ALBERT'S RELIABILITY ALLOCATION TECHNIQUE WITH RELAXED ASSUMPTION ON EFFORT FUNCTION

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CHAPTER 1

Introduction

Reliability is defined as the probability that a system will perform its intended function for a specified interval under stated conditions. As it relates to equipments, components, and parts, reliability is one of the important characteristic by which the usefulness of a system is judged. Reliability often given as function of time but we are assuming constant values. In calculating the reliability of a given system it is necessary to specify the system configuration which describes how the equipment is connected, and the rules of operation. The present work is based on a theorem by A. Albert [1], who introduced a technique of increasing the reliability of a series system with minimum effort.

Let n components be connected in series configuration and let R_i be the reliability of i^{th} component. The reliability of the system is given by

$$R_s = \prod_{i=1}^{n} R_i \qquad 0 \leq R_i \leq 1 \tag{1.1}$$

Let $\bar{R}$ be the required reliability of the system, where $\bar{R} > R_s$. It is then required to increase at least one of the values of the R_i so that the required reliability $\bar{R}$ will be obtained in accordance with Equation (1.1). Accomplishing such an increase requires a certain "effort", which is to be allotted in some way among the components (under the conditions assumed by Albert, which are discussed below). The technique of increasing R_s to $\bar{R}$ with minimum effort is as follows:

(A) Order the known reliabilities $R_1, R_2, \ldots, R_n$ in nondecreasing order (we assume now that such an ordering is implicit in the notation) so that

$$R_1 \leq R_2 \leq R_3 \leq \cdots \leq R_n \tag{1.2}$$

(B) Increase each of the reliabilities $\underline{R_1, R_2, \ldots, R_{K_0}}$ to the same value $\bar{R}_0$; but do not attempt to increase the reliabilities $R_{K_0+1}, \ldots, R_n$. The number K_0 is determined as

K_0 = Maximum value of j such that

$$R_j < \left(\frac{\bar{R}}{\prod_{i=j+1}^{n+1} R_i} \right)^{1/j} = r_j \quad \text{(say)} \tag{1.3}$$

Why need specify r_j?

where $R_{n+1} = 1$ by definition.

The number $\bar{R}_0$ is determined as

$$\bar{R}_0 = \left(\frac{\bar{R}}{\prod_{j=K_0+1}^{n+1} R_j} \right)^{1/k_0} \tag{1.4}$$

(C) It is evident that the system reliability will then be $\bar{R}$, since

$$\text{New reliability} = \bar{R}_0^{\,k_0} R_{k_0+1} \cdots R_n = \bar{R}_0^{\,k_0} \prod_{j=k_0+1}^{n+1} R_j \tag{1.5}$$

From using equation (1.4) we immediately obtain

New reliability = $\bar{R}$

The foregoing technique is based on the existence of an effort function $G_i(x,y)$ which measures the amount of effort (weight, volume, cost, manpower, etc) needed to increase the reliability from x to y for i^{th} component. The effort function is assumed to satisfy the following requirements:

1. $G(x,y) \geq 0$, which means that increasing the reliability from level x to higher level y will always need at least zero effort.
2. $G(x,y)$ is nondecreasing in y for fixed x and nonincreasing in x for fixed y; eg.,

   $$G(0.35, 0.65) \leq G(0.35, 0.75)$$

   and $$G(0.25, 0.65) \leq G(0.35, 0.65)$$

transitive 3. If $\underline{x} \leq \underline{y} \leq \underline{z}$, $G(x,y) + G(y,z) = G(x,z)$,

   which states that the amount of effort to increase the reliability from x to z is equal to the sum of effort to increase the reliability from x to y, then from y to z namely, $G(x,y)$ is additive.
4. $G(0,x)$ has a derivative $h(x)$ such that $xh(x)$ is strictly increasing in $(0 < x < 1)$.

If $\bar{R}$ is the minimum requirement of the system reliability and $x_i = R_i^o$ the optimal i^{th} stage reliability, then we can readily define the effort function minimization problem as:

$$\text{Minimize} \sum_{i=1}^{N} G_i(R_i, x_i)$$

Subject to;

$$\prod_{i=1}^{n} x_i \geq \bar{R} \; .$$

There are a number of functions which obey the stated criteria; all have the form

$$G(R, x) = H(x) - H(R) \tag{1.6}$$

The function used in this work is

$$G(R_i, x_i) = a_i \ \ln\left(\frac{1-R_i}{1-x_i}\right) \qquad 0 \leq R_i, x_i \leq 1 \tag{1.7}$$

Albert assumed $a_i = a$, so that all components have same effort coefficients. ?

if not ?

Example:

Let $(R_1, R_2, R_3, R_4) = (0.6, 0.7, 0.8, 0.9)$, so that

$$R_s = \prod_{j=1}^{4} R_j = 0.302 \tag{1.8}$$

Suppose the required value of system reliability to be $\bar{R} = 0.45$. We use equation (1.3) to determine k_o to do this we calculate the quantities:

$$r_4 = \left(\frac{0.45}{1}\right)^{1/4} = 0.819, \tag{1.9}$$

so that $r_4 < R_4$. Therefore the 4^{th} component is good enough.

$$r_3 = \left(\frac{0.45}{(0.9)x1}\right)^{1/3} = 0.794, \tag{1.10}$$

so that $r_3 < R_4$. Therefore the 3^{th} component is good enough.

$$r_2 = \left(\frac{0.45}{(0.8)x(0.9)x1}\right)^{1/2} = 0.791 \tag{1.11}$$

so that $r_2 > R_2$. Therefore the 2^{nd} component is not good enough. Since 2 is the largest subscript j such that $R_j < r_j$, then $k_o = 2$, which means that to achieve the desired system reliability of 0.45, the minimum effort results from raising the reliability of the 1^{st} and 2^{nd} components from 0.6 and 0.7 to the same level $x_2 = 0.791$; the remaining components are left unchanged. The resulting reliability of the entire system is, as required,

$$\bar{R} = (0.791)^2 \times 0.8 \times 0.9 = 0.45 \tag{1.12}$$

The total effort required, from equation (1.6), is (assuming $a_i = 1$),

$$e_1 = \sum_{i=1}^{4} a_i \ \ell n \left(\frac{1-R_i}{1-x_i}\right) = 1.00 \tag{1.13}$$

Suppose that we did not consider the selection of k_o by equation (1.3), but arbitrarily decided to set $k_o = 1$ and use equation (1.4), we would then obtain

$$\bar{R}_o = \left(\frac{0.4}{\prod_{j=2}^{4} R_j \times 1}\right)^{1/1} = 0.893 \tag{1.14}$$

and we would have

$$\bar{R} = 0.893 \times 0.7 \times 0.8 \times 0.9 = 0.45 \tag{1.15}$$

as desired, with total effort of

$$e_2 = \sum_{i=1}^{4} a_i \ \ell n \left(\frac{1-R_i}{1-x_i}\right) = 1.32 \tag{1.16}$$

we see that here the effort to increase reliability has not been allotted in an optimum manner; i.e., more effort has been used than is necessary $(e_2 > e_1)$.

The objective of this study is to investigate how the reliability allocation changes if the component cost coefficients a_i in equation (1.6) are not equal. This will be done for:

a) A simple series configuration.

b) A simple parallel configuration.

c) A elementary mixed (complex) configuration.

The optimization technique employed for obtaining the optimal allocation is the Generalized Reduced Gradient (GRG), which solves problems having a nonlinear objective function with linear or nonlinear constraints. It is an elaborate extension of hill-climbing gradient technique, and has been coded in FORTRAN in a program named GREG.

CHAPTER 2

Optimal Reliability Allocation of a Series System

2.1 Introduction

When every one of group of components (subsystems) must function properly for the system to succeed, they are said to function in series. A system consisting of a series arrangement of n components is shown in Figure 2.1.

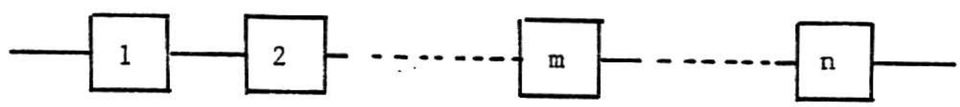


Fig. 2.1 Simple serial system.

Thus in Figure 2.1 we have

System success = $[C_1 \text{ success}] \cap [C_2 \text{ success}] \cap \ldots \cap [C_n \text{ success}]$

so that

$$R_s = P\{[C_1 \text{ success}] \cap [C_2 \text{ success}] \cap \ldots \cap [C_n \text{ success}]\}$$

$$= P[C_1 \text{ success}]\, P[C_2 \text{ success}] \ldots P[C_n \text{ success}]$$

or

$$R_s = R_1 R_2 \ldots R_n = \prod_{i=1}^{n} R_i \tag{2.1}$$

2.2 Formulation of A_n Optimization Problem

The problem of minimizing the effort for the series system given in Figure 2.1, subject to the desired system reliability $\bar{R}$ can be stated as

$$\text{Minimize:} \quad \sum_{i=1}^{n} a_i \ \ln \left(\frac{1-R_i}{1-x_i}\right) \tag{2.2}$$

Subject to

$$\prod_{i=1}^{n} x_i \geq \bar{R} \tag{2.3}$$

The solution of the above constrained non-linear programming problem can be obtained by the GREG technique.

2.3 Analytical Solution for n = 3.

To illustrate the analytical solution to the problem of minimizing effort for a series system consisting of 3 components, we solve the following:

$$\text{Minimize:} \quad e = \sum_{i=1}^{3} a_i \ \ln \left(\frac{1-R_i}{1-x_i}\right) \tag{2.4}$$

Subject to

$$g = \prod_{i=1}^{3} x_i \geq \bar{R} \tag{2.5}$$

From the equation (2.5) x_1 can be found and eliminated from the objective function. Thus, writing

$$x_1 = \frac{\bar{R}}{x_2 x_3} \tag{2.6}$$

we get

$$e = a_1 \ \ell n \ (1 - R_1) - a_1 \ \ell n \ (1 - \frac{\bar{R}}{x_2 x_3}) + a_2 \ \ell n \ (1 - R_2) -$$

$$a_2 \ \ell n \ (1 - x_2) + a_3 \ \ell n \ (1 - R_3) - a_3 \ \ell n \ (1 - x_3) \tag{2.7}$$

This may now be minimized by calculus methods, since there are no bounds imposed on x_2 or x_3 due to the original restriction. Hence, in this modified problem,

$$\frac{\partial e}{\partial x_2} = -a_1 \left(\frac{\dfrac{\bar{R}}{x_2^2 \cdot x_3}}{1 - \dfrac{\bar{R}}{x_2 \cdot x_3}} \right) - a_2 \frac{(-1)}{1-x_2} \ \underline{\underline{\text{set}}} \ 0 \tag{2.8}$$

$$\frac{\partial e}{\partial x_3} = -a_1 \left(\frac{\dfrac{\bar{R}}{x_2 \cdot x_3^2}}{1 - \dfrac{\bar{R}}{x_2 \cdot x_3}} \right) - a_3 \frac{(-1)}{1-x_3} \ \underline{\underline{\text{set}}} \ 0 \tag{2.9}$$

Now we simplfy equations (2.8 and 2.9); from equation (2.8) we have

$$(1 - x_2) \ a_1 \bar{R} - (x_2 x_3 - \bar{R}) \ a_2 x_2 = 0 \tag{2.10}$$

And from equation (2.9) we have

$$(1 - x_3)\, a_1\bar{R} - (x_2x_3 - \bar{R})\, a_3x_3 = 0. \tag{2.11}$$

We may form equation (2.10) as

$$x_3 = \frac{(1 - x_2)\, a_1\bar{R} + a_2\bar{R}x_2}{a_2\, x_2^2} \tag{2.12}$$

From equations (2.11) and (2.12), we obtain

$$\left(1 - \frac{(1-x_2)a_1\bar{R} + a_2\bar{R}x_2}{a_2\, x_2^2}\right) a_1\bar{R} - \left(\frac{(1-x_2)a_1\bar{R} + a_2\bar{R}x_2}{a_2x_2} - \bar{R}\right) x$$

$$\left(\frac{(1-x_2)a_1\bar{R} + a_2\bar{R}x_2}{a_2\, x_2^2}\right) a_3 = 0 \tag{2.13}$$

which can be written as

$$\left(a_2x_2^2 - (1-x_2)a_1\bar{R} - a_2\bar{R}x_2\right) a_1\bar{R} - \left((1-x_2)a_1\bar{R}x_2 + a_2\bar{R}x_2^2\right.$$

$$\left. - a_2\bar{R}x_2^2\right) x \left((1-x_2)\, a_1a_3\bar{R} + a_2a_3\bar{R}x_2\right) = 0 \tag{2.14}$$

so that

$$(a_2a_3\bar{R} - a_1a_3\bar{R})x_2^3 + (2a_1a_3\bar{R} - a_2a_3\bar{R} + a_2)\, x_2^2 +$$

$$(a_1\bar{R} - a_1a_3\bar{R} - a_2\bar{R})\, x_2 - a_1\bar{R} = 0 \tag{2.15}$$

Now we may rewrite equation (2.11) as

$$x_2 = \frac{(1-x_3)a_1\bar{R} + a_3\bar{R}x_3}{a_3\ x_3^2} \tag{2.16}$$

From equations (2.10) and (2.16), we find x_3 from

$$\left(1 - \frac{(1-x_3)\ a_1\bar{R} + a_3\bar{R}x_3}{a_3\ x_3^2}\right) a_1\bar{R} - \left(\frac{(1-x_3)a_1\bar{R} + a_3\bar{R}x_3}{a_3\ x_3} - \bar{R}\right) x$$

$$\left(\frac{(1-x_3)a_1\bar{R} + a_3\bar{R}x_3}{a_3\ x_3^2}\right) a_2 = 0 \tag{2.17}$$

so that

$$(a_2a_3\bar{R} - a_1a_2\bar{R})\ x_3^3 + (2a_1a_2\bar{R} - a_2a_3\bar{R} + a_3)\ x_3^2$$

$$+ (a_1\bar{R} - a_1a_2\bar{R} - a_3\bar{R})\ x_3 - a_1\bar{R} = 0 \tag{2.18}$$

Therefore the optimum values of x_2 and x_3 can obtained from equations (2.15) and (2.18), and by substituting these optimum values in equation (2.6) we can find x_1. The corresponding optimal (i.e. minimized) effort is obtained from equation (2.4).

2.3.1 A Numerical Example.

The nonlinear programming problem formulated in the preceding section is restated again and the objective is to minimize

$$e = \sum_{i=1}^{3} a_i\ \ell n\ \left(\frac{1-R_i}{1-x_i}\right) \tag{2.19}$$

subject to the constraint

$$g = \prod_{i=1}^{3} x_i \geq \bar{R} \tag{2.20}$$

The constants a_1, a_2 and a_3, and $\bar{R}$, are as follows:

$a_1 = 1.2, \quad a_2 = 1.1, \quad a_3 = 1.0,$

$\bar{R} = 0.74$

The problem is formulated in equations (2.15) and (2.18) and solved by the Newton root finding program (Appendix 1).

From equation (2.15),

$$0.074x_2^3 - 2.062x_2^2 + 0.814x_2 + 0.888 = 0 \tag{2.21}$$

which has $x_2 = 0.9016$.

Also from equation (2.18),

$$0.1628x_3^3 - 2.1396x_3^2 + 0.8288x_3 + 0.8880 = 0 \tag{2.22}$$

which has $x_3 = 0.9074$.

By substituting the values of x_2 and x_3 into equation (2.6), we found $x_1 = 0.9045$. The corresponding effort is $e = 2.844$

2.4 Further Numerical Examples

The problem of minimizing the reliability of the series system, subject to a single constraint, can be stated as follows

$$\text{Minimize} \quad e = + a_1 \ln \left(\frac{1-R_1}{1-x_1}\right) + a_2 \ln \left(\frac{1-R_2}{1-x_2}\right)$$

$$+ a_3 \ln \left(\frac{1-R_3}{1-x_3}\right) \tag{2.23}$$

Subject to

$$x_1 \cdot x_2 \cdot x_3 \geq \bar{R} \tag{2.24}$$

$$R_i \leq x_i \leq 1 \qquad i = 1,2,3 \tag{2.25}$$

The numerical values of the parameters $\underline{a} = \{a_1, a_2, a_3\}$, are given in Table 2.1, and $\bar{R}$ was determined as follows. $\bar{R}$ values were chosen to be uniformly distributed over the range from R_s to 1. The range $\Delta = 1 - R_s$ was divided into 5 equal intervals yielding $\bar{R} = R_s + \frac{k}{n}\Delta$, k = 1,2,3,4. Then for

$$k = \frac{1}{5}, \qquad \bar{R} = 0.36 + 1/5(0.64)$$
$$= 0.49$$

$$k = \frac{2}{5}, \qquad \bar{R} = 0.36 + 2/5(0.64)$$
$$= 0.62$$

$$k = \frac{3}{5}, \qquad \bar{R} = 0.36 + 3/5(0.64)$$
$$= 0.74$$

$$k = \frac{4}{5}, \qquad \bar{R} = 0.36 + 4/5(0.64)$$
$$= 0.87$$

In the present example, the resulting $\bar{R}$ set was thus {0.49, 0.62, 0.74, 0.87}.

Now for this set of $\bar{R}$ values, for $\underline{a}$ as given in Table 2.1, and the given initial reliability values of (0.5, 0.8, 0.9) for the three series components, we apply the GREG computer code. Specifically, the problem will be reformulated as

$$\text{Minimize} \quad e = -a_1 \ln \left(\frac{1-0.5}{1 - x_1}\right) - a_2 \ln\left(\frac{1-0.8}{1 - x_2}\right)$$

$$- a_3 \ln \left(\frac{1-0.9}{1- x_3}\right) \tag{2.26}$$

Subject to

$$-x_1 \cdot x_2 \cdot x_3 + \bar{R} \leq 0 \tag{2.27}$$

$$R_i \leq x_i \leq 1 \qquad\qquad i = 1,2,3 \tag{2.28}$$

Four external, user-supplied subroutines (Appendix 2), are used in which PHIX defines the objective function, CPHI defines the constraint functions, JACOB defines the gradient of constraint functions, and GRADF1 defines the gradient of the objective function.

The optimal solutions which were obtained are presented in Tables 2.1 - 2.5. Table 2.1 presents the optimal solutions according to different $\underline{a}$ (effort coefficients) and $\bar{R}$ (desired system reliability). Tables 2.2 - 2.5 present optimal solutions according to different effort coefficient ($\underline{a}$) for the $\bar{\underline{R}}$ = [0.49, 0.62, 0.74, 0.87] respectively.

The results given in Table 2.1 show that the component reliabilities R_i and the total effort are monotonically increasing for increasing $\bar{R}$. They are also sensitive to the values of effort coefficients ($\underline{a}$).

In Figures 2.2 to 2.5, the reliability allocation for each component is depicted for the various $\bar{R}$ values. They show that the worst component is improved the most, then the next worst, etc. For example, Figure 2.5 shows that in case 1 we must increase from $[R_1, R_2, R_3] = [0.5, 0.8, 0.9]$ to $[x_1, x_2, x_3] = [0.95, 0.95, 0.95]$ in order to meet our system demand. For case 2, we must raise the component reliabilities to $[x_1, x_2, x_3] = [0.93, 0.96, 0.97]$ to get the desired system reliability. We see the increases for R_1, R_2 and R_3 in case 1 and 2, are not identical, even though $\bar{R} = 0.87$ in both cases. This is because for case 2 the effort coefficients, for C_1 and C_2 are 2.0 and 1.1 times the corresponding effort coefficients for case 1; therefore we obtain different allocations of component reliability. The effort coefficients for case 3 are the reverse of those for case 2. This causes a symmetric result in the reliability allocation. The same symmetric situation occurs between cases 3 and 4. The results do not hold for all values of $\bar{R}$.

TABLE 2.1

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS AND SYSTEM RELIABILITY

EFFORT COEFFICIENTS $\underline{a}$	$\overline{R}$	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
		0.5	0.8	0.9	
(1.00,1.00,1.00)	0.49	0.681	0.800	0.900	0.448
	0.62	0.830	0.830	0.900	1.241
	0.74	0.905	0.905	0.905	2.443
	0.87	0.955	0.955	0.955	4.574
(1.11,1.10,1.00)	0.49	0.681	0.800	0.900	0.497
	0.62	0.829	0.831	0.900	1.376
	0.74	0.903	0.903	0.906	2.695
	0.87	0.953	0.953	0.958	5.083
(1.12,1.10,1.00)	0.49	0.681	0.800	0.900	0.502
	0.62	0.829	0.831	0.900	1.387
	0.74	0.902	0.904	0.908	2.711
	0.87	0.953	0.954	0.958	5.107
(1.13,1.10,1.00)	0.49	0.681	0.800	0.900	0.506
	0.62	0.828	0.832	0.900	1.398
	0.74	0.902	0.904	0.908	2.727
	0.87	0.952	0.954	0.958	5.131
(1.20,1.10,1.00)	0.49	0.681	0.800	0.900	0.538
	0.62	0.824	0.836	0.900	1.472
	0.74	0.898	0.906	0.910	2.839
	0.87	0.951	0.955	0.959	5.294
(1.30,1.10,1.00)	0.49	0.681	0.800	0.900	0.583
	0.62	0.818	0.842	0.900	1.574
	0.74	0.893	0.908	0.912	2.994
	0.87	0.948	0.956	0.960	5.523
(1.50,1.10,1.00)	0.49	0.681	0.800	0.900	0.672
	0.62	0.809	0.852	0.900	1.771
	0.74	0.882	0.914	0.918	3.292
	0.87	0.944	0.958	0.962	5.969

TABLE 2.1 (CONTINUED)

EFFORT COEFFICIENTS $\underline{a}$	$\overline{R}$	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
		0.5	0.8	0.9	
(2.00,1.10,1.00)	0.49	0.681	0.800	0.900	0.896
	0.62	0.790	0.872	0.900	2.226
	0.74	0.867	0.922	0.926	3.982
	0.87	0.935	0.963	0.966	7.022
(1.00,1.10,1.11)	0.49	0.681	0.800	0.900	0.448
	0.62	0.837	0.823	0.900	1.256
	0.74	0.909	0.904	0.900	2.517
	0.87	0.957	0.953	0.953	4.916
(1.00,1.10,1.12)	0.49	0.681	0.800	0.900	0.448
	0.62	0.837	0.823	0.900	1.256
	0.74	0.909	0.904	0.900	2.517
	0.87	0.958	0.954	0.952	4.914
(1.00,1.10,1.13)	0.49	0.681	0.800	0.900	0.448
	0.62	0.837	0.823	0.900	1.256
	0.74	0.909	0.905	0.900	2.518
	0.87	0.958	0.954	0.952	4.921
(1.00,1.10,1.20)	0.49	0.681	0.800	0.900	0.448
	0.62	0.837	0.823	0.900	1.256
	0.74	0.907	0.906	0.900	2.519
	0.87	0.958	0.955	0.952	4.972
(1.00,1.10,1.30)	0.49	0.681	0.800	0.900	0.448
	0.62	0.837	0.823	0.900	1.256
	0.74	0.911	0.903	0.900	2.517
	0.87	0.960	0.956	0.948	5.040
(1.00,1.10,1.50)	0.49	0.681	0.800	0.900	0.448
	0.62	0.837	0.823	0.900	1.256
	0.74	0.907	0.906	0.900	2.519
	0.87	0.962	0.958	0.944	5.164
(1.00,1.10,2.00)	0.49	0.681	0.800	0.900	0.448
	0.62	0.837	0.823	0.900	1.256
	0.74	0.911	0.903	0.900	2.517
	0.87	0.966	0.963	0.935	5.413

TABLE 2.2

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS AND $\bar{R}$ = 0.49

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
	0.5	0.8	0.9	
(1.00,1.00,1.00)	0.681	0.800	0.900	0.448
(1.11,1.10,1.00)	0.681	0.800	0.900	0.497
(1.12,1.10,1.00)	0.681	0.800	0.900	0.502
(1.13,1.10,1.00)	0.681	0.800	0.900	0.506
(1.20,1.10,1.00)	0.681	0.800	0.900	0.536
(1.30,1.10,1.00)	0.681	0.800	0.900	0.583
(1.50,1.10,1.00)	0.681	0.800	0.900	0.672
(2.00,1.10,1.00)	0.681	0.800	0.900	0.896
(1.00,1.10,1.11)	0.681	0.800	0.900	0.448
(1.00,1.10,1.12)	0.681	0.800	0.900	0.448
(1.00,1.10,1.13)	0.681	0.800	0.900	0.448
(1.00,1.10,1.20)	0.681	0.800	0.900	0.448
(1.00,1.10,1.30)	0.681	0.800	0.900	0.448
(1.00,1.10,1.50)	0.681	0.800	0.900	0.448
(1.00,1.10,2.00)	0.681	0.800	0.900	0.448

TABLE 2.3

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS AND $\bar{R}$ = 0.62

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
	0.5	0.8	0.9	
(1.00,1.00,1.00)	0.830	0.830	0.900	1.241
(1.11,1.10,1.00)	0.829	0.831	0.900	1.376
(1.12,1.10,1.00)	0.829	0.831	0.900	1.387
(1.13,1.10,1.00)	0.828	0.832	0.900	1.398
(1.20,1.10,1.00)	0.824	0.836	0.900	1.472
(1.30,1.10,1.00)	0.818	0.842	0.900	1.574
(1.50,1.10,1.00)	0.809	0.852	0.900	1.771
(2.00,1.10,1.00)	0.790	0.872	0.900	2.226
(1.00,1.10,1.11)	0.837	0.823	0.900	1.256
(1.00,1.10,1.12)	0.837	0.823	0.900	1.256
(1.00,1.10,1.13)	0.837	0.823	0.900	1.256
(1.00,1.10,1.20)	0.837	0.823	0.900	1.256
(1.00,1.10,1.30)	0.837	0.823	0.900	1.256
(1.00,1.10,1.30)	0.837	0.823	0.900	1.256
(1.00,1.10,1.50)	0.837	0.823	0.900	1.256
(1.00,1.10,2.00)	0.837	0.823	0.900	1.256

TABLE 2.4

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS AND $\bar{R}$ = 0.74

EFFORT COEFFICIENTS <u>a</u>	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
	0.5	0.8	0.9	
(1.00,1.00,1.00)	0.905	0.905	0.905	2.443
(1.11,1.10,1.00)	0.903	0.903	0.906	2.695
(1.12,1.10,1.00)	0.902	0.904	0.908	2.711
(1.13,1.10,1.00)	0.902	0.904	0.908	2.727
(1.20,1.10,1.00)	0.898	0.906	0.910	2.839
(1.30,1.10,1.00)	0.893	0.908	0.912	2.994
(1.50,1.10,1.00)	0.882	0.914	0.918	3.292
(2.00,1.10,1.00)	0.867	0.922	0.926	3.982
(1.00,1.10,1.11)	0.909	0.904	0.900	2.517
(1.00,1.10,1.12)	0.909	0.904	0.900	2.517
(1.00,1.10,1.13)	0.909	0.905	0.900	2.518
(1.00,1.10,1.20)	0.907	0.906	0.900	2.519
(1.00,1.10,1.30)	0.911	0.903	0.900	2.517
(1.00,1.10,1.50)	0.907	0.906	0.900	2.519
(1.00,1.10,2.00)	0.911	0.903	0.900	2.517

TABLE 2.5

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS AND $\bar{R}$ = 0.87

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
	0.5	0.8	0.9	
(1.00,1.00,1.00)	0.955	0.955	0.955	4.574
(1.11,1.10,1.00)	0.953	0.953	0.958	5.083
(1.12,1.10,1.00)	0.953	0.954	0.958	5.107
(1.13,1.10,1.00)	0.952	0.954	0.958	5.131
(1.20,1.10,1.00)	0.951	0.955	0.959	5.294
(1.30,1.10,1.00)	0.948	0.956	0.960	5.523
(1.50,1.10,1.00)	0.944	0.958	0.962	5.969
(2.00,1.10,1.00)	0.935	0.963	0.966	7.022
(1.00,1.10,1.11)	0.957	0.953	0.953	4.916
(1.00,1.10,1.12)	0.958	0.954	0.952	4.914
(1.00,1.10,1.13)	0.958	0.954	0.952	4.921
(1.00,1.10,1.20)	0.958	0.955	0.952	4.972
(1.00,1.10,1.30)	0.960	0.956	0.948	5.040
(1.00,1.10,1.50)	0.962	0.958	0.944	5.164
(1.00,1.10,2.00)	0.966	0.963	0.935	5.413

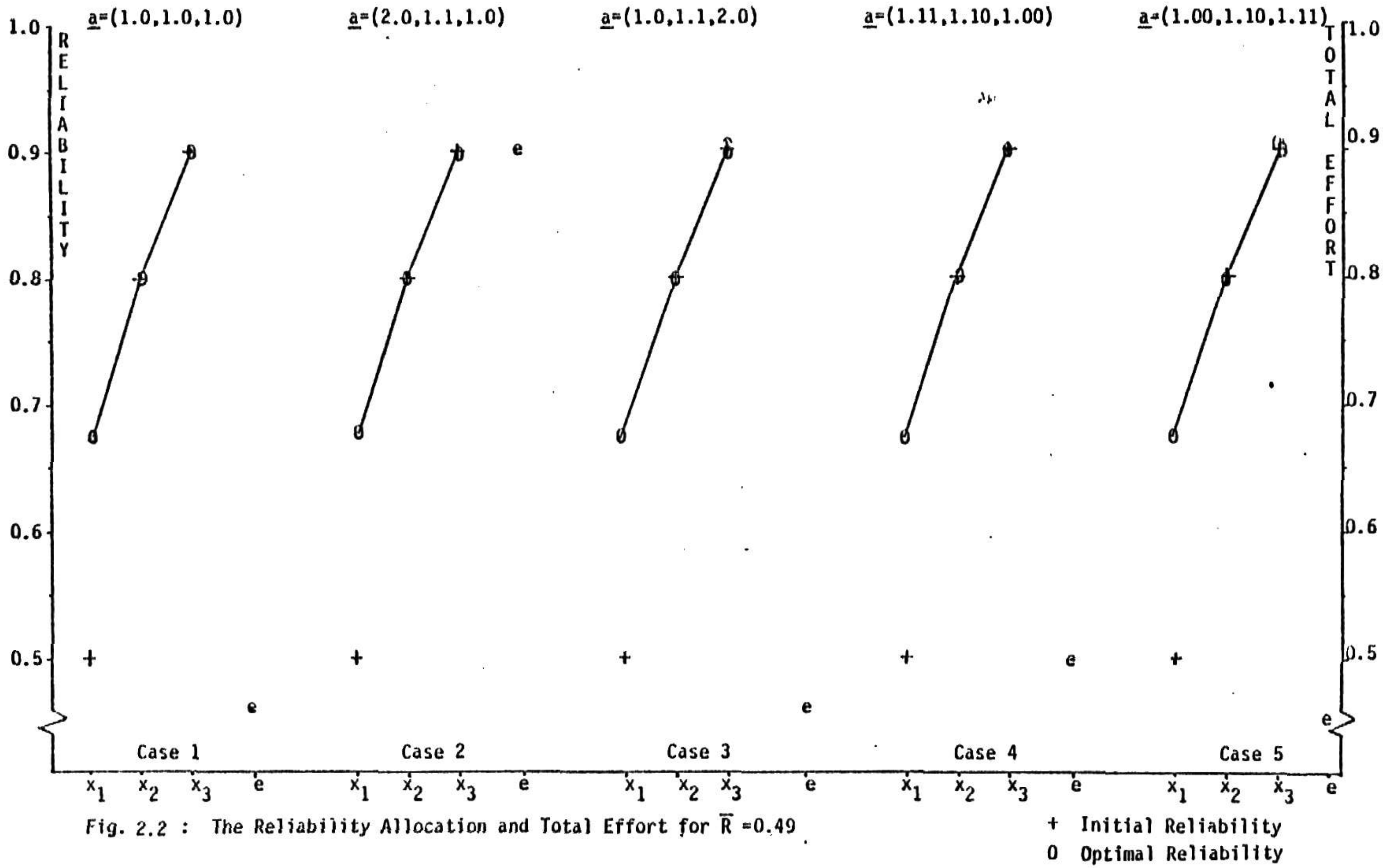


Fig. 2.2 : The Reliability Allocation and Total Effort for $\overline{R}$ =0.49

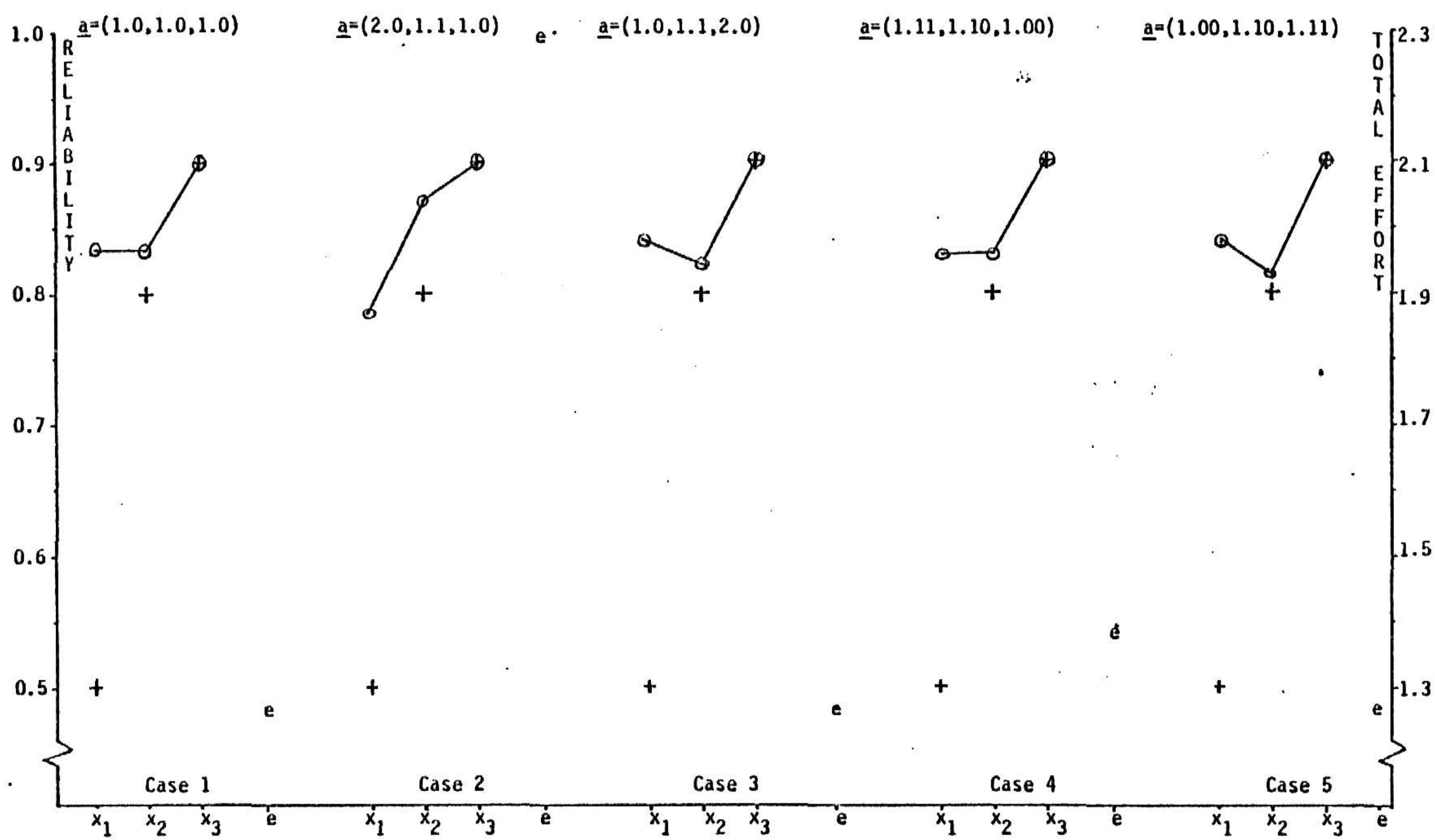


Fig. 2.3 : The Reliability Allocation and Total Effort for $\overline{R} = 0.62$.

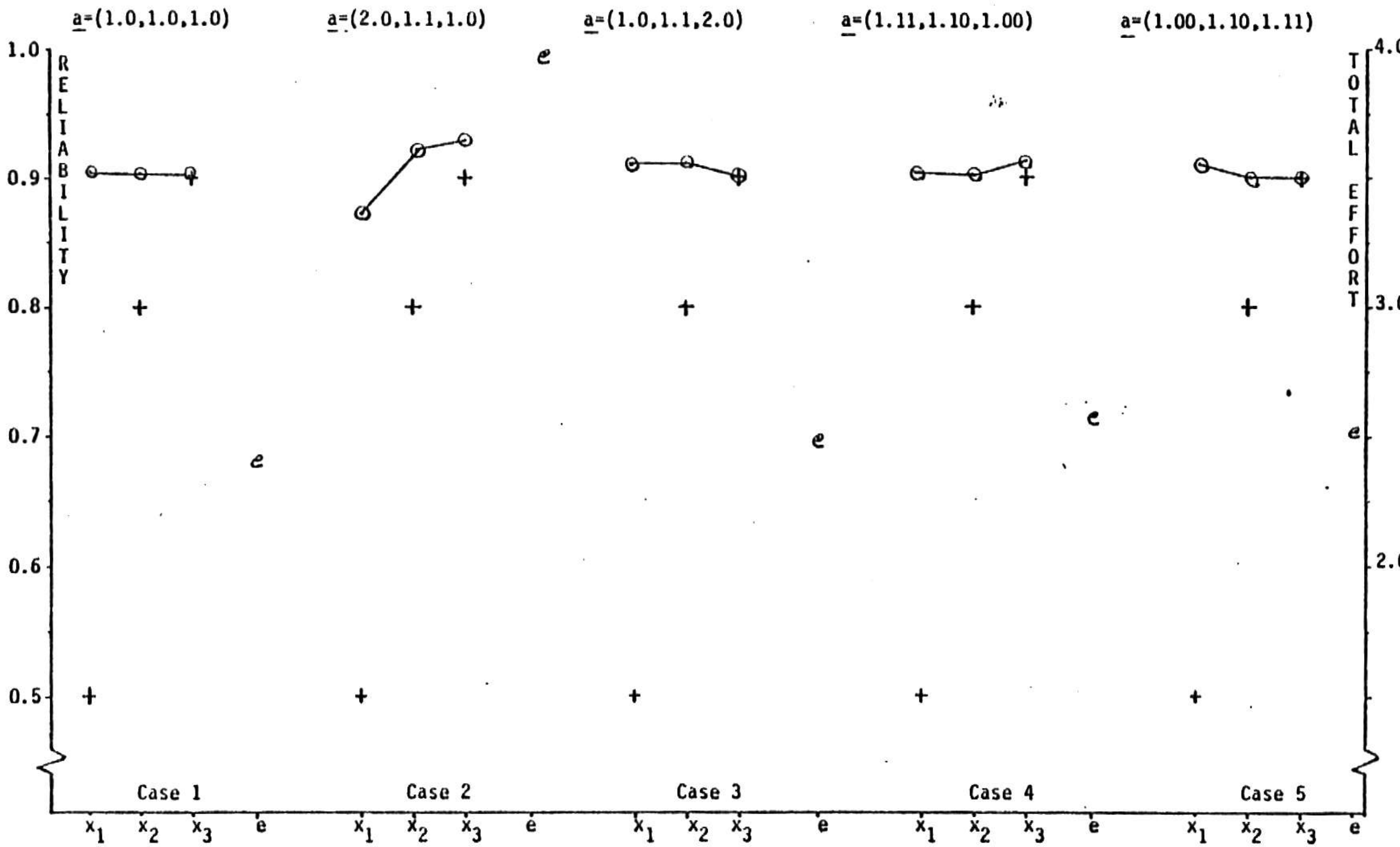


Fig. 2.4 : The Reliability Allocation and Total Effort for $\overline{R}$ =0.74

\+ Initial Reliability
0 Optimal Reliability

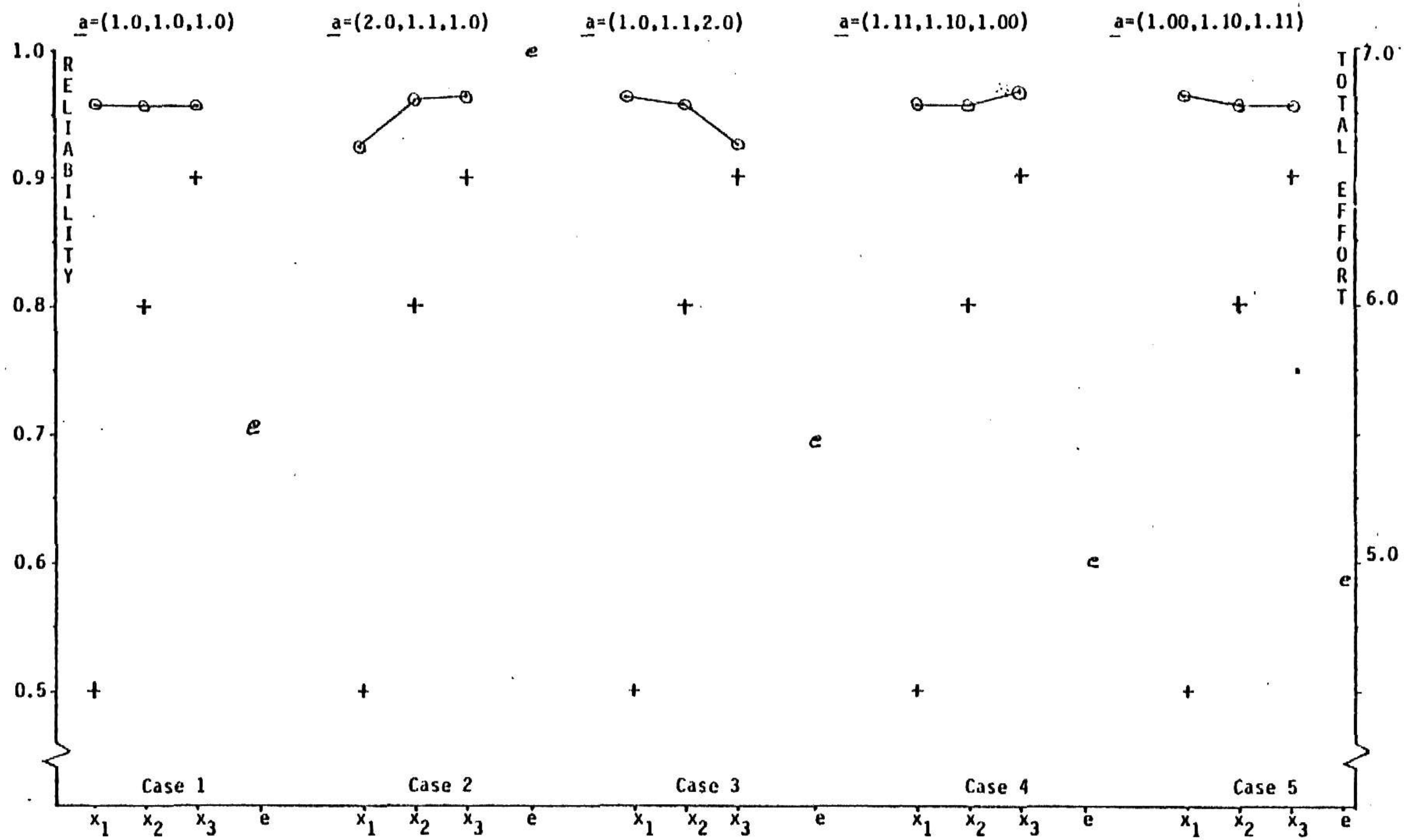


Fig. 2.5 : The Reliability Allocation and Total Effort for $\overline{R}$ = 0.87

+ Initial Reliability

0 Optimal Reliability

CHAPTER 3

Optimal Reliability Allocation for Parallel System

3.1 *Introduction*

System or subsystem reliability can sometimes be increased by including redundant components so that success is achieved as long as at least one component is operating satisfactorily. Such components are said to be in parallel. A parallel system of n components (or subsystems) is illustrated in Figure 3.1.

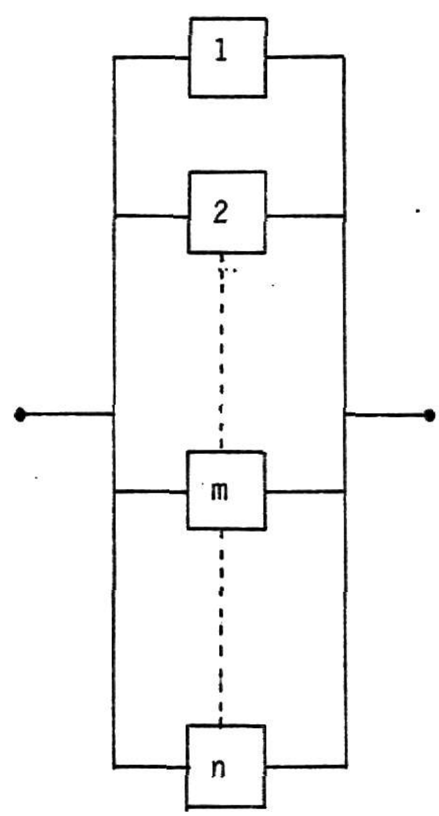


Fig. 3.1 Simple parallel system

It is easily shown that all of the properties given in Chapter 2 for series systems can be dualized to give the corresponding properties for parallel systems by simply replacing any event by its complementary event.

Thus in Figure 3.1 we have

$$\text{System success} = [C_1 \text{ success}] \cup [C_2 \text{ success}] \cup \ldots \cup [C_n \text{ success}]$$

$$R_s = P\left\{\overline{[\overline{C_1 \text{ success}}] \cap [\overline{C_2 \text{ success}}] \cap \ldots \cap [\overline{C_n \text{ success}}]}\right\}$$

$$= 1 - P\left\{[\overline{C_1 \text{ success}}] \cap [\overline{C_2 \text{ success}}] \cap \ldots \cap [\overline{C_n \text{ success}}]\right\}$$

$$= 1 - P[\overline{C_1 \text{ success}}]\, P[\overline{C_2 \text{ success}}] \ldots P[\overline{C_n \text{ success}}]$$

or $$R_s = 1 - \prod_{i=1}^{n} (1 - R_i) \qquad (3.1)$$

3.2 Formulation of the Optimization Problem

The problem of minimizing the effort for the parallel system given in Figure 3.1, subject to desired system reliability $\bar{R}$ can be stated as

$$\text{Minimize:} \quad \sum_{i=1}^{n} a_i \ln \left(\frac{1-R_i}{1-x_i}\right) \qquad (3.2)$$

Subject to

$$1 - \prod_{i=1}^{n} (1 - x_i) \geq \bar{R} \qquad (3.3)$$

The solution of the above constrained non-linear programming problem can be obtained by the GREG technique.

3.3 A Numerical Example

The problem of minimizing the reliability of the parallel system, subject to a single constraint, can be stated as follows

$$\text{Minimize} \quad e = + a_1 \, \ell n \left(\frac{1-R_1}{1-x_1}\right) + a_2 \, \ell n \left(\frac{1-R_2}{1-x_2}\right)$$

$$+ a_3 \, \ell n \left(\frac{1-R_3}{1-x_3}\right) \tag{3.4}$$

Subject to

$$1 - \prod_{i=1}^{3} (1 - x_i) \geq \bar{R} \tag{3.5}$$

$$R_i \leq x_i \leq 1 \qquad i = 1,2,3 \tag{3.6}$$

The numerical values of the cost parameters $\underline{a}$, are given in Table 3.1, and $\bar{R}$ was determined as in Section 2.4 of Chapter 2; in the present example, the resulting $\bar{R}$ set was {0.92, 0.94, 0.96, 0.98}. Now for this set $\bar{R}$, for $\underline{a}$ as given in Table 3.1 and the given initial reliabilities, we apply the GREG computer code, to solve this example. This problem will be reformulated to

$$\text{Minimize} \quad e = - a_1 \, \ell n \left(\frac{1-0.2}{1-x_1}\right) - a_2 \, \ell n \left(\frac{1-0.6}{1-x_2}\right)$$

$$- a_3 \, \ell n \left(\frac{1-0.7}{1-x_3}\right) \tag{3.7}$$

Subject to

$$\prod_{i=1}^{3} (1 - x_i) - (1 - \bar{R}) \leq 0 \tag{3.8}$$

$$R_i \leq x_i \leq 1 \qquad i = 1, 2, 3 \tag{3.9}$$

Four external, user-supplied subroutines (Appendix 3), are used in which PHIX defines the objective function, GRADF1 defines the gradient of the objective function, CPHI defines the constraint functions and JACOB defines the gradient of constraint functions.

The optimal solutions obtained are presented in Tables 3.1 - 3.5. Table 3.1 shows the optimal solutions according to different $\underline{a}$ (effort coefficient) and $\bar{R}$ (desired system reliability). Tables 3.2 - 3.5 present the optimal solutions according to different effort coefficients ($\underline{a}$) for the $\bar{R}$ = [0.92, 0.94, 0.96, 0.98] respectively.

The results of Table 3.1 show that the component reliability x_i and total effort are monotonically increasing with increasing $\bar{R}$; they are also sensitive to the value of effort coefficients ($\underline{a}$).

Figures 3.2 to 3.5 depict the reliability allocation for each component for various $\bar{R}$. They show that the best component is changed the most, then the next best, etc. For example, in case 1 of Figure 3.5 we must increase from $[R_1, R_2, R_3]$ = [0.2, 0.6, 0.7] to $[x_1, x_2, x_3]$ = [0.27, 0.74, 0.90] to meet our system demand, whereas for case 2, we need only increase R_3 to the level x_3 = 0.94. Notice that the amount of reliability allocation for case 1 and 2 are not identical, even though both have the same required system reliability ($\bar{R}$ = 0.98), because in case 2 the effort coefficients, for C_1 and C_2 are 2.0 and 1.1 times those of C_1 and C_2 for case 1. The effort coefficients for case 3 are symmetrical with case 2, but there is no sign of symmetry in the reliability allocation. Case 4 and 5 are symmetric in the effort coefficients, but the reliability allocations are not symmetric.

TABLE 3.1

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS AND SYSTEM RELIABILITY

EFFORT COEFFICIENTS $\underline{a}$	$\overline{R}$	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
		0.2	0.6	0.7	
(1.00,1.00,1.00)	0.92	0.212	0.623	0.731	0.182
	0.94	0.228	0.656	0.775	0.470
	0.96	0.246	0.693	0.827	0.876
	0.98	0.267	0.739	0.896	1.569
(1.11,1.10,1.00)	0.92	0.200	0.600	0.750	0.182
	0.94	0.200	0.600	0.813	0.470
	0.96	0.200	0.600	0.875	0.876
	0.98	0.200	0.600	0.938	1.569
(1.12,1.10,1.00)	0.92	0.200	0.600	0.750	0.183
	0.94	0.200	0.600	0.813	0.470
	0.96	0.200	0.600	0.875	0.876
	0.98	0.200	0.600	0.938	1.569
(1.13,1.10,1.00)	0.92	0.200	0.600	0.750	0.182
	0.94	0.200	0.600	0.813	0.470
	0.96	0.200	0.600	0.875	0.876
	0.98	0.200	0.600	0.938	1.569
(1.20,1.10,1.00)	0.92	0.200	0.600	0.750	0.182
	0.94	0.200	0.600	0.812	0.470
	0.96	0.200	0.600	0.875	0.876
	0.98	0.200	0.600	0.937	1.569
(1.30,1.10,1.00)	0.92	0.200	0.600	0.750	0.182
	0.94	0.200	0.600	0.813	0.470
	0.96	0.200	0.600	0.875	0.876
	0.98	0.200	0.600	0.938	1.569
(1.50,1.10,1.00)	0.92	0.200	0.600	0.750	0.182
	0.94	0.200	0.600	0.813	0.470
	0.96	0.200	0.600	0.875	0.876
	0.98	0.200	0.600	0.938	1.569

TABLE 3.1 (CONTINUED)

EFFORT COEFFICIENTS $\underline{a}$	$\overline{R}$	R_1^0	R_2^0	R_3^0	TOTAL EFFORT
		0.2	0.6	0.7	
(2.00,1.10,1.00)	0.92	0.200	0.600	0.750	0.182
	0.94	0.200	0.600	0.813	0.470
	0.96	0.200	0.600	0.875	0.876
	0.98	0.200	0.600	0.937	1.569
(1.00,1.10,1.11)	0.92	0.333	0.600	0.700	0.182
	0.94	0.500	0.600	0.700	0.470
	0.96	0.667	0.600	0.700	0.876
	0.98	0.833	0.600	0.700	1.569
(1.00,1.10,1.12)	0.92	0.333	0.600	0.700	0.183
	0.94	0.500	0.600	0.700	0.470
	0.96	0.667	0.600	0.700	0.876
	0.98	0.833	0.600	0.700	1.569
(1.00,1.10,1.13)	0.92	0.333	0.600	0.700	0.182
	0.94	0.500	0.600	0.700	0.470
	0.96	0.667	0.600	0.700	0.876
	0.98	0.833	0.600	0.700	1.569
(1.00,1.10,1.20)	0.92	0.333	0.600	0.700	0.182
	0.94	0.500	0.600	0.700	0.470
	0.96	0.667	0.600	0.700	0.876
	0.98	0.833	0.600	0.700	1.569
(1.00,1.10,1.30)	0.92	0.333	0.600	0.700	0.182
	0.94	0.500	0.600	0.700	0.470
	0.96	0.667	0.600	0.700	0.876
	0.98	0.833	0.600	0.700	1.569
(1.00,1.10,1.50)	0.92	0.333	0.600	0.700	0.182
	0.94	0.500	0.600	0.700	0.470
	0.96	0.667	0.600	0.700	0.876
	0.98	0.833	0.600	0.700	1.569
(1.00,1.10,2.00)	0.92	0.333	0.600	0.700	0.182
	0.94	0.500	0.600	0.700	0.470
	0.96	0.667	0.600	0.700	0.876
	0.98	0.833	0.600	0.700	1.569

TABLE 3.2

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS AND $\bar{R}$ = 0.92

EFFORT COEFFICIENTS <u>a</u>	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
	0.2	0.6	0.7	
(1.00,1.00,1.00)	0.212	0.623	0.731	0.182
(1.11,1.10,1.00)	0.200	0.600	0.750	0.182
(1.12,1.10,1.00)	0.200	0.600	0.750	0.182
(1.13,1.10,1.00)	0.200	0.600	0.750	0.182
(1.20,1.10,1.00)	0.200	0.600	0.750	0.182
(1.30,1.10,1.00)	0.200	0.600	0.750	0.182
(1.50,1.10,1.00)	0.200	0.600	0.750	0.182
(2.00,1.10,1.00)	0.200	0.600	0.750	0.182
(1.00,1.10,1.11)	0.333	0.600	0.700	0.182
(1.00,1.10,1.12)	0.333	0.600	0.700	0.182
(1.00,1.10.1.13)	0.333	0.600	0.700	0.182
(1.00,1.10,1.20)	0.333	0.600	0.700	0.182
(1.00,1.10,1.30)	0.333	0.600	0.700	0.182
(1.00,1.10,1.50)	0.333	0.600	0.700	0.182
(1.00,1.10,2.00)	0.333	0.600	0.700	0.182

TABLE 3.3

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS AND $\bar{R}$ = 0.94

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
	0.2	0.6	0.7	
(1.00,1.00,1.00)	0.228	0.656	0.775	0.470
(1.11,1.10,1.00)	0.200	0.600	0.813	0.470
(1.12,1.10,1.00)	0.200	0.600	0.813	0.470
(1.13,1.10,1.00)	0.200	0.600	0.813	0.470
(1.20,1.10,1.00)	0.200	0.600	0.813	0.470
(1.30,1.10,1.00)	0.200	0.600	0.813	0.470
(1.50,1.10,1.00)	0.200	0.600	0.813	0.470
(2.00,1.10,1.00)	0.200	0.600	0.813	0.470
(1.00,1.10,1.11)	0.500	0.600	0.700	0.470
(1.00,1.10,1.12)	0.500	0.600	0.700	0.470
(1.00,1.10,1.13)	0.500	0.600	0.700	0.470
(1.00,1.10,1.20)	0.500	0.600	0.700	0.470
(1.00,1.10,1.30)	0.500	0.600	0.700	0.470
(1.00,1.10,1.50)	0.500	0.600	0.700	0.470
(1.00,1.10,2.00)	0.500	0.600	0.700	0.470

TABLE 3.4

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS AND $\bar{R}$ = 0.96

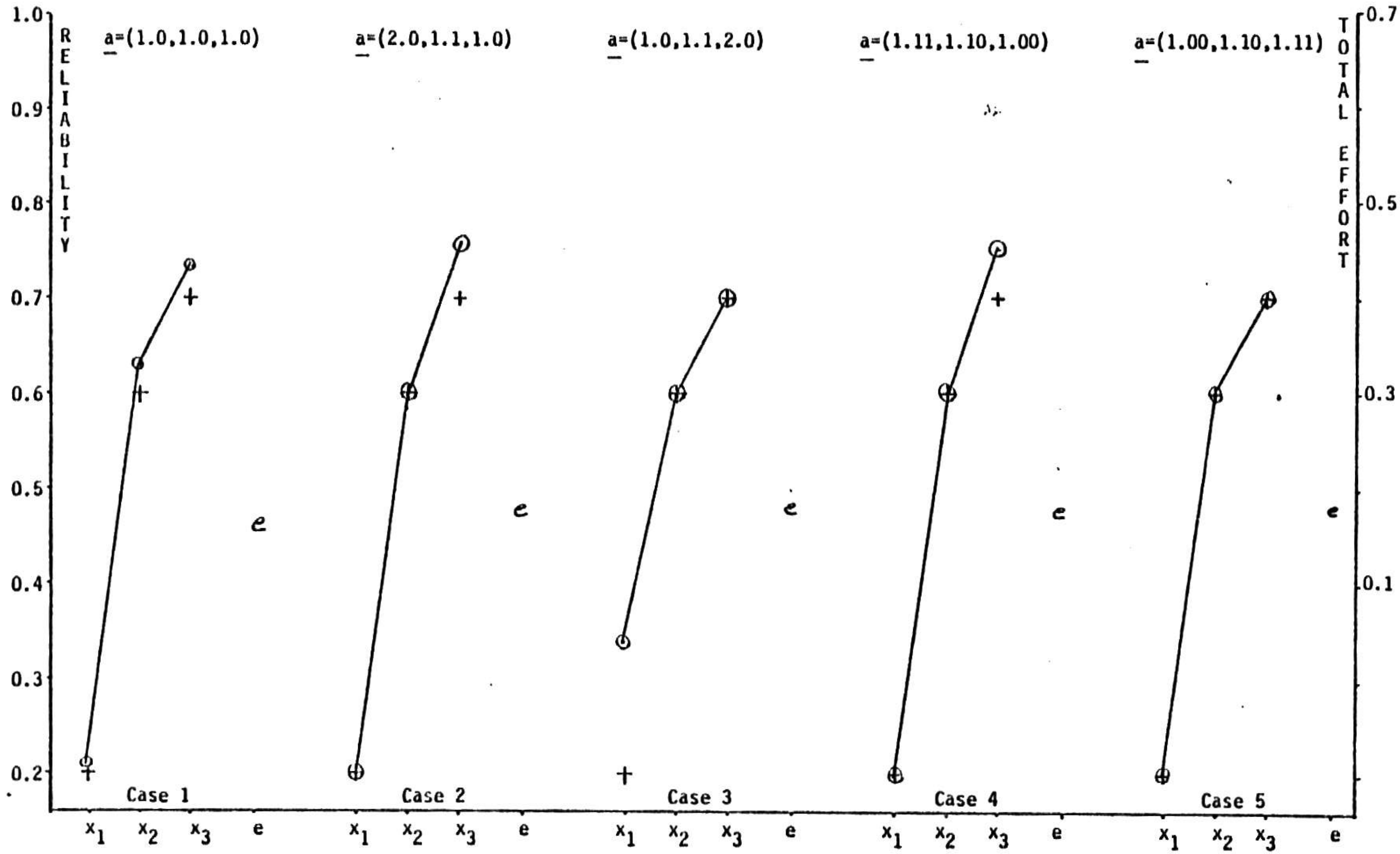
EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
	0.2	0.6	0.7	
(1.00,1.00,1.00)	0.246	0.693	0.827	0.876
(1.11,1.10,1.00)	0.200	0.600	0.875	0.876
(1.12,1.10,1.00)	0.200	0.600	0.875	0.876
(1.13,1.10,1.00)	0.200	0.600	0.875	0.876
(1.20,1.10,1.00)	0.200	0.600	0.875	0.876
(1.30,1.10,1.00)	0.200	0.600	0.875	0.876
(1.50,1.10,1.00)	0.200	0.600	0.875	0.876
(2.00,1.10,1.00)	0.200	0.600	0.875	0.876
(1.00,1.10,1.11)	0.667	0.600	0.700	0.876
(1.00,1.10,1.12)	0.667	0.600	0.700	0.876
(1.00,1.10,1.13)	0.667	0.600	0.700	0.876
(1.00,1.10,1.20)	0.667	0.600	0.700	0.876
(1.00,1.10,1.30)	0.667	0.600	0.700	0.876
(1.00,1.10,1.50)	0.667	0.600	0.700	0.876
(1.00,1.10,2.00)	0.667	0.600	0.700	0.876

TABLE 3.5

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

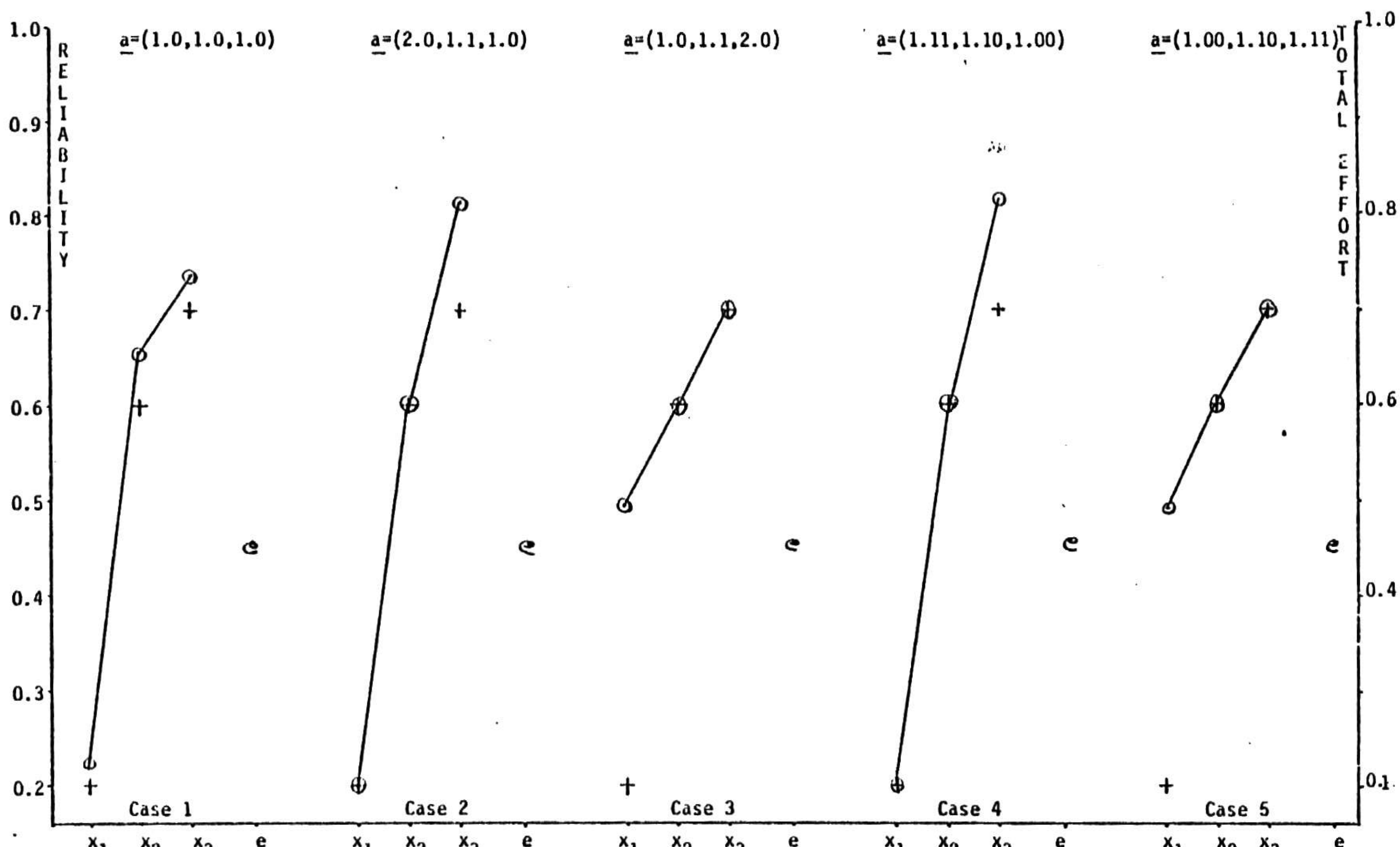
COEFFICIENTS AND $\bar{R}$ = 0.98

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
	0.2	0.6	0.7	
(1.00,1.00,1.00)	0.267	0.739	0.896	1.569
(1.11,1.10,1.00)	0.200	0.600	0.938	1.569
(1.12,1.10,1.00)	0.200	0.600	0.938	1.569
(1.13,1.10,1.00)	0.200	0.600	0.938	1.569
(1.20,1.10,1.00)	0.200	0.600	0.938	1.569
(1.30,1.10,1.00)	0.200	0.600	0.938	1.569
(2.00,1.10,1.00)	0.200	0.600	0.937	1.569
(1.00,1.10,1.11)	0.833	0.600	0.700	1.569
(1.00,1.10,1.12)	0.833	0.600	0.700	1.569
(1.00,1.10,1.13)	0.833	0.600	0.700	1.569
(1.00,1.10,1.20)	0.833	0.600	0.700	1.569
(1.00,1.10,1.30)	0.833	0.600	0.700	1.569
(1.00,1.10,1.50)	0.833	0.600	0.700	1.569
(1.00,1.10,2.00)	0.833	0.600	0.700	1.569

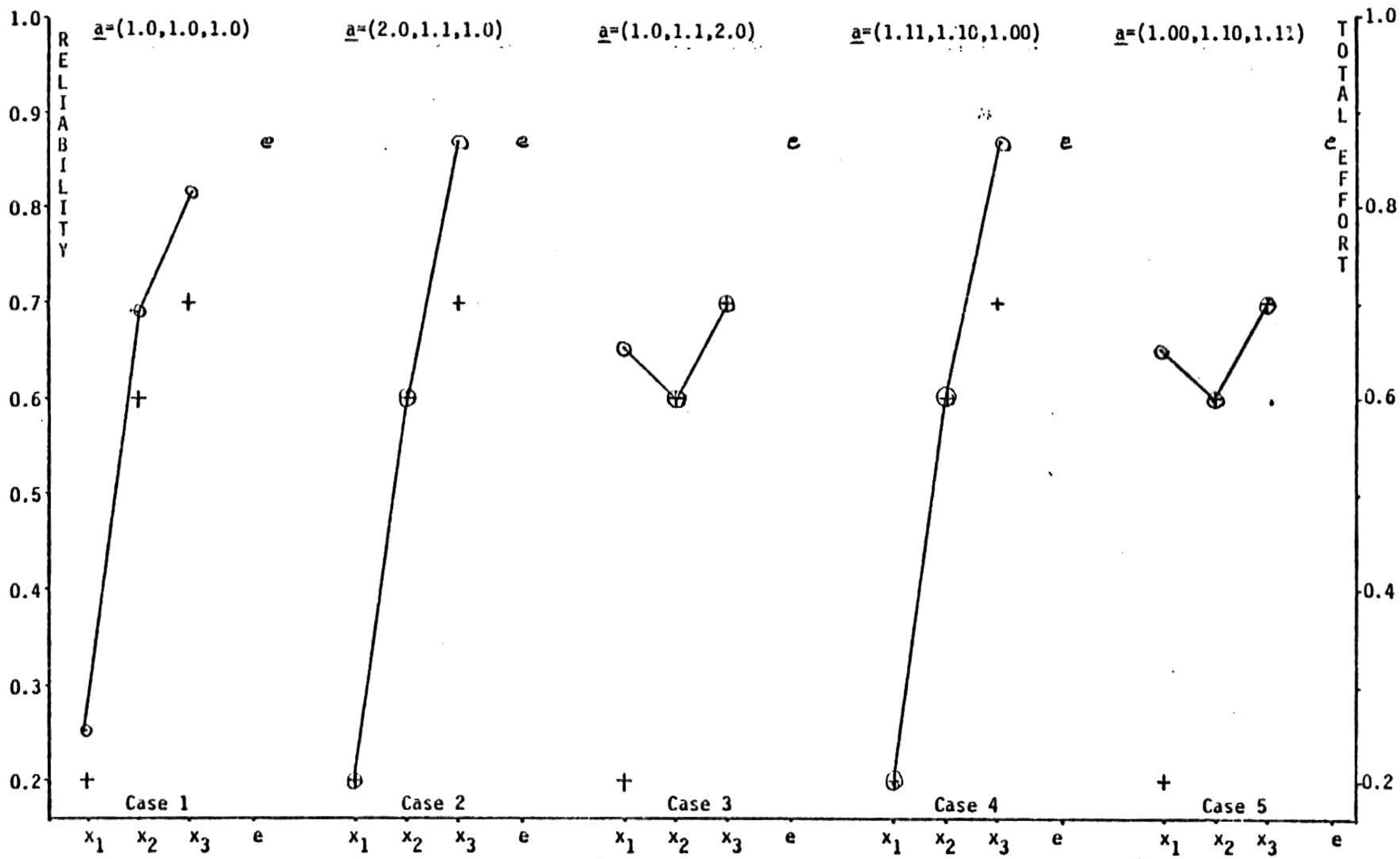


Fiag. 3.2 : The Reliability Allocation and Total Effort Required for $\bar{R} = 0.92$

+ Initial Reliability
0 Optimal Reliability



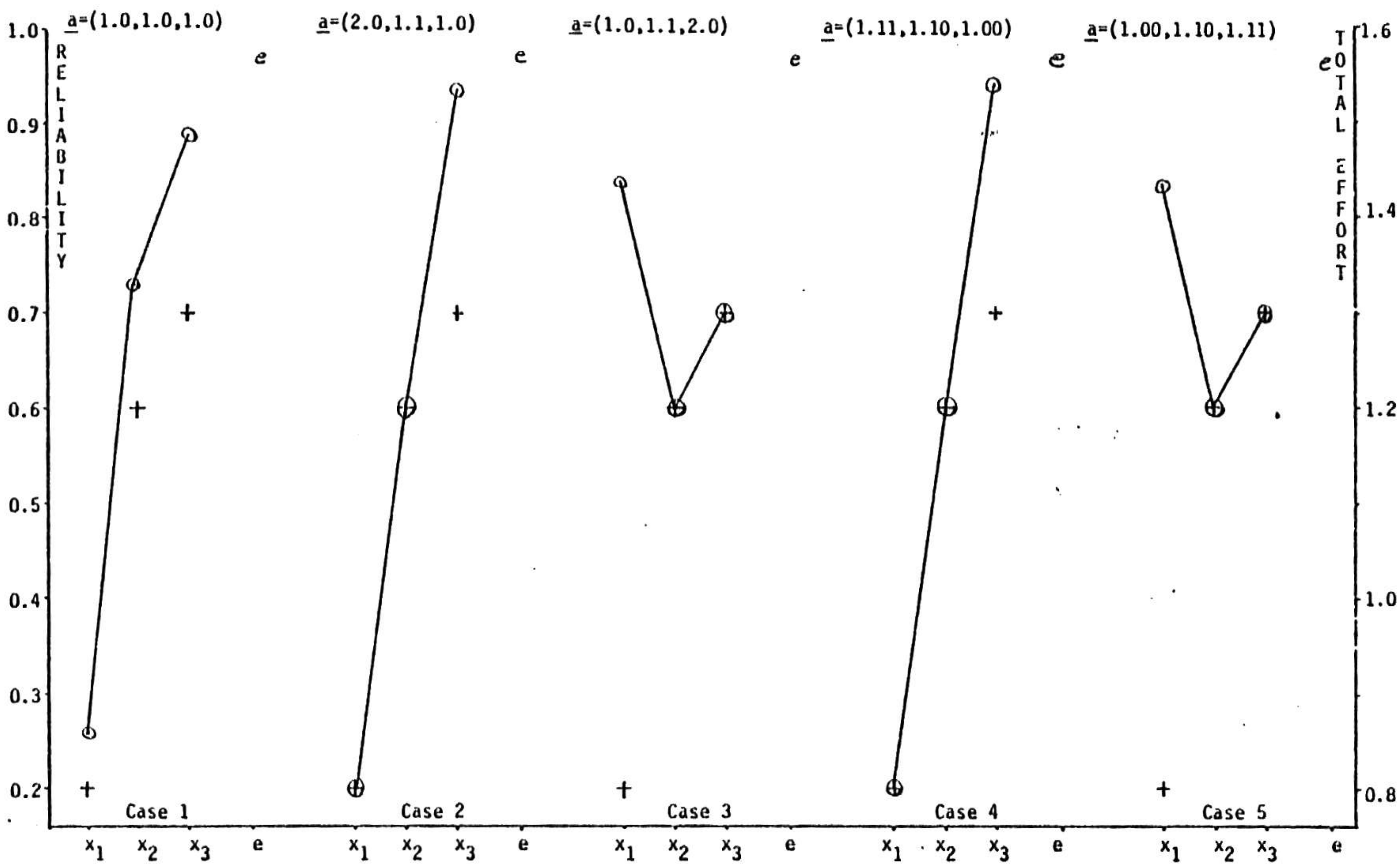
Fiag. 3.3 : The Reliability Allocation and Total Effort Required for $\bar{R} = 0.94$



Fiag. 3.4 : The Reliability Allocation and Total Effort Required for $\bar{R} = 0.96$

+ Initial Reliability
0 Optimal Reliability

BLANK SHEET



Fiag. 3. 5 : The Reliability Allocation and Total Effort Required for $\bar{R}$ = 0.98

\+ Initial Reliability
0 Optimal Reliability

CHAPTER 4

Optimal Reliability Allocation for a Complex System

4.1 Introduction

In previous chapters, the system considered had redundant components in simple series or parallel configuration. The problem becomes considerably more difficult when the redundant component of the system can not be reduced to series or parallel configurations; then it is called a complex system. The case treated here is depicted in Figure 4.1.

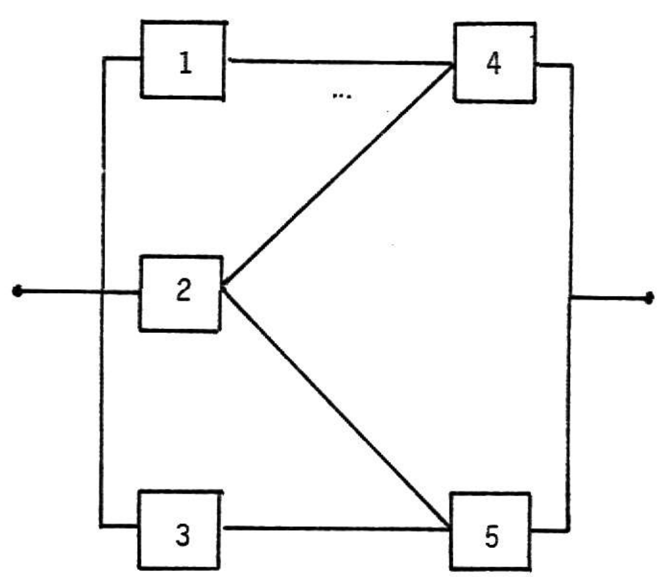


Fig. 4.1. A schematic diagram of a complex system.

In performing the reliability analysis of a complex system, it is almost impossible to treat the system in its entirety. The logical approach is to decompose the system into functional entities composed of components, subsystems or units. In the example used here the following paths of energy flow are possible: C_1C_4, C_2C_4, C_2C_5, C_3C_5. A method of decomposing the reliability structure of a complex system into simplex substructures can be developed through successive application of a conditional probability theorem. The technique begins by selection of a KEYSTONE COMPONENT which appears to bind together the reliability structure of the problem. The reliability may then be expressed in terms of the keystone component k. Thus if k is a component upon whose state, whether good or bad, the system reliability depends, we say that the reliability of system R (system), is

$$R_s = P\text{ (system good given component k is good)} \quad P(\text{k is good})$$
$$+ P\text{ (system good given component k is bad)} \quad P(\text{k is bad}) \quad (4.1)$$

In figure (4.1) component C_2 is selected for the key component; thus we have the expression for system reliability

$$R_s = P(\text{system good}|C_2) \quad P(C_2) + P(\text{system fails}|\bar{C}_2) \quad P(\bar{C}_2) \quad (4.2)$$

where C_i indicates that the $i^{\underline{th}}$ component is good

$\bar{C}_i$ indicates that the $i^{\underline{th}}$ component is bad.

If component C_2 is good the system can fail only if both C_4 and C_5 fail. The system reliability, given C_2 is good, is

$$R_s \text{ (if } C_2 \text{ is good)} = (R_4 + R_5 - R_4 \cdot R_5) \cdot (R_2) \quad (4.3)$$

If on the other hand C_2 is bad the system fails only if the two paths C_1C_4 and C_3C_5 fail. Thus the system reliability for C_2 bad, is

$$R_s \text{ (if } C_2 \text{ is bad)} = (R_1R_4 + R_3R_5 - R_1R_3R_4R_5) \cdot (1-R_2) \tag{4.4}$$

Now by summing equations 4.3 and 4.4 we obtain the reliability of the system, as

$$R_s = P(C_4 \text{ or } C_5 \text{ good}).\ P(C_2 \text{ good}) + P(C_1C_4 \text{ or } C_3C_5).\ P(C_2 \text{ bad}) \tag{4.5}$$

$$R_s = (R_4+R_5 - R_4R_5)(R_2) + (R_1 .\ R_4 + R_3R_5 - R_1R_3R_4R_5)(1 - R_2) \tag{4.6}$$

4.2 Formulation of the Optimization Problem

The problem of minimizing the effort for the complex system given in Figure 4.1, subject to desired system reliability $\bar{R}$ can be stated as

$$\text{Minimize} \quad \sum_{i=1}^{5} a_i \; \ell n \left(\frac{1-R_i}{1 - x_i}\right) \tag{4.7}$$

Subject to

$$(x_4 + x_5-x_4x_5)(x_2) + (x_1x_4 + x_3x_5 - x_1x_3x_4x_5)(1-x_2) \geq \bar{R} \tag{4.8}$$

The solution of the above constrained non-linear programming problem can be obtained by the GREG technique.

4.3 Numerical Examples

The following test problems were solved by the computer program using the method of GREG for Figure 4.1 .

1. Assume C_1 as best component and all others equal.
2. Assume C_2 as best component and all others equal.
3. Assume C_5 as best component and all others equal.

4.3.1 First Example

The nonlinear programming problem formulated in the preceding section is restated, when the C_1 is best component and the objective is to minimize

$$\text{Minimize} \quad e = + a_1 \ln \left(\frac{1-R_1}{1-x_1}\right) + a_2 \ln \left(\frac{1-R_2}{1-x_2}\right)$$

$$+ a_3 \ln \left(\frac{1-R_3}{1-x_3}\right) + a_4 \ln \left(\frac{1-R_4}{1-x_4}\right)$$

$$+ a_5 \ln \left(\frac{1-R_5}{1-x_5}\right) \tag{4.9}$$

Subject to

$$(x_4+x_5-x_4x_5)(x_2) + (x_1x_4 + x_3x_5 - x_1x_3x_4x_5)(1-x_2) \geq \bar{R} \tag{4.10}$$

$$R_i \leq x_i \leq 1 \qquad i = 1,2, \ldots, 5 \tag{4.11}$$

The numerical values of parameters $\underline{a}$, are given in Table 4.1, and assume $\underline{\bar{R}}$ = [0.90, 0.93, 0.95, 0.97] value with initial value $[R_1, R_2, R_3, R_4, R_5]$ = [0.9, 0.7, 0.7, 0.7, 0.7],

The problem is formulated in GREG format as follows:

$$\text{Minimize} \quad e = - a_1 \ln \left(\frac{1-0.9}{1-x_1}\right) - a_2 \ln \left(\frac{1-0.7}{1-x_2}\right)$$

$$- a_3 \ln \left(\frac{1-0.7}{1-x_3}\right) - a_4 \ln \left(\frac{1-0.7}{1-x_4}\right)$$

$$- a_5 \ln \left(\frac{1-0.7}{1-x_3}\right) \tag{4.12}$$

Subject to

$$- (x_4 + x_5 - x_4x_5)(x_2)$$

$$- (x_1x_4 + x_3x_5 - x_1x_3x_4x_5)(1 - x_2) + \bar{R} \leq 0 \tag{4.13}$$

$$R_i \leq x_i \leq 1 \qquad i = 1,2, \ldots, 5 \tag{4.14}$$

Four external, user-supplied subroutines will be used (Appendix 4). The optimal solutions which were obtained are presented in Tables 4.1 to 4.5. Table 4.1 presents the optimal solutions according to different $\underline{a}$ (effort coefficient) and $\bar{R}$ (desired system reliability). Tables 4.2 to 4.5 present the optimal solutions according to different effort coefficient ($\underline{a}$) for the $\underline{\bar{R}}$ = [0.90, 0.93, 0.95, 0.97] respectively.

Table 4.5 shows the reliability allocation for each component when $\bar{R}$ = 0.97. Note that in all cases the increase on reliability occurred for the keystone component C_2 and the two other components, C_4 and C_5, which follow the keystone component. The C_1 and C_3 remain unchanged. The optimal solutions for cases 1 to 3 are identical, and for case 4 and 5 we have different reliability allocations and optimal effort this is because the effort coefficient is different for the best component in the various cases.
The roles of R_1 and R_3 can be enterchanged for alternate solutions, because of the symmetry of the configuration.

TABLE 4.1

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS AND SYSTEM RELIABILITY

EFFORT COEFFICIENTS $\underline{a}$	$\bar{R}$	R_1^0	R_2^0	R_3^0	R_4^0	R_5^0	TOTAL EFFORT
		0.9	0.7	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.90	0.900	0.700	0.700	0.756	0.700	0.208
	0.93	0.900	0.700	0.700	0.843	0.700	0.646
	0.95	0.900	0.700	0.700	0.900	0.700	1.101
	0.97	0.900	0.793	0.700	0.950	0.820	2.668
(1.11,1.00,1.00,1.00,1.00)	0.90	0.900	0.700	0.700	0.756	0.700	0.208
	0.93	0.900	0.700	0.700	0.843	0.700	0.646
	0.95	0.900	0.700	0.700	0.900	0.700	1.101
	0.97	0.900	0.795	0.700	0.946	0.832	2.670
(2.00,1.00,1.00,1.00,1.00)	0.90	0.900	0.700	0.700	0.756	0.700	0.208
	0.93	0.900	0.700	0.700	0.843	0.700	0.646
	0.95	0.900	0.700	0.700	0.900	0.700	1.101
	0.97	0.900	0.795	0.700	0.947	0.829	2.670
(1.00,1.11,1.11,1.11,1.11)	0.90	0.900	0.700	0.700	0.756	0.700	0.231
	0.93	0.900	0.700	0.700	0.843	0.700	0.717
	0.95	0.900	0.700	0.700	0.900	0.700	1.222
	0.97	0.900	0.794	0.700	0.947	0.828	2.964
(1.00,2.00,2.00,2.00,2.00)	0.90	0.900	0.700	0.700	0.756	0.700	0.416
	0.93	0.900	0.700	0.700	0.843	0.700	1.291
	0.95	0.900	0.700	0.700	0.900	0.700	2.201
	0.97	0.953	0.700	0.700	0.961	0.721	4.999

TABLE 4.2

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R}$ = 0.90

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.9	0.7	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.900	0.700	0.700	0.756	0.700	0.208
(1.11,1.00,1.00,1.00,1.00)	0.900	0.700	0.700	0.756	0.700	0.208
(2.00,1.00,1.00,1.00,1.00)	0.900	0.700	0.700	0.756	0.700	0.208
(1.00,1.11,1.11,1.11,1.11)	0.900	0.700	0.700	0.756	0.700	0.231
(1.00,2.00,2.00,2.00,2.00)	0.900	0.700	0.700	0.756	0.700	0.416

TABLE 4.3

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R} = 0.93$

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.9	0.7	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.900	0.700	0.700	0.843	0.700	0.646
(1.11,1.00,1.00,1.00,1.00)	0.900	0.700	0.700	0.843	0.700	0.646
(2.00,1.00,1.00,1.00,1.00)	0.900	0.700	0.700	0.843	0.700	0.646
(1.00,1.11,1.11,1.11,1.11)	0.900	0.700	0.700	0.843	0.700	0.717
(1.00,2.00,2.00,2.00,2.00)	0.900	0.700	0.700	0.843	0.700	1.291

TABLE 4.4

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R} = 0.95$

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.9	0.7	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.900	0.700	0.700	0.900	0.700	1.101
(1.11,1.00,1.00,1.00,1.00)	0.900	0.700	0.700	0.900	0.702	1.101
(2.00,1.00,1.00,1.00,1.00)	0.900	0.700	0.700	0.900	0.700	1.101
(1.00,1.11,1.11,1.11,1.11)	0.900	0.700	0.700	0.900	0.700	1.222
(1.00,2.00,2.00,2.00,2.00)	0.900	0.700	0.700	0.900	0.700	2.201

TABLE 4.5

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R}$ = 0.97

EFFORT COEFFICIENTS

$\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.9	0.7	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.900	0.793	0.700	0.950	0.820	2.668
(1.11,1.00,1.00,1.00,1.00)	0.900	0.795	0.700	0.946	0.832	2.670
(2.00,1.00,1.00,1.00,1.00)	0.900	0.795	0.700	0.947	0.829	2.670
(1.00,1.11,1.11,1.11,1.11)	0.900	0.794	0.700	0.947	0.828	2.964
(1.00,2.00,2.00,2.00,2.00)	0.953	0.700	0.700	0.961	0.721	4.999

4.3.2 Second Example

Consider Figure 4.1 when the C_2 is the best component ($R_2 = 0.9$, $R_i = 0.7 \quad i = 1,3,4,5$). The formulation for GREG is

$$\text{Minimize} \quad e = - a_1 \ln \left(\frac{1-0.7}{1-x_1}\right) - a_2 \ln \left(\frac{1-0.9}{1-x_2}\right)$$

$$- a_3 \ln \left(\frac{1-0.7}{1-x_3}\right) - a_4 \ln \left(\frac{1-0.7}{1-x_4}\right)$$

$$- a_5 \ln \left(\frac{1-0.7}{1-x_5}\right) \qquad (4.15)$$

Subject to

$$- (x_4 + x_5 - x_4x_5)(x_2) - (x_1x_4 + x_3x_5 - x_1x_3x_4x_5)(1-x_2)$$

$$+ \bar{R} \leq 0 \qquad (4.16)$$

$$R_i \leq x_i \leq 1 \qquad i = 1, 2, \ldots, 5 \qquad (4.17)$$

Four external, user-supplied subroutines will be used (Appendix 4). The optimal solutions which were obtained are presented in Tables 4.6 to 4.10.

In Table 4.10, the reliability allocation is shown for each component when $\bar{R} = 0.98$. The optimal solutions for cases 1 to 3 are almost identical. That is, the most expensive component is also the best component; the optimal effort is e = 2.440. For cases 4 and 5, we have different reliability allocations and optimal total efforts.

$$\text{Minimize} \quad e = - a_1 \ln \left(\frac{1-0.7}{1-x_1}\right) - a_2 \ln \left(\frac{1-0.7}{1-x_2}\right)$$

$$- a_3 \ln \left(\frac{1-0.7}{1-x_3}\right) - a_4 \ln \left(\frac{1-0.7}{1-x_4}\right)$$

$$- a_5 \ln \left(\frac{1-0.9}{1-x_5}\right) \tag{4.18}$$

Subject to

$$- (x_4 + x_5 - x_4x_5)(x_2) - (x_1x_4 + x_3x_5 - x_1x_3x_4x_5)\ (1-x_2) + \bar{R} \leq 0 \tag{4.19}$$

$$R_i \leq x_i < 1 \qquad i = 1, 2, \ldots, 5 \tag{4.20}$$

Four external, user-supplied subroutines will be used (Appendix 4). The optimal solutions are presented in Tables 4.11 to 4.15.

In Table 4.15, the reliability allocations are shown for the various component when $\bar{R} = 0.98$. Note that all cases the increase occurs for the keystone component C_2 and the two other components C_4 and C_5 which follow the keystone component. The C_1 and C_3 remain unchanged. The optimal solutions for case 1 to 3 are identical. For cases 4 and 5, we have different reliability allocations and optimal efforts (Notice the difference in the values of the effort coefficients). The roles of R_4 and R_5 can be enterchanged for alternate solutions, because of the symmetry of the configuration.

TABLE 4.6

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS AND SYSTEM RELIABILITY

EFFORT COEFFICIENTS $\underline{a}$	$\bar{R}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
		0.7	0.9	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.91	0.700	0.900	0.700	0.726	0.733	0.205
	0.93	0.700	0.900	0.700	0.760	0.775	0.511
	0.96	0.700	0.900	0.700	0.838	0.842	1.258
	0.98	0.700	0.914	0.700	0.884	0.921	2.439
(1.00,1.11,1.00,1.00,1.00)	0.91	0.700	0.900	0.700	0.726	0.732	0.205
	0.93	0.700	0.900	0.700	0.760	0.775	0.511
	0.96	0.700	0.900	0.700	0.826	0.851	1.259
	0.98	0.700	0.909	0.700	0.894	0.918	2.450
(1.00,2.00,1.00,1.00,1.00)	0.91	0.700	0.900	0.700	0.726	0.732	0.205
	0.93	0.700	0.900	0.700	0.760	0.775	0.511
	0.96	0.700	0.900	0.700	0.826	0.853	1.259
	0.98	0.700	0.900	0.716	0.895	0.923	2.459
(1.11,1.00,1.11,1.11,1.11)	0.91	0.700	0.900	0.700	0.726	0.733	0.227
	0.93	0.700	0.900	0.700	0.760	0.775	0.568
	0.96	0.700	0.900	0.700	0.829	0.850	1.397
	0.98	0.700	0.920	0.700	0.887	0.913	2.681
(2.00,1.00,2.00,2.00,2.00)	0.91	0.700	0.900	0.700	0.726	0.732	0.409
	0.93	0.700	0.900	0.700	0.764	0.771	1.022
	0.96	0.700	0.903	0.700	0.821	0.855	2.518
	0.98	0.700	0.947	0.700	0.870	0.900	4.503

TABLE 4.7

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R}$ = 0.91

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.7	0.9	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.700	0.900	0.700	0.726	0.733	0.205
(1.00,1.11,1.00,1.00,1.00)	0.700	0.900	0.700	0.726	0.732	0.205
(1.00,2.00,1.00,1.00,1.00)	0.700	0.900	0.700	0.726	0.732	0.205
(1.11,1.00,1.11,1.00,1.00)	0.700	0.900	0.700	0.726	0.733	0.227
(2.00,1.00,2.00,2.00,2.00)	0.700	0.900	0.700	0.726	0.732	0.409

T A B L E 4.8

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS FOR $\bar{R} = 0.93$

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.7	0.9	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.700	0.900	0.700	0.760	0.775	0.511
(1.00,1.11,1.00,1.00,1.00)	0.700	0.900	0.700	0.760	0.775	0.511
(1.00,2.00,2.00,2.00,2.00)	0.700	0.900	0.700	0.760	0.775	0.511
(1.11,1.00,1.11,1.11,1.11)	0.700	0.900	0.700	0.760	0.775	0.568
(2.00,1.00,2.00,2.00,2.00)	0.700	0.900	0.700	0.764	0.771	1.022

TABLE 4.9

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R} = 0.96$

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.7	0.9	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.700	0.900	0.700	0.838	0.842	1.258
(1.00,1.11,1.00,1.00,1.00)	0.700	0.900	0.700	0.826	0.851	1.259
(1.00,2.00,1.00,1.00,1.00)	0.700	0.900	0.700	0.826	0.853	1.259
(1.11,1.00,1.11,1.11,1.11)	0.700	0.900	0.700	0.829	0.850	1.397
(2.00,1.00,2.00,2.00,2.00)	0.700	0.903	0.700	0.821	0.855	2.518

TABLE 4.10

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R} = 0.98$

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.7	0.9	0.7	0.7	0.7	
(1.00,1.00,1.00,1.00,1.00)	0.700	0.914	0.700	0.884	0.921	2.439
(1.00,1.11,1.00,1.00,1.00)	0.700	0.909	0.700	0.894	0.918	2.450
(1.00,2.00,1.00,1.00,1.00)	0.700	0.900	0.716	0.895	0.923	2.459
(1.11,1.00,1.11,1.11,1.11)	0.700	0.900	0.700	0.887	0.913	2.681
(2.00,1.00,2.00,2.00,2.00)	0.700	0.900	0.700	0.870	0.900	4.503

4.3.3 Third Example

Consider again the example of Figure 4.1 when C_5 is the best component ($R_5 = 0.9$, $R_i = 0.7$ $\quad i = 1,2,3,4$). The formulation for GREG is

$$\text{Minimize} \quad e = -a_1 \ln\left(\frac{1-0.7}{1-x_1}\right) - a_2 \ln\left(\frac{1-0.7}{1-x_2}\right)$$

$$- a_3 \ln\left(\frac{1-0.7}{1-x_3}\right) - a_4 \ln\left(\frac{1-0.7}{1-x_4}\right)$$

$$- a_5 \ln\left(\frac{1-0.9}{1-x_5}\right) \tag{4.18}$$

Subject to

$$-(x_4 + x_5 - x_4x_5)(x_2) - (x_1x_4 + x_3x_5 - x_1x_3x_4x_5)(1-x_2) + \bar{R} \leq 0 \tag{4.19}$$

$$R_i \leq x_i < 1 \qquad i = 1, 2, \ldots, 5 \tag{4.20}$$

Four external, user-supplied subroutines will be used (Appendix 4). The optimal solutions are presented in Tables 4.11 to 4.15.

In Table 4.15, the reliability allocations are shown for the various component when $\bar{R} = 0.98$. Note that all cases the increase occurs for the keystone component C_2 and the two other components C_4 and C_5 which follow the keystone component. The C_1 and C_3 remain unchanged. The optimal solutions for case 1 to 3 are identical. For cases 4 and 5, we have different reliability allocations and optimal efforts (Notice the difference in the values of the effort coefficients). The roles of R_4 and R_5 can be enterchanged for alternate solutions, because of the symmetry of the configuration.

TABLE 4.11

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT COEFFICIENTS AND SYSTEM RELIABILITY

EFFORT COEFFICIENTS $\underline{a}$	$\overline{R}$	R_1^0	R_2^0	R_3^0	R_4^0	R_5^0	TOTAL EFFORT
		0.7	0.7	0.7	0.7	0.9	
(1.00,1.00,1.00,1.00,1.00)	0.94	0.700	0.772	0.700	0.745	0.900	0.439
	0.95	0.700	0.805	0.700	0.783	0.900	0.757
	0.97	0.700	0.877	0.700	0.863	0.900	1.678
	0.98	0.700	0.915	0.700	0.904	0.902	2.433
(1.00,1.00,1.00,1.00,1.11)	0.94	0.700	0.770	0.700	0.748	0.900	0.439
	0.95	0.700	0.802	0.700	0.787	0.900	0.758
	0.97	0.700	0.877	0.700	0.863	0.900	1.678
	0.98	0.700	0.915	0.700	0.907	0.900	2.432
(1.00,1.00,1.00,1.00,2.00)	0.94	0.700	0.772	0.700	0.754	0.900	0.439
	0.95	0.700	0.805	0.700	0.784	0.900	0.757
	0.97	0.700	0.877	0.700	0.864	0.900	1.678
	0.98	0.700	0.916	0.700	0.906	0.900	2.433
(1.11,1.00,1.00,1.00,1.00)	0.94	0.700	0.770	0.700	0.747	0.900	0.487
	0.95	0.700	0.803	0.700	0.786	0.900	0.841
	0.97	0.700	0.876	0.700	0.848	0.911	1.860
	0.98	0.700	0.915	0.700	0.857	0.937	2.680
(2.00,2.00,2.00,2.00,1.00)	0.94	0.700	0.743	0.700	0.700	0.934	0.730
	0.95	0.700	0.783	0.700	0.700	0.946	1.269
	0.97	0.700	0.870	0.700	0.700	0.967	2.785
	0.98	0.700	0.912	0.700	0.700	0.978	3.995

TABLE 4.12

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R} = 0.94$

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.7	0.7	0.7	0.7	0.9	
(1.00,1.00,1.00,1.00,1.00)	0.700	0.772	0.700	0.745	0.900	0.439
(1.00,1.00,1.00,1.00,1.11)	0.700	0.770	0.700	0.748	0.900	0.439
(1.00,1.00,1.00,1.00,2.00)	0.700	0.772	0.700	0.754	0.900	0.439
(1.11,1.11,1.11,1.11,1.00)	0.700	0.770	0.700	0.747	0.900	0.487
(2.00,2.00,2.00,2.00,1.00)	0.700	0.743	0.700	0.700	0.934	0.730

TABLE 4.13

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R} = 0.95$

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.7	0.7	0.7	0.7	0.9	
(1.00,1.00,1.00,1.00,1.00)	0.700	0.805	0.700	0.783	0.900	0.757
(1.00,1.00,1.00,1.00,1.11)	0.700	0.802	0.700	0.787	0.900	0.758
(1.00,1.00,1.00,1.00,2.00)	0.700	0.805	0.700	0.784	0.900	0.757
(1.11,1.11,1.11,1.11,1.00)	0.700	0.803	0.700	0.786	0.900	0.841
(2.00,2.00,2.00,2.00,1.00)	0.700	0.784	0.700	0.700	0.946	1.269

TABLE 4.14

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R} = 0.97$

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.7	0.7	0.7	0.7	0.9	
(1.00,1.00,1.00,1.00,1.00)	0.700	0.877	0.700	0.863	0.900	1.678
(1.00,1.00,1.00,1.00,1.11)	0.700	0.877	0.700	0.863	0.900	1.678
(1.00,1.00,1.00,1.00,2.00)	0.700	0.877	0.700	0.864	0.900	1.678
(1.11,1.11,1.11,1.11,1.00)	0.700	0.876	0.700	0.848	0.911	1.860
(2.00,2.00,2.00,2.00,1.00)	0.700	0.870	0.700	0.700	0.967	2.785

TABLE 4.15

OPTIMAL SOLUTIONS FOR DIFFERENT EFFORT

COEFFICIENTS FOR $\bar{R} = 0.98$

EFFORT COEFFICIENTS $\underline{a}$	R_1^o	R_2^o	R_3^o	R_4^o	R_5^o	TOTAL EFFORT
	0.7	0.7	0.7	0.7	0.9	
(1.00,1.00,1.00,1.00,1.00)	0.700	0.915	0.700	0.904	0.902	2.433
(1.00,1.00,1.00,1.00,1.11)	0.700	0.915	0.700	0.907	0.900	2.432
(1.00,1.00,1.00,1.00,2.00)	0.700	0.916	0.700	0.906	0.900	2.433
(1.11,1.11,1.11,1.11,1.00)	0.700	0.915	0.700	0.857	0.937	2.680
(2.00,2.00,2.00,2.00,1.00)	0.700	0.912	0.700	0.700	0.978	3.995

CHAPTER 5

Discussion and Conclusion

5.1 Introduction.

The purpose of this investigation was to find out how robust Albert's procedure is when the components do not all have the same effort function and/or when the configuration is not series.

In performing the reliability analysis in the previous chapters, all the results depend on the structure assumed for $G(x,y) = a \ln\left(\frac{1-x}{1-y}\right)$. It is of interest to determine what the results would have been for

$G(x,y) = a \ln\left(\frac{1+y}{1+x}\right)$ and $G(x,y) = a(\sqrt{y} - \sqrt{x})$. To answer this question the following test problems were solved.

5.2 Problem No. 1

$$\text{Case 1} \quad G_i(x,y) = a_i \ln\left(\frac{1-R_i}{1-x_i}\right) \qquad i = 1,2,3 \tag{5.1}$$

$$\text{Case 2} \quad G_i(x,y) = a_i \ln\left(\frac{1+y_i}{1+x_i}\right) \qquad i = 1,2,3 \tag{5.2}$$

$$\text{Case 3} \quad G_i(x,y) = a_i \left(\sqrt{y_i} - \sqrt{x_i}\right) \qquad i = 1,2,3 \tag{5.3}$$

Case 1 is the effort function we used in the previous chapter. We use the above cases to minimize the effort for three components in a series system with the values of the constants as:

$R_1 = 0.7, \quad R_2 = 0.8, \quad R_3 = 0.9,$

$\bar{R} = 0.65, \quad a_i = 1.0 \quad i = 1,2,3.$

The optimal solutions which obtained are presented in Table 5.1.

Table 5.1 Optimal Solution

	R_1^o	R_2^o	R_3^o	TOTAL EFFORT
Initial value	0.7	0.8	0.9	
Case 1.	0.85	0.85	0.90	0.979
Case 2.	0.85	0.85	0.90	0.112
Case 3.	0.85	0.85	0.90	0.113

The solutions are identical, indicating that the solution is not very sensitive to the structure of the effort function.

5.3 Problem No. 2

In this problem we minimize the effort for six components in a series system (with the same G(x,y) structure as introduced in problem No. 1), with the value of the constants as:

$R_1 = 0.75, \quad R_2 = 0.80, \quad R_3 = 0.87$

$R_4 = 0.90, \quad R_5 = 0.95, \quad R_6 = 0.99$

$\bar{R} = 0.53 \quad a_i = 1.0 \quad i = 1,2, \ldots, 6$

The optimal solutions are presented in Table 5.2.

Table 5.2 Optimal Solution.

	$\underline{R_1^o}$	$\underline{R_2^o}$	$\underline{R_3^o}$	$\underline{R_4^o}$	$\underline{R_5^o}$	$\underline{R_6}$	TOTAL EFFORT
Initial value	0.75	0.80	0.87	0.90	0.95	0.99	
Case 1.	0.85	0.85	0.87	0.90	0.95	0.99	0.778
Case 2.	0.84	0.84	0.88	0.91	0.95	0.99	0.082
Case 3.	0.84	0.84	0.88	0.91	0.95	0.99	0.082

The solutions are almost identical, indicating that the solution is not very sensitive to the structure of the effort function.

5.4 Conclusion

From the results presented in this study the following conclusions can be drawn:

1. The optimal reliability allocation depends on:
   - I) Series Configuration: $\bar{R}$ and $\underline{a}$.
   - II) Parallel Configuration: $\bar{R}$ (not $\underline{a}$).
   - III) Complex Configuration: $\bar{R}$ and $\underline{a}$.

2. The change of reliability for the:
   - I) Series Configuration: worst component changes the most.
   - II) Parallel configuration: best component changes the most.
   - III) Complex Configuration: keystone component changes the most.

3. The numerical value of the optimal efforts:
   - I) Series configuration: depends on $\underline{a}$.
   - II) Parallel configuration: does not depend on $\underline{a}$ (since the only components changed were those with $a_i = 1$).

III Complex configuration: depends on <u>a</u>.

Therefore, the assumption that the effort coefficients are the same for all components is a rather strange one.

The present work does not pretend to be an exhaustive investigation in the problem. Many more cases could have been run, but this was not done because this is a preliminary study and the results are expensive (each case cost about $2 to obtain).

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APPENDIX I

```
$JOB
C      PROGRAM FOR ROOT FINDING
C
C      DATA INPUT:
C
C.     CARD 1. MAX.ITERATIONS *** FORMAT(I3)
C
C      CARD 2. INTIAL POINT,ACCURACY IN X, ACCURACY IN FX
C              STOPPING CRITERION) *** FORMAT(3F10.4)
C
C      PROGRAM HANDLES ALL FUNCTIONS OF THE TYPE Y=F(X)
C
C      STOPPING CRITERION ON BOTH X AND F(X) ARE USED
C
C      NEWTON METHOD
C         CHECKS THE CONVERGENCE OF THE PROCESS
C         INDICATES IF DERIVATIVE IS ZERO FOR ANY X
C
C
C      USER SUPPLIED FUNCTION SUBPROGRAM
C      TO DEFINE THE FUNCTION F(X)
       FUNCTION F(X)
C
C***********************************************************
C
C      THE EXPRESSION GOES HERE
C
       F=(0.074)*X**3-(2.062)*X**2+0.814*X+0.888                      R3   ROOT
C
C***********************************************************
       RETURN
       END
C
C
C      USER SUPPLIED FUNCTION SUBPROGRAM
C    . TO DEFINE THE DERIVATIVE OF THE FUNCTION F(X)

       FUNCTION DF(X)
C
C************************************************************
C
C      EXPRESSION GOES HERE
C
       DF=(0.222)*X**2-(4.124)*X+0.814                                R3   ROOT
C
C************************************************************
C
       RETURN
       END
```

```
      EXTERNAL   F,DF
      COMMON     MAXITR,X1,E1,E2
    1 READ (5,105,END=999) MAXITR
      READ (5,115) X1,E1,E2
      WRITE (6,206)
      WRITE (6,216)
      WRITE (6,226)
      CALL      ROOT (F,DF,XF)
      WRITE (6,236) XF
  206 FORMAT ('1')
  216 FORMAT (' ',T25,'***** NEWTON SINGLE-VARIABLE ROOT FINDING PROCEDU
     1RE  *****')
  226 FORMAT ('-',T5,'ITERATION',T20,'X(N-1)',T39,'F(X)',T56,'DF(X)',T73
     1          ,'X(N)',T87,'X(N)-X(N-1)'/)
  236 FORMAT ('-','THE SOLUTION IS:   ',F10.4)
  105 FORMAT (I3)
  115 FORMAT (3F10.4)
      GO TO 1
  999 STOP
      END

      SUBROUTINE  ROOT (F,DF,XF)
      COMMON  MAXITR,X1,E1,E2
      X0=X1
   10 DO 30 I=1,MAXITR
         IF (DF(X0).EQ.0.0) GO TO 40
         X=X0-(F(X0)/DF(X0))
         FX=F(X0)
         DFX=DF(X0)
         P=X-X0
         WRITE (6,106) I,X0,FX,DFX,X,P
         IF (ABS(FX)-ABS(F(X0)).GT.0.0) GO TO 50
         IF (ABS(F(X0)).GT.E2)   GO TO 20
         IF (ABS(P).GT.E1)       GO TO 20
         XF=X0
         GO TO 888
   20    X0=X
   30 CONTINUE
C
   40 XF=X0
      WRITE (6,116)
      GO TO 888
   50 WRITE (6,126)
C
  106 FORMAT ('0',I12,7E17.7)
  116 FORMAT ('-','***** THE DERIVATIVE IS ZERO AT THIS X',/'0',
     1          '***** DISREGARD THE SOLUTION GIVEN BELOW')
  126 FORMAT ('-','***** THE NEWTON METHOD DOES NOT CONVERGE'/' ',
     1          'TO THE SOLUTION FOR GIVEN INITIAL POINT'/' ',
     2          'REASSIGN THE INITIAL POINT')
C
  888 RETURN
      END
```

***** NEWTON SINGLE-VARIABLE ROOT FINDING PROCEDURE *****

ITERATION	X(N-1)	F(X)	DF(X)	X(N)	X(N)-X(N-1)
1	0.8000000E 00	0.2574093E 00	-0.2343115E 01	0.9098575E 00	0.1098575E 00
2	0.9098575E 00	-0.2264434E-01	-0.2754471E 01	0.9016363E 00	-0.8221209E-02
3	0.9016363E 00	-0.1249909E-03	-0.2723872E 01	0.9015904E 00	-0.4589558E-04
4	0.9015904E 00	-0.2384186E-06	-0.2723701E 01	0.9015903E 00	-0.1192093E-06

THE SOLUTION IS: 0.9016

CORE USAGE OBJECT CODE= 2680 BYTES,ARRAY AREA= 32 BYTES,TOTAL AREA AVAILABLE= 159840 BYTES

DIAGNOSTICS NUMBER OF ERRORS= 0, NUMBER OF WARNINGS= 0, NUMBER OF EXTENSIONS= 0

COMPILE TIME= 0.23 SEC,EXECUTION TIME= 0.04 SEC, 7.47.21 MONDAY 13 AUG 79 WATFIV - JAN 1976 V1L5

APPENDIX II

```
$JOB
C                                                                     PHIX  10
C                                                                     PHIX  20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * PHIX  30
C                                                                     PHIX  40
C     SUBROUTINE PHIX                                                 PHIX  50
C                                                                     PHIX  60
C     PURPOSE                                                         PHIX  70
C        THE EXTERNAL, USER-SUPPLIED SUBROUTINE "PHIX" DEFINES THE    PHIX  90
C        OBJECTIVE FUNCTION TO THE  GREG  PROGRAM.                    PHIX 100
C                                                                     PHIX 110
C     DESCRIPTION OF PARAMETERS                                       PHIX 120
C        AI    - THE EFFORT COEFFIENT FOR I TH COMPONENT.             PHIX 130
C        RI    - THE RELIABILITY OF I TH COMPONENT.                   PHIX 140
C                                                                     PHIX 150
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * PHIX 160
C                                                                     PHIX 170
C                                                                     PHIX 180
      SUBROUTINE PHIX
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(190),IVC(50),IVA(100)
      COMMON      A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON      NV,NC,NK,NEG,NIN,NTV,NVI,NEV,NEVL,NIO,NIN1,NIN2,NIN3,NI
     1N4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IR1,IS,IS1,IT,IBP,ICDB,JCDB,KC
     2DB,KFIL,KLIN,KREN,KD,FIL,PHI,PSI,PSI3,TB,TD,TC,EPSIL,EPSILO,EPSIL2
      COMMON     KFONC,KGRAD,KCONT,KINV1,KINV2,KCDBA,KJACO,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCDBA1,KREN11,KREN21
      COMMON      IDIREC,DELTFI,ETA,JKO,LC,YSORT
      PHI=-A1*ALOG((1-R1)/(1-XC(1)))-A2*ALOG((1-R2)/(1-XC(2)))
     1    -A3*ALOG((1-R3)/(1-XC(3)))
      RETURN
      END
```

```
C                                                                       GRAD  10
C                                                                       GRAD  20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * GRAD  30
C                                                                       GRAD  40
C     SUBROUTINE GRADFI                                                 GRAD  50
C                                                                       GRAD  60
C     PURPOSE                                                           GRAD  70
C        THIS SUBROUTINE DEFINES THE GRADIENT OF THE OBJECTIVE FUNCTION GRAD  80
C        IN TERMS OF THE ARRAY XC(J), J=1, ....,NV. THE COMPONENT VALUESGRAD  90
C        ARE STRORED IN THE VECTOR ARRAY C(J), J=1, ....,NV.            GRAD 100
C                                                                       GRAD 110
C     DESCRIPTION OF PARAMETER.                                         GRAD 120
C        AI    - THE EFFORT COEFFIENT FOR I TH COMPNENTS.               GRAD 130
C                                                                       GRAD 140
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * GRAD 150
C                                                                       GRAD 160
C                                                                       GRAD 170
      SUBROUTINE GRADFI
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON     A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON     NV,NC,NK,NEG,NIN,NTV,NV1,NEV,NEVL,NIO,NIN1,NIN2,NIN3,NI
     1N4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IR1,IS,IS1,IT,IBP,ICDB,JCDB,KC
     2DB,KFIL,KLIN,KREN,KD,FI1,PHI,PSI,PSI3,TB,TD,TC,EPSIL,EPSILO,EPSIL2
      COMMON     KFUNC,KGRAD,KCONT,KINV1,KINV2,KCDBA,KJACO,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCDBA1,KREN11,KREN21
      COMMON     IDIREC,DELTFI,ETA,JKO,LC,YSORT
      C(1)=A1/(XC(1)-1.0)
      C(2)=A2/(XC(2)-1.0)
      C(3)=A3/(XC(3)-1.0)
      RETURN
      END
```

```
C                                                                      CPHI  10
C                                                                      CPHI  20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * CPHI  30
C                                                                      CPHI  40
C        SUBROUTINE CPHI                                               CPHI  50
C                                                                      CPHI  60
C        PURPOSE                                                       CPHI  70
C           CPHI DEFINES THE CONSTRAINT FUNCTIONS ( .LE. OR .EQ. 0 ).  CPHI  80
C           THE VALUES ARE STORED IN THE VECTOR ARRAY VC(I), I=1, ....., NC,CPHI  90
C           AND IN TERMS OF THE ORIGINAL PROBLEM VARIABLES, XC(J), J=1, ...CPHI 100
C           ,NV.                                                       CPHI 110
C                                                                      CPHI 120
C        DESCRIPTION OF PARAMETER.                                     CPHI 130
C           RI    - THE RELIABILITY OF I TH, COMPONENTS.               CPHI 140
C                                                                      CPHI 150
C        REMARKS                                                       CPHI 160
C           THE CONSTRAINTS MUST BE ORDERED WITH INEQUALITIES FIRST AND  CPHI 170
C           EQUALITIES SECOND.                                         CPHI 180
C                                                                      CPHI 190
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * CPHI 200
C                                                                      CPHI 210
C                                                                      CPHI 220
      SUBROUTINE CPHI
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON     A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON     NV,NC,NK,NEG,NIN,NTV,NVI,NEV,NEVL,NIO,NIN1,NIN2,NIN3,NI
     1N4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IR1,IS,IS1,IT,IBP,ICDB,JCDB,KC
     2DB,KFIL,KLIN,KREN,KD,FI1,PHI,PS1,PS13,TB,TD,TC,EPSIL,EPSIL0,EPSIL2
      COMMON     KFONC,KGRAD,KCONT,KINV1,KINV2,KCDBA,KJACG,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCDBA1,KREN11,KREN21
      COMMON     IDIREC,DELIF1,ETA,JKC,LC,YSORT
      VC(1)=-XC(1)*XC(2)*XC(3)+RS
      VC(2)=XC(1)-1.
      VC(3)=XC(2)-1.
      VC(4)=XC(3)-1.
      VC(5)=R1-XC(1)
      VC(6)=R2-XC(2)
      VC(7)=R3-XC(3)
      RETURN
      END
```

```
C                                                                       JACOB 10
C                                                                       JACOB 20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * JACOB 30
C                                                                       JACOB 40
C     SUBROUTINE JACOB                                                  JACOB 50
C                                                                       JACOB 60
C     PURPOSE                                                           JACOB 70
C        THE SUBROUTINE "JACOB" DEFINES THE GRADIENTS OF THE CONSTRAINT JACOB 80
C        FUNCTIONS. THE PARTIAL DERIVATIVE OF I/DXJ IS STORED IN THE    JACOB 90
C        MATRIX ARRAY A(I,J). THE ROWS OF THE MATRIX REPRESENT EACH     JACOB100
C        CONSTRAINT FUNCTION, FI(X), I=1, ..., NC, IN THE SAME ORDER AS JACOB110
C        SEQUENCED IN CPHI. THE PARTIAL DERIVATIVES ARE REPRESENTED IN  JACOB120
C        TERMS OF THE "FORTRAN" VARIABLE XC(J), J= 1,..., NV.           JACOB130
C                                                                       JACOB140
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * JACOB150
C                                                                       JACOB160
C                                                                       JACOB170
      SUBROUTINE JACOB
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON      A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON      NV,NC,NK,NEG,NIN,NTV,NVI,NEV,NEVL,NIO,NIN1,NIN2,NIN3,NI
     IN4,NVNIN1,NVNIN2,NVNIN3,INDEX,II,IR,IR1,IS,IS1,IT,IBP,ICOB,JCOB,KC
     2OB,KFIL,KLIN,KREN,KD,FI1,PHI,PSI,PSI3,TB,TD,IC,EPSIL,EPSIL0,EPSIL2
      COMMON      KFUNC,KGRAD,KCONT,KINV1,KINV2,KCOBA,KJACO,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCOBA1,KREN11,KREN21
      COMMON      IDIREC,DELTFI,ETA,JKO,LC,YSORT
      A(1,1)=-XC(2)*XC(3)
      A(1,2)=-XC(1)*XC(3)
      A(1,3)=-XC(1)*XC(2)
      A(2,1)=1.
      A(2,2)=0.
      A(2,3)=0.
      A(3,1)=0.
      A(3,2)=1.
      A(3,3)=0.
      A(4,1)=0.
      A(4,2)=0.
      A(4,3)=1.
      A(5,1)=-1.
      A(5,2)=0.
      A(5,3)=0.
      A(6,1)=0.
      A(6,2)=-1.
      A(6,3)=0.
      A(7,1)=0.
      A(7,2)=0.
      A(7,3)=-1.
      RETURN
      END
```

APPENDIX III

```
$JOB
C                                                                       PHIX  10
C                                                                       PHIX  20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * PHIX  30
C                                                                       PHIX  40
C     SUBROUTINE PHIX                                                   PHIX  50
C                                                                       PHIX  60
C     PURPOSE                                                           PHIX  70
C        THE EXTERNAL, USER-SUPPLIED SUBROUTINE "PHIX" DEFINES THE      PHIX  90
C        OBJECTIVE FUNCTION TO THE  GREG  PROGRAM.                      PHIX 100
C                                                                       PHIX 110
C     DESCRIPTION OF PARAMETERS                                         PHIX 120
C        AI    - THE EFFORT COEFFICIENT FOR I TH COMPONENT.             PHIX 130
C        RI    - THE RELIABILITY OF I TH COMPONENT.                     PHIX 140
C                                                                       PHIX 150
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * PHIX 160
C                                                                       PHIX 170
C                                                                       PHIX 180
      SUBROUTINE PHIX
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(10))
      COMMON      A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON      NV,NC,NK,NEG,NIN,NTV,NVI,NEV,NEVL,NIO,NIN1,NIN2,NIN3,NI
     IN4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IK1,IS,IS1,IT,IBP,ICDB,JCDB,KC
     2DB,KFIL,KLIN,KREN,KD,FI1,PHI,PSI,PS13,TB,TD,TC,EPSIL,EPSILO,EPSIL2
      COMMON     KFONC,KGRAD,KCONT,KINV1,KINV2,KCDBA,KJACO,KMAX1,KMAX2,K
     IREN1,KREN2,KINV,KCDBA1,KREN11,KREN21
      COMMON      IDIREC,DELTFI,ETA,JKO,LC,YSORT
      PHI=-A1*ALOG((1-R1)/(1-XC(1)))-A2*ALOG((1-R2)/(1-XC(2)))
     1    -A3*ALOG((1-R3)/(1-XC(3)))
      RETURN
      END
```

```
C                                                                       GRAD  10
C                                                                       GRAD  20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * GRAD  30
C                                                                       GRAD  40
C     SUBROUTINE GRADF1                                                 GRAD  50
C                                                                       GRAD  60
C     PURPOSE                                                           GRAD  70
C        THIS SUBROUTINE DEFINES THE GRADIENT OF THE OBJECTIVE FUNCTION GRAD  80
C        IN TERMS OF THE ARRAY XC(J), J=1, .....,NV. THE COMPONENT VALUESGRAD  90
C        ARE STRORED IN THE VECTOR ARRAY C(J), J=1, .....,NV.           GRAD 100
C                                                                       GRAD 110
C     DESCRIPTION OF PARAMETER.                                         GRAD 120
C        AI    - THE EFFORT COEFFICIENT FOR I TH COMPNENTS.             GRAD 130
C                                                                       GRAD 140
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * GRAD 150
C                                                                       GRAD 160
C                                                                       GRAD 170
      SUBROUTINE GRADF1
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON     A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON     NV,NC,NK,NEG,NIN,NIV,NV1,NEV,NEV1,NIO,NIN1,NIN2,NIN3,NI
     1N4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IR1,IS,IS1,IT,IBP,ICOB,JCOB,KC
     2OB,KFIL,KLIN,KREN,KD,FI1,PHI,PSI,PSI3,TB,TD,TC,EPSIL,EPSIL0,EPSIL2
      COMMON    KFONC,KGRAD,KCONT,KINV1,KINV2,KCOBA,KJACO,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCOBA1,KREN11,KREN21
      COMMON     IDIREC,DELTFI,ETA,JKO,LC,YSORT
      C(1)=A1/(XC(1)-1.0)
      C(2)=A2/(XC(2)-1.0)
      C(3)=A3/(XC(3)-1.0)
      RETURN
      END
```

```
C                                                                       CPHI  10
C                                                                       CPHI  20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * CPHI  30
C                                                                       CPHI  40
C     SUBROUTINE CPHI                                                   CPHI  50
C                                                                       CPHI  60
C     PURPOSE                                                           CPHI  70
C        CPHI DEFINES THE CONSTRAINT FUNCTIONS ( .LE. OR .EQ. 0 ).      CPHI  80
C        THE VALUES ARE STORED IN THE VECTOR ARRAY VC(I), I=1, ...., NC,CPHI  90
C        AND IN TERMS OF THE ORIGINAL PROBLEM VARIABLES, XC(J), J=1, ...CPHI 100
C        ,NV.                                                           CPHI 110
C                                                                       CPHI 120
C     DESCRIPTION OF PARAMETER.                                         CPHI 130
C        RI    - THE RELIABILITY OF I TH, COMPONENTS.                   CPHI 140
C                                                                       CPHI 150
C     REMARKS                                                           CPHI 160
C        THE CONSTRAINTS MUST BE ORDERED WITH INEQUALITIES FIRST AND    CPHI 170
C        EQUALITIES SECOND.                                             CPHI 180
C                                                                       CPHI 190
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * CPHI 200
C                                                                       CPHI 210
C                                                                       CPHI 220
      SUBROUTINE CPHI
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON      A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON      NV,NC,NK,NEG,NIN,NTV,NVI,NEV,NEVL,NTO,NIN1,NIN2,NIN3,NI
     1N4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IR1,IS,IS1,IT,IBP,ICDB,JCDB,KC
     2DB,KFIL,KLIN,KREN,KD,FI1,PHI,PS1,PS13,TB,TD,TC,EPSIL,EPSILO,EPSIL2
      COMMON     KFONC,KGRAD,KCONT,KINV1,KINV2,KCOBA,KJACO,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCOBA1,KREN11,KREN21
      COMMON      IDIREC,DELTFI,ETA,JKO,LC,YSORT
      VC(1)=(1.0-XC(1))*(1.0-XC(2))*(1.0-XC(3))-(1.0-RS)
      VC(2)=XC(1)-1.
      VC(3)=XC(2)-1.
      VC(4)=XC(3)-1.
      VC(5)=R1-XC(1)
      VC(6)=R2-XC(2)
      VC(7)=R3-XC(3)
      RETURN
      END
```

```
C                                                                      JACOB  10
C                                                                      JACOB  20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * JACOB  30
C                                                                      JACOB  40
C     SUBROUTINE JACOB                                                 JACOB  50
C                                                                      JACOB  60
C     PURPOSE                                                          JACOB  70
C        THE SUBROUTINE "JACOB" DEFINES THE GRADIENTS OF THE CONSTRAINT JACOB  80
C        FUNCTIONS. THE PARTIAL DERIVATIVE DFI/DXJ IS STORED IN THE    JACOB  90
C        MATRIX ARRAY A(I,J). THE ROWS OF THE MATRIX REPRESENT EACH    JACOB 100
C        CONSTRAINT FUNCTION, FI(X), I=1, ..., NC, IN THE SAME ORDER AS JACOB 110
C        SEQUENCED IN CPHI. THE PARTIAL DERIVATIVES ARE REPRESENTED IN JACOB 120
C        TERMS OF THE "FORTRAN" VARIABLE XC(J), J= 1,..., NV.          JACOB 130
C                                                                      JACOB 140
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * JACOB 150
C                                                                      JACOB 160
C                                                                      JACOB 170
      SUBROUTINE JACOB
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON     A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON     NV,NC,NK,NEG,NIN,NIV,NVI,NEV,NEVL,NI0,NIN1,NIN2,NIN3,NI
     1N4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IR1,IS,IS1,IT,IBP,ICDB,JCDB,KC
     2DB,KFIL,KLIN,KREN,KD,FI1,PHI,PS1,PS13,TB,TD,TC,EPSIL,EPSILO,EPSIL2
      COMMON     KFONC,KGRAD,KCONT,KINV1,KINV2,KCDBA,KJACO,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCDBA1,KREN11,KREN21
      COMMON     IDIREC,DELTFI,ETA,JKO,LC,YSORT
      A(1,1)=-(1.-XC(2))*(1.-XC(3))
      A(1,2)=-(1.-XC(1))*(1.-XC(3))
      A(1,3)=-(1.-XC(1))*(1.-XC(2))
      A(2,1)=1.0
      A(2,2)=0.
      A(2,3)=0.
      A(3,1)=0.
      A(3,2)=1.0
      A(3,3)=0.
      A(4,1)=0.
      A(4,2)=0.
      A(4,3)=1.0
      A(5,1)=-1.0
      A(5,2)=0.
      A(5,3)=0.
      A(6,1)=0.
      A(6,2)=-1.0
      A(6,3)=0.
      A(7,1)=0.
      A(7,2)=0.
      A(7,3)=-1.0
      RETURN
      END
```

APPENDIX IV

```
$JOB
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C      SUBROUTINE PHIX
C
C      PURPOSE
C         THE EXTERNAL, USER-SUPPLIED SUBROUTINE "PHIX" DEFINES THE
C         OBJECTIVE FUNCTION TO THE  GRLG  PROGRAM.
C
C      DESCRIPTION OF PARAMETERS
C         AI    - THE EFFORT COEFFIENT FOR I TH COMPONENT.
C         RI    - THE RELIABILITY OF I TH COMPONENT.
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
      SUBROUTINE PHIX
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON     A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON     NV,NC,NK,NEG,NIN,NTV,NVI,NEV,NLVL,NTO,NIN1,NIN2,NIN3,NI
     1N4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IR1,IS,IS1,IT,IBP,ICDB,JCDB,KC
     2DB,KFIL,KLIN,KREN,KD,FIL,PHI,PSI,PSI3,TB,TD,TC,EPSIL,EPSIL0,EPSIL2
      COMMON     KFONC,KGRAD,KCONT,KINV1,KINV2,KCDBA,KJACO,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCDBA1,KREN11,KREN21
      COMMON     IDIREC,DELTFI,ETA,JKO,LC,YSORT
      PHI=-A1*ALOG((1-R1)/(1-XC(1)))-A2*ALOG((1-R2)/(1-XC(2)))
     1    -A3*ALOG((1-R3)/(1-XC(3)))-A4*ALOG((1-R4)/(1-XC(4)))
     2    -A5*ALOG((1-R5)/(1-XC(5)))
      RETURN
      END
```

```
C                                                                       GRAD  10
C                                                                       GRAD  20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * GRAD  30
C                                                                       GRAD  40
C     SUBROUTINE GRADFI                                                 GRAD  50
C                                                                       GRAD  60
C     PURPOSE                                                           GRAD  70
C        THIS SUBROUTINE DEFINES THE GRADIENT OF THE OBJECTIVE FUNCTION GRAD  80
C        IN TERMS OF THE ARRAY XC(J), J=1, .....NV. THE COMPONENT VALUESGRAD  90
C        ARE STRORED IN THE VECTOR ARRAY C(J), J=1, ....,NV.            GRAD 100
C                                                                       GRAD 110
C     DESCRIPTION OF PARAMETER.                                         GRAD 120
C        AI    - THE EFFORT COEFFIENT FOR I TH COMPNENTS.               GRAD 130
C                                                                       GRAD 140
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * GRAD 150
C                                                                       GRAD 160
C                                                                       GRAD 170
      SUBROUTINE GRADFI
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON     A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON     NV,NC,NK,NEG,NIN,NTV,NV1,NEV,NEVL,NTO,NIN1,NIN2,NIN3,NI
     IN4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IR1,IS,IS1,IT,IBP,ICOB,JCOB,KC
     2OB,KFIL,KLIN,KREN,KD,FI1,PHI,PS1,PS13,TD,ID,IC,EPSIL,EPSILO,EPSIL2
      COMMON    KFUNC,KGRAD,KCONT,KINV1,KINV2,KCOBA,KJACO,KMAX1,KMAX2,K
     IREN1,KREN2,KINV,KCOBA1,KREN11,KREN21
      COMMON     IDIREC,DELTFI,ETA,JKO,LC,YSORT
      C(1)=A1/(XC(1)-1)
      C(2)=A2/(XC(2)-1)
      C(3)=A3/(XC(3)-1)
      C(4)=A4/(XC(4)-1)
      C(5)=A5/(XC(5)-1)
      RETURN
      END
```

```
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C     SUBROUTINE CPHI
C
C     PURPOSE
C        CPHI DEFINES THE CONSTRAINT FUNCTIONS ( .LE. OR .EQ. 0 ).
C        THE VALUES ARE STORED IN THE VECTOR ARRAY VC(I), I=1, ....., NC,
C        AND IN TERMS OF THE ORIGINAL PROBLEM VARIABLES, XC(J), J=1, ...
C        ,NV.
C
C     DESCRIPTION OF PARAMETER.
C        RI   - THE RELIABILITY OF I TH, COMPONENTS.
C
C     REMARKS
C        THE CONSTRAINTS MUST BE ORDERED WITH INEQUALITIES FIRST AND
C        EQUALITIES SECOND.
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
      SUBROUTINE CPHI
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON     A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON     NV,NC,NK,NEG,NIN,NTV,NVI,NEV,NEVL,NIO,NIN1,NIN2,NIN3,NI
     1N4,NVNIN1,NVNIN2,NVNIN3,INDEX,I1,IR,IRI,IS,IS1,IT,IBP,ICDB,JCDB,KC
     2DB,KFIL,KLIN,KREN,KD,FII,PHI,PSI,PSI3,TB,TD,TC,EPSIL,EPSILO,EPSIL2
      COMMON    KFLNC,KGRAD,KCONT,KINV1,KINV2,KCDBA,KJACO,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCDBA1,KREN11,KREN21
      COMMON     IDIREC,DELTFI,ETA,JKO,LC,YSORT
      VC(1)=-(XC(4)+XC(5)-XC(4)*XC(5))*XC(2)-(XC(1)*XC(4)
     1      +XC(4)*XC(5)-XC(1)*XC(3)*XC(4)*XC(5))*(1-XC(2))+RS
      VC(2) =XC(1)-1.
      VC(3) =XC(2)-1.
      VC(4) =XC(3)-1.
      VC(5) =XC(4)-1.
      VC(6) =XC(5)-1.
      VC(7) =R1-XC(1)
      VC(8) =R2-XC(2)
      VC(9) =R3-XC(3)
      VC(10)=R4-XC(4)
      VC(11)=R5-XC(5)
      RETURN
      END
```

```
C                                                                       JACOB  10
C                                                                       JACOB  20
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *  JACOB  30
C                                                                       JACOB  40
C     SUBROUTINE JACOB                                                  JACOB  50
C                                                                       JACOB  60
C     PURPOSE                                                           JACOB  70
C        THE SUBROUTINE "JACOB" DEFINES THE GRADIENTS OF THE CONSTRAINT JACOB  80
C          FUNCTIONS. THE PARTIAL DERIVATIVE DFI/DXJ IS STORED IN THE   JACOB  90
C          MATRIX ARRAY A(I,J). THE ROWS OF THE MATRIX REPRESENT EACH   JACOB 100
C          CONSTRAINT FUNCTION, FI(X), I=1, ..., NC, IN THE SAME ORDER AS JACOB 110
C          SEQUENCED IN CPHI. THE PARTIAL DERIVATIVES ARE REPRESENTED IN JACOB 120
C          TERMS OF THE "FORTRAN" VARIABLE XC(J), J= 1,..., NV.         JACOB 130
C                                                                       JACOB 140
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *  JACOB 150
C                                                                       JACOB 160
C                                                                       JACOB 170
      SUBROUTINE JACOB
      DIMENSION A(50,100),ALFA(50,50),X(150),XC(150),XI(150),XS(150)
     1,Y(150),C(150),VC(50),IBAS(50),IHB(100),IVC(50),IVA(100)
      COMMON     A,ALFA,X,XC,XI,XS,Y,C,VC,IBAS,IHB,IVC,IVA,IVB
      COMMON     NV,NC,NK,NEG,NIN,NIV,NVL,NEV,NEVL,NIO,NIN1,NIN2,NIN3,NI
     1N4,NVNIN1,NVNIN2,NVNIN3,INDEX,II,IR,IR1,IS,IS1,IT,IBP,ICOB,JCOB,KC
     2OB,KFIL,KLIN,KREN,KO,FIL,PHI,PSI,PS1,TB,TD,TC,EPSIL,EPSIL0,EPSIL2
      COMMON     KFONC,KGRAD,KCONT,KINV1,KINV2,KCOBA,KJACO,KMAX1,KMAX2,K
     1REN1,KREN2,KINV,KCOBA1,KREN11,KREN21
      COMMON     IDIREC,DELTT1,ETA,JKO,LC,YSORT
      A(1,1)=-XC(4)+XC(3)*XC(4)*XC(5)+XC(2)*XC(4)
     1       -XC(2)*XC(3)*XC(4)*XC(5)
      A(1,2)=-XC(4)-XC(5)+XC(4)*XC(5)+XC(1)*XC(4)
     1       +XC(4)*XC(5)-XC(1)*XC(3)*XC(4)*XC(5)
      A(1,3)=+XC(1)*XC(4)*XC(5)-XC(1)*XC(2)*XC(4)*XC(5)
      A(1,4)=-XC(2)+XC(2)*XC(5)-XC(1)-XC(5)
     1       +XC(1)*XC(3)*XC(5)+XC(1)*XC(2)+XC(2)*XC(5)
     2       -XC(1)*XC(2)*XC(3)*XC(5)
      A(1,5)=-XC(2)+XC(2)*XC(4)-XC(4)+XC(1)*XC(3)*XC(4)
     1       +XC(2)*XC(4)-XC(1)*XC(2)*XC(3)*XC(4)
      A(2,1)=+1
      A(2,2)=0.0
      A(2,3)=0.0
      A(2,4)=0.0
      A(2,5)=0.0
      A(3,1)=0.0
      A(3,2)=1.
      A(3,3)=0.
      A(3,4)=0.
      A(3,5)=0.
      A(4,1)=0.
      A(4,2)=0.
      A(4,3)=1.
      A(4,4)=0.
      A(4,5)=0.
      A(5,1)=0.
      A(5,2)=0.
      A(5,3)=0.
      A(5,4)=1.
      A(5,5)=0.
      A(6,1)=0.
      A(6,2)=0.
```

```
A(6,3)=0.
A(6,4)=0.
A(6,5)=1.
A(7,1)=-1.
A(7,2)=0.
A(7,3)=0.
A(7,4)=0.
A(7,5)=0.
A(8,1)=0.
A(8,2)=-1.
A(8,3)=0.
A(8,4)=0.
A(8,5)=0.
A(9,1)=0.
A(9,2)=0.
A(9,3)=-1.
A(9,4)=0.
A(9,5)=0.
A(10,1)=0.
A(10,2)=0.
A(10,3)=0.
A(10,4)=-1.
A(10,5)=0.
A(11,1)=0.
A(11,2)=0.
A(11,3)=0.
A(11,4)=0.
A(11,5)=-1.
RETURN
END
```

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ALBERT'S RELIABILITY ALLOCATION TECHNIQUE WITH RELAXED ASSUMPTION ON EFFORT FUNCTION

BY

NADER AFZALI

B.S., Kansas State University, 1977

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The major purpose of this investigation was to discover how robust Albert's procedure is when the components do not all have the same effort function. This was done for:

a) A simple series configuration.

b) A simple parallel configuration.

c) An elementary mixed (complex) configuration.

The study also examined the sensitivity of the results to the structure of the effort function.

The major conclusions were:

1. The optimal reliability allocation depends on:

I) Series Configuration: $\overline{R}$ and $\underline{a}$.

II) Parallel Configuration: $\overline{R}$ (not $\underline{a}$).

III) Complex Configuration: $\overline{R}$ and $\underline{a}$.

2. The change of reliability for the:

I) Series Configuration: worst component changes the most.

II) Parallel Configuration: best component changes the most.

III) Complex Configuration: Keystone component changes the most.

3. The numerical value of the optimal effort.

I) Series configuration: depends on $\underline{a}$.

II) Parallel configuration: does not depend on $\underline{a}$ (since the only components changed were those with $a_i = 1$).

III) Complex configuration: depends on $\underline{a}$.

4. The solution was not very sensitive to the structure of the effort function.

Therefore, the assumption that the effort coefficients are the same for all components is not justified.