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INTRODUCTION: The subject of this report is the "simulation of a multiwarehouse inventory system." The particular system which is considered is a production distribution system. The system consists of customers, one retail sector, one distributor sector and a factory with a factory warehouse. In such a system, environmental changes lead to a decision that results in action which affects the environment and thereby influences future decisions. Such a system is generally defined as an information feed back system. This is a concept of servomechanism developed during and after world war II. The operation of an automatic temperature controlled heater provides a good example. When the temperature of the room decreases beyond a certain limit the heater starts automatically and the temperature of the room increases. Again when it increases beyond a certain point the heater stops automatically, and the process starts again. In this case the temperature of the room varies between an upper and lower limit and it tries to be constant. The process is continuous, and a new environment leads to a new decision, which keeps the system in continuous motion. A very accurate and detail study of an industrial production distribution system by simulation is not always possible. The complexity of the system and the human link Which plays an important role in decision making are difficult to incorporate in a model. But study of such a model provides an excellent tool to understand the dynamic behavior
of a practical situation.
This study might lead to a sound enterprise design and to create successful management policies and organizational structures. To develop a model of the system and to determine by simulation with various alternatives policies, structures, decisions and delays are interrelated and how a little distortion in one of them affects the whole system is the goal of this report. Simulation will also indicate the nature of resulting distortions, in the system and how long these distortions last before the system becomes stable again.

The particular study carried out in this report is taken directly from the work of J. W. Forrester's Industrial Dynamics. The book is the result of Forrester's years of experience and study. The work started in 1956, and after 5 years of joint work by the author and his associates, the book was published in its present form. Part of Forrester's efforts were directed to developing a simulation language called "DYNAMO". This language is similar to Fortran, the main difference lies in the handling of subscripted variable. The "DYNAMO" compilier is available from SHARE for the IBM 709 or 7090 computer. It will be seen, from the results of this study, that fortran is also a suitable language for the study of industrial dynamics and that fruitful problems may be simulated on the IBM 1410 computer.

This type of study is an application of the developments in the areas of information feed back theory, decision theory, modeling and digital computers. The theory of information feed back system as defined earlier provides a basis for understanding the goal-seeking and self correcting interplay between the parts of a business system. Decision theory forms a basis for understanding the place of decision making in industry. The experimantal model approach can be applied to complex systems. The progress of computers resulted in making an effective simulation faster and more accurate. The advent of computing machines has made it more feasible to deal with complex systems, hence the digital computer is a practical, and economical tool for the vast amount of computation required.

Our first step is to identify the problem. The main functions of the enterprise are the processes of production and distribution. It is necessary to know three kinds of information about the system. The organizational structure of the system, the delays in decisions and actions, and the policies governing purchases and inventories. Figure 1 shows the kind of system considered for this report. This is the same production-distribution system considered primarily by Mr. Forrester in his book Industrial Dynamics. As shown in the figure there is a flow of goods from the factory to the factory warehouse, from the factory warehouse to the distributor's warehouse and then to the retailer's

## PRODUCNION DISTRIBUTION SYSTEN


5. Order filling delay.

Figure-1
warehouse and finally to the customers. There is another flow of orders from customers to retailers, then to distributors, then to factory warehouse and finally to the manufacturing unit. We can see that there are three stages where an inventory of goods and orders exist. They are the factory warehouse, the distributor's warehouse and the retailer's warehouse.

Now the complete system can be comprehensively described as follows: Customers send their requests to retailers for goods. After receiving an order, the retailers check their inventory for a desired level, the order is processed through some clerical work and after some handling delay the goods are shipped to the customers. There is always a shipping and communicating delay between the two stages. Accordingly, retailers after checking their inventory and flow of customer orders send their replacement order to the distributors. There is a mailing delay between the two stages to allow the order to travel from one place to another. Since this is a continuous process there is always, at a particular instant, a rate of flow of orders and a rate of flow of goods from one place to another. At any instant there will always be orders on the way to the respective stages as well as some quantity of goods flowing in the other directions. In this problem such orders and goods are described as pipeline orders and pipeline goods. Distributors send their requests to the factory warehouse. The relation of placing orders
and receiving goods is similar between the distributors and the factory warehouse, and the retailers and the distributors. The factory warehouse is supplied goods from the factory production. By checking their inventory and the flow of orders from distributors, the factory control unit adjusts the factory production rate. Since there is a lead time at the factory, due to scheduling, job assignment and other preparation activities, the rate of production asked for by the factory warehouse is achieved after a few weeks of delay. This delay is considered as lead time at the factory. Since the system is dynamic, the change in the rate of production is continuous i.e. there are no sudden changes of the production rate which is atypical of the usual practice. At first, the maximum possible rate of production is kept very high, but then to study the effect of a situation where only a $20 \%$ increase in production is allowed, the maximum capacity is reduced.

The interconnecting equations and delays are set up in a manner that a little change at one place will affect the whole situation, but this effect will not be at once. It will take place according to the delays and relation between the stages. Even if the distortion in the system at one place is momentary, its effect might last for a long time. This will be realized in the first case when a step decrease in demand changes the production rate considerably and the effect of this change lasts more than 40 weeks.

This problem is simplified for the primary study of a system through which only flows of materials and orders are considered. The other important factors of industrial activity, money, personnel, capital equipment and information are not taken into account. Also forecasting, the widely used and powerful tool for sales prediction, is not used. Forecasting methods can be added once the more basic part of the system is understood. This type of study can be interpreted as the response of the system to deviations between the actual and forecast sales. Forecasting is a major subject in its own right and can easily be added later after we have achieved an understanding of the simple structure of our system.

OBJECTIVE: As mentioned previously the major objective behind this type of study is to simulate the behavior of industrial systems, to show how policies, decisions, structures and delays are interrelated to growth and stability. Such a model is a systematic way to express our descriptive knowledge about industrial activity. The model shows us how the behavior of the system results from the interactions of its component parts. After erecting a model we can see the results of some changes in the system, such as decreasing or removing some of the delays, or setting some capacity constraints for the production rate or the warehouse inventory. These types of results are valuable in the design of a. manufacturing unit or warehouses. The results also help in removing unnecessary paper work or reducing some of the major delays.

The major benefit of such a model is that proposed changes can be tried in the model and the best of them used as a guide to better management.

IXPIANATION OF TERMS: For simplicity, all the variables are assigned a code name to represent them in short terms. This is to avoid unnecessarily long names in develoving the equations.

DI- Delta time is the small solution interval between two solutions of all the equations. This is not the system parameter as such but it is the parameter of the simulation procedure. This is a constant and its value is assigned at 0.1 week.
$D H R, D H D$ and $D H F$ are the minimum order handing constant delays at the retailers, the distributors and the factory sector respectively.

The values assumed for our system simulation are:
DHR = 1 week
$D H D=1$ week
DHF = 1 week
DUR, DUD and DUF represent the delays in handing unfilled orders which are caused by out of stock items at retail, distributor and factory warehouses respectively. The total delay in filling an order is the minimum delay plus a ratio of desired inventory to actual inventory multiplied by the delay due to out of stock items.

$$
\begin{aligned}
& \text { DUR }=.4 \text { week } \\
& \text { DUD }=.6 \text { week } \\
& \text { DUF }=1.0 \text { week }
\end{aligned}
$$

are the values assigned to these constants.

AIR, AID and AIF are the number of weeks of average sales which could be supplied out of the normal level of inventory at the retail, distributor and factory warehouses. These constants relate the level of a desired inventory to the average rate of demand. When divided into 52 weeks, they represent the turnover rate of the inventory at each sector. The constants are:

AIR $=8$ weeks
AID $=6$ weeks
AIF $=4$ weeks
$D R R, D R D$ and $D R F$ : The present demand rate is averaged into a smoothed demand rate by using an exponential smoothing. These constants are the exponential smoothing time constants. Their meaning is that the importance of each successive past value decreases by the same ratio. This ratio is one divided by this constant. The value of the constant is 8 weeks and it is called delay in smoothing orders in the retail, distributor and factory sector respectively.
$D R R=8$ weeks
$D R D=8$ weeks
DRF = 8 weeks
$D I R, D I D$ and $D I F$ are the time constants representing the rates at which retailer's, distributors and factory warehouse people will act on inventory and pipeline deficit situations. This represents the management ability to deal with a deficit
condition, hence the system is sensitive to these constants. At the beginning we shall select value of these constants equal to 4 weeks. We shall see the effect of changing the value on the system, later on.

> DIR $=4$ weeks
> DID $=4$ weeks
> DIF $=4$ weeks
$D O R, D C D$ and $D C F$ are the clearical delays in processing purchase orders. We shall assume that it is the smallest at the factory and equal to 1 week, at the distributor 2 weeks and at the retailers 3 weeks. So:

$$
\begin{aligned}
& D C R=1 \text { week } \\
& D C D=2 \text { weeks } \\
& D C F=3 \text { weeks }
\end{aligned}
$$

DMR and DMD are the time constants required to mail the order from the retailer to the distributor and from the distributor to the factory.
$D M R=0.5$ weeks
DMD $=0.5$ weeks
DTR and DTD represents the time required to ship the goods from the distributor to the retailer and from the factory to the distributor.

$$
\operatorname{DTR}=1 \text { week }
$$

DTD = 2 weeks
DPF- Factory lead time of 6 weeks is assumed because of the time required to change the production rate and scheduling
at the factory.
ALF- represents the maximum allowable rate of manufacturing goods at the factory.

RRI- is the initial rate of demand at retail before simulating the system, 1000 units/week.

UOR, UOD and UOF represent the inventory of unfilled orders in units at retail, distributor and factory respectively.
$R R R, R R D$ and $R R F$ are the terms for the rate of demand in units per week at retail, distributor and factory respectively.

SSR, SSD and SSF are the rate of shipments in units per week shipped from retail, distributor and factory warehouse respectively.

AIAR, AIAD and AIAF are the actual level of the inventory in units at retail, distributor and factory warehouses. $S R R, S R D$ and $S R F$ are the rate of shipments in units per week received at retail, distributor and factory sector. $D F R, D F D$ and $D F F$ are the variables representing total delays in filling orders at retail, distributor and factory warehouse.

ANIR, ANID and ANIF are the shipping rate limits, such that there is no possibility of a negative inventory level at the retail, distributor and factory warehouse. Thus the actual shipping rate will be the smaller of $S S R$ and ANIR for the retail sector.
$A I D R, A I D D$ and $A I D F$ are the desired level of inventory in units at the retail, distributor and factory sector.

RSR, RSD and RSF are the smoothed values of the demand rate (receiving orders rate) at retail, distributor and factory.
$P D R$ and $P D D$ represents the purchasing rate decision at retail and distributor.
$A I D R, A L D D, A L D F$ are the number of orders in units desired in the pipeline in transit to supply the retail, distributor and factory warehouse respectively.
$A L A R, A L A D$ and $A L A F$ are the number of orders in units actually in transit to supply the retail, distributor and factory warehouse.

UNR, UND and UNF are the number of units of normally unfilled orders at the retail, distributor and factory warehouse. The term "normally unfilled orders" is defined as those orders waiting for processing, when the desired and actual inventory levels are the same.
$C P R, C D D$ and $C P F$ are the number of orders (units) in the clerical process at the retail, distributor and factory sector respectively.

PMR and PMD represent the number of purchase orders (units) in the mail from the retail and from distributor sectors respectively.

AMTR, AMTD are the number of goods (units) in transit to restock the retail and the distributor respectively.

PSR, PSD represent the purchase order rate in units per week sent from retail to the distributor and from the dis-
tributor to the factory.
ANWF is the rate of production in units/week wanted at the factory.

AMOF is the rate of manufacturing orders received at the factory.

OPF is the number of orders (units) in production at the factory.

RRFI, RRDI and RRI are the previous values of RRF, RRD and $R R R$ respectively at one time unit (DT) earlier.

DEVELOPNENT OF EQUATIONS: Before we can put the model into a computer for simulation, it is necessary that the equations connecting the various phases of the system are defined. It is very difficult to define these equations in an exact form because the nature of decisions, the respons to delays and value of information is not the same at all times and at all places. They are also affected by the people in management who take these decisions. Such an industrial system is never controlled by equation type rules or mechanical devices. Some other factors also need to be considered. The effect of all factors for a particular decision may not be a linear or exponential or any other mathematical function. The delays and lead time cannot be constant all of the time. They will have an unknown distribution with some average value. The average values are selected for our constants. In general, the value of these constants will be different for each different enterprise. This is a very general model, we have tried to choose plausible values for the constants and thus define legitimate equations. This type of difficulty does not necessarily reduce the value of a model and its development. The equations developed are from the general nature of the decision processes in modern industry. There is a natural tendency for management to keep step with the rate of demand and the level of inventory.

As an example retailers will order more goods from
distributors if the normal level of their inventory is low and the rate of demand from their customers increases steadily or suddenly. In our model the amount they should order is defined linearly by an equation. In the opposite case, if the actual inventory is more than the desired inventory, they naturally will order less goods.

In this model because of the continuous nature of the process, at every moment there will be a change in the inventory level, rate of demend and rate of manufacturing. The time interval chosen for simulation is 0.1 week, a fraction of the length of time represented in any third order delay in the system. The various types of exponential delays are explained later.

Since the three main sectors, retailers, distributor and factory have similar ordering and inventory policies, we will develop equations for the factory sector in some detail and afterwards will write down the retailers and distributor equations with an explanation wherever required.

In this development of equations I represents level, those variables that measure quantities will be included in this manner, its unit will be the number of units. The principal levels at the factory sector are:

1. The number of orders received from the distributor but not yet filled.
2. Inventory of goods in stock.
3. The level of orders and goods which are in transit
from one location to another.
Similarly $R$ represents rates of flow of goods or orders. Decisions control the rate of flow. The dimension of rates is units/week. The principal rates at the factory sector are:
4. Rate of orders coming in from the distributors.
5. Rate of shipments going out to the distributors.
6. Rate of manufacturing orders into the factory.
7. Rate of shipments coming from the factory.

The major delays in the system are:

1. Delay in filling orders at each sector.
2. Delay in making a decision and clerical preparation for out going orders.
3. Delay in transmitting orders from one place to another.
4. Delay in shipment of goods from one sector to the other.

It should be kept in mind that the equations are based on the verbal description of the system. They merely describe the most significant relationship. Since there is a limit to the depth of the system study, the factors of highest priority are selected in developing the equations.

For differentiating the level of the same variable between two solution intervals, we have selected three letters $J, K$, and $I . K$ represents the most recent value, J represents the value one deIta time before the most recent and I re-
presents the next value one delta time unit after the most recent value. Also for rates JK and KL are selected for the same purpose. i.e. KI shows the value of a rate from now and up to one delta time in the future. JK represents the value of rate between now and one delta time in the past. It is assumed that the values are constant for the one delta time interval. Since $D T$ is very small compared to the period of simulation we can describe the process as a continuous one. This also can be seen from the curves plotted for the various cases.

The first equation we will develop for the factory sector, is the actual inventory at the factory warehouse. The actual inventory at the end of a time period will be equal to the inventory at the beginning of the time period, plus the rate of incoming goods minus the rate of outgoing goods for the time period multiplied by the delta time. The description when put into an equation takes the following form:
1-I $\quad$ AIAF $\cdot K=A I A F \cdot J \div D T(S R F \cdot J K-S S F \cdot J K)$
The left hand side of the equation is the inventory level at the factory warehouse at the present ( $K$ ). The first term on right hand side is the level of inventory one delta time unit before and the terms in parenthesis give the addition or reduction rate of the inventory. The dimensions of the equation will be met as follows:

$$
\text { Units }=\text { Units }+ \text { week (Units/week - Units/week) }
$$

Similar equations at the distributor and retail sector will be:

2-I $\quad A I A D \cdot K=A I A D \cdot J+D T(S R D \cdot J K-S S D \cdot J K)$
This equation describes the nature of calculating the level of inventory at the distributor. The equation for retail sector's actual inventory level is:

3-I $\quad A I A R \cdot K=A I A R \cdot J+D T(S R R \cdot J K-S S R \cdot J K)$
Where

$$
\begin{aligned}
\text { AIAR•K }= & \text { Actual inventory level at present. } \\
\text { AIAR•J }= & \text { Actual inventory level one delta time } \\
& \text { unit before. } \\
D T= & \text { Delta time, solution interval. } \\
S R R \cdot J K= & \text { Rate of shipments received at retail } \\
& \text { between time now and one delta time period } \\
& \text { before. } \\
\text { SSR•JK = } & \text { Rate of shipments sent from retailers } \\
& \text { to the customers between time now and one } \\
& \text { delta time period before. }
\end{aligned}
$$

Now the other equation to be developed is the inventory of the number of unfilled orders at the factory. This is the number (units) of orders already received from the distributors but whose demand has not yet been met. Since the rate of incoming orders is a variable depending upon many factors, it is likely that some time will be required to fill the order, thus an inventory of unfilled orders is possible at a.ll the sectors. For the factory sector the
equation will be:
4-I $\mathrm{UOF} \cdot \mathrm{K}=\mathrm{UOF} \cdot \mathrm{J}+\mathrm{DT}(R R F \cdot J K-S S F \cdot J K)$
Where
UOF•K $=$ The level of unfilled orders at the present.
$U O F \cdot J=$ The level of unfilled orders one delta time unit before.

RRF•JK $=$ The rate of orders received at the factory during time DT.

SSF•JK $=$ The rate of shipments sent from the factory warehouse to the distributor during time DT.

Similar equations for the distributor and the retailer
sectors are:
5-I UOD•K = UOD•J + DT (RRD•JK - SSD•JK)
6-I $U O R \cdot K=U O R \cdot J+D T(R R R \cdot J K-S S R \cdot J K)$
In these equations the balance on hand at a given point in time is found by a repetitive addition and subtraction of the inflow and outflow rates. This form is possible, since the equations reflect the results of the previous decisions. On the otherhand, equations for the rates are not as obvious and straight forward. The decisions of the management play an important role in deciding the rate of orders, rate of shipments and rate of production wanted etc.. Thus the equations which represents the decision must be formulated so that they remain plausible and adequate over the allowable ranges of the variables. It is natural that these decisions are not the same for the complete
range of the system, often the equations will be nonlinear. Hence a rate equation will be developed by considering important factors that affect the rate. To keep the rate equations simple, it will be assumed that an equation is the same for any boundry existing in the system. This assumption certainly narrows the boundry of the system. For our purpose of simulating the system, it is assumed that the above boundary condition holds.

In the previous equation 6 , we used a term SSF for the rate of shipments sent from the factory warehouse. Defining SSF and putting it into an equation, means that the rate of goods delivery is controlled by the resulting state of the system and not by a free managerial decision. The rate of shipments sent from the factory warehouse to the distributors largely depend upon the level of unfilled orders, which are received from the distributors but not yet filled. We shall assume that the relationship is linear, that is, for the same conditions if the number of unfilled orders is doubled the rate of shipments will also be doubled. In addition, if there are no orders, there will be no shipping. The other determining factor to consider is the level of actual inventory on hand. Since it is assumed that out of stock conditions do not exist, the rate of shipments will be controlled in such a way that it never exceedes the negative inventory limit rates. This negative inventory limit rate represented by a term ANIF is the maximum limit for
the rate of shipments. The negative inventory limit for the factory sector is defined as:
$7-R \quad A N I F \cdot K I=A I A F \cdot K / D T$
Where
$\mathrm{ANIF} \cdot \mathrm{KI}=$ The maximum allowable rate of shipments to be sent from the factory warehouse to the distributors, for the time interval beginning now and extending one unit, DT, into the future.
$A I A F \cdot K=$ The actual level of inventory in the factory warehouse at the present time, hence we cannot ship more than we have on hand.

Similar equations apply for the distributor and the retail sector.

For the distributor sector it will be:
$8-R \quad A N I D \cdot K I=A I A D \cdot K / D T$
9-R $\quad$ ANIR•KI $=A I A R \cdot K / D T$
After establishing the upper limit of the shipping rate we can develop an equation for the actual shipping rate. For simplification, the basis for determining the shipping rate will be the number of unfilled orders and the delay in filling orders. The latter term is a variable represented by DFF.K. Later on it will be seen that the delay in filling orders is a function of the actual inventory. Our equation for the shipping rate at the factory will be:

10-R $\quad S S F \cdot K I=U O F \cdot K / D F F \cdot K$
The equation for the distributor sector is:
11-R $S S D \cdot K I=U O D \cdot K / D F D \cdot K$
Similarly for the retailer:
12-R $S S R \cdot K I=U O R \cdot K / D F R \cdot K$
These equations have the form of a first order exponential delay. In this type of delay, if the input is an impulse, the output decreases or increases exponentially depending upon the nature of the impulse: First order exponential delays are represented by only one equation. But the second and third order delays need two and three equations respectively.

The shipping rates determined by equations 10, 11 and 12 are compared with the negative inventory rates determined by equations 7, 8 and 9 and the ones which are less, are the rates applied to the system.

Now a relationship for the delay in filling orders needs a little consideration. We shall assume that the overall ability to fill orders will slowly decrease as the total actual inventory falls. Figure 2 shows the general shape of a relationship that exists between the delay in filling orders and the actual and desirable level of inventory for this system.

The amount of delay experienced on the average in filling orders will be inversely related to the ability to fill orders. i.e. due to a low actual inventory, the ability to fill orders is reduced hence more delay in filling orders.

At the same time since fewer and fewer orders can be filled from inventory, a larger and larger backlog of orders will be waiting to be filled from incoming shipments. When there is a very large inventory i.e. the ratio of desired inventory to actual inventory tends to zero the delay in filling orders is approximately the minimum order handing delay. This can be seen from Figure 2. While the ratio is more than one i.e. actual inventory is less than the desirable inventory, the delay in filling orders increases due to the out of stock conditions. The delay in filling orders is adjusted in such a way that there is no actual out of stock condition. The self correcting characteristic of the model causes this to happen. The delay increases to infinity when there is no inventory, so there is no possibility of making a shipment. We shall define an equation for the delay in filling orders to be equal to the minimum order handing delay plus a constant multiplied by the ratio of the desired over the actual inventories.

For the factory warehouse:
13-I $D F F \cdot K=D H F+D U F(A I D F \cdot K / A I A F \cdot K)$
At the distributor sector:
14-I $D F D \cdot K=D H D+D U D(A I D D \cdot K / A I A D \cdot K)$
Similarly at the retailer sector:
15-I $D F R \cdot K=D H R+D U R(A I D R \cdot K / A I A R \cdot K)$
Now the concept of a desired inventory level needs to be defined. This is an ideal level of inventory which is


Ficure-2
desirable to supply the average rate of demand for a specified time. This time is defined as the proportionality constant between the inventory level and average sales rate. It is called a proportionality constant because inventories are proportional to the sales rate. If the present sales are 100 units per week and our proportionality constant is 5 weeks than the desired level of inventory will be 500 units, but if the sales rate changes to 80 units per week then our new desired level will be 400 units.

The concept of a desired level of inventory is used for two purposes. First we use it in determining the ability to fill orders, hence the delay in filling orders and secondly it is one of the inputs to the ordering decision rule for generating purchase orders for new stock. The relationship of inventory changes to changes in the average level of sales, is a source of amplification for the fluctuations of industrial activity. In a warehouse, usually it is the practice to carry the inventory at a sufficiently high level to cover the expected sales for a stated period of time. This level increases or decreases 2.5 the rate of demand changes. Hence We shall define the desired inventory at the factory as:

16-I $\quad A I D F \cdot K=(A I F)(R S F \cdot K)$
AIF is the proportionality constant between the inventory and the average demand at the factory warehouse. If this constant is divided into 52 weeks, it would give the annual inventory turn over rate.

The similar equations for the distributor sector and the retail sector are:

17-I $\quad$ AIDD•K $=(A I D)(R S D \cdot K)$
18-I $\quad A I D R \cdot K=(A I R)(R S R \cdot K)$
This relationship is not necessarily true of every actual industrial situation. The equations are correct in the sense that they are the relationships that will exist in the model being studied.

RSR, RSD and RSF are the smoothed rates of the demands. It is necessary to smooth the actual sales data by averaging to obtain a figure upon which the inventory and ordering policies are based. This is required because the actual demand or sales rate fluctuate from day to day.

The averaging time period depends on the noise content of sales data. For noisy sales data, a long averaging time is needed, and a long averaging period causes the estimate of the average to lag more behind the true events then a short period. Hence the full response of any sales fluctuations will be delayed. Also the nature of the output will depend upon the input and smoothing the procedure. Realistic data require smoothing. Smoothing causes delay, and the delay affects the system behavior and stability.

The flow rates in an industrial or economic system are usually irregular. The decisions that generate the flows are influenced by many minor events. The causes of irregularity are the individuality of human actions, the uncertainty of
labor, the weather, holidays and week ends, error in data analysis, decisions made on an undefined and varying basis, discount prices that encourage large orders, set up costs and the nature of the production processes that cause seperate production lots rather than continuous production, national and international news event that changes public trends toward purchases.

Recognizing significant changes in this data, while ignoring the superimposed, meaningless fluctuations is called smoothing. Smoothing of incoming information occurs to some extent at every decision making point in the system. Eventhough smoothing introduces inherent delays in decisions, it is essential for filtering out short-period noise. Smoothing changes the sensitivity of the system to different periodicities that may exist in data fluctuations.

As smoothing is a process of taking a series of past values and averaging them in some manner to represent present values, there can be many methods depending upon the weight given to past data. The two general and widely used methods are moving average and exponential smoothing. The moving average assigns the same weight to each past value up to a fixed time spand. e.g. Average sales for the past eight weeks is a moving average.

In exponential smoothing data are given progressively less weight as they become older. i.e. the importance of each successive past value falls by the same ratio. In
smoothing the sales rate at the factory warehouse, distributor and retailers; a first order exponential smoothing is applied.

The equation for the factory sector will be:
19-R $R S F \cdot K L=R S F \cdot J K+(D T)(R R F \cdot J K-R S F \cdot J K) / D R F$
Where
$\mathrm{RSF} \cdot \mathrm{KI}=$ The requisitions rate smoothed for one DT period beginning now. RSF•JK $=$ The previous smoothed value one period ago.

DRF = The smoothing time constant. RRF•JK $=$ The rate of orders received at the factory during past delta time DT.

Similar equations for the distributors and retailers will be:
20-R $R S D \cdot K I=R S D \cdot J K+(D T)(R R D \cdot J K-R S D \cdot J K) / D R D$
21-R $R S R \cdot K I=R S R \cdot J K+(D T)(R R R \cdot J K-R S R \cdot J K) / D R R$
The constants $D R R, D R D$ and $D R F$ give the fraction of the difference that is to be corrected each time. In this type of smoothing the most recent data has the greatest weight.

The next step is to develop the decision criteria for the placing of purchase orders from the retailer to the distributor and from the distributor to the factory. We shall assume the same criteria for both of the sectors. The
equation describing the purchase rate will be for the generation of the outgoing orders rate. This decision will be one sided, because whatever the ordering rate may be, the delivery of goods will be determined by the availibility of goods.

In placing a purchase order for replacement goods, certain sources of information are used. Various subsidiary decisions are reached, requisitions are authorized, and clerical delays are encountered in making up the actual orders. They require data collection, proposals, recommendations and review. In this general study, the fine details of the seperate steps in decision making and the seperate delays in the process are not included. We have handled delays seperately from decisions. Afterwards the delays and the decisions are grouped together to establish a relationship. It is assumed that the decision is made first on the basis of the available information and then the output of the decision is delayed to represent the total delay encountered in the information sources and the decision process. Actually there are some delays before and some delays after the actual decision point.

Our equation will define the decision about the purchasing rate and then the flow of orders will be delayed by an amount equal to the total delay involved in the processing of raw data, in the making a purchasing decision, and in the clerical steps of a purchasing office in writing purchase orders. Also it is assumed that any forecasting tech-
niques are not being used for the rate of orders.
First, the rate of purchasing will depend on the rate of sales. The orders are necessary for the correction of differences between the actual and the desired inventory. The difference between the actual number of unfilied orders and the normal level of unfilled orders, it a decisive factor in a purchasing rate. The normal level of unfilled orders is the unfilled orders at the time when the actual inventory and the desired inventory are the same.

When there is a fixed time for orders and goods to travel through the pipeline from the retailer to the distributor and return, from the distributor to the factory and return, it is necessary that the total of the orders plus goods in the pipeline be proportional to the level of business activity. If the orders are not inserted in the pipeline for this purpose, the deficits will come out of the inventories in the system. All these factors are considered in the equation for the purchasing rate.

The purchasing rate equation for the retail sector is:
$22-R \quad \mathrm{PDR} \cdot \mathrm{KI}=\mathrm{RRR} \cdot J \mathrm{~K}+\frac{1}{D I R}\left(\begin{array}{l}(\mathrm{AIDR} \cdot \mathrm{K}-\mathrm{AIAR} \cdot \mathrm{K})+ \\ (\mathrm{ALDR} \cdot \mathrm{K}-\mathrm{AIAR} \cdot \mathrm{K})+ \\ (\mathrm{UOR} \cdot \mathrm{K}-\mathrm{UNR} \cdot \mathrm{K})\end{array}\right)$
A similar equation for the distributors is:
$23-R \quad \mathrm{PDD} \cdot \mathrm{KI}=\mathrm{RRD} \cdot \mathrm{JK}+\frac{1}{\operatorname{DID}}\left(\begin{array}{l}(\mathrm{AIDD} \cdot \mathrm{K}-A I A D \cdot K)+ \\ (\mathrm{AIDD} \cdot \mathrm{K}-A T A D \cdot K)+ \\ (\mathrm{UOD} \cdot \mathrm{K}-U N D \cdot K)\end{array}\right)$
The delays DIR and DID are the time constants representing the rate at which the retailers and distributors, on the
average, act on inventory and pipeline deficit situations. It is assumed that they would not respond immediately to the full extent of any theoritical difference between desired and actual inventory. A value of four weeks for the constant DIR and DID would give a purchasing rate that corrects any deficit in the term in brackets at the rate of one-quarter of the deficit per week. This is one of the important parameter in determining the system's dynamic performance.

The two inventory terms give the difference between the desired and the actual inventory. If the level of these inventories is not the same, a correcting term will be introduced into the purchasing order rate equation.

The desired level of orders in the pipeline is proportional to the level of the smoothed sales. The difference between desired and actual level of orders also contributes to the decision policy for purchasing.

The terms $U O R$ and UNR are included so that under steady state conditions the actual inventory level at the retailer will remain equal to the desired inventory. Under normal steady-state conditions the two pipeline terms will cancel each other, since the desired and actual inventory are the same. If the unfilled order term UOR were present without the canceling normal level of unfilled orders UNR, the inventory terms would differ from one another by enough to equal the unfilled order term UOR. Under steady state conditions i.e. constant rate of demand for a long time, the
actual inventory will approach the desired inventory. The interaction betwern Enventory, pipeline and unfilled order terms is important. Suppose that a deficit exists in the actual inventory, AIAR, with respect to the desired inventory, AIDR, the result will be an ordering rate that will place orders in process in the pipeline term, ALAR. When sufficient orders are initiated to meet inventory needs, the additional orders in the ALAR will remove the difference between ALAR and ALDR. As the goods ordered for inventory are delivered, they will be transformed from orders in the pipeline ALAR to inventory AIAR, so that the ordering rate equation continues in balance. The equation thus avoids duplicate reordering of inventory in succeeding time periods before the goods are delivered.

As the retailers order from the factory warehouse, the factory sector satisfies these orders by adjusting the manufacturing rate. In a real situation, the manufacturing decision is affected by various practical considerations determined by physical production facilities. Te assume that the manufacturing rate is continuously variable from zero up to some maximum rate. This assumption is valid because most production lines and equipment can operate over a range of production capacities. In the simulation example a case is tried for a range of production rate between 0 and 1200.

Even if the rate of manufacturing wanted is more than
the maximum capacity, the output rate is the maximum Iimit. Thus the production rate is tied to the maximum capacity only. In the computer program the desired rate is compared with the maximum allowable rate, and the smaller one is accepted as the production rate. The equation for desired rate of production is similar to the purchasing order rate for retailers and distributors as discussed before.

Where DIF is the delay in inventory and pipeline adjustment at the factory in weeks.

The terms ALDF and ALAF are the desired and actual level of orders and goods in the transit pipeline from the factory warehouse to the factory and return. Since the warehouse and factory are at the same location, the actual inventory of goods and orders in the pipeline are the orders in the clerical process at the factory plus the orders in production at the factory. It is assumed that there is neither delay in moving the manufactured goods to the warehouse, nor is there any mail delay.

So the equation for the actual level of pipeline orders at the factory is:
25-I $\quad A I A F \cdot K=C P F \cdot K+O P F \cdot K$
There $C P R$ and $O P F$ are the clerical in process orders and orders in production at the factory respectively.

The equations for retailers and distributors are not
similar because the actual content of the supply pipeline consists of the sum of the orders and the goods in the clerical process, purchase orders in the mail, goods in transit to the place of ordering and the number of unfilled orders where the orders are placed. The pipeline orders and goods on the way for retail sector are defined as:

26-I $A L A R \cdot K=C P R \cdot K+P M R \cdot K+U O D \cdot K+A M T R \cdot K$ Where

$$
\begin{aligned}
C P R= & \text { Clerical in process orders. } \\
\mathrm{PMR} \mathrm{=} & \text { Orders in mail from retailer to the } \\
& \text { distributor. } \\
\mathrm{UOD}= & \text { The number of unfilled orders at the } \\
& \text { distributor. } \\
\text { AMTR = } & \text { Goods on the way or in transit to retailers } \\
& \text { from distributors. }
\end{aligned}
$$

The similar equation for the distributor sector is:
27-I $A J A D \cdot K=C P D \cdot K+P M D \cdot K+U O F \cdot K+A M T D \cdot K$
Now the desired level of goods and orders in transit at the factory depends upon a smoothed rate of orders received at the factory and the delay in clerical processing time required for the orders and the production lead time at the factory. So the desired number of orders and goods in the pipeline will be equal to:

28-I $\quad A I D F \cdot K=(R S F \cdot K)(D C F+D P F)$
DPF is the lead time required for the scheduling of a change in production rate. That is, if the rate of pro-
duction wanted today is 500 units/week, and lead time is six weeks then the rate of shipments received at the warehouse will be 500 units/week after six weeks.

The equation for pipeline orders and goods for the retail sector will depend mainly upon the rate of orders received at retail, the delay in clerical orders processing (DCR), delay in order mailing from retail to the distributor (DMR), delay in filling orders at the distributor (DFD•K) and delay in transportation of goods from the distributor to retail (DTR).
$D C R, D M R$ and $D T R$ are constants but $D F D$, the delay in filling orders, is a variable as defined previously. Total delay multiplied by the smoothed rate of orders received at retail gives the desired level of orders and goods in the pipeline to supply the retailer. The equation for the retail sector is:

29-I $A I D R \cdot K=(R S R \cdot K)(D C R \neq D N R+D F D \cdot K+D T R)$
The similar equation for the distributor sector is:
30-I $\quad A L D D \cdot K=(R S D \cdot K)(D C D+D M D+D F F \cdot K+D T D)$
The UNR, UND and UNF terms in the equation for the purchasing and manufacturing decisions are the normal level of unfilled orders at retail, distributor and factory respectively. The normal level of unfilled orders is equal to the average sales rate multiplied by the normal delay in filling orders. Normal delay is defined as the delay in filling orders when the desired and the actual inventory
levels are the same. This is a minimum order handing delay plus a constant delay in unfilled orders at the normal inventory level.

Hence the equation for the normal level of unfilled orders at the factory sector is:
31-I UNF•K $=(R S F \cdot K)(D H F+D U F)$
The similar equations for the distributor sector and retailers are:
32-I UND•K $=(R S D \cdot K)(D H D+D U D)$
33-I UNR•K = (RSR•K) (DHR + DUR)
Previously we developed an equation for the desired purchasing rate but the rate of purchase orders sent is not the same as the desired purchase rate. The reason for this is the delay in clerical order processing before sending the orders. This delay DCR between the decision to purchase and the actual purchasing rate is considered a third order delay. A third order delay is obtained by feeding the flow through two or more cascaded first order delays. A first order delay and a third order delay have the same total average delay but the response to change in the flow rate is different. The total delay is distributed one-third in each first order section. So the third order exponential delay is defined by three pairs of equations.

The response curves of first, second and third order exponential delays are shown in figure 3.

We chose a third order delay because the shape of the

Input


Figure 3-a First-order exponential delay.

Output


Input


Output


Figure $3-b$ Second-order exponential delay.

Input


Outout


Figure $3-c$ Thiri-order exponential delay.
output response satisfies the characteristic of the actual shipping process, where these delays are used.

Now the equations which represent the rate of purchase orders sent from retail with third order clerical delays are:

34-R PSR1•KI $=$ PSR1•JK $+3 D T(P D R \cdot J K-P S R 1 \cdot J K) / D C R$
34A-R PSR2•KI $=$ PSR2•JK $+3 D T(P S R 1 \cdot J K-P S R 2 \cdot J K) / D C R$
$34 B-R \quad P S R \cdot K I=P S R \cdot J K+3 D T(P S R 2 \cdot J K-P S R \cdot J K) / D C R$
The similar equations for the distributor sector are:
35-R PSD1•KI $=P S D 1 \cdot J K+3 D T(P D D \cdot J K-P S D 1 \cdot J K) / D C D$ $35 A-R \quad P S D 2 \cdot K I=P S D 2 \cdot J K+3 D T(P S D 1 \cdot J K-P S D 2 \cdot J K) / D C D$ $35 B-\mathrm{R} \quad \mathrm{PSD} \cdot \mathrm{KI}=\mathrm{PSD} \cdot \mathrm{JK}+3 \mathrm{DT}(\mathrm{PSD2} \cdot \mathrm{JK}-\mathrm{PSD} \cdot J K) / D C D$ Similarly for the factory sector we have:

36-R AMOF1•KI $=A M O F 1 \cdot J K+3 D T(A M D F \cdot J K-A M O F 1 \cdot J K) / D C F$ 36A-R AMOF2•KI $=A M O F 2 \cdot J K+3 D T(A M O F 1 \cdot J K-A M O F 2 \cdot J K) / D C F$ $36 B-R \quad A M O F \cdot K I=A M O F \cdot J K+3 D T(A M O F 2 \cdot J K-A M O F \cdot J K) / D C F$ Now the orders in clerical processing at the factory sector for the present time period are equal to the orders in process one time unit before plus the difference in the rate of manufacturing decision and rate of manufacturing orders into the factory multiplied by DT. So the equation becomes:

37-I $\quad C P F \cdot K=C P F \cdot J+D T(A M W F \cdot J K-A M O F \cdot J K)$
Similarly the level of orders in the clerical process at the retailers and the distributors are defined as:

38-I $C P R \cdot K=C P R \cdot J+D T(P D R \cdot J K-P S R \cdot J K)$
39-I $C P D \cdot K=C P D \cdot J+D T(P D D \cdot J K-P S D \cdot J K)$
$P D R$ and $P D D$ are the purchasing decision rates and $P S R$ and PSD are the actual rates of the purchase orders sent from the retailers and the distributors respectively.

The output of the clerical processing delay is the input to the mailing delay. Since there is no mail delay at the factory sector there will be two equations defining purchase orders in the mail from retail to distributor and from distributor to factory. For a continuous process with a DT time interval the purchase orders in the mail, at any moment, equals the purchase orders in the mail one time unit before plus the difference of the rate of purchase orders sent and the rate of orders received at the distributor or factory multiplied by delta time, DT.

The equation for the retail sector is:
40-I $\quad P M R \cdot K=P M R \cdot J+D T(P S R \cdot J K-R R D \cdot J K)$
And at the distributors it is:
41-I $\quad P M D \cdot K=P M D \cdot J+D T(P S D \cdot J K-R R F \cdot J K)$
RRD and RRF are the rate of orders received at the distributor and factory respectively. RRR, the rate of orders received at retail, is a quantity which is generated by the computer according to the type of distortion we want to give to the system. This is the only variable which changes the nature of the process in simulation runs. This is a sales rate to customers which in practice determines the behavior of inventory and production rate and other quantities of the production distribution system.

The quantities which determine the rate of orders received at the distributor and the factory are the purchase orders sent from the retail and the distributor and the delay in mail from the retail to the distributor and from the distributor to the factory. Mail delay is considered a third order delay, thus the rate of orders received is defined by three equations.

For the distributors:
42-R RRD1•K工 $=R R D 1 \cdot J K+3 D T(P S R \cdot J K-R R D 1 \cdot J K) / D N R$ 42A-R RRD2•KI $=R R D 2 \cdot J K+3 D T(R R D 1 \cdot J K-R R D 2 \cdot J K) / D M R$ $42 B-R \quad R R D \cdot K I=R R D \cdot J K+3 D T(R R D 2 \cdot J K-R R D \cdot J K) / D M R$

The similar three equations for the rate of orders received at the factory warehouse are:

43-R $\quad R R F 1 \cdot K J=R R F \cdot J K+3 D T(P S D \cdot J K-R R F 1 \cdot J K) / D M D$
43A-R RRF2•KI $=\mathrm{RRF2} \cdot \mathrm{JK}+3 \mathrm{DT}(\mathrm{RRF1} \cdot \mathrm{JK}-\mathrm{RRF2} \cdot \mathrm{JK}) / \mathrm{DND}$
43B-R $\quad R R F \cdot K L=R R F \cdot J K+3 D T(R R F 2 \cdot J K-R R F \cdot J K) / D M D$
As before we developed equations for the number of orders in the mail, similarly we can develop equations for the amount of goods on the way from the distributors to the retailers and from the factory warehouse to the distributors. The material in transit to retailers at a particular time equals the material in transit one time unit before plus the difference of the rate of shipments sent from the distributors and the rate of shipments received at the retail multiplied by DT. The equation for the retailers will be: 44-I $\quad A M P R-K=A M T R \cdot J+D T(S S D \cdot J K-S R R \cdot J K)$

Similarly for distributors:
45-I $\quad$ AMTD $\cdot K=A M T D \cdot J+D T(S S F \cdot J K-S R D \cdot J K)$
The corresponding equation of a little different type at the factory sector is for the number of orders in production at the factory. This will equal the number of orders one time unit before added to the difference of the rate of orders in manufacturing and shipments received at the factory warehouse multiplied by time DT.

The equation will be:
46-L OPF•K = OPF•J + DT (AMOF•JK - SRF•JK)
Lastly the quantities that need to be defined are the rate of shipments received at the retailers from the distributors and at the distributors from the factory warehouse and at the factory warehouse from the manufacturing unit. The delay encountered in getting shipments at the retailer, is a delay in transportation of the goods from the distributor, at the distributor it is a delay in transportation of goods from the factory warehouse and at the factory warehouse it is lead time delay at the factory.

Shipments received at retail are defined by three equations relating shipments sent from distributor and third order delay in transportation of goods from distributors (DTR). The equations are:

47-R $\quad S R R 1 \cdot K L=S R R 1 \cdot J K+3 D T(S S D \cdot J K-S R R 1 \cdot J K) / D T R$
47A-R $\quad$ SRR2•KL $=S R R 2 \cdot J K+3 D T(S R R 1 \cdot J K-S R R 2 \cdot J K) / D T R$
$47 B-R \quad S R R \cdot K I=S R R \cdot J K+3 D T(S R R 2 \cdot J K-S R R \cdot J K) / D T R$

Similarly shipments received at the distributor are defined as:
48-R SRD1•KI $=S R D 1 \cdot J K+3 D T(S S R \cdot J K-S R D 1 \cdot J K) / D T D$
48A-R SRD2•KI $=S R D 2 \cdot J K+3 D T(S R D 1 \cdot J K-S R D 2 \cdot J K) / D T D$
$43 B-R \quad S R D \cdot K L=S R D \cdot J K+3 D T(S R D 2 \cdot J K-S R D \cdot J K) / D T D$
The rate of shipments received at the factory warehouse is also related by a third order lead time delay to the manufacturing order rate into the factory.

The three equations define the rate of shipments received at the warehouse:
49-R $\quad \mathrm{SRF1} \cdot \mathrm{KI}=\mathrm{SRF1} \cdot J K+3 D T\left(A \mathrm{NOF}^{2} \cdot J K-S R F 1 \cdot J K\right) / D P F$
49A-R $\quad \mathrm{SRF2} \cdot \mathrm{KI}=\mathrm{SRF2} \cdot \mathrm{JK}+3 \mathrm{DT}(\mathrm{SRF1} \cdot \mathrm{JK}-\mathrm{SRF2} \cdot \mathrm{JK}) / D P F$
49B-R $\quad \mathrm{SRF} \cdot \mathrm{KI}=\mathrm{SRF} \cdot \mathrm{JK}+3 \mathrm{DT}(\mathrm{SRF} 2 \cdot \mathrm{JK}-\mathrm{SRF} \cdot \mathrm{JK}) / D P F$
All these equations give the mathematical description of the system. All variables except $R R R$, rate of orders received at retail are defined. As stated before $R R R$ will be generated according to the nature of the system we want to study.

The flow diagrams representing these equations are drawn on pages $40 \mathrm{a}, 40 \mathrm{~b}$, and 40 c . These diagrams do not represent computer flow diagrams but they show the flow of information, orders and goods. The solid line (___) shows the flow of goods, the dotted line ( - - - ) represents flow of orders and the dotted line with circles between two dashes (-0-0-) represent the flow of information. Arrow suggests the direction. The short names are used to describe


Pigure-4 Flow diagram of retail sector.


Figure-5 Flow diagram of distributor sector.


Figure-6 Flow diagram of factory sector.
the variables. The three flow diagrams of the retailer, distributor and factory sector give a good picture of each sector.

INITIAI CONDITIONS The equations relating flow of information, goods, orders and delays are developed in previous section, also the flow diagrams representing this information for retail, distributor and factory sector were shown. This furnishes a complete picture of the production distribution system model that is considered here. Before starting the actual computer simulation, it is necessary that $a l l$ of the equations are stated for the FORTRAN compiler. Since the flow of orders starts from customers to retailers we shall arrange the equations for retailers first, then distributors and finally the factory sector. For each sequence of the equations, it was kept in mind that all of the requirements of the variables must be satisfied. The previous period values were stored whenever necessary to use them. Three equations were given for third order delays. The general program for the computer simulation of this model is in appendix 1. Before simulating the system it is necessary that all the variables whose last period value is required are assigned initial values. This system has a self correcting property i.e. the system variables always try to be in a steady state depending upon the demand rate. Hence initially we shall assign the values of the variables in such a way that the complete system behavior is in a steady state. This means that if there is no change in the demand by the customers, there will not be any change in any of the variables. Thus the desired and the actual in-
ventory levels at the retailers, the distributors and the factory warehouse will be the same. All of the rates of ordering at each sector and the rate of production at the factory will be equal to the customers' demand. If the curves of all of the variable rates and levels are drawn, versus time all of the curves will be a straight line and parallel to the time axis. This shows that there is no change in the customers demand rate.

Initial values are assigned in terms of the customers' demand, so that steady state condition for any particular value of the demand rate can be achieved. This initial value of the demand rate is given a name RRI.

Now all of the variables representing the rate of goods or orders per week were assigned the same value RRI. The variables for rate equations are:

RRR: Rate of orders received at retail.
RSR: Smoothed rate of requisition at retail (exponential smoothing is employed).

SSR: Rate of shipments sent from retail.
PDR: Purchasing decision rate wanted at retail.
PSR: Actual purchasing orders sent from retail.
SRR: Rate of shipments received at retail.
PSR1, PSR2, SRR1, SRR2, the additional dummy variables added for the third order delay were also assigned the same initial values as PSR and SRR.

RRD: Rate of orders received at the distributor.

SSD: Rate of shipments sent from the distributor.
PDD: Decision of purchasing rate wanted at the distributor.
RSD: Smoothed rate of orders received at the distributor.
PSD: Actual purchasing orders sent from the distributors.
SRD: Rate of shipments received at the distributors.
Also the dummy variables SRD1, SRD2, PSD1, PSD2, RRD1 and RRD2 are added for representing third order delays and were assigned the same values.

RRF: Rate of orders received at the factory warehouse.
SSF: Rate of shipments sent from the warehouse.
RSF: Smoothed rate of requisitions at the factory warehouse.
SRF: Rate of shipments received at the factory warehouse.
AMOF: Rate of manufacturing orders into the factory. Also AMOF1, AMOF2, RRF1, RRF2, SRF1 and SRF2, the dummy variables for third order delays, were assigned the same values.

After assigning the initial values to the rate variables we need to assign values for level variables.

They are:
UOR: Number of unfilled orders at retail.
This value was obtained by multiplying the rate of orders received by the sum of the minimum order handing delay and a constant representing delay when the desired and actual inventory level are the same.

$$
U O R=R R I(D H R+D U R)
$$

Similar equations were also used for calculating initial
values of the number of unfilled orders at the distributor and the factory sector.

$$
\begin{aligned}
& U O D=R R I(D H D+D U D) \\
& U O F=R R I(D H D+D U F)
\end{aligned}
$$

The initial value of the actual level of inventories at retail, distributor and the factory warehouse were calculated from equating the actual level to the desired level. This assumption is valid because of the steady state condition of the system.

The respective initial values for the inventory levels are:

$$
\begin{aligned}
& \text { AIAR }=(\text { AIR })(\text { RRI }) \\
& \text { AIAD }=(\text { AID })(R R I) \\
& \text { AIAF }=(A I F)(R R I)
\end{aligned}
$$

The initial levels of goods in transit i.e. on the Way to respective location were calculated by multiplying the demand rate and the delay in transportation of goods from one location to another.

The equations are:
$A M T R=(D T R)(R R I)$
Goods in transit from the distributors to the retailers.
$A M T D=(D T D)(R R I)$
Goods in transit from the factory warehouse to the distributors' warehouse.

The similar equations apply for the level of orders
in the mail from one location to another. In this case the mailing delay will be used instead of transportation delay. The respective equations for orders in the mail from retailers to distributors and from distributors to factory warehouse are:

$$
\begin{aligned}
& \mathrm{PMR}=(\mathrm{DMR})(\mathrm{RRI}) \\
& \mathrm{PMD}=(\mathrm{DMD})(\mathrm{RRI})
\end{aligned}
$$

At the factory there will be some orders in production. OPF is the number of orders in production. Its initial. value was given as:

$$
O P F=(D P F)(R R I)
$$

Where DPF is the delay in the production lead time.
For calculating initial value of orders in process with the clerks at the retailers, distributors and factory sectors, similar equations were developed. In that case the delay in clerical processing took the place of the delay in mailing or lead time. Hence initial values of clerical in process orders at the respective sectors are:

$$
\begin{aligned}
& C P R=(D C R)(R R I) \\
& C P D=(D C D)(R R I) \\
& C P F=(D C F)(R R I)
\end{aligned}
$$

The above equations assign the initial values of the variables to put the system into existence in a steady state. Without the initial values of such a closed system it is impractical, or at least very difficult to achieve a satisfactory solution.

During definition of the various constants and variables, the values of the constants were also given. These values of the parameters and delays are chosen for a very general production distribution system. These values might vary considerably for the different corporations. The actual values for a particular system needs extensive data gathering and analysis. Even after that analysis one cannot get the actual constant. The mean value with some kind of distribution can be concluded after much labor. These values are changeable. In some kinds of business the orders are placed by a telegram or by a long distance telephone call which considerably reduces the mailing delay. As previously stated, this is a way of understanding the actual system, one can try different values of the parameters and study the change in the situation due to the change in the parameters. These changes are not equally sensitive for all the parameters. One might have considerable effect on the situation and the other might have very little effect. Also in practice, the complexity of the system structure, the existence of delays distributed throughout, the decisions that introduce amplification, and the time constants that arise from human memory and action and life span all combine to produce a system behavior that is independent of reasonable changes in most of the parameters.

It is natural that the values of parameters introduced in this system study are subject to change in various circumstances.

SIMULATION RUNS OF THE MODE NOW the equations are developed relating the different parameters of the system, and initial and constant values of the parameters have been assigned. This system is a closed loop system with only one opening. This opening is the demand of the customers at retail. The orders flow from the customers to the retailers and from the retailers to the distributors and from the distributors to the factory warehouse. At the factory sector, a decision is made for the production rate and the prepared goods are stored in the warehouse. The goods flow from the factory and warehouse, to the distributors, to the retailers and finally to the customers. Now to understand the nature of the system, we will simulate it for various alternatives. The first variable which can be given a perturbation is the demand rate at retail. The change in values for the constant parameters such as clerical delays, solution interval and maximum production capacity give another area for simulation. The following 10 cases are considered.

1. Response of the system when demand is decreased suddenly by 10 percent.
2. Response of the system of case 1 with reduced delays of inventory and pipeline adjustment.
3. Response of the system of case 1 with doubling the delays of inventory and pipeline adjustment.
4. Response of the system when the demand increases
linearly by 10 percent in 10 weeks.
5. Response of the system when demand suddenly increases by 10 percent with double the solution interval.
6. Response of the system when demand is seasonal that is, demand increases and decreases sinusoidally by $\pm 10$ percent over a one year period.
7. Response of the system of case 6 with reduced clerical delays.
8. Response of the system of case 6 with a maximum production capacity constraint ( 150 percent of the mean demand).
9. Response of the system when the demand is random, that is uniformly distributed between $\pm 10$ percent of the mean.
10. Response of the system of case 9 with a maximum production capacity constraint (120 percent of the mean demand).

Now in case 1 demand decreases suddenly by 10 percent. This is the case when there is a steady state condition with the demand at 1000 units per week is distorted by giving an impulse of a 10 percent reduction in demand.

In the computer program, the instruction was given such that, for the four weeks the demand is 1000 units per week and during fifth week due to some reasons the demand becomes 900 units per week.

The purpose behind this distortion in the system is to study the long-run effect on the different order rates and inventory levels at the various sectors and the change required in the manufacturing rate, as well as the time required for the system to become adjusted to the new rate of demand. When the demand decreases at the retallers there will be a higher level of inventory and a smaller number of unfilled orders at the retailers. Thus the retailers will order a smaller quantity from the distributors, hence a similar situation will arise at the distributors, so they will reduce their demand on the factory warehouse. The executives at the factory will make a decision to reduce the production rate. The decisions at all sectors will be affected by the time required in making the decision and the lead time required for adjusting the scheduling at the factory. The effect of lead time can be explained by an example: Suppose the present manufacturing rate is 1000 units per week and due to changes in the situation, a decision is made to reduce the production rate to 940 units per week. Let us assume that the lead time is 6 weeks at the factory. The decision is to reduce the rate by 60 units/week. If there is no other decision to be made and if there is no other effect, the production rate will be 990 for the first week, 980 for the second week, 970 for the third week, 960 for the fourth week, 950 for the fifth week and 940 for the sixth week and onwards. This is due
to the continuity assumption for the system. Also due to this assumption the new decisions are made every one tenth of a week (DT), hence there is a continuous change in the variables.

Now, the actual reduction does not appear for some time, thus the inventory at the factory warehouse during that time continues to increase. The rate of shipping will also increases, ultimately the inventory level increases at each sector. When the factory production is reduced considerably, not necessarily by 10 percent, the levels of the inventory starts reducing which causes an increased ordering rate at each sector. This oscillating behavior of the system continues for some time, each time the increase and decrease are damped a little more. Due to the self correcting nature of the system, the system tries to attain a steady state with a mean demand of 900 units per week.

The results of this type of behavior were plotted with time versus different rates and levels of the system. For plotting these curves a special subprogram was included in the main program and the curves were plotted in letters $A$, $B, C, D, E, F, G, H, I, J$ for the 10 different variables of the system. The program was developed so that the horizontal axis of the output paper represented units per week for a rate and number of units for level. 125 columns of the printer were utilized. Hence the limit on the horizontal axis
was 125 columns. The vertical axis of the output paper represented time in weeks. At each week the variables were plotted and the results were destroyed so that the space is available for the next week readings. Due to this characteristic of the program it was possible to simulate and plot for almost any number of weeks. A 100 weeks period was chosen for simulation which is quite reasonable for the type of study carried out here.

The plotting results of case 1 are shown in appendix 2 .
The 10 percent sudden reduction in demand after 4 weeks affected all the variables. The rate of orders received at the distributor decreased by 23 percent after 8 weeks and then the rate increases to be stable with a new rate of 900 units per week. The increase is not significant because the rate of the orders sent from the retailers is the very first one to be affected by a change in the demand at the retailers. The rate of orders received at the factory warehouse starts decreasing and reaches its minimum value of 40 percent less than original after 10 weeks and then starts increasing up to 8 percent more than required, again it decreases and becomes stable at 900 units per week.

Now the change in the production rate wanted at the factory decreases about 60 percent after 12 weeks and then it increases up to 13 percent more than required after 25 weeks. It takes about 45 to 50 weeks to become stable at the new desired production rate.

The rate of shipments received at the factory warehouse, that is the rate of the actual manufactured goods received at the warehouse also decreases by 50 percent and than again increases and than becomes stable. It is noted that the shipping rate lags about six weeks behind the desired production rate. It is also observed that the minimum production rate wanted is actually never achieved because when the decision is taken, it takes six weeks to reach that rate, meantime during the six weeks the situation changes which again alters the production rate. It is also noted that the minimum production rate wanted is achieved at the l2th week and the minimum shipping rate is reached at the 18 th week, exactly six weeks later.

The inventory of unfilled orders is not affected as much because as soon as the demand drops, the retailers cut down their demand to the distributors and hence they can adjust to their new environment in a short period so the number of unfilled orders reduces as the rate of demand decreases. The actual inventory of goods at retail increases slightly due to the reduction in demand but then decreases to the new level of inventory. Due to the reduction in the rate of demand from the distributors the actual level of inventory at the distributors also increases considerably. This 11 percent increase at the 12 th week causes a reduced rate of demand on the factory warehouse. When this shipping rate comes into effect the level at the distributors starts decreasing. This 20 percent decrease
again starts to increase toward the new desired level of inventory. The decisions always try to equate the desired and actual level of inventory.

The similar effects are experienced by the factory warehouse. Since the factory is remote from the distortion at the retailers the effect is experienced later and the distortion in the situation at the factory is greater than at the retailers or the distributors. The actual inventory level at the factory increases by 22 percent and then decreases by 30 percent at the 27 th week, afterwards it increases and becomes stable at the desired level after 45 to 50 weeks. As the inventory level increases at the beginning the number of unfilled orders decreases by 44 percent and then again increases and decreases to become stable with the new normal level of unfilled orders.

All the effects described above can be actually seen in the plotting of the curves in appendix 2.

The second case of the simulation run is the first case with a reduction in the delays of the inventory and pipeline adjustment. The constants DIR, DID and DIF determines the rapidity with which orders are placed to adjust inventories and pipeline content. For this system these constants actually affect the purchasing rate decision. Let us take an example.

The equation for the purchasing rate decision at retail is:

$$
\mathrm{PDR} \cdot \mathrm{KI}=\mathrm{RRR} \cdot \mathrm{JK}+\left(\frac{1}{\mathrm{DIR}}\right)\left(\begin{array}{l}
(\mathrm{AIDR} \cdot \mathrm{~K}-\mathrm{AIAR} \cdot \mathrm{~K})+ \\
(\mathrm{ALDR} \cdot \mathrm{~K}-\mathrm{AIAR} \cdot \mathrm{~K})+ \\
(\mathrm{UOR} \cdot \mathrm{~K}-\mathrm{UNR} \cdot \mathrm{~K})
\end{array}\right)
$$

From the equation $1 t$ is clear that if we decrease or increase the constant DIR the purchasing rate decision will increase or decrease respectively. These two cases are plotted in appendix 3 and 4. It is concluded that as the inventory adjustment policy is lengthened, the system changes from a highly oscillatory response with a short period of fluctuation toward a more stable mode of operation and a longer period between peaks. This conclusion is clearly seen from plots of appendix 2, 3, and 4.

The fourth case considered in this report is the 10 percent linear increase in demand in 10 weeks. This is usually the case when effective advertisement impresses the consumers. It is assumed that after 10 weeks the demand is stable at its final value. It is natural that the response of the system will be slower than the sudden increase in demand. Inventory levels fall considerably but recover slowly because of the long time constants of the inventory rebuilding that have affected the system policies. For computing the Iinear demand trend the instruction was given such that at every solution time an increase of one unit was made in the demand. The solution interval is 0.1 week so 10 units were increased for 10 weeks per week.

$$
R R R=R R R+1
$$

The fifth case is actually not to study the system
behavior but to judge the effect of a change in the solution interval in simulating the system. The choice of solution interval depends on the relation between the levels and flow rates in the system. In the system the rate equations were developed as follows.

$$
\begin{aligned}
& \text { LEV } \cdot K=\text { LEV } \cdot J+D T(I N \cdot J K-O U T \cdot J K) \\
& \text { Rate }=\frac{\text { Level }}{\text { Delay }}
\end{aligned}
$$

Exponential delays are assumed. But this exponential curve can be obtained only when the solution interval is very small. When DT is half of the delay the level rises to half of its final value, and the output rate also increases to half of its value. Half of the remaining difference between output and input rates is closed at each time interval. When the solution interval equals the delay time, the level and output rate reach their final values. A very small solution interval takes a long time to compute for a simulation of 100 weeks. As suggested by Forrester, the solution interval should be a small fraction, approximately one sixth of the length of time represented in third order delays of clerical and mailing delays. The amplitude of the curves increases as the solution interval increases. Hence a larger solution interval may give deeper curves than actually is the case. Thus we can conclude, a larger solution interval might tend towards less reasonable results, Which ultimately reduces the value of the information gained by the simulation study.

The sixth simulation run is the response of the system to an unexpected rise and fall ( $\pm 10 \%$ ) in the demand over a one year period. This is a typical case of seasonal demand, i.e. during the first 25 weeks the demand increases sinusoidally and then for the remaining 25 weeks decreases sinusoidally. The demand rate equation for generating a sinusoidal demand is given as:

$$
R R R=R R R+100 \operatorname{SINE}\left(\frac{\text { TIME IN WEEKS X 2 }}{50}\right)
$$

The above equation will cause the change in demand at all sectors as shown in appendix 6. Before the simulation run, the system was in a steady state with 1000 units per week as the retail demand. A sudden change in the characteristic of the demand pattern causes much distortion in the other parameters. As an example, the actual rate of production increases steadily by 45 percent at the end of 24 weeks and then it decreases by 65 percent of its steady state value at the end of 45 weeks. This is followed by an steady increase of 75 to 80 percent of its original value after 70 weeks. The increase and decrease in parameters continue. The inventory levels of the orders and goods at the different sectors also experiences a large fluctuation. This study shows some unusual results. This is due to our assumption that each sector does not consider the position of the other sector. If the executives at the factory can predict a reasonably close demand, they need not increase or decrease their production as much as indicated.

This also suggests that the people at the production level should be in direct contact with the consumers or at least the retailers and even they should use various forecasting techniques to predict the demand and hence to adjust themselves according to the changes.

In the seventh case we have repeated case 6 with reduced clerical delays. One objective of the system study is to evaluate changes that might be made in an industrial organization. A change frequently proposed is to substitute machine processes for manual clerical operations in order to speed up the flow of information. In this case we reduced clerical delays to one third of the original delays. The effect of this change is that the variables start to response earlier and there is a little reduction in the upper and lower limits of the variables. The clerical processing delays are a very small part of the system structure hence the effect is not very large.

The eighth case is case 6 with an upper limit placed on the production capacity, only a 50 percent increase is allowed. If the desired production rate is higher than 1500 units per week only 1500 units per week is allowed for the production rate. In such a highly responsive system the upper limit caused very unusual results. In the second period the unfilled orders at the factory warehouse increases by about 400 percent. Due to our limited scale of horizontal axis on the plotting program this change caused disturbances
in the output after 75 weeks.
The ninth case has a random demand as the input to the system. The random number generator subroutine of the computing center was employed for generating random numbers. The uniform distribution of the demand was assumed for the demand between 900 and 1100. The simple rule used is:

If we want samples from a uniform distribution between $A$ and $A+B$, the $n$-th sample will be:

$$
\mathrm{Xn}_{n}=\mathrm{PnB}+\mathrm{A}
$$

Where
Rn- Random number generated.
The results of this case were not more distortive than previous cases. The results were not smooth. There was not a definite period and a definite amplitude of the curves. The fluctuation of the variables never becomes stable even If the smoothed demand is around 1000 units per week. This type of observations indicate the maximum reserve capacity needed for the warehouses and production facility.

The last case was similar to a previous case with a limited production rate. This is the usual case, factory production cannot be increased beyond a certain limit. This case indicates that random demand is possible without unlimited production capacity. Also due to the limited production capacity the production unit operates at its maximum capacity for a long period. This type of situation avoids unnecessary changes in the production rate which are
not economical and practical.
All of the above 10 simulation runs provide good material to begin to understand the dynamic behavior of the system. Several other cases can be considered, to learn in detail about other parameters in the system, also new variables of accounting, advertisement and forecasting techniques may be included.

CONCLUSION: In the attempt to simulate the system, and to try different alternatives, the main aim was to collect information about the system response as a whole and the system parameters in particular. It is this information which will lead the executives of a system to take decisions. The decisions may vary from the design of inventory warehouses to the changes in employe level for achieving desirable changes in the production rate. The managers are interested in more useful information on which to base decisions. Greater usefulness can be achieved by improving existing information sources and by basing decisions upon new and different sources that have not been used in the past. Better information is worth the value that we attach to the industrial performance.

In this study we have seen how changes in parameters affect the system and the extent of the effect on the system. In our simulation run we see that if the factory executives know the sales rate at retail they can plan better, well in advance and without causing such tremendous disturbances. The confidence in the estimation of retail sales is also important.

From the results of the simulation runs it becomes clear that the manager must be able to provide both the required investment money and the warehouse capacity for the fluctuating inventory. This type of study also provides information before hand for the designing of inventory ware-
houses. After fudging the effect of the purchasing rate and variables affecting the decision, a reasonable and effective purchasing policy can be developed. In a practical situation when a corporation wants to launch a massive advertisement for its product they must be ready and well prepared for the situation which may arise after a period of effective advertisement. As an example they must be ready to produce more according to the requirement, otherwise the inventory levels of the distributors and retailers will be considerably reduced which will cause unnecessary delay in the shipping of goods to the consumers.

This type of simulation helps in any expansion program for the corporation. This can be done by constructing a model of the present system and then by changing it for the necessary expansions of the system. The simulation of the changed system will clearly indicate the areas that require more attention and more investment and where unnecessary costs can be reduced.

It is not easy to construct a model of the existing system, it requires sharp judgement and years of experience. The main task is in selecting the values of the different parameters and in stating their relationships. Eventhough accurate results are not possible, the results obtained by such a study have considerable value. The results and value of the information depend mostly upon the people who use them. Hence it is necessary that the people who make decisions
should also be acquainted with the ways in which the models are constructed and simulated. These things play an important role in the final decisions.

Finally it is concluded that the solution to better system behavior lies not in more information at one place but in a change in operating policies at each sector. The results of the various simulation runs offer ample evidence of the feasibility of performing such studies on small computers such as the IBM 1410 with a FORTRAN compiler.

APPENDIX 1. SIMULATION PROGRAM FOR STEP DECREASE IN DEMAND.

```
    MONSS JOB SIMULATION OF MULTIWAREHOUSE INVENTORY SYSTEM
    MONSS COMT 3OMINUTES, O5PAGES
    MONSS ASGNMUB,I2
    NONSS ASGN MGO,16
    NONES MODE GO,TEST
    MONSS EXEQ FORTRAN,,,,,,,ANIL
        OINENSIONA(20),B(125)
        FORMAT(20AI)
        FORMAT(F6.0,125A1)
        FORMAT(11F4.1,2F6.0)
        FORMAT(13F5.1)
        FORMAT(F5.0,10F6.0,F5.1)
        FORMAT!IHK,47HCASE- STEP DECREASE IN DEMAND FROM 1000 T0 900)
```

$1 C 0$ FORMATIIHK,47H CURVES OF TIME IN WEEKS VS A,B,C,D,E,F,G,H,I, J /IHK
1,3EH A=RATE OF ORDERS RECEIVED AT RETAIL /IHK,41H B=RATE OF ORDERS
2 RECEIVED AT DISTRIBUTOR /IHK, $47 H$ C=RATE OF ORDERS RECEIVED AT FAC
3TORY WAREHOUSE /IHK,39H D=MANUFACTURING RATE WANTED AT FACTORY / IH
4K,50Hi E=RATE OF SHIPMENTS RECEIVED AT FACTORY WAREHOUSE /1HK,41H F
$5=$ = NVENTORY OF UNFILLED ORDERS AT RETAIL)
101 FORMAT $11 H K, 38 H$ G=ACTUAL INVENTORY OF GOODS AT RETAIL/1HK,43H H=AC
ITLAL INVENTORY OF GOODS AT DISTRIGUTOR /IHK,49H I=ACTUAL INVENTORY
2 OF GOODS AT FACTORY WAREHOUSE /IHK, $42 \mathrm{H} J=I N V E N T O R Y$ OF UNFILLED OR
3DERS AT CACTORY /IHK,89H SCALE 16 UNITS/WK $=1$ COLUMN (SPACE) FOR R
4ATE AND 30 UNITS $=1$ COLUMN FOR INVENTORY LEVEL)
102 FORMAT: $105 \mathrm{X}, 1 \mathrm{H}, 25 \mathrm{HI111111111111111111111111/14X,1H,116H1111111}$
111122222222233333333334444444444555555555566666666667777777777888
288888889999999999000000000011111111111222222/5X,1H,125H1234567890
3123456789012345678901234567890123456789012345678901234567890123456
47890123456789012345678901234567890123456789012345 )
READ (1,11) DT,DHR,DHD, OHF,DUR,DUD,DUF,AIR,AID,AIF,DRR,DRD,DRF
READII,2:1 DIR,DID,DIF,DCR,DCD,DCF,DMR,DMD,DTR,DTD,DPF,RRI,ALF
READ(1,12)(A(I),I=1,20)
: RRITE(3,99)
WRITE(3,100)
裙TE(三,101)
YRITE(3,102)
RTMX=2600.
P.TVN=0.
$A L V X=10000$.
ALAN $=0$.
RRFI =RRI
RRDI =RRI
RRRI=RRI

```
SSR=RRI
SSD=RRI
PDR=RRI
PDD=RRI
SSF=RRI
CPR=DCR*RR!
PMR=DMR*RRI
CPD=DCD*RRI
PMD=DMD*RRI
OPF=DPF*RRI
CPF=DCF*RRI
RSR=RRI
RSD=RRI
RSF=RRI
RRD=RRI
PSR=RRI
SRR=RRI
RRF=RRI
PSD=RRI
SRD=RRI
SRF=RRI
RRR=RRI
AMOF=RRI
SRF2=RRI
SRFI=RRI
SRD2=RRI
SRDI=RRI
PSD2=RRI
PSDI=RRI
RRF2=RRI
RRF1=RRI
SRR2=RRI
SRRI=RRI
PSR2=RRI
PSR1=RRI
RRD2=RRI
RRDI=RRI
AMWF=RRI
RRRI=RRI
AMOFI=RRI
AMOF2=RRI
UこF=RSF*(DHF+DUF)
U'CD=RSD*(DHD+DUD)
UOR=RSR*(DHR+DUR)
AIAF=AIF*RRI
AIAD=AID*RRI
```

```
    AMTR=DTR*RRI
    AMTD=DTD*RRI
    AIAR=AIR*RRI
    Y=0.
    I=0
80 I=I+I
    0050 L=1,10
    Y=Y+1.
    IF(Y-40.) 40,40,30
30 RRR=900.
40 UこR=UCR+?T*(RRRI-SSR)
    AIAR=AIf 2+DT*(SRR-SSR)
    RSR=RSR+DT*(RRRI-RSR)
    AIDR=AIR*RSR
    DFR=DHR+DUR*AIDR/AIAR
    ANIR=AIAR/DT
    SSR=UCR/OFR
    IF(SSR-ANIR)I,\,2
2 SSR=ANIR
1 CONTINUE
    UNR=RSR*(DHR+DUR)
    CPR=CPR+DT*(PDR-PSR)
    PMR=PMK +DT*(PSR-RRD)
    AMTR=AMTR+DT*(SSD-jRR)
    UOD=UOD+DT*(RRD-SSD)
    ALAR=CPR+PMR+UCD+AMTR
    RSD=RSD+DT*(RRD-RSD)/DRD
    AIDD=AID*RSD
    AIAD=AIAD+DT*(SRD-SSD)
    DFD=DHD+DUD*AIDD/AIAD
    ALDR=RSR*(DCR+DMR+DFD+DTR)
    RRD=RRD+3.*DT*(RRD 2-RRD)/DMR
    RRD2=RRD2+3.*DT*(RRD1-RRD2)/DMR
    RRDI=RRDI+3.*DT*(PSR-RRDI)/DMR
    PSR=PSR+3.*DT*(PSR2-PSR)/DCR
    PSR2=PSR2+3**DT*(PSR1-PSR2)/DCR
    PSR1=PSR1+3.*DT*(PDR-PSR1)/DCR
    SRR=SRR+3**DT*(SRR2-SRR)/DTR
    SRR2=SRR2+3.*DT*(SRR1-SRR2)/DTR
    SRRI=SRR1+3.*DT*(SSD-SRRI)/DTR
    PDR=RRRI-(AIDR-AIAR+ALDR-ALAR+UCR-UNR)/DIR
    SSD=UUD/DFD
    ANID=AIAD/DT
    IF(SSD-ANID) 3,3,4
4SSD=ANID
```

3 CONTINUE
$A I A F=A I A F+D T *(S R F-S S F)$
$R S F=R S F+D T *(R R F-R S F) / D R F$
$A I D F=A I F * R S F$
$D F F=D H F+D U F \approx A I D F / A I A F$
$A L D D=R S D *(D C D+D M D+D F F+D T D)$
$C P D=C P D+D T *(P D D-P S D)$
$U \cap F=U Q F+D T *(R R F-S S F)$
$A M T D=A M T D+D T *(S S F-S R D)$
$P M D=P M D+D T *(P S D-R R F)$
$A L A D=C P D+P M D+U N F+A M T D$
UND $=$ RSD* (DHD + DUD)
$R R F=R R F+3 . * D T *(R R F 2-R R F) / D M D$
RRF2=RRF2+30*DT*(RRF1-RRF2)/DMD
RRF1=RRF1+3.*DT*(PSD-RRF1)/DMD
$P S D=P S D+3 . * D T *(P S D 2-P S D) / D C D$
PSD2 $=$ PSD2+3.*DT*(PSD1-PSD2)/DCD
PSDI=PSD1+3.*DT*(PDD-PSD1)/DCD
$S R D=S R D+3 \cdot * D T *(S R D 2-S R D) / D T D$
SRD2 $=$ SRD $2+3$. $\because D T *(S R D 1-S R D 2) / D T D$
SRDI=SRDI+3.*DT*(SSF-SRDI)/DTD
$P D D=R R D I+(A I D D-A I A D+A L D D-A L A D+U C D-U N D) / D I D$
SSF=UCF J JFF
ANIF=AIAF/DT
IF (SSF-ANIF)5,5,6
6 SSF=ANIF
5 CONTINUE
$U N F=R S F *(D H F+D U F)$
$C P F=C P F+D T *(A M W F-A M O F)$
$\triangle P F=O P F+D T *(A M O F-S R F)$
ALAF $=C P F+O P F$
$A L D F=R S F *(D C F+D P F)$
$S R F=S R F+3 . * D T *(S R F 2-S R F) / D P F$
SRF2 $=$ SFF $2+3$ •*DT*(SRFI-SRF2)/DPF
SRF1=SRF1+3.*DT*(AイMF-SRF1)/DPF
$A M O F=A M O F+3 . * D T *(A M O F 2-A M O F) / D C F$
AMOF2=AMOF2+3•*DT*(AMOF1-AMOF2)/DCF
AMOF1=AMOF1+3•*DT*(AMWF-AMOF1)/DCF
$A M W F=R R F I+(A I D F-A I A F+A L D F-A L A F+U C F-U N F) / D I F$
IF(ALF-AMWF)7,7,8
7 AMWF=ALF
8 CONTINUE
RRFI =RRF
RRDI $=$ RRD
RRRI = RRR

```
    X=I
    Dこ22J=1,125
    22 B(J)=A(1)
    I2=125.*(RRR-RTMN)/(RTMX-RTMN)+.5
    13=125.*(RRD-RTMN)/(RTMX-RTMN)+.5
    I4=125.*(RRF-RTMN)/(RTMX-RTMN)+.5
    I5=125.*(AMWF-RTMN)/(RTMX-RTMN)+.5
    I6=125.*(SRF-RTMN)/(RTMX-RTMN)+.5
    I7= 125.*(UNR-ALMN)/(ALMX-ALMN)+.5
    I 8=125.*(AIAR-ALMN)/(ALMX-ALMN)+.5
    I 9=125.*(AIAD-ALMN)/(ALMX-ALMN)+.5
    I10=125.*(AIAF-ALMN)/(ALMX-ALMN )+.5
    I11=125.*(UCF-ALMN)/(ALMX-ALMN)+.5
    B(I2)=A(2)
    B(I3)=A(3)
    B(I4)=A(4)
    B(I5)=A(5)
    B(I6)=A(6)
    B(17)=A(7)
    B(I8)=A(8)
    B(I9)=A(9)
    B(I10)=A(10)
    B(I11)=A(11)
    WRITE(3,13)X,(B(M),M=1,125)
    IF(I-100)90,90,70
    90 Gこ Tこ 80
    7 0 ~ S T C P ~
        END
        MONSS EXEQ LINKLOAD
        CALL ANIL
        MONSS EXEQ ANIL,MJB
.1 1.0 1.0 1.0 .4 .0 1.0 8.0 6.0 4.0 8.0 8.0 8.0
4.0 4.0 4.0 1.0 0.6 0.3 0.5 0.5 1.0 2.0 6.0 1000. 9999.
ABCDEFGHIJ
    MONSS JOB ACT$S A.T.PATEL I.E.DEPT. 0313C40409
```


# Appendix 2. Results of step decrease in demand Appendix 3. Results of step decrease in demand with reduced inventory and pipeIne adjustment delays <br> Appendix 4. Results of step decrease in demand with increased inventory and pipeline ad justinent delays 

CURVES OF TIME IN WEEKS VS A,B,C,D,E,F,G,H,I,J
A=RATE OF ORDERS RECEIVED AT RETAIL
B=RATE OF ORDERS RECEIVED AT DIStributor
C=RATE OF ORDERS RECEIVED AT FACTORY WAREHOUSE
D=MANUFACTURING RATE WANTED AT FACTORY
E=RATE OF SHIPMENTS RECEIVED AT FACTORY WAREHOUSE
F=INVENTORY OF UNFILLED ORDERS AT RETAIL
G=ACTUAL INVENTORY OF GOODS AT RETAIL
H=ACTUAL INVENTORY OF GOODS AT DISTRIBUTOR
I=ACTUAL INVENTORY OF GOODS AT FACTORY WAREHOUSE
$J=I N V E N T O R Y$ OF UNFILLED ORDERS AT FACTORY


CURVES OF TIME IN WEEKS VS $A, B, C, D, E, F, G, H, I, J$
$A=$ RATE OF ORDERS RECEIVED AT RETAIL
$B=$ RATE OF ORDERS RECEIVED AT DISTRIBUTOR
L- RATE OF ORDERS RECEIVED AT FACTORY WAREHOUSE
D=MANUFACTURING RATE WANTED AT FACTORY
E=RATE OF SHIPMENTS RECEIVED AT FACTORY WAREHOUSE
F=INVENTORY OF UNFILLED ORDERS AT RETAIL
G=ACTUAL INVENTORY OF GOODS AT RETAIL
H=ACTUAL INVENTORY OF GOOOS AT DISTRIBUTOR
I =ACTUAL INVENTORY OF GOODS AT FACTORY WAREHOUSE
J=INVENTORY OF IJNFILLED ORDERS AT FACTORY
SCALE 16 UNITS/WR = 1 COLUMN (SPACE) FOR RATE AND 80 UNITS $=1$ COLUMN FOR INVENTORY LEVEL



SUDDEN 10 PERCENT DECREASE IN DEMAND WITH REDUCED INVENTORY AND PIPELINE ADJUSTMENT DELAYS
CURVES OF TIME IN WEEKS VS $A, B, C, D, E, F, G, H, I, J$
A=RATE OF ORDERS RECEIVED AT RETAIL
$B=$ RATE OF ORDERS RECEIVED AT DISTRIBUTOR
$C=$ RATE OF ORDERS RECEIVED AT FACTORY WAREHOUSE
D=MANUFACTURING RATE WANTED AT FACTORY
E=RATE OF SHIPMENTS RECEIVED AT FACTORY WAREHOUSE
F=INVENTORY OF UNFILLED ORDERS AT RETAIL
G = ACTUAL INVENTORY OF GOODS AT RETAIL
H=ACTUAL INVENTORY OF GOODS AT DISTRIBUTOR
I = ACTUAL INVENTORY OF GOODS AT FACTORY WAREHOUSE
J=INVENTORY OF UNFILLED ORDERS AT FACTORY
SCALE 16 UNITS/WK = 1 COLUMN (SPACE) FOR RATE AND 80 UNITS $=1$ COLUMN FOR INVENTORY LEVEL


Appendix 5. Results of Linear Inerease
In demand
Appendix 6. Results of step increase in
demand with double the
solution interval.
Appendix 7. Resulte of seasonal demand

CURVES OF TIME IN WEEKS VS $A, B, C, D, E, F, G, H, I, J$
$A=$ RATE OF CRDERS RECEIVED AT RETAIL
B=RATE OF ORDERS RECEIVED AT DISTRIBUTOR
C=RATE OF ORDERS RECEIVED AT FACTORY WAREHOUSE
D=MANUFACTURING RATE WANTED AT FACTORY
E=RATE OF SHIPMENTS RECEIVED AT FACTORY WAREHOUSE
$F=I N V E N T C R Y$ OF UNFILLED ORDERS AT RETAIL
G=ACTUAL INVENTORY OF GOCDS AT RETAIL H=ACTUAL INVENTORY OF GOODS AT DISTRIBUTOR

I = ACTUAL INVENTORY OF GOODS AT FACTORY WAREHOUSE
$J=I N V E N T C R Y$ OF UNFILLED ORDERS AT FACTORY
SCALE 16 UNITS/WK = 1 COLUMN (SPACE) FOR RATE AND 80 UNITS $=1$ COLUMN FOR INVENTORY LEVEL

CURVES OF TIME IN WEEKS VS $A, B, C, D, E, F, G, H, I, J$
$A=$ RATE OF ORDERS RECEIVED AT RETAIL
$B=R A T E$ OF ORDERS RECEIVED AT DISTRIBUTOR
$C=R A T E$ OF ORDERS RECEIVED AT FACTORY WAREHOUSE
D=MANUFACTURING RATE WANTED AT FACTORY
$E=$ RATE OF SHIPMENTS RECEIVED AT FACTORY WAREHOUSE
$F=I N V E N T O R Y$ OF UNFILLED ORDERS AT RETAIL.
G=ACTUAL INVENTORY OF GOODS AT RETAIL
H=ACTUAL INVENTORY OF GOCDS AT DISTRIBUTOR
I=ACTUAL INVENTORY CF GOODS AT FACTORY WAREHOUSE
$J=I N V E N T O R Y$ OF UNFILLED ORDERS AT FACTORY
SCALE 16 UNITS/WK = 1 COLUMN (SPACE) FOR RATE AND 80 UNITS = 1 COLUMN FOR INVENTORY LEVEL



## $\square \rightarrow$

Appendix 8, Results of seasonal demand with reduced clerical delays
Appendix 9. Results of seasonal demand with production capacity constraint

RESPONSE OF SYSTEM TO A 10 PERCENT UNEXPECTED RISE AND FALL IN DEMAND OVER A ONE YEAR PERIOD WITH REDUGED CLERICAL OELAYS
CURVES OF TIME IN WEEKS VS $A, B, C, D, E, F, G, H, I, J$
$A=$ RATE OF ORDERS RECEIVED AT RETAIL
B=RATE OF ORDERS RECEIVED AT DISTRIBUTOR
C=RATE OF ORDERS RECEIVED AT FACTORY WAREHOUSE
D=MANUFACTURING RATE WANTED AT FACTORY
E=RATE OF SHIPMENTS RECEIVED AT FACTORY WAREHOUSE
= INVENTORY OF UNFILLED ORDERS AT RETAIL
G=ACTUAL INVENTORY OF GOODS AT RETAIL
H=ACTUAL INVENTORY OF GOODS AT DISTRIBUTOR
I=ACTUAL INVENTORY OF GOODS AT FACTORY WAREHOUSE
=INVENTORY OF UNFILLED ORDERS AT FACTORY


SINUSOIDAL DEMAND OVER A ONE YEAR PERIOD WITH MAXIMUM PRODUCTION CAPACITY 50 PERCENT HIGHER THAN MEAN
CURVES OF TIME IN WEEKS VS A,B,C,D,E,F,G,H,I,J
$A=$ RATE OF ORDERS RECEIVED AT RETAIL
$B=$ RATE OF ORDERS RECEIVED AT DISTRIBUTOR
C= RATE OF ORDERS RECEIVED AT FACTORY WAREHOUSE
D= MANUFACTURING RATE WANTED AT FACTORY
E=RATE OF SHIPMENTS RECEIVED AT FACTORY WAREHOUSE
F= INVENTORY OF UNFILLED ORDERS AT RETAIL
$G=A C T U A L$ INVENTORY OF GOODS AT RETAIL
H=ACTUAL INVENTORY OF GOODS AT DISTRIBUTOR
I =ACTUAL INVENTORY OF GOODS AT FACTORY WAREHOUSE
$J=I N V E N T O R Y$ OF UNFILLED ORDERS AT FACTORY
SCALE 16 UNITS/WK $=1$ COLUMN (SPACE) FOR RATE AND 12OUNITS $=1$ COLUMN FOR INVENTORY LEVEL



# Appendix 10. Results of random demand <br> Appendix 11. Results of random demand with production capacity constraint 



DEMAND IS UNIFORMLY DISTRIBUTED BETWEEN 900 AND 1100
CURVES OF TIME IN WEEKS VS $A, B, C, D, E, F, G, H, I, J$
$A=$ RATE OF ORDERS RECEIVED AT RETAIL
$B=$ RATE OF ORDERS RECEIVED AT DISTRIBUTOR
C=RATE OF ORDERS RECEIVED AT FACTORY WAREHOUSE
$D=M A N U F A C T U R I N G$ RATE WANTED AT FACTORY
E=RATE OF SHIPMENTS RECEIVED AT FACTORY WAREHOUSE
F=INVENTORY OF UNFILLED CRDERS AT RETAIL
G=ACTUAL INVENTORY OF GOODS AT RETAIL
H=ACTUAL INVENTORY OF GOODS AT DISTRIBUTOR
I = ACTUAL INVENTORY OF GOODS AT FACTORY WAREHOUSE
J=INVENTORY OF UNFILLED ORDERS AT FACTORY
SCALE 16 UNITS/WK = 1 COLUMN (SPACE) FOR RATE AND 80 UNITS $=1$ COLUMN FOR INVENTORY LEVEL





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ANIIKUMAR T. PATEI
B. S., Sardar Vallabhbhai Vidyapeeth, India, 1965
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AN ABSTRACT OF A MASTER'S REPORT

# submitted in partial fulfillment of the 


MASTER OF SCIENCE
Department of Industrial Engineering
KANSAS STATE UNIVERSITY
Manhattan, Kansas
1967

## ABSTRACT:

This report is a simulation study of a production distribution system. The particular system considered in the report is taken directly from part of Mr . Forrester's work in his book Industrial Dynamics. The system consists of customers, one retail sector, one distributor sector and a factory sector. For simplicity, only the flows of materials and orders are considered.

The various stages of the system are interconnected by a system of equations which define the inventory policy, order placing and the decision making procedures of the system. The system behavior is continuous and dynamic. The distortion at one stage affects the behavior of the whole system. In this study 10 different cases are considered in which the system is distorted by changing the rate of customer demand or by changing some of the delays. The results are directly plotted with time versus the different rates and levels of the system.

The simulation study of such a model provides an excellent tool to understand the dynamic behavior of a practical situation. It is helpful in creating successful management policies and organizational structures.

This type of simulation study was done by Forrester using the "DYNAMO" language on an IBM 709 or 7090 computer. One of the aims of this report is to check the feasibility of this study by using the FORTRAN language on an IBM 1410
computer. It is seen from the results of this study, that FORTRAN is a suitable language for the study of industrial dynamics and that fruitful problems may be simulated on the IBM 1410 computer.

