. Here a non-zero value for any of the pseudo variables represents an overfulfillment of requirements.

After executing the indicated operation, the relations (3) become:

$$-1.4\lambda_1 + 0.1\lambda_2 + 1.1\lambda_3 \dots + 0.1\lambda_{25} + \lambda_{26} = 0$$

$$-1.5\lambda_1 - 2.0\lambda_2 - 1.0\lambda_3 \dots - 1.6\lambda_{25} + \lambda_{27} = 0$$

$$-2.0\lambda_1 - 0.5\lambda_2 - 0.2\lambda_3 \dots + 0\lambda_{25} + \lambda_{28} = 0$$

$$-0.5\lambda_1 + 0\lambda_2 + 0.5\lambda_3 \dots -0.5\lambda_{25} + \lambda_{29} = 0$$

$$-5\lambda_1 - 3\lambda_2 + 0\lambda_3 \dots + 1\lambda_{25} + \lambda_{30} = 0$$

$$-0.5\lambda_1 + 0\lambda_2 - 2.0\lambda_3 \dots + 0\lambda_{25} + \lambda_{31} = 0$$

$$-0.03\lambda_1 - 0.01\lambda_2 + 0.01\lambda_3 \dots - 0.03\lambda_{25} + \lambda_{32} = 0 \qquad (4)$$

$$-30\lambda_1 + 10\lambda_2 + 30\lambda_3 \dots + 0\lambda_{25} + \lambda_{33} = 0$$

$$-1.0\lambda_1 + 0\lambda_2 + 0.5\lambda_3 \dots + 0\lambda_{25} + \lambda_{34} = 0$$

$$-1.0\lambda_1 + 0.5\lambda_2 - 0.5\lambda_3 \dots + 0\lambda_{25} + \lambda_{35} = 0$$

$$-2.5\lambda_1 - 1.0\lambda_2 - 0.5\lambda_3 \dots -1.0\lambda_{25} + \lambda_{36} = 0$$

$$-1\lambda_1 - 2\lambda_2 + 1\lambda_3 \dots -2\lambda_{25} + \lambda_{37} = 0$$

The same procedure can be followed for the equations in set (2). This set of relations become:

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_{38} = 30$$

$$\lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{39} = 20$$

$$\lambda_{15} + \lambda_{16} + \lambda_{17} + \lambda_{18} + \lambda_{40} = 15 \qquad (5)$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_{14} + \lambda_{41} = 70$$

$$\lambda_{15} + \lambda_{16} + \lambda_{17} + \ldots + \lambda_{25} + \lambda_{42} = 30$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_{24} + \lambda_{25} + \lambda_{43} = 100$$

where $\lambda_{38}, \lambda_{39}, \ldots, \lambda_{43}$ represent nonnegative unknowns.

However, the value of these λ_i are known to be zero since the relations (2) were stated in the form of equations. These unknowns were added to facilitate computations. In Table 3, the artificial activities (P_{38}-P_{43}) which correspond to these unknowns will be given a value of M. The value M, will be defined to be so large that it dominates everything else; therefore, none of these variables will appear in the solution.

The sets of equations (4) and (5) yield 18 equations in 43 unknowns which represent the basic set of technological and policy restrictions. A feasible solution (or program) must provide a set of nonnegative values which will simultaneously satisfy all of these relations.

The problem is to minimize the cost per bushel subject to these restrictions. Such a function is formed by introducing the appropriate set of prices for each of the bins of wheat. Referring to the set of prices in Table 1, the following (linear) functional may be constructed:

$$(6)\ z = 2.22\lambda_1 + 2.17\lambda_2 + 2.15\lambda_3 + 2.02\lambda_4 + \ldots + 2.19\lambda_{25} + M\lambda_{38} + M\lambda_{39} + \ldots + M\lambda_{43}$$

where z is merely a representation (in dollars per bushel) of the cost of the mill-mix obtained from the mixture of bins A, B, C, D, . . . ,Y.

Table 5. Program Matrix for mill-mix.

Structural Vectors																									Slack Vectors												Artificial Vectors						Requirements
2.22	2.17	2.15	2.02	2.20	2.18	2.15	2.00	2.09	2.20	2.14	2.21	2.10	2.16	2.30	2.26	2.21	2.15	2.20	2.18	2.16	2.23	2.20	2.24	2.19													M	M	M	M	M	M	
P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{38}	P_{39}	P_{40}	P_{41}	P_{42}	P_{43}	
-1.4	0.1	1.1	2.1	-0.6	.0	1.1	2.1	0.1	-0.9	2.1	-0.4	1.1	2.1	-1.4	-0.9	-0.4	1.1	0.1	0.6	1.1	-0.4	0.1	-0.4	0.1	1																		
-1.5	-2.0	-1.0	-0.5	-1.0	-0.5	-2.5	-1.0	-2.5	-0.7	0.1	-1.1	0.2	-0.8	-1.7	-1.2	-0.6	-0.3	-1.7	-0.6	-0.4	-1.3	-0.7	-0.5	-1.6		1																	
-2.0	-0.5	-0.2	1.5	-0.5	-1.0	0.3	-0.4	1.0	-1.2	-0.5	-0.8	1.8	-1.0	-0.5	1.0	0.8	-0.5	1.5	.0	0.8	1.5	-0.2	1.1	.0			1																
-0.5	.0	0.5	.0	1.0	.0	-0.5	-1.0	.0	-0.5	.0	-1.0	.0	-0.5	0.5	-1.0	.0	-0.5	.0	-0.5	-0.5	-1.0	.0	-0.5	-0.5				1															
-5.0	-5.0	.0	5.0	-5.0	-2.0	10.0	2.0	-3.0	.0	-1.0	.0	2.0	5.0	-1.0	-2.0	-1.0	5.0	5.0	.0	2.0	.0	1.0	5.0	1.0					1														
-0.5	.0	-2.0	2.0	0.6	-0.2	2.1	-1.5	-0.3	0.5	2.6	-1.5	-0.7	0.8	-1.0	2.0	-0.3	2.3	0.7	-1.0	.0	3.0	1.3	0.5	.0						1													
-0.09	-0.01	0.01	.0	0.09	0.02	-0.04	0.01	-0.02	0.06	0.02	-0.03	0.01	-0.02	-0.03	0.01	-0.01	-0.02	0.03	-0.03	-0.01	-0.04	0.01	-0.02	-0.03							1												
-30.0	10.0	30.0	40.0	-10.0	10.0	20.0	30.0	-20.0	-30.0	50.0	-30.0	20.0	30.0	-30.0	-20.0	-10.0	20.0	10.0	10.0	30.0	-20.0	.0	-10.0	.0								1											
-1.0	.0	0.5	1.5	.0	-0.5	-1.0	1.5	-0.5	.0	2.0	.0	1.0	0.5	-2.0	-1.0	-1.0	1.0	1.5	1.0	1.5	-1.0	0.5	-0.5	.0									1										
-1.0	0.5	-0.5	.0	-0.5	-1.0	1.0	2.0	.0	0.5	2.0	.0	0.5	.0	-1.0	.0	1.0	1.5	.0	1.0	-0.5	1.0	1.5	.0	.0										1									
-2.5	-1.0	-0.5	2.0	-1.6	-0.7	-1.0	2.5	2.0	-1.5	-0.5	.0	-1.0	-0.5	.0	1.0	-1.0	2.0	0.5	1.0	.0	1.5	-0.5	.0	-1.0											1								
-1.0	-2.0	1.0	2.0	-1.0	-1.0	-2.0	3.0	-1.0	-2.0	.0	.0	-2.0	-1.0	-1.0	-2.0	-2.0	-2.0	-1.0	-2.0	.0	-2.0	-2.0	.0	-2.0												1							
1	1	1	1																																		1						30
				1	1	1	1																															1					20
														1	1	1	1																						1				15
1	1	1	1	1	1	1	1	1	1	1	1	1	1																											1			70
														1	1	1	1	1	1	1	1	1	1	1																	1		30
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1																		1	100

A complete statement of the functional would require introduction of the slack variables $\lambda_{26}, \lambda_{27}, \ldots, \lambda_{37}$ into the right-hand member of ~~(7)~~ (6). For compactness, these variables are not introduced explicitly. These variables must enter at zero price and, therefore, need never to be written explicitly in the functional. Over-fulfillments of requirements do not result in addition to the cost of the mix.

The prices used in the functional are not necessarily the market prices that were paid for the wheat. Replacement cost is usually considered when blending a mill-mix rather than the actual cost of the wheat on hand. In this problem replacement cost was considered.

The problem is to minimize the linear functional, Z, as given in (6), subject to the side conditions of (4) and (5). Since only nonnegative solutions are admissible, the solutions must satisfy the additional restriction:

$$\lambda_i \geq 0 \quad (i = 1, 2, 3, \ldots, 43) \qquad (7)$$

where these terms represent the level of "real activities" (λ_1 through λ_{25}) and the level of "slack" and "artificial activities" (λ_{26} through λ_{43}). Thus, all solutions must be within the positive orthant. Geometrically, the solutions form a convex polyhedron, i.e., a convex set with a finite number of corners. The 43 conditions (7) plus the 18 conditions (4) and (5) and the statement of the linear form (6), which is to be minimized subject to these conditions, constitute the complete model for solution of the mill-mix problem.

Computational Procedure

To facilitate systematic computations the coefficients of the 18 equations and 43 unknowns were arrayed in a matrix form, as shown in Table 3, where P_o is the vector whose components represent the requirements specified on the right-hand side of equations (4) and (5).

Since suitable computational equipment was not available, the solution to this problem was not computed. Of course, there may be more than one solution which will minimize Z.

The number of equations and unknowns in this problem could have been reduced. This would have permitted the use of present equipment; but, since a reduction in the number of variables results in a non-realistic problem, it was felt that further computation was unnecessary. A solution was obtained for a problem proposed by Edgar S. Miller, having fewer restrictions and requirements; thus giving a comparison of a mill-mix problem solved by linear programming and by another method.

EDGAR S. MILLER'S MILL-MIX PROBLEM

Statement of Problem

Edgar S. Miller's mill-mix problem offered an opportunity to check a solution obtained by the use of linear programming with that of another method. The mix obtained by programming was a lower cost mix as is shown later.

This same problem was sent to the operative firms who were sent the questionnaire. They were asked to solve this problem using their present methods. The purpose of this was to check the solution obtained by linear programming with solutions obtained by methods used in the milling industry at the present time. None of the solutions submitted by the milling firms were equal or less than that of linear programming. Some of these firms were influenced by their flour quality policy and, therefore, did not use Miller's requirements--but set their own. Thus, their solutions were not too comparable with the programming solution.

All of the milling representatives either said or implied that Miller's problem was not realistic in that many more factors were considered when blending wheat for a mill-mix. This led to the development of what was hoped a realistic problem which has already been discussed.

Edgar S. Miller states his mill-mix problem in Studies in Practical Milling as follows:

> All bakers do not want flours of equal strength (that is, with equal percentages of protein). Usually, however, the operative making such flours is required to meet certain definite specifications which include protein percentages. He cannot merely meet these, however; in addition he must exert the greatest effort to insure against undesirable properties not revealed by the protein estimation. These are not concerned wholly with the gluten quality of the flour; they have to do with properties more or less indefinable but best described in the language of the operative miller under the general head of 'quality.' Precautions must be taken against various types of 'unsoundness,' including mustiness, immaturity and heat damage--even the slight heat damage often ignored in grading, such as is revealed by darkened and brittle germs. Whether facilities for pre-milling are provided or not, a tentative blend should be made up, employing the best judgment possible with all information at hand as a basis.

Assuming that the protein and moisture contents, as well as the test weights, of the various available wheats are known, a useful table showing these can be devised. In addition to the more or less positive information just enumerated, it is of some value to set down observations made with the aid of the senses of sight, feel and smell. Let us see what can be done with such a table constructed, for convenience, to include grain in ten different bins.

WORK TABLE

Bin No.	Test Wt.	Protein Percent	Moisture Percent	Observations	Cost
1	58	11.9	12.2	Ordinary, Sound	$1.02
2	55	12.4	10.6	Ordinary to dark, Shriveled berries. Dark germs.	.99
3	57	13.1	14.5	Slight heat damage. Bleached. Use sparingly.	1.02
4	59	12.9	12.2	Bright and sound	1.05
5	60	11.4	10.6	Yellowish, Sound,Bright	1.00
6	58	11.9	12.5	Ordinary, Sound, Bright	1.02
7	58	13.6	13.0	Suspicious. Some immature. Use with care	
8	57.5	13.8	12.5	Very good stuff	1.07
9	58	12.6	13.0	Ordinary. Some Durum	1.03
10	56.5	12.5	14.5	Shriveled. Trace of Mustiness. Careful	.98

While there is no absolute insurance that the quantity of protein in the flour milled from any given wheat blend will be exactly so many points lower than that of the wheat, we must make an assumption based upon past experiences--our own or those of someone else. Just for the sake of illustration, suppose that we are called upon to build a mill-mix from which we can produce a short patent with a minimum of 11.25% protein. This will call for a mill-mix with a protein content of about 12.6%, taking the protein of patent as 1.25% to 1.40% below that of the wheat. We are, of course, anxious to hold the cost of our mill-mix as low as possible. Looking over the table, it will be seen that we cannot merely draw certain percentages of high, medium and low protein wheats and arrive at the lowest possible cost consistent with the characteristics we must have in our flour. Observe, for example, the wheat in Bin 9. For its test weight and degree of soundness its price is comparatively low and its protein equal to our requirements. It contains some durum, however, and its moisture content is rather high. The lowest priced wheat is in Bin 10, and its protein content is nearly up to what we shall want in the mill-mix, but here we have high moisture, low test weight, probably a considerable amount of shriveled berries and a trace of unsoundness. We must be careful with such wheat.

Common procedure among operative millers is to ascertain first of all the figures for average protein per cent, cost, test weight and percentages of moisture of the wheat available, and then exercise judgment in arranging the percentages of each for the mill-mix. In the case under consideration we have 10 bins. If we add all the protein, test weights, moisture and cost figures and divide by 10 we will have the average of all, and our final figures will represent those of a mill-mix made up of 10% of wheat from each bin. For test weight we will have 57.6 pounds; for protein, 12.61%; for moisture, 12.51% and the cost will be $1.02 per bushel.

It would not be advisable, however, to use 10% of the wheats from Bins 3, 7, 9 and 10. We must find some means of cutting down there without the lowering of the protein in the blend. Suppose we try this:

			PROTEIN		
	%		%		
Bin No. 1,	20	X	11.9	=	238.0
Bin No. 2,	10	X	12.4	=	124.0
Bin No. 3,	4	X	13.1	=	52.4
Bin No. 4,	25	X	12.9	=	322.5
Bin No. 5,	8	X	11.4	=	91.2
Bin No. 6,	6	X	11.9	=	71.4
Bin No. 7,	2	X	13.6	=	27.2
Bin No. 8,	15	X	13.8	=	207.0
Bin No. 9,	7	X	12.6	=	88.2
Bin No.10,	3	X	12.5	=	37.5
	100				1259.4

Dividing by 100% we get $\frac{1,259.4}{100}$ = 12.594%

The percentage of protein having been found ample, the next thing to do is to ascertain the cost of this proposed mill-mix. If it is too high, some changes will have to be made, but since these changes will necessitate the inclusion of larger quantities of cheaper and less desirable wheats than we have chosen, the facts should be presented to the management. We arrive at the

> cost of our tentative mill-mix in the same manner used for calculating the protein.[1]

The cost of the mill-mix was calculated to be 102.99 cents, or nearly $1.03. The test weight was 57.95 pounds, and the moisture content of the mix was 12.208 per cent.

Miller's concluding statement concerning the mill-mix just formulated is as follows:

> Assembling the figures and drawing upon the comments relative to the various wheats used, we now have a tentative mill-mix costing $1.03 per bushel. Its apparent soundness characteristics are good. It carries about the percentage of protein that will be required, and its test weight and moisture content are favorable. It is readily apparent that the cost can be lowered only by making some characteristic factor less favorable.[2]

Miller's Problem Solved by Linear Programming

The protein content, moisture content, and test weight of Miller's final blend were used as the basic requirements to be met in the solution obtained by the method of linear programming. That is, the mill-mix obtained by the use of programming was to have a protein content of not less than 12.594 per cent, a test weight of not less than 57.95 pounds, and a moisture content not greater than 12.208 per cent. Limitations were also put on the use of the wheats from Bins 3, 7, 9, and 10. To make the two problems comparable, the maximum per cent that could be used from each of these bins in the programming solution was not to exceed that used in Miller's mix. However, if there was a more

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[1] Miller, *op. cit.*, p. 139-141.

[2] *Ibid.*, p. 142.

efficient blend of the grains which would meet the requirements mentioned above, the programming solution would not need to contain wheat from these bins having undesirable characteristics.

A mathematical statement of this problem was not formulated in this thesis since one was stated for the problem mentioned previously. The solution to Miller's problem obtained by using linear programming is shown in Table 4.

Table 4. Linear programming solution of Edgar S. Miller's mill-mix problem.

Bins used	Percentage of mix
2	17.4881
3	4.0000
5	37.0896
7	2.0000
8	36.4242
10	3.0000
Total	100.

This blend would yield a mill-mix with a protein content of 12.594 per cent, a test weight of 57.95 pounds, and a moisture content of 11.613 per cent. The protein content and test weight are identical with those of Miller's but the moisture content is 0.595 per cent less. This would be advantageous to the miller since he could add more water to the mix (moisture content is usually raised to 16 per cent before milling) which would result in greater flour extraction (by weight). Table 5 shows a comparison of the two solutions. The percentage of wheat used from each bin for the two blends is quite different. The programming mix did not use any wheat from bins 1, 4, 6, and 9.

Table 5. Comparison of Edgar S. Miller's solution with that of linear programming.

Bin No.	Protein %	Total weight lbs.	Moisture %	Price $	Miller's mix %	Programming mix %
1	11.9	58.0	12.2	1.02	20	
2	12.4	55.0	10.6	0.99	10	17.4881
3	13.1	57.0	14.5	1.02	4	4.0000
4	12.9	59.0	12.2	1.05	25	
5	11.4	60.0	10.6	1.00	8	37.0896
6	11.9	58.0	12.5	1.02	6	
7	13.6	58.0	13.6	1.02	2	2.0000
8	13.8	57.5	12.5	1.07	15	36.4242
9	12.6	58.0	13.0	1.03	7	
10	12.5	56.5	14.5	0.98	3	3.0000
					100	100
					12.594	Pro. 12.594
					57.95	T.W. 57.95
					12.208	Mois. 11.613
					102.99¢	Price 102.44¢

More important than the factors just mentioned is the cost of the new mill-mix. The cost of this programming mix was 102.44 cents or .55 cents a bushel less than Miller's blend. This would result in a considerable saving to a milling firm. For example, if a firm milled 10 million bushels of wheat a year, a saving of $55,000 could be attributed to the use of linear programming (at the rate of a saving of .55 cents a bushel).

Miller's statement, "It is readily apparent that the cost can be lowered only by making some characteristic factor less favorable"[1] was shown to be in error by the solution obtained by programming. Cost was lowered without making any factor less

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[1] Loc. cit.

favorable in the mix. The best solution "readily apparent" to Miller using one technique was proved not to be the best solution when using programming. For this particular mill-mix problem, linear programming was more capable of blending a mix at less cost than was the method used by Miller.

GRAIN MERCHANDISING PROBLEM

Purchase of Poor Quality Wheat

All wheat that a mill firm purchases may not be suitable for flour milling purposes. Such things as impurities, poor baking characteristics, poor varieties, etc., may be reasons for wheat to be undesirable for flour. Of course, a miller will usually not purchase such wheat if he knows it has these poor qualities, but sometimes he is unaware of the presence of these traits, or it may result from storing in his own elevators. Some millers may use a limited amount of this poor quality wheat in their blending, depending on the factor present or the extent of damage. Whatever the case, he will probably sell a large percentage of this wheat in the open market. Thus, the problem arises of how to market this inferior wheat. Should this poor quality wheat be mixed with other wheat or should it be sold separately? In other words, which method would be most profitable to the miller? The technique of linear programming can again be called upon to yield the solution to this problem if certain information is available.

Example of a Grain Merchandising Problem

For illustrative purposes, suppose a miller has five bins of wheat, each containing 1000 bushels, which he prefers to sell on the cash market rather than to have milled for flour. The miller has the problem of determining if it would be more profitable to sell each bin separately or to mix them in some manner. Selling on the cash market will mean there will be certain requirements to be met by the wheat in order to meet a certain grade. These grade requirements are listed in the U. S. Department of Agriculture's "Handbook of Official Grain Standards of the United States."

The price buyers will pay for wheat is not entirely determined by the factors that determine grades. Protein content and variety are two additional factors that influence price at central markets. The level of protein content was the only factor considered in this problem other than grade requirements in order to keep it simple enough to work by hand calculators.

Table 6 lists the five bins of Hard Red Winter wheat that were to be sold on the cash market with measurements of the factors that determine grade plus other related data.

The assumption was made that the miller knew the market price for each of these bins of wheat if sold separately; likewise, he knew the price of specified grades of wheat at various protein levels. Table 7 lists the prices that could be received for wheat grading No. 1 and No. 2 containing different levels of protein. To keep the number of variables as low as possible only three protein levels were considered under each of these two grades.

Table 6. Grade factor measurements and other related data for bins of wheat to be sold on the cash market.

Bins	Test wt. lbs.	Damaged kernels	Foreign material		Wheats of other classes	Moisture content %	Protein content %	Grade*	Bushels available for sale	Quoted market price
			Total %	Matter except other grain %						
1	59.5	All meet standard of Grade 1	0.3	0.2	All meet standard of Grade 1	16.0	13.0	Sample DH	1000	$2.17
2	61.0		0.2	0.1		12.5	13.6	No. 1 DH	1000	2.34
3	56.0		0.3	0.2		12.9	11.7	No. 3 DH	1000	2.12
4	60.2		1.4	1.0		13.5	10.5	No. 2 DH	1000	2.09
5	61.0		0.2	0.1		13.0	12.8	No. 1 DH	1000	2.27

* Dark and hard represented by D and H.

Table 7. Market price for hard red winter wheat grading No. 1 or No. 2 at various protein levels.

Grade	Protein level (per cent)	Quoted market price
1	13	$2.31
1	12	2.18
1	11	2.10
2	13	2.26
2	12	2.15
2	11	2.08

Wheat is graded and designated in central cash markets according to the respective grade requirements of the numerical grades and sample grade of its appropriate class or subclass. Table 8 gives the grade requirements that must be met by a mixture of the wheats if it is found profitable to sell wheat grading No. 1 or No. 2 at a given protein level.[1] If these requirements cannot be met without an increase in total income, no mixture will be made.

Table 8. Grade requirements for hard red winter wheat.

		Maximum limits of					
		Damaged kernels (wheat and other grains)		Foreign material		Wheats of other classes	
Grade	Minimum test weight per bushel	Total	Heat-damaged	Total	Matter except other grain	Total	Durum and/or Red Durum
No.	Pounds	Per cent					
1	60	2	0.1	1	0.5	5	1
2	58	4	0.2	2	1.0	10	2

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[1] *Handbook of Official Grain Standards of the United States*, p. 7.

In this example, cost of mixing was not considered as a determining factor. Of course, in actual operation, mixing cost is one of the factors that must be confronted when solving such a problem. There are also other considerations that must be taken into account such as mixing facilities, available labor, etc., that have a bearing on the grain merchandising problem. Each firm will have its own unique problems.

From Tables 6, 7, and 8 a linear programming problem was formulated. As previously stated, the objective was to maximize the income from the sale of these five bins of wheat. This problem was expressed in 36 equations with 66 unknowns. The mathematical statement of this problem is similar to the one previously stated; therefore, none was made here. The program matrix is shown in Table 9 from which the solution was obtained.

One solution to this problem was acquired after calculating 18 tables. That is, a maximum solution was obtained in the eighteenth computation. By means of the criteria developed by Dantzig[1] it is possible at each stage of the calculations, provided degeneracy is not present, to determine whether (a) further calculations are required, (b) a maximum has been obtained, or (c) no finite maximum exists. When condition (a) occurs (and degeneracy is not present) the solutions themselves provide a simple means of improving the value of the functional at the next stage of calculations. It was possible to improve the profit

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[1] G. B. Dantzig, "Maximization of a Linear Form Where Variables are Subject to a System of Linear Inequalities," loc. cit.

Table 1. Program matrix for grain merchandising problem.

Structural Vectors																														Slack Vectors																																				Requirements Vectors
.14	-.03	.10	.02	.06	.06	-.08	.14	.27	-.01	.70	-.16	.06	.09	-.0[illegible]	-.70	-.29	.03	.06	-.12	-.07	-.24	-.02	.02	-.27	-.02	-.26	-.04	-.02	-.35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{38}	P_{39}	P_{40}	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}	P_{50}	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	P_{56}	P_{57}	P_{58}	P_{59}	P_{60}	P_{61}	P_{62}	P_{63}	P_{64}	P_{65}	P_{66}	P_0
0.6	-0.2	4.0	-0.2	-0.2																										1																																				
-0.7	-0.8	-0.7	0.4	-0.3																											1																																			
-0.3	-0.4	-0.2	0.1	-0.9																												1																																		
2.0	-1.1	-1.1	-0.5	-1.0																													1																																	
.0	-0.9	1.3	0.3	0.1																														1																																
					-2.1	-0.0	2.0	-0.2	-2.0																										1																															
					-2.7	-1.3	-1.7	-0.8	-1.3																											1																														
					-0.8	-0.9	-0.9	.0	-0.1																												1																													
					4.0	-1.3	-0.2	-0.9	-0.9																													1																												
					.0	-0.8	1.3	2.1	0.1																														1																											
										0.1	-0.2	2.0	-0.1	-0.9																										1																										
										-0.7	-0.8	-0.7	0.4	-0.8																											1																									
										-0.3	-0.4	-0.3	0.5	-0.4																												1																								
										2.0	-1.3	-1.1	-0.5	-0.9																													1																							
										-1.0	-0.9	0.3	2.0	-0.2																														1																						
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				1					1					1					1					1					1																																			1		1,000
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1																																				1	1,000

functional upon completion of the first 17 computations, but the eighteenth calculation showed it to be a finite maximum. This optimal program is displayed in Table 10.

Table 10. Optimal program for grain merchandising problem.

	Quantity used for various blends (bu.)							
	Grade 1 13%	Grade 1 12%	Grade 1 11%	Grade 2 13%	Grade 2 12%	Grade 2 11%	Excess capacity	Total capacity
Bin	Protein						Bushels	
1	1000.0	0	0	0	0	0	0.0	1000.0
2	1000.0	0	0	0	0	0	0.0	1000.0
3	257.6	0	0	0	0	0	742.4	1000.0
4	0.0	0	0	0	0	0	1000.0	1000.0
5	530.3	0	0	0	0	0	469.7	1000.0
Total	2787.9	0	0	0	0	0	2212.1	5000.0

The program consists of a 2787.9-bushel blend of grade 1 wheat at a 13 per cent protein level. All of bins 1 and 2 will go into this mix plus 257.6 bushels of bin 3 and 530.3 bushels of bin 5. None of bin 4 will be used. No other mix will be made with the remaining wheat. It will be most profitable to sell the remaining 2212.1 bushels separately. The profit resulting from the blend of grade 1 wheat at a 13 per cent protein level is $180.16. In other words, this particular miller would increase his total income by $180.16 from the sale of these five bins of wheat if he mixes bins 1, 2, 3, and 5 according to Table 10 and then sells the remaining wheat in each bin separately.

Costs resulting from mixing were not considered in this problem, however, in reality costs would be a factor. If for example, the cost of mixing was one-half cent per bushel, the cost of blending 2787.9 bushels would be $13.94. This would still be $166.22 over the income that could have been realized by selling each bin separately. This is a sizable sum for such a small quantity. For the bushels actually mixed, this would amount to an increase of six cents per bushel over the price that would have been received had there been no blending.

This example demonstrates another use for the application of linear programming. This problem does not need to be limited to a miller's problem. Anyone in the grain merchandising business might have had these five bins of wheat for sale. His objective, of course, would be to maximize his profits. Given certain information, linear programming may be used to maximize his profits or to minimize his costs. Linear programming has many possibilities.

SUMMARY

Linear programming is a method for considering a number of variables simultaneously and calculating the best possible solution of a given problem within the stated limitations. This is a precise statement of every manufacturer's main problem. In deciding what particular items to manufacture, and in what quantities, a great number of complex factors must be taken into account. Capacity of machines, the cost and salability of the items produced, etc., are some factors that must be considered. To make matters worse, each subdivision of a manufacturer's problem has its own complexities. For instance, he may have to choose among a number of possible raw materials for making a particular commodity. All of the factors and decisions may interlock and react upon one another in ways that cannot be seen. The best any management can hope to achieve under such circumstances is a reasonably workable compromise. With linear programming, however, it becomes possible to locate the optimum solutions among all the available ones.

The attention of this study was directed toward the milling industry. Linear programming was applied to several problems that confront management of this industry. First, a mill-mix was formulated that would yield a flour for "Harina" baking purposes. Second, a grain merchandising problem was deduced for wheat purchased by millers for milling purposes but which was found undesirable for this purpose. This meant that the wheat had to be resold in the open market.

Edgar S. Miller solved a mill-mix problem in his Studies in Practical Milling by a method other than linear programming. This problem offered an opportunity to check a solution obtained by the use of programming with that of another method. The cost of the programming mix was .55 cents a bushel less than Miller's blend. In the milling industry where a saving of a quarter or half cent per bushel can mean a profit or loss, linear programming may be a valuable tool for solving mill-mix problems.

In the grain merchandising problem the profit which resulted from mixing portions of the bins of wheat available for sale--rather than selling each bin separately--was $180.16. This example demonstrated another possible application of linear programming.

It was shown that linear programming can be used to solve such blending problems in the milling industry. This same technique may be applied to other phases of milling.

A secondary objective of this study was to determine whether linear programming was a better tool than present methods used in the blending of wheat for a mill-mix at minimum cost. From correspondence with various millers it was assumed they were not using this technique. No concrete statement can be made that linear programming is better since milling firms will not say definitely how they determine their mill-mixes. Also, no realistic problems for which millers have solutions were solved by linear programming since such problems were not available. Of course, it is shown by programming that the solution so obtained cannot be exceeded by any other solution regardless of the technique used. Due to the complexity of such problems, it is reasonably safe to conclude that linear programming will find the optimal solution while other methods will yield solutions which are more or less a compromise. If the use of programming results in the saving of one-half cent per bushel, a sizable sum would be saved by the industry over a year's time.

In many real-life problems the factors that must be considered are sometimes numerous and difficult to identify. It is as important to determine the truly pertinent factors as it is to construct the correct mathematical model for dealing with them. To evaluate the various features of a problem and determine which factors should be included in the model requires a thorough understanding of the operations of the industry.

The use of linear programming as an economic tool is being adapted for use by many different phases of business. Programming is being applied to many types of problems within each business. The future looks bright for further development and for wider application of this technique.

Of course, linear programming is not a cure-all. This technique has its limitations as does any other mathematical procedure. The calculations which have to be performed are quite often long and tedious, even though they are straight-forward. For problems containing many unknowns, high speed digital computers should be employed. Computers are available which are capable of handling as many as 100 variables and 50 equations.[1]

One of the toughest problems facing the mathematical analyst is how to put unmeasurable factors into mathematical terms. For instance, how does one rate consumer satisfaction in mathematical terms in order that it can be put into an equation? The mathematician must know what information is needed and what factors may be discarded in order to keep problems from becoming too complex.

The major purpose of this study was to show the applicability of linear programming to blending problems in the milling industry. It was demonstrated that programming may be applied to such problems. Calculation of a program matrix can easily be made if adequate computational facilities are available. Therefore,

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[1] A. Acrivos, "Linear Programming, How Does It Work?" Chemical Engineering, August 1956, p. 216.

little would be gained by pushing the present calculations further for the blending problems that were presented. The important step now is to extend the techniques of programming to the area of operations.

ACKNOWLEDGMENTS

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REFERENCES

Acrivos, A. "Linear Programming, How Does It Work? Chemical Engineering, August 1956, 63:215-216.

Charnes, A., W. W. Cooper, and A. Henderson. An Introduction to Linear Programming. New York: John Wiley & Sons, 1953.

Charnes, A., W. W. Cooper, and B. Mellon. "Blending Aviation Gasolines--A Study in Programming Interdependent Activities in an Integrated Oil Company." Econometrica, April 1952, 20:135-159.

Dantzig, G. B. Activity Analysis of Production and Allocation. T. C. Koopmans, ed. New York: John Wiley & Sons, 1951, Chaps. XXI and XXIII.

__________. "Maximization of a Linear Form Where Variables Are Subject to a System of Linear Inequalities." Headquarters, USAAF, November 1, 1919.

Dorfman, Robert. Application of Linear Programming to the Theory of the Firm. Berkeley: University of California Press, 1951.

Heady, Earl O. Economics of Agricultural Production and Resource Use. New York: Prentice-Hall, Inc., 1952.

Lockwood, J. F. (assisted by Anthony Simon). Flour Milling. Liverpool, New York: Northern Co., 1949.

Miller, Edgar S. Observations on Milling. Chicago: National Miller, 1923.

__________. Studies in Practical Milling. Minneapolis: Miller Co., 1941.

Scott, James Hay. Flour Milling Processes. London: Chapman & Hall, 1951.

Smith, Leslie. Flour Milling Technology. Liverpool, England: Northern Co., 1945.

Swanson, Charles Oscar. Wheat and Flour Quality. Minneapolis: Burgess Co., 1941.

U. S. Department of Agriculture. Handbook of Official Grain Standards of the United States. Washington: Government Printing Office, 1955.

APPENDICES

Appendix A

KANSAS STATE COLLEGE

Manhattan, Kansas

Agricultural Experiment Station
Department of Agricultural Economics

WHEAT BLENDING FOR MILL-MIX

Please check either Yes or No for the following questions. Further explanation of your answers would be appreciated. Room has been left for your comments at the last of each section.

I. Is each bin of wheat that's available for a mill-mix tested for these factors (before a blend is made)?

	YES	NO
A. Protein content	______	______
B. Gluten quality	______	______
1. Strength of grist	______	______
a. Mixing time	______	______
b. Mixing tolerance of flour	______	______
2. Gas production	______	______
3. Gas retention	______	______
4. Water absorption by flour	______	______
C. Moisture content	______	______
D. Test weight	______	______
E. Ash content	______	______

	YES	NO
F. Impurities		
1. Mustiness	______	______
2. Immaturity	______	______
3. Heat damage	______	______
4. Foreign material	______	______
5. If the percentage of these impurity factors are known for each bin, are limitations put on the percent of wheat from that bin that can be used in mix?	______	______
G. Degree of infestation	______	______
H. Flour-yielding capacity	______	______
I. By-products remaining after extraction of flour (from that particular wheat)	______	______

J. Other information: ____

____

____

____

K. Remarks: ____

____

____

____

II. If a small quantity of flour is extracted for testing purposes from each wheat available for a mill-mix, are the following factors tested for from that flour?

	YES	NO
A. Color	______	______
B. Granularity	______	______
C. Mixing time	______	______

	YES	NO
D. Mixing tolerance	______	______
E. Ash content of flour	______	______
F. Flour absorption	______	______
G. Loaf volume	______	______
H. Loaf texture	______	______

I. Other information: ______________________________

J. Remarks: ______________________________

III. Is a baking test run on flour from each bin of wheat to determine its baking characteristics? Yes______ No______

Remarks: ______________________________

IV. Is a baking test only run on flour from wheats that have already been blended as a possible mill-mix that will yield flour with the desired characteristics? Yes______ No______

Remarks: ______________________________

V. As a matter of policy is there a minimum in the number of bins that can be utilized in a mill-mix in order to attain minimum quality variation within the mix and between mixes from day to day? Yes______ No______

Remarks: ____

____

____

VI. Is the price of the different bins of wheat considered as a factor when blending wheat for a mill-mix? Yes_____ No_____

If your answer is yes, do you consider price when purchased or replacement price?______________________

Remarks: ____

____

____

VII. If there are other tests made that have not been listed that are essential to the blending of a mill-mix please list below.

A.

B.

C.

Remarks: ____

____

____

____

Appendix B: Identification of Symbols

The symbol, λ, is read as "Llamba."

The notation λ_1 represents the amount (to be determined) of bin A going into mix; λ_2 represents the amount (to be determined) of bin B going into mix; and so on.

The sign = means "is (or are) equal to."

The sign $\geq$ is read "is equal to or greater than."

The sign $\leq$ is read "is equal to or less than."

The notation, P_i, refers to any column vector in the program matrix.

The notation, P_o, refers to the requirement vector in the program matrix.

The notation, c_i, is the unit price (in dollars per bushel), appearing in equation (6). The unit price of each bin of wheat also appears in the program matrix above its corresponding vector.

APPLICATION OF LINEAR PROGRAMMING TO MILLING PROBLEMS WHICH INVOLVE BLENDING OF WHEAT

by

JOSEPH ELDON UNGER

B. S., Kansas State College
of Agriculture and Applied Science, 1953

AN ABSTRACT OF A THESIS

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MASTER OF SCIENCE

Department of Economics and Sociology

KANSAS STATE COLLEGE
OF AGRICULTURE AND APPLIED SCIENCE

1957

Within recent years remarkable progress has been made in the development of the technique known as linear programming. A powerful and flexible set of tools for theoretical and empirical research is provided through the methods of programming. The scope of economics can be extended by the use of these tools which may be adapted to the solution of a wide variety of practical problems.

The primary purpose of this study was to investigate applications of linear programming in the blending of wheat for the milling industry. The technique was used in the blending of wheat for a mill-mix and in the blending of wheat to offer for sale in the cash market. A secondary objective of this study was to determine whether linear programming is a better tool than methods currently used in the blending of wheat for a minimum cost mill-mix.

This study was conducted in three stages. First, a realistic mill-mix problem was formulated from data obtained from millers and grain merchandisers. Second, the mill-mix problem stated in Edgar S. Miller's Studies in Practical Milling was solved by linear programming. Third, a grain merchandising problem was set up to show that linear programming could be used in solving such problems.

In order to show that linear programming could be used in solving mill-mix problems, a realistic problem had to be formulated since there were no problems of this nature available. This was accomplished from correspondence with millers and grain merchandisers.

The variables included in the realistic mill-mix problem were ones which it is believed the milling industry as a whole would consider when blending a mix. These variables are: (1) protein content, (2) moisture content, (3) test weight, (4) dough development time, (5) mixing tolerance, (6) water absorption by flour, (7) ash content of flour, (8) loaf volume, (9) loaf texture, (10) color, (11) yield of straight flour, (12) infestation, (13) variety, and (14) price. These variables were the ones which were most generally agreed upon in questionnaires sent to millers and grain merchandisers and in interviews with Professor Eugene Farrell and Professor Max Milner of the Milling Department at Kansas State College.

A mathematical statement of this problem was formulated in this study. Each of the variables had stipulated minimal or maximal specifications that had to be met by the mill-mix. These specifications, and other known information, were used to form inequations which were reduced to equations by the introduction of slack variables. The problem was to minimize the cost per bushel subject to these restrictions. Since suitable computational equipment was not available, the solution to this problem was not computed. The number of equations and unknowns could have been reduced. This would have permitted the use of present equipment; but, since a reduction in the number of variables results in a non-realistic problem, it was felt that further computation was unnecessary.

A problem proposed in Edgar S. Miller's Studies in Practical Milling offered an opportunity to check a solution obtained by another method with that of linear programming. The cost of Miller's mill-mix was 102.99 cents per bushel while the linear programming mix was calculated to be 102.44 cents per bushel. Thus, the programming mix was .55 cents a bushel less than Miller's blend. For this particular mill-mix problem, linear programming was more capable of blending a mix at less cost than was the method used by Miller. In the milling industry where a saving of a quarter or half cent per bushel can mean a profit or loss, linear programming may be a valuable tool for solving such problems.

Linear programming was also applied to a grain merchandising problem in this study. This example was to demonstrate another possible application of programming. A problem was formulated to show that programming could be used in mixing bins of wheat which are available for sale. Blending may result in a greater profit than selling each bin separately. From this example it was shown that linear programming can be used to solve such blending problems in the milling industry.

No categorical statement can be made that linear programming is a better tool than present methods used in the blending of wheat for a mill-mix at minimum cost since milling firms will not say definitely how they determine their mixes. Also, no realistic problems for which millers had solutions were solved by programming since such problems were not available. Of course, it is shown by programming that the solution so obtained cannot be exceeded by any other solution regardless of the technique used. Due to the complexity of such blending problems, it is reasonably safe to conclude that linear programming will find the optimal solution while other methods will yield solutions which are more or less a compromise. If the use of programming results in the saving of a quarter or half cent per bushel, a sizable sum would be saved by an industry over a year's time.

The major purpose of this study was to show the applicability of linear programming to blending problems in the milling industry. It was demonstrated that programming may be applied to such problems. Calculation of a program matrix can easily be made if adequate computational facilities are available.