THE POWER OF CONOVER'S K-SAMPLE SLIPPAGE TEST

by

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1. INTRODUCTION.

In testing a hypothesis $\theta = \theta_o$ against an alternative $\theta = \theta_1$ there are two types of error which can be made. The type I error is made if the hypothesis $\theta = \theta_o$ is rejected when it is in fact true. A type II error is made if the hypothesis $\theta = \theta_o$ is accepted when the alternative against which it is being tested, $\theta = \theta_1$, is in fact true. The power of the test at $\theta = \theta_1$ is the probability that the test will reject the hypothesis $\theta = \theta_o$ if in fact $\theta = \theta_1$. Or the power of the test may be stated in other words as 1 minus the probability of a type II error.

The purpose of this report is to examine the power of Conover's k-Sample Slippage test. The study made is a Monte Carlo study on the IBM 1410 computer. This report will begin by giving a general survey of power studies made of some nonparametric tests and will be followed by a general discussion of Conover's k-Sample Slippage test, the different types of hypotheses which can be tested, how to use the table for testing these hypotheses, and an analysis of the power of Conover's k-Sample Slippage test based on the Monte Carlo study which was done on the computer. A print out of the computer program and comments concerning the program appear in the appendix of this report.

2. SURVEY OF RECENT POWER STUDIES OF SOME NONPARAMETRIC TESTS.

The most pressing need in the theory and practice of nonparametric tests at this time seems to be a need for results concerning the power of such tests, particularly those based on ranks. This would provide a basis for comparing the many different tests proposed, as well as for determining the sample size necessary to distinguish significant departures from a hypothesis with a reasonable degree of certainty.

The chief problem one is faced with when investigating the power of a nonparametric test is the choice of suitable alternatives. Even in the simplest problem the variety of alternatives is so great that it is clearly impossible to consider all of them. In the past, investigators have concentrated on alternatives postulating normal distributions for the random variables in question. These alternatives, which unfortunately are rather difficult to handle, must, of course, be studied if one wishes to find out how nonparametric methods compare with procedures based on normal theory. On the other hand when comparing different rank tests, one is no longer tied to normal alternatives, but it would seem proper to make the comparisons in terms of nonparametric classes of alternatives.

Lehmann (1953) looked at the power of rank tests against certain types of alternatives, and optimum properties of Wilcoxon's one and two-sample tests and of the rank correlation tests for independence.

Bateman (1948) derived the distribution of the longest run under the hypothesis of randomness and looked at the power function when the alternative hypothesis is that of positive dependence in the sequence both for a simple Markoff chain and when the structure of dependence is more complex.

The computation of power under normality is simplest for small samples and small levels of significance.

Dixon (1953) studied power and power efficiency of four nonparametric tests (rank-sum, maximum deviation, median, and total number of runs) for detecting differences in means of two samples drawn from normal populations with equal variance. The cases considered are for equal sample sizes of three, four and five observations and alternatives $\delta = |\mu_1 - \mu_2| / \sigma$.

The conclusion of the above study for the four nonparametric tests considered resulted in high power efficiences for very small samples and small α, when compared with the t-test for normal alternatives. Power efficiency decreases slightly for more distant alternatives. As the level of significance increases, the power efficiency of the rank sum test increases slightly whereas the power efficiencies of the median and maximum deviation tests decrease.

The local power efficiencies for the rank sum test are very high. For all cases considered they are greater than $3/\Pi$, the limiting local power for large samples.

Dixon (1953) also looked at power functions of the sign test and power efficiency for normal alternatives. Power functions were tabulated for the sign test for various sample sizes and α near .05 and .01. Several of these power functions were compared with the power function of the t-test for samples from normal populations by means of a power efficiency function. The results indicated decreasing power efficiency for increasing sample size, for increasing level of significance and for increasing alternatives.

Epstein (1955) made comparisons of some nonparametric tests against normal alternatives with an application to life testing.

The hypothesis set up was for equal means or,

$$H_o: \mu_1 = \mu_2$$

under the assumption of normal distributions with equal variance.

The hypothesis was tested on the basis of samples of size ten drawn from each population. The performance and relative merits of four non-parametric test procedures were studied experimentally.

The following table summarizes the experimental findings for the 200 pairs of samples, where each sample is of size ten. Samples correspond to the cases where $d = |(\mu_1-\mu_2)/\sigma| = 1,2,3$.

TABLE 1

OBSERVED PROBABILITY OF ACCEPTING $H_o(d = 0)$ BASED ON 200 PAIRS OF SAMPLES, EACH OF SIZE TEN

$d = \left\|\frac{\mu_1-\mu_2}{\sigma}\right\|$	Rank Sum	Run	Exceedance			Maximum Deviation		
			r=1	r=2	r=3	r=3	r=6	r=10
0	.935	.965	.95	.96	.96	.955	.945	.945
1	.485	.795	.655	.65	.60	.575	.555	.555
2	.015	.275	.16	.12	.10	.065	.045	.045
3	0	.02	.025	0	0	0	0	0

The following remarks are pertinent:

(i) As r increases, there appears to be a slight improvement in the power of exceedance and maximum deviation tests. It happens that the truncated maximum deviation test for r=6 has the same experimental O.C. curve as the untruncated maximum deviation test for the particular samples being reported in this paper.

(ii) The maximum deviation test has slightly better power than the exceedance test for the particular samples being reported in this paper.

(iii) Ranked in order of power we have: Rank sum, best; run test, worst; exceedance and maximum deviation tests in between.

In order to be able to make more positive and more general statements, particularly in (i) and (ii), we would need much more in the way of experimental evidence. To settle the question completely awaits the theoretical treatment of what seems to be a complicated analytical problem.

3. DISCUSSION OF CONOVER'S K-SAMPLE SLIPPAGE TEST.

Conover's k-Sample Slippage test is presented in a paper by Conover (1964). Conover's k-Sample Slippage test is a nonparametric or a distribution free test statistic. It is analagous to the one-way analysis of variance. The greatest advantage it has over the analysis of variance is that it is a quick and easy-to-compute statistical test. Exactly what Conover's k-Sample Slippage test gains or loses in the way of power at this point cannot be stated with certainty but some interesting results have been compiled from the Monte Carlo study.

Let $X_{1j}, X_{2j}, \ldots, X_{nj}$; $j = 1, 2, \ldots, k$, represent k random samples drawn from populations with distribution functions $F_j(X)$, $j = 1, 2, \ldots, k$, respectively. The null hypothesis to be tested can be stated as:

H_o: $F_1(x) = F_2(x) = \ldots + F_k(x)$ (i.e. all the samples were drawn from identical populations)

versus the alternative hypothesis

H_1: $F_1(x;\theta_1) = F_2(x;\theta_2) = \ldots = F_k(x;\theta_k)$, where $\theta_i \neq \theta_j$ for at least one pair (i,j).

3.1 Assumptions.

The analysis of variance is the most powerful test under the following assumptions:

(1) Each of the "observable" random variables is normally distributed.

(2) Each of the k random samples is normally distributed with common standard deviations or,

$\sigma = \sigma_1 = \sigma_2 \; . \; . \; . = \sigma_k \; .$

The assumption for using Conover's k-Sample Slippage test is that the distribution functions being considered are all continuous. The purpose of this assumption is to eliminate ties. In practice tied values do occur, and can be handled in a manner described later.

3.2 Test Statistic.

To evaluate the test statistic for Conover's test, first order each of the k-samples within itself from the greatest to the least in the usual manner. Then order the samples among themselves on the basis of the greatest random variable in each sample. The result is shown in Figure 1. The ordered random variable Y_{ij} represents the random variable of rank i within its own sample, and the sample containing Y_{ij} has rank j among all k samples.

$$
\begin{array}{ccccccccc}
Y_{1,1} & > & Y_{1,2} & > & Y_{1,3} & > & \cdots & > & Y_{1,k} \\
\vee & & \vee & & \vee & & & & \vee \\
Y_{2,1} & & Y_{2,2} & & Y_{2,3} & & \cdots & & Y_{2,k} \\
\vee & & \vee & & \vee & & & & \vee \\
Y_{3,1} & & Y_{3,2} & & Y_{3,3} & & \cdots & & Y_{3,k} \\
\vee & & \vee & & \vee & & & & \vee \\
\cdots & & \cdots & & \cdots & & \cdots & & \cdots \\
\vee & & \vee & & \vee & & & & \vee \\
Y_{n,1} & & Y_{n,2} & & Y_{n,3} & & \cdots & & Y_{n,k}
\end{array}
$$

FIGURE 1

The test statistic m for Conover's k-Sample Slippage test is the number of values in the sample of rank 1 that exceed the top value or extreme value in the sample of rank k.

3.3 Significance.

Tables of critical values of m for up to 20 samples and for selected values of n ranging from 2 to ∞ have been calculated at the .05, .01 and .001 significance level.

Using the table for the appropriate values of k and n, and the desired level of significance α, let $j = k$. Reject H_o in favor of H_1 if m equals or exceeds the value in the table. Do not reject H_o if the value of m is less than the value in the table.

If H_o is rejected, it is sometimes desirable to determine which sample or samples are significantly better than the others. Two methods of doing this are suggested.

3.4 Method 1.

If it is desired to determine which sample or samples come from populations having the greatest location parameters, let m be the number of values from the sample of rank 1 that exceeds the extreme value from the sample of rank 2, and enter the table as before except that now $j = 2$. If m is less than the critical value as given in the table, repeat the procedure for $j = 3, 4$, and so on, comparing the values from the sample of rank 1 with the top value from the sample of rank 3, 4, and so on, until the value of m equals or exceeds the critical value given in the table. At this point a significant difference between the sample of rank 1 and the sample of rank j is established, and the first $j - 1$ samples can be considered as having come from populations having the greatest location parameters.

3.5 Method 2.

If it is desired to eliminate the population or populations with the smallest location parameters, compare the sample of rank 1 successively with the samples of rank $k - 1$, $k - 2$, etc., in the manner described under method 1, until the test statistic m is less than the critical value in the table. If this first occurs while comparing the sample of rank 1 with the sample of rank j, the samples of rank $j + 1, \ldots, k$ can be considered to have come from populations with the smallest location parameters.

It is possible for method 1 and method 2 to yield different results.

3.6 Ties.

In case one or more values from the sample of rank 1 exactly equal the top value from the sample of rank j, the method of determining m described above results in a conservative test; that is, the actual level of significance is smaller than level of significance indicated in the tables. If there is difficulty in determining the ranks of the samples because of ties in their extremes, it is suggested that these ties be resolved by comparing the next highest values from the samples in question, and assigning ranks correspondingly. This procedure may be continued until the tie is broken.

The above discussion assumes each sample contains the same number of observations. Our interest was in looking at the power of Conover's k-Sample Slippage test for equal sample size.

An example will follow to illustrate the use of Conover's k-Sample Slippage test. In the example reference will be made to table 2 which will appear after the example. This table is only a segment of the tables which have been produced for Conover's k-Sample Slippage test. For complete tables see Conover (1964).

3.7 Example.

The following data, from soil tests conducted by Kansas State University, represent nitrogen gains (all negative) under different rotation plans. It is desired to determine whether any significant differences exist among the six different plans. The first step is to determine the extreme in each sample, denoted by an asterisk, and the second step is to record the sample ranks on the basis of these extremes. The results are as follows:

Rotation Plans

	1	2	3	4	5	6
	-.048	-.036	-.048*	-.040	-.006	-.019
	-.061	-.029*	-.060	-.044	-.020	-.026
	-.042*	-.037	-.057	-.033*	-.028	-.008*
	-.043	-.036	-.054	-.048	-.012	-.032
	-.047	-.029	-.056	-.037	-.009	-.011
	-.055	-.038	-.059	-.043	-.010	-.018
	-.051	-.037	-.049	-.039	-.005*	-.031
rank	5	3	6	4	1	2

The test statistic

m = number of values from the sample of rank 1 that exceeds the extreme value from the sample of rank 6

$= 7$

is greater than or equal to the critical value 7 obtained in the table for $k = 6$, $j = 6$, $n = 7$, $\alpha = .01$. Hence the null hypothesis of no differences among plans is rejected at the .01 level of significance.

To determine which rotation plans should be retained for further testing, the samples of ranks 1 and 2 are compared. The test statistic is now

m = the number of values from the sample of rank 1 exceeding the top value in the sample rank 2

$= 2$

which is less than the value 3 found in the table under $k = 6$, $j = 2$, $n = 7$, $\alpha = .05$, and hence the decision of no significant difference is reached. Next the samples of rank 1 and 3 are compared in a similar way and $m = 7$, which is equal to or greater than the critical value of 6 found under $j = 3$, $\alpha = .001$. Hence only rotation plans 5 and 6 are considered for further analysis.

If it is desired to eliminate the rotation plans which have the smallest location parameters, compare the sample of rank 1 with the sample of rank $j = 5$. The test statistic is now

m = the number of values from the sample of rank 1 exceeding the top value or extreme value in the sample of rank 5

= 7

which is greater than or equal to the value 6 under $k = 6$, $j = 5$, $n = 7$, $\alpha = .001$ and hence the decision of significant difference is reached.

The above procedure can be continued for $j = 4, 3$ and in both cases the decision of significant difference is reached.

For $j = 2$, the decision of no significant difference is reached. Hence, rotation plans having rank 3, 4, 5, 6 can be considered to have come from populations with the smallest location parameters.

Note: In both cases the result was the same but they need not yield the same result.

If it is desired to compare the samples of ranks r and s, for $r \neq 1$ and $s > r$, the tables are still valid, if entered with the "reduced number of samples" $k' = k - r + 1$, and the "reduced sample rank" $j' = s - r + 1$, as illustrated in the following. Suppose in the above example it is desired to compare the sample of rank 2 with the sample of rank 3. Here $r = 2$ and $s = 3$. Let

m' = the number of values from the sample of rank r exceeding the top value in the sample of rank s

= 5

for this example. Looking in the table given in Conover (1964) for $k' = k - r + 1 = 5$ samples, under $j' = s - r + 1 = 2$, the critical value is seen to be $m' = 5$ for $\alpha = .001$. Since the value of m' actually obtained equals or exceeds the critical value, it can be stated, with some confidence, that rotation plan 6 is better than the next best rotation plan, plan 2.

TABLE 2. CRITICAL VALUES FOR CONOVER'S K-SAMPLE SLIPPAGE TEST

$k = 6$ $\alpha = .05$

n =	2	3	4	5	6	7	8	...
j								
2	—	3	3	3	3	3	3	...
3	—	3	3	3	4	4	4	...
4	—	—	4	4	4	4	4	...
5	—	—	4	5	5	5	5	...
6	—	—	—	5	6	6	6	...

$k = 6$ $\alpha = .01$

n =	2	3	4	5	6	7	8	...
j								
2	—	3	4	4	4	4	4	...
3	—	—	4	4	4	4	5	...
4	—	—	4	5	5	5	5	...
5	—	—	—	5	6	6	6	...
6	—	—	—	—	6	7	7	...

$k = 6$ $\alpha = .001$

n =	2	3	4	5	6	7	8	...
j								
2	—	—	4	5	5	5	5	...
3	—	—	—	5	5	6	6	...
4	—	—	—	—	6	6	6	...
5	—	—	—	—	—	7	7	...
6	—	—	—	—	—	—	8	...

4. DISCUSSION OF DISTRIBUTIONS STUDIED.

4.1 Rectangular.

The first distribution considered was the rectangular distribution, denoted by R.

If X is R(0,1) then $E(X) = \frac{1}{2}$, $Var(X) = 1/12$

Hence aX is R(0,a) and $E\ (aX) = a/2$; $Var(ax) = a^2/12$

so the coefficient of variation $= \frac{\sqrt{Var}}{mean} = 1/\sqrt{3} =$ constant

Six rectangular populations were set up by letting a = 1, 1.05, 1.10, 1.15, 1.20, and 1.25.

The observations or random variables for each distribution were generated by obtaining a four digit random number, dividing this number by 10,000 and multiplying by the corresponding factor given above.

4.2 Exponential.

The second distribution studied was the exponential distribution:

$$f(x) = \theta \cdot e^{-x\theta} \quad \theta > 0$$

$$E(X) = \frac{1}{\theta}\ ;\ Var(X) = \frac{1}{\theta^2}$$

so the coefficient of variation $= \frac{\sqrt{Var}}{mean} = 1$

Six exponential populations were set up by letting θ = 1, 1.2, 1.4, 1.6, 1.8, and 2.0. The observations or random variables were generated by considering the following transformation.

Any density for a continuous variable X may be transformed to the uniform density:

$$f(y) = 1 \quad 0 < y < 1$$

by letting $Y = F(X)$ where $F(x)$ is the cumulative distribution of X

$$F(x) = \int_0^x \theta e^{-\theta t}\,dt = 1 - e^{-\theta x}$$

Hence $Y = F(X)$ is $R(0,1)$

or

$$Y = 1 - \theta^{-\theta X}$$

$$1-Y = \theta^{-\theta X}$$

(1) $\qquad -\frac{1}{\theta}\ln(1-Y) = X$ which is exponential.

Hence we proceed to draw a value of the uniform $R(0,1)$ random variable, call it Y and substitute this value into equation (1) for each θ in order to obtain each value of the exponential random variable.

4.3 Chi-square.

The third population considered was the chi-square population with k degrees of freedom for k = 2, 2, 2, 3, 3, 3.

$$\text{Coefficient of Variation} = \frac{\sqrt{\text{Var}}}{\text{mean}} = \frac{\sqrt{2k}}{k}$$

If X_i is a normal (0,1) random variable and if the X_i are mutually independent then

$Y = X_1^2 + X_2^2 + \ldots + X_k^2$ is a chi-square random variable with k degrees of freedom.

In order to get normal (0,1) random variables the transformation developed by Box and Muller (1958) was used. If U1 and U2 are two independent random uniform or R(0,1) deviates then RN1 and RN2 are two independent random normal deviates where

$$RN1 = \sqrt{-2 \ln U1} \cdot \cos(2\Pi U2)$$

$$RN2 = \sqrt{-2 \ln U1} \cdot \sin(2\Pi U2)$$

This transformation was used in distributions 4.4 and 4.5 to follow.

4.4 Normal with Unequal Variances.

The fourth population studied was the normal population with mean b = 5.0, 5.2, 5.4, 5.6, 5.8, and 6.0 with corresponding standard deviations σ = 1.0, 1.2, 1.4, 1.6, 1.8, and 2.0 .

For each population the coefficient of variation

$$= \frac{\sqrt{Var}}{mean} = \frac{\sigma}{b} \quad .$$

The following transformation was used to obtain the observed random variables.

If Y is N(0,1), then

$aY + b$ is $N(b, \sigma = a)$.

4.5 Normal with Equal Variance.

The last population studied satisfied the conditions for which the analysis of variance is most powerful. The populations were normal with mean 5.0, 5.2, 5.4, 5.6, 5.8, 6.0 and all with equal standard deviation $\sigma = 1$.

For each of the populations described above the sample size $n = 35$.

The value of n equals 35 was chosen to give critical values for Conover's k-Sample Slippage test as close as possible to α = .05, .01, .001. The actual α levels for Conover's test were α = .0485, .0044, .0007.

For the corresponding analysis of variance interpolation over degrees of freedom was used to give exact α levels of .05, .01, .001.

For each distribution, except the chi-square, 100 runs or trials were made on the computer. For the chi-square 25 runs were made.

TABLE 3. RESULTS OF MONTE CARLO STUDY

	α = .05 Level		α = .01 Level		α = .001 Level	
	Accepted	Rejected	Accepted	Rejected	Accepted	Rejected
		Population 4.1:	Rectangular			
A.O.V.	76	24	91	9	99	1
C.S.T.	36	64	79	21	96	4
		Population 4.2:	Exponential			
A.O.V.	32	68	52	48	78	22
C.S.T.	60	40	84	16	90	10
		Population 4.3:	Chi-square			
A.O.V.	7	18	11	14	19	6
C.S.T.	21	4	25	0	25	0
		Population 4.4:	Normal with unequal variance			
A.O.V.	34	66	57	43	87	13
C.S.T.	12	88	36	64	54	46
		Population 4.5:	Normal with equal variance			
A.O.V.	0	100	9	91	24	76
C.S.T.	53	47	85	15	94	6

Notation

A.O.V. represents analysis of variance.

C.S.T. represents Conover's k-Sample Slippage test.

5. SUMMARY OF MONTE CARLO STUDY.

Table 4 below shows the estimate of power for the analysis of variance and Conover's k-Sample Slippage test for each of the populations considered and for each of the three α levels.

The results of the Monte Carlo study are:

(1) For the rectangular distribution the power of Conover's test at the .01 level appeared to be approximately equal to the power of the analysis of variance at the .05 level.

(2) For the exponential distribution the power of Conover's test at the .01 level appeared to be approximately equal to the power of the analysis of variance at the .001 level.

(3) For the chi-square distribution the power of Conover's test appeared to be approximately equal to the power of the analysis of variance at the .001 level.

(4) For the normal distribution with unequal variances the power of Conover's test at the .001 level appeared to be approximately equal to the power of the analysis of variances at the .01 level.

(5) For the normal distribution with equal variance the power of Conover's test appeared to have less power at the .05 level than the analysis of variance did at the .001 level.

TABLE 4. ESTIMATION OF POWER FOR A.O.V. VERSUS C.S.T.

	α = .05		α = .01		α = .001	
	A.O.V.	C.S.T.	A.O.V.	C.S.T.	A.O.V.	C.S.T.
Population 4.1	.240	.640	.090	.210	.01	.04
Population 4.2	.680	.400	.480	.160	.220	.100
Population 4.3	.720	.160	.640	0.0	.240	0.0
Population 4.4	.660	.880	.430	.640	.130	.460
Population 4.5	1.00	.470	.910	.150	.760	.060

6. TOTAL COMPUTER TIME USED.

All of the computing was done on the IBM 1410 computer. Table 5 shows the approximate computer time involved.

TABLE 5. COMPUTER TIME

	Number of Runs	Approximate Time
Population 4.1	100	1 Hour
Population 4.2	100	1 1/4 Hours
Population 4.3	25	1 2/3 Hours
Population 4.4	100	2 1/2 Hours
Population 4.5	100	2 Hours

7. ACKNOWLEDGEMENT.

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9. APPENDIX.

The program listing which immediately follows is the program which was used to assemble population where the distributions are normal with equal variance 1 (section 4.5). The program is written in Fortran IV version for the IBM 1410 computer. Included are comments within the program to indicate to some degree what the program is doing at the different stages. If the comments are followed closely then this program can easily be used or can be easily modified to consider any statistical population in which a study is being made to compare Conover's k-Sample Slippage test to the analysis of variance for power.

There were two random number generators used in this Monte Carlo study. The first random number generated used was developed by Koh (1966). The author developed the second random number generator and decided to use it as a check for consistancy of results. Both showed to yield very consistent results. However, Koh's random number generator appeared to be about twice as fast. The process used in the second random number generator was to select arbitrarily an 8 digit number, call it A, and one which did not end in zero, five, or six. The number A is squared which yields a 15 or 16 digit number. (see figure 2)

$$A = \text{xxxxxxxx.0}$$

$$A^2 = \text{xxxxxxxx}\underline{\text{xxxx}}\text{xxxx.0}$$

FIGURE 2

The first random number will be the four digits underlined. To obtain a second random number take A times the last 8 digits (that is the first 8 digits in A^2 beginning with the unit digit) in A^2.

One may repeat this process obtaining random numbers, 3, 4, and so on.

```
      MON$$      JOB  MASTERS REPORT
      MON$$      COMT 45,80 PAGES,,CASH STATISTICS
      MON$$      ASGN MJB,12
      MON$$      ASGN MGO,16
      MON$$      MODE GO,TEST
      MON$$      EXEQ FORTRAN,,,17,17,,,CASH
C
C
C       THIS PROGRAM ASSEMBLES 6 NORMAL POPULATIONS WITH MEANS OF 5.0,5.2,
C       5.4,5.6,5.8,AND 6.0 WITH EQUAL VARIANCE  OF 1.0 THERE WILL BE 35 V
C       ALUES IN EACH POPULATION.THE ANALYSIS OF VARIANCE TABLE IS PRINTED
C       OUT FIRST THEN THE 35 VALUES ARE PRINTED FOR EACH POPULATION.THE L
C       AST VALUE TO APPEAR IN EACH POPULATION WILL BE THE EXTREME OR THE
C       HIGHEST VALUE IN THAT POPULATION.THE RESULTS,OR THE ACCEPTANCE O
C       R REJECTION BY THE ANALYSIS OF VARIANCE AND CONOVERS K SAMPLE SILL
C       IPAGE TEST FOR THE THREE LEVELS.05,.01,.001 RESPECTIVELY APPEAR NE
C       XT
        DIMENSION X(38,6),UTTS(7),KR(7)
        DIMENSION ALPHA(4),MNX(4),NREJ(4),MREJ(4),CRIT(4)
        DIMENSION XMAX(10)
C
C
   50 FORMAT(1H1,10X,33HANALYSIS  OF  VARIANCE,  NUMBER  ,I3,49H,  FOR
     1A  COMPLETELY  RANDOMIZED  DESIGN  WITH  ,I2,2X,10HTREATMENTS)
   51 FORMAT(////,18X,19HSOURCE OF VARIATION,8X,18HDEGREES OF FREEDOM,10
     1X,14HSUM OF SQUARES,10X,12HMEAN SQUARES,///,23X,9HTREATMENT,19X,I3
     1,18X,F15.5,7X,F15.5,//,25X,5HERROR,21X,I3,18X,F15.5,7X,F15.5,//,25
     1X,5HTOTAL,21X,I3,18X,F15.5)
   52 FORMAT(35I2)
   53 FORMAT (F10.5)
   54 FORMAT(////,18X,2HF(,I2,1H,,I3,4H) = ,F10.5)
   55 FORMAT(10X,6F10.5)
   56 FORMAT(////,40X,18HTEST STATISTICS M=I4)
   57 FORMAT(////,20X,27HTEST STATISTIC M REJECTS AT,F4.3,5X,5HLEVEL)
   58 FORMAT(////,20X,27HTEST STATISTIC M ACCEPTS AT,F4.3,5X,5HLEVEL)
  111 FORMAT(////,25X,9H.05 LEVEL,25X,9H.01 LEVEL,30X,10H.001 LEVEL)
  110 FORMAT(//,20X,2I6)
   59 FORMAT(////,20X,35HTHE ANALYSIS OF VARIANCE REJECTS AT,F4.3,5X,5HL
     1EVEL)
   60 FORMAT(////,20X,35HTHE ANALYSIS OF VARIANCE ACCEPTS AT,F4.3,5X,5HL
     1EVEL)
  112 FORMAT(////,3X,13HCONOVERS TEST,4X,8HACCEPTS ,I3,13H AND REJECTS ,
     1I3,7X,8HACCEPTS ,I3,13H AND REJECTS ,I3,8X,8HACCEPTS ,I3,13H AND R
     2EJECTS,I3)
  113 FORMAT(/,2X,16HANALYSIS OF VAR.,2X,8HACCEPTS ,I3,13H AND REJECTS ,
     1I3,7X,8HACCEPTS ,I3,13H AND REJECTS ,I3,8X,8HACCEPTS ,I3,13H AND R
     2EJECTS,I3)
        DO 942 I=1,4
        NREJ(I)=0
        MREJ(I)=0
  942 CONTINUE
        IDX=0
C
C       AXY1 DENOTES THE INITIAL VALUE USED IN THE RANDOM NUMBER GENERATOR
C       AXY1 IS AN 8 DIGIT FLOATING POINT NUMBER.AXY1 MUST BE CHANGED FOR
C       EACH RUN OF THIS PROGRAM IN ORDER TO GET NEW RANDOM NUMBERS.
C
        AXY1=10909981.0
C
C       NWW DENOTES THE NUMBER OF RUNS PER POPULATION DESIRED.FOR THIS POP
C       ULATION  NWW=15 WILL TAKE ABOUT 30 MINUTES ON THE COMPUTER.
C
        NWW=12
        ID=88
        AY1=AXY1
        PI=3.1415927
        NREPS=210
        NR=35
        NT=6
        DO 7 I=1,6
    7 KR(I)=35
        XNWW=10.0
        DO 999 IKK=1,NWW
        ID=ID+1
```

```
    5 USCRFR=0.
      UTLSS=0.
      UTRTSS=0.
  833 CONTINUE
      XFACT1=5.0
C
C
C     THE STATEMENTS FROM HERE TO 10 CONTINUE ARE THE BODY OF THE PROGRA
C     M WHICH INCLUDES THE RANDOM NUMBER GENERATOR FORM WHICH THE DISTRI
C     BUTIONS ARE ASSEMBLED. THE POPULATIONS VALUES ARE STORED IN AN ARR
C     AY X(35,6).THE 35 INDICATES 35 OBSERVATIONS PER POPULATION AND THE
C     6 INDICATES THE NUMBER OF POPULATIONS.
C
C
      DO 10 J=1,NT
      UTTS(J)=0.0
      DO 11 JK=1,18
      DO 639 IWX=1,2
      AXY1=AXY1*AY1
      NXY1=AXY1/100000000.0
      AXY2=NXY1*100000000
      AXY1=AXY1-AXY2
      NRN1=AXY1/10000.0
      IF(IWX.EQ.2)GO TO 640
      U1=NRN1
      U1=U1+.50
      U1=U1/10000.0
      GO TO 639
  640 U2=NRN1
      U2=U2+.50
      U2=U2/10000.0
  639 CONTINUE
C
C     THE NEXT  4  STATEMENTS PERFORM WHAT IS KNOWN AS THE DIRECT METHOD
C     OF PRODUCING TWO RANDOM NORMAL DEVIATES  FROM TWO RANDOM UNIFORM
C     DEVIATES DEVELOPED BY BOX AND MULLER IN 1958.
C
      XAB=SQRT(-2.0*ALOG(U1))
      XAC=COS(2.0*PI*U2)
      RN1=XAB*XAC
      RN2=XAB*SIN(2.0*PI*U2)
      X(2*JK,J)=RN2+XFACT1
      X(2*JK-1,J)=RN1+XFACT1
   11 CONTINUE
      XFACT1=XFACT1+.20
   10 CONTINUE
C
C     AT THIS POINT WE BEGIN THE CALCULATIONS FOR THE ANALYSIS OF VARIAN
C     CE.
C
      DO 20J=1,NT
      NR=KR(J)
      DO20I=1,NR
      UTTS(J)=UTTS(J)+X(I,J)
      UTLSS=UTLSS+X(I,J)**2
      IF(I-NR)20,15,30
   15 R=NR
      UTRTSS=UTRTSS+((UTTS(J)**2)/R)
      USCRFR=USCRFR+UTTS(J)
   20 CONTINUE
      NDFTT=NT-1
      NDFER=NREPS-NT
      NDFTL=NREPS-1
      REPS=NREPS
      CRFR=(USCRFR**2)/REPS
      TRTSS=UTRTSS-CRFR
      TTLSS=UTLSS-CRFR
      ERRSS=TTLSS-TRTSS
      DFTT=NDFTT
      TRTMS=TRTSS/DFTT
      DFERR=NDFER
      ERRMS=ERRSS/DFERR
      F=TRTMS/ERRMS
      WRITE (3,50) ID,NT
```

```
      WRITE(3,51)NDFTT,TRTSS,TRTMS,NDFER,ERRSS,ERRMS,NDFTL,TTLSS
      WRITE(3,54)NDFTT,NDFER,F
C
C     AT THIS POINT WE BEGIN DETERMING  THE VALUE OF THE TEST STATISTIC
C     M FOR CONOVERS K SAMPLE SLIPPAGE TEST.
C
      DO 21 I=1,NT
      XMAX(I)=X(1,I)
      DO 21 J=1,34
      IF(XMAX(I).GT.X(J+1,I))GO TO 21
      XMAX(I)=X(J+1,I)
   21 CONTINUE
      NKW=1
      DO 23 I=1,5
      IF(XMAX(NKW).GT.XMAX(I+1))GO TO 23
      NKW=I+1
   23 CONTINUE
      MKW=1
      DO 25 I=1,5
      IF(XMAX(MKW).LT.XMAX(I+1))GO TO 25
      MKW=I+1
   25 CONTINUE
      MCOUNT=0
      DO 27 I=1,35
      IF(X(I,NKW).LT.XMAX(MKW))GO TO 27
      MCOUNT=MCOUNT+1
   27 CONTINUE
      DO 28 I=1,35
      WRITE(3,55)(X(I,J),J=1,6)
   28 CONTINUE
      WRITE(3,55)(XMAX(K),K=1,6)
      WRITE(3,56)MCOUNT
      ALPHA(1)=.05
      ALPHA(2)=.01
      ALPHA(3)=.001
      CRIT(1)=2.26
      CRIT(2)=3.11
      CRIT(3)=4.26
      MNX(1)=7
      MNX(2)=10
      MNX(3)=12
      DO 150 I=1,3
      IF(MCOUNT.GE.MNX(I))GO TO 130
      WRITE(3,58)ALPHA(I)
      GO TO 135
  130 WRITE(3,57)ALPHA(I)
      MREJ(I)=MREJ(I)+1
  135 IF(F.GE.CRIT(I))GO TO 140
      WRITE(3,60)ALPHA(I)
      GO TO 150
  140 WRITE(3,59) ALPHA(I)
      NREJ(I)=NREJ(I)+1
  150 CONTINUE
      IDX=0
  999 CONTINUE
      NDIFF1=NWW-MREJ(1)
      NDIFF2=NWW-MREJ(2)
      NDIFF3=NWW-MREJ(3)
      NDIFF4=NWW-NREJ(1)
      NDIFF5=NWW-NREJ(2)
      NDIFF6=NWW-NREJ(3)
      WRITE(3,111)
      WRITE(3,112)NDIFF1,MREJ(1),NDIFF2,MREJ(2),NDIFF3,MREJ(3)
      WRITE(3,113)NDIFF4,NREJ(1),NDIFF5,NREJ(2),NDIFF6,NREJ(3)
   30 STOP
      END
    MON$$        EXEQ  LINKLOAD
                 CALL  CASH
    MON$$        EXEQ  CASH,MJB
    MON$$        JOB   ACT$$CASH                    STATISTICS0332U40333
```

THE POWER OF CONOVER'S K-SAMPLE SLIPPAGE TEST

by

WILLIAM STEPHEN CASH

B. S., University of Dayton, 1965

AN ABSTRACT OF A MASTER'S REPORT

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Conover's k-Sample Slippage Test is a non-parametric test which is analogous to the one-way analysis of variance for testing the hypothesis that k samples could have come from populations with equal means, or location parameters. The main advantage of Conover's k-Sample Slippage Test over the analysis of variance is that it is a quick and easy-to-compute statistical test.

The power of the test at $\theta = \theta_1$ is the probability that the test will reject the hypothesis $\theta = \theta_o$ if in fact $\theta = \theta_1$. The purpose of this report is to investigate the power of Conover's k-Sample Slippage Test versus the power of the analysis of variance. The study made is a Monte Carlo study on the IBM 1410 computer. Five distributions were considered:

(4.1) Rectangular; $R(0,a)$ $a = 1.0, 1.05, 1.10, 1.15, 1.20, 1.25$

(4.2) Exponential; $f(x) = \theta e^{-\theta x}$ $\theta = 1.0, 1.2, 1.4, 1.6, 1.8, 2.0$

(4.3) Chi-square; degrees of freedom $k = 2, 2, 2, 3, 3, 3$

(4.4) Normal; $N(b,\sigma = a)$ $b = 5.0, 5.2, 5.4, 5.6, 5.8, 6.0$, and $a = 1.0, 1.2, 1.4, 1.6, 2.0$, respectively.

(4.5) Normal; $N(b,\sigma = 1)$ $b = 5.0, 5.2, 5.4, 5.6, 5.8, 6.0$

The hypothesis tested for each of the distributions can be stated as follows:

$$H_o: \; F_1(X) = F_2(X) = F_3(X) = F_4(X) = F_5(X) = F_6(X)$$

versus

$$H_1: \; F_1(X;\theta_1) = F_2(X;\theta_2) = F_3(X;\theta_3) = F_4(X;\theta_4) = F_5(X;\theta_5) = F_6(X;\theta_6)$$

$$\theta_i \neq \theta_j \text{ for at least one pair } (i,j)$$

A sample size of $n = 35$ observations was chosen for each population.

For each distribution, except the chi-square, 100 runs or trials were made on the computer. For the chi-square distribution 25 runs were made.

The results of the Monte Carlo study are:

(1) For the rectangular distribution the power of Conover's test at the .01 level appeared to be approximately equal to the power of the analysis of variance at the .05 level.

(2) For the exponential distribution the power of Conover's test at the .01 level appeared to be approximately equal to the power of the analysis of variance at the .001 level.

(3) For the chi-square distribution the power of Conover's test appeared to be approximately equal to the power of the analysis of variance at the .001 level.

(4) For the normal distribution with unequal variances the power of Conover's test at the .001 level appeared to be approximately equal to the power of the analysis of variances at the .01 level.

(5) For the normal distribution with equal variance the power of Conover's test appeared to have less power at the .05 level than the analysis of variance did at the .001 level.