GAS FLOWS IN TUBES WITH CONSTANT HEAT FLUX AND WITH CONSTANT RATIO OF WALL AND STAGNATION TEMPERATURE

by

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NOMENCLATURE

a	dimensionless parameter, defined by eq. (28)
	or dimensionless constant, $a = (k-1)/2$
b	dimensionless parameter, defined by eq. (27)
	or dimensionless parameter, $b = (t_w + 1)k/(t_w - 1)$
A	tube cross-sectional area, ft^2
A_w	tube wetted area, ft^2
C_p	constant-pressure specific heat, BTU/slug-R
C_v	constant-volume specific heat, BTU/slug-R
D	equivalent hydraulic diameter, $D = 4Ax/A_w$, ft
e_e	expansion efficiency, defined by eq. (A-2)
f	local friction coefficient, defined by eq. (18), dimensionless
$\overline{f}$	mean value of friction coefficient
G	mass velocity, slug/sec-ft^2
h	coefficient of convective heat transfer, BTU/sec-ft^2-R
J	mechanical equivalent of heat, 778 ft-lb_f/BTU
k	ratio of specific heats, $k = C_p / C_v$
L	length of flow passage, ft
L_{max}	maximum length for continuous flow, ft
m	dimensionless term, defined as $m = M^2$
M	Mach number
n	thermodynamic variable for apolytropic process, $dp/p + ndv/v=0$
p	static pressure, lb_f/ft^2 abs.
p_o	stagnation pressure, lb_f/ft^2 abs.

Q	heat flow per unit mass, BTU/slug
R	gas constant, ft-lb_f/slug-R
r	recovery factor, defined as $r = (T_{aw} - T) / (T_o - T)$
s	specific entropy, BTU/slug-R
T	static temperature, R
T_{aw}	adiabatic wall temperature, R
T_o	stagnation temperature, R
T_w	temperature of tube wall, R
$\overline{T}_o$	dimensionless stagnation temperature, defined as $\overline{T}_o = T_o/T_o^*$
T_o^*	stagnation temperature at sonic section, R
U	dimensionless velocity, defined as $U^2 = V^2 / aRT_o^*$
u	specific internal energy, BTU/slug
V	axial velocity, ft/sec
v	specific volume, ft^3/slug
W	work per unit mass, ft-lb_f/slug
w	rate of mass flow, slug/sec
x	tube length, ft
$\overline{x}$	dimensionless length, defined as $\overline{x} = 4\overline{f}x/D$
X_{max}	maximum length for continuous flow, ft
$\overline{X}_{max}$	dimensionless length, defined as $\overline{X}_{max} = 4\overline{f}X_{max}/D$

Subscripts

$(\ )_1$	signifies property at initial section
$(\ )_2$	signifies property at a section other than the initial

Superscript

$(\ )^*$ signifies properties at sonic state

Greet Letters

ρ density, slug/ft^3

ϕ defined by eq. (A-3)

τ_w wall shearing stress, lb_f/sq.ft.

INTRODUCTION

Gas flow in tubes is a complicated problem. This subject has been analyzed by many investigators. Shapiro and Hawthorne (2) first presented a generalized equation of one-dimensional flow of a compressible fluid within a tube including the effects of:

i change in tube area,

ii wall friction,

iii drag of internal bodies,

iv external heat exchange,

v chemical reaction,

vi change in phase of the fluid,

vii change in molecular weight and specific heat.

Unfortunately, no exact solution has been found for the general case.

Noyes (3) presented an exact solution of compressible flow in a constant-area tube with the combined effects of friction and heat transfer for the case of constant heat flux.

Analysis of gas flow in tubes for various conditions has been undertaken in the Department of Mechanical Engineering, Kansas State University, Manhattan, Kansas under the direction of Professor Wilson Tripp. Chen (6) presented three cases of subsonic heating for gas flowing in a constant-area duct; constant heat flux, constant wall temperature and exponential longitudinal fluid temperature distribution. Chang (7) presented polytropic gas flow in a constant-area duct under the simultaneous effects of friction and heat transfer.

This thesis presents some aspects of this general analysis. It contains two exact solutions for the cases of a perfect gas flowing in a constant-area duct with the effects of heat transfer and friction. One is for constant heat flux per unit length along the duct. The other is for constant ratio of wall temperature and stagnation temperature along the duct based on the assumptions of validity of Reynolds analogy and the recovery factor being unity. The case of constant heat flux only includes heating for both subsonic and supersonic processes. It is also shown in the text and graphs that the Fanno-line process and Rayleigh-line process are two particular cases in each of the aforesaid two cases.

Besides the investigation of the usual properties of fluid along the passages for various flow conditions, the apolytropic variable "n" (defined by $dp/p + ndv/v = 0$) and the entropy changes are also surveyed. The development of the apolytropic variable "n" is presented in the Appendix A which is a part of the analysis under the direction of Dr. Wilson Tripp.

Two numerical examples are presented which show that the changes of properties along the passages are readily found from the graphs. In addition, the solution of the examples demonstrates the technique of using the equations in the thesis to solve practical problems.

METHOD OF ANALYSIS

The thermodynamic characteristics and the properties of the perfect gas are considered for flow in tubes under the simultaneous effects of fluid-flow friction and heat transfer. The differential equations which describe the general case of gas flow in tubes can not be integrated. For some special cases these differential equations can be integrated. For the two special cases investigated in this thesis (constant heat flux and constant ratio of wall and stagnation temperature) the equations are integrable.

The following assumptions are made:

- i The flow is steady and one-dimensional, i.e. all properties are uniform over each section.
- ii Changes of stream properties are continuous.
- iii The fluid is a perfect gas, i.e. the perfect gas equations are applicable, specific heats are constant.
- iv Body forces are negligible.
- v The flow passage is a constant-area tube.
- vi Heat transfer is instantaneous and complete in the radial direction with no heat transfer along the tube axis.

FUNDAMENTAL EQUATIONS

The fundamental principles used are expressed in the following list:

- i equation of state for a perfect gas,
- ii law of conservation of mass,
- iii law of conservation of energy,

iv Newton's second law of motion.

ANALYSIS

The state equation for a perfect gas is

$$p = \rho RT \qquad \text{or} \qquad pv = RT \tag{1}$$

and its logarithmic differential form is

$$dp/p = d\rho/\rho + dT/T$$

or

$$dp/p + dv/v = dT/T \ . \tag{2}$$

The equation of conservation of mass is

$$G = w/A = \rho V = \text{constant} \tag{3}$$

or in logarithmic differential form is

$$d\rho/\rho + dV/V = 0$$

or

$$-dv/v + dV/V = 0 \ . \tag{4}$$

The Mach number is

$$M^2 = V^2 \ / \ kRT \tag{5}$$

or in logarithmic differential form is

$$dM^2 \ / \ M^2 = dV^2 \ / \ V^2 - dT/T \ . \tag{6}$$

The energy equation for an adiabatic process is

$$V^2 + 2C_pT = \text{constant} \ . \tag{7}$$

The definition of stagnation temperature is

$$T_o = V^2 \ / \ 2C_p + T \ . \tag{8}$$

Combining equations (5) and (8), one obtains

$$T_o = T \left(1 + \frac{k-1}{2} M^2 \right) \tag{9}$$

and its logarithmic differential form is

$$\frac{dT_o}{T_o} = \frac{dT}{T} + \frac{\frac{k-1}{2}\, dM^2}{1 + \frac{k-1}{2} M^2} \quad . \tag{10}$$

The thermodynamic variable, n, for the apolytropic process is given by

$$\frac{dp}{p} + n\,\frac{dv}{v} = 0 \qquad \text{or} \qquad \frac{dp}{p} - n\,\frac{d\rho}{\rho} = 0 \quad . \tag{11}$$

Combining (11) and (2), one obtains

$$-\frac{n}{n-1}\,\frac{dT}{T} + \frac{dp}{p} = 0 \quad . \tag{12}$$

Eliminating the terms of V and from equations (1), (3) and (5), one obtains

$$\frac{dp}{p} = \frac{1}{2}\left(\frac{dT}{T} - \frac{dM^2}{M^2}\right) \quad . \tag{13}$$

Combining equations (12) and (13), we obtain

$$\frac{dT}{T} = -\frac{n-1}{n+1}\,\frac{dM^2}{M^2} \quad . \tag{14}$$

The control surface for the flow with friction and heat transfer is shown in Figure I.

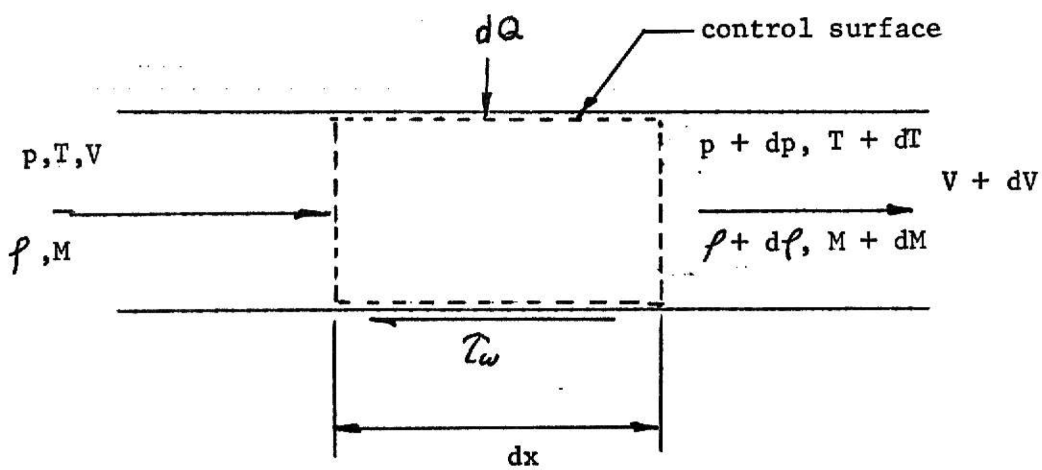

Fig. I Control surface for analysis of diabatic, frictional and constant-area flow.

The energy equation is

$$dQ = C_p\,dT + d\left(\frac{V^2}{2}\right) = C_p\,dT_o \quad . \tag{15}$$

Substituting $C_p = \frac{k}{k-1} R$ and equation (5), one obtains

$$\frac{dT_o}{T} = \frac{dT}{T} + \frac{k-1}{2} M^2 \frac{dV^2}{V^2} \quad . \tag{16}$$

The momentum equation is

$$\rho AVdV + Adp + \tau_w dA_w = 0 \quad . \tag{17}$$

The definition of the friction coefficient is

$$f = \frac{\tau_w}{\frac{1}{2}\rho V^2} \quad . \tag{18}$$

The definition of hydraulic diameter for a circular pipe is

$$D \equiv \frac{4A}{dA_w/dx} \quad . \tag{19}$$

Combining equations (16), (17), (18) and the relation of $\rho V^2 = kpM^2$, one obtains

$$\frac{dp}{p} + \frac{kM^2}{2} 4f \frac{dx}{D} + \frac{kM^2}{2} \frac{dV^2}{V^2} = 0 \quad . \tag{20}$$

Combining equations (2), (4), (6), (9), (16) and (20), we obtain

$$\frac{dM^2}{M^2} = \frac{(1+kM^2)(1+\frac{k-1}{2}M^2)}{1-M^2} \frac{dT_o}{T_o} + \frac{kM^2(1+\frac{k-1}{2}M^2)}{1-M^2} 4f \frac{dx}{D} \quad . \tag{21}$$

Equation (21) will be applied in the following two cases.

CASE I CONSTANT HEAT FLUX

Constant heat flux means that the rate of heat transfer per unit length of the tube is constant, i.e. the change of stagnation temperature along the flow passage has a linear relation with its length. This case can be realized by wrapping resistance wire uniformly around the tube.

Let

$$\overline{T}_o = T_o/T_o^* = 1 - a\overline{X}_{max} \tag{22}$$

and

$$d\overline{x} = - d\overline{X}_{max} = 4fdx/D \quad . \tag{23}$$

Let

$$a = - \frac{d\overline{T}_o}{d\overline{X}_{max}} = \frac{d\overline{T}_o}{d\overline{x}} = - \frac{1}{T_o^*} \frac{dT_o}{d\overline{x}} \tag{24}$$

and

$$b = \frac{2k}{a(k + 1)} \quad . \tag{25}$$

Substituting equation (2) into equation (16), one obtains

$$dV^2 + 2R(dT + Td\rho/\rho) - V^2 d\overline{X}_{max} = 0 \quad . \tag{26}$$

In a manner similar to the development used by Noyes (3), let

$$U^2 = bV^2/kRT^* \tag{27}$$

(when $M = 1$, $V^2 = kRT^*$ i.e. $U^{*2} = b$), and let

$$U^2 = \frac{V^2}{aRT_o^*} \quad . \tag{28}$$

Combining equation (9) with equation (27), one obtains

$$T_o = T + (k - 1)U^2T^*/2b. \tag{29}$$

Introducing $T_o^* = (k + 1)T^*/2$ into equation (29), one obtains

$$T/T^* = (T_o - \frac{k - 1}{k + 1}\frac{U^2}{b})\frac{k + 1}{2} \tag{30}$$

or in logarithmic differential form

$$dT/T^* = \frac{k + 1}{2}d\overline{T}_o - \frac{k - 1}{2b}dU^2 . \tag{31}$$

Combining equations (26), (27) and (31), and simplifying, one obtains

$$\frac{d\overline{X}_{max}}{dU^2} - \frac{\overline{X}_{max}}{U^2(U^2 + 2)} = \frac{U^2 - b}{abU^2(U^2 + 2)} . \tag{32}$$

Equation (32) is a typical linear differential equation of first order of the form

$$dy/dx + py = Q \tag{33}$$

where p and Q are functions of x only.

The integration factor is $e^{\int pdx}$.

The solution of equation (33) is

$$ye^{\int pdx} = \int Qe^{\int pdx} dx + c .$$

Following the above procedure, equation (32) in integrated to (detail of integration shown in Appendix B)

$$1 - a\overline{X}_{max} = \frac{\sqrt{1 + 2/b}}{\sqrt{1 + 2/U^2}} + \frac{1}{b\sqrt{1 + 2/U^2}} \ln \frac{b(1 + 1/b + \sqrt{1 + 2/b})}{U^2(1 + 1/U^2 + \sqrt{1 + 2/U^2})} . \tag{34}$$

Combining equations (22) and (34), one obtains

$$\overline{T}_o = \sqrt{\frac{1 + 2/b}{1 + 2/U^2}} + \frac{1/b}{\sqrt{1 + 2/U^2}} \ln \frac{b(1 + 1/b + \sqrt{1 + 2/b})}{U^2(1 + 1/U^2 + \sqrt{1 + 2/U^2})} . \tag{35}$$

The integration limits are from the sonic section to any arbitrary section, i.e. $U^2 = U^{*2} = b$ to $U^2 = U^2$ and $\overline{X}_{max} = 0$ to $\overline{X}_{max} = \overline{X}_{max}$.

Combining equations (27) and (9), we obtain

$$M^2 = \frac{U^2}{\frac{k+1}{2} b\overline{T}_o - \frac{k-1}{2} U^2} . \tag{36}$$

Substituting equations (22), (23) and (24) into equation (21), we obtain

$$\frac{dM^2}{M^2} = \left(\frac{(1 + kM^2)\left(1 + \frac{k-1}{2} M^2\right)}{1 - M^2} + \frac{kM^2\left(1 + \frac{k-1}{2} M^2\right.}{1 - M^2} \frac{\overline{T}_o}{a} \right) \frac{d\overline{T}_o}{\overline{T}_o} . \tag{37}$$

Substituting equation (10) into equation (37), one obtains

$$\frac{2}{n + 1} = \frac{a(1 - M^2) + a\,(1 + kM^2) + k\overline{T}_o M^2}{\left(1 + \frac{k-1}{2} M^2\right)[a(1 + kM^2) + k\overline{T}_o M^2]} . \tag{38}$$

For $a = 0$

then $\overline{T}_o = T_o/T_o^* = 1$,

which is the case for the Fanno line; equation (38) will reduce to

$$n = 1 + (k - 1)M^2 . \tag{39}$$

For

$$a \longrightarrow \infty$$

then

$$\overline{T}_o = T_o/T_o^* \longrightarrow 0 ,$$

which is the case for the Rayleigh line; equation (38) will reduce to

$$n = kM^2 \ . \tag{40}$$

CASE II CONSTANT RATIO OF WALL TEMPERATURE AND STAGNATION TEMPERATURE

In this case the ratio of local wall temperature of the tube and the local stagnation temperature inside the tube is a constant everywhere along the flow passage. Heat exchangers, such as an air preheater in the furnace of a steam power plant, arranged so that the air and stack gases are in counter flow are examples of this case.

Referring to Figure I, the energy equation can be written as

$$\begin{aligned} w dQ &= \frac{\pi}{4} D^2 \rho V C_p dT_o \\ &= \pi h D dx (T_w - T_{aw}) \ . \end{aligned} \tag{41}$$

The definition of the recovery factor is

$$r = (T_{aw} - T)/(T_o - T) \ .$$

Assume that $r = 1$

then $T_{aw} = T_o$.

Introducing the relation $T_{aw} = T_o$ into equation (41), one obtains

$$\frac{dT_o}{T_w - T_o} = \frac{4h}{\rho V C_p} \frac{dx}{D} \ . \tag{42}$$

From Reynolds analogy (p. 243 of Reference (1)), one obtains

$$h/\rho V C_p = f/2 \ . \tag{43}$$

Substituting equation (43) into equation (42), one obtains

$$\frac{dT_o}{T_w - T_o} = 2fdx/D \tag{44}$$

Defining

$$t_w = T_w/T_o \tag{45}$$

equation (44) can be rewritten as

$$4fdx/D = \frac{2}{t_w - 1} \frac{dT_o}{T_o} \tag{46}$$

Substituting equation (46) into equation (21), one obtains

$$\frac{1 - M^2}{1 + (k - 1)M^2/2} \frac{dM^2}{M^2} = [1 + kM^2(1 + \frac{2}{t_w - 1})] \frac{dT_o}{T_o} \quad . \tag{47}$$

Defining

$$m = M^2 \tag{48}$$

$$a = (k - 1)/2 \tag{49}$$

$$b = (t_w + 1)k/(t_w - 1) \tag{50}$$

and substituting equations (48), (49) and (50) into equation (47), we obtain

$$\frac{1 - m}{1 + am} \frac{dm}{m} = (1 + bm)\ dT_o/T_o \quad .$$

Integrating, one obtains

$$\int_m^1 \frac{(1 - m)dm}{m(1 + am)(1 + bm)} = \int_{T_o}^{T_o^*} \frac{dT_o}{T_o}$$

where the integration limits are from an arbitrary section to the sonic section.

By the above integration, the following equation is obtained

$$\frac{T_o}{T_o^*} = \frac{m\left(\frac{1+a}{1+am}\right)^{\frac{1+a}{a-b}}}{\left(\frac{1+b}{1+bm}\right)^{\frac{1+b}{a-b}}} \quad . \tag{51}$$

Substituting equation (9) into equation (51), we obtain

$$\frac{T}{T^*} = \frac{m\left(\frac{1+a}{1+am}\right)^{\frac{1+2a-b}{a-b}}}{\left(\frac{1+b}{1+bm}\right)^{\frac{1+b}{a-b}}} \quad . \tag{52}$$

Combining equations (1), (3) and (5), the following equation is obtained:

$$p/p^* = \frac{1}{M}\left(\frac{T}{T^*}\right)^{\frac{1}{2}} \quad .$$

Substituting equation (52) into the above relation, one obtains

$$\frac{p}{p^*} = \frac{\left(\frac{1+a}{1+am}\right)^{\frac{1+2a-b}{2(a-b)}}}{\left(\frac{1+a}{1+bm}\right)^{\frac{1+b}{2(a-b)}}} \quad . \tag{53}$$

From the thermodynamic relation

$$s - s^* = C_p \ln \frac{T}{T^*} - R \ln \frac{p}{p^*} \tag{54}$$

and substituting equations (52) and (53) into equation (54), we obtain

$$\frac{s - s^*}{R} = \ln \frac{m^{\frac{2a+1}{2a}}\left(\frac{1+a}{1+am}\right)^{\frac{(1+a)(1+2a-b)}{2a(a-b)}}}{\left(\frac{b+1}{1+bm}\right)^{\frac{(1+a)(1+b)}{2a(a-b)}}} \quad . \tag{55}$$

Substituting equation (14) into equation (47), obtains

$$\frac{2n}{n - 1} = \frac{(1 - M^2) - [1 + (k - 1)M^2][1 + kM^2 + 2kM^2/(t_w - 1)]}{(1 - M^2) - [(k - 1)M^2/2][1 + kM^2 + 2kM^2/(t_w - 1)]} \quad . \tag{56}$$

For the case of $t_w = 1$

we get the Fanno-line process; equation (56) becomes

$$n = 1 + (k - 1)M^2 \quad . \tag{57}$$

For the case of $t_w \longrightarrow \infty$

we have the Rayleigh-line process in which the effect of heat transfer is large and the effect of friction can be neglected.

Equation (56) reduces to

$$n = kM^2 \; . \tag{58}$$

Equations (57) and (58) are the same as equations (39) and (40), respectively.

NUMERICAL EXAMPLES

EXAMPLE I CONSTANT HEAT FLUX

Air is flowing steadily in a constant-area smooth pipe, diabatically and frictionally, under the effect of constant heat flux. The inside diameter of the pipe is one inch and the friction factor $4\overline{f} = 0.015$. The specific heat ratio $k = 1.4$. At one section of the pipe air is at $M_1 = 0.2$, $T_{o1} = 500$ R and $p_1 = 20$ psia. Ten feet from this section $T_{o2} = 700$ R. The problem is to determine all the physical properties at the choking section ($M = 1$) and at the section where $M = 0.5$.

Solution:

From equations (27) and (28), we obtain

$$b = \frac{2k}{a(k - 1)}$$

and

$$a = \frac{1}{T_o^*} \frac{dT_o}{dx}$$

Combining the above two equations we obtain

$$b = \frac{2k}{k + 1} \frac{T_o^*}{T_{o1}} \frac{T_{o1}}{dT_o/\overline{dx}} = \frac{2k}{k + 1} \frac{T_{o1}}{dT_o/\overline{dx}} \frac{1}{\overline{T}_{o1}}$$

Define

$$c = \frac{2k}{k + 1} \frac{T_{o1}}{dT_o/\overline{dx}}$$

then $\quad b = c/ T_{o1}$

Equation (35) can be written as

$$\bar{T}_{o1} = \frac{\sqrt{1+\frac{2}{b}}}{\sqrt{1+\frac{2}{U_1^2}}} - \frac{1}{b}\frac{1}{\sqrt{1+\frac{2}{U_1^2}}}\ln\frac{b(1+\frac{1}{b}+\sqrt{1+\frac{2}{b}})}{U_1(1+\frac{1}{U_1^2}+\sqrt{1+\frac{2}{U_1^2}})} .$$

Substituting $b = c/T_{o1}$ into the above, one obtains

$$c\sqrt{1+\frac{2}{U_1}} + \ln U_1\,(1+\frac{1}{U_1}+\sqrt{1+\frac{2}{U_1}})$$

$$= b\sqrt{1+\frac{2}{b}} + \ln b\,(1+\frac{1}{b}+\sqrt{1+\frac{2}{b}}) . \qquad \text{(ii)}$$

$$\Delta\bar{x} = 4\bar{f}\,\frac{x}{D} = 0.015\,\frac{10}{1/12} = 1.8$$

$$\frac{dT_o}{d\bar{x}} = \frac{T_o}{\Delta\bar{x}} = \frac{700-500}{1.8} = 111.111$$

$$a = \frac{1}{T_o^*}\frac{dT_o}{d\bar{x}}$$

$$aT_o^* = 111.111$$

$$T_1 = T_{o1}/(1 - \frac{k-1}{2}M_1^2) = 496 \text{ R}$$

$$V_1 = M_1\sqrt{kRT_1} = 218.45 \text{ ft/sec}$$

$$U_1^2 = \frac{V_1^2}{aRT_o^*} = \frac{M_1^2kRT_1}{aRT_o^*} = \frac{M_1^2kT_1}{111.11R} = 0.25$$

$$c = \frac{2k}{k+1}\,\frac{T_{o1}}{dT_o/d\bar{x}} = 5.25$$

Substituting the known values U_1 and c into equation (ii), one obtains

$$b\sqrt{1+\frac{2}{b}} + \ln b(1+\frac{1}{b}+\sqrt{1+\frac{2}{b}}) = 16.44 .$$

Solving the above equation by trial and error we obtain

$$b = 12.2$$

and

$$a = \frac{2k}{b(k-1)} = 0.09556 \quad .$$

With the heat parameter "a" obtained, the other values may be readily found from Figures I-V of Appendix C

At $M_1 = 0.2$

$$\frac{T_{o1}}{T_o^*} = 0.43 \qquad \therefore T_o^* = 1162.7 \text{ R}$$

$$\frac{T_1}{T^*} = 0.512 \qquad \therefore T^* = 968.9 \text{ R}$$

$$\frac{p_1}{p^*} = 3.5775 \qquad \therefore p^* = 5.59 \text{ psia}$$

$$\frac{s_1 - s^*}{R} = -3.618$$

$$4fL_{max1}/D = 5.964433 \qquad \therefore L_{max1} = 33.13'$$

$$n_1 = 0.1627 \qquad n^* = 1.4$$

From Figures I-V of Appendix C

At $M_2 = 0.5$

$$\frac{T_{o2}}{T_o^*} = 0.921 \qquad \therefore T_{o2} = 1070.85 \text{ R}$$

$$\frac{T_2}{T_1^*} = 1.0528 \qquad \therefore T_2 = 1020 \text{ R}$$

$$\frac{p_2}{p^*} = 2.052 \qquad \therefore p_2 = 11.47 \text{ psia}$$

$(s_2 - s^*)/R = - 0.538$

$4\overline{f}\, L_{max2}/D = 0.824$ $\qquad L_{max_2} = 4.58'$

$n_2 = 0.812$

$(s_2 - s_1)/R = 3.08$

$x_2 - x_1 = L_{max_1} - L_{max_2} = 5.14D/4\overline{f} = 5.14/(12 \times 0.015) = 28.55'$

EXAMPLE II CONSTANT RATIO OF WALL-STAGNATION TEMPERATURE

Air is flowing steadily in a constant-area tube with friction and heat transfer for the case of constant ratio of wall temperature and stagnation temperature. The inside diameter of the pipe is one inch. At one section of the pipe air is at $M_1 = 0.2$, $T_{o1} = 500$ R, $T_{w1} = 750$ R and $p_1 = 20$ psia. The problem is to determine all the physical properties at $M_2 = 0.5$, and $M = 1$ where the flow is choked.

Solution:

Assume that Reynolds analogy is valid and the recovery factor is unity. The ratio of wall temperature and stagnation temperature is

$t_w = T_{w1}/T_{o1} = 750/500 = 1.5$

At

$M_1 = 0.2$

$$T_1 = T_{o1} \,/\, (1 + \frac{k-1}{2} M_1^2) = 496 \text{ R}$$

From the working charts (Figures VI to XIV) the following data are obtained:

$$\frac{T_{o1}}{T_o^*} = 0.335 \qquad \therefore\ T_o^* = 1492.5\ R$$

$$\frac{T_1}{T^*} = 0.3988 \qquad \therefore\ T^* = 1243.73\ R$$

$$\frac{p_1}{p^*} = 3.1575 \qquad \therefore\ p^* = 6.33\ \text{psia}$$

$$n_1 = 0.152 \qquad n^* = 1.4$$

$$\frac{s_1 - s^*}{R} = -\ 4.3674$$

$$4f\,\frac{L_{max1}}{D} = 4.375 \qquad \therefore\ L_{max1} = 24.30'$$

At $M_2 = 0.5$

$$\frac{T_{o2}}{T_o^*} = 0.8576 \qquad \therefore\ T_{o2} = 1280\ R$$

$$\frac{T_2}{T^*} = 0.98 \qquad \therefore\ T_2 = 1219\ R$$

$$\frac{p_2}{p^*} = 1.9801 \qquad \therefore\ p_2 = 12.534\ \text{psia}$$

$$\frac{s_2 - s^*}{R} = -0.7533 \qquad \therefore\ \frac{s_2 - s_1}{R} = 3.6141$$

$$\therefore\ L_{1-2} = L_{max1} - L_{max2} = 20.89'$$

$$n_2 = 0.65$$

DISCUSSION

CASE I CONSTANT HEAT FLUX

In this case the sonic section is used as a reference. Figures I to V show a family of curves for various values of the heat parameter "a". If a problem is given, such as example I in this thesis, the friction coefficient and the rate of heat transfer are known, the heat parameter "a" will be fixed, and all of the properties can be found from the plotted curves.

Noyes (3) presented an exact solution for constant heat flux in 1961. He selected an arbitrary numerical value of the initial Mach Number, M_1, as a reference and from this selection calculated the properties at other sections. He defined $\overline{T}_o = T_o/T_{o1} = 1 + m\overline{x}$, where "m" is Noyes' heat-flux parameter and T_{o1} is the value of T_o at the arbitrarily selected initial Mach Number, M_1. By this method he constructed a chart of local Mach Number, M, versus the friction-distance parameter, $4\overline{f}x/D$, in which there were two families of curves for the heat flux parameter, m; one family of curves for $k = 1.4$ and the other for $k = 1.3$. If M_1 is chosen to have a different numerical value, the numerical value of the heat parameter, m, will change and another chart must be constructed for this new value of M_1. The heat parameter "a" presented in this thesis is a function of friction coefficient and rate of heat transfer. In this thesis Figure V is (similar to Noyes' chart) a plot of the local Mach Number, M, as a function of the friction-distance parameter $4fL_{max}/D$, except Noyes' uses $4\overline{f}x/D$ and this thesis uses $4\overline{f}L_{max}/D$. However, a Noyes' chart is good for only one value of the initial Mach Number, M, while Figure V in this thesis applies for all values of M_1. It may be considered that the exact solution for the case of constant heat flux in this thesis is an improvement over Noyes' method.

CASE II CONSTANT RATIO OF WALL TEMPERATURE AND STAGNATION TEMPERATURE

Assumptions of unity of the recovery factor and validity of Reynolds analogy were made for this case. The unity of recovery factor gives simplification to this problem. The Reynolds analogy gives the relation between the convective heat transfer coefficient "h" and the friction factor, "f". With these assumptions equation (21) is simplified to equation (47), which is then integrated to form equation (51).

All the properties of the fluid along the flow passages are shown in Figures I to XIV. From the figures it will be seen that the heating process for constant heat flux, for both subsonic and supersonic regions, has the same characteristics as the case of constant ratio of wall temperature and stagnation temperature. In both cases the processes are the combination of Fanno and Rayleigh processes. Fanno and Rayleigh processes are two particular processes in this thesis. When the value of the heat parameter "a" is zero, the process is a Fanno-line process. As the value of the parameter increases, the effect of heat transfer will increase. When the heat transfer effect approaches infinity, the friction still exists but becomes negligible and the process may be treated solely as a heat transfer process without friction; i.e. Rayleigh-line process. When the value of t_w of case II is unity, the process will be affected entirely by friction. The process then will be a Fanno-line process. When the value of t_w is larger than unity, there is a heating process. When t_w is smaller than unity, it is a cooling process.

When t_w is infinite, the process will be a Rayleigh-line process because the friction, by comparison, will be negligible.

CONCLUSIONS

Two theoretical exact solutions were investigated for one-dimensional compressible fluid flow in a constant-area duct under the effects of friction and heat transfer. One is the constant heat flux process of heating, the other is constant ratio of wall temperature and stagnation temperature. The formulas derived for the case of constant heat flux are applicable to heating process for both subsonic and supersonic flows. The formulas derived for the case of constant ratio of wall temperature and stagnation temperature are good for both heating and cooling as well as for subsonic and supersonic flows. Two assumptions are made: the validity of Reynolds analogy and the unity of the recovery factor.

The heating process for constant heat flux has the same characteristics as the process for constant ratio of wall temperature and stagnation temperature. The Rayleigh and Fanno lines are two typical processes of constant heat flux and constant ratio of wall temperature and stagnation temperature.

Figures I and XI are T/T^* versus $(s - s^*)/R$ diagrams. It can be seen that, for both cases, the entropy increases for both subsonic and supersonic flow to the limit of sonic section when the fluid is heated. The entropy may increase or decrease when the fluid flow is cooling, for both subsonic and supersonic flow, for the case of constant ratio of wall temperature and stagnation temperature. When there is friction and no heat flow the entropy increases. When there is no friction and there is cooling the entropy decreases. If the increase of entropy owing to the existance of friction is compensated by the decrease of entropy owing to cooling (heat out), the process is called an irreversible isentropic process.

Figures IV and XIV are n versus M diagrams. It is found from the figures that at M = 0, n = 0, except for the Fanno line where M = 0, n = 1. When M = 1, n is always 1.4. As to heating regions (i.e. $t_w > 1$, $a > 0$) for the two cases, when the flow is subsonic, the values of n always increase from zero to k = 1.4 and M always increases from zero to unity (except for the Fanno-line process, where n increases from unity to k). When the flow is supersonic, the values of n always decrease to the value of k and M always decreases.

As regards the cooling process, for the case of constant ratio of wall temperature and stagnation temperature, for subsonic flow, and when M decreases, the values of n always increase from $-\infty$ to zero. For subsonic flow, and when M increases, the value of n changes from ∞ to k in the manner shown on Figure XIV. In the supersonic region, M always decreases, and the values of n change from ∞ to k in the manner shown on Figure XIV.

Figure XII represents the T_o/T_o^* versus $(s - s^*)/R$ diagram, on which constant Mach-number lines, constant friction-distance parameter, $\overline{X}_{max}$, lines and constant pressure lines were plotted. Figure XI is the T_o/T_o^* - M diagram on which constant $\overline{X}_{max}$ and constant pressure lines were plotted. These two figures show the fluid-property changes and details of the process for the combinations of friction and heat transfer that can exist for the case of constant ratio of wall temperature and stagnation temperature.

The cooling process for the case of constant heat flux has been partially investigated by the writer of this thesis. It is recommended that the above case be analyzed further by those who are interested in this subject.

ACKNOWLEDGEMENT

The writer of the thesis would like first to express his sincere thanks to Dr. Wilson Tripp. The writer owes his interest in this topic, the equations derived and the preparation of this thesis to Dr. Tripp. In addition, appreciation is also made to Dr. Tripp's careful and patient examination of this thesis.

The writer is indebted to Dr. H. S. Walker for his help in computer programming which made possible the construction of the plotted curves.

Finally, a word of appreciation must be expressed to Miss Shelley Yu-Hsing Yang for her helpful typing of the thesis.

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APPENDIX A

Derivation of the Thermodynamic Apolytropic Process

The first law of thermodynamics for quasistatic process is

$$dQ = du + dW \quad . \tag{A - 1}$$

If this process is an irreversible, diabatic process for a perfect gas, equation (A - 1) becomes

$$dQ = C_v dT + e_e pdv/J \quad . \tag{A - 2}$$

Defining

$$\phi = \frac{e_e pdv/J}{dQ} \tag{A - 3}$$

i.e.

$$dQ = \frac{e_e pdv/J}{\phi} \tag{A - 4}$$

and substituting equation (A - 4) into equation (A - 2), we obtain

$$\frac{e_e pdv/J}{\phi} = \frac{R}{k - 1} dT + \frac{e_e pdv}{J} \quad . \tag{A - 5}$$

Substituting the equation of state, RdT = pdv + vdp into equation (A - 5), the following equation is obtained

$$\frac{e_e pdv}{\phi} = \frac{pdv + vdp}{k - 1} + e_e pdv \quad . \tag{A - 6}$$

Equation (A - 6) may be written as

$$1 + e_e(k - 1)\ (1 - 1/\phi)\ pdv + vdp = 0 \quad . \tag{A - 7}$$

Let

$$n = 1 + e_e(k - 1)(1 - 1/\phi),$$

then equation (A - 7) becomes $dp/p + ndv/v = 0$. (A - 8)

If n is constant then equation (A - 8) may be integrated as

$$pv^n = \text{constant}$$

which is the polytropic process, and is shown on the graphs below for the case of a perfect gas.

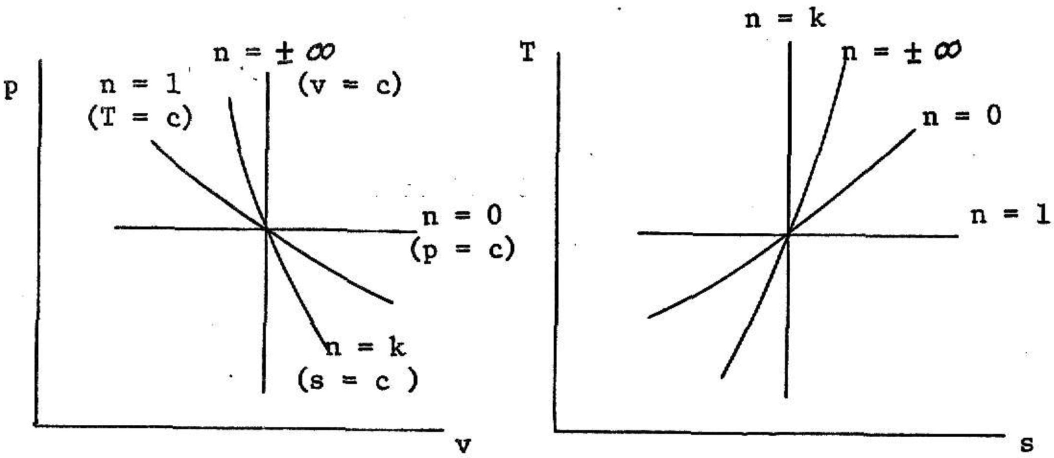


Fig. II p - v and T - s diagrams for polytropic processes.

If n is a variable then equation (A - 8) can not be integrated as shown above and the process is an apolytropic process.

APPENDIX B

Equation (32) on p. 8 is

$$\frac{d\overline{X}_{max}}{dU^2} - \frac{\overline{X}_{max}}{U^2(U^2+2)} = \frac{U^2 - b}{abU^2(U^2+2)} \quad . \qquad (B-1)$$

It is a linear differential equation and its integration factor is

$$e^{-\int \frac{dU^2}{U^2(U^2+2)}} = e^{\int [\frac{dU^2}{2(U^2+2)} - \frac{dU^2}{2U^2}]}$$

$$= e^{[\ln 1/(U^2+2) - \ln 1/U^2]} = \sqrt{1 + 2/U^2} \; .$$

Equation (B - 1) is integrated as

$$\overline{X}_{max}\sqrt{1 + 2/U^2} = \frac{(U^2 - b)\sqrt{1 + 2/U^2}\, dU^2}{abU^2(U^2+2)} + c \qquad (B-2)$$

where

$$\frac{(U^2 - b) \quad 1 + 2/U^2 \quad dU^2}{abU^2(U^2+2)}$$

$$= \frac{1}{ab}\frac{2dU^2}{(U^2+2)^{1/2}} - \frac{1}{a}\frac{2dU^2}{U^2(U^2+2)^{1/2}} \quad . \qquad (B-3)$$

From equation (200.10), p. 50 of Reference (12), we obtain

$$\frac{1}{ab}\frac{2dU^2}{(U^2+2)^{1/2}} = \frac{1}{ab} 2\ln[U + (U^2+2)^{1/2}]$$

$$= \frac{1}{ab}\ln 2U^2(1 + 1/U^2 + \sqrt{1 + 2/U^2}) \; . \qquad (B-4)$$

From equation (222.01), p. 55 of Reference (12), one obtains

$$\frac{1}{a}\int\frac{dU^2}{U^2(U^2+2)^{1/2}} = -\frac{1}{a}\sqrt{(U^2+2)/U^2}$$

$$= -\frac{1}{a}\sqrt{1+2/U^2}\ . \qquad \text{(B - 5)}$$

Substituting equations (B - 4) and (B - 5) into equation (B - 3), then substituting this combination into equation (B - 2), we get

$$\overline{X}_{max}\sqrt{1+2/U^2} = \frac{1}{ab}\ \ln 2U^2\left(1+1/U^2+\sqrt{1+2/U^2}\right) + \frac{1}{a}\sqrt{1+2/U^2} + c$$

(B - 6)

In equation (B - 6) the constant of integration, c, is obtained by letting

$U^2 = U^{*2} = b$ when $\overline{X}_{max} = 0$.

When this is done equation (B - 6) becomes

$$1 - a\overline{X}_{max} = \frac{\sqrt{1+2/b}}{\sqrt{1+2/U^2}} + \frac{1/b}{\sqrt{1+2/U^2}}\ \ln\frac{b\left(1+1/b+\sqrt{1+2/b}\right)}{U^2\left(1+1/U^2+\sqrt{1+2/U^2}\right)}\ .$$

(B - 7)

APPENDIX C

GRAPHS

All curves plotted are based on air, i.e. $k = 1.4$, $R = 53.35$ ft-lb_f/lb_m R, gravitational acceleration = 32.174 ft/sec^2.

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

THIS IS AS RECEIVED FROM CUSTOMER.

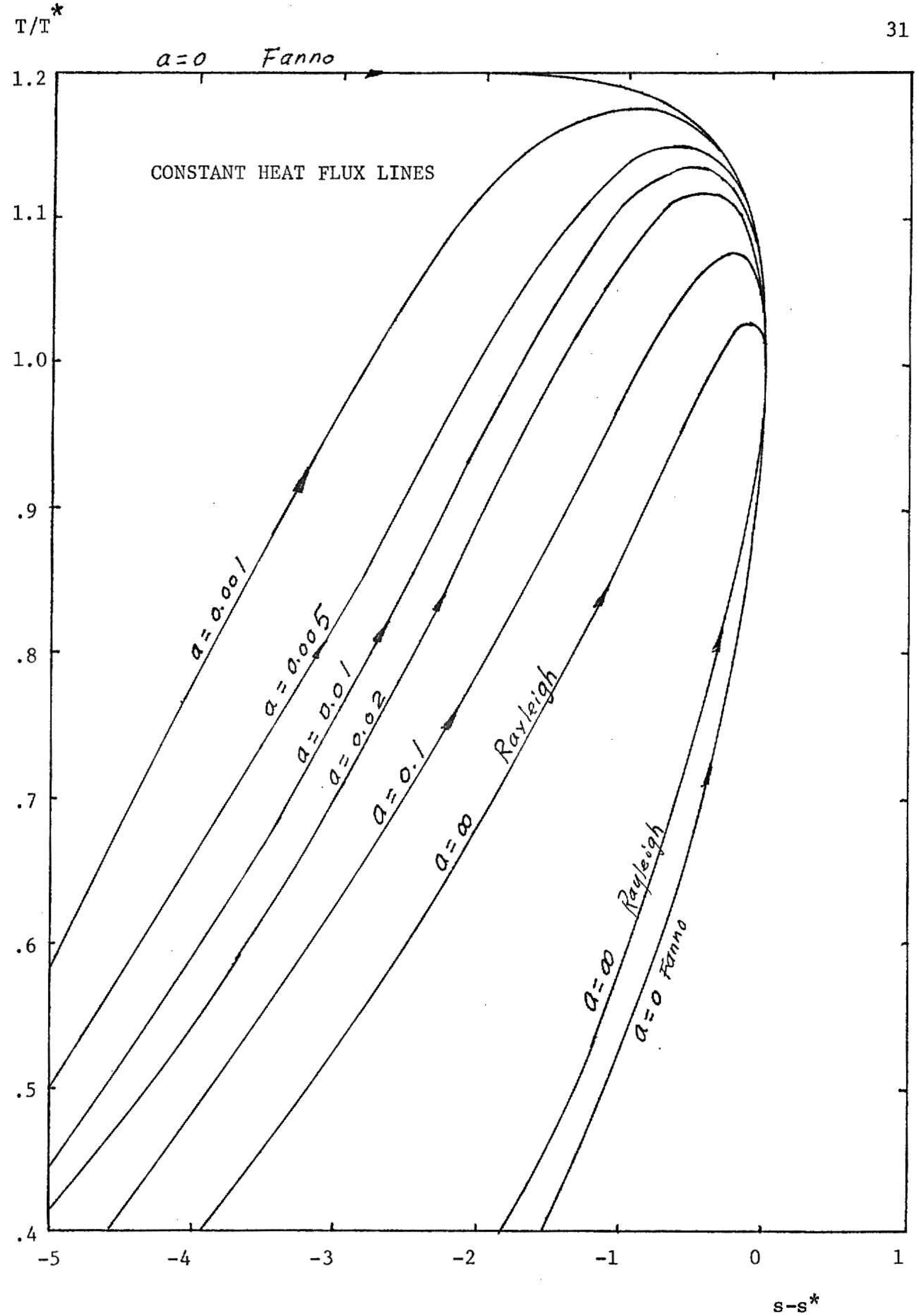


Fig. I T/T^* VERSUS $(s-s^*)/R$

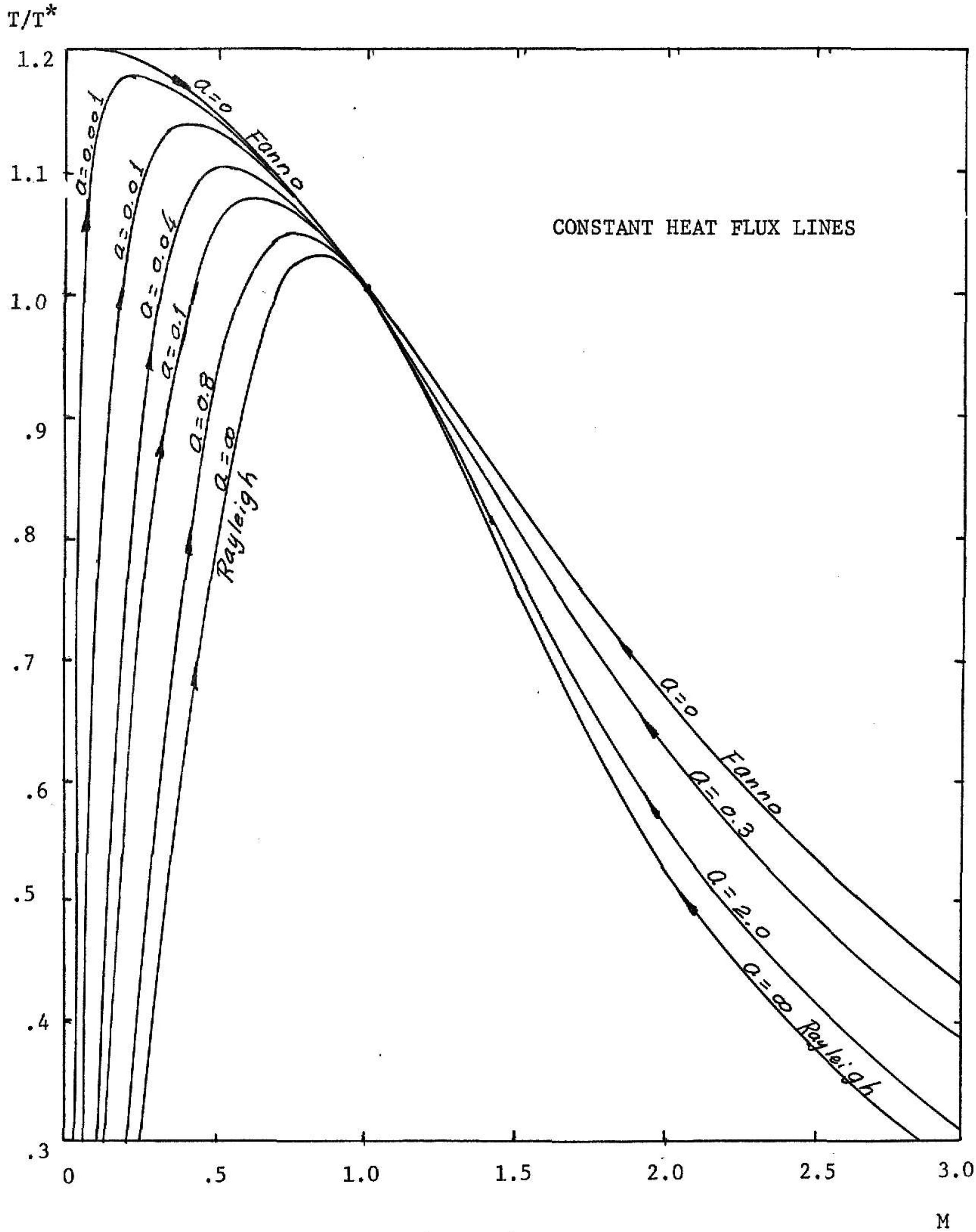


Fig. II T/T^* versus M

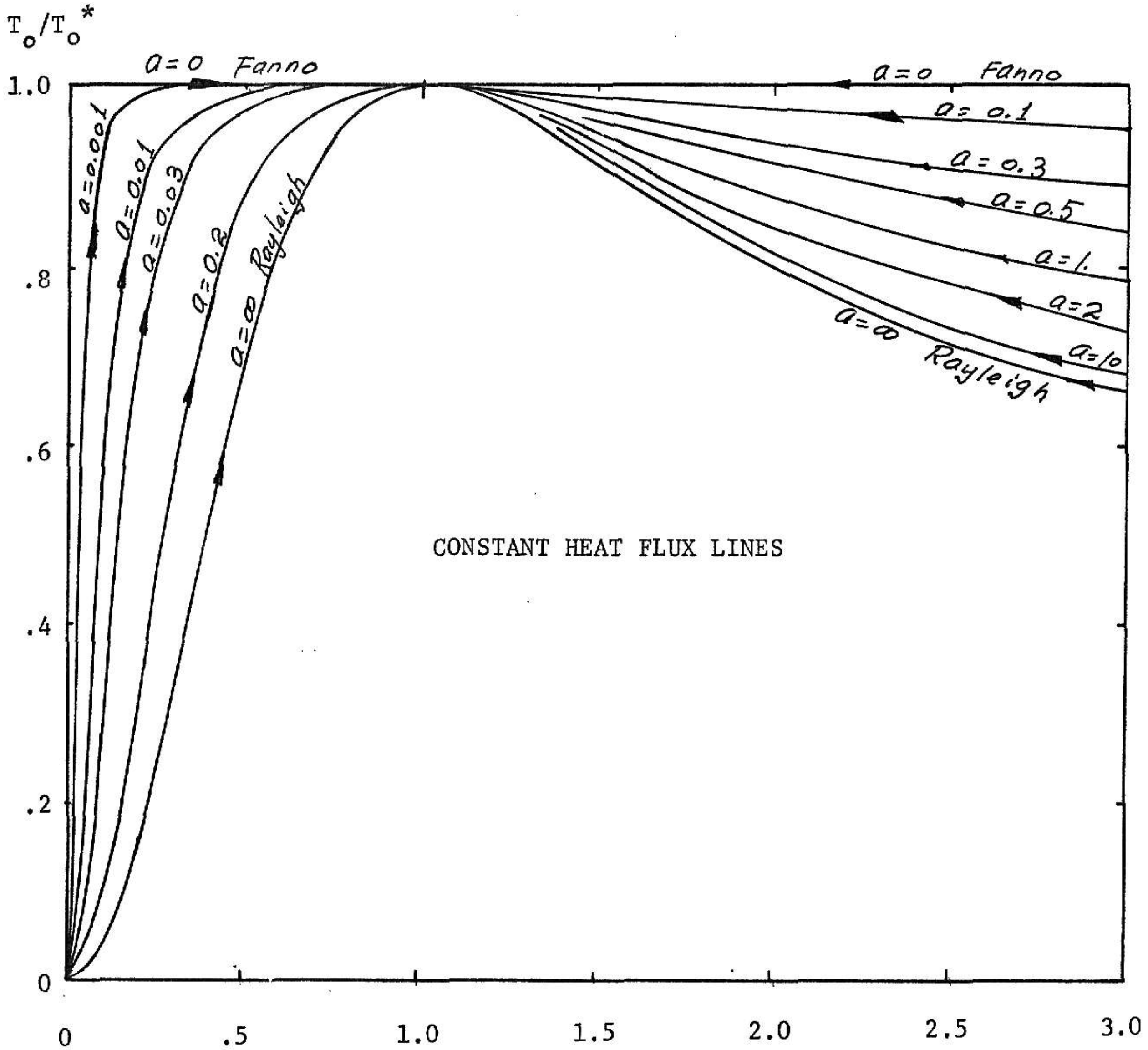
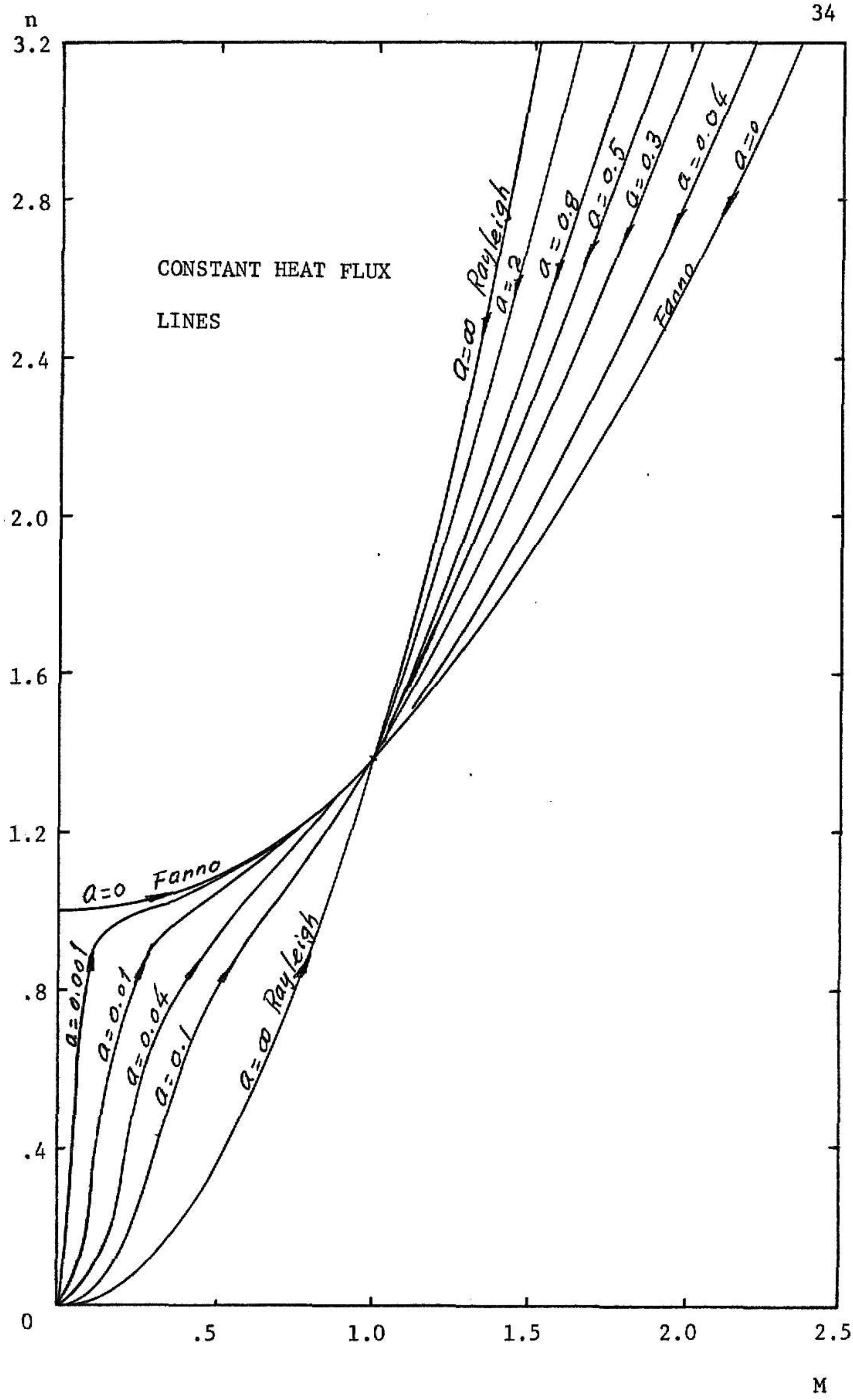


Fig. III T_o/T_o^* versus M



Fig, IV n versus M

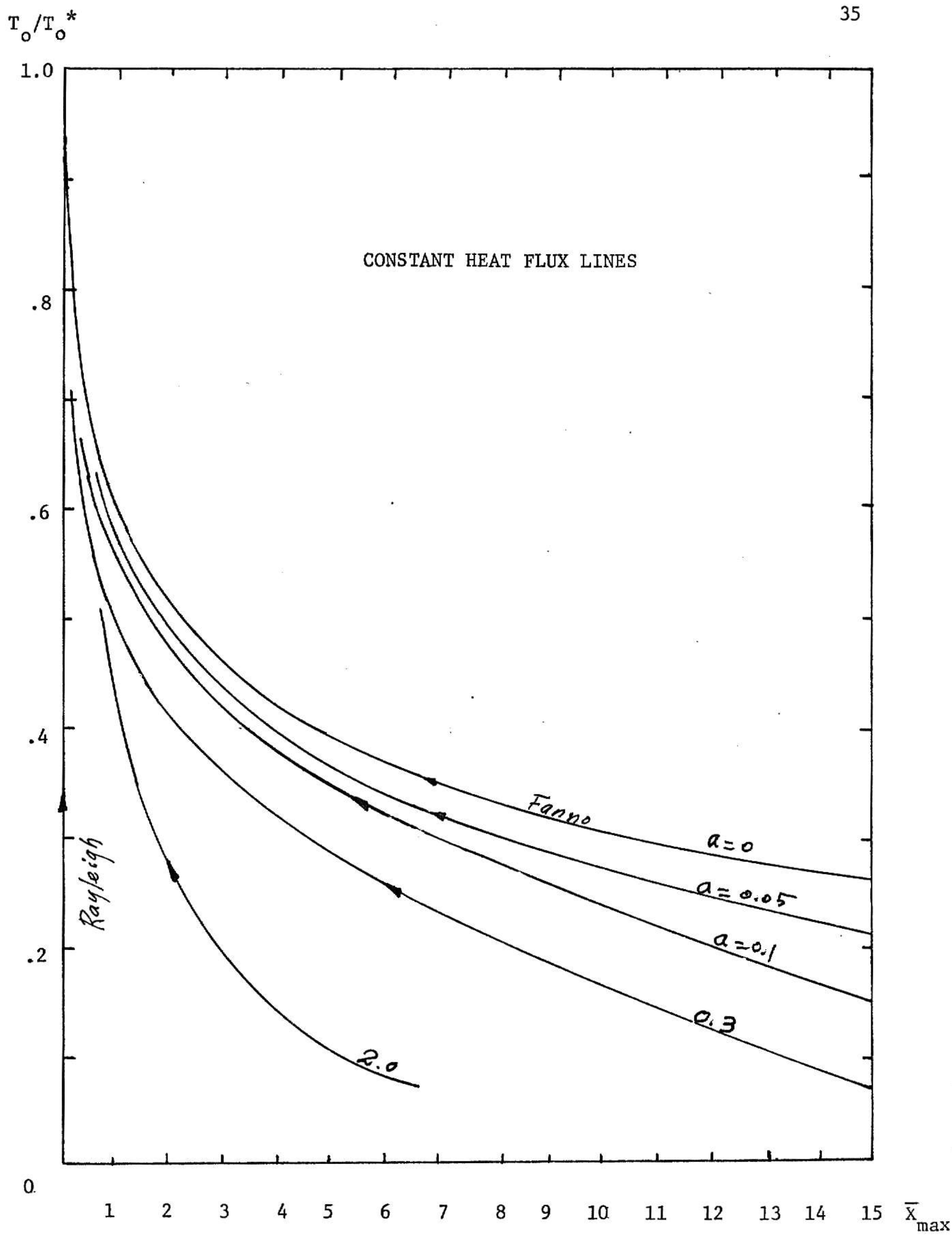


Fig. V T_o/T_o^* versus $\overline{X}_{max}$

($\overline{X}_{max} = 4\overline{f}L_{max}/D$)

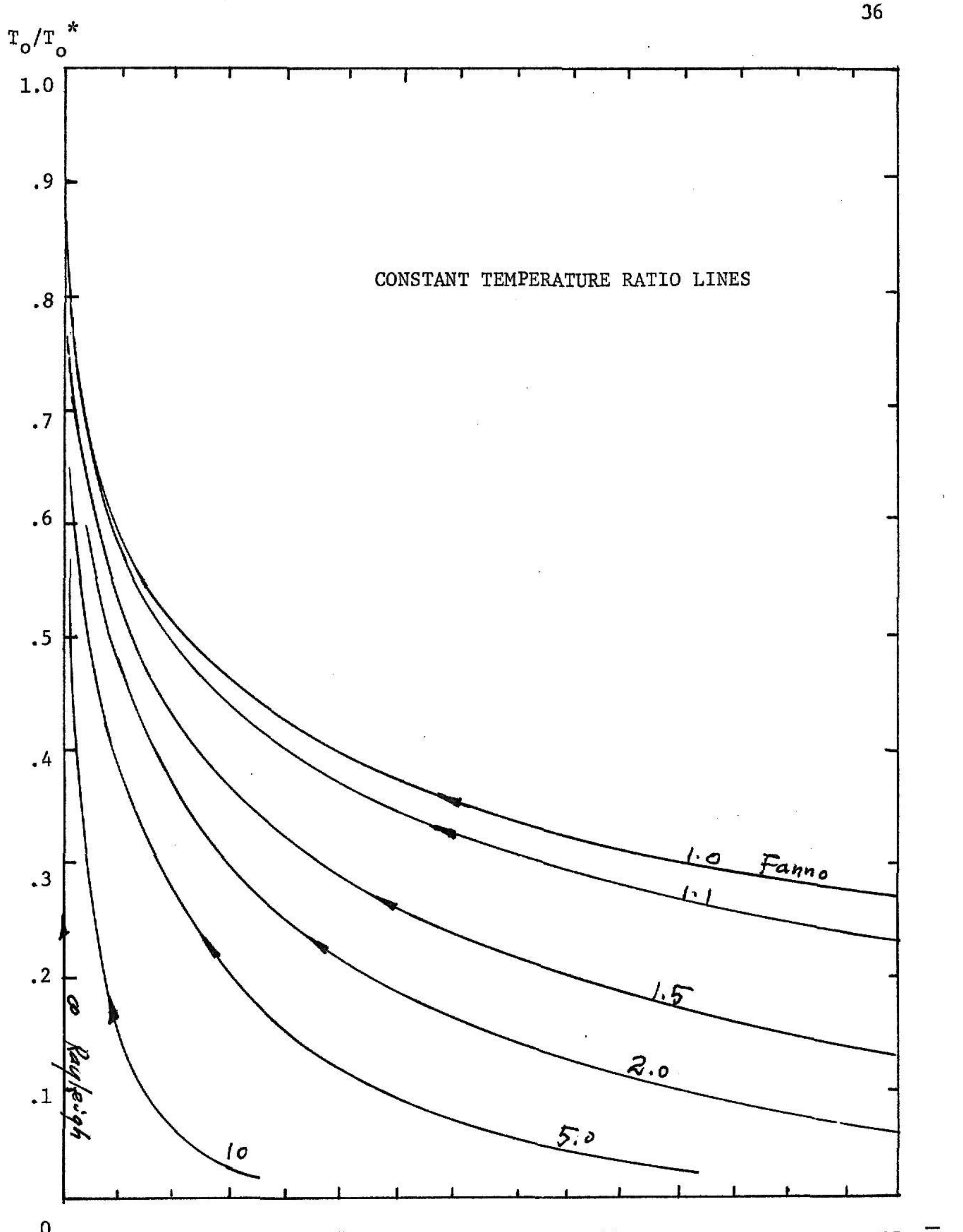


Fig. VI T_o/T_o^* versus $\overline{X}_{max}$

($\overline{X}_{max} = 4\bar{f}L_{max}/D$)

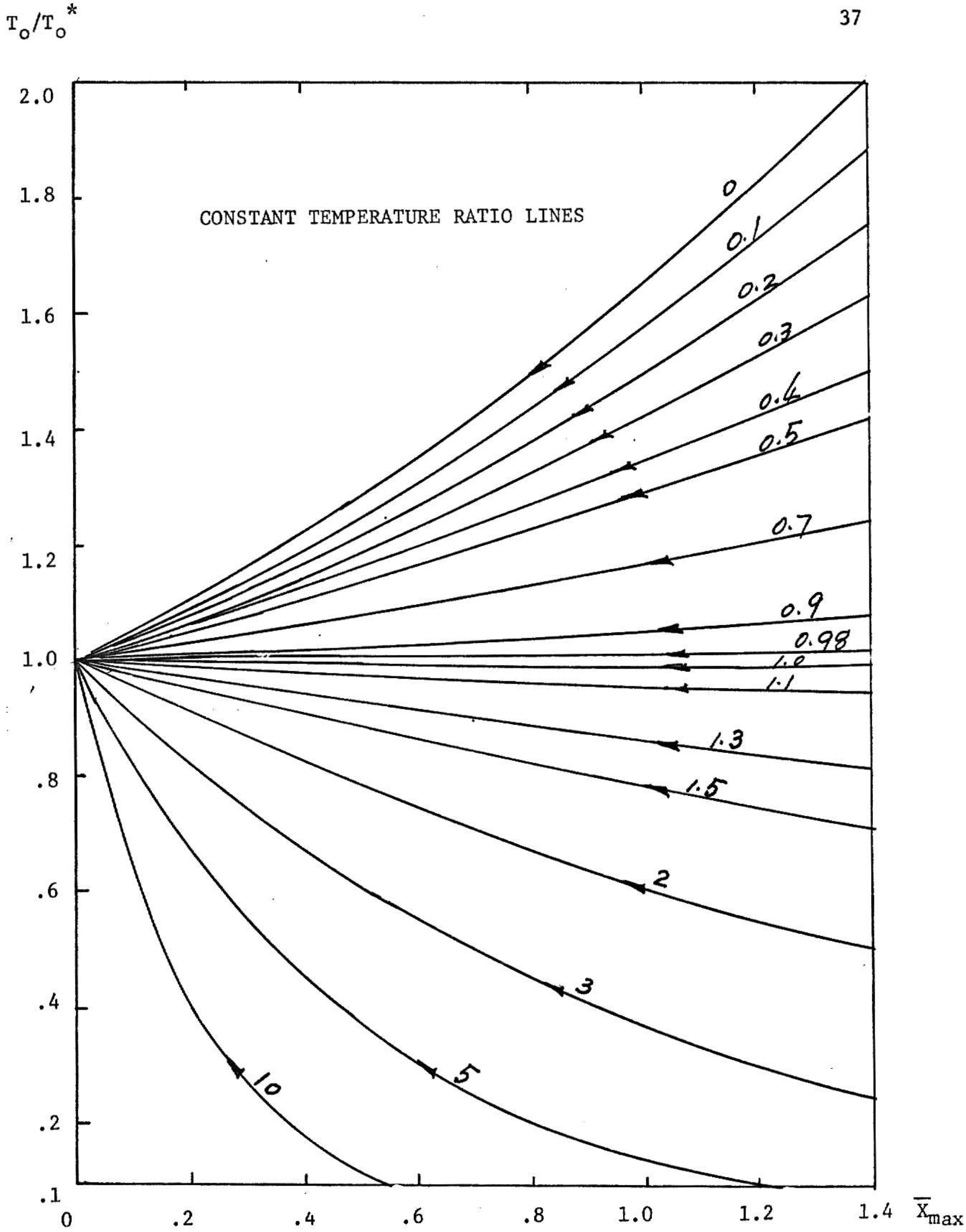


Fig. VII T_o/T_o^* versus $\overline{X}_{max}$

($\overline{X}_{max} = 4\bar{f}L_{max}/D$)

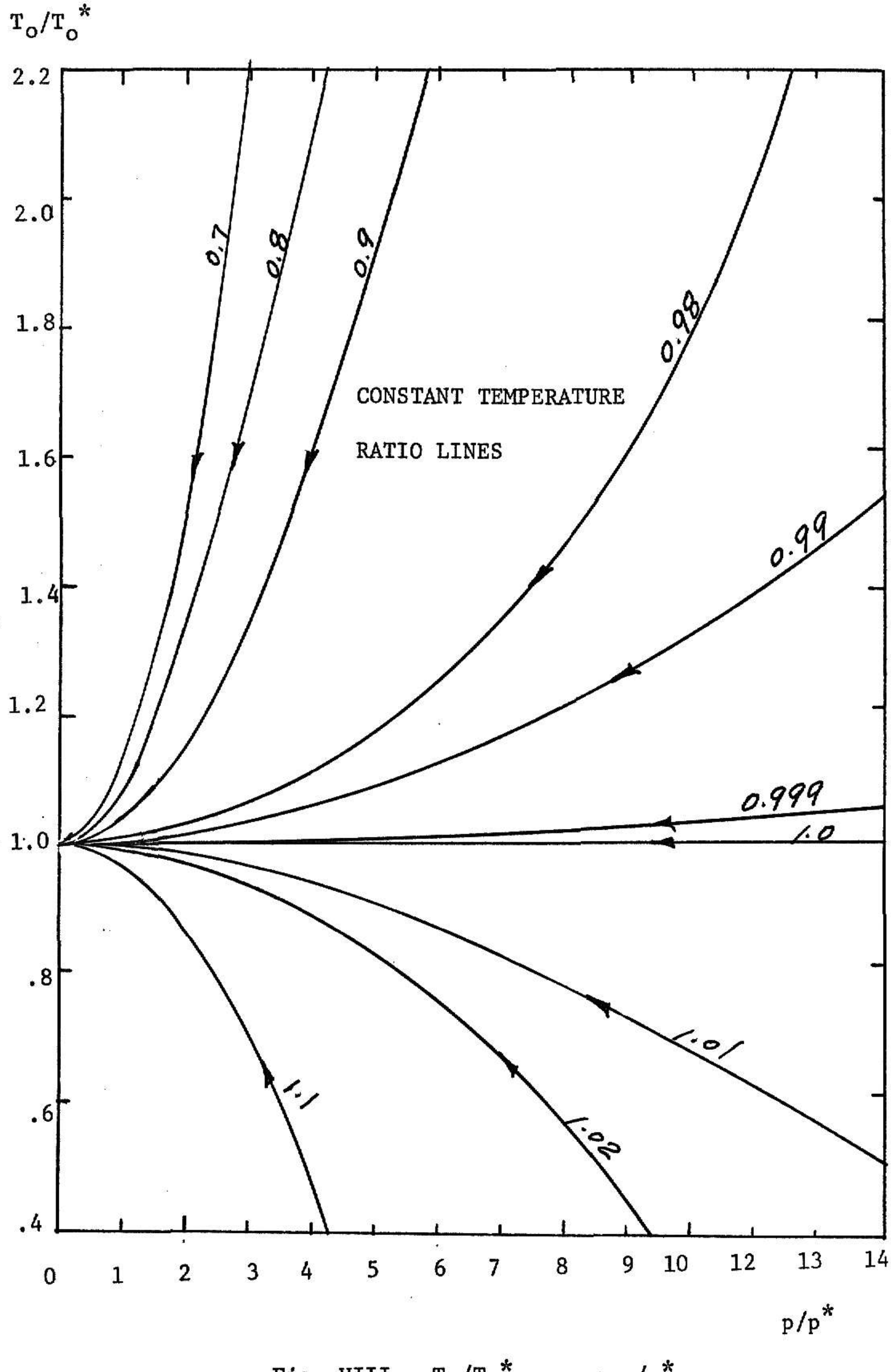


Fig. VIII T_o/T_o^* versus p/p^*

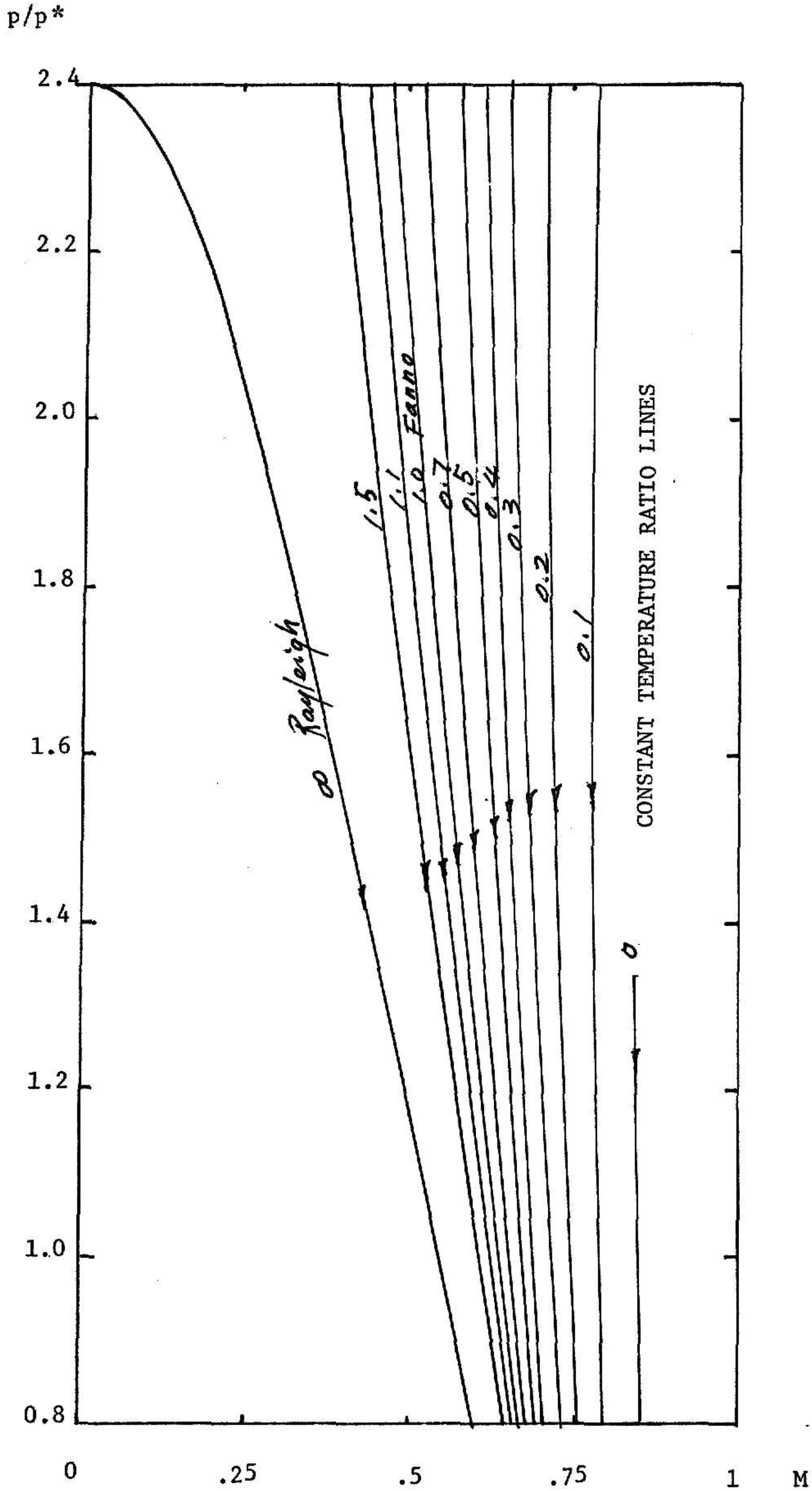


Fig. IX p/p^* versus M

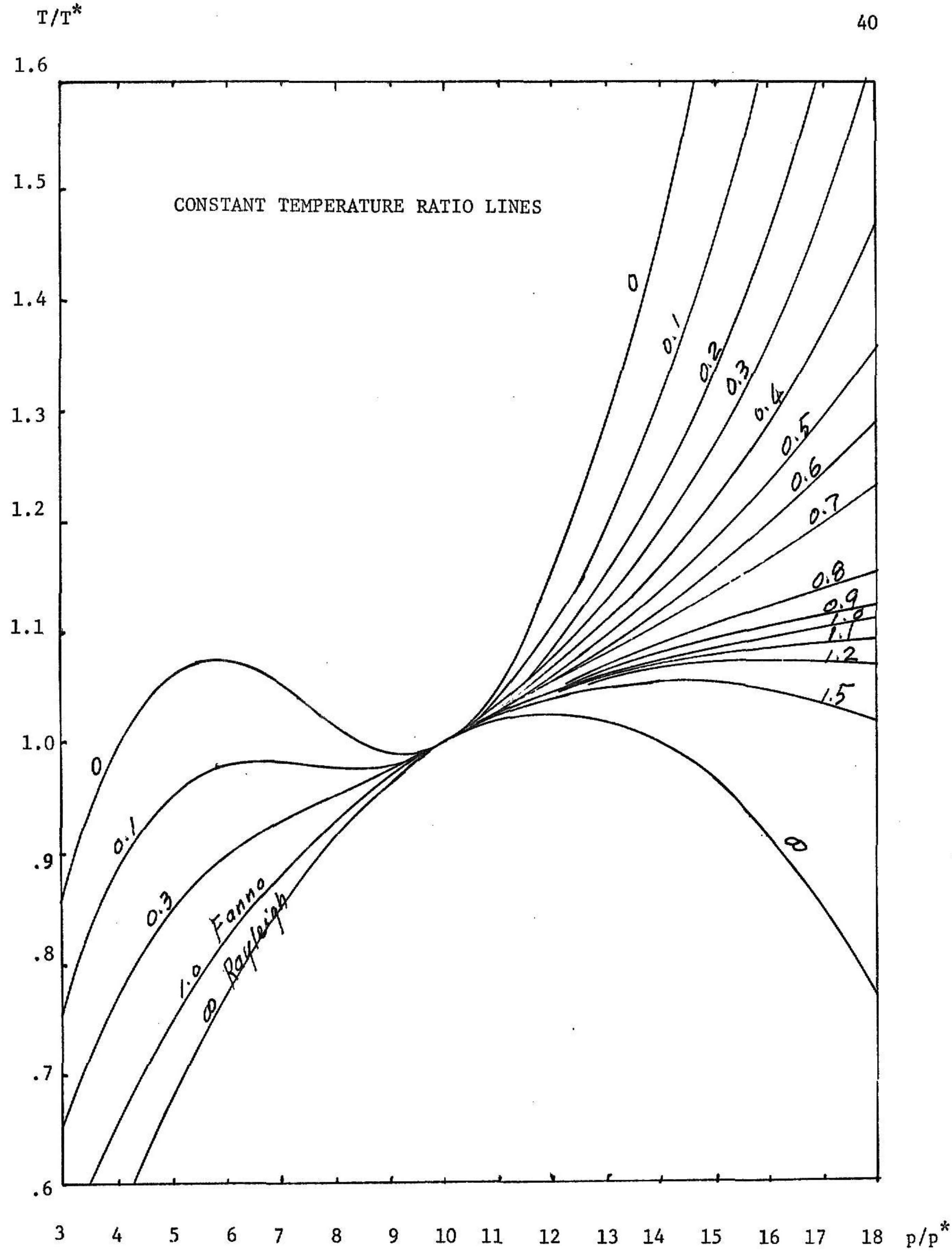


Fig. X T/T* versus p/p*

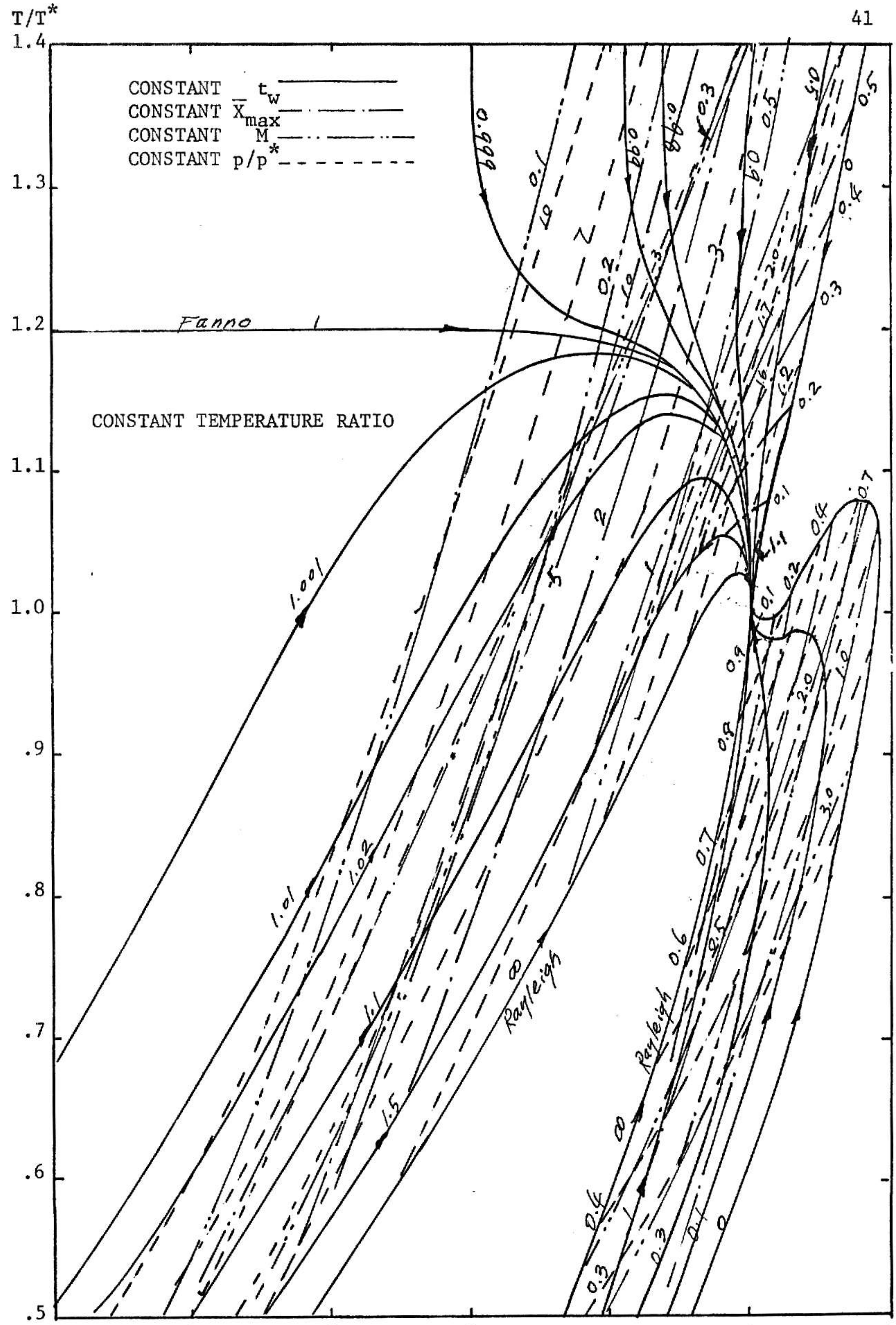


Fig. XI T/T^* versus $(s-s^*)/R$

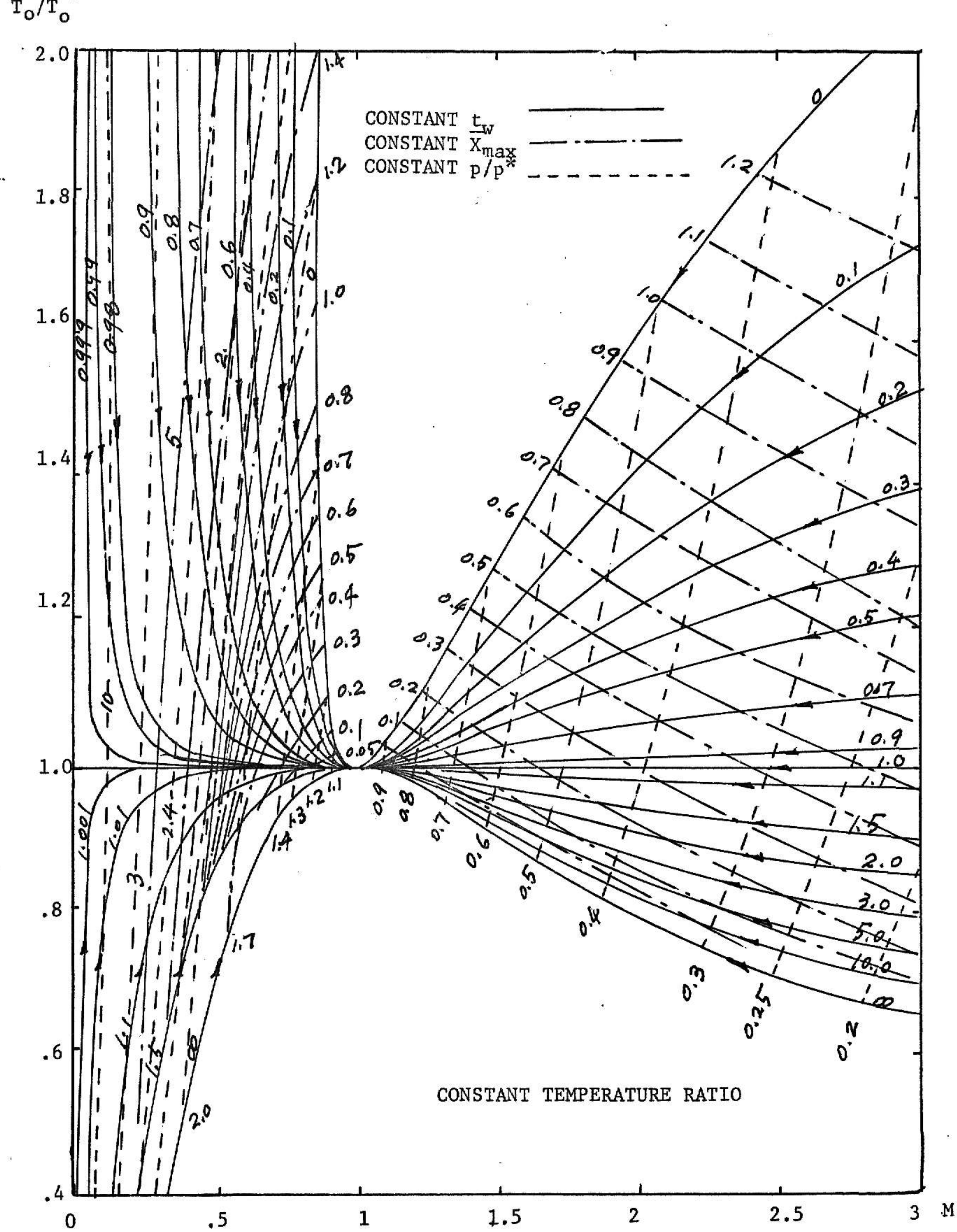


Fig. XII T_o/T_o^* versus M

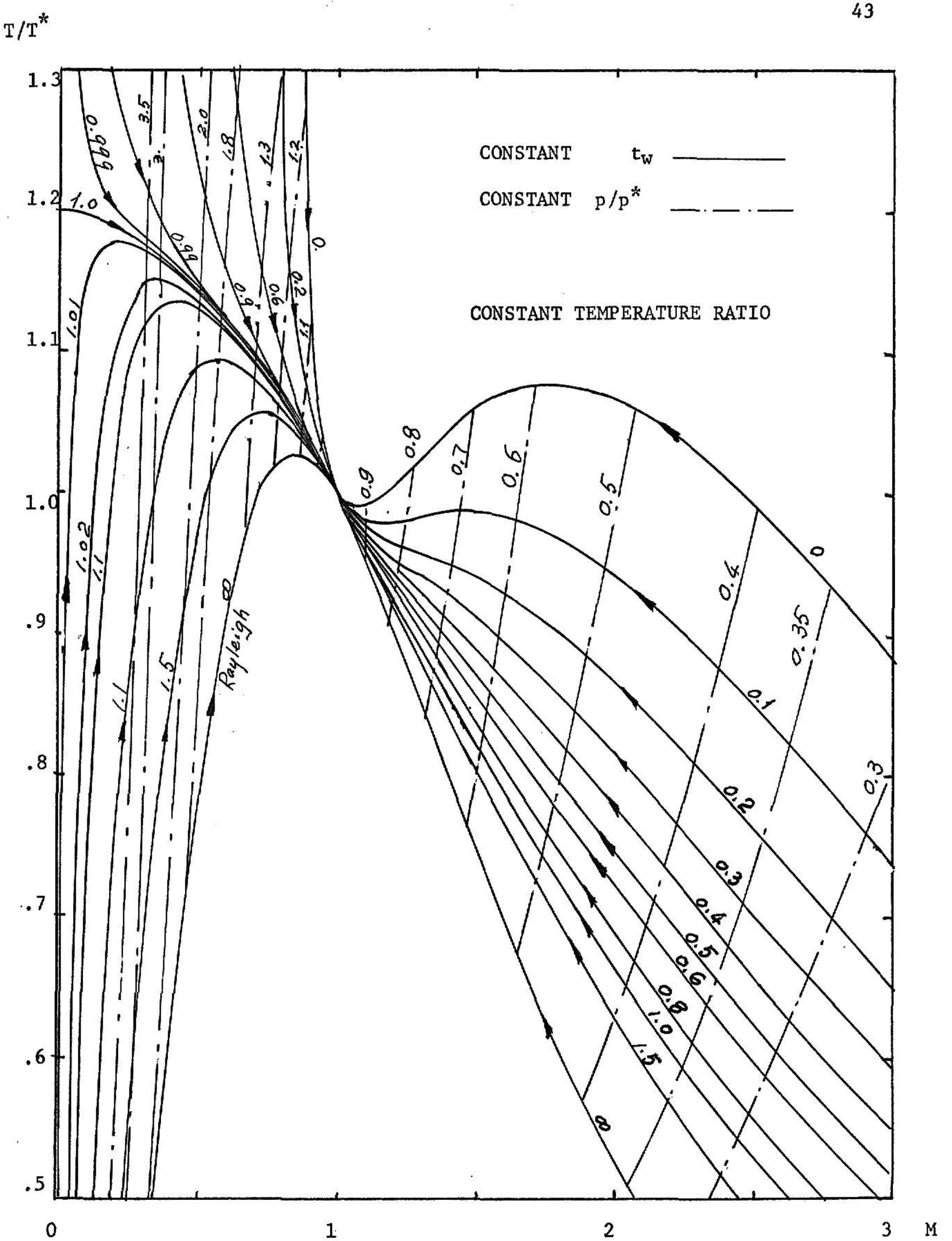


Fig. XIII T/T^* versus M

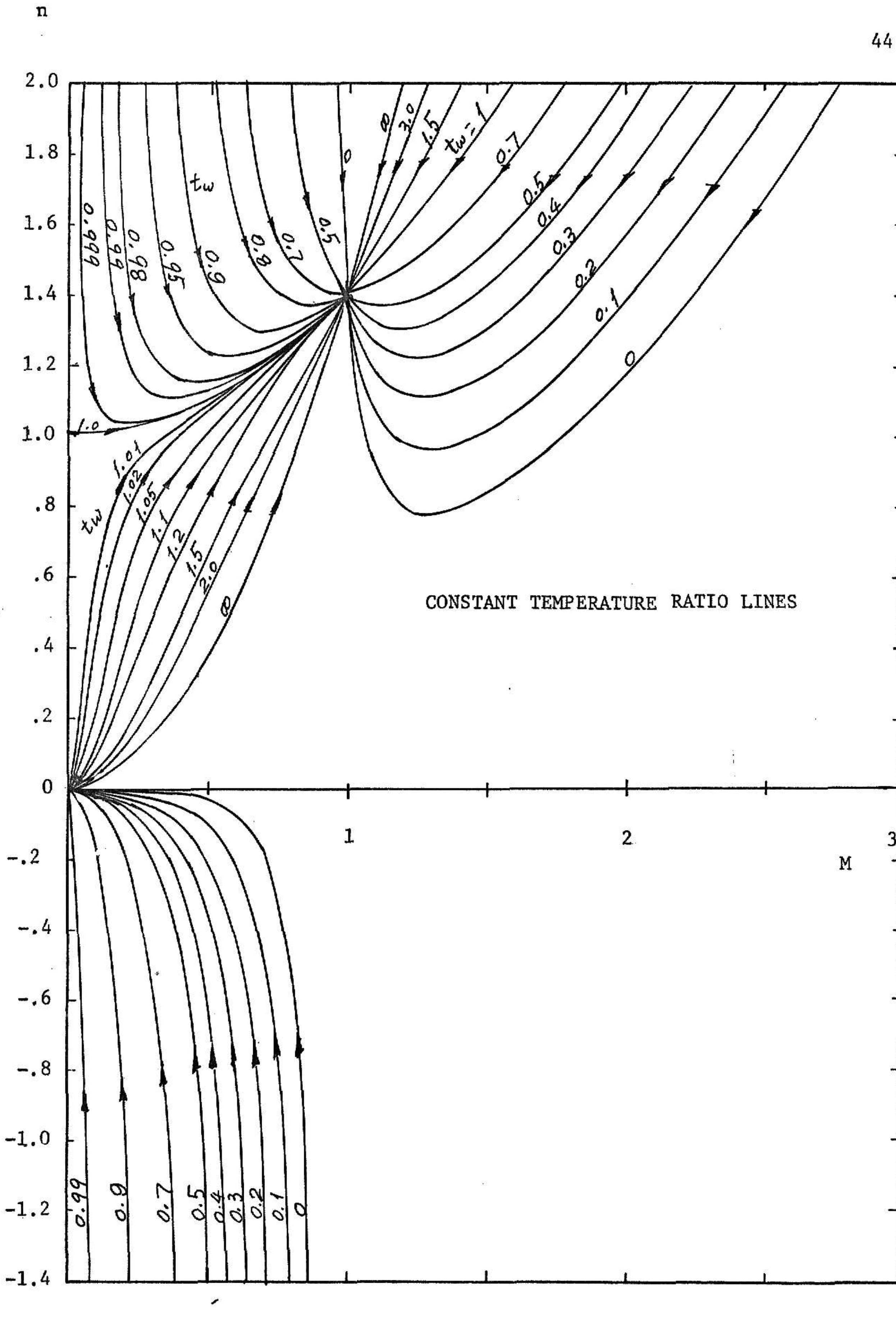


Fig. XIV n versus M

GAS FLOWS IN TUBES WITH CONSTANT HEAT FLUX AND WITH CONSTANT RATIO OF WALL AND STAGNATION TEMPERATURE

by

YI-AN CHEN

B. S., Taiwan Cheng Kung University, 1965

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requirements for the degree

MASTER OF SCIENCE

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1972

ABSTRACT

No general solution can be found for one-dimensional steady flow of a compressible fluid flowing in a constant area duct with the simultaneous effects of friction and heat transfer. This thesis presents two exact solutions for two particular cases of fluid flow; one is constant heat flux for heating, for both subsonic and supersonic flow. The other is constant ratio of local wall temperature and local stagnation temperature of the fluid along the flow passage. The validity of Reynolds analogy and the unity of recovery factor are assumed for the case of constant ratio of wall temperature and stagnation temperature. This thesis is an extension of Chen's (5) and Chang's (6) research, as well as a part of the research of the Department of Mechanical Engineering, Kansas State University, Manhattan, Kansas under the direction of Dr. Wilson Tripp.

The investigation of the two exact solutions were expressed in detail in the thesis. In addition, all the properties of the fluid along the flow passage were plotted. The ratio of the wall temperature and stagnation temperature, "t_w", and the heat parameter "a" (which is proportional to the rate of stagnation temperature increase with tube length.) for the case of constant heat flux are functions of friction and the rate of heat transfer.

If a gas problem is given and can be classified to either one of constant heat flux or constant ratio of wall temperature and stagnation temperature, then "a" or "t_w" may be evaluated. All the properties along the flow passage were plotted in the thesis.

Two numerical examples were presented for the purpose of showing the

application of the derived equations. Besides the usual properties of the fluid, the thermodynamic characteristics of the apolytropic variable, n, (defined by $dp/p + ndv/v = 0$) along the flowing passage have been investigated and plotted.